

**Unit 9 Test B**  
**PCHA 2022-23 / Dr. Kessner**

**No calculator! Have fun!**

1. Evaluate the following limits, evaluating left and right side limits where applicable.

a.  $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x - 3}$

b.  $\lim_{x \rightarrow 3} \frac{x - 3}{x - 3}$

c.  $\lim_{x \rightarrow 3} \frac{1}{x - 3}$

d.  $\lim_{x \rightarrow \infty} \frac{1}{x - 3}$

e.  $\lim_{h \rightarrow 0} \frac{(3 + h)^4 - 3^4}{h}$

2. For the following functions find the derivative using one of the limit definitions.

a. Suppose that a little bird or a mathematician tells you that  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$

and also that  $\cos(u + v) = \cos u \cos v - \sin u \sin v$ .

Find the derivative of  $f(x) = \cos x$  using a limit definition.

b. Find the derivative of  $g(x) = -1/x$ , using a limit definition.

**3.** Using the various rules for differentiation, calculate the derivatives of the following functions.

a.  $p(x) = x^4 + 4^x + x^{-4} + 4^{-x}$

b.  $q(x) = \sin 4x + \log_4 x + \ln 4x$

c.  $r(x) = e^{\sin x} + \sin e^x$

d.  $s(x) = \sqrt{x^2 + 1}$

e.  $t(x) = \frac{1}{1 + e^{-x}}$

4. a. Use implicit differentiation to show that

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

- b. Let  $f(x) = \tan^{-1}(e^x)$ . Find  $f'(x)$ .

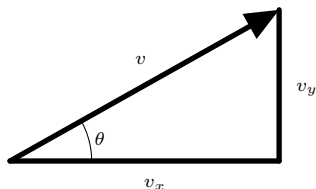
- c. Let  $g(x) = \tan^{-1}(\sqrt{x})$ . Find  $g'(x)$ .

5. Recall that you can model projectile motion with parametric equations:

$$x(t) = x_0 + v_x t$$

$$y(t) = y_0 + v_y t - 16t^2$$

where  $(x_0, y_0)$  is the initial position of the object, and  $v_x$  and  $v_y$  are the components of the initial velocity vector  $v$ :



Suppose that you launch a rocket from the ground, at an angle of  $45^\circ$ , with an initial speed of  $96\sqrt{2}$  ft/sec.

- a. Write equations for  $x(t)$  and  $y(t)$ .
- b. Find  $x'(t)$  and  $y'(t)$ .
- c. Find  $x''(t)$  and  $y''(t)$ .
- d. Using the derivatives you found above, find the maximum height of the rocket.
- e. When does the rocket hit the ground, and how far has it traveled in the x-direction?