KEY

Unit 10 Group Work
PCHA 2022-23 / Dr. Kessner

No calculator! Have fun!

1. Graph the given function on the specified interval. Find all critical points. Identify any points where there is a local min/max, and verify with a derivative test. Identify the absolute max and min. If either fails to exist, state the condition of the Extreme Value Theorem that is *not* satisfied.

a. $g(x) = x^2 - 9$ on $[-3, 3]$

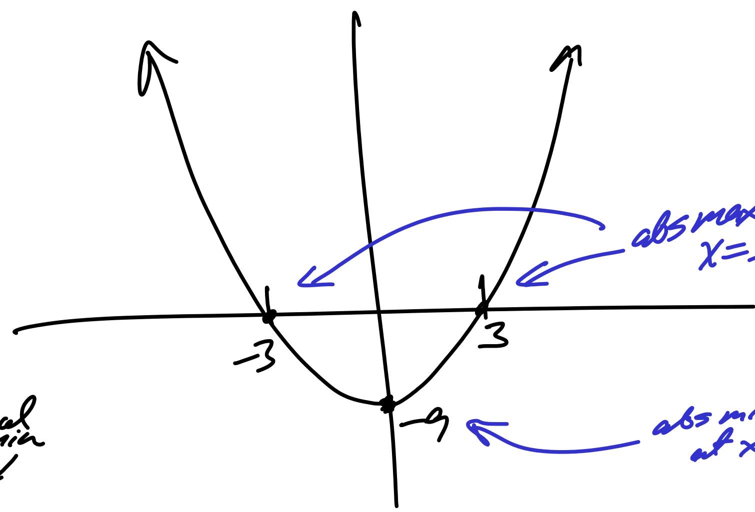
$g'(x) = 2x$

critical pts:

$x = 0$

$g''(x) = 2$

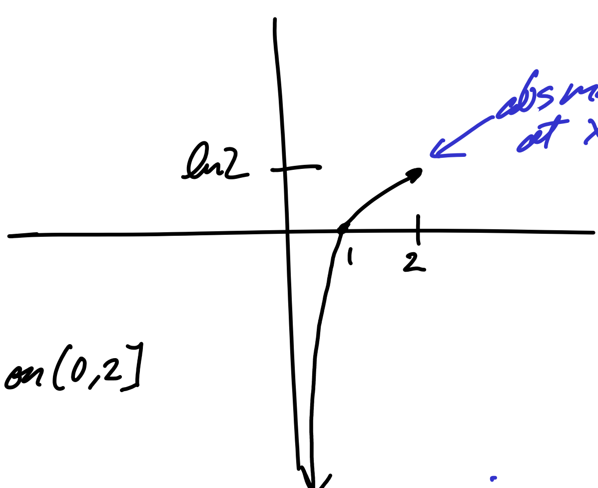
$g''(0) = 2 > 0$ local min



abs max at $x = \pm 3$

abs min at $x = 0$

b. $h(x) = \ln x$ on $(0, 2]$



abs max at $x=2$

$h'(x) = \frac{1}{x} > 0$ always increasing

$\Rightarrow$ shows $x=2$ abs max

(or 1st deriv test $\Rightarrow$ local max)

$h'(x) = \frac{1}{x}$

no critical pts on $(0,2]$

no abs min

$(0,2]$ not closed interval $\Rightarrow$ EVT does not apply

c. $m(x) = \tan x$ on $[0, \pi]$

no abs. min/max

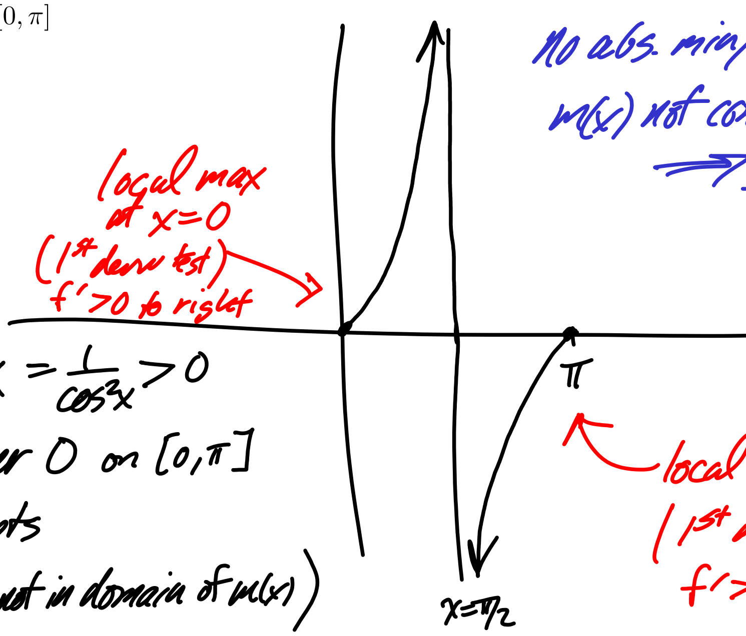
$m(x)$ not continuous on $[0,\pi]$

$\Rightarrow$ EVT does not apply

local max at $x=0$

(1st deriv test)

$f'>0$ to right



$m'(x) = \sec^2 x = \frac{1}{\cos^2 x} > 0$

$m'(x)$ never 0 on $[0,\pi]$

no critical pts

($x = \pi/2$ is not in domain of $m(x)$)

local max at $x=\pi$

(1st deriv test)

$f'>0$ from left

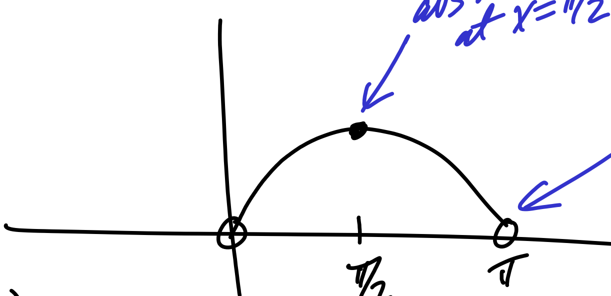
d. $n(x) = \sin x$ on $(0, \pi)$

abs max at $x=\pi/2$

no abs min

$(0,\pi)$ not closed interval

$\Rightarrow$ EVT does not apply



$n'(x) = \cos x$

$\cos(\pi/2) = 0 \Rightarrow x = \pi/2$ critical pt.

$n''(x) = -\sin x$

$n''(\pi/2) = -1 < 0$ ∩ local max at $x = \pi/2$

2. For each of the given functions find all antiderivatives.

a. $p'(x) = x^4$

$p(x) = \frac{1}{5}x^5 + C$

b. $q'(x) = \cos 2x$

$q(x) = \frac{1}{2}\sin 2x + C$

c. $r'(x) = \dfrac{1}{x}$

$r(x) = \ln x + C$

d. $s'(x) = \dfrac{1}{x^2}$

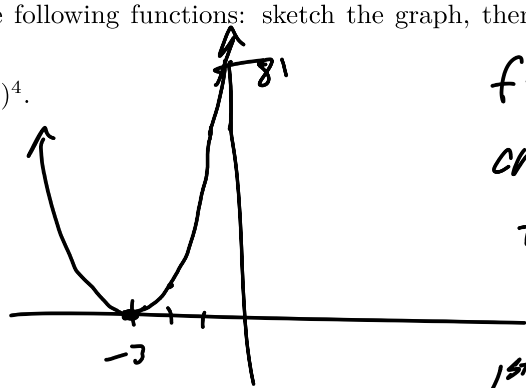
$s(x) = -\frac{1}{x} + C$

e. $t'(x) = e^{\frac{x}{3}}$

$t(x) = 3e^{x/3} + C$

3. For each of the following functions: sketch the graph, then find all local extrema and verify with a derivative test.

a. $f(x) = (x+3)^4$.



$f'(x) = 4(x+3)^3$

critical pt at $x=-3$

$f''(x) = 12(x+3)^2$

$f''(-3) = 0 \rightarrow$ 2nd deriv. test inconclusive

1st deriv test : $f'>0$ for $x>-3$, $f'<0$ for $x<-3$] local min at $x=-3$

b. $g(x) = x^3 - 9x$.

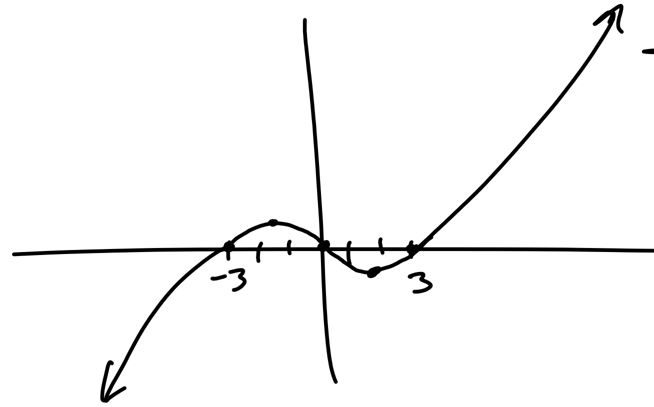
$= x(x^2-9)$

$= x(x+3)(x-3)$

$g'(x) = 3x^2 - 9$

$= 3(x^2-3)$

critical pts $x = \pm\sqrt{3}$



2nd deriv test:

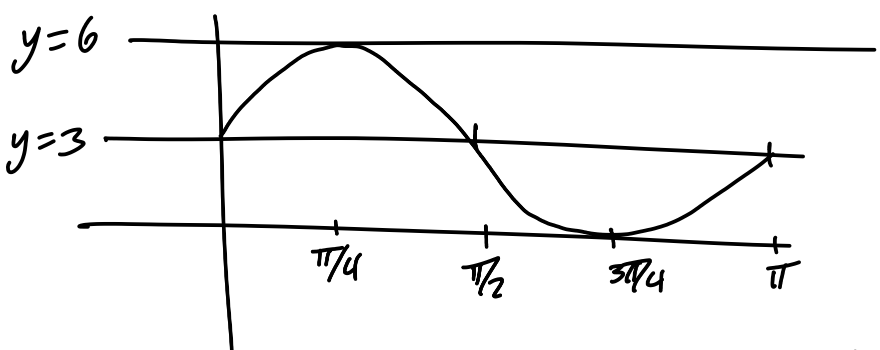
$g''(x) = 6x$

$g''(\sqrt{3}) > 0$ local min

$g''(-\sqrt{3}) < 0$ local max

c. $h(x) = 3 + 3\sin 2x$. You may restrict your attention to the first period of the function. But as an extra challenge, identify *all* local extrema (not just the first period), including derivative tests to show which are minima and which are maxima.

period $\frac{2\pi}{2} = \pi$



$h'(x) = 6\cos 2x$

$h'(x) = 0 \Rightarrow \cos 2x = 0$

$2x = \frac{\pi}{2} + k\pi$

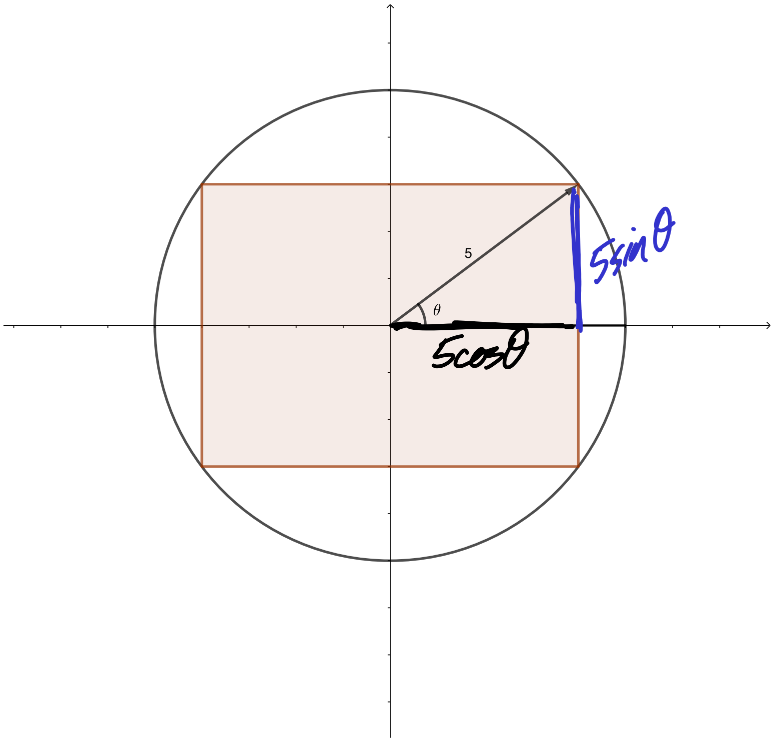
$\boxed{x = \pi/4 + k\pi/2 \text{ critical pts}}$

$h''(x) = -12\sin 2x$

$h''\left(\frac{\pi}{4} + \pi k\right) = -12\underbrace{\sin\frac{\pi}{2}}_{1} < 0$ local max at $x = \frac{\pi}{4} + \pi k$

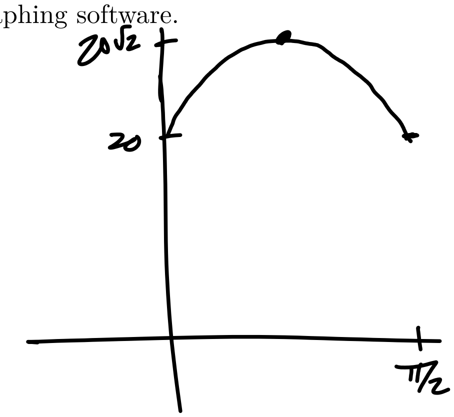
$h''\left(\frac{3\pi}{4} + \pi k\right) = -12\underbrace{\sin\frac{3\pi}{2}}_{-1} > 0$ local min at $x = \frac{3\pi}{4} + \pi k$

4. Consider the following rectangle inscribed in a circle of radius 5. Note that the perimeter of the rectangle changes as the angle θ changes.



a. Write an equation for the perimeter $P(\theta)$ of the rectangle as a function of θ. Challenge: draw a sketch of the graph of $P(\theta)$ on $[0, \frac{\pi}{2}]$ by hand, and then check with graphing software.

$$P(\theta) = 4 \cdot 5\sin\theta + 4 \cdot 5\cos\theta$$
$$= 20\sin\theta + 20\cos\theta$$



b. Find the absolute min and max of the perimeter, for θ in $[0, \frac{\pi}{2}]$. Why must there be an absolute minimum and maximum?

$$P'(\theta) = 20\cos\theta - 20\sin\theta$$

critical pts: $P'(\theta) = 0$

$$20\cos\theta - 20\sin\theta = 0$$
$$\cos\theta = \sin\theta$$
$$\tan\theta = 1$$
$$\theta = \pi/4$$

$P(0) = 20 = P(\pi/2)$ abs min

$P(\pi/4) = 20\sqrt{2}$ abs max

abs min/max exist because P is continuous on closed interval $[0, \pi/2]$