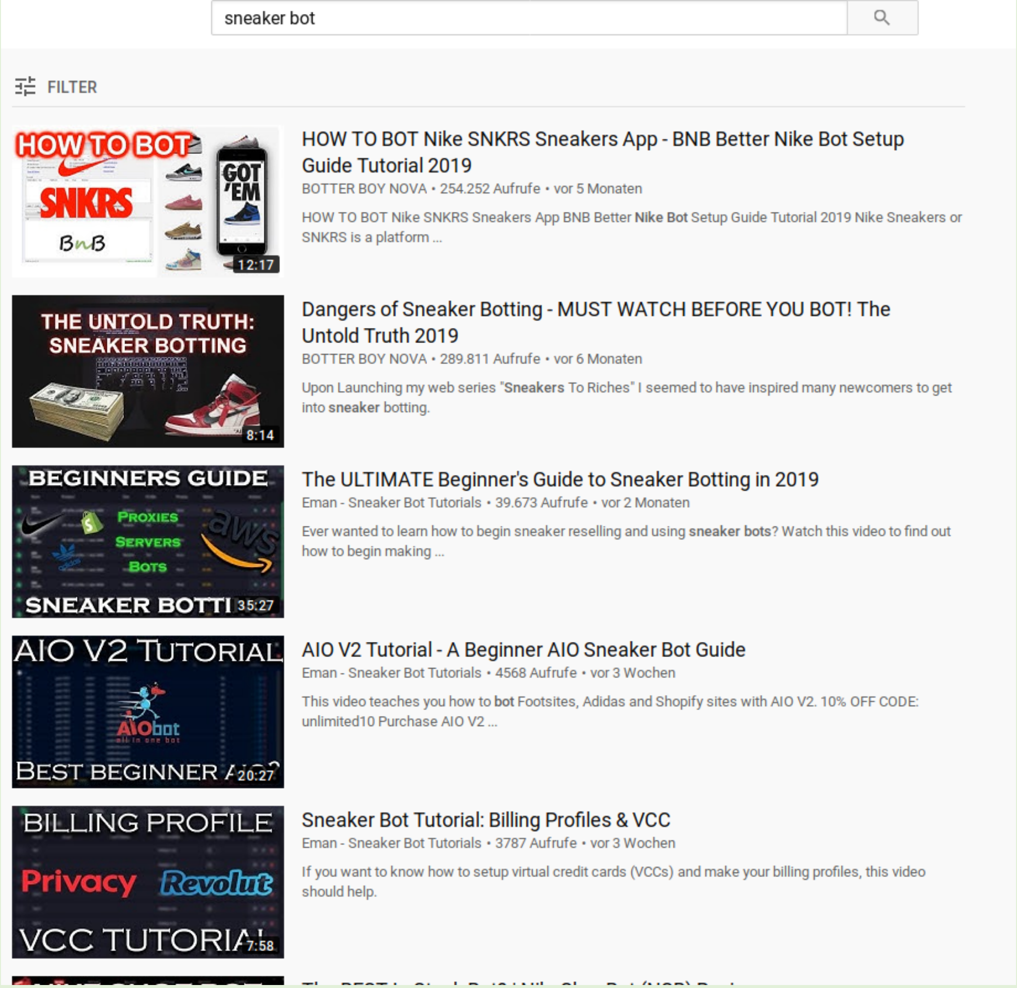
Artificial Intelligence

Algorithms and Applications with Python

Chapter 2

Dr. Dominik Jung
dominik.jung@jung-isec.de

Business Case: Stop Automated Nike Shopping Agents



Business Case: Stop Automated Nike Shopping Agents

01 | Executive Summary

Limited sneakers can be resold profitably. Professional resellers use AI-based agents that can make automated purchases. Today, there are many AI-based agents and web-crawler that are able to detect and buy limited sneakers or other limited offers in web-shops, while normal customers often go away empty-handed. Furthermore, these kind of information systems produce a lot of web traffic (especially if they are very bad designed), and most shop-owners do not want them. A sneaker shop in Frankfurt has now outwitted the software. With a simple but ingenious idea.

02 | Solution

- Most simple AI agents use simple decision rules for decision-making.
- Hence, they lack of “true” intelligence and can be fooled easily by targeting on the decision-rules of such systems

03 | References

- https://www.n-tv.de/mediathek/videos/wirtschaft/Skate-Shop-verkauft-Fotos-statt-Schuhe-article21229466.html

Take-Aways

- Shopping agents can be used to automate human tasks (find cheap offers)
- However, they can not replace humans if it gets difficult

Outline

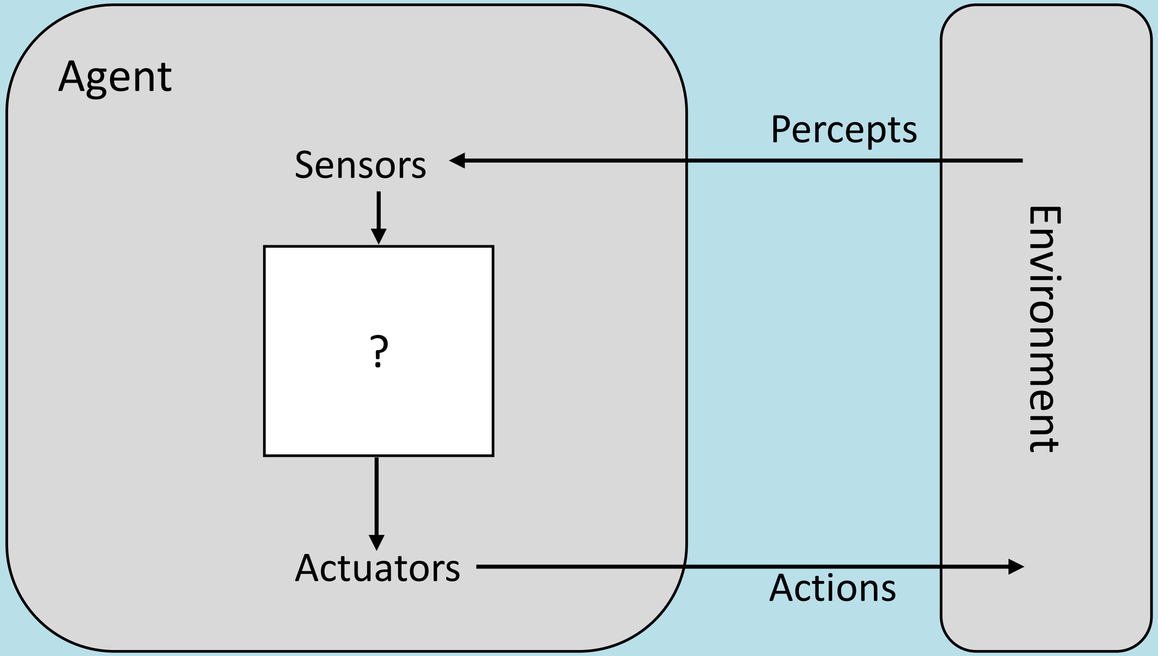
▶ **What we will learn:**
- We define the concept of rational agents (≈ intelligent agents)
- Characteristics of artificial agents (perfect or otherwise), the diversity of environments, and the resulting menagerie of agent types
- We discuss how AI problems can be modelled as search-problems, and how they can be solved by searching

Image source: ↗ Pixabay (2019) / ↗ CC0

▶ **Duration:**
- 180 min

▶ **Relevant for Exam:**
- 2.1 – 2.5

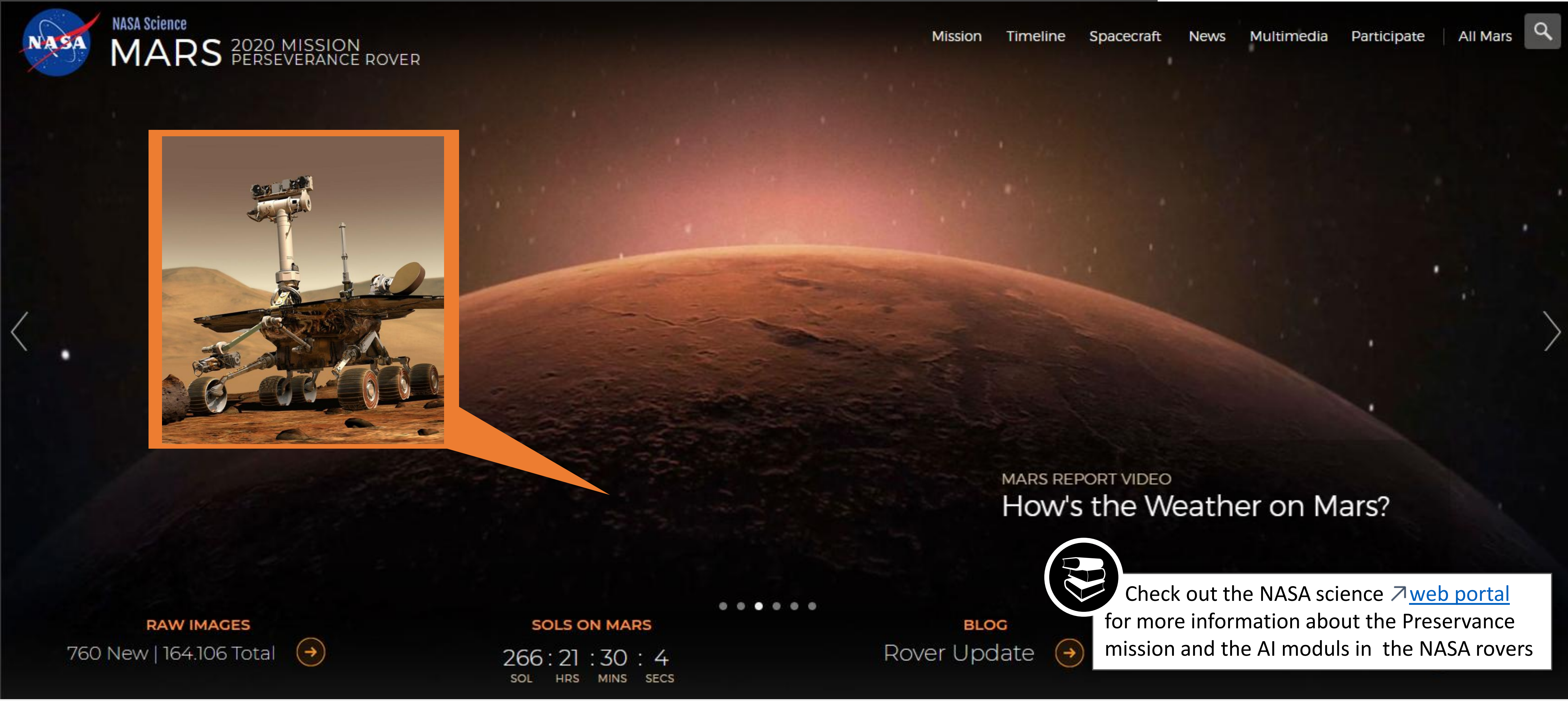
2.1 „Intelligent“ Agents (Russel, Norvig)



D *An agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators (*Rusell & Norvig, 2016*)*

Adapted from Rusell, S., & Norvig, P. (2016) | Image source: ↗ Pixabay (2019) / ↗ CC0

2.1 Robotic Agents: NASA Perseverance Rover (2020)



Check out the NASA science ↗[web portal](https://mars.nasa.gov/mars2020) for more information about the Preservance mission and the AI moduls in the NASA rovers

Image source: NASA (2022) ↗ https://mars.nasa.gov/mars2020 ; ↗ Pixabay (2019) / ↗ CC0

2.1 Software Agent: OpenAI Operator (2025)

OpenAI Introduction into Operator
https://openai.com/index/introducing-operator/

Image source: OpenAI (2025)

2.1 Agent in this Lecture

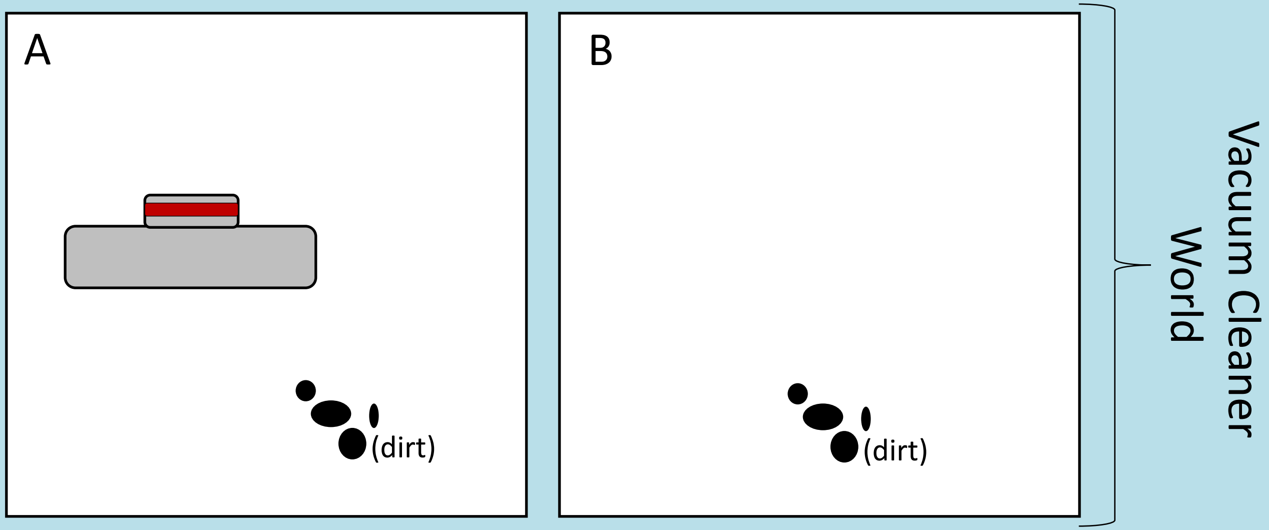
- Examples of agents
  - A web shopping program
  - An automated factory module
  - A traffic control system
  - NASA's Perseverance Rover

- Main focus in this lecture: **Software agents** that gathers information about an environment and takes actions based on that

Image source: ↗ Pixabay (2019) / ↗ CC0

How do you design an intelligent agent?

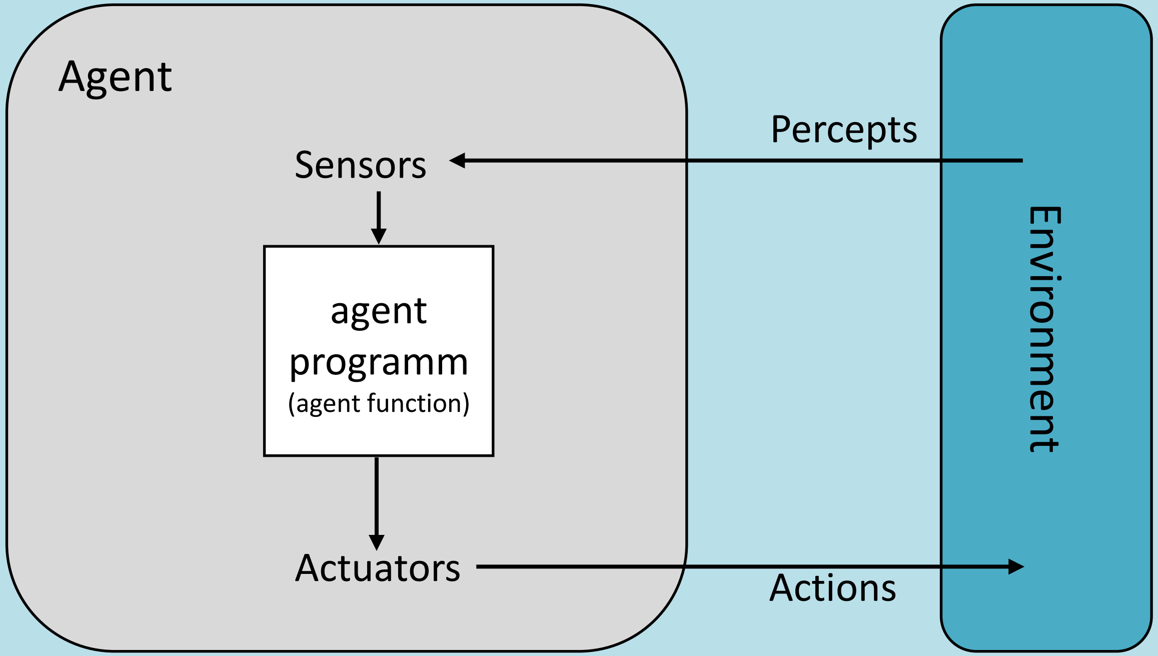
2.1 Example: Vacuum Cleaner World - Tabulation



- We use a very simple example: the vacuum-cleaner world with just two squares (A and B)
- The vacuum cleaner agent perceives which square it is and whether there is dirt or not
- It can move (left or right), suck up the dirt or do nothing.
- One simple function is: If the current square is dirty, then suck

Adapted from Rusell, S., & Norvig, P. (2016) | Image sources: Dominik Jung (2019)

2.1 Step 1 – Specifying the Task Environment



- How does the task and the environment in which the task should be solved look like?

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Specifying the Task Environment

Agent Type	Performance Measure	Environment	Actuators	Sensors
Taxi driver	Safe, fast, legal, comfortable trip, maximize profits	Roads, other traffic, pedestrians, customers	Steering, accelerator, brake, signal, horn, display	Cameras, sonar, speedometer, GPS, odometer, accelerometer, engine sensors, keyboard

- Norvig and Russel propose to characterize the task environment based on the four characteristics: Performance, Environment, Actuators and Sensors (PEAS)
- In designing an agent, the first step must always be to specify the task environment as fully as possible.

Table and example adapted from Rusell, S., & Norvig, P. (2016)

2.1 Further Properties of Task Environments

- Fully observable or partially observable
- Single agent or multiagent
- Deterministic or stochastic
- Episodic or sequential
- Static or dynamic
- Discrete or continuous
- Known or unknown

Task Environment	Observable	Agents	Deterministic	Episodic	Static	Discrete
Crossword puzzle						
Chess with a clock						
Poker						
Backgammon						
Taxi driving						
Medical diagnostics						
Image Analysis						
Part-Picking Robot						
Refinery Controller						
Interactive English Tutor						

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Environment Types in Reality

The simplest environment is

- Fully observavle
- Deterministic
- Episodic
- Static
- Discrete
- Single-agent

Most real situations are

- Partially observable
- Stochastic
- Sequential
- Dynamic
- Continous
- Multi-agent

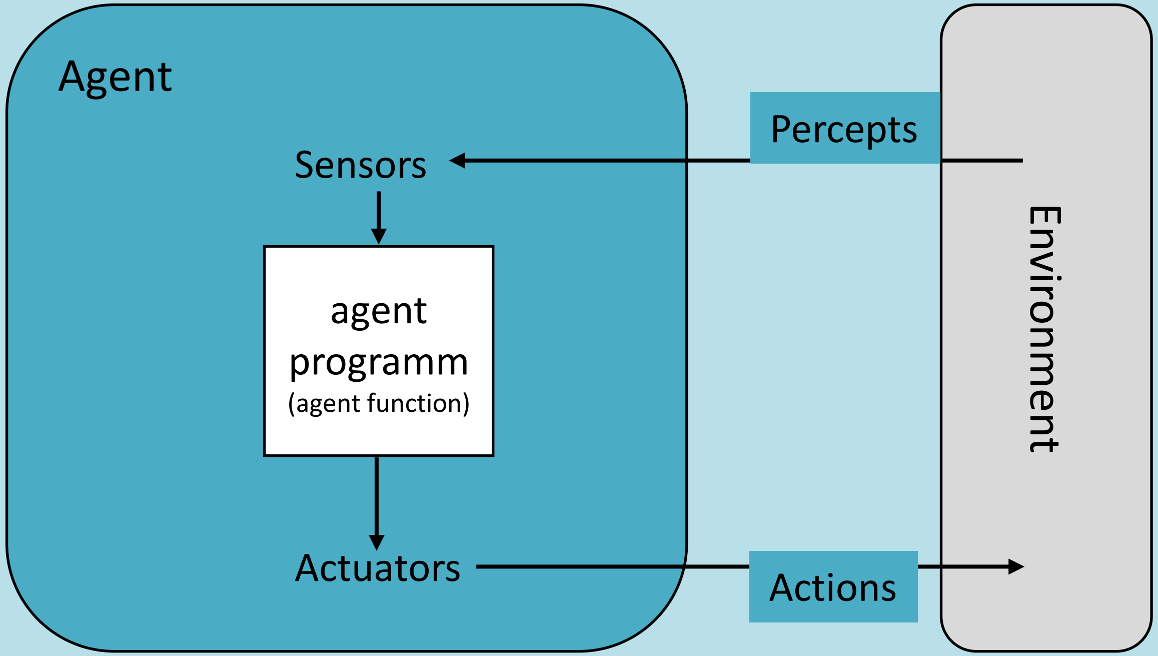
2.1 How to Get Task Environments in Development?

- We will use simulators that provide different environments to test our agents (see lectorials)

- The simulator takes one or more agents as input, provides each agent with the correct perceptions, accepts the agent's actions, and updates the environment based on the actions and possibly other influences

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Step 2 – Specifying the Agent



Agent = architecture + program

- How should the agent act, what would be „intelligent behaviour“?

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 What Makes Agents Intelligent?

Percept Sequence	Action
[A, Clean]	Right
[A, Dirty]	Suck
[B, Clean]	Left
[B, Dirty]	Suck
[A, Clean], [A, Clean]	Right
[A, Clean], [A, Dirty]	Suck
...	...
[A, Clean], [A, Clean], [A, Clean]	Right
[A, Clean], [A, Clean], [A, Dirty]	Suck

Adapted from Rusell, S., & Norvig, P. (2016)

- An agent's behavior is described by the agent function that maps any given percept sequence to an action.
- We could imagine this as a table matching each possible percepts and actions (remember chinese room argument?)
- Internally, the agent function is implemented by an agent program

2.1 Good Behaviour: Conceptionalizing Rationality in AI

- For a vacuum cleaner: What does it mean to do the right thing?

- What is rational, at any given time, depends on four things:
  - The performance measure that defines the criterion of success.
  - The agent's prior knowledge of the environment.
  - The actions that the agent can perform.
  - The agent's percept sequence to date

D

Rational Agent

*For each possible percept sequence, a rational agent should select an action that is expected to maximize its performance measure, given the evidence provided by the percept sequence and whatever built-in knowledge the agent has (*Rusell & Norvig, 2016, p.37*)*

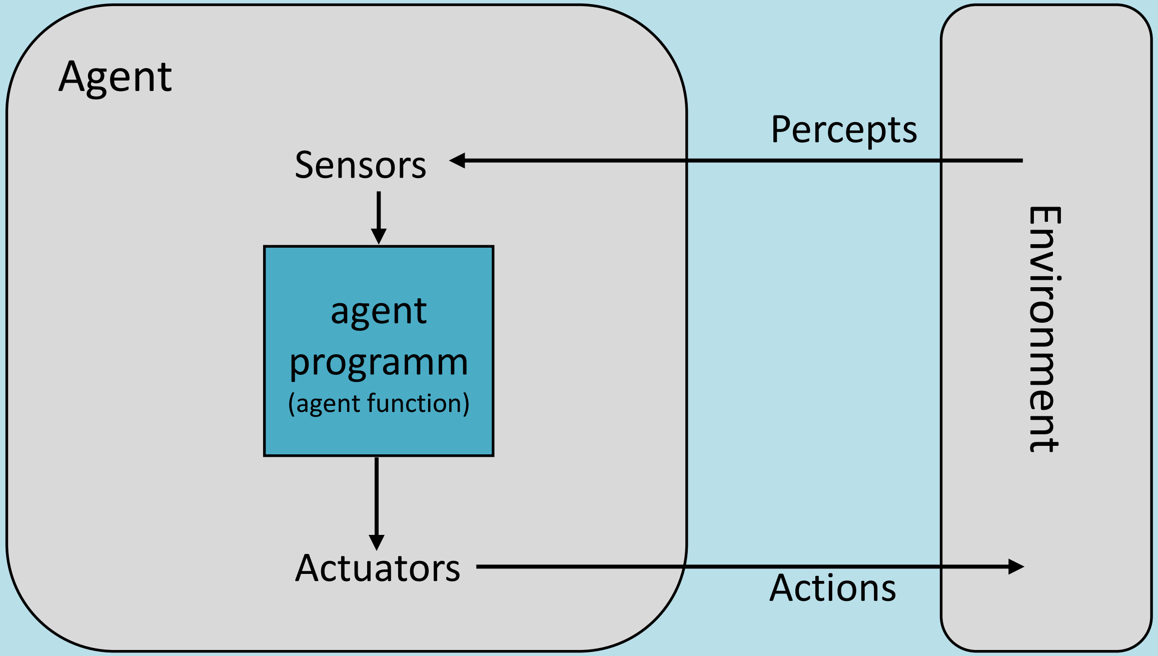
Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Omniscience, Learning, and Autonomy

- We need to be careful to distinguish between rationality and omniscience
- Rationality maximizes **expected** performance, while perfection maximizes **actual** performance
- Norvig & Russels' definition of rationality does not require omniscience, because the rational choice depends only on the percept sequence **to date**
- Their definition requires a rational agent not only to gather information but also to **learn** as much as possible from what it perceives.

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Step 3 – Specifying the Agents Behaviour



Agent = architecture + program

- How can we implement the „intelligent“ behaviour?

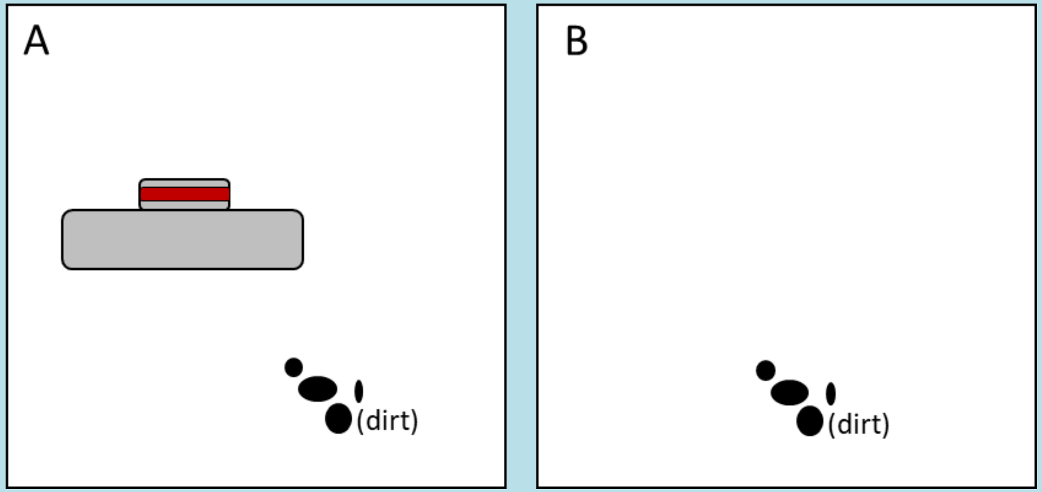
Adapted from Rusell, S., & Norvig, P. (2016) | Image source: ↗ Pixabay (2019) / ↗ CC0

2.1 Structure of Agents (Architecture)

- The job of AI specialists is to design an agent program that implements the agent function/the mapping from percepts to actions

- This program runs on some sort of computing device with physical sensors and actuators (architecture)

- *Agent = architecture + program*



Adapted from Rusell, S., & Norvig, P. (2016)

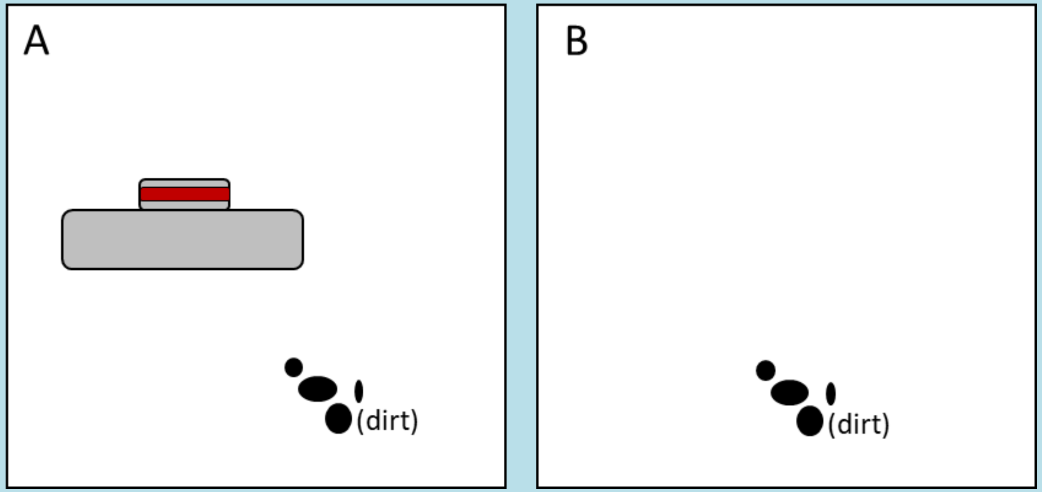
2.1 Structure of Agents (Architecture)

Algorithm: Reflex-Vacuum Agent

$\boldsymbol{if}\ status = dirty\ \boldsymbol{then}$
    $return\ suck$
$\boldsymbol{end}$

$\boldsymbol{else\ if}\ location = A\ \boldsymbol{then}$
    $return\ right$
$\boldsymbol{end}$

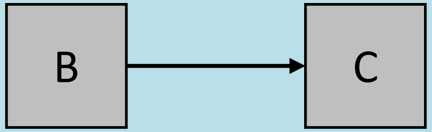
$\boldsymbol{else\ if}\ location = B\ \boldsymbol{then}$
    $return\ left$
$\boldsymbol{end}$



Adapted from Rusell, S., & Norvig, P. (2016)

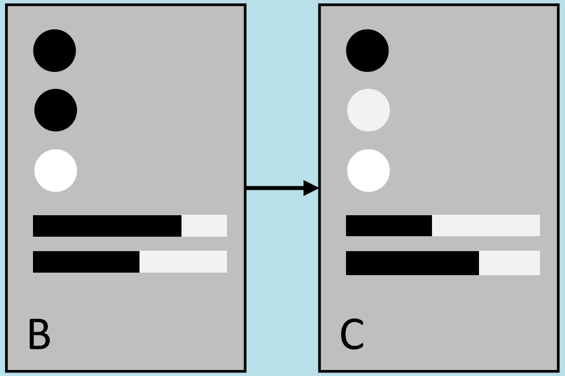
2.1 Components of Agents

Atomic Representation



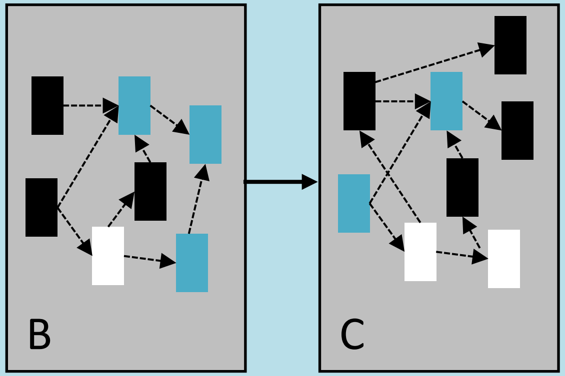
- a state (such as B or C) is a black box with no internal structure

Factored Representation



- a state consists of a vector of attribute values; values can be Boolean, real-valued, or one of a fixed set of symbol

Structured Representation



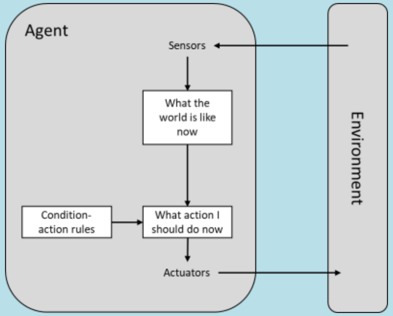
- a state includes objects, each of which may have attributes of its own as well as relationships to other objects.

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Subtypes of Agents (Russell & Norvig, 2016)

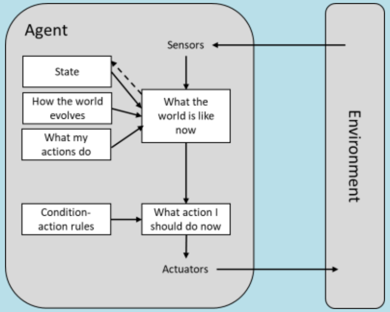
Rational Agents

Simple Reflex Agent



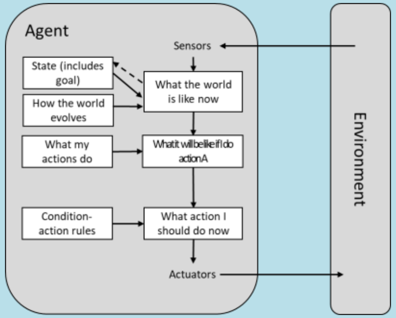
- Agents select actions on the basis of the current percept
- Work only when the correct decision can be made on the basis of the current percept

Model-based Agent



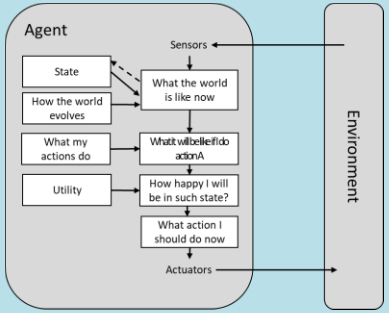
- Agents select actions on the basis of the current state-model / state description

Goal-based Agent



- Agents select actions on the basis of current state, and some sort of goal information that describes situations that are desirable

Utility Agent



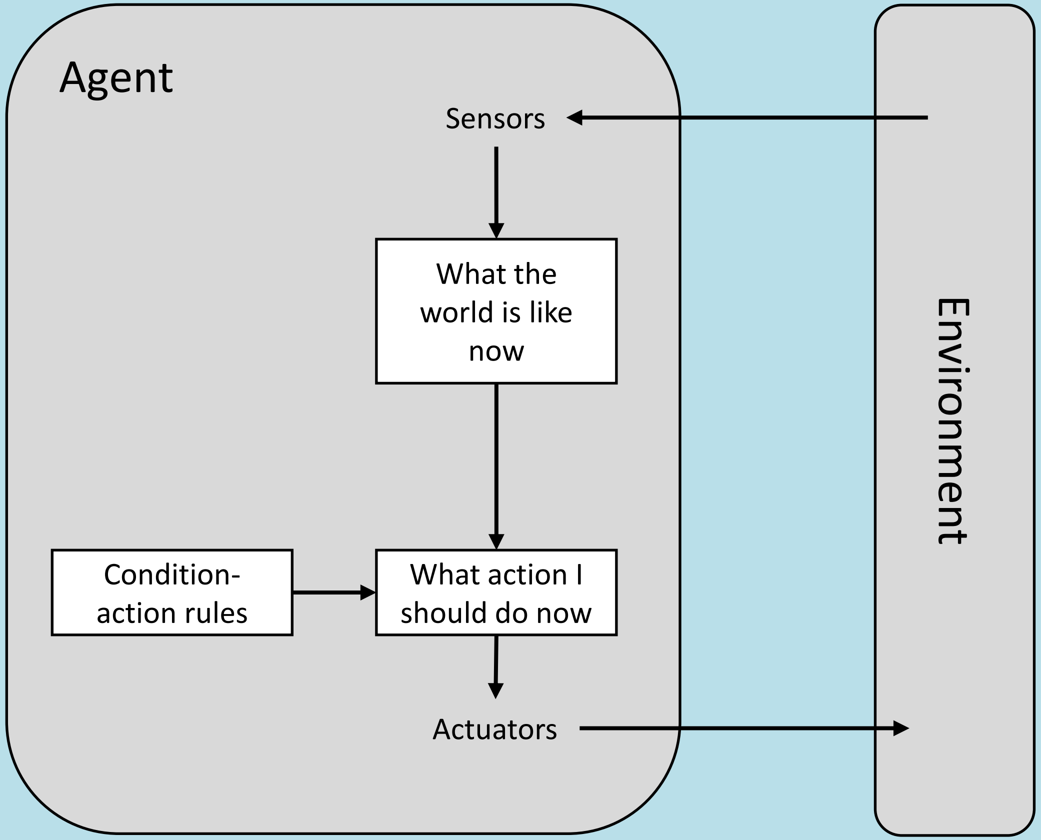
- Agents select actions by maximizing the utility function (if the external performance measure and the utility function are in agreement)

Learning Agent	All the agents can be generalized as learning agent

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Simple Reflex Agent

- Select actions on the basis of the current percept, ignoring the rest of the percept history



Adapted from Rusell, S., & Norvig, P. (2016)

Algorithm: Simple Reflex Agent

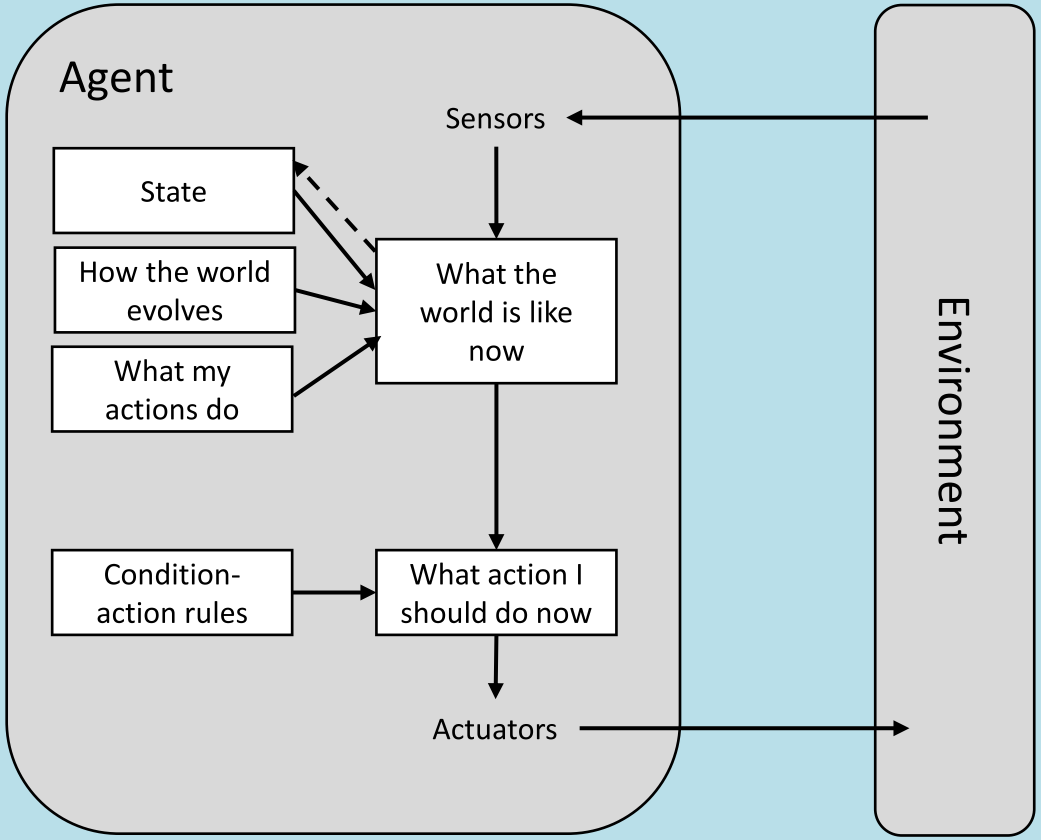
persistent: *rules*, a set of condition–action rules

state ← *INTERPRET-INPUT(percept)*
rule← *RULE-MATCH(state, rules)*
action ← *rule.ACTION*

return *action*

2.1 Model-based Reflex Agent

- Keep track of the part of the world it can't see now



Adapted from Rusell, S., & Norvig, P. (2016)

Algorithm: Model-based Reflex Agent

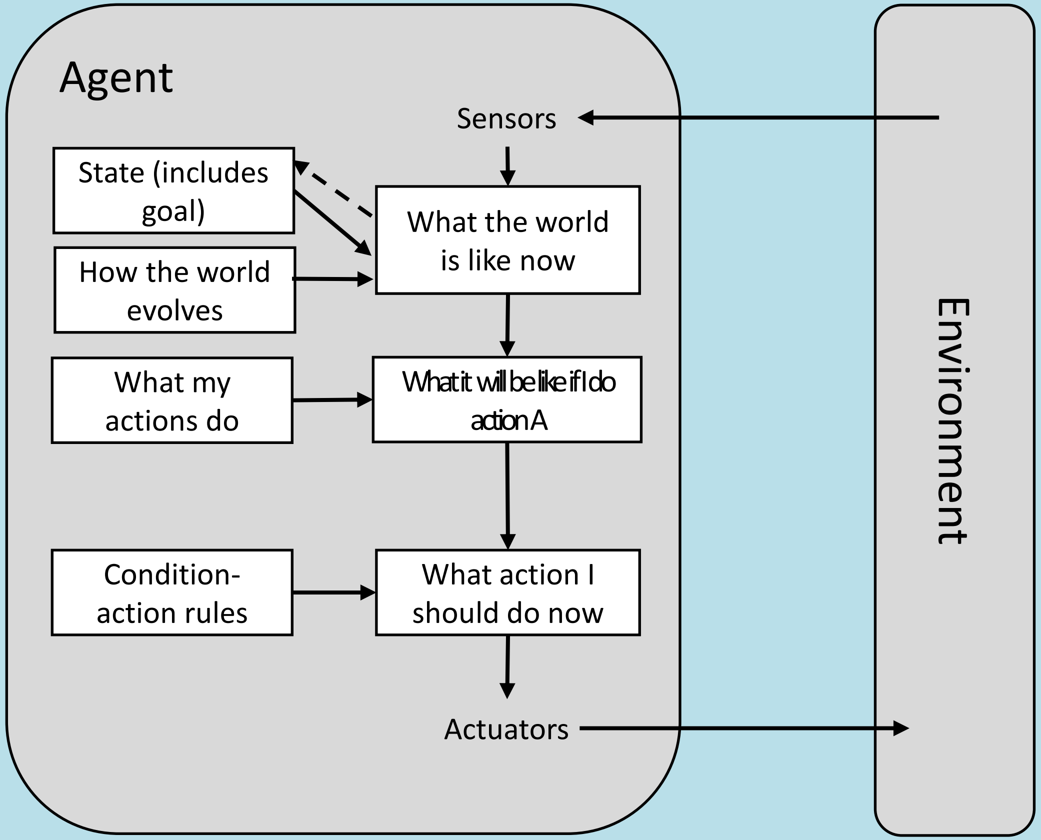
persistent:
state, the agent's current conception of the world state
model, a description of how the next state depends on current state and action
rules, a set of condition–action rules
action, the most recent action, initially

state ← UPDATE-STATE(state, action, percept, model)
rule← RULE-MATCH(state, rules)
action ← rule.ACTION

return *action*

2.1 Goal-based Agents

- Besides state description the agent uses goal information in its decision process



Adapted from Rusell, S., & Norvig, P. (2016)

Algorithm: Goal-based Agent

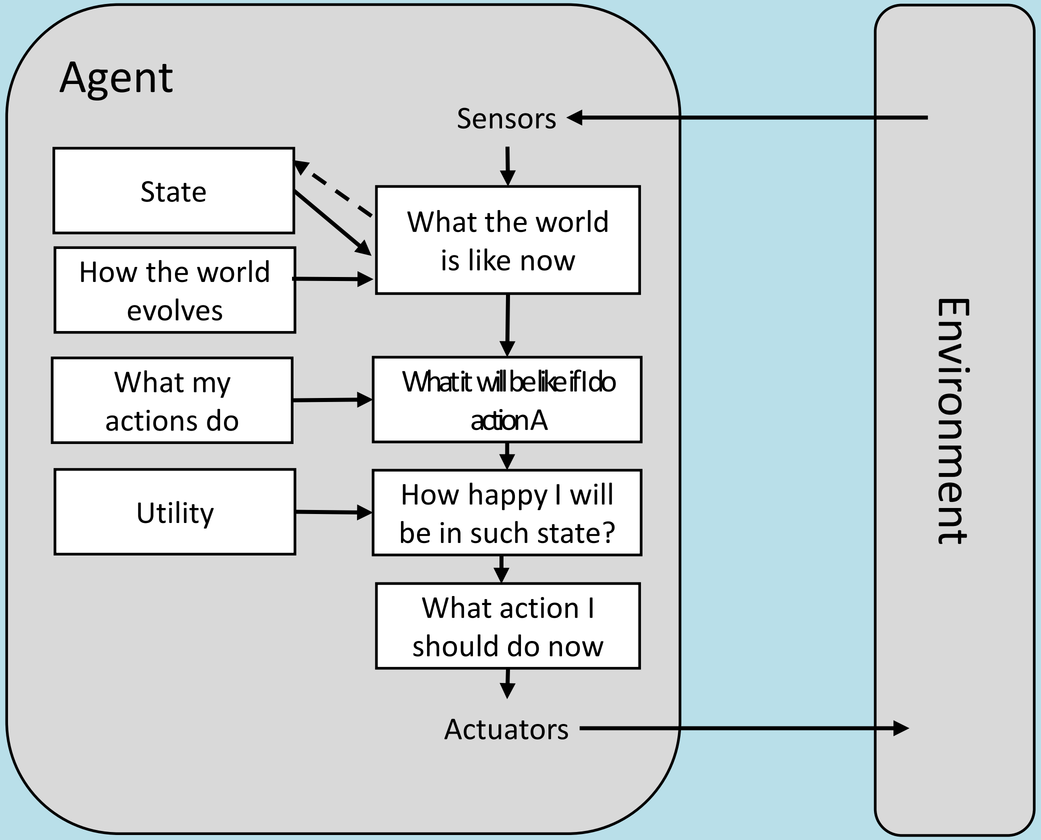
persistent:
state, the agent's current conception of the world state
model, a description of how the next state depends on current state and action
rules, a set of condition–action rules
action, the most recent action, initially
goal, desired result of the agents behaviour

state ← UPDATE-STATE(state, action, percept, model, goal)
rule← RULE-MATCH(state, rules)
action ← rule.ACTION

return *action*

2.1 Utility-based Agent

▶ Utility function to describe desired behavior



Algorithm: Goal-based Agent

persistent:
state, the agent’s current conception of the world state
model, a description of how the next state depends on current state and action
rules, a set of condition–action rules
action, the most recent action, initially
Utility function, performance measure

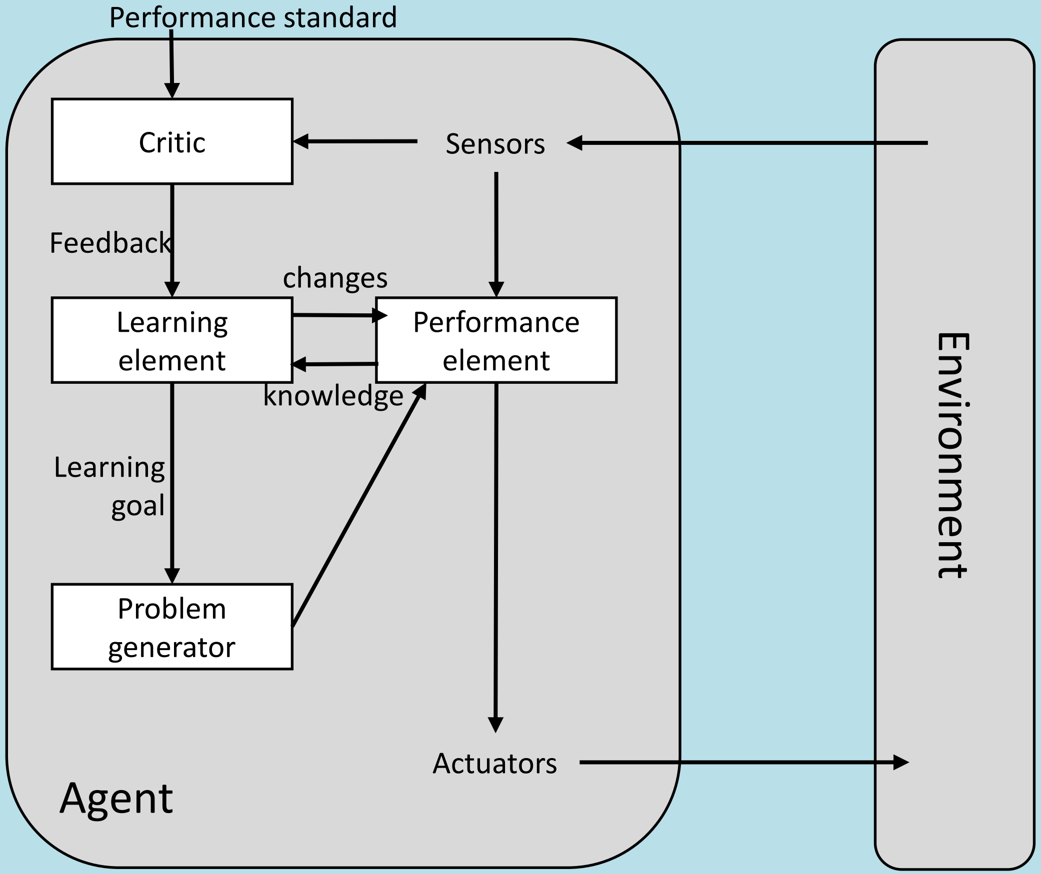
state ← UPDATE-STATE(state, action, percept, model, utility function)
rule← RULE-MATCH(state, rules)
action ← rule.ACTION

return *action*

Adapted from Rusell, S., & Norvig, P. (2016)

2.1 Conclusion: The Learning Agent

- Agent improves its performance by learning optimal outputs



Adapted from Rusell, S., & Norvig, P. (2016)

- Turing proposed four conceptual components: Learning element, performance element, critic and problem generator
- The design of the learning element depends very much on the design of the performance element
- The critic tells the learning element how well the agent is doing with respect to a fixed performance standard

Your turn!

Task

Please explain:

- What is the difference between an agents program and function?
- Could you model a hand-held calculator as an agent that chooses the action of displaying "`3`" when given the percept sequence "`1 + 2 =`" ? Is this correct based on the definitions of Russel & Norvig?

Outline

2 Search, Problem Solving, and Planning

- 2.1 Intelligent Agents
- **2.2 Solving Problems by Searching**
- 2.3 Beyond Classical Search
- 2.4 Adversarial Search and Game Theory
- 2.5 Constraint Satisfaction Problems

Image source: ↗ Pixabay (2019) / ↗ CC0

▶ **What we will learn:**
- We define the concept of rational agents (≈ intelligent agents)
- Characteristics of artificial agents (perfect or otherwise), the diversity of environments, and the resulting menagerie of agent types
- We discuss how AI problems can be modelled as search-problems, and how they can be solved by searching

▶ **Duration:**
- 225 min

▶ **Relevant for Exam:**
- 2.1 – 2.5

2.2 Why We Need Goals: Example Roadtrip Planning

- **Example**: US Nationalpark roadtrip with your new Porsche
- Next step: How to formulate AI Problems and solve them

Adapted from Rusell, S., & Norvig, P. (2016) | Image source: ↗ Pixabay (2019) / ↗ CC0

2.2 Why We Need Goals: Example Roadtrip Planning

D *We will consider a **goal** to be a set of world states—exactly those states in which the goal is satisfied* (Rusell & Norvig, 2016, p.65*)*

Our problem can be defined formally by different aspects like:

- Initial state or start
- Driving costs
- **Visit Top 5 parks**
- **Music playlist**
- Possible driving routes, allowed streets
- What each action does
- List with all parks to check if we got them all

→ **Simplify Goal**: Be in Shenandoah

Image sources: ↗ US NationalParks (2015) by Mwierschkec↗ CC BY-SA 4.0; ↗ View from Skyline Drive (2019) by Steevven1 ↗ CC BY-SA 4.0; ↗ Schluchten des Grand Canyon (2006) by Tenji ↗ CC BY-SA 3.0
And yes I am working on a roadtrip playlist, you can check it out here (↗ Spotify), feel free to give any suggestions

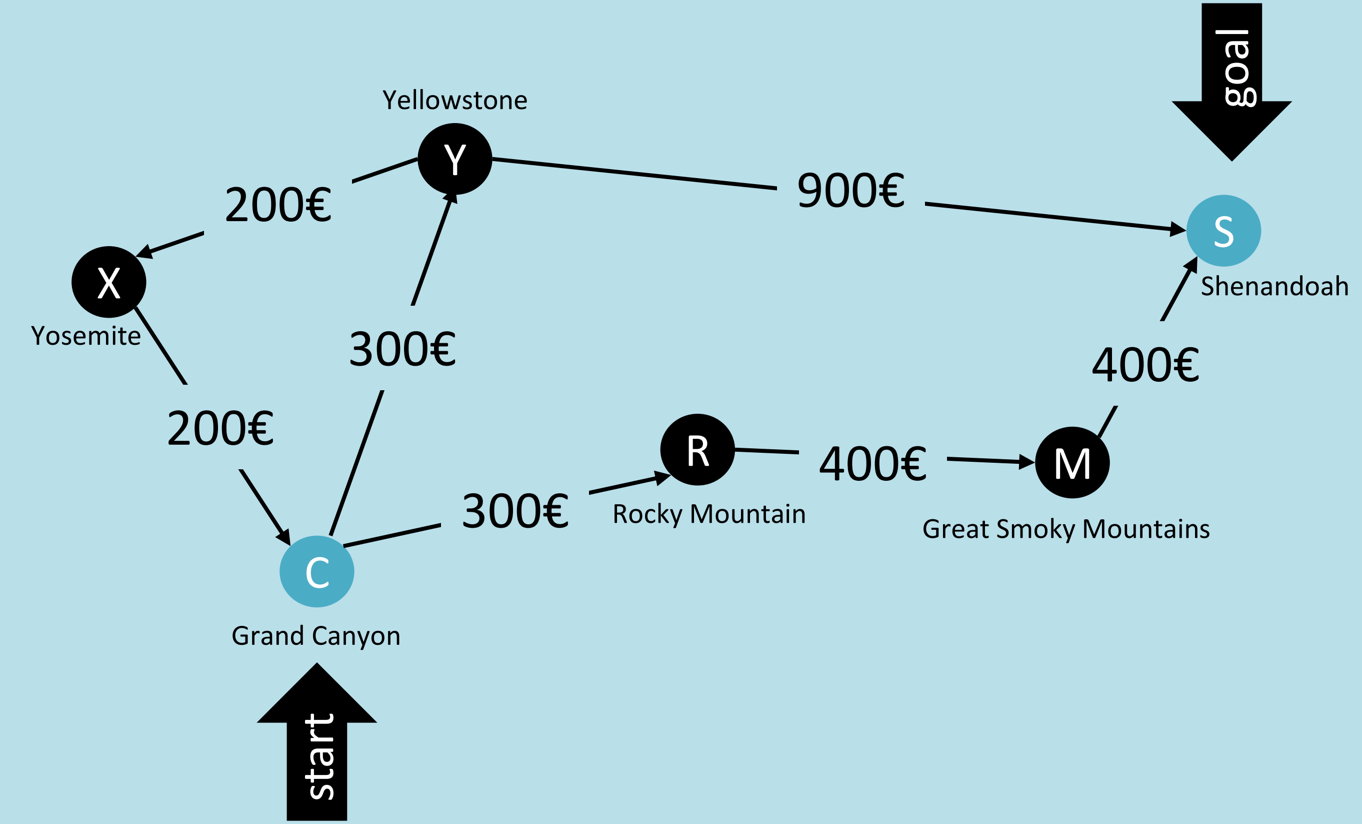
2.2 Well-defined Problems and Solutions

Adapted from Rusell, S., & Norvig, P. (2016)

Let us now try to model our problem in a way a computer can solve it:

- In a **first step**, we want to build an agent that finds the **optimal route from start to end** (route-finding problem)
- Our **second step** is to have an agent that finds the **optimal route for all/Top 5 mainland US parks** (touring-problem)

2.2 Model the Roadtrip as an AI problem



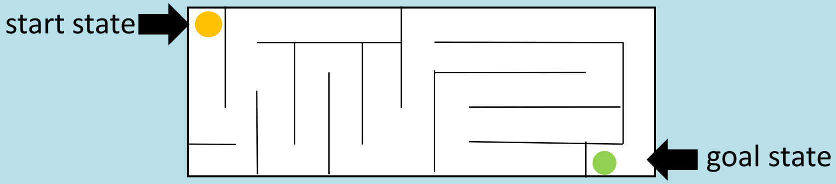
Goal: Be in Shenandoah

Other aspects:

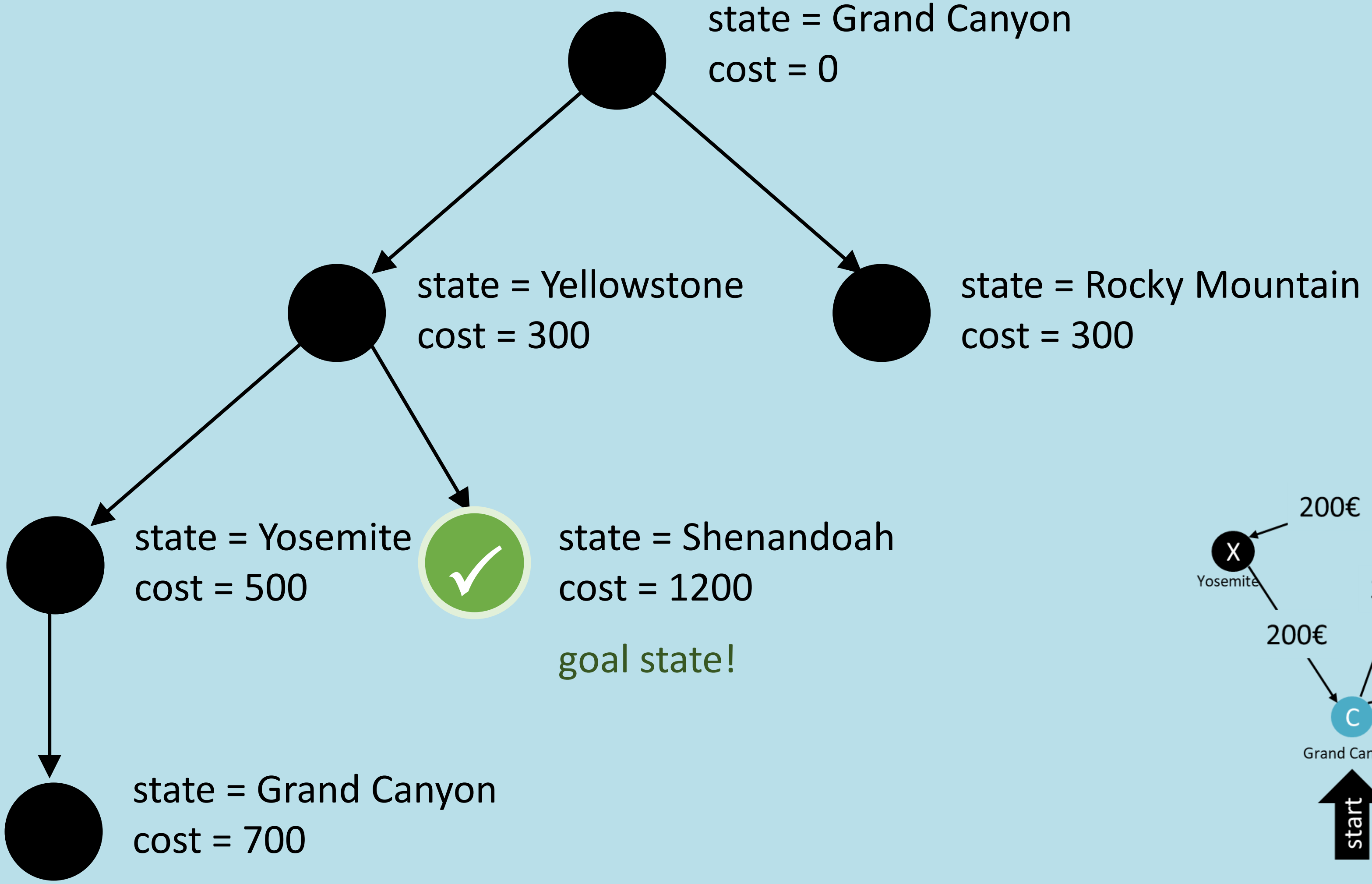
- Initial state or start: Grand Canyon
- Consider Driving costs
- Visit Top 5 parks

2.2 Possible Solution: Model Roadtrip-Problem as Search

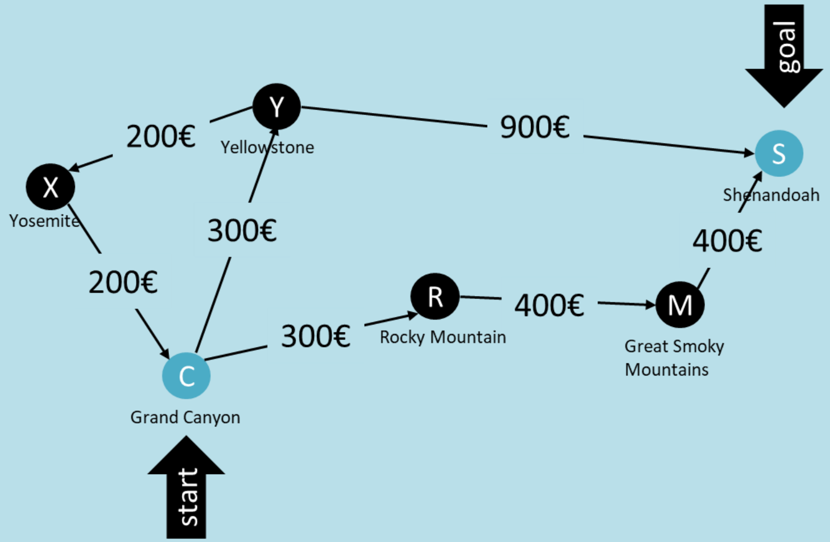
- Model problems as search problems
- Our agent does not have to execute all actions in real life while searching for solution
- We want to find a sequence of actions that will lead us to a desired state
  - We want to minimize number of actions
  - We want to minimize total cost of actions (more general speaking)



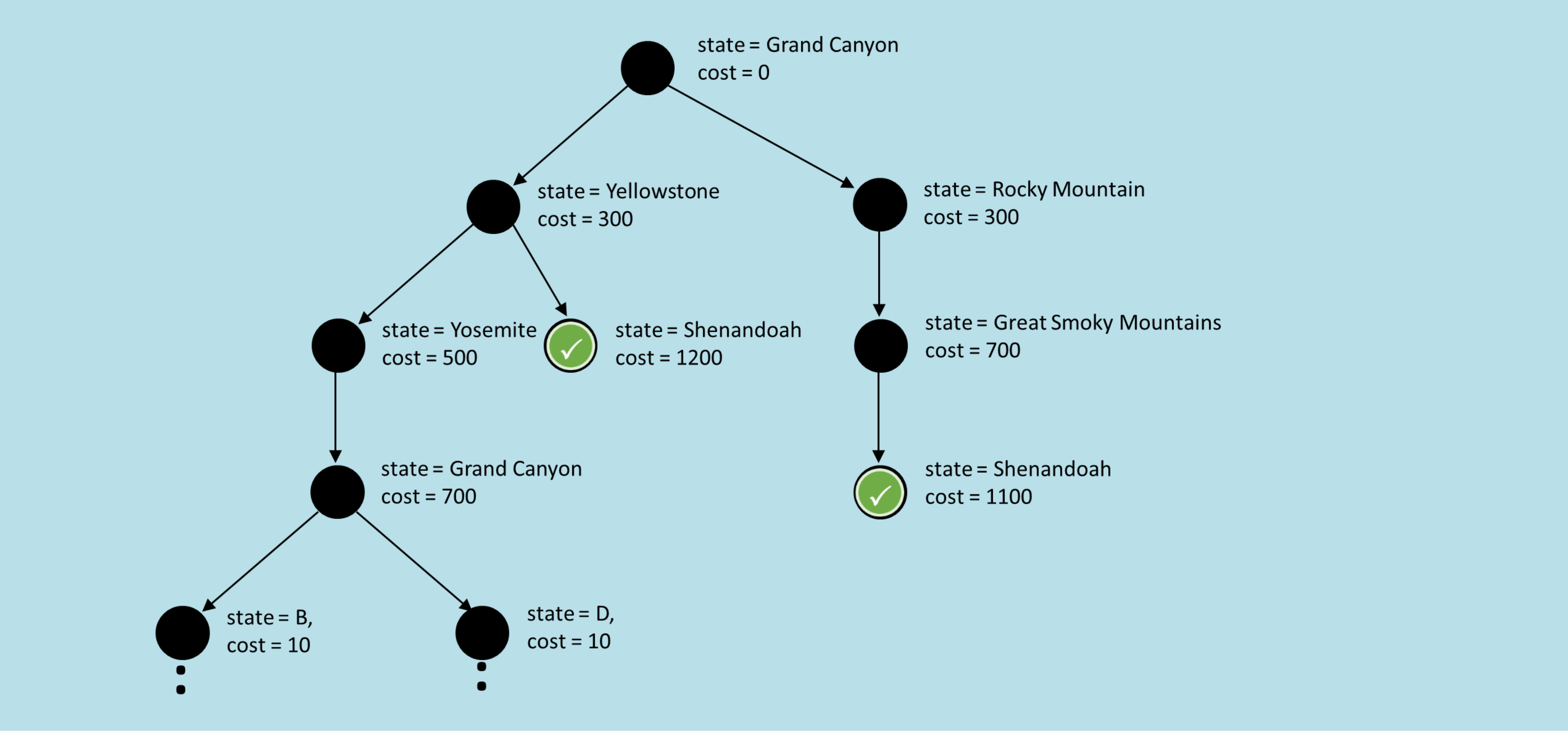
2.2 Model Problem as Search-tree ▶ Step 1



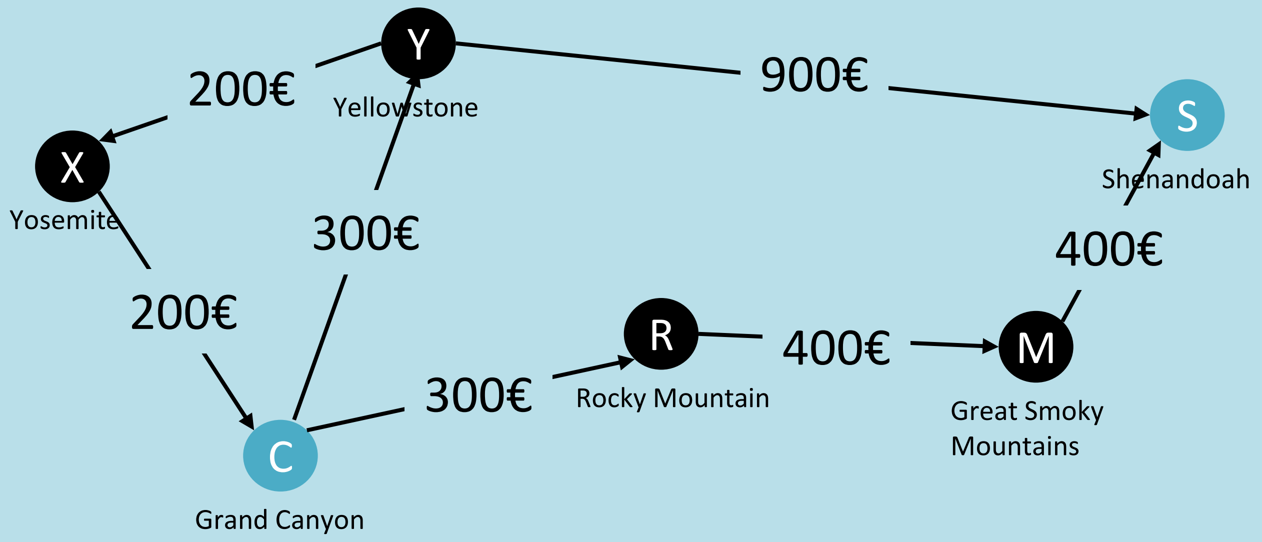
Search tree nodes and states are not the same thing!



2.2 Model Problem as Search-tree ▶ Step 2



2.2 From Rout-Finding to Touring-Problems: Visit TOP 5 Nationalparks

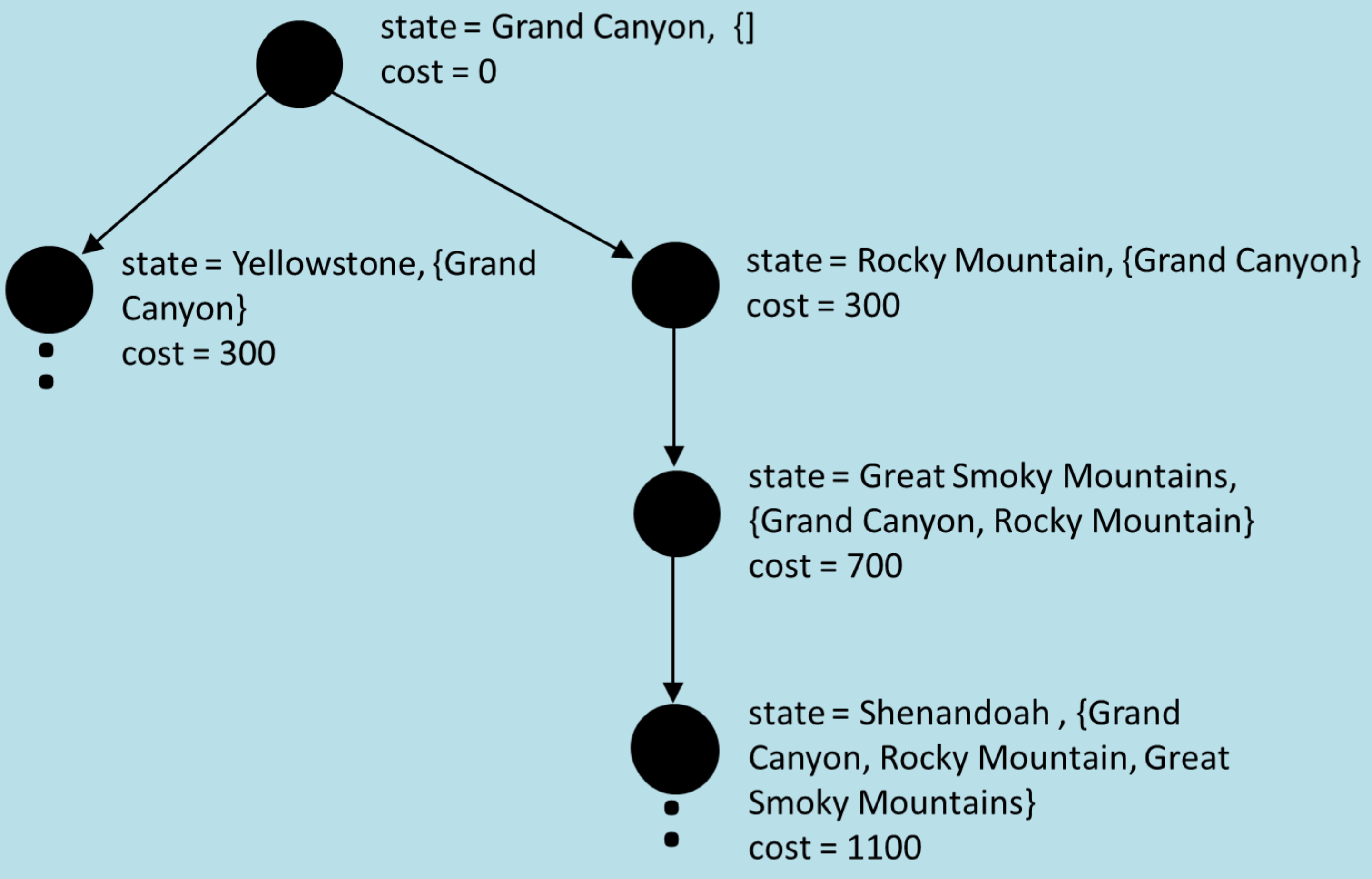


Goal: Visit all vertices on the graph

- As before, the actions correspond to trips between adjacent parks.
- The state space is quite different: Each state must include not just the current location but also the set of parks the agent has visited.

- **Problem**: large number of states

2.2 Full Search Tree



X Tree gets incredible big, due to large number of states: $n \cdot 2^{n-1}$

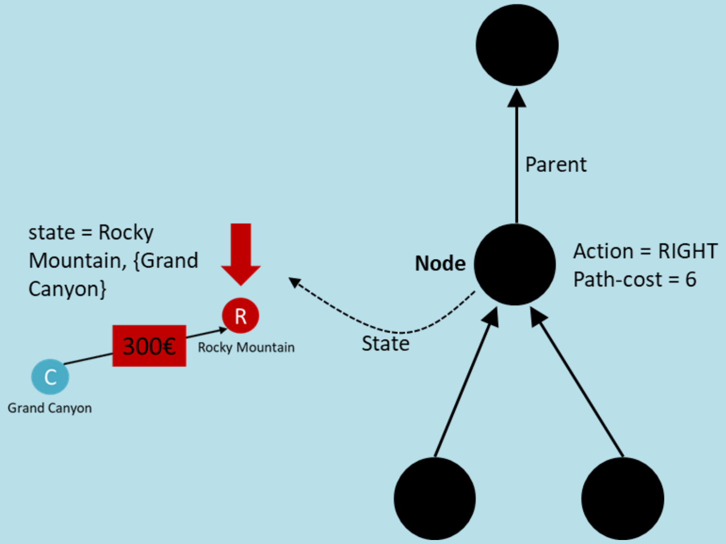
2.2 Summary Key Concepts in Search

Set of **states** that we can be in

- Including an initial state
- and goal states (equivalently, a goal test)

For every state, a set of **actions** that we can take

- Each action results in a new state
- Typically defined by successor function



Transition model: Given a state, produces all states that can be reached from it

Cost function that determines the cost of each action (or path = sequence of actions)

Solution: path from initial state to a goal state (optimal solution: solution with minimal cost)

Adapted from Rusell, S., & Norvig, P. (2016)

2.2 Problem-Solving Agents

- We can formalize this concepts into a general concept of a simple problem-solving agent
- For that purpose, we say that AI problems should be defined formally by five components:
  - initial state
  - description of actions
  - what each action does (transition model)
  - goal test
  - path cost function

operators

Algorithm: Simple Problem-Solving Agent

persistent:
seq, an action sequence, initially empty
state, a description of the current world state
goal, a goal, initially null
problem, a problem formulation

state ← *UPDATE-STATE(state, percept)*
If *seq* is empty ***then***
    goal← *FORMULATE-GOAL(state)*
    problem← *FORMULATE-PROBLEM(state, goal)*
    seq← *SEARCH(problem)*
    If *seq* = *failure* ***then return*** null action
action ← *FIRST(seq)*
seq← *REST(seq)*

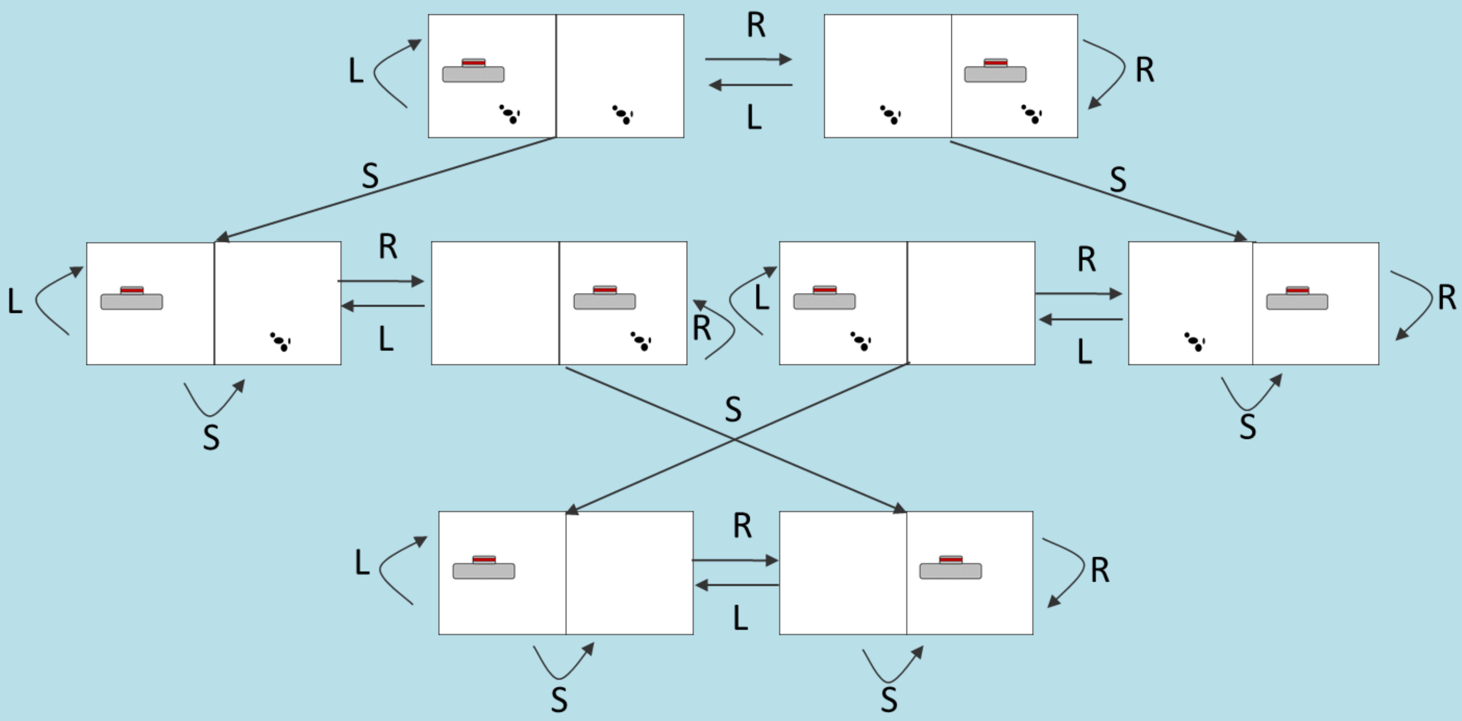
return *action*

2.2 What We Did so Far - Formulating Problems for AI

- In the preceding section we proposed a simple formulation of the problem of planning an US nationalpark trip
- This formulation seems reasonable, but it is still an abstract mathematical simplification of a much more complex problem
- Many considerations are not considered in our state descriptions because they are irrelevant to the problem of finding a route.
- The process of removing detail from a representation is called abstraction.
- Toy vs. real-world problem

Adapted from Rusell, S., & Norvig, P. (2016) | And yes I am working on a roadtrip playlist, you can check it out here (↗ Spotify), feel free to give any suggestions

2.2 Example: Vacuum-Cleaner



Adapted from Rusell, S., & Norvig, P. (2016);

- **States**: The state is determined by both the agent location and the dirt locations.
- **Initial state**: Any state can be designated as the initial state.
- **Actions**: *Left*, *Right*, and *Suck*.
- **Transition model**: The actions have their expected effects, except that moving *Left* in the leftmost square, moving *Right* in the rightmost square, and *Suck*ing in a clean square have no effect.
- **Goal test**: This checks whether all the squares are clean.
- **Path cost**: Each step costs 1, so the path cost is the number of steps in the path.

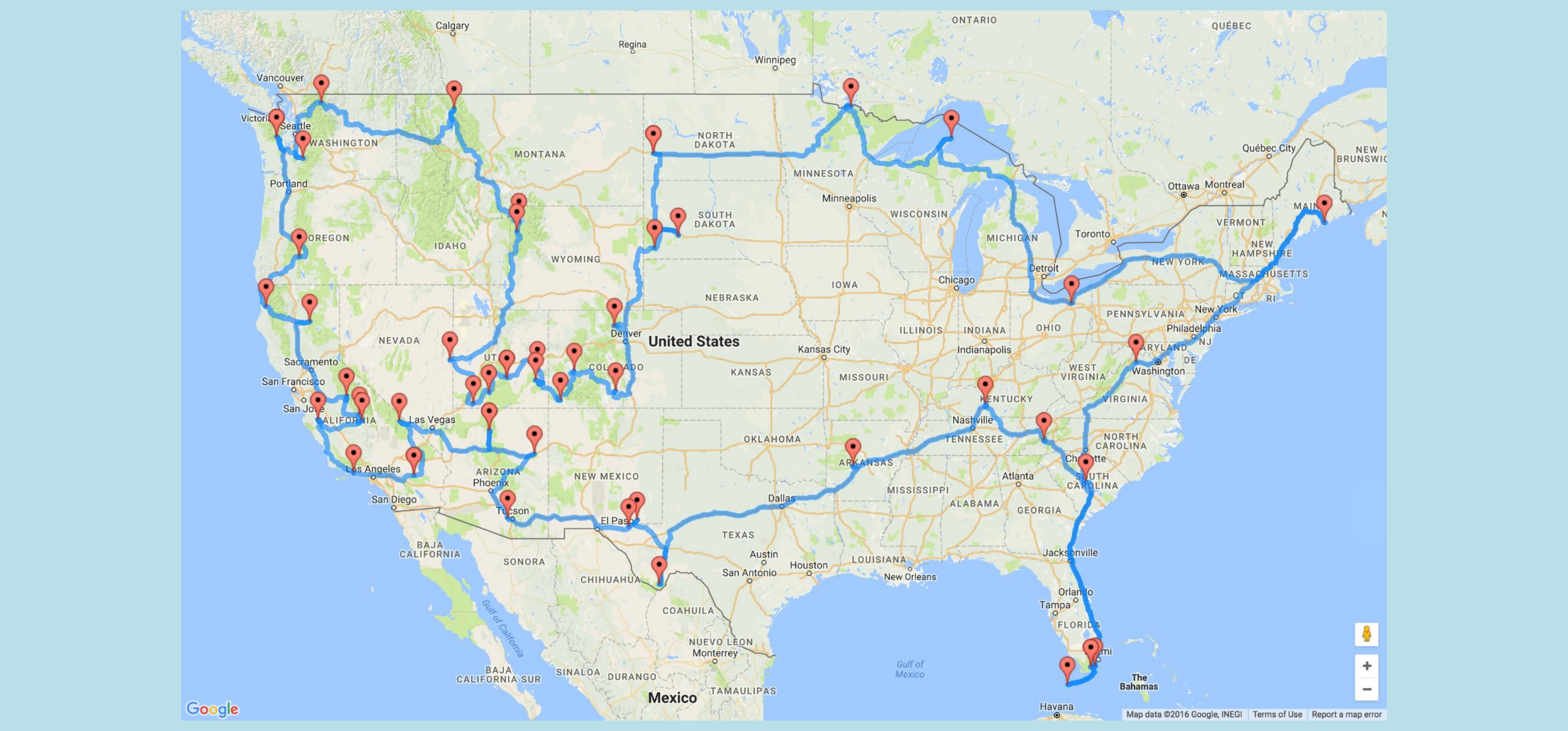
2.2 Other Popular Real-world Problems

- We tried to find
  - the **optimal route from start to end** (route-finding problem)
  - **An optimal route for to visit all TOP 5 US parks** (touring-problem)

- The **traveling salesperson problem** (TSP) is a touring problem in which each city must be visited exactly once. The aim is to find the *shortest* tour.

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Traveling Salesperson Problem of all 47 Mainland US Nationalparks



Credits for this route and Image source: ↗ [Dr. Randal Olson](#) (2019)

2.2 Other Popular Real-world Problems

- The **traveling salesperson problem** (TSP) is a touring problem in which each city must be visited exactly once. The aim is to find the *shortest* tour.

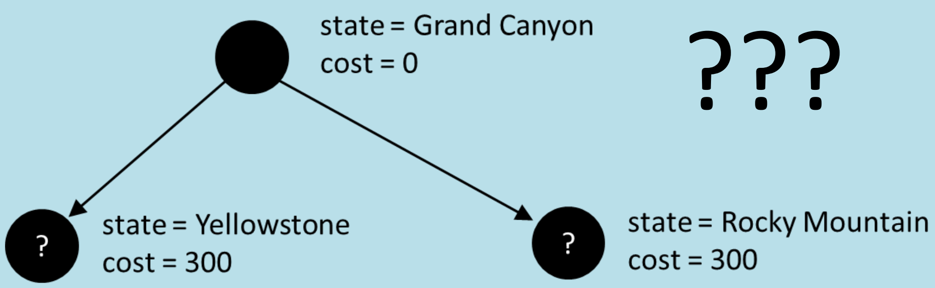
- A **VLSI layout** problem requires positioning millions of components and connections on a chip to minimize area, minimize circuit delays, minimize stray capacitances, and maximize manufacturing yield

- **Protein design**: in which the goal is to find a sequence of amino acids that will fold into a three-dimensional protein with the right properties to cure some disease.

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Generic Search Algorithm

- Recap: We will consider the problem of designing **goal-based** agents in **observable, deterministic, discrete, known** environments
- **Key question in search**: Which of the generated nodes do we expand next?



Algorithm: Tree Search

initialize the frontier using the initial state of problem

loop do
  if the frontier is empty **then return** failure
  choose a leaf node and remove it from the frontier

  if the node contains a goal state **then return** the corresponding solution

expand the chosen node, adding the resulting nodes to the frontier

Adapted from Rusell, S., & Norvig, P. (2016)

2.2 Overview: Fundamental Search Algorithms

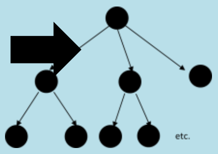
D

Uninformed Search

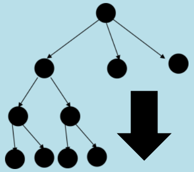
Strategies have no additional information about states beyond that provided in the problem definition

Uninformed Search

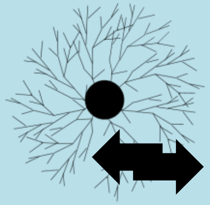
- Breadth-first (BFS)
- Depth-first search (DFS)
- Bidirectional



- Root node is expanded first, then all the successors of the root node are expanded next, then their successors, and so on.



- Always expands the deepest node in the current frontier of the search tree.



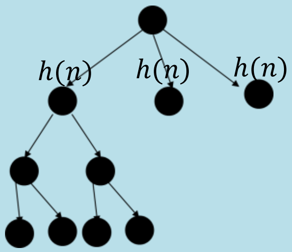
- Always expands the deepest node in the current frontier of the search tree.

Informed Search

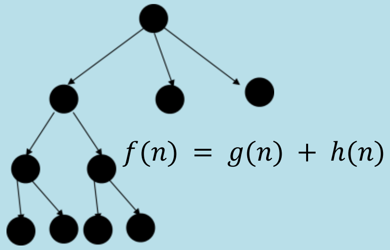
Uses problem-specific knowledge beyond the definition of the problem itself—can find solutions more efficiently than can an uninformed strategy.

Informed Search

- Greedy best-first
- A-Star (A*)



- Expand the node that is closest to the goal, on the grounds that this is likely to lead to a solution quickly.



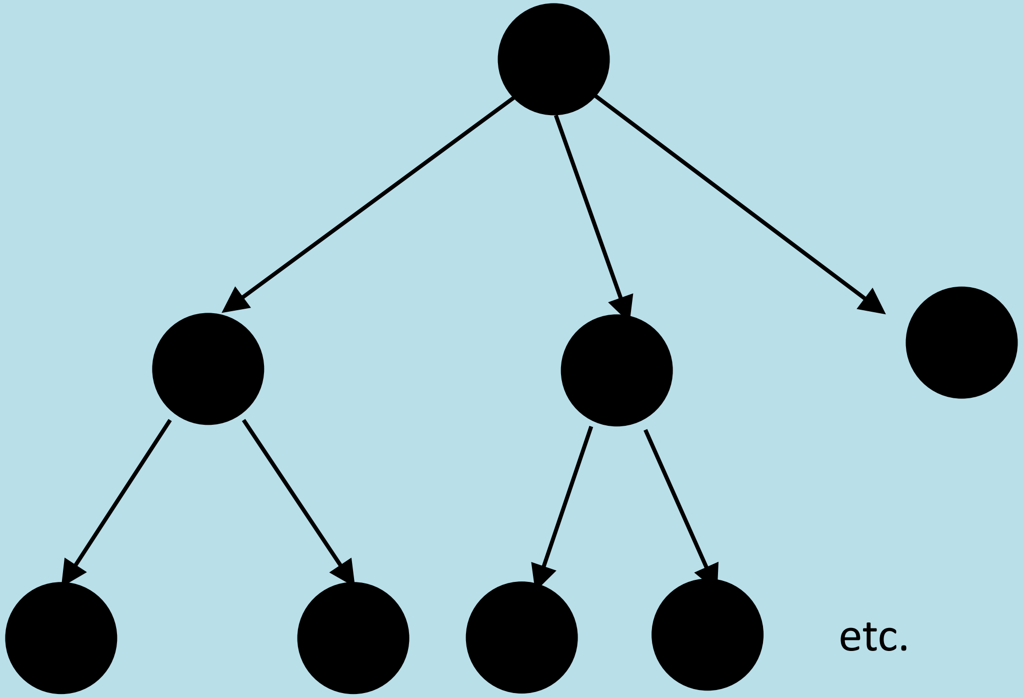
- Cluster observations into (distinct/different) groups

2.2 Uninformed Search

- Given a state, we only know whether it is a goal state or not
- Cannot say one non-goal state looks better than another non-goal state
- Can only traverse state space blindly in hope of somehow hitting a goal state at some point
  - Also called blind search
  - Blind does **not** imply unsystematic!

Adapted from Rusell, S., & Norvig, P. (2016);

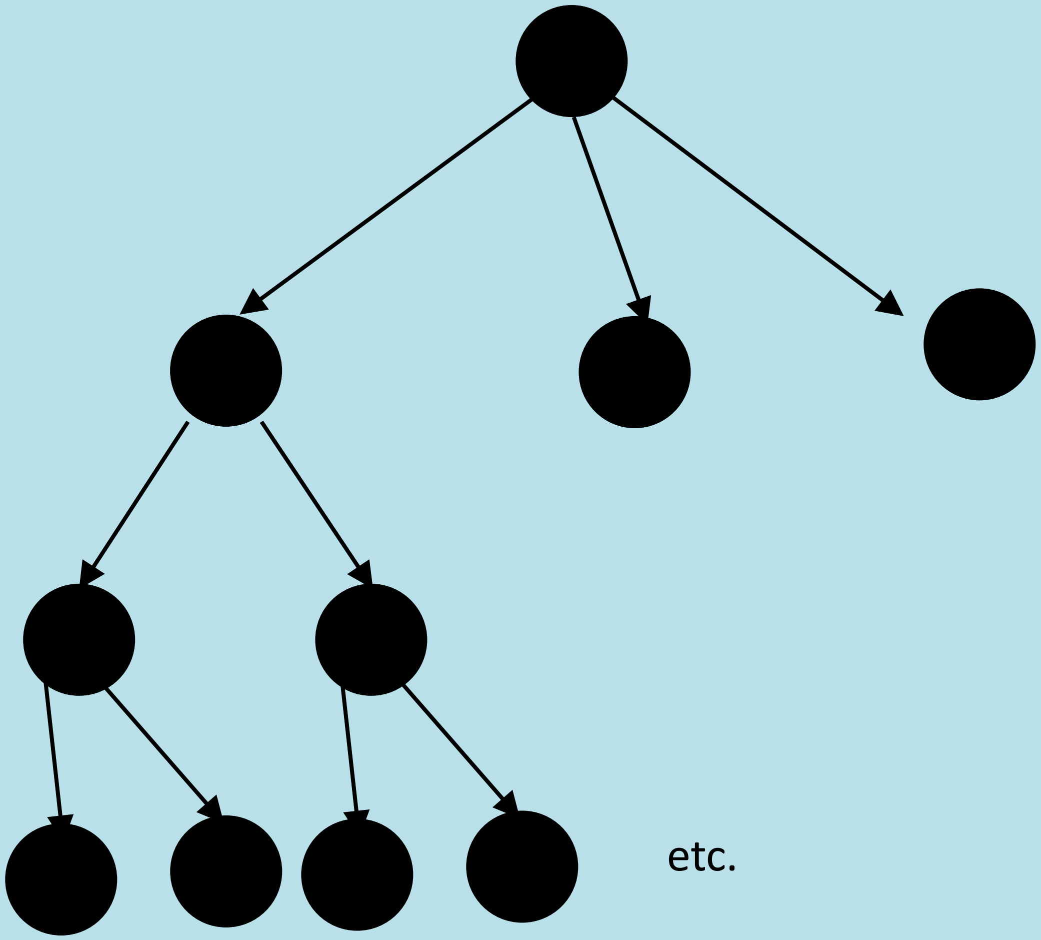
2.2 Breadth-first Search



Adapted from Rusell, S., & Norvig, P. (2016);

- Nodes are expanded in the same order in which they are generated
- Fringe can be maintained as a First-In-First-Out (FIFO) queue
- BFS is complete: if a solution exists, one will be found
- BFS finds a shallowest solution
- Not necessarily an optimal solution
- If every node has b successors (the branching factor), first solution is at depth d, then fringe size will be at least bd at some point
- This much space (and time) required !!!

2.2 Depth-first Search



- Always expand node at the deepest level of the tree, e.g. one of the most recently generated nodes
- When hit a dead-end, backtrack to last choice
- Fringe can be maintained as a Last-In-First-Out (LIFO) queue (aka. a stack)

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Expansion: Combining Properties of BFS and DFS

- Limited depth DFS: just like DFS, except never go deeper than some depth d
- Iterative deepening DFS:
  - Call limited depth DFS with depth 0;
  - If unsuccessful, call with depth 1;
  - If unsuccessful, call with depth 2;
  - etc.
- Complete, finds shallowest solution
- Space requirements of DFS
- May seem wasteful timewise because replicating effort
- Really not that wasteful because almost all effort at deepest level
- $db + (d-1)b^2 + (d-2)b^3 + \ldots + 1bd \text{ is } O(b^d) \text{ for } b > 1$

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Let’s Start Thinking About Cost

- **Path costs**: a function that assigns a cost to path, typically by summing the costs of the individual operators in the path. We want to minimize the cost.

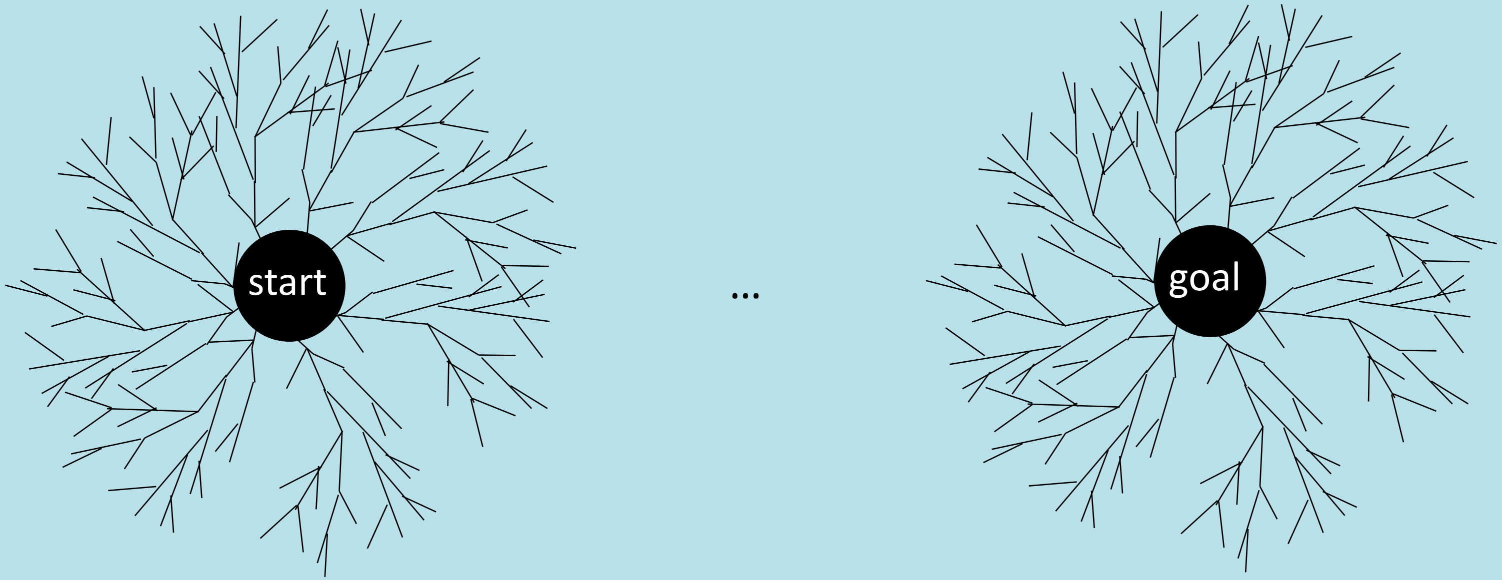
- **Search costs**: The computational time and space (memory) required to find the solution

- There is a trade-off between path costs and search cost, in real-world-problems we can not build full search trees, we have to find best solution in the time available

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Bidirectional Search

- Even better: search from both the start and the goal, in parallel!



If the shallowest solution has depth d and branching factor is b on both sides, requires only $O(b^{d/2})$ nodes to be explored!

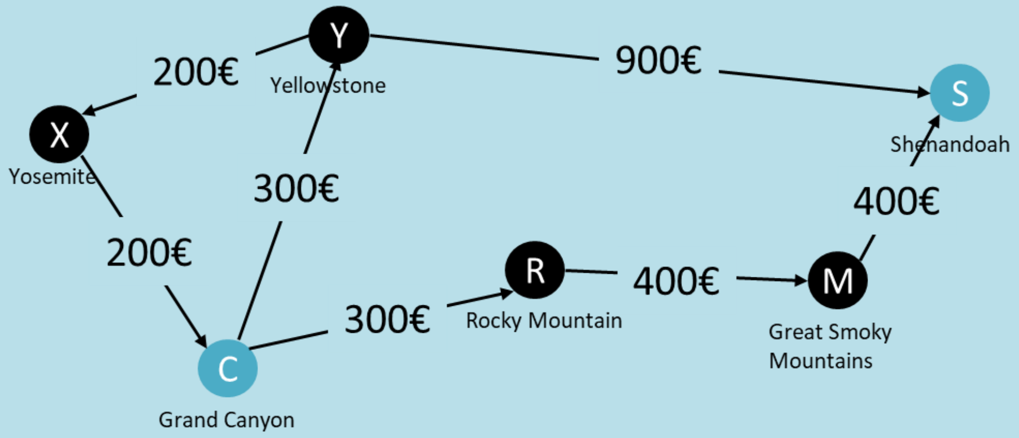
Adapted from Rusell, S., & Norvig, P. (2016);

- So far, have assumed that no non-goal state looks better than another
- Use knowledge to build search trees:
  - Even without knowing the road structure, some locations seem closer to the goal than others
  - Some states of a problem seem closer to the goal than others
- Makes sense to expand closer-seeming nodes first

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Idea: Use an Criterion to Identify which Node to Expand First

- Heuristic function h(n) gives an estimate of the distance from n to the goal (with h(n)=0 for goal nodes)
- E.g. straight-line distance for traveling problem (less costs for fuel etc.)



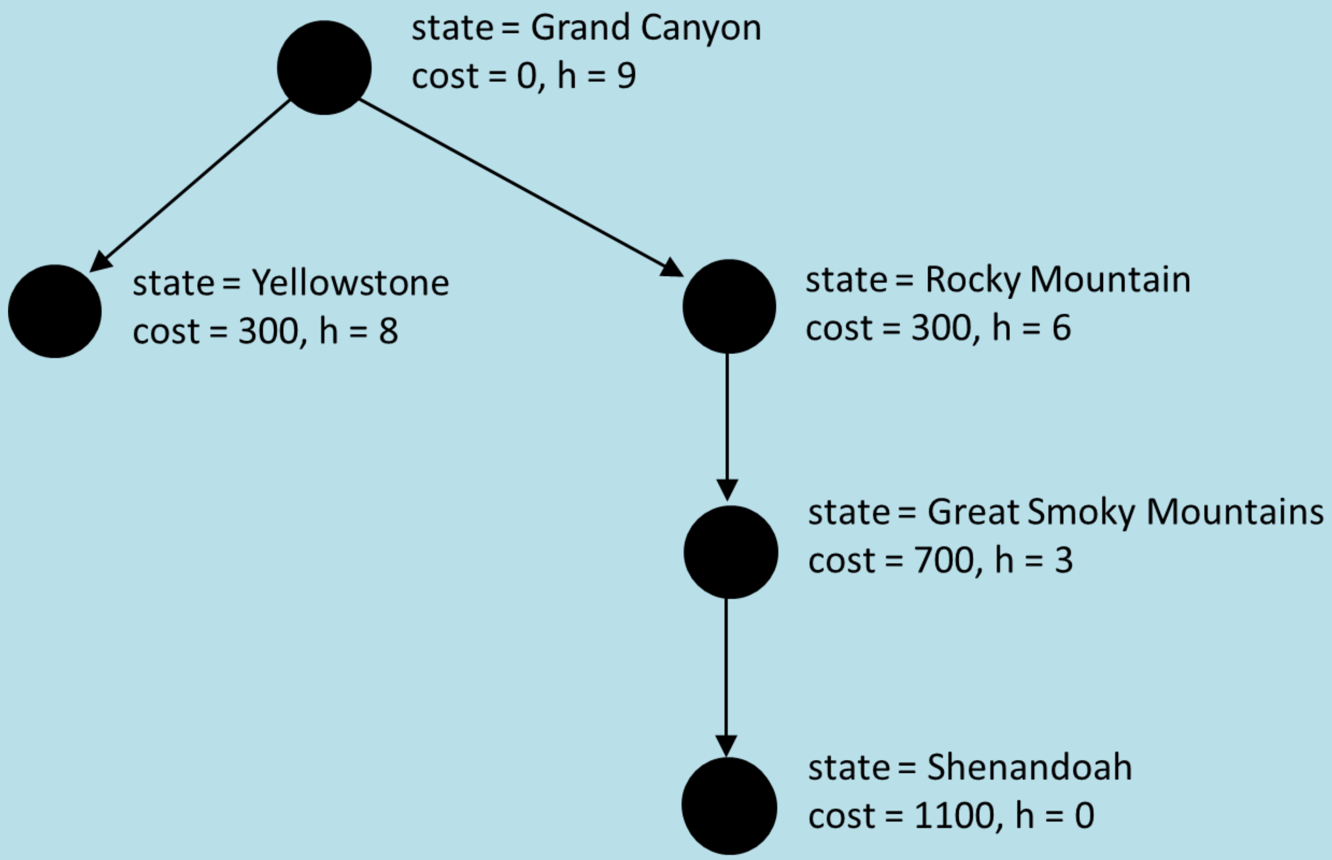
- We assume: h(C) = 9, h(Y) = 8, h(X) = 9, h(R) = 6, h(M) = 3, h(S) = 0
- Can use heuristic to decide which nodes to expand first

Adapted from Rusell, S., & Norvig, P. (2016) | Image sources: ↗ US NationalParks (2015) by Mwierschkec↗ CC BY-SA 4.0;

2.2 Greedy Best-First Search

- Greedy best-first search: expand nodes with lowest h values first

- Can find fast an optimal solution



Adapted from Rusell, S., & Norvig, P. (2016);

2.2 A* search: Minimizing the Total Estimated Solution Cost

- Evaluate nodes by combining g(n), the cost to reach the node, and h(n), the cost to get from the node to the goal:

$$f(n) = g(n) + h(n)$$

- Since $g(n)$ gives the path cost from the start node to node n, and $h(n)$ is the estimated cost of the cheapest path from n to the goal, we have:

$$f(n) = estimated\ cost\ of\ the\ cheapest\ solution\ through\ n$$

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 A* Search and Admissibility

- A heuristic is admissible if it never overestimates the distance to the goal
- If n is the optimal solution reachable from n', then $g(n) \geq g(n') + h(n')$
- Straight-line distance is admissible: can't hope for anything better than a straight road to the goal
- Admissible heuristic means that A* is always optimistic

Adapted from Rusell, S., & Norvig, P. (2016);

2.2 Optimality of A*

- If the heuristic is admissible, A* is optimal (in the sense that it will never return a suboptimal solution)
- Proof:
  - Suppose a suboptimal solution node n with solution value C > C* is about to be expanded (where C* is optimal)
  - Let n* be an optimal solution node (perhaps not yet discovered)
  - There must be some node n’ that is currently in the fringe and on the path to n*
  - We have $g(n) = C > C* = g(n*) \geq g(n’) + h(n’)$
  - But then, n’ should be expanded first (contradiction)

Adapted from Rusell, S., & Norvig, P. (2016);

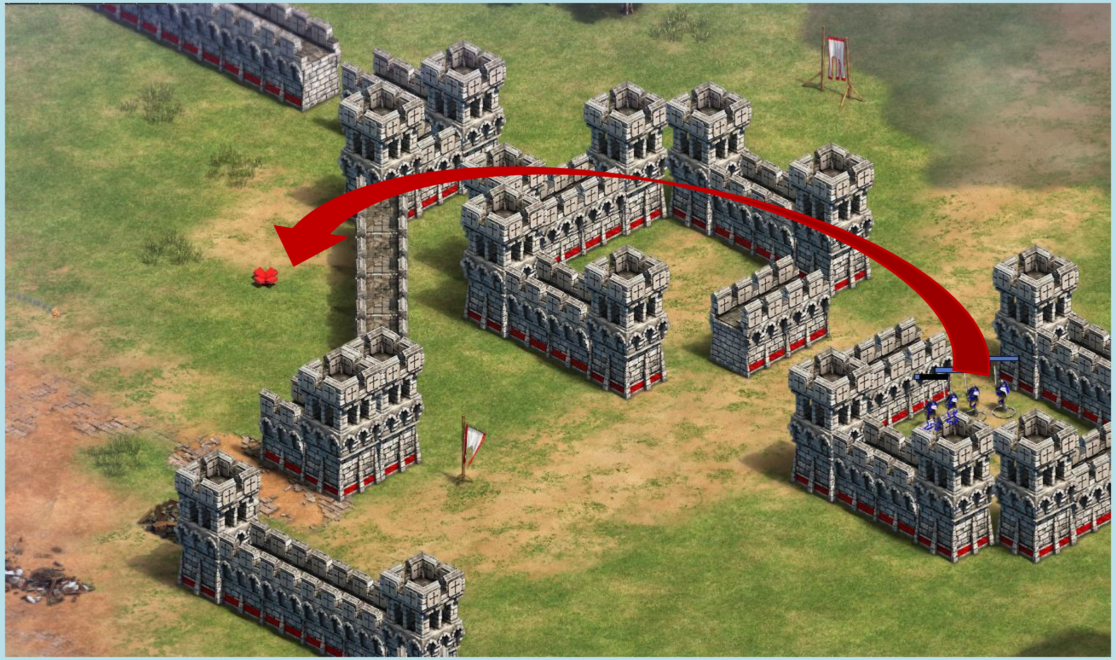
2.2 Example: Age of Empires 2 Pathfinding

freeaoe / src / ai / location.hh

Code Blame 334 lines (285 loc) · 8.38 KB

```
/// Add \a width columns, in place.
inline position&
operator+= (position& res, position::counter_type width)
{
  res.columns (width);
  return res;
}

/// Add \a width columns.
inline position
operator+ (position res, position::counter_type width)
{
  return res += width;
}

/// Subtract \a width columns, in place.
inline position&
operator-= (position& res, position::counter_type width)
{
  return res += -width;
}

/// Subtract \a width columns.
inline position
operator- (position res, position::counter_type width)
{
  return res -= width;
}

/// Compare two position objects.
inline bool
operator== (const position& pos1, const position& pos2)
{
  return (pos1.line == pos2.line
          && pos1.column == pos2.column
          && (pos1.filename == pos2.filename
              || (pos1.filename && pos2.filename
                  && *pos1.filename == *pos2.filename)));
}
```

- The map in AOE2 is a grid of squares which computes distances as Manhattan distance
- Some of the squares are not accessible (water, trees, buildings, squares with units in them etc.)

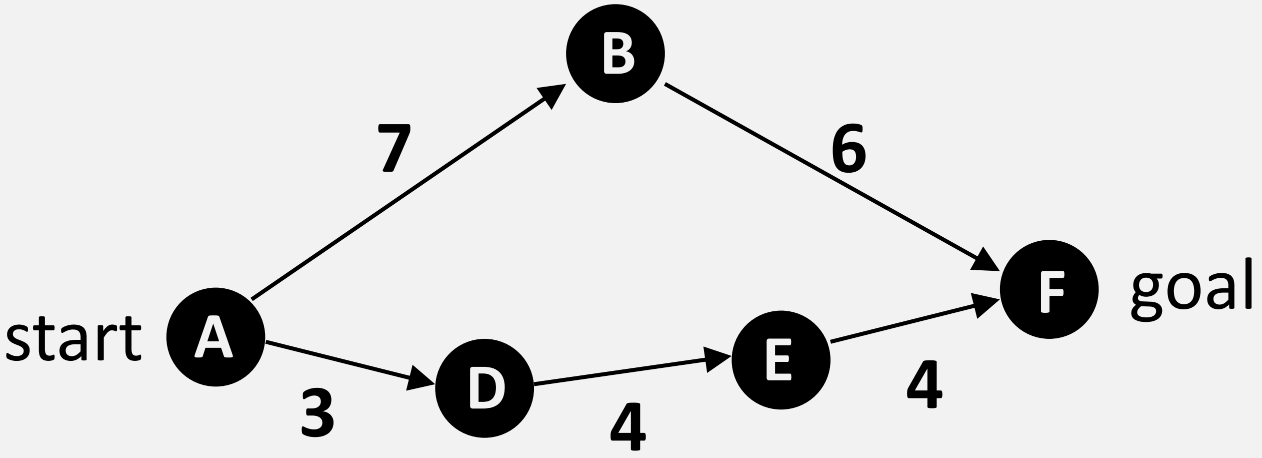


Source: https://github.com/sandsmark/freeaoe/blob/master/src/ai/location.hh

Your turn!

Task

Given the following route map with the following distance heuristics h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0. Try to solve this map with the greedy algorithm and discuss the results with your neighbors!



Outline

Image source: ↗ Pixabay (2019) / ↗ CC0

▶ **What we will learn:**

- We define the concept of rational agents (≈ intelligent agents)
- Characteristics of artificial agents (perfect or otherwise), the diversity of environments, and the resulting menagerie of agent types
- We discuss how AI problems can be modelled as search-problems, and how they can be solved by searching

▶ **Duration:**

- 225 min

▶ **Relevant for Exam:**

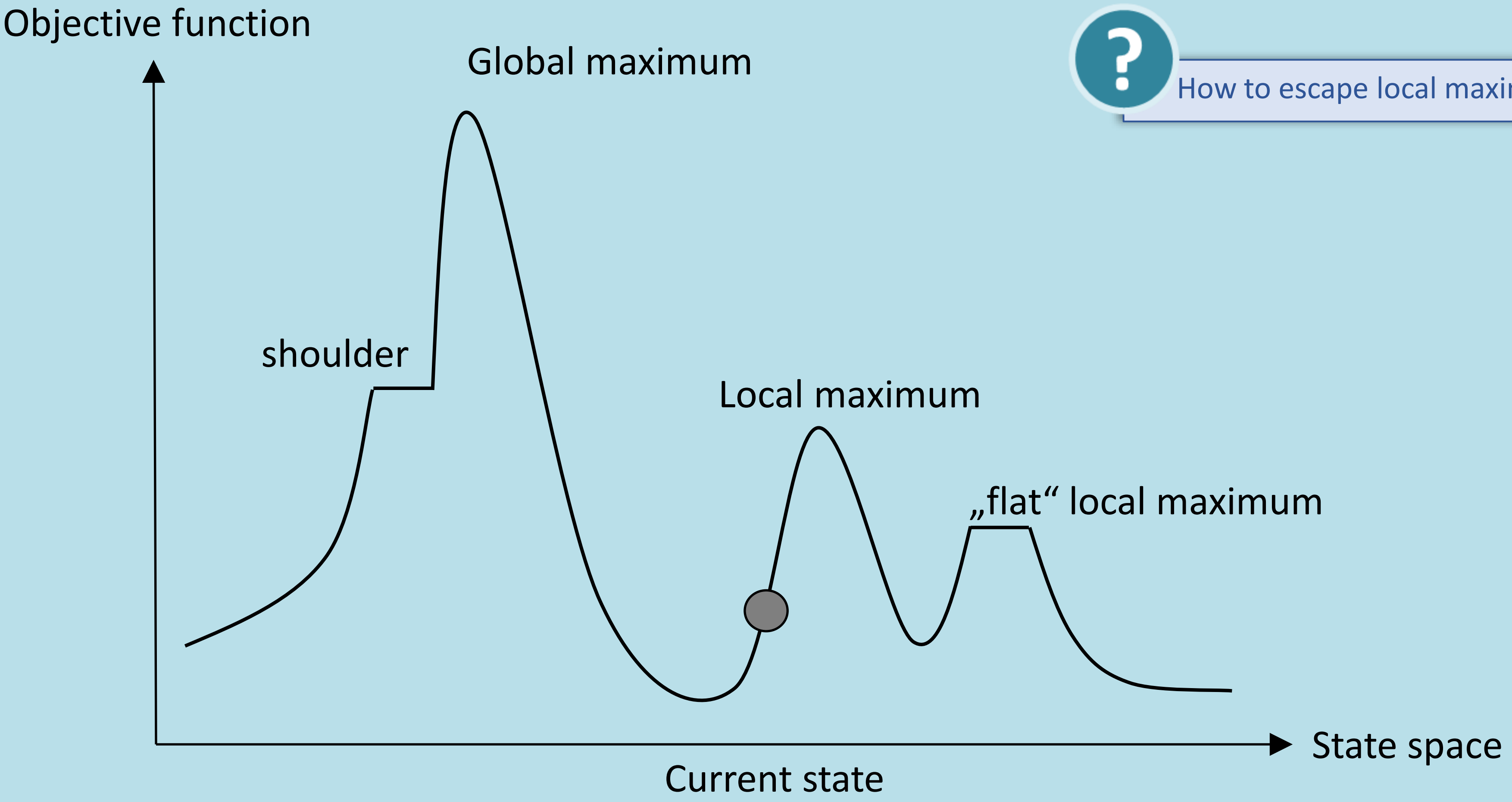
- 2.1 – 2.5

- **Previous chapter**: Path to goal is solution to problem

- But sometimes…
  - The start state may not be specified
  - The path to the goal doesn't matter

- In such cases, we can use local search algorithms that keep a single "current" state and gradually try to improve it

2.3 The State Space "Landscape"



How to escape local maxima?

Image adapted from Rusell, S., & Norvig, P. (2016);

2.3 How to Escape Local Maxima: Trivial Algorithms

- Random Sampling
  - Generate a state randomly
- Random Walk
  - Randomly pick a neighbor of the current state
- Both algorithms asymptotically complete

2.2 Overview: Beyond Classical Search

D

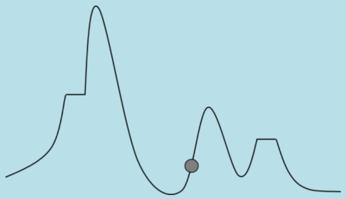
Local Search

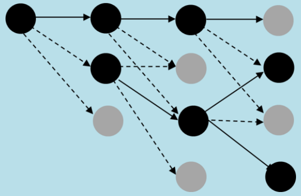
Local search algorithms are such algorithms, which use a single current node (rather than multiple paths) and generally move only to neighbors of that node

Local Search

Hilll Climbing

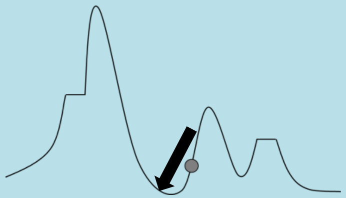
Beam





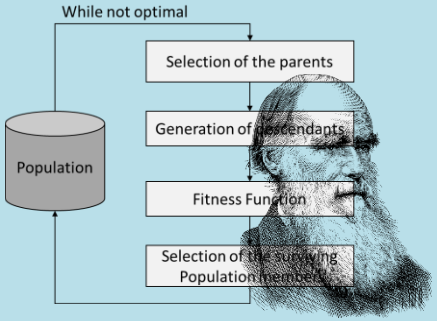
- Continually move in the direction of increasing value
- expanding the most promising node in a limited set

Simulated Anhealing



- Escape local maxima by allowing some "bad" moves but gradually decrease their frequency

Genetic Algorithms



- Always expands the deepest node in the current frontier of the search tree.

Adapted from Rusell, S., & Norvig, P. (2016) | | Image source: ↗ Pixabay (2019) / ↗ CC0

2.3 Local Search Algorithms

- If the path to the goal does not matter → simplify algorithms and ignore paths
- **Local search** algorithms are such algorithms, which use a single **current node** (rather than multiple paths) and generally move only to neighbors of that node
- Typically, the paths followed by the search are not retained. Although local search algorithms are not systematic, they have two key advantages:
  - + they use very little memory - usually a constant amount
  - + they can often find reasonable solutions in large or infinite (continuous) state spaces for which systematic algorithms are unsuitable

Adapted from Rusell, S., & Norvig, P. (2016);

2.3 Hill-Climbing Search

Algorithm: Hill Climbing

current ← *MAKE-NODE(problem.INITIAL-STATE)*

While STOP != TRUE

  $neighbor \leftarrow a\ highest{-}valued\ succesor\ of\ current$

  If *neighbor.value* ≤ *current.VALUE*

    Then return *current. STATE*

  current ← *neighbor*

- **Idea**: simply a loop that continually moves in the direction of increasing value—that is, uphill
- The algorithm does not maintain a search tree, so the data structure for the current node need only record the state and the value of the objective function.
- Hill climbing does not look ahead beyond the immediate neighbors of the current state

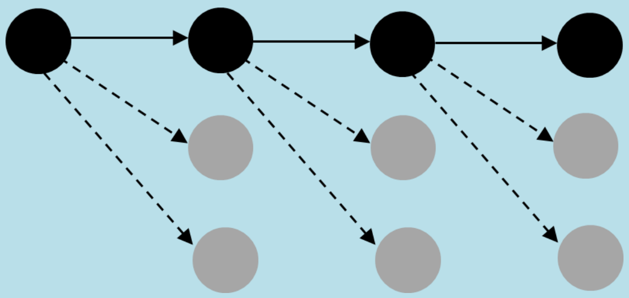
Adapted from Rusell, S., & Norvig, P. (2016)

2.3 Local Beam Search

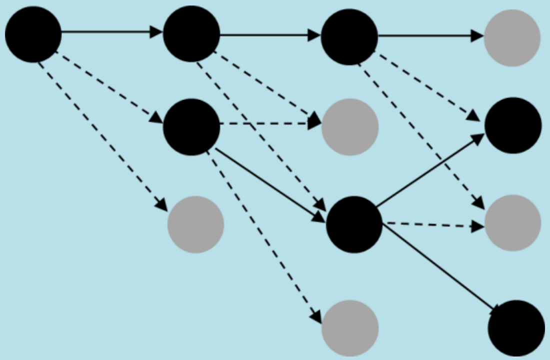
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat

? Is this the same as running k greedy searches in parallel?

Greedy search



Beam search



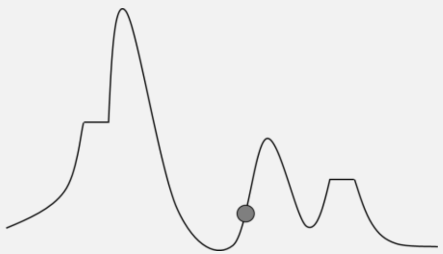
Adapted from Rusell, S., & Norvig, P. (2016);

2.3 Simulated Annealing Search

Algorithm: Simulated Annealing

Initialize current to starting state

For i = 1 to ∞
   If T(i) = 0 *return current*
   Let *next = random successor of current*
   Let *Δ = value(next) – value(current)*
   If *Δ > 0* ***then let*** *current = next*
   Else *let current = next with probability exp(Δ/T(i))*



- **Idea**: Escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- Probability of taking downhill move decreases with number of iterations, steepness of downhill move
- Controlled by annealing schedule
- Inspired by tempering of glass, metal

Adapted from Rusell, S., & Norvig, P. (2016)

2.3 Conclusion: Simulated Annealing Search

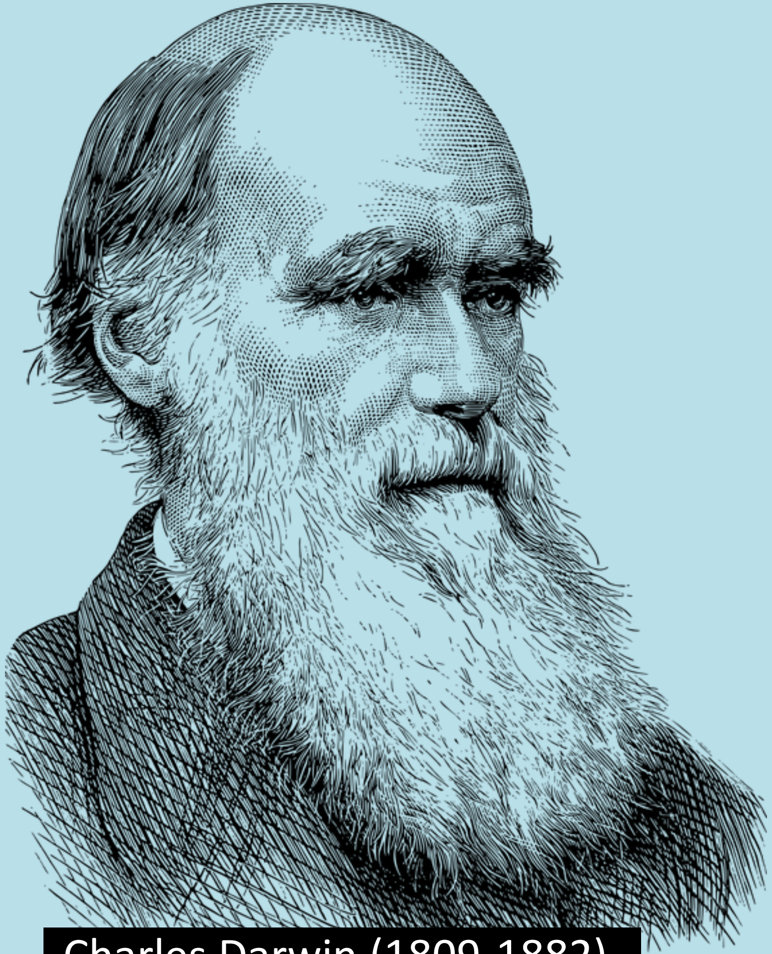
- We can mathematically prove that a slow decrease in temperature will find a global optimum with probability approaching one

- However:
  - This usually takes impractically long
  - The more downhill steps you need to escape a local optimum, the less likely you are to make all of them in a row

- **State-of-the-Art**: General family of Markov Chain Monte Carlo (MCMC) algorithms for exploring complicated state spaces

Adapted from Rusell, S., & Norvig, P. (2016);

2.3 Genetic/Evolutionary Algorithms



Charles Darwin (1809-1882)

- Biological evolutionary model according to Darwin:
  Selection = driving force of evolution
- Transfer to Computer Science: Evolution as optimization of complex, artificial systems
- Build machines that adapt to an defined working environment

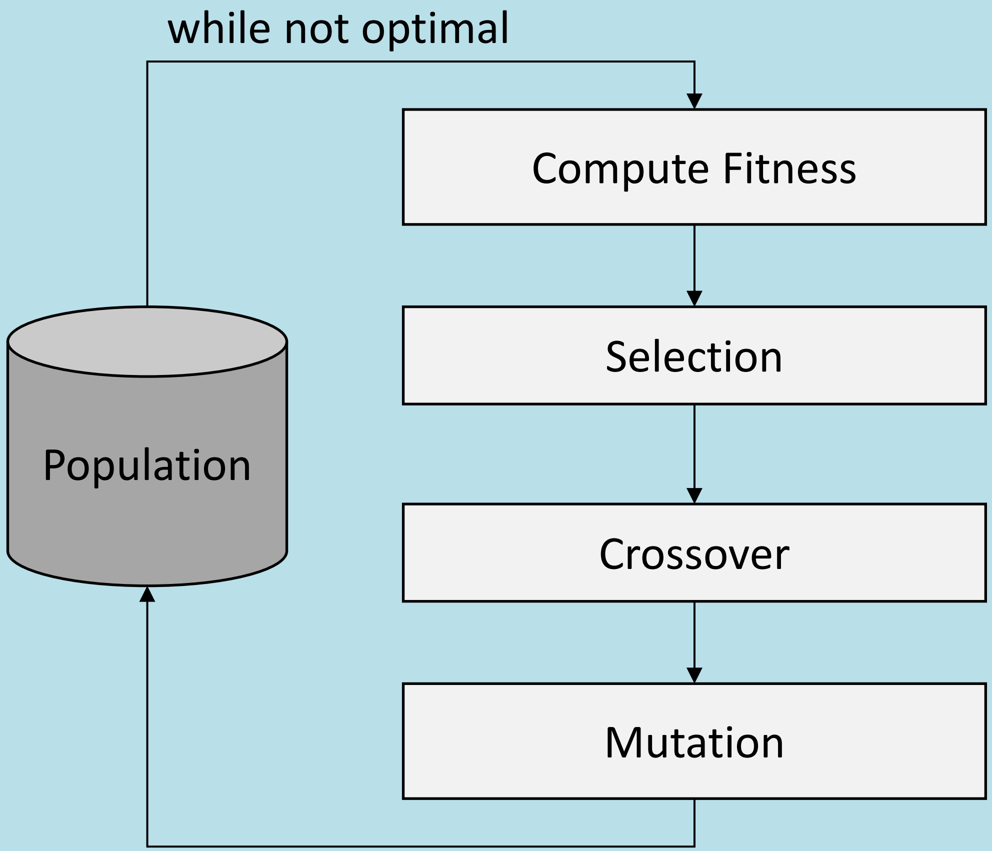
Adapted from Rusell, S., & Norvig, P. (2016) | | Image source: ↗ [Pixabay](#) (2019) / ↗ [CC0](#)

2.3 Wording in Evolutionary Algorithms

▪ Individual	Possible solution, hypothesis, or configuration
▪ Population and generation	Set of solutions or hypothesis
▪ Generation of descendants	Generation of new hypotheses. Methods: recombination (Cross-over) and mutation
▪ Changed successor, child offspring	New hypothesis or configuration
▪ Fitness function	Hypothesis quality, criterion to be optimized
▪ Selection of the best	Selection of the hypotheses that the create the best problem solution

Adapted from Rusell, S., & Norvig, P. (2016);

2.3 Basic Algorithm



Algorithm: Genetic Algorithm

Inputs: population, fitness_func

new_population <- empty set
Repeat
    For i = 1 to** SIZE(population) **do
        X ← RANDOM-SELECTION(population, fitness_func)
        Y ← RANDOM-SELECTION(population, fitness_func)
        Child ← REPRODUCE(x,y)
        ***If(random prob) then** child ← MUTATE(child)*
            Add child to new_population
    Population ← new_population
***Until** some individuals optimal or time elapse*

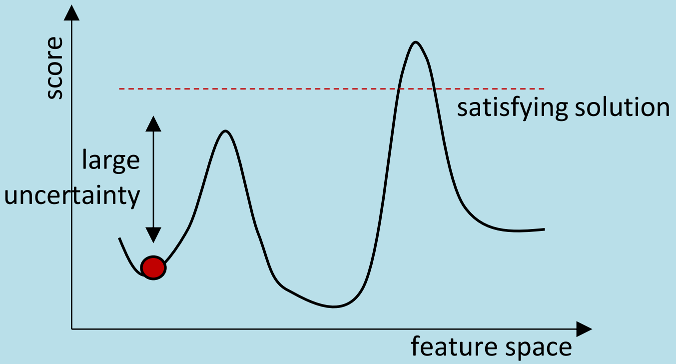
***Return** the best individuals in pop, according fitness_func*

*Reproduce returns an new individual

Adapted from Rusell, S., & Norvig, P. (2016);

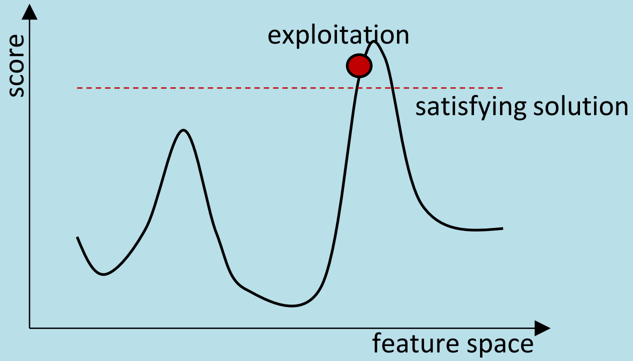
2.3 Generation of Descendants

Exploration



- Exploring the local space, local optimization

Exploitation



- Exploitation of the hypothesis space

- The stronger and more random changes are, the lower is the probability of producing better offsprings
- With local improvement methods, the risk of local minima is given
- level of exploration must be in accordance with the current fitness of the generation can be selected (e.g.: initially high then falling)

Adapted from Rusell, S., & Norvig, P. (2016);

Mutation

The offspring is descended from one parent, with mutation of the gene

Mutation:

- All bits of a sequence are independently inverted with a certain probability
- For a certain (or random) number of bits the indices are selected randomly
- Remove a partial sequence and insert it at another Place
- Inverted insertion of the partial sequence

parent	1	1	0	NA
		Invertion		Insertion
offspring	1	0	0	1

Crossover / Recombination

Mix properties of two or more parents

Crossover:

- Discrete recombination

father	1	1	0
mother	1	0	0
offspring	1	1	0

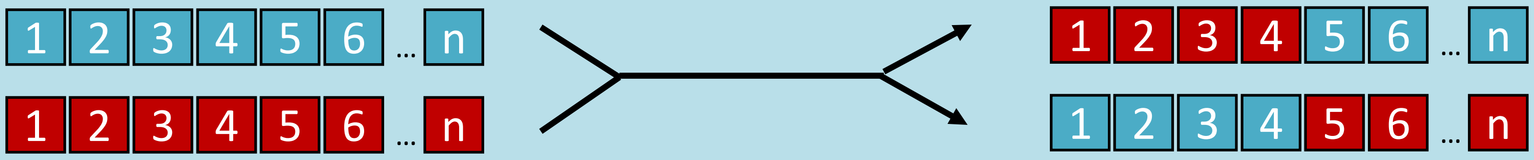
- Intermediate recombination (continuous representation)

$$parent_1 \coloneqq x \text{ and } parent_2 \coloneqq y,$$
$$offspring_i \coloneqq (x_i + y_i)\,/\,2$$

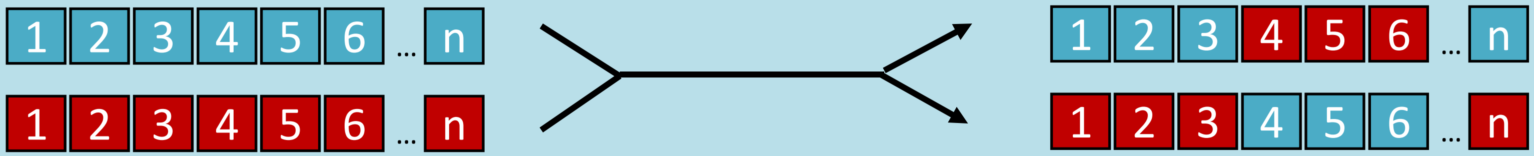
Adapted from Rusell, S., & Norvig, P. (2016);

2.3 Crossover/Recombination

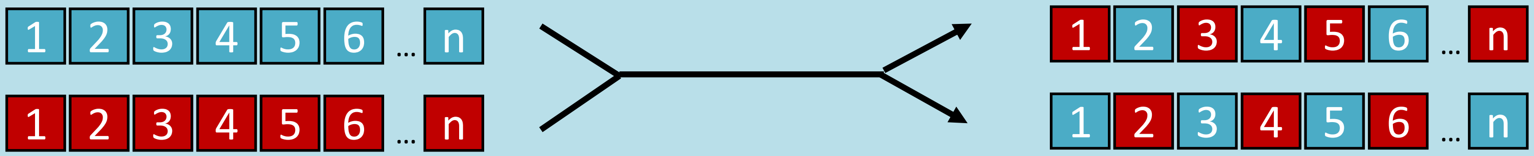
Single-point Crossover



Two-point Crossover



Uniform Crossover



2.3 Selection

Two different types of selection:

- of the parents for respective production of offspring (Mating)
- of the population in each iteration

Problems:

- Genetic drift: Individuals reproduce more than others by chance
- Crowding, outlier problem: "fit" individuals and similar offspring dominate the Population

→Development of individuals is slowed down

→Diversity of the population is restricted

Solution:

- Different population models and selection methods
- Optimize population size

2.3 Population Models

- Island model (local)
  The evolution runs largely separately, only sometimes individuals exchanged

- Neighborhood model (near surroundings)
  Descendants may only be produced by individuals who are have the best fitness in their neighborhood

- A simple set (global)
  The best in the world are developing rapidly, others Lines of development are suppressed

2.3 Population Members

Population size:

- Should it remain constant? (μ)
- How many newly produced offspring? (λ)
- How many parents should be used? (ρ)
- How are they determined?

Member selection:

stochastically selected $\Rightarrow$ the best μ individuals

- (μ, λ) Strategy: Selection refers only to the Offspring (better exploration)
- (μ + λ) Strategy:

  Selection also involves parents (the Best are considered, search for Elites → Exploitation, cheap with good calculable fitness functions)

2.3 Substitute Population Members

Substitution rule for members:

- Descendants replace all parents (Generation mode)
- Offspring replaced a part of the parents
- Offspring replaced parents that are most similar to them
- Geographical Replacement
- Best individual survives (Elitist - Mode)

Rule of thumb:

The best quarter of the population should be three quarters of the descendants

2.3 Selection Methods - Fitness Based Selection

Fitness Based Selection: $P(X) \approx \frac{f(x)}{\sum_{x' \in Pop.} f(x')}$ exactly $P(X) = \frac{\lambda}{\mu} \cdot \frac{f(x)}{\sum_{x' \in Pop} f(x')}$

P(x) : Probability of selection of individual x
λ : Number of descendants
μ : Population size
f : Fitness – Function

- depending on the value of the fitness function
- e.g. during Evolution only minor changes in $f(x)$ and thus in *P(x)*

2.3 Selection Methods - Ranking Based Selection

Ranking Based Selection : $P(x) \approx \frac{g(r(x))}{\sum_{x' \epsilon Pop.} g(r(x'))}$ with

$P(x)$: Probability of selection of individual x
$\mathrm{r}(x)$: Ranking of x in the current population according to fitness -Function
g : function increasing monotonically with the quality of the rank greater than 0

- Exponential: $\boxed{g(x) = a^{-x}}$
- Hyperbolic: $\boxed{g(x) = x^{-a}}$
- the best k: $\mathrm{g}(x) = \begin{cases} {}^{1}/_{k}\,, & x \leq 0 \\ 0, & else \end{cases}$

- less dependent on the amount of fitness
- better adaptation of exploration / exploitation

2.3 Selection Method – Tournament Selection

Tournament Selection (tournament)

- select n (=2) individuals for each individual to be created
- reward (increase rating) of it, according to the fitness the best individual
- select individuals with highest rating

- little dependent on the amount of fitness

Choosing the selection method

- often application-specific

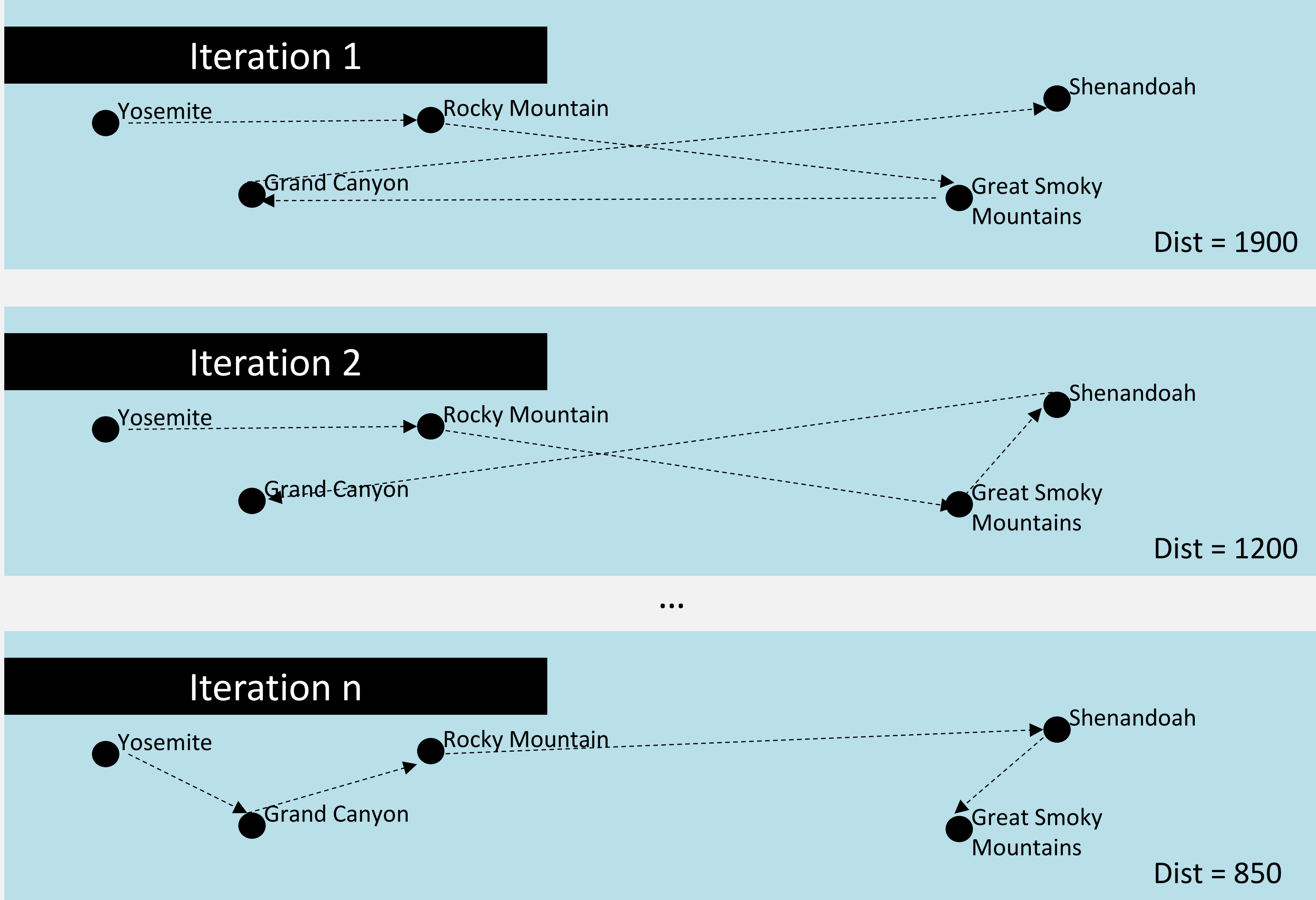
2.3 Use Genetic Algorithms to Solve our Travelling Salesman Problem

Find a path:

- start in Yosemite
- each national park is visited exactly once
- the travelled distance is minimal

Image sources: ↗ [US NationalParks](#) (2015) by Mwierschkec↗ [CC BY-SA 4.0](#)

2.3 Use Genetic Algorithms to Solve our Travelling Salesman Problem I



Find a path:

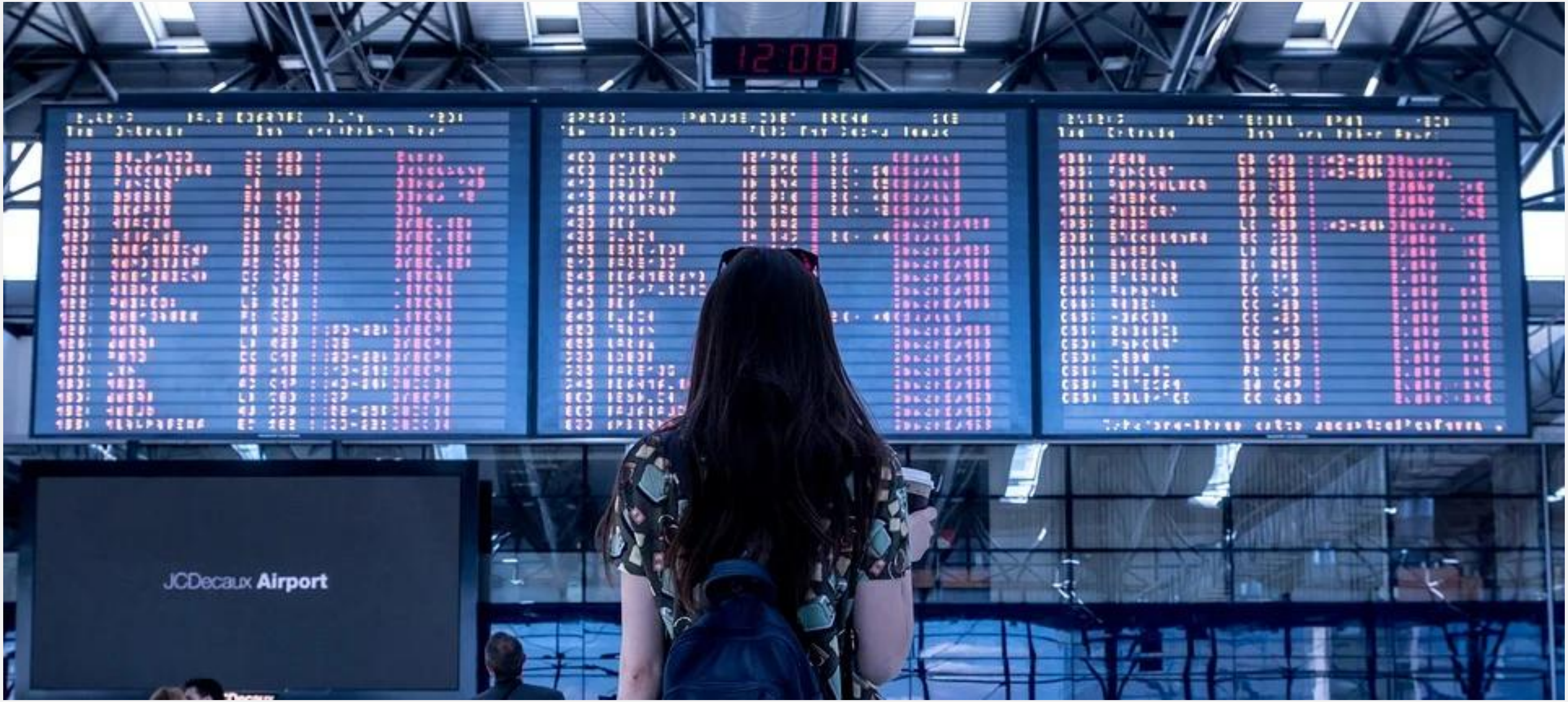
- each national park is visited exactly once
- the travelled distance is minimal

[YOS ROC GRE GRA SHE]

↓ Inversion

[YOS ROC GRE SHE GRA]

2.3 Application of Genetic Algorithms



Your turn!

Task

Please explain

- What is the difference between the steps mutation and crossover in the context of genetic algorithms?

2.4 Outline

Image source: ↗ Pixabay (2019) / ↗ CC0

▶ **What we will learn:**

- We define the concept of rational agents (≈ intelligent agents)
- Characteristics of artificial agents (perfect or otherwise), the diversity of environments, and the resulting menagerie of agent types
- We discuss how AI problems can be modelled as search-problems, and how they can be solved by searching

▶ **Duration:**

- 225 min

▶ **Relevant for Exam:**

- 2.1 – 2.5

2.4 “2/3 of the average” game

- Everyone writes down a number between 0 and 100
- Person closest to 2/3 of the average wins
- Example:
  - A says 50
  - B says 10
  - C says 90
  - Average(50, 10, 90) = 50
  - 2/3 of average = 33.33
  - A is closest (|50-33.33| = 16.67), so A wins

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 What is game theory?

- Game theory studies settings where multiple parties (agents) each have
  - different preferences (utility functions),
  - different actions that they can take
- Each agent’s utility (potentially) depends on all agents’ actions
  - What is optimal for one agent depends on what other agents do
    - Very circular!
- Game theory studies how agents can rationally form beliefs over what other agents will do, and (hence) how agents should act
  - Useful for acting as well as predicting behavior of others

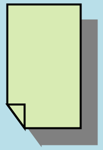
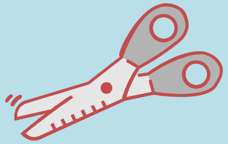
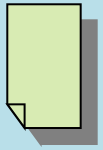
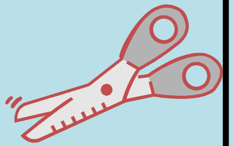
Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Rock-paper-scissors

Zero-sum game: the utilities in each entry sum to 0 (or a constant)
Three-player game would be a 3D table with 3 utilities per entry, etc.

Column player aka. player 2 (simultaneously) chooses a column

Row player aka. player 1 chooses a row

			
	0, 0	-1, 1	1, -1
	1, -1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

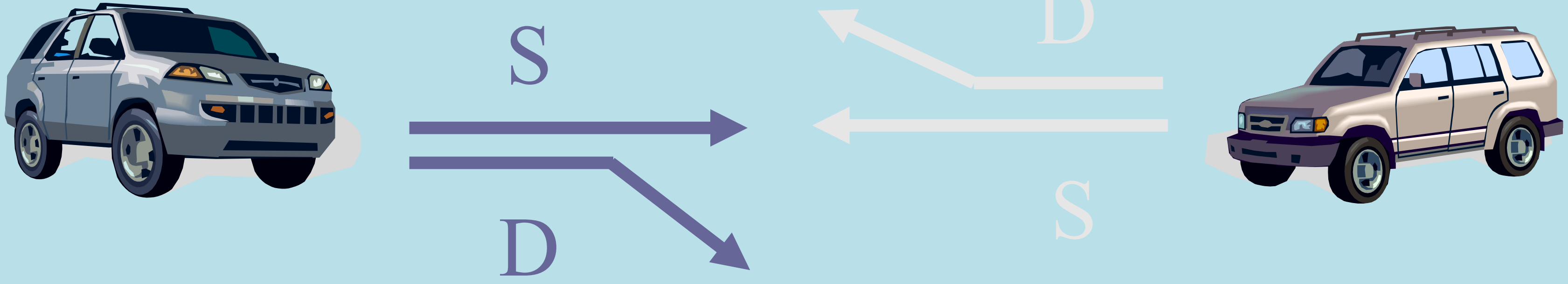
A row or column is called an action or (pure) strategy

Row player's utility is always listed first, column player's second

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 “Chicken”

- Two players drive cars towards each other
- If one player goes straight, that player wins
- If both go straight, they both die



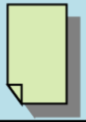
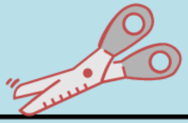
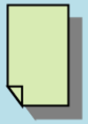
	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

not zero-sum

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Rock-paper-scissors – Seinfeld variant

MICKEY: All right, rock beats paper!
(Mickey smacks Kramer's hand for losing)
KRAMER: I thought paper covered rock.
MICKEY: Nah, rock flies right through paper.
KRAMER: What beats rock?
MICKEY: (looks at hand) Nothing beats rock.

			
	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

Adapted from Rusell, S., & Norvig, P. (2016);

- Player i's strategy s_i strictly dominates s_i' if
  - for any s_{-i}, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$
- s_i weakly dominates s_i' if
  - for any s_{-i}, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$; and
  - for some s_{-i}, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

-i = "the player(s) other than i"

strict dominance

weak dominance

0, 0	1, -1	1, -1
-1, 1	0, 0	-1, 1
-1, 1	1, -1	0, 0

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Prisoner’s Dilemma

- Pair of criminals has been caught
- District attorney has evidence to convict them of a minor crime (1 year in jail); knows that they committed a major crime together (3 years in jail) but cannot prove it
- Offers them a deal:
  - If both confess to the major crime, they each get a 1 year reduction
  - If only one confesses, that one gets 3 years reduction

	confess	don’t confess
confess	-2, -2	0, -3
don’t confess	-3, 0	-1, -1

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 “Should I buy an SUV?”

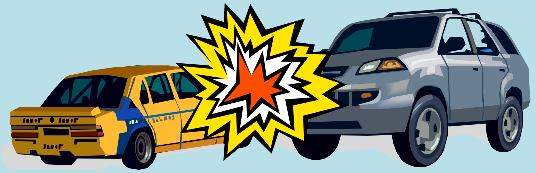
purchasing cost

cost: 5

cost: 3

accident cost

cost: 5  cost: 5

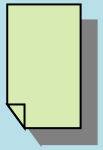
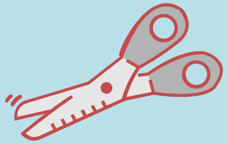
cost: 8  cost: 2

cost: 5  cost: 5

		
	-10, -10	-7, -11
	-11, -7	-8, -8

Adapted from Rusell, S., & Norvig, P. (2016);

- Mixed strategy for player i = probability distribution over player i’s (pure) strategies

- E.g. 1/3  1/3  , 1/3 
- Example of dominance by a mixed strategy:

1/2	3, 0	0, 0
1/2	0, 0	3, 0
	1, 0	1, 0

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Checking for dominance by mixed strategies I

- Linear program for checking whether strategy s_i* is strictly dominated by a mixed strategy:
- maximize ε
- such that:
  - for any s_{-i}, $\Sigma_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}) + \varepsilon$
  - $\Sigma_{s_i} \mathbf{p}_{s_i} = 1$

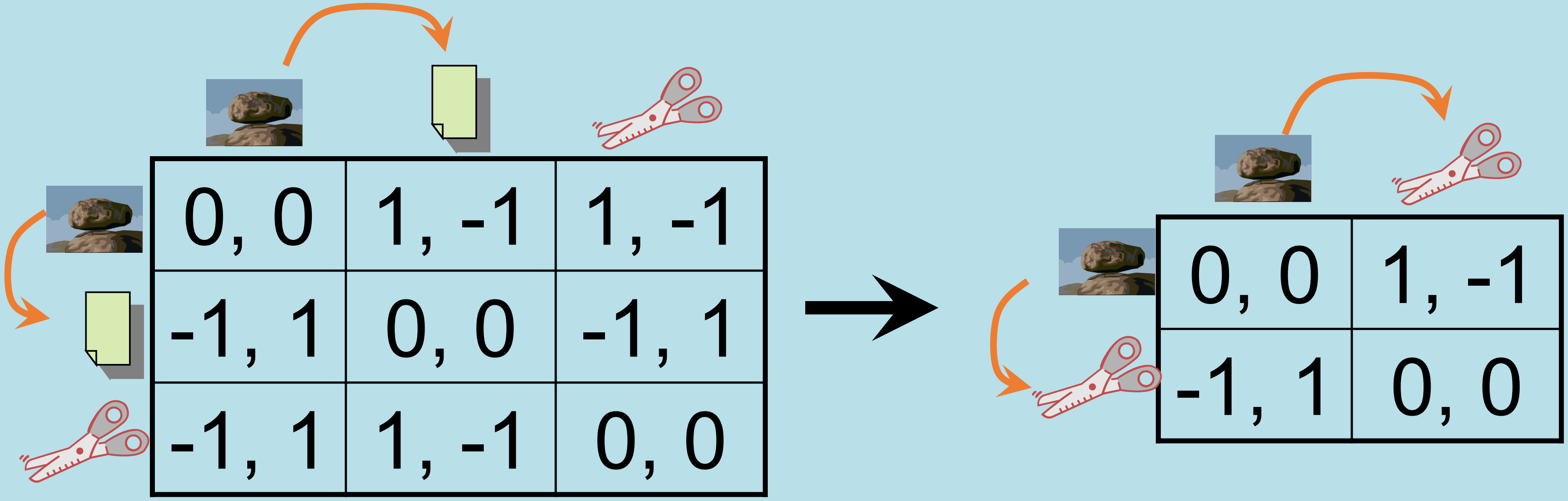
Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Checking for dominance by mixed strategies II

- Linear program for checking whether strategy s_i* is weakly dominated by a mixed strategy:
- maximize $\Sigma_{s_{-i}}(\Sigma_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i})) - u_i(s_i^*, s_{-i})$
- such that:
  - for any s_{-i}, $\Sigma_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i})$
  - $\Sigma_{s_i} \mathbf{p}_{s_i} = 1$

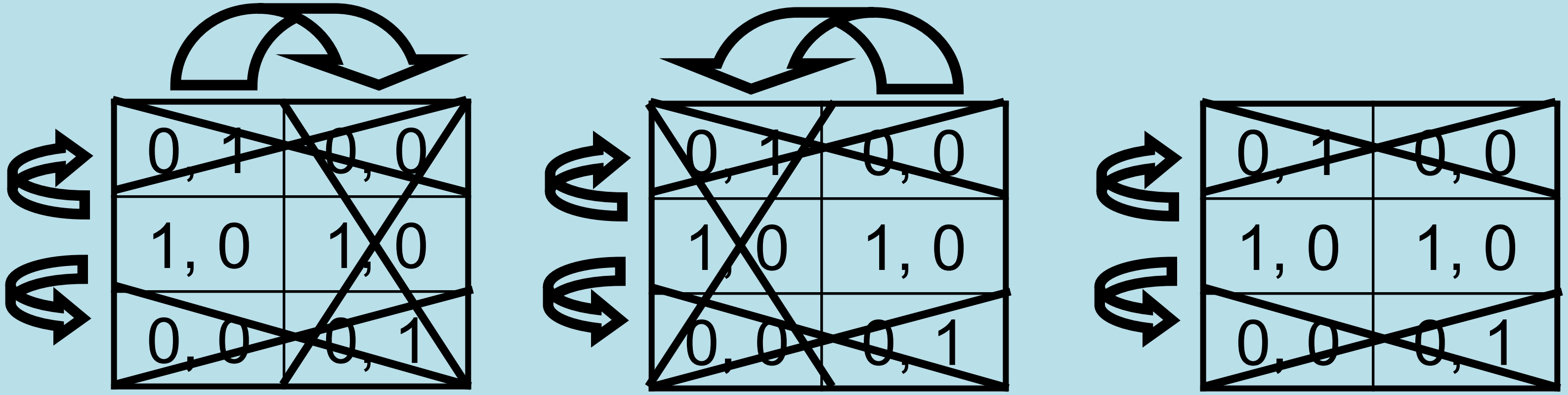
Adapted from Rusell, S., & Norvig, P. (2016);

- Iterated dominance: remove (strictly/weakly) dominated strategy, repeat
- Iterated strict dominance on Seinfeld's RPS:



Adapted from Rusell, S., & Norvig, P. (2016);

Iterated weak dominance is path-dependent: sequence of eliminations may determine which solution we get (if any)
(whether or not dominance by mixed strategies allowed)



Iterated strict dominance is path-independent: elimination process will always terminate at the same point
(whether or not dominance by mixed strategies allowed)

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 “2/3 of the average” game revisited

100

dominated

(2/3)*100

dominated after removal of (originally) dominated strategies

(2/3)*(2/3)*100

…

0

Adapted from Rusell, S., & Norvig, P. (2016);

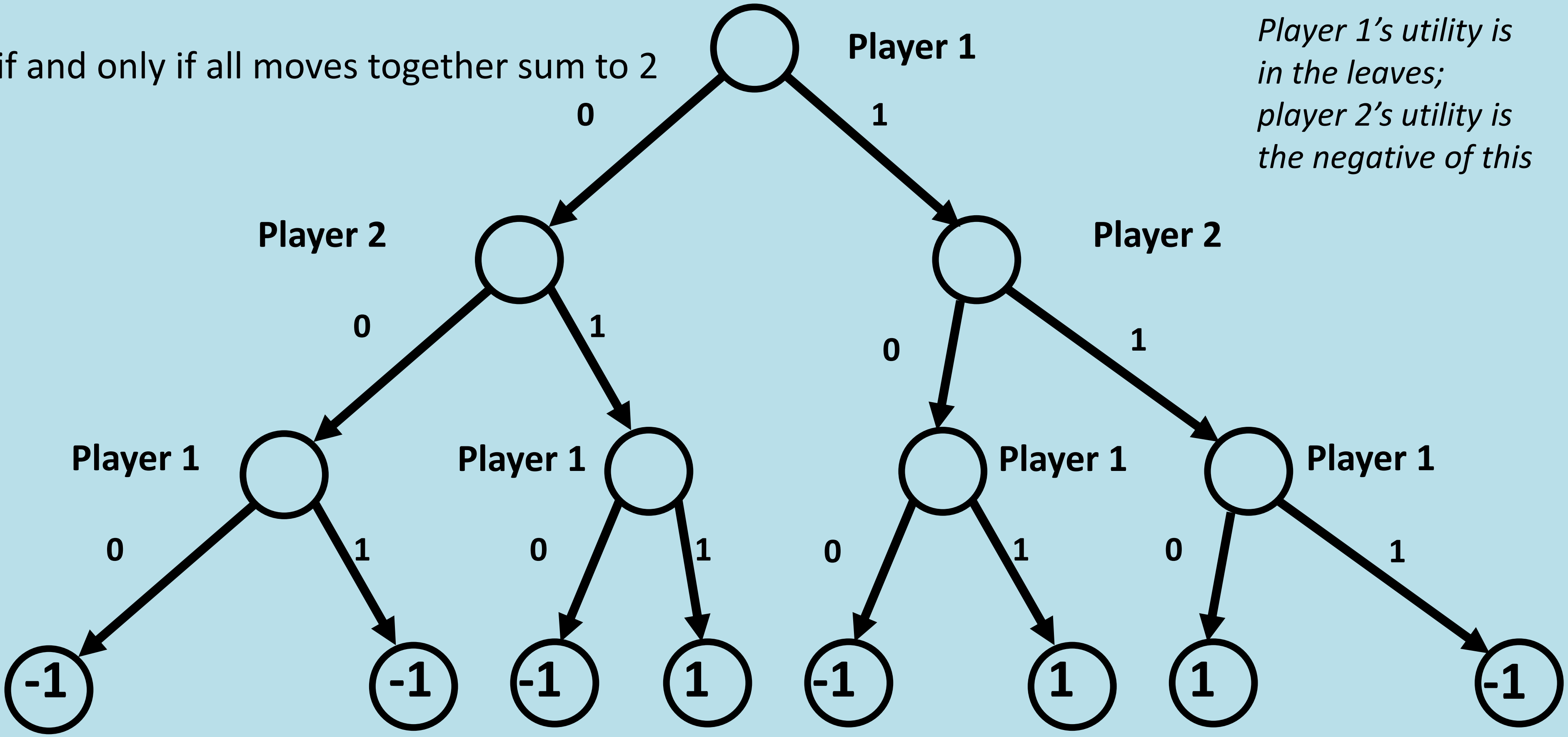
2.4 Game playing

- Rich tradition of creating game-playing programs in AI
- Many similarities to search
- Most of the games studied
  - have two players,
  - are zero-sum: what one player wins, the other loses
  - have perfect information: the entire state of the game is known to both players at all times
- E.g., tic-tac-toe, checkers, chess, Go, backgammon, …
- Will focus on these for now
- Recently more interest in other games
  - Esp. games without perfect information; e.g., poker
    - Need probability theory, game theory for such games

Adapted from Rusell, S., & Norvig, P. (2016);

2.4“Sum to 2” game

- Player 1 moves, then player 2, finally player 1 again
- Move = 0 or 1
- Player 1 wins if and only if all moves together sum to 2

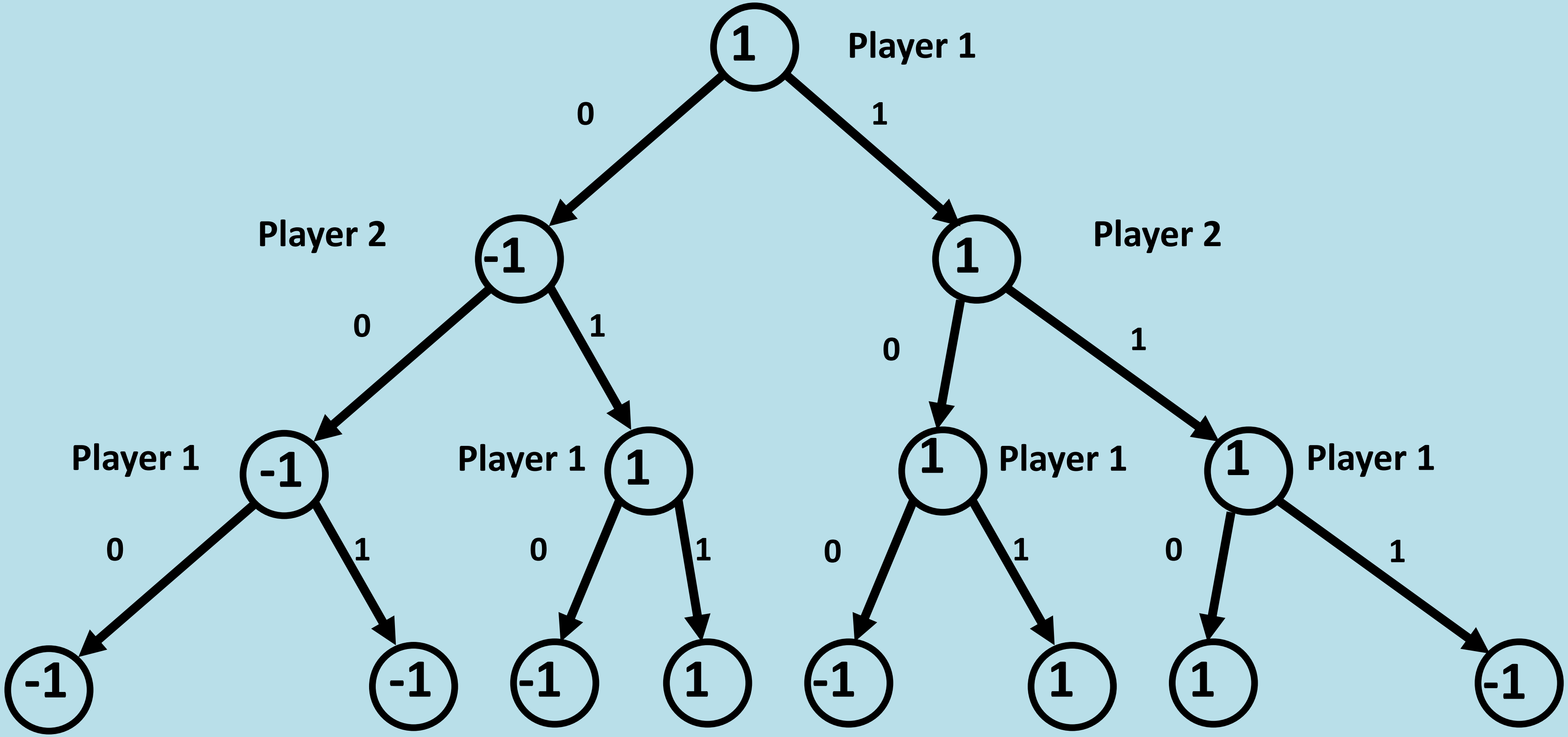


Player 1’s utility is in the leaves; player 2’s utility is the negative of this

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Backward induction (aka. minimax)

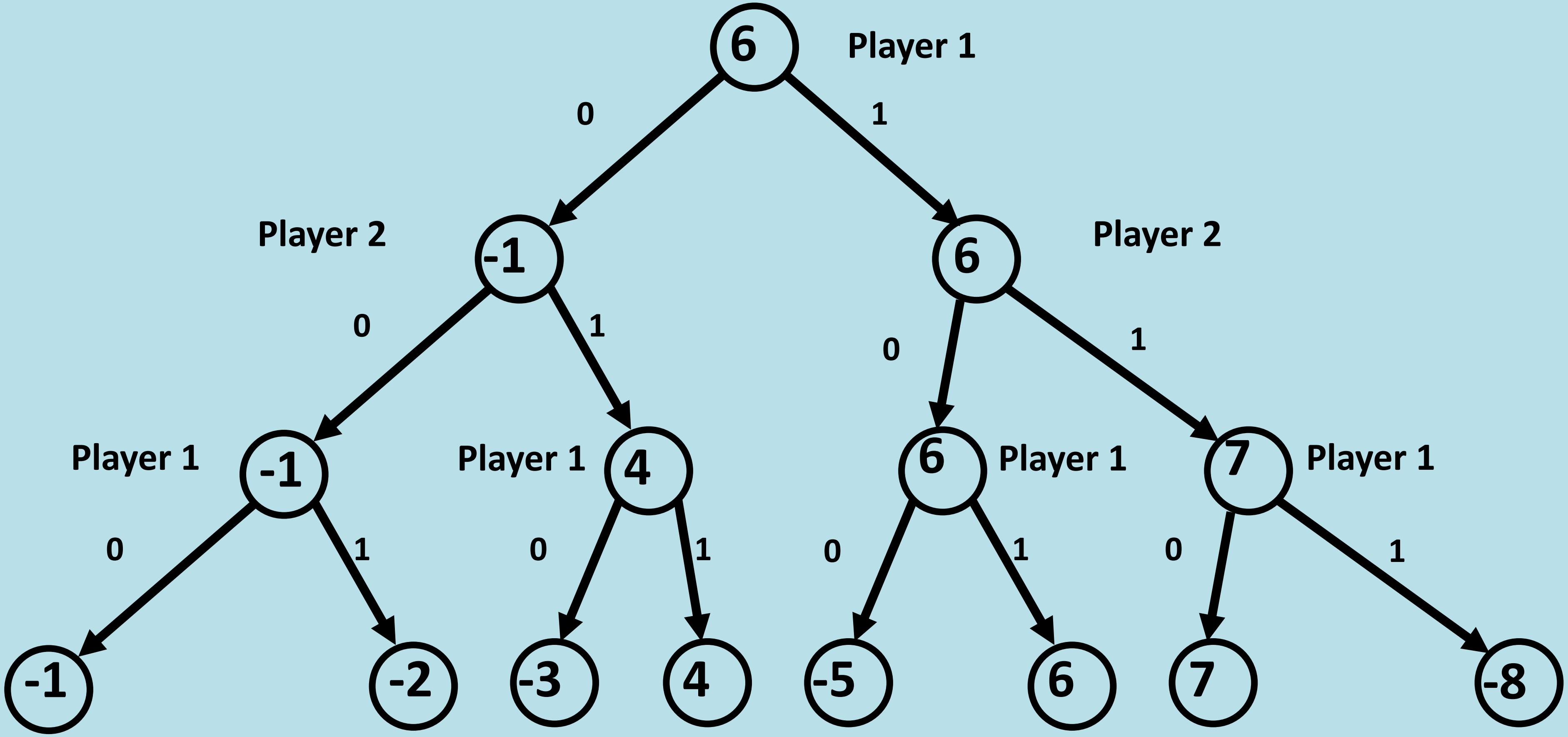
- From leaves upward, analyze best decision for player at node, give node a value
  - Once we know values, easy to find optimal action (choose best value)



Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Modified game

- From leaves upward, analyze best decision for player at node, give node a value



Adapted from Rusell, S., & Norvig, P. (2016);

2.4 A recursive implementation

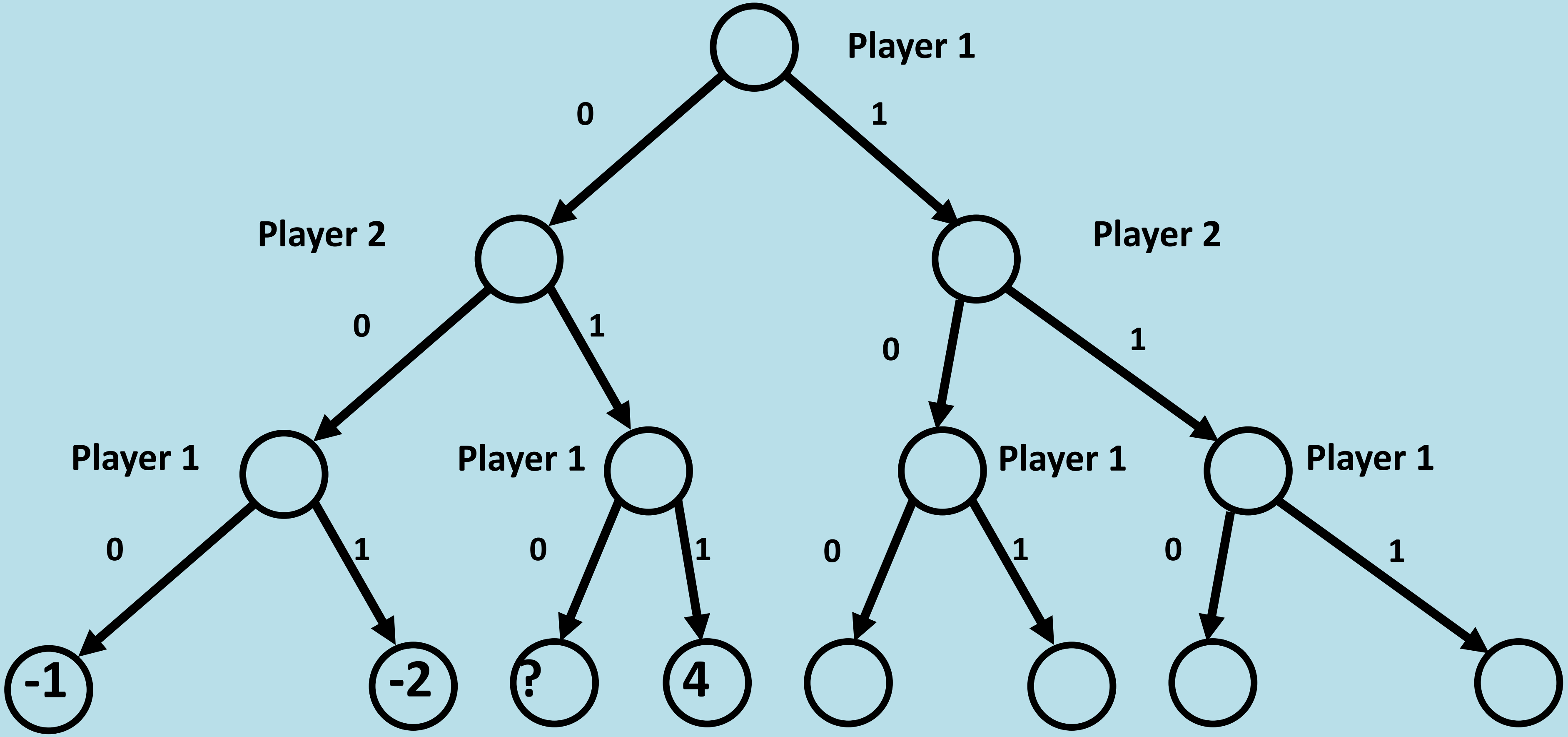
```
Value(state)

If state is terminal, return its value

If (player(state) = player 1)
    v := -infinity
    For each action
        v := max(v, Value(successor(state, action)))
    Return v

Else
    v := infinity
    For each action
        v := min(v, Value(successor(state, action)))
    Return v
```

Space? Time?

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Do we need to see all the leaves?

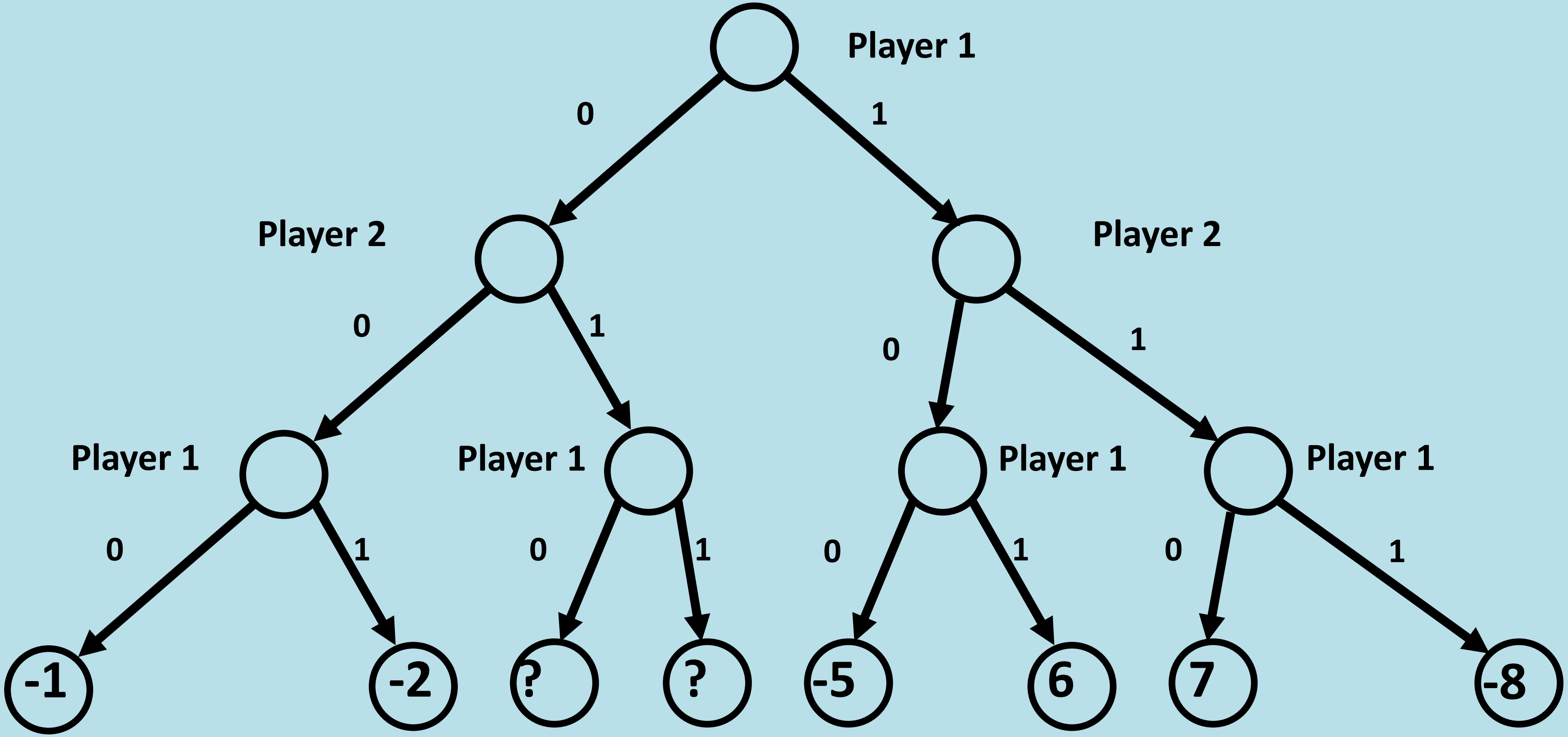
- Do we need to see the value of the question mark here?



Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Do we need to see all the leaves?

- Do we need to see the values of the question marks here?



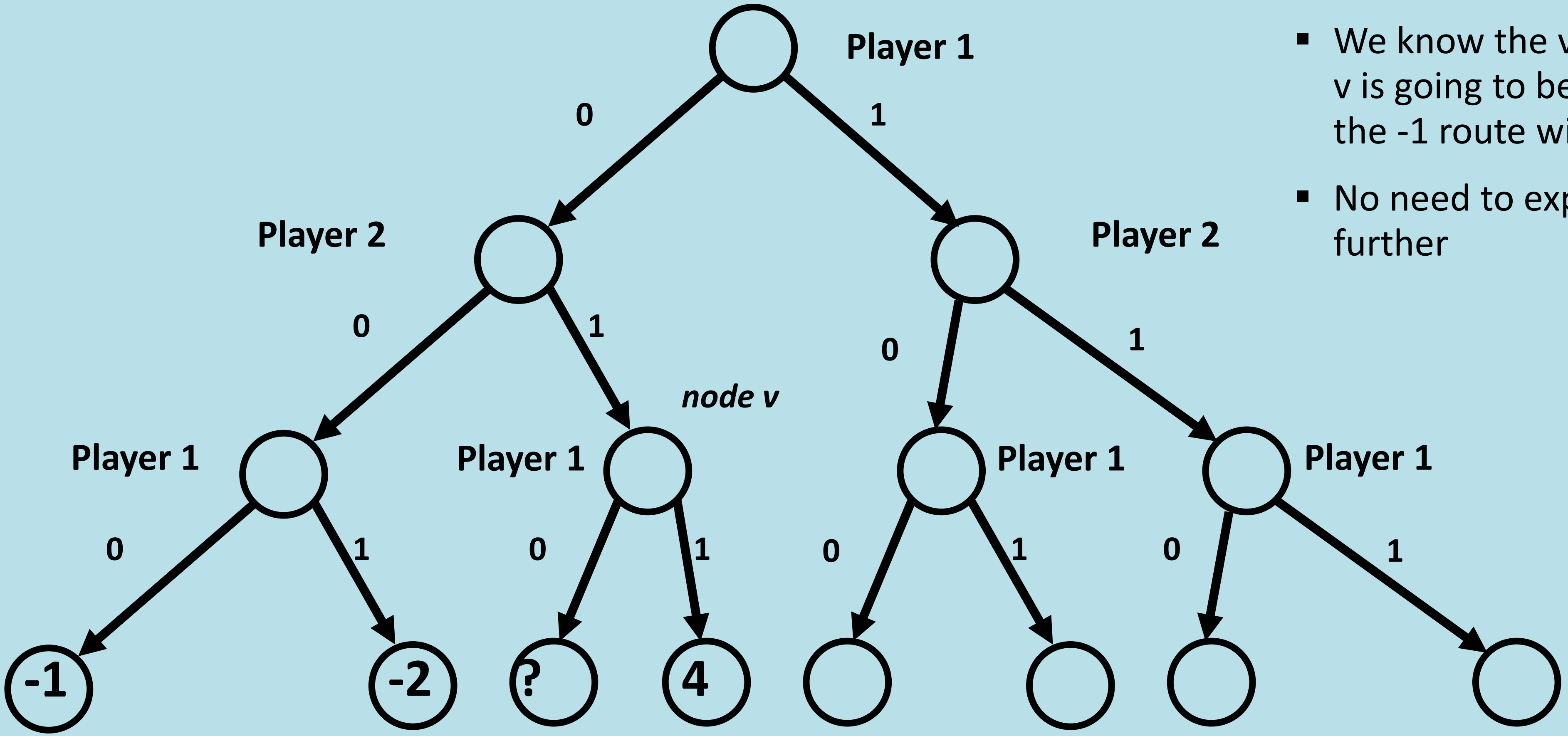
Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Alpha-beta pruning

- Pruning = cutting off parts of the search tree (because you realize you don't need to look at them)
  - When we considered A* we also pruned large parts of the search tree
- Maintain alpha = value of the best option for player 1 along the path so far
- Beta = value of the best option for player 2 along the path so far

Adapted from Rusell, S., & Norvig, P. (2016);

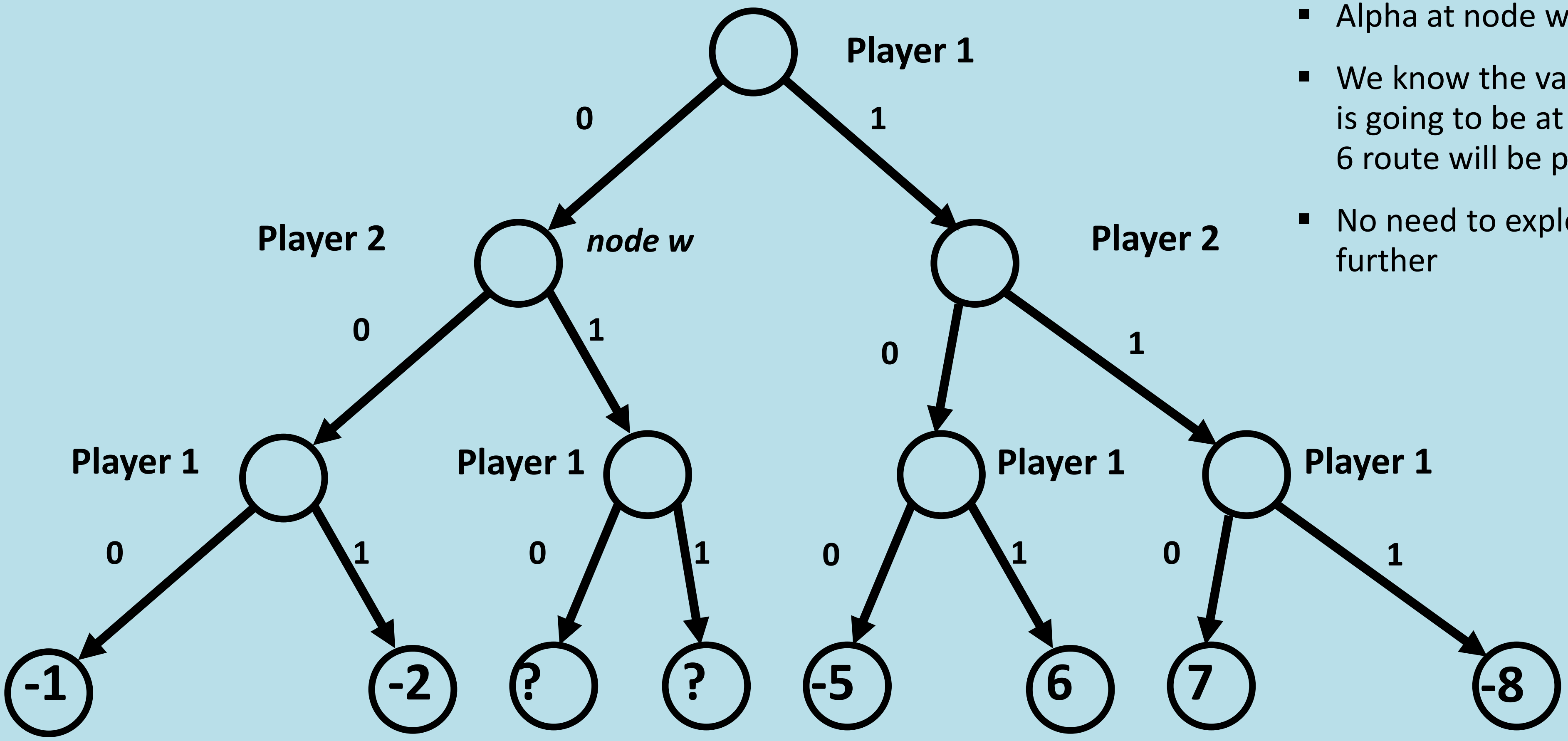
2.4 Pruning on beta



- Beta at node v is -1
- We know the value of node v is going to be at least 4, so the -1 route will be preferred
- No need to explore this node further

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Pruning on alpha



- Alpha at node w is 6
- We know the value of node w is going to be at most -1, so the 6 route will be preferred
- No need to explore this node further

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Modifying recursive implementation to do alpha-beta pruning

```
Value(state, alpha, beta)
If state is terminal, return its value
If (player(state) = player 1)
    v := -infinity
    For each action
        v := max(v, Value(successor(state, action), alpha, beta))
        If v >= beta, return v
        alpha := max(alpha, v)
    Return v
Else
    v := infinity
    For each action
        v := min(v, Value(successor(state, action), alpha, beta))
        If v <= alpha, return v
        beta := min(beta, v)
    Return v
```

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Benefits of alpha-beta pruning

- Without pruning, need to examine $O(b^m)$ nodes
- With pruning, depends on which nodes we consider first
- If we choose a random successor, need to examine $O(b^{3m/4})$ nodes
- If we manage to choose the best successor first, need to examine $O(b^{m/2})$ nodes
  - Practical heuristics for choosing next successor to consider get quite close to this
- Can effectively look twice as deep!
  - Difference between reasonable and expert play

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Repeated states

- As in search, multiple sequences of moves may lead to the same state
- Again, can keep track of previously seen states (usually called a transposition table in this context)
  - May not want to keep track of **all** previously seen states…

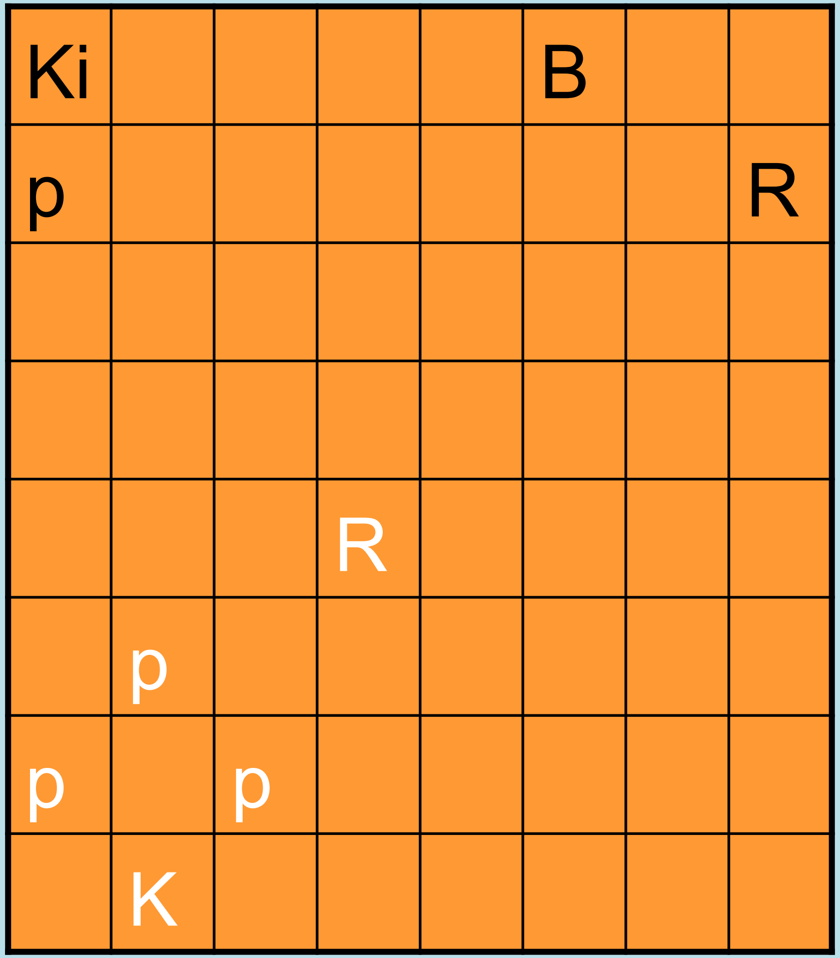
Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Using evaluation functions

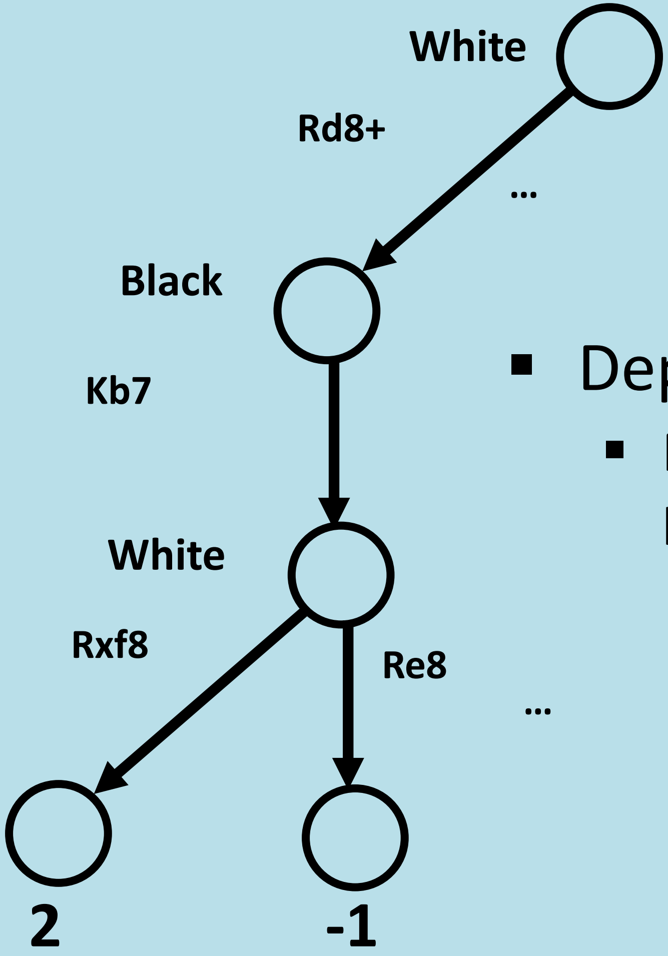
- Most games are too big to solve even with alpha-beta pruning
- Solution: Only look ahead to limited depth (nonterminal nodes)
- Evaluate nodes at depth cutoff by a heuristic (aka. evaluation function)
- E.g., chess:
  - Material value: queen worth 9 points, rook 5, bishop 3, knight 3, pawn 1
  - Heuristic: difference between players' material values

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Chess example



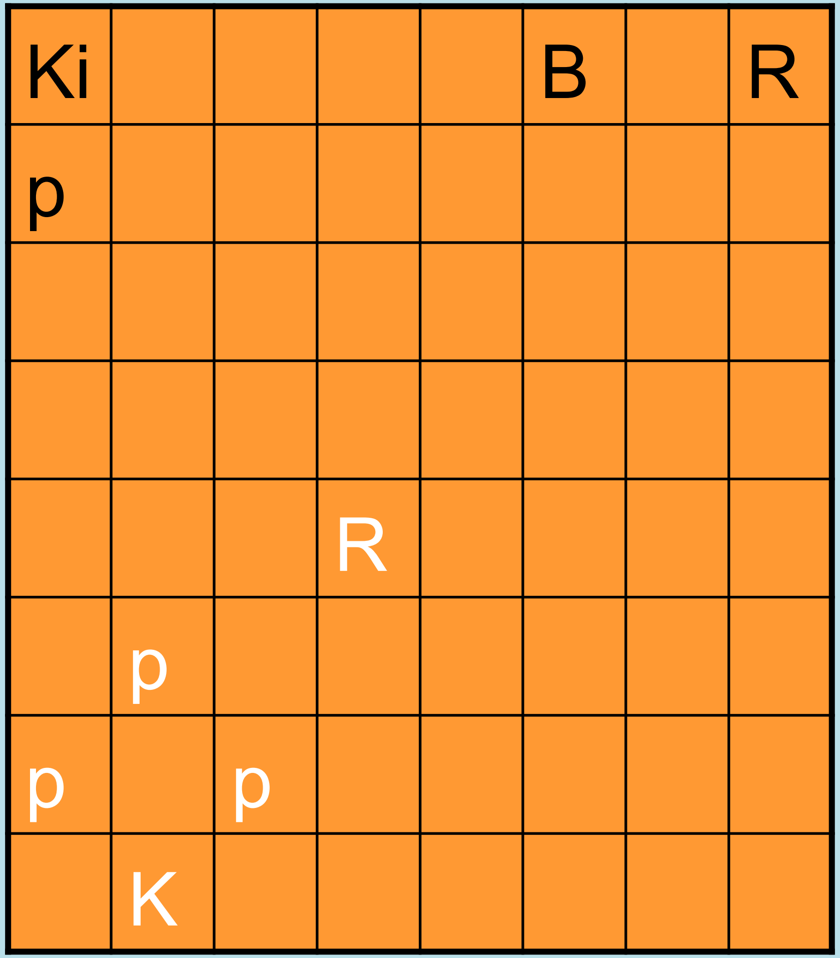
White to move

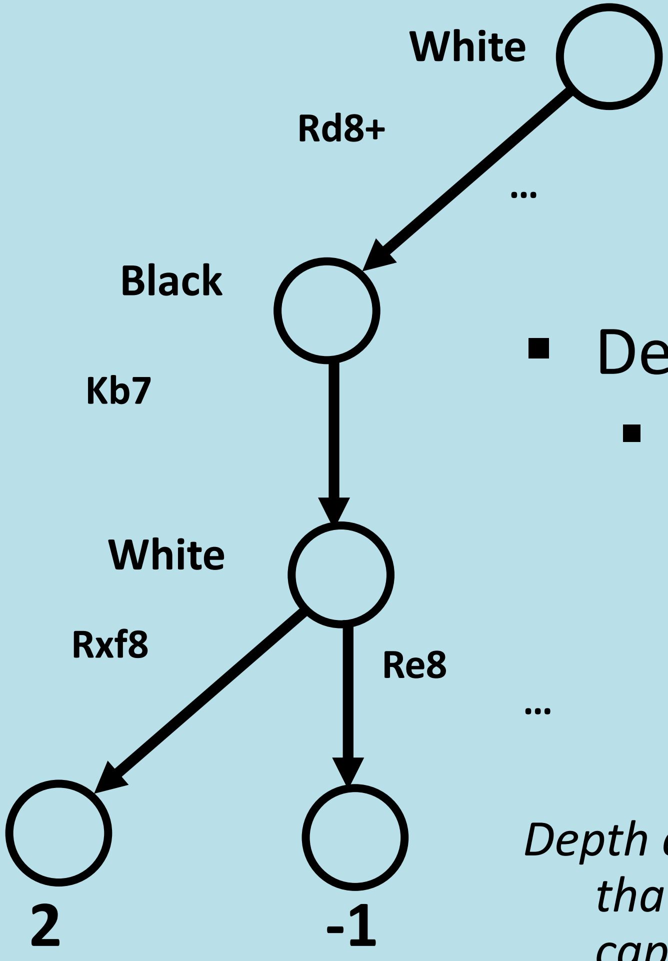


- Depth cutoff: 3 ply
  - Ply = move by one player

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Chess (bad) example





- Depth cutoff: 3 ply
  - Ply = move by one player

Depth cutoff obscures fact that white R will be captured

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Addressing this problem

- Try to evaluate whether nodes are quiescent
  - Quiescent = evaluation function will not change rapidly in near future
  - Only apply evaluation function to quiescent nodes
- If there is an “obvious” move at a state, apply it before applying evaluation function

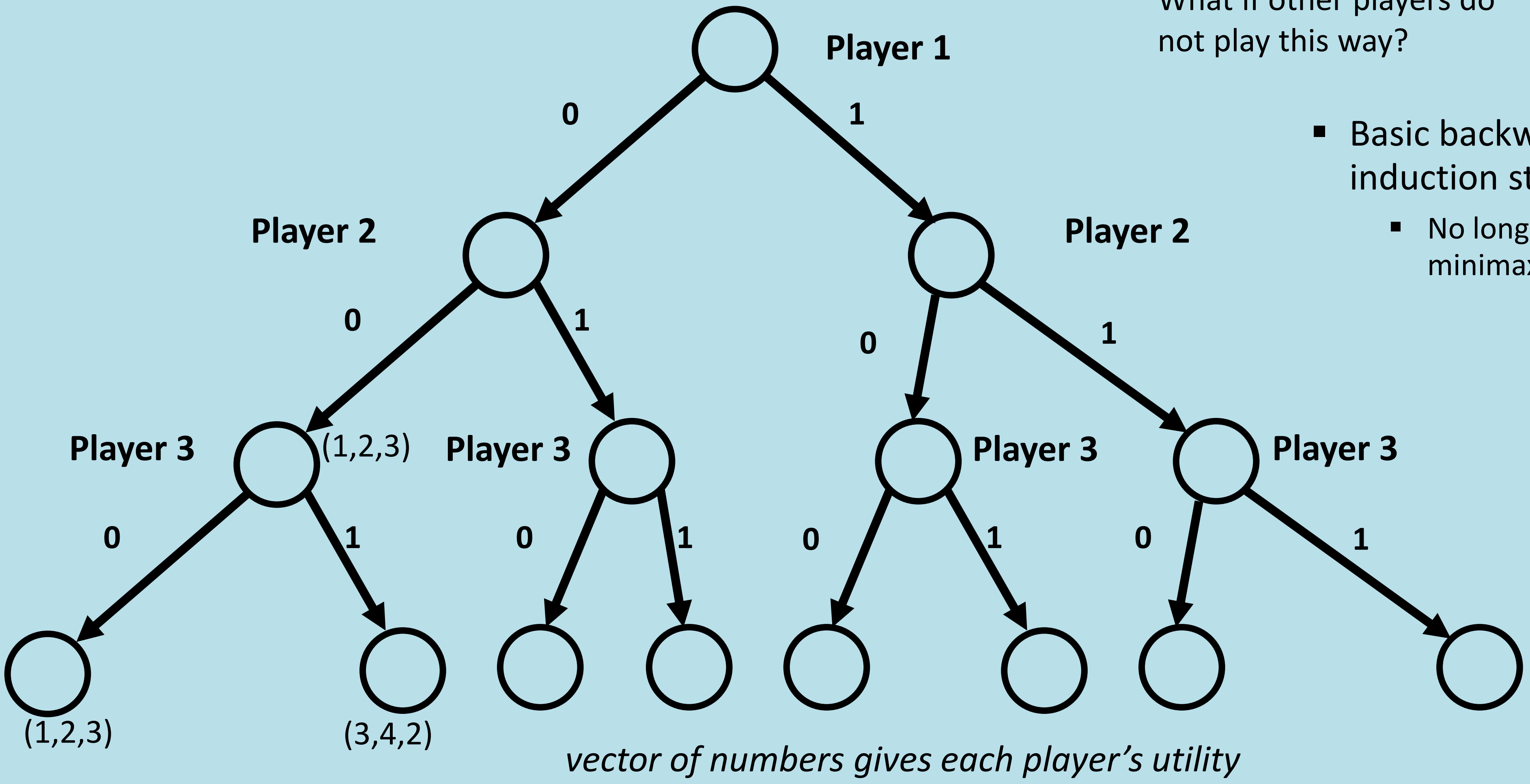
Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Playing against suboptimal players

- Minimax is optimal against other minimax players
- What about against players that play in some other way?

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Many-player, general-sum games of perfect information



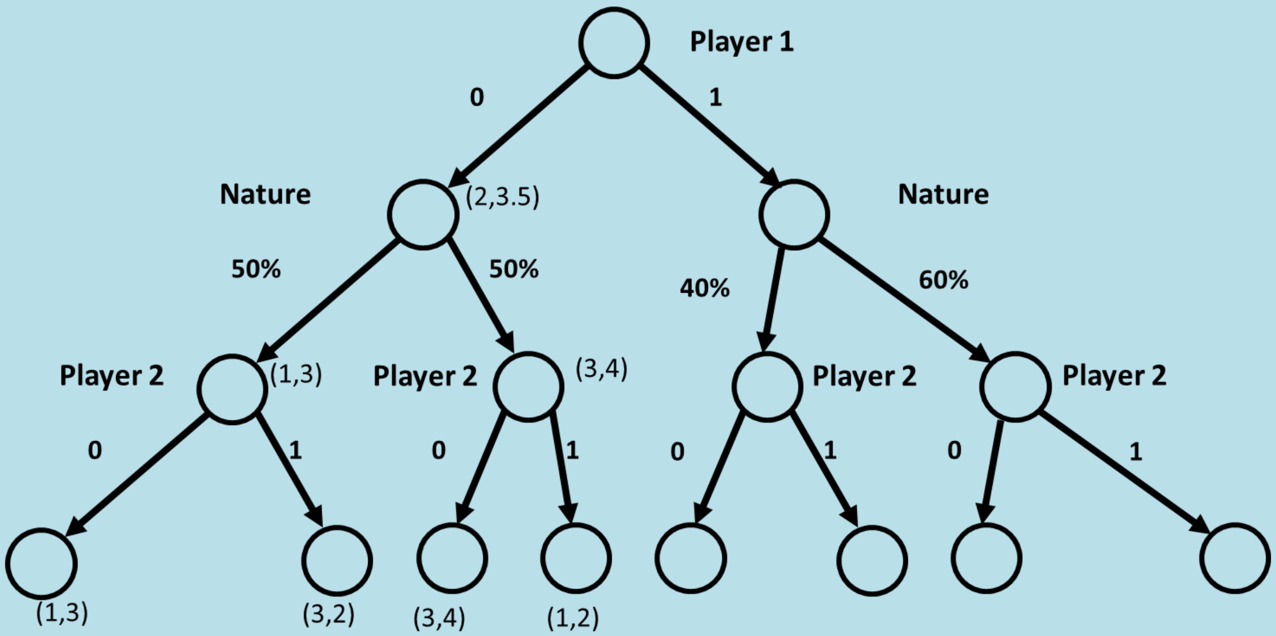
What if other players do not play this way?

- Basic backward induction still works
  - No longer called minimax

Adapted from Rusell, S., & Norvig, P. (2016);

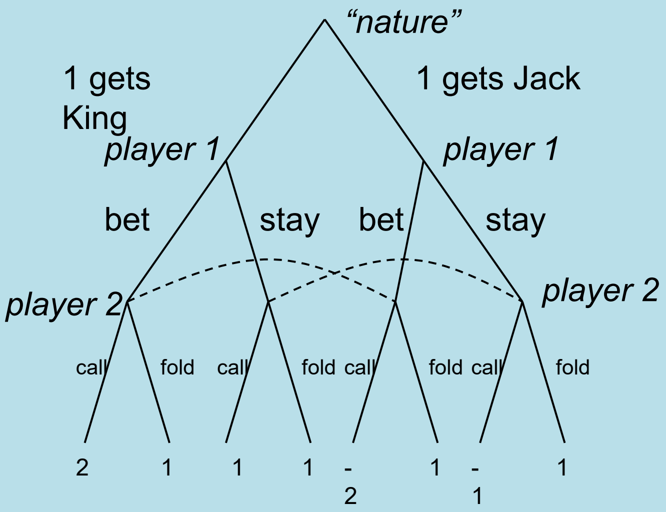
2.4 Games with random moves by "Nature"

- E.g., games with dice (Nature chooses dice roll)
- Backward induction still works…
  - Evaluation functions now need to be cardinally right (not just ordinally)
  - For two-player zero-sum games with random moves, can we generalize alpha-beta? How?



Adapted from Rusell, S., & Norvig, P. (2016);

- Players cannot necessarily see the whole current state of the game
  - Card games
- Ridiculously simple poker game:
  - Player 1 receives King (winning) or Jack (losing),
  - Player 1 can bet or stay,
  - Player 2 can call or fold
- Dashed lines indicate indistinguishable states
- Backward induction does **not** work, need random strategies for optimality! (more later in course)



Adapted from Rusell, S., & Norvig, P. (2016);

- Suppose my strategy is “bet on King, stay on Jack”
  - What will you do?
  - What is your expected utility?
- What if my strategy is “always bet”?
- What if my strategy is “always bet when given King, 10% of the time bet when given Jack”?

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 The state of the art for some games Until (2010)

- Chess:
  - 1997: IBM Deep Blue defeats Kasparov
  - … there is still debate about whether computers are really better
- Checkers:
  - Computer world champion since 1994
  - … there was still debate about whether computers are really better…
  - until 2007: checkers solved optimally by computer
- Go:
  - Computers still not very good
  - Branching factor really high
  - Some recent progress
- Poker:
  - Competitive with top humans in some 2-player games
  - 3+ player case much less well-understood

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Is this of any value to society?

- Some of the techniques developed for games have found applications in other domains
  - Especially “adversarial” settings
- Real-world strategic situations are usually not two-player, perfect-information, zero-sum, …
- But game theory does not need any of those
- Example application: security scheduling at airports

Adapted from Rusell, S., & Norvig, P. (2016);

2.4 Classroom task

Your turn!

Task

Please explain

- The Seinfeld variant of rock-paper-scissors.
- how minimax algorithms are used in decision-making for two-player zero-sum games, and describe one limitation of the basic minimax approach.

Outline

2 Search, Problem Solving, and Planning

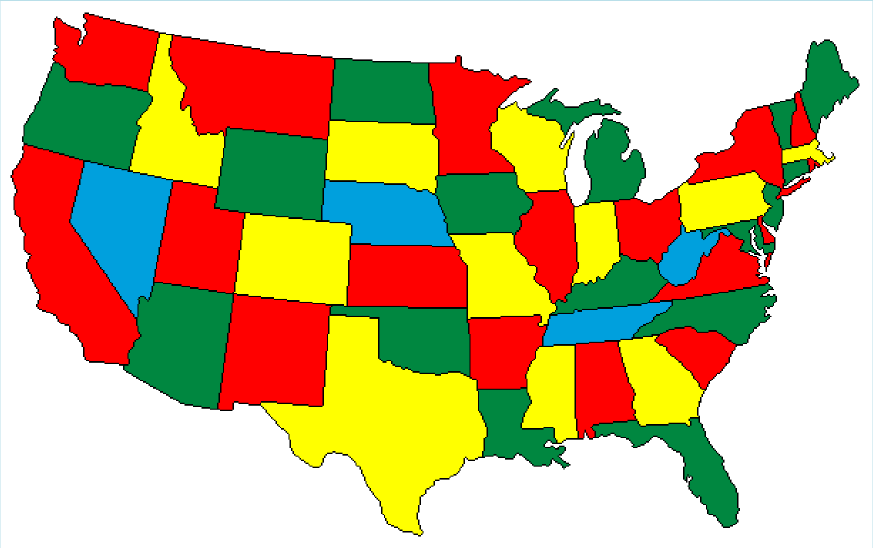
▶ **What we will learn:**
- We define the concept of rational agents (≈ intelligent agents)
- Characteristics of artificial agents (perfect or otherwise), the diversity of environments, and the resulting menagerie of agent types
- We discuss how AI problems can be modelled as search-problems, and how they can be solved by searching

Image source: ↗ Pixabay (2019) / ↗ CC0

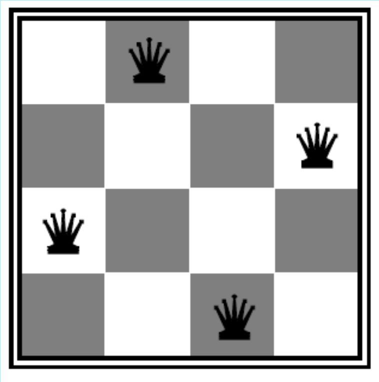
▶ **Duration:**
- 225 min

▶ **Relevant for Exam:**
- 2.1 – 2.5

2.5 Constraint Satisfaction Problems



8			4		6			7
						4		
	1					6	5	
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2					9	
		1						
3			9		2			5



Adapted from Rusell, S., & Norvig, P. (2016);

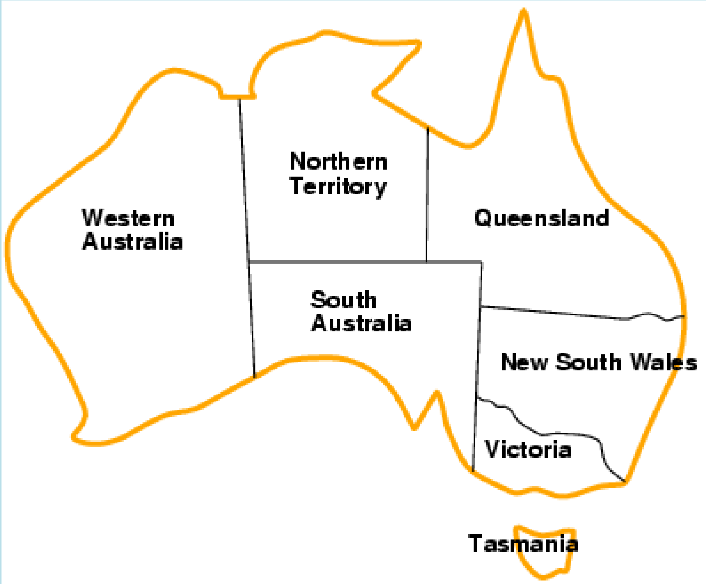
2.5 Constraint satisfaction problems (CSPs)

- Standard search problem:
  - **State** is a "black box" – any data structure that supports successor function, heuristic function, and goal test
- Constraint satisfaction problem:
  - **State** is defined by variables X_i with values from domain D_i
  - **Goal test** is a set of constraints specifying allowable combinations of values for subsets of variables
- A simple example of a formal representation language
- Allows useful general-purpose algorithms with more power than standard search algorithms

Adapted from Rusell, S., & Norvig, P. (2016);

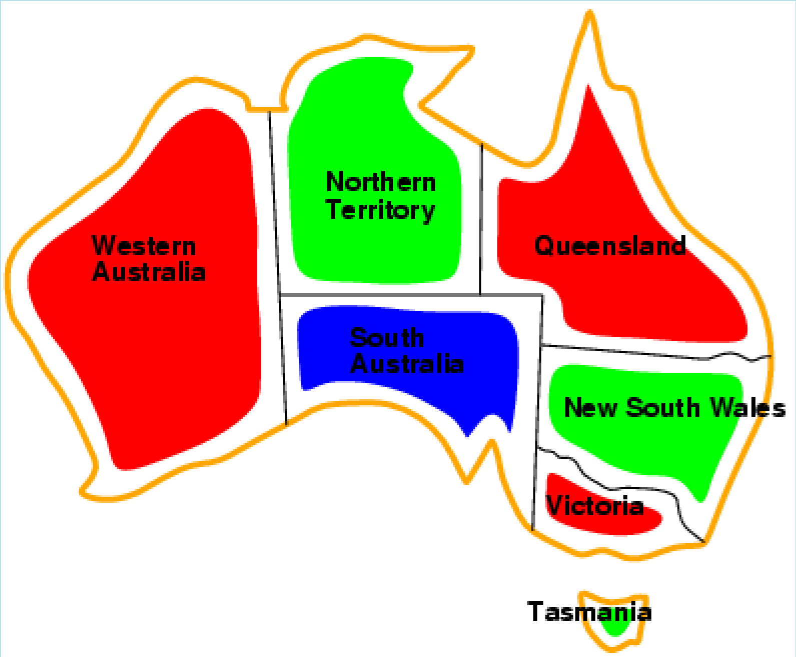
2.5 Example: Map Coloring

- **Variables:** WA, NT, Q, NSW, V, SA, T
- **Domains:** {red, green, blue}
- **Constraints:** adjacent regions must have different colors e.g., WA ≠ NT, or (WA, NT) in {(red, green), (red, blue), (green, red), (green, blue), (blue, red), (blue, green)}



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example: Map Coloring



- **Solutions** are *complete* and *consistent* assignments, e.g., WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example: N-Queens

- **Variables:** X_{ij}
- **Domains:** {0, 1}
- **Constraints:**

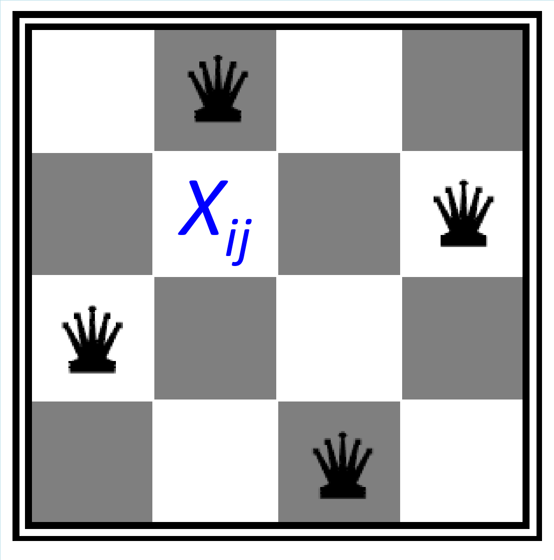
$\Sigma_{i,j}\, X_{ij} = N$

$(X_{ij}, X_{ik}) \in \{(0, 0), (0, 1), (1, 0)\}$

$(X_{ij}, X_{kj}) \in \{(0, 0), (0, 1), (1, 0)\}$

$(X_{ij}, X_{i+k,\, j+k}) \in \{(0, 0), (0, 1), (1, 0)\}$

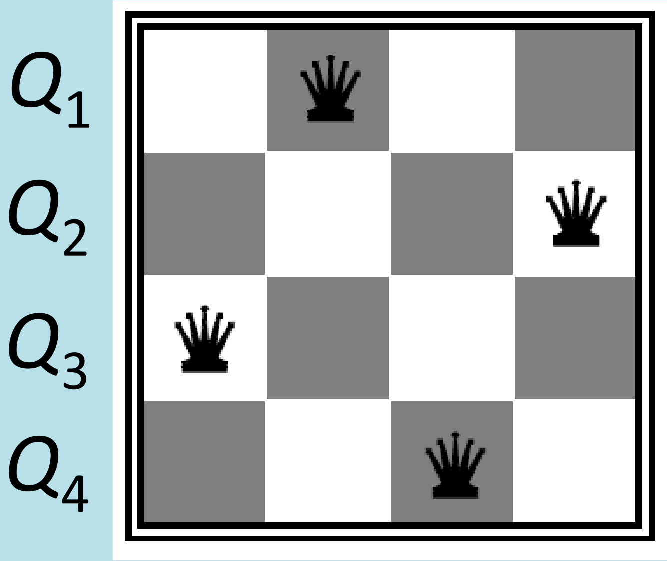
$(X_{ij}, X_{i+k,\, j-k}) \in \{(0, 0), (0, 1), (1, 0)\}$



Adapted from Rusell, S., & Norvig, P. (2016);

- **Variables:** Q_i
- **Domains:** $\{1, \ldots, N\}$
- **Constraints:**

  $\forall\ i, j$ non-threatening (Q_i, Q_j)



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example: Cryptarithmetic

- **Variables:** T, W, O, F, U, R
  X_1, X_2
- **Domains**: {0, 1, 2, …, 9}
- **Constraints:**
  Alldiff(T, W, O, F, U, R)
  $O + O = R + 10 * X_1$
  $W + W + X_1 = U + 10 * X_2$
  $T + T + X_2 = O + 10 * F$
  $T \neq 0, F \neq 0$

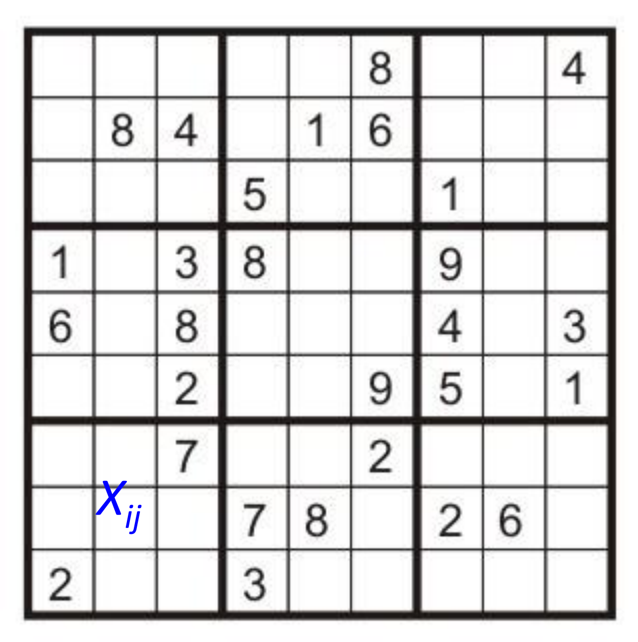
$$\begin{array}{cccc} & X_2 & X_1 & \\ & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example: Sudoku

- **Variables:** X_{ij}
- **Domains:** {1, 2, …, 9}
- **Constraints:**

  Alldiff(X_{ij} in the same *unit*)



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Real-world CSPs

- Assignment problems
  - e.g., who teaches what class
- Timetable problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling

- More examples of CSPs: http://www.csplib.org/

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Standard search formulation (incremental)

- **States**:
  - Values assigned so far
- **Initial state:**
  - The empty assignment { }
- **Successor function:**
  - Choose any unassigned variable and assign to it a value that does not violate any constraints
    - Fail if no legal assignments
- **Goal test:**
  - The current assignment is complete and satisfies all constraints

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Standard search formulation (incremental)

- What is the depth of any solution?
  - *n* (with *n* variables assigned)
  - This is the good news (why?)
- Given that there are *m* possible values for any variable, how many paths are there in the search tree?
  - $n! \cdot m^n$
  - This is the bad news
- How can we reduce the branching factor?

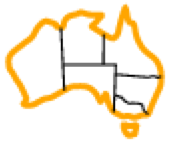
Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Backtracking search

- In CSP's, variable assignments are **commutative**
  - For example, *[WA = red then NT = green]* is the same as *[NT = green then WA = red]*
- We only need to consider assignments to a single variable at each level (i.e., we fix the order of assignments)
  - Then there are only m^n leaves
- Depth-first search for CSPs with single-variable assignments is called **backtracking search**

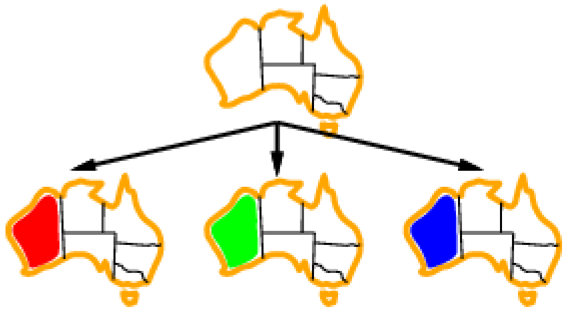
Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example



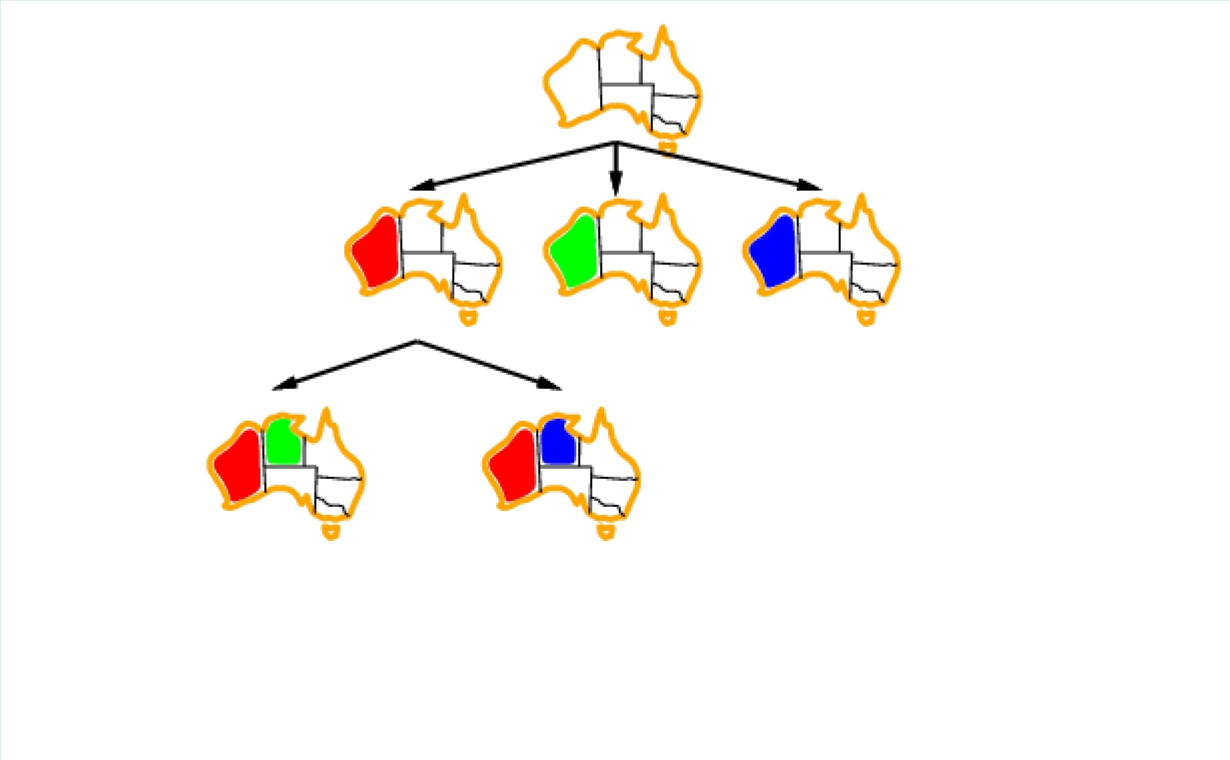
Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example



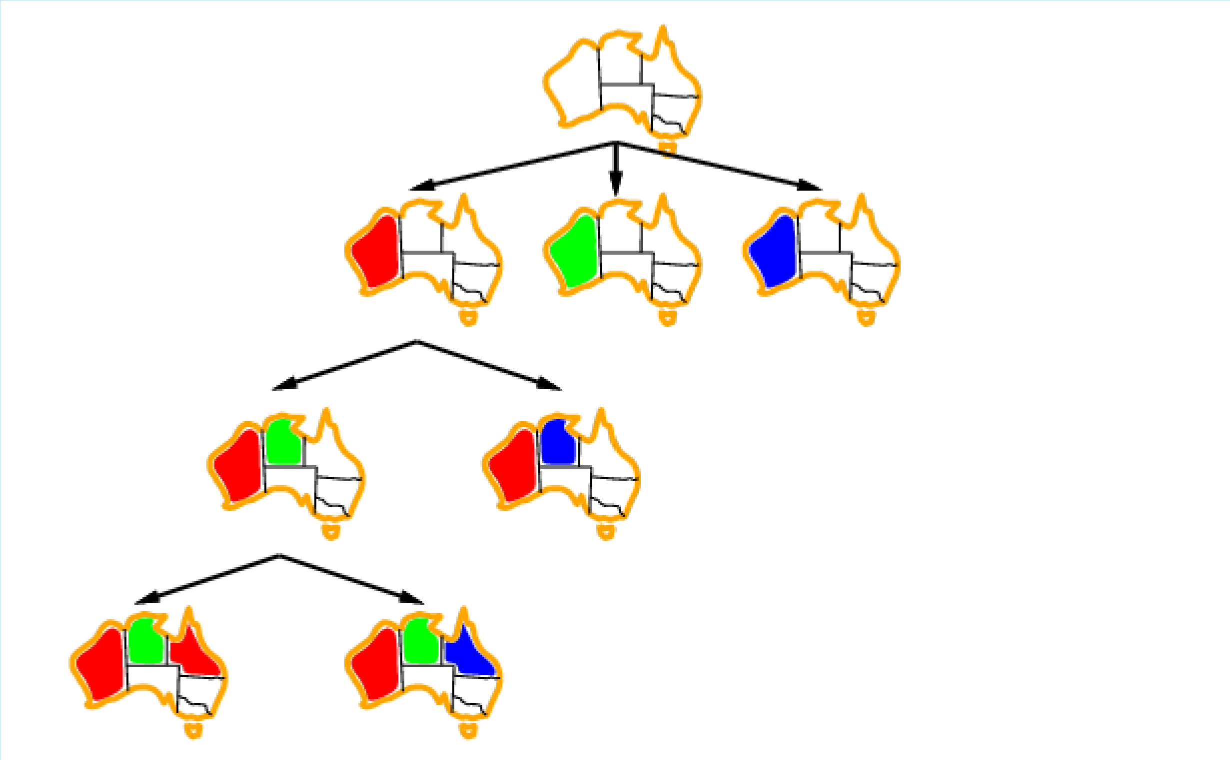
Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Example



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Backtracking search algorithm

function RECURSIVE-BACKTRACKING(*assignment*, *csp*)
  if *assignment* is complete **then return** *assignment*
  var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*], *assignment*, *csp*)
  for each *value* **in** ORDER-DOMAIN-VALUES(*var*, *assignment*, *csp*)
    if *value* is consistent with *assignment* given CONSTRAINTS[*csp*]
      add {*var* = *value*} to *assignment*
      result ← RECURSIVE-BACKTRACKING(*assignment*, *csp*)
      if *result* ≠ *failure* **then return** *result*
      remove {*var* = *value*} from *assignment*
  return *failure*

- Improving backtracking efficiency:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Which variable should be assigned next?

- **Most constrained variable:**
  - Choose the variable with the fewest legal values
  - A.k.a. **minimum remaining values** (MRV) heuristic

Adapted from Rusell, S., & Norvig, P. (2016);

- **Most constrained variable:**
  - Choose the variable with the fewest legal values
  - A.k.a. **minimum remaining values** (MRV) heuristic

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Which variable should be assigned next?

- **Most constraining variable:**
  - Choose the variable that imposes the most constraints on the remaining variables
  - Tie-breaker among most constrained variables

Adapted from Rusell, S., & Norvig, P. (2016);

- **Most constraining variable:**
  - Choose the variable that imposes the most constraints on the remaining variables
  - Tie-breaker among most constrained variables

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Given a variable, in which order should its values be tried?

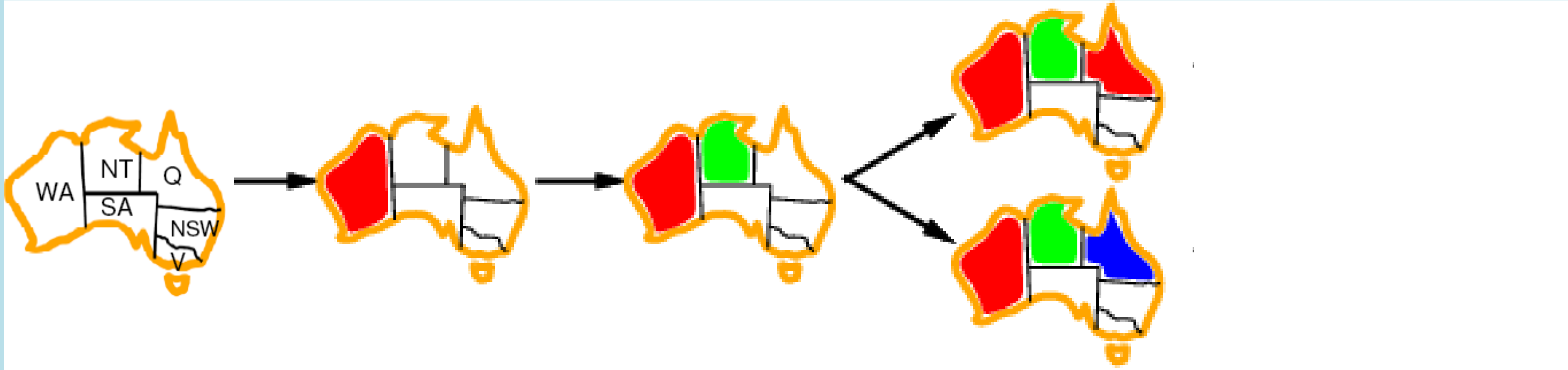
- Choose the **least constraining value**:
  - The value that rules out the fewest values in the remaining variables

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Given a variable, in which order should its values be tried?

- Choose the **least constraining value**:
  - The value that rules out the fewest values in the remaining variables

Which assignment for Q should we choose?



Adapted from Rusell, S., & Norvig, P. (2016);

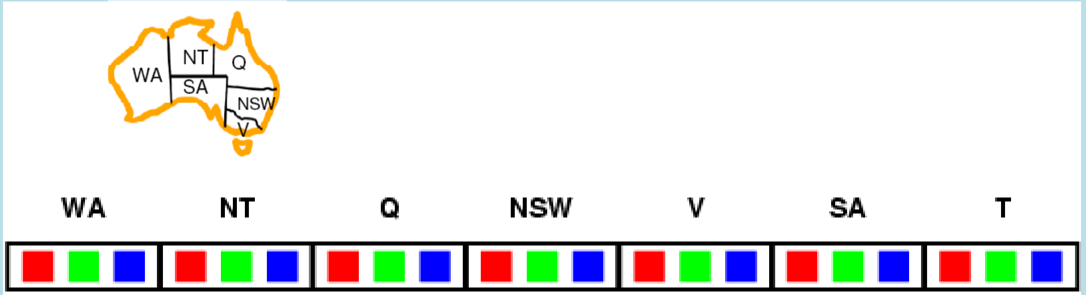
2.5 Early detection of failure: Forward checking

- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Early detection of failure: Forward checking

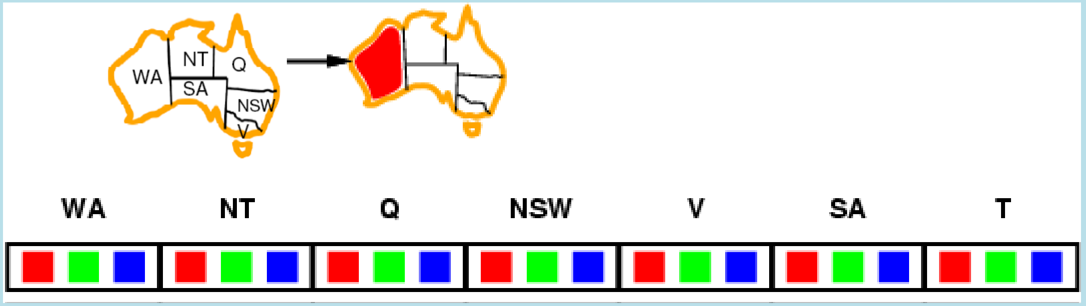
- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Early detection of failure: Forward checking

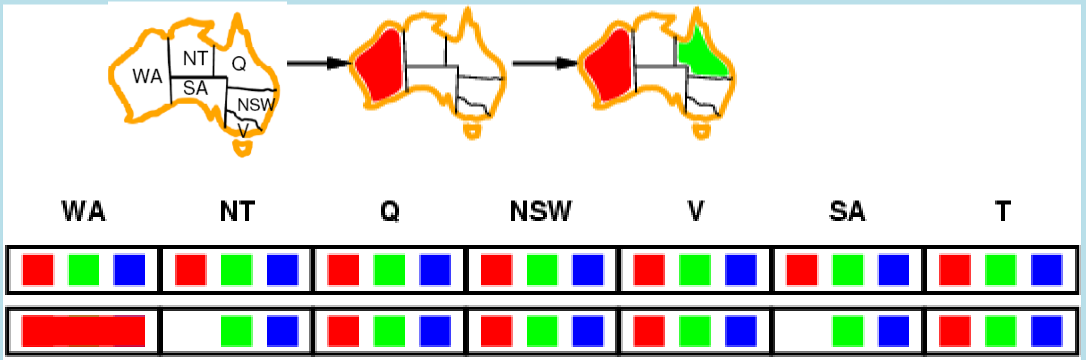
- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Early detection of failure: Forward checking

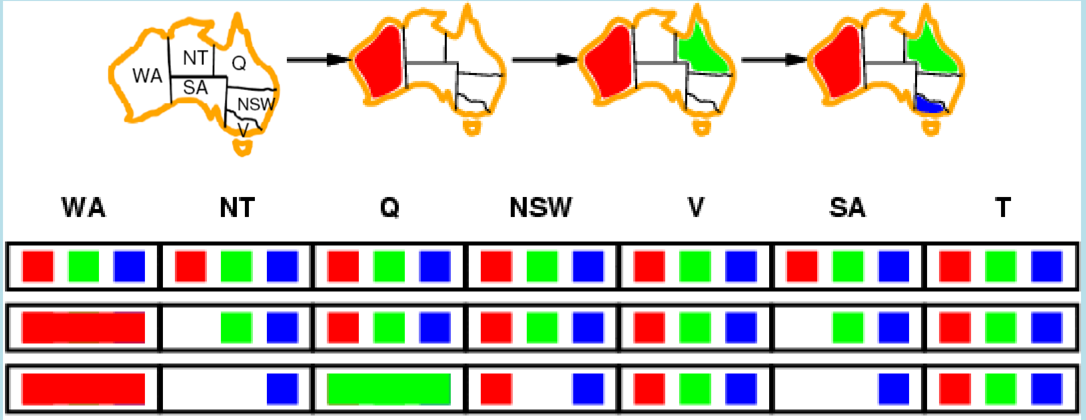
- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Early detection of failure: Forward checking

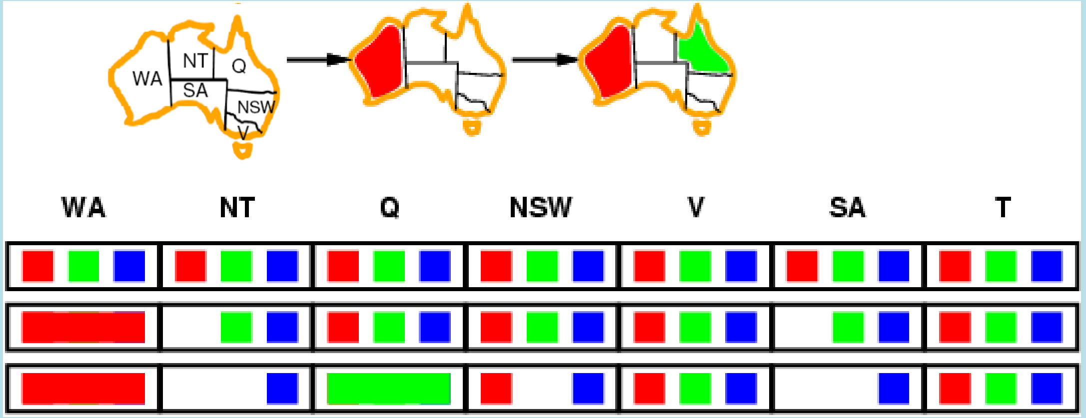
- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Constraint propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures

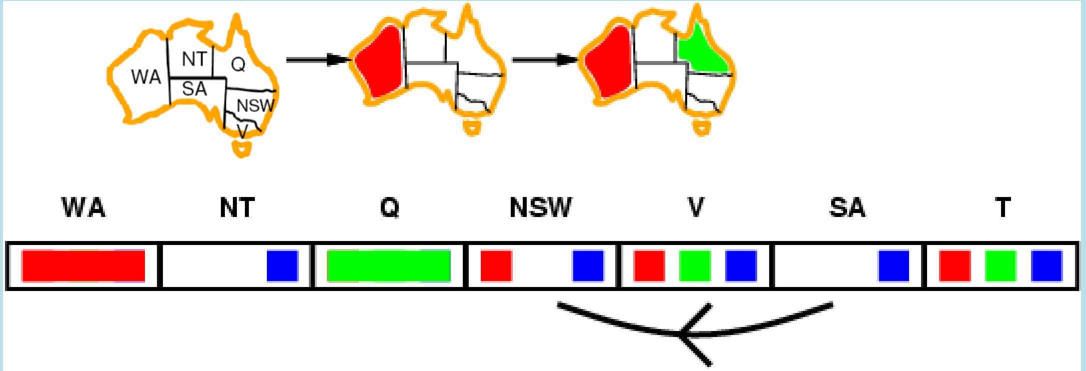


- NT and SA cannot both be blue!
- **Constraint propagation** repeatedly enforces constraints locally

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Arc consistency

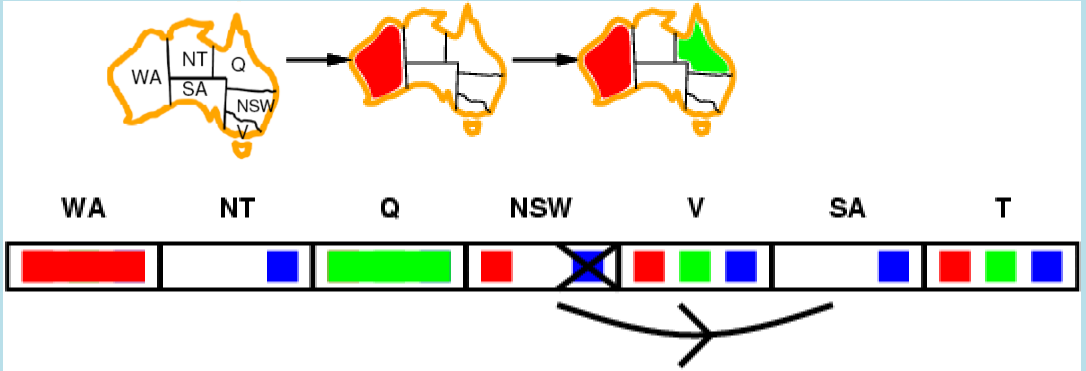
- Simplest form of propagation makes each pair of variables **consistent:**
  - $X \rightarrow Y$ is consistent iff for every value of X there is some allowed value of Y



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Arc consistency

- Simplest form of propagation makes each pair of variables **consistent:**
  - *X* → *Y* is consistent iff for every value of *X* there is some allowed value of *Y*

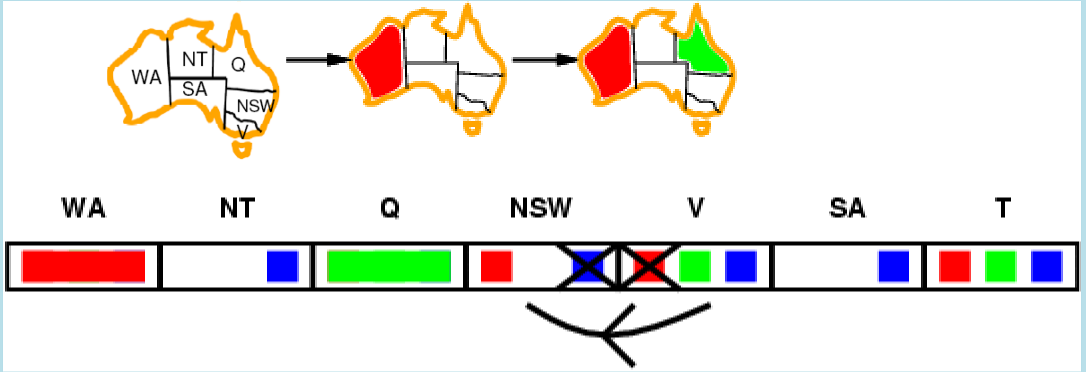


- If *X* loses a value, all pairs *Z* → *X* need to be rechecked

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Arc consistency

- Simplest form of propagation makes each pair of variables **consistent:**
  - *X* → *Y* is consistent iff for every value of *X* there is some allowed value of *Y*

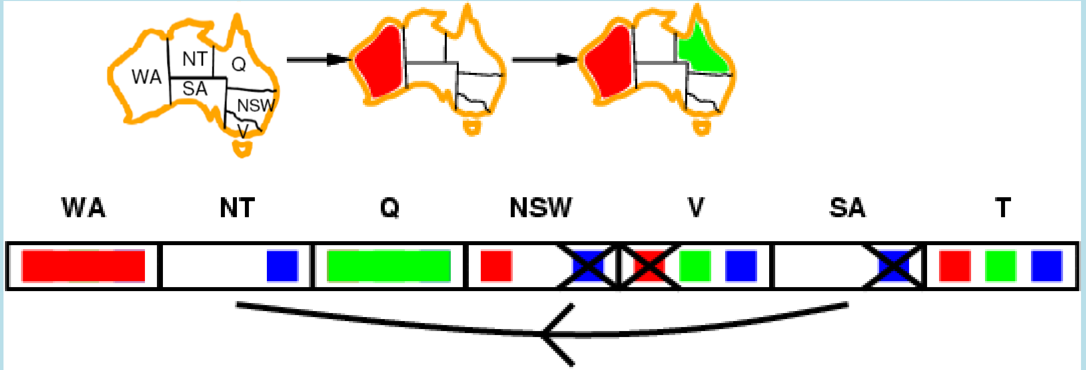


- If *X* loses a value, all pairs *Z* → *X* need to be rechecked

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Arc consistency

- Simplest form of propagation makes each pair of variables **consistent:**
  - *X* → *Y* is consistent iff for every value of *X* there is some allowed value of *Y*



- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment

Adapted from Rusell, S., & Norvig, P. (2016);

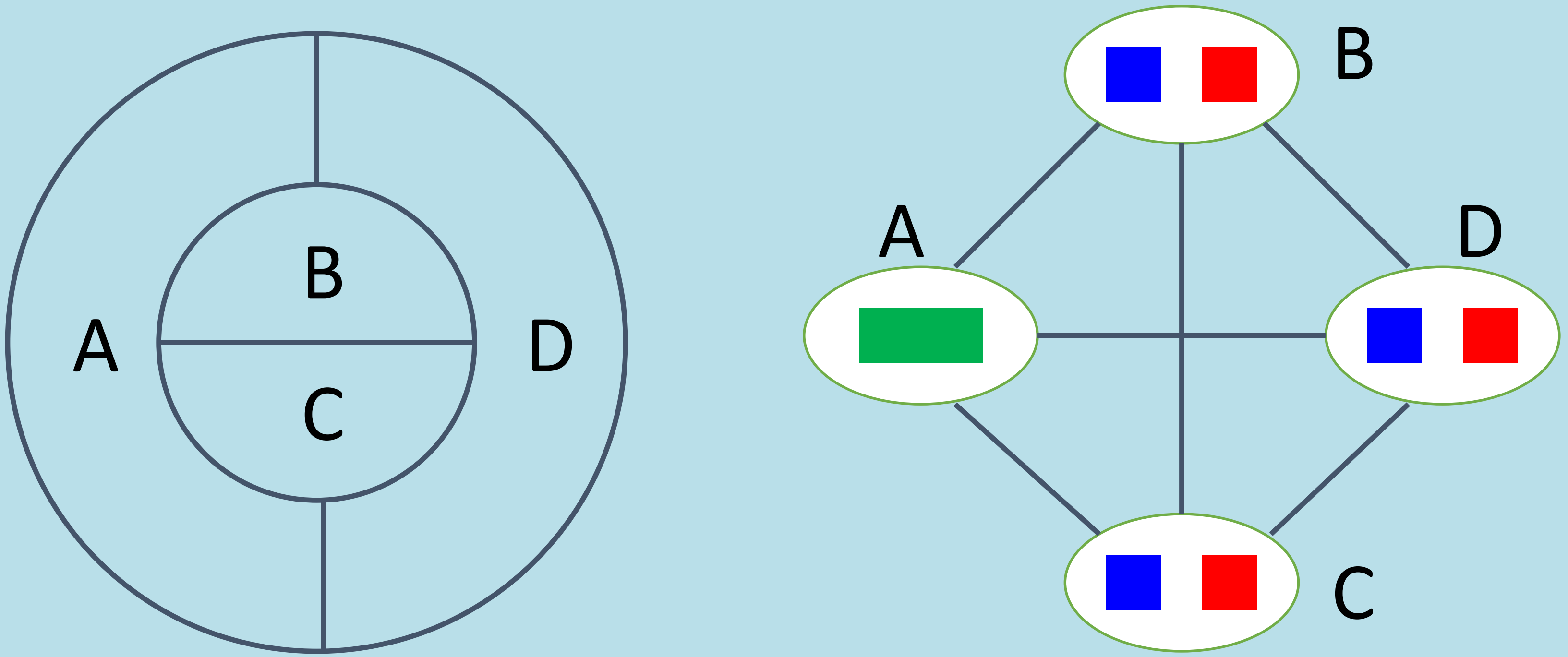
2.5 Arc consistency algorithm AC-3

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X_1, X_2, ..., X_n}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty
      (X_i, X_j) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(X_i, X_j) then
         for each X_k in NEIGHBORS[X_i] do
            add (X_k, X_i) to queue
```

```
function REMOVE-INCONSISTENT-VALUES( X_i, X_j) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[X_i]
      if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint X_i ↔ X_j
         then delete x from DOMAIN[X_i];  removed ← true
   return removed
```

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Does arc consistency always detect the lack of a solution?



- There exist stronger notions of consistency (path consistency, k-consistency), but we won't worry about them

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Backtracking search with inference

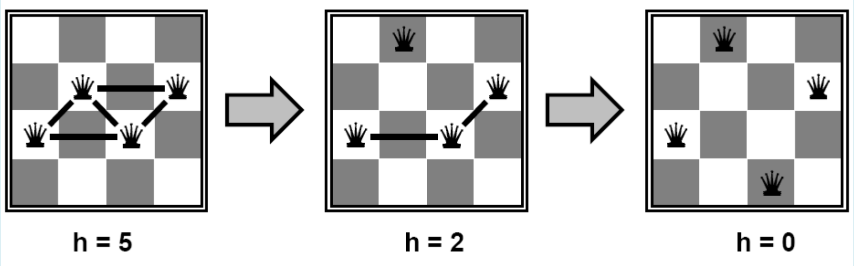
```
function RECURSIVE-BACKTRACKING(assignment, csp)
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp)
        if value is consistent with assignment given CONSTRAINTS[csp]
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

- Do inference (forward checking or constraint propagation) here

Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Local search for CSPs

- Hill-climbing, simulated annealing typically work with “complete” states, i.e., all variables assigned
- To apply to CSPs:
  - Allow states with unsatisfied constraints
  - Attempt to improve states by reassigning variable values
- Variable selection:
  - Randomly select any conflicted variable
- Value selection by min-conflicts heuristic:
  - Choose value that violates the fewest constraints
  - I.e., hill-climb with *h(n)* = total number of violated constraints



Adapted from Rusell, S., & Norvig, P. (2016);

2.5 Summary

- CSPs are a special kind of search problem:
  - States defined by values of a fixed set of variables
  - Goal test defined by constraints on variable values
- **Backtracking** = depth-first search where successor states are generated by considering assignments to a single variable
  - **Variable ordering** and **value selection** heuristics can help significantly
  - **Forward checking** prevents assignments that guarantee later failure
  - **Constraint propagation** (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- Local search can be done by iterative min-conflicts

Adapted from Rusell, S., & Norvig, P. (2016);

Your turn!

Task

- What is the difference between a consistent assignment and a complete assignment in the context of CSPs, and why is consistency important for solving CSPs efficiently?
- How does backtracking search work in CSPs, and what is one technique used to improve its efficiency (e.g., forward checking or arc consistency)?

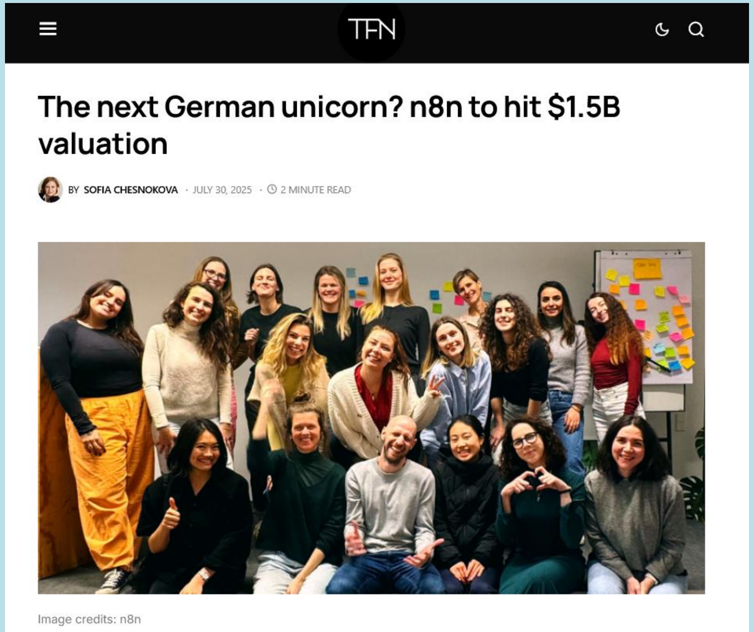
Exercises

Workbook Exercises

- Please read the chapters 2-4 from Rusell, S. & Norvig, P. (2016). Work through the exercises of the related chapters. Start with the exercises related to algorithms we discussed during lecture.

Coding Exercises

- Explore one of the emerging AI agent frameworks or toolboxes (e.g., OpenAI, Gemini, n8n, Meta's Agentic Framework, etc.) and design your own simple AI agent that performs a specific real-world task.Be prepared to give a short demo of your agent in the next lecture.



References

Literature

1. Rusell, S., & Norvig, P. (2016). *Artificial Intelligence: A Modern Approach.* Global Edition.

News articles

1. NTV (2019): Sneaker-Bots ausgetrickst - Skate-Shop verkauft Fotos statt Schuhe. Online available at: https://www.n-tv.de/mediathek/videos/wirtschaft/Skate-Shop-verkauft-Fotos-statt-Schuhe-article21229466.html

Images

All images that were not marked other ways are made by myself, or licensed ↗CC0 from ↗Pixabay.

References

Further reading

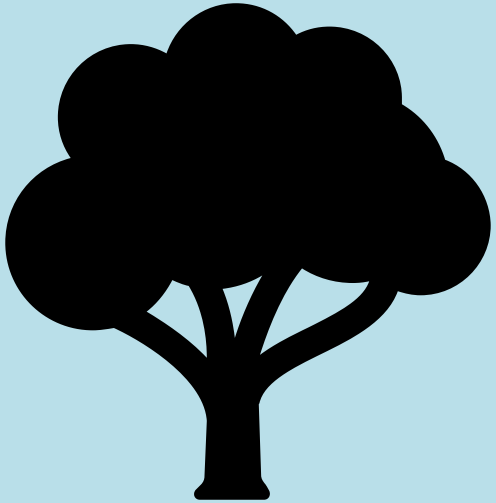
- If you are also interested in visiting all US national parks, I can recommend to take a look at the blog article from *University of Penn data scientist Dr. Randal Olson* (↗http://www.randalolson.com). He created a roadtrip of the optimal way to visit all 47* U.S. National Parks in the mainland United States. Take a look at his Python projects (↗Github).
- I can you also recommend the following Golem articel (↗https://www.golem.de/) to read more about the application of the A* algorithm in pathfinding in games.

**before Gateway Arch in 2018 became a national park*

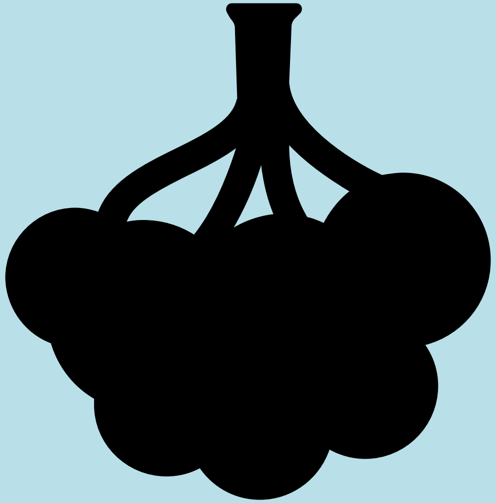
Glossary

Agent	*An agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators (Rusell & Norvig, 2016)*
Agent program	*Implements the agents function (Rusell & Norvig, 2016, p.59)*
Performance measure	*Evaluates the behavior of the agent in an environment. A rational agent acts so as to maximize the expected value of the performance measure, given the percept sequence it has seen so far (Rusell & Norvig, 2016, p.59)*
Rationality/Rational agent	*For each possible percept sequence, a rational agent should select an action that is expected to maximize its performance measure, given the evidence provided by the percept sequence and whatever built-in knowledge the agent has (Rusell & Norvig, 2016, p.37)*
Path costs	A function that assigns a cost to path, typically by summing the costs of the individual operators in the path
Search costs	The computational time and space (memory) required to find the solution
Task environment	*External environment of an agent including the performance measure, the external environment, the actuators, and the sensors (Rusell & Norvig, 2016, p.59)*

Trees



How normal
people see trees



How computer
scientists see trees