Typed Quote/Antiquote

Embedding languages in Haskell

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Domain Specific Languages

A Domain Specific Language (DSL) is:

- A language for a particular domain
- Counterpart of a general-purpose language
- Not always a programming language
- For example: SQL, shell scripting, HTML, Make

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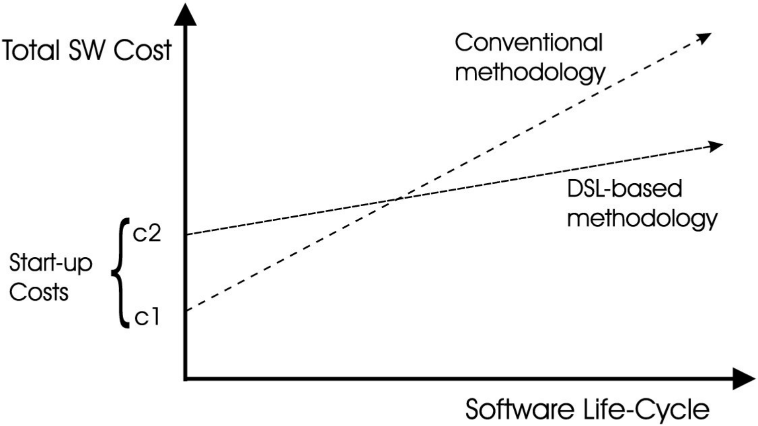
Programs written in a DSL are:

- Easier to write
- Easier to reason about
- Easier to learn for a domain expert
- Easier to modify

The payoff of DSL technology



Domain Specific Embedded Languages (DSELs)

Inherit the infrastructure of another language

- Macro expansion of DSL to another language
- Generating software from a specification in a DSL
- Embed a DSL in another language

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Embed a DSL in Haskell

- Currying
- Higher-order functions
- Polymorphism
- Type classes (overloading)

Embedding concrete syntax in Haskell

Concrete syntax for natural number

$$
\begin{aligned}
test = {} & quote \\
& \quad tick \\
& \quad tick \\
& \quad tick \\
& end \\
& + \\
& quote \\
& \quad tick \\
& end
\end{aligned}
$$

How can we implement this?

How we going to implement this?

Function application is left-assiociative

$$(((quote\ tick)\ tick)\ tick)\ end$$

How we going to implement this?

Function application is left-assiociative

$$(((\mathit{quote}\ \mathit{tick})\ \mathit{tick})\ \mathit{tick})\ \mathit{end}$$

If Haskell had postfix function application...

$$\begin{aligned}
&\mathit{quote} :: \quad \mathit{Int} \\
&\mathit{quote} \qquad = 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{tick} :: \mathit{Int} \to \mathit{Int} \\
&\mathit{tick}\quad i \quad = i + 1
\end{aligned}$$

$$\begin{aligned}
&\mathit{end} :: \mathit{Int} \to \mathit{Int} \\
&\mathit{end}\quad i \quad = i
\end{aligned}$$

Postfix function application

Introduce an postfix operator

$$(\triangleleft) :: \alpha \rightarrow (\alpha \rightarrow r) \rightarrow r$$

$$x \triangleleft f = f\ x$$

$$test = quote \triangleleft tick \triangleleft tick \triangleleft tick \triangleleft end$$

Postfix function application

Introduce an postfix operator

$$(\triangleleft) :: \alpha \to (\alpha \to r) \to r$$
$$x \triangleleft f = f\ x$$
$$test = quote \triangleleft tick \triangleleft tick \triangleleft tick \triangleleft end$$

Pushing the operator into the terminals

$$\begin{aligned} quote &:: (Int \to r) \to r \\ quote &= (\triangleleft)\ 0 \end{aligned}$$

$$\begin{aligned} tick &:: Int \to (Int \to r) \to r \\ tick\ \ i &= (\triangleleft)\ (i + 1) \end{aligned}$$

$$\begin{aligned} end &:: Int \to Int \\ end\ \ i &= i \end{aligned}$$

Finishing touch

The CPS monad

$$\textbf{type}\ CPS\ \alpha = \forall\ r.(\alpha \to r) \to r$$

$$\begin{aligned}
lift &:: \alpha \to CPS\ \alpha \\
lift\ \ a &= \lambda f \to f\ a \\
quote &:: CPS\ Int \\
quote &= lift\ \ 0 \\
tick &:: Int \to CPS\ Int \\
tick\ \ i &= lift\ \ (i+1) \\
end &:: Int \to Int \\
end\ \ i &= i
\end{aligned}$$

$$test = quote\ tick\ tick\ tick\ end$$

Evaluation steps

The quotation is evaluated like this

$$\begin{aligned}
\mathit{quote} &= \mathit{lift}\ 0 \\
\mathit{tick}\ i &= \mathit{lift}\ (i+1) \\
\mathit{end}\ i &= i
\end{aligned}$$

$$\begin{array}{llll}
\mathit{quote} & & & \mathit{tick}\ \mathit{tick}\ \mathit{tick}\ \mathit{end} \\
\mathit{lift} & 0 & \mathit{tick} & \mathit{tick}\ \mathit{tick}\ \mathit{end} \\
\mathit{tick}\ 0 & & & \mathit{tick}\ \mathit{tick}\ \mathit{end} \\
\mathit{lift} & (0+1) & \mathit{tick} & \mathit{tick}\ \mathit{end} \\
\mathit{tick}\ 1 & & & \mathit{tick}\ \mathit{end} \\
\mathit{lift} & (1+1) & \mathit{tick} & \mathit{end} \\
\mathit{tick}\ 2 & & & \mathit{end} \\
\mathit{lift} & (2+1) & \mathit{end} & \\
\mathit{end}\ 3 & & & \\
3 & & &
\end{array}$$

Ready for the next step

What we have

- Framework for postfix function application
- Threading a state through the terminals

Ready for the next step

What we have

- Framework for postfix function application
- Threading a state through the terminals

The next step

- Embed postfix notation
- Embed prefix notation

Postfix and Prefix notation

Postfix notation

- Functions follow their arguments
- 3 5 + 2 *
- Also known as Reverse Polish Notation (RPN)
- Stack-based implementation

Postfix and Prefix notation

Postfix notation

- Functions follow their arguments
- 3 5 + 2 *
- Also known as Reverse Polish Notation (RPN)
- Stack-based implementation

Prefix notation

- Functions precedes their arguments
- * + 3 5 2
- Also known as Polish notation
- Haskell data contructors

Example

Financial Combinators (S. Peyton-Jones et al.)

$$
\begin{aligned}
\textbf{data}\ \mathit{Contract} &= \mathit{Zero}\\
&\mid \mathit{One}\\
&\mid \mathit{Give}\ \mathit{Contract}\\
&\mid \mathit{Or}\ \ \ \mathit{Contract}\ \mathit{Contract}
\end{aligned}
$$

Example

Financial Combinators (S. Peyton-Jones et al.)

$$
\begin{aligned}
\textbf{data}\ \mathit{Contract} = \ & \mathit{Zero} \\
\mid\ & \mathit{One} \\
\mid\ & \mathit{Give}\ \ \mathit{Contract} \\
\mid\ & \mathit{Or}\quad \mathit{Contract}\ \mathit{Contract}
\end{aligned}
$$

Concrete syntax

$$
\begin{aligned}
\mathit{contract} = \ & \mathit{quote} \\
& \quad \mathit{one}\ \mathit{zero}\ \mathit{or}\ \mathit{give} \\
& \mathit{end}
\end{aligned}
$$

Should evaluate to: $\mathit{Give}\ (\mathit{Or}\ \mathit{One}\ \mathit{Zero})$

Systematic translation

Redefine quotation

$$
\begin{aligned}
&\mathit{quote} :: \mathit{CPS}\ ()\\
&\mathit{quote} = \mathit{lift}\quad ()\\
&\mathit{end}\quad :: ((), \alpha) \to \alpha\\
&\mathit{end}\qquad ((), a)\ =\ a
\end{aligned}
$$

Systematic translation

Redefine quotation

$$
\begin{aligned}
&quote :: CPS\ ()\\
&quote = lift\ \ ()\\
&end\ \ :: ((), \alpha) \rightarrow \alpha\\
&end\ \ \ ((), a)\ = a
\end{aligned}
$$

Translation of constructors

Introduce for each contructor $C :: \tau_1 \rightarrow \ldots \rightarrow \tau_n \rightarrow \tau$, a postfix function (terminal) c:

$$
\begin{aligned}
&c :: (((st, \tau_1), \ldots), \tau_n) \rightarrow CPS\ (st, \tau)\\
&c\ \ (((st, t_1), \ldots), t_n) = lift\ \ (st, C\ t_1 \ldots t_n)
\end{aligned}
$$

Example translation

Example constructors

$$
\begin{array}{lll}
\mathit{Zero}, \mathit{One} & :: & \mathit{Contract} \\
\mathit{Give} & :: & \mathit{Contract} \to \mathit{Contract} \\
\mathit{Or} & :: & \mathit{Contract} \to \mathit{Contract} \to \mathit{Contract}
\end{array}
$$

Translated postfix functions (terminals)

$$
\begin{array}{llll}
\mathit{zero} :: \mathit{st} & \to & \mathit{CPS} & (\mathit{st}, \mathit{Contract}) \\
\mathit{zero} \quad \mathit{st} & = & \mathit{lift} & (\mathit{st}, \mathit{Zero}) \\
\mathit{give} :: (\mathit{st}, \mathit{Contract}) & \to & \mathit{CPS} & (\mathit{st}, \mathit{Contract}) \\
\mathit{give} \quad (\mathit{st}, c) & = & \mathit{lift} & (\mathit{st}, \mathit{Give}\ c) \\
\mathit{or} :: ((\mathit{st}, \mathit{Contract}), \mathit{Contract}) & \to & \mathit{CPS} & (\mathit{st}, \mathit{Contract}) \\
\mathit{or} \quad ((\mathit{st}, c1), c2) & = & \mathit{lift} & (\mathit{st}, \mathit{Or}\ c1\ c2)
\end{array}
$$

Concrete example, types

The types correspond to the stack layout

$$
\begin{array}{lll}
\mathit{quote} & :: & \ \mathit{CPS}\ () \\
\mathit{zero}, \mathit{one} & :: \mathit{st} & \rightarrow \mathit{CPS}\ (\mathit{st}, \mathit{Contract}) \\
\mathit{give} & :: (\mathit{st}, \mathit{Contract}\ ) & \rightarrow \mathit{CPS}\ (\mathit{st}, \mathit{Contract}) \\
\mathit{or} & :: ((\mathit{st}, \mathit{Contract}), \mathit{Contract}) & \rightarrow \mathit{CPS}\ (\mathit{st}, \mathit{Contract}) \\
\mathit{end} & :: ((), \alpha) & \rightarrow \alpha
\end{array}
$$

$$
\begin{array}{ll}
\mathit{quote} & :: \mathit{CPS}\ () \\
\mathit{quote}\ \mathit{one} & :: \mathit{CPS}\ ((), \mathit{Contract}) \\
\mathit{quote}\ \mathit{one}\ \mathit{zero} & :: \mathit{CPS}\ (((), \mathit{Contract}), \mathit{Contract}) \\
\mathit{quote}\ \mathit{one}\ \mathit{zero}\ \mathit{or} & :: \mathit{CPS}\ ((), \mathit{Contract}) \\
\mathit{quote}\ \mathit{one}\ \mathit{zero}\ \mathit{or}\ \mathit{give} & :: \mathit{CPS}\ ((), \mathit{Contract}) \\
\mathit{quote}\ \mathit{one}\ \mathit{zero}\ \mathit{or}\ \mathit{give}\ \mathit{end} & :: \mathit{Contract}
\end{array}
$$

Concrete example, evaluation

Evaluation steps

$$
\begin{aligned}
&\mathit{quote} && = \mathit{lift}\ ()\\
&\mathit{zero}\ \mathit{st} && = \mathit{lift}\ (\mathit{st}, \mathit{Zero})\\
&\mathit{one}\ \ \mathit{st} && = \mathit{lift}\ (\mathit{st}, \mathit{One})\\
&\mathit{give}\ (\mathit{st}, c) && = \mathit{lift}\ (\mathit{st}, \mathit{Give}\ c)\\
&\mathit{or}\ \ \ ((\mathit{st}, c1), c2) && = \mathit{lift}\ (\mathit{st}, \mathit{Or}\ c1\ \ c2)\\
&\mathit{end}\ \ ((), a) && = a
\end{aligned}
$$

$$
\begin{array}{ll}
\mathit{quote} & \mathit{one}\ \ \mathit{zero}\ \mathit{or}\ \ \ \mathit{give}\ \mathit{end}\\
\mathit{one}\ \ () & \mathit{zero}\ \mathit{or}\ \ \ \mathit{give}\ \mathit{end}\\
\mathit{zero}\ ((), \mathit{One}) & \mathit{or}\ \ \ \mathit{give}\ \mathit{end}\\
\mathit{or}\ \ \ (((), \mathit{One}), \mathit{Zero}) & \mathit{give}\ \mathit{end}\\
\mathit{give}\ ((), \mathit{Or}\ \ \mathit{One}\ \ \mathit{Zero}) & \mathit{end}\\
\mathit{end}\ \ ((), \mathit{Give}\ (\mathit{Or}\ \ \mathit{One}\ \ \mathit{Zero})) & \\
\mathit{Give}\ (\mathit{Or}\ \ \mathit{One}\ \ \mathit{Zero}) &
\end{array}
$$

Prefix notation

Embedding prefix notation

- Systematic translation for embedding data types
- State is a stack of pending arguments
- Stack is represented by a function
- Running example: Financial Combinators

Prefix notation

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- Stack is represented by a function
- Running example: Financial Combinators

Concrete syntax

$$
\begin{aligned}
\mathit{contract} = {} & \mathit{quote} \\
& \quad \mathit{give}\ \mathit{or}\ \mathit{zero}\ \mathit{one} \\
& \mathit{end}
\end{aligned}
$$

Should evaluate to: $\mathit{Give}\ (\mathit{Or}\ \mathit{Zero}\ \mathit{One})$

Systematic translation

Redefine quotation

$$
\begin{aligned}
&\mathit{quote} :: \mathit{CPS}\ (\alpha \to \alpha)\\
&\mathit{quote} = \mathit{lift}\quad \mathit{id}\\
&\mathit{end}\quad :: \alpha \to \alpha\\
&\mathit{end}\qquad a\ = a
\end{aligned}
$$

Systematic translation

Redefine quotation

$$\begin{aligned} quote &:: CPS\ (\alpha \to \alpha) \\ quote &= lift \quad id \end{aligned}$$

$$\begin{aligned} end &:: \alpha \to \alpha \\ end \quad a &= a \end{aligned}$$

Translation of constructors

Introduce for each contructor $C :: \tau_1 \to \ldots \to \tau_n \to \tau$, a prefix function (terminal) c:

$$\begin{aligned} c &:: (\tau \to \alpha) \to CPS\ (\tau_1 \to \ldots \to \tau_n \to \alpha) \\ c \quad ctx &= lift \quad (\lambda t_1 \quad \ldots \quad t_n \to ctx\ (C\ t_1 \ldots t_n)) \end{aligned}$$

Example translation

Example constructors

$$
\begin{array}{ll}
\mathit{Zero}, \mathit{One} & :: \mathit{Contract} \\
\mathit{Give} & :: \mathit{Contract} \rightarrow \mathit{Contract} \\
\mathit{Or} & :: \mathit{Contract} \rightarrow \mathit{Contract} \rightarrow \mathit{Contract}
\end{array}
$$

Translated prefix functions (terminals)

$$
\begin{array}{lllll}
\mathit{zero} & :: (\mathit{Contract} \rightarrow a) & \rightarrow \mathit{CPS}\ a & & \\
\mathit{zero} & \mathit{ctx} & = \mathit{lift} & (\mathit{ctx}\ \mathit{Zero}) & \\
\mathit{give} & :: (\mathit{Contract} \rightarrow a) & \rightarrow \mathit{CPS}\ (\mathit{Contract} \rightarrow a) & & \\
\mathit{give} & \mathit{ctx} & = \mathit{lift} & (\lambda c \rightarrow \mathit{ctx}\ (\mathit{Give}\ c)) & \\
\mathit{or} & :: (\mathit{Contract} \rightarrow a) & \rightarrow \mathit{CPS}\ (\mathit{Contract} \rightarrow \mathit{Contract} \rightarrow a) & & \\
\mathit{or} & \mathit{ctx} & = \mathit{lift} & (\lambda c1\ c2 \rightarrow \mathit{ctx}\ (\mathit{Or}\ c1\ c2)) &
\end{array}
$$

Concrete example, types

The types correspond to the stack layout

$$
\begin{aligned}
\mathit{quote} &:: & \mathit{CPS}\ (\alpha \to \alpha) \\
\mathit{zero}, \mathit{one} &:: (\mathit{Contract} \to a) \to & \mathit{CPS}\ a \\
\mathit{give} &:: (\mathit{Contract} \to a) \to & \mathit{CPS}\ (\mathit{Contract} \to a) \\
\mathit{or} &:: (\mathit{Contract} \to a) \to & \mathit{CPS}\ (\mathit{Contract} \to \mathit{Contract} \to a) \\
\mathit{end} &:: \alpha \to \alpha
\end{aligned}
$$

$$
\begin{aligned}
\mathit{quote} &:: \mathit{CPS}\ (\alpha \to \alpha) \\
\mathit{quote}\ \mathit{give} &:: \mathit{CPS}\ (\mathit{Contract} \to \mathit{Contract}) \\
\mathit{quote}\ \mathit{give}\ \mathit{or} &:: \mathit{CPS}\ (\mathit{Contract} \to \mathit{Contract} \to \mathit{Contrac} \\
\mathit{quote}\ \mathit{give}\ \mathit{or}\ \mathit{zero} &:: \mathit{CPS}\ (\mathit{Contract} \to \mathit{Contract}) \\
\mathit{quote}\ \mathit{give}\ \mathit{or}\ \mathit{zero}\ \mathit{one} &:: \mathit{CPS}\ (\mathit{Contract}) \\
\mathit{quote}\ \mathit{give}\ \mathit{or}\ \mathit{zero}\ \mathit{one}\ \mathit{end} &:: \mathit{Contract}
\end{aligned}
$$

Concrete example, evaluation

Evaluation steps

$$
\begin{aligned}
&\mathit{quote} &&= \mathit{lift}\ \mathit{id}\\
&\mathit{zero}\ \mathit{ctx} &&= \mathit{lift}\ (\mathit{ctx}\ \mathit{Zero})\\
&\mathit{one}\ \ \mathit{ctx} &&= \mathit{lift}\ (\mathit{ctx}\ \mathit{One})\\
&\mathit{give}\ \mathit{ctx} &&= \mathit{lift}\ (\lambda c \rightarrow \mathit{ctx}\ (\mathit{Give}\ c))\\
&\mathit{or}\ \ \ \ \mathit{ctx} &&= \mathit{lift}\ (\lambda c1\ c2 \rightarrow \mathit{ctx}\ (\mathit{Or}\ c1\ c2))\\
&\mathit{end}\ \ a &&= a
\end{aligned}
$$

$$
\begin{array}{ll}
\mathit{quote} & \mathit{give}\ \mathit{or}\ \ \mathit{zero}\ \mathit{one}\ \mathit{end}\\
\mathit{give}\ (\lambda c \rightarrow c) & \mathit{or}\ \ \mathit{zero}\ \mathit{one}\ \mathit{end}\\
\mathit{or}\ \ (\lambda c \rightarrow \mathit{Give}\ c) & \mathit{zero}\ \mathit{one}\ \mathit{end}\\
\mathit{zero}\ (\lambda c1\ c2 \rightarrow \mathit{Give}\ (\mathit{Or}\ c1\ c2)) & \mathit{one}\ \mathit{end}\\
\mathit{one}\ (\lambda c2 \rightarrow \mathit{Give}\ (\mathit{Or}\ \mathit{Zero}\ c2)) & \mathit{end}\\
\mathit{end}\ (\mathit{Give}\ (\mathit{Or}\ \mathit{Zero}\ \mathit{One})) & \\
\mathit{Give}\ (\mathit{Or}\ \mathit{Zero}\ \mathit{One}) &
\end{array}
$$

Ready for the real work

Embed languages described by context-free grammars

- Grammars in Greibach Normal Form (GNF)
- LL(1) grammars
- LR(0) grammars

Ready for the real work

Embed languages described by context-free grammars

- Grammars in Greibach Normal Form (GNF)
- LL(1) grammars
- LR(0) grammars

A grammars is in GNF if:

- All production are of the form $N \longrightarrow aB_1 \dots B_n$
- It cannot generate the empty word (ϵ).

Example syntax

New syntax for Financial Combinators

quote give one dollar or give one euro
or give zero
end

Abstract syntax

$$
\begin{array}{lcl}
\textbf{type } \mathit{Contracts} & = & [\mathit{Contract}] \\
\textbf{data } \mathit{Contract} & = & \mathit{Give}\ \mathit{Contract} \\
 & | & \mathit{One}\ \ \mathit{Currency} \\
 & | & \mathit{Zero} \\
\textbf{data } \mathit{Currency} & = & \mathit{Euro} \mid \mathit{Dollar}
\end{array}
$$

Corresponding grammar

Grammar and equivalent grammar in GNF

$$
\begin{array}{lcl}
\mathsf{S} & \rightarrow & \mathsf{quote\ CS\ end} \\
\mathsf{CS} & \rightarrow & \mathsf{C\ or\ CS} \\
 & | & \mathsf{C} \\
\mathsf{C} & \rightarrow & \mathsf{give\ C} \\
 & | & \mathsf{one\ CU} \\
 & | & \mathsf{zero} \\
\mathsf{CU} & \rightarrow & \mathsf{euro} \\
 & | & \mathsf{dollar}
\end{array}
$$

Corresponding grammar

Grammar and equivalent grammar in GNF

$$
\begin{array}{lcl}
\mathsf{S} & \longrightarrow & \mathsf{quote\ CS\ end} \\
\mathsf{CS} & \longrightarrow & \mathsf{C\ or\ CS} \\
 & \mid & \mathsf{C} \\
\mathsf{C} & \longrightarrow & \mathsf{give\ C} \\
 & \mid & \mathsf{one\ CU} \\
 & \mid & \mathsf{zero} \\
\mathsf{CU} & \longrightarrow & \mathsf{euro} \\
 & \mid & \mathsf{dollar}
\end{array}
\quad \Rightarrow \quad
\begin{array}{lcl}
\mathsf{S} & \longrightarrow & \mathsf{quote\ C\ CS} \\
\mathsf{CS} & \longrightarrow & \mathsf{or\ C\ CS} \\
 & \mid & \mathsf{end} \\
\mathsf{C} & \longrightarrow & \mathsf{give\ C} \\
 & \mid & \mathsf{one\ CU} \\
 & \mid & \mathsf{zero} \\
\mathsf{CU} & \longrightarrow & \mathsf{euro} \\
 & \mid & \mathsf{dollar}
\end{array}
$$

Systematic translation, step 1

A datatype for each non-terminal: N

$$\textbf{newtype}\ N\ \alpha = N\ (S \rightarrow \alpha)$$

Where S is the type of the semantic value.

Systematic translation, step 1

A datatype for each non-terminal: N

$$\textbf{newtype}\ N\ \alpha = N\ (S \rightarrow \alpha)$$

Where S is the type of the semantic value.

Datatypes for the example grammar

$$\begin{array}{lllll}
\textbf{newtype}\ S & a = S & (Contracts \rightarrow a) \\
\textbf{newtype}\ CS & a = CS & (Contracts \rightarrow a) \\
\textbf{newtype}\ C & a = C & (Contract \rightarrow a) \\
\textbf{newtype}\ CU & a = CU & (Currency \rightarrow a)
\end{array}$$

Systematic translation, step 2

For each production: $N \longrightarrow aB_1 \dots B_n$

We introduce the following function:

$$
\begin{array}{llllll}
a :: (N\ \alpha) & \rightarrow CPS\ (B_1\ (& & \dots & (B_n\ \alpha)\dots)) \\
a\ \ \ (N\ ctx) & =\ lift\ \ \ (B_1\ (\lambda v1 & \rightarrow & \dots \rightarrow & B_n\ (\lambda vn \rightarrow ctx\ (f\ \ v1\ \dots\ vn)))
\end{array}
$$

Where f is the semantic function.

Example translation

Productions of the example grammar

$$
\begin{array}{lcl}
S & \rightarrow & \text{quote C CS} \\
CS & \rightarrow & \text{or C CS} \\
 & | & \text{end} \\
C & \rightarrow & \text{give C} \\
 & | & \text{zero}
\end{array}
$$

Translated parsing functions (terminals)

$$
\begin{array}{lll}
\mathit{quote} & & = \mathit{lift}\ (C\ (\lambda c \rightarrow CS\ (\lambda cs \rightarrow c : cs))) \\
\mathit{or} & (CS\ \mathit{ctx}) & = \mathit{lift}\ (C\ (\lambda c \rightarrow CS\ (\lambda cs \rightarrow \mathit{ctx}\ (c : cs)))) \\
\mathit{end} & (CS\ \mathit{ctx}) & = \mathit{ctx}\ [\,] \\
\mathit{give} & (C\ \mathit{ctx}) & = \mathit{lift}\ (C\ (\lambda c \rightarrow \mathit{ctx}\ (\mathit{Give}\ c))) \\
\mathit{zero} & (C\ \mathit{ctx}) & = \mathit{lift}\ (\mathit{ctx}\ \mathit{Zero})
\end{array}
$$

LL(1) grammars

Syntax we want to embed

$$\begin{aligned} contract = {} & quote\ give\ (one\ dollar\ and\ one\ euro) \\ & \quad or \quad give\ zero \\ & end \end{aligned}$$

Abstract syntax

$$\begin{aligned} \textbf{data}\ Contract &= Or \quad Contract\ Contract \\ &\mid And\ Contract\ Contract \\ &\mid Give\ Contract \\ &\mid One\ Currency \\ &\mid Zero \\ \textbf{data}\ Currency &= Euro \mid Dollar \end{aligned}$$

Corresponding grammar

Grammar and equivalent LL(1) grammar

S	$\rightarrow$	AC or S
	$\mid$	AC
AC	$\rightarrow$	C and AC
	$\mid$	C
C	$\rightarrow$	(S)
	$\mid$	give C
	$\mid$	one CU
	$\mid$	zero
CU	$\rightarrow$	euro
	$\mid$	dollar

Corresponding grammar

Grammar and equivalent LL(1) grammar

$$
\begin{array}{lcl c lcll}
\text{S} & \rightarrow & \text{AC or S} & & \text{S} & \rightarrow & \text{AC S'} & \\
 & | & \text{AC} & & \text{S'} & \rightarrow & \text{or AC S'} & | \quad \epsilon \\
\text{AC} & \rightarrow & \text{C and AC} & & \text{AC} & \rightarrow & \text{C AC'} & \\
 & | & \text{C} & \Rightarrow & \text{AC'} & \rightarrow & \text{and C AC'} & | \quad \epsilon \\
\text{C} & \rightarrow & \text{(S)} & & \text{C} & \rightarrow & \text{(S)} & \\
 & | & \text{give C} & & & & \text{give C} & \\
 & | & \text{one CU} & & & \rightarrow & \text{one CU} & \\
 & | & \text{zero} & & & | & \text{zero} & \\
\text{CU} & \rightarrow & \text{euro} & & \text{CU} & \rightarrow & \text{euro} & \\
 & | & \text{dollar} & & & | & \text{dollar} &
\end{array}
$$

Architecture of an LL parser

The parsing table

	S	AC	C	CU
give	S $\rightarrow$AC S'	AC$\rightarrow$C AC'	C$\rightarrow$give C	
one	S $\rightarrow$AC S'	AC$\rightarrow$C AC'	C$\rightarrow$one CU	
euro				CU$\rightarrow$euro

Architecture of an LL parser

The parsing table

	S	AC	C	CU
give	S →AC S'	AC→C AC'	C→give C	
one	S →AC S'	AC→C AC'	C→one CU	
euro				CU→euro

The parse process

Step	Stack	Input buffer
1	S, end	give one euro
2	AC, S', end	give one euro
3	C, AC', S', end	give one euro
4	give, C, AC', S', end	give one euro
5	C, AC', S', end	one euro
n	end	end

Translation to Haskell

The state

- The state is a stack of pending symbols
- Represent a symbol by a data type
- *quote* pushes the start symbol on the stack
- $\mathit{quote} = \mathit{lift}\ (S\ (\lambda x \rightarrow x))$

Translation to Haskell

The state

- The state is a stack of pending symbols
- Represent a symbol by a data type
- *quote* pushes the start symbol on the stack
- $quote = lift\ (S\ (\lambda x \rightarrow x))$

Some of the data types

For the terminal *give*: **newtype** $G\ a = G\ a$
For the non-terminals S, AC, and C

$$
\begin{array}{lll}
\textbf{newtype}\ S & a = S & (Contract \rightarrow a) \\
\textbf{newtype}\ AC & a = AC & (Contract \rightarrow a) \\
\textbf{newtype}\ C & a = C & (Contract \rightarrow a)
\end{array}
$$

Translation to Haskell

Terminals must both

- Encode the parsing of a terminal
- Encode the expansion rules of the parsing table
- Overloading!

Translation to Haskell

Terminals must both

- Encode the parsing of a terminal
- Encode the expansion rules of the parsing table
- Overloading!

Recognition of *give*

Introduce a class for *give*

$$\begin{array}{l}\textbf{class}\ \mathit{Give}\ \mathit{old}\ \mathit{new} \mid \mathit{old} \rightarrow \mathit{new}\ \textbf{where}\\ \quad \mathit{give} :: \mathit{old} \rightarrow \mathit{CPS}\ \mathit{new}\end{array}$$

And an instance, parsing *give*

$$\begin{array}{l}\textbf{instance}\ \mathit{Give}\ (G\ a)\ a\ \textbf{where}\\ \quad \mathit{give}\ (G\ \mathit{ctx}) = \mathit{lift}\ \mathit{ctx}\end{array}$$

Encoding of the expansion rules

Entries in the parsing table

	S	AC	C
give	S $\rightarrow$AC S'	AC$\rightarrow$C AC'	C$\rightarrow$give C

Translated instance declarations

$$
\begin{array}{l}
\textbf{instance } \mathit{Give}\ (S\ a)\quad (C\ (AC'\ (S'\ a)))\ \textbf{where} \\
\quad \mathit{give}\ (S\ \mathit{ctx})\quad = \mathit{give}\ (AC\ (\lambda t \rightarrow S'\ (\lambda e' \rightarrow \mathit{ctx}\ (e'\ t)))) \\
\textbf{instance } \mathit{Give}\ (AC\ a)\ (C\ (AC'\ a))\ \textbf{where} \\
\quad \mathit{give}\ (AC\ \mathit{ctx}) = \mathit{give}\ (C\ (\lambda f \rightarrow AC'\ (\lambda t' \rightarrow \mathit{ctx}\ (t'\ f)))) \\
\textbf{instance } \mathit{Give}\ (C\ a)\quad (C\ a)\ \textbf{where} \\
\quad \mathit{give}\ (C\ \mathit{ctx})\quad = \mathit{give}\ (G\ (C\ (\lambda c \rightarrow \mathit{ctx}\ (\mathit{Give}\ c))))
\end{array}
$$

Conclusion

What we have done

- Developed a postfix framework
- Embedded postfix and prefix notation
- Embedded languages described by LL(1) and GNF grammars
- Type-checker guarantees correct syntax embedding

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Practical concerns

- Type-error messages are unreadable
- Haskell compilers are not designed for this
- Lexical syntax of functions limits the concrete syntax