Remarks

rial in this chapter is covered in most introductory textbooks to prob-
he early classical texts by DeGroot (1975), Subrahmaniam (1979), and
hly accessible introductions into this material. More advanced treat-
ller 1968; Casella and Berger 1990; Tanner 1996), and in (Devroye et al.
he robot environment interaction paradigm is common in robotics. It
erspective by Russell and Norvig (2002).

e sensor that can measure ranges from $0m$ and $3m$. For
hat actual ranges are distributed uniformly in this in-
ly, the sensor can be faulty. When the sensor is faulty,
s a range below $1m$, regardless of the actual range in
ement cone. We know that the prior probability for a
$p = 0.01$.

ueried its sensor N times, and every single time the
is below $1m$. What is the posterior probability of a
$= 1, 2, \ldots, 10$. Formulate the corresponding proba-

place where days are either sunny, cloudy, or rainy.
function is a Markov chain with the following tran-

		tomorrow will be...		
		sunny	cloudy	rainy
	sunny	.8	.2	0
...	cloudy	.4	.4	.2
	rainy	.2	.6	.2

nny day. What is the probability of the following
y2 = *cloudy*, Day3 = *cloudy*, Day4 = *rainy*?

can randomly generate sequences of "weathers"
on function.

o determine the stationary distribution of this
tationary distribution measures the probability
l be sunny, cloudy, or rainy.

d-form solution to calculating the stationary dis-
state transition matrix above?

(e) What is the entropy of the stationary distribution?

(f) Using Bayes rule, compute the probability table of yesterday's weather given today's weather. (It is okay to provide the probabilities numerically, and it is also okay to rely on results from previous questions in this exercise.)

(g) Suppose we added seasons to our model. The state transition function above would only apply to the Summer, whereas different ones would apply to Winter, Spring, and Fall. Would this violate the Markov property of this process? Explain your answer.

3. Suppose that we cannot observe the weather directly, but instead rely on a sensor. The problem is that our sensor is noisy. Its measurements are governed by the following measurement model:

		our sensor tells us...		
		sunny	cloudy	rainy
the actual weather is...	sunny	.6	.4	0
	cloudy	.3	.7	0
	rainy	0	0	1

(a) Suppose Day 1 is sunny (this is known for a fact), and in the subsequent four days our sensor observes *cloudy, cloudy, rainy, sunny*. What is the probability that Day 5 is indeed sunny as predicted by our sensor?

(b) Once again, suppose Day 1 is known to be sunny. At Days 2 through 4, the sensor measures *sunny, sunny, rainy*. For each of the Days 2 through 4, what is the most likely weather on that day? Answer the question in two ways: one in which only the data available to the day in question is used, and one in hindsight, where data from future days is also available.

(c) Consider the same situation (Day 1 is sunny, the measurements for Days 2, 3, and 4 are *sunny, sunny, rainy*). What is the most likely sequence of weather for Days 2 through 4? What is the probability of this most likely sequence?

4. In this exercise we will apply Bayes rule to Gaussians. Suppose we are a mobile robot who lives on a long straight road. Our location x will simply be the position along this road. Now suppose that initially, we believe to be at location $x_{\text{init}} = 1{,}000m$, but we happen to know that this estimate

2.7 Bibliographical Remarks

The basic statistical material in this chapter is covered in most introductory textbooks to probability and statistics. Some early classical texts by DeGroot (1975), Subrahmaniam (1979), and Thorp (1966) provide highly accessible introductions into this material. More advanced treatments can be found in (Feller 1968; Casella and Berger 1990; Tanner 1996), and in (Devroye et al. 1996; Duda et al. 2000). The robot environment interaction paradigm is common in robotics. It is discussed from the AI perspective by Russell and Norvig (2002).

2.8 Exercises

1. A robot uses a range sensor that can measure ranges from $0m$ and $3m$. For simplicity, assume that actual ranges are distributed uniformly in this interval. Unfortunately, the sensor can be faulty. When the sensor is faulty, it constantly outputs a range below $1m$, regardless of the actual range in the sensor's measurement cone. We know that the prior probability for a sensor to be faulty is $p = 0.01$.

   Suppose the robot queried its sensor N times, and every single time the measurement value is below $1m$. What is the posterior probability of a sensor fault, for $N = 1, 2, \ldots, 10$. Formulate the corresponding probabilistic model.

2. Suppose we live at a place where days are either sunny, cloudy, or rainy. The weather transition function is a Markov chain with the following transition table:

		tomorrow will be...		
		sunny	cloudy	rainy
today it's...	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2

   (a) Suppose Day 1 is a sunny day. What is the probability of the following sequence of days: Day2 = *cloudy*, Day3 = *cloudy*, Day4 = *rainy*?

   (b) Write a simulator that can randomly generate sequences of "weathers" from this state transition function.

   (c) Use your simulator to determine the stationary distribution of this Markov chain. The stationary distribution measures the probability that a random day will be sunny, cloudy, or rainy.

   (d) Can you devise a closed-form solution to calculating the stationary distribution based on the state transition matrix above?