Trigonometric integrals (7.2)

Integral	$u =$	$du =$	Identity to use	Works when
$\int \sin^m x \cos^n x \, dx$	$\sin x$	$\cos x \, dx$	$\sin^2 x + \cos^2 x = 1$	n is odd
	$\cos x$	$-\sin x \, dx$		m is odd
$\int \tan^m x \sec^n x \, dx$	$\tan x$	$\sec^2 x \, dx$	$\tan^2 x + 1 = \sec^2 x$	n is even
	$\sec x$	$\sec x \tan x \, dx$		m is odd

If both $\sin x$ and $\cos x$ appear with even exponents, use

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$	$\cos^2 x = \dfrac{1 + \cos 2x}{2}$	$\sin x \cos x = \dfrac{\sin 2x}{2}$

Trigonometric substitutions (7.3)

Integral has	$x =$	$\sqrt{\cdots} =$	$dx =$
$\sqrt{a^2 - x^2}$	$a \sin\theta$	$a \cos\theta$	$a \cos\theta \, d\theta$
$\sqrt{a^2 + x^2}$	$a \tan\theta$	$a \sec\theta$	$a \sec^2\theta \, d\theta$
$\sqrt{x^2 - a^2}$	$a \sec\theta$	$a \tan\theta$	$a \sec\theta \tan\theta \, d\theta$

If $\sqrt{\cdots}$ is multiplied by x to an odd power, use $u = \cdots$ instead of trig sub.

How to complete the square

- Given: $x^2 + 6x + 5$
- Let b be the coefficient of x divided by 2. In this example $b = 6/2 = 3$.
- Replace the two terms with x by $(x + b)^2 - b^2$. Here I replace $x^2 + 6x$ with $(x + 3)^2 - 9$.
- The result: $(x + 3)^2 - 9 + 5$, which simplifies to $(x + 3)^2 - 4$. The square is complete!
- *(if given $3x^2 + 18x + 15$, factor out 3 to get $3(x^2 + 6x + 5)$, then proceed as above)*

If this was a trig substitution problem, you would sub $x + 3 = 2\sec\theta$.

In a partial fraction problem you want to know if the quadratic is reducible or not. If after completing the square you have a sum, then the quadratic is irreducible. If you have a difference, like $(x + 3)^2 - 4$ in my example, then it's reducible. Use the formula $a^2 - b^2 = (a - b)(a + b)$ to reduce it: $(x + 3)^2 - 4 = (x + 3 - 2)(x + 3 + 2) = (x + 1)(x + 5)$.