Practice Exam 2

1. A point has coordinates $x = -2$ and $y = -2$. Find its polar coordinates r and θ.

2. (a) Eliminate the parameter to find a Cartesian equation of the curve $x = 1 - t^2$, $y = 2t$.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

3. Find the length of the curve $y = \sqrt{x^3}$ from the point $(0, 0)$ to the point $(4, 8)$.

4. Find the surface area obtained by rotating the curve $y = 4 - x^2$, $0 \le x \le 2$, around the y-axis.

5. Use the Comparison Theorem to determine whether the integral is convergent or divergent:
(a) $\displaystyle\int_2^\infty \frac{x+3}{x^2 - \sqrt{x}}\, dx$;
(b) $\displaystyle\int_2^\infty \frac{1}{\sqrt{x^3+3}}\, dx$

6. (a) Set up, but **do not evaluate**, an integral for the length of the curve

$$x = e^{3t}, \quad y = \sqrt{t}, \quad \text{where } 1 \le t \le 2.$$

(b) Also set up an integral for the surface area obtained by rotating this curve about the y-axis.

7. Find the area of the region that lies inside the polar curve $r = 3$ and outside the polar curve $r = 3 - \cos\theta$.

8. Evaluate the improper integral, if it is convergent.
(a) $\displaystyle\int_1^\infty xe^{-x^2}\, dx$;
(b) $\displaystyle\int_0^1 x^2 \ln x\, dx$.

9. Find an equation of the tangent line to the parametric curve

$$x = 4t - \sin(2t), \quad y = 1 - \cos(2t)$$

at the point where $t = \pi/4$.

10. Find a Cartesian equation for the polar curve $r = 2\sin\theta + 2\cos\theta$, and identify this curve.

11. Sketch the polar curve $r = \dfrac{1}{1 + \cos\theta}$.