MAT 513 Spring 2010. Final Exam: list of topics.

The number(s) in parentheses indicate the relevant chapter(s).

Theoretical knowledge

- Algebraic properties of complex numbers, including $\operatorname{Re} z$, $\operatorname{Im} z$, $\bar{z}$, $|z|$. Triangle inequality and reverse triangle inequality. (1)
- Topology of sets: interior, boundary, exterior. Limit points. Open, closed, connected sets. Continuity. (2)
- Power series, radius of convergence. (3)
- Relation between $\mathbb{C}$-differentiability (p.64), $\mathbb{R}$-differentiability (multivariable calculus), and partial derivatives. Cauchy-Riemann equations. Differentiation of power series. (4)
- Definition and properties of exp, sin, cos. Properties of log and arg, multivaluedness, branches. (5, 7.1-2-3)
- Integration: Fundamental Theorem, Estimation Lemma. (6)
- Cauchy's theorem: $\int_\gamma f = 0$, where γ is a closed contour in a star-shaped domain Ω, and f is differentiable in Ω. (8)
- Taylor series of a differentiable function, formula for coefficients, Cauchy's estimate. (10)
- Maximum principle, a.k.a. Maximum Modulus theorem. (10)
- Taylor series at a zero of f, identity theorem (10).
- Laurent series, formula for coefficients, classification of isolated singularities and behavior of f near them. (11)
- Residue theorem and applications: evaluation of integrals, Rouche's theorem. (12)

Procedural knowledge

- Verify identities and inequalities involving complex numbers.
- Determine the topological properties of a given subset of $\mathbb{C}$.
- Find the radius (radii) of convergence of a given power series (Taylor or Laurent), either by Ratio Test or by analysis of singularities of the function it represents.
- Determine the $\mathbb{C}$-differentiability of a given function.
- Evaluate a path integral: by definition, by the Fundamental Theorem (i.e., using an anti-derivative), or using Cauchy's theorem.
- Estimate a path integral using the Estimation Lemma.
- Represent a given function by a power series (Taylor or Laurent).
- Find and classify the singularities of a function.
- Compute residues and use them for evaluation of improper integrals.
- Use Rouche's theorem to find the number of zeroes of a given function.