# Circular Arc - Constant Stress Block

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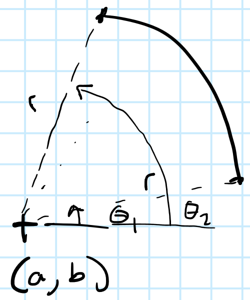

$x = a + r\cos\theta$

$y = b + r\sin\theta$

PARAMETRIC $\theta$

$\theta(t) = \theta_1 + t(\theta_2 - \theta_1)$, $0 \le t \le 1$

GREENS THEOREM:
$$\int P\,dx + Q\,dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$dx = -r(\theta_2 - \theta_1)\sin(\theta(t))\,dt$$
$$dy = r(\theta_2 - \theta_1)\cos(\theta(t))\,dt$$

CONSTANT STRESS:

$P = \int f\,dA \Rightarrow$ $Q = fx$, $P = 0$ $\Rightarrow \frac{\partial Q}{\partial x} = f$, $\frac{\partial P}{\partial y} = 0$ $\quad f - 0 = f \;\therefore$ USE $Q = f \cdot x$

$= \int f x\,dy \rightarrow$ SUB. PARAMETRIC

$$\int_0^1 f\,(a + r\cos(\theta(t)))\cdot(r(\theta_2 - \theta_1)\cos(\theta(t))\,dt$$

FROM MAXIMA: $\frac{1}{4} f r \left(r\sin(2\cdot\theta_2) + 4\cdot a\cdot\sin(\theta_2) + 2\cdot r\cdot\theta_2 - r\cdot\sin(2\cdot\theta_1) - 4\cdot a\cdot\sin(\theta_1) - 2\cdot r\cdot\theta_1\right) = P$

CHECK $a,b=0$; $f=1$

| | | |
|---|---|---|
| $-\pi \rightarrow \pi$ | YIELDS | $\pi r^2$ ✓ |
| $0 \rightarrow \pi$ | | $\frac{1}{2}\pi r^2$ ✓ |
| $0 \rightarrow 2\pi$ | | $\pi r^2$ ✓ |
| $-\pi \rightarrow 0$ | | $\frac{1}{2}\pi r^2$ ✓ |

$M_x = \int f\cdot y\,dA \Rightarrow$ $Q = fyx$, $P = 0$ $\rightarrow \frac{\partial Q}{\partial x} = fy$, $\frac{\partial P}{\partial y} = 0$ $\;\therefore$ EQUIV.

$$\int f\,x\,y\,dy$$

SUB PARAMETRIC $\rightarrow \int_0^1 f\,\underbrace{(a + r\cos(\theta(t)))}_{x}\,\underbrace{(b + r\sin(\theta(t)))}_{y}\,\underbrace{(r(\theta_2 - \theta_1)\cos(\theta(t)))\,dt}_{dy}$

FROM MAXIMA: $-\frac{fr}{12}\Big[r^2\cos(3\theta_2) - 3br\sin(2\theta_2) + 3ar\cos(2\theta_2)$
$- 12ab\sin(\theta_2) + 3r^2\cos(\theta_2) - 6br\theta_2$
$- r^2\cos(3\theta_1) + 3br\sin(2\theta_1)$
$- 3ar\cos(2\theta_1) + 12ab\sin(\theta_1) - 3r^2\cos(\theta_1)$
$+ 6br\theta_1\Big] = M_x$

CHECK $a,b=0$ $f=1$

| | |
|---|---|
| $-\pi \rightarrow \pi$ | $0$ ✓ |
| $-\pi \rightarrow 0$ | $-\frac{2}{3}R^3$ ✓ |
| $0 \rightarrow \pi$ | $\frac{2}{3}R^3$ ✓ |
| $0 \rightarrow 2\pi$ | $0$ ✓ |

NOTE FOR PROPER SIGN CONV.

$M_y = -\int f x\,dA \Rightarrow$ $Q = 0$, $P = fxy$ $\quad \frac{\partial Q}{\partial x} = 0$, $\frac{\partial P}{\partial y} = fx$ $\Rightarrow (0 - fx) = -fx \;\therefore$ EQUIV.

$$\int fxy\,dx$$

SUB PARAMETRIC $\rightarrow \int_0^1 f\,\underbrace{(a + \cos(\theta(t)))}_{x}\,\underbrace{(b + \sin(\theta(t)))}_{y}\,\underbrace{(-r(\theta_2 - \theta_1)\sin(\theta(t)))\,dt}_{dx}$

$\frac{fr}{12}\Big[r^2\sin(3\theta_2) + 3ar\sin(2\theta_2) + 3br\cos(2\theta_2) - 3r^2\sin(\theta_2) + 12ab\cos(\theta_2) - 6ar\theta_2$
$- r^2\sin(3\theta_1) - 3ar\sin(2\theta_1) - 3br\cos(2\theta_1) + 3r^2\sin(\theta_1) - 12ab\cos(\theta_1) + 6ar\theta_1\Big] = M_y$