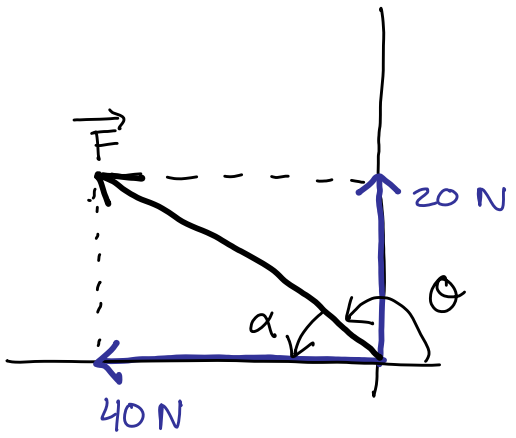
11.2 #9

(a)



Let $\vec{F}$ be the resultant force and θ the angle it makes with the positive x-axis.

$$\|\vec{F}\| = \sqrt{40^2+20^2} = \sqrt{2000} = 20\sqrt{5}$$

$$\tan\alpha = \frac{20}{40} = \frac{1}{2} \Rightarrow \alpha = \tan^{-1}\left(\tfrac{1}{2}\right)$$

So $\theta = \pi - \tan^{-1}\left(\tfrac{1}{2}\right)$

Another way to find θ:

$$\vec{F} = \|\vec{F}\|\langle\cos\theta, \sin\theta\rangle \iff \langle -40, 20\rangle = 20\sqrt{5}\langle\cos\theta, \sin\theta\rangle$$

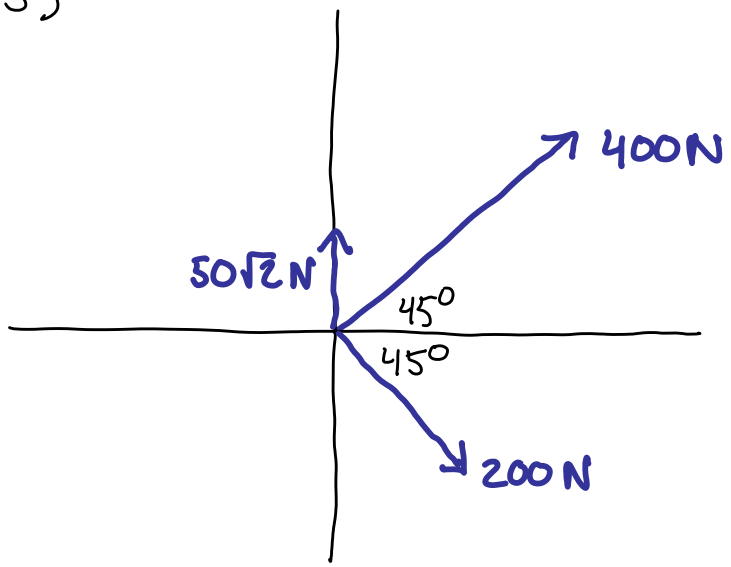
$$\iff -40 = 20\sqrt{5}\cos\theta$$

$$\iff \cos\theta = -\frac{2}{\sqrt{5}}$$

$$\iff \theta = \cos^{-1}\left(-\frac{2}{\sqrt{5}}\right)$$

[Note that we can write $\theta = \pi - \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ as well.]

(b)



Let $\vec{F}$ be the resultant force and θ the angle it makes with the positive x-axis.

$$\vec{F} = \langle 0, 50\sqrt{2} \rangle + 400 \langle \cos 45°, \sin 45° \rangle + 200 \langle \cos(-45°), \sin(-45°) \rangle$$

$$= \langle 0, 50\sqrt{2} \rangle + 400 \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle + 200 \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$$

$$= \langle 300\sqrt{2}, 150\sqrt{2} \rangle$$

So $\|\vec{F}\| = \sqrt{(300\sqrt{2})^2 + (150\sqrt{2})^2} = \sqrt{180{,}000 + 45{,}000} = \sqrt{225{,}000} = 150\sqrt{10}$

Now $\vec{F} = \|\vec{F}\| \langle \cos\theta, \sin\theta \rangle$

so $\cos\theta = \frac{300\sqrt{2}}{150\sqrt{10}} = \frac{2}{\sqrt{5}} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Note: $\theta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ or $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ as well.