Polar Coordinates: Tangent Lines, Arc Length, & Area

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 10.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Know how to compute the slope of the tangent line to a polar curve at a given point.
- Be able to find the arc length of a polar curve.
- Be able to Calculate the area enclosed by a polar curve or curves.

PRACTICE PROBLEMS:

For problems 1-3, find the slope of the tangent line to the polar curve for the given value of θ.

1. $r = \theta$; $\theta = \dfrac{\pi}{6}$

   $\boxed{\dfrac{\sqrt{3}\pi + 6}{6\sqrt{3} - \pi}}$

2. $r = 3 + 2\sin\theta$; $\theta = \dfrac{\pi}{6}$

   $\boxed{-5\sqrt{3}\text{; Detailed Solution: Here}}$

3. $r = 1 - \sin 2\theta$; $\theta = \pi$

   $\boxed{-\dfrac{1}{2}}$

4. Consider the circle $r = 3\cos\theta$. Find all values of θ in $0 \le \theta < \pi$ for which the curve has either a horizontal or vertical tangent line.

   Vertical Tangent Lines when $\theta = 0$ and $\theta = \dfrac{\pi}{2}$;
   Horizontal Tangent Lines when $\theta = \dfrac{\pi}{4}$ or $\theta = \dfrac{3\pi}{4}$.

For problems 5-7, fnd the arc length of the given curves

5. The entire circle $r = 4\sin\theta$.

   $\boxed{4\pi}$

6. The spiral $r = e^{-\theta}$ for $\theta \geq 0$.

   $\boxed{\sqrt{2}}$

7. The entire cardioid $r = 1 + \cos\theta$. (Hint: It may be useful to use symmetry and the identity $\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$)

   $\boxed{\text{8; Detailed Solution: Here}}$

For problems 8-16, find the area of each of the specified regions.

8. The region in the 1st quadrant within the circle $r = 3\cos\theta$

   $\boxed{\dfrac{9\pi}{8}}$

9. The region enclosed by the cardioid $r = 3 + 3\sin\theta$

   $\boxed{\dfrac{27\pi}{2}}$

10. The region inside the circle $r = 3$ but outside the cardioid $r = 1 + \cos\theta$

    $\boxed{\dfrac{15\pi}{2}}$

11. The region inside the circle $r = 3$ but outside the cardioid $r = 2 + 2\cos\theta$

    $\boxed{\dfrac{9\sqrt{3}}{2} + 2\pi}$

12. The region outside the circle $r = 3$ but inside the cardioid $r = 2 + 2\cos\theta$

    $\boxed{\dfrac{9\sqrt{3}}{2} - \pi}$

13. The region in common between the two circles $r = 3\sin\theta$ and $r = 3\cos\theta$

    $\boxed{-\dfrac{9}{4} + \dfrac{9\pi}{8}}$

14. The region inside the circle $r = 2$ and to the right of the line $r = \sec\theta$

    $\boxed{\dfrac{4\pi}{3} - \sqrt{3}}$

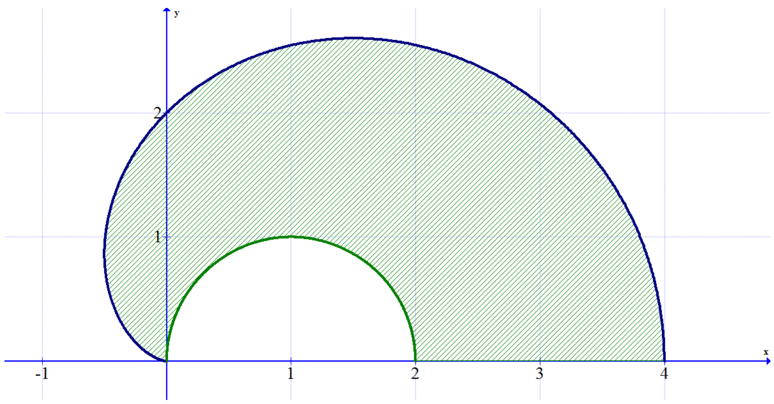
15. The region enclosed by the rose $r = 3\cos 2\theta$

    $\boxed{\dfrac{9\pi}{2}}$

16. The region enclosed by the rose $r = 2\sin 3\theta$

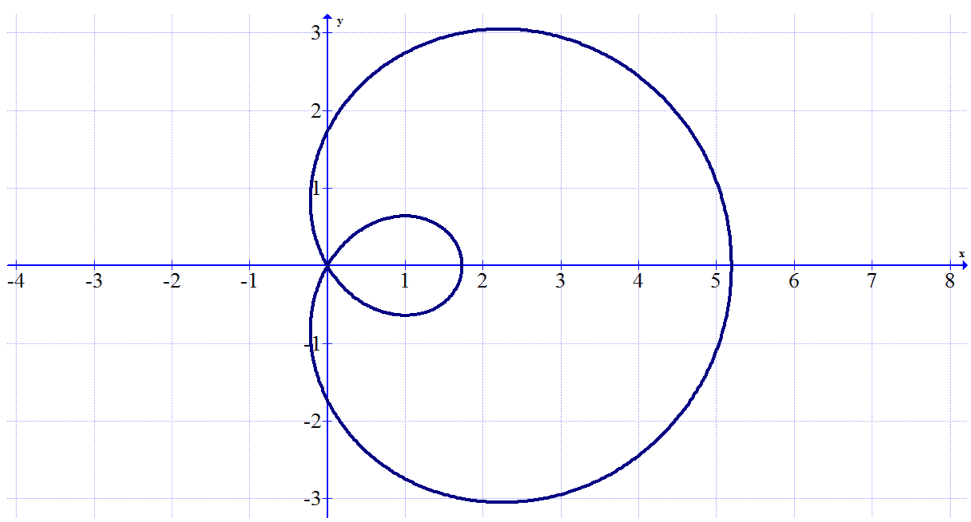
$\boxed{\pi}$

17. Find the area of the shaded region (shown below) which is enclosed between the circle $r = 2\cos\theta$ and the cardioid $r = 2 + 2\cos\theta$.



$\boxed{\frac{5\pi}{2}}$; Detailed Solution: Here

18. Consider the limaçon $r = \sqrt{3} + 2\sqrt{3}\cos\theta$



(a) Compute the area enclosed by the inner loop of the limacon.

$-\frac{9\sqrt{3}}{2} + 3\pi$; Detailed Solution: Here

(b) Compute the area enclosed between the outer and inner loops of the limacon.

$9\sqrt{3} + 3\pi$; Detailed Solution: Here