

Chapter 3.5 Practice Problems

EXPECTED SKILLS:

- Be able to compute the local linear approximation of a function at a specific value.
- Know how to use the local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-4, calculate the Local Linear Approximation, $L(x)$, for the given function at the specified value of x_0 . Also, sketch $f(x)$ and $L(x)$ over the indicated interval

1. $f(x) = x^2 + 1$, $x_0 = 1$, $[0, 2]$
2. $f(x) = \frac{1}{2x - 1}$, $x_0 = 1$, $[0, 2]$
3. $f(x) = e^{-x}$, $x_0 = 0$, $[-1, 1]$
4. $f(x) = \tan(x)$, $x_0 = \frac{\pi}{4}$, $\left[0, \frac{\pi}{2}\right]$

For problems 5-10, use an appropriate local linear approximation to approximate the following values.

5. $(5.05)^3$
6. $\sqrt{101}$
7. $\sqrt[3]{28}$
8. $e^{0.9}$
9. $\cos 0.1$
10. $\sin 61^\circ$
11. Show that $(1 - x)^5 \approx 1 - 5x$ for x near 0.
12. Show that $\ln(2x) \approx 2x - 1$ for x near $\frac{1}{2}$.
13. Consider $f(x) = (x + 1)^{13}$.
 - (a) Compute the Local Linear Approximation of $f(x)$ at $x_0 = 0$.
 - (b) Using your approximation, estimate $(0.99)^{13}$.
 - (c) Using your approximation, estimate $(1.01)^{13}$.

14. Let $f(x) = x^2$.

(a) Calculate the Local Linear Approximation, $L(x)$, for $f(x)$ at $x = a$.

(b) Does $L(x)$ overestimate or underestimate $f(x)$ near $x = a$? Explain.

15. **Multiple Choice:** Which of the following is the best local linear approximation for $f(x) = \tan(x)$ near $x = \frac{\pi}{4}$?

(a) $1 + \left(x - \frac{\pi}{4}\right)$

(b) $1 + \frac{1}{2} \left(x - \frac{\pi}{4}\right)$

(c) $1 + \sqrt{2} \left(x - \frac{\pi}{4}\right)$

(d) $1 + 2 \left(x - \frac{\pi}{4}\right)$

(e) $2 + 2 \left(x - \frac{\pi}{4}\right)$