13.6 #14

$-\nabla f(2,-1,1)$ is a vector in the direction in which f decreases most rapidly at $(2,-1,1)$.

$$\left.\begin{aligned} f_x(x,y,z) &= 3x^2yz^2 \Longrightarrow f_x(2,-1,1) = -12 \\ f_y(x,y,z) &= x^3z^2 \Longrightarrow f_y(2,-1,1) = 8 \\ f_z(x,y,z) &= 2x^3yz \Longrightarrow f_z(2,-1,1) = -16 \end{aligned}\right\} \Longrightarrow -\nabla f(2,-1,1) = \langle 12,-8,16\rangle$$

So $\vec{u} = \dfrac{-\nabla f(2,-1,1)}{\|-\nabla f(2,-1,1)\|} = \dfrac{4\langle 3,-2,4\rangle}{4\sqrt{9+4+16}} = \left\langle \dfrac{3}{\sqrt{29}}, \dfrac{-2}{\sqrt{29}}, \dfrac{4}{\sqrt{29}} \right\rangle$

The rate of change in this direction is

$$-\|\nabla f(2,-1,1)\| = -4\sqrt{29}$$