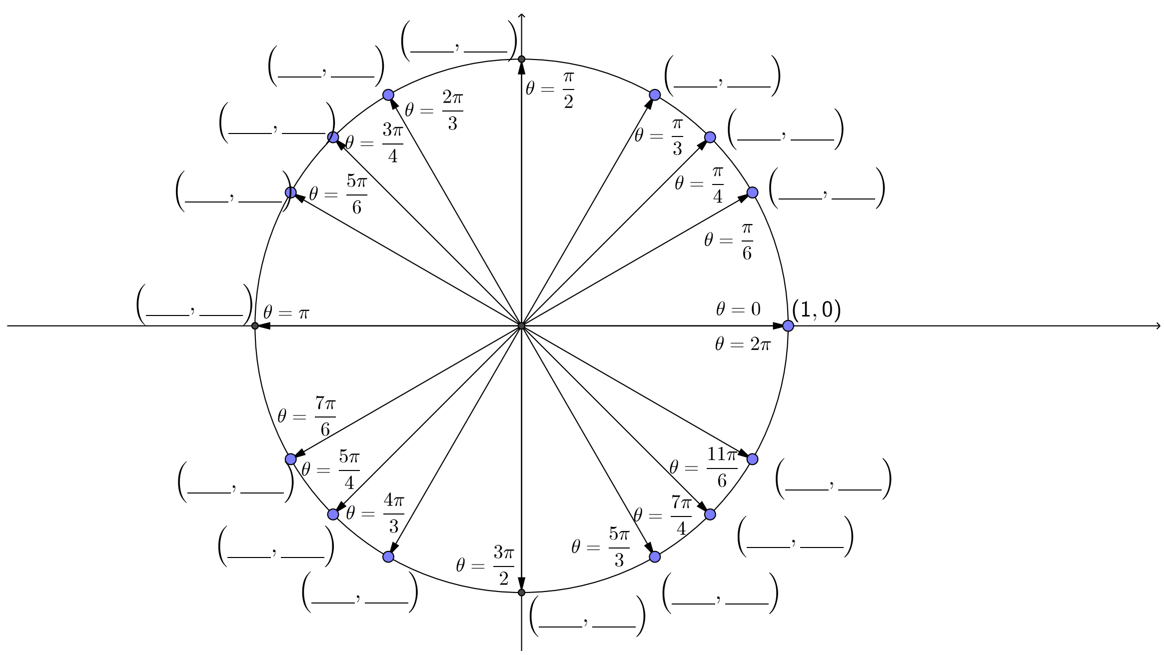
Chapter 3.2: Definition of Trigonometric Functions

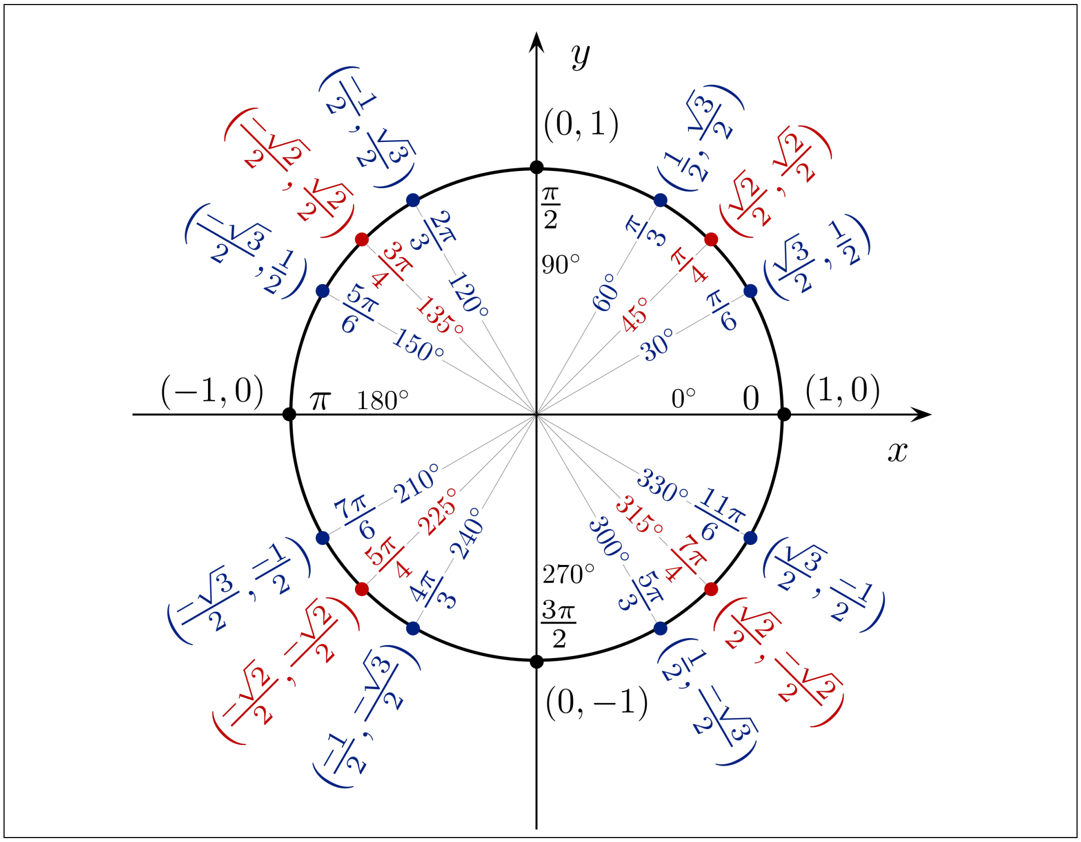
Expected Skills:

- Be able to define $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$.
- Be able to determine the domain and range of the 6 trigonometric functions.
- Be able to evaluate the 6 trig functions (if defined) at the quadrantal angles or angles related to 30°, 45°, and 60°.
- Be able to use the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ and other given information to evaluate all 6 trigonometric functions.

Practice Problems:

1. Label all of the indicated points on the unit circle, shown below. Also, convert all of the angles from radian measurement to degree measurement.





2. Use your results from question (1) to evaluate each of the following without using a calculator.

   (a) $\sin 225^\circ$

   $\boxed{-\dfrac{1}{\sqrt{2}}}$

   (b) $\cos 240^\circ$

   $\boxed{-\dfrac{1}{2}}$

   (c) $\tan 30^\circ$

   $\boxed{\dfrac{1}{\sqrt{3}}}$

   (d) $\sec \dfrac{11\pi}{6}$

   $\boxed{\dfrac{2}{\sqrt{3}}}$

   (e) $\cot \dfrac{\pi}{2}$

   $\boxed{0}$

(f) $\sin\left(-\frac{4\pi}{3}\right)$

$\boxed{\frac{\sqrt{3}}{2}}$

(g) $\csc(-690^\circ)$

$\boxed{2}$

(h) $\cos\frac{23\pi}{3}$

$\boxed{\frac{1}{2}}$

3. Use your results from question (1) to find all solutions in the interval $[0, 2\pi)$ to the following equations.

(a) $\sin\theta = 1$

$\boxed{\theta = \frac{\pi}{2}}$

(b) $\cos\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3} \text{ or } \theta = \frac{5\pi}{3}}$

(c) $\sec\theta = 2$

$\boxed{\theta = \frac{\pi}{3} \text{ or } \theta = \frac{5\pi}{3}}$

(d) $\csc\theta = \sqrt{2}$

$\boxed{\theta = \frac{\pi}{4} \text{ or } \theta = \frac{3\pi}{4}}$

(e) $\tan\theta = 0$

$\boxed{\theta = 0 \text{ or } \theta = \pi}$

4. Repeat question (3) providing all solutions in the interval $[2\pi, 4\pi)$.

(a) $\sin\theta = 1$

$\boxed{\theta = \frac{5\pi}{2}}$

(b) $\cos\theta = \frac{1}{2}$

$\boxed{\theta = \frac{7\pi}{3} \text{ or } \theta = \frac{11\pi}{3}}$

(c) $\sec\theta = 2$

$\boxed{\theta = \dfrac{7\pi}{3} \text{ or } \theta = \dfrac{11\pi}{3}}$

(d) $\csc\theta = \sqrt{2}$

$\boxed{\theta = \dfrac{9\pi}{4} \text{ or } \theta = \dfrac{11\pi}{4}}$

(e) $\tan\theta = 0$

$\boxed{\theta = 2\pi \text{ or } \theta = 3\pi}$

5. Suppose $\sin\theta = \dfrac{5}{13}$ and $\dfrac{\pi}{2} \le \theta \le \pi$. Compute the values of $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$.

$\boxed{\cos\theta = -\dfrac{12}{13},\ \tan\theta = -\dfrac{5}{12},\ \sec\theta = -\dfrac{13}{12},\ \csc\theta = \dfrac{13}{5},\ \text{and } \cot\theta = -\dfrac{12}{5}}$

6. Suppose $\cos\theta = \dfrac{5}{13}$ and $\tan\theta < 0$. Compute the values of $\sin\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$.

$\boxed{\sin\theta = -\dfrac{12}{13},\ \tan\theta = -\dfrac{12}{5},\ \sec\theta = \dfrac{13}{5},\ \csc\theta = -\dfrac{13}{12},\ \text{and } \cot\theta = -\dfrac{5}{12}}$

7. Recall that for any angle θ it follows that $\cos^2\theta + \sin^2\theta = 1$.

(a) Suppose $\theta \neq \pi k$ (where k is an integer). Divide the original identity by $\cos^2\theta$ to derive a trigonometric identity involving tangent and secant.

Suppose $\theta \neq \pi k$ (where k is an integer). Then, it follows that $\cos\theta \neq 0$ and we may divide by $\cos\theta$.

$$\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 & \\
\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} & \text{Divide both sides by } \cos^2\theta \\
1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 &= \left(\frac{1}{\cos\theta}\right)^2 & \\
1 + \tan^2\theta &= \sec^2\theta & \text{By definition of tangent and secant}
\end{aligned}$$

Thus, $1 + \tan^2\theta = \sec^2\theta$.

(b) Suppose $\theta \neq (2k+1)\dfrac{\pi}{2}$ (where k is an integer). Divide the original identity by $\sin^2\theta$ to derive a trigonometric identity involving cotangent and cosecant.

$\boxed{\cot^2\theta + 1 = \csc^2\theta}$

8. Fill in the following table:

	Domain	Range
$\sin\theta$		
$\cos\theta$		
$\tan\theta$		
$\csc\theta$		
$\sec\theta$		
$\cot\theta$		

	Domain	Range
$\sin\theta$	$(-\infty,\infty)$	$[-1,1]$
$\cos\theta$	$(-\infty,\infty)$	$[-1,1]$
$\tan\theta$	$\theta \neq (2k+1)\frac{\pi}{2}$	$(-\infty,\infty)$
$\csc\theta$	$\theta \neq \pi k$	$(-\infty,-1]\cup[1,\infty)$
$\sec\theta$	$\theta \neq (2k+1)\frac{\pi}{2}$	$(-\infty,-1]\cup[1,\infty)$
$\cot\theta$	$\theta \neq \pi k$	$(-\infty,\infty)$

Where k is any integer.

9. For each of the following functions, determine the domain.

(a) $f(\theta) = \dfrac{\theta}{1-\tan\theta}$

$\theta \neq \dfrac{\pi}{4} + \pi k$ and $\theta \neq \dfrac{\pi}{2} + \pi k$ where k is any integer.

(b) $f(\theta) = \sqrt{\sin\theta}$

$$\bigcup_{k=-\infty}^{\infty} [2k\pi, (2k+1)\pi] = \ldots [-2\pi, -\pi] \cup [0,\pi] \cup [2\pi, 3\pi] \ldots$$