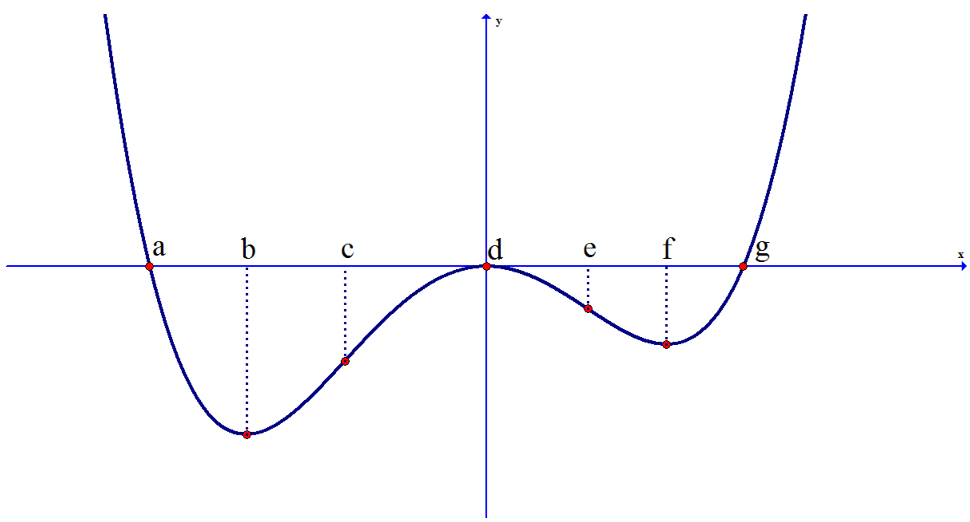
Chapter 4.1 & 4.2 (Part 1) Practice Problems

EXPECTED SKILLS:

- Understand how the signs of the first and second derivatives of a function are related to the behavior of the function.
- Know how to use the first and second derivatives of a function to find intervals on which the function is increasing, decreasing, concave up, and concave down.
- Be able to find the critical points of a function, and apply the First Derivative Test and Second Derivative Test (when appropriate) to determine if the critical points are relative maxima, relative minima, or neither
- Know how to find the locations of inflection points.

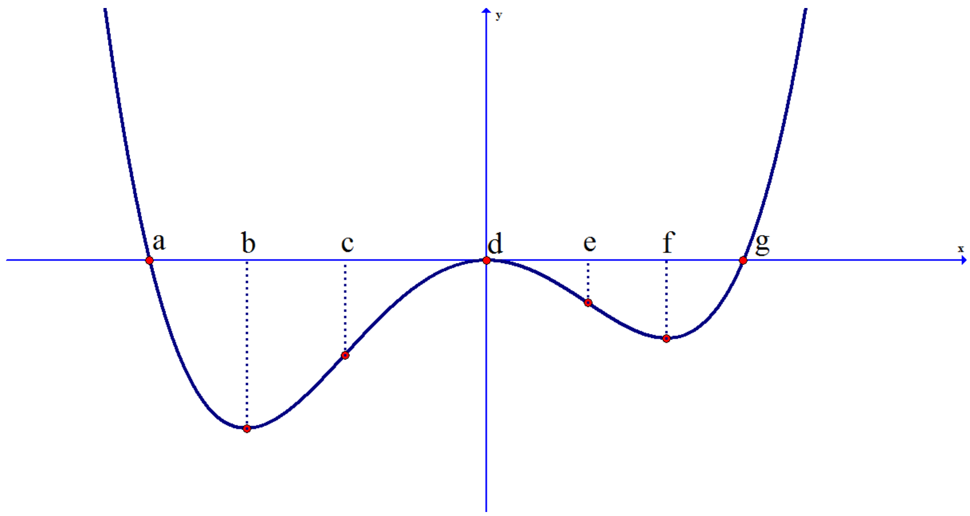
PRACTICE PROBLEMS:

1. Consider the graph of $y = f(x)$, shown below.



   (a) Determine the interval(s) where $f(x)$ is increasing.
   (b) Determine the interval(s) where $f(x)$ is decreasing.
   (c) Determine the interval(s) where $f(x)$ is concave up.
   (d) Determine the interval(s) where $f(x)$ is concave down.
   (e) Determine the value(s) of x where $f(x)$ has relative (local) extrema. Classify each as the location of a relative maximum or a relative minumum.
   (f) Determine the value(s) of x where $f(x)$ has an inflection point.

2. The graph of **the derivative** of $y = f(x)$ is shown below.



(a) Determine the interval(s) where $f(x)$ is increasing.

(b) Determine the interval(s) where $f(x)$ is decreasing.

(c) Determine the interval(s) where $f(x)$ is concave up.

(d) Determine the interval(s) where $f(x)$ is concave down.

(e) Determine the value(s) of x where $f(x)$ has relative (local) extrema. Classify each as the location of a relative maximum or a relative minumum.

(f) Determine the value(s) of x where $f(x)$ has an inflection point.

3. Sketch the graph of a continuous function, $y = f(x)$, which is decreasing on $(-\infty, \infty)$, has an inflection point at $x = 1$, and is concave down on $(1, \infty)$.

4. Sketch the graph of a continuous function, $y = f(x)$, which is decreasing on $(-\infty, 1)$, has a relative minimum at $x = 1$, and does not have any inflection points.

5. Sketch the graph of a continuous function $y = f(x)$ which satisfies all of the following conditions:

- Domain of $f(x)$ is $(-\infty, \infty)$
- $f(-1) = -2$, $f(0) = f(7) = 3$, and $f(5) = 9$
- $f'(x) < 0$ on $(-\infty, -1) \cup (5, 7)$ and $f'(x) > 0$ on $(-1, 0) \cup (0, 5) \cup (7, \infty)$
- $f''(x) < 0$ on $(0, 7) \cup (7, \infty)$ and $f''(x) > 0$ on $(-\infty, 0)$

6. Consider the function that you sketched in question 5. At which value(s) of x must $f'(x) = 0$? At which value(s) of x must $f'(x)$ fail to exist?

For problems 7-15, calculate each of the following:

(a) The intervals on which $f(x)$ is increasing
(b) The intervals on which $f(x)$ is decreasing
(c) The intervals on which $f(x)$ is concave up
(d) The intervals on which $f(x)$ is concave down
(e) All points of inflection. Express each as an ordered pair (x, y)

7. $f(x) = x^3 - 2x + 3$
8. $f(x) = \dfrac{x}{x-2}$
9. $f(x) = \sin x$ on $[0, 2\pi]$
10. $f(x) = (4x - 1)^4$
11. $f(x) = xe^x$
12. $f(x) = \arctan(2x)$
13. $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$
14. $f(x) = \dfrac{\ln x}{x}$
15. $f(x) = 2x + 3x^{2/3}$

For problems 16-20, compute the critical points of the given function. Then use the First Derivative Test to determine all relative (local) extrema. Express each extremum as an ordered pair (x, y).

16. $f(x) = x^2 - 16$
17. $f(x) = (2x + 3)^3$
18. $f(x) = \dfrac{3x}{x^2 + 1}$
19. $f(x) = e^x - x$
20. $f(x) = x^3 - x^5$

For problems 21-22, use the Second Derivative Test to determine the relative (local) extrema. Express each as an ordered pair (x, y).

21. $f(x) = \sin(3x)$ on $[0, \pi]$

22. $f(x) = \sec(3x)$ on $[0, \pi]$

For problems 23-27, determine the critical points. Classify each as a relative extremum, relative minimum, or neither. Express all relative extrema as ordered pairs (x, y).

23. $f(x) = \sin^2 x$ on $[0, 2\pi]$
24. $f(x) = \dfrac{x^3}{3} + x^2 + x + 3$
25. $f(x) = xe^x$
26. $f(x) = 2x + 3x^{2/3}$
27. $f(x) = \dfrac{\ln x}{x}$

HINT: For problems 25-27, it may be helpful to use your work from earlier in the assignment.