#6

$$a_n = \frac{e^n - e^{-n}}{e^n + e^{-n}}$$

Note that every term in the sequence is positive since the numerator is $e^n - \frac{1}{e^n} > 0$ and the denominator is $e^n + e^{-n} > 0$.

So $$\frac{a_{n+1}}{a_n} = \frac{e^{n+1} - e^{-n-1}}{e^{n+1} + e^{-n-1}} \cdot \frac{e^n + e^{-n}}{e^n - e^{-n}}$$

$$= \frac{e^{2n+1} + e - e^{-1} - e^{-2n-1}}{e^{2n+1} - e + e^{-1} - e^{-2n-1}}$$

Let $A = e^{2n+1} - e^{-2n-1}$

$$= \frac{A + e - \frac{1}{e}}{A + \frac{1}{e} - e}$$

Since $e - \frac{1}{e} > \frac{1}{e} - e$, the numerator $A + e - \frac{1}{e}$ is greater than the denominator $A + \frac{1}{e} - e$.

Also, $a_{n+1} > 0$ and $a_n > 0$ (see above), so

$$\frac{a_{n+1}}{a_n} > 1 \implies a_{n+1} > a_n.$$

So the sequence is (strictly) increasing.