

Polar Coordinates: Tangent Lines, Arc Length, & Area

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 10.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Know how to compute the slope of the tangent line to a polar curve at a given point.
- Be able to find the arc length of a polar curve.
- Be able to Calculate the area enclosed by a polar curve or curves.

PRACTICE PROBLEMS:

For problems 1-3, find the slope of the tangent line to the polar curve for the given value of θ .

1. $r = \theta$; $\theta = \frac{\pi}{6}$
2. $r = 3 + 2 \sin \theta$; $\theta = \frac{\pi}{6}$
3. $r = 1 - \sin 2\theta$; $\theta = \pi$
4. Consider the circle $r = 3 \cos \theta$. Find all values of θ in $0 \leq \theta < \pi$ for which the curve has either a horizontal or vertical tangent line.

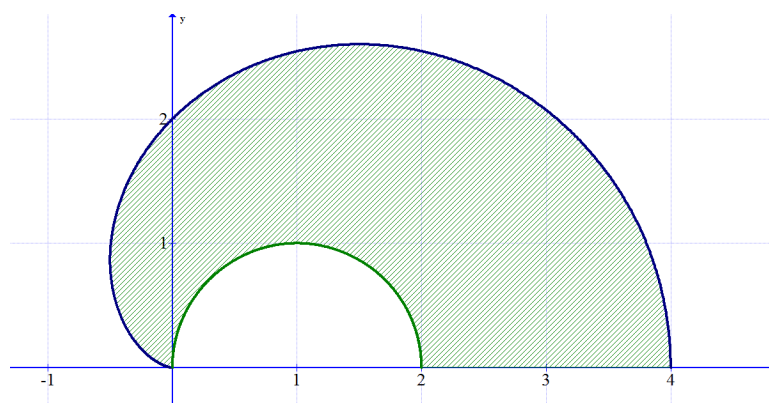
For problems 5-7, find the arc length of the given curves

5. The entire circle $r = 4 \sin \theta$.
6. The spiral $r = e^{-\theta}$ for $\theta \geq 0$.
7. The entire cardioid $r = 1 + \cos \theta$. (Hint: It may be useful to use symmetry and the identity $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$)

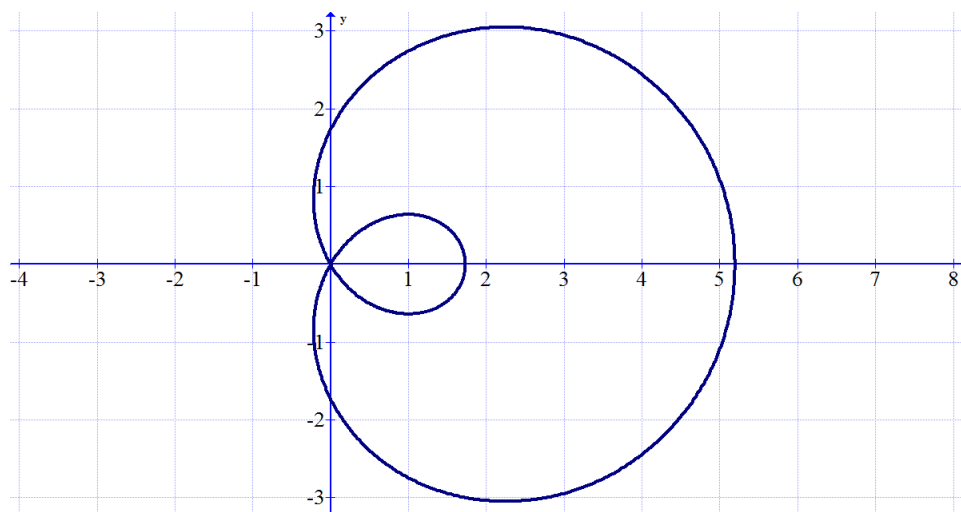
For problems 8-16, find the area of each of the specified regions.

8. The region in the 1st quadrant within the circle $r = 3 \cos \theta$
9. The region enclosed by the cardioid $r = 3 + 3 \sin \theta$
10. The region inside the circle $r = 3$ but outside the cardioid $r = 1 + \cos \theta$

11. The region inside the circle $r = 3$ but outside the cardioid $r = 2 + 2 \cos \theta$
12. The region outside the circle $r = 3$ but inside the cardioid $r = 2 + 2 \cos \theta$
13. The region in common between the two circles $r = 3 \sin \theta$ and $r = 3 \cos \theta$
14. The region inside the circle $r = 2$ and to the right of the line $r = \sec \theta$
15. The region enclosed by the rose $r = 3 \cos 2\theta$
16. The region enclosed by the rose $r = 2 \sin 3\theta$
17. Find the area of the shaded region (shown below) which is enclosed between the circle $r = 2 \cos \theta$ and the cardioid $r = 2 + 2 \cos \theta$.



18. Consider the limaçon $r = \sqrt{3} + 2\sqrt{3} \cos \theta$



- (a) Compute the area enclosed by the inner loop of the limaçon.
- (b) Compute the area enclosed between the outer and inner loops of the limaçon.