7.5 #11

$$\int \frac{1}{x^2(x-1)^2}\,dx$$

$$\frac{1}{x^2(x-1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}$$

$$1 = Ax(x-1)^2 + B(x-1)^2 + Cx^2(x-1) + Dx^2$$

$x=0$: $1 = B$

$x=1$: $1 = D$

$$1 = Ax^3 - 2Ax^2 + Ax + Bx^2 - 2Bx + B + Cx^3 - Cx^2 + Dx^2$$

$$1 = (A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B)x + B$$

① $A+C=0$ ② $-2A+B-C+D=0$ ③ $A-2B=0$

By ③ $A = 2B = 2(1) = 2$

By ① $C = -A = -2$

$$\text{So} \int \frac{1}{x^2(x-1)^2}\,dx$$

$$= \int \frac{2}{x}\,dx + \int \frac{1}{x^2}\,dx + \int \frac{-2}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

$$= 2\ln|x| - \frac{1}{x} - 2\ln|x-1| - \frac{1}{(x-1)} + C$$