

Chapter 5.1: L'Hôpital's Rule & Indeterminate Forms

Expected Skills:

- Know how to use L'Hôpital's Rule to help compute limits involving indeterminate forms of $\frac{0}{0}$ and $\frac{\infty}{\infty}$
- Be able to compute limits involving indeterminate forms $\infty - \infty$, $0 \cdot \infty$, 0^0 , ∞^0 , and 1^∞ by manipulating the limits into a form where L'Hôpital's Rule is applicable.

Practice Problems:

For problems 1-27, calculate the indicated limit. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate). Make sure that L'Hôpital's rule applies before using it. And, whenever you apply L'Hôpital's rule, indicate that you are doing so.

1. $\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{x^2 - 7x + 12}$

2. $\lim_{x \rightarrow 0} \frac{\tan 3x}{\ln(1+x)}$

3. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}$

4. $\lim_{x \rightarrow 0} \frac{\sin(6x)}{\sin(3x)}$

5. $\lim_{x \rightarrow 1^-} \frac{x-1}{x^2 - 2x + 1}$

6. $\lim_{x \rightarrow \infty} \frac{e^{-x}}{x^{-2}}$

7. $\lim_{x \rightarrow 0} \frac{\ln(\cos 3x)}{5x^2}$

8. $\lim_{x \rightarrow 1} \frac{\tan^{-1} x - \frac{\pi}{4}}{x-1}$

9. $\lim_{x \rightarrow 0^+} \frac{8\sqrt{x} - 1}{1 - 5\sqrt{x}}$

10. $\lim_{x \rightarrow 0^+} \frac{5 \sin x}{\sqrt{x}}$

11. $\lim_{x \rightarrow -\infty} \frac{x^3 + 4x - 5}{5x^2 - 5x - 89}$
12. $\lim_{x \rightarrow 0} \frac{\sin^{-1}(2x)}{\tan^{-1}(3x)}$
13. $\lim_{x \rightarrow 1} \frac{\ln x^2}{x^2 - 9}$
14. $\lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$
15. $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\ln(x - \frac{\pi}{2})}{\tan x}$
16. $\lim_{x \rightarrow 1^-} \frac{x - 1}{\arccos x}$
17. $\lim_{x \rightarrow +\infty} \frac{e^{\sqrt{x}}}{x}$
18. $\lim_{x \rightarrow +\infty} x e^{-6x}$
19. $\lim_{x \rightarrow +\infty} \frac{\sqrt{4 + 3x^2}}{2 + 2x}$
20. $\lim_{x \rightarrow 0^+} x \csc 3x$
21. $\lim_{x \rightarrow +\infty} [\ln(x + 2) - \ln(3x + 5)]$
22. $\lim_{x \rightarrow \infty} 3^x 7^{-x}$
23. $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x} - x)$
24. $\lim_{x \rightarrow 0^+} \tan x \sec x$
25. $\lim_{x \rightarrow 0^+} x^{1/x}$
26. $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{5x}$
27. $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{-x}$

28. Which of the following are indeterminate forms?

$$\begin{array}{cccc} \frac{0}{0} & \frac{0}{\infty} & \frac{\infty}{0} & \frac{\infty}{\infty} \\ \infty - \infty & \infty + \infty & 0 \cdot \infty & \infty \cdot \infty \\ 0^0 & \infty^0 & 0^\infty & 1^\infty & \infty^\infty & \infty^1 \end{array}$$

29. Calculate each of the following limits:

(a) $\lim_{x \rightarrow 0^+} (1 + 3^x)^{1/x}$

(b) $\lim_{x \rightarrow 0^-} (1 + 3^x)^{1/x}$

(c) $\lim_{x \rightarrow +\infty} (1 + 3^x)^{1/x}$

(d) $\lim_{x \rightarrow -\infty} (1 + 3^x)^{1/x}$

30. Show that $\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = 0$ for any positive integer n .

31. Find the value(s) of the constant k which make $f(x) = \begin{cases} \frac{\sin x - 1}{x - \frac{\pi}{2}} & \text{if } x \neq \frac{\pi}{2} \\ k & \text{if } x = \frac{\pi}{2} \end{cases}$ continuous at $x = \frac{\pi}{2}$.

32. Find all values of k and m such that $\lim_{x \rightarrow 1} \frac{k + m \ln x}{x - 1} = 5$

33. **Multiple Choice:** What is $\lim_{x \rightarrow 1^+} \frac{x}{\ln(x)}$?

(a) 0

(b) 1

(c) e

(d) e^{-1}

(e) $+\infty$

34. **Multiple Choice:** What is $\lim_{x \rightarrow 0} \frac{e^x - 1}{\tan(x)}$?

- (a) -1
- (b) 0
- (c) 1
- (d) 2
- (e) The limit does not exist.

35. **Multiple Choice:** If $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} g(x) = +\infty$ and $f'(x) = 1$ and $g'(x) = e^x$, what is $\lim_{x \rightarrow +\infty} \frac{f(x)}{g(x)}$?

- (a) -1
- (b) 0
- (c) 1
- (d) e
- (e) The limit does not exist.