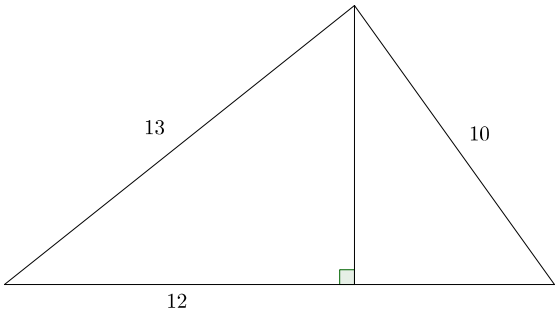
Chapters 2.1-2.3: Pythagorean Theorem, Distance Formula, & Circles

Expected Skills:

- Given the lengths of two sides of a right triangle, be able to use the Pythagorean Theorem to determine the length of the remaining side.
- Be able to calculate the distance between two points in the plane.
- Be able to write an equation of a circle which satisfies some given conditions. Also, be able to identify the center and radius of a circle.
- Be able to find the point on a curve which is closest to or farthest from a given point P.

Practice Problems:

1. For the triangle below, determine the lengths of the two unlabeled sides.



2. Televisions are advertised by the length of the screen's diagonal. A television has a rectangular screen with a height of 36 inches and a length of 64 inches. How should this television be advertised?

3. A ladder of length 25 feet is leaning against a vertical wall. The ladder is initially 7 feet from the wall; but, it is being pushed towards the wall at a constant rate of 2 feet per second. This causes the top of the ladder to slide up the wall.

   (a) How high above the ground is the top of the ladder initially?

   (b) How high above the ground is the top of the ladder after 1 second has elapsed?

4. Let T be the triangle with vertices $A(0,0)$, $B(8,0)$, and $C(4,4\sqrt{3})$.

   (a) Show that T is an equilateral triangle.

(b) Show that the area of an equilateral triangle with sides of length s is $A = \frac{\sqrt{3}}{4}s^2$.

(c) What is the area of T?

5. What is the area of a square inscribed in a unit circle?

6. Find an equation of the circle which has a center of $(3, 5)$ and which has a radius of 6.

7. Suppose $A(1, 5)$ and $B(3, -2)$ are endpoints of a diameter of a circle. Find an equation of this circle.

8. The set of points in the xy-plane which satisfy $x^2 - 2x + y^2 + 10y = -17$ forms a circle.

   (a) What are the center and radius of this circle?

   (b) Does the origin lie inside or outside this circle? Explain.

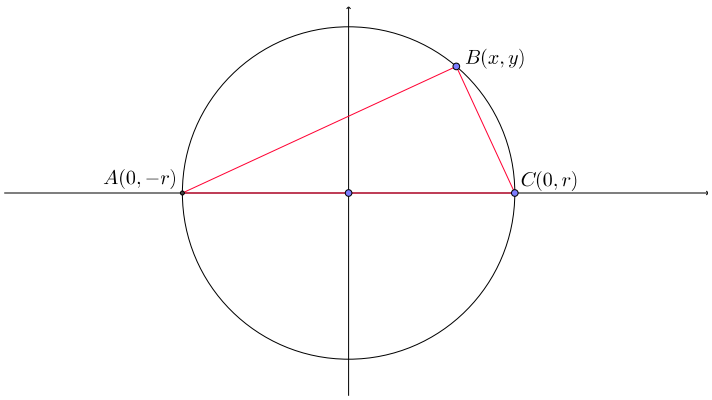
9. Consider the square which is centered at the origin and has sides of length 2 which are parallel to the coordinate axes.

   (a) Find an equation of the circle which is inscribed within this square.

   (b) Find an equation of the circle which is circumscribed around this square.

10. Find equations of the two tangent circles of equal radii which have centers $C_1(-2, 5)$ and $C_2(3, 4)$, respectively.

11. Consider the circle of radius r shown below:



   (a) Calculate d_{AB}, d_{BC}, and d_{AC}.

   (b) Use your result from part (a) to argue that $\triangle ABC$ is a right triangle. (This is Thales' Theorem.)

12. Consider the curve $y = \sqrt{x}$ on $[0, 10]$ and the point $P(9.5, 0)$

    (a) Find the point on the curve which is closest to P.

    (b) Find the point on the curve which is farthest from to P.

13. Consider the curve $y = x^2$ for $0 \leq x \leq 2$ and the point $P(3, 0)$.

    (a) Find the point on the curve which is closest to P.

    (b) Find the point on the curve which is farthest from to P.