Vectors & Vector Arithmetic

SUGGESTED REFERENCE MATERIAL:

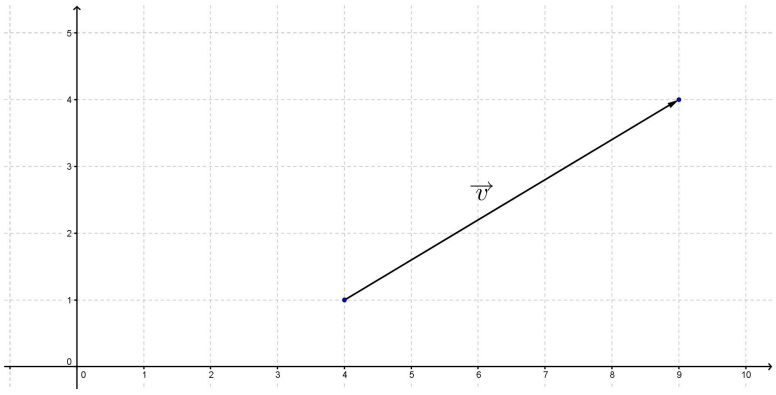
As you work through the problems listed below, you should reference Chapter 11.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

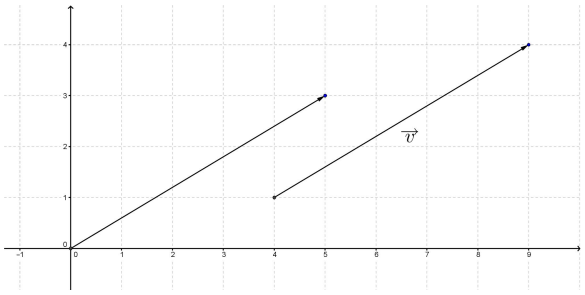
- Be able to perform arithmetic operations on vectors and understand the geometric consequences of the operations.
- Know how to compute the magnitude of a vector and normalize a vector.
- Be able to use vectors in the context of geometry and force problems.

PRACTICE PROBLEMS:

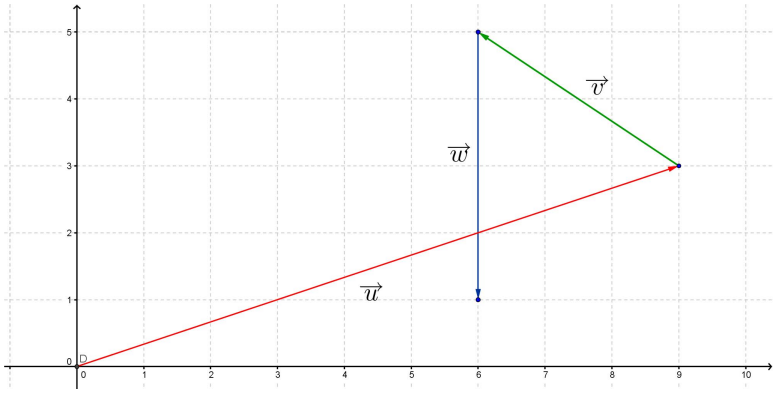
1. Find the components of vector $\overrightarrow{v}$ and sketch an equivalent vector with its initial point at the origin.



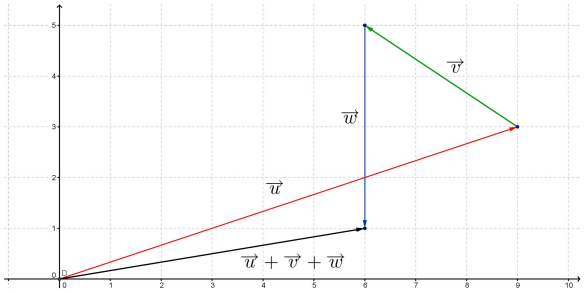
$\overrightarrow{v} = \langle 5, 3 \rangle$



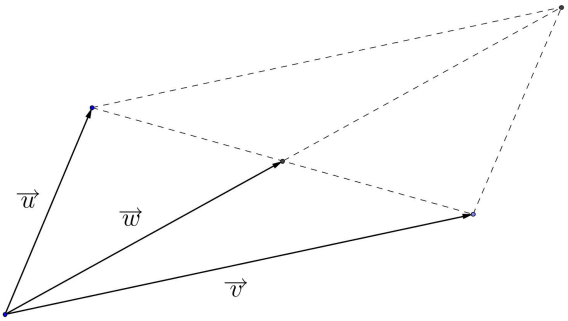
2. Sketch the vector $\overrightarrow{u} + \overrightarrow{v} + \overrightarrow{w}$ and express it in component form.



$$\overrightarrow{u} + \overrightarrow{v} + \overrightarrow{w} = \langle 6, 1 \rangle$$

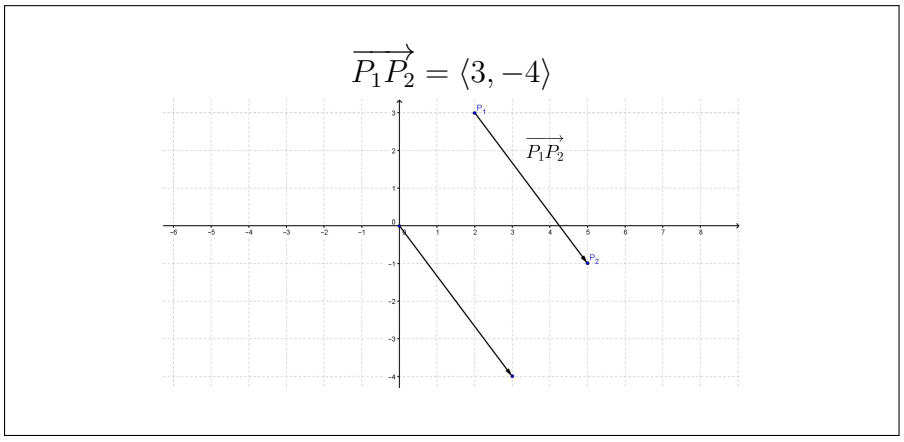


3. The figure below is a parallelogram. Express $\overrightarrow{w}$ in terms of $\overrightarrow{u}$ and $\overrightarrow{v}$.



$$\overrightarrow{w} = \frac{1}{2}\left(\overrightarrow{u} + \overrightarrow{v}\right)$$

4. Consider the points $P_1(2,3)$ and $P_2 = (5,-1)$. Find the components of the vector $\overrightarrow{P_1P_2}$. Sketch P_1, P_2, $\overrightarrow{P_1P_2}$, and an equivalent vector with its initial point at the origin.



5. Consider the points $P_1(1,2,3)$ and $P_2(5,4,6)$. Find the components of the vector $\overrightarrow{P_1P_2}$.

$\boxed{\overrightarrow{P_1P_2} = \langle 4, 2, 3 \rangle}$

6. Let $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, and $\mathbf{w} = 7\mathbf{i} + 4\mathbf{j} + \mathbf{k}$. Compute each of the following:

   (a) $2\mathbf{u} - 3\mathbf{w}$

   $\boxed{-15\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}}$

   (b) $\|\mathbf{u} + \mathbf{v}\|$

   $\boxed{\sqrt{41}}$

   (c) $\|\mathbf{u}\| + \|\mathbf{v}\|$

   $\boxed{\sqrt{14} + \sqrt{29}}$

   (d) $\|2\mathbf{u}\|$

   $\boxed{2\sqrt{14}}$

   (e) $\left\| \dfrac{1}{\|\mathbf{v}\|}\mathbf{v} \right\|$

   $\boxed{1}$

7. For each of the following, find a vector which satisfies the given conditions.

   (a) A unit vector which is in the opposite direction of $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$

   $\boxed{-\dfrac{3}{5}\mathbf{i} - \dfrac{4}{5}\mathbf{j}}$; Detailed Solution: Here

(b) A unit vector which is in the same direction as the vector from $P_1(1,0,5)$ to $P_2(3,-1,2)$

$\boxed{\left\langle \frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}} \right\rangle}$; Video Solution: Here

(c) A vector which is in the opposite direction of $\overrightarrow{v} = \langle 1,2,3 \rangle$ and whose maginitude is half that of $\overrightarrow{v}$.

$\boxed{\left\langle -\frac{1}{2}, -1, -\frac{3}{2} \right\rangle}$; Detailed Solution: Here

(d) A vector which is in the same direction of $\mathbf{w} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and which has a length of $\sqrt{5}$

$\boxed{\frac{\sqrt{5}}{\sqrt{14}}\mathbf{i} - \frac{2\sqrt{5}}{\sqrt{14}}\mathbf{j} + \frac{3\sqrt{5}}{\sqrt{14}}\mathbf{k}}$; Detailed Solution: Here

(e) A vector in 2-space which makes an angle of $\theta = \frac{\pi}{6}$ with the positive x-axis and which has a magnitude of 4.

$\boxed{\left\langle 2\sqrt{3}, 2 \right\rangle}$; Detailed Solution: Here

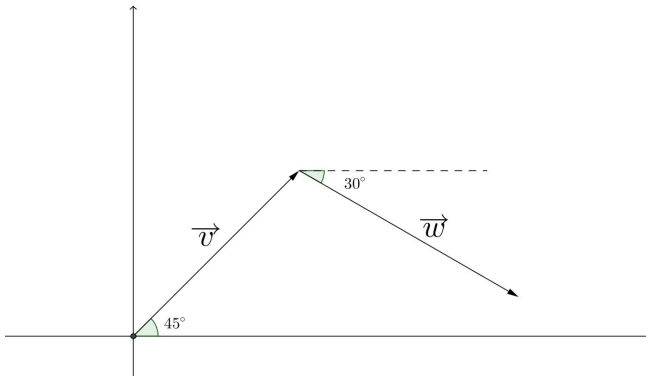
(f) A vector in 2-space which makes an angle of $\theta = 210°$ with the positive x-axis and which has a length of 2.

$\boxed{\left\langle -\sqrt{3}, -1 \right\rangle}$; Detailed Solution: Here

8. Find the value(s) of a so that the vectors $\overrightarrow{v} = \langle a^2, 6 \rangle$ and $\overrightarrow{w} = \langle 4a, 2 \rangle$ are parallel.

$\boxed{a = 0 \text{ or } a = 12}$

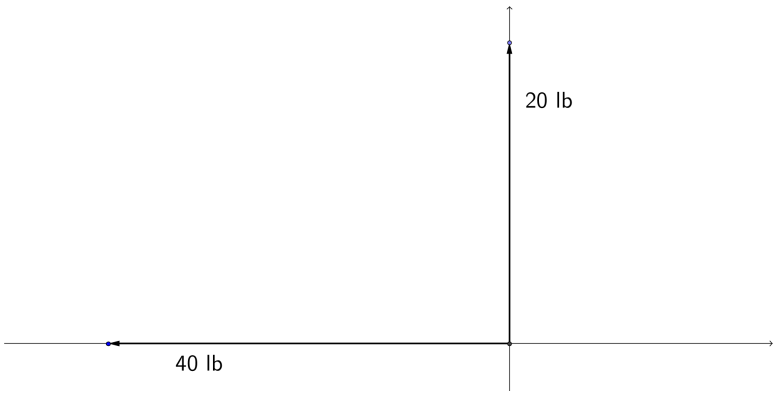
9. Vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, shown below, are unit vectors. Find the components of $\overrightarrow{v} + \overrightarrow{w}$.



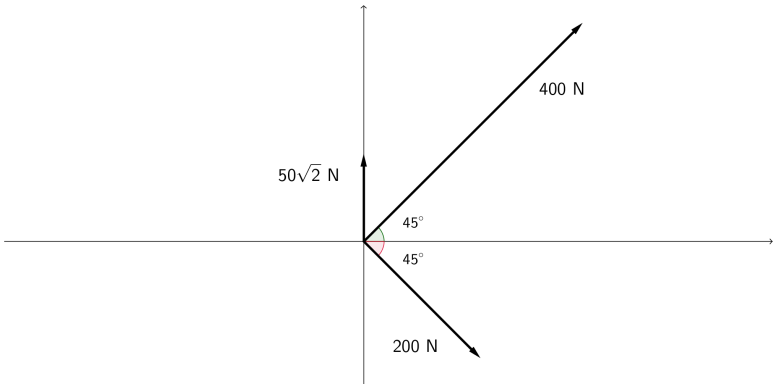
$\boxed{\left\langle \frac{\sqrt{2}+\sqrt{3}}{2}, \frac{\sqrt{2}-1}{2} \right\rangle}$; Detailed Solution: Here

10. For each of the following, find the magnitude of the resultant force and the angle that it makes with the positive x-axis.

(a)



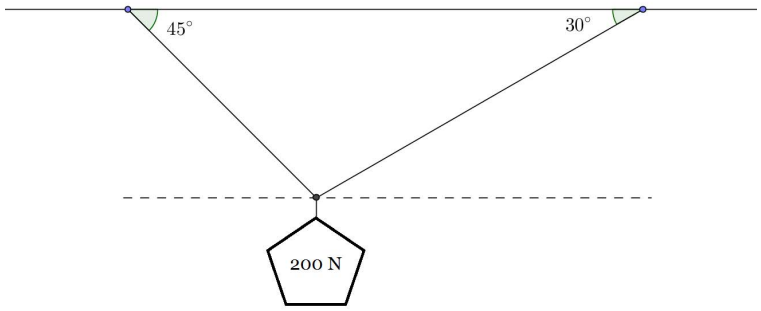
(b)



(a) The magnitude is $20\sqrt{5}$ lb at an angle of $\pi - \tan^{-1}\left(\frac{1}{2}\right)$ radians counterclockwise with the positive x axis.

(b) The magnitude is $150\sqrt{10}$ N at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ radians counterclockwise with the positive x axis.

11. A weight of 200 Newtons (N) is being supported by two wires, as shown below. Find the tension in each wire.



Let F_1 be the wire which makes an angle of 45° clockwise with the ceiling and F_2 be the wire which makes an angle of 30° counterclockwise with the ceiling. Then $\|F_2\| = \dfrac{400}{1+\sqrt{3}}$ N and $\|F_1\| = \dfrac{\sqrt{3}}{\sqrt{2}} \cdot \dfrac{400}{1+\sqrt{3}}$ N.