10.1 #17

$$x = e^t, \quad y = \frac{2}{3}e^{\frac{3t}{2}}, \quad \ln 2 \le t \le \ln 3$$

$$\frac{dx}{dt} = e^t \implies \left(\frac{dx}{dt}\right)^2 = e^{2t}$$

$$\frac{dy}{dt} = e^{\frac{3t}{2}} \implies \left(\frac{dy}{dt}\right)^2 = e^{3t}$$

$$L = \int_{\ln 2}^{\ln 3} \sqrt{e^{2t} + e^{3t}}\, dt = \int_{\ln 2}^{\ln 3} \sqrt{e^{2t}(1+e^t)}\, dt$$

$$= \int_{\ln 2}^{\ln 3} e^t \sqrt{1+e^t}\, dt$$

$$u = 1 + e^t \implies du = e^t dt$$

$$x = \ln 3 \implies u = 1 + 3 = 4$$

$$x = \ln 2 \implies u = 1 + 2 = 3$$

$$= \int_3^4 \sqrt{u}\, du = \frac{2}{3}u^{\frac{3}{2}}\Big|_3^4 = \frac{2}{3}\left(8 - 3\sqrt{3}\right) = \frac{16}{3} - 2\sqrt{3}$$