Quadric Surfaces

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.7 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute & traces of quadic surfaces; in particular, be able to recognize the resulting conic sections in the given plane.
- Given an equation for a quadric surface, be able to recognize the type of surface (and, in particular, its graph).

PRACTICE PROBLEMS:

For problems 1-9, use traces to identify and sketch the given surface in 3-space.

1.
$$4x^2 + y^2 + z^2 = 4$$

$$2. -x^2 - y^2 + z^2 = 1$$

$$3. \ 4x^2 + 9y^2 - 36z^2 = 36$$

4.
$$z = 4x^2 + y^2$$

5.
$$z^2 = x^2 + y^2$$

$$6. \ x^2 + y^2 - z^2 = 1$$

7.
$$z = 4 - x^2 - y^2$$

$$8. \ 3x^2 + 4y^2 + 6z^2 = 12$$

$$9. -4x^2 - 9y^2 + 36z^2 = 36$$

10. Identify each of the following surfaces.

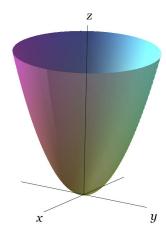
(a)
$$16x^2 + 4y^2 + 4z^2 - 64x + 8y + 16z = 0$$

(b)
$$-4x^2 + y^2 + 16z^2 - 8x + 10y + 32z = 0$$

- 11. Consider the paraboloid $z = x^2 + y^2$
 - (a) Compute equations for the traces in the $z=0,\,z=1,\,z=2,$ and z=3 planes.

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- (b) Sketch all the traces that you found in part (a) on the same coordinate axes.
- (c) Compute equations for the traces in the y = 0, y = 1, y = 2, and y = 3 planes.
- (d) Sketch all the traces that you found in part (c) on the same coordinate axes.
- (e) Below is the graph of $z = x^2 + y^2$. On the graph of the surface, sketch the traces that you found in parts (a) and (c).



For problems 12-13, find an equation of the trace of the surface in the indicated plane. Describe the graph of the trace.

- 12. Surface: $8x^2 + y^2 + z^2 = 9$; Plane: z = 1
- 13. Surface: $-4x^2 4y^2 + 9z^2 = 35$; Plane $x = \frac{1}{2}$

For problems 14-15, sketch the indicated region.

- 14. The region bounded below by $z = \sqrt{x^2 + y^2}$ and bounded above by $z = 2 x^2 y^2$.
- 15. The region bounded below by $2z = x^2 + y^2$ and bounded above by z = y.
- 16. Match each equation to an appropriate graph from the table below.

(a)
$$x^2 - y + z^2 = 0$$

(b)
$$4x^2 - 9y^2 + 36z^2 = -36$$

(c)
$$4x^2 + 4y^2 + 4z^2 = 36$$

(d)
$$x^2 + z^2 = 16$$

(e)
$$x^2 + z - y^2 = 0$$

(f)
$$4x^2 - 36y^2 + 9z^2 = 36$$

