7.4 #5

$$\int \frac{x^2}{\sqrt{1-2x^2}}\,dx = \int \frac{x^2}{\sqrt{1-(\sqrt{2}x)^2}}\,dx$$

(*) $\sqrt{2}x = \sin\theta \Rightarrow x = \frac{1}{\sqrt{2}}\sin\theta \Rightarrow \theta = \sin^{-1}(\sqrt{2}x)$

(**) $\sqrt{1-2x^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$

$dx = \frac{1}{\sqrt{2}}\cos\theta$

So $$\int \frac{x^2}{\sqrt{1-2x^2}}\,dx = \int \frac{\frac{1}{2}\sin^2\theta}{\cos\theta}\cdot\frac{1}{\sqrt{2}}\cos\theta\,d\theta = \frac{1}{2\sqrt{2}}\int \sin^2\theta\,d\theta$$

$$= \frac{1}{2\sqrt{2}}\int \frac{1}{2}(1-\cos 2\theta)\,d\theta = \frac{1}{4\sqrt{2}}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$= \frac{1}{4\sqrt{2}}\left[\theta - \frac{1}{\cancel{2}}(\cancel{2})\sin\theta\cos\theta\right] + C$$

$$= \frac{1}{4\sqrt{2}}\left[\sin^{-1}(\sqrt{2}x) - (\sqrt{2}x)(\sqrt{1-2x^2})\right] + C$$

$$= \frac{1}{4\sqrt{2}}\sin^{-1}(\sqrt{2}x) - \frac{1}{4}x\sqrt{1-2x^2} + C$$

$\sin\theta = \sqrt{2}x$ (*) above

$\cos\theta = \sqrt{1-2x^2}$ (**) above

or use right triangle: