Chapter 4.2: Exponential & Logarithmic Functions of Base b

Expected Skills:

- Be able to specify the domain and range of $f(x) = b^x$ and $f(x) = \log_b x$.
- Be able to graph $f(x) = b^x$ and $f(x) = \log_b x$, labeling all intersections with the coordinate axes and all asymptotes.
- Be able to solve equations involving logarithm or exponential functions.
- Be able to differentiate exponential and logarithmic functions; also, be able to solve application problems such as tangent line, rates of change, local/absolute extrema, and curve sketching.

Practice Problems:

1. Evaluate each of the following without a calculator.

   (a) $\log_4 16$

   $\boxed{2}$

   (b) $\ln \dfrac{1}{\sqrt[5]{e}}$

   $\boxed{-\dfrac{1}{5}}$

   (c) $\log_{43} 1$

   $\boxed{0}$

   (d) $\log_{16} 2$

   $\boxed{\dfrac{1}{4}}$

2. Use the properties of logarithms to expand (as much as possible) the expression as a sum, difference, and/or constant multiple of logarithms. (Assume that all variables are positive.)

   (a) $\log_5 (5x^2\sqrt{y})$

   $\boxed{1 + 2\log_5 x + \dfrac{1}{2}\log_5 y}$

   (b) $\log_6 \dfrac{x^3}{y^2 z^4}$

   $\boxed{3\log_6 x - 2\log_6 y - 4\log_6 z}$

3. Determine the domain of the following functions. Express your answer in interval notation.

   (a) $f(x) = \dfrac{\ln(x-1)}{x-5}$

   $\boxed{(1,5) \cup (5,\infty)}$

   (b) $f(x) = \dfrac{\sqrt{4-x}}{2^x - 3}$

   $\boxed{(-\infty, \log_2 3) \cup (\log_2 3, 4]}$

   (c) $f(x) = \dfrac{x-1}{2 - \log_4 x}$

   $\boxed{(0,16) \cup (16,\infty)}$

4. Solve the given equation for x. Where appropriate, you may leave your answers in logarithmic form.

   (a) $\left(3^{x-5}\right) - 4 = 11$

   $\boxed{x = \dfrac{\ln 15}{\ln 3} + 5}$

   (b) $2\log_5(3x) = 4$

   $\boxed{x = \dfrac{25}{3}}$

   (c) $\log_3 x + \log_3(x-8) = 2$

   $\boxed{x = 9}$

   (d) $\log_8 2x + \log_8(x+4) = 2$

   $\boxed{x = 4}$

5. In a research experiment the population of a certain species is given by $P(t) = 15(7^t)$, where t is the number of weeks since the beginning of the experiment.

   (a) How large was the population at the beginning of the experiment?

   $\boxed{15}$

   (b) How long will it take for the population to reach 300? You may leave your answer in logarithmic form.

   $\boxed{\dfrac{\ln 20}{\ln 7} \text{ weeks}}$

6. Calculate $\frac{dy}{dx}$.

   (a) $y = \log_2(3x - 1)$

   $\boxed{\frac{3}{(3x-1)\ln 2}}$

   (b) $y = \frac{\log x}{2 - \log x}$

   $\boxed{\frac{2}{x\ln(10)(2-\log x)^2}}$

7. Use the change of base formula to express the following function in terms of the natural log. Then, calculate $\frac{dy}{dx}$. (Assume $x > 0$.)

   (a) $y = \log_{x^2}(e)$

   $\boxed{y = \frac{1}{2\ln x};\ \frac{dy}{dx} = -\frac{1}{2x(\ln x)^2}}$

   (b) $y = \log_{3x}(x)$

   $\boxed{y = \frac{\ln x}{\ln 3x};\ \frac{dy}{dx} = \frac{\ln 3}{x(\ln 3x)^2}}$

8. Find an equation of the tangent line to the graph of $f(x) = 3^{2x}$ at the point where $x = \log_3 4$.

   $\boxed{y - 16 = 32(\ln 3)(x - \log_3 4)}$