Differential Equations & First-Order Separable Equations

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Be able to verify that a given function is a solution to a differential equation.
- Be able to solve first-order separable equations by separating and integrating.
- Be able to solve initial-value problems for first-order separable equations.

PRACTICE PROBLEMS:

- 1. Verify that $y = x^2 + 1$ is a solution to the differential equation $y \frac{dy}{dx} = (x 1)^2$.
- 2. Find the value(s) of the constant A for which $y = e^{Ax}$ is a solution to the differential equation y'' + 5y' 6y = 0.

For problems 3-9, use separation of variables to solve the given differential equation. If possible, express your answer as an explicit function of x.

$$3. \ \frac{dy}{dx} = \frac{x^2 - 1}{y^2}$$

4.
$$\frac{dy}{dx} - \sqrt{xy} \ln x = 0$$

$$5. \ y' = yx^2$$

$$6. \frac{dy}{dx} - e^{-y}\sec^2 x = 0$$

7.
$$\frac{dy}{dx} = xy^3$$

8.
$$\frac{dy}{dx} = \frac{1}{(x^2 - 5x + 6)y}$$

9.
$$\left(\frac{\sqrt{x}y'}{2+y}\right) = 1$$
, for $x \neq 0$.

For problems 10-11, find the solution of the differential equation which satisfies the initial condition.

10.
$$\frac{dy}{dx} = \frac{x^2 - 2}{y}, y(0) = 1$$

11.
$$\frac{dy}{dx} = \frac{\ln x}{xy^2}$$
, $y(e) = 1$

12. Find an equation of the curve that passes through the point (0,1) and whose slope at (x,y) is xe^y .