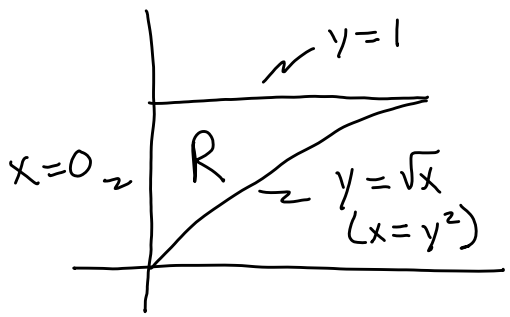
14.2 #13



Volume is $\int_0^1 \int_0^{y^2} x e^{\frac{x}{y^2}} \, dx \, dy$

[we cannot use dydx since we cannot find an antiderivative of $x e^{\frac{x}{y^2}}$ with respect to y]

$\int_0^{y^2} x e^{\frac{x}{y^2}} \, dx$ requires integration by parts

$u = x \quad dv = e^{\frac{x}{y^2}} dx$

$du = dx \quad v = y^2 e^{\frac{x}{y^2}}$ (remember, we are treating y as a constant)

$$x y^2 e^{\frac{x}{y^2}} \Big|_0^{y^2} - \int_0^{y^2} y^2 e^{\frac{x}{y^2}} \, dx$$

$$x y^2 e^{\frac{x}{y^2}} \Big|_0^{y^2} - y^4 e^{\frac{x}{y^2}} \Big|_0^{y^2}$$

$$= (y^4 e - 0) - (y^4 e - y^4)$$

$$= y^4$$

Now $\int_0^1 y^4 \, dy = \frac{1}{5} y^5 \Big|_0^1 = \frac{1}{5}$