Chapter 4.2 (Part 2) & 4.3 Practice Problems

EXPECTED SKILLS:

- Be able to use the extrema along with the end behavior (i.e. dominant term) of polynomials to sketch the graph of polynomial functions.
- Know how to determine if the graph of a function has a cusp or vertical tangent line at a point (i.e. the function is not differentiable at that point). And, be able to use this information, along with extrema, intercepts, and asymptotes, to sketch the graph of a function.

PRACTICE PROBLEMS:

For problems 1-12, sketch the given functions. Label the coordinates of all critical points, inflection points, x-intercepts, y-intercepts, and holes. Also label all horizontal asymptotes and vertical asymptotes

1. $f(x) = x^2(x^2 - 4)$
2. $f(x) = x^3 + 7x^2 + 8x - 16$
   (HINT: $f(1) = 0$)
3. $f(x) = \dfrac{x}{x+2}$
4. $f(x) = \dfrac{x^2 + x}{x^2 - 1}$
5. $f(x) = \dfrac{x}{x^2 + 2}$
6. $f(x) = \dfrac{x}{x^2 - 4}$
7. $f(x) = xe^{2x}$
8. $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$
9. $f(x) = \dfrac{\ln x}{x}$
10. $f(x) = x^{2/3}(x + 15)$
11. $f(x) = 4x - \tan x$ on $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
12. $f(x) = \sin^2(x)$ on $[0, 2\pi]$

13. Consider the graphs of $f(x) = x^{1/3}$ and $g(x) = x^{2/3}$. $x_0 = 0$ is a critical point for both $f(x)$ and $g(x)$ since 0 is in the domain of each function but $f'(0)$ and $g'(0)$ are both undefined. How does the behavior of $f(x)$ differ from that of $g(x)$ at this critical point?

14. Consider a general quadratic curve $f(x) = ax^2 + bx + c$, where $a \neq 0$. Show that $f(x)$ cannot have any inflection points.

15. Consider a general quartic curve $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where $a \neq 0$.

    (a) What is the largest number of distinct inflection points that $f(x)$ could have?

    (b) What condition on the coefficients a, b, c, d, and e is necessary for the number of distinct inflection points to be maximized?