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First note that  $\sum_{k=1}^{\infty} \cos(k\pi) k e^{-k} = \sum_{k=1}^{\infty} (-1)^k k e^{-k}$

$$\text{Now } \sum_{k=1}^{\infty} |(-1)^k k e^{-k}| = \sum_{k=1}^{\infty} k e^{-k}$$

$$\text{Ratio Test: } \lim_{k \rightarrow +\infty} \frac{k+1}{e^{k+1}} \cdot \frac{e^k}{k} = \lim_{k \rightarrow +\infty} \left[ \left( \frac{k+1}{k} \right) \cdot \frac{1}{e} \right] = 1 \cdot \frac{1}{e} < 1$$

So  $\sum_{k=1}^{\infty} k e^{-k}$  converges by the Ratio Test.

[Alternatively, use the Integral Test:

see Convergence Tests homework #11.]

So  $\sum_{k=1}^{\infty} \cos(k\pi) k e^{-k}$  converges absolutely.