

Chapter 2.1 Practice Problems

EXPECTED SKILLS:

- Be able to compute the average rate of change of a function over an interval; i.e., be able to find the slope of the secant line through two points on the graph of a function.
- Be comfortable using a limit to compute the instantaneous rate of change of a function (for arbitrary and specific values); i.e., know how to find the slope of the tangent line to a function.

PRACTICE PROBLEMS:

1. Find the average rate of change of the given function on the given interval.

(a) $f(x) = x^2$ on $[0, 2]$

$\boxed{2}$

(b) $f(x) = x^3 - 3x + 5$ on $[-2, 2]$

$\boxed{1}$

(c) $f(x) = \frac{1}{x}$ on $[1, 2]$

$\boxed{-\frac{1}{2}}$

2. Find the instantaneous rate of change of the given function at the given point.

(a) $f(x) = x^2 - 1$ at $x = 3$

$\boxed{6}$

(b) $f(x) = x^3$ at $x = 2$

$\boxed{12}$

(c) $f(x) = \sqrt{x}$ at $x = 9$

$\boxed{\frac{1}{6}}$

(d) $f(x) = \frac{1}{x^2}$ at $x = 1$

$\boxed{-2}$

3. A ball is thrown straight up in the air (from the ground) and its position in feet above the ground after t seconds is given by: $f(t) = -8t^2 + 16t$. Answer the following questions about the path of the ball.

(a) At what time t does the ball hit the ground?

After 2 seconds

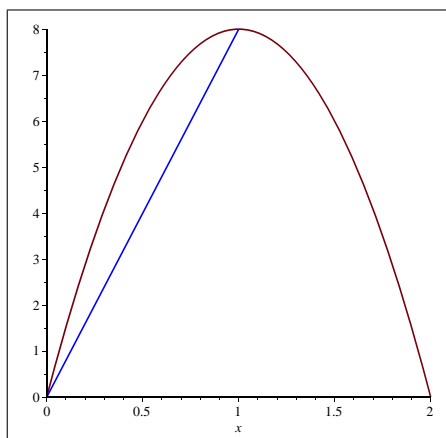
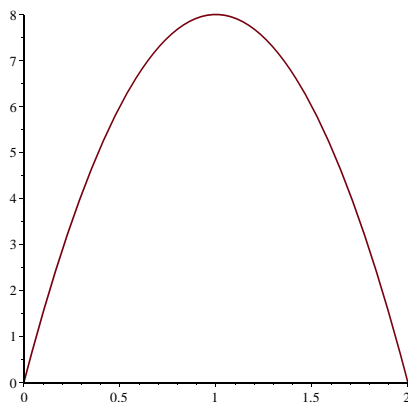
(b) What is the average velocity of the ball from $t = 0$ to $t = 1$?

8 ft/sec

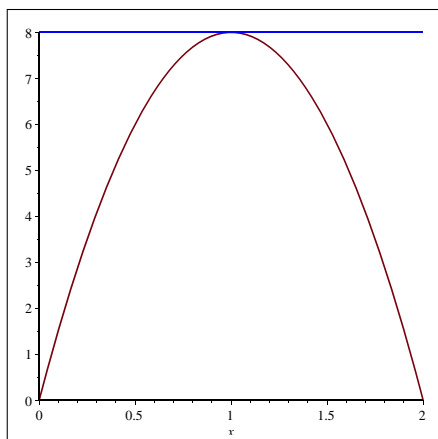
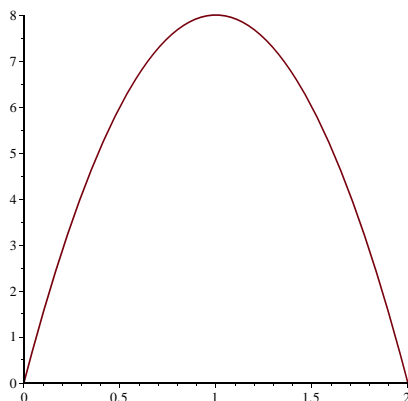
(c) What is the instantaneous velocity of the ball at $t = 1$ second?

0 ft/sec

(d) Below is the graph of $f(t)$. Sketch the secant line whose slope is the average velocity of the ball on $[0, 1]$.



- (e) Below is the graph of $f(t)$. Sketch the tangent line whose slope is the instantaneous velocity of the ball at $t = 1$ second.



4. If a rock is thrown upward on the moon with an initial velocity of 3.244 (m/s), its height (in meters) after t seconds is given by $H(t) = -0.811t^2 + 3.244t$.

- (a) Find the velocity of the rock at $t = a$ seconds.

$$\boxed{-1.622a + 3.244 \text{ m/s}}$$

- (b) Find the velocity of the rock at $t = 1$ second.

$$\boxed{1.622 \text{ m/s}}$$

- (c) When will the rock hit the surface of the moon?

$$\boxed{\text{At } t = 4 \text{ seconds}}$$

- (d) Compute the velocity of the rock at the moment when it hits the surface of the moon.

$$\boxed{-3.244 \text{ m/s}}$$