13.3 #4

(a) $\frac{\partial z}{\partial x} = 2x$

The requested slope is $\frac{\partial z}{\partial x}(1,1) = 2$

(b) From (a) we know the slope of the tangent line to S at (1,1,4) in the x-direction is 2,

i.e. for every one unit "run" in x there is a two unit "rise" in z. So a vector parallel to the tangent line is $\langle 1, 0, 2 \rangle$.

Tangent line: $\begin{cases} x = 1 + t \\ y = 1 \\ z = 4 + 2t \end{cases}$

(c) $\frac{\partial z}{\partial y} = 6y$

The requested slope is $\frac{\partial z}{\partial y}(1,1) = 6$.

(d) From (c) we know the slope of the tangent line to S at (1,1,4) in the y-direction is 6, i.e. for every one unit "run" in y there is a six unit "rise" in z. So a vector parallel to the tangent line is $\langle 0,1,6 \rangle$.

Tangent line: $\begin{cases} x = 1 \\ y = 1+t \\ z = 4+6t \end{cases}$

(e) A normal to the tangent plane is

$$\langle 1,0,2 \rangle \times \langle 0,1,6 \rangle = -2\vec{i} - 6\vec{j} + \vec{k}$$

Tangent plane: $-2(x-1) - 6(y-1) + 1(z-4) = 0$