Chapter 1.2 Practice Problems

EXPECTED SKILLS:

- Know the basic properties of limits; i.e., be familiar with how limits "interact" with sums, differences, products, and other operations. See Theorem 1.2.2.
- Given the formula of a function $y = f(x)$, be able to determine the limit of $f(x)$ as x approaches some finite value (as both a one-sided and two sided limit).
- Be able to determine when such a limit does not exist, and if appropiate, indicate if the behavior of the function is increasing or decreasing without bound.
- Be familiar with the indeterminate forms of $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$. And, know how to use algebraic techniques such as factoring and rationalizing to help compute these types of limits.

PRACTICE PROBLEMS:
In each problem, compute the limit. If the limit doesn't exist write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

1. Given that $\lim_{x\to 1} f(x) = 4$ and $\lim_{x\to 1} g(x) = 2$, determine each of the following limits:

   (a) $\lim_{x\to 1} (f(x) + g(x))$

   $\boxed{6}$

   (b) $\lim_{x\to 1} (5f(x) - g(x))$

   $\boxed{18}$

   (c) $\lim_{x\to 1} \left(\frac{f(x)}{g(x)}\right)$

   $\boxed{2}$

2. $\lim_{x\to 1} \left(x^2 + 1\right)$

   $\boxed{2}$

3. $\lim_{x\to 4} 1$

   $\boxed{1}$

4. $\lim_{x\to -1} (x+1)(x^3)$

   $\boxed{0}$

5. $\lim_{x \to 5^-} \left(\frac{x^2 - 6x}{x^3 - 1} \right)$

$\boxed{-\frac{5}{124}}$

6. $\lim_{x \to -1} \left(\frac{x^2 - 1}{x + 1} \right)$

$\boxed{-2}$

7. $\lim_{x \to 2^-} \left(\frac{x^2 - 4x + 4}{x - 2} \right)$

$\boxed{0}$

8. $\lim_{x \to 3^+} \left(\frac{x^2 + 2x - 15}{x - 3} \right)$

$\boxed{8}$

9. $\lim_{x \to 1} \left(\frac{x^3 - 3x^2 - x + 3}{x^2 - 1} \right)$

$\boxed{-2}$

10. $\lim_{x \to 16} \left(\frac{\sqrt{x} - 4}{x - 16} \right)$

$\boxed{\frac{1}{8}}$

11. $\lim_{x \to 0} \left(\frac{|x|}{x} \right)$

$\boxed{\text{DNE}}$

12. $\lim_{x \to 4^-} \left(\frac{x}{x - 4} \right)$

$\boxed{-\infty}$

13. $\lim_{x \to 4^+} \left(\frac{x}{x - 4} \right)$

$\boxed{+\infty}$

14. $\lim_{x \to 4} \left(\frac{x}{x - 4} \right)$

$\boxed{\text{DNE}}$

15. $\lim_{x \to -2} \left(\frac{1}{x-2} \right)$

$\boxed{-\frac{1}{4}}$

16. $\lim_{x \to -2^-} \left(\frac{x}{x^2+2x} \right)$

$\boxed{-\infty}$

17. $\lim_{x \to -2^+} \left(\frac{x}{x^2+2x} \right)$

$\boxed{+\infty}$

18. $\lim_{x \to 3} \left(\frac{x^3}{|x-3|} \right)$

$\boxed{+\infty}$

19. $\lim_{x \to 1^-} \left(\frac{x-1}{x^2-2x+1} \right)$

$\boxed{-\infty}$

20. $\lim_{x \to 1^+} \left(\frac{x-1}{x^2+2x-3} \right)$

$\boxed{\frac{1}{4}}$

21. Let $f(n) = \begin{cases} n^2+1, & \text{if } n \leq -1 \\ 3n+1, & \text{if } n > -1 \end{cases}$. Compute $\lim_{n \to -1} f(n)$

$\boxed{\text{DNE}}$

22. Let $f(x) = \begin{cases} 3x^3+2x-3, & \text{if } x < 1 \\ 100, & \text{if } x = 1 \\ \frac{x^2-1}{x-1}, & \text{if } x > 1 \end{cases}$. Determine $\lim_{x \to 1} f(x)$

$\boxed{2}$

23. Let $f(p) = \begin{cases} 3p-1, & \text{if } p < 3 \\ p^3-4p-7, & \text{if } p > 3 \end{cases}$. Find $\lim_{p \to 3} f(p)$

$\boxed{8}$

24. Let $f(x) = \begin{cases} x^2 + 2ax + a^2, & \text{if } x < 4 \\ 432, & \text{if } x = 4 \\ x^2 - 7, & \text{if } x > 4 \end{cases}$. Find the value(s) of a such that $\lim_{x \to 4} f(x)$ exists.

$\boxed{-1 \text{ or } -7}$

25. Let x_0 be a fixed real number. Compute $\lim_{h \to 0} \dfrac{(x_0 + h)^2 - x_0^2}{h}$

$\boxed{2x_0}$

26. Let x_0 be a fixed real number. Compute $\lim_{x \to x_0} \dfrac{x^2 - x_0^2}{x - x_0}$

$\boxed{2x_0}$

27. Let x_0 be a fixed, positive real number. Compute $\lim_{h \to 0} \dfrac{\sqrt{x_0 + h} - \sqrt{x_0}}{h}$

$\boxed{\dfrac{1}{2\sqrt{x_0}}}$