Limits at Infinity & Integration Review

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Determine limits at infinity.
- Evaluate integrals using various techniques such as substitution, parts, and partial fractions.
- Evaluate improper integrals using limits.

PRACTICE PROBLEMS:

For problems 1 - 10, evaluate the limit.

1. $\lim_{x\to+\infty}\left(\frac{50000}{x}\right)$

   $\boxed{0}$

2. $\lim_{x\to+\infty}\left(\frac{4x-3x^5}{2x^5+4x^3+x^2+5}\right)$

   $\boxed{-\frac{3}{2}}$

3. $\lim_{x\to+\infty}\left(\frac{\sqrt{4+3x^2}}{2+7x}\right)$

   $\boxed{\frac{\sqrt{3}}{7}}$

4. $\lim_{x\to+\infty}\left(\sqrt{x^2+8x-5}-x\right)$

   $\boxed{4}$

5. $\lim_{x\to+\infty}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$

   $\boxed{1}$

6. $\lim_{x\to+\infty}\left(xe^{-x}\right)$

   $\boxed{0}$

7. $\lim_{x\to+\infty}\left(\frac{\cos x}{x}\right)$

$\boxed{0}$

8. $\lim_{x\to+\infty}\left(\arctan\left(\frac{1}{x}\right)-\arctan(x)\right)$

$\boxed{-\frac{\pi}{2}}$

9. $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}$

$\boxed{e}$

10. $\lim_{x\to+\infty}(1+3^{x})^{1/x}$

$\boxed{3}$

For problems 11 - 22, evaluate the integral. If the integral is improper, determine what it converges to or show that it diverges.

11. $\int_{\sqrt{e}}^{e}\frac{1}{4x}\,dx$

$\boxed{\frac{1}{8}}$

12. $\int\frac{\sqrt{\ln x}}{x}\,dx$

$\boxed{\frac{2}{3}(\ln x)^{3/2}+C}$

13. $\int_{0}^{\sqrt{\pi}}x\sin(x^{2})\,dx$

$\boxed{1}$

14. $\int x^{2}\sin x\,dx$

$\boxed{2x\sin x-x^{2}\cos x+2\cos x+C}$

15. $\int\tan x\,dx$

$\boxed{-\ln|\cos x|+C=\ln|\sec x|+C}$

16. $\displaystyle\int \frac{1}{x^2+9}\,dx$

$\boxed{\dfrac{1}{3}\arctan\left(\dfrac{x}{3}\right)+C}$

17. $\displaystyle\int_1^{+\infty} \frac{x}{x^2+9}\,dx$

$\boxed{\text{Diverges to } +\infty}$

18. $\displaystyle\int \frac{dx}{x^3-x}$

$\boxed{\dfrac{1}{2}\ln|x-1|+\dfrac{1}{2}\ln|x+1|-\ln|x|+C}$

19. $\displaystyle\int_0^{1/2} \frac{3}{\sqrt{1-x^2}}\,dx$

$\boxed{\dfrac{\pi}{2}}$

20. $\displaystyle\int_1^{+\infty} xe^{-x}\,dx$

$\boxed{\text{Converges to } \dfrac{2}{e}}$

21. $\displaystyle\int \sqrt{x}\ln x\,dx$

$\boxed{\dfrac{2}{3}x^{3/2}\ln x-\dfrac{4}{9}x^{3/2}+C}$

22. $\displaystyle\int_5^{+\infty} \frac{dx}{x^2-3x-4}$

$\boxed{\text{Converges to } \dfrac{1}{5}\ln 6}$