#9

Consider the ODE $\frac{dy}{dt} = k\left(1 - \frac{y}{L}\right)y$ to be

a function of y, i.e. $f(y) = k\left(1 - \frac{y}{L}\right)y = k\left(y - \frac{y^2}{L}\right)$

We want to maximize $f(y)$, so using Calc I techniques:

$$f'(y) = k\left(1 - \frac{2y}{L}\right) = 0 \implies y = \frac{L}{2}$$

We can confirm this yields a maximum with the Second Derivative Test:

$f''(y) = \left(-\frac{2}{L}\right) < 0$, so $y = \frac{L}{2}$ maximizes $f(y)$

and thus is when the population grows the fastest.