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$$\lim_{k \rightarrow +\infty} \sqrt[k]{\frac{k}{7^k}} = \lim_{k \rightarrow +\infty} \frac{k^{\frac{1}{k}}}{7} = \frac{1}{7} \lim_{k \rightarrow +\infty} k^{\frac{1}{k}}$$

Now $\lim_{k \rightarrow +\infty} k^{\frac{1}{k}}$ gives ∞^0 , which is indeterminate.

$$\text{Let } y = k^{\frac{1}{k}} \Rightarrow \lim_{k \rightarrow +\infty} \ln y = \lim_{k \rightarrow +\infty} \frac{1}{k} \ln k \quad \begin{array}{l} \rightarrow \infty \\ \rightarrow \infty \end{array}$$

$$\text{L'Hopital} \Rightarrow \lim_{k \rightarrow +\infty} \frac{\frac{1}{k}}{1} = 0$$

$$\text{So } \lim_{k \rightarrow +\infty} y = e^0 = 1 \Rightarrow \lim_{k \rightarrow +\infty} \sqrt[k]{\frac{k}{7^k}} = \frac{1}{7} < 1$$

So by the Root Test, $\sum_{k=1}^{\infty} \frac{k}{7^k}$ converges.