Cylindrical & Spherical Coordinates

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.8 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to convert between rectangular, cylindrical, and spherical coordinates (Table 11.8.1).

- Be able describe simple surfaces in terms of cylindrical and spherical coordinates (Table 11.8.2).

PRACTICE PROBLEMS:

1. Consider the point $(r, \theta, z) = \left(2, \dfrac{\pi}{2}, 1\right)$ in cylindrical coordinates.

   (a) Convert this point to rectangular coordiinates.
   (b) Convert this point to spherical coordinates.

2. Consider the point $(r, \theta, z) = \left(1, \dfrac{\pi}{4}, -4\right)$ in cylindrical coordinates.

   (a) Convert this point to rectangular coordiinates.
   (b) Convert this point to spherical coordinates.

3. Consider the point $(\rho, \theta, \phi) = \left(5, \dfrac{\pi}{3}, \dfrac{2\pi}{3}\right)$ in spherical coordinates.

   (a) Convert this point to rectangular coordiinates.
   (b) Convert this point to cylindrical coordinates.

4. Consider the point $(x, y, z) = \left(1, -\sqrt{3}, -2\right)$ in rectangular coordinates.

   (a) Convert this point to cylindrical coordiinates.
   (b) Convert this point to spherical coordinates.

For problems 5-10, each of the given surfaces is expressed in rectangular coordinates. Express the equation of the surface in (a) cylindrical coordinates and (b) spherical coordinates.

5. $x^2 + y^2 + z^2 = 16$

6. $x^2 + y^2 + z^2 = 3z$

7. $z = \sqrt{2x^2 + 2y^2}$

8. $x^2 + y^2 = 9$

9. $x + 3y + 5z = 4$

10. $z = 2$

For problems 11-15, each of the given surfaces is expressed in cylindrical coordinates. Express the equation of the surface in rectangular coordinates.

11. $r = 5$

12. $\theta = \dfrac{\pi}{2}$

13. $r = 6 \sin \theta$

14. $z = r \sin \theta$

15. $r^2 \sin 2\theta = z$

For problems 16-19, each of the given surfaces is expressed in spherical coordinates. Express the equation of the surface in rectangular coordinates.

16. $\rho = 4$

17. $\phi = \dfrac{\pi}{3}$

18. $\rho = 4 \cos \phi$

19. $\rho = 3 \sec \phi$

For problems 20-21, describe in words all points in 3-space which satisfy the given inequalities.

20. In cylindrical coordinates: $\left\{ (r, \theta, z) : 0 \leq r \leq 2, 0 \leq \theta \leq \dfrac{\pi}{3}, 0 \leq z \leq 1 \right\}$

21. In spherical coordinates: $\left\{ (\rho, \theta, \phi) : 1 \leq \rho \leq 3, 0 \leq \theta \leq \dfrac{\pi}{2}, 0 \leq \phi \leq \dfrac{\pi}{4} \right\}$