11.5 #10

L_1: $x = 1-2t_1$, $y = 14+t_1$, $z = 5-t_1$

L_2: $x = t_2$, $y = 4+3t_2$, $z = 3+t_2$

L_1 is parallel to $V_1 = \langle -2, 1, -1 \rangle$

L_2 is parallel to $V_2 = \langle 1, 3, 1 \rangle$

Now $\vec{V}_1$ is not parallel to $\vec{V}_2$ since $\vec{V}_1$ is not a constant multiple of $\vec{V}_2$.

[You could also show $\vec{V}_1 \times \vec{V} \neq \vec{0}$, so the vectors are not parallel.]

Thus L_1 is not parallel to L_2.

Does L_1 intersect L_2?

If yes, we will find the point of intersection.

If no, the lines are skew.

① $1-2t_1 = t_2$

② $14+t_1 = 4+3t_2$

③ $5-t_1 = 3+t_2$

② + ③ $\Rightarrow 19 = 7+4t_2 \Rightarrow 12 = 4t_2 \Rightarrow t_2 = 3$

Plug $t_2 = 3$ into ② $\Rightarrow 14+t_1 = 4+3(3)$

$t_1 = -1$

Plug $t_1 = -1$ and $t_2 = 3$ into ① and see if the equation is satisfied.

LHS of ①: $1-2t_1 = 1-2(-1) = 3$

RHS of ①: $t_2 = 3$

same, so lines intersect

Point of intersection: $x = 3$

$y = 14+t_1 = 13$

$z = 5-t_1 = 6$

$(3,13,6)$