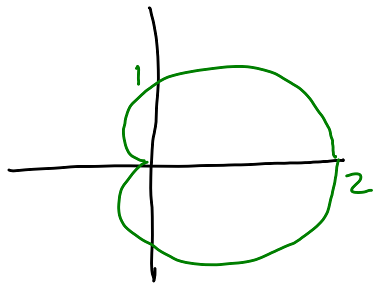
10.3 #7

(7) $r = 1 + \cos\theta$

$$r^2 = 1 + 2\cos\theta + \cos^2\theta$$



$$\frac{dr}{d\theta} = -\sin\theta \implies \left(\frac{dr}{d\theta}\right)^2 = \sin^2\theta$$

$$L = \int_0^{2\pi} \sqrt{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{2(1+\cos\theta)}\, d\theta = \sqrt{2}\int_0^{2\pi} \sqrt{1+\cos\theta}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{1+\cos\theta}\, d\theta \quad \text{(by symmetry)}$$

$$\left[\text{Recall: } \cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta) \iff 2\cos^2\theta = 1 + \cos 2\theta \iff 2\cos^2\left(\tfrac{\theta}{2}\right) = 1 + \cos\theta\right]$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{2\cos^2\left(\tfrac{\theta}{2}\right)}\, d\theta = 4\int_0^{\pi} \left|\cos\left(\tfrac{\theta}{2}\right)\right| d\theta \quad \left[\begin{array}{l}\cos\left(\tfrac{\theta}{2}\right) \geq 0 \text{ on } [0,\pi] \\ \text{so absolute value is not needed}\end{array}\right]$$

$$= 4\int_0^{\pi} \cos\left(\tfrac{\theta}{2}\right) d\theta = 8\sin\left(\tfrac{\theta}{2}\right)\Big|_0^{\pi} = 8[1-0] = 8$$