Quadric Surfaces

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.7 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.
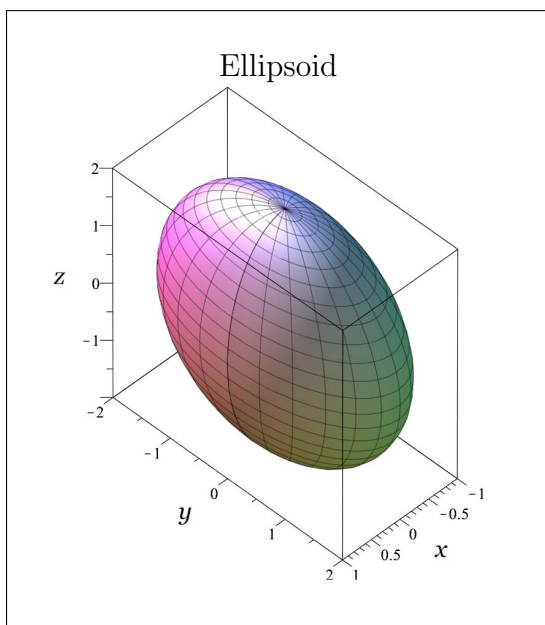
EXPECTED SKILLS:

- Be able to compute & traces of quadic surfaces; in particular, be able to recognize the resulting conic sections in the given plane.

- Given an equation for a quadric surface, be able to recognize the type of surface (and, in particular, its graph).
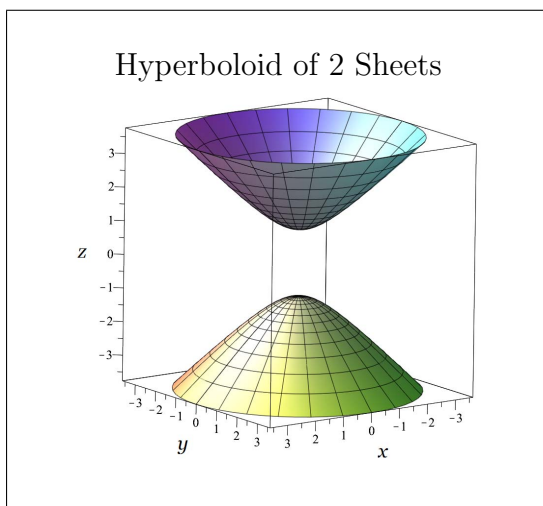
PRACTICE PROBLEMS:

For problems 1-9, use traces to identify and sketch the given surface in 3-space.
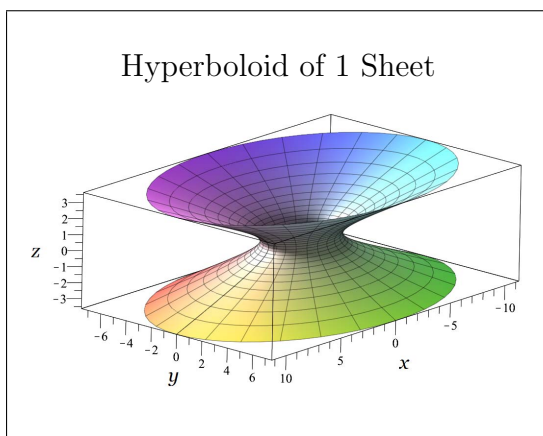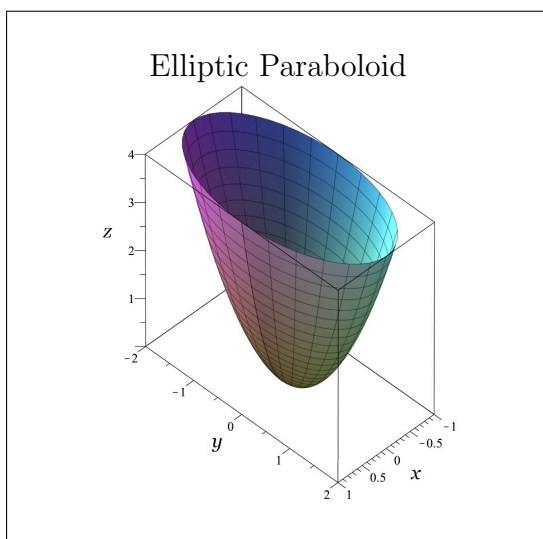
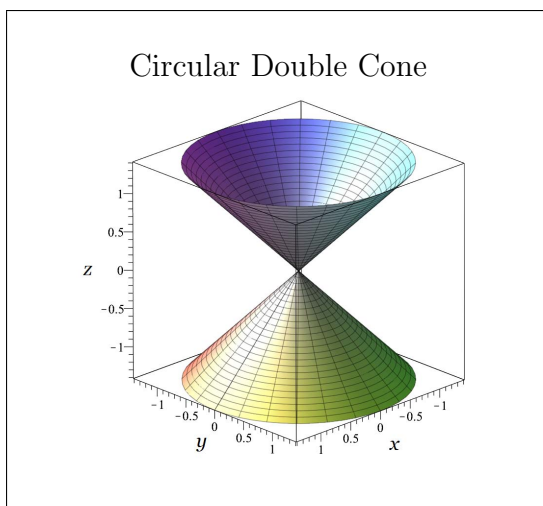1. $4x^2 + y^2 + z^2 = 4$

Ellipsoid

2. $-x^2 - y^2 + z^2 = 1$

Hyperboloid of 2 Sheets

3. $4x^2 + 9y^2 - 36z^2 = 36$

Hyperboloid of 1 Sheet

4. $z = 4x^2 + y^2$

Elliptic Paraboloid

5. $z^2 = x^2 + y^2$

Circular Double Cone
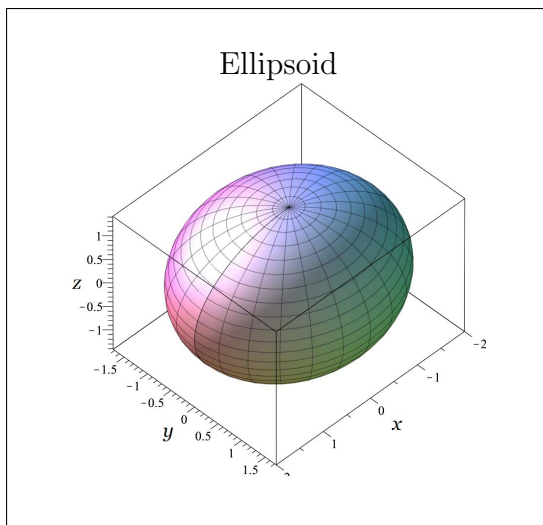
6. $x^2 + y^2 - z^2 = 1$
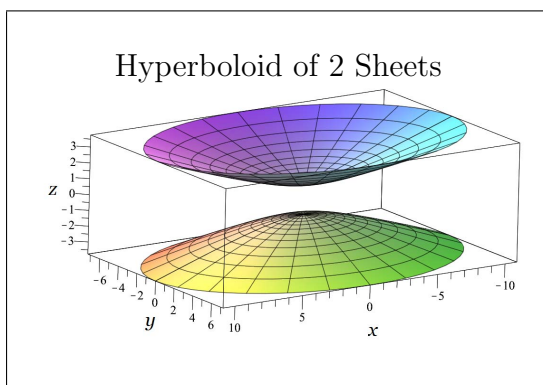
Hyperboloid of 1 Sheet

7. $z = 4 - x^2 - y^2$

Circular Paraboloid
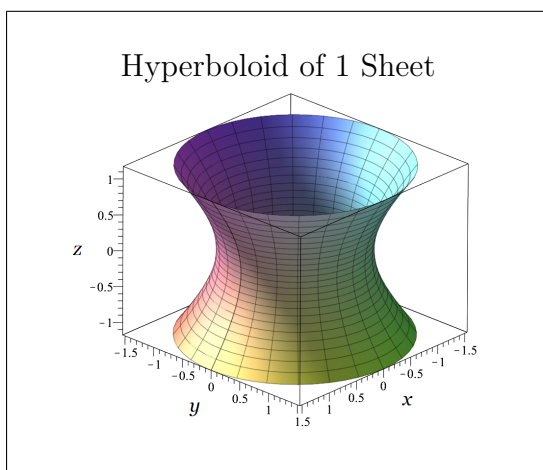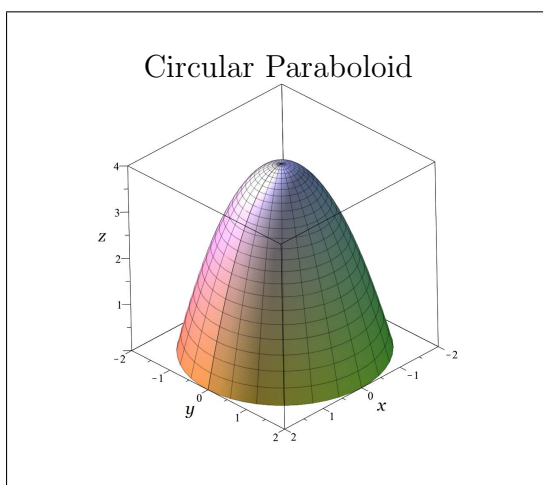
8. $3x^2 + 4y^2 + 6z^2 = 12$

9. $-4x^2 - 9y^2 + 36z^2 = 36$

10. Identify each of the following surfaces.

(a) $16x^2 + 4y^2 + 4z^2 - 64x + 8y + 16z = 0$

After completing the square, we can rewrite the equation as:

$$16(x-2)^2 + 4(y+1)^2 + 4(z+2)^2 = 84$$

This is an ellipsoid which has been shifted. Specifically, it is now centered at $(2, -1, -2)$.

(b) $-4x^2 + y^2 + 16z^2 - 8x + 10y + 32z = 0$

After completing the square, we can rewrite the equation as:

$$-4(x-1)^2 + (y+5)^2 + 16(z+1)^2 = 37$$

This is a hyperboloid of 1 sheet which has been shifted. Specifically, its central axis is parallel to the x-axis. In fact, the equation of its central axis is $\vec{\ell}(t) = \langle 1, -5, -1 \rangle + t\langle 1, 0, 0 \rangle$.

11. Consider the paraboloid $z = x^2 + y^2$

   (a) Compute equations for the traces in the $z = 0$, $z = 1$, $z = 2$, and $z = 3$ planes.

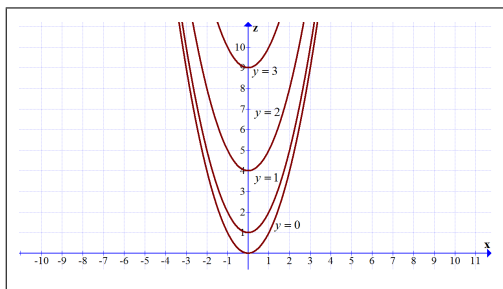 | Plane | Trace |
   |-------|-------|
   | $z = 0$ | Point $(0,0)$ |
   | $z = 1$ | Circle $x^2 + y^2 = 1$ |
   | $z = 2$ | Circle $x^2 + y^2 = 2$ |
   | $z = 3$ | Circle $x^2 + y^2 = 3$ |

   (b) Sketch all the traces that you found in part (a) on the same coordinate axes.
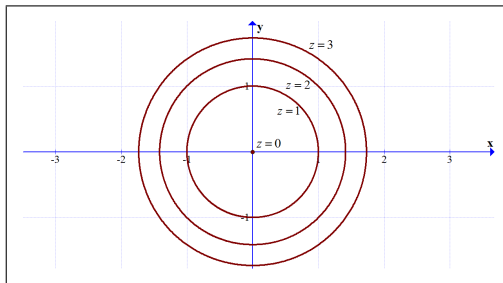
   (c) Compute equations for the traces in the $y = 0$, $y = 1$, $y = 2$, and $y = 3$ planes.

Plane	Trace
$y = 0$	Parabola $z = x^2$
$y = 1$	Parabola $z = x^2 + 1$
$y = 2$	Parabola $z = x^2 + 4$
$y = 3$	Parabola $z = x^2 + 9$

   (d) Sketch all the traces that you found in part (c) on the same coordinate axes.
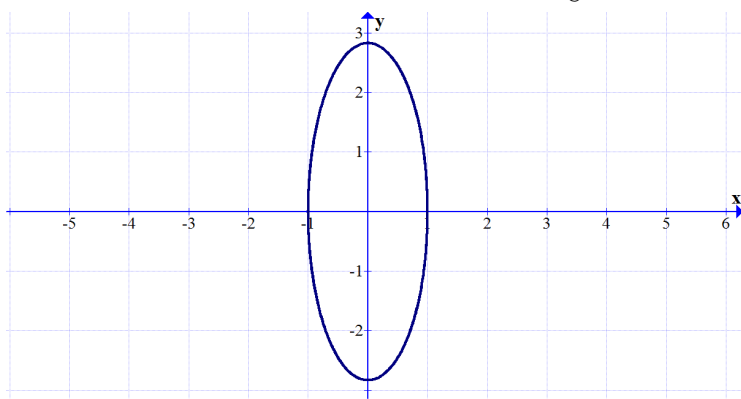

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(e) Below is the graph of $z = x^2 + y^2$. On the graph of the surface, sketch the traces that you found in parts (a) and (c).
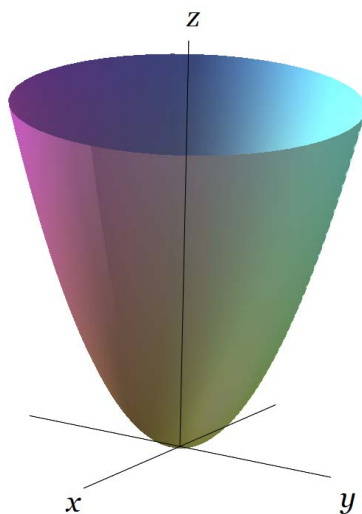
For problems 12-13, find an equation of the trace of the surface in the indicated plane. Describe the graph of the trace.
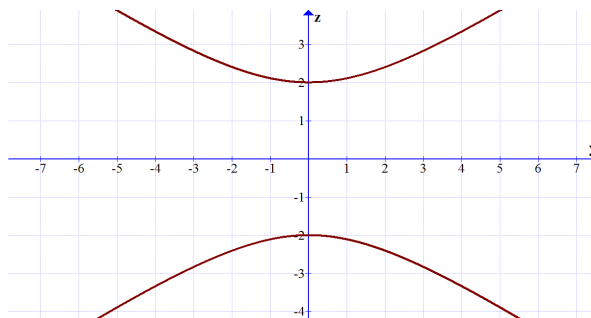
12. Surface: $8x^2 + y^2 + z^2 = 9$; Plane: $z = 1$

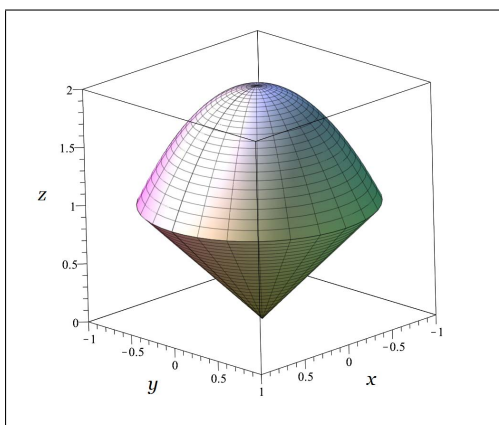The trace in the $z = 1$ plane is the ellipse $x^2 + \dfrac{y^2}{8} = 1$, shown below.

13. Surface: $-4x^2 - 4y^2 + 9z^2 = 35$; Plane $x = \dfrac{1}{2}$

The trace in the $x = \dfrac{1}{2}$ plane is the hyperbola $-\dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$, shown below.
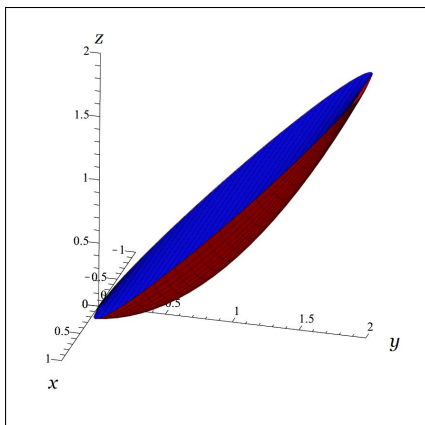

For problems 14-15, sketch the indicated region.

14. The region bounded below by $z = \sqrt{x^2 + y^2}$ and bounded above by $z = 2 - x^2 - y^2$.

15. The region bounded below by $2z = x^2 + y^2$ and bounded above by $z = y$.

16. Match each equation to an appropriate graph from the table below.

(a) $x^2 - y + z^2 = 0$

(b) $4x^2 - 9y^2 + 36z^2 = -36$
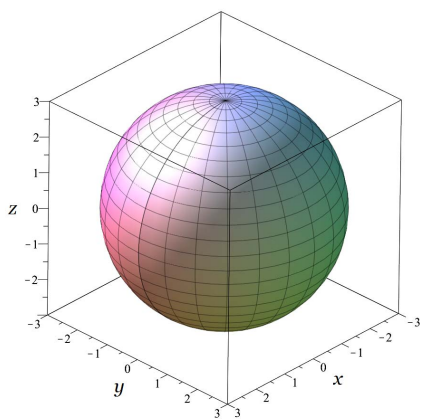
(c) $4x^2 + 4y^2 + 4z^2 = 36$

(d) $x^2 + z^2 = 16$
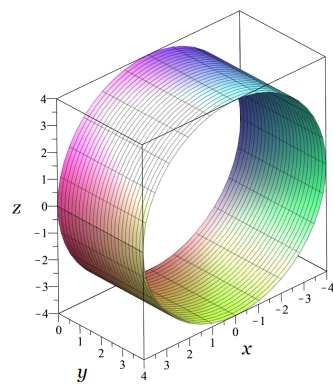
(e) $x^2 + z - y^2 = 0$

(f) $4x^2 - 36y^2 + 9z^2 = 36$

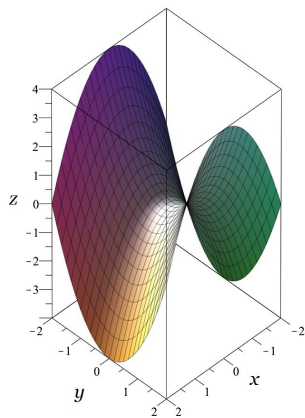| Equation	Graph
a	V
b	III
c	I
d	IV
e	II
f	VI

(I)

(IV)

(II)

(V)

(III)

(VI)

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