7.4 #14

$$V = \int_0^1 \pi \left(\frac{1}{x^2+1}\right)^2 dx = \text{... by problem (2) ...}$$

$$= \pi\left[\left(\tfrac{1}{2}\tan^{-1}1 + \frac{1}{2(1+1)}\right) - \left(\tfrac{1}{2}\tan^{-1}0 + 0\right)\right] = \pi\left[\tfrac{1}{2}\left(\tfrac{\pi}{4}\right) + \tfrac{1}{4}\right] = \frac{\pi^2}{8} + \frac{\pi}{4}$$

OR

$$V = \pi \int_0^1 \left(\frac{1}{x^2+1}\right)^2 dx$$

$x = \tan\theta \Rightarrow \theta = \tan^{-1}x$; $x = 1 \Rightarrow \theta = \frac{\pi}{4}$; $x = 0 \Rightarrow \theta = 0$

$x^2+1 = \tan^2\theta + 1 = \sec^2\theta$; $dx = \sec^2\theta\, d\theta$

$$= \pi \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{(\sec^2\theta)^2}\, d\theta = \pi \int_0^{\frac{\pi}{4}} \frac{1}{\sec^2\theta}\, d\theta = \pi \int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$$

$$= \pi \int_0^{\pi/4} \tfrac{1}{2}(1+\cos 2\theta)\, d\theta = \frac{\pi}{2}\left[\theta\Big|_0^{\frac{\pi}{4}} + \tfrac{1}{2}\sin 2\theta\Big|_0^{\frac{\pi}{4}}\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} + \tfrac{1}{2}\left(\sin\tfrac{\pi}{2} - \sin 0\right)\right] = \frac{\pi}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi^2}{8} + \frac{\pi}{4}$$