Relative and Absolute Extrema

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.8 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to use partial derivatives to find critical points (possible locations of maxima or minima).
- Know how to use the Second Partials Test for functions of two variables to determine whether a critical point is a relative maximum, relative minimum, or a saddle point.
- Be able to solve word problems involving maxima and minima.
- Know how to compute absolute maxima and minima on closed regions.

PRACTICE PROBLEMS:

For problems 1-10, identify all critical points of the given function. Then, classify each as the location of a relative maximum, relative minimum, or saddle point.

1. $g(x,y) = x^2 + y^2 - 3x - 4y + 6$
2. $f(x,y) = x^2 + 4y^2 - 4y - 2$
3. $g(x,y) = 4x^2 - 3y^2 + 8x - 9y - 4$
4. $f(x,y) = x^3 - 3x + y^2 - 6y$
5. $h(x,y) = x^2 - 5xy + y^2$
6. $f(x,y) = 3x + y^2 - e^x$
7. $f(x,y) = x^6 + y^6$
8. $f(x,y) = x^2y - 6y^2 - 3x^2$
9. $f(x,y) = x^3 - 3xy + \frac{1}{2}y^2$
10. $f(x,y) = \frac{1}{3}x^3 - 2x + x^2 + 2xy + y^2$
11. Consider $h(x,y) = 3\sqrt{x^2 + y^2} + 6$

(a) Explain why the Second Partials Test may not be used to locate the relative extrema/saddle points of $h(x, y)$.

(b) Locate all relative maxima, relative minima, and saddle points, if any.

For problems 12-15, find the absolute extrema of the given function on the specified region R.

12. $f(x, y) = 5 - 4y - 2x$; R is the closed triangular region in the xy-plane with vertices $(3, 0)$, $(0, 1)$, and $(1, 2)$.

13. $f(x, y) = x^2 - 4xy + 5y^2 - 8y$; R is the closed triangular region with vertices $(0, 0)$, $(3, 0)$, and $(3, 3)$.

14. $g(x, y) = x^2 - y^2 - 2x$; R: is the closed region in the xy-plane bounded by the graphs $y = x^2$ and $y = 4$.

15. $f(x, y) = x^2+xy+y^2$; R is the closed square region defined by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$.

16. A closed rectangular box with a volume of 16 ft^3 is made from two kinds of materials. The top and bottom are made of material costing \$0.10 per square foot and the sides are made of material costing \$0.05 per square foot. Find the dimensions of the box so the cost is minimized.

17. Determine the dimensions of a rectangular box, open at the top, which has a volume of 32ft^3 and requires the least amount of material for construction.

18. Consider the region R which satisfies all of the following constraints: $x \geq 0$, $y \geq 0$, $x + 2y \leq 6$, $2x + y \leq 6$.

    (a) On the same set of axes, sketch R. Also sketch the level curves $f(x, y) = k$ of $f(x, y) = x + y - 1$ for $k = -1, 0, 1, 2, 3$.

    (b) At which point will $f(x, y)$ achieve an absolute maximum value? And, what is this maximum value?

    (c) At which point will $f(x, y)$ achieve an absolute minimum value? And, what is this minimum value?