Convergence Tests: Divergence, Integral, and p-Series Tests

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Recognize series that cannot converge by applying the Divergence Test.

- Use the Integral Test on appropriate series (all terms positive, corresponding function is decreasing and continuous) to make a conclusion about the convergence of the series.

- Recognize a p-series and use the value of p to make a conclusion about the convergence of the series.

- Use the algebraic properties of series.

PRACTICE PROBLEMS:

For problems $1 - 9$, apply the Divergence Test. What, if any, conlcusions can you draw about the series?

1. $\displaystyle\sum_{k=1}^{\infty}(-1)^k$

2. $\displaystyle\sum_{k=1}^{\infty}(-1)^k\,\frac{1}{k}$

3. $\displaystyle\sum_{k=3}^{\infty}\frac{\ln k}{k}$

4. $\displaystyle\sum_{k=1}^{\infty}\frac{\ln 6k}{\ln 2k}$

5. $\displaystyle\sum_{k=1}^{\infty}ke^{-k}$

6. $\displaystyle\sum_{k=1}^{\infty}\frac{e^k - e^{-k}}{e^k + e^{-k}}$

7. $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^k$

8. $\displaystyle\sum_{k=1}^{\infty}(\sqrt{k^2 + 8k - 5} - k)$

9. $\displaystyle\sum_{k=2}^{\infty}(\sqrt{k^2 + 3} - \sqrt{k^2 - 4})$

For problems $10 - 20$, determine if the series converges or diverges by applying the Divergence Test, Integral Test, or noting that the series is a p-series. Explicitly state what test you are using. If you use the Integral Test, you must first verify that the test is applicable. If the series is a p-series, state the value of p.

10. $\displaystyle\sum_{k=3}^{\infty}\frac{\ln k}{k}$

11. $\displaystyle\sum_{k=1}^{\infty}ke^{-k}$

12. $\displaystyle\sum_{k=1}^{\infty}\left(\arctan\left(\frac{1}{k}\right) - \arctan(k)\right)$

13. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{\sqrt[4]{k + 15}}$

14. $\displaystyle\sum_{k=1}^{\infty}\pi^k e^{-k}$

15. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{4k^2}$

16. $\displaystyle\sum_{k=2}^{\infty}\frac{k^2}{4k^2 + 9}$

17. $\displaystyle\sum_{k=2}^{\infty}\frac{k}{4k^2 + 9}$

18. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{4k^2 + 9}$

19. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{4k^2 - 9}$

20. $\displaystyle\sum_{k=10}^{\infty} 15k^{-0.999}$

For problems 21 & 22, use algebraic properties of series to find the sum of the series.

21. $\displaystyle\sum_{k=1}^{\infty} \left[\frac{1}{6^k} - \left(\frac{1}{k} - \frac{1}{k+1} \right) \right]$

22. $\dfrac{1}{2} + 2 - \dfrac{1}{4} + \dfrac{4}{7} + \dfrac{1}{8} + \dfrac{8}{49} - \dfrac{1}{16} + \dfrac{16}{343} + \cdots$

[Hint: See <u>Infinite Series</u> problems #11 & #12.]