#18

All the terms in the series are positive.

$f(x) = \frac{1}{4x^2+9}$ is continuous on $[2,+\infty)$ and is decreasing on $[2,+\infty)$

since $f'(x) = \frac{-8x}{(4x^2+9)^2} < 0$ on $[2,+\infty)$. So we may apply

the Integral Test.

$$\int_2^{+\infty} \frac{1}{4x^2+9}\,dx = \lim_{t\to+\infty} \int_2^t \frac{1}{4x^2+9}\,dx$$

$$\int_2^t \frac{1}{4x^2+9}\,dx = \int_2^t \frac{1}{9\left(\frac{4}{9}x^2+1\right)}\,dx = \frac{1}{9}\int_2^t \frac{1}{\left(\frac{2}{3}x\right)^2+1}\,dx \qquad u=\frac{2}{3}x \quad du=\frac{2}{3}\,dx$$

$$= \frac{1}{9} \cdot \frac{3}{2} \int_{\frac{4}{3}}^{\frac{2}{3}t} \frac{1}{u^2+1}\, du = \frac{1}{6} \arctan u \Big|_{\frac{4}{3}}^{\frac{2}{3}t}$$

$$= \frac{1}{6}\left(\arctan\left(\frac{2}{3}t\right) - \arctan\frac{4}{3}\right)$$

$$\lim_{t \to +\infty} \frac{1}{6}\left(\arctan\left(\frac{2}{3}t\right) - \arctan\frac{4}{3}\right) = \frac{1}{6}\left[\frac{\pi}{2} - \arctan\frac{4}{3}\right]$$

Since $\int_{2}^{+\infty} \frac{1}{4x^2+9}\, dx$ converges, $\sum_{k=2}^{+\infty} \frac{1}{4k^2+9}$ converges.