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(a) $Mx'' + kx = 0$, M, k are positive constants

Auxiliary equation: $Mm^2 + k = 0$

$$m = \frac{0 \pm \sqrt{0^2 - 4Mk}}{2M} = \frac{2\sqrt{M}\sqrt{k}\,i}{2M} = 0 \pm \sqrt{\frac{k}{M}}\,i$$

Solution to ODE:

$$x(t) = c_1 e^{(0)t} \cos\left(\sqrt{\tfrac{k}{M}}\,t\right) + c_2 e^{(0)t} \sin\left(\sqrt{\tfrac{k}{M}}\,t\right)$$

$$x(t) = c_1 \cos\left(\sqrt{\tfrac{k}{M}}\,t\right) + c_2 \sin\left(\sqrt{\tfrac{k}{M}}\,t\right)$$

(b) $M = 3, k = 75 \Rightarrow \sqrt{\frac{k}{M}} = \sqrt{25} = 5$

Natural length = 1 meter and at $t = 0$ spring has been stretched to 1.5 meters, so

$$x(0) = 1.5 - 1 = 0.5$$

Initial velocity is $0 \Rightarrow x'(0) = 0$

$$x(0) = 0.5 \Rightarrow 0.5 = C_1(\cos 0) + C_2(\sin 0)$$

$$0.5 = C_1 + 0$$

$$C_1 = 0.5$$

$$x'(t) = -0.5(5)\sin(5t) + C_2(5)\cos(5t)$$

$$x'(0) = 0 \Rightarrow 0 = 0 + C_2(1) \Rightarrow C_2 = 0$$

So $x(t) = 0.5\cos(5t)$.