

Chapter 2.6 Practice Problems

EXPECTED SKILLS:

- Know how to use the chain rule to calculate derivatives of compositions of functions.

PRACTICE PROBLEMS:

For problems 1-16, calculate the derivative of the given function.

1. $f(x) = (x^3 + 4)^{-3}$

$$\boxed{-9x^2(x^3 + 4)^{-4}}$$

2. $f(x) = (x^2 + 2x)^6$

$$\boxed{6(2x + 2)(x^2 + 2x)^5}$$

3. $f(x) = \sqrt{x^3 - 2}$

$$\boxed{\frac{3x^2}{2\sqrt{x^3 - 2}}}$$

4. $f(x) = \tan\left(\frac{1}{x^2}\right)$

$$\boxed{-2x^{-3} \sec^2\left(\frac{1}{x^2}\right)}$$

5. $f(x) = \sec 2x$

$$\boxed{2 \sec(2x) \tan(2x)}$$

6. $f(x) = \cos^3 3x$

$$\boxed{-9 \sin(3x) \cos^2(3x)}$$

7. $f(x) = \left(x^5 - \frac{1}{x^2}\right)^4$

$$\boxed{4 \left(x^5 - \frac{1}{x^2}\right)^3 \left(5x^4 + \frac{2}{x^3}\right)}$$

8. $f(x) = \frac{x^2 - 3}{(3x - 5)^3}$

$$\boxed{\frac{2x}{(3x - 5)^3} - \frac{9(x^2 - 3)}{(3x - 5)^4}}$$

9. $f(x) = (x^2 + 2x)^5(x^2 - 4x)^3$

$$5(2x + 2)(x^2 + 2x)^4(x^2 - 4x)^3 + 3(2x - 4)(x^2 - 4x)^2(x^2 + 2x)^5$$

10. $f(x) = \sin\left(\frac{\pi}{x}\right)$

$$-\pi x^{-2} \cos\left(\frac{\pi}{x}\right)$$

11. $f(x) = \sin(\sin 2x)$

$$2 \cos(\sin 2x) \cos 2x$$

12. $f(x) = \tan^2(x^2 - 1)$

$$4x \tan(x^2 - 1) \sec^2(x^2 - 1)$$

13. $f(x) = \frac{2}{(x^5 + 4x^3 - 4x)^3}$

$$\frac{-6(5x^4 + 12x^2 - 4)}{(x^5 + 4x^3 - 4x)^4}$$

14. $f(x) = \left(\frac{x^2 - 1}{x^2 + 1}\right)^3$

$$\frac{12x(x^2 - 1)^2}{(x^2 + 1)^4}$$

15. $y = 4x^2 \csc 5x$

$$8x \csc(5x) - 20x^2 \csc(5x) \cot(5x)$$

16. $y = \tan(4 + x^2 \sin 3x)$

$$(3x^2 \cos 3x + 2x \sin 3x) \sec^2(4 + x^2 \sin 3x)$$

17. Use the given table to calculate each of the following quantities:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-2	-5	3	9
2	5	-3	4	-2
3	-1	6	7	-6
4	3	1	-2	5
5	4	7	1	8

(a) $\left. \frac{d}{dx}[f(g(x))] \right|_{x=2}$

$\boxed{-2}$

(b) $(f \circ g)'(1)$

$\boxed{54}$

(c) $\left. \frac{d}{dx}[f(3x)] \right|_{x=1}$

$\boxed{18}$

(d) $\left. \frac{d}{dx} \left[g \left(\sqrt{2} \sin \left(\frac{\pi}{4} x \right) \right) \right] \right|_{x=3}$

$\boxed{-\frac{9\pi}{4}}$

(e) $h'(2)$ if $h(x) = x^2 f(g(x))$

$\boxed{4}$

For problems 18-20, calculate $\frac{d^2 y}{dx^2}$.

18. $y = \sin 3x$

$\boxed{-9 \sin 3x}$

19. $y = x \left(1 + \frac{1}{x} \right)^2$

$\boxed{2x^{-3}}$

20. $y = \frac{1}{1 - 2x}$

$\boxed{\frac{8}{(1 - 2x)^3}}$

21. Suppose that $f(x)$ is a twice differentiable function and define $g(x) = x^3 f(2x)$. Compute $g''(x)$ in terms of f , f' , and f''

$\boxed{g''(x) = 4x^3 f''(2x) + 12x^2 f'(2x) + 6x f(2x)}$

22. Let $f(x) = \frac{5}{(x^2 + 1)^3}$. Compute an equation of the tangent line to the graph of $f(x)$ at $x = 0$.

$\boxed{y = 5}$

23. Where does the tangent line to $y = (5x + 7)^3$ at the point $(-1, 8)$ cross the x -axis?

$$x = -\frac{17}{15}$$

24. Find all points on the graph of $y = \sin^2 x$ where the tangent lines are parallel to the line $y = x$.

$$\frac{\pi}{4} + \pi k \text{ where } k \text{ is any integer}$$

25. What is the 100th derivative of $y = \sin(2x)$?

$$2^{100} \sin 2x$$

26. **Multiple Choice:** The derivative of $y = x^2 \cos\left(\frac{1}{x}\right)$ is

(a) $2x \cos\left(\frac{1}{x}\right) - x^2 \sin\left(\frac{1}{x}\right)$

(b) $\frac{2}{x} \sin\left(\frac{1}{x}\right)$

(c) $-2x \sin\left(\frac{1}{x}\right)$

(d) $2x \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$

(e) $\sin\left(\frac{1}{x}\right)$

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