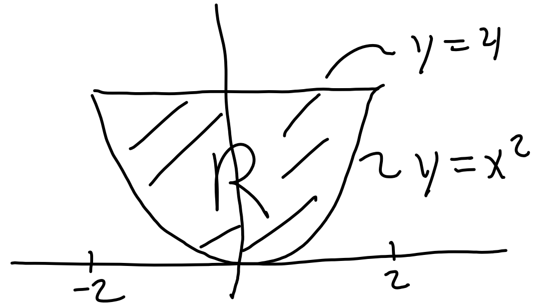
13.8 #14

$g(x,y) = x^2 - y^2 - 2x$



$g_x(x,y) = 2x - 2 = 0$

$g_y(x,y) = -2y = 0$

There is a critical point at $(1,0)$, but it is not in R.

Along $y=x^2$: $g(x,y) = g(x,x^2) = x^2 - x^4 - 2x = u(x) \quad -2 \le x \le 2$

$u'(x) = 2x - 4x^3 - 2 = 0$

$\iff 4x^3 - 2x + 2 = 0 \iff 2(2x^3 - x + 1) = 0$

$\iff 2(2x^2 - 2x + 1)(x+1) = 0$

$\iff x = -1$

$u(-1) = 2 \quad [\text{This is } g(-1,1) = 2]$

$u(-2) = -8 \quad [\text{This is } g(-2,4) = -8]$

$u(2) = -16 \quad [\text{This is } g(2,4) = -16]$

Along $y=4$: $g(x,y) = g(x,4) = x^2 - 16 - 2x = v(x) \quad -2 \le x \le 2$

$v'(x) = 2x - 2 = 0 \implies x = 1$

$v(1) = -17$ [This is $g(1,4) = -17$]

$v(-2)$ and $v(2)$ were computed earlier since they correspond to $g(-2,4)$ and $g(2,4)$.

So there is an absolute minimum of -17 at $(1,4)$ and an absolute maximum of 2 at $(-1,1)$.