Alternating Series; Absolute/Conditional Convergence

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Determine if an alternating series converges using the Alternating Series Test.
- Analyze the absolute values of the terms of a series and determine if it converges.
- Use any of the previously discussed convergence tests to determine if a series with negative terms converges absolutely, converges conditionally, or diverges.
- Find a partial sum that approximates a convergent alternating series to some specified accuracy.

PRACTICE PROBLEMS:

For problems 1 – 3, show that the series converges by verifying that it satisfies the hypotheses of the Alternating Series Test, or show that the series does not satisfy the hypotheses of the Alternating Series Test. If the latter, then use some other test to determine if the series converges or diverges.

1. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64} + \ldots$
2. $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{k^2}{2+k^3}$
3. $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1+2k}{3k}$

For problems 4 – 13, determine if the series converges absolutely, converges conditionally, or diverges.

4. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} - \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \frac{1}{512} - \frac{1}{1024} - \ldots$
5. $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{2+k^3}$
6. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{1}{4k^2+9}$

7. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \arcsin\left(\frac{1}{k}\right)$

8. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$

9. $\displaystyle\sum_{k=1}^{\infty} \cos(k\pi) k e^{-k}$

10. $\displaystyle\sum_{k=1}^{\infty} (-1)^k e^{(1/k)}$

11. $\displaystyle\sum_{k=3}^{\infty} (-1)^k \frac{\sqrt{k}}{k-2}$

12. $\displaystyle\sum_{k=1}^{\infty} \frac{\sin k}{k^4 + 4^k}$

13. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{(2k+1)!}{200^k}$

14. $\displaystyle\sum_{k=3}^{\infty} \sin\left[(2k+1)\frac{\pi}{2}\right] \frac{\ln k}{k}$

For problems 15 – 17, the series converge to some sum S. Find the smallest value of n so that the n-th partial sum s_n will guarantee the approximation of S to the required accuracy.

15. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$; $|S - s_n| < 0.001$

16. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[4]{k+7}}$; $|S - s_n| < 0.2$

17. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k k}$; $|S - s_n| < 0.0005$

18. Is the 300^{th} partial sum of $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k k}$ an overestimate or an underestimate of the sum of the series? Justify your answer.