Chapter 3.5 Practice Problems

EXPECTED SKILLS:

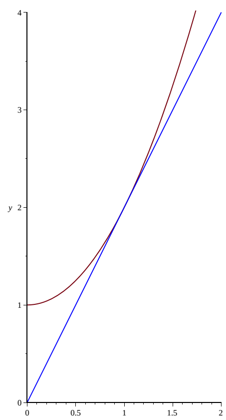
- Be able to compute the local linear approximation of a function at a specific value.
- Know how to use the local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-4, calculate the Local Linear Approximation, $L(x)$, for the given function at the specified value of x_0. Also, sketch $f(x)$ and $L(x)$ over the indicated interval

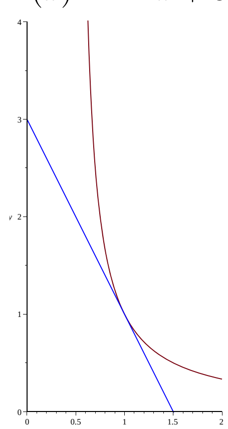
1. $f(x) = x^2 + 1$, $x_0 = 1$, $[0, 2]$

   $L(x) = 2x$

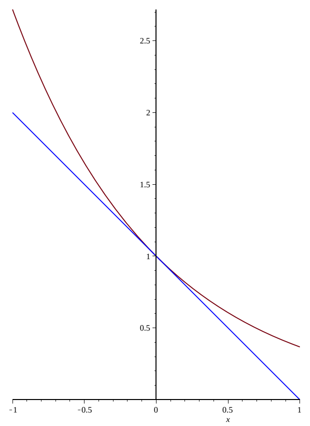
2. $f(x) = \dfrac{1}{2x - 1}$, $x_0 = 1$, $[0, 2]$

   $L(x) = -2x + 3$

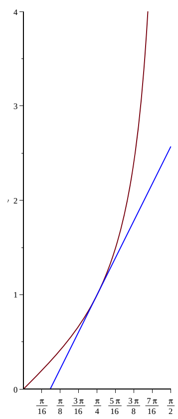
3. $f(x) = e^{-x}$, $x_0 = 0$, $[-1, 1]$

   $L(x) = -x + 1$

4. $f(x) = \tan(x)$, $x_0 = \dfrac{\pi}{4}$, $\left[0, \dfrac{\pi}{2}\right]$

   $L(x) = 2x - \dfrac{\pi}{2} + 1$

For problems 5-10, use an appropriate local linear approximation to approximate the following values.

5. $(5.05)^3$

   $\boxed{128.75}$

6. $\sqrt{101}$

   $\boxed{10.05}$

7. $\sqrt[3]{28}$

   $\boxed{\dfrac{82}{27}}$

8. $e^{0.9}$

$\boxed{\dfrac{9}{10}e}$

9. $\cos 0.1$

$\boxed{1}$

10. $\sin 61°$

$\boxed{\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{360}}$

11. Show that $(1-x)^5 \approx 1 - 5x$ for x near 0.

12. Show that $\ln(2x) \approx 2x - 1$ for x near $\dfrac{1}{2}$.

13. Consider $f(x) = (x+1)^{13}$.

    (a) Compute the Local Linear Approximation of $f(x)$ at $x_0 = 0$.

    $\boxed{(x+1)^{13} \approx 1 + 13x \text{ for } x \text{ near } 0}$

    (b) Using your approximation, estimate $(0.99)^{13}$.

    $\boxed{0.87}$

    (c) Using your approximation, estimate $(1.01)^{13}$

    $\boxed{1.13}$

14. Let $f(x) = x^2$.

    (a) Calculate the Local Linear Approximation, $L(x)$, for $f(x)$ at $x = a$.

    $\boxed{L(x) = 2ax - a^2}$

    (b) Does $L(x)$ overestimate or underestimate $f(x)$ near $x = a$? Explain.

    $L(x)$ is always an underestimate of $f(x)$ for x near a. Graphically, this should make sense since the tangent line to $f(x) = x^2$ is always below its graph.

    To show that $L(x)$ is always an underestimate, we will show that $L(x) \leq f(x)$ for all values of x. In other words, we will show that $L(x) - f(x) \leq 0$ for all x. So, we compute:

$$\begin{aligned} L(x) - f(x) &= 2ax - a^2 - x^2 \\ &= -(x^2 - 2ax + a^2) \\ &= -(x-a)^2 \\ &\leq 0 \end{aligned}$$

15. **Multiple Choice:** Which of the following is the best local linear approximation for $f(x) = \tan(x)$ near $x = \dfrac{\pi}{4}$?

(a) $1 + \left(x - \dfrac{\pi}{4}\right)$

(b) $1 + \dfrac{1}{2}\left(x - \dfrac{\pi}{4}\right)$

(c) $1 + \sqrt{2}\left(x - \dfrac{\pi}{4}\right)$

(d) $1 + 2\left(x - \dfrac{\pi}{4}\right)$

(e) $2 + 2\left(x - \dfrac{\pi}{4}\right)$

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