

#6

$$\tan x = \frac{\sin x}{\cos x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots} = \frac{x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \dots}{1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \dots}$$

Long division:

$$\begin{array}{r} x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots \\ 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \dots \overline{) \begin{array}{l} x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \dots \\ - (x - \frac{1}{2}x^3 + \frac{1}{24}x^5 - \dots) \\ \hline \frac{1}{3}x^3 - \frac{1}{30}x^5 + \dots \\ - (\frac{1}{3}x^3 - \frac{1}{6}x^5 + \dots) \\ \hline \frac{2}{15}x^5 + \dots \\ - (\frac{2}{15}x^5 + \dots) \\ \hline \end{array}} \end{array}$$

$$\text{So } \tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$$