Chapter 3.5: Trigonometric Equations

Expected Skills:

- Be able to solve trigonometric equations.

Practice Problems:

For problems 1-7, find all values of θ which satisfy the following equations.

1. $2\sin x + \sqrt{3} = 0$

   $\boxed{\theta = \frac{4\pi}{3} + 2\pi k \text{ or } \theta = \frac{5\pi}{3} + 2\pi k, \text{ where } k \text{ is any integer.}}$

2. $2\cos^2\theta - \cos\theta - 1 = 0$

   $\boxed{\theta = \frac{2\pi}{3} + 2\pi k,\ \theta = \frac{4\pi}{3} + 2\pi k, \text{ or } \theta = 2\pi k, \text{ where } k \text{ is any integer.}}$

3. $\sin(2\theta) + \cos\theta = 0$

   $\boxed{\theta = \frac{\pi}{2} + \pi k,\ \theta = \frac{7\pi}{6} + 2\pi k, \text{ or } \theta = \frac{11\pi}{6} + 2\pi k, \text{ where } k \text{ is any integer.}}$

4. $\sin\theta - \cos(2\theta) = 0$

   $\boxed{\theta = \frac{3\pi}{2} + 2\pi k,\ \theta = \frac{\pi}{6} + 2\pi k, \text{ or } \theta = \frac{5\pi}{6} + 2\pi k, \text{ where } k \text{ is any integer.}}$

5. $\sec^2\theta - 2 = 0$

   $\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{2}k, \text{ where } k \text{ is any integer.}}$

6. $|\tan\theta| = \sqrt{3}$

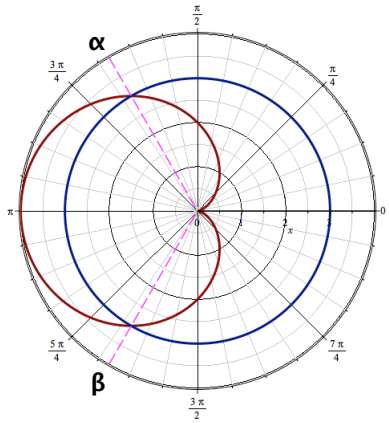
   $\boxed{\theta = \frac{\pi}{3} + \pi k \text{ or } \theta = \frac{2\pi}{3} + \pi k, \text{ where } k \text{ is any integer.}}$

7. $\tan 4\theta = -1$

   $\boxed{\theta = \frac{3\pi}{16} + \frac{\pi}{4}k, \text{ where } k \text{ is any integer.}}$

In Math 122, you will study the polar coordinate system. In the polar coordinate system, we identify the location of a point in the plane as an ordered pair (r, θ) where r is the distance of the point from the origin and θ is the angle from the positive x-axis. In this coordinate system, we often describe curves by expressing r as a function of θ, $r = f(\theta)$.

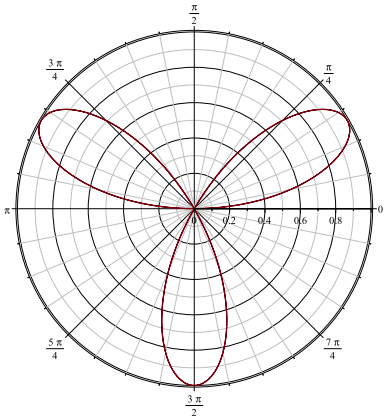
8. The curve $r = 2 - 2\cos\theta$ is called a cardioid, shown in red below. The curve $r = 3$ is the blue circle shown below.



Find the angles α and β at which these curves intersect where $0 < \alpha < \beta < 2\pi$

$$\boxed{\alpha = \frac{2\pi}{3} \text{ and } \beta = \frac{4\pi}{3}}$$

9. The curve $r = \sin(3\theta)$ describes the rose, shown below.



(a) Find all values of θ in the interval $[0, \pi]$ at which the curve passes through the origin. (Hint: At these points, $r = 0$.)

$$\boxed{\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi}$$

(b) Find all values of θ in the interval $[0, \pi]$ which correspond to the "tips" of the rose petals. (Hint: At these points, either $r = 1$ or $r = -1$.)

$$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}$$