Chapter 3.3 Practice Problems

EXPECTED SKILLS:

- Know how to compute the derivatives of exponential functions.
- Be able to compute the derivatives of the inverse trigonometric functions, specifically, $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ and $\sec^{-1} x$.
- Know how to apply logarithmic differentiation to compute the derivatives of functions of the form $(f(x))^{g(x)}$, where f and g are non-constant functions of x.

PRACTICE PROBLEMS:

For problems 1-16, differentiate. In some cases it may be better to use logarithmic differentiation.

1. $y = e^{6x}$
2. $g(x) = xe^{2x}$
3. $f(x) = 5^{x^2}$
4. $y = e^x \cos x$
5. $g(x) = e^{x^2(x-1)}$
6. $f(x) = \dfrac{1 - e^{2x}}{1 - e^x}$
7. $f(x) = \dfrac{\ln x}{e^x + 3x}$
8. $f(x) = \ln(e^x + 5)$
9. $y = x^{x^2}$
10. $f(x) = e^{\cos^2 2x + \sin^2 2x}$
11. $h(x) = \exp\left(\dfrac{1}{1 - \ln x}\right)$
12. $f(x) = (\ln x)^{e^x}$
13. $y = \cos^{-1}(3x)$
14. $y = \arcsin(x^2)$
15. $y = \dfrac{\arctan(e^x)}{x^3}$

16. $y = x^{\cos x}$

17. Compute an equation of the line which is tangent to the graph of $y = e^{3x}$ at the point where $x = \ln 2$.

18. Compute an equation of the line which is tangent to the graph of $f(x) = \cos^{-1} x$ at the point where $x = \dfrac{1}{2}$.

19. Find all value(s) of x at which the tangent lines to the graph of $f(x) = \tan^{-1}(4x)$ are perpendicular to the line which passes through $(0, 1)$ and $(2, 0)$.

20. Find a linear function $T_1(x) = mx + b$ which satisfies both of the following conditions:

    - $T_1(x)$ has the same y-intercept as $f(x) = e^{2x}$.
    - $T_1(x)$ has the same slope as $f(x) = e^{2x}$ at the y-intercept.

21. Compute an equation of the line which is tangent to the curve $e^{xy^2} + y = x^4$ at $(-1, 0)$.

22. The equation $y'' + 5y' - 6y = 0$ is called a differential equation because it involves an unknown function y and its derivatives. Find the value(s) of the constant A for which $y = e^{Ax}$ satisfies this equation.

23. Evaluate $\lim\limits_{h \to 0} \dfrac{\sin^{-1}\left(\frac{\sqrt{3}}{2} + h\right) - \frac{\pi}{3}}{h}$ by interpreting the limit as the derivative of a function a particular point.

24. **Multiple Choice:** Which of the following is the equation of the tangent line to the graph of $f(x) = \tan^{-1}(2x)$ at the point where $x = 0$?

    (a) $y = x$

    (b) $y = x + 1$

    (c) $y = x - 1$

    (d) $y = 2x$

    (e) $y = 2x - 1$

25. **Multiple Choice:** Consider the curve defined implicitly by $\sin x = e^y$ for $0 < x < \pi$. What is $\dfrac{dy}{dx}$ in terms of x?

(a) $-\tan x$

(b) $-\cot x$

(c) $\cot x$

(d) $\tan x$

(e) $\csc x$

26. Consider the following two hyperbolic functions:

Hyperbolic Sine	Hyperbolic Cosine
$\sinh x = \dfrac{e^x - e^{-x}}{2}$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$

(a) Compute $\lim\limits_{x \to \infty} \sinh x$

(b) Compute $\lim\limits_{x \to -\infty} \sinh x$

(c) Compute $\lim\limits_{x \to \infty} \cosh x$

(d) Compute $\lim\limits_{x \to -\infty} \cosh x$

(e) Compute the x and y intercepts, if any, for $y = \sinh x$.

(f) Compute the x and y intercepts, if any, for $y = \cosh x$.

(g) Solve $\sinh x = 1$ for x.

(h) Show that $\cosh^2 x - \sinh^2 x = 1$

(i) Show that $\dfrac{d}{dx}(\sinh x) = \cosh x$

(j) Show that $\dfrac{d}{dx}(\cosh x) = \sinh x$