14.1 #8

We should integrate with respect to y first.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \int_{0}^{1} x\sec^2(xy)\sec^2 x \, dy\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left[x\sec^2 x \left(\frac{1}{x}\right) \tan(xy) \Big|_0^1 \right] dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 x \,(\tan x - \tan 0)\, dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 x \tan x \, dx$$

$u = \tan x$ $\quad du = \sec^2 x\, dx$

$x = \frac{\pi}{3} \Rightarrow u = \tan\frac{\pi}{3} = \sqrt{3}$

$x = \frac{\pi}{4} \Rightarrow u = \tan\frac{\pi}{4} = 1$

$$= \int_{1}^{\sqrt{3}} u\, du = \frac{1}{2}u^2 \Big|_1^{\sqrt{3}} = \frac{1}{2}(3-1) = 1$$