13.4 #11

$f(x,y) = xy + y^2$

$\quad f(x_0, y_0) = x_0 y_0 + y_0^2$

$\quad f_x(x,y) = y \implies f_x(x_0, y_0) = y_0$

$\quad f_y(x,y) = x + 2y \implies f_y(x_0, y_0) = x_0 + 2y_0$

Now at (x_0, y_0) we are told that

$\quad L(x,y) = -15 + 3x + 8y.$

So $\quad -15 + 3x + 8y = x_0 y_0 + y_0^2 + y_0(x - x_0) + (x_0 + 2y_0)(y - y_0)$

The only way the left-hand and right-hand sides are equal is

if the coefficients of x and y on both sides are equal.

So $\quad 3 = y_0$ and $8 = x_0 + 2y_0 \iff 8 = x_0 + 6 \iff x_0 = 2$

$\quad$ So $(x_0, y_0) = (2, 3).$