Chapter 3.6 Practice Problems

EXPECTED SKILLS:

- Know how to use L'Hopital's Rule to help compute limits involving indeterminate forms of $\frac{0}{0}$ and $\frac{\infty}{\infty}$
- Be able to compute limits involving indeterminate forms $\infty - \infty$, $0 \cdot \infty$, 0^0, ∞^0, and 1^∞ by manipulating the limits into a form where L'Hopital's Rule is applicable.

PRACTICE PROBLEMS:

For problems 1-27, calculate the indicated limit. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate). Make sure that L'Hopital's rule applies before using it. And, whenever you apply L'Hopital's rule, indicate that you are doing so.

1. $\displaystyle\lim_{x\to 3} \frac{x^2+4x-21}{x^2-7x+12}$

2. $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{\ln(1+x)}$

3. $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{x^2}$

4. $\displaystyle\lim_{x\to 0} \frac{\sin(6x)}{\sin(3x)}$

5. $\displaystyle\lim_{x\to 1^-} \frac{x-1}{x^2-2x+1}$

6. $\displaystyle\lim_{x\to \infty} \frac{e^{-x}}{x^{-2}}$

7. $\displaystyle\lim_{x\to 0} \frac{\ln(\cos 3x)}{5x^2}$

8. $\displaystyle\lim_{x\to 1} \frac{\tan^{-1} x - \frac{\pi}{4}}{x-1}$

9. $\displaystyle\lim_{x\to 0^+} \frac{8^{\sqrt{x}}-1}{1-5^{\sqrt{x}}}$

10. $\displaystyle\lim_{x\to 0^+} \frac{5\sin x}{\sqrt{x}}$

11. $\lim_{x\to-\infty} \dfrac{x^3+4x-5}{5x^2-5x-89}$

12. $\lim_{x\to 0} \dfrac{\sin^{-1}(2x)}{\tan^{-1}(3x)}$

13. $\lim_{x\to 1} \dfrac{\ln x^2}{x^2-9}$

14. $\lim_{x\to\infty} \dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

15. $\lim_{x\to\frac{\pi}{2}^+} \dfrac{\ln\left(x-\frac{\pi}{2}\right)}{\tan x}$

16. $\lim_{x\to 1^-} \dfrac{x-1}{\arccos x}$

17. $\lim_{x\to+\infty} \dfrac{e^{\sqrt{x}}}{x}$

18. $\lim_{x\to+\infty} xe^{-6x}$

19. $\lim_{x\to+\infty} \dfrac{\sqrt{4+3x^2}}{2+2x}$

20. $\lim_{x\to 0^+} x\csc 3x$

21. $\lim_{x\to+\infty} [\ln(x+2)-\ln(3x+5)]$

22. $\lim_{x\to\infty} 3^x 7^{-x}$

23. $\lim_{x\to\infty} \left(\sqrt{x^2-x}-x\right)$

24. $\lim_{x\to 0^+} \tan x \sec x$

25. $\lim_{x\to 0^+} x^{1/x}$

26. $\lim_{x\to\infty} \left(1+\dfrac{2}{x}\right)^{5x}$

27. $\lim_{x\to\infty} \left(1-\dfrac{1}{x}\right)^{-x}$

28. Which of the following are indeterminate forms?

$$\frac{0}{0} \quad \frac{0}{\infty} \quad \frac{\infty}{0} \quad \frac{\infty}{\infty}$$
$$\infty - \infty \quad \infty + \infty \quad 0 \cdot \infty \quad \infty \cdot \infty$$
$$0^0 \quad \infty^0 \quad 0^\infty \quad 1^\infty \quad \infty^\infty \quad \infty^1$$

29. Calculate each of the following limits:

(a) $\lim_{x \to 0^+} (1 + 3^x)^{1/x}$

(b) $\lim_{x \to 0^-} (1 + 3^x)^{1/x}$

(c) $\lim_{x \to +\infty} (1 + 3^x)^{1/x}$

(d) $\lim_{x \to -\infty} (1 + 3^x)^{1/x}$

30. Show that $\lim_{x \to \infty} \frac{x^n}{e^x} = 0$ for any positive integer n.

31. Find the value(s) of of the constant k which make $f(x) = \begin{cases} \dfrac{\sin x - 1}{x - \frac{\pi}{2}} & \text{if} \quad x \neq \dfrac{\pi}{2} \\ k & \text{if} \quad x = \dfrac{\pi}{2} \end{cases}$ continuous at $x = \dfrac{\pi}{2}$.

32. Find all values of k and m such that $\lim_{x \to 1} \frac{k + m \ln x}{x - 1} = 5$

33. **Multiple Choice:** What is $\lim_{x \to 1^+} \frac{x}{\ln(x)}$?

(a) 0

(b) 1

(c) e

(d) e^{-1}

(e) $+\infty$

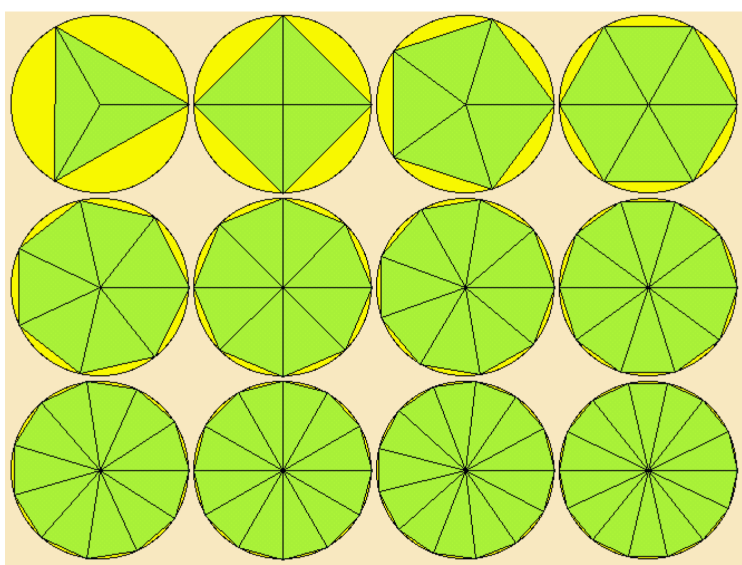
34. **Multiple Choice:** What is $\lim_{x\to 0} \frac{e^x - 1}{\tan(x)}$?

(a) -1
(b) 0
(c) 1
(d) 2
(e) The limit does not exist.

35. **Multiple Choice:** If $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} g(x) = +\infty$ and $f'(x) = 1$ and $g'(x) = e^x$, what is $\lim_{x\to+\infty} \frac{f(x)}{g(x)}$?

(a) -1
(b) 0
(c) 1
(d) e
(e) The limit does not exist.

36. A Regular Polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length). The diagram below shows several regular polygons inscribed within a circle of radius r.



(a) Let A_n be the area of a regular n-sided polygon inscribed within a circle of radius r. Divide the polygon into n congruent triangles each with a central angle of $\frac{2\pi}{n}$ radians, as shown in the diagram above for several different values of n. Show that $A_n = \frac{1}{2}r^2 \sin\left(\frac{2\pi}{n}\right)n$.

(b) What can you conclude about the area of the n-sided polygon as the number of sides of the polygon, n, approaches infinity? In other words, compute $\lim_{n\to\infty} A_n$.