Polynomial Approximations of Functions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Find and use the local linear and local quadratic approximations of a function $f(x)$ at a specified $x = x_0$.
- Determine the Maclaurin polynomials of various degrees for a function $f(x)$, and use sigma notation to write the n-th Maclaurin polynomial.
- Determine the Taylor polynomials of various degrees for a function $f(x)$ at a specified $x = x_0$, and use sigma notation to write the n-th Taylor polynomial.

PRACTICE PROBLEMS:

1. Consider the function $f(x) = \sqrt{x}$.
   (a) Find the local linear approximation $p_1(x)$ and the local quadratic approximation $p_2(x)$ to $f(x)$ at $x = 4$.
   (b) Approximate $\sqrt{4.1}$ using your answers in part (a).

For problems 2 – 4, use the appropriate local linear and local quadratic approximations to approximate the following values.

2. $\sin 0.1$
3. $\sqrt[3]{28}$
4. $\tan 44^\circ$
5. Suppose that the values of $f(x)$ and its first four derivatives at $x = 0$ are as follows:

   $f(0) = 5 \quad f'(0) = -2 \quad f''(0) = 0 \quad f'''(0) = -1 \quad f^{(4)}(0) = 12$

   Based on this information, list out as many Maclaurin polynomials for $f(x)$ as possible.
6. Find the 4th Maclaurin polynomial $p_4(x)$ for the function $f(x) = 2x^4 - x^3 + 6$.

For problem 7, find the Macluarin polynomials $p_0(x), p_1(x), p_2(x), p_3(x)$, and $p_4(x)$. Then write the n-th Maclaurin polynomial $p_n(x)$ using sigma notation.

7. $f(x) = \ln(1 + x)$

For problems 8 & 9, find the Taylor polynomials $p_0(x), p_1(x), p_2(x), p_3(x)$, and $p_4(x)$ about $x = x_0$. Then write the n-th Taylor polynomial $p_n(x)$ at $x = x_0$ using sigma notation.

8. $f(x) = \dfrac{1}{1-x};\ x_0 = 2$

9. $f(x) = e^{2x};\ x_0 = \ln 3$