The Definite Integral

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 5.5 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to evaluate the definite integral of a function over a given interval using geometry.
- Be familiar with the interpretation of the definite integral of a function over an interval as the net signed area between the graph of the function and the x-axis.
- Know how to use linearity properties of the definite integral to evaluate scalar multiples, sums, and differences of integrable functions.

PRACTICE PROBLEMS:

For problems 1 & 2, use the given values of a and b to express the given limit as a definite integral. Do not evaluate the limits or integrals.

1. $\displaystyle\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \frac{1}{1+(x_k^*)^2} \Delta x_k,\ a=-1,\ b=1.$

2. $\displaystyle\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \cos{(x_k^*)} \Delta x_k,\ a=0,\ b=\pi.$

For problems 3-9, sketch the region whose net signed area is represented by the given definite integral. Evaluate the given integral using an appropriate formula from geometry.

3. $\displaystyle\int_0^7 (x+1)\,dx$

4. $\displaystyle\int_{-7}^7 x\,dx$

5. $\displaystyle\int_{-1}^4 6\,dx$

6. $\displaystyle\int_{-4}^2 |x-1|\,dx$

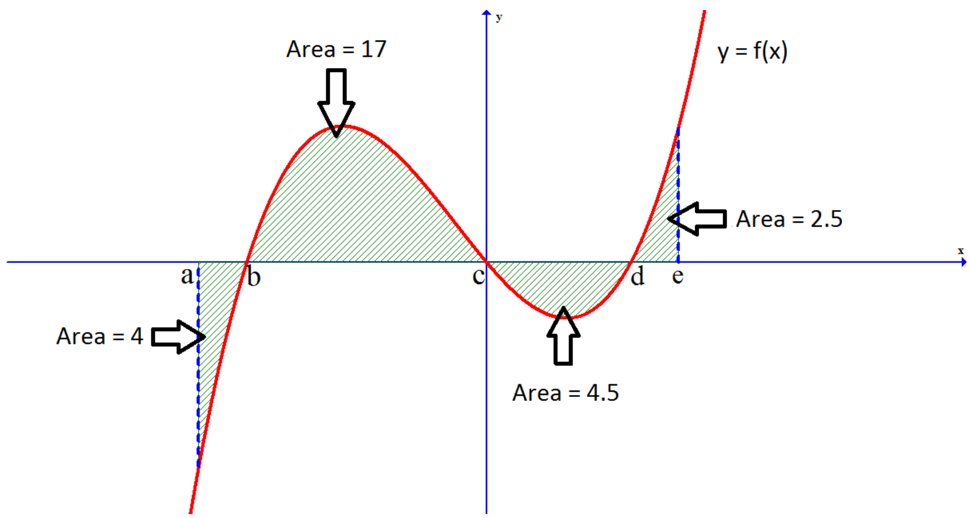
7. $\displaystyle\int_{-2}^{2} \sqrt{4-x^2}\,dx$

8. $\displaystyle\int_{-2}^{0} \left(3x+5\sqrt{4-x^2}\right)dx$

9. $\displaystyle\int_{4}^{8} \sqrt{8x-x^2}\,dx$

10. Let $f(x) = \begin{cases} 3x & \text{if } x \le 2 \\ 6 & \text{if } x > 2 \end{cases}$. Compute $\displaystyle\int_{-1}^{5} f(x)\,dx$.

11. For each of the following, use the areas shown to evaluate the given definite integral.



(a) $\displaystyle\int_{b}^{c} f(x)\,dx$

(b) $\displaystyle\int_{c}^{d} f(x)\,dx$

(c) $\displaystyle\int_{a}^{e} f(x)\,dx$

(d) $\displaystyle\int_{b}^{a} f(x)\,dx$

12. Again consider the graph of $y = f(x)$ shown in problem 11. Compute $\displaystyle\int_{a}^{e} |f(x)|\,dx$ and $\left|\displaystyle\int_{a}^{e} f(x)\,dx\right|$. Which is larger?

13. Suppose that $\int_{-1}^{3} f(x)\,dx = 6$ and $\int_{-1}^{3} g(x)\,dx = -8$. Compute $\int_{-1}^{3} (f(x) + 4g(x))\,dx$.

14. Suppose that $\int_{0}^{8} f(x)\,dx = 3$ and $\int_{4}^{8} f(x)\,dx = 10$. Compute $\int_{0}^{4} f(x)\,dx$.

15. Suppose that $\int_{-2}^{9} f(x)\,dx = 4$ and $\int_{-2}^{6} f(x)\,dx = 11$. Compute $\int_{9}^{6} f(x)\,dx$.

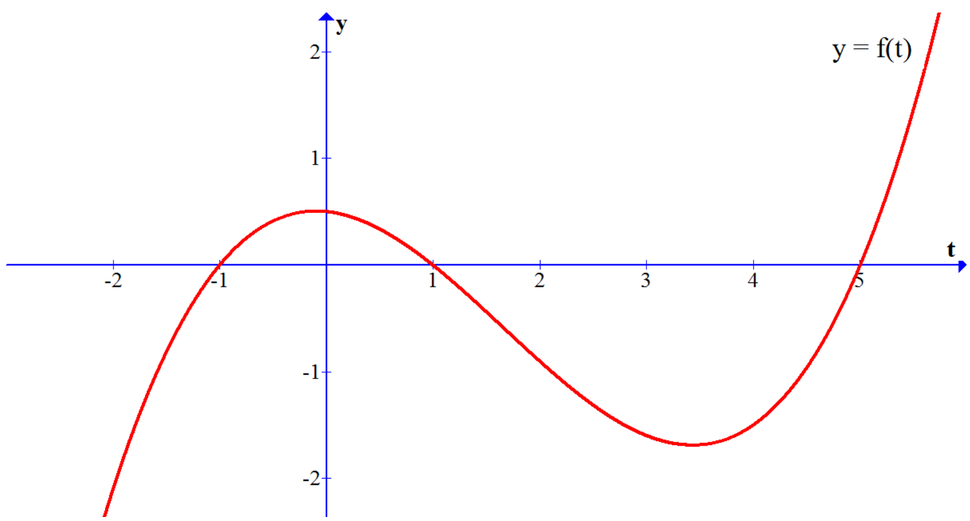
16. Express each of the following in terms of $\int_{0}^{\pi} \sin x\,dx$. **Do not evaluate any of the integrals.** Hint: Draw a graph and consider the net signed area.

   (a) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x\,dx$.

   (b) $\int_{\frac{3\pi}{2}}^{2\pi} \cos x\,dx$.

   (c) $\int_{\pi}^{\frac{3\pi}{2}} \sin x\,dx$.

17. Suppose $F(x) = \int_{0}^{x} f(t)\,dt$, where $f(t)$ is shown below.



Arrange the following quantities in order from least to greatest. $F(0)$, $F(1)$, $F(5)$, $F(1) - F(5)$, $F(5) - F(1)$

18. The following Riemann Sum was derived by dividing an interval $[a, b]$ into n subintervals of equal width and then choosing x_k^* to be the right endpoint of each subinterval.

$$\lim_{n\to+\infty} \sum_{k=1}^{n} \left(1 + \frac{4}{n}k\right) \frac{4}{n}$$

(a) What is the interval, $[a, b]$?

(b) Convert the Riemann Sum to an equivalent definite integral.

(c) Using the definite integral from part (b) and an appropriate formula from geometry, evaluate the limit.