# #19

All the terms in the series are positive.

$f(x) = \dfrac{1}{4x^2-9}$ is continuous on $[2,+\infty)$ and is decreasing on $[2,+\infty)$

since $f'(x) = \dfrac{-8x}{(4x^2-9)^2} < 0$ on $[2,+\infty)$.

So we may apply the Integral Test.

$$\int_2^{+\infty} \frac{1}{4x^2-9}\,dx = \lim_{t\to+\infty} \int_2^{t} \frac{1}{4x^2-9}\,dx$$

$$\frac{1}{4x^2-9} = \frac{1}{(2x-3)(2x+3)} = \frac{A}{2x-3} + \frac{B}{2x+3}$$

$$1 = A(2x+3) + B(2x-3)$$

$$x = \frac{3}{2}: \quad 1 = 6A \Rightarrow A = \frac{1}{6}$$

$$x = -\frac{3}{2}: \quad 1 = -6B \Rightarrow B = -\frac{1}{6}$$

So $$\int_2^t \frac{1}{4x^2-9}\,dx = \int_2^t \frac{\frac{1}{6}}{2x-3}\,dx - \int_2^t \frac{\frac{1}{6}}{2x+3}\,dx$$

$$= \frac{1}{6}\cdot\frac{1}{2}\ln|2x-3|\Big|_2^t - \frac{1}{6}\cdot\frac{1}{2}\ln|2x+3|\Big|_2^t$$

$$= \frac{1}{12}\ln\left|\frac{2x-3}{2x+3}\right|\Bigg|_2^t = \frac{1}{12}\left[\ln\left|\frac{2t-3}{2t+3}\right| - \ln\frac{1}{7}\right]$$

Now $$\lim_{t\to+\infty} \frac{1}{12}\left[\ln\left|\frac{2t-3}{2t+3}\right| - \ln\frac{1}{7}\right] = \frac{1}{12}\left[\ln 1 - \ln\frac{1}{7}\right] = -\frac{1}{12}\ln\frac{1}{7}$$

Since the integral $\int_2^{+\infty} \frac{1}{4x^2-9}\,dx$ converges,

the series $\sum_{k=2}^{+\infty} \frac{1}{4k^2-9}$ converges as well.