Local Linear Approximation

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.4 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute the local linear approximation for a function of two or more variables at a given point.
- Be able to use a local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-5, find the local linear approximation $L(x, y)$ of the given function at the specified point.

1. $f(x, y) = x^2 - y^2$; $P(1, 2)$

   $\boxed{L(x, y) = 3 + 2x - 4y}$

2. $f(x, y) = \dfrac{x + y}{x - y}$; $P(2, 1)$

   $\boxed{L(x, y) = 3 - 2x + 4y}$

3. $f(x, y) = e^x \sin y$; $P\left(\ln 3, \dfrac{\pi}{2}\right)$

   $\boxed{L(x, y) = 3 - 3\ln 3 + 3x}$

4. $f(x, y) = \ln(x^2 - y^2)$; $P\left(2, \sqrt{3}\right)$

   $\boxed{L(x, y) = -2 + 4x - 2\sqrt{3}y}$

5. $f(x, y) = \tan^{-1}\left(\dfrac{x}{y}\right)$; $P(1, 1)$

   $\boxed{L(x, y) = \dfrac{\pi}{4} + \dfrac{1}{2}x - \dfrac{1}{2}y}$; Detailed Solution: Here

6. Find the local linear approximation of the function $f(x,y) = \sqrt{32 - 3x^2 - y^2}$ at $(1,2)$ and use it to approximate $f(0.98, 2.01)$.

$f(x,y) \approx L(x,y) = \dfrac{32}{5} - \dfrac{3}{5}x - \dfrac{2}{5}y$ for (x,y) near $(1,2)$.

So, $f(0.98, 2.01) \approx L(0.98, 2.01) = \dfrac{626}{125}$

7. Suppose that $f(x,y)$ is a differentiable function at the point $(2,3)$ with $f(2,3) = 1$, $f_x(2,3) = 5$, and $f_y(2,3) = -2$. Estimate $f(1.98, 3.01)$.

$f(x,y) \approx L(x,y) = -3 + 5x - 2y$ for (x,y) near $(2,3)$.
So, $f(1.98, 3.01) \approx L(1.98, 3.01) = 0.88$

8. Find the local linear approximation $L(x,y,z)$ to $f(x,y,z) = 3x^2 - 2y^2 + xz^3$ at the point $P(-1,2,1)$.

$L(x,y,z) = 8 - 5x - 8y - 3z$

9. Verify that $e^x \cos y \approx 1 + x$ for (x,y) near $(0,0)$.

Show that the local linear approximation to $f(x,y) = e^x \cos y$ at the point $(0,0)$ is $L(x,y) = 1 + x$

10. Verify that $(x+y)^3 \approx -16 + 12x + 12y$ for (x,y) near $(1,1)$.

Show that the local linear approximation to $f(x,y) = (x+y)^3$ at the point $(1,1)$ is $L(x,y) = -16 + 12x + 12y$

11. At a particular point $P(x_0, y_0)$, the local linear approximation of $f(x,y) = xy + y^2$ is $L(x,y) = -15 + 3x + 8y$. What is the point P?

$(2,3)$; Detailed Solution: Here