Monotone Sequences

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Understand what it means for a sequence to be increasing, decreasing, strictly increasing, strictly decreasing, eventually increasing, or eventually decreasing.
- Use an approriate test for monotonicity to determine if a sequence is increasing or decreasing.
- Show that a sequence must converge to a limit by showing that it is montone and appropriately bounded.

PRACTICE PROBLEMS:

1. Give an example of a convergent sequence that is not a monotone sequence.
2. Give an example of a sequence that is bounded from above and bounded from below but is not convergent.

For problems 3 and 4, determine if the sequence is increasing or decreasing by calculating $a_{n+1} - a_n$.

3. $\left\{ \dfrac{1}{4^n} \right\}_{n=1}^{+\infty}$
4. $\left\{ \dfrac{2n-3}{3n-2} \right\}_{n=1}^{+\infty}$

For problems 5 amd 6, determine if the sequence is increasing or decreasing by calculating $\dfrac{a_{n+1}}{a_n}$.

5. $\left\{ \dfrac{1}{4^n} \right\}_{n=1}^{+\infty}$
6. $\left\{ \dfrac{e^n - e^{-n}}{e^n + e^{-n}} \right\}_{n=1}^{+\infty}$

For problems 7 and 8, determine if the sequence is increasing or decreasing by calculating the derivative a_n'.

7. $\left\{\dfrac{1}{4^n}\right\}_{n=1}^{+\infty}$

8. $\left\{\dfrac{\ln(2n)}{\ln(6n)}\right\}_{n=1}^{+\infty}$

For problems 9 – 17, use an appropriate test for monotonicity to determine if the sequence increases, decreases, eventually increases, or eventually decreases.

9. $\left\{\dfrac{3n}{2n+1}\right\}_{n=1}^{+\infty}$

10. $\left\{n-\dfrac{1}{n}\right\}_{n=1}^{+\infty}$

11. $\left\{\dfrac{n^2}{n!}\right\}_{n=1}^{+\infty}$

12. $\left\{\dfrac{2n+1}{(2n)!}\right\}_{n=1}^{+\infty}$

13. $\left\{\dfrac{e^{\sqrt{n}}}{n}\right\}_{n=1}^{+\infty}$

14. $\left\{e^n\pi^{-n}\right\}_{n=1}^{+\infty}$

15. $\left\{\dfrac{3^{(n^2)}}{(1000)^n}\right\}_{n=1}^{+\infty}$

16. $\left\{\dfrac{n!}{n^n}\right\}_{n=1}^{+\infty}$

17. $\left\{n^3e^{-n}\right\}_{n=1}^{+\infty}$

18. In the previous set of assigned problems it was shown that **if** the sequence

$$\sqrt{30}, \sqrt{30+\sqrt{30}}, \sqrt{30+\sqrt{30+\sqrt{30}}}, \ldots$$

converged to a limit, that limit was 6. Now we will show that the sequence is bounded above and increasing; thus, it must converge.

(a) Define the sequence recursively.

(b) Show that the sequence has an upper bound of 6.

(c) Show that the sequence is increasing by computing $a_{n+1}^2 - a_n^2$.