The Comparison, Limit Comparison, Ratio, & Root Tests

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Use the following tests to make a conclusion about the convergence of series with no negative terms:

  - Comparison Test

  - Limit Comparison Test

  - Ratio Test

  - Root Test

PRACTICE PROBLEMS:

For problems 1 & 2, apply the Comparison Test to determine if the series converges. Clearly state to which other series you are comparing.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3^k + 5}$

2. $\displaystyle\sum_{k=3}^{\infty} \frac{1}{3(k-2)}$

For problems 3 & 4, apply the Limit Comparison Test to determine if the series converges. Clearly state to which other series you are comparing.

3. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3(k+2)}$

4. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{3^k - 5}$

For problems 5 – 7, apply the Ratio Test to determine if the series converges. If the Ratio Test is inconclusive, apply a different test.

5. $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k!}$

6. $\displaystyle\sum_{k=0}^{\infty} \frac{1}{3^k}$

7. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3(k+2)}$

For problems 8 – 10, apply the Root Test to determine if the series converges. If the Root Test is inconclusive, apply a different test.

8. $\displaystyle\sum_{k=0}^{\infty} \frac{1}{3^k}$

9. $\displaystyle\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^k$

10. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{7^k}$

For problems 11 – 22, apply the Comparison Test, Limit Comparison Test, Ratio Test, or Root Test to determine if the series converges. State which test you are using, and if you use a comparison test, state to which other series you are comparing to.

11. $\displaystyle\sum_{k=1001}^{\infty} \frac{1}{\sqrt[3]{k} - 10}$

12. $\displaystyle\sum_{k=1}^{\infty} \frac{4k^2 + 5k}{\sqrt{10 + k^5}}$

13. $\displaystyle\sum_{k=0}^{\infty} \frac{2k + 1}{(2k)!}$

14. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2 \cos^2 k}{2 + k^5}$

15. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{(5k)}}$

16. $\displaystyle\sum_{k=0}^{\infty} \frac{2 + 2^k}{3 + 3^k}$

17. $\displaystyle\sum_{k=0}^{\infty} \frac{6^{(k+1)}}{k!}$

18. $\displaystyle\sum_{k=0}^{\infty} \frac{6^k + k}{k! + 6}$ [Hint:Use the result from the previous problem.]

19. $\displaystyle\sum_{k=2}^{\infty} \frac{k^3 - 2}{\left(k^2 + 1\right)^2}$

20. $\displaystyle\sum_{k=1}^{\infty} \frac{\arctan k}{k^{1.5}}$

21. $\displaystyle\sum_{k=1}^{\infty} \frac{7k}{k^2 + |\sin k|}$

22. $\displaystyle\sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^2}$