Parametric Equations of Lines

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.5 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to find the parametric equations of a line that satisfies certain conditions by finding a point on the line and a vector parallel to the line.
- Know how to determine whether two lines in space are parallel, skew, or intersecting. And, if the lines intersect, be able to determine the point of intersection.
- Know how to determine where a line intersects a surface.

PRACTICE PROBLEMS:

For problems 1-4, compute parametric equations of the line which satisfies the given conditions.

1. The line which passes through the point $(1, 0, -1)$ and is parallel to $\vec{v} = \langle 1, -2, 0 \rangle$.

   $\boxed{x = 1 + t, y = -2t, z = -1}$

2. The line which passes through points $A(3, -6, 6)$ and $B(2, 0, 7)$.

   $\boxed{x = 3 - t, y = -6 + 6t, z = 6 + t}$

3. The line which passes through the point $(-1, 2, 4)$ and is parallel to $L_1 = \begin{cases} x = 3 - 4t \\ y = 3 + 2t \\ z = t \end{cases}$

   $\boxed{x = -1 - 4t, y = 2 + 2t, z = 4 + t}$

4. The line which passes through the point $(-2, 1, 4)$ and is parallel to both the xy-plane and the xz-plane.

   $\boxed{x = -2 + t, y = 1, z = 4\text{; Detailed Solution: Here}}$

5. Is the line which passes through points $A_1(1, 2, 3)$ and $B_1(5, 8, 9)$ parallel to the line which passes through points $A_2(-2, 5, 3)$ and $B_2(4, 14, 12)$?

   $\boxed{\text{Yes.}}$

6. Find the coordinates of the point at which the line $L_1 = \begin{cases} x = 3 - 6t \\ y = 3 + 3t \\ z = t \end{cases}$ intersects the given plane:

   (a) The xy-plane.

   $\boxed{(x, y, z) = (3, 3, 0)}$

   (b) The xz-plane.

   $\boxed{(x, y, z) = (9, 0, -1)}$

   (c) The yz-plane.

   $\boxed{(x, y, z) = \left(0, \frac{9}{2}, \frac{1}{2}\right)}$

7. Find the coordinates of the points in 3-space where the line $L_1 = \begin{cases} x = t \\ y = 1 + t \\ z = 1 - t \end{cases}$ intersects the sphere $x^2 + y^2 + z^2 = 29$.

   $\boxed{(x, y, z) = (3, 4, -2) \text{ and } (x, y, z) = (-3, -2, 4)\text{; Detailed Solution: Here}}$

For problems 8-11, determine whether the given lines intersect, are parallel, or are skew. If the lines intersect, find the point of intersection.

8. $L_1 : x = 2 + 3t, y = 1 - 2t, z = 4 + 5t$
   $L_2 : x = 3 - 6t, y = -2 + 4t, z = -1 - 10t$

   $\boxed{\text{The lines are parallel.}}$

9. $L_1 : x = 1, y = t, z = 2 - t$
   $L_2 : x = 2 + 3t, y = 4 - 3t, z = t$

   $\boxed{\text{The lines are skew.}}$

10. $L_1 : x = 1 - 2t, y = 14 + t, z = 5 - t$
    $L_2 : x = t, y = 4 + 3t, z = 3 + t$

    $\boxed{\text{The lines intersect at the point } (x, y, z) = (3, 13, 6)\text{; Detailed Solution: Here}}$

11. $L_1 : x = 2 + 5t, y = 4 - t, z = t + 1$
    $L_2 : x = 3 + 6t, y = 1 - t, z = t$

    $\boxed{\text{The lines are skew.}}$

12. Verify that the following lines are parallel. Then compute the distance between them. (Hint: See HW 11.3 #10 or 11.4 #6.)

$$L_1 : x = 5 + 3t, y = 3 + 9t, z = 0$$

$$L_2 : x = 1 + t, y = 3t, z = 1$$

The lines are parallel because $\langle 3, 9, 0\rangle = 3\langle 1, 3, 0\rangle$. The distance between the lines is $d = \sqrt{\frac{91}{10}}$

13. Two bugs are walking along lines in 3-space. At time t, bug 1's position is the point (x, y, z) on the line $L_1 = \begin{cases} x = 1 + 2t \\ y = 3 + 5t \\ z = 5 + 2t \end{cases}$ and bug 2's position is the point (x, y, z) on the line $L_2 = \begin{cases} x = t \\ y = 11 - t \\ z = 4 + t \end{cases}$

    (a) Compute the distance between the bugs' initial positions.

    Bug 1's initial position is $(x, y, z) = (1, 3, 5)$ and Bug 2's initial position is $(x, y, z) = (0, 11, 4)$. The distance between these two points is $\sqrt{66}$

    (b) At which point in space will the bugs' paths intersect? (Note: the paths may not intersect at the same moment in time.)

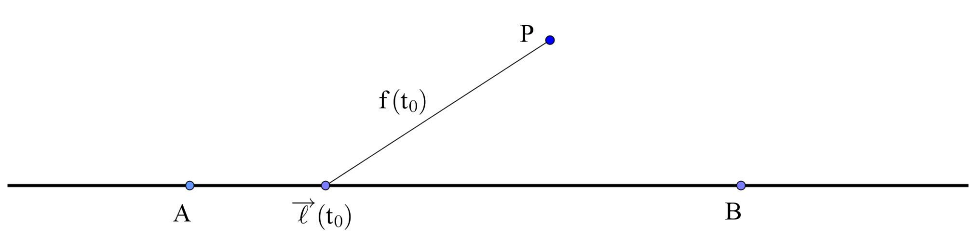
    The paths intersect at the point $(x, y, z) = (3, 8, 7)$

14. Consider the point $P(5, 3, 0)$ and the line L which contains points $A(1, 0, 1)$ and $B(2, 3, 1)$. This problem will show you another way to find the distance d between the point P and the line L.

    (a) Compute an equation of line L.

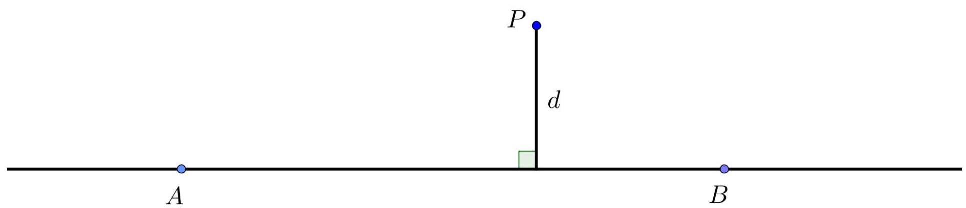
    $\overrightarrow{\ell}(t) = \langle 1 + t, 3t, 1\rangle$

    (b) Compute a function $f(t)$ which gives the distance from the point P to an arbitrary point on the line.



Your answer to this part depends on your parametric equations from part (a). Using the parameterization given the distance from P to an arbitrary point on line L is given by $f(t) = \sqrt{(4-t)^2 + (3-3t)^2 + 1}$.

(c) The distance from the point P to line L is the shortest distance. Calculate the value of t which minimizes the distance from the point P to line L; that is, calculate the value of t which minimizes $f(t)$ from part (b).



$t = \dfrac{13}{10}$; again, this depends on your parameterization of the line.

(d) Compute the distance from the point $P(5, 3, 0)$ to line L by calculating the distance from this P to the point on your the line which corresponds to your value of t from part (c). Verify your answer with HW 11.3 #10(b).

$d = \sqrt{\dfrac{91}{10}}$