Double Integrals Over General Regions

SUGGESTED REFERENCE MATERIAL:

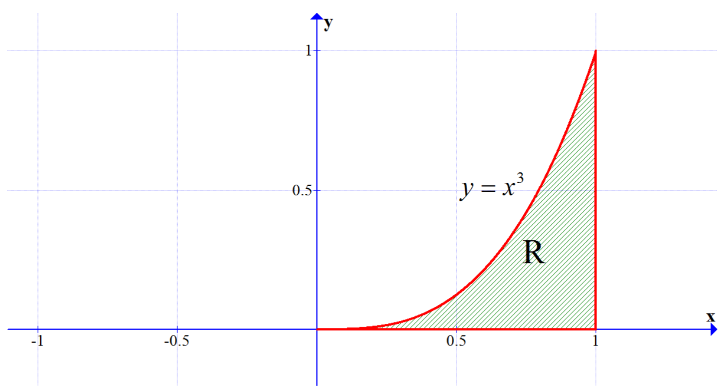
As you work through the problems listed below, you should reference Chapter 14.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute double integral calculations over rectangular regions using partial integration.
- Know how to inspect an integral to decide if the order of integration is easier one way (y first, x second) or the other (x first, y second).
- Kow how to use a double integral to calculate the volume under a surface or find the area or a region in the xy-plane.
- Know how to reverse the order of integration to simplify the evaluation of a double integral.

PRACTICE PROBLEMS:

1. Consider the region R shown below which is enclosed by $y = x^3$, $y = 0$ and $x = 1$.



Fill in the missing limits of integration.

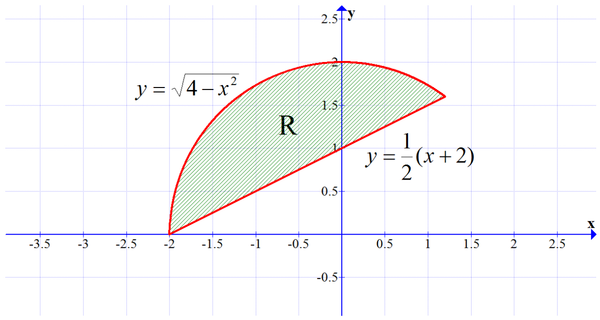
(a) $\displaystyle\iint\limits_R f(x,y)\,dA = \int_{\square}^{\square}\int_{\square}^{\square} f(x,y)\,dy\,dx$

$$\boxed{\iint\limits_R f(x,y)\,dA = \int_0^1 \int_0^{x^3} f(x,y)\,dy\,dx}$$

(b) $\displaystyle\iint_R f(x,y)\,dA = \int_{\square}^{\square}\int_{\square}^{\square} f(x,y)\,dx\,dy$

$$\boxed{\iint_R f(x,y)\,dA = \int_0^1 \int_{\sqrt[3]{y}}^1 f(x,y)\,dx\,dy}$$

2. Consider the region R shown below which is enclosed by $y = \sqrt{4-x^2}$ and $y = \frac{1}{2}(x+2)$.



(a) Set up $\displaystyle\iint_R f(x,y)\,dA$ with the order of integration as $dy\,dx$

$$\boxed{\int_{-2}^{6/5} \int_{\frac{1}{2}(x+2)}^{\sqrt{4-x^2}} f(x,y)\,dy\,dx}$$

(b) Set up $\displaystyle\iint_R f(x,y)\,dA$ with the order of integration as $dx\,dy$

$$\boxed{\int_0^{8/5} \int_{-\sqrt{4-y^2}}^{2y-2} f(x,y)\,dx\,dy + \int_{8/5}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x,y)\,dx\,dy}$$

For problems 3-7, evaluate the iterated integral. For some problems, it may be helpful to switch the order of integration.

3. $\displaystyle\int_1^2 \int_{-x}^{x} \left(y^2 + 3xy + x^2\right)\,dy\,dx$

$\boxed{10}$

4. $\displaystyle\int_0^{\pi/3} \int_0^{\sin x} y\cos x\,dy\,dx$

$\boxed{\dfrac{\sqrt{3}}{16}}$

5. $\displaystyle\int_0^1 \int_0^{x^3} \sqrt{1-x^4}\, dy\, dx$

$\boxed{\dfrac{1}{6}}$

6. $\displaystyle\int_0^1 \int_y^1 \sqrt{1-x^2}\, dx\, dy$

$\boxed{\dfrac{1}{3}}$

7. $\displaystyle\int_0^{\sqrt{\pi}/2} \int_{2y}^{\sqrt{\pi}} \sin\left(x^2\right) dx\, dy$

$\boxed{\dfrac{1}{2}}$

8. Evaluate $\displaystyle\iint_R (4x-3y)\, dA$ where R is the region enclosed by the circle $x^2+y^2=1$.

$\boxed{0}$

9. Evaluate $\displaystyle\iint_R xy^2\, dA$ where R is the trianglar region enclosed by $y=3x$, $y=\dfrac{x}{2}$, and $y=1$.

$\boxed{\dfrac{7}{18}}$

10. Let R be the region enclosed by $y=x^2$ and $y=2x+3$.

(a) Set up a double integral (or double integrals) with the order of integration as $dy\, dx$ which represents the area of R.

$\boxed{\displaystyle\int_{-1}^3 \int_{x^2}^{2x+3} 1\, dy\, dx}$

(b) Set up a double integral (or double integrals) with the order of integration as $dx\, dy$ which represents the area of R.

$\boxed{\displaystyle\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} 1\, dx\, dy + \int_1^9 \int_{\frac{1}{2}y-\frac{3}{2}}^{\sqrt{y}} 1\, dx\, dy}$

(c) Compute the area of R.

$\boxed{\dfrac{32}{3}}$

11. Use a double integral to find the volume of the solid in the first octant which is enclosed by the surface $3x + 6y + 2z = 12$ and the coordinate planes.

    $\boxed{8}$

12. Consider the solid that enclosed by the cylinder $\dfrac{x^2}{9} + y^2 = 1$ and the planes $z = 0$ and $x + 2y + z = 4$. Use a double integral to compute the volume of this wedge.

    $\boxed{12\pi}$; Detailed Solution: Here

13. Let R be the region in the first quadrant of the xy plane which is enclosed by $y = \sqrt{x}$, $x = 0$ and $y = 1$. Compute the volume of the solid which is bounded above by $z = xe^{x/y^2}$ and has R as its base.

    $\boxed{\dfrac{1}{5}}$; Detailed Solution: Here