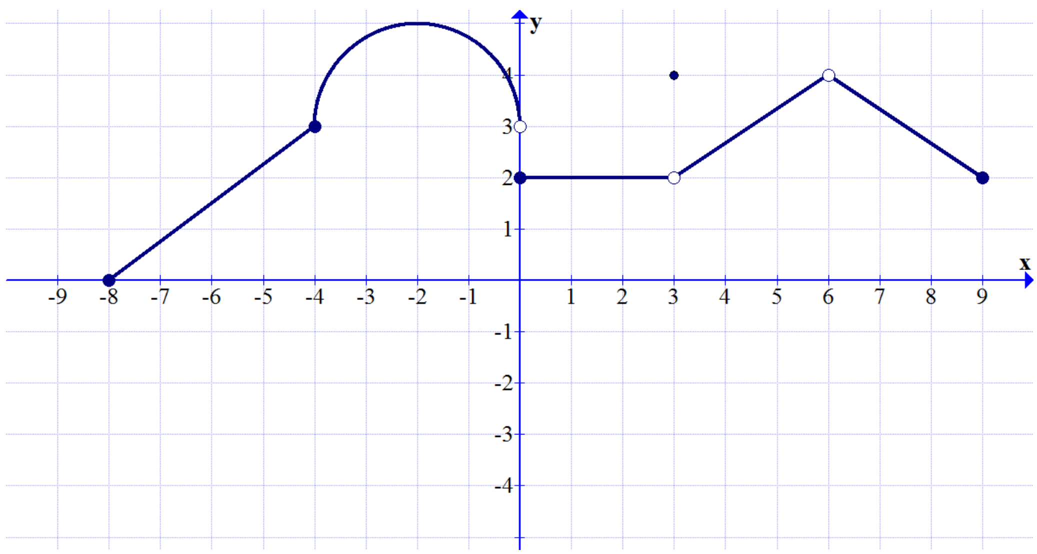
Chapter 1.5 Practice Problems

EXPECTED SKILLS:

- Know what it means for a function to be continuous at a specific value and on an interval.
- Find values where a function is not continuous; specifically, you should be able to do this for polynomials, rational functions, exponential and logarithmic functions, and other elementary functions.
- Determine the values for which a piecewise function is discontinuous, if any such values exist.
- Use the Intermediate Value Theorem to show the existence of a solution to an equation.

PRACTICE PROBLEMS:

Use the graph of $f(x)$, shown below, to answer questions 1-3



1. For which values of x is $f(x)$ discontinuous?

2. At each point of discontinuity, explain why $f(x)$ is discontinuous.

3. Determine whether $f(x)$ is continuous on the given interval. If not, explain why.

   (a) $[-8, -4]$
   (b) $[-8, 0]$
   (c) $[-8, 0)$

(d) $[-2, 1]$
(e) $(3, 6)$
(f) $[3, 6)$
(g) $(6, 9]$
(h) $[6, 9]$

4. For each of the following, sketch the graph of a function, $y = f(x)$, which satisfies the given characteristic. (There are many possible answers for each)

   (a) $f(x)$ is continuous everywhere except at $x = 1$.

   (b) $f(x)$ is continuous everywhere except at $x = -2$ where the $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^+} f(x)$.

   (c) $f(x)$ is continuous everywhere except at $x = 0$, where $f(0) = 2$.

5. Sketch the graph of a function which satisfies the following criteria:

   - The domain of $f(x)$ is $[1, 3]$
   - $f(x)$ is continuous on $[1, 2]$ and $(2, 3]$.
   - $f(x)$ is not continuous on $[1, 3]$

For problems 6-15, determine the value(s) of x where the given function has a point of discontinuity, if any such values exist.

6. $f(x) = |x|$

7. $f(x) = x^2 - x - 5$

8. $f(x) = \dfrac{x}{x-1}$

9. $f(x) = \sqrt[3]{x-1}$

10. $f(x) = \dfrac{x^2 + 3x - 10}{x - 7}$

11. $f(x) = \dfrac{x^2 - 4}{x - 2}$

12. $f(x) = \dfrac{1}{x^2 - 2} + \dfrac{x^3 - 1}{2x^2 - 1}$

13. $f(x) = \begin{cases} x^2 - 1, & \text{if } x < 2 \\ \dfrac{3}{x-1}, & \text{if } x \geq 2 \end{cases}$

14. $f(x) = \begin{cases} 5 + \dfrac{1}{x}, & \text{if } x < -1 \\ 3x^2 + 2x + 3, & \text{if } x > -1 \end{cases}$

15. $f(x) = \begin{cases} x^2 - 3x + 4, & \text{if } x \leq 1 \\ x^4 - 4x^3 - 2x^2 + 6, & \text{if } x > 1 \end{cases}$

16. Find the value(s) of k such that $f(x)$ is continuous everywhere:

$$f(x) = \begin{cases} x^2 - 7, & \text{if } x \leq 2 \\ 4x^3 - 3kx + 2, & \text{if } x > 2 \end{cases}$$

17. Find the value(s) of k and m such that $f(x)$ is continuous everywhere:

$$f(x) = \begin{cases} 2x + 8m, & \text{if } x \leq -2 \\ mx + k, & \text{if } -2 < x \leq 2 \\ -3x^2 + 8x - 2k, & \text{if } x > 2 \end{cases}$$

18. **Multiple Choice:** Where is $f(x) = \dfrac{\sqrt{x-2}}{x^2 - x}$ continuous?

    (a) $x \neq 0$ and $x \neq 1$
    (b) $x \leq 2$ where $x \neq 0$ and $x \neq 1$
    (c) $x \leq 2$
    (d) $x \geq 2$
    (e) $|x| > 2$

19. Consider the following definitions:

    - **Definition:** A function $f(x)$ has a removable discontinuity at $x = a$ if $\lim_{x \to a} f(x)$ exists but $f(x)$ is not continuous at $x = a$. This could be because $f(a)$ is undefined or because $\lim_{x \to a} f(x) \neq f(a)$.
    - **Definition:** A function $f(x)$ has a jump discontinuity at $x = a$ if $\lim_{x \to a^-} f(x)$ exists and $\lim_{x \to a^+} f(x)$ exists, but $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

    For each of the follwing, determine the value(s) of x where the given function has a point of discontinuity. Classify each discontinuity as a removable discontinuity, a jump discontinuity, or neither.

    (a) $f(x) = \dfrac{x^2 - 4}{x - 2}$

(b) $f(x) = \dfrac{x-1}{x-4}$

(c) $f(x) = \begin{cases} x^2 - 3x + 4, & \text{if } x \leq 1 \\ x^4 - 4x^3 - 2x^2 + 6, & \text{if } x > 1 \end{cases}$

(d) $f(x) = \dfrac{x-1}{x^2 - 4x + 3}$

20. **Multiple Choice:** Consider the function:

$$f(x) = \begin{cases} x^2 & \text{if } x < -2 \\ 4 & \text{if } -2 < x \leq 1 \\ 6 - x & \text{if } x > 1 \end{cases}$$

Which of the following statements is true about $f(x)$?

(a) $f(x)$ is continuous everywhere.

(b) If $f(-2)$ were defined to be 4, then $f(x)$ would be continuous everywhere.

(c) The only discontinuity of $f(x)$ occurs when $x = -2$.

(d) The only discontinuity of $f(x)$ occurs when $x = 1$.

(e) The only discontinuities of $f(x)$ occur when $x = -2$ and $x = 1$.

21. Show that the equation $x^3 - x^2 + 3x - 1 = 1$ has at least one solution in $(0, 1)$.

22. Show that $f(x) = x^3 - 9x + 5$ has at least one x-intercept in $(1, 10)$.

23. Use the intermediate value theorem to show that $x^3 - 2x^2 - 2x + 1 = 0$ has at least **TWO** solutions in $[0, 5]$.