Integration by Parts

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 7.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to use integration by parts to evaluate various integrals, including integrands involving products of functions, isolated logarithmic functions, or isolated inverse trigonometric functions.

PRACTICE PROBLEMS:

For problems 1-12, evaluate the given integral.

1. $\int xe^{4x}\,dx$

   $\boxed{\frac{x}{4}e^{4x} - \frac{1}{16}e^{4x} + C}$

2. $\int x^2 \cos(x)\,dx$

   $\boxed{x^2 \sin x + 2x\cos x - 2\sin x + C}$

3. $\int x^2 \ln(x)\,dx$

   $\boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$

4. $\int \frac{\ln x}{x^4}\,dx$

   $\boxed{-\frac{1}{3}x^{-3}\ln x - \frac{1}{9}x^{-3} + C}$

5. $\int \arcsin(x)\,dx$

   $\boxed{x\arcsin x + \sqrt{1-x^2} + C}$

6. $\int x\sec^2(x)\,dx$

   $\boxed{x\tan x + \ln|\cos x| + C}$

7. $\int \ln(x^2+10)\,dx$

$\boxed{x\ln(x^2+10) - 2x + 2\sqrt{10}\arctan\left(\frac{x}{\sqrt{10}}\right) + C}$; Detailed Solution: Here

8. $\int e^{2x}\cos(3x)\,dx$

$\boxed{\frac{2}{13}e^{2x}\cos(3x) + \frac{3}{13}e^{2x}\sin(3x) + C}$; Detailed Solution: Here

9. $\int x\arctan(x)\,dx$

$\boxed{\frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + C}$

10. $\int x^3\cos(x^2)\,dx$

$\boxed{\frac{1}{2}\cos(x^2) + \frac{1}{2}x^2\sin(x^2) + C}$

11. $\int_0^{\pi} 3x\sin(x)\,dx$

$\boxed{3\pi}$; Detailed Solution: Here

12. $\int_0^1 x^2e^x\,dx$

$\boxed{e-2}$

13. Suppose that u and v are differentiable functions of x with $\int_{x=0}^{x=1} v\,du = 3$ and the following functional values.

x	$u(x)$	$v(x)$
0	5	2
1	7	-4

Use this information to compute $\int_{x=0}^{x=1} u\,dv$.

$\boxed{-41}$

14. Evaluate $\int \sin\sqrt{x}\,dx$ by first making an appropriate substitution and then applying integration by parts.

$\boxed{2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right) + C}$

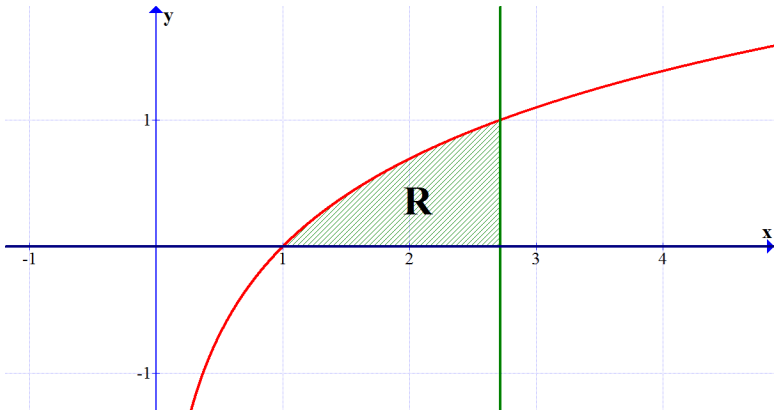
15. Evaluate $\int \left(\sin^{-1} x\right)^2\,dx$

$\boxed{x\left(\sin^{-1} x\right)^2 - 2x + 2\sqrt{1-x^2}\left(\sin^{-1} x\right) + C}$

16. Find the area of the region which is enclosed by $y = \ln x$, $y = 1$, and $x = e^2$.

$\boxed{e}$

17. Let R be the region enclosed by the graphs of $y = \ln x$, $x = e$, and the x-axis (as shown below).



Find the volume of the solid that results from revolving R around the line $y = -1$.

$\boxed{\pi e}$

18. Let f be a differentiable function. Use integration by parts to show:

$$\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$$

Consider $\int f(x)\,dx$. Let $u = f(x)$ and $dv = dx$. Then, $du = f'(x)dx$ and $v = x$. And, integration by parts yields $\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$, as desired.