Chapter 3.8: Graphs of the Trigonometric Functions
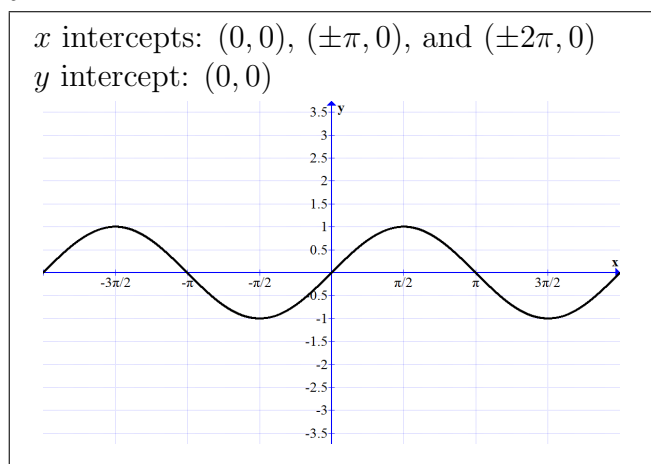
Expected Skills:

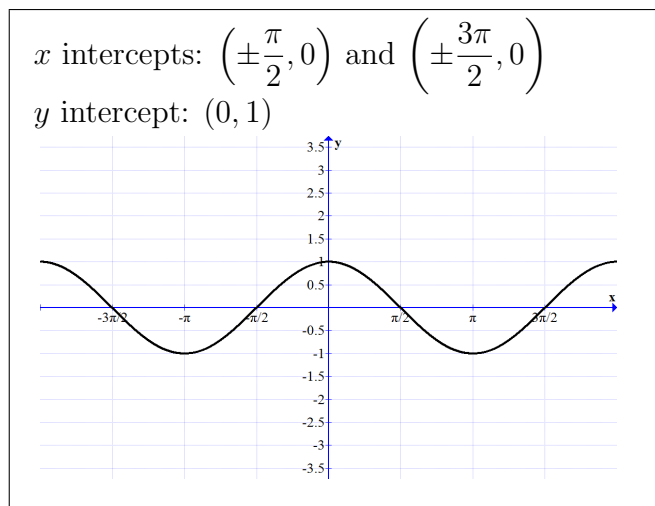- Be able to graph the trigonometric functions.

Practice Problems:

1. Sketch the given function on the interval $[-2\pi, 2\pi]$. Label all asymptotes, intercepts with the coordinate axes, and local extrema.
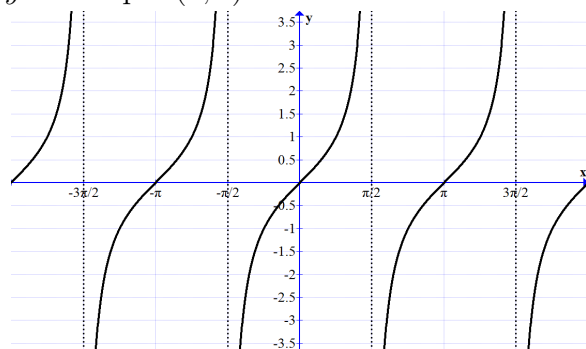
    (a) $y = \sin x$

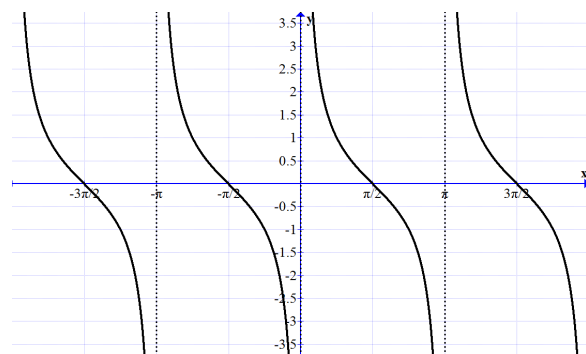 x intercepts: $(0,0)$, $(\pm\pi, 0)$, and $(\pm 2\pi, 0)$
    y intercept: $(0,0)$

    (b) $y = \cos x$

    x intercepts: $\left(\pm\dfrac{\pi}{2}, 0\right)$ and $\left(\pm\dfrac{3\pi}{2}, 0\right)$
    y intercept: $(0,1)$

(c) $y = \tan x$

x intercepts: $(0,0)$, $(\pm\pi, 0)$ and $(\pm 2\pi, 0)$
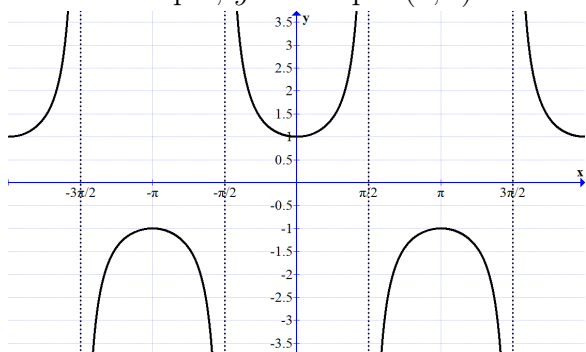y intercept: $(0,0)$

(d) $y = \cot x$

x intercepts: $\left(\pm\dfrac{\pi}{2}, 0\right)$ and $\left(\pm\dfrac{3\pi}{2}, 0\right)$
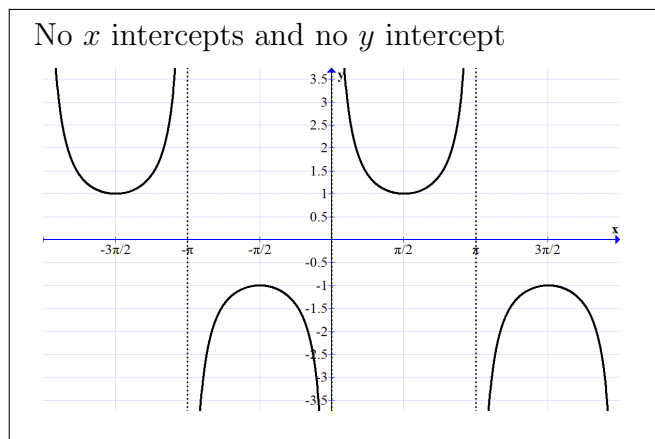
No y intercept

(e) $y = \sec x$

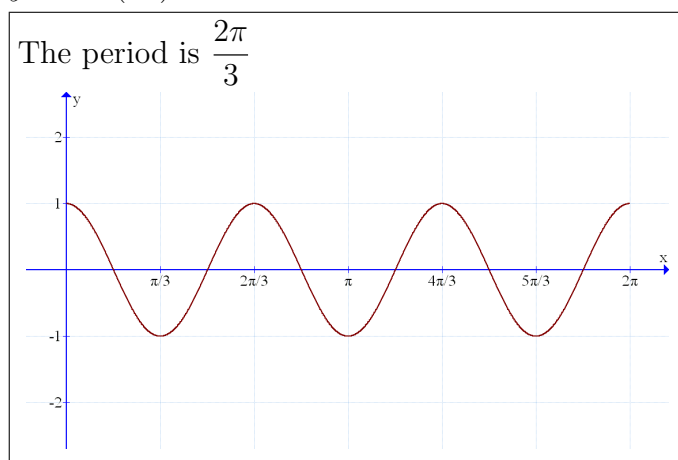No x intercepts, y intercept: $(0,1)$

(f) $y = \csc x$

No x intercepts and no y intercept

2. Sketch each of the following functions on the interval $[0, 2\pi]$. Label all asymptotes, intercepts with the coordinate axes, and local extrema. Also, determine the period.
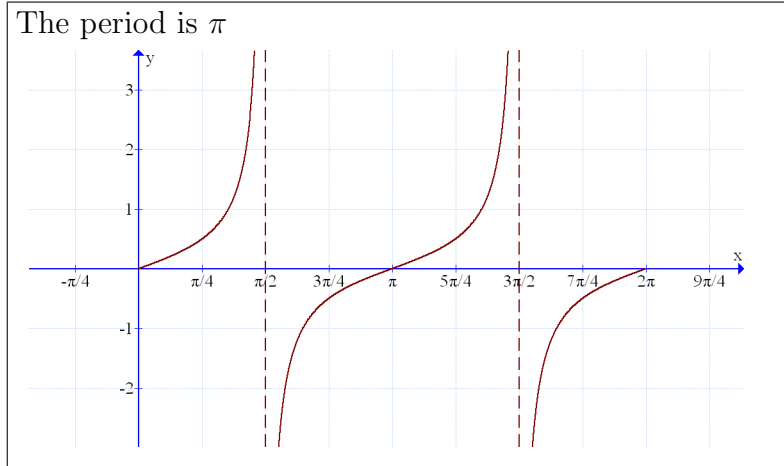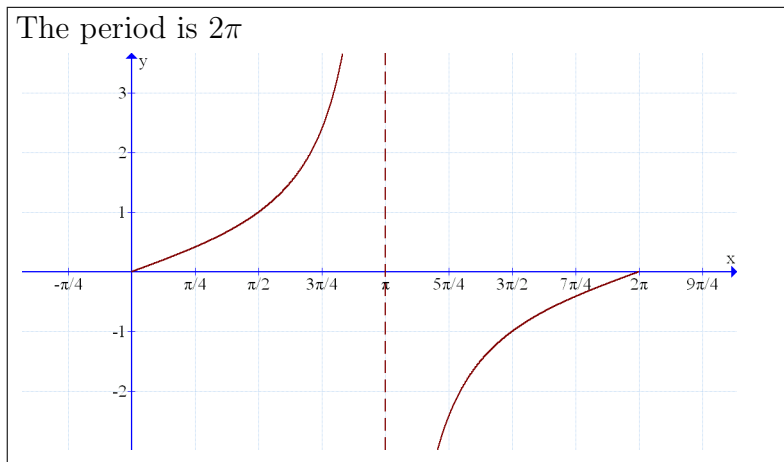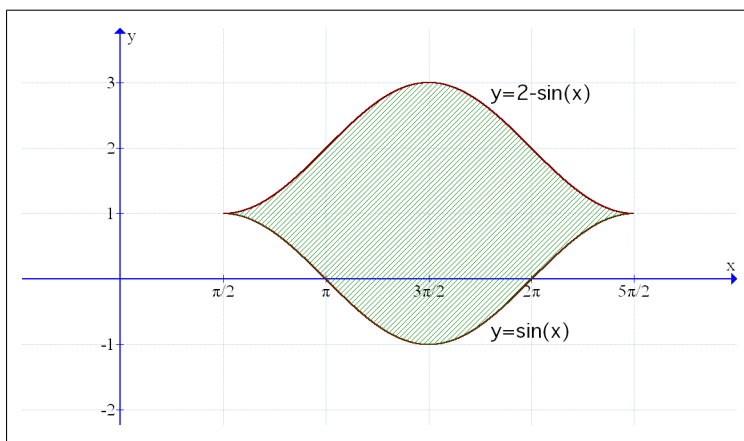
(a) $y = \cos(3x)$

The period is $\dfrac{2\pi}{3}$

(b) $y = 3\sin\left(\dfrac{x}{2}\right)$

The period is 4π

(c) $y = \dfrac{1}{2}\tan(x)$
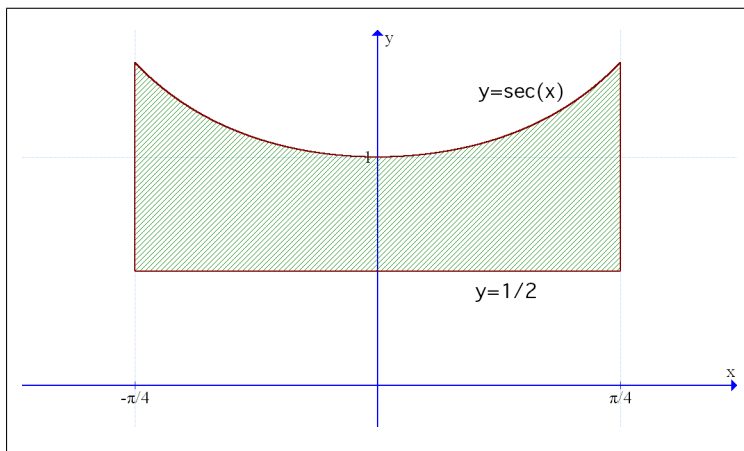
The period is π

(d) $y = \tan\left(\dfrac{x}{2}\right)$

The period is 2π

3. Sketch the region in the xy-plane which is enclosed by $y = \sin x$ and $y = 2 - \sin x$ for $\dfrac{\pi}{2} \leq x \leq \dfrac{5\pi}{2}$.

4. Sketch the region in the xy-plane which is enclosed by $y = \sec x$ and $y = \dfrac{1}{2}$ for $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$.

5. Sketch $f(x) = \sin^2(x)$ on the interval $[0, 2\pi]$. Label all asymptotes, intercepts with the coordinate axes, and local extrema.

Hint: $f'(x) = 2 \sin x \cos x$ and $f''(x) = 4 \cos^2 x - 2$