Integration by Substitution

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 5.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Know how to simplify a "complicated integral" to a known form by making an appropriate substitution of variables.

PRACTICE PROBLEMS:

For problems 1-21, evaluate the given indefinite integral and verify that your answer is correct by differentiation.

1. $\displaystyle\int 3x^2(x^3+3)^3\,dx$

2. $\displaystyle\int \frac{5}{5x+3}\,dx$

3. $\displaystyle\int 2x\cos\left(x^2\right)dx$

4. $\displaystyle\int 4x(x^2+6)^2\,dx$

5. $\displaystyle\int \sec(4x)\tan(4x)\,dx$

6. $\displaystyle\int (3x-5)^9\,dx$

7. $\displaystyle\int e^{-2x}\,dx$

8. $\displaystyle\int \frac{\sin x\cos x}{1+\sin^2 x}\,dx$

9. $\displaystyle\int \csc\left(\frac{x}{2}\right)\cot\left(\frac{x}{2}\right)dx$

10. $\displaystyle\int -3x^3\sqrt{1-x^4}\,dx$

11. $\displaystyle\int \left(\frac{e^{\sqrt{x}}}{\sqrt{x}}-\frac{1}{4}\cos 4x\right)dx$

12. $\displaystyle\int \frac{1}{2+4x^2}\,dx$

13. $\displaystyle\int \frac{4x}{(3+x^2)^2}\,dx$

14. $\displaystyle\int x^2\sqrt{4-x}\,dx.$

15. $\displaystyle\int \frac{1}{\sqrt{\frac{3}{4}+x-x^2}}\,dx$ (HINT: Complete the square)

16. $\displaystyle\int \frac{e^{3/x}}{x^2}\,dx$

17. $\displaystyle\int \frac{e^x}{e^{2x}+1}dx$

18. $\displaystyle\int (\sin 4x)(\cos 4x)^{2/3}dx$

19. $\displaystyle\int \csc^2(3x)\tan^2(3x)+x^2e^{x^3}\,dx$

20. $\displaystyle\int \frac{1}{x\ln x}\,dx$

21. $\displaystyle\int \frac{\cos^{-1}x}{\sqrt{1-x^2}}\,dx$

22. Use an appropriate trigonometric identity followed by a reasonable substitution to evaluate $\displaystyle\int \tan x\,dx$

23. It can be shown that $\displaystyle\frac{32x^2+77x+49}{(3x+1)(4x+5)^2}=\frac{2}{3x+1}-\frac{1}{(4x+5)^2}$. Use this fact to evaluate $\displaystyle\int \frac{32x^2+77x+49}{(3x+1)(4x+5)^2}\,dx.$

24. Using the substitution $x=\sin\theta$ for $-\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$, evaluate $\displaystyle\int\sqrt{1-x^2}\,dx$. Express your answer completely in terms of the variable x.

HINT - The following trigonometric identities will be helpful: $\sin^2\theta+\cos^2\theta=1$, $\cos^2\theta=\frac{1}{2}(1+\cos(2\theta)$, and $\sin(2\theta)=2\sin\theta\cos\theta$