# Area Between Two Curves

SUGGESTED REFERENCE MATERIAL:

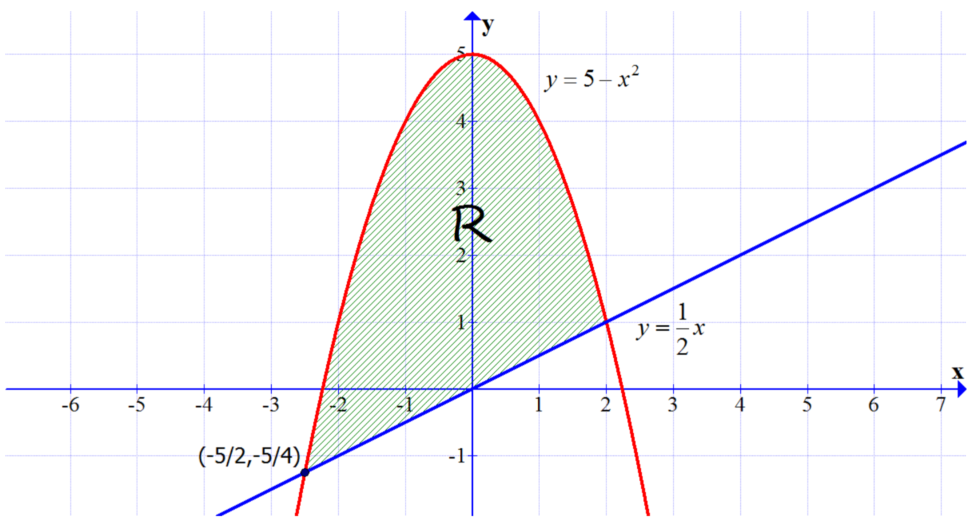
As you work through the problems listed below, you should reference Chapter 6.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to find the area between the graphs of two functions over an interval of interest.
- Know how to find the area enclosed by two graphs which intersect.

PRACTICE PROBLEMS:

1. Let $R$ be the shaded region shown below.

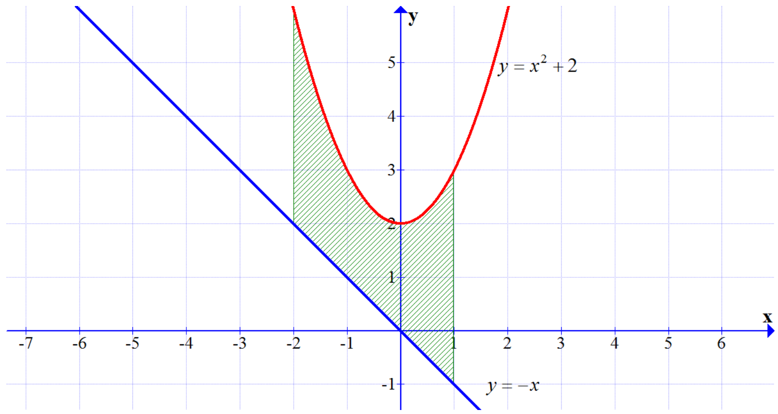



(a) Set up but do not evaluate an integral (or integrals) <u>in terms of $x$</u> that represent(s) the area of $R$.

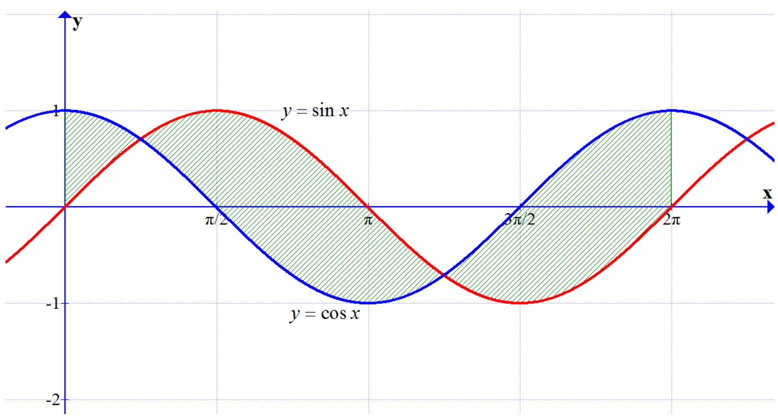
(b) Set up but do not evaluate an integral (or integrals) <u>in terms of $y$</u> that represent(s) the area of $R$.

**For problems 2-4, compute the area of the shaded region.**

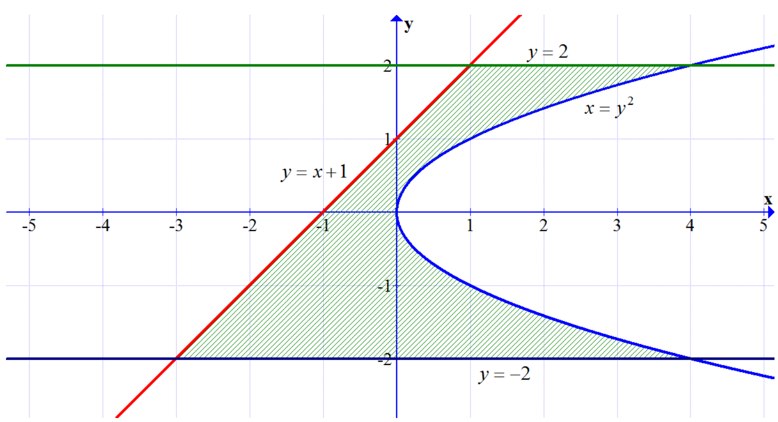
2.




3.




4.



**For problems 5-13, compute the area of the region which is enclosed by the given curves.**

5. $y = 4x$, $y = 6x^2$
6. $y = 2x^2$, $y = x^2 + 2$
7. $y = x^{2/3}$, $y = x^4$, in the first quadrant
8. $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, $x = 4$
9. $y = \sin x$, $y = 2 - \sin x$, $\dfrac{\pi}{2} \leq x \leq \dfrac{5\pi}{2}$
10. $y = e^{5x}$, $y = e^{8x}$, $x = 1$
11. $x = 4 - y^2$, $x = y^2 - 4$
12. $y = x^4$, $y = |x|$
13. $y = x^2$, $y = \dfrac{2}{x^2 + 1}$

14. Let $R$ be the region enclosed by $y = x$, $y = 8x$, and $y = 4$.

    (a) Compute the area of $R$ by evaluating an integral (or integrals) in terms of $x$.

    (b) Compute the area of $R$ by evaluating an integral (or integrals) in terms of $y$.

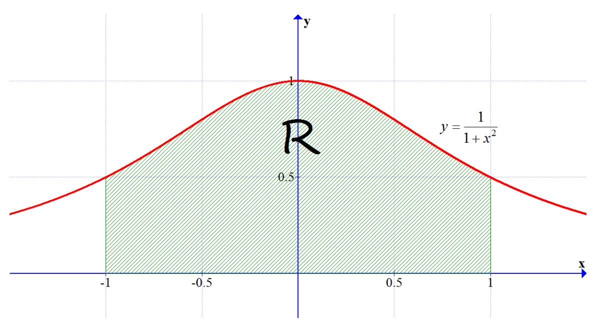
15. Use an integral (or integrals) to compute the area of the triangle in the $xy$-plane which has vertices $(0, 0)$, $(2, 3)$, and $(-1, 6)$.

16. Consider the 2D ice cream cone topped with a delicious scoop of ice cream that is enclosed by $y = 6|x|$ and $y = 16 - x^2$.

    (a) Compute the area enclosed within the ice cream cone (including the scoop portion).

    (b) After a bite is taken from the top, the remaining area is enclosed by $y = 6|x|$, $y = 16 - x^2$, and $y = x^2 + 12$. Compute the area of the remaining portion.

17. Consider the region R shown below:




The area of the region $R$ is equivalent to $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx$.

    (a) Using the substitution $u = \tan^{-1} x$, express the given integral (including the limits of integration) in terms of the variable $u$.

    (b) Sketch a region whose area is equivalent to your integral from part (a). Label this region $S$.

    (c) Evaluate the original integral and your integral from part (a). Conclude that the area of region $R$ is equal to the area of region $S$.

    (Note: This is an example of how changing coordinate systems can simplify a problem. We will discuss this idea more in Math 200 and Math 201.)