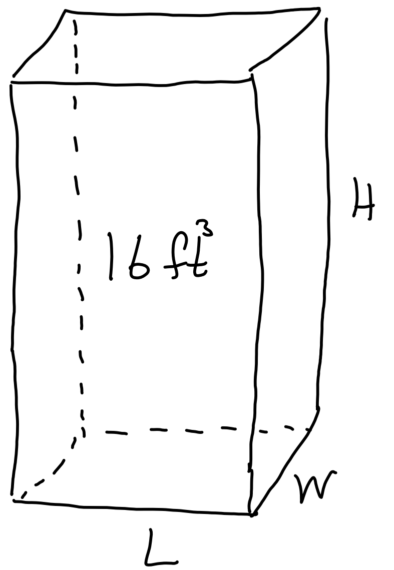
13.8 #16



Top/bottom: \$0.10/ ft^2 Sides: \$0.05/ ft^2

Minimize cost

$$C = 2(0.1)LW + 2(0.05)LH + 2(0.05)WH$$

Know $LWH = 16 \Rightarrow H = \dfrac{16}{LW}$

So $C(L,W) = 0.2LW + \dfrac{1.6}{W} + \dfrac{1.6}{L}$

For the sake of familiar notation, we will use

$$f(x,y) = 0.2xy + \frac{1.6}{x} + \frac{1.6}{y}$$

$$f_x(x,y) = 0.2y - \frac{1.6}{x^2} = 0 \iff 2x^2y - 16 = 0$$

$$f_y(x,y) = 0.2x - \frac{1.6}{y^2} = 0 \iff 2xy^2 - 16 = 0$$

So $x^2y = 8 = xy^2 \Rightarrow y = x \Rightarrow x^3 = 8 \Rightarrow \underbrace{x=2,\ y=2}_{\text{critical point}}$

$$f_{xx}(x,y) = \frac{3.2}{x^3}$$

$$f_{yy}(x,y) = \frac{3.2}{y^3}$$

$$f_{xy}(x,y) = 0.2$$

$$D(x,y) = \left(\frac{3.2}{x^3}\right)\left(\frac{3.2}{y^3}\right) - (0.2)^2$$

$$D(2,2) = (0.4)(0.4) - (0.2)^2 > 0$$

and $f_{xx}(2,2) = 0.4 > 0$ so there is a relative minimum at $(2,2)$.

If $L = 2$ and $W = 2$ then $H = \frac{16}{LW} = 4$

Dimensions of box with smallest cost: 2ft x 2ft x 4ft