

Vectors & Vector Arithmetic

SUGGESTED REFERENCE MATERIAL:

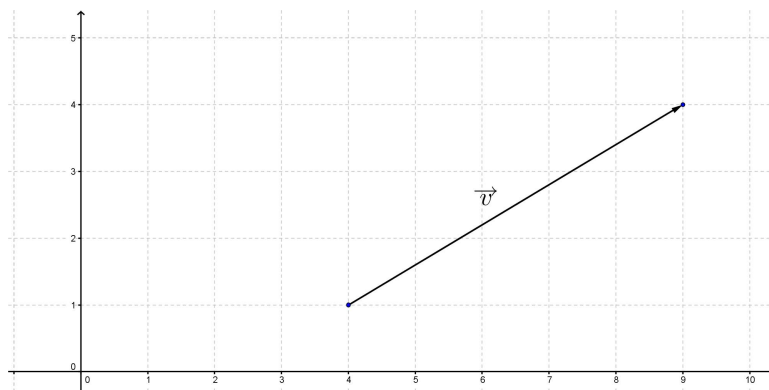
As you work through the problems listed below, you should reference Chapter 11.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

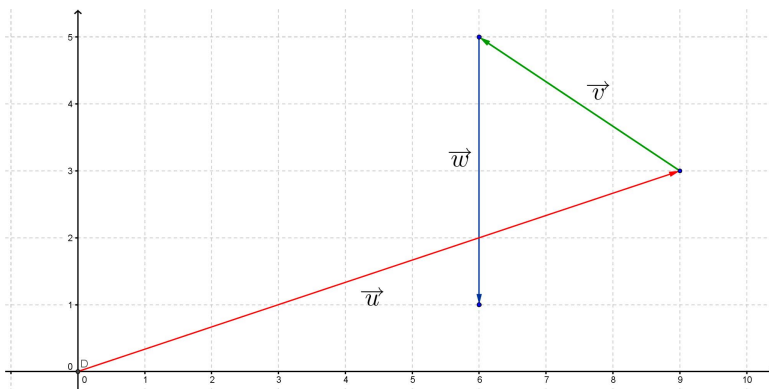
- Be able to perform arithmetic operations on vectors and understand the geometric consequences of the operations.
- Know how to compute the magnitude of a vector and normalize a vector.
- Be able to use vectors in the context of geometry and force problems.

PRACTICE PROBLEMS:

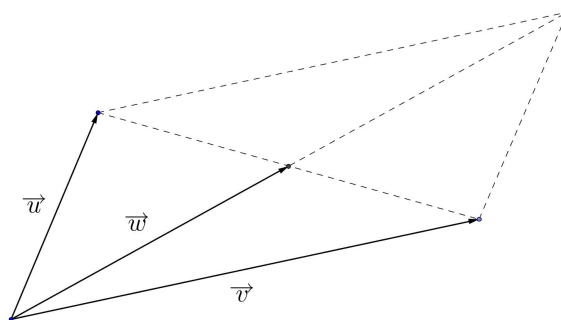
1. Find the components of vector \vec{v} and sketch an equivalent vector with its initial point at the origin.



2. Sketch the vector $\vec{u} + \vec{v} + \vec{w}$ and express it in component form.

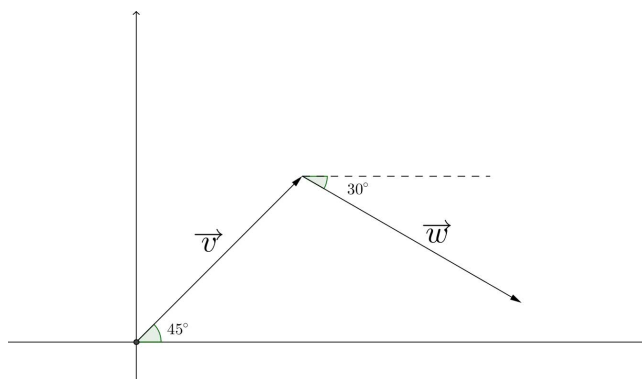


3. The figure below is a parallelogram. Express \vec{w} in terms of \vec{u} and \vec{v} .



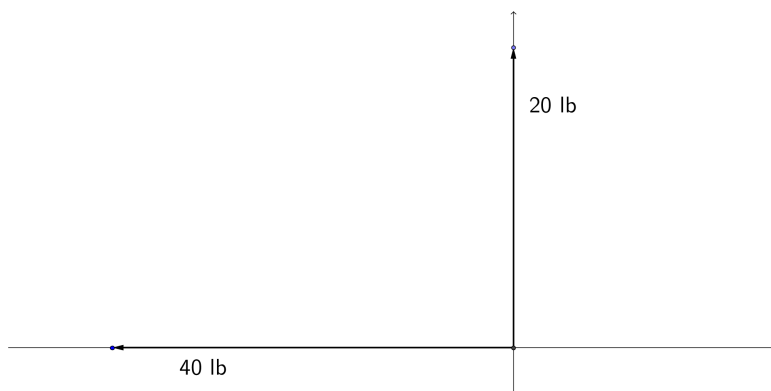
4. Consider the points $P_1(2, 3)$ and $P_2 = (5, -1)$. Find the components of the vector $\overrightarrow{P_1P_2}$. Sketch P_1 , P_2 , $\overrightarrow{P_1P_2}$, and an equivalent vector with its initial point at the origin.
5. Consider the points $P_1(1, 2, 3)$ and $P_2(5, 4, 6)$. Find the components of the vector $\overrightarrow{P_1P_2}$.
6. Let $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, and $\mathbf{w} = 7\mathbf{i} + 4\mathbf{j} + \mathbf{k}$. Compute each of the following:
- $2\mathbf{u} - 3\mathbf{w}$
 - $\|\mathbf{u} + \mathbf{v}\|$
 - $\|\mathbf{u}\| + \|\mathbf{v}\|$
 - $\|2\mathbf{u}\|$
 - $\left\| \frac{1}{\|\mathbf{v}\|} \mathbf{v} \right\|$
7. For each of the following, find a vector which satisfies the given conditions.
- A unit vector which is in the opposite direction of $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$
 - A unit vector which is in the same direction as the vector from $P_1(1, 0, 5)$ to $P_2(3, -1, 2)$
 - A vector which is in the opposite direction of $\vec{v} = \langle 1, 2, 3 \rangle$ and whose magnitude is half that of \vec{v} .
 - A vector which is in the same direction of $\mathbf{w} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and which has a length of $\sqrt{5}$
 - A vector in 2-space which makes an angle of $\theta = \frac{\pi}{6}$ with the positive x -axis and which has a magnitude of 4.
 - A vector in 2-space which makes an angle of $\theta = 210^\circ$ with the positive x -axis and which has a length of 2.

8. Find the value(s) of a so that the vectors $\vec{v} = \langle a^2, 6 \rangle$ and $\vec{w} = \langle 4a, 2 \rangle$ are parallel.
9. Vectors \vec{v} and \vec{w} , shown below, are unit vectors. Find the components of $\vec{v} + \vec{w}$.

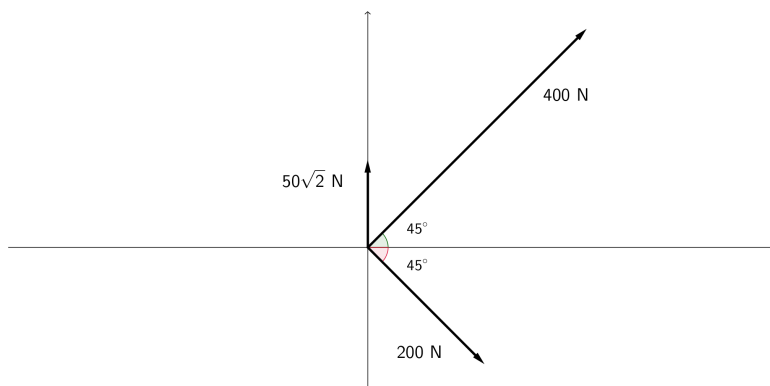


10. For each of the following, find the magnitude of the resultant force and the angle that it makes with the positive x -axis.

(a)



(b)



11. A weight of 200 Newtons (N) is being supported by two wires, as shown below. Find the tension in each wire.

