13.6 #6

Let $u = \langle a, b \rangle$ where $\|\vec{u}\| = 1$, i.e. $\sqrt{a^2 + b^2} = 1$.

Given $D_{\vec{u}} f(1, \pi) = f_x(1, \pi) a + f_y(1, \pi) b = 0$.

Now $f_x(x,y) = y \sec^2(xy) \implies f_x(1, \pi) = \pi \sec^2(\pi) = \pi$

$f_y(x,y) = x \sec^2(xy) \implies f_y(1, \pi) = 1 \sec^2(\pi) = 1$

So $\pi a + 1b = 0$ and $\sqrt{a^2 + b^2} = 1 \iff a^2 + b^2 = 1$

$\Updownarrow$

$b = -\pi a \implies a^2 + \pi^2 a^2 = 1$

$\Downarrow$

$a = \dfrac{1}{\sqrt{1+\pi^2}}, \quad a = \dfrac{-1}{\sqrt{1+\pi^2}}$

$b = \dfrac{-\pi}{\sqrt{1+\pi^2}}, \quad b = \dfrac{\pi}{\sqrt{1+\pi^2}}$

Solution: $\vec{u} = \left\langle \dfrac{1}{\sqrt{1+\pi^2}}, \dfrac{-\pi}{\sqrt{1+\pi^2}} \right\rangle$ or $\vec{u} = \left\langle \dfrac{-1}{\sqrt{1+\pi^2}}, \dfrac{\pi}{\sqrt{1+\pi^2}} \right\rangle$