Chapter 3.6 Practice Problems

EXPECTED SKILLS:

- Know how to use L'Hopital's Rule to help compute limits involving indeterminate forms of $\frac{0}{0}$ and $\frac{\infty}{\infty}$
- Be able to compute limits involving indeterminate forms $\infty - \infty$, $0 \cdot \infty$, 0^0, ∞^0, and 1^∞ by manipulating the limits into a form where L'Hopital's Rule is applicable.

PRACTICE PROBLEMS:

For problems 1-27, calculate the indicated limit. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate). Make sure that L'Hopital's rule applies before using it. And, whenever you apply L'Hopital's rule, indicate that you are doing so.

1. $\displaystyle\lim_{x\to 3} \frac{x^2+4x-21}{x^2-7x+12}$

   $\boxed{-10}$

2. $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{\ln(1+x)}$

   $\boxed{3}$

3. $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{x^2}$

   $\boxed{0}$

4. $\displaystyle\lim_{x\to 0} \frac{\sin(6x)}{\sin(3x)}$

   $\boxed{2}$

5. $\displaystyle\lim_{x\to 1^-} \frac{x-1}{x^2-2x+1}$

   $\boxed{-\infty}$

6. $\displaystyle\lim_{x\to \infty} \frac{e^{-x}}{x^{-2}}$

   $\boxed{0}$

7. $\displaystyle\lim_{x\to 0} \frac{\ln(\cos 3x)}{5x^2}$

   $\boxed{-\frac{9}{10}}$

8. $\displaystyle\lim_{x\to 1}\frac{\tan^{-1}x-\frac{\pi}{4}}{x-1}$

$\boxed{\dfrac{1}{2}}$

9. $\displaystyle\lim_{x\to 0^+}\frac{8^{\sqrt{x}}-1}{1-5^{\sqrt{x}}}$

$\boxed{-\dfrac{3\ln 2}{\ln 5}}$

10. $\displaystyle\lim_{x\to 0^+}\frac{5\sin x}{\sqrt{x}}$

$\boxed{0}$

11. $\displaystyle\lim_{x\to -\infty}\frac{x^3+4x-5}{5x^2-5x-89}$

$\boxed{-\infty}$

12. $\displaystyle\lim_{x\to 0}\frac{\sin^{-1}(2x)}{\tan^{-1}(3x)}$

$\boxed{\dfrac{2}{3}}$

13. $\displaystyle\lim_{x\to 1}\frac{\ln x^2}{x^2-9}$

$\boxed{0}$

14. $\displaystyle\lim_{x\to \infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$

$\boxed{1}$

15. $\displaystyle\lim_{x\to \frac{\pi}{2}^+}\frac{\ln\left(x-\frac{\pi}{2}\right)}{\tan x}$

$\boxed{0}$

16. $\displaystyle\lim_{x\to 1^-}\frac{x-1}{\arccos x}$

$\boxed{0}$

17. $\displaystyle\lim_{x\to +\infty}\frac{e^{\sqrt{x}}}{x}$

$\boxed{+\infty}$

18. $\lim\limits_{x\to+\infty} xe^{-6x}$

$\boxed{0}$

19. $\lim\limits_{x\to+\infty} \dfrac{\sqrt{4+3x^2}}{2+2x}$

$\boxed{\dfrac{\sqrt{3}}{2}}$

20. $\lim\limits_{x\to 0^+} x\csc 3x$

$\boxed{\dfrac{1}{3}}$

21. $\lim\limits_{x\to+\infty} [\ln(x+2)-\ln(3x+5)]$

$\boxed{\ln\left(\dfrac{1}{3}\right)}$

22. $\lim\limits_{x\to\infty} 3^x 7^{-x}$

$\boxed{0}$

23. $\lim\limits_{x\to\infty} \left(\sqrt{x^2-x}-x\right)$

$\boxed{-\dfrac{1}{2}}$

24. $\lim\limits_{x\to 0^+} \tan x \sec x$

$\boxed{0}$

25. $\lim\limits_{x\to 0^+} x^{1/x}$

$\boxed{0}$

26. $\lim\limits_{x\to\infty} \left(1+\dfrac{2}{x}\right)^{5x}$

$\boxed{e^{10}}$

27. $\lim\limits_{x\to\infty} \left(1-\dfrac{1}{x}\right)^{-x}$

$\boxed{e}$

28. Which of the following are indeterminate forms?

$$\frac{0}{0} \quad \frac{0}{\infty} \quad \frac{\infty}{0} \quad \frac{\infty}{\infty}$$
$$\infty - \infty \quad \infty + \infty \quad 0 \cdot \infty \quad \infty \cdot \infty$$
$$0^0 \quad \infty^0 \quad 0^\infty \quad 1^\infty \quad \infty^\infty \quad \infty^1$$

$\boxed{\frac{0}{0},\ \frac{\infty}{\infty},\ \infty - \infty,\ 0 \cdot \infty,\ 0^0,\ \infty^0,\ 1^\infty}$

29. Calculate each of the following limits:

(a) $\lim\limits_{x \to 0^+} (1 + 3^x)^{1/x}$

$\boxed{+\infty}$

(b) $\lim\limits_{x \to 0^-} (1 + 3^x)^{1/x}$

$\boxed{0}$

(c) $\lim\limits_{x \to +\infty} (1 + 3^x)^{1/x}$

$\boxed{3}$

(d) $\lim\limits_{x \to -\infty} (1 + 3^x)^{1/x}$

$\boxed{1}$

30. Show that $\lim\limits_{x \to \infty} \frac{x^n}{e^x} = 0$ for any positive integer n.

$\lim\limits_{x \to \infty} \frac{x^n}{e^x}$ is of the indeterminate form $\frac{\infty}{\infty}$, so, we may apply L'Hopital's Rule:

$$\lim_{x \to \infty} \frac{x^n}{e^x} = \lim_{x \to \infty} \frac{nx^{n-1}}{e^x}$$

This new limit is also of the indeterminate form $\frac{\infty}{\infty}$, so, we may again apply L'Hopital's Rule:

$$\lim_{x \to \infty} \frac{nx^{n-1}}{e^x} = \lim_{x \to \infty} \frac{n(n-1)x^{n-2}}{e^x}$$

In fact, we repeat the process until we end up with the following limit:

$$\lim_{x \to \infty} \frac{n(n-1)(n-2)\ldots(2)(1)}{e^x}$$

which equals 0. Thus, $\lim\limits_{x \to \infty} \frac{x^n}{e^x} = 0$

31. Find the value(s) of of the constant k which make $f(x) = \begin{cases} \dfrac{\sin x - 1}{x - \frac{\pi}{2}} & \text{if } x \neq \dfrac{\pi}{2} \\ k & \text{if } x = \dfrac{\pi}{2} \end{cases}$ continuous at $x = \dfrac{\pi}{2}$.

$\boxed{k = 0}$

32. Find all values of k and m such that $\lim_{x \to 1} \dfrac{k + m \ln x}{x - 1} = 5$

$\boxed{k = 0 \text{ and } m = 5}$

33. **Multiple Choice:** What is $\lim_{x \to 1^+} \dfrac{x}{\ln(x)}$?

(a) 0

(b) 1

(c) e

(d) e^{-1}

(e) $+\infty$

$\boxed{\text{E}}$

34. **Multiple Choice:** What is $\lim_{x \to 0} \dfrac{e^x - 1}{\tan(x)}$?

(a) -1

(b) 0

(c) 1

(d) 2

(e) The limit does not exist.

$\boxed{\text{C}}$

35. **Multiple Choice:** If $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} g(x) = +\infty$ and $f'(x) = 1$ and $g'(x) = e^x$, what is $\lim\limits_{x\to+\infty} \dfrac{f(x)}{g(x)}$?

(a) -1

(b) 0

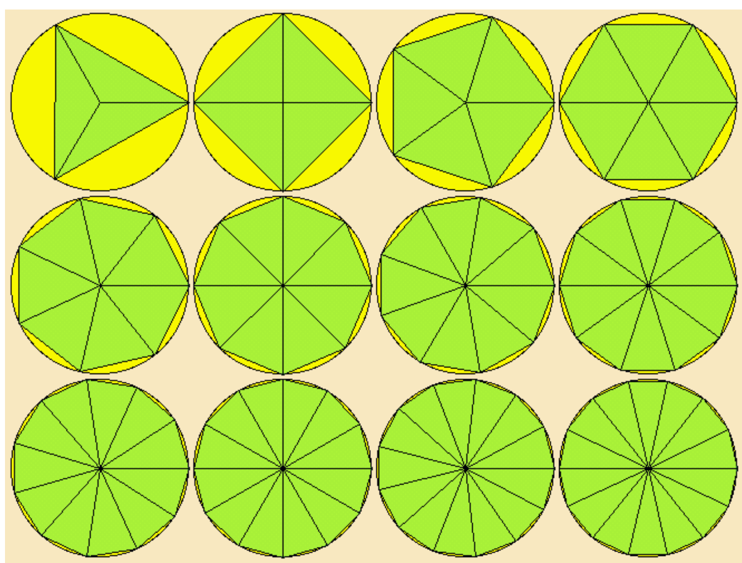
(c) 1

(d) e

(e) The limit does not exist.

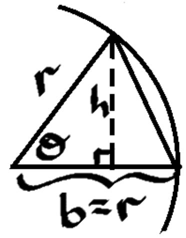
$\boxed{\text{B}}$

36. A Regular Polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length). The diagram below shows several regular polygons inscribed within a circle of radius r.



(a) Let A_n be the area of a regular n-sided polygon inscribed within a circle of radius r. Divide the polygon into n congruent triangles each with a central angle of $\dfrac{2\pi}{n}$ radians, as shown in the diagram above for several different values of n. Show that $A_n = \dfrac{1}{2}r^2 \sin\left(\dfrac{2\pi}{n}\right)n$.

We begin by examining one of the n triangles, pictured below.



The base of the triangle has a length of r. And, the height of the triangle is $r \sin \theta$, where θ is the central angle, $\frac{2\pi}{n}$. Thus, the area of one triangle is:

$$A = \frac{1}{2}(r)\left(r \sin\left(\frac{2\pi}{n}\right)\right) = \frac{1}{2}r^2 \sin\left(\frac{2\pi}{n}\right)$$

But, the polygon is composed of n such triangles. So, the area of a regular n-sided polygon inscribed in the circle of radius r is:

$$A_n = \frac{1}{2}r^2 \sin\left(\frac{2\pi}{n}\right) n$$

(b) What can you conclude about the area of the n-sided polygon as the number of sides of the polygon, n, approaches infinity? In other words, compute $\lim_{n\to\infty} A_n$.

$$\boxed{\lim_{n\to\infty} A_n = \pi r^2}$$