10.1 #13

$x = 2\tan t, \quad y = \sec t, \quad 0 \le t \le \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec t \tan t}{2\sec^2 t} = \frac{1}{2}\frac{\tan t}{\sec t} = \frac{1}{2}\sin t$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{1}{2}\cos t}{2\sec^2 t} = \frac{1}{4}\cos^3 t$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} = \frac{1}{4}\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{2\sqrt{2}}{32} = \frac{\sqrt{2}}{16}$$