

Multivariable Functions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to describe and sketch the domain of a function of two or more variables.
- Know how to evaluate a function of two or more variables.
- Be able to compute and sketch level curves & surfaces.

PRACTICE PROBLEMS:

1. For each of the following functions, describe the domain in words. Whenever possible, draw a sketch of the domain as well.

(a) $f(x, y) = \sqrt{10 - x^2 - y^2}$

(b) $f(x, y) = \arcsin(2x + y)$

(c) $f(x, y, z) = \ln(36 - 4x^2 - 9y^2 - 36z^2)$

(d) $f(x, y, z) = \sqrt{6 - 2x - 3y - z}$

2. Let $f(x, y) = 2xe^{3y}$. Compute the following.

(a) $f(4, 0)$

(b) $f(1, \ln 2)$

3. Suppose $f(x, y) = \int_x^y (t^2 - 1) dt$. Compute the following.

(a) $f(-1, 2)$

(b) $f(0, 2)$

4. Suppose $f(x_1, x_2, \dots, x_n) = x_1 + 2x_2 + 3x_3 + \dots + nx_n$. Determine $f(1, 1, \dots, 1)$.

5. Consider $f(x, y) = x^2 + y^2$. Compute $f(x(t), y(t))$ if $x(t) = 1 + t$ and $y(t) = 2 - 3t$

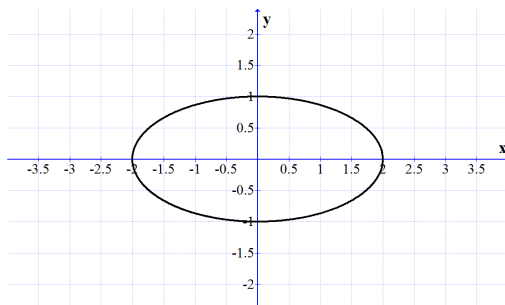
6. Sketch the level curves $f(x, y) = k$, for the specified values of k .

(a) $z = 2x - y$; $k = -2, -1, 0, 1, 2$

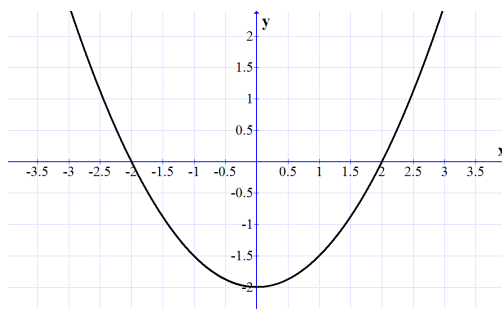
(b) $z = y^2 - x^2$; $k = -2, -1, 0, 1, 2$

7. **Multiple Choice:** Which of the following graphs is the level curve of $f(x, y) = x^2 + 4y^2$ which passes through $P(-2, 0)$?

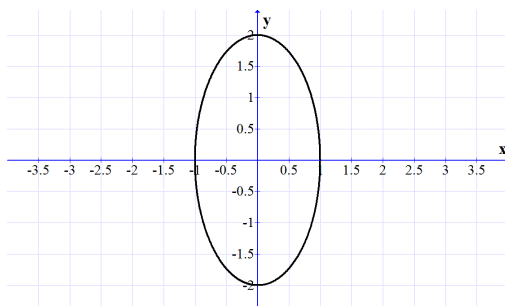
(a)



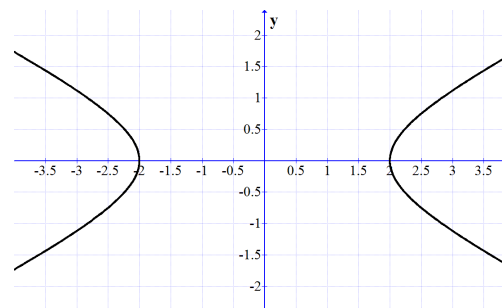
(d)



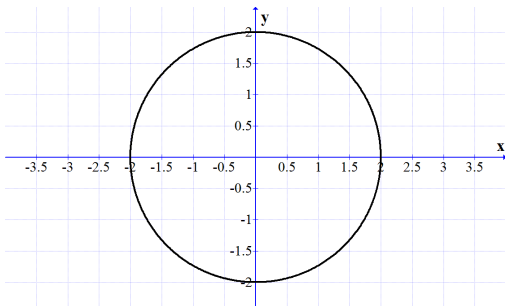
(b)



(e)



(c)



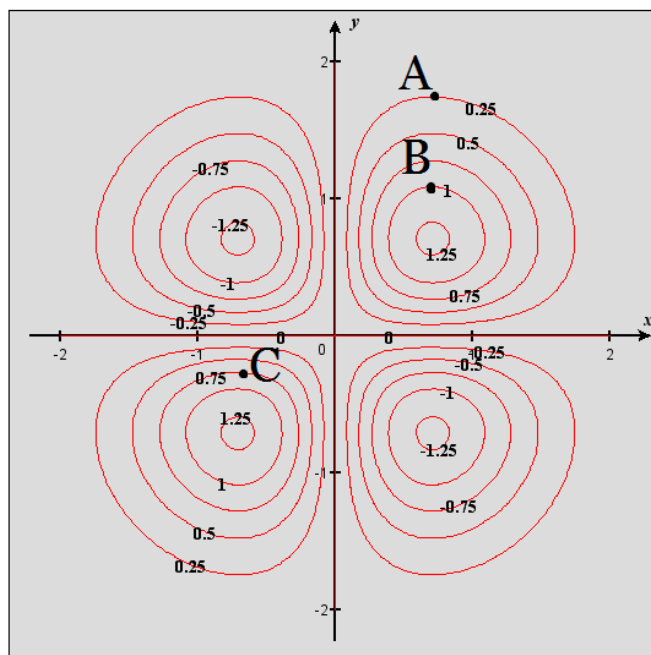
8. Suppose $f(x, y, z) = x^2 + y^2 - z^2$. For each of the following, sketch the level surface $f(x, y, z) = k$ corresponding to the indicated value of k .

(a) $k = 1$

(b) $k = 0$

(c) $k = -1$

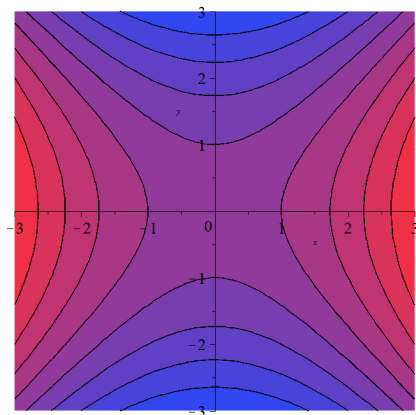
9. Consider the contour map shown below.



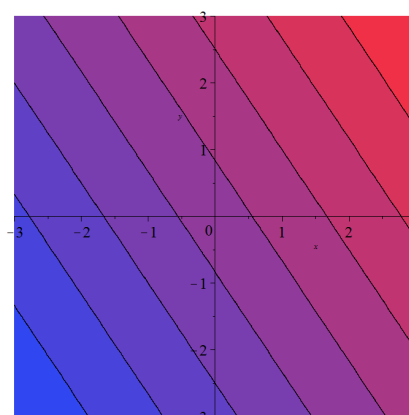
- If a person were walking straight from point A to point B , would s/he be walking uphill or downhill?
- Is the slope steeper at point B or point C ?
- Starting at C and moving so that x remains constant and y decreases, will the elevation begin to increase or decrease?
- Starting at B and moving so that y remains constant and x increases, will the elevation begin to increase or decrease?

10. **Matching:** Each of the following contour plots were drawn on the window $[-3, 3] \times [-3, 3]$ in the xy -plane. Points with larger z -values are shaded in blue. Those with smaller z -values are shaded in red. Match each contour map (a-f) to an appropriate graph (I-VI).

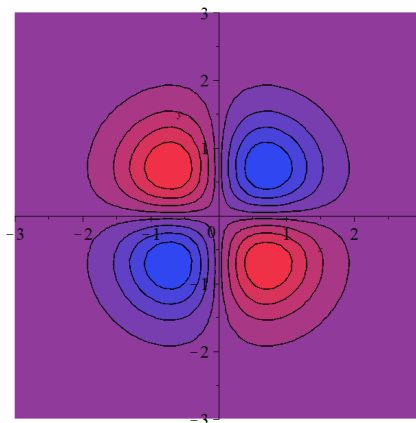
(a)



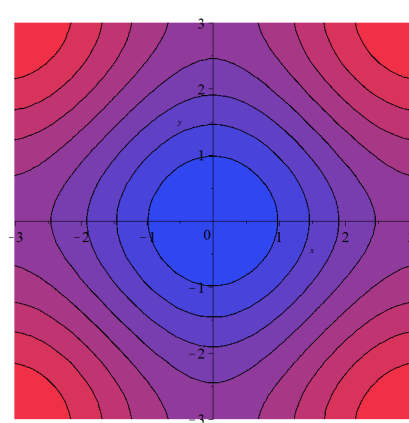
(d)



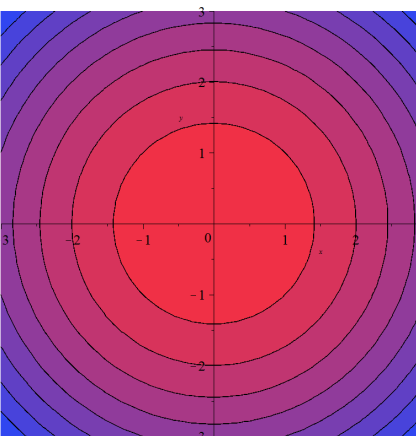
(b)



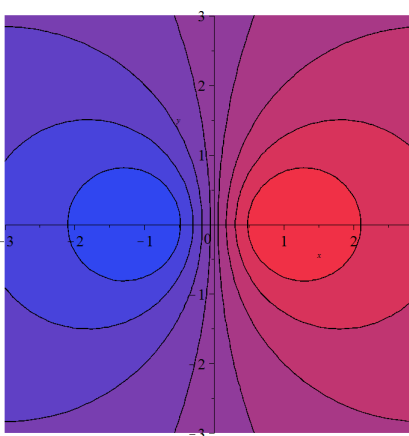
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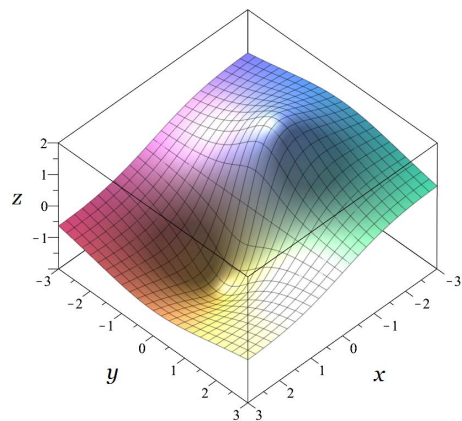
(c)



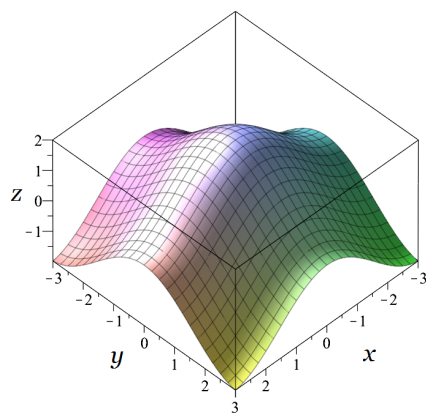
(f)



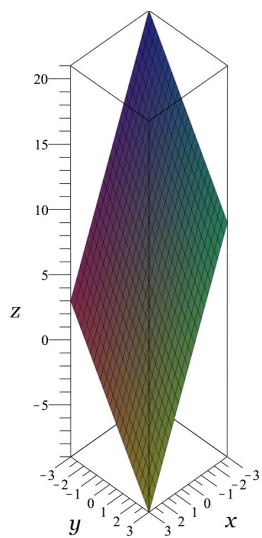
(I)



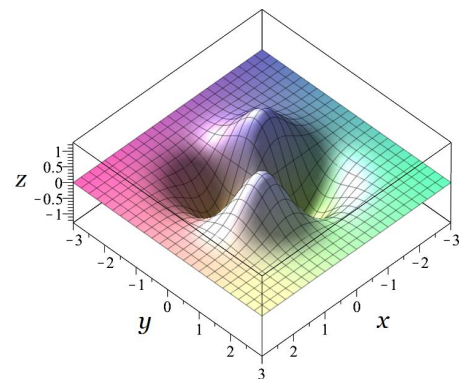
(II)



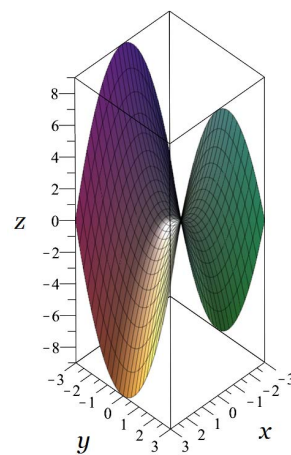
(III)



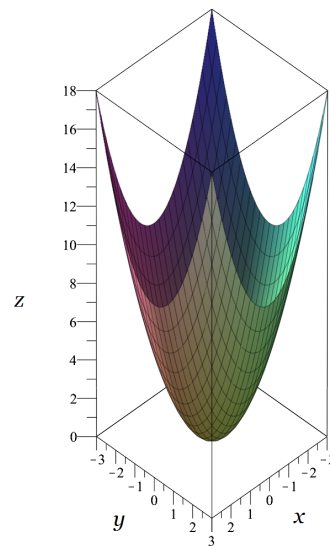
(IV)



(V)

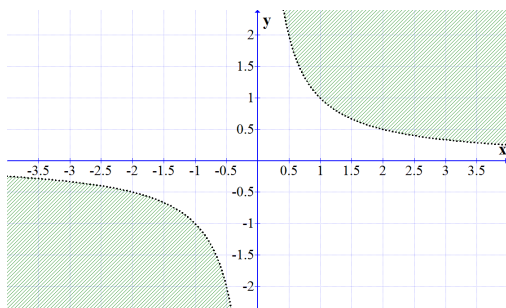


(VI)

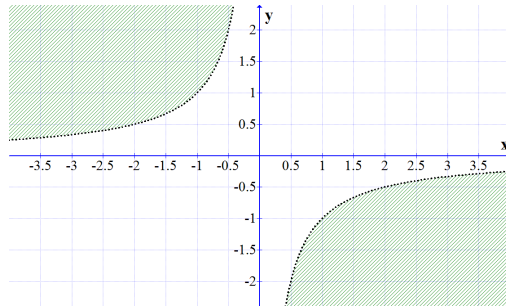


11. **Multiple Choice:** Which of the following is a sketch of the domain of $f(x, y) = \ln(xy - 1) + e^{x^2y} - y^8$?

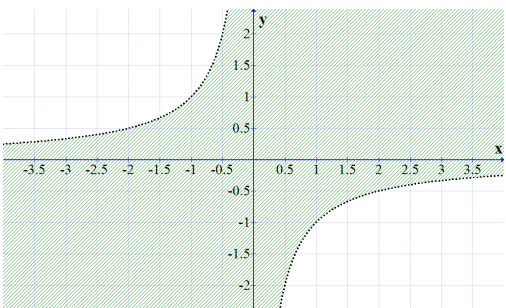
(a)



(d)



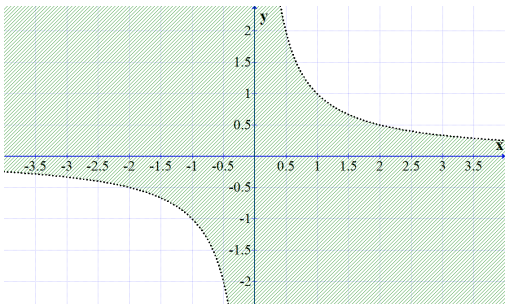
(b)



(e)



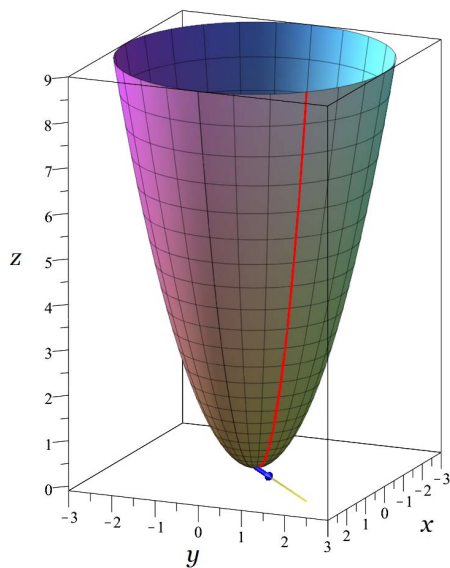
(c)



12. Suppose that $f(x, y) = x^2 + y^2$. And, consider line L with parametric equations:
 $x(t) = \frac{\sqrt{2}}{2}t$, $y(t) = \frac{\sqrt{2}}{2}t$, $z(t) = 0$.

Notice that this line in the xy -plane is parallel to the unit vector $\vec{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right\rangle$.

- (a) Now consider the image below. The red curve along $f(x, y)$ is the curve that results from evaluating the function at points along L . In other words, it is $f(x(t), y(t))$, where $x(t)$ and $y(t)$ are taken from the parametric equations of L . Compute $f(x(t), y(t))$.



- (b) Notice that your answer from part (a) is a single variable function of the variable t . Call it $g(t)$. Compute $\left. \frac{d}{dt} (g(t)) \right|_{t=1}$ and interpret your answer geometrically.