$$f(x_{1}y) = \frac{1}{4} = \frac{1}{4}$$

$$f(x_{1}y) = \frac{1}{1 + \frac{x^{2}}{y^{2}}} \cdot \frac{1}{y} \implies f(x_{1}y_{1}) = \frac{1}{1 + 1} \cdot \frac{1}{1} = \frac{1}{2}$$

$$f(x_{1}y_{1}) = \frac{1}{1 + \frac{x^{2}}{y^{2}}} \cdot \frac{1}{y} \implies f(y_{1}y_{1}) = \frac{1}{1 + 1} \cdot \frac{1}{1} = \frac{1}{2}$$

Now at (1,1)

$$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$$

So
$$L(x,y) = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{2}(y-1)$$

$$= \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{2} - \frac{1}{2}y + \frac{1}{2}$$

$$= \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{2}y$$