#11

$$\frac{dy}{dt} = \text{rate in} - \text{rate out}$$

rate in: $3 \frac{\text{lb}}{\text{gal}} \cdot 8 \frac{\text{gal}}{\text{min}} = 24 \frac{\text{lb}}{\text{min}}$

rate out: $\frac{y}{100+(8-6)t} \frac{\text{lb}}{\text{gal}} \cdot 6 \frac{\text{gal}}{\text{min}}$

$$= \frac{6y}{100+\underbrace{2t}} = \frac{3y}{50+t} \frac{\text{lb}}{\text{min}}$$

every t minutes there are
2 more gallons of solution in the tank

(a) So IVP is
$$\begin{cases} \dfrac{dy}{dt} = 24 - \dfrac{3y}{50+t} \\ y(0) = 7 \end{cases}$$

(b) $\dfrac{dy}{dt} + \dfrac{3y}{50+t} = 24$

$$\mu(t) = e^{\int \frac{3}{50+t}\,dt} = e^{3\ln|50+t|} = e^{3\ln(50+t)} \quad [50+t>0]$$

$$= e^{\ln(50+t)^3} = (50+t)^3$$

So $(50+t)^3\left[\dfrac{dy}{dt} + \dfrac{3y}{50+t}\right] = 24(50+t)^3$

$$\frac{d}{dt}\left[y(50+t)^3\right] = 24(50+t)^3$$

$$y(50+t)^3 = \int 24(50+t)^3\,dt = 6(50+t)^4 + C$$

$$y = 6(50+t) + C(50+t)^{-3}$$

Now $y(0) = 7 \Rightarrow C = -293(50)^3$

So $y(t) = 6(50+t) - 293(50)^3(50+t)^{-3}$

(c) Initially the solution is 100 gallons.

2 gallons enter the tank every minute.

After 50 minutes the tank is at its 200 gallon capacity.

$$y(50) = 600 - 293(50)^3(100)^{-3}$$

$$= 600 - 293\left(\frac{1}{2}\right)^3$$

$$= \frac{4800 - 293}{8} = \boxed{\frac{4507}{8}} \text{ pounds of salt}$$