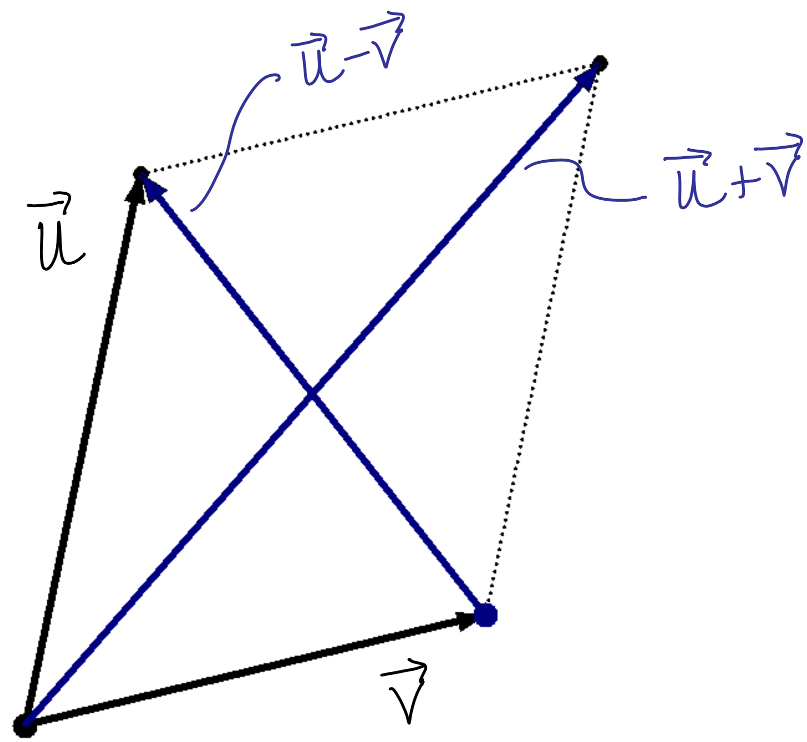
11.3 #9

We can think of this in two different ways.

<u>Geometrically</u>: Note that $\|\vec{u}-\vec{v}\|$ and $\|\vec{v}-\vec{u}\|$ are the lengths of the diagonals of the parallelogram formed by $\vec{u}$ and $\vec{v}$.



The diagonals of a parallelogram have equal length only if the parallelogram is a square, which means $\vec{u}$ is orthogonal to $\vec{v}$.

Algebraically

$$\|\vec{u}+\vec{v}\| = \|\vec{u}-\vec{v}\|$$

$$\iff \|\vec{u}+\vec{v}\|^2 = \|\vec{u}-\vec{v}\|^2$$

$$\iff (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$$

$$\iff \vec{u}\cdot\vec{u}+2(\vec{u}\cdot\vec{v})+(\vec{v}\cdot\vec{v}) = \vec{u}\cdot\vec{u}-2(\vec{u}\cdot\vec{v})+(\vec{v}\cdot\vec{v})$$

$$\iff \vec{u}\cdot\vec{v} = -(\vec{u}\cdot\vec{v})$$

$$\iff 2(\vec{u}\cdot\vec{v}) = 0$$

$$\iff \vec{u}\cdot\vec{v} = 0$$

$\iff$ $\vec{u}$ and $\vec{v}$ are orthogonal