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We begin with the Ratio Test for Absolute Convergence.

$$\lim_{k\to+\infty}\left|\frac{(-5)^{k+1}\,x^{k+1}}{\sqrt{k+11}}\cdot\frac{\sqrt{k+10}}{(-5)^k\,x^k}\right|$$

$$=\lim_{k\to+\infty}\left|(-5)\,x\,\sqrt{\frac{k+10}{k+11}}\right| = 5|x|\,(1) = 5|x|$$

So the series converges if $5|x|<1$,

i.e. if $|x|<\frac{1}{5}$, or $-\frac{1}{5}<x<\frac{1}{5}$.

The test fails if $5|x|=1$, or $x=\pm\frac{1}{5}$, so

we must check these separately.

$x = -\frac{1}{5}$: $\displaystyle\sum_{k=0}^{\infty} \frac{(-5)^k(-\frac{1}{5})^k}{\sqrt{k+10}} = \sum_{k=0}^{\infty} \frac{1}{\sqrt{k+10}} = \sum_{k=10}^{\infty} \frac{1}{\sqrt{k}}$

which diverges (p-series, $p = \frac{1}{2} < 1$).

$x = \frac{1}{5}$: $\displaystyle\sum_{k=0}^{\infty} \frac{(-5)^k(\frac{1}{5})^k}{\sqrt{k+10}} = \sum_{k=0}^{\infty} (-1)^k \frac{1}{\sqrt{k+10}}$

Now $\displaystyle\lim_{k\to+\infty} \frac{1}{\sqrt{k+10}} = 0.$

Also, let $a_k = \dfrac{1}{\sqrt{k+10}}$. So $a_{k+1} = \dfrac{1}{\sqrt{k+11}}$.

Now $a_{k+1} < a_k$, i.e. $a_{k+1} - a_k < 0$. So

$\{a_k\}_0^{+\infty}$ is decreasing, and thus by the

Alternating Series Test $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{1}{\sqrt{k+10}}$ converges.

Conclusion: $\sum_{k=0}^{\infty} \frac{(-5)^k x^k}{\sqrt{k+10}}$ has an

interval of convergence of $\left(-\frac{1}{5}, \frac{1}{5}\right]$ and a

radius of convergence $R = \frac{1}{5}$.