13.8 #8

$f(x,y) = x^2y - 6y^2 - 3x^2$

$f_x(x,y) = 2xy - 6x = 0$ (1)

$f_y(x,y) = x^2 - 12y = 0$ (2)

(2) $\Rightarrow y = \frac{1}{12}x^2 \xrightarrow{(1)} \frac{1}{6}x^3 - 6x = 0$

$\Rightarrow x^3 - 36x = 0$

$\Rightarrow x(x^2 - 36) = 0$

$x = 0, \quad x = -6 \quad x = 6$

$y = 0, \quad y = 3 \quad y = 3$

Three critical points : $(0,0), (-6,3), (6,3)$

$f_{xx}(x,y) = 2y - 6$

$f_{yy}(x,y) = -12$

$f_{xy}(x,y) = 2x$

$D(x,y) = (2y-6)(-12) - (2x)^2$

$D(0,0) = 72 - 0 > 0$ and $f_{xx}(0,0) = -6 < 0$

so there is a relative maximum at $(0,0)$

$D(6,3) = D(-6,3) = 0 - 144 < 0$

so there are saddle points at $(6,3)$ and $(-6,3)$