Rectangular Coordinates, Spheres, & Cylindrical Surfaces

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to determine the location of a point in space using rectangular coordinates.

- Be able to find the distance between and the midpoint of two points in space.

- Know the standard equation of a sphere and be able to find the center and radius of a sphere.

- Be able to sketch cylindrical surfaces.

PRACTICE PROBLEMS:

Problems 1-3 refer to the rectangular box, shown below. The base of the rectangular box is in the xy-plane.
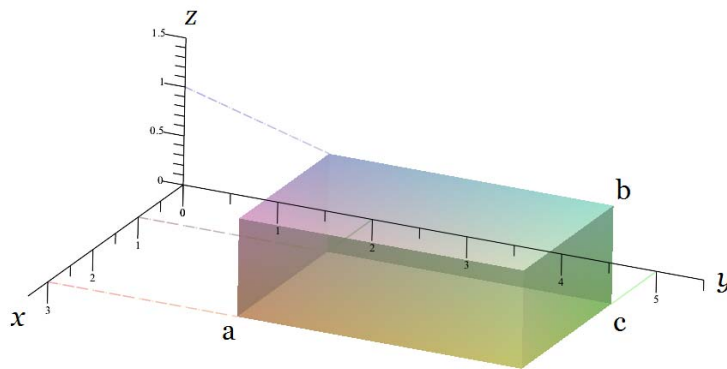
1. Find the coordinates of the eight corners of the box

2. Compute the midpoint of the diagonal which extends from vertex a to vertex b.

3. Consider the triangle with vertices a, b, and c.

    (a) Compute the length of each of the three sides.

(b) Verify that the triangle is a right triangle.

(c) Compute the angle between the diagonal which extends from vertex a to vertex b and the line segment which extends from vertex a to vertex c.

4. Consider the triangle with vertices $A(5, -2, -1)$, $B(7, 0, 3)$, and $C(9, -4, 1)$.

   (a) Show that the triangle is an equilateral triangle.

   (b) Compute the area of the triangle.

5. Find an equation of the sphere whose center is $(3, 0, 2)$ and which has a diameter of 6.

6. Find an equation of the sphere whose center is $(4, 2, -1)$ and which passes through the origin.

7. Find an equation of the sphere which contains points $A(1, 3, 2)$ and $B(4, 3, 7)$ and the distance between A and B is equal to the diameter of the sphere.

8. Does the origin lie inside of the sphere $(x - 1)^2 + (y + 2)^2 + (z + 3)^2 = 13$? Justify your answer.

9. Consider the cube with a center at the origin which has sides of length 2 that are parallel to the coordinate planes.

   (a) Compute an equation of the sphere which is inscribed in this cube.

   (b) Compute an equation of the sphere which is circumscribed around the cube.

10. Find equations of the tangent spheres of equal radii whose centers are $(2, 3, 1)$ and $(5, -3, 2)$, respectively.

11. Sketch the following surfaces in space.

    (a) $3x + 4y = 12$

    (b) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

    (c) $z = x^2$

    (d) $z = e^y$

12. Describe all points in space whose coordinates satisfy the following inequality
$$x^2 + z^2 - 4x - 8z + 13 > 0$$

13. Consider the surface $x^2 + y^2 + z^2 - 4x - 12y - 8z = k$, where k is a real number. For which values of k will the surface be a sphere?