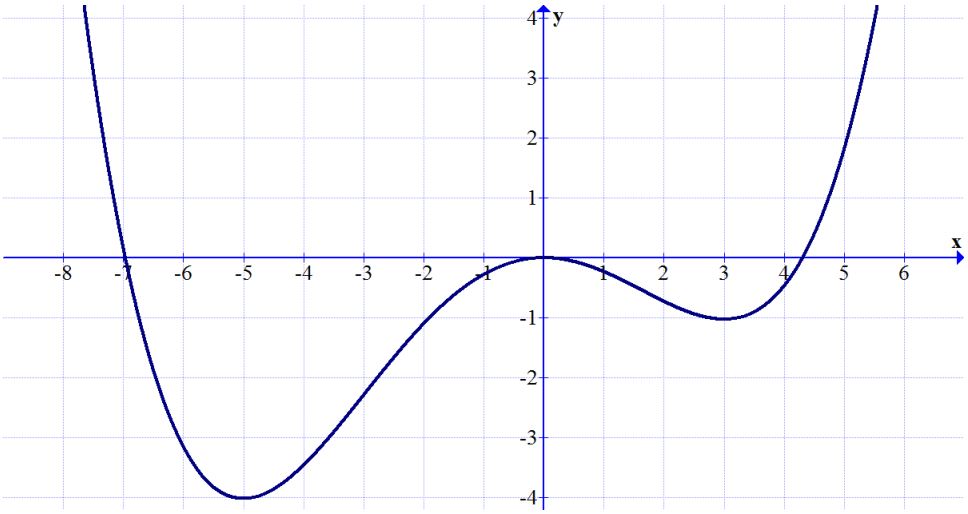
Chapter 4.4 Practice Problems

EXPECTED SKILLS:

- Be able to find the absolute maxima and minima of a function, and where they occur, over a given interval.
- Be able to state and apply the Extreme Value Theorem, where appropriate.

PRACTICE PROBLEMS:

1. Consider the graph of $y = f(x)$, shown below. For each of the following, compute the absolute maximum and absolute minimum values of $f(x)$ on the given interval, if they exist. (Make reasonable assumptions about the behavior of the function outside of the shown interval.)



(a) $(-\infty, \infty)$

No absolute maximum; Absolute minimum of -4 when $x = -5$

(b) $[-7, 5]$

Absolute maximum of 2 when $x = 5$; Absolute minimum of -4 when $x = -5$

(c) $[-6, -2]$

Absolute maximum of -1 when $x = -2$; Absolute minimum of -4 when $x = -5$

(d) $[-7.5, -6]$

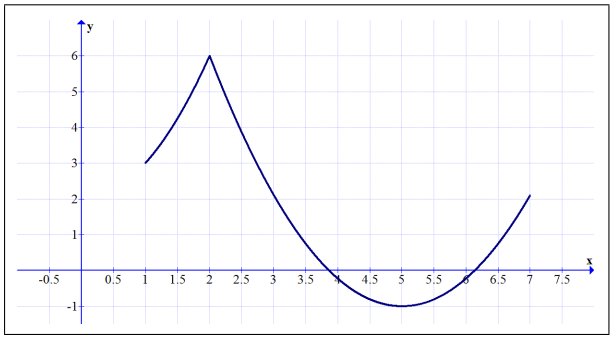
Absolute maximum of 3 when $x = -7.5$; Absolute minimum of -3 when $x = 6$

(e) $(-4, 1)$

Absolute maximum of 0 when $x = 0$; No absolute minimum

2. Sketch the graph of a continuous function, $y = f(x)$, which has all of the following properties:

   - $f(x)$ has a domain of $[1, 7]$
   - $f(x)$ has an absolute maximum of 6 when $x = 2$ and an absolute minumum of -1 when $x = 5$.
   - $f''(x) > 0$ for all x in the domain of $f(x)$, with the exception of $x = 2$ where $f''(x)$ DNE.



For each of the following, find the absolute maximum and minimum values of $f(x)$ on the given interval.

3. $f(x) = x^2 + 3x - 4$ on $[-3, 3]$.

   absolute maximum of 14 when $x = 3$; absolute minimum of $\dfrac{-25}{4}$ when $x = \dfrac{3}{2}$

4. $f(x) = (2x + 1)^3$ on $[-1, 4]$.

   absolute maximum of 729 of $x = 4$; absolute minimum of -1 when $x = -1$

5. $f(x) = \dfrac{x - 3}{(x - 4)^2}$ on $[-4, 1]$.

   absolute minimum of $-\dfrac{2}{9}$ when $x = 1$, absolute maximum of $-\dfrac{7}{64}$ when $x = -4$

6. $f(x) = \cos x - \sin x$ on $[-\pi, \pi]$.

   absolute maximum of $\sqrt{2}$ when $x = -\dfrac{\pi}{4}$, absolute minimum of $-\sqrt{2}$ when $x = \dfrac{3\pi}{4}$

7. $f(x) = \sqrt{1 - x^2}$ on $[-1, 1]$

   absolute minimum of 0 when $x = -1$ and when $x = 1$; absolute maximum of 1 when $x = 0$

8. $f(x) = |x - 3|$ on $[-5, 5]$

   absolute minimum of 0 when $x = 3$, absolute maximum of 8 when $x = -5$

9. $f(x) = x^{\frac{1}{3}}(x - 5)^2$ on $[1, 10]$

   absolute minimum of 0 when $x = 5$, absolute maximum of $25 \cdot \sqrt[3]{10}$ when $x = 10$

10. $f(x) = \tan x + \sin x$ on $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

    absolute minimum of $-1 - \frac{\sqrt{2}}{2}$ when $x = \frac{\pi}{4}$

    absolute maximum of $1 + \frac{\sqrt{2}}{2}$ when $x = -\frac{\pi}{4}$

11. $f(x) = 3x^2 - 4x + 9$ on $(-\infty, \infty)$

    no absolute maximum, absolute minimum of $\frac{23}{3}$ when $x = \frac{3}{2}$

12. $f(x) = -x^2 + 5x - 10$ on $(-\infty, \infty)$

    no absolute minimum, absolute maximum at $-\frac{15}{4}$ when $x = \frac{5}{2}$

13. $f(x) = \frac{x - 2}{x + 5}$ on $(-\infty, \infty)$

    none

14. $f(x) = x^2 e^{-2x}$ on $(-\infty, \infty)$

    no absolute maximum, absolute minimum of 0 when $x = 0$