Area Between Two Curves

SUGGESTED REFERENCE MATERIAL:

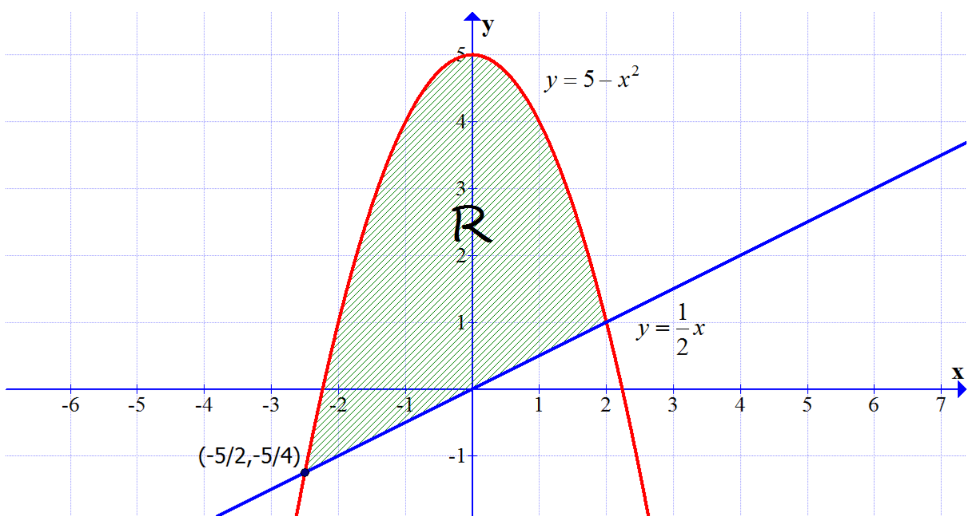
As you work through the problems listed below, you should reference Chapter 6.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to find the area between the graphs of two functions over an interval of interest.
- Know how to find the area enclosed by two graphs which intersect.

PRACTICE PROBLEMS:

1. Let R be the shaded region shown below.



(a) Set up but do not evaluate an integral (or integrals) in terms of x that represent(s) the area of R.

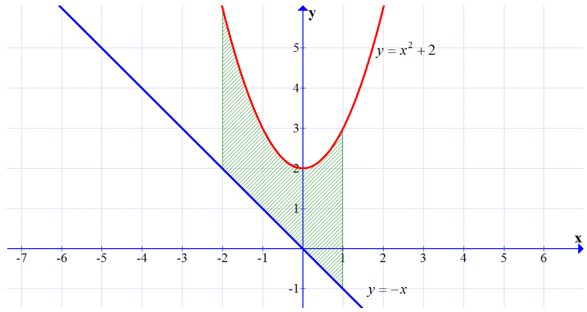
$$\boxed{\int_{-\frac{5}{2}}^{2} \left(-x^2 - \frac{1}{2}x + 5\right)\, dx}$$

(b) Set up but do not evaluate an integral (or integrals) in terms of y that represent(s) the area of R.

$$\boxed{\int_{-\frac{5}{4}}^{1} \left(2y + \sqrt{5-y}\right)\, dy + 2\int_{1}^{5} \sqrt{5-y}\, dy}$$

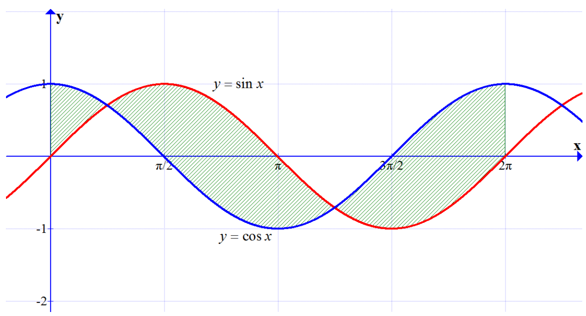
For problems 2-4, compute the area of the shaded region.

2.



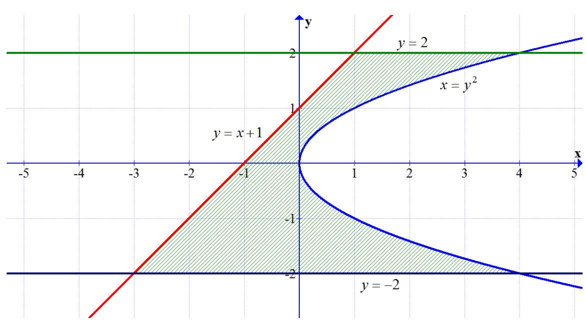
$$\boxed{A = \frac{15}{2}}$$

3.



$$\boxed{A = 4\sqrt{2}}$$

4.



$\boxed{A = \frac{28}{3}\text{; Detailed Solution: Here}}$

For problems 5-13, compute the area of the region which is enclosed by the given curves.

5. $y = 4x$, $y = 6x^2$

   $\boxed{\dfrac{8}{27}}$

6. $y = 2x^2$, $y = x^2 + 2$

   $\boxed{\dfrac{8\sqrt{2}}{3}}$

7. $y = x^{2/3}$, $y = x^4$, in the first quadrant

   $\boxed{\dfrac{2}{5}\text{; Detailed Solution: Here}}$

8. $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, $x = 4$

   $\boxed{-\dfrac{3}{4} + 2\ln 2}$

9. $y = \sin x$, $y = 2 - \sin x$, $\dfrac{\pi}{2} \leq x \leq \dfrac{5\pi}{2}$

   $\boxed{4\pi}$

10. $y = e^{5x}$, $y = e^{8x}$, $x = 1$

    $\boxed{\dfrac{3}{40} + \dfrac{1}{8}e^8 - \dfrac{1}{5}e^5}$

11. $x = 4 - y^2$, $x = y^2 - 4$

    $\boxed{\dfrac{64}{3}\text{; Detailed Solution: Here}}$

12. $y = x^4$, $y = |x|$

    $\boxed{\dfrac{3}{5}}$

13. $y = x^2$, $y = \dfrac{2}{x^2 + 1}$

    $\boxed{\pi - \dfrac{2}{3}}$

14. Let R be the region enclosed by $y = x$, $y = 8x$, and $y = 4$.

    (a) Compute the area of R by evaluating an integral (or integrals) <u>in terms of x</u>.

    $$\boxed{\int_0^{\frac{1}{2}} 7x\,dx + \int_{\frac{1}{2}}^{4} (4 - x)\,dx = 7}$$

    (b) Compute the area of R by evaluating an integral (or integrals) <u>in terms of y</u>.

    $$\boxed{\int_0^4 \frac{7}{8} y\,dy = 7}$$

15. Use an integral (or integrals) to compute the area of the triangle in the xy-plane which has vertices $(0, 0)$, $(2, 3)$, and $(-1, 6)$.

    $$\boxed{\frac{15}{2}}$$

16. Consider the 2D ice cream cone topped with a delicious scoop of ice cream that is enclosed by $y = 6|x|$ and $y = 16 - x^2$.

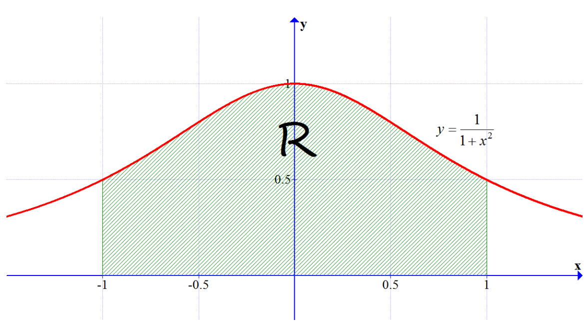
    (a) Compute the area enclosed within the ice cream cone (including the scoop portion).

    $$\boxed{\frac{104}{3}}$$

    (b) After a bite is taken from the top, the remaining area is enclosed by $y = 6|x|$, $y = 16 - x^2$, and $y = x^2 + 12$. Compute the area of the remaining portion.

    $$\boxed{\frac{104}{3} - \frac{16\sqrt{2}}{3}}$$

17. Consider the region R shown below:

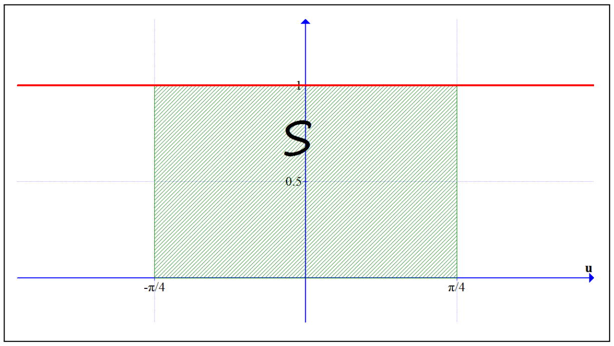


The area of the region R is equivalent to $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx$.

(a) Using the substitution $u = \tan^{-1} x$, express the given integral (including the limits of integration) in terms of the variable u.

$$\boxed{\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, du}$$

(b) Sketch a region whose area is equivalent to your integral from part (a). Label this region S.



(c) Evaluate the original integral and your integral from part (a). Conclude that the area of region R is equal to the area of region S.

(Note: This is an example of how changing coordinate systems can simplify a problem. We will discuss this idea more in Math 200 and Math 201.)

$$\boxed{\frac{\pi}{2}}$$