Integration by Substitution

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 5.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Know how to simplify a "complicated integral" to a known form by making an appropriate substitution of variables.

PRACTICE PROBLEMS:

For problems 1-21, evaluate the given indefinite integral and verify that your answer is correct by differentiation.

1. $\displaystyle\int 3x^2(x^3+3)^3\,dx$

   $\boxed{\dfrac{1}{4}(x^3+3)^4+C}$

2. $\displaystyle\int \frac{5}{5x+3}\,dx$

   $\boxed{\ln|5x+3|+C}$

3. $\displaystyle\int 2x\cos(x^2)\,dx$

   $\boxed{\sin(x^2)+C}$

4. $\displaystyle\int 4x(x^2+6)^2\,dx$

   $\boxed{\dfrac{2}{3}(x^2+6)^3+C}$

5. $\displaystyle\int \sec(4x)\tan(4x)\,dx$

   $\boxed{\dfrac{1}{4}\sec(4x)+C}$

6. $\displaystyle\int (3x-5)^9\,dx$

   $\boxed{\dfrac{1}{30}(3x-5)^{10}+C}$

7. $\displaystyle\int e^{-2x}\,dx$

$\boxed{-\frac{1}{2}e^{-2x} + C}$

8. $\displaystyle\int \frac{\sin x \cos x}{1+\sin^2 x}\,dx$

$\boxed{\frac{1}{2}\ln\left(1+\sin^2 x\right) + C}$

9. $\displaystyle\int \csc\left(\frac{x}{2}\right)\cot\left(\frac{x}{2}\right)\,dx$

$\boxed{-2\csc\left(\frac{x}{2}\right) + C}$

10. $\displaystyle\int -3x^3\sqrt{1-x^4}\,dx$

$\frac{1}{2}(1-x^4)^{\frac{3}{2}} + C$; Video Solution: https://www.youtube.com/watch?v=6EqONIlA0Vc

11. $\displaystyle\int \left(\frac{e^{\sqrt{x}}}{\sqrt{x}} - \frac{1}{4}\cos 4x\right)\,dx$

$\boxed{2e^{\sqrt{x}} - \frac{1}{16}\sin(4x) + C}$

12. $\displaystyle\int \frac{1}{2+4x^2}\,dx$

$\frac{1}{2\sqrt{2}}\arctan(\sqrt{2}x) + C$; Video Solution: http://www.youtube.com/watch?v=HVA-eDtKsG4

13. $\displaystyle\int \frac{4x}{(3+x^2)^2}\,dx$

$\boxed{-2(3+x^2)^{-1} + C}$

14. $\displaystyle\int x^2\sqrt{4-x}\,dx.$

$\boxed{-\frac{32}{3}(4-x)^{3/2} + \frac{16}{5}(4-x)^{5/2} - \frac{2}{7}(4-x)^{7/2} + C}$

15. $\displaystyle\int \frac{1}{\sqrt{\frac{3}{4}+x-x^2}}\,dx$ (HINT: Complete the square)

$\boxed{\arcsin\left(x-\frac{1}{2}\right)+C\text{; Detailed Solution: Here}}$

16. $\displaystyle\int \frac{e^{3/x}}{x^2}\,dx$

$\boxed{-\frac{1}{3}e^{3/x}+C\text{; Detailed Solution: Here}}$

17. $\displaystyle\int \frac{e^x}{e^{2x}+1}dx$

$\boxed{\tan^{-1}\left(e^x\right)+C}$

18. $\displaystyle\int (\sin 4x)(\cos 4x)^{2/3}dx$

$\boxed{-\frac{3}{20}(\cos 4x)^{5/3}+C}$

19. $\displaystyle\int \csc^2(3x)\tan^2(3x)+x^2e^{x^3}\,dx$

$\boxed{\frac{1}{3}\tan(3x)+\frac{1}{3}e^{x^3}+C}$

20. $\displaystyle\int \frac{1}{x\ln x}\,dx$

$\boxed{\ln|\ln x|+C}$

21. $\displaystyle\int \frac{\cos^{-1}x}{\sqrt{1-x^2}}\,dx$

$\boxed{-\frac{1}{2}\left(\cos^{-1}x\right)^2+C}$

22. Use an appropriate trigonometric identity followed by a reasonable substitution to evaluate $\displaystyle\int \tan x\,dx$

$\boxed{\ln|\sec x|+C}$

23. It can be shown that $\dfrac{32x^2+77x+49}{(3x+1)(4x+5)^2} = \dfrac{2}{3x+1} - \dfrac{1}{(4x+5)^2}$. Use this fact to evaluate $\displaystyle\int \frac{32x^2+77x+49}{(3x+1)(4x+5)^2}\,dx$.

$$\boxed{\frac{2}{3}\ln|3x+1| + \frac{1}{4(4x+5)} + C}$$

24. Using the substitution $x = \sin\theta$ for $-\dfrac{\pi}{2} \leq \theta \leq \dfrac{\pi}{2}$, evaluate $\displaystyle\int \sqrt{1-x^2}\,dx$. Express your answer completely in terms of the variable x.

HINT - The following trigonometric identities will be helpful: $\sin^2\theta + \cos^2\theta = 1$, $\cos^2\theta = \dfrac{1}{2}(1+\cos(2\theta)$, and $\sin(2\theta) = 2\sin\theta\cos\theta$

$$\boxed{\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + C}$$