Trigonometric Substitution

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 7.4 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to evaluate integrals that involve particular expressions (see Table 7.4.1) by making the appropriate trigonometric substitution.
- Know how to evaluate integrals that involve quadratic expressions by first completing the square and then making the appropriate substitution.

PRACTICE PROBLEMS:

For problems 1-12, evaluate the given integral. Notice that it may not be necessary to use a trigonometric substitution for all problems.

1. $\displaystyle\int \sqrt{3-x^2}\,dx$
2. $\displaystyle\int \frac{1}{(x^2+1)^2}\,dx$
3. $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx$
4. $\displaystyle\int \frac{x}{\sqrt{1-4x^2}}\,dx$
5. $\displaystyle\int \frac{x^2}{\sqrt{1-2x^2}}\,dx$
6. $\displaystyle\int_1^{\sqrt{3}} x\sqrt{x^2+1}\,dx$
7. $\displaystyle\int_{\sqrt{2}}^{2} \frac{\sqrt{4-x^2}}{x^2}\,dx$
8. $\displaystyle\int \frac{1}{x^2\sqrt{x^2+16}}\,dx$
9. $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

10. $\displaystyle\int_{-\sqrt{5}}^{\sqrt{15}} \frac{1}{x^2+5}\,dx$

11. $\displaystyle\int \frac{1}{4x^2-2x+17/4}\,dx$

12. $\displaystyle\int \frac{1}{\sqrt{-x^2+4x-3}}\,dx$

13. Compute the area enclosed within the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$.

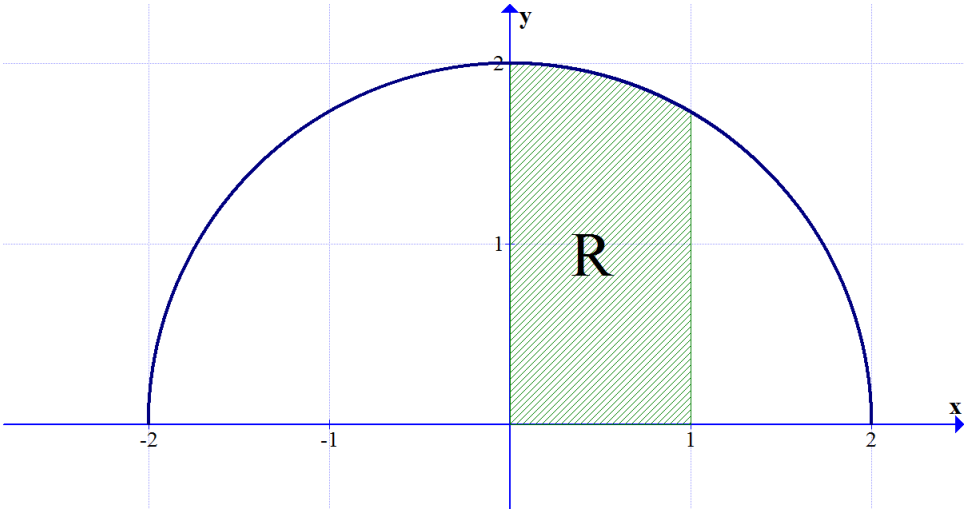
14. Let R be the region in the xy-plane which is enclosed by $y=\dfrac{1}{x^2+1}$, $y=0$, $x=0$ and $x=1$. Calculate the volume of the solid which results from revolving R around the x-axis. (Hint: see number 2 above.)

15. Compute the length of the curve $y=x^2$ on the interval $\left[-\dfrac{\sqrt{3}}{2},\dfrac{1}{2}\right]$. (Hint: See problem 25 (a) in the "Trigonometric Integrals (Chapter 7.3)" homework.)

16. (a) Evaluate $\displaystyle\int \frac{\sqrt{x^2+1}}{x}\,dx$. (Hint: $\displaystyle\int \frac{\sec^3\theta}{\tan\theta}\,d\theta=\sec\theta-\ln\left|\csc\theta+\cot\theta\right|+C$)

    (b) Compute the length of the curve $y=\ln x$ on the interval $[1,3]$. (Hint: Use part a.)

17. Consider the region R which is enclosed by $y=\sqrt{4-x^2}$, $y=0$, $x=0$, and $x=1$, in the first quadrant.



    (a) By evaluating an appropriate integral, compute the area of R.

(b) Verify your answer geometrially by combining the area of the sector and the area of the triangle, shown below.