Chapter 3.2 Practice Problems

EXPECTED SKILLS:

- Be able to compute the derivatives of logarithmic functions.
- Know how to use logarithmic differentiation to help find the derivatives of functions involving products and quotients.

PRACTICE PROBLEMS:

For problems 1-16, calculate $\frac{dy}{dx}$.

1. $y = \ln(x^2)$

   $\boxed{\frac{2}{x}}$

2. $y = \frac{1}{\ln(3x)}$

   $\boxed{-\frac{1}{x[\ln(3x)]^2}}$

3. $y = x^2 \ln x$

   $\boxed{x + 2x\ln x}$

4. $y = \ln\left(\frac{1}{x}\right)$

   $\boxed{-\frac{1}{x}}$

5. $y = \ln|x^3|$

   $\boxed{\frac{3}{x}}$

6. $y = \ln(x^2+1)^2$

   $\boxed{\frac{4x}{x^2+1}}$

7. $y = \left[\ln(x^2+1)\right]^2$

   $\boxed{\frac{4x\ln(x^2+1)}{x^2+1}}$

8. $y = \sqrt{\ln 2x}$

$\boxed{\dfrac{1}{2x\sqrt{\ln(2x)}}}$

9. $y = \log_2(3x-1)$

$\boxed{\dfrac{3}{(3x-1)\ln 2}}$

10. $y = \tan(\ln x)$

$\boxed{\dfrac{1}{x}\sec^2(\ln x)}$

11. $y = \ln(\ln x)$

$\boxed{\dfrac{1}{x\ln x}}$

12. $y = \dfrac{\log x}{2-\log x}$

$\boxed{\dfrac{2}{x\ln(10)(2-\log x)^2}}$

13. $y = \ln|\sec x|$

$\boxed{\tan x}$

14. $y = \ln|\sec x + \tan x|$

$\boxed{\sec x}$

15. $y = \ln(x^x)$

$\boxed{1+\ln(x)}$

16. $y = \ln\left(\dfrac{2x+1}{\sqrt{x}(3x-4)^{10}}\right)$

$\boxed{\dfrac{2}{2x+1} - \dfrac{1}{2x} - \dfrac{30}{3x-4}}$

17. Use logarithmic differentiation to calculate $\dfrac{dy}{dx}$ if $y = \dfrac{2x+1}{\sqrt{x}(3x-4)^{10}}$

$\boxed{\dfrac{2x+1}{\sqrt{x}(3x-4)^{10}}\left(\dfrac{2}{2x+1} - \dfrac{1}{2x} - \dfrac{30}{3x-4}\right)}$

18. Recall the change of base formula: $\log_b x = \dfrac{\ln x}{\ln b}$

   (a) Remind yourself of why this is true.

   $\boxed{\begin{array}{l}\text{Proof: Suppose } y = \log_b x. \text{ This is equivalent to the exponential equation } b^y = x. \\ \text{Now, we take the natural log of both sides, which gives us } \ln(b^y) = \ln x. \\ \text{Using a property of logarithms, we see that this is equivalent to } y \ln b = \ln x. \\ \text{Finally, we solve for } y \text{ by dividing both sides of the equation by } \ln b \text{, a non-zero} \\ \text{constant since } b > 0 \text{ and } b \neq 1. \\ \text{Thus, } y = \dfrac{\ln x}{\ln b}.\end{array}}$

   (b) Compute y' if $y = \log_{x^2}(e)$

   $\boxed{-\dfrac{1}{2x(\ln x)^2}}$

   (c) Compute $\dfrac{dy}{dx}$ if $y = \log_{3x}(x)$

   $\boxed{\dfrac{\ln 3}{x(\ln 3x)^2}}$

19. Compute an equation of the line which is tangent to the graph of $f(x) = \ln(x^2 - 3)$ at the point where $x = 2$.

   $\boxed{y = 4x - 8}$

20. Find the value(s) of x at which the tangent line to the graph of $y = \ln(x^2 + 11)$ is perpendicular to $y = -6x + 5$.

   $\boxed{x = 1 \text{ and } x = 11}$

21. Find the value(s) of x at which the tangent line to the graph of $y = -\ln x$ passes through the origin.

   $\boxed{x = e}$

22. Calculate $\dfrac{d^2y}{dx^2}$ if $y = \ln(3x^2 + 2)$.

   $\boxed{\dfrac{12 - 18x^2}{(3x^2 + 2)^2}}$

23. **Multiple Choice:** Let $y = \ln(\cos x)$. Which of the following is $\dfrac{dy}{dx}$?

(a) $(\ln x)(-\sin x) + (\cos x)(\ln x)$

(b) $-\tan x$

(c) $\cot x$

(d) $\sec x$

(e) $\dfrac{1}{\ln(\cos x)}$

$\boxed{\text{B}}$

24. **Multiple Choice:** Let $h(x) = \ln[(f(x))^2 + 1]$. Suppose that $f(1) = -1$ and $f'(1) = 1$. Find $h'(1)$.

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

$\boxed{\text{B}}$

25. Consider the triangle formed by the tangent line to the graph of $y = -\ln x$ at the point $P(t, -\ln t)$, the horizontal line which passes through P, and the y-axis. Find a function $A(t)$ which gives the area of this triangle.

$\boxed{A(t) = \frac{1}{2}t}$