Chapter 1.2 Practice Problems

EXPECTED SKILLS:

- Know the basic properties of limits; i.e., be familiar with how limits "interact" with sums, differences, products, and other operations. See Theorem 1.2.2.
- Given the formula of a function $y = f(x)$, be able to determine the limit of $f(x)$ as x approaches some finite value (as both a one-sided and two sided limit).
- Be able to determine when such a limit does not exist, and if appropiate, indicate if the behavior of the function is increasing or decreasing without bound.
- Be familiar with the indeterminate forms of $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$. And, know how to use algebraic techniques such as factoring and rationalizing to help compute these types of limits.

PRACTICE PROBLEMS:
In each problem, compute the limit. If the limit doesn't exist write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

1. Given that $\lim_{x\to 1} f(x) = 4$ and $\lim_{x\to 1} g(x) = 2$, determine each of the following limits:
   (a) $\lim_{x\to 1} (f(x) + g(x))$
   (b) $\lim_{x\to 1} (5f(x) - g(x))$
   (c) $\lim_{x\to 1} \left(\frac{f(x)}{g(x)}\right)$
2. $\lim_{x\to 1} \left(x^2 + 1\right)$
3. $\lim_{x\to 4} 1$
4. $\lim_{x\to -1} (x+1)(x^3)$
5. $\lim_{x\to 5^-} \left(\frac{x^2 - 6x}{x^3 - 1}\right)$
6. $\lim_{x\to -1} \left(\frac{x^2 - 1}{x + 1}\right)$
7. $\lim_{x\to 2^-} \left(\frac{x^2 - 4x + 4}{x - 2}\right)$

8. $\lim_{x\to 3^+}\left(\dfrac{x^2+2x-15}{x-3}\right)$

9. $\lim_{x\to 1}\left(\dfrac{x^3-3x^2-x+3}{x^2-1}\right)$

10. $\lim_{x\to 16}\left(\dfrac{\sqrt{x}-4}{x-16}\right)$

11. $\lim_{x\to 0}\left(\dfrac{|x|}{x}\right)$

12. $\lim_{x\to 4^-}\left(\dfrac{x}{x-4}\right)$

13. $\lim_{x\to 4^+}\left(\dfrac{x}{x-4}\right)$

14. $\lim_{x\to 4}\left(\dfrac{x}{x-4}\right)$

15. $\lim_{x\to -2}\left(\dfrac{1}{x-2}\right)$

16. $\lim_{x\to -2^-}\left(\dfrac{x}{x^2+2x}\right)$

17. $\lim_{x\to -2^+}\left(\dfrac{x}{x^2+2x}\right)$

18. $\lim_{x\to 3}\left(\dfrac{x^3}{|x-3|}\right)$

19. $\lim_{x\to 1^-}\left(\dfrac{x-1}{x^2-2x+1}\right)$

20. $\lim_{x\to 1^+}\left(\dfrac{x-1}{x^2+2x-3}\right)$

21. Let $f(n)=\begin{cases} n^2+1, & \text{if } n\le -1 \\ 3n+1, & \text{if } n>-1 \end{cases}$. Compute $\lim_{n\to -1} f(n)$

22. Let $f(x)=\begin{cases} 3x^3+2x-3, & \text{if } x<1 \\ 100, & \text{if } x=1 \\ \dfrac{x^2-1}{x-1}, & \text{if } x>1 \end{cases}$. Determine $\lim_{x\to 1} f(x)$

23. Let $f(p) = \begin{cases} 3p - 1, & \text{if } p < 3 \\ p^3 - 4p - 7, & \text{if } p > 3 \end{cases}$. Find $\lim_{p \to 3} f(p)$

24. Let $f(x) = \begin{cases} x^2 + 2ax + a^2, & \text{if } x < 4 \\ 432, & \text{if } x = 4 \\ x^2 - 7, & \text{if } x > 4 \end{cases}$. Find the value(s) of a such that $\lim_{x \to 4} f(x)$ exists.

25. Let x_0 be a fixed real number. Compute $\lim_{h \to 0} \dfrac{(x_0 + h)^2 - x_0^2}{h}$

26. Let x_0 be a fixed real number. Compute $\lim_{x \to x_0} \dfrac{x^2 - x_0^2}{x - x_0}$

27. Let x_0 be a fixed, positive real number. Compute $\lim_{h \to 0} \dfrac{\sqrt{x_0 + h} - \sqrt{x_0}}{h}$