(a) $r=0 \implies 0=\sqrt{3}+2\sqrt{3}\cos\theta$

$\implies \cos\theta = -\frac{1}{2} \implies \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \tfrac{1}{2}(\sqrt{3}+2\sqrt{3}\cos\theta)^2\,d\theta$$

$$= \cancel{2}\int_{\frac{2\pi}{3}}^{\pi} \cancel{\tfrac{1}{2}}(\sqrt{3}+2\sqrt{3}\cos\theta)^2\,d\theta \quad \text{by symmetry}$$

$$= \int_{\frac{2\pi}{3}}^{\pi} (3+12\cos\theta+12\cos^2\theta)\,d\theta = \int_{\frac{2\pi}{3}}^{\pi} (3+12\cos\theta+6(1+\cos 2\theta))\,d\theta$$

$$= 9\theta\Big|_{\frac{2\pi}{3}}^{\pi} + 12\sin\theta\Big|_{\frac{2\pi}{3}}^{\pi} + 3\sin 2\theta\Big|_{\frac{2\pi}{3}}^{\pi}$$

$$= 9\left(\tfrac{\pi}{3}\right)+12\left(0-\tfrac{\sqrt{3}}{2}\right)+3\left(0-\left(-\tfrac{\sqrt{3}}{2}\right)\right) = 3\pi-6\sqrt{3}+\frac{3\sqrt{3}}{2} = 3\pi-\frac{9\sqrt{3}}{2}$$

(b) $A = \int_0^{2\pi} \frac{1}{2}(\sqrt{3}+2\sqrt{3}\cos\theta)^2 \, d\theta \quad - \; 2(\text{area within inner loop})$

part (a)

$$= \int_0^{2\pi} \frac{1}{2}(3+12\cos\theta+12\cos^2\theta)\,d\theta \; - 2\left(3\pi - \frac{9\sqrt{3}}{2}\right)$$

$$= \int_0^{2\pi}\left(\frac{3}{2}+6\cos\theta+3(1+\cos 2\theta)\right)d\theta \; - 6\pi + 9\sqrt{3}$$

$$= \frac{9}{2}\theta \Big|_0^{2\pi} + 6\sin\theta\Big|_0^{2\pi} + \frac{3}{2}\sin 2\theta\Big|_0^{2\pi} \; - 6\pi + 9\sqrt{3}$$

$$= \frac{9}{2}(2\pi) + 6(0) + \frac{3}{2}(0) - 6\pi + 9\sqrt{3}$$

$$= 3\pi + 9\sqrt{3}$$