Multivariable Functions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

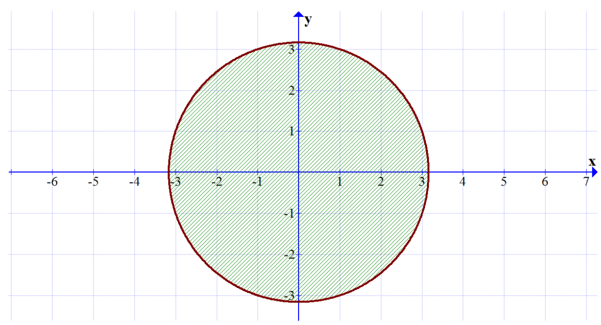
- Be able to describe and sketch the domain of a function of two or more variables.
- Know how to evaluate a function of two or more variables.
- Be able to compute and sketch level curves & surfaces.

PRACTICE PROBLEMS:

1. For each of the following functions, describe the domain in words. Whenever possible, draw a sketch of the domain as well.

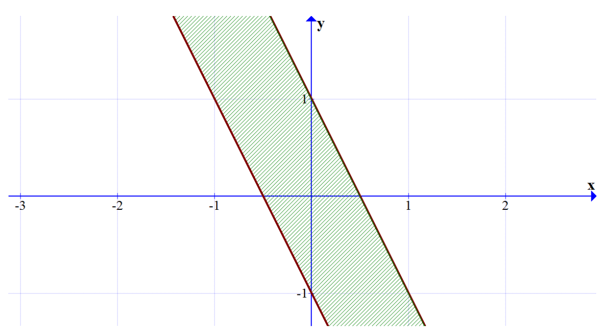
   (a) $f(x, y) = \sqrt{10 - x^2 - y^2}$

   The domain is all points in the xy-plane which are on or inside of $x^2 + y^2 = 10$, the circle with a radius of $\sqrt{10}$ centered at the origin.

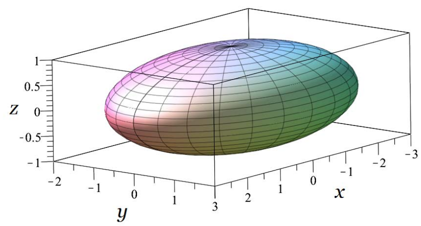
   (b) $f(x, y) = \arcsin(2x + y)$

   The domain is all points in the xy plane which are between the lines $y = -2x - 1$ and $y = -2x + 1$, including the points on the lines.

(c) $f(x,y,z) = \ln(36 - 4x^2 - 9y^2 - 36z^2)$

All point in 3-space which are inside of (but not on) the ellipsoid $\dfrac{x^2}{9} + \dfrac{y^2}{4} + z^2 = 1$.



(d) $f(x,y,z) = \sqrt{6 - 2x - 3y - z}$

All points in 3-space which are on or below the plane $2x + 3y + z = 6$

2. Let $f(x,y) = 2xe^{3y}$. Compute the following.

(a) $f(4,0)$

$\boxed{8}$

(b) $f(1, \ln 2)$

$\boxed{16}$

3. Suppose $f(x,y) = \displaystyle\int_x^y (t^2 - 1)\, dt$. Compute the following.

(a) $f(-1,2)$

$\boxed{0}$

(b) $f(0,2)$

$\boxed{\dfrac{2}{3}}$

4. Suppose $f(x_1, x_2, \ldots, x_n) = x_1 + 2x_2 + 3x_3 + \cdots + nx_n$. Determine $f(1,1,\ldots,1)$.

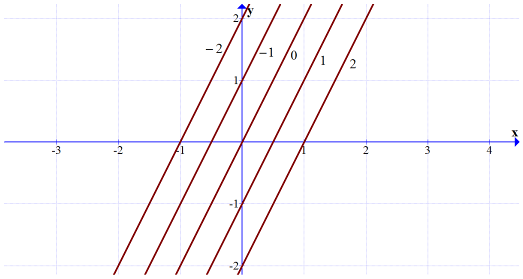
$\boxed{\dfrac{n(n+1)}{2}}$

5. Consider $f(x,y) = x^2 + y^2$. Compute $f(x(t), y(t))$ if $x(t) = 1 + t$ and $y(t) = 2 - 3t$

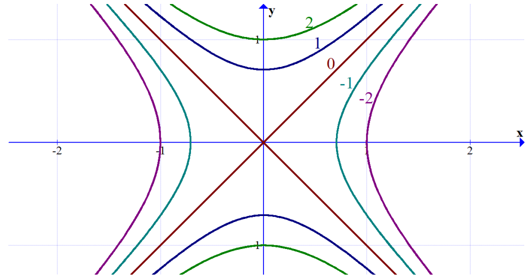
$\boxed{10t^2 - 10t + 5}$

6. Sketch the level curves $f(x, y) = k$, for the specified values of k.

(a) $z = 2x - y$; $k = -2, -1, 0, 1, 2$

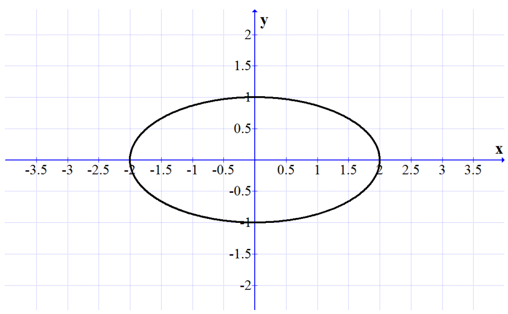


(b) $z = y^2 - x^2$; $k = -2, -1, 0, 1, 2$

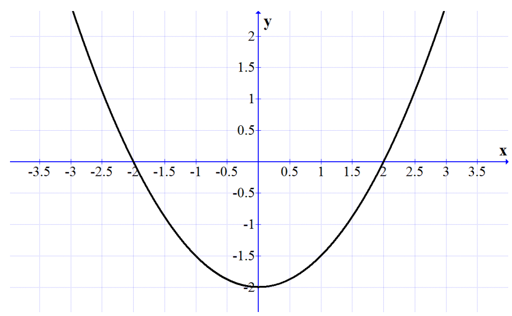


7. **Multiple Choice:** Which of the following graphs is the level curve of $f(x, y) = x^2+4y^2$ which passes through $P(-2, 0)$?

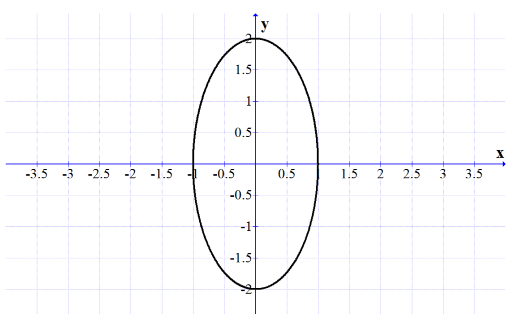
$\boxed{\text{(a)}}$



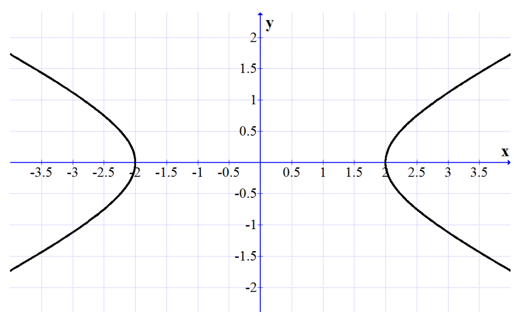
(d)



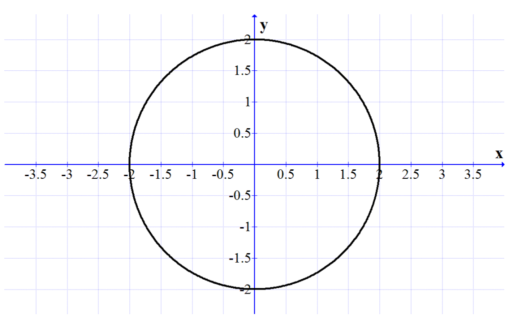
(b)



(e)



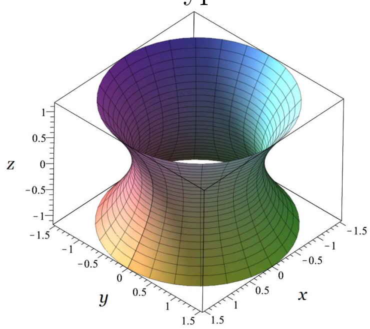
(c)



8. Suppose $f(x, y, z) = x^2 + y^2 - z^2$. For each of the following, sketch the level surface $f(x, y, z) = k$ corresponding to the indicated value of k.

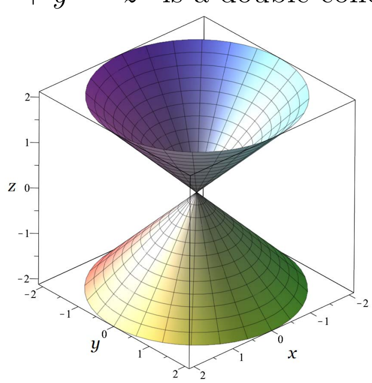
(a) $k = 1$

$x^2 + y^2 - z^2 = 1$ is a hyperboloid of 1 sheet.



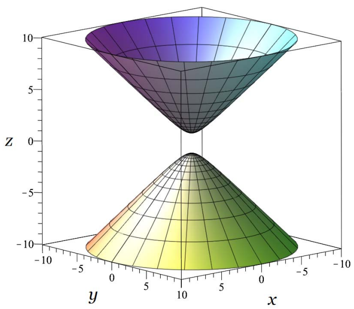
(b) $k = 0$

$x^2 + y^2 = z^2$ is a double cone.

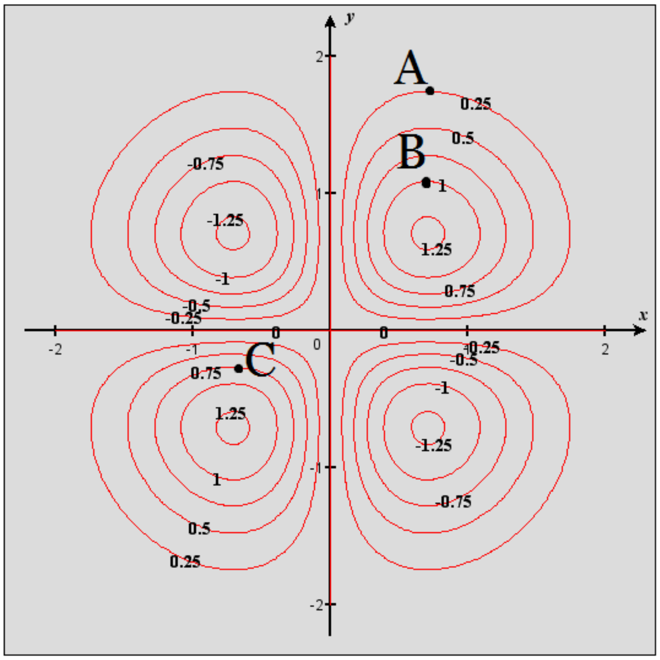


(c) $k = -1$

$-x^2 - y^2 + z^2 = 1$ is a hyperboloid of 2 sheets.



9. Consider the contour map shown below.



(a) If a person were walking straight from point A to point B, would s/he be walking uphill or downhill?

$\boxed{\text{Uphill}}$

(b) Is the slope steeper at point B or point C?

$\boxed{\text{Point } C}$

(c) Starting at C and moving so that x remains contant and y decreases, will the elevation begin to increase or decrease?

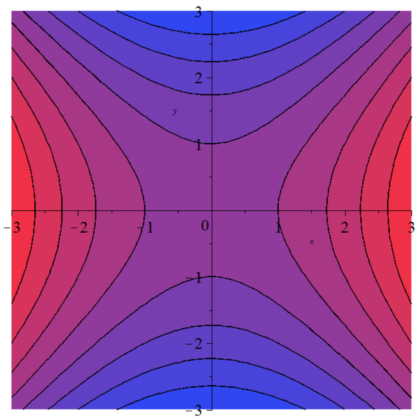
$\boxed{\text{Increase}}$

(d) Starting at B and moving so that y remains contant and x increases, will the elevation begin to increase or decrease?

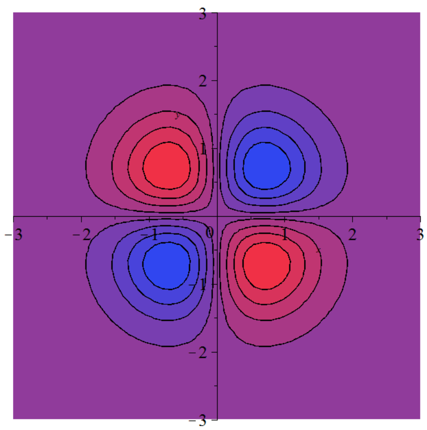
$\boxed{\text{Decrease}}$

10. **Matching:** Each of the following contour plots were drawn on the window $[-3, 3] \times [-3, 3]$ in the xy-plane. Points with larger z-values are shaded in blue. Those with smaller z-values are shaded in red. Match each contour map (a-f) to an appropriate graph (I-VI).

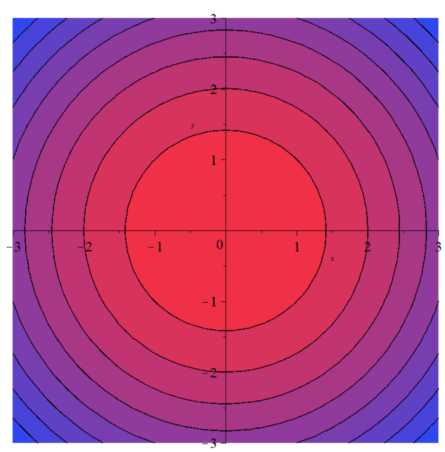
(a)



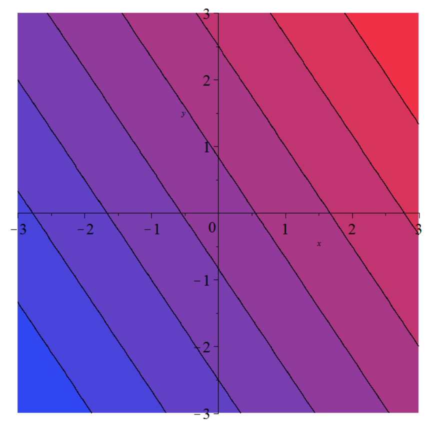
(b)



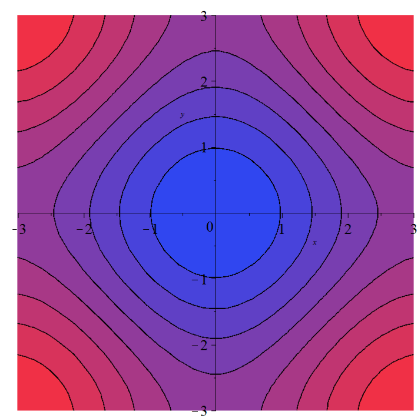
(c)



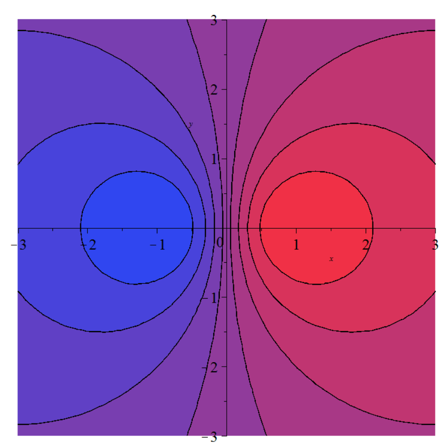
(d)



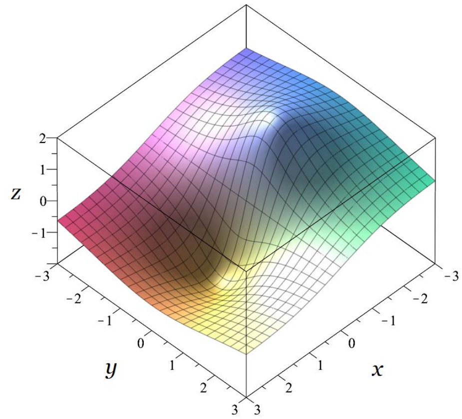
(e)



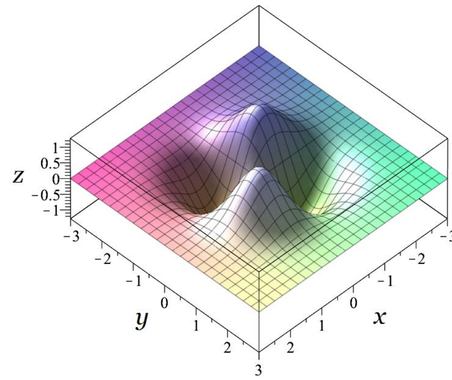
(f)



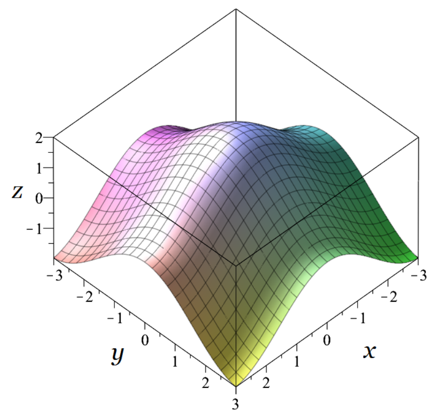
(I)



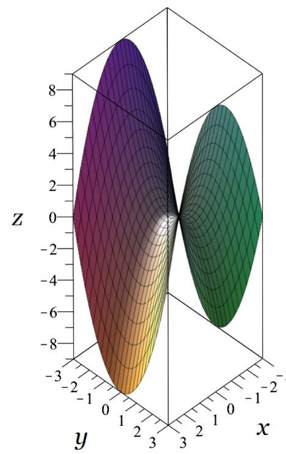
(IV)



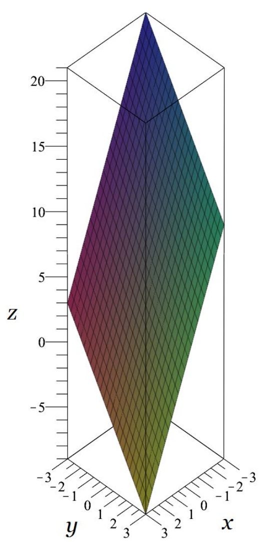
(II)



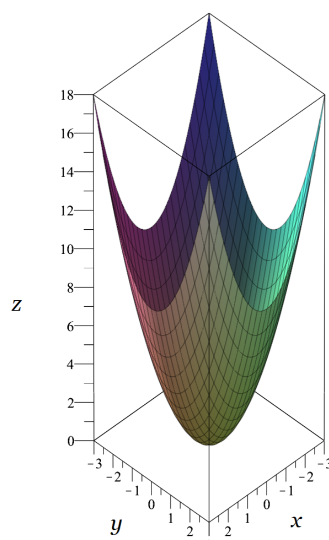
(V)



(III)



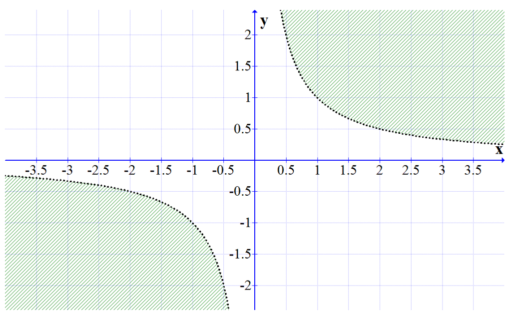
(VI)



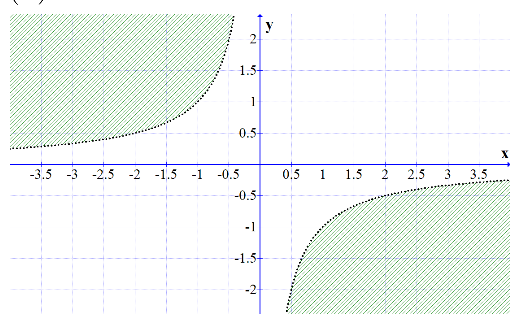
Contour Plot	Graph
a	V
b	IV
c	VI
d	III
e	II
f	I

11. **Multiple Choice:** Which of the following is a sketch of the domain of $f(x, y) = \ln(xy - 1) + e^{x^2 y} - y^8$?

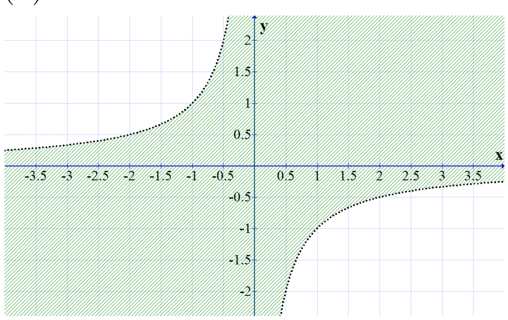
(a)



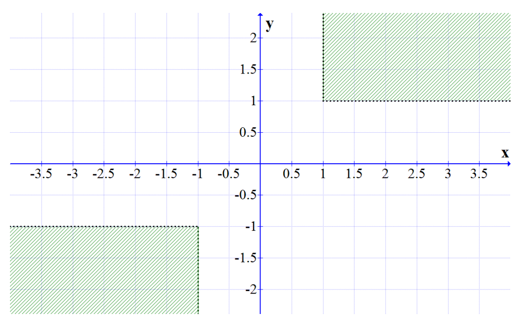
(d)



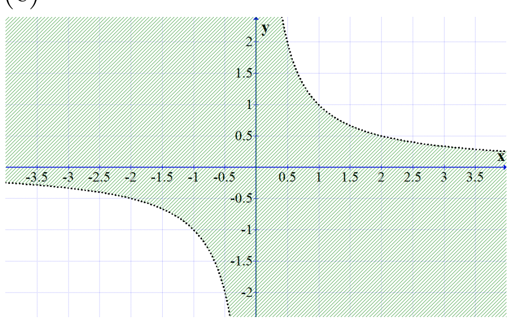
(b)



(e)



(c)

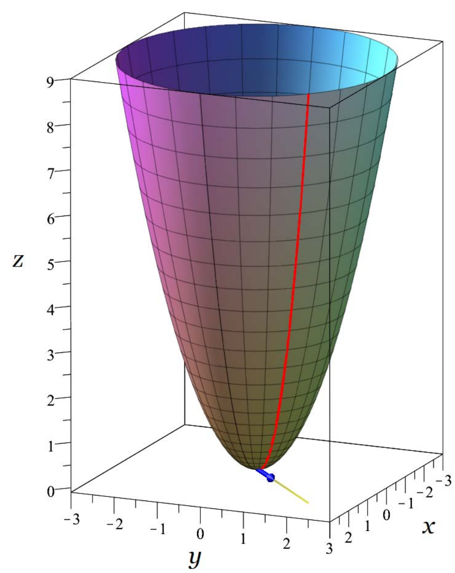


$\boxed{\text{(a)}}$

12. Suppose that $f(x, y) = x^2 + y^2$. And, consider line L with parametric equations: $x(t) = \dfrac{\sqrt{2}}{2}t$, $y(t) = \dfrac{\sqrt{2}}{2}t$, $z(t) = 0$.

Notice that this line in the xy-plane is parallel to the unit vector $\overrightarrow{u} = \left\langle \dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}, 0 \right\rangle$.

(a) Now consider the image below. The red curve along $f(x, y)$ is the curve that results from evaluating the function at points along L. In other words, it is $f(x(t), y(t))$, where $x(t)$ and $y(t)$ are taken from the parametric equations of L. Compute $f(x(t), y(t))$.



$$\boxed{f(x(t), y(t)) = f\left(\frac{\sqrt{2}}{2}t, \frac{\sqrt{2}}{2}t\right) = t^2}$$

(b) Notice that your answer from part (a) is a single variable function of the variable t. Call it $g(t)$. Compute $\dfrac{d}{dt}(g(t))\Big|_{t=1}$ and interpret your answer geometrically.

$\dfrac{d}{dt}(g(t))\Big|_{t=1} = 2$. This can be thought of as the instantaneous rate of change of $f(x, y) = x^2 + y^2$ at the point $(x, y, z) = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}, 1\right)$ in the direction of $\overrightarrow{u}$; or, equivalently, this can be thought of the slope of the tangent line to the red curve on the surface at the point $(x, y, z) = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}, 1\right)$.