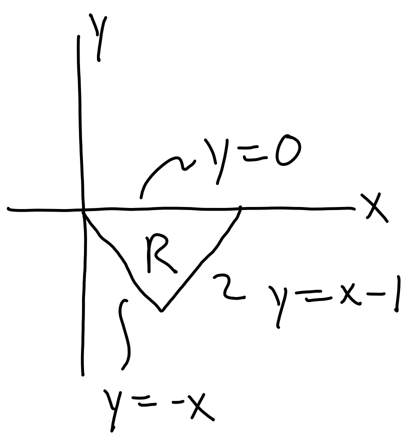
14.7 #8

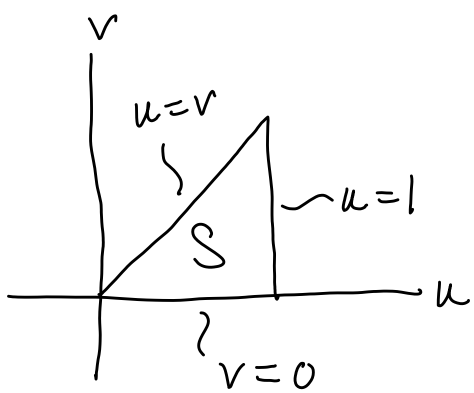


$u = x - y\,,\quad v = x + y$

So $y = x - 1 \iff x - y = 1 \iff u = 1$

$y = -x \iff x + y = 0 \iff v = 0$

$y = 0 \iff u = x, v = x \iff u = v$



$$\iint_R e^{(x-y)^2}\,dA = \iint_S e^{u^2} \underbrace{\left|\frac{\partial(x,y)}{\partial(u,v)}\right|}_{\text{We need to compute the Jacobian}} dA$$

$u = x - y\,,\quad v = x + y \iff x = \frac{1}{2}u + \frac{1}{2}v,\ y = \frac{1}{2}v - \frac{1}{2}u$

So $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$$\iint_S e^{u^2} \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dA = \frac{1}{2} \int_0^1 \int_0^u e^{u^2} \, dv \, du$$

$$= \frac{1}{2} \int_0^1 u e^{u^2} du \qquad \text{Let } t = u^2 \Rightarrow dt = 2u\,du$$

$$= \frac{1}{4} \int_0^1 e^t dt = \frac{1}{4}(e-1)$$