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(a) Since the graph of  $y(x)$  goes through the origin and  $(\ln 2, 64)$  we know  $y(0) = 0$  and  $y(\ln 2) = 64$ .

So the BVP is

$$\begin{cases} 25y = 10y' - y'' \\ y(0) = 0 \\ y(\ln 2) = 64 \end{cases}$$

$$(b) \quad 25y = 10y' - y'' \iff y'' - 10y' + 25y = 0$$

Auxiliary equation:  $m^2 - 10m + 25 = 0$

$$(m-5)^2 = 0 \implies m=5$$

Solution to ODE:

$$y = c_1 e^{5x} + c_2 x e^{5x} \quad [\text{This is \# 5}]$$

$$y(0) = 0 \Rightarrow 0 = C_1(1) + C_2(0) \\ \Rightarrow C_1 = 0$$

$$y(\ln 2) = 64 \Rightarrow 64 = 0 + C_2(\ln 2) e^{5 \ln 2}$$

$$e^{5 \ln 2} = (e^{\ln 2})^5 = 2^5 = 32$$

$$\text{So } 64 = 32(\ln 2) C_2 \Rightarrow C_2 = \frac{2}{\ln 2}$$

$$\text{Solution to BVP: } y = \frac{2}{\ln 2} x e^{5x}$$