Chapter 4.8 Practice Problems

EXPECTED SKILLS:

- Understand the hypotheses and conclusion of Rolle's Theorem or the Mean Value Theorem.
- Be able to find the value(s) of "c" which satisfy the conclusion of Rolle's Theorem or the Mean Value Theorem.

PRACTICE PROBLEMS:

1. For each of the following, verify that the hypotheses of Rolle's Theorem are satisfied on the given interval. Then find all value(s) of c in that interval that satisfy the conclusion of the theorem.

   (a) $f(x) = x^2 - 4x - 11$; $[0, 4]$

   (b) $f(x) = \sin x$; $[0, 2\pi]$

2. Let $f(x) = \dfrac{1}{x^2}$

   (a) Show that there is no point c in the interval $(-1, 1)$ such that $f'(c) = 0$, even though $f(-1) = f(1) = 1$.

   (b) Explain why the result from part (a) does not contradict Rolle's Theorem.

3. For each of the following, verify that the hypotheses of the Mean Value Theorem are satisfied on the given interval. Then find all value(s) of c in that interval that satisfy the conclusion of the theorem.

   (a) $f(x) = x^2 - 4x$; $[1, 5]$

   (b) $f(x) = x - \cos x$; $[0, 2\pi]$

4. Let $f(x) = x^{2/3}$

   (a) Show that there is no point c in $(-8, 1)$ such that $f'(c)$ will be equal to the slope of the secant line through $(-8, f(-8))$ and $(1, f(1))$.

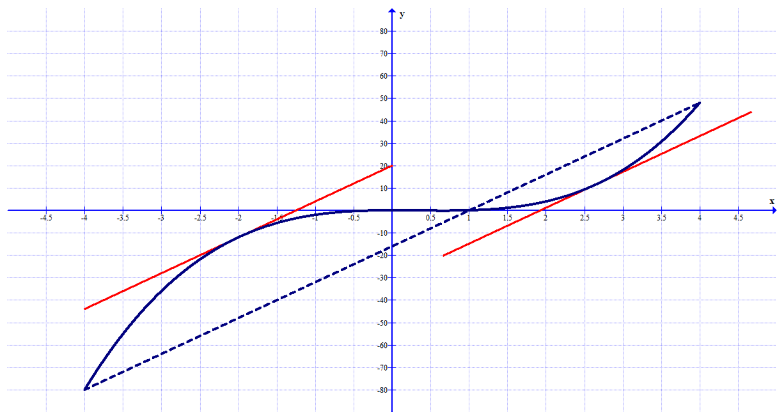
   (b) Explain why the result from part (a) does not contradict the Mean Value Theorem.

5. Consider $f(x) = x^3 - x^2$.

   (a) Find the value(s) of c which satisfy the conclusion of the Mean Value Theorem on $[-4, 4]$.

   (b) At each value of c found in part (a), calculate an equation of the line which is tangent to the graph of $f(x)$.

   (c) On the axes provided below, sketch the tangent lines which you found in part (b).



6. Consider the quadratic function $f(x) = c_1x^2 + c_2x + c_3$, where $c_1 \neq 0$. Show that the number c in the conclusion of the mean value theorem is always the midpoint of the given interval $[a, b]$.

7. **Theorem:** Suppose that $f'(x) = 0$ for all x in some open interval I. Then, $f(x)$ is constant on the interval.

   Prove this theorem. (HINT: Consider any two numbers a and b in the interval I, where $a < b$. Show that $f(a) = f(b)$ on the interval I.)

8. **Definition:** A function $F(x)$ is an <u>antiderivative</u> of $f(x)$ if $\dfrac{d}{dx}[F(x)] = f(x)$. For example, since $\dfrac{d}{dx}\left[x^2 + 6\right] = 2x$, we say that $F(x) = x^2 + 6$ is an antiderivative of $f(x) = 2x$.

   (a) List some other antiderivatives of $2x$.

   (b) **Theorem:** Suppose $g'(x) = f'(x)$ for all x in an open interval I. Then, for some constant c, $g(x) = f(x) + c$ for all x in the interval I.

   Prove this theorem. (HINT: Define a new function $h(x) = g(x) - f(x)$ and appeal to the theorem in problem 7.)

   (c) Let $f(x) = \sin^{-1}(x)$ and $g(x) = -\cos^{-1}(x)$. Verify that $f'(x) = g'(x)$ and find the constant C such that $\sin^{-1}(x) = -\cos^{-1}(x) + C$.