#7

Let $a_k = \arcsin\left(\frac{1}{k}\right)$

$$a_k' = \frac{1}{\sqrt{1-\left(\frac{1}{k}\right)^2}} \cdot \frac{-1}{k^2} < 0 \quad \text{for } k > 1$$

So $\left\{\arcsin\left(\frac{1}{k}\right)\right\}_{k=1}^{+\infty}$ is decreasing.

Also, $\lim\limits_{k \to +\infty} \arcsin\left(\frac{1}{k}\right) = \arcsin 0 = 0.$

Thus by the Alternating Series Test,

$\sum\limits_{k=1}^{\infty} (-1)^k \arcsin\left(\frac{1}{k}\right)$ converges.

Now consider $\sum_{k=1}^{\infty} \left| (-1)^k \arcsin\left(\frac{1}{k}\right) \right| = \sum_{k=1}^{\infty} \arcsin\left(\frac{1}{k}\right)$.

$$\lim_{k \to +\infty} \frac{\arcsin\left(\frac{1}{k}\right)}{\frac{1}{k}} = \lim_{k \to +\infty} \frac{\frac{1}{\sqrt{1-\left(\frac{1}{k}\right)^2}} \cdot \cancel{-\frac{1}{k^2}}}{\cancel{-\frac{1}{k^2}}} = 1$$

which is finite and nonzero.

Since $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges (Harmonic Series),

$\sum_{k=1}^{\infty} \arcsin\left(\frac{1}{k}\right)$ diverges by the Limit Comparison Test,

and thus $\sum_{k=1}^{\infty} (-1)^k \arcsin\left(\frac{1}{k}\right)$ converges conditionally.