Trigonometric Integral

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 7.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Know antiderivatives for all six elementary trigonometric functions.
- Be able to evaluate integrals that involve powers of sine, cosine, tangent, and secant by using appropriate trigonometric identities.

PRACTICE PROBLEMS:

1. Fill in the following table

$\int \sin x \, dx =$	
$\int \cos x \, dx =$	
$\int \tan x \, dx =$	
$\int \cot x \, dx =$	
$\int \sec x \, dx =$	
$\int \csc x \, dx =$	

2. $\int_{\pi/4}^{\pi/3} \cot 2x dx$

<u>Powers of Sines & Cosines:</u> For each of the following, evaluate the given integral.

3. $\int \sin(x) \cos^3(x) \, dx$
4. $\int \sin^3(x) \cos^4(x) \, dx$
5. $\int \sqrt{\sin x} \cos^3(x) \, dx$

6. $\int \sin^2 x \, dx$

7. $\int \sin^3 (bx) \, dx$, where b is a non-zero constant

8. $\int \sin^2 x \cos^2 x \, dx$

9. $\int_{\pi/4}^{\pi/2} \cos^3 x \, dx$

10. $\int \cos^4 5x \, dx$

11. Consider the trigonmetric identity $\sin(A + B) = \sin A \cos B + \cos A \sin B$

    (a) Use this identity to derive an identity for $\sin(A - B)$ in terms of $\sin A$, $\cos A$, $\sin B$, and $\cos B$.

    (b) Use the given identity and your answer for part (a) to derive the following identity:

$$\sin A \cos B = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$$

12. Consider the trigonmetric identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$

    (a) Use this identity to derive an identity for $\cos(A - B)$ in terms of $\sin A$, $\cos A$, $\sin B$, and $\cos B$.

    (b) Use the given identity and your answer for part (a) to derive the following identity:

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

    (c) Use the given identity and your answer for part (a) to derive the following identity:

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

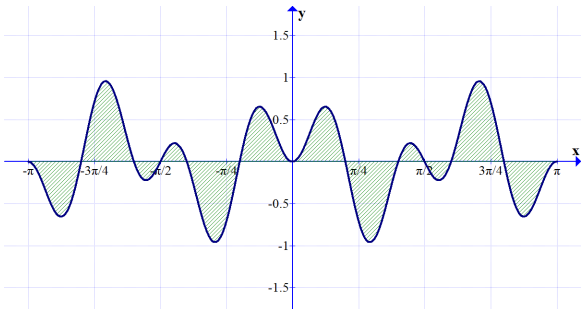
For problems 13-16, use an appropriate identity from problem 11 or 12 to evaluate the given integral.

13. $\int \sin(2x) \cos\left(\frac{x}{2}\right) dx$

14. $\int \cos(3x) \cos(4x) \, dx$

15. $\int \sin(5x) \cos(2x) \, dx$

16. The graph of $f(x) = \sin 2x \sin 5x$ on the interval $[-\pi, \pi]$ is shown below.



Compute the net signed area between the graph of $f(x)$ and the x-axis on the interval $[-\pi, \pi]$

Powers of Tangents & Secants: For each of the following, evaluate the given integral.

17. $\int \tan^2 3x \, dx$

18. $\int_0^{\pi/4} \tan^3(x) \sec^3(x) \, dx$

19. $\int \tan(x) \sec^3(x) dx$

20. $\int \tan^3(x) \sec^4(x) \, dx$

21. $\int \tan^5(2x) \sec^2(2x) \, dx$

22. $\int \tan(x) \sec^{5/2}(x) \, dx$

23. $\int \sec^4 x \, dx$

24. Consider $\int_{\pi/2}^{\pi} \sec x \, dx$

    (a) Explain why this integral is improper.

    (b) Evaluate the given integral. If it diverges, explain why.

25. (a) Use integration by parts to evaluate $\int \sec^3(x)\,dx$. (Hint: $\sec^3 x = \sec^2 x \sec x$ and $\tan^2 x = \sec^2 x - 1$)

    (b) Use part (a) to evaluate $\int \tan^2(x)\sec(x)\,dx$

26. Let R be the region bounded between the graphs of $y = \sin x$ and $y = \cos x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

    (a) Compute the area of R.

    (b) Compute the volume of the solid which results from revolving R around the x-axis.

27. Find the length of the curve $y = \ln(\sin x)$ on the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.