Sequences

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Find the general term of a sequence.
- Determine whether a sequence converges, and if so, what it converges to. This may require techniques such as L'Hopital's Rule and The Squeeze Theorem.

PRACTICE PROBLEMS:

For problems 1 – 8, rewrite the sequence by placing the general term inside braces.

1. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \ldots$
2. $\frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, -\frac{1}{256}, \ldots$
3. $0, 1, 2^3, 3^4, 4^5, \ldots$
4. $3, 2, 1, 0, -1, -2, -3, -4, -5, \ldots$
5. $1, \frac{1}{e}, e^2, \frac{1}{e^3}, e^4, \frac{1}{e^5}, \ldots$
6. $\frac{1}{1\cdot 2}, \frac{1}{1\cdot 2\cdot 3\cdot 4}, \frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}, \frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}, \ldots$
7. $\frac{3}{1\cdot 2}, \frac{5}{1\cdot 2\cdot 3\cdot 4}, \frac{7}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}, \frac{9}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}, \ldots$
8. $0, 1, 0, -1, 0, 1, 0, -1, 0, 1\ldots$ [Hint: Think about a trigonometric function.]
9. For each of the sequences in problems 1 – 8, determine if the sequence converges, and if so, what it converges to. If it diverges, determine if the general term approaches $+\infty$, $-\infty$, or neither.

For problems 10 – 35, determine if the sequence converges, and if so, what it converges to. If it diverges, determine if the general term approaches $+\infty$, $-\infty$, or neither.

10. $\{5\}_{n=1}^{+\infty}$

11. $\{5n\}_{n=0}^{+\infty}$

12. $\left\{5-5n^3\right\}_{n=1}^{+\infty}$

13. $\left\{\dfrac{4n-3n^5}{2n^5+4n^3+n^2+5}\right\}_{n=1}^{+\infty}$

14. $\left\{(-1)^n\ \dfrac{n^3+n^2+n+1}{n^3+1}\right\}_{n=1}^{+\infty}$

15. $\left\{\dfrac{n^4-3n^3-2n}{4n^2+19}\right\}_{n=0}^{+\infty}$

16. $\left\{\dfrac{1-10n^2}{n^2-4n^3}\right\}_{n=1}^{+\infty}$

17. $\left\{(-1)^{n+1}\ \dfrac{1-10n^2}{n^2-4n^3}\right\}_{n=1}^{+\infty}$

18. $\left\{\dfrac{\sqrt{4+3n^2}}{2+7n}\right\}_{n=1}^{+\infty}$

19. $\left\{e^{1/n}\right\}_{n=1}^{+\infty}$

20. $\left\{\dfrac{e^{-n}}{n^{-2}}\right\}_{n=1}^{+\infty}$

21. $\left\{\dfrac{e^n-e^{-n}}{e^n+e^{-n}}\right\}_{n=1}^{+\infty}$

22. $\left\{\dfrac{e^{\sqrt{n}}}{n}\right\}_{n=1}^{+\infty}$

23. $\left\{e^n\sin(e^{-n})\right\}_{n=1}^{+\infty}$

24. $\left\{e^n\pi^{-n}\right\}_{n=1}^{+\infty}$

25. $\left\{\ln\left(\dfrac{1}{n}\right)\right\}_{n=1}^{+\infty}$

26. $\left\{\dfrac{\ln(6n)}{\ln(2n)}\right\}_{n=1}^{+\infty}$

27. $\{\ln(n+2)-\ln(3n+5)\}_{n=0}^{+\infty}$

28. $\left\{\sqrt{n^2+8n-5}-n\right\}_{n=1}^{+\infty}$

29. $\left\{\sqrt{n^2-n}+n\right\}_{n=0}^{+\infty}$

30. $\left\{\sqrt{n^2-n}-n\right\}_{n=1}^{+\infty}$

31. $\left\{\frac{\cos n}{n}\right\}_{n=1}^{+\infty}$

32. $\left\{\arccos\left(\frac{n^2}{3n-n^2}\right)\right\}_{n=1}^{+\infty}$

33. $\left\{\arctan\left(\frac{1}{n}\right)-\arctan(n)\right\}_{n=1}^{+\infty}$

34. $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{+\infty}$

35. $\left\{(1+3^n)^{1/n}\right\}_{n=1}^{+\infty}$

36. $\left\{\left(\frac{4}{n}\right)^{2/n}\right\}_{n=1}^{+\infty}$

37. Consider the sequence $\sqrt{30}, \sqrt{30+\sqrt{30}}, \sqrt{30+\sqrt{30+\sqrt{30}}}, ...$

   (a) Define the sequence recursively.

   (b) Assuming the sequence converges to some limit L, find L.

38. Consider the sequence $\{a_n\}_{n=1}^{+\infty}$ that has the following recursive definition: $a_{n+1} = 10 - a_n$ for integers $n \geq 1$.

   (a) Assuming the sequence converges to some limit L, find L.

   (b) How must a_1 be defined to ensure that the sequence converges? Justify your answer.

39. The Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, 21, ...$ begins with two 1's and thereafter each term in the sequence is the the sum of previous two terms.

   (a) Define the Fibonacci sequence recursively.

(b) Clearly the Fibonacci sequence diverges to $+\infty$, but consider the ratio of successive terms $\dfrac{a_{n+1}}{a_n}$ for $n \geq 1$, i.e

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \dots$$

Assuming this "ratio sequence" converges to some limit L, find L.