Cylindrical & Spherical Coordinates

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.8 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to convert between rectangular, cylindrical, and spherical coordinates (Table 11.8.1).
- Be able describe simple surfaces in terms of cylindrical and spherical coordinates (Table 11.8.2).

PRACTICE PROBLEMS:

1. Consider the point $(r, \theta, z) = \left(2, \frac{\pi}{2}, 1\right)$ in cylindrical coordinates.

   (a) Convert this point to rectangular coordiinates.

   $\boxed{(x, y, z) = (0, 2, 1)}$

   (b) Convert this point to spherical coordinates.

   $$\boxed{(\rho, \theta, \phi) = \left(\sqrt{5}, \frac{\pi}{2}, \cos^{-1}\frac{1}{\sqrt{5}}\right)}$$

2. Consider the point $(r, \theta, z) = \left(1, \frac{\pi}{4}, -4\right)$ in cylindrical coordinates.

   (a) Convert this point to rectangular coordiinates.

   $$\boxed{(x, y, z) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -4\right)}$$

   (b) Convert this point to spherical coordinates.

   $$\boxed{(\rho, \theta, \phi) = \left(\sqrt{17}, \frac{\pi}{4}, \cos^{-1}\left(-\frac{4}{\sqrt{17}}\right)\right)}$$

3. Consider the point $(\rho, \theta, \phi) = \left(5, \frac{\pi}{3}, \frac{2\pi}{3}\right)$ in spherical coordinates.

   (a) Convert this point to rectangular coordiinates.

   $\boxed{(x, y, z) = \left(\frac{5\sqrt{3}}{4}, \frac{15}{4}, -\frac{5}{2}\right)}$

   (b) Convert this point to cylindrical coordinates.

   $\boxed{(r, \theta, z) = \left(\frac{5\sqrt{3}}{2}, \frac{\pi}{3}, -\frac{5}{2}\right)}$

4. Consider the point $(x, y, z) = \left(1, -\sqrt{3}, -2\right)$ in rectangular coordinates.

   (a) Convert this point to cylindrical coordiinates.

   $\boxed{(r, \theta, z) = \left(2, \frac{5\pi}{3}, -2\right)}$

   (b) Convert this point to spherical coordinates.

   $\boxed{(\rho, \theta, \phi) = \left(\sqrt{8}, \frac{5\pi}{3}, \frac{3\pi}{4}\right)}$

For problems 5-10, each of the given surfaces is expressed in rectangular coordinates. Express the equation of the surface in (a) cylindrical coordinates and (b) spherical coordinates.

5. $x^2 + y^2 + z^2 = 16$

   $\boxed{\text{(a)}r^2 + z^2 = 16;\ \text{(b)}\ \rho = 4}$

6. $x^2 + y^2 + z^2 = 3z$

   $\boxed{\text{(a)}r^2 + z^2 = 3z;\ \text{(b)}\ \rho = 3\cos\phi}$

7. $z = \sqrt{2x^2 + 2y^2}$

   $\boxed{\text{(a)}z = \sqrt{2}r;\ \text{(b)}\ \phi = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)}$

8. $x^2 + y^2 = 9$

   $\boxed{\text{(a)}r = 3;\ \text{(b)}\ \rho\sin\phi = 3}$

9. $x + 3y + 5z = 4$

   $\boxed{\text{(a)}r\cos\theta + 3r\sin\theta + 5z = 4;\ \text{(b)}\ \rho\cos\theta\sin\phi + 3\rho\sin\theta\sin\phi + 5\rho\cos\phi = 4}$

10. $z = 2$

$\boxed{\text{(a)} z = 2; \text{ (b) } \rho\cos\phi = 2}$

For problems 11-15, each of the given surfaces is expressed in cylindrical coordinates. Express the equation of the surface in rectangular coordinates.

11. $r = 5$

$\boxed{x^2 + y^2 = 25}$

12. $\theta = \dfrac{\pi}{2}$

$\boxed{x = 0, \text{ where } y \geq 0}$

13. $r = 6\sin\theta$

$\boxed{x^2 + (y-3)^2 = 9}$

14. $z = r\sin\theta$

$\boxed{z = y}$

15. $r^2 \sin 2\theta = z$

$\boxed{z = 2xy}$

For problems 16-19, each of the given surfaces is expressed in spherical coordinates. Express the equation of the surface in rectangular coordinates.

16. $\rho = 4$

$\boxed{x^2 + y^2 + z^2 = 16}$

17. $\phi = \dfrac{\pi}{3}$

$\boxed{z = \dfrac{1}{\sqrt{3}}\sqrt{x^2 + y^2}}$

18. $\rho = 4\cos\phi$

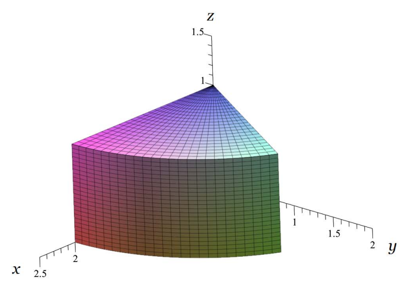
$\boxed{x^2 + y^2 + (z-2)^2 = 4}$

19. $\rho = 3\sec\phi$

$\boxed{z = 3}$

For problems 20-21, describe in words all points in 3-space which satisfy the given inequalities.

20. In cylindrical coordinates: $\left\{(r, \theta, z) : 0 \leq r \leq 2, 0 \leq \theta \leq \frac{\pi}{3}, 0 \leq z \leq 1\right\}$

> All points in the first octant which are on or inside of the circular cylinder $x^2 + y^2 = 4$ between the planes $z = 0$, $z = 1$, $y = 0$ and $y = \sqrt{3}x$.
>
> 
>

21. In spherical coordinates: $\left\{(\rho, \theta, \phi) : 1 \leq \rho \leq 3, 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq \frac{\pi}{4}\right\}$

> All points in the first octant which are on and between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$, on and between the planes $x = 0$ and $y = 0$, and on or within the cone $z = \sqrt{x^2 + y^2}$.