Tangent Planes & Normal Lines

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.7 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute an equation of the tangent plane at a point on the surface $z = f(x, y)$.
- Given an implicitly defined level surface $F(x, y, z) = k$, be able to compute an equation of the tangent plane at a point on the surface.
- Know how to compute the parametric equations (or vector equation) for the normal line to a surface at a specified point.
- Be able to use gradients to find tangent lines to the intersection curve of two surfaces. And, be able to find (acute) angles between tangent planes and other planes.

PRACTICE PROBLEMS:

For problems 1-4, find two unit vectors which are normal to the given surface S at the specified point P.

1. $S : 2x - y + z = -7$; $P(-1, 2, -3)$
2. $S : x^2 - 3y + z^2 = 11$; $P(-1, -2, 2)$
3. $S : z = y^4$; $P(3, -1, 1)$
4. $S : z = 2 - x^2 \cos(xy)$; $P\left(-1, \frac{\pi}{2}, 2\right)$

For problems 5-9, compute equations of the tangent plane and the normal line to the given surface at the indicated point.

5. $S : \ln(x + y + z) = 2$; $P(-1, e^2, 1)$
6. $S : x^2 + y^2 + z^2 = 1$; $P\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$
7. $S : z = \arcsin\left(\frac{x}{y}\right)$; $P\left(-1, -\sqrt{2}, \frac{\pi}{4}\right)$

8. $S : x^2 - xy + z^2 = 9$; $P(2, 2, 3)$

9. $S : z = x\cos(x + y)$; $P\left(\frac{\pi}{2}, \frac{\pi}{3}, -\frac{\sqrt{3}\pi}{4}\right)$

10. Consider the surfaces $S_1 : x^2 + y^2 = 25$ and $S_2 : z = 2 - x$

    (a) Find an equation of the tangent line to the curve of intersection of S_1 and S_2 at the point $(3, 4, -1)$.

    (b) Find the acute angle between the planes which are tangent to the surfaces S_1 and S_2 at the point $(3, 4, -1)$.

11. Consider the surfaces $S_1 : z = x^2 - y^2$ and $S_2 : y^2 + z^2 = 10$

    (a) Find an equation of the tangent line to the curve of intersection of S_1 and S_2 at the point $(2, 1, 3)$.

    (b) Find the acute angle between the planes which are tangent to the surfaces S_1 and S_2 at the point $(2, 1, 3)$.

12. Find all points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 72$ where the tangent plane is parallel to the plane $4x + 4y + 12z = 3$.

13. Find all points on the hyperboloid of 1 sheet $x^2 + y^2 - z^2 = 9$ where the normal line is parallel to the line which contains points $A(1, 2, 3)$ and $B(7, 6, 5)$.

14. Two surfaces are called **orthogonal** at a point of intersection if their normal lines are perpendicular at that point. Show that the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z^2 = x^2 + y^2$ are orthogonal at all points of intersection. (HINT: Assume that the surfaces intersect at the arbitary point (x_0, y_0, z_0).)

15. Show that every plane which is tangent to the cone $z^2 = x^2 + y^2$ must pass through the origin. (HINT: Compute the equation of the plane which is tangent to the surface at the point $P_0(x_0, y_0, z_0)$ and see what happens.)