Chapter 3.4: Trigonometric Identities

Expected Skills:

- Be able to derive Pythagorean Identities relating tangent/secant or cotangent/cosecant from $\sin^2\theta + \cos^2\theta = 1$.
- Given the identities $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ be able to derive the double angle formulas and power reducing formulas (as described in the course notes).
- Be able to use the Law of Cosines to relate the sides lengths of a triangle with one of the angles.

Practice Problems:

1. Find the exact values of each of the following:
   (a) $\sin 15^\circ$ and $\cos 15^\circ$.
   (b) $\sin 165^\circ$ and $\cos 165^\circ$.
   (c) $\sin 195^\circ$ and $\cos 195^\circ$.
2. Express $\cos\alpha\cos\beta$ in terms of $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$.
   Hint: write out the sum and difference identities for cosine and combine them appropriately.
3. Express $\sin\alpha\sin\beta$ in terms of $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$.
4. Express $\sin\alpha\cos\beta$ in terms of $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$.
5. Suppose $\tan\alpha = \dfrac{3}{4}$, $\tan\beta = 8$ where $0 < \alpha < \dfrac{\pi}{2}$ and $0 < \beta < \dfrac{\pi}{2}$. Evaluate $\sin(\alpha+\beta)$.
6. Derive the following identity: $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.
7. Suppose $\tan\alpha = \dfrac{3}{4}$ and $\tan\beta = 8$. Use the result of the previous exercise to evaluate $\tan(\alpha+\beta)$.
8. Suppose $\cos\theta = \dfrac{3}{5}$ and $\sin\theta < 0$. Evaluate each of the following:
   (a) $\sin(2\theta)$
   (b) $\cos(2\theta)$
   (c) $\tan(2\theta)$

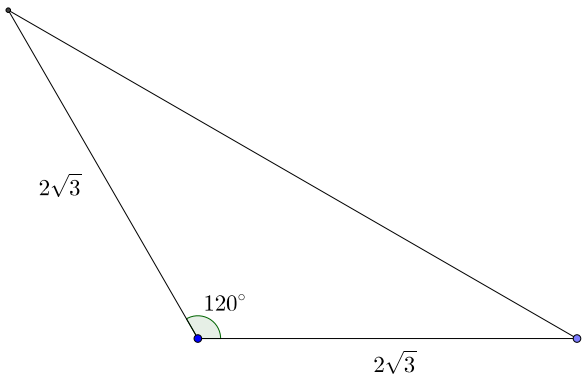
9. Rewrite $\sin^4\theta$ as an equivalent expression which does not have any trigonometric functions with powers greater than 1.

10. One hand of a very large clock is 3 feet long and the other is 4 feet long.

    (a) What is the distance between their tips at the moment when the clock strikes 3:00 pm?

    (b) What is the distance between their tips at the moment when the clock strikes 1:00 pm?

11. Consider the following triangle:



    (a) Calculate the area of this triangle using the following theorem:

    Heron's Formula: The area of a triangle with sides of length a, b, and c is $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$

    (b) Calculate the area of this triangle using the formula $A = \dfrac{1}{2}bh$.