Chapter 2.2 Practice Problems

EXPECTED SKILLS:

- Know how to compute the derivative of a function using the limit definition.
- Understand the geometric interpretation of a derivative (as the slope of a tangent line), and be able to use the derivative to help find the equation of a tangent line.
- Understand the physics interpretation of the derivative (as instantaneous velocity).
- Understand how the graph of a function affects the derivative.
- If given the graph of a function, be able to make a reasonable sketch of its derivative function.

PRACTICE PROBLEMS:

1. For each of the following problems, use the definition of the derivative to calculate $f'(x)$.

   (a) $f(x) = 3x$
   $\boxed{3}$

   (b) $f(x) = 2x^2 - x$
   $\boxed{4x - 1}$

   (c) $f(x) = 3\sqrt{x}$
   $\boxed{\dfrac{3}{2\sqrt{x}}}$

   (d) $f(x) = \dfrac{1}{\sqrt{x}}$
   $\boxed{-\dfrac{1}{2x^{3/2}}}$

   (e) $f(x) = \dfrac{1}{x-1}$
   $\boxed{\dfrac{-1}{(x-1)^2}}$

2. Find an equation of the tangent line to the graph of the given function at the specified value of x.

   (a) $f(x) = x^3$ at $x = 2$
   $\boxed{y = 12x - 16}$

(b) $f(x) = x^2 - 1$ at $x = -1$

$\boxed{y = -2x - 2}$

3. Find an equation of the line which is tangent to the graph of $y = f(x)$ when $x = 3$ if $f(3) = 7$ and $f'(3) = -1$.

$\boxed{y = -x + 10}$

4. Suppose the tangent line to the graph of $y = f(x)$ at the point $(1, 2)$ also passes through the point $(7, 5)$. Compute $f(1)$ and $f'(1)$.

$\boxed{f(1) = 2;\ f'(1) = \frac{1}{2}}$

5. Suppose $f(x)$ is a function such that $f'(x) = x^2 - 4$.

(a) For which value(s) of x will the graph of $f(x)$ have horizontal tangent lines?

$\boxed{x = 2 \text{ or } x = -2}$

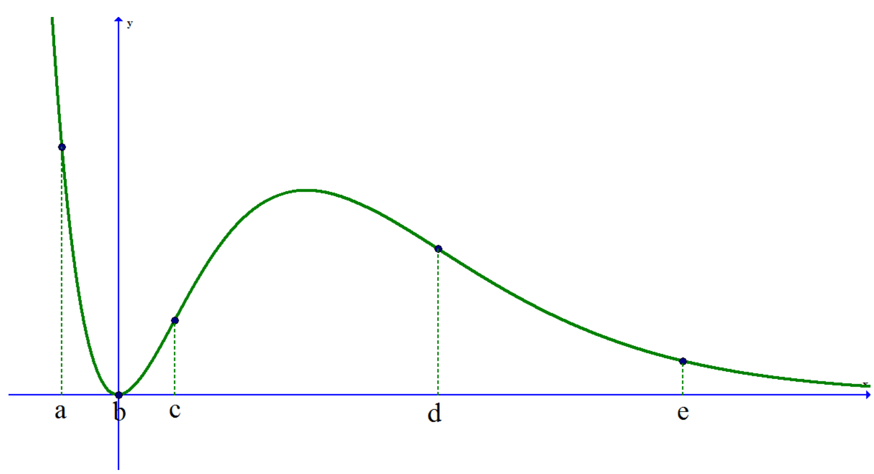
(b) For which value(s) of x will the tangent line to the graph of $f(x)$ be parallel to the line $y = 5x - 37$?

$\boxed{x = 3 \text{ or } x = -3}$

(c) For which value(s) of x will the tangent line to the graph of $f(x)$ be perpendicular to the line $y = 2x + \pi$?

$\boxed{x = \sqrt{\frac{7}{2}} \text{ or } x = -\sqrt{\frac{7}{2}}}$

6. Consider the graph of $y = f(x)$ shown below.



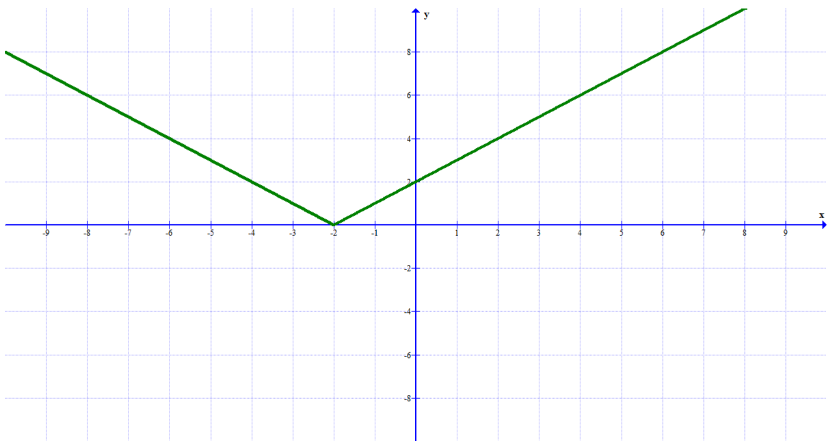
Arrange $f'(a)$, $f'(b)$, $f'(c)$, $f'(d)$, and $f'(e)$ in increasing order.

$\boxed{f'(a) < f'(d) < f'(e) < f'(b) < f'(c)}$

7. For each of the following, sketch the graph of the given function and determine where the function is not differentiable. Explain.

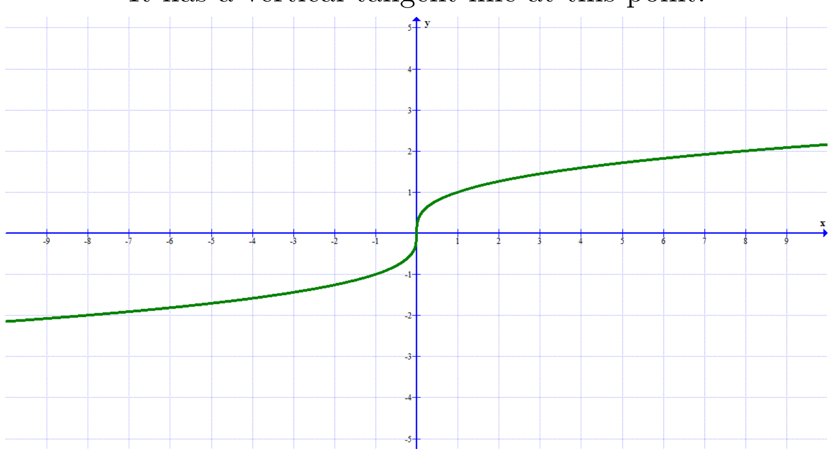
   (a) $f(x) = |x + 2|$

   $f(x)$ is not differentiable when $x = -2$.
   The graph has a "sharp corner" at this point.



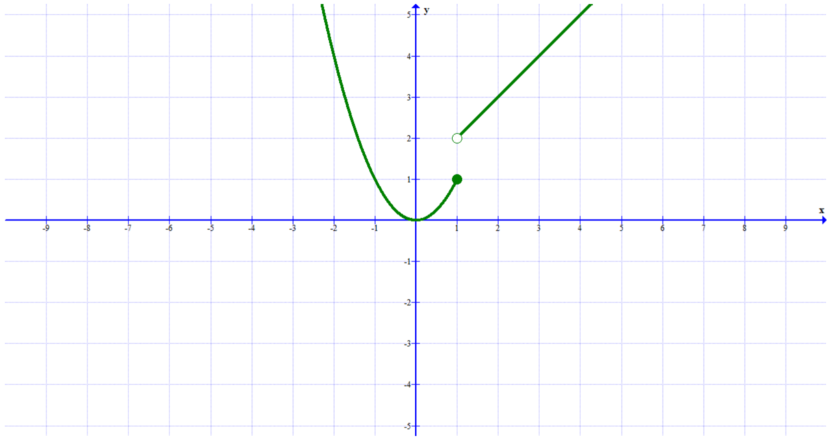
   (b) $f(x) = \sqrt[3]{x}$

   $f(x)$ is not differentiable at $x = 0$.
   It has a vertical tangent line at this point.



(c) $f(x) = \begin{cases} x+1 & \text{if} \quad x > 1 \\ x^2 & \text{if} \quad x \leq 1 \end{cases}$

$f(x)$ is not differentiable at $x = 1$.
The graph is not continuous at this point.



8. **Multiple Choice:** If the function $y = f(x)$ is not differentiable at $x = 0$, then which of the following MUST be true?

   (a) $f(0)$ is undefined.

   (b) $f(x)$ is NOT continuous when $x = 0$.

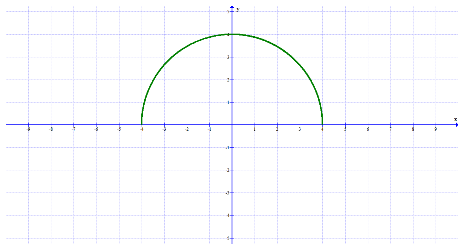
   (c) There is a horizontal tangent line to the graph of $y = f(x)$ when $x = 0$.

   (d) There is a vertical tangent line to the graph of $y = f(x)$ when $x = 0$.

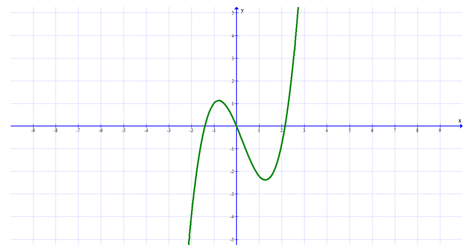
   (e) $\lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$ does not exist.

9. Match each of the graphs for functions (a)-(d) with the appropriate graph of its derivative (i)-(iv).

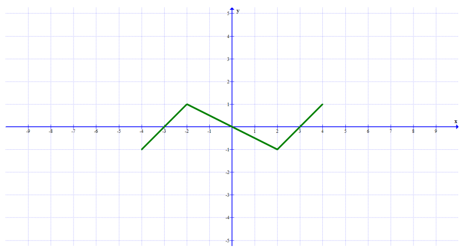
(a)



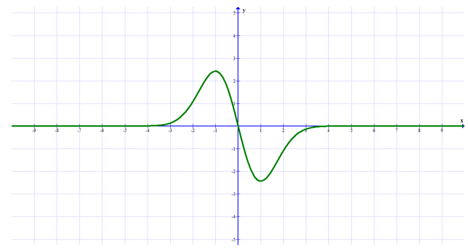
(i)



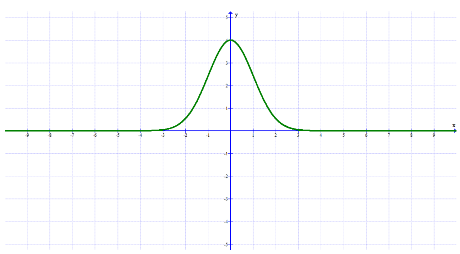
(b)



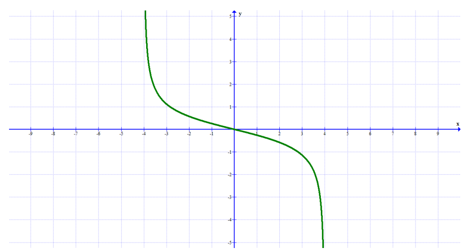
(ii)



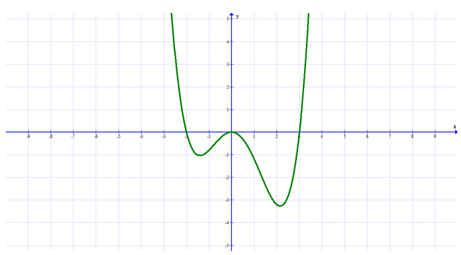
(c)



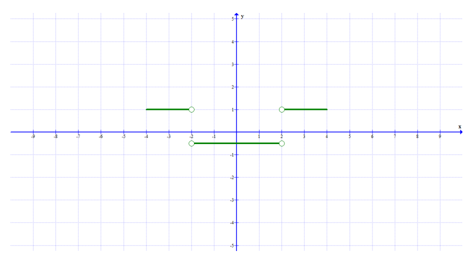
(iii)



(d)



(iv)

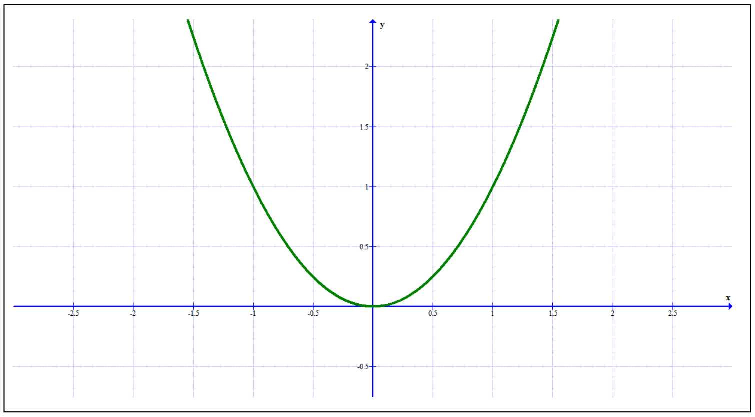


Answer:

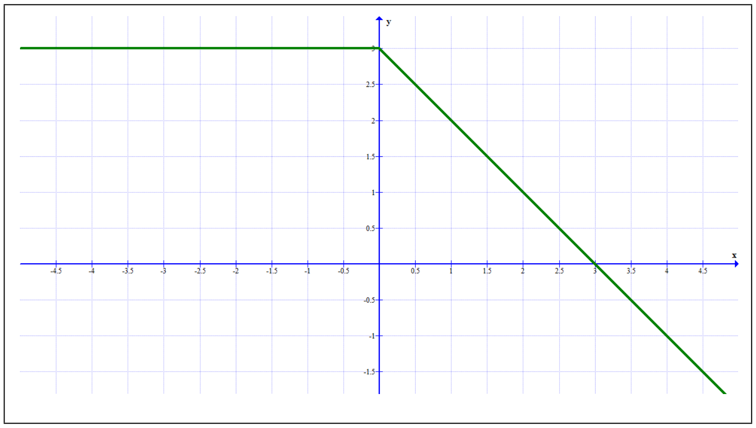
$f(x)$	$f'(x)$
(a)	(iii)
(b)	(iv)
(c)	(ii)
(d)	(i)

10. Sketch a function $y = f(x)$ with the given characteristics. (There are many possible answers.)

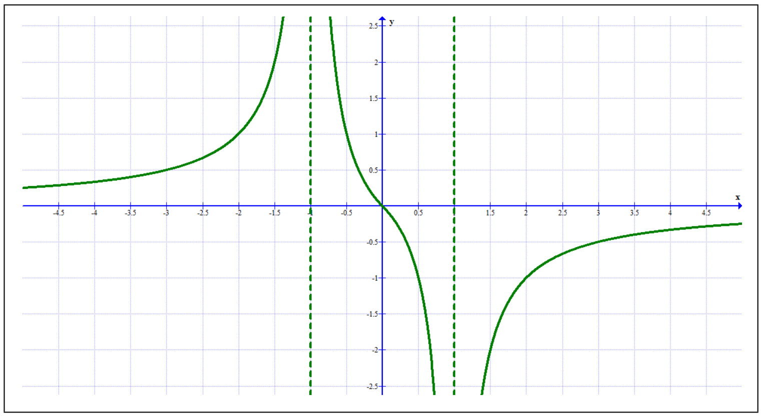
(a) $f'(x) < 0$ when $x < 0$; $f'(x) > 0$ when $x > 0$; and $f(0) = 0$.



(b) $f'(x) = 0$ when $x < 0$; $f'(x) < 0$ when $x > 0$; and $f(-1) = 3$; $f'(0)$ DNE.



(c) $f'(x) > 0$ when $x < -1$ and when $x > 1$; $f'(x) < 0$ when $-1 < x < 1$.



(d) $f(x)$ has a vertical tangent line when $x = 1$; $f'(x) > 0$ for $x < 1$; $f(x)$ is not differentiable when $x = -1$.