Chapters 2.3 & 2.4 Practice Problems

EXPECTED SKILLS:

- Know that the derivative of a constant function is 0.

- Be able to compute the derivatives of sums, differences, and constant multiples of functions.

- Know how to compute the derivatives of power functions by using the Power Rule.

- Be able to compute higher order derivatives.

- Know how to compute the derivatives of products and quotients of differentiable functions by correctly applying the product and quotient rules, respectively.

PRACTICE PROBLEMS:

1. Differentiate.

   (a) $f(x) = 167.9$

   (b) $f(x) = x^4 + 6$

   (c) $y = x^2 - 4x$

   (d) $y = 2x - 4x^3$

   (e) $f(x) = -x^5 + 3x^4 - 2x^3 + 3$

   (f) $y = \sqrt{x}$

   (g) $f(x) = \sqrt[3]{x^7}$

   (h) $f(x) = \pi^3 + e^2 - \sqrt{2}$

   (i) $y = \dfrac{3}{x} - \dfrac{4}{5x^2}$

   (j) $f(x) = \left(4x^2 - \dfrac{1}{3x}\right)\left(\dfrac{1}{x^2}\right)$

2. Find $\left.\dfrac{dy}{dx}\right|_{x=1}$ if $y = (x^2 - 1)(3x^2 + 2)$

3. Let $f(0) = 3$, $f'(0) = 7$, $g(0) = 2$, and $g'(0) = 5$. Compute each of the following:

   (a) $(f + g)'(0)$

   (b) $(fg)'(0)$

   (c) $\left(\dfrac{f}{g}\right)'(0)$

4. Consider $h(x) = x^2 f(x)$. Compute $h'(3)$ if $f(3) = 1$ and $f'(3) = 4$.

5. Consider $y = \sqrt{x} f(x)$. Compute $\dfrac{dy}{dx}\bigg|_{x=16}$ if $f(16) = 2$ and $f'(16) = 5$.

6. Compute an equation of the line which is tangent to $y = \dfrac{x^3}{f(x)}$ at $x = 2$ if $f(2) = 3$ and $f'(2) = 4$

7. Find all value(s) of x for which $f(x) = x^3 - 4x^2$ has horizontal tangent lines.

8. Find all value(s) of x at which the tangent line(s) to $y = x^2 + 3$ are parallel to $y = 7x$.

9. Find all value(s) of x for which the tangent line(s) to $y = \dfrac{x+2}{x+1}$ are perpendicular to $y = x$.

10. Find all value(s) of x at which the tangent line to $f(x) = \dfrac{1}{x+3}$ passes through the origin.

11. Let $f(x) = x^3 - 3x + 1$. Determine $f''(x)$

12. Consider $y = \dfrac{1}{x}$. What is y''?

13. Suppose $f(x) = \dfrac{3x}{x-3}$. Compute $\dfrac{d^2 y}{dx^2}$

14. Let $f(x) = \dfrac{x+1}{x}$. Determine $f''(x)$.

15. Consider $y = x^{80} - 17x^5 + 100x^2 - 1$. What is $\dfrac{d^{81} y}{dx^{81}}$.

16. Let $f(x) = x^3$. Compute all value(s) of x in $(1, 3)$ at which the tangent line to the graph of $f(x)$ is parallel to the secant line which cuts the curve at $x = 1$ and $x = 3$.

17. **Multiple Choice:** Let $f(x) = ax^2 + bx + c$, where a, b, and c are constants. Also, suppose that $f(2) = 4$, $f'(2) = -3$, and $f''(2) = -2$. Find $\dfrac{a+b+c}{3}$.

    (a) -2
    (b) -1
    (c) 0
    (d) 1
    (e) 2

18. Let n be a positive integer and $f(x) = \dfrac{1}{x^n}$. Use the quotient rule to show that $f'(x) = -nx^{-n-1}$.

19. Let $f(x) = \dfrac{2}{x}$

   (a) Compute an equation of the tangent line to $f(x)$ at $x = 1$.

   (b) Sketch $f(x)$ and the segment of the tangent line (from part a) which is in the first quadrant. Label the x and y intercepts.

   (c) Show that the point of tangency bisects the segment of the tangent line (sketched in part b). i.e., show that the point of tangency is the midpoint of the segment of the tangent line which was sketched in part b.

20. Show that the area of the triangle formed by any tangent line to the graph of $y = \dfrac{k}{x}$ and the coordinate axes is $2k$, for a fixed positive constant k.