Chapter 2.6 Practice Problems

EXPECTED SKILLS:

- Know how to use the chain rule to calculate derivatives of compositions of functions.

PRACTICE PROBLEMS:

For problems 1-16, calculate the derivative of the given function.

1. $f(x) = (x^3 + 4)^{-3}$
2. $f(x) = (x^2 + 2x)^6$
3. $f(x) = \sqrt{x^3 - 2}$
4. $f(x) = \tan\left(\dfrac{1}{x^2}\right)$
5. $f(x) = \sec 2x$
6. $f(x) = \cos^3 3x$
7. $f(x) = \left(x^5 - \dfrac{1}{x^2}\right)^4$
8. $f(x) = \dfrac{x^2 - 3}{(3x - 5)^3}$
9. $f(x) = (x^2 + 2x)^5(x^2 - 4x)^3$
10. $f(x) = \sin\left(\dfrac{\pi}{x}\right)$
11. $f(x) = \sin(\sin 2x)$
12. $f(x) = \tan^2(x^2 - 1)$
13. $f(x) = \dfrac{2}{(x^5 + 4x^3 - 4x)^3}$
14. $f(x) = \left(\dfrac{x^2 - 1}{x^2 + 1}\right)^3$
15. $y = 4x^2 \csc 5x$
16. $y = \tan(4 + x^2 \sin 3x)$

17. Use the given table to calculate each of the following quantities:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-2	-5	3	9
2	5	-3	4	-2
3	-1	6	7	-6
4	3	1	-2	5
5	4	7	1	8

(a) $\left.\dfrac{d}{dx}[f(g(x))]\right|_{x=2}$

(b) $(f \circ g)'(1)$

(c) $\left.\dfrac{d}{dx}[f(3x)]\right|_{x=1}$

(d) $\left.\dfrac{d}{dx}\left[g\left(\sqrt{2}\sin\left(\dfrac{\pi}{4}x\right)\right)\right]\right|_{x=3}$

(e) $h'(2)$ if $h(x) = x^2 f(g(x))$

For problems 18-20, calculate $\dfrac{d^2y}{dx^2}$.

18. $y = \sin 3x$

19. $y = x\left(1 + \dfrac{1}{x}\right)^2$

20. $y = \dfrac{1}{1-2x}$

21. Suppose that $f(x)$ is a twice differentiable function and define $g(x) = x^3 f(2x)$. Compute $g''(x)$ in terms of f, f', and f''

22. Let $f(x) = \dfrac{5}{(x^2+1)^3}$. Compute an equation of the tangent line to the graph of $f(x)$ at $x = 0$.

23. Where does the tangent line to $y = (5x+7)^3$ at the point $(-1, 8)$ cross the x-axis?

24. Find all points on the graph of $y = \sin^2 x$ where the tangent lines are parallel to the line $y = x$.

25. What is the 100th derivative of $y = \sin(2x)$?

26. **Multiple Choice:** The derivative of $y = x^2 \cos\left(\frac{1}{x}\right)$ is

(a) $2x \cos\left(\frac{1}{x}\right) - x^2 \sin\left(\frac{1}{x}\right)$

(b) $\frac{2}{x} \sin\left(\frac{1}{x}\right)$

(c) $-2x \sin\left(\frac{1}{x}\right)$

(d) $2x \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$

(e) $\sin\left(\frac{1}{x}\right)$