

Local Linear Approximation

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.4 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute the local linear approximation for a function of two or more variables at a given point.
- Be able to use a local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-5, find the local linear approximation $L(x, y)$ of the given function at the specified point.

1. $f(x, y) = x^2 - y^2$; $P(1, 2)$
2. $f(x, y) = \frac{x + y}{x - y}$; $P(2, 1)$
3. $f(x, y) = e^x \sin y$; $P\left(\ln 3, \frac{\pi}{2}\right)$
4. $f(x, y) = \ln(x^2 - y^2)$; $P(2, \sqrt{3})$
5. $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$; $P(1, 1)$
6. Find the local linear approximation of the function $f(x, y) = \sqrt{32 - 3x^2 - y^2}$ at $(1, 2)$ and use it to approximate $f(0.98, 2.01)$.
7. Suppose that $f(x, y)$ is a differentiable function at the point $(2, 3)$ with $f(2, 3) = 1$, $f_x(2, 3) = 5$, and $f_y(2, 3) = -2$. Estimate $f(1.98, 3.01)$.
8. Find the local linear approximation $L(x, y, z)$ to $f(x, y, z) = 3x^2 - 2y^2 + xz^3$ at the point $P(-1, 2, 1)$.
9. Verify that $e^x \cos y \approx 1 + x$ for (x, y) near $(0, 0)$.
10. Verify that $(x + y)^3 \approx -16 + 12x + 12y$ for (x, y) near $(1, 1)$.
11. At a particular point $P(x_0, y_0)$, the local linear approximation of $f(x, y) = xy + y^2$ is $L(x, y) = -15 + 3x + 8y$. What is the point P ?