Chapter 2.2 Practice Problems

EXPECTED SKILLS:

- Know how to compute the derivative of a function using the limit definition.
- Understand the geometric interpretation of a derivative (as the slope of a tangent line), and be able to use the derivative to help find the equation of a tangent line.
- Understand the physics interpretation of the derivative (as instantaneous velocity).
- Understand how the graph of a function affects the derivative.
- If given the graph of a function, be able to make a reasonable sketch of its derivative function.

PRACTICE PROBLEMS:

1. For each of the following problems, use the definition of the derivative to calculate $f'(x)$.

   (a) $f(x) = 3x$
   (b) $f(x) = 2x^2 - x$
   (c) $f(x) = 3\sqrt{x}$
   (d) $f(x) = \dfrac{1}{\sqrt{x}}$
   (e) $f(x) = \dfrac{1}{x-1}$

2. Find an equation of the tangent line to the graph of the given function at the specified value of x.

   (a) $f(x) = x^3$ at $x = 2$
   (b) $f(x) = x^2 - 1$ at $x = -1$

3. Find an equation of the line which is tangent to the graph of $y = f(x)$ when $x = 3$ if $f(3) = 7$ and $f'(3) = -1$.

4. Suppose the tangent line to the graph of $y = f(x)$ at the point $(1, 2)$ also passes through the point $(7, 5)$. Compute $f(1)$ and $f'(1)$.

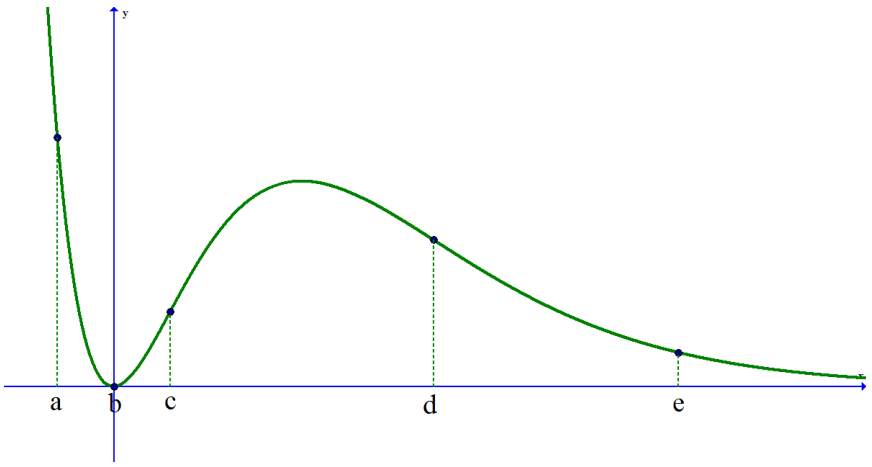
5. Suppose $f(x)$ is a function such that $f'(x) = x^2 - 4$.

   (a) For which value(s) of x will the graph of $f(x)$ have horizontal tangent lines?

(b) For which value(s) of x will the tangent line to the graph of $f(x)$ be parallel to the line $y = 5x - 37$?

(c) For which value(s) of x will the tangent line to the graph of $f(x)$ be perpendicular to the line $y = 2x + \pi$?

6. Consider the graph of $y = f(x)$ shown below.



Arrange $f'(a)$, $f'(b)$, $f'(c)$, $f'(d)$, and $f'(e)$ in increasing order.

7. For each of the following, sketch the graph of the given function and determine where the function is not differentiable. Explain.

(a) $f(x) = |x + 2|$

(b) $f(x) = \sqrt[3]{x}$

(c) $f(x) = \begin{cases} x + 1 & \text{if } x > 1 \\ x^2 & \text{if } x \leq 1 \end{cases}$

8. **Multiple Choice:** If the function $y = f(x)$ is not differentiable at $x = 0$, then which of the following MUST be true?

(a) $f(0)$ is undefined.

(b) $f(x)$ is NOT continuous when $x = 0$.

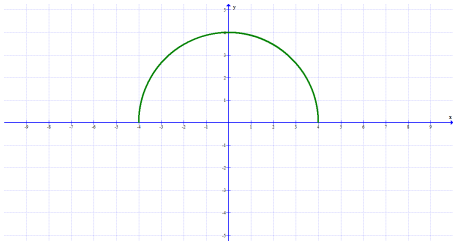
(c) There is a horizontal tangent line to the graph of $y = f(x)$ when $x = 0$.

(d) There is a vertical tangent line to the graph of $y = f(x)$ when $x = 0$.

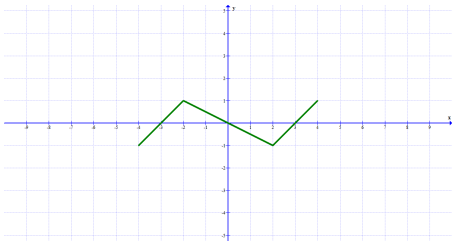
(e) $\lim_{h \to 0} \dfrac{f(0 + h) - f(0)}{h}$ does not exist.

9. Match each of the graphs for functions (a)-(d) with the appropriate graph of its derivative (i)-(iv).

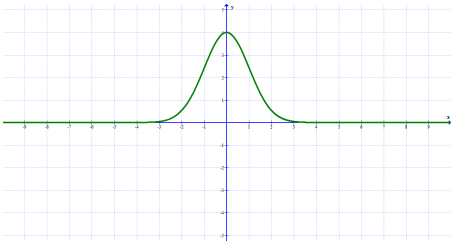
(a)



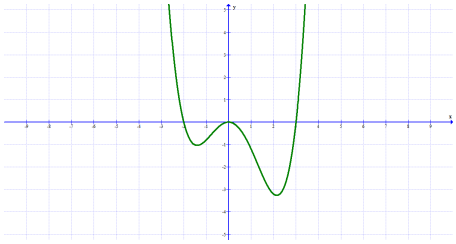
(b)



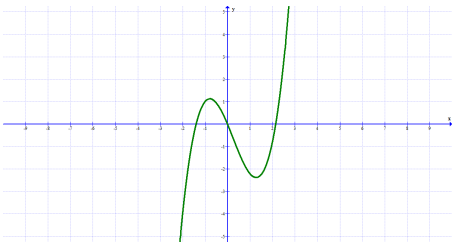
(c)



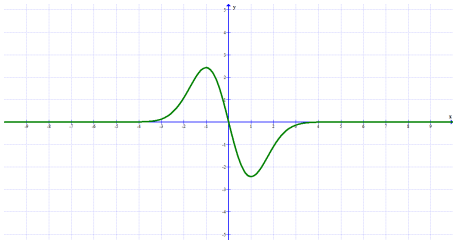
(d)



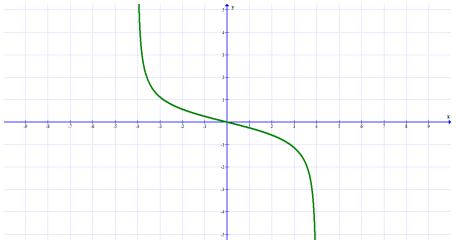
(i)



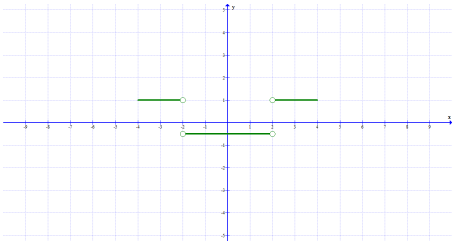
(ii)



(iii)



(iv)



10. Sketch a function $y = f(x)$ with the given characteristics. (There are many possible answers.)

    (a) $f'(x) < 0$ when $x < 0$; $f'(x) > 0$ when $x > 0$; and $f(0) = 0$.

    (b) $f'(x) = 0$ when $x < 0$; $f'(x) < 0$ when $x > 0$; and $f(-1) = 3$; $f'(0)$ DNE.

    (c) $f'(x) > 0$ when $x < -1$ and when $x > 1$; $f'(x) < 0$ when $-1 < x < 1$.

    (d) $f(x)$ has a vertical tangent line when $x = 1$; $f'(x) > 0$ for $x < 1$; $f(x)$ is not differentiable when $x = -1$.