Limits at Infinity & Integration Review

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Determine limits at infinity.
- Evaluate integrals using various techniques such as substitution, parts, and partial fractions.
- Evaluate improper integrals using limits.

PRACTICE PROBLEMS:

For problems 1 - 10, evaluate the limit.

1. $\lim_{x\to+\infty}\left(\frac{50000}{x}\right)$
2. $\lim_{x\to+\infty}\left(\frac{4x-3x^5}{2x^5+4x^3+x^2+5}\right)$
3. $\lim_{x\to+\infty}\left(\frac{\sqrt{4+3x^2}}{2+7x}\right)$
4. $\lim_{x\to+\infty}\left(\sqrt{x^2+8x-5}-x\right)$
5. $\lim_{x\to+\infty}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$
6. $\lim_{x\to+\infty}\left(xe^{-x}\right)$
7. $\lim_{x\to+\infty}\left(\frac{\cos x}{x}\right)$
8. $\lim_{x\to+\infty}\left(\arctan\left(\frac{1}{x}\right)-\arctan(x)\right)$
9. $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x$
10. $\lim_{x\to+\infty}\left(1+3^x\right)^{1/x}$

For problems 11 - 22, evaluate the integral. If the integral is improper, determine what it converges to or show that it diverges.

11. $\displaystyle\int_{\sqrt{e}}^{e} \frac{1}{4x}\,dx$

12. $\displaystyle\int \frac{\sqrt{\ln x}}{x}\,dx$

13. $\displaystyle\int_{0}^{\sqrt{\pi}} x\sin(x^2)\,dx$

14. $\displaystyle\int x^2 \sin x\,dx$

15. $\displaystyle\int \tan x\,dx$

16. $\displaystyle\int \frac{1}{x^2+9}\,dx$

17. $\displaystyle\int_{1}^{+\infty} \frac{x}{x^2+9}\,dx$

18. $\displaystyle\int \frac{dx}{x^3-x}$

19. $\displaystyle\int_{0}^{1/2} \frac{3}{\sqrt{1-x^2}}\,dx$

20. $\displaystyle\int_{1}^{+\infty} xe^{-x}\,dx$

21. $\displaystyle\int \sqrt{x}\ln x\,dx$

22. $\displaystyle\int_{5}^{+\infty} \frac{dx}{x^2-3x-4}$