The Indefinite Integral

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 5.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Given a differentiation rule, be able to construct the associated indefinite integration rule.
- Know how to integrate power functions (including polynomials), exponential functions, & trigonometric functions.

PRACTICE PROBLEMS:

For problems 1 and 2, compute the indicated derivative and state a corresponding integration formula.

1. $\dfrac{d}{dx}\left[\dfrac{1}{(2x+3)^2}\right]$

2. $\dfrac{d}{dx}[x\ln x - x]$

For problems 3-18, evaluate given indefinite integral and check your answer by differentiation.

3. $\displaystyle\int \left(\frac{1}{2}x + x^2\right)\,dx$

4. $\displaystyle\int \left(\sqrt{x^7} + e\right)\,dx$

5. $\displaystyle\int \left(\frac{1}{x^3} + 3x^3\right)\,dx$

6. $\displaystyle\int \left(3x^{-2/3} + x^{-1/2} + 5x\right)\,dx$

7. $\displaystyle\int \left(4x^{4/3} - 7\sqrt{x}\right)\,dx$

8. $\displaystyle\int 3\cos x\,dx$

9. $\displaystyle\int -7\sec^2 x\,dx$

10. $\displaystyle\int \left(-\frac{1}{x} + e^x\right)\,dx$

11. $\displaystyle\int (1-x^2)(x^3+4)\,dx$

12. $\displaystyle\int \frac{x^2 - 3x^5}{x^3}\,dx.$

13. $\displaystyle\int \frac{-2\sin x}{\cos^2 x}\,dx$

14. $\displaystyle\int \frac{1}{\sqrt{4-4x^2}}\,dx$

15. $\displaystyle\int \left(6\cos x + 9\csc^2 x\right)\,dx$

16. $\displaystyle\int (\sin x - 3\sec x\tan x)\,dx$

17. $\displaystyle\int 2^x\,dx$

18. $\displaystyle\int \frac{x^2}{x^2+1}\,dx$ (HINT: Use polynomial division)

19. Consider $\displaystyle\int \cot^2 x\,dx.$

    (a) Using the fact that $\sin^2 x + \cos^2 x = 1$, derive the identity $\cot^2 x = \csc^2 x - 1$.

    (b) Use the identity that you derived in part (a) to evaluate the original integral.

For problems 20 and 21, find a function $y = y(x)$ which satisfies the given Initial Value Problem.

20. $\begin{cases} \dfrac{dy}{dx} = \dfrac{1}{9x^2} \\ y(1) = \dfrac{1}{2} \end{cases}$

21. $\begin{cases} \dfrac{dy}{dx} = -2e^x \\ y(0) = -5 \end{cases}$

22. A ball is thrown straight up in the air from an initial height of s_0 feet above the ground with an initial speed of v_0 ft/sec. Then $s(t)$ gives the height (in feet) above the ground at time t, $v(t) = s'(t)$ gives the velocity (in ft/sec) of the ball at time t, and $a(t) = s''(t)$ gives the acceleration (in ft/sec^2) of the ball at time t. Assuming that acceleration is constant, $-g$ ft/sec^2, determine $v(t)$ and $s(t)$.