

Chapter 3.9: Inverse Trigonometric Functions

Expected Skills:

- Be able to specify the domain and range of $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$. Also be able to graph these functions.
- Be able to evaluate an inverse trigonometric function at a ratio which is related to the common angles of $0^\circ - 30^\circ - 45^\circ - 60^\circ - 90^\circ$.
- Be able to evaluate limits involving inverse trigonometric functions.
- Be able to differentiate $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$. Also be able to use the derivative to solve application problems.

Practice Problems:

1. For each of the following functions, state the domain and the range.

(a) $f(x) = \sin^{-1} x$

(b) $f(x) = \cos^{-1} x$

(c) $f(x) = \tan^{-1} x$

2. Evaluate each of the following. (Do not use a calculator. And remember the ranges from problem 1.)

(a) $\arcsin \frac{\sqrt{3}}{2}$

(b) $\arcsin \left(-\frac{\sqrt{3}}{2} \right)$

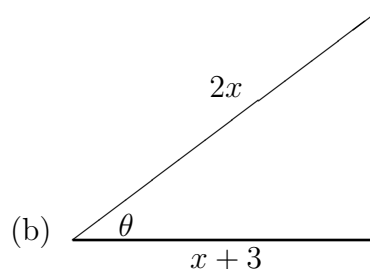
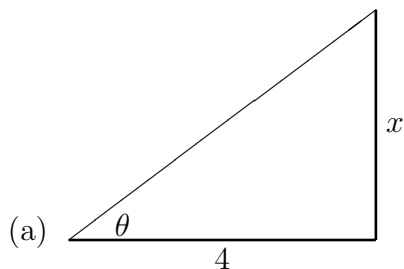
(c) $\arccos \frac{\sqrt{3}}{2}$

(d) $\arccos \left(-\frac{\sqrt{3}}{2} \right)$

(e) $\arctan \frac{\sqrt{3}}{3}$

(f) $\arctan \left(-\frac{\sqrt{3}}{3} \right)$

3. Use an inverse trigonometric function to express θ as a function of x :



4. Find the exact value of each expression.

(a) $\sin \left(\tan^{-1} \left(\frac{3}{4} \right) \right)$

(b) $\sec \left(\arctan \left(-\frac{3}{5} \right) \right)$

(c) $\sin \left(\arccos \left(-\frac{2}{3} \right) \right)$

(d) $\csc \left(\cos^{-1} \left(\frac{\sqrt{3}}{2} \right) \right)$

5. Find the exact value of each expression. Remember the ranges from problem (1)!

(a) $\sin^{-1} \left(\sin \left(\frac{\pi}{3} \right) \right)$

(b) $\sin^{-1} \left(\sin \left(\frac{2\pi}{3} \right) \right)$

(c) $\cos^{-1} \left(\cos \left(\frac{\pi}{4} \right) \right)$

(d) $\cos^{-1} \left(\cos \left(-\frac{\pi}{4} \right) \right)$

(e) $\tan^{-1} \left(\tan \left(\frac{\pi}{6} \right) \right)$

(f) $\tan^{-1} \left(\tan \left(\frac{5\pi}{6} \right) \right)$

6. For each of the following, find all solutions in the interval $[0, 2\pi]$. Give the exact values, not decimal approximations.

(a) $(\sin x - 1)(4 \sin x - 3) = 0$

(b) $3 \tan x = 1$

(c) $5 \cos^2 x + 11 \cos x + 2 = 0$

(d) $3 \tan x = -1$

7. Evaluate the following limits. If a limit does not exist, write $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \rightarrow \infty} \arccos \left(\frac{-x^2}{x^2 + 3x} \right)$

(b) $\lim_{x \rightarrow 0} \arctan \left(\frac{1}{x^2} \right)$

(c) $\lim_{h \rightarrow 0} \frac{\sin^{-1} \left(\frac{\sqrt{3}}{2} + h \right) - \frac{\pi}{3}}{h}$

(**Hint:** Interpreting the limit as the derivative of a function at a particular point.)

8. Calculate $\frac{dy}{dx}$

(a) $y = (\tan^{-1} x)^3$

(b) $y = 3x^2 \sin^{-1}(4x)$

9. Compute an equation of the line which is tangent to the graph of $f(x) = \cos^{-1} x$ at the point where $x = \frac{1}{2}$.

10. Find all value(s) of x at which the tangent lines to the graph of $f(x) = \tan^{-1}(4x)$ are perpendicular to the line which passes through $(0, 1)$ and $(2, 0)$.

11. Let $f(x) = \arctan x^2$.

(a) Find all intervals on which $f(x)$ is increasing and those on which $f(x)$ is decreasing.

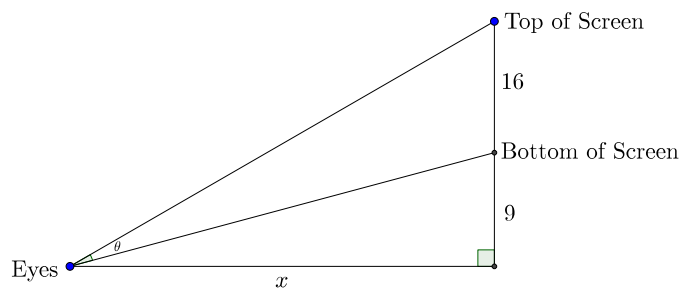
(b) Locate all local extrema. Express each as an ordered pair (x, y) .

(c) Find all intervals on which $f(x)$ is concave up and those on which $f(x)$ is concave down.

(d) Locate all points of inflection. Express each as an ordered pair (x, y) .

(e) Sketch $f(x)$.

12. The screen at the front of a movie theater is 16 feet high and positioned 9 feet above eye level. How far away from the front of the room should you sit in order to have the “best” view ? (HINT: Find the largest possible angle θ in diagram shown below.)



13. Find the area of the shaded region by adding together the area of the sector and the area of the triangle.

