Rectangular Coordinates, Spheres, & Cylindrical Surfaces

SUGGESTED REFERENCE MATERIAL:

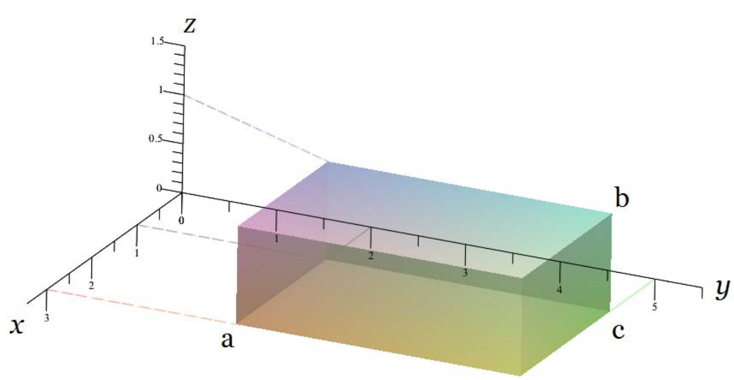
As you work through the problems listed below, you should reference Chapter 11.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to determine the location of a point in space using rectangular coordinates.
- Be able to find the distance between and the midpoint of two points in space.
- Know the standard equation of a sphere and be able to find the center and radius of a sphere.
- Be able to sketch cylindrical surfaces.

PRACTICE PROBLEMS:

Problems 1-3 refer to the rectangular box, shown below. The base of the rectangular box is in the xy-plane.



1. Find the coordinates of the eight corners of the box

   $\boxed{(1,2,0),(3,2,0),(3,5,0),(1,5,0),(1,2,1),(3,2,1),(3,5,1),(1,5,1)}$

2. Compute the midpoint of the diagonal which extends from vertex a to vertex b.

   $\boxed{\left(2,\frac{7}{2},\frac{1}{2}\right)}$

3. Consider the triangle with vertices a, b, and c.

   (a) Compute the length of each of the three sides.

   The diagonal from vertex a to vertex b has length $\sqrt{14}$;
   The line segment from vertex a to vertex c has length $\sqrt{13}$;
   The line segment from vertex c to vertex b has length 1.

   (b) Verify that the triangle is a right triangle.

   Notice that $\left(\sqrt{13}\right)^2 + 1^2 = \left(\sqrt{14}\right)^2$. So, the sides of the triangle (which are not collinear) satisfy the Pythagorean Theorem. Thus, the triangle is a right triangle.

   (c) Compute the angle between the diagonal which extends from vertex a to vertex b and the line segment which extends from vertex a to vertex c.

   $\cos^{-1}\left(\dfrac{\sqrt{13}}{\sqrt{14}}\right)$

4. Consider the triangle with vertices $A(5,-2,-1)$, $B(7,0,3)$, and $C(9,-4,1)$.

   (a) Show that the triangle is an equilateral triangle.

   The length of all sides of the triangle is $\sqrt{24}$; Detailed Solution: Here

   (b) Compute the area of the triangle.

   $6\sqrt{3}$ square units; Detailed Solution: Here

5. Find an equation of the sphere whose center is $(3,0,2)$ and which has a diameter of 6.

   $(x-3)^2 + y^2 + (z-2)^2 = 9$

6. Find an equation of the sphere whose center is $(4,2,-1)$ and which passes through the origin.

   $(x-4)^2 + (y-2)^2 + (z+1)^2 = 21$

7. Find an equation of the sphere which contains points $A(1,3,2)$ and $B(4,3,7)$ and the distance between A and B is equal to the diameter of the sphere.

   $\left(x-\frac{5}{2}\right)^2 + (y-3)^2 + \left(z-\frac{9}{2}\right)^2 = \frac{17}{2}$; Video Solution: Here

8. Does the origin lie inside of the sphere $(x-1)^2 + (y+2)^2 + (z+3)^2 = 13$? Justify your answer.

   No. The distance from the origin to the center $(1,-2,-3)$ is $\sqrt{14}$ which is greater than the radius, $\sqrt{13}$.

9. Consider the cube with a center at the origin which has sides of length 2 that are parallel to the coordinate planes.

   (a) Compute an equation of the sphere which is inscribed in this cube.

   $\boxed{x^2 + y^2 + z^2 = 1\text{; Detailed Solution: Here}}$

   (b) Compute an equation of the sphere which is circumscribed around the cube.

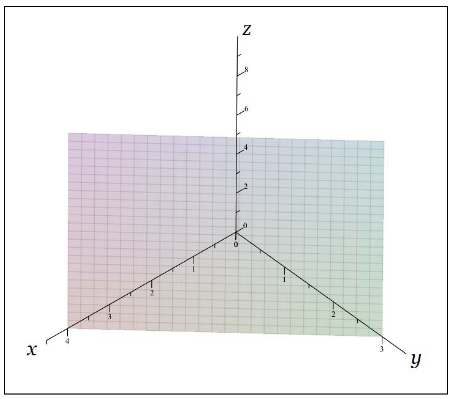
   $\boxed{x^2 + y^2 + z^2 = 3\text{; Detailed Solution: Here}}$

10. Find equations of the tangent spheres of equal radii whose centers are $(2, 3, 1)$ and $(5, -3, 2)$, respectively.

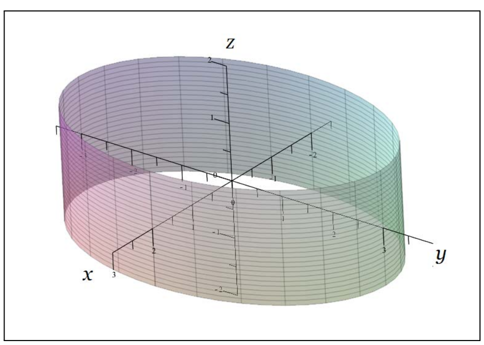
$$\boxed{(x-2)^2 + (y-3)^2 + (z-1)^2 = \frac{23}{2} \text{ and } (x-5)^2 + (y+3)^2 + (z-2)^2 = \frac{23}{2}}$$

11. Sketch the following surfaces in space.

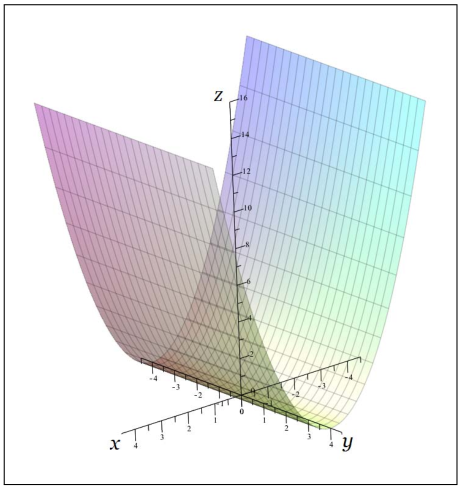
   (a) $3x + 4y = 12$



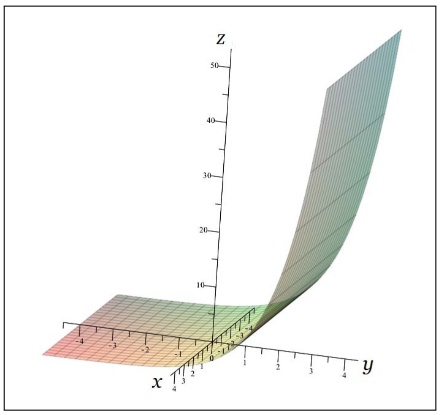
   (b) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$



(c) $z = x^2$



(d) $z = e^y$



12. Describe all points in space whose coordinates satisfy the following inequality

$$x^2 + z^2 - 4x - 8z + 13 > 0$$

$\boxed{\text{All points outside of the cylinder } (x-2)^2 + (z-4)^2 = 7\text{; Detailed Solution: Here}}$

13. Consider the surface $x^2 + y^2 + z^2 - 4x - 12y - 8z = k$, where k is a real number. For which values of k will the surface be a sphere?

$\boxed{k > -56}$