Chapter 2.5 Practice Problems

EXPECTED SKILLS:

- Know the derivatives of the 6 elementary trigonometric functions.
- Be able to use these derivatives in the context of word problems.

PRACTICE PROBLEMS:

1. Fill in the given table:

$f(x)$	$f'(x)$
$\sin x$	
$\cos x$	
$\tan x$	
$\cot x$	
$\sec x$	
$\csc x$	

2. Use the definition of the derivative to show that $\dfrac{d}{dx}(\cos x) = -\sin x$
   Hint: $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
3. Use the quotient rule to show that $\dfrac{d}{dx}(\cot x) = -\csc^2 x$.
4. Use the quotient rule to show that $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$.
5. Evaluate $\lim\limits_{h\to 0} \dfrac{\tan\left(\frac{\pi}{3}+h\right) - \tan\left(\frac{\pi}{3}\right)}{h}$ by interpreting the limit as the derivative of a function at a particular point.

For problems 6-14, differentiate

6. $f(x) = 2\cos x + 4\sin x$
7. $f(x) = 5\cos x + \cot x$
8. $g(x) = 4\csc x + 2\sec x$
9. $f(x) = \sin x \cos x$
10. $f(x) = \dfrac{\sin^2 x}{\cos x}$

11. $f(x) = x^3 \sin x$

12. $f(x) = \sec^2 x + \tan^2 x$

13. $f(x) = \dfrac{x + \sec x}{1 + \cos x}$

For problems 14-17, compute $\dfrac{d^2y}{dx^2}$

14. $f(x) = \tan x$

15. $f(x) = \sin x$

16. $f(x) = \cos^2 x$

17. $f(x) = \sin^2 x + \cos^2 x$

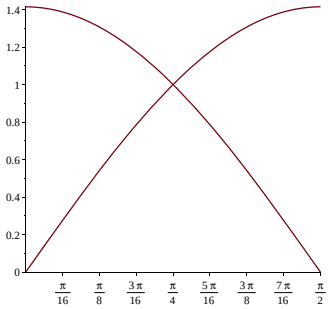
For problems 18-19, find all values of x in the interval $[0, 2\pi]$ where the graph of the given function has horizontal tangent lines.

18. $f(x) = \sin x \cos x$

19. $g(x) = \csc x$

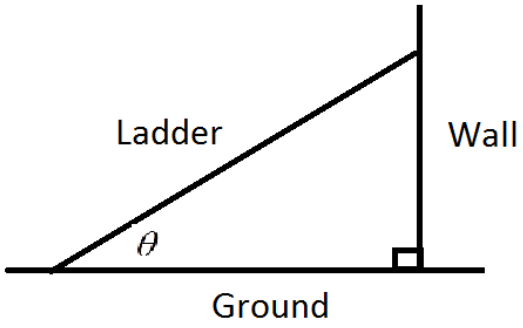
20. Compute an equation of the line which is tangent to the graph of $f(x) = \dfrac{\cos x}{x}$ at the point where $x = \pi$.

21. Consider the graphs of $f(x) = \sqrt{2}\cos(x)$ and $g(x) = \sqrt{2}\sin(x)$ shown below on the interval $\left[0, \dfrac{\pi}{2}\right]$.



Show that the graphs of $f(x)$ and $g(x)$ intersect at a right angle when $x = \dfrac{\pi}{4}$. (Hint: Show that the tangent lines to f and g at $x = \dfrac{\pi}{4}$ are perpendicular to each other.)

22. A 15 foot ladder leans against a vertical wall at an angle of θ with the horizontal, as shown in the figure below. The top of the ladder is h feet above the ground. If the ladder is pushed towards the wall, find the rate at which h changes with respect to θ at the instant when $\theta = 30°$. Express your answer in **feet/degree**.



23. Use the Intermediate Value Theorem to show that there is at least one point in the interval $(0, 1)$ where the graph of $f(x) = \sin x - \frac{1}{3}x^3$ will have a horizontal tangent line.

24. **Multiple Choice:** At how many points on the interval $[-\pi, \pi]$ is the tangent line to the graph of $y = 2x + \sin x$ parallel to the secant line which passes through the graph endpoints of the interval?

    (a) 0

    (b) 1

    (c) 2

    (d) 3

    (e) None of these