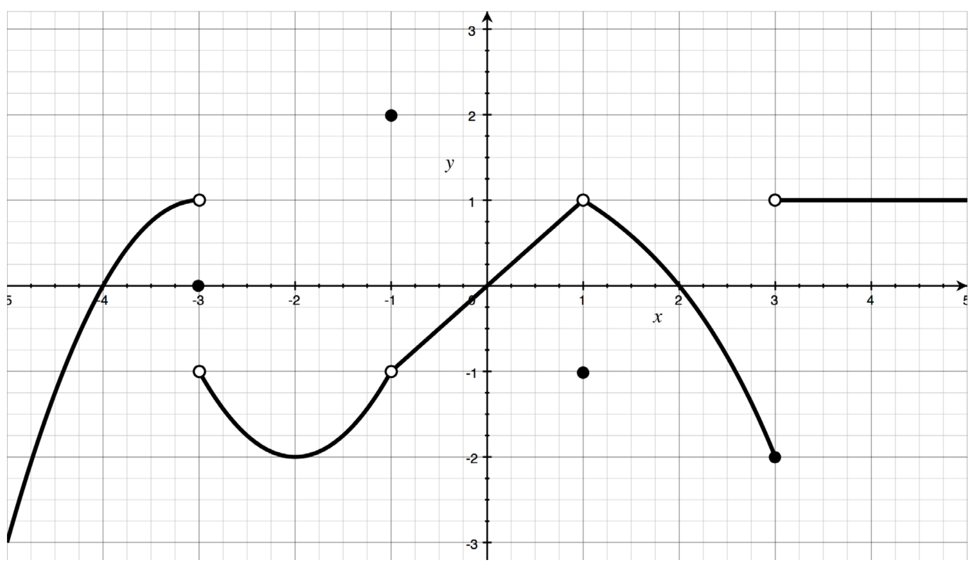
Chapter 1.1 Practice Problems

EXPECTED SKILLS:

- Given the graph of a function $y = f(x)$, be able to determine the limit of $f(x)$ as x approaches some finite value (as both a one-sided and two-sided limit).
- Know how to determine when such a limit does not exist, and if appropriate, indicate whether the behavior of the function increases or decreases without bound.

PRACTICE PROBLEMS:

Questions 1-5 refer to the function $F(x)$, which is illustrated below.



1. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

   (a) $\lim\limits_{x\to 1^-} F(x)$

   $\boxed{1}$

   (b) $\lim\limits_{x\to 1^+} F(x)$

   $\boxed{1}$

   (c) $\lim\limits_{x\to 1} F(x)$

   $\boxed{1}$

   (d) $F(1)$

   $\boxed{-1}$

2. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

   (a) $\lim_{x\to 3^-} F(x)$

   $\boxed{-2}$

   (b) $\lim_{x\to 3^+} F(x)$

   $\boxed{1}$

   (c) $\lim_{x\to 3} F(x)$

   $\boxed{\text{DNE because } \lim_{x\to 3^-} F(x) \neq \lim_{x\to 3^+} F(x)}$

   (d) $F(3)$

   $\boxed{-2}$

3. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

   (a) $\lim_{x\to 0^-} F(x)$

   $\boxed{0}$

   (b) $\lim_{x\to 0^+} F(x)$

   $\boxed{0}$

   (c) $\lim_{x\to 0} F(x)$

   $\boxed{0}$

   (d) $F(0)$

   $\boxed{0}$

4. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

   (a) $\lim_{x\to -1^-} F(x)$

   $\boxed{-1}$

   (b) $\lim_{x\to -1^+} F(x)$

   $\boxed{-1}$

   (c) $\lim_{x\to -1} F(x)$

   $\boxed{-1}$

(d) $F(-1)$

$\boxed{2}$

5. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim\limits_{x \to -3^-} F(x)$

$\boxed{1}$

(b) $\lim\limits_{x \to -3^+} F(x)$

$\boxed{-1}$

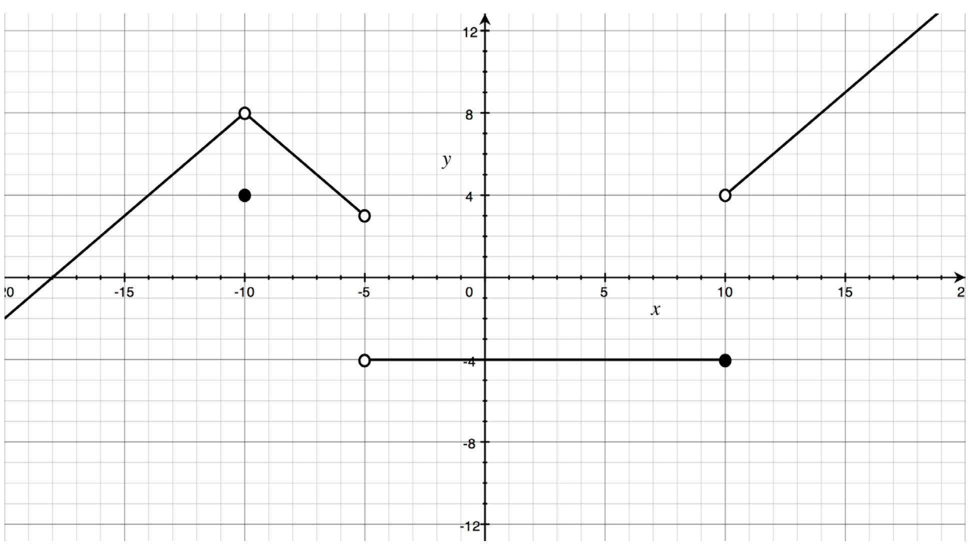
(c) $\lim\limits_{x \to -3} F(x)$

$\boxed{\text{DNE because } \lim\limits_{x \to -3^-} F(x) \neq \lim\limits_{x \to -3^+} F(x)}$

(d) $F(-3)$

$\boxed{0}$

Questions 6-9 refer to the graph of $G(x)$, which is illustrated below.



6. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim\limits_{x \to -10^-} G(x)$

$\boxed{8}$

(b) $\lim_{x \to -10^+} G(x)$

$\boxed{8}$

(c) $\lim_{x \to -10} G(x)$

$\boxed{8}$

(d) $G(-10)$

$\boxed{4}$

7. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim_{x \to -5^-} G(x)$

$\boxed{3}$

(b) $\lim_{x \to -5^+} G(x)$

$\boxed{-4}$

(c) $\lim_{x \to -5} G(x)$

$\boxed{\text{DNE because } \lim_{x \to -5^-} G(x) \neq \lim_{x \to -5^+} G(x)}$

(d) $G(-5)$

$\boxed{\text{Undefined}}$

8. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim_{x \to 0^-} G(x)$

$\boxed{-4}$

(b) $\lim_{x \to 0^+} G(x)$

$\boxed{-4}$

(c) $\lim_{x \to 0} G(x)$

$\boxed{-4}$

(d) $G(0)$

$\boxed{-4}$

9. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim_{x \to 10^-} G(x)$

$\boxed{-4}$

(b) $\lim_{x \to 10^+} G(x)$

$\boxed{4}$

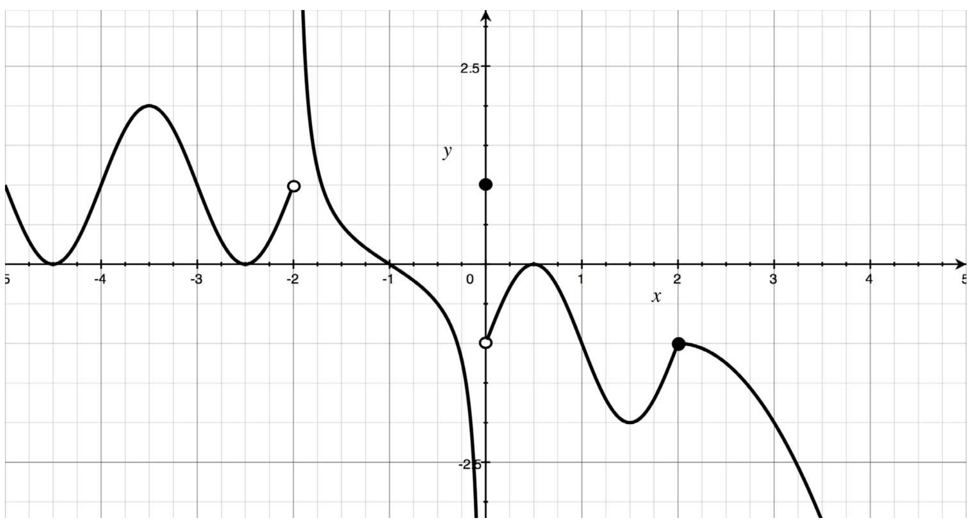
(c) $\lim_{x \to 10} G(x)$

$\boxed{\text{DNE because } \lim_{x \to 10^-} G(x) \neq \lim_{x \to 10^+} G(x)}$

(d) $G(10)$

$\boxed{-4}$

Questions 10-12 refer to the graph of $H(x)$, which is illustrated below.



10. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim_{x \to -2^-} H(x)$

$\boxed{1}$

(b) $\lim_{x \to -2^+} H(x)$

$\boxed{+\infty}$

(c) $\lim_{x \to -2} H(x)$

$\boxed{\text{DNE}}$

(d) $H(-2)$

$\boxed{\text{Undefined}}$

11. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim_{x\to 0^-} H(x)$

$\boxed{-\infty}$

(b) $\lim_{x\to 0^+} H(x)$

$\boxed{-1}$

(c) $\lim_{x\to 0} H(x)$

$\boxed{\text{DNE}}$

(d) $H(0)$

$\boxed{1}$

12. Compute each of the following quantities. If a limit does not exist, write $+\infty$, $-\infty$, or DNE (whichever is most appropriate).

(a) $\lim_{x\to 2^-} H(x)$

$\boxed{-1}$

(b) $\lim_{x\to 2^+} H(x)$

$\boxed{-1}$

(c) $\lim_{x\to 2} H(x)$

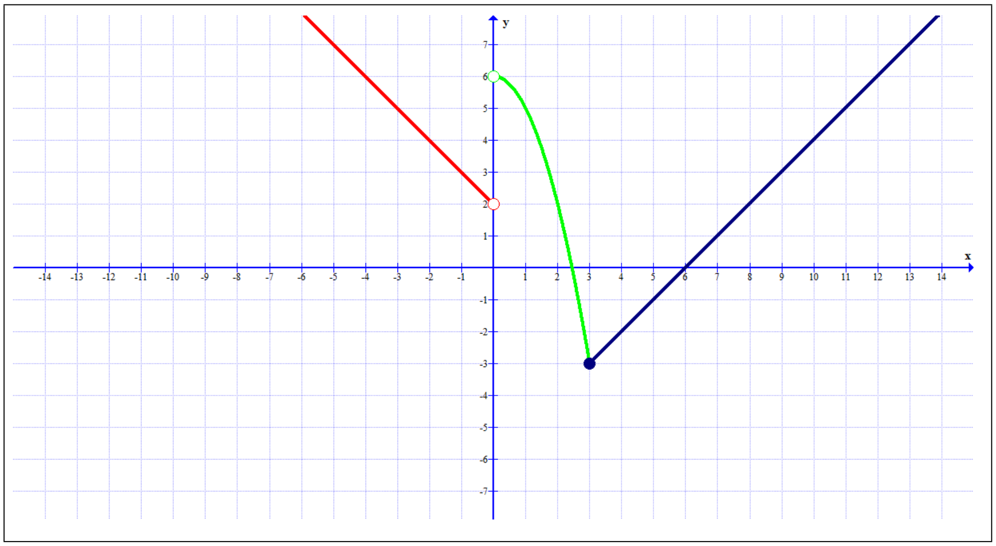
$\boxed{-1}$

(d) $H(2)$

$\boxed{-1}$

13. Let $f(x) = \begin{cases} 2 - x & \text{if } x < 0 \\ 6 - x^2 & \text{if } 0 < x < 3 \\ x - 6 & \text{if } x \geq 3 \end{cases}$

Sketch the graph of $f(x)$ and use your graph to compute each of the following:



(a) $\lim_{x \to 0^-} f(x)$

$\boxed{2}$

(b) $\lim_{x \to 0^+} f(x)$

$\boxed{6}$

(c) $\lim_{x \to 0} f(x)$

$\boxed{\text{DNE because } \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)}$

(d) $f(0)$

$\boxed{\text{Undefined}}$

(e) $\lim_{x \to 3^-} f(x)$

$\boxed{-3}$

(f) $\lim_{x \to 3^+} f(x)$

$\boxed{-3}$

(g) $\lim_{x \to 3} f(x)$

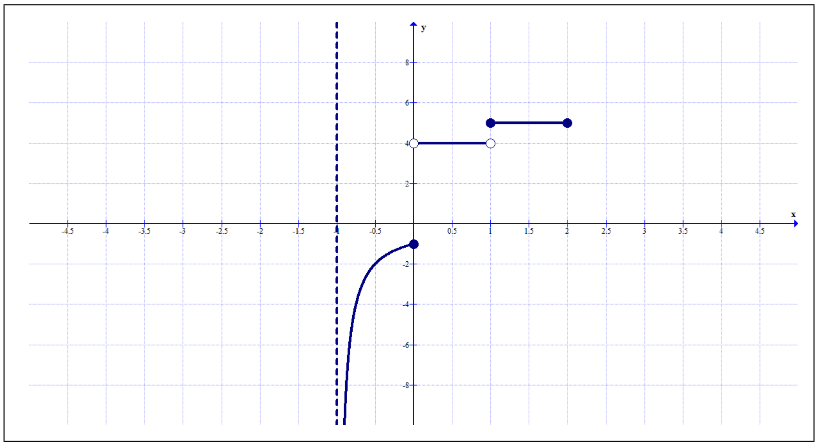
$\boxed{-3}$

(h) $f(3)$

$\boxed{-3}$

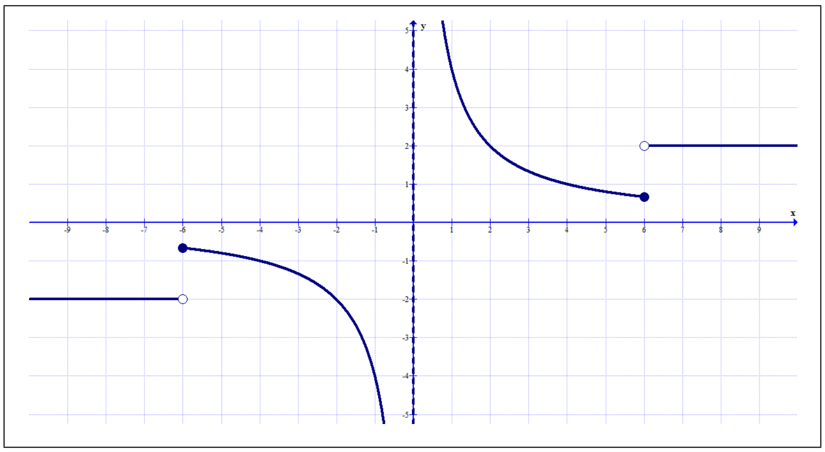
14. Sketch the graph of a function $y = f(x)$ which satisfies the following conditions. (There are many possible answers.)

- The domain is $(-1, 2]$.
- $f(1) = f(2) = 5$
- $\lim_{x \to 1^-} f(x) = 4$
- $\lim_{x \to -1^+} f(x) = -\infty$



15. Sketch the graph of a function $y = f(x)$ which satisfies the following conditions. (There are many possible answers.)

- $f(-x) = -f(x)$
- $\lim_{x \to 0^+} f(x) = +\infty$
- $\lim_{x \to 1^-} f(x) = 4$
- $\lim_{x \to 6^-} f(x) \neq \lim_{x \to 6^+} f(x)$.



16. For each of the following, determine whether the given statement is true or false. If the statement is false, give a specific counterexample.

(a) If $f(x)$ is not defined at $x = c$, then $\lim_{x \to c} f(x)$ DNE.

False. For example, consider $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$. Then, even though $f(0)$ is undefined, we have $\lim_{x \to 0} f(x) = 1$.

(b) If $\lim_{x \to a^-} f(x) = L$, then $\lim_{x \to a} f(x) = L$.

False. For example, consider $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ 2 & \text{if } x > 0 \end{cases}$. Then, $\lim_{x \to 0^-} f(x) = 1$; but, $\lim_{x \to 0} f(x)$ DNE because $\lim_{x \to 0^+} f(x) = 2 \neq 1$.