# Differential Equations & First-Order Separable Equations

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Be able to verify that a given function is a solution to a differential equation.
- Be able to solve first-order separable equations by separating and integrating.
- Be able to solve initial-value problems for first-order separable equations.

PRACTICE PROBLEMS:

1. Verify that $y = x^2 + 1$ is a solution to the differential equation $y - \dfrac{dy}{dx} = (x-1)^2$.

   Differentiating $y = x^2 + 1$ with respect to $x$ yields $y' = 2x$. Thus,

   $$y - \frac{dy}{dx} = (x^2 + 1) - (2x) = (x-1)^2$$

2. Find the value(s) of the constant $A$ for which $y = e^{Ax}$ is a solution to the differential equation $y'' + 5y' - 6y = 0$.

   $A = -6$ and $A = 1$

**For problems 3-9, use separation of variables to solve the given differential equation. If possible, express your answer as an explicit function of $x$.**

3. $\dfrac{dy}{dx} = \dfrac{x^2 - 1}{y^2}$

   $y = \sqrt[3]{x^3 - 3x + C}$

4. $\dfrac{dy}{dx} - \sqrt{xy}\ln x = 0$

   $y = \left(\dfrac{1}{3}x^{3/2}\ln x - \dfrac{2}{9}x^{3/2} + C\right)^2$, $y = 0$; Detailed Solution: Here

5. $y' = yx^2$

   $y = Ce^{x^3/3}$

6. $\dfrac{dy}{dx} - e^{-y}\sec^2 x = 0$

   $\boxed{y = \ln(\tan x + C)}$

7. $\dfrac{dy}{dx} = xy^3$

   $\boxed{y = \pm\sqrt{\dfrac{1}{C - x^2}},\ y = 0}$

8. $\dfrac{dy}{dx} = \dfrac{1}{(x^2 - 5x + 6)y}$

   $\boxed{y = \pm\sqrt{2\ln\left|\dfrac{x-3}{x-2}\right| + C}}$

9. $\left(\dfrac{\sqrt{x}y'}{2+y}\right) = 1$, for $x \neq 0$.

   $\boxed{y = Ce^{2\sqrt{x}} - 2,\ C \neq 0}$

**For problems 10-11, find the solution of the differential equation which satisfies the initial condition.**

10. $\dfrac{dy}{dx} = \dfrac{x^2 - 2}{y},\ y(0) = 1$

    $\boxed{y = \sqrt{\dfrac{2x^3}{3} - 4x + 1}}$

11. $\dfrac{dy}{dx} = \dfrac{\ln x}{xy^2},\ y(e) = 1$

    $\boxed{y = \sqrt[3]{\dfrac{3(\ln x)^2 - 1}{2}}}$

12. Find an equation of the curve that passes through the point $(0, 1)$ and whose slope at $(x, y)$ is $xe^y$.

    $\boxed{y = -\ln\left(\dfrac{1}{e} - \dfrac{x^2}{2}\right)}$; Detailed Solution: Here