#4

$$\frac{dy}{dx} - \sqrt{xy}\ln x = 0$$

$$\frac{dy}{dx} = \sqrt{xy}\ln x = \sqrt{x}\ln x\sqrt{y}$$

If $y \neq 0$: $\frac{dy}{\sqrt{y}} = \sqrt{x}\ln x\,dx$ Note that $y=0$ is a solution.

$$\int \frac{dy}{\sqrt{y}} = \int \sqrt{x}\ln x\,dx$$

LHS: $2\sqrt{y}$

RHS: Integration by parts

$u = \ln x$, $dv = \sqrt{x}\,dx \Rightarrow du = \frac{1}{x}dx$, $v = \frac{2}{3}x^{\frac{3}{2}}$

$$\int \sqrt{x}\ln x = \frac{2}{3}x^{\frac{3}{2}}\ln x - \int \frac{2}{3}x^{\frac{1}{2}}dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C$$

So $2\sqrt{y} = \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C$

Solutions: $y = \left(\frac{1}{3}x^{\frac{3}{2}}\ln x - \frac{2}{9}x^{\frac{3}{2}} + C\right)^2$

and $y = 0$