7.8 # 17 $\displaystyle\int_1^{+\infty} \frac{1}{x^p}\,dx$

If $p=1$: $\displaystyle\lim_{t\to+\infty}\int_1^{t} \frac{1}{x}\,dx = \lim_{t\to+\infty} \ln|x|\Big|_1^{t}$

$\displaystyle = \lim_{t\to+\infty}\left[\ln|t| - \ln|1|\right] = +\infty$ Diverges

If $p\neq 1$: $\displaystyle\lim_{t\to+\infty}\int_1^{t} \frac{1}{x^p}\,dx = \lim_{t\to+\infty}\int_1^{t} x^{-p}\,dx$

$\displaystyle = \lim_{t\to+\infty} \frac{x^{1-p}}{1-p}\Big|_1^{t} = \lim_{t\to+\infty} \frac{1}{1-p}\left[t^{1-p} - 1\right]$

If $1-p>0$ (or $p<1$) $\displaystyle\lim_{t\to+\infty} t^{1-p} = +\infty$ and integral diverges

If $1-p<0$ (or $p>1$) $\displaystyle\lim_{t\to+\infty} t^{1-p} = 0$ and integral converges

So $\displaystyle\int_1^{+\infty} \frac{1}{x^p}\,dx$ diverges if $p\leq 1$ and converges (to $\frac{1}{p-1}$) if $p>1$.