13.7 #11

(a) $z = x^2 - y^2 \iff x^2 - y^2 - z = 0$

$f(x,y,z) = x^2 - y^2 - z$

$g(x,y,z) = y^2 + z^2$

So $\nabla f(x,y,z) = \langle 2x, -2y, -1 \rangle \implies \nabla f(2,1,3) = \langle 4, -2, -1 \rangle$

and $\nabla g(x,y,z) = \langle 0, 2y, 2z \rangle \implies \nabla g(2,1,3) = \langle 0, 2, 6 \rangle$

$$\langle 4,-2,-1\rangle \times \langle 0,2,6\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2 & -1 \\ 0 & 2 & 6 \end{vmatrix} = -10\vec{i} - 24\vec{j} + 8\vec{k}$$

is parallel to the required tangent line, and thus so is any scalar multiple of it, e.g. $\langle 5, 12, -4 \rangle$

Answer: $\begin{cases} x = 2 + 5t \\ y = 1 + 12t \\ z = 3 - 4t \end{cases}$

(b) The angle between planes is the angle between the respective normal vectors. Normal vectors to S_1 and S_2 are the gradient vectors from (a), $\langle 4,-2,-1\rangle$ and $\langle 0,2,6\rangle$.

Let θ be an angle between them, $0 \le \theta \le \pi$.

So $\cos\theta = \dfrac{0-4-6}{\sqrt{16+4+1}\,\sqrt{0+4+36}} = \dfrac{-10}{\sqrt{21}\sqrt{40}}$

Since $\cos\theta < 0$, θ is not acute, so the acute angle between the tangent planes is

$$\pi - \theta = \pi - \arccos\left(\frac{-10}{\sqrt{21}\sqrt{40}}\right)$$