Chapter 4.2: Exponential & Logarithmic Functions of Base b

Expected Skills:

- Be able to specify the domain and range of $f(x) = b^x$ and $f(x) = \log_b x$.

- Be able to graph $f(x) = b^x$ and $f(x) = \log_b x$, labeling all intersections with the coordinate axes and all asymptotes.

- Be able to solve equations involving logarithm or exponential functions.

- Be able to differentiate exponential and logarithmic functions; also, be able to solve application problems such as tangent line, rates of change, local/absolute extrema, and curve sketching.

Practice Problems:

1. Evaluate each of the following without a calculator.

   (a) $\log_4 16$

   (b) $\ln \dfrac{1}{\sqrt[5]{e}}$

   (c) $\log_{43} 1$

   (d) $\log_{16} 2$

2. Use the properties of logarithms to expand (as much as possible) the expression as a sum, difference, and/or constant multiple of logarithms. (Assume that all variables are positive.)

   (a) $\log_5 \left(5x^2 \sqrt{y}\right)$

   (b) $\log_6 \dfrac{x^3}{y^2 z^4}$

3. Determine the domain of the following functions. Express your answer in interval notation.

   (a) $f(x) = \dfrac{\ln(x-1)}{x-5}$

   (b) $f(x) = \dfrac{\sqrt{4-x}}{2^x - 3}$

   (c) $f(x) = \dfrac{x-1}{2 - \log_4 x}$

4. Solve the given equation for x. Where appropriate, you may leave your answers in logarithmic form.

   (a) $\left(3^{x-5}\right) - 4 = 11$

   (b) $2\log_5(3x) = 4$

   (c) $\log_3 x + \log_3(x-8) = 2$

   (d) $\log_8 2x + \log_8(x+4) = 2$

5. In a research experiment the population of a certain species is given by $P(t) = 15(7^t)$, where t is the number of weeks since the beginning of the experiment.

   (a) How large was the population at the beginning of the experiment?

   (b) How long will it take for the population to reach 300? You may leave your answer in logarithmic form.

6. Calculate $\dfrac{dy}{dx}$.

   (a) $y = \log_2(3x - 1)$

   (b) $y = \dfrac{\log x}{2 - \log x}$

7. Use the change of base formula to express the following function in terms of the natural log. Then, calculate $\dfrac{dy}{dx}$. (Assume $x > 0$.)

   (a) $y = \log_{x^2}(e)$

   (b) $y = \log_{3x}(x)$

8. Find an equation of the tangent line to the graph of $f(x) = 3^{2x}$ at the point where $x = \log_3 4$.