Double Integrals in Polar Coordinates

SUGGESTED REFERENCE MATERIAL:

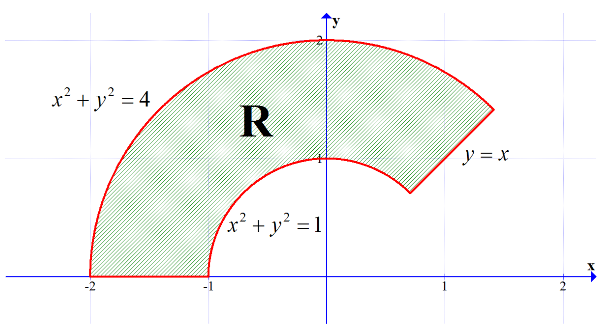
As you work through the problems listed below, you should reference Chapter 14.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to convert rectangular double integrals to polar double integrals, including converting the limits of integration, the function to be integrated, and the differential dA to $r\,dr\,d\theta$.

PRACTICE PROBLEMS:

1. Consider the region R shown below which is enclosed by $x^2+y^2=1$, $x^2+y^2=4$, $y=x$ and the x axis.



Fill in the missing limits of integration: $\displaystyle\iint_R f(x,y)\,dA = \int_{\square}^{\square}\int_{\square}^{\square} f(r,\theta)r\,dr\,d\theta.$

$$\boxed{\iint_R f(x,y)\,dA = \int_{\pi/4}^{\pi}\int_1^2 f(r,\theta)r\,dr\,d\theta}$$

For problems 2-6, evaluate the iterated integral by converting to polar coordinates.

2. $\displaystyle\int_0^4\int_0^{\sqrt{16-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

$$\boxed{\frac{32\pi}{3}}$$

3. $\displaystyle\int_0^{3/\sqrt{2}} \int_x^{\sqrt{9-x^2}} \left(x^2+y^2\right)^2 \, dy\, dx$

$\boxed{\dfrac{243}{8}\pi}$

4. $\displaystyle\int_0^{2} \int_0^{\sqrt{2x-x^2}} xy\, dy\, dx$

$\boxed{\dfrac{2}{3}}$

5. Evaluate $\displaystyle\iint_R (x-y)\, dA$ where $R = \{(x,y) : 4 \le x^2+y^2 \le 16 \text{ and } y \le x\}$

$\boxed{\dfrac{112}{3}\sqrt{2}}$

6. Evaluate $\displaystyle\iint_R e^{-(x^2+y^2)}\, dA$ where $R = \{(x,y) : x^2+y^2 \le 3 \text{ and } 0 \le y \le \sqrt{3}x\}$

$\boxed{\dfrac{\pi}{6}\left(1-\dfrac{1}{e^3}\right)}$

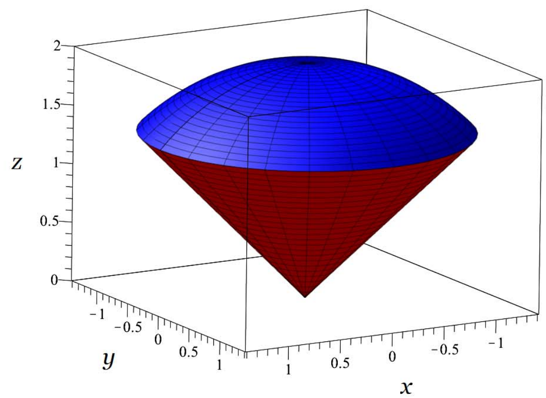
7. Use a double integral in polar coordinates to calculate the area of the region which is inside of the cardioid $r = 2+2\cos\theta$ and outside of the circle $r = 3$.

$\boxed{\dfrac{9\sqrt{3}}{2}-\pi}$

8. Use a double integral in polar coordinates to calculate the area of the region which is common to both circles $r = 3\sin\theta$ and $r = \sqrt{3}\cos\theta$.

$\boxed{\dfrac{5\pi}{8}-\dfrac{3\sqrt{3}}{4}\text{; Detailed Solution: Here}}$

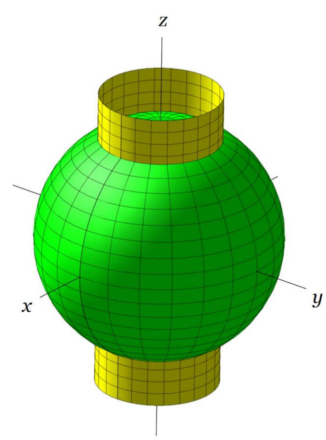
9. Consider the top which is bounded above by $z = \sqrt{4 - x^2 - y^2}$ and bounded below by $z = \sqrt{x^2 + y^2}$, as shown below.



Use a double integral in polar coordinates to calculate the volume of the top.

$\boxed{\dfrac{16\pi}{3} - \dfrac{8\pi\sqrt{2}}{3}}$

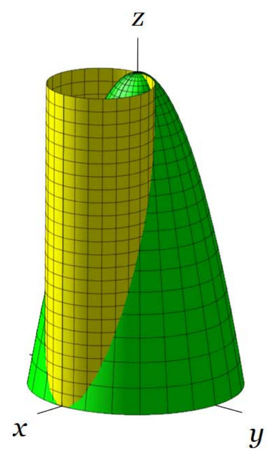
10. Consider the surfaces $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 = 4$, shown below.



Calculate the volume of the solid which is inside of $x^2 + y^2 + z^2 = 16$ but outside of $x^2 + y^2 = 4$.

$\boxed{32\pi\sqrt{3}}$; Detailed Solution: Here

11. Calculate the volume of the solid which is bounded above by $z = 9 - x^2 - y^2$, bounded below by $z = 0$, and contained within $x^2 - 3x + y^2 = 0$.



$$\boxed{\frac{405\pi}{32}}$$