The Indefinite Integral

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 5.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Given a differentiation rule, be able to construct the associated indefinite integration rule.
- Know how to integrate power functions (including polynomials), exponential functions, & trigonometric functions.

PRACTICE PROBLEMS:

For problems 1 and 2, compute the indicated derivative and state a corresponding integration formula.

1. $\dfrac{d}{dx}\left[\dfrac{1}{(2x+3)^2}\right]$

   $$\boxed{\frac{d}{dx}\left[\frac{1}{(2x+3)^2}\right] = \frac{-4}{(2x+3)^3} \implies \int \frac{-4}{(2x+3)^3}\,dx = \frac{1}{(2x+3)^2} + C}$$

2. $\dfrac{d}{dx}[x\ln x - x]$

   $$\boxed{\frac{d}{dx}[x\ln x - x] = \ln x \implies \int \ln x\,dx = x\ln x - x + C}$$

For problems 3-18, evaluate given indefinite integral and check your answer by differentiation.

3. $\displaystyle\int \left(\frac{1}{2}x + x^2\right)\,dx$

   $$\boxed{\frac{1}{4}x^2 + \frac{1}{3}x^3 + C}$$

4. $\displaystyle\int \left(\sqrt{x^7} + e\right)\,dx$

   $$\boxed{\frac{2}{9}x^{9/2} + ex + C}$$

5. $\displaystyle\int \left(\frac{1}{x^3} + 3x^3\right)\, dx$

$\boxed{\dfrac{-1}{2}x^{-2} + \dfrac{3}{4}x^4 + C}$

6. $\displaystyle\int \left(3x^{-2/3} + x^{-1/2} + 5x\right)\, dx$

$\boxed{9x^{1/3} + 2x^{1/2} + \dfrac{5}{2}x^2 + C}$

7. $\displaystyle\int \left(4x^{4/3} - 7\sqrt{x}\right)\, dx$

$\boxed{\dfrac{12}{7}x^{7/3} - \dfrac{14}{3}x^{3/2} + C}$

8. $\displaystyle\int 3\cos x\, dx$

$\boxed{3\sin x + C}$

9. $\displaystyle\int -7\sec^2 x\, dx$

$\boxed{-7\tan x + C}$

10. $\displaystyle\int \left(-\frac{1}{x} + e^x\right)\, dx$

$\boxed{-\ln|x| + e^x + C}$

11. $\displaystyle\int (1-x^2)(x^3+4)\, dx$

$\boxed{-\dfrac{1}{6}x^6 + \dfrac{1}{4}x^4 - \dfrac{4}{3}x^3 + 4x + C}$

12. $\displaystyle\int \frac{x^2 - 3x^5}{x^3}\, dx.$

$\ln|x| - x^3 + C$; Video Solution: https://www.youtube.com/watch?v=JCbEFor0NYY

13. $\displaystyle\int \frac{-2\sin x}{\cos^2 x}\, dx$

$\boxed{-2\sec x + C}$

14. $\displaystyle\int \frac{1}{\sqrt{4-4x^2}}\,dx$

$\boxed{\frac{1}{2}\sin^{-1}x + C\text{; Detailed Solution: Here}}$

15. $\displaystyle\int \left(6\cos x + 9\csc^2 x\right)\,dx$

$\boxed{6\sin x - 9\cot x + C}$

16. $\displaystyle\int \left(\sin x - 3\sec x\tan x\right)\,dx$

$\boxed{-\cos x - 3\sec x + C}$

17. $\displaystyle\int 2^x\,dx$

$\boxed{\frac{2^x}{\ln 2} + C}$

18. $\displaystyle\int \frac{x^2}{x^2+1}\,dx$ (HINT: Use polynomial division)

$\boxed{x - \arctan x + C\text{; Video Solution: https://www.youtube.com/watch?v=kTLkMO8l2Ak}}$

19. Consider $\displaystyle\int \cot^2 x\,dx$.

(a) Using the fact that $\sin^2 x + \cos^2 x = 1$, derive the identity $\cot^2 x = \csc^2 x - 1$.

Provided that $x \neq \pi \cdot k$, where k is any integer, we have:

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} &= \frac{1}{\sin^2 x} \\ 1 + \cot^2 x &= \csc^2 x \\ \cot^2 x &= \csc^2 x - 1 \end{aligned}$$

(b) Use the identity that you derived in part (a) to evaluate the original integral.

$\boxed{\displaystyle\int \cot^2 x\,dx = -\cot x - x + C.}$

For problems 20 and 21, find a function $y = y(x)$ which satisfies the given Initial Value Problem.

20. $$\begin{cases} \dfrac{dy}{dx} = \dfrac{1}{9x^2} \\ \\ y(1) = \dfrac{1}{2} \end{cases}$$

$$\boxed{y = -\frac{1}{9}x^{-1} + \frac{11}{18}}$$

21. $$\begin{cases} \dfrac{dy}{dx} = -2e^x \\ \\ y(0) = -5 \end{cases}$$

$\boxed{y = -2e^x - 3}$; Video Solution: https://www.youtube.com/watch?v=kvFRPT4nTIM

22. A ball is thrown straight up in the air from an initial height of s_0 feet above the ground with an initial speed of v_0 ft/sec. Then $s(t)$ gives the height (in feet) above the ground at time t, $v(t) = s'(t)$ gives the velocity (in ft/sec) of the ball at time t, and $a(t) = s''(t)$ gives the acceleration (in ft/sec^2) of the ball at time t. Assuming that acceleration is constant, $-g$ ft/sec^2, determine $v(t)$ and $s(t)$.

$$\boxed{v(t) = -gt + v_0,\ s(t) = -\frac{1}{2}gt^2 + v_0t + s_0}$$