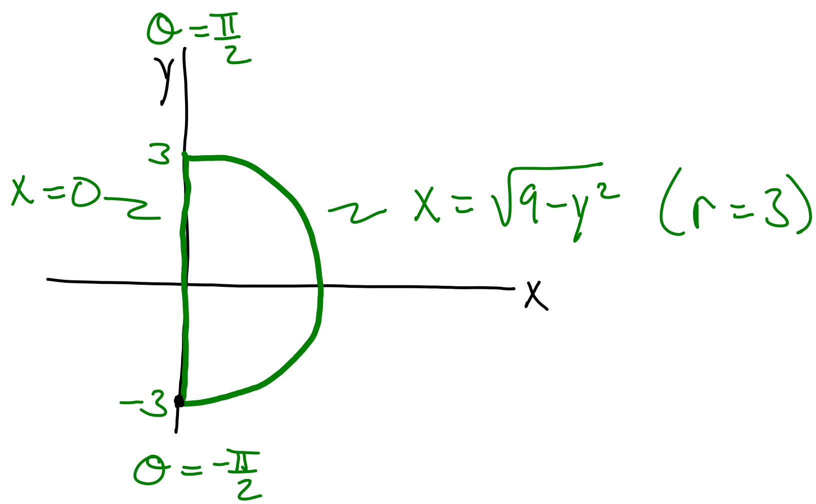
14.5, 14.6 #11

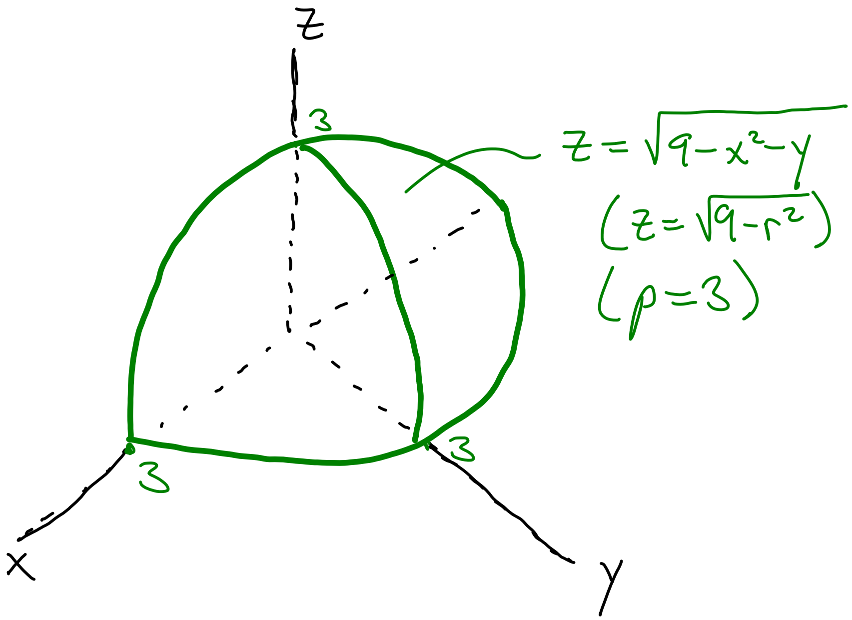
$$\int_{-3}^{3}\int_{0}^{\sqrt{9-y^2}}\int_{0}^{\sqrt{9-x^2-y^2}} (x^2+y^2)\,dz\,dx\,dy$$

We will sketch the solid G over which we are integrating. Since dz is the innermost ("height") differential, we examine the projection of G on the xy-plane, which is bounded by $y=-3$, $y=3$, $x=0$, and $x=\sqrt{9-y^2}$.



Bottom of G: $z=0$ Top of G: $z=\sqrt{9-x^2-y^2}$

So G is $\frac{1}{4}$ of a hemisphere.



Cylindrical: $\displaystyle\iiint_G r^2\,dz\,r\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^3\int_0^{\sqrt{9-r^2}} r^3\,dz\,dr\,d\theta$

Spherical: $\displaystyle\iiint_G \left((\rho\sin\phi\cos\theta)^2+(\rho\sin\theta\sin\theta)^2\right)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^3 \rho^4\sin^3\phi\,d\rho\,d\phi\,d\theta$$

[Since limits of integration are all constants, the order of integration can easily be changed, e.g. see posted answer on website.]