#39

(a) $a_1 = 1,\ a_2 = 1,\ a_{n+2} = a_n + a_{n+1},\quad n \geq 1$

(b) We assumed $\lim\limits_{n\to+\infty} \dfrac{a_{n+1}}{a_n} = L$, so $\lim\limits_{n\to+\infty} \dfrac{a_{n+2}}{a_{n+1}} = L$.

Now $\dfrac{a_{n+2}}{a_{n+1}} = \dfrac{a_n + a_{n+1}}{a_{n+1}} = \dfrac{a_n}{a_{n+1}} + 1$.

So $\lim\limits_{n\to+\infty} \dfrac{a_{n+2}}{a_{n+1}} = \lim\limits_{n\to+\infty} \left(\dfrac{a_n}{a_{n+1}} + 1 \right)$.

$$L = \frac{1}{L} + 1$$

$$L^2 = 1 + L$$

$$L^2 - L - 1 = 0$$

By quadratic formula, $L = \dfrac{1 \pm \sqrt{5}}{2}$. Since all the terms in the sequence are positive, $L = \dfrac{1+\sqrt{5}}{2}$.