13.7 #13

A direction vector for the line containg A and B is $\overrightarrow{AB} = \langle 6,4,2\rangle$. Let (x,y,z) be a point on $x^2+y^2-z^2=9$

where the normal line is parallel to $\langle 6,4,2\rangle$.

Let $f(x,y,z) = x^2+y^2-z^2 \Rightarrow \nabla f(x,y,z) = \langle 2x, 2y, -2z\rangle$

Since $\nabla f(x,y,z)$ is a direction vector for the normal line,

we have $\langle 2x, 2y, -2z\rangle = k\langle 6,4,2\rangle$ for some scalar k.

So $x=3k$, $y=2k$, $z=-k$ and since $x^2+y^2-z^2=9$ we have

$$(3k)^2+(2k)^2-(-k)^2=9 \iff 12k^2=9 \iff k^2=\frac{3}{4} \iff k=\pm\frac{\sqrt{3}}{2}$$

$$k=\frac{\sqrt{3}}{2} \Rightarrow (x,y,z) = \left(\frac{3\sqrt{3}}{2}, \sqrt{3}, -\frac{\sqrt{3}}{2}\right)$$

$$k=-\frac{\sqrt{3}}{2} \Rightarrow (x,y,z) = \left(-\frac{3\sqrt{3}}{2}, -\sqrt{3}, \frac{\sqrt{3}}{2}\right)$$