Chapter 1.6 Practice Problems

EXPECTED SKILLS:

- Know where the trigonometric and inverse trigonometric functions are continuous.
- Be able to use $\lim_{x\to 0} \frac{\sin x}{x} = 1$ or $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$ to help find the limits of functions involving trigonometric expressions, when appropriate.
- Understand the squeeze theorem and be able to use it to compute certain limits.

PRACTICE PROBLEMS:

Evaluate the following limits. If a limit does not exist, write DNE, $+\infty$, or $-\infty$ (whichever is most appropriate).

1. $\lim_{x\to \frac{\pi}{4}} \sin(2x)$
2. $\lim_{\theta\to \pi} (\theta \cos\theta)$
3. $\lim_{x\to 0^+} \csc x$
4. $\lim_{x\to \frac{\pi}{2}^+} \tan x$
5. $\lim_{x\to \frac{\pi}{2}^-} \tan x$
6. $\lim_{x\to \frac{\pi}{4}} \sec x$
7. $\lim_{x\to 0} \left(\frac{\sin x}{3x}\right)$
8. $\lim_{x\to 0} \left(\frac{\sin 3x}{3x}\right)$
9. $\lim_{x\to 0} \left(\frac{\sin x}{|x|}\right)$
10. $\lim_{x\to 0} \left(\frac{1-\cos x}{4x}\right)$
11. $\lim_{x\to 0^-} \left(\frac{\cos x}{x}\right)$
12. $\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)$

13. $\lim\limits_{x \to 0} \left(\dfrac{\tan 2x}{x} \right)$

14. $\lim\limits_{x \to 0} \left(\dfrac{1 - 3\cos x}{3x} \right)$

15. $\lim\limits_{x \to \infty} \arccos \left(\dfrac{-x^2}{x^2 + 3x} \right)$

16. $\lim\limits_{x \to 0} \left(\dfrac{3x^2}{1 - \cos^2 x} \right)$

17. $\lim\limits_{x \to 0} \left(\dfrac{\tan 5x}{\sin 9x} \right)$

18. **Multiple Choice:** Evaluate $\lim\limits_{x \to 0} \dfrac{\tan^2 x}{x^2}$

    (a) -1
    (b) 0
    (c) 1
    (d) $-\infty$
    (e) $+\infty$

For problems 19-23, evaluate the following limits by first making an appropriate substition. If the limit does not exist, write DNE, $+\infty$, or $-\infty$ (whichever is most appropriate).

19. $\lim\limits_{x \to \infty} \left(e^x \sin\left(e^{-x}\right) \right)$

20. $\lim\limits_{x \to 1} \left(\dfrac{\sin\left(\ln x^5\right)}{\ln x} \right)$

21. $\lim\limits_{x \to \frac{\pi}{2}^+} e^{\sec x}$

22. $\lim\limits_{x \to 0} \sin \left(\dfrac{1}{x^2} \right)$

23. $\lim\limits_{x \to 0^+} \tan^{-1}(\ln x)$

For problem 24-28, determine the value(s) of x where the given function is continuous.

24. $f(x) = \csc x$

25. $f(x) = e^{\sin x}$

26. $f(x) = \dfrac{1}{1 - 2\cos x}$ on $[0, 2\pi]$

27. $f(x) = \sin^{-1} x$

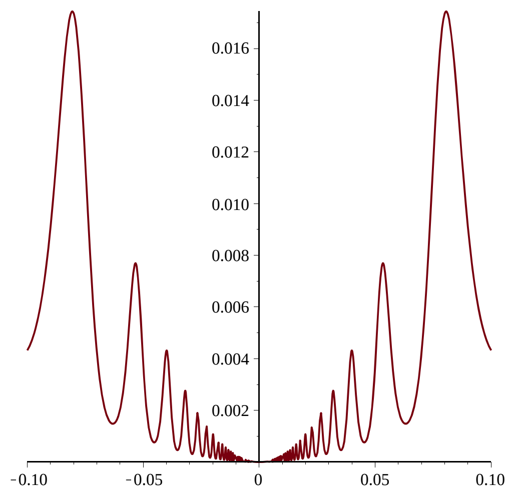
28. $f(x) = \begin{cases} \cos x & \text{if} \quad x < \dfrac{\pi}{4} \\ \sin x & \text{if} \quad x \geq \dfrac{\pi}{4} \end{cases}$

29. Find all non-zero value(s) of k so that $f(x) = \begin{cases} \dfrac{3\sin(kx)}{x} & \text{if} \quad x > 0 \\ 6k^2 + 5x & \text{if} \quad x \leq 0 \end{cases}$ is continuous at $x = 0$.

30. Use the Intermediate Value Theorem to prove that there is at least one solution to $\cos x = x^2$ in $(0, 1)$.

31. Let $f(x)$ be a function which satisfies $5x - 6 \leq f(x) \leq x^2 + 3x - 5$ for all $x \geq 0$. Compute $\lim\limits_{x \to 1} f(x)$.

32. The graph of $f(x) = x^2 e^{\cos(1/x)}$ is shown below on $[-0.1, 0.1]$:



Make a conjecture about $\lim\limits_{x \to 0} f(x)$ and then use the Squeeze Theorem to show this is true.

33. Let x be a fixed real number. Compute $\lim_{h \to 0} \dfrac{\sin(x+h) - \sin x}{h}$. (Hint: The identity $\sin(A+B) = \sin A \cos B + \cos A \sin B$ will be useful.)