

# Power Series

## SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

## EXPECTED SKILLS:

- Use sigma notation to write the Maclaurin series for a function  $f(x)$ .
- Use sigma notation to write the Taylor series for a function  $f(x)$  about a specified  $x = x_0$ .
- Find the interval of convergence and the radius of convergence of a power series.
- Find the domain of a function that is expressed as a power series.

## PRACTICE PROBLEMS:

**For problems 1 & 2, use sigma notation to write the Maclaurin series for the given function.**

1.  $f(x) = \ln(1 + x)$
2.  $f(x) = x \cos x$

**For problems 3 & 4, use sigma notation to write the Taylor series for the given function about  $x = x_0$ .**

3.  $f(x) = e^{2x}; x_0 = \ln 3$
4.  $f(x) = \sin x; x_0 = \frac{\pi}{2}$

**For problems 5 – 13, find the interval of convergence and the radius of convergence  $R$  for the power series.**

5.  $x + x^2 + x^3 + x^4 + \dots$
6.  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$
7.  $\sum_{k=0}^{\infty} \frac{x^k}{k!}$
8.  $\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$

$$9. \sum_{k=0}^{\infty} \frac{(-5)^k x^k}{\sqrt{k+10}}$$

$$10. \sum_{k=0}^{\infty} [(2k)! (2x+1)^k]$$

$$11. \sum_{k=0}^{\infty} \left[ \left( \frac{2}{7} \right)^k (x+4)^{k+1} \right]$$

$$12. \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$13. \sum_{k=2}^{\infty} \frac{(x-3)^k}{k \ln k}$$

For problems 14 – 16, a function is represented as a power series. Find the domain of the function.

$$14. f(x) = \sum_{k=0}^{\infty} [(-1)^{k+1} (x-2)^k]$$

$$15. f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$16. f(x) = \sum_{k=0}^{\infty} \frac{e^{(k^2)} x^k}{k!}$$