$$\begin{cases} \frac{dy}{dx} = xe^{y} \\ y(0) = 1 \end{cases}$$

$$\frac{dy}{e^{y}} = x dx$$

$$\int e^{-y} dy = \int x dx$$

$$-e^{-y} = \frac{1}{2}x^{2} + C$$

$$y(0) = 1 \implies -e^{-1} = \frac{1}{2}(0)^{2} + C \implies C = -\frac{1}{e}$$

$$-e^{-y} = \frac{1}{2}x^{2} - \frac{1}{e}$$

$$e^{-y} = \frac{1}{e} - \frac{1}{2}x^{2}$$

$$-y = \ln(\frac{1}{e} - \frac{1}{2}x^{2})$$

$$y = -\ln(\frac{1}{e} - \frac{1}{2}x^{2})$$