7.3 # 27

$$y = \ln(\sin x) \implies \frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$$

$$L = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sqrt{1+\cot^2 x}\, dx = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sqrt{\csc^2 x}\, dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \csc x\, dx = \ln|\csc x - \cot x| \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \ln\left|\csc\tfrac{3\pi}{4} - \cot\tfrac{3\pi}{4}\right| - \ln\left|\csc\tfrac{\pi}{4} - \cot\tfrac{\pi}{4}\right|$$

$$= \ln\left|\sqrt{2} - (-1)\right| - \ln\left|\sqrt{2}-1\right| = \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$ ← This answer is fine.

$$= \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1}\right) = \ln(2+2\sqrt{2}+1) = \ln\left(\frac{4+4\sqrt{2}+2}{2}\right)$$

$$= \ln(4+4\sqrt{2}+2) - \ln 2 = \ln(2+\sqrt{2})^2 - \ln 2 = \underbrace{2\ln(2+\sqrt{2}) - \ln 2}$$

answer online (from Maple)