Polar Coordinates

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 10.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to describe points and curves in both polar and rectangular form, and be able to convert between the two coordinate systems.
- Know the formulas for the basic shapes in polar coordinates: circles, lines, limacons, cardioids, rose curves, and spirals.

PRACTICE PROBLEMS:

For problems 1-6, compute the rectangular coordinates of the points whose polar coordinates are given.

1. $\left(-1, \frac{\pi}{3}\right)$

   $\boxed{\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$

2. $\left(3, \frac{2\pi}{3}\right)$

   $\boxed{\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)}$

3. $(5, -\pi)$

   $\boxed{(-5, 0)}$

4. $\left(-2, \frac{9\pi}{4}\right)$

   $\boxed{\left(-\sqrt{2}, -\sqrt{2}\right)}$

5. $\left(6, \frac{11\pi}{6}\right)$

   $\boxed{\left(3\sqrt{3}, -3\right)}$

For problems 7-11, find two pairs of polar coordinates for the point whose rectangular coordinates are given. The first pair should satisfy $r \geq 0$ and $0 \leq \theta < 2\pi$. The second pair should satisfy $r \geq 0$ and $-2\pi < \theta \leq 0$.

7. $(-5, -5)$

   I. $\left(5\sqrt{2}, \frac{5\pi}{4}\right)$; II. $\left(5\sqrt{2}, -\frac{3\pi}{4}\right)$

8. $(-3, 3)$

   I. $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$; II. $\left(3\sqrt{2}, -\frac{5\pi}{4}\right)$

9. $(0, 3)$

   I. $\left(3, \frac{\pi}{2}\right)$; II. $\left(3, -\frac{3\pi}{2}\right)$

10. $\left(\sqrt{3}, -1\right)$

    I. $\left(2, \frac{11\pi}{6}\right)$; II. $\left(2, -\frac{\pi}{6}\right)$; Detailed Solution: Here

11. $\left(-4\sqrt{3}, -4\right)$

    I. $\left(8, \frac{7\pi}{6}\right)$; II. $\left(8, -\frac{5\pi}{6}\right)$

12. Consider the point with rectangular coordinates $\left(1, \sqrt{3}\right)$.

    (a) Find a pair of polar coordinates which satisfy $r \geq 0$ and $0 \leq \theta < 2\pi$

    $(r, \theta) = \left(2, \frac{\pi}{3}\right)$

    (b) Find a pair of polar coordinates which satisfy $r \leq 0$ and $0 \leq \theta < 2\pi$

    $(r, \theta) = \left(-2, \frac{4\pi}{3}\right)$

    (c) Find a pair of polar coordinates which satisfy $r \geq 0$ and $-2\pi < \theta \leq 0$

    $(r, \theta) = \left(2, -\frac{5\pi}{3}\right)$;

(d) Find a pair of polar coordinates which satisfy $r \leq 0$ and $-2\pi < \theta \leq 0$

$\boxed{(r, \theta) = \left(-2, -\frac{2\pi}{3}\right)}$

For problems 13-17, identify the curve by transforming the polar equation into rectangular coordinates.

13. $r = 1$

    $\boxed{\text{circle, } x^2 + y^2 = 1}$

14. $r = 2\cos\theta$

    $\boxed{\text{circle, } (x-1)^2 + y^2 = 1}$

15. $r\sin\theta = 2$

    $\boxed{\text{line, } y = 2}$

16. $r = 3\cos\theta - 2\sin\theta$

    $\boxed{\text{circle, } \left(x - \frac{3}{2}\right)^2 + (y+1)^2 = \frac{13}{4}\text{; Detailed Solution: Here}}$

17. $r = 6\sec\theta$

    $\boxed{\text{line, } x = 6}$

For problems 18-21, express the given equation in polar coordinates.

18. $y = 2$

    $\boxed{r = 2\csc\theta}$

19. $x = 3$

    $\boxed{r = 3\sec\theta\text{; Detailed Solution: Here}}$

20. $x^2 + y^2 = 10$

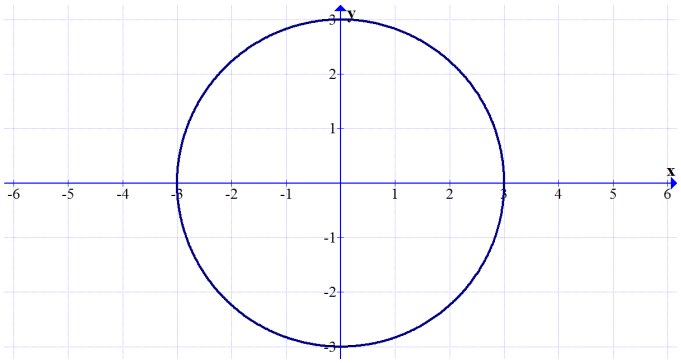
    $\boxed{r = \sqrt{10}}$

21. $x^2 + y^2 + 8y = 0$

    $\boxed{r = -8\sin\theta}$

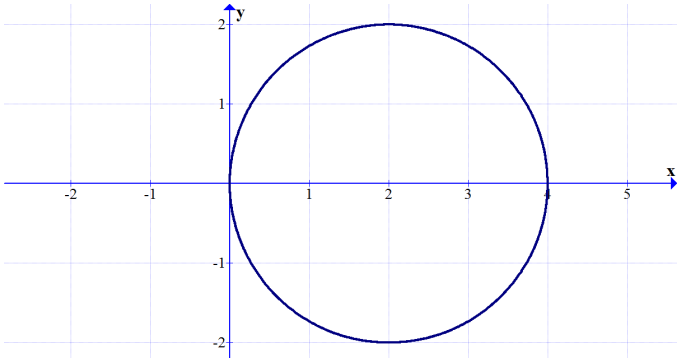
For problems 22-24, find an equation in polar coordinates for each of the given graphs.

22. Circle:



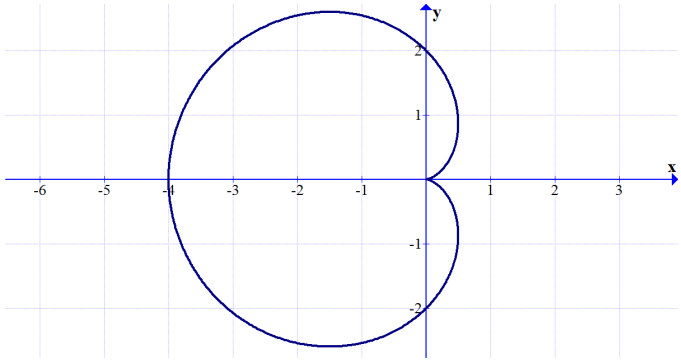
$\boxed{r = 3}$

23. Circle



$\boxed{r = 4\cos\theta}$

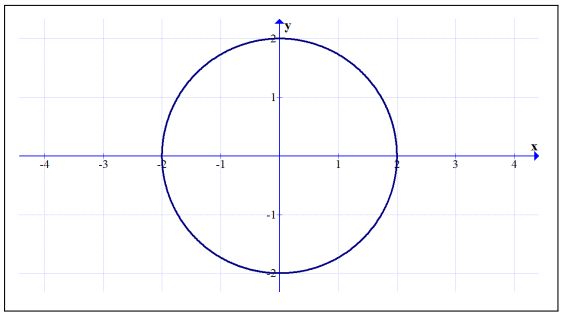
24. Cardioid



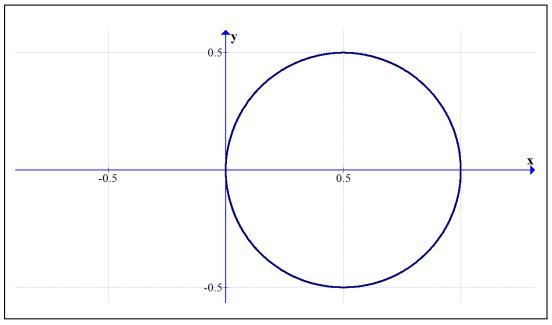
$\boxed{r = 2(1 - \cos\theta)}$

For problems 25-34, sketch the curve in polar coordinates.

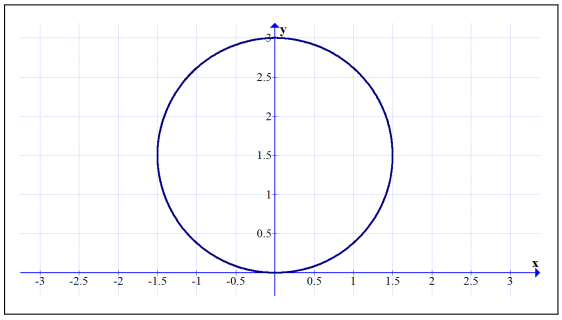
25. $r = 2$



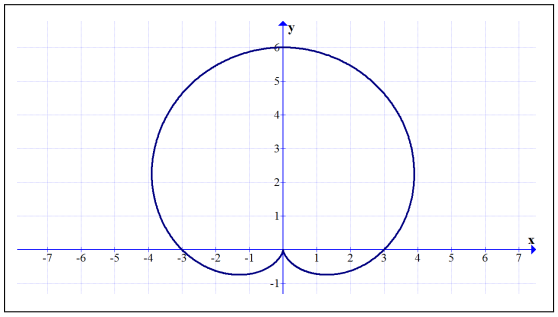
26. $r = \cos\theta$



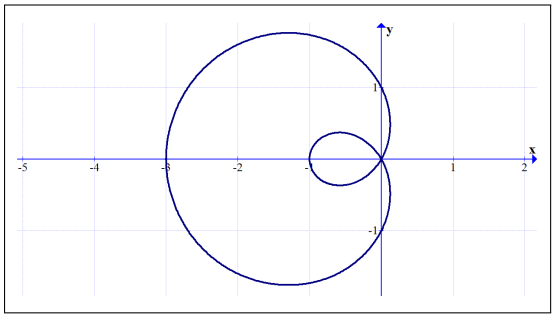
27. $r = 3\sin\theta$



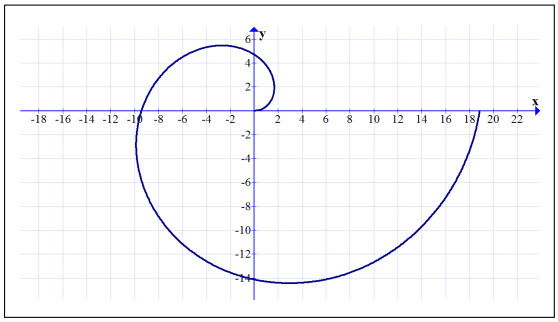
28. $r = 3 + 3\sin\theta$



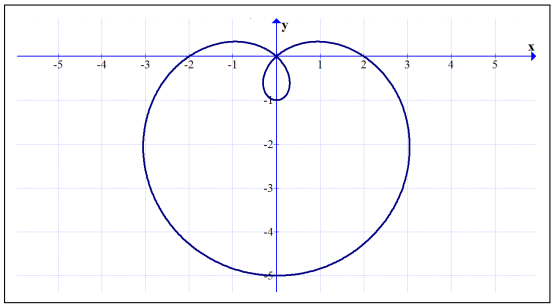
29. $r = 1 - 2\cos\theta$



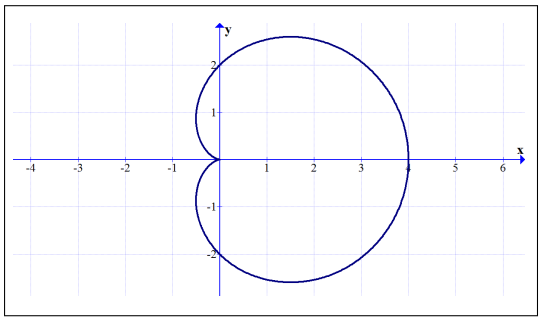
30. $r = 3\theta,\ 0 \le \theta \le 2\pi$



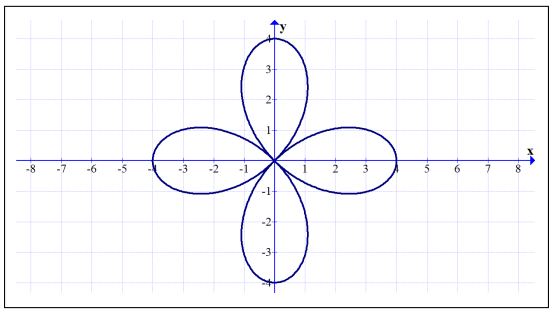
31. $r = 2 - 3\sin\theta$



32. $r = 2(1 + \cos\theta)$



33. $r = 4\cos 2\theta$



34. $r = -3\sin 3\theta$