#12

Idea: As $k \to +\infty$, k^2 will dominate k in the numerator

and k^5 will dominate 10 in the denominator, so

$$\sum_{k=1}^{\infty} \frac{4k^2+5k}{\sqrt{10+k^5}} \text{ should "act like" } \sum_{k=1}^{\infty} \frac{k^2}{\sqrt{k^5}} = \sum_{k=1}^{\infty} \frac{1}{k^{1/2}}$$

So

$$\lim_{k\to+\infty} \frac{\dfrac{4k^2+5k}{\sqrt{10+k^5}}}{\dfrac{1}{k^{1/2}}} = \lim_{k\to+\infty} \frac{4k^{\frac{5}{2}}+5k^{\frac{3}{2}}}{\sqrt{10+k^5}} \cdot \frac{\dfrac{1}{k^{5/2}}}{\dfrac{1}{k^{5/2}}}$$

$$= \lim_{k\to+\infty} \frac{4+\cancelto{0}{\dfrac{5}{k}}}{\sqrt{\cancelto{0}{\dfrac{10}{k^5}}+1}} = 4, \text{ which is finite and nonzero.}$$

Since $\sum_{k=1}^{\infty} \frac{1}{k^{1/2}}$ diverges (p-series, $p=\frac{1}{2}$),

$$\sum_{k=1}^{\infty} \frac{4k^2+5k}{\sqrt{10+k^5}}$$ diverges as well.

Note: It is perfectly reasonable to compare the series to $\sum_{k=1}^{\infty} \frac{4}{k^{1/2}}$, i.e. keep the leading coefficient.

The limit would then be 1, but the conclusion would be the same.