Double Integrals Over Rectangular Regions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 14.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute double integral calculations over rectangular regions using partial integration.
- Know how to inspect an integral to decide if the order of integration is easier one way (y first, x second) or the other (x first, y second).
- Kow how to use a double integral as the volume under a surface or find the area or a region in the xy-plane.

PRACTICE PROBLEMS:

For problems 1-4, evaluate the given iterated integral.

1. $\displaystyle\int_0^1 \int_0^2 \left(3x^3 - y^2 + 2\right)\, dx\, dy$

   $\boxed{\dfrac{46}{3}}$

2. $\displaystyle\int_0^2 \int_1^3 x^2 y\, dy\, dx$

   $\boxed{\dfrac{32}{3}}$

3. $\displaystyle\int_0^{\ln 4} \int_0^{\ln 5} e^{x+y}\, dy\, dx$

   $\boxed{12}$

4. $\displaystyle\int_0^{\pi} \int_1^2 x \sin y\, dx\, dy$

   $\boxed{3}$

5. Consider $f(x,y) = x^2 + y^2$ and $R : [0,4] \times [0,4]$.

   (a) Estimate the volume bounded between the graph of $f(x,y)$ and the xy-plane over the region R using 4 subrectagles of equal area and choosing the lower left hand corners as the sample points.

   $\boxed{64}$

   (b) Estimate the volume bounded between the graph of $f(x,y)$ and the xy-plane over the region R using 4 subrectagles of equal area and choosing the upper right hand corners as the sample points.

   $\boxed{320}$

   (c) Estimate the volume bounded between the graph of $f(x,y)$ and the xy-plane over the region R using 4 subrectagles of equal area and choosing the middle of the rectangle as the sample points.

   $\boxed{160}$

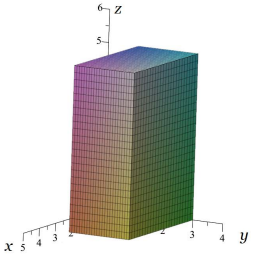
   (d) Compute the exact volume of the solid bounded between $f(x,y)$ and the xy-plane over the region R using an appropriate double integral.

   $\boxed{\frac{512}{3}}$

6. Each of the following iterated integrals represents the volume of a solid. Make a sketch of a solid whose volume is represented by the integral.

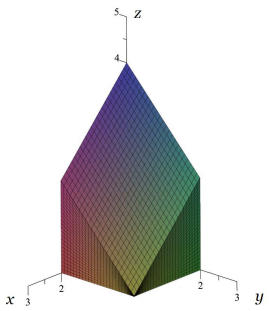
   (a) $\displaystyle\int_0^4 \int_1^3 5 \, dy \, dx$

   This value of this integral can be thought of as the volume between the $z = 5$ plane and the xy-plane over the rectangle $[0,4] \times [1,3]$.



(b) $\displaystyle\int_0^2 \int_0^2 (4 - x - y)\, dx\, dy$

This value of this integral can be thought of as the volume between the plane $z = 4 - x - y$ and the xy-plane over the square $[0, 2] \times [0, 2]$.



7. Use a double integral to find the volume of the solid which is bounded by the circular paraboloid $z = x^2 + y^2$ and the planes $z = 0$, $x = 0$, $x = 4$, $y = 0$, and $y = 2$.

   $\boxed{\dfrac{160}{3}}$

8. Consider the rectangle R in the xy-plane which has vertices $(0, 1)$, $(0, 4)$, $(3, 1)$, and $(3, 4)$.

   (a) Use a double integral to compute the area of R.

   $$\boxed{A = \int_0^3 \int_1^4 1\, dy\, dx = \int_1^4 \int_0^3 1\, dx\, dy = 9}$$

   (b) Verify your answer from part (a) by using an appropriate formula from geometry.

   $\boxed{A = bh = 3 \cdot 3 = 9}$

9. By choosing a convenient order or integration, evaluate $\displaystyle\iint_R x \sec^2(xy) \sec^2 x\, dA$ where $R = \left\{(x, y) : \dfrac{\pi}{4} \le x \le \dfrac{\pi}{3}, 0 \le y \le 1\right\}$

   $\boxed{1;\ \text{Detailed Solution: Here}}$