13.6 #4

First we compute $\nabla g(0,1,-2)$.

$$g_x(x,y,z) = \frac{z}{x+y} \Rightarrow g_x(0,1,-2) = -2$$

$$g_y(x,y,z) = \frac{z}{x+y} \Rightarrow g_y(0,1,-2) = -2$$

$$g_z(x,y,z) = \ln(x+y) \Rightarrow g_z(0,1,-2) = \ln 1 = 0$$

So $\nabla g(0,1,-2) = \langle -2,-2,0 \rangle$

Next we find a unit vector $\vec{u}$ in the direction from P to Q.

$$\overrightarrow{PQ} = \langle 1,2,4 \rangle \Rightarrow \vec{u} = \frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \frac{\langle 1,2,4 \rangle}{\sqrt{1+4+16}} = \left\langle \frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}} \right\rangle$$

So $D_{\vec{u}}\, g(0,1,-2) = \nabla g(0,1,-2) \cdot \vec{u}$

$$= \frac{-2}{\sqrt{21}} - \frac{4}{\sqrt{21}} = \frac{-6}{\sqrt{21}}$$