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$$\lim_{n\to+\infty} (1+3^n)^{\frac{1}{n}}$$ ∞^0 is an indeterminate form

Let $y = (1+3^n)^{\frac{1}{n}} \Longrightarrow \ln y = \frac{1}{n}\ln(1+3^n)$

We want to compute $\lim\limits_{n\to+\infty} y$, but first we

compute $\lim\limits_{n\to+\infty} \ln y = \lim\limits_{n\to+\infty} \dfrac{\ln(1+3^n)}{n}$ $\begin{matrix}\longrightarrow +\infty \\ \longrightarrow +\infty\end{matrix}$

Use L'Hopital's Rule:

$$= \lim_{n\to+\infty} \frac{\frac{1}{1+3^n}\cdot 3^n \cdot \ln 3}{1} = \lim_{n\to+\infty} \frac{3^n \ln 3}{1+3^n}$$

Divide by 3^n: $\quad = \lim\limits_{n\to+\infty} \dfrac{\ln 3}{\cancel{\frac{1}{3^n}}^{\nearrow 0} + 1} = \ln 3$

So $\lim_{n \to +\infty} \ln y = \ln 3$

Thus $\lim_{n \to +\infty} y = \lim_{n \to +\infty} e^{\ln y} = e^{\ln 3} = 3.$

So the sequence converges to 3.