11.4 #7

(a) Volume $V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$

Now $\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 0 \\ -1 & 3 & -2 \end{vmatrix} = \vec{i} \begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & 0 \\ -1 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & 4 \\ -1 & 3 \end{vmatrix}$

$= -8\vec{i} + 6\vec{j} + 13\vec{k} = \langle -8, 6, 13 \rangle$

and $\vec{u} \cdot (\vec{v} \times \vec{w}) = \langle 1, 2, 3 \rangle \cdot \langle -8, 6, 13 \rangle = -8 + 12 + 39 = 43$

So $V = |43| = 43$.

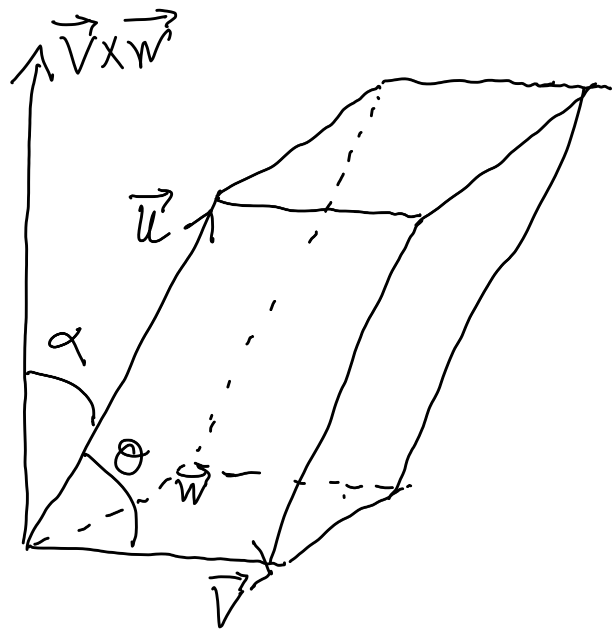
OR use scalar triple product $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 0 \\ -1 & 3 & -2 \end{vmatrix}$

to compute $\vec{u} \cdot (\vec{v} \times \vec{w})$.

(b) Area $A = \|\vec{v} \times \vec{w}\|$

$$= \|\langle -8, 6, 13 \rangle\| = \sqrt{64 + 36 + 169} = \sqrt{269}$$

(c) Consider the concept picture



We want θ.
So first calculate α, the angle between $\vec{u}$ and $\vec{v} \times \vec{w}$.

$$\cos \alpha = \frac{\vec{u} \cdot (\vec{v} \times \vec{w})}{\|\vec{u}\| \, \|\vec{v} \times \vec{w}\|} = \frac{43}{\sqrt{1+4+9}\,\sqrt{269}} = \frac{43}{\sqrt{14}\,\sqrt{269}}$$

$$\alpha = \cos^{-1}\left(\frac{43}{\sqrt{14}\,\sqrt{269}}\right) \implies \theta = \frac{\pi}{2} - \cos^{-1}\left(\frac{43}{\sqrt{14}\,\sqrt{269}}\right)$$