

## Chapter 4.3: Hyperbolic Functions

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### Expected Skills:

- Be able to define  $\sinh x$  and  $\cosh x$  in terms of exponential functions.
- Be able to determine the domain, range, and graph of  $\sinh x$  and  $\cosh x$ .
- Be able to justify properties and solve equations involving the hyperbolic functions.
- Be able to compute limits and derivatives involving the hyperbolic functions.

### Practice Problems:

1. Consider  $f(x) = \sinh x$ .
  - (a) Compute  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$
  - (b) Determine whether the graph of  $f(x)$  has any curvilinear asymptotes.
  - (c) Compute the  $x$  and  $y$  intercepts of  $f(x)$ .
  - (d) Solve  $\sinh x = 1$  for  $x$ .
  - (e) Show that  $\frac{d}{dx}(\sinh x) = \cosh x$
  - (f) Find all intervals on which  $f(x)$  is increasing and those on which  $f(x)$  is decreasing.
  - (g) Find all intervals on which  $f(x)$  is concave up and those on which  $f(x)$  is concave down.
  - (h) Determine the coordinates of all local extrema (max/min) and all inflection points.
  - (i) Sketch the graph of  $f(x)$
2. In order to verify the identity  $\sinh 2x = 2 \sinh x \cosh x$  compute the following by appealing to the appropriate definitions.
  - (a)  $\sinh 2x$
  - (b)  $2 \sinh x \cosh x$
3. We define the **Hyperbolic Tangent** function to be  $f(x) = \tanh x = \frac{\sinh x}{\cosh x}$ .
  - (a) Express  $f(x)$  in terms of exponential functions.
  - (b) What is the domain of  $f(x)$ ?
  - (c) Compute  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$ .

- (d) Determine whether the graph of  $f(x)$  has any curvilinear asymptotes.
  - (e) Find the coordinates of all  $x$  and  $y$  intercepts of  $f(x)$ .
  - (f) Find all  $x$  for which  $f(x) = \frac{1}{2}$ .
4. Compute an equation of the line which is tangent to  $f(x) = \cosh x$  at the point where  $x = \ln 2$ .
5. Suppose  $y = x \cosh x$ . Compute  $\frac{d^2y}{dx^2}$ .