Lecture 16: Solving and estimating directional dynamic games

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Roadmap: dynamic games

1. Games in family/life-cycle decision making
   John Rust
2. "Collusion on the beach" a model of leapfrogging investments
   John Rust, Fedor Iskhakov
3. Experiment with the model
   John Rust, Fedor Iskhakov, Bertel Schjerning
4. State recursion algorithm
5. Recursive lexicographical search (RLS) algorithm
6. Full solution for the leapfrogging game
   Fedor Iskhakov
7. Structural estimation of directional dynamic games
   with Nested RLS method

Games in family/life-cycle decision making

- Most empirical work on life-cycle model uses a *single agent* modeling approach — *the unitary model*
- For family decision making, the *collective model* is often used where the family is assumed to act as a single decision maker but it maximizes a weighted average of the husband's and wife's utility functions
- This effectively converts the solution of a multi-agent decision problem to a single agent decision problem
- Other approaches: model family decision making as a *non-cooperative game*
- Other work even models *individual decision making as a non-cooperative game* – as "game against your future selves" or as "games between your alter-egos"

The Collective Model vs Unitary Model

- Introduced by Chiapporri 1992 "in which agents are characterized by their own (possibly altruistic) preferences, and household decisions are only assumed to be Pareto efficient"
- Mazzocco (2007) "The theoretical and empirical literature on household intertemporal decisions has traditionally assumed that households behave as single agents. One of the main drawbacks of this approach is that the effect of intra-household commitment on intertemporal decisions cannot be analyzed and tested."
- The "Unitary Model" — "each household behaves as a single agent independently of the number of decision makers. This is equivalent to the assumption that the utility functions of the individual members can be collapsed into a unique utility function which fully describes the preferences of the entire household."

$$\max_{\{c_t, q_t, s_t\}} E\left\{\sum_{t=0}^{T} \beta^t u(c_t, q_t)\right\} \tag{1}$$

subject to

$$c_t + p_t q_t + s_t \leq y_{h,t} + y_{w,t} + R_t s_{t-1} \tag{2}$$

The Collective Model vs Unitary Model

$$\max_{\{c_{ht}, c_{wt}, q_t, s_t\}} \mu_h(z) E\left\{\sum_{t=0}^{T} u_h(c_{ht}, q_t)\right\} + \mu_w(Z) E\left\{\sum_{t=0}^{T} u_w(c_{wt}, q_t)\right\} \quad (3)$$

subject to

$$c_{ht} + c_{wt} + p_t q_t + s_t \leq y_{ht} + y_{wt} + R_t s_{t-1} \quad (4)$$

- $\mu_h(z)$ and $\mu_w(z)$ are positive "Pareto weights" on the welfare of the husband and wife, respectively, which may depend on time invariant variables z that determine the "type" of the family.
- Can recast the collective model as a unitary model using a *family utility function* $u_f(c, q)$ given by

$$u_f(c, q) = \max_{c_h, c_w} \mu_h(z) u_h(c_h, q) + \mu_w(z) u_w(c_w, q) \quad (5)$$

subject to

$$c_h + c_w \leq c \quad (6)$$

Non-Cooperative Models of Family Behavior

- Eckstein and Lifschitz (2015) *IER* "Household Interaction and the Labor Supply of Married Women"
- Compared three types of households
- *Classical Household* Husband is a Stackelberg leader and wife is a Stackelberg follower
- *Modern Household* Husband and wife are symmetric but make their labor supply decisions independently and simultaneously each period and their choices are a dynamic Nash equilibrium where each member of the couple maximize their own discounted utilties but taking into account the labor supply strategy of their spouse.
- *Cooperative Household* maximizes a collective utility function.
- Note: the Nash equilibria in the Modern Household could be Pareto-inefficient, so the Cooperative Household is able to coordinate and commit to Pareto-efficient decision rules

Non-Cooperative Models of Family Behavior

- Eckstein and Lifchitz estimated their model using PSID data. They treated the *type* of each household as unobserved heterogeneity.
- So they solved the model 3 times under each of the solution concepts for the family described above, and estimated the probability that a household was one of these three types.
- Specifically they used SMM and used logit probabilities that are a function of age and time-invariant household characteristics to determine the simulated proportions of each type of household in their simulations
- They found a model with 3 types can be indentified and results in the best fit to the data. "The estimation results indicate that 57% of the 1983-4 cohort of newlywed couples are of the Classical type, and the hypothesis that all households are Classical is rejected. The proportion of Modern households is 25% and that of Cooperative households is 18%."

Non-Cooperative Models of Family Behavior

- “We find that the labor supply of men is not affected by the type of game, whereas the employment rate for women is lower in Classical households than in Modern households by about 12 percentage points and is higher in Cooperative households than in Modern households by 4 percentage points.”
- “In other words, the social norms reflected in a Nash symmetric game and in the collective game lead to an increase in the labor supply of women in Modern and Cooperative households while leaving that of their husbands almost unchanged.”
- “The results support the hypothesis that some of the increase in married female labor supply observed in recent decades may be due to changes in social norms that affected the way couples decide on their joint labor supply. ”

Dealing with potential multiple equilibria

- In dynamic simultaneous move games there is a possibility of many equilibria. Example: entry games in IO: there is a "coordination problem" and one firm may not want to enter if it believes others are going to enter, but will enter if it believes others will not enter.
- In Eckstein and Lifschitz paper, they did not find multiple Nash equilibria for their Modern households who are playing discrete choice, simultaneous move dynamic games.
- But if multiple equilibria arise, one way to make them go away is model the game as one of *alternating moves* instead of *simultaneous moves*
- Example: Bowlus and Seitz (2006) *IER* "Domestic Violence, Employment and Divorce"
- "Men and women make decisions sequentially in the model. Women make marital status decisions taking into account expectations of abuse given their spouse's characteristics and past behavior, and men decide whether to abuse taking into account the likelihood their wives will divorce them."

Alternating move game with unique equilibrium

- The timing in the model is as follows. Women make decisions in every odd period and men make decisions in every even period.
- Individuals receive a constant level of utility for the period in which they make decisions and for the subsequent period in which their spouses make decisions. One full period for a couple therefore consists of one odd and one even period.
- All agents are single in the first period. All single women meet a potential spouse in every odd period. Women move first and decide whether to work (h) or not (n) and whether to be married (m) or single (s).
- Denote the choice set for women $I = \{sn, sh, mn, mh\}$. After observing their wife's employment choice, the husband decides whether to be abusive (a) or not (na) in the marriage. Denote the choice set for husbands as $J = \{a, na\}$.
- "The evidence presented on the importance of abuse in the divorce decision highlights the fact that many women observed in representative data respond to domestic violence by leaving the relationship."

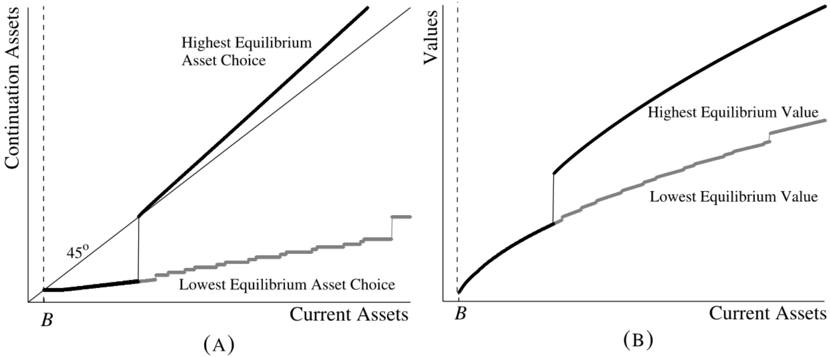
Multiple equilibria for Sophisticated hyperbolic discounters

- Bernheim, Ray and Sveltekin (2015) "Poverty and Self-Control"
- Individuals choose a consumption path to maximize

$$u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t) \tag{7}$$

- Note the *time inconsistency* in planning when $\beta \neq \delta$: the "time t self" puts less weight on the future utility when $\beta < 1$ but assumes that after their momentary "splurge" to consume more today at the expense of their future selves, they will return and behave as a time-consistent planner
- But tomorrow never comes, so at time $t+1$ the time $t+1$ does the same thing and splurges at the expense of their future selves.
- A "sophsticated" hyperbolic discounter realizes this time iconsistency and takes it into account. When they do, this becomes a game with an infinity of players: the "time 0" self, "time 1" self, ...

Multiple equilibria in the game against one's future selves



Unique equilibrium in "dual self" model of self-control

- Fudenberg and Levine (2006) *AER* "A Dual-Self Model of Self-Control"
- "In our model, the patient long-run self and a sequence of myopic short-run selves share the same preferences over stage-game outcomes; they differ only in how they regard the future. Specifically, we imagine that the short-run myopic self has 'baseline preferences' in the stage game that depends only on the outcome in the current stage. That is, the short-run players are completely myopic."
- "The stage game is played in two phases. In the first phase, the long-run self chooses a self-control action that influences the utility function of the myopic self. That is, at some reduction in utility (for both selves), the long-run self can choose preferences other than the baseline preferences."
- "In the second phase of the stage game, after the short-run player preferences have been chosen, the short-run player takes the final decision."

Unique equilibrium in "dual self" model of self-control

- Consumption example: short run self has utility $u(y,0,s) = \log((1-s)y)$ from saving at rate $s \in (0,1)$ when the long run self exercises no self-control cost. In such case the myopic short-run self would save nothing and obtain utility $u(y,0,0) = \log(y)$, but leave nothing for the long run self.
- So the long run self disciplines the short run self by imposing a "mental cost" that incentivizes the short run to choose a positive saving rate s. But doing this costs the long-run self $C(y,s) = \gamma \log(1-s)$.
- The long run self chooses a saving rate s to maximize lifetime utility from the consumptions chosen by the short run selves net of the costs of incentivizing them

$$\max_s \sum_{t=0}^{\infty} \delta^t \left[u(y,0,s) - C(y,s)\right] \tag{8}$$

- The solution is s^* given by $s^* = \delta/(1+\gamma(1-\delta))$.

Bertrand Price Competition with Leapfrogging Investments

- We extend the standard static textbook model of Bertrand price competition by allowing duopolists to undertake cost-reducing investments in discrete time
- Technological progress is exogenous and stochastic
- Each firm has a binary decision to acquire the state of the art production technology
- Even though this is a small extension of the classic static model of Bertrand price competition, surprisingly little is known about Bertrand competition in the presence of production cost uncetainty, especially in dynamic settings
- We show how to compute <u>all</u> equilibria of this game and show that this dynamic model of Bertrand price competition has surprisingly rich, complex, and counter-intuitive equilibrium outcomes.

How do you find *all* Markov Perfect Equilibria?

The Markov Perfect Equilibrium (MPE) concept of Maskin and Tirole (1988) is now a widely used in *empirical IO.* However computing MPE remains a daunting computation problem

Quote (Hörner *et. al. Econometrica* 2011)

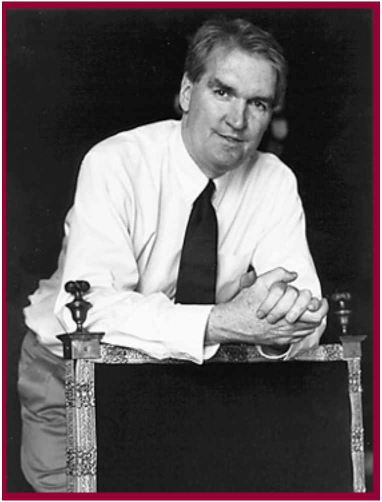
"Dynamic games are difficult to solve. In repeated games, finding some equilibrium is easy, as any repetition of a stage-game Nash equilibrium will do. This is not the case in stochastic games. The characterization of even the most elementary equilibria for such games, namely (stationary) Markov equilibria, in which continuation strategies depend on the current state only, turns out to be often challenging."

Finding even a single MPE is challenging!

- How do people "find" MPEs?
- Theorists: Guess and Verify
- Applied people: Iterate on the player Bellman equations
- Pakes and McGuire (1994): some of the earliest work on computing MPE. Proposed a deterministic, iterative algorithm to compute MPE. Found a curse of dimensionality in trying to solve MPE model of firm dynamics with even moderate numbers of firms
- Pakes and McGuire (2001): Proposed a *stochastic algorithm* to approximate an MPE, in an attempt to break this curse of dimensionality

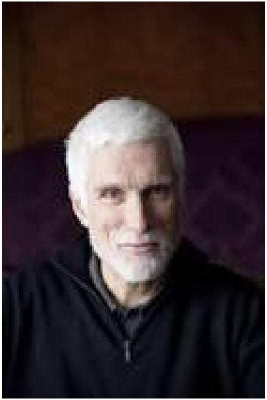
Eric Maskin: taught my game theory class at MIT

One of my classmates: Tim Kehoe



Another classmate: David Levine

Another classmate: Drew Fudenberg



Another classmate: Jean Tirole

Motivation: Collusion on the beach

Peter Brown: Amcor Managing Director

Harry Debney: Visy CEO

Russell Jones: Chairman of Amcor

Richard Pratt: Owner of Visy

The Australian cardboard market

- The Australian market for *cardboard* (CFP) is essentially a duopoly
- Between 2000 and 2005 the two firms, *Visy* and *Amcor* colluded to raise the price of CFP
- I was hired to estimate the damage caused by the collusion, which requires predicting what CFP prices would have been in the absence of collusion
- My opinion was that the "but-for" CFP prices are those predicted by Bertrand price competition in the short run, with *leapfrogging investments* by the two firms over the longer run as they vie for low-cost leadership

Amcor's New B9 Paper Mill

Main Mill Site, Botany Bay Road, Botany Bay NSW

Source: Amcor

B9 is an example of leapfrogging

- Amcor's existing paper plant was over 50 years old
- "The B9 paper machine, so named as it is the ninth paper machine to operate at the company's Botany site, will produce more than 400,000 tonnes of paper annually when operating at full capacity and will deliver significant environmental benefits."
- Cost: $500 million, the largest single investment in Amcor's 144 year history. "Largest and most innovative recycled paper machine of its kind in Australasia"
- "The machine is 330 metres long, and 22 metres high, and produces 1.6 km of paper per minute and reduces water consumption by 26%, energy usage by 34% and the amount of waste sent to landfill by 75%" (Nigel Garrard, Amcor CEO)

But collusion caused B9 to be abandoned

- Amcor had planned B9 back in 1999, and at that time internal studies estimated huge rate of return for this investment because it would enable it to leapfrog Visy to become the low-cost producer of CFP in Australia.
- Amcor and Visy were locked in a price war that started in 1999, around the time the Amcor Board authorized the B9 investment.
- However when Visy and Amcor started to collude in 2000, the B9 project was curiously scrapped. B9 was not actually started until 2011, well after the end of the collusion in 2005. B9 only came online in February 2013.

Justification for Bertrand pricing

- cardboard is a highly standardized product
- the consumers of cardboard are firms that are highly rational and interested in buying inputs at least possible cost
- further, firms acquire these inputs via *tenders* that create strong incentives for Bertrand-like price cutting
- In the case, we lacked good data on *aggregate demand* for cardboard facing Amcor and Visy before and after collusion
- but there was good data on their *costs of production*
- cardboard is made on production lines with machinery that is well-approximated as constant returns to scale with constant marginal costs

A cardboard corrugator

Technological progress via cost-reducing investments

- in this industry, Amcor and Visy do minimal amounts of R&D since there is limited scope for new product innovations to replace cardboard
- however the firms do spend considerable amounts on *cost reducing investments*
- these investments consist of building new plants or upgrading existing plants with the latest technology and machinery for producing cardboard
- rather than developing these machines themselves, Amcor and Visy purchase these machines from other companies that specialize in doing the R&D and product development to develop the machines that produce cardboard at the least possible cost

Leapfrogging by Amcor lead to a price war

- the proximate cause of the collusion between Amcor and Visy was a price war in cardboard
- a key input to cardboard is *paper* and Amcor had a severe cost disadvantage relative to Visy due to its outdated paper production plant, with machines that had not been replaced/upgraded in decades
- Visy on the other hand, has aggressively invested in the latest and most cost-efficient technology and maintained a persistent edge as the low cost leader
- however Amcor planned to invest in a new paper mill, B9, enabling it to produce CFP at substantially lower costs, thereby leapfrogging Visy to become the low cost leader in Australia

Are price wars evidence of tacit collusion?

- The economic experts defending Amcor and Visy dismissed theory of Bertrand competition and leap frogging investments as naive and out of touch with reality
- They claim that there is a huge body of research and empirical work in IO that supports a theory *tacit collusion* for repeatedly interacting duopolists
- In particular, they claimed that duopolists could achieve via tacit collusion the same discounted profits as they could via *explicit collusion.*
- This implies that the damage is *zero.*
- But if this is the case, and if tacit collusion is *legal,* why would Amcor and Visy have had any incentive to do illegal explicit collusion?

Paucity of empirical support for tacit collusion

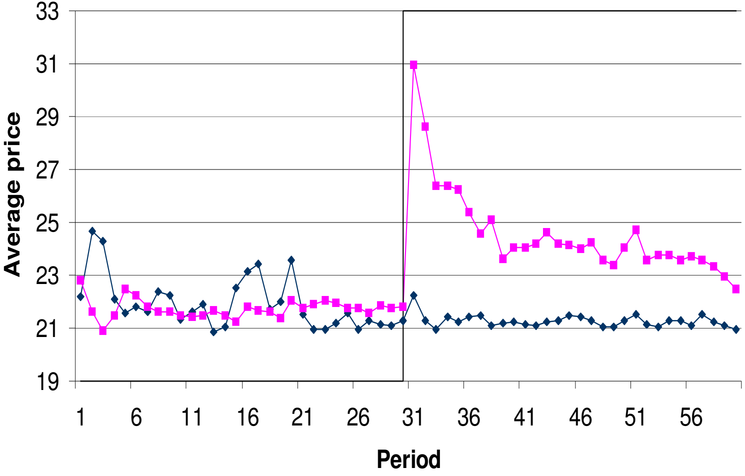
- Tacit collusion is hard to “observe” by the very fact that it is tacit
- We need good data on costs and demands to calculate what the cartel price would be
- Most of the empirical work on tacit collusion comes from laboratory experiments
- Hundreds of experiments done on tacit collusion have found that it is extremely difficult to “grow” tacit collusion in laboratory settings
- There are very few “field studies” that find evidence of tacit collusion outside of Breshnahan’s (1987) JIE paper, “Competition and Collusion in the American Automobile Industry: the 1955 Price War”

Conclusions of meta-study of over 500 experiments

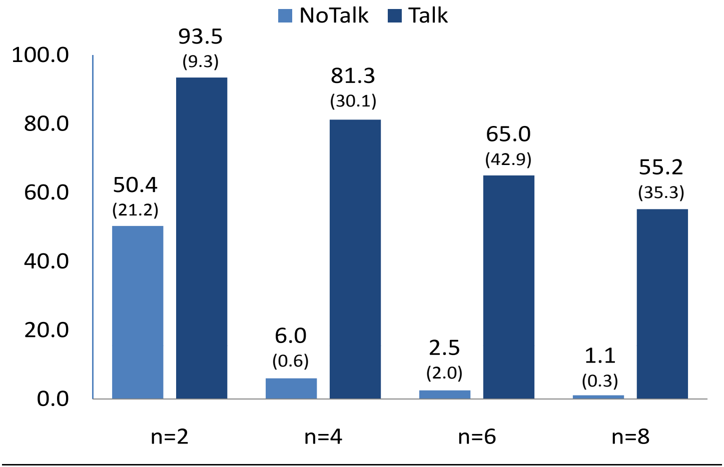
- Christoph Engel (2007) "Tacit Collusion The Neglected Experimental Evidence"
- Econometric meta-analysis of 510 laboratory experiments finds no systematic evidence supporting tacit collusion
- D. Engelmann and W. Müller (2008) "Collusion through price ceilings? A search for a focal point effect"
- "Note that the Folk Theorem (see for example Tirole, 1988) predicts that infinitely many prices can occur as outcomes of collusive equilibria in infinitely repeated games if the discount factor is sufficiently high. This suggests a coordination problem when firms attempt to collude." (p. 2)

Results of a laboratory duopoly

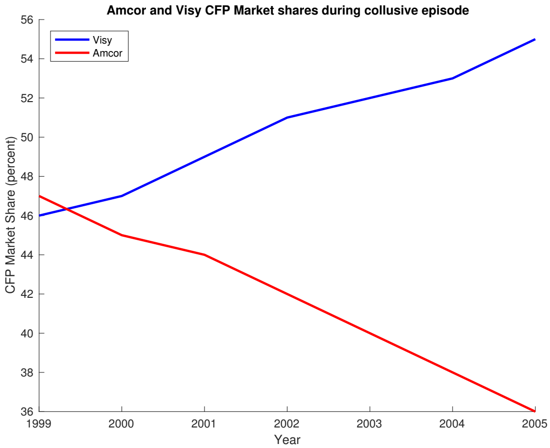
(Note: the Bertrand price is 21, the maximum cartel price is 48 and 28 is the price ceiling)



Explicit communication is necessary for collusion



Amcor and Visy collusive market shares



The Bertrand Investment Paradox

Why should Bertrand competitors undertake cost-reducing investments?

- Suppose a pair of duopolists simultaneously invest in the state of the art low cost production technology with marginal cost c
- Bertrand price competition following these investments will lead to a price of $p = c$ and *zero profits for each firm*
- If each firm earns zero profits *ex post*, why would either have incentive to invest *ex ante*?

The investment stage game is an anti-coordination game. Can the firms dynamically coordinate their investments in equilibrium, in order to avoid "bad" simultaneous investment outcomes?

The Riordan and Salant Conjecture

- In their 1994 *Journal of Industrial Economics* article, Riordan and Salant proved that in continuous time, if duopolists move alternately and technological progress is deterministic, then investment preemption is the only possible equilibrium outcome
- Further, they show this equilibrium is *completely inefficient* due to the excessively frequent investments of the preempting firm, a result they call rent dissipation
- They conjectured that their result does not depend on the alternating move assumption and that preemption (as opposed to leapfrogging) will be the generic equilibrium outcome in models of Bertrand price competition with cost-reducing investements.

Solution to the Bertrand Investment Paradox

We show:

- Endogenous coordination is possible in equilibrium
  - leapfrogging (alternating investments) is possible
  - We show that the Riordan and Salant conjecture is wrong: leapfrogging, not preemption, is the generic outcome
- Price paths are piecewise flat and non-increasing
  - *Price wars* occur when the high cost firm leapfrogs its rival to become the new low-cost leader
  - These price wars lead to *permanent* price declines, unlike the conventional interpretation of price wars as punishments for periodic breakdowns in tacit collusion
- Equilibria are generally inefficient due to overinvestment
  - duplicative investments
  - excessively frequent investments

Computing all equilibria

Our findings are based on the computation of all Markov perfect equilibria of this dynamic game

- New solution approach consisting of:
  1. State recursion algorithm for finding stage equilibria
  2. Recursive Lexicographic equilibrium Search (RLS) algorithm for finding all MPE paths
- Traditional solution approach (value function iterations, i.e. time recursion) fails in this model due to multiplicity of equilibria
  - Implementation of the Bellman operator induces an equilibrium selection rule
  - Not a contraction mapping, convergence is not guaranteed
- Danger of imposing symmetry
  - Most of MPE equilibria we find are asymmetric