

Who Gets Placed Where and Why?

An Empirical Framework for Foster Care Placement

Alejandro Robinson-Cortés

University of Exeter

Annual Conference in Dynamic Structural Econometrics: Market Design

MIT, August, 2022

Motivation

Foster care

System that provides **temporary care** for children removed from home by child-protective services

Motivation

Foster care

System that provides **temporary care** for children removed from home by child-protective services

In the U.S.

- Up to **5.91% (1 out of 17)** of children are placed in foster care
- On any given day, nearly **450,000** children are in foster care
- On average, children stay **19 months** in foster care (median = 14 months)
- Exit foster care: **reunification** (55%), **adoption** (35%), **emancipation** (10%)

Why market design in foster care?

- **Goal:** Study **how matching** between children and foster parents **is done**, and how to **improve** it

Why market design in foster care?

- **Goal:** Study **how matching** between children and foster parents **is done**, and how to **improve** it

Motivating Problem

Many foster **children go through several foster homes** before exiting foster care

- **Prevalent problem:** $56.1\% > 1$, $\text{avg} = 2.56$ (U.S., 2015)
- **Placement disruptions** are **detrimental** for children
- Social workers (say they) try to **minimize disruptions**
 - Do what is best for children, and minimize workload

This paper

1. How is it done?

- No explicit systematic matching algorithm → revealed preference exercise
- Formulate and estimate structural model of placement assignment in foster care
- How do social workers weigh duration and disruptions when assigning children to foster homes
- Model accounts for sample selection due to unobservable heterogeneity

2. How to improve it?

- Use model estimates to study new policies aimed at improving outcomes
- Keep estimated preferences fixed
- Improve placement outcomes by increasing market thickness through:
 - Temporal aggregation (delaying assignments)
 - Geographical centralization (centralizing regional offices)

This paper

1. How is it done?

- **No** explicit systematic matching algorithm → **revealed preference** exercise
- Formulate and estimate **structural model** of **placement assignment in foster care**
- How do social workers weigh **duration** and **disruptions** when assigning children to foster homes
- Model accounts for **sample selection** due to **unobservable heterogeneity**

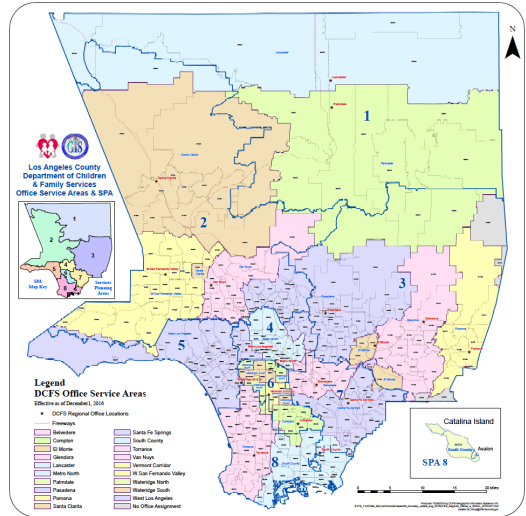
2. How to improve it?

- Use model estimates to study **new policies aimed at improving outcomes**
- Keep **estimated preferences fixed**
- Improve placement outcomes by **increasing market thickness** through:
 - **Temporal aggregation** (delaying assignments)
 - **Geographical centralization** (centralizing regional offices)

Los Angeles County, CA

Details

- Foster care administered at the **county level**
- County with **most foster children** in the U.S.
 - On any given day, **17,000** children in foster care
 - **40** children assigned to a foster home everyday
 - **19** regional offices (color-coded)
- **Data** Confidential administrative records from LA's child-protective services agency
- **Sample** Every placement assigned in Jan–Feb 2011 (2,087 children; 2,358 placements)
 - Observe outcomes until 2016



This paper

1. How is it done?

- **No** explicit systematic matching algorithm → **revealed preference** exercise
- Formulate and estimate **structural model** of **placement assignment in foster care**
- How do social workers weigh **duration** and **disruptions** when assigning children to foster homes
- Model accounts for **sample selection** due to **unobservable heterogeneity**

2. How to improve it?

- Use model estimates to study **new policies aimed at improving outcomes**
- Keep **estimated preferences fixed**
- Improve placement outcomes by **increasing market thickness** through:
 - **Temporal aggregation** (delaying assignments)
 - **Geographical centralization** (centralizing regional offices)

Main Findings

- Within regional offices, social workers do a **“fair job”** assigning children to foster homes
 - Placements **more likely to be disrupted are less likely to be assigned**
 - Social workers **minimize disruptions and the time children stay in foster care**
 - Why “fair”? There is **room for improvement**

Main Findings

- Within regional offices, social workers do a **“fair job”** assigning children to foster homes
 - Placements **more likely to be disrupted are less likely to be assigned**
 - Social workers **minimize disruptions and the time children stay in foster care**
 - Why “fair”? There is **room for improvement**
- \uparrow market thickness by **delaying assignments does not improve outcomes substantially**
- **Decentralization** into regional offices **is sub-optimal**: if system were centralized...
 - Avg. $\mathbb{P}(\text{disruption}) \downarrow 4.2 \text{ \%pts} \implies 8\% \downarrow$ placements per child before exiting foster care
 - **Why?** 54% less **distance between foster homes and schools**

Main Findings

- Within regional offices, social workers do a **“fair job”** assigning children to foster homes
 - Placements **more likely to be disrupted are less likely to be assigned**
 - Social workers **minimize disruptions and the time children stay in foster care**
 - Why “fair”? There is **room for improvement**
- \uparrow market thickness by **delaying assignments does not improve outcomes substantially**
- **Decentralization** into regional offices **is sub-optimal**: if system were centralized...
 - Avg. $\mathbb{P}(\text{disruption}) \downarrow 4.2 \text{ \%pts} \implies 8\% \downarrow$ placements per child before exiting foster care
 - **Why?** 54% less **distance between foster homes and schools**
- **Moral** *Social workers do a decent job at matching; exogenous institutions cause inefficiencies*
- **Policy Conclusion** *Improve coordination between regional offices, do not delay assignments*

Related Literature

- **Foster Care and Adoption**

- **Matching** Baccara, Collard-Wexler, Felli, and Yariv (2014); Slauch, Akan, Kesten, and Ünver (2015); MacDonald (2019); Olberg, Dierks, Seuken, Slauch, and Ünver (2021)
- **Foster care outcomes** Doyle Jr. and Peters (2007); Doyle Jr. (2007, 2008, 2013); Doyle Jr. and Aizer (2018); Bald, Doyle Jr., Gross, and Jacob (2022); Gross and Baron (2022); Bald, Chyn, Hastings, and Machelett (2022)

- **Empirical Market Design**

- **Medical Match** Agarwal (2015)
- **School Choice** Abdulkadiroğlu, Agarwal, and Pathak (2017); Agarwal and Somaini (2018);
- **Kidney Exchange** Agarwal, Ashlagi, Azevedo, Featherstone, and Karaduman (2017); Agarwal, Ashlagi, Rees, Somaini, and Waldinger (2019);

- **Empirical Decentralized Matching**

- **Marriage** Choo and Siow (2006); Galichon and Salanié (2015);

Outline

1. Data Description
2. Model
3. Estimation Results
4. Identification and Estimation
5. Counterfactual Policy Analysis

Description of markets and excess supply

- **Market** = **day** × **regional office** × **relatives**
- Foster homes are observed conditional on being matched
 - Excess supply is **not observed, but relatively small**
 - Children are left **unmatched** only if there are **no foster homes available**

Description of markets and excess supply

- **Market** = **day** × **regional office** × **relatives**
- Foster homes are observed conditional on being matched
 - Excess supply is **not observed, but relatively small**
 - Children are left **unmatched** only if there are **no foster homes available**
- Description of markets
 - **Sample period** = 58 days | **Regional offices** = 19 | **Office-days** = 1,102
 - Office-days with ≥ 1 **child without a relative** = 90.7%
 - At least one **unmatched child** in 88.1% of these office-days
 - 85% children are matched on same day they need a placement
 - Avg. **waiting time** (of those who wait) = 6.5 days
 - **Takeaway** Most children matched right away; unmatched children in almost all office-days

Summary Statistics

	n	mean	sd	median
<i>Termination Reasons</i>				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
<i>Duration</i>				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
<i>Placement Characteristics</i>				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Summary Statistics

	n	mean	sd	median
<i>Termination Reasons</i>				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
<i>Duration</i>				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
<i>Placement Characteristics</i>				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Summary Statistics

	n	mean	sd	median
<i>Termination Reasons</i>				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
<i>Duration</i>				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
<i>Placement Characteristics</i>				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Summary Statistics

	n	mean	sd	median
<i>Termination Reasons</i>				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
<i>Duration</i>				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
<i>Placement Characteristics</i>				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Summary Statistics

	n	mean	sd	median
<i>Termination Reasons</i>				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
<i>Duration</i>				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
<i>Placement Characteristics</i>				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Model

Model Overview

- **Unit of observation:** day within a regional office (“**market**”)
- **Empirical model:**

$$\underbrace{(M, \mathbf{T}, \mathbf{R})}_{\text{Endogenous}} \quad | \quad \underbrace{(C, H, \mathbf{X}, \mathbf{Y})}_{\text{Exogenous}}$$

- M = **matching** between children and foster homes
 - $\mathbf{T} = (T_{ch})_{(c,h) \in M}$ **duration** of placements
 - $\mathbf{R} = (R_{ch})_{(c,h) \in M}$ **termination reason** of placements
 - $R_{ch} \in \{ \text{disruption, permanency, emancipation} \}$
 - permanency \equiv reunification or adoption
- C = set of **children**
 - H = set of **foster homes**
 - \mathbf{X} = children **characteristics** (age, school zip-code)
 - \mathbf{Y} = foster homes **characteristics** (type, zip-code)

Model Overview

- **Unit of observation:** day within a regional office (“**market**”)
- **Empirical model:**

$$\underbrace{(M, \mathbf{T}, \mathbf{R})}_{\text{Endogenous}} \quad | \quad \underbrace{(C, H, \mathbf{X}, \mathbf{Y})}_{\text{Exogenous}}$$

- M = **matching** between children and foster homes
- $\mathbf{T} = (T_{ch})_{(c,h) \in M}$ **duration** of placements
- $\mathbf{R} = (R_{ch})_{(c,h) \in M}$ **termination reason** of placements
 - $R_{ch} \in \{ \text{disruption, permanency, emancipation} \}$
 - permanency \equiv reunification or adoption
- C = set of **children**
- H = set of **foster homes**
- \mathbf{X} = children **characteristics** (age, school zip-code)
- \mathbf{Y} = foster homes **characteristics** (type, zip-code)

Model Overview

- **Unit of observation:** day within a regional office (“**market**”)
- **Empirical model:**

$$\underbrace{(M, \mathbf{T}, \mathbf{R})}_{\text{Endogenous}} \quad | \quad \underbrace{(C, H, \mathbf{X}, \mathbf{Y})}_{\text{Exogenous}}$$

- M = **matching** between children and foster homes
 - $\mathbf{T} = (T_{ch})_{(c,h) \in M}$ **duration** of placements
 - $\mathbf{R} = (R_{ch})_{(c,h) \in M}$ **termination reason** of placements
 - $R_{ch} \in \{ \text{disruption, permanency, emancipation} \}$
 - permanency \equiv reunification or adoption
- C = set of **children**
 - H = set of **foster homes**
 - \mathbf{X} = children **characteristics** (age, school zip-code)
 - \mathbf{Y} = foster homes **characteristics** (type, zip-code)

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) V(c, h) \right\}$$

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) \boxed{V(c, h)} \right\}$$

- Payoff of placing child c in home h

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E}[u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E} [u(T, R; T_{em,c}) | \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
- $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
- μ_R = utility of termination reason R
- φ_R = mg. utility of duration conditional on termination reason R
- $T_{em,c}$ = time until emancipation (18 – child's age)

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E}[u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
- $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
- $\omega_{ch} = (\omega_d, \omega_{ex}) \sim N(0, \Sigma_\omega)$ placement's unobservable characteristics
- $(\mathbf{x}_c, \mathbf{y}_h)$ = child- and home-observable characteristics

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h
 - Outcome-based payoff: $\pi(c, h) = \mathbb{E} [u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
 - $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
 - $\omega_{ch} = (\omega_d, \omega_{ex}) \sim N(0, \Sigma_\omega)$ placement's unobservable characteristics
 - Child-specific taste variation: $\varepsilon_{cy} \sim N(0, \Sigma_\varepsilon)$
 - Home-specific taste variation: $\eta_{xh} \sim N(0, \Sigma_\eta)$

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E} [u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
- $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
- $\omega_{ch} = (\omega_d, \omega_{ex}) \sim N(0, \Sigma_\omega)$ placement's unobservable characteristics
- Child-specific taste variation: $\varepsilon_{cy} \sim N(0, \Sigma_\varepsilon)$
- Home-specific taste variation: $\eta_{xh} \sim N(0, \Sigma_\eta)$

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E} [u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
- $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
- $\omega_{ch} = (\omega_d, \omega_{ex}) \sim N(0, \Sigma_\omega)$ placement's unobservable characteristics
- Child-specific taste variation: $\varepsilon_{cy} \sim N(0, \Sigma_\varepsilon)$
- Home-specific taste variation: $\eta_{xh} \sim N(0, \Sigma_\eta)$

- **Placement Outcome:** $(T, R) \mid (\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) \sim \text{Burr Competing Risks Duration Model}$

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E}[u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
- $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
- $\omega_{ch} = (\omega_d, \omega_{ex}) \sim N(0, \Sigma_\omega)$ placement's unobservable characteristics
- Child-specific taste variation: $\varepsilon_{cy} \sim N(0, \Sigma_\varepsilon)$
- Home-specific taste variation: $\eta_{xh} \sim N(0, \Sigma_\eta)$

- **Placement Outcome:** $(T, R) \mid (\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) \sim$ Burr Competing Risks Duration Model
- **Placement Data:** $(T, R) \mid (M(c, h) = 1, \mathbf{x}_c, \mathbf{y}_h) \sim$ Mixed Outcome Distribution

1. Social Workers' Matching Problem

- **Matching** between children and foster homes assigned according to:

$$M = \arg \max_{\tilde{M} \in \mathbb{M}(C, H)} \left\{ \sum_{c \in C, h \in H} \tilde{M}(c, h) [\pi(c, h) + \varepsilon_{cy_h} + \eta_{x_ch}] \right\}$$

- Payoff of placing child c in home h

- Outcome-based payoff: $\pi(c, h) = \mathbb{E} [u(T, R; T_{em,c}) \mid \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h]$
- $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
- $\omega_{ch} = (\omega_d, \omega_{ex}) \sim N(0, \Sigma_\omega)$ placement's unobservable characteristics
- Child-specific taste variation: $\varepsilon_{cy} \sim N(0, \Sigma_\varepsilon)$
- Home-specific taste variation: $\eta_{xh} \sim N(0, \Sigma_\eta)$

- **Placement Outcome:** $(T, R) \mid (\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) \sim$ Burr Competing Risks Duration Model
- **Placement Data:** $(T, R) \mid (M(c, h) = 1, \mathbf{x}_c, \mathbf{y}_h) \sim$ Mixed Outcome Distribution
- **Matching Data:** $M \mid (C, H, \mathbf{X}, \mathbf{Y}) \sim$ Mixed Probit

2. Competing Risks Duration Model of Placement Outcomes

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min \{ T_R : R \in \mathcal{R} \} \quad \& \quad R = \arg \min \{ T_R : R \in \mathcal{R} \}.$$

2. Competing Risks Duration Model of Placement Outcomes

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min \{T_R : R \in \mathcal{R}\} \quad \& \quad R = \arg \min \{T_R : R \in \mathcal{R}\}.$$

Burr Hazard Rates

For $R \in \{d, ex\}$, conditional on $(\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h)$, T_R follows a **Burr distribution** with hazard rate:

$$\lambda_R(T | \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) = \frac{k_R(\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) \alpha_R T^{\alpha_R - 1}}{1 + \gamma_R^2 k_R(\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) T^{\alpha_R}}$$

where $\alpha_R > 0$, $\gamma_R \geq 0$, and $k_R(\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h) = \exp(\omega_{R,ch} + g(\mathbf{x}_c, \mathbf{y}_h)\beta_R)$.

Note 1: α_R and γ_R determine the shape (duration-dependence) of the hazard rate

Note 2: $\lambda_R(T | \omega_{ch}, \mathbf{x}_c, \mathbf{y}_h)$ is increasing in $k_R(\omega_{ch}, \mathbf{x}_c, \mathbf{y}_h)$

Note 3: Unobservable characteristics (ω_d, ω_{ex}) are **frailty terms**

Identification and Estimation

Identification and Estimation

- **Identification** Details

- **Exogenous variation** in $(C, H, \mathbf{X}, \mathbf{Y})$ across markets identifies distribution of ω (Akerberg and Botticini 2002; Sørensen 2007).
 - Intuition akin to traditional **sample selection** models (Heckman 1979)

- **Estimation: Simulated Maximum Likelihood** Details

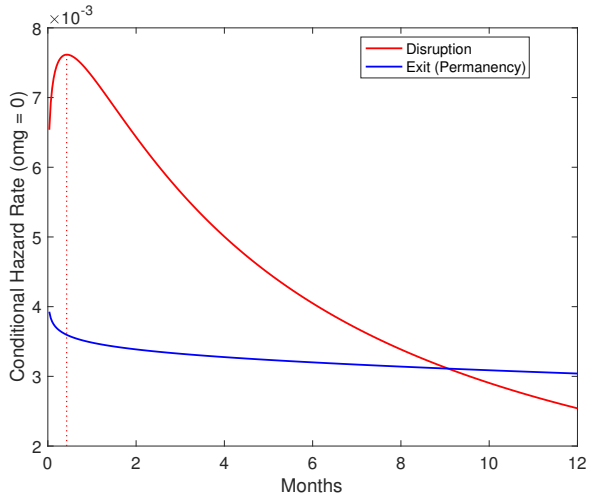
- Let $\mathbf{Z}_i \equiv (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i)$. Integrate joint conditional likelihood:

$$\begin{aligned} (M_i, \mathbf{T}_i, \mathbf{R}_i) | \mathbf{Z}_i &\sim \int (M_i, \mathbf{T}_i, \mathbf{R}_i) | (\mathbf{Z}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i) \\ &\sim \int (M_i | \mathbf{Z}_i, \boldsymbol{\Omega}_i) (\mathbf{T}_i, \mathbf{R}_i | M_i, \mathbf{Z}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i), \end{aligned}$$

where $\boldsymbol{\Omega}_i = (\omega_{ch})_{(c,h) \in C_i \times H_i} \sim G \equiv \times_{c,h} N(0, \boldsymbol{\Sigma}_\omega)$.

Estimation Results

Estimated Hazard Rates

[Back](#)[Parameter Estimates](#)[Model Fit](#)

Matching Utility Estimates

Matching Utility—Parameter Estimates			
	Disruption	Permanency	Emancipation
μ_R — <i>MgU. Term. Reason</i>	-2.908*** (0.6972)	2.449** (1.091)	-2.057*** (0.7183)
φ_R — <i>MgU. Duration</i>	-0.3549*** (0.1005)	-0.5265*** (0.167)	0† (0)
$\bar{\varphi}_R$ — <i>MgU. Emanc. Time</i>	0.3093*** (0.06172)	-0.1179 (0.09607)	0.009985 (0.01364)
Number of markets (n)	1467		
<i>SMLL</i>	-17005.86		

Note: $u = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$. Standard errors in parenthesis. Significance level of parameters: *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.1$. † indicates fixed parameter (not estimated).

Matching Utility Estimates

Matching Utility—Parameter Estimates			
	Disruption	Permanency	Emancipation
μ_R — <i>MgU. Term. Reason</i>	-2.908*** (0.6972)	2.449** (1.091)	-2.057*** (0.7183)
φ_R — <i>MgU. Duration</i>	-0.3549*** (0.1005)	-0.5265*** (0.167)	0† (0)
$\bar{\varphi}_R$ — <i>MgU. Emanc. Time</i>	0.3093*** (0.06172)	-0.1179 (0.09607)	0.009985 (0.01364)
Number of markets (n)	1467		
<i>SMLL</i>	-17005.86		

Note: $u = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$. Standard errors in parenthesis. Significance level of parameters: *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.01$. † indicates fixed parameter (not estimated).

- Placements more likely to be disrupted are less likely to be assigned

Matching Utility Estimates

Matching Utility—Parameter Estimates			
	Disruption	Permanency	Emancipation
μ_R — <i>MgU. Term. Reason</i>	-2.908*** (0.6972)	2.449** (1.091)	-2.057*** (0.7183)
φ_R — <i>MgU. Duration</i>	-0.3549*** (0.1005)	-0.5265*** (0.167)	0† (0)
$\bar{\varphi}_R$ — <i>MgU. Emanc. Time</i>	0.3093*** (0.06172)	-0.1179 (0.09607)	0.009985 (0.01364)
Number of markets (n)	1467		
<i>SMLL</i>	-17005.86		

Note: $u = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$. Standard errors in parenthesis. Significance level of parameters: *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.01$. † indicates fixed parameter (not estimated).

- Placements more likely to be disrupted are less likely to be assigned
- Social workers minimize the time children stay in foster care

Matching Utility Estimates

Matching Utility—Parameter Estimates			
	Disruption	Permanency	Emancipation
μ_R — <i>MgU. Term. Reason</i>	-2.908*** (0.6972)	2.449** (1.091)	-2.057*** (0.7183)
φ_R — <i>MgU. Duration</i>	-0.3549*** (0.1005)	-0.5265*** (0.167)	0† (0)
$\bar{\varphi}_R$ — <i>MgU. Emanc. Time</i>	0.3093*** (0.06172)	-0.1179 (0.09607)	0.009985 (0.01364)
Number of markets (n)	1467		
<i>SMLL</i>	-17005.86		

Note: $u = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$. Standard errors in parenthesis. Significance level of parameters: *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.01$. † indicates fixed parameter (not estimated).

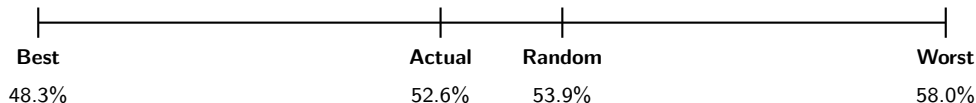
- Placements more likely to be disrupted are less likely to be assigned
- Social workers minimize the time children stay in foster care
- Social workers reveal preferences over children's age conditional on termination reason

How good are social workers at minimizing disruptions?

- Simulate assignments under alternative matching policies (change parameters in utility function)

How good are social workers at minimizing disruptions?

- Simulate assignments under alternative matching policies (change parameters in utility function)
- Average predicted disruption probability across assigned placements:



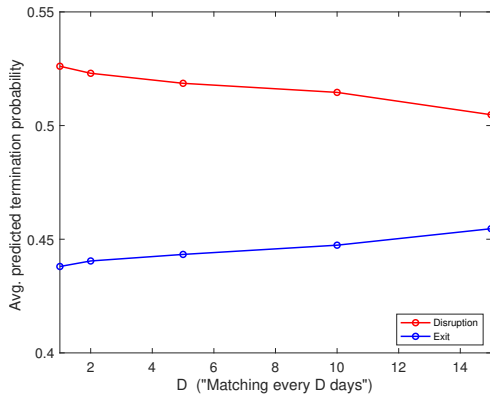
- Social workers
 - could do worse (up to a 10.3% increase)
 - do better than random (2.5% increase)
 - but could also do better (up to 8.2% decrease)

Counterfactual Policy Analysis

Counterfactual Policy Analysis

- Increasing **market thickness** by aggregating markets
 - **Centralization** Pool regional offices together into a single county-wide market
 - **Temporal aggregation** Assign placements within regional offices every $D \geq 1$ days
 - **Benchmark** Pool regional offices together and match everyone at once ($D = \infty$)
- Assume zero costs of information aggregation
 - Obtain **upper bound of gains** from greater market thickness

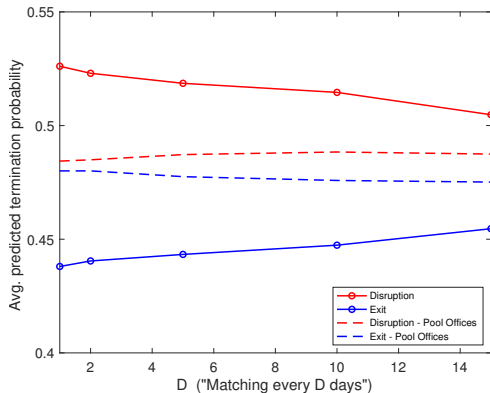
Temporal and Spatial Aggregation



Notes:

- y-axis = avg. termination probability
- x-axis = temporal aggregation

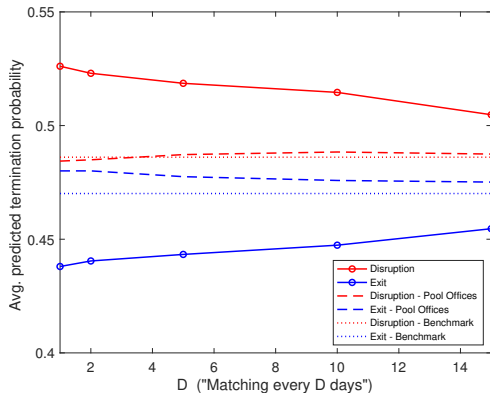
Temporal and Spatial Aggregation



Notes:

- y-axis = avg. termination probability
- x-axis = temporal aggregation
- dashed lines = spatial aggregation

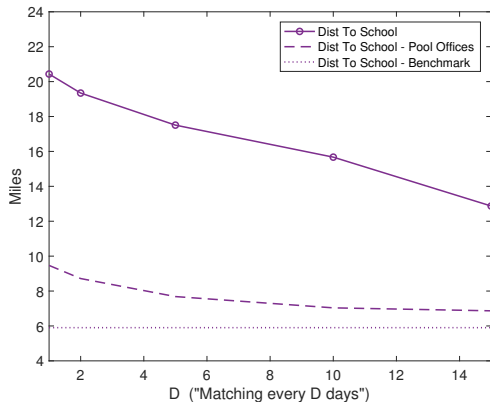
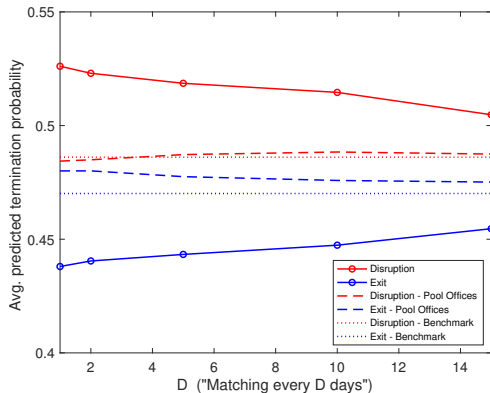
Temporal and Spatial Aggregation



Notes:

- y-axis = avg. termination probability
- x-axis = temporal aggregation
- dashed lines = spatial aggregation
- dotted lines = maximum market thickness

Temporal and Spatial Aggregation



Notes:

- y-axis = avg. termination probability (left), avg. distance to school (right)
- x-axis = temporal aggregation
- dashed lines = spatial aggregation
- dotted lines = maximum market thickness

Conclusion

- **Objective** Formulate and estimate **structural model** of **foster care placement**
- Social workers do a **“fair job”** at minimizing disruptions
 - $\uparrow \mathbb{P}(\text{disruption}) \implies \downarrow \mathbb{P}(\text{placement})$
 - Better than random, but there is room for improvement
- **However,...**
 - Regional offices **coordinate sub-optimally** with one another.
 - There are **gains from centralizing the assignment of placements** across LA County
 - $\mathbb{P}(\text{disruption}) \downarrow 4.2 \text{ \%pts} \implies 8\% \downarrow$ fewer foster homes per child
 - 54% less distance between foster homes and schools
- **What do we learn?**
 - Social workers do a **fair job** at matching, but **exogenous institutions cause inefficiencies**
 - **Policy recommendation** Improve coordination between regional offices, do not delay assignments

Motivation (Sources)

[Back](#)

In the U.S.

- **5.91% (1 out of 17)** of children are placed in foster care
 - Estimated share of children from total population who spent at least a day in foster care before their 18th birthday, 2000–2011 (Wildeman and Emanuel 2014)
- Every year, more than **half a million children** go through foster care
 - 2013 (638,041) through 2017 (690,548)
 - Source: U.S. Department of Health and Human Services (AFCARS Report, 2018)
- On any given day, nearly **450,000** children are in foster care
 - 10/30/2013 (400,39) through 10/30/2017 (442,995) (AFCARS Report, 2018)
- On average, children stay **19 months** in foster care (median = 14 months)
 - Average and median length of stay across children who exited during FY 2017 (AFCARS Report, 2018)
- Exit foster care: **reunification** (55%), **adoption** (35%), **emancipation** (10%)
 - Discharge reasons across children who exited during FY 2017 (AFCARS Report, 2018)

Why market design in foster care? (Sources)

[Back](#)

- **Prevalent problem:** $56.1\% > 1$, $\text{avg} = 2.56$ (U.S., 2015) (Source: AFCARS)
- **Placement disruptions** detrimental for children's development
 - ↑ emergency and mental-health services (Rubin et al. 2004; Rubin, Alessandrini, Feudtner, Localio, and Hadley 2004)
 - ↑ behavioral and attachment problems (Gauthier, Fortin, and Jéliu 2004; Rubin, O'Reilly, Luan, and Localio 2007)
 - affect children's bodily capacity to regulate cortisol (stress hormone) (Fisher, Ryzin, and Gunnar 2011)
- Also, associated with **worse outcomes in adult life:**
 - More and longer placements \Rightarrow ↑ depression, smoking, drug use, criminal convictions (Dregan and Gulliford 2012)

Why structural model?

Back

- **Main Challenge**

- **Objective:** Recover **preferences over outcomes** from data on which **matchings were chosen**
- Placement outcomes (duration and disruptions) are **lotteries**
- ⇒ Need to estimate **conditional distribution of outcomes**

- **Problem** Possible selection on **unobservables**

- Unobservables → Expected match outcomes → Matching → Observed outcomes are selected
- **Endogeneity** when estimating distribution of outcomes conditional on observables

- **Solution**

- **Structural model** of **matching** and **placement outcomes**, with **unobserved heterogeneity**
- **Identification** Exogenous variation across dates and regions at which children enter foster care

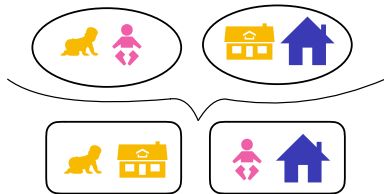
Market Thickness

[Back](#)

Office-day 1

Children

Foster homes



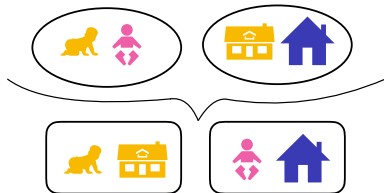
Market Thickness

[Back](#)

Office-day 1

Children

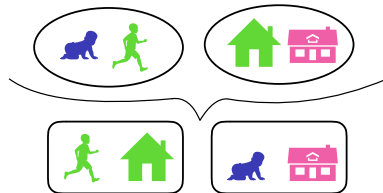
Foster homes



Office-day 2

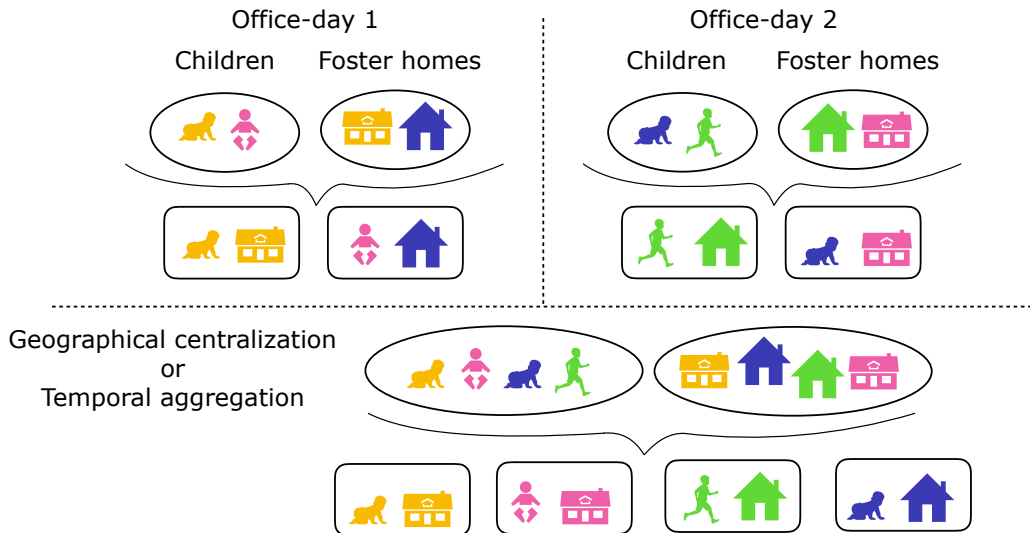
Children

Foster homes



Market Thickness

[Back](#)



Background and Data

[Back](#)[Markets and Excess Supply](#)[Summary Statistics](#)

- **Data** Confidential county records (accessed through court order) from the Los Angeles County Department of Children and Family Services (DCFS)
- **Dataset** Records of all children who went through foster care between 2006 and 2016 (FY)
 - 112,755 **children** | 129,084 **foster care episodes** | 266,887 **placements**
 - Avg. episodes per child = 1.14
 - Avg. placements per episode = 2.09
 - Avg. episode duration = 14.02 months (median = 10.32 months)
 - Avg. placement duration = 7.39 months (median = 3.67 months)
- **Sample** Every placement assigned between January 1, 2011, and February 28, 2011
 - 2,087 **children** | 2,358 **placements**
 - **Children characteristics** Age, school zip-code
 - **Foster homes characteristics** Type (county, agency, group-home, relative), zip-code

Description of markets and excess supply

Back

- **Market** = **day** × **regional office** × **relatives**
- Foster homes are observed conditional on being matched
 - Excess supply is **not observed, but relatively small**
 - Children are left **unmatched** only if there are **no foster homes available**
- Description of markets
 - **Sample period** = 58 days | **Regional offices** = 19 days | **Office-days** = 1102
 - Office-days with ≥ 1 **child without a relative** = 90.7%
 - At least one **unmatched child** in 88.1% of these office-days
 - 85% children are matched on same day they need a placement
 - Avg. **waiting time** (of those who wait) = 6.5 days
 - **Takeaway** Most children matched right away, but unmatched children in almost all office-days

Summary Statistics

[Back](#)[Full Dataset](#)

	n	mean	sd	median
<i>Termination Reasons</i>				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
<i>Duration</i>				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
<i>Placement Characteristics</i>				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Summary Statistics (sample and full dataset)

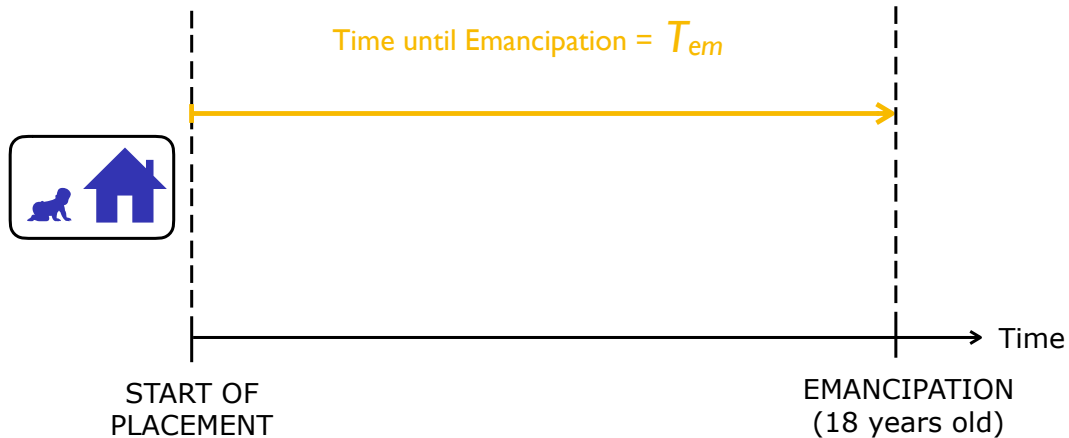
[Back](#)

	mean	sd	mean-full	sd-full
<i>Termination Reasons</i>				
Disruption	0.51	0.5	0.49	0.5
Permanency	0.42	0.49	0.37	0.48
Reunification	0.31	0.46	0.26	0.44
Adoption	0.12	0.32	0.11	0.31
Emancipation	0.052	0.2	0.048	0.21
Censored	0.015	0.12	0.090	0.27
<i>Duration</i>				
Duration (months)	8.37	11.28	8.12	10.66
Duration—Disrup	5.4	7.96	4.86	7.38
Duration—Perm	9.97	9.99	10.4	9.90
Duration—Emanc	12.94	14.3	13.23	15.93
Duration—Cens	47.89	27.88	13.99	17.28
<i>Placement Characteristics</i>				
Child's Age	8.69	5.97	8.55	5.91
County Foster Home	0.086	0.27	0.09	0.29
Agency Foster Home	0.43	0.5	0.36	0.48
Group Home	0.12	0.32	0.11	0.32
Relative Home	0.37	0.48	0.43	0.5
Distance Plac-School (mi.)	18.13	23.77	15.75	23.31
No School	0.25	0.43	0.33	0.47

Note: Distance measures at zip-code level, computed using Google Maps API.

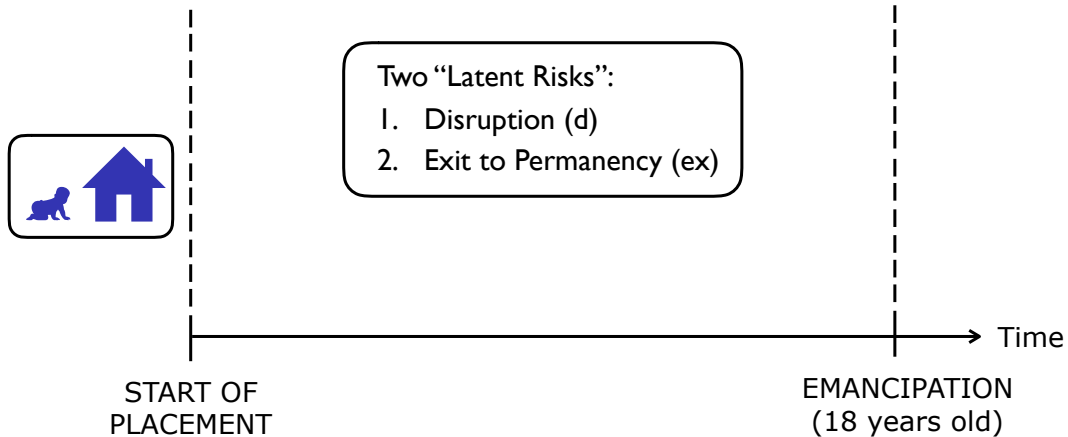
2. Competing Risks Duration Model of Placement Outcomes

[Back](#)



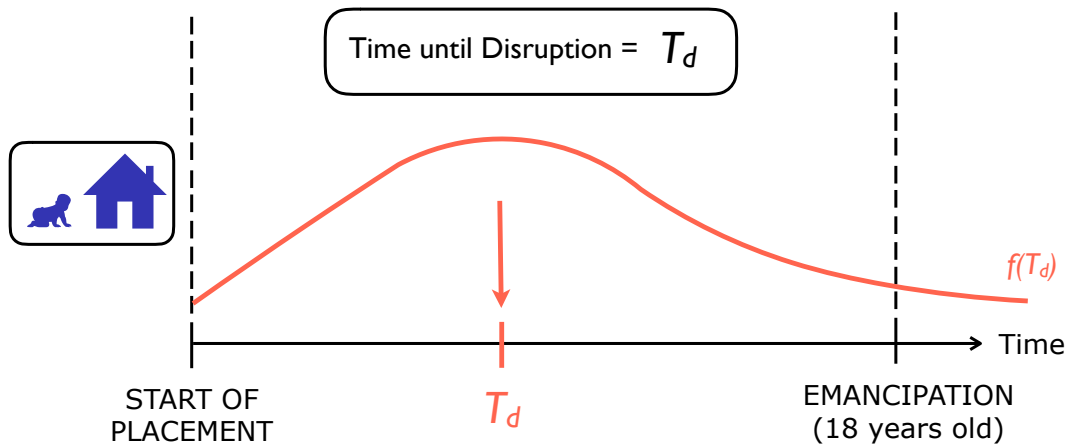
2. Competing Risks Duration Model of Placement Outcomes

[Back](#)



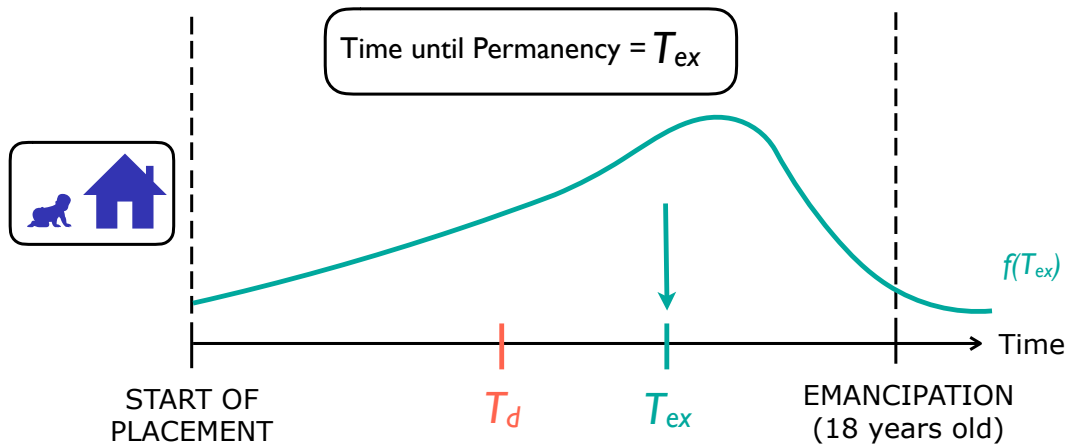
2. Competing Risks Duration Model of Placement Outcomes

[Back](#)



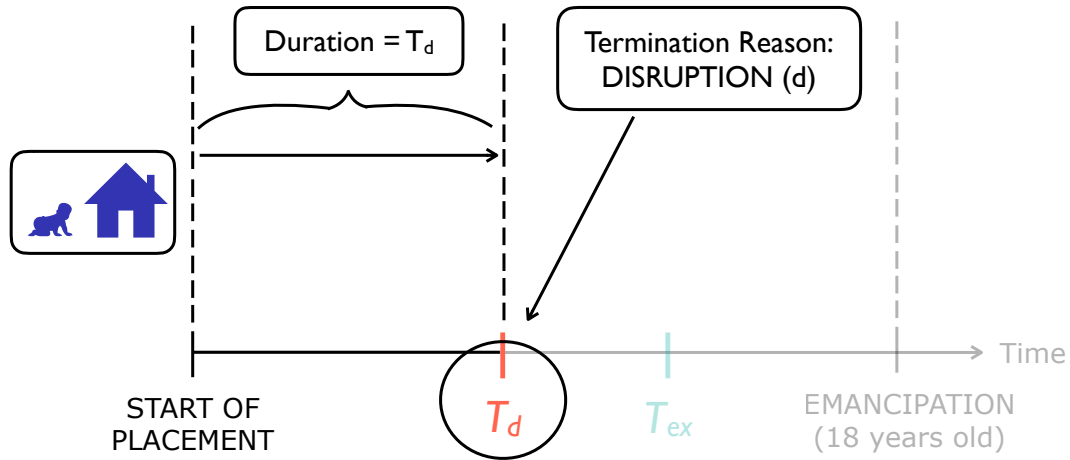
2. Competing Risks Duration Model of Placement Outcomes

[Back](#)



2. Competing Risks Duration Model of Placement Outcomes

Back



2. Competing Risks Duration Model of Placement Outcomes

[Back](#)

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min \{ T_R : R \in \mathcal{R} \} \quad \& \quad R = \arg \min \{ T_R : R \in \mathcal{R} \}.$$

- Need to specify the **conditional outcome distribution**: $(T, R) \mid \mathcal{I}_{ch}$
 - \mathcal{I}_{ch} = central planner's information about (prospective) placement (c, h)

2. Competing Risks Duration Model of Placement Outcomes

[Back](#)

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min \{T_R : R \in \mathcal{R}\} \quad \& \quad R = \arg \min \{T_R : R \in \mathcal{R}\}.$$

Assumption: Normal Mixing Distribution

The **central planner's information** of a placement is $\mathcal{I}_{ch} = (\mathbf{x}_c, \mathbf{y}_h, \boldsymbol{\omega}_{ch})$ where:

$\boldsymbol{\omega}_{ch} = (\omega_d, \omega_{ex})$ are unobservable **frailty terms** (or random effects)

$$\boldsymbol{\omega}_{ch} \sim N(0, \boldsymbol{\Sigma}_{\omega})$$

Note: “Frailty term” means that ω_R shifts the hazard rate of T_R

2. Competing Risks Duration Model of Placement Outcomes

[Back](#)

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min \{T_R : R \in \mathcal{R}\} \quad \& \quad R = \arg \min \{T_R : R \in \mathcal{R}\}.$$

Assumption: Burr Hazard Rates

- 3a. For $R \in \{d, ex\}$, conditional on \mathcal{I}_{ch} , T_R follows a **Burr distribution** with hazard rate:

$$\lambda_R(T|\mathcal{I}_{ch}) = \frac{k_R(\mathcal{I}_{ch})\alpha_R T^{\alpha_R-1}}{1 + \gamma_R^2 k_R(\mathcal{I}_{ch}) T^{\alpha_R}}$$

where $\alpha_R > 0$, $\gamma_R \geq 0$, and $k_R(\mathcal{I}_{ch}) = \exp(\omega_{R,ch} + g(\mathbf{x}_c, \mathbf{y}_h)\beta_R)$.

Note 1: α_R and γ_R determine the shape (duration-dependence) of the hazard rate $\lambda_R(T|\mathcal{I}_{ch})$

Note 2: $\lambda_R(T|\mathcal{I}_{ch})$ is increasing in $k_R(\mathcal{I}_{ch})$

- 3b. Latent durations are independent conditional on \mathcal{I}_{ch} , $\omega_{ch} \perp \varepsilon_c$, and $\omega_{ch} \perp \eta_h$.

Identification and Estimation

[Back](#)

- **Identification** [Details](#)

- **Exogenous variation** in $(C, Y, \mathbf{X}, \mathbf{Y})$ across markets identifies distribution of ω (Akerberg and Botticini 2002; Sørensen 2007).
 - Intuition akin to traditional **sample selection** models (Heckman 1979)

- **Estimation: Simulated Maximum Likelihood** [Details](#)

- Let $\mathbf{Z}_i \equiv (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i)$. Integrate joint conditional likelihood:

$$\begin{aligned}(M_i, \mathbf{T}_i, \mathbf{R}_i) | \mathbf{Z}_i &\sim \int (M_i, \mathbf{T}_i, \mathbf{R}_i) | (\mathbf{Z}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i) \\ &\sim \int (M_i | \mathbf{Z}_i, \boldsymbol{\Omega}_i) (\mathbf{T}_i, \mathbf{R}_i | M_i, \mathbf{Z}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i),\end{aligned}$$

where $\boldsymbol{\Omega}_i = (\omega_{ch})_{(c,h) \in C_i \times H_i} \sim G \equiv \times_{c,h} N(0, \boldsymbol{\Sigma}_\omega)$.

Average Partial Effects on Expected Outcomes

[Back](#)

Average Partial Effects					
	$\mathbb{P}(\text{Disrup})$	$\mathbb{P}(\text{Permanency})$	$\mathbb{E}(\log T \mid \text{Disrup})$	$\mathbb{E}(\log T \mid \text{Exit})$	$\mathbb{E}(\log T)$
<i>Age At Plac.</i>	0.0139	-0.0115	-0.0406	-0.022	-0.0401
<i>County-FH</i>	0.317	-0.266	-0.969	-0.628	-0.927
<i>Agency-FH</i>	0.320	-0.272	-1.221	-0.874	-1.174
<i>Group Home</i>	0.165	-0.158	0.287	0.450	0.339
<i>Distance To School (zip)</i>	0.00401	-0.00376	-0.007978	-0.00309	-0.00736
<i>No School</i>	0.1136	-0.09686	-0.5244	-0.3653	-0.5212
Number of placements	2358				

Note: Average partial effects of placement characteristics on expected outcomes. Averages taken across the sample of assigned placements in the data. The partial effects with respect to continuous variables is taken by considering a marginal change of one unit.

Average Partial Effects on Expected Outcomes

[Back](#)

Average Partial Effects					
	$\mathbb{P}(\text{Disrup})$	$\mathbb{P}(\text{Permanency})$	$\mathbb{E}(\log T \mid \text{Disrup})$	$\mathbb{E}(\log T \mid \text{Exit})$	$\mathbb{E}(\log T)$
<i>Age At Plac.</i>	0.0139	-0.0115	-0.0406	-0.022	-0.0401
<i>County-FH</i>	0.317	-0.266	-0.969	-0.628	-0.927
<i>Agency-FH</i>	0.320	-0.272	-1.221	-0.874	-1.174
<i>Group Home</i>	0.165	-0.158	0.287	0.450	0.339
<i>Distance To School (zip)</i>	0.00401	-0.00376	-0.007978	-0.00309	-0.00736
<i>No School</i>	0.1136	-0.09686	-0.5244	-0.3653	-0.5212
Number of placements	2358				

Note: Average partial effects of placement characteristics on expected outcomes. Averages taken across the sample of assigned placements in the data. The partial effects with respect to continuous variables is taken by considering a marginal change of one unit.

Average Partial Effects on Expected Outcomes

[Back](#)

Average Partial Effects					
	$\mathbb{P}(\text{Disrup})$	$\mathbb{P}(\text{Permanency})$	$\mathbb{E}(\log T \mid \text{Disrup})$	$\mathbb{E}(\log T \mid \text{Exit})$	$\mathbb{E}(\log T)$
<i>Age At Plac.</i>	0.0139	-0.0115	-0.0406	-0.022	-0.0401
<i>County-FH</i>	0.317	-0.266	-0.969	-0.628	-0.927
<i>Agency-FH</i>	0.320	-0.272	-1.221	-0.874	-1.174
<i>Group Home</i>	0.165	-0.158	0.287	0.450	0.339
<i>Distance To School (zip)</i>	0.00401	-0.00376	-0.007978	-0.00309	-0.00736
<i>No School</i>	0.1136	-0.09686	-0.5244	-0.3653	-0.5212
Number of placements	2358				

Note: Average partial effects of placement characteristics on expected outcomes. Averages taken across the sample of assigned placements in the data. The partial effects with respect to continuous variables is taken by considering a marginal change of one unit.

Average Partial Effects on Expected Outcomes

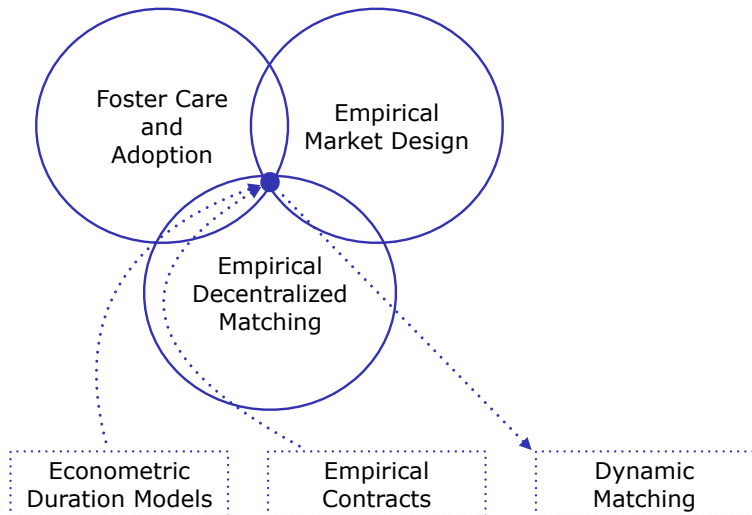
[Back](#)

Average Partial Effects					
	$\mathbb{P}(\text{Disrup})$	$\mathbb{P}(\text{Permanency})$	$\mathbb{E}(\log T \mid \text{Disrup})$	$\mathbb{E}(\log T \mid \text{Exit})$	$\mathbb{E}(\log T)$
<i>Age At Plac.</i>	0.0139	-0.0115	-0.0406	-0.022	-0.0401
<i>County-FH</i>	0.317	-0.266	-0.969	-0.628	-0.927
<i>Agency-FH</i>	0.320	-0.272	-1.221	-0.874	-1.174
<i>Group Home</i>	0.165	-0.158	0.287	0.450	0.339
<i>Distance To School (zip)</i>	0.00401	-0.00376	-0.007978	-0.00309	-0.00736
<i>No School</i>	0.1136	-0.09686	-0.5244	-0.3653	-0.5212
Number of placements	2358				

Note: Average partial effects of placement characteristics on expected outcomes. Averages taken across the sample of assigned placements in the data. The partial effects with respect to continuous variables is taken by considering a marginal change of one unit.

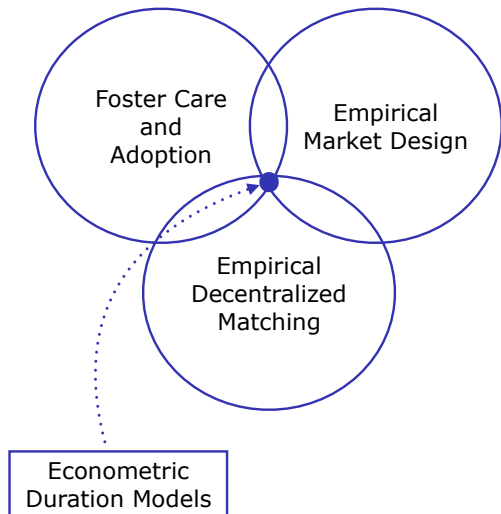
Related Literature (extended)

[Back](#)



Related Literature (extended)

Back



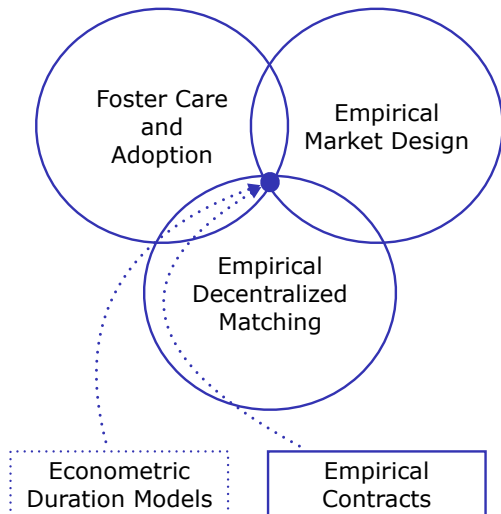
Competing Risks Duration Models

- ♦ Heckman and Honoré 1989
- ♦ Lancaster 1990
- ♦ Kalbfleisch and Prentice 2002

Borrow econometric methods and identification techniques

Related Literature (extended)

[Back](#)



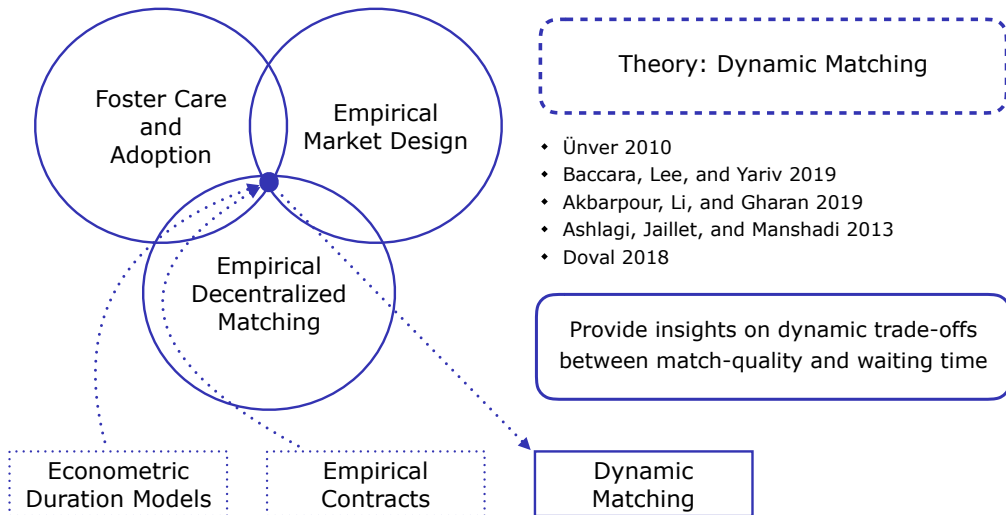
Empirical Contracting Models

- ♦ Akerberg and Botticini 2002
- ♦ Sørensen 2007
- ♦ Ewens, Gorbenko, and Korteweg 2019

Use similar identification strategy for selection on unobservables

Related Literature (extended)

Back

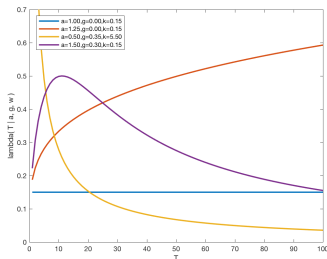


Burr Distribution

[Back](#)

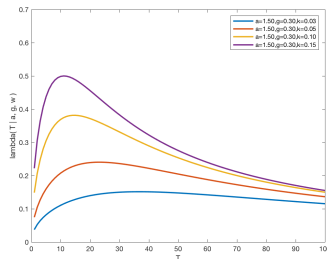
- The random variable $T \in \mathbb{R}_+$ has a Burr distribution with parameters $\alpha > 0$, $\gamma \geq 0$, and $k > 0$, if its hazard function takes the following form:

$$\lambda(T) = \frac{k\alpha T^{\alpha-1}}{1 + \gamma^2 k T^\alpha}.$$



Left: Examples of Burr hazard functions for different values of α , γ .

Particular cases: Exponential ($\alpha = 1$, $\gamma = 1$), Weibull ($\gamma = 0$), and Log-Logistic ($\gamma = 1$)



Right: Examples of hazard functions for different values of k .

Data Generating Process (DGP)

[Back](#)

- Need to identify the distribution of the **endogenous** (“left-hand side”) variables

$$(M_i, \mathbf{T}_i, \mathbf{R}_i),$$

conditional on the **exogenous** (“right-hand side”) ones

$$(C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i).$$

- Also, need to identify distribution of the **unobserved heterogeneity** (“mixing distribution”)

$$(M_i, \mathbf{T}_i, \mathbf{R}_i) | (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i) \sim \int (M_i, \mathbf{T}_i, \mathbf{R}_i) | (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i),$$

where $\boldsymbol{\Omega}_i = (\omega_{ch})_{(c,h) \in C_i \times H_i}$.

Identification

Back

1. Duration Distribution (hazard rates and unobserved heterogeneity)

- **Mixed competing risks** with covariates identified **non-parametrically** (Heckman and Honoré 1989).
- Distribution of ω across observed outcomes is **conditional on being matched**: $\omega_{ch} | M(c, h) = 1$.
- **Exogenous variation** in $(C, Y, \mathbf{X}, \mathbf{Y})$ across markets identifies distribution of ω (Akerberg and Botticini 2002; Sørensen 2007).
 - Intuition akin to traditional **sample selection** models (Heckman 1979)

2. Matching Distribution (multinomial probit)

- **Utility index** $\sum_{c,h} M(c, h)\pi(c, h)$ **linear** in utility parameters $(\mu_R, \varphi_R, \bar{\varphi}_R)_{R \in \mathcal{R}}$.
- Distribution of **individual shocks** ε_c and η_y can be backed out from **composite error** v_M
- Exploit variation in $(C, Y, \mathbf{X}, \mathbf{Y})$ across markets, and observing **unmatched children**.

Estimation

[Back](#)

- Estimate via **Simulated Maximum Likelihood**.
- Collect all the parameters of the model:

$$\boldsymbol{\theta}_T = (\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\beta}); \quad \boldsymbol{\theta}_M = (\boldsymbol{\mu}, \boldsymbol{\varphi}, \bar{\boldsymbol{\varphi}}, \boldsymbol{\Sigma}_\epsilon, \boldsymbol{\Sigma}_\eta); \quad \boldsymbol{\theta} = [\boldsymbol{\Sigma}_\omega, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M].$$

- The likelihood of observing $(M_i, \mathbf{T}_i, \mathbf{R})$, conditional on $\boldsymbol{\Omega}_i = (\boldsymbol{\omega}_{ch})_{(c,h) \in C_i \times H_i}$, is given by:

$$\mathcal{L}(M_i, \mathbf{T}_i, \mathbf{R}_i | \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) = \mathcal{L}_M(M_i | \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) \prod_{(c,h) \in M_i} \mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} | \boldsymbol{\omega}_{ch}, \boldsymbol{\theta}_T),$$

where:

$$\mathcal{L}_M(M_i | \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) = \text{probit choice probability}$$

$$\mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} | \boldsymbol{\omega}_{ch}, \boldsymbol{\theta}_T) = \text{Burr competing risks conditional likelihood}$$

Estimation

Back

- Let $G = \times_{c,h} G_{ch}$ denote the distribution of $\boldsymbol{\Omega}_i$, i.e., $G_{ch} \equiv N(0, \boldsymbol{\Sigma}_\omega)$. Then,

$$\mathcal{L}(M_i, \mathbf{T}_i, \mathbf{R}_i | \boldsymbol{\theta}) = \int \mathcal{L}_M(M_i | \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) \prod_{(c,h) \in M_i} \mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} | \boldsymbol{\omega}_{ch}, \boldsymbol{\theta}_T) dG(\boldsymbol{\Omega}_i | \boldsymbol{\Sigma}_\omega).$$

- The log-likelihood of the data is $\ell(\boldsymbol{\theta}) = \sum_{i=1}^n \log \mathcal{L}(M_i, \mathbf{T}_i, \mathbf{R}_i | \boldsymbol{\theta})$.
- Simulated analog of \mathcal{L} :

$$\mathcal{L}^{S_v, S_\omega}(M_i, \mathbf{T}_i, \mathbf{R}_i | \boldsymbol{\theta}) = \frac{1}{S_v} \frac{1}{S_\omega} \sum_{s_v=1}^{S_v} \sum_{s_\omega=1}^{S_\omega} \mathcal{L}_M^{s_v}(M_i | \boldsymbol{\Omega}_i^{s_\omega}, \boldsymbol{\theta}) \prod_{(c,h) \in M_i} \mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} | \boldsymbol{\omega}_{ch}^{s_\omega}, \boldsymbol{\theta}_T, \boldsymbol{\Sigma}_\omega),$$

where $\mathcal{L}_M^{s_v}$ is the simulated probit choice probability using a logit-kernel (Train 2009).

- The SMLE of $\boldsymbol{\theta}$ is given by: $\hat{\boldsymbol{\theta}}_{SMLE} = \arg \max_{\boldsymbol{\theta}} \sum_{i=1}^n \log \mathcal{L}^{S_v, S_\omega}(M_i, \mathbf{T}_i, \mathbf{R}_i | \boldsymbol{\theta})$
- $\hat{\boldsymbol{\theta}}_{SMLE} \stackrel{a}{=} \hat{\boldsymbol{\theta}}_{MLE}$ (**consistent**, **asymptotically normal** and **efficient**) if $n, S_v, S_\omega \rightarrow \infty$, and $\sqrt{n}/\min(S_v, S_\omega) \rightarrow 0$ (Gourieroux and Monfort 1997).

Aggregate payoff function

Probit Model

Identification

- The aggregate payoff of matching $M \in \mathbb{M}(C, H)$ is a **linear function of the utility function parameters**:

$$\begin{aligned} \sum_{c,h} M(c, h) \pi(c, h) = \sum_{R \in \mathcal{R}} \left\{ \left[\sum_{c,h} M(c, h) \mathbb{P}(R | \mathcal{I}_{ch}) \right] \mu_R \right. \\ \left. + \left[\sum_{c,h} M(c, h) \mathbb{P}(R | \mathcal{I}_{ch}) \mathbb{E}(\log T | R, \mathcal{I}_{ch}) \right] \varphi_R \right. \\ \left. + \left[\sum_{c,h} M(c, h) \mathbb{P}(R | \mathcal{I}_{ch}) \log T_{em,c} \right] \bar{\varphi}_R \right\}, \end{aligned}$$

Expected placement outcomes

Estimation

APEs

- **Termination probabilities** and **expected log-duration**:

$$\mathbb{P}(R|\mathcal{I}_{ch}) = \int_0^{T_{em,c}} \bar{F}(T|\mathcal{I}_{ch}) \lambda_R(T|\mathcal{I}_{ch}) dT$$
$$\mathbb{E}(\log T | R, \mathcal{I}_{ch}) = \int_0^{T_{em,c}} \log T \left[\frac{\bar{F}(T|\mathcal{I}_{ch}) \lambda_R(T|\mathcal{I}_{ch})}{\mathbb{P}(R|\mathcal{I}_{ch})} \right] dT,$$

where $\bar{F}(T|\mathcal{I}_{ch})$ denotes the **conditional survival function** of T , given by

$$\bar{F}(T|\mathcal{I}_{ch}) = \exp \left\{ - \sum_{R \in \mathcal{R}_0} \gamma_R^{-2} \log [1 + \gamma_R^2 k_R(\mathcal{I}_{ch}) T^{\alpha_R}] \right\}.$$

- The integrals above have **no closed-form solution**. They need to be computed numerically.

Conditional Hazard Functions

[Back](#)

	Disruption	Exit
$Var(\omega_R)$	0.873*** (0.2912)	0.02955 (0.02867)
$Cov(\omega_d, \omega_{ex})$	0.1573* (0.08908)	0.1573* (0.08908)
Age At Plac.	0.09872*** (0.01767)	-0.01615 (0.01047)
County-FH	2.217*** (0.332)	-0.02375 (0.2101)
Agency-FH	2.983*** (0.2556)	0.4547*** (0.1237)
Group Home	-2.077** (0.9188)	-1.987*** (0.5642)
Age At Plac. \times County-FH	-0.02272 (0.0261)	0.01804 (0.01636)
Age At Plac. \times Agency-FH	-0.07878*** (0.0194)	-0.01007 (0.0124)
Age At Plac. \times Group Home	0.2569*** (0.06179)	0.1419*** (0.03894)
Distance To School (zip)	0.02052*** (0.002471)	-0.006059*** (0.001724)
No School	0.9007*** (0.1603)	0.1222 (0.08942)
Constant	-8.996*** (0.5408)	-6.082*** (0.2132)
Alpha (α_R)	1.091*** (0.07551)	0.9665*** (0.03427)
Gamma (γ_R)	0.9527*** (0.1183)	0.2222 (0.2361)
Number of placements	2358	

Note: Estimated parameters of unobserved heterogeneity (Σ_ω) and conditional hazard rates (θ_T). Standard errors in parenthesis. Significance level of parameters: *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.1$.

Model Fit

[Back](#)

Goodness of Fit and Estimation Parameters

	Predicted	Sample
$\mathbb{P}(\textit{Disruption})$	0.514	0.5093
$\mathbb{P}(\textit{Exit})$	0.4303	0.4237
$\mathbb{P}(\textit{Emanc/Cens})$	0.05568	0.06701
$\mathbb{E}(\log T \mid \textit{Disruption})$	4.482	4.141
$\mathbb{E}(\log T \mid \textit{Exit})$	4.721	4.994
$\mathbb{E}(\log T \mid \textit{Emanc/Cens})$	7.19	5.534
$\mathbb{E}(\log T)$	4.615	4.596
Number of markets (n)	1467	
Number of assigned placements	2358	
Number of prospective placements	8900	
S_{MLL}	-17005.86	
S_{ω}	50	
S_{ψ}	50	
$\dim(\theta)$	39	

Note: Average predicted outcomes and sample average outcomes. Averages taken across the sample of assigned placements in the data. The number of assigned placements in the data is equal to $\sum_i \sum_{c,h} M_i(c, h)$. The number of prospective placements is equal to $\sum_i \sum_{c,h} |C_i| \times |H_i|$. S_{MLL} gives the value of the simulated log-likelihood at the estimated vector of parameters. S_{ω} , S_{ψ} , and ψ are the parameters of the simulated log-likelihood. $\dim(\theta)$ refers to the number of parameters estimated.