

Empirical Mechanism Design

Paulo Somaini

Stanford

DSE 2022

Introduction

- Competitive Markets:

- ▶ Price-taking behavior
- ▶ Homogeneous goods & perfect information
- ▶ Walrasian auctioneer

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 - ✓ prices clear the market
 - ✓ first and second welfare theorems
- These conditions fail in real-world markets
- Study of market failures central in many fields in economics
 - ▶ Industrial organization: monopoly, collusion
 - ▶ Contract Theory: principal-agent with hidden types and actions
 - ▶ Labor: monopsony
 - ▶ Macro: price rigidity, agency problems
 - ▶ Market Design: congestion, auctions, matching, allocation mechanisms

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- How does the market clear?
 - ▶ prices: price discovery in auctions [yesterday's lecture]
 - ▶ priority cutoffs: matching and school choice [today's lectures]
 - ▶ wait-time: queuing in organ allocation, public housing [tomorrow!]

Market Design: Objectives

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- Practicality and implementability
 - ▶ ideal of strategy-proofness
 - ▶ rules easy to codify and explain

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 - ▶ Estimate economic primitives
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 - ▶ Evaluate alternative designs
- Practice: interpersonal skills, networking
 - ▶ learn market institutions
 - ▶ propose and implement new designs

Empirical Market Design

- Complementary to theory in evaluation of trade-offs
 - ▶ Testing theoretical predictions
 - ▶ Quantify tradeoffs
 - ▶ Analysis when theory is ambiguous
 - ▶ Document effect of designs, market failures
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 - ▶ Well-understood rules
 - ▶ Administrative data

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- Empirical Approaches useful also in other areas of economics
 - ▶ Estimation of heterogeneous preferences and demand, e.g.,
 - ★ What do parents value in a school?
 - ★ What are the preferences of individuals for public housing?
 - ▶ Analysis of policy interventions, e.g.,
 - ★ Impact of financial aid reforms given admission mechanisms
 - ★ What are the effects of more generous public housing program?

School Choice

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 - ▶ Theory: properties mechanisms for student allocation
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- I will focus on the empirical tools

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- Many different algorithms implemented/studied (example):
 - ▶ Deferred acceptance (a.k.a Gale and Shapley alg)
 - ▶ Immediate acceptance (a.k.a Boston Mechanism)
 - ▶ Top-Trading Cycles
 - ▶ Serial Dictator

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- Use data on reports for strategy-proof mechanisms
- Use data on reports + behavior for non-strategy proof mechanisms

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 - ▶ Market does not clear on prices: $D(p)$ is not the full picture
- Interpretation of data from mechanisms
- Find tractable statistical-econometric tools

Preferences

- Students indexed by i , schools/programs indexed by j

$$v_{ij} = v(\mathbf{x}_{ij}, \zeta_j, \varepsilon_{ij}) - d_{ij},$$

- ▶ \mathbf{x}_{ij} observable characteristics
- ▶ ζ_j school quality, unobserved to the econometrician but observed to students
- ▶ ε_{ij} i.i.d preference shock
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 - ▶ Simple linear model

$$v_{ij} = \overbrace{\mathbf{x}_{ij}\beta + \zeta_j}^{\delta_{ij}} - d_{ij} + \varepsilon_{ij}$$

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$$v_{ij} = \overbrace{\mathbf{x}_{ij}\beta + \zeta_j}^{\delta_{ij}} - d_{ij} + \varepsilon_{ij}$$

- ▶ Random coefficient models: multiple preference shocks $\varepsilon_{ij} = (\gamma_i, \omega_{ij})$

$$v_{ij} = \overbrace{\mathbf{x}_{ij}\beta + \zeta_j + \mathbf{x}_{ij}(\bar{\gamma}\mathbf{z}_i + \gamma_i)}^{\delta_{ij}} - d_{ij} + \omega_{ij}$$

Outline

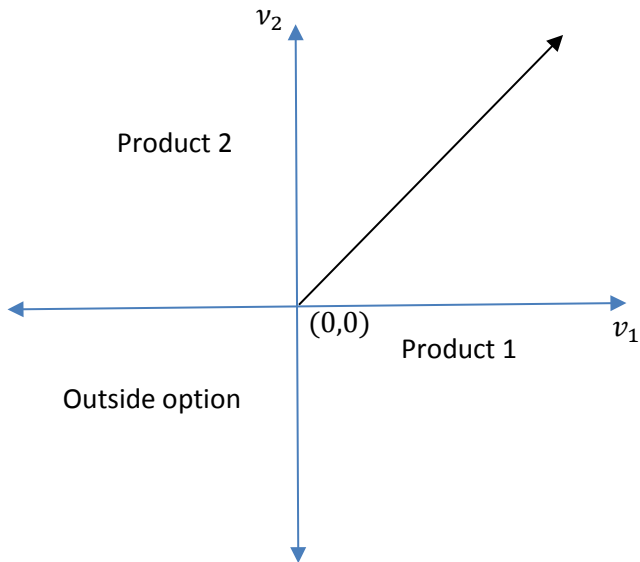
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- 2 Discrete Choice Models**
- 3 Stability
- 4 Truthful Reports
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Discrete Choice Models

$$v_{ij} = v(\mathbf{x}_{ij}, \zeta_j, \varepsilon_{ij}) - d_{ij},$$

- Consumer preferences for product
- Each consumer chooses the good with the maximum indirect utility
- The numeraire is usually price
- There are a variety of methods to estimate these models
- Rely on revealed preferences relations derived from observed choices

Revealed Preferences – Discrete Choice



Estimation Approaches – Discrete Choice

- Method of Moments (endogenous prices) [Berry 1994; Berry, Levinsohn and Pakes, 1995]
- Bayesian - Monte Carlo Markov Chain [Rossi, McCulloch and Allenby, 1996]
- Maximum Score [Manski, 1985]
- Moment Inequality [Ciliberto and Tamer, 2009; Pakes, 2010; Chernohukov, Hong and Tamer, 2007]
- Maximum Likelihood [McFadden, 1974; Train, 2004]
 - ▶ In the simple linear model, with extreme value shocks: $\frac{\varepsilon_{ij}}{\sigma} \sim EV1$

$$v_{ij} = \overbrace{x_{ij}\beta + \zeta_j}^{\delta_{ij}} - d_{ij} + \varepsilon_{ij}$$

$$P(i \text{ chooses } j | x_{ij}; \beta) = \frac{\exp\left(\frac{1}{\sigma}(\delta_{ij} - d_{ij})\right)}{\sum_k \exp\left(\frac{1}{\sigma}(\delta_{ik} - d_{ik})\right)}$$

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 - ★ $R_i \in \mathcal{R}$ student i 's submitted rank-order list
 - ★ R_{ik} is the school ranked in position k
 - ▶ students priorities:
 - ★ $\mathbf{t}_i = (t_{i1}, \dots, t_{iJ})$ is student i 's priority, t_{ij} has finitely many values
 - ★ Tie-breaker: ν_{ij}

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- Step 4: Schools consider new and previously held applicants and rank them according to priority. **Provisionally** hold applicants until exhausting capacity and definitively reject the rest
- Step 5 Repeat Steps 3-4 until each student (a) is tentatively held by some school; or (ii) has been rejected by all ranked schools

Properties of DA

- Report-Specific Priority + Cutoff representation:

- ▶ Score: $e_{ij} = f_j(R_i, t_i, v_{ij})$
- ▶ Cutoff p_j for school j
- ▶ Each student is placed in the highest ranked school in

$$S(\mathbf{e}_i, \mathbf{p}) = \{j : e_{ij} > p_j\}$$

- ▶ DA: score does not depend on R_i

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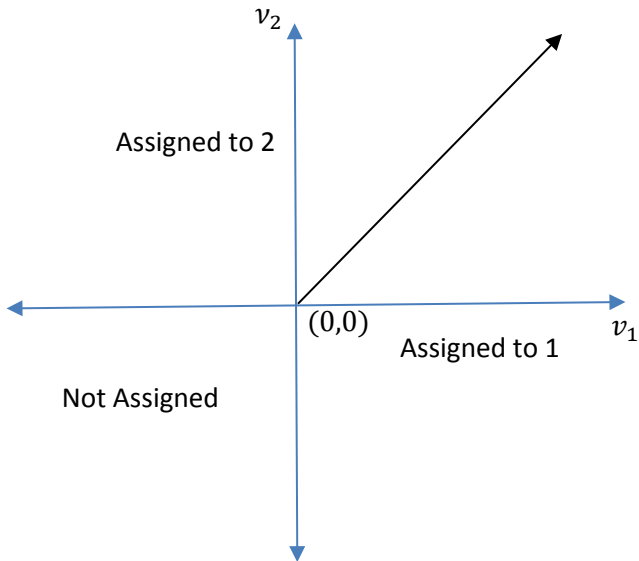
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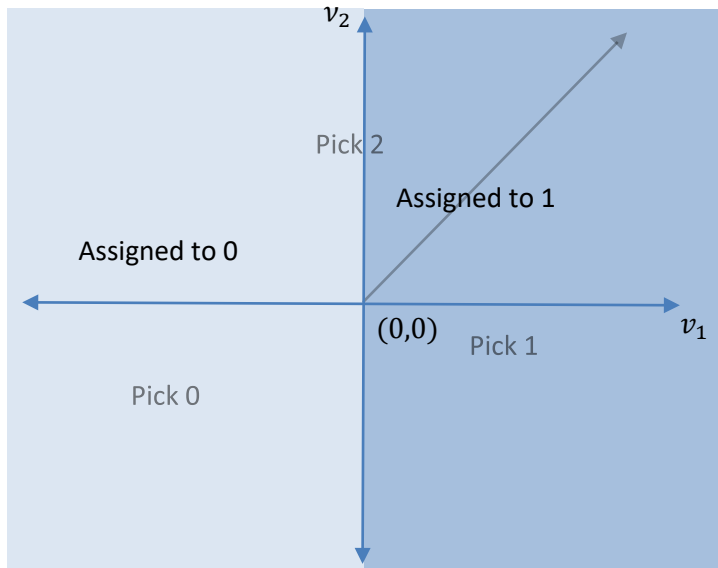
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- Levels the playfield between sophisticates and naives

Revealed Preferences – Stability – Full Choice Set



Revealed Preferences – Stability – Restricted Choice Set



Stability – Estimation Methods

- Logit models: build likelihood for $\mathbb{P}(i \text{ is assigned to } j | \delta_i, d_i; \theta)$:

$$\frac{\exp\left(\frac{1}{\sigma}(\delta_{ij} - d_{ij})\right)}{\sum 1 \{k \in S(\mathbf{e}_i, \mathbf{p})\} \exp\left(\frac{1}{\sigma}(\delta_{ik} - d_{ik})\right)}$$

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- Akyol and Krishna (2017) for high-schools in Turkey
- Bucarey (2018) for colleges in Chile

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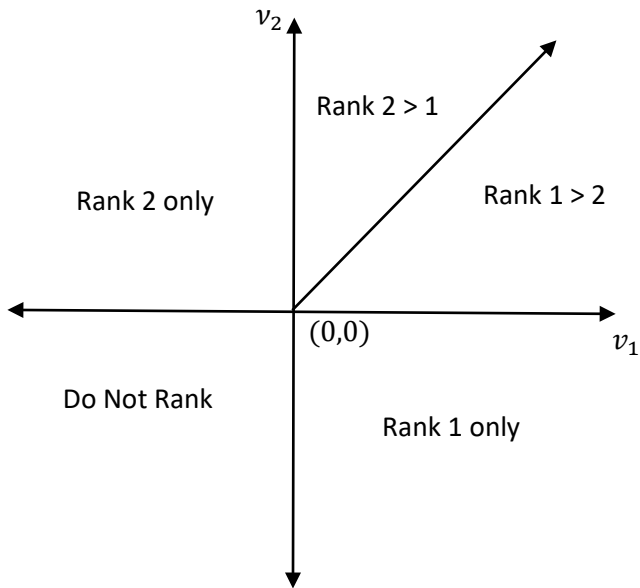
		Change in average:			
		Utility	Utility Net of Price	Sticker Tuition	Received Scholarship
<i>A. Common Price Coefficient Model</i>					
Family Income					
	Poorest Quintile	-\$3,396	-\$1,180	-\$567	\$1,137
	Second Quintile	-\$4,586	-\$1,454	-\$243	\$1,458
	Third Quintile	-\$2,994	-\$1,109	-\$524	\$1,274
	Fourth Quintile	-\$1,247	-\$776	\$630	\$2,736
	Richest Quintile	-\$96	-\$490	\$1,460	\$3,484
Test Scores					
	Lowest Quartile	-\$8,533	-\$2,485	-\$2,184	\$24
	Top Quartile	\$1,955	\$178	\$3,328	\$4,515
<i>B. Income-heterogeneous Price Coefficient Model</i>					
Family Income					
	Poorest Quintile	-\$6,530	-\$1,078	-\$506	\$1,271
	Second Quintile	-\$3,684	-\$990	-\$323	\$1,379
	Third Quintile	-\$1,461	-\$778	-\$25	\$1,629
	Fourth Quintile	\$404	-\$572	\$675	\$3,070
	Richest Quintile	\$1,486	-\$332	\$1,204	\$3,832
Test Scores					
	Lowest Quartile	-\$10,980	-\$2,178	-\$2,160	\$34
	Top Quartile	\$5,480	\$614	\$2,509	\$5,038

Notes: This table compares the average of the variable in each column for the free tuition case and the baseline. Utilities are expressed in dollar equivalent.

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Revealed Preferences – Truthtelling



Truth-telling – Estimation Methods

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- Abdulkadiroglu, Agarwal and Pathak, 2017: NYC High School
- Ajayi and Sidibe, 2022: High Schools in Ghana

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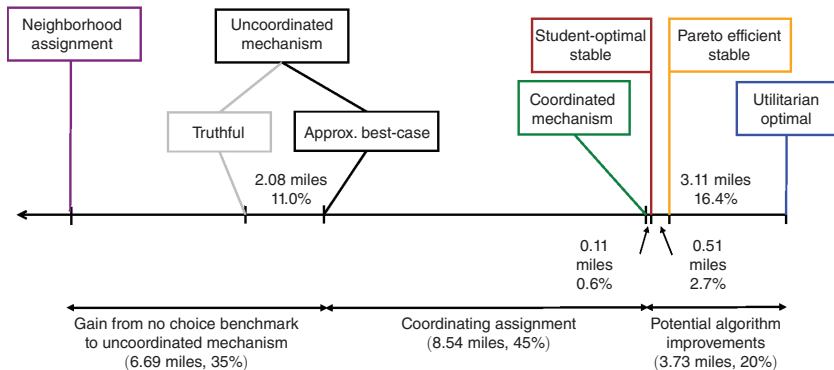
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Abdulkadiroglu et. al. 2017:

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 - ▶ Calculate aggregate welfare and distributional consequences



- Centralized (Coordinated) mechanisms perform better!

Outline

- 1 Introduction
- 2 Discrete Choice Models
- 3 Stability
- 4 Truthful Reports
- 5 Strategic Reports**
- 6 Conclusion

Manipulable Mechanisms

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- a.k.a Boston Mechanism

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- Infer revealed preference relations from ranked lists
- How should we treat schools that are not ranked on the list?
 - ▶ Only infer relations from ranked lists?
 - ▶ Assume any ranked school is preferred to any non-ranked school?
 - ▶ Assume non-ranked schools are worse than the outside option?
 - ▶ Default back to using stability [Fack et al, 2019]?
- Alternatively, take the skipping strategy more seriously
 - ▶ Require analysis attainability of each school
 - ▶ Popular schools are harder to get
 - ▶ Not ranking a unpopular school implies dislike [Hwang, 2014]

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- Step 5 Repeat Steps 3-4 until each student (a) has been accepted by some school; or (ii) has been rejected by all ranked schools

Listening to parents

- ... if I understand the impact of Gale Shapley, and I've tried to study it and I've met with BPS staff... I understood that in fact the random number... [has] preference over your choices... [Recording from the BPS Public Hearing, 6-8-05]

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- I'm troubled that you're considering a system that takes away the little power that parents have to prioritize... what you call this strategizing as if strategizing is a dirty word... [Recording from the BPS Public Hearing, 05-11-04].

Were they right?

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- Preferences are important!

Choice under uncertainty

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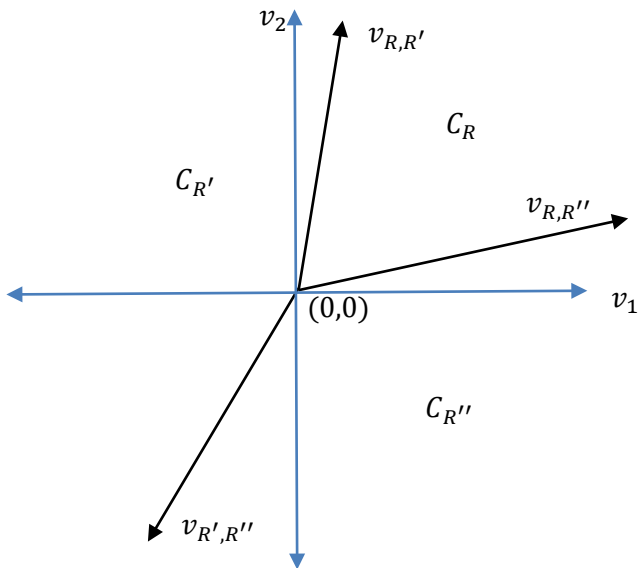
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- Thus, $\mathbf{v}_i \in C_{R_i}$ where C_{R_i} is a cone $\{\mathbf{v}_i \in \mathbb{R}^J : \mathbf{v}_i \cdot \Delta L_{R_i} \geq 0\}$

Revealed Preferences – Strategic Behavior



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- No easy way to compute the likelihood function
- Bayesian methods do not require computation of the likelihood function
 - ✓ Obtain the MLE without computing the likelihood function
 - ▶ Useful for discrete choice models [Rossi, McCulough, Allenby, 1995]
 - ▶ Can be adapted for choices over lotteries [Agarwal and Somaini, 2018]

Bayesian Methods

- Frequentist approach: $\log \mathcal{L}(\theta; data) := \log f(data|\theta)$
 - ▶ θ is a parameter
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- Bayesian Approach: $f(\theta|data) = \frac{f(data|\theta)f(\theta)}{f(data)}$
 - ▶ θ is random vector
 - ▶ $f(\theta)$ is the prior [knowledge about θ]
 - ▶ $f(\theta|data)$ is the posterior
 - ▶ $f(data) = \int f(data|\theta) f(\theta) d\theta$
 - ▶ The posterior contains all the information we want!
 - ▶ Asymptotically Gaussian posterior irrespective of prior [Bernstein von-Mises Theorem]

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 - ✓ There are other algorithms: e.g., Metropolis-Hastings, Hamiltonian Monte Carlo
- For k large enough $\theta^k = (\theta_1^k, \theta_2^k) \sim f(\theta|data)$
 - ✓ Generalizes to $\theta = (\theta_1, \theta_2, \dots, \theta_d)$

Gibbs Sampler in School Choice

- Recall

$$v_{ij} = x_{ij}\beta_j - d_{ij} + \varepsilon_{ij}$$

where $\varepsilon_{ij} \sim N(0, \Sigma)$, $\beta \sim N(\bar{\beta}, \Sigma_\beta)$ and $\Sigma \sim IW(u_0, S)$

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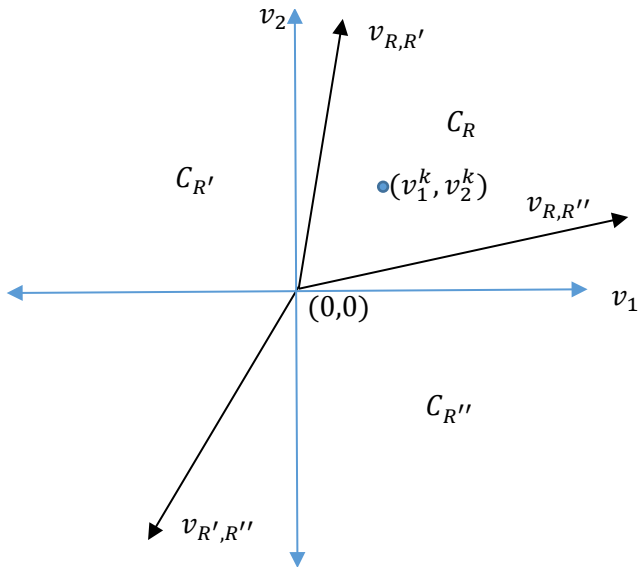
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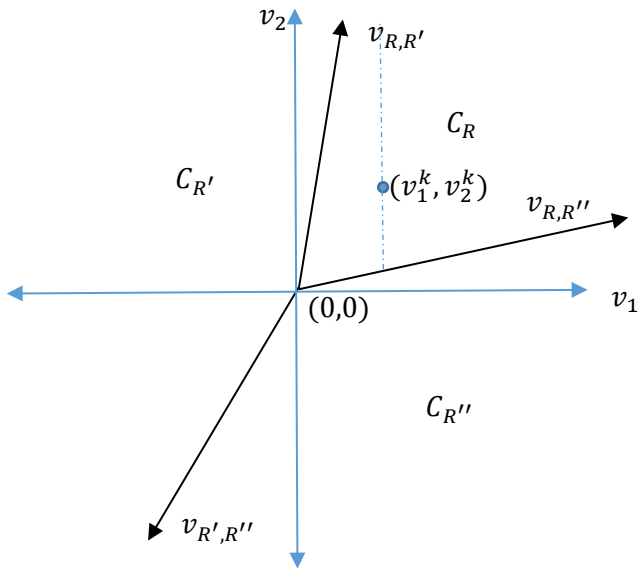
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Updating v^{k+1}



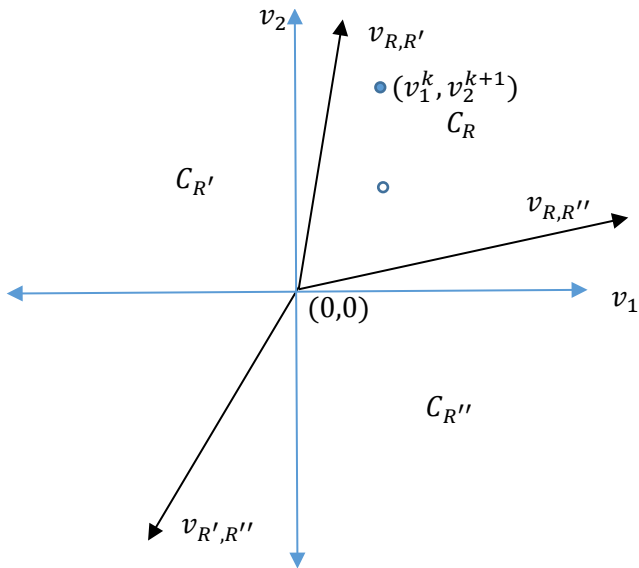
We start from the previous vector v^k

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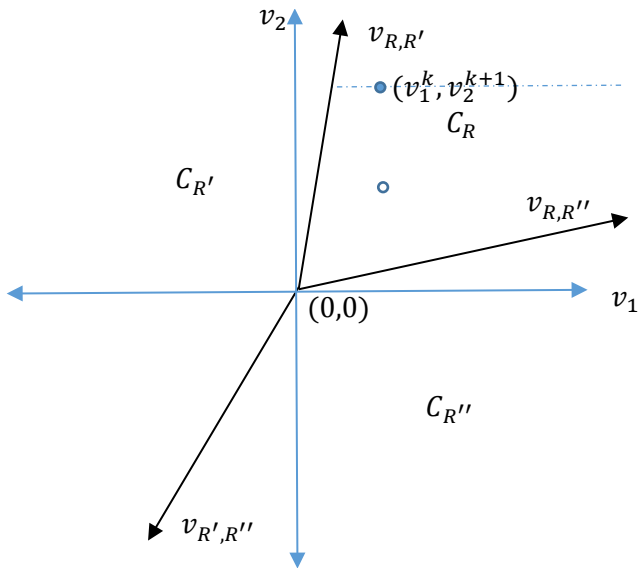
Draw v_2^{k+1} given $v_1^k, \beta^{k+1}, \Sigma^{k+1}$ from a truncated normal

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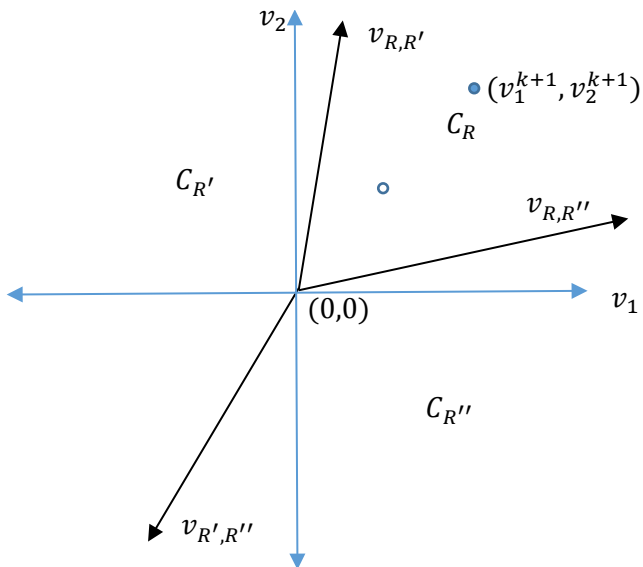
New draw of v_2^{k+1}

Updating v^{k+1}



Now, draw v_1^{k+1} given $v_2^{k+1}, \beta^{k+1}, \Sigma^{k+1}$ from a truncated normal

Updating v^{k+1}



Now, draw of v_1^{k+1} . We have v^{k+1} !

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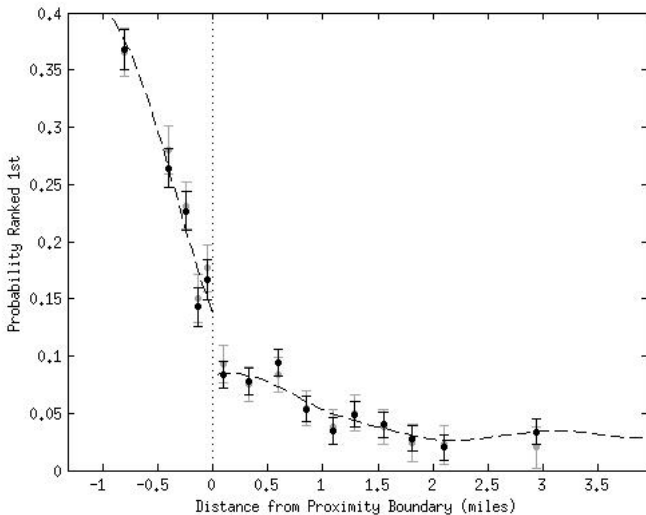
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- Compare performance of DA vs IA

Elementary Schools and Students

Year	2004	2005	2006	2007	2008	Average
<i>Panel A: District Characteristics</i>						
Schools	13	13	13	13	13	13
Programs	24	25	25	27	27	25.6
Seats	473	456	476	508	438	470
Students	412	432	397	457	431	426
Free/Reduced Lunch	32%	38%	37%	29%	32%	34%
Paid Lunch	68%	62%	63%	71%	68%	66%
<i>Panel B: Student's Ethnicity</i>						
White	47%	47%	45%	49%	49%	47%
Black	27%	22%	24%	22%	23%	24%
Asian	17%	18%	15%	13%	18%	16%
Hispanic	9%	11%	10%	9%	9%	10%
<i>Panel C: Language spoken at home</i>						
English	72%	73%	73%	78%	81%	76%
Spanish	3%	4%	4%	4%	3%	3%
Portuguese	0%	1%	1%	1%	1%	1%
<i>Panel D: Distances(miles)</i>						
Closest School	0.43	0.67	0.43	0.47	0.45	0.49
Average School	1.91	1.93	1.93	1.93	1.89	1.92

Strategic Behavior

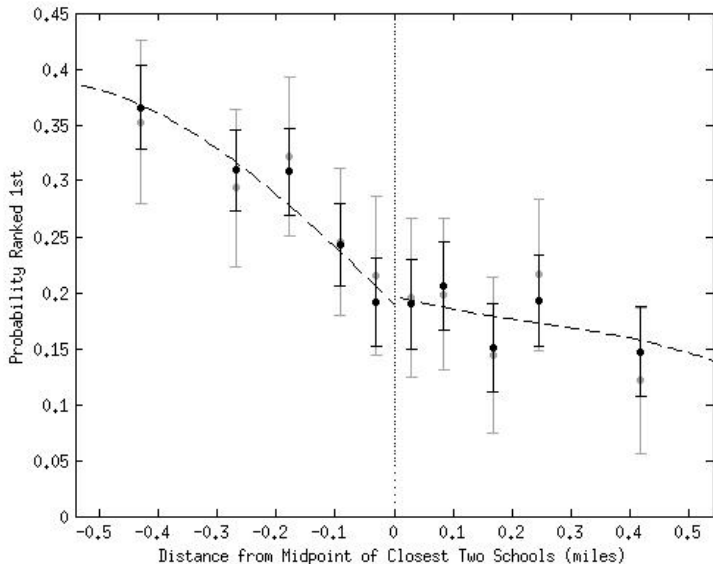
Top Rank: Proximity Boundary



✓ Difficult to explain entire response with residential sorting

Strategic Behavior

Placebo with Two Closest Schools



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 - ▶ Own lottery draw $v_i \sim U[0, 1]$
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- Estimate assignment probabilities by resampling R_{-i}, t_{-i} , B times

$$\hat{L}_{R,t,j} = \frac{1}{B} \sum_{b=1}^B \int 1\{f_j(R, t, v) \geq p_j^b \text{ and } f_k(R, t, v) < p_k^b \text{ if } k \neq j\} dv$$

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 - ▶ Market clearing cutoff p^* depend on all agents' actions and lotteries
- Estimate assignment probabilities by resampling R_{-i}, t_{-i} , B times

$$\hat{L}_{R,t,j} = \frac{1}{B} \sum_{b=1}^B \int 1\{f_j(R, t, v) \geq p_j^b \text{ and } f_k(R, t, v) < p_k^b \text{ if } k \neq j\} dv$$

- ▶ **Idea:** Resampling approximation to beliefs about assignment probabilities

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- ✓ Paper establishes consistency and asymptotic normality in a large market
 - ▶ Large number of students, fixed schools with increasing capacity
 - ▶ Target: Equilibrium of a limit game

Deferred Acceptance vs. Cambridge Mechanism

	Truthful			Rational Expectations		
	All Students	Paid Lunch	Free Lunch	All Students	Paid Lunch	Free Lunch
<i>Panel A: Deferred Acceptance</i>						
Assigned to First Choice	67.7	58.2	86.6	67.9	58.1	87.5
Assigned to Second Choice	12.1	14.2	8.1	15.5	18.6	9.4
Assigned to Third Choice	5.7	8.2	0.8	5.2	7.1	1.3
<i>Panel B: Cambridge Mechanism</i>						
Assigned to First Choice	79.0	74.5	87.8	72.3	63.9	88.8
Assigned to Second Choice	6.5	6.8	6.0	14.7	18.1	7.9
Assigned to Third Choice	3.1	4.0	1.4	3.9	5.1	1.3
<i>Panel C: Deferred Acceptance vs Cambridge</i>						
Mean Utility DA - Cambridge	-0.004 (0.017)	-0.010 (0.025)	0.008 (0.006)	-0.072 (0.011)	-0.109 (0.015)	0.003 (0.013)
Std. Utility DA - Cambridge	0.230	0.280	0.047	0.171	0.142	0.197
Percent DA > Cambridge	26.8	26.0	28.3	16.5	14.2	21.1
Percent DA \approx Cambridge	31.9	26.2	43.0	30.3	27.1	36.6
Percent DA < Cambridge	41.4	47.8	28.7	53.2	58.7	42.3
Percent with Justified Envy	9.93	12.69	4.46	5.6	5.1	6.4

✓ Approach evaluates assignments, ignoring potential costs of strategizing

DA vs. Cambridge w/ Biased Beliefs

	Coarse Beliefs			Adaptive Expectations		
	All Students	Paid Lunch	Free Lunch	All Students	Paid Lunch	Free Lunch
<i>Panel A: Deferred Acceptance</i>						
Assigned to First Choice	69.7	61.0	87.1	68.4	56.9	89.1
Assigned to Second Choice	11.9	13.7	8.5	13.6	17.3	7.1
Assigned to Third Choice	4.9	6.7	1.2	5.1	7.3	1.1
<i>Panel B: Cambridge Mechanism</i>						
Assigned to First Choice	73.9	67.3	86.9	72.3	63.0	88.9
Assigned to Second Choice	10.2	11.1	8.3	12.1	15.3	6.4
Assigned to Third Choice	3.5	4.6	1.5	3.7	4.9	1.4
<i>Panel C: Deferred Acceptance vs Cambridge</i>						
Mean Utility DA - Cambridge	-0.045 (0.011)	-0.074 (0.013)	0.013 (0.016)	-0.049 (0.028)	-0.097 (0.035)	0.037 (0.040)
Std. Utility DA - Cambridge	0.174	0.146	0.207	0.213	0.142	0.282
Percent DA > Cambridge	22.6	21.3	25.1	19.1	16.5	23.9
Percent DA \approx Cambridge	30.6	26.5	38.7	31.6	26.2	41.4
Percent DA < Cambridge	46.9	52.2	36.2	49.3	57.4	34.7
Percent with Justified Envy	7.1	7.8	5.6	6.7	8.0	4.4

✓ Advantage of the Cambridge mechanism are sensitive to agent information

Outline

- 1 Introduction
- 2 Discrete Choice Models
- 3 Stability
- 4 Truthful Reports
- 5 Strategic Reports
- 6 Conclusion**

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- Big welfare effects of centralized mechanisms
- IA can increase our measure of welfare if students strategize correctly
- Difference between the mechanisms is smaller if beliefs are biased
- Similar results in Barcelona [Calsamiglia, Guell and Fu, 2018]
- Manipulable mechanism do badly if beliefs are wrong like in New Haven [Kapor, Nielsen and Zimmerman, 2018]

Exciting avenues for research

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