Teacher Labor Market Equilibrium and the Distribution of Student Achievement

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Motivation and questions

Teachers matter for student outcomes. [e.g., Chetty, Friedman and Rockoff (2014)]

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Questions:

- How equitable and efficient is the current allocation of teachers to schools? Why?
- What policies would make the allocation more equitable and/or efficient?

What we show: current allocation poses a puzzle

Use data from a large district in NC. Look at elementary schools.

The current allocation is equitable: [e.g., Sass et al (2012), Mansfield (2015), Angrist et al (2021),...]

- Advantaged and disadvantaged students have teachers with similar value-added (VA).
- Surprising because teachers prefer schools with advantaged students.

The current allocation is inefficient: potential gains of 0.05σ.

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Potential explanations:

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2. Teacher preferences: joint distribution of preferences and VA
   - Marginal not average
3. Principal preferences: do not hire better teachers measured by VA [e.g., Ballou (1996)]
4. Match effects: some teachers have strong comp. adv. w/ disadv. students

How we answer this question

Estimate an equilibrium model using data on teacher transfer system linked to test score data.

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- Estimate comparative advantage using teacher-student links and test score data
- Estimate market clearing, and teacher/principal preferences with weak assumptions [compare to Boyd et al (2013), Biasi et al (2022), Bobba et al (2022), Tincani (2021)...]

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- Estimate market clearing, and teacher/principal preferences with weak assumptions [compare to Boyd et al (2013), Biasi et al (2022), Bobba et al (2022), Tincani (2021)...]
- Equity explanation: principal behavior drives the equilibrium
- Inefficiency explanation: lack of differentiated wages

Outline of the talk

1. Assignment model
   - Define equilibrium and first-best problems
2. Data
3. Student achievement: Production model
4. How applications / vacancies work
5. Teacher and principal preferences
6. Understanding equilibrium/counterfactuals
7. Bonuses

Overview of the environment

Setting has several important features.

- **Timing**: April to August, clears on a rolling basis
- **Decentralized**: teachers apply, principals decide on interviews and offers
- **Wages**: do not vary with assignment

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We model how teacher-school assignments are formed and the value of each assignment.

- **Agent types**: school principals and teachers
- **Output measures**: student test scores and teacher (non-wage) utility
- **Equilibrium concept**: pairwise stability

Utilities and Output

For teacher j and school k:

- Teacher utility:

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- Student output:

$$VA_{jk}$$

Decentralized equilibrium

Schools and teachers meet and match

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- **Stable** allocations: no teacher and school pair, not already matched, would prefer to jointly deviate and match (Roth & Sotomayor, 1992, Definition 2.3)
- Find range of equilibria using deferred acceptance algorithm (Roth & Sotomayor, 1992, Theorem 2.12)

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  - Understand joint distribution of $\tilde{u}_{jk}$ and VA_{jk}
- Principal preferences/selection: principals do not exploit excess demand
  - Understand joint distribution of $\tilde{v}_{jk}$ and VA_{jk}
- Match effects
  - Structure of VA_{jk}

District's first-best problem

Let $\phi : \mathcal{J} \to \mathcal{K}$ be a one-to-one assignment.

$$\max_{\phi \in \Phi} \Big\{ \underbrace{\lambda}_{\text{weight on students}} \underbrace{\sum_{j \in \mathcal{J}} VA_{j\phi(j)}}_{\text{student output}} + \underbrace{\sum_{j \in \mathcal{J}} \tilde{u}_{j\phi(j)}}_{\text{teacher non-wage utility}} \Big\}$$

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- Varying $\lambda \Rightarrow$ traces out the PPF
- Slope of PPF $\Rightarrow$ technological trade-off between student achievement and teacher utility
- Equity objectives: vary weight on student types in VA

Outline

1. Assignment model
   - Define decentralized equilibrium and first best problems
2. Data
   - Applications / vacancies data for identifying preferences
3. Student achievement: Production model
4. How applications / vacancies work
5. Teacher and principal preferences
6. Understanding equilibrium/counterfactuals
7. Bonuses

Data

Application and vacancy data (**more novel**):

- Large urban school district in NC, starting in 2010
- Flags for principal "positive assessment," interviewed, hired
- Include records of timing of (some) events

$\Rightarrow$ estimate teacher/principal preferences directly, don't infer from equilibrium outcomes

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Student-level data (**more standard**):

- For the whole state of NC back to 2007
- Student demographics
- Test scores (we focus on math)

Teacher-level data (**more standard**):

- Experience, demographics, classroom assignment

Focus on elementary schools

Outline

1. Assignment model
2. Data
   - Applications / vacancies data for identifying preferences
3. Student achievement: Production model
   - Current allocation balanced; potential equity/efficiency gains
4. How applications / vacancies work
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Measuring value-added

Two-dimensional value-added: (Aucejo et al, 2019; Delgado, 2020; Bau, 2021, Biasi, Fu & Stromme, 202)

- Economically disadvantaged (National School Lunch program definition)
- Non-economically disadvantaged

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- Non-economically disadvantaged

High-level:

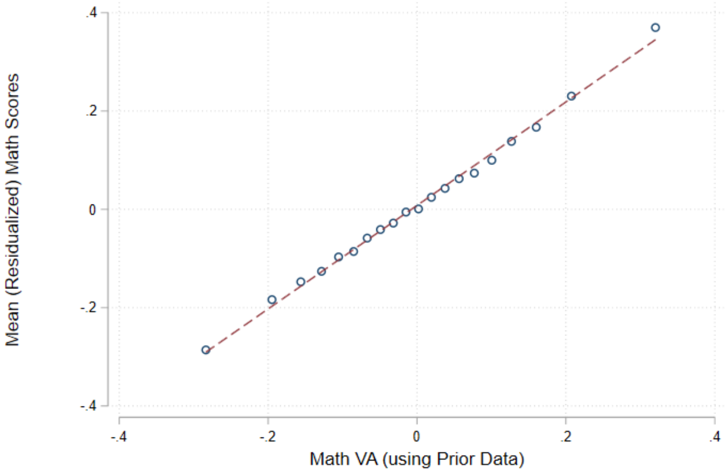
- Follow Chetty, Friedman, and Rockoff (2014) in allowing drift
- Follow Delgado (2020) in allowing correlation across types, using prior data only

$$\widehat{VA}_{jkt} = n_{k0t}\hat{\mu}_{j0t} + n_{k1t}\hat{\mu}_{j1t} + f(Z_{jt}; \hat{\alpha})$$

- Type-specific VA, weighted by number of students of each type in school k in year t

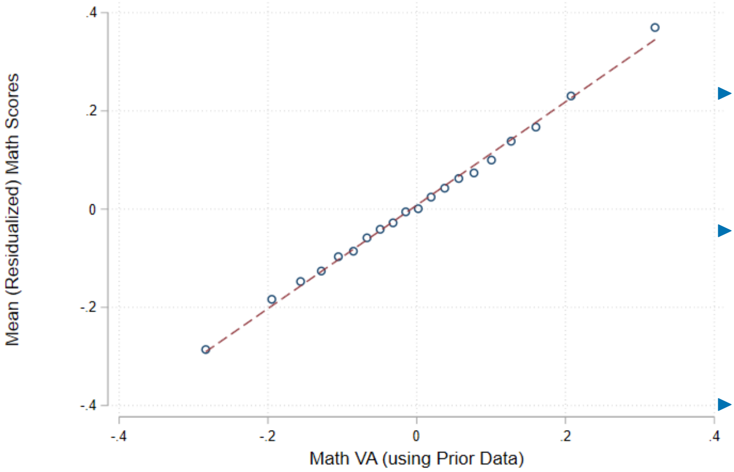
▸ Details ▸ Other match specifications ▸ Experience ▸ Structural ▸ Drift ▸ Joint distribution ▸ LR test

Validation of value-added



- Coefficient of 1.052 (0.007)

Validation of value-added



- Coefficient of 1.052 (0.007)
- Predictive for transfers: 0.972 ‣ Link
  ‣ Event study
- Similar across large composition changes ‣ Link and class size changes ‣ Link
- Reject homogeneous model ‣ Structural
  ‣ LR test

Properties of the current allocation: student achievement

	Adv	Disadv
Student performance (level scores)		
Math	0.70	-0.16
Student performance (gain scores)		
Math	0.07	0.07
Mean math value-added		
Baseline, Adv	0.01	0.02
Baseline, Dis	0.02	0.02
Homogeneous	0.02	0.01

- Large absolute gaps
- No difference in test score growth
- No/small difference in VA of teachers
- No sorting on comparative advantage
- Observables poorly predict VA (R^2 is less than 0.025)

First-best allocations with different targets and instruments

	Per-Student Gains (σ)	Adv	Dis	Dis-Adv
Alternate Allocations				
Best	0.054	0.095	0.018	-0.077
Alternate Policies				
Replace Bottom 5% of Teachers	0.012	0.015	0.009	-0.006
Targeting Disadvantaged Students				
Max Disadvantaged VA	0.016	-0.049	0.075	0.124

- Baseline gap: 0.86
- Standard deviation of teacher value-added: 0.14

‣ Transfer match ‣ Partial reassignments ‣ Simulation ‣ Composition and class size ‣ School class size autocorr ‣ Teacher class size autocorr ‣ School composition autocorr ‣ Teacher class composition autocorr

Outline

1. Assignment model
2. Data
3. Student achievement: Production model
    - Equitable allocation in VA
    - Surprising given teacher labor market facts
    - Two-dimensional value-added, with large potential gains
4. How applications / vacancies work Link
    - Choice sets are all active positions
    - Teachers apply non-strategically
5. Teacher and principal preferences
6. Understanding equilibrium/counterfactuals
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Model of teacher applications

Position (p)

- Associated with school $k = k(p)$
- Exogenous posting and closing dates
- May be assigned to at most one teacher
- Outside option $(p = 0)$ is not teaching in this public district

Teacher (j)

- Utility $\tilde{u}_{jpt}$ for each position
- Exogenous market entry and exit dates
- May be assigned to at most one position
- Current assignment: $c = p(j, t-1)$

For position p in teacher j's choice set, **submit an application** $(a_{jpt} = 1)$ if:

$$a_{jpt} = \mathbf{1}\{\tilde{u}_{jpt} > \max\{\tilde{u}_{jct}, \tilde{u}_{j0t}\}\}$$

Parameterization of utilities

$$\tilde{u}_{jpt} = -\gamma d_{jpt} + \pi_j VA_{jpt} + \beta_j X_{pt} + \eta_{jt} + \epsilon_{jpt}$$

- d_{jpt}: commute time (minutes)

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- η_{jt}: teacher char. (gender, race, mean VA), Mundlak-Chamberlain device, RE

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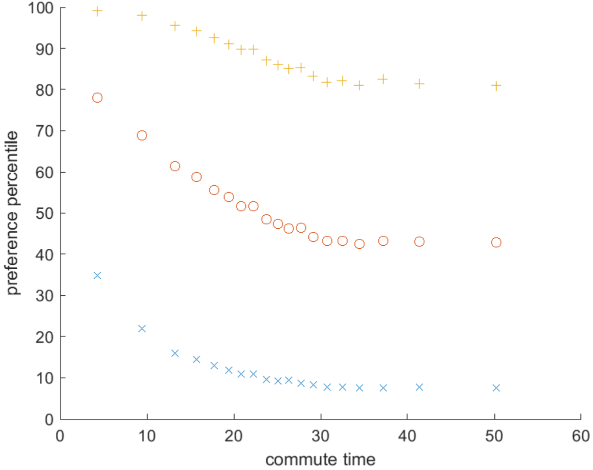
- ϵ_{jpt}: iid Type I EV error

▸ Commute time ▸ Title I gap

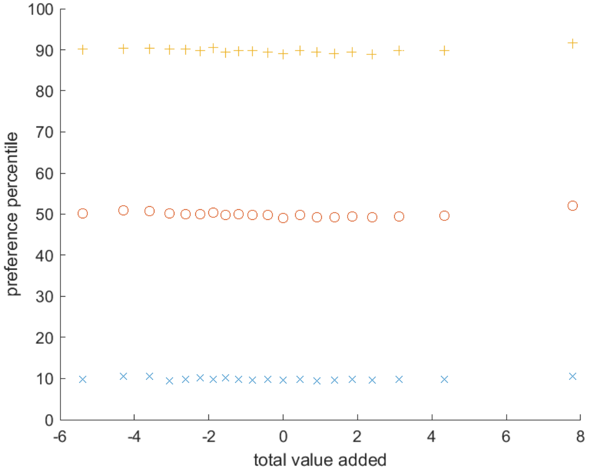
Teacher preferences: commute time



- Pointwise 90th, mean, and 10th percentile
- Teachers prefer schools with a shorter commute
- Massive heterogeneity
- Doesn't show causal effect of commute time

‣ Other coefficients ‣ Teacher characteristics coefficients

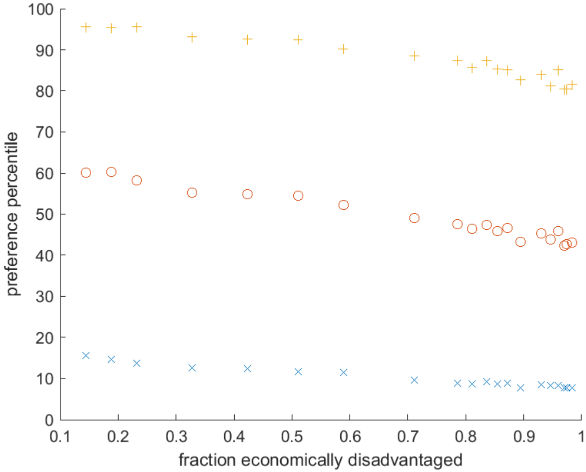
Teacher preferences: value-added



- Teachers have weak preferences over output (coefficient/se: 0.081 (0.008))
- Large amounts of heterogeneity
- Doesn't show causal effect of output

‣ Other coefficients ‣ Teacher characteristics coefficients

Teacher preferences: fraction disadvantaged



- Doesn't show causal effect
- Teachers prefer schools with smaller share of disadvantaged students
- Substantial heterogeneity, possibility for marginal $\neq$ average
- $\Rightarrow$ model quantifies this possibility

‣ Other coefficients ‣ Teacher characteristics coefficients

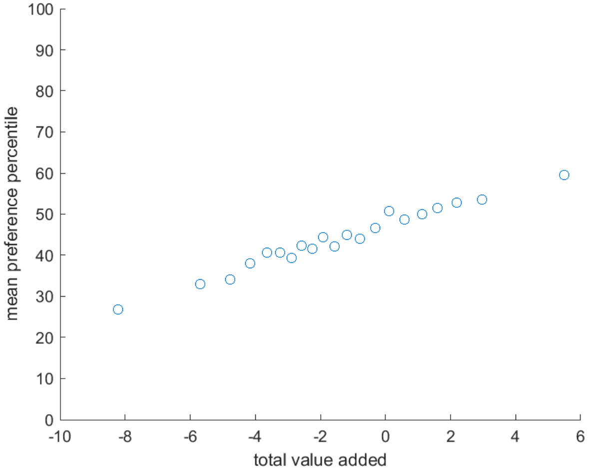
Principal ratings

The principal for position p gives teacher j a positive rating if $\tilde{v}_{jpt} > 0$.

$$\tilde{v}_{jpt} = \alpha_p W_{jpt} + \underbrace{\kappa_{pt}}_{\text{random effect}} + \underbrace{\upsilon_{jpt}}_{\text{iid Type I EV}}$$

- W_{jpt} includes...
  - teacher's value-added
  - absolute advantage
  - experience, Masters, Black, Hispanic, female
  - Black x fraction Black, Hispanic x fraction Hispanic
- α_p: separate coefficients by p's Title I status

Principal preferences: value-added



- Principals value output
- Doesn't perfectly predict ranking
- Doesn't show causal effect of output
- $\Rightarrow$ hard to evaluate magnitudes, equilibrium model quantifies

‣ Other coefficients 1 ‣ Other coefficients 2

Observables explain little of principal's ratings

Change in pseudo-R^2 from adding:	Non-Title I	Title I
Demographics (DG)	0.008	0.004
Teacher Characteristics (TC)	0.031	0.018
Value Added (VA)	0.006	0.003

- Demographics: race and gender, interacted with school demo.
- Teacher char.: experience, licensing, certification, and Praxis scores
- Recall, observables predict VA poorly
- Teacher char. go furthest

Outline

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2. Data
3. Student achievement: Production model
4. How applications / vacancies work
5. Teacher and principal preferences
    - Distance and student composition matter for teachers
    - Magnitudes hard to interpret for equilibrium—need a model!
6. Understanding equilibrium/counterfactuals
7. Bonuses

Putting the pieces together

Sample:

- Teachers who apply in 2015-2016, and for whom we have prior data to estimate value-added (both in district, and from the rest of the state)
- Vacancies posted in 2015-2016. Drop randomly until market is balanced

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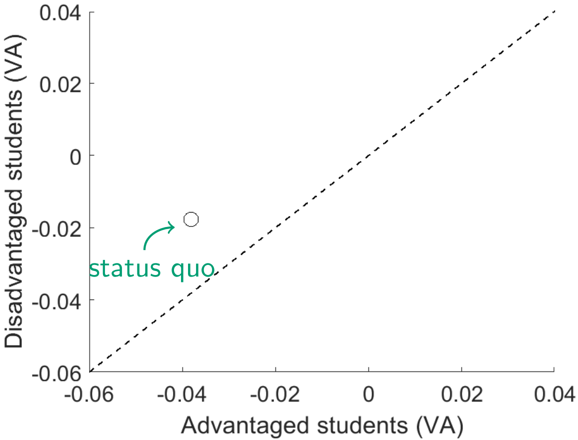
Teacher and principal preferences:

- Take "best guess" of random coefficients given choices and choice set
- Simulate error draws

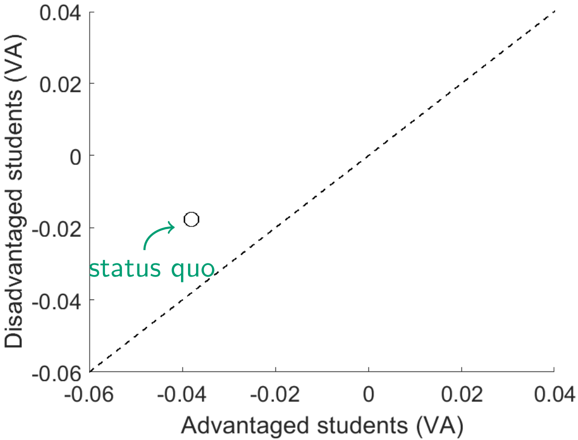
Fit – AA Fit – Black Fit – Experience

Distributional: status quo



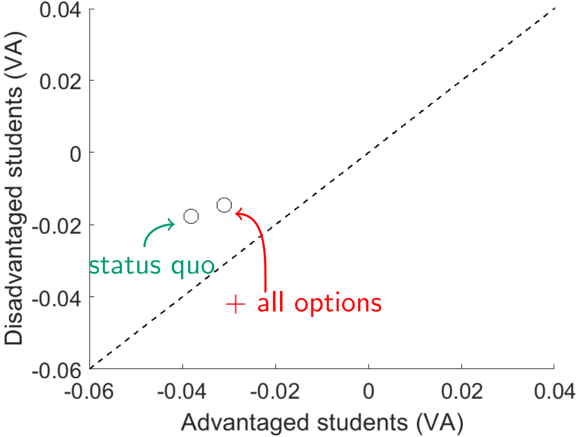
- Status quo: estimated teacher and principal preferences, empirical timing, teacher propose
- Model fits well: favors econ disadv. (!) (puzzling)

Distributional: status quo + equilibrium selection



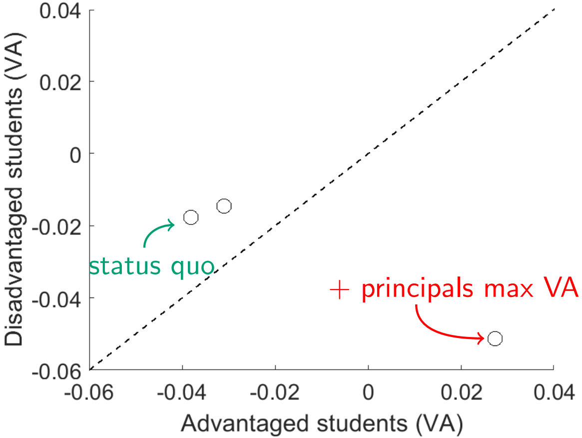
- Status quo: favors econ disadv. (!) (puzzling)
- Change to school propose: no change
- $\Rightarrow$ equilibrium selection not an explanation

Distributional: remove timing constraints



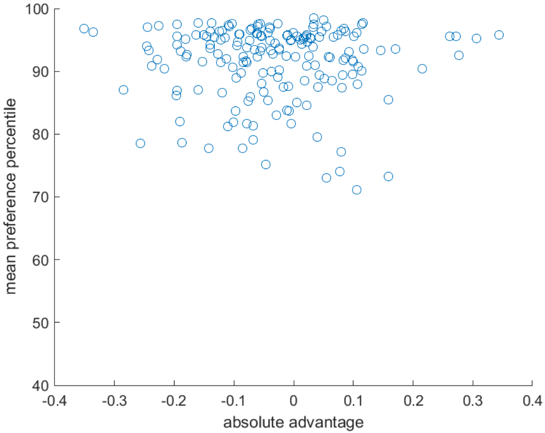
- + all options: little change
- ⇒ timing not an explanation

Distributional: and principals max VA



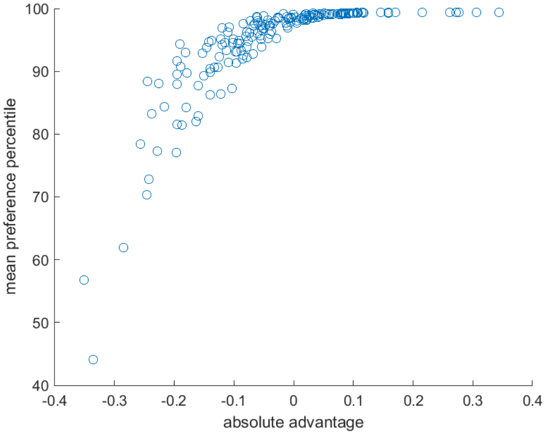
- + principals max VA: strongly favors econ. adv.
- Allocation you might expect based on teacher pref
- ⇒ **principals are the explanation**
- "Random" principal preferences push back on teacher prefs

Teacher choice by absolute advantage – Status quo



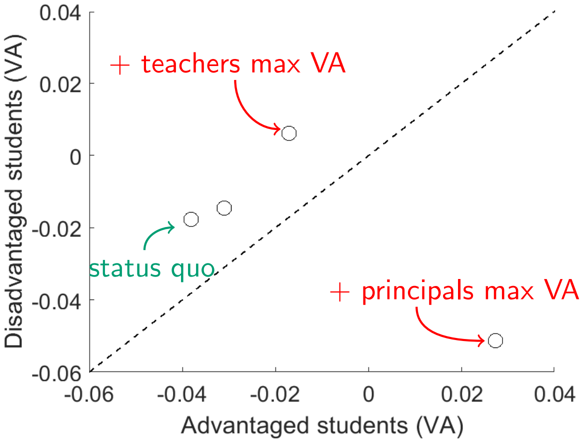
- Higher absolute advantage teachers don't get much more ability to choose

Teacher choice by absolute advantage – Principals maximize VA



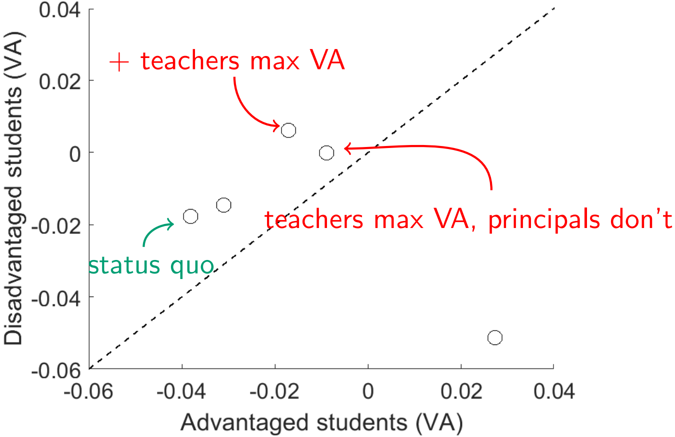
- Higher absolute advantage teachers now get much more ability to choose

Distributional: **teachers** and principals max VA



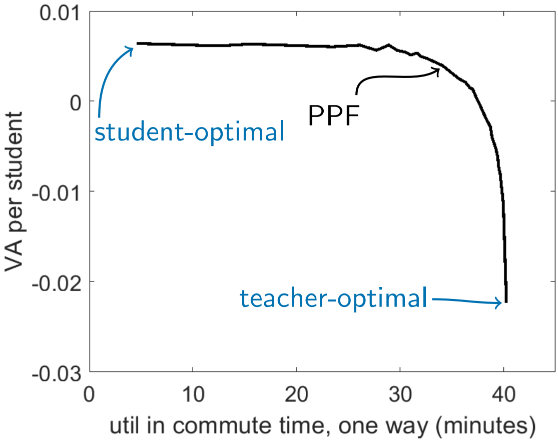
- + teachers max VA: flips back toward econ. disadv.
- ⇒ for equity, need force pushing back on prefs of strongest teachers
- Change teacher prefs, or principal prefs

Distributional: **teachers** max VA



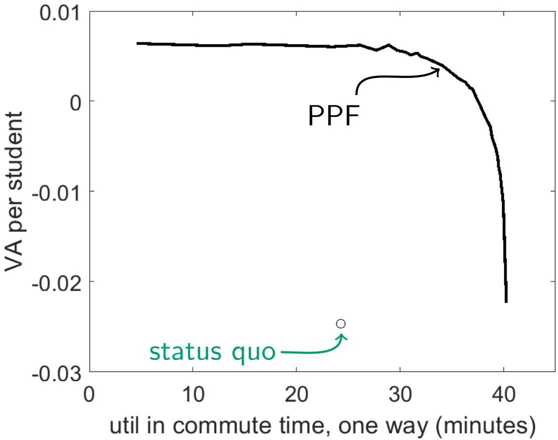
- teachers max VA: w/status quo principal prefs "changing" teacher prefs generates good outcome
- For equity: "random" principal prefs almost as good as compensating teachers to only value output
- $\Rightarrow$ teacher preference heterogeneity doesn't generate good outcome

PPF



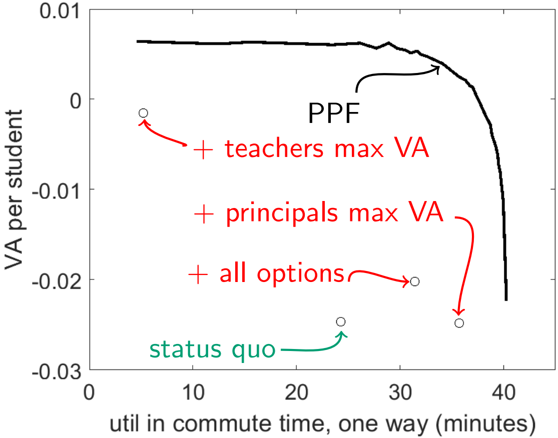
- PPF is very flat/steep near the corners

PPF and status quo allocation



- Status quo: teacher and principal preferences, options based on timing, teacher propose

All potential allocations



- Status quo: teacher and principal preferences, options based on timing, teacher propose
- + all options: small gains
- + principals max VA: small losses
- + teachers max VA: large gains

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1. Assignment model
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3. Student achievement: Production model
4. How applications / vacancies work
5. Teacher and principal preferences
6. Understanding equilibrium/counterfactuals
    - Principal preferences help counteract teacher prefs
    - "Fixing" principal preferences bad for equity/efficiency
    - Gains rely on compensating teachers
7. Bonuses Link Conclusion

Conclusion

Test scores and linkages

Applications and vacancies

Output at each school

Preferences

Matching framework

$\Rightarrow$ Basic puzzle about the current allocation:

- Teachers do not like teaching in schools with disadvantaged students.
- Disadvantaged students do not have worse teachers than advantaged students.

$\Rightarrow$ Principals' (imperfect) selection rules play a surprisingly positive role.

$\Rightarrow$ Design of bonuses to teachers depends on principals' rules.

Structural parameters

	Estimates	Standard Errors
σ_{ϵ_1}	0.450	0.000
σ_{ϵ_2}	0.470	0.000
σ_{θ_1}	0.110	0.007
σ_{θ_2}	0.088	0.015
correlation$(\theta_{c0t}, \theta_{c1t})$	0.657	0.162
σ_{μ_1}	0.249	0.007
σ_{μ_2}	0.243	0.015
correlation(μ_{j0t}, μ_{j1t})	0.859	0.035

▸ Model ▸ Validation

Experience returns

	1	2	3	4	5	6	7+
Estimate	0.056	0.077	0.083	0.088	0.088	0.091	0.070
Standard Error	0.004	0.004	0.005	0.005	0.005	0.005	0.005

▸ Back

Posting early and late

Posts in April	Posts in July: No	Yes	Total
No	8	15	23
Yes	10	88	98
Total	18	103	121

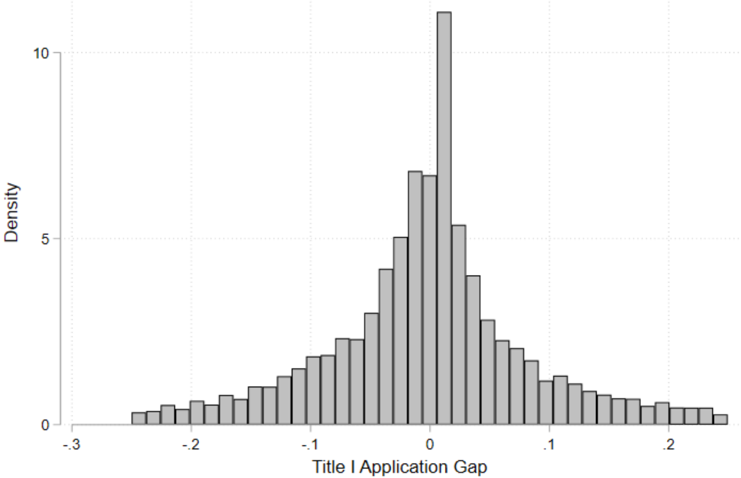
▸ Back

Application evaluations, outcomes, and timing

Panel A: Outcomes within job postings								
	Hired	Declined	Interview	Positive	Middle	Negative	Withdrew	No comment
Share with outcome	0.824	0.114	0.002	0.080	0.017	0.075	0.048	0.983
Number with outcome	2,046	283	4	199	41	187	118	2,442
Panel B: Day of application relative to hired application date								
	Obs	Mean	10th	25th	50th	75th	90th	Std. dev.
All applications	434,095	0.0	-18.0	-6.8	-0.9	5.3	18.2	16.78
No evaluation record	423,021	0.1	-17.6	-6.6	-0.9	5.3	17.9	16.53
Evaluation or outcome	11,074	-2.1	-32.1	-15.0	-4.1	7.6	31.2	24.54

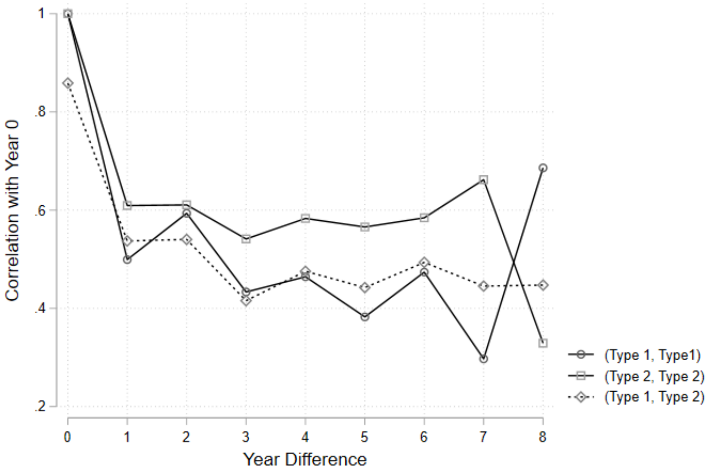
Back

Application gap by Title I status



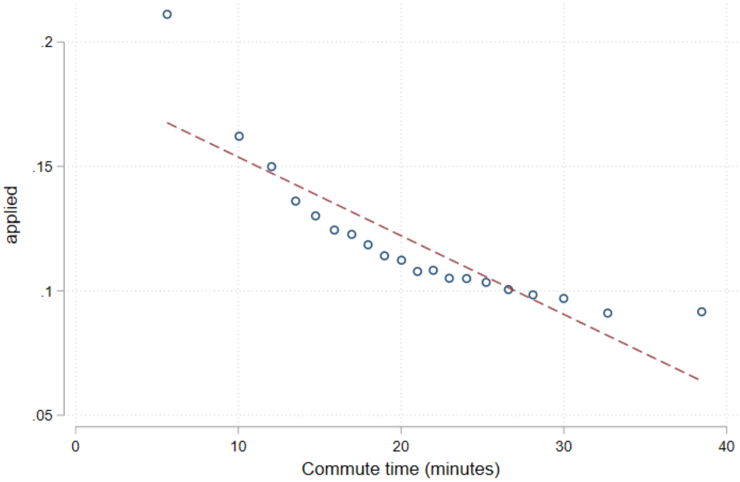
Back

Drift correlations



‣ Model

Applications by commute time



‣ Back

Assumptions (to generate moment restrictions)

1. Exogeneity and stationarity of classroom and student level shocks
   - $\mathbb{E}[\theta_{c0lt}|t] = \mathbb{E}[\theta_{c1lt}] = 0$; $Var\left(\theta_{c0lt}\right) = \sigma^2_{\theta_{0l}}$, $Var\left(\theta_{c1lt}\right) = \sigma^2_{\theta_{1l}}$, $Cov\left(\theta_{c0lt}, \theta_{c1lt}\right) = \sigma_{\theta_{0l}\theta_{1l}}$ for all t
   - $\mathbb{E}\left[\tilde{\epsilon}_{ilt}|t\right] = 0$; $Var\left(\tilde{\epsilon}_{ilt}\right) = \sigma^2_{\epsilon m}$ for $m = 0, 1$ for all t
2. Joint stationarity of teacher effects
   - $\mathbb{E}\left[\mu_{j0lt}|t\right] = \mathbb{E}\left[\mu_{j1sl}|t\right] = 0$;
     $Var\left(\mu_{j0lt}\right) = \sigma^2_{\mu_{0l}}$, $Var\left(\mu_{j1lt}\right) = \sigma^2_{\mu_{1l}}$, $Cov\left(\mu_{j0lt}, \mu_{j1lt}\right) = \sigma_{\mu_{0l}\mu_{1l}}$
   - $Cov\left(\mu_{j0lt}, \mu_{j0l,t+s}\right) = \sigma_{\mu_{0l}s}$, $Cov\left(\mu_{j1lt}, \mu_{j1l,t+s}\right) = \sigma_{\mu_{1l}s}$; $Cov\left(\mu_{j0lt}, \mu_{j1l,t+s}\right) = \sigma_{\mu_{0l}\mu_{1l}s}$
3. Independence of drift and school effects
   - Let $\bar{\mu}_{jml}$ be teacher j's mean value-added for student type m in subject l. Let k be j's assigned school in year t. Then: $\left(\mu_{jmlt} - \bar{\mu}_{jml}\right) \perp \mu_{kl}$ for $m = 0, 1$.

Back to presentation

Implementation details

- Estimate coefficients on X (student char.) using within classroom student-type variation. Form residuals.
- Estimate experience effects (dummied out 0 to 6, and single category after 7) and school effects, controlling for teacher effects on these residuals. Form residuals.
- Construct teacher-year-student type value added as best linear predictors of these residuals

Teacher j, in subject l, school k and year t is:

$$\hat{\mu}_{jklt} = n_{k0t}\hat{\mu}_{j0lt} + n_{k1t}\hat{\mu}_{j1lt} + n_{kt}f(Z_{jt}; \hat{\alpha}_l)$$

- Type-specific VA, weighted by number of students of each type in school k in year t

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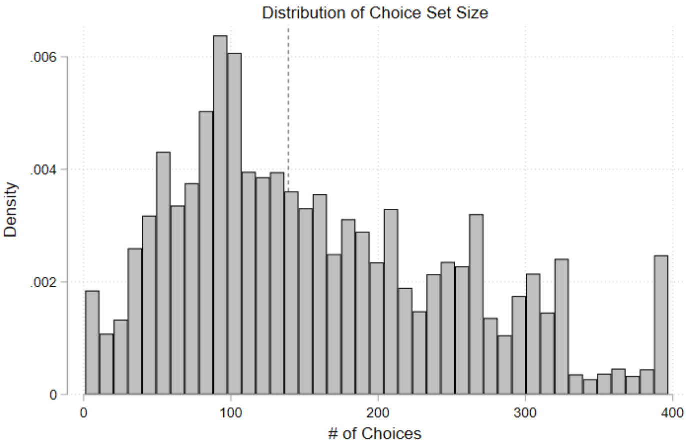
Choice set sizes



- Teachers have large sets of vacancies to apply to
  - Early/late
  - Events

Back

Teacher preference estimates – school characteristics

	Estimate	Standard Error
Constant	2.032	4.453
Commute Time	-0.073	0.001
Commute Time Missing	-1.660	0.223
Value Added	0.081	0.008
St Dev Value Added RC	0.128	0.007
School Characteristics and Interactions		
Fraction Disadvantaged	-1.188	0.136
Fraction Black	-0.452	0.132
Fraction Hispanic	0.441	0.144
Fraction Above Median Achievement	0.163	0.149
Abs Adv x Fraction Disadvantaged	-0.797	1.029
Abs Adv x Fraction Black	-1.635	1.025
Abs Adv x Fraction Hispanic	2.487	1.074
Abs Adv x Fraction Above Median Achievement	-1.997	1.185
Black x Fraction Black	1.072	0.130
Hispanic x Fraction Hispanic	0.491	0.771
St Dev Fraction Disadvantaged RC	1.591	0.034
St Dev Fraction Black RC	1.296	0.054
St Dev Fraction Hispanic RC	0.637	0.065
St Dev Fraction Above Median Achievement RC	1.397	0.045

▸ Main Coefficients

Teacher preference estimates – teacher characteristics

	Estimate	Standard Error
Teacher Characteristics		
VA Non-Disadvantaged Students	0.746	0.307
VA Disadvantaged Students	0.937	0.331
In District	-0.509	0.061
Black	-0.095	1.043
Hispanic	6.017	3.762
Female	0.284	0.064
Experience 2-3	0.070	0.083
Experience 4-6	-0.268	0.082
Experience 7+	-0.141	0.074
St Dev Random Effect	1.687	0.030
Chamberlain-Mundlak Device		
Fraction Disadvantaged Mean	-1.903	3.182
Commute Time Mean	0.032	0.004
Commute Time Missing Mean	1.231	0.249
Value Added Mean	-0.489	0.295
Fraction Black Mean	-2.786	2.707
Fraction Hispanic Mean	0.041	2.457
Fraction Above Median Achievement Mean	-0.986	4.718
Abs Adv x Fraction Disadvantaged Mean	-37.628	19.086
Abs Adv x Fraction Black Mean	36.183	18.362
Abs Adv x Fraction Hispanic Mean	15.838	19.942
Abs Adv x Fraction Above Median Achievement Mean	-16.346	6.488
Black x Fraction Black Mean	-2.200	2.412
Hispanic x Fraction Hispanic Mean	-20.462	14.686
Number of Students Mean	0.009	0.023

▸ Main Coefficients

Principal preference estimates – part 1

	Estimate	Standard Error
Constant	-4.363	0.127
St Dev Random Effect	1.531	0.022
Title I	0.521	0.156
Value Added	0.092	0.026
Value Added x Title I	0.038	0.034
Experience 2-3	0.351	0.128
Experience 2-3 x Title I	-0.005	0.163
Experience 4-6	0.271	0.117
Experience 4-6 x Title I	0.035	0.160
Experience 7+	0.097	0.089
Experience 7+ x Title I	-0.344	0.120
Experience Missing	-0.342	0.060
Experience Missing x Title I	0.371	0.086
Masters	0.188	0.098
Masters x Title I	0.124	0.125

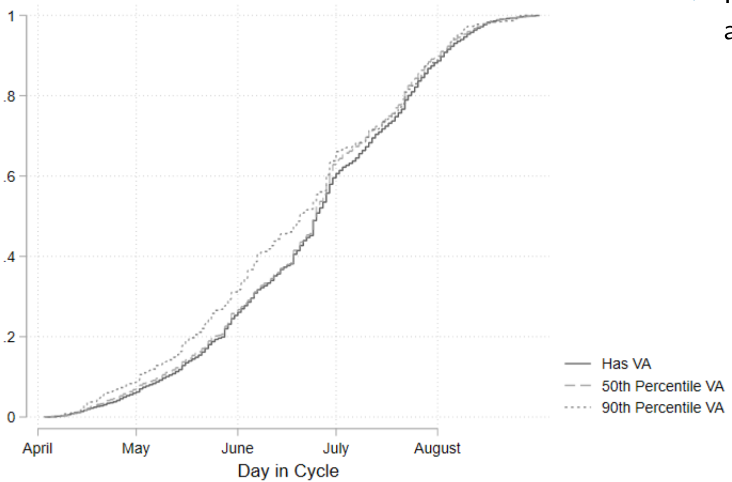
▸ Main coefficients ▸ Other coefficients

Principal preference estimates – part 2

	Estimate	Standard Error
Black	-1.035	0.227
Black x Title I	1.722	0.453
Black x Fraction Black	0.396	0.267
Black x Fraction Black x Title I	-0.253	0.511
Hispanic	-0.690	0.454
Hispanic x Title I	0.450	0.561
Hispanic x Fraction Hispanic	2.259	2.219
Hispanic x Fraction Hispanic x Title I	-1.833	2.345
Female	0.053	0.106
Female x Title I	0.031	0.129
Gender Missing	-0.327	0.230
Gender Missing x Title I	-0.197	0.277
Race Missing	-0.530	0.210
Race Missing x Title I	0.374	0.247
VA Missing	0.490	0.089
VA Missing x Title I	-0.230	0.124

‣ Main coefficients ‣ Prior coefficients

Market timing



- Higher VA teachers arrive slightly earlier
  - Back

Validation of value-added forecasts

	Mean Res	Mean Res	Mean Res
VA (Heterog)	1.052***		1.060***
	(0.00650)		(0.00681)
Post Transfer		-0.00243	0.00576**
		(0.00367)	(0.00280)
VA * Post Transfer			-0.0885***
			(0.0212)
Constant	0.00810***	0.00779***	0.00745***
	(0.000835)	(0.00174)	(0.000883)
Subject	Math	Math	Math
Mean DV	0.00764	0.00754	0.00764
Clusters	21514	21834	21514
N	74552	75459	74552

▸ Back

Same-race and same-gender match effects

	Student Res
Black Teacher - Black Student	0.00225
	(0.00164)
Hispanic Teacher - Hispanic Student	-0.00556
	(0.00549)
Female Teacher - Female Student	0.00478***
	(0.000550)
Fixed Effects	Teacher, School
Mean DV	0.0000115
Clusters	37940
N	5158740

▸ Back

Match effects across transfers

	(1) Math	(2) Reading
Share ED	-0.017***	0.009**
	(0.006)	(0.004)
Share ED $\times$ Better with ED	0.031***	0.017***
	(0.008)	(0.005)
Student obs	1,198,194	1,205,088
Mean VA difference	0.003	0.057
SD VA difference	0.043	0.914
fixed effects	Teacher	Teacher

Standard errors are clustered by teacher appear in parentheses
* $p < .1$, ** $p < .05$, *** $p < .01$

Back

Validation across big composition changes

	Mean Res	Mean Res
VA – below 25th (disadv)	1.030***	
	(0.0127)	
VA – 25th-75th (disadv)	1.054***	
	(0.00877)	
VA – above 75th (disadv)	1.075***	
	(0.0121)	
VA – below 10th (disadv)		0.990***
		(0.0223)
VA – 10th-90th (disadv)		1.058***
		(0.00698)
VA – above 90th (disadv)		1.066***
		(0.0228)
Constant	0.00810***	0.00810***
	(0.000835)	(0.000835)
Subject	Math	Math
Mean DV	0.00764	0.00764
p-value on Equal Coeff	.035	.012
Clusters	21514	21514
N	74552	74552

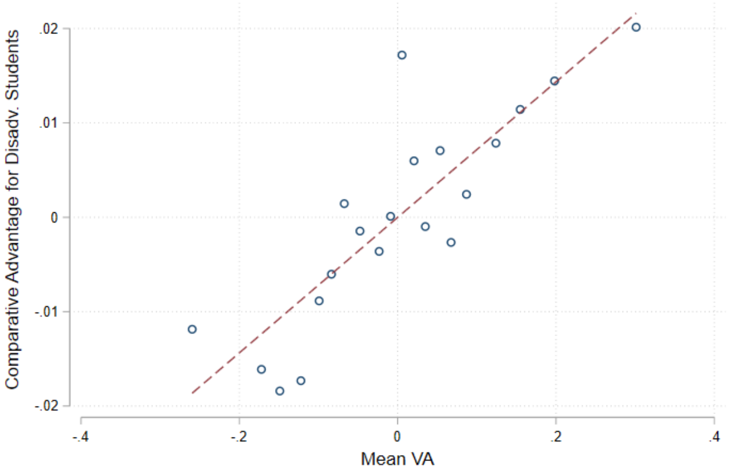
▸ Back

Validation across big class size changes

	Mean Res	Mean Res
VA – below 25th (size)	1.052***	
	(0.0128)	
VA – 25th-75th (size)	1.057***	
	(0.00920)	
VA – above 75th (size)	0.978***	
	(0.0115)	
VA – below 10th (size)		1.011***
		(0.0224)
VA – 10th-90th (size)		1.066***
		(0.00713)
VA – above 90th (size)		0.961***
		(0.0188)
Constant	0.00819***	0.00800***
	(0.000845)	(0.000843)
Subject	Math	Math
Mean DV	0.00764	0.00764
p-value on Equal Coeff	0	0
Clusters	21514	21514
N	74552	74552

▸ Back

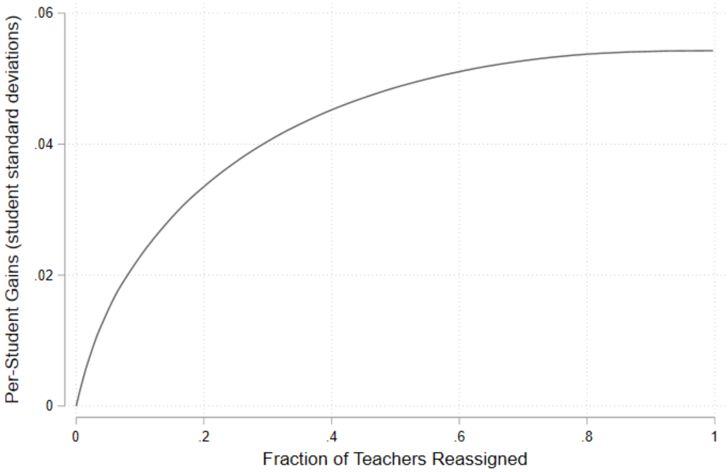
Relationship between absolute and comparative advantage



- High absolute advantage teachers tend to have higher comparative advantage with disadvantaged students

▸ Back

Gains as a function of number of new assignments



- Large gains even if only 20% – 60% of teachers are reassigned

▸ Back

Likelihood ratio test

$\bar{A}_{jcmt}$: teacher-classroom-student type mean residuals

$$\begin{pmatrix} \bar{A}_{jc1t} \\ \bar{A}_{jc'2t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{\mu_1} + \sigma^2_{\theta_1} + \frac{\sigma^2_{\epsilon_1}}{N_{jc0t}} & \sigma_{\mu_1\mu_2} \\ \sigma_{\mu_1\mu_2} & \sigma^2_{\mu_2} + \sigma^2_{\theta_2} + \frac{\sigma^2_{\epsilon_2}}{N_{jc2t}} \end{pmatrix} \right) \tag{1}$$

- H_0: baseline model
- H_1: $\sigma^2_{\mu_1} = \sigma^2_{\mu_2}$, $\sigma^2_{\theta_1} = \sigma^2_{\theta_2}$, $\sigma^2_{\epsilon_1} = \sigma^2_{\epsilon_2}$, and $\sigma_{\mu_1\mu_2} = 0$

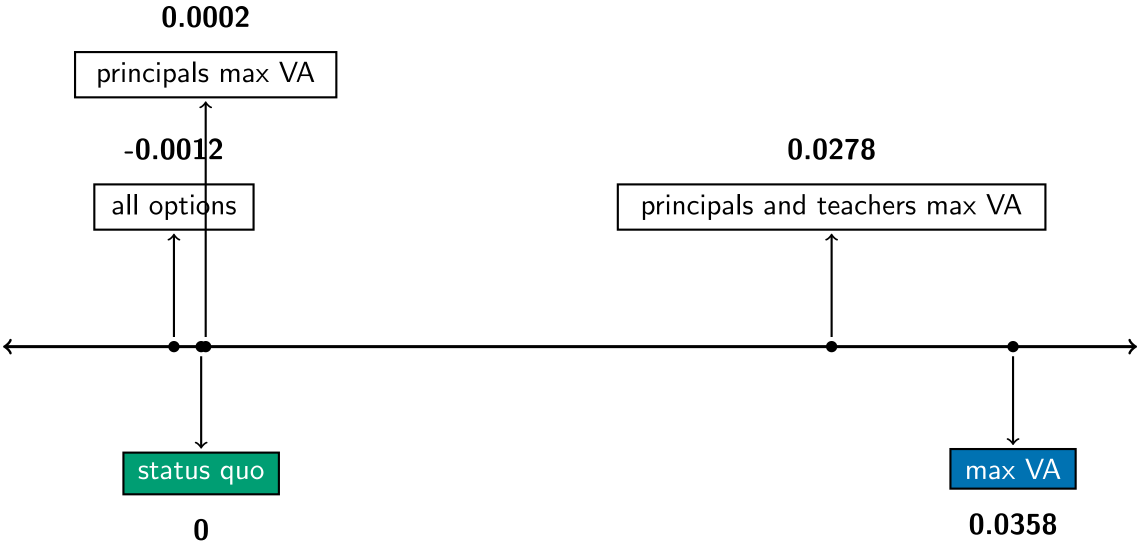
Reject homogeneous model in favor of heterogeneous model (test stat $= 610$)

▸ Model ▸ Validation

	Posting			Applying			Hiring		
	Vacs	Share	Share TI	Apps	Share	Share TI	Apps	Share	Share TI
April	295	16.24	0.62	24799	7.13	0.50	393	13.23	0.69
May	392	21.57	0.52	70248	20.21	0.50	585	19.70	0.63
June	502	27.63	0.52	108776	31.29	0.51	827	27.85	0.60
July	451	24.82	0.42	94171	27.09	0.50	755	25.42	0.50
August	167	9.19	0.46	44673	12.85	0.51	358	12.05	0.57
Total	1807	100		342667	100		2918	2918	

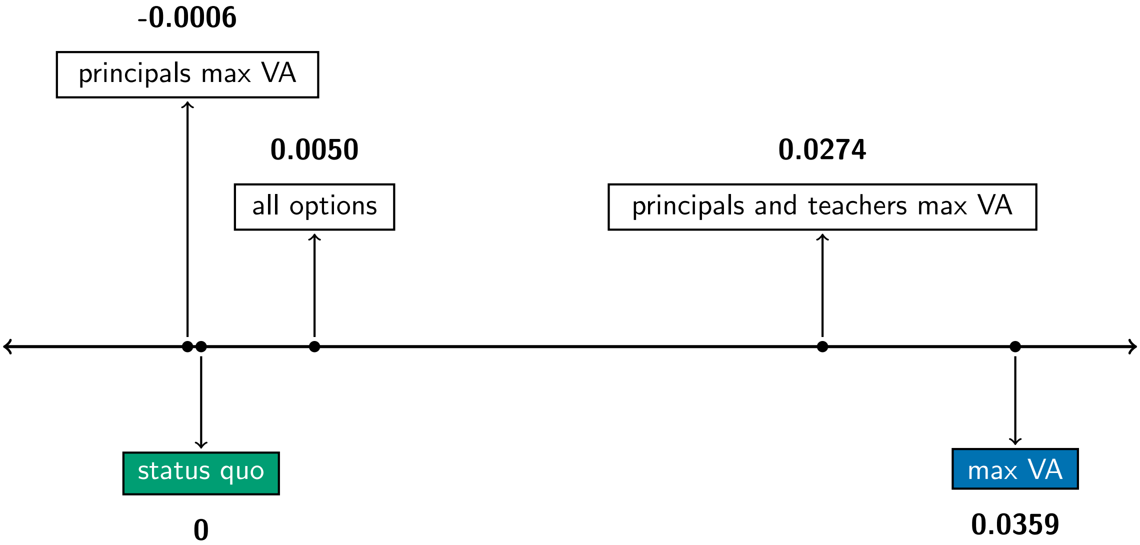
Back

Output across different allocations – White / non-White VA



Back

Output across different allocations – Lagged Achievement VA



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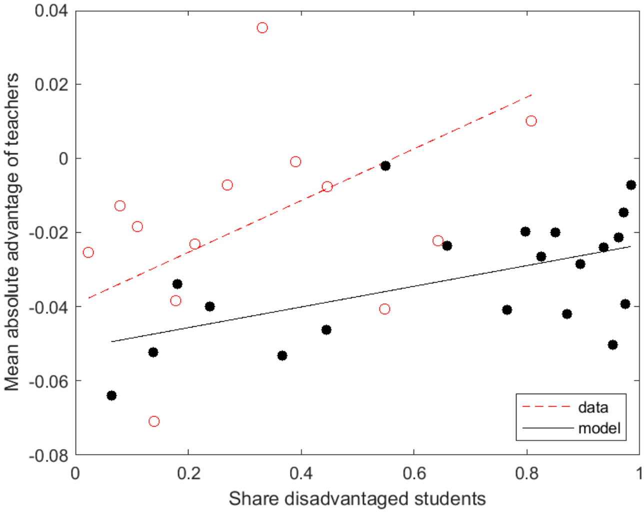
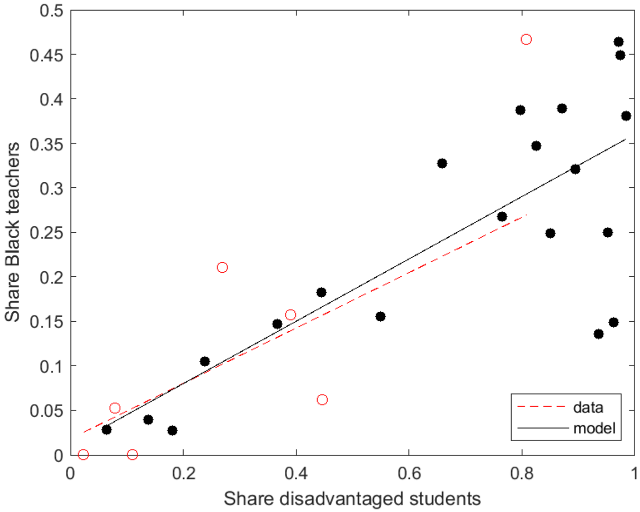
Fit – Absolute Advantage

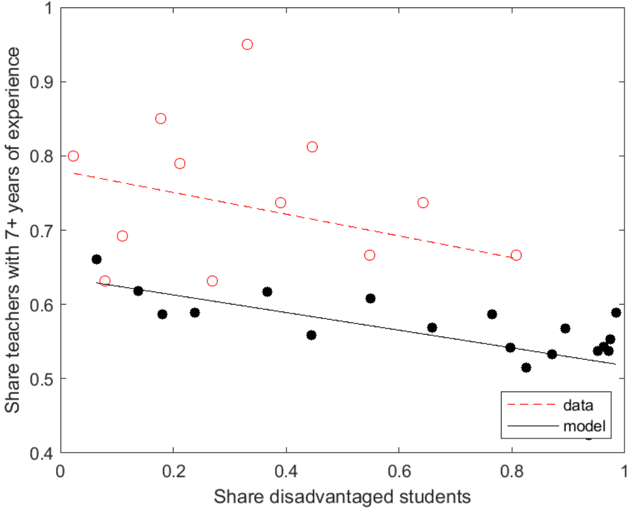


Fit – Fraction black teachers

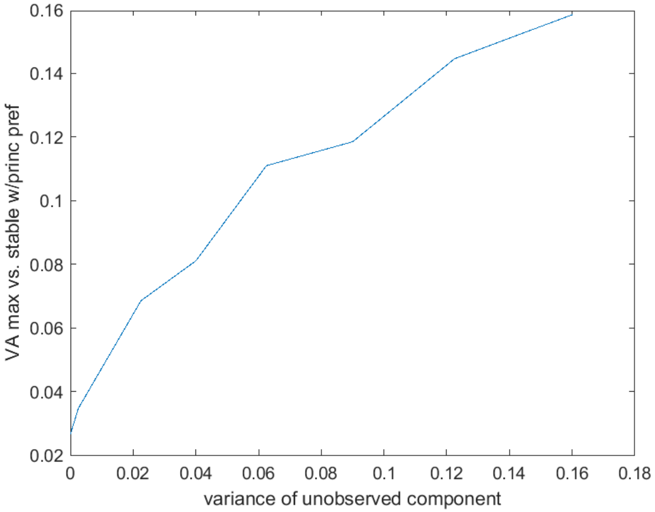


▸ Back

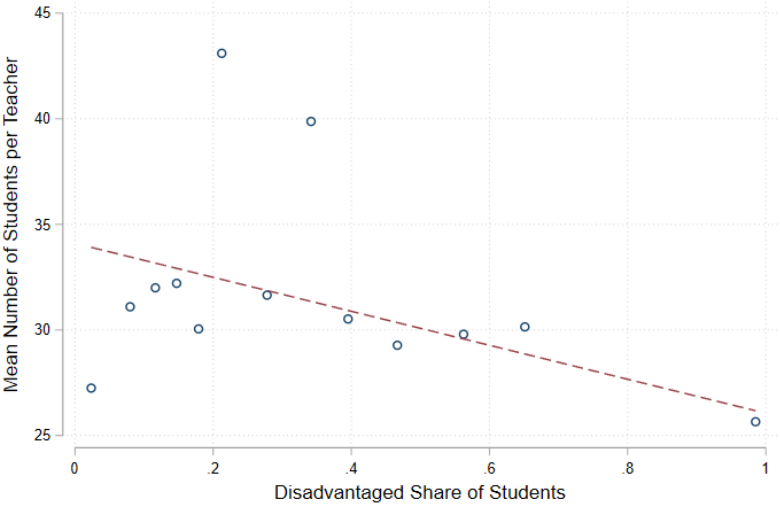
Fit – Experienced teachers



Simulation of additional unobserved value added components

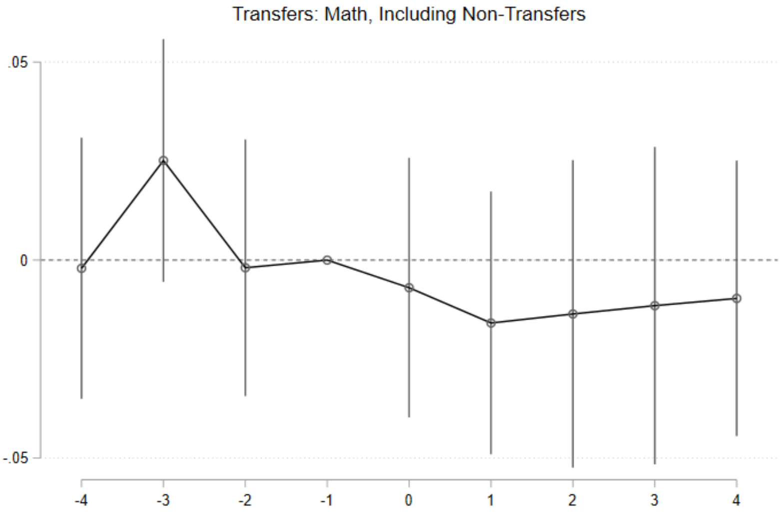


Relationship between student composition and class size



▸ Back

Transfer event study: math residual scores



‣ Back

Autocorrelation in school's mean class size

Table: Cross-correlation table

Variables	Class Size t	Class Size t-1	Class Size t-2	Class Size t-3	Class Size t-4
Class Size t	1.0000				
Class Size t-1	0.7329	1.0000			
	(0.0000)				
Class Size t-2	0.6248	0.6966	1.0000		
	(0.0000)	(0.0000)			
Class Size t-3	0.4093	0.5261	0.6598	1.0000	
	(0.0000)	(0.0000)	(0.0000)		
Class Size t-4	0.3722	0.3746	0.4365	0.5796	1.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	

Nb. obs. : 247

Back

Autocorrelation in teacher's mean class size, controlling for school mean

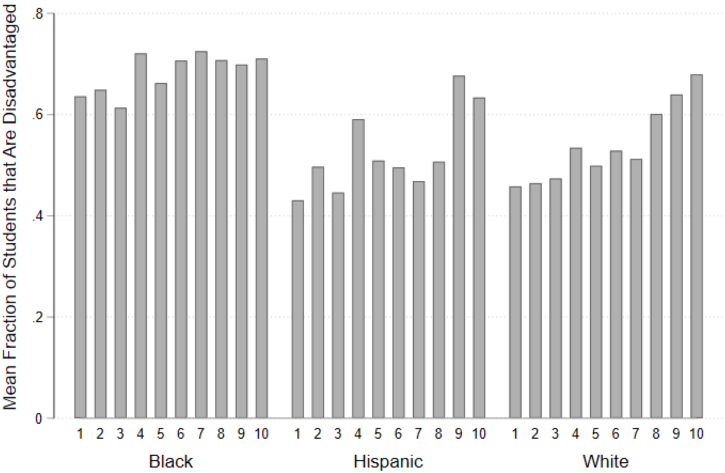
Table: Cross-correlation table

Variables	(Res.) Size t	(Res.) Size t-1	(Res.) Size t-2	(Res.) Size t-3	(Res.) Size t-4
(Res.) Size t	1.0000				
(Res.) Size t-1	0.3668	1.0000			
	(0.0000)				
(Res.) Size t-2	0.2688	0.3717	1.0000		
	(0.0000)	(0.0000)			
(Res.) Size t-3	0.2900	0.1272	0.2699	1.0000	
	(0.0000)	(0.0186)	(0.0000)		
(Res.) Size t-4	0.1173	0.1438	0.0698	0.3098	1.0000
	(0.0301)	(0.0077)	(0.1978)	(0.0000)	

Nb. obs. : 342

Back

Student composition by teachers' commute time



Black teachers' closest schools are more likely to have disadvantaged students ▸ Back

Autocorrelation in school's composition

Table: Cross-correlation table

Variables	Frac Disadv t	Frac Disadv t-1	Frac Disadv t-2	Frac Disadv t-3	Frac Disadv t-4
Frac Disadv t	1.0000				
Frac Disadv t-1	0.9602	1.0000			
	(0.0000)				
Frac Disadv t-2	0.9430	0.9555	1.0000		
	(0.0000)	(0.0000)			
Frac Disadv t-3	0.9363	0.9370	0.9496	1.0000	
	(0.0000)	(0.0000)	(0.0000)		
Frac Disadv t-4	0.9435	0.9467	0.9554	0.9775	1.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	

Nb. obs. : 247

▸ Back

Autocorrelation in teacher's classroom composition, controlling for school mean

Table: Cross-correlation table

Variables	(Res.) Dis t	(Res.) Dis t-1	(Res.) Dis t-2	(Res.) Dis t-3	(Res.) Dis t-4
(Res.) Dis t	1.0000				
(Res.) Dis t-1	0.3170	1.0000			
	(0.0000)				
(Res.) Dis t-2	0.2898	0.3200	1.0000		
	(0.0000)	(0.0000)			
(Res.) Dis t-3	0.1524	0.2076	0.3723	1.0000	
	(0.0047)	(0.0001)	(0.0000)		
(Res.) Dis t-4	0.0921	0.0512	0.2203	0.3925	1.0000
	(0.0889)	(0.3450)	(0.0000)	(0.0000)	

Nb. obs. : 342

Back

Teacher bonuses

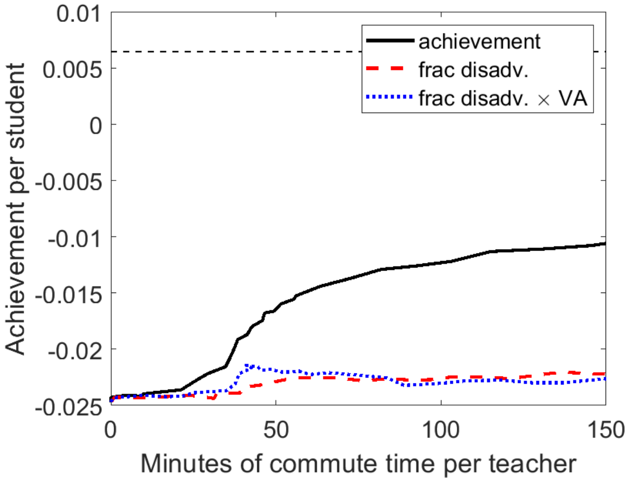
$$u_{jpt} = \tilde{u}_{jpt} + \gamma \underbrace{\left(b_0 + b_1 z_{jpt}\right)}_{\text{bonus}}$$

- z_{jpt}: object on which bonuses are based
- Vary b_1 to trace out different bonuses sizes
- For each b_1, solve for teacher-optimal stable equilibrium assignments: $p^*(j)$
- Choose b_0 to minimize cost while holding *all* teachers harmless
- Total cost per teacher: $b_0 + b_1 z_{jp^*(j)t}$

Consider bonuses over three objects (z_{jpt}):

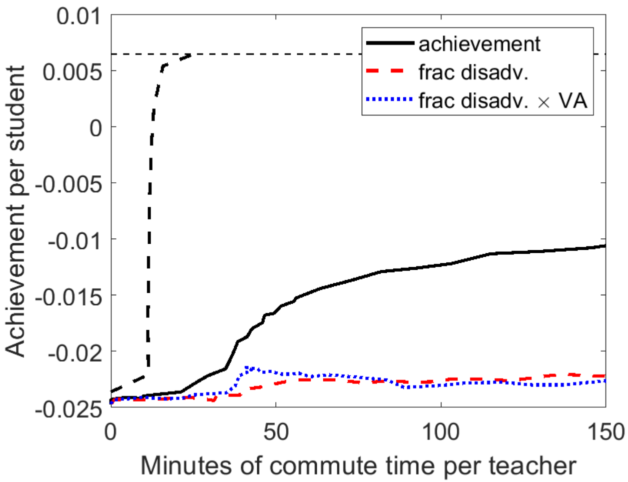
1. Total output: $\sum_m n_{k(p)m} \hat{\mu}_{jm}$
2. Fraction disadvantaged: $p_{k(p)1t}$
3. Absolute advantage x fraction disadvantaged: $(p_{0t}\hat{\mu}_{j0t} + (1 - p_{0t})\hat{\mu}_{j1t})p_{k(p)1t}$

Student achievement with teacher bonuses



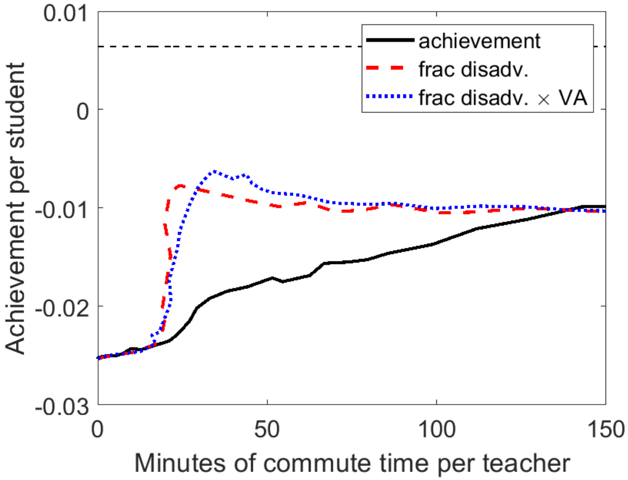
- Targeting achievement more cost-effective than targeting disadvantaged students
- Achievement gains are very costly

Student achievement with teacher bonuses



- Targeting achievement more cost-effective than targeting disadvantaged students
- Achievement gains are very costly
- **Bonuses based on preferences are much more cost-effective**

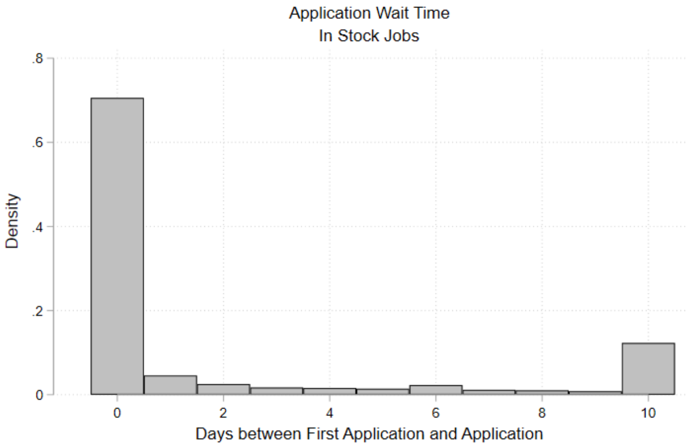
Student achievement with teacher and principal bonuses



- When principals hire according to VA, targeting disadvantaged students is more cost-effective
- $\Rightarrow$ optimal teacher bonus depends on whether principals receive bonuses

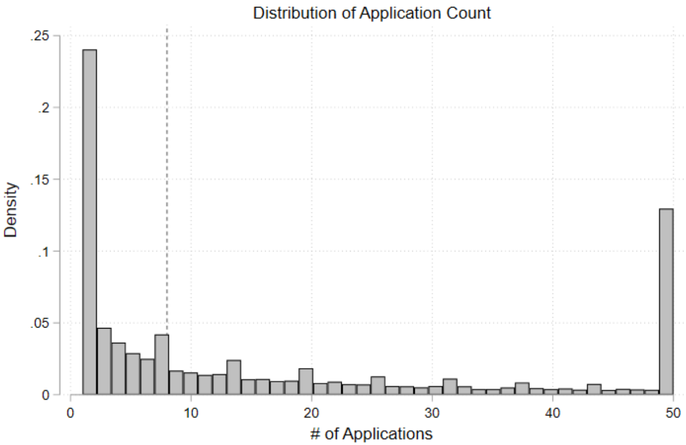
Back

Application time to positions "in stock"



- Teachers apply immediately to jobs already posted
- Similar picture for jobs "in flow"
- $\Rightarrow$ no strategic delay
- $\Rightarrow$ we define choice sets as all jobs that were available on day of 1st application until day of last application
  - Median size: ≈ 120
  - ▸ Distribution

Application set sizes



- Teachers apply to many positions
- ⇒ possible to identify preference heterogeneity

▸ Back

How the market works

- Principals post vacancies when they learn a position is open.
- In-district teachers typically have a one-week head start in early April. Details on timing
- Apply in a centralized system. Agree not to "disintermediate" (MC of an app. ≈ 0).
- Principals review and rate applications, and interview and make offers on a rolling basis.
- Offers explode in 24 hours.

Back Analysis

Student achievement specification

$$A_{it}^{*} = \beta_s X_{it} + f(Z_{jt}; \alpha) + \mu_{jmt} + \mu_k + \mu_t + \theta_{cmt} + \tilde{\epsilon}_{it}$$

- Student i, type m ("adv", "disadv"), school k, classroom c, teacher j, year t
- A_{it}^{*}: test score
- X_{it}: student determinants of achievement (lagged test scores,....)
- Z_{jt}: teacher experience
- μ_{jmt}: teacher-student type-year effects
- μ_k: school factors (principal)
- μ_t: year effects
- θ_{cmt}: classroom-student type specific shocks

Substantive restriction:

- Nothing with a $j - k$ subscript; i.e., "match" effects depend on composition of students

Back

Measuring value-added

$$A^*_{it} = \beta_s X_{it} + f(Z_{jt};\alpha) + \mu_{jmt} + \mu_k + \mu_t + \theta_{cmt} + \tilde{\epsilon}_{it}$$

- Student i, type m ("adv", "disadv"), school k, classroom c, teacher j, year t
- Z_{jt}: teacher experience
- μ_{jmt}: teacher-student type-year effects

Student types:

- Economically disadvantaged (National School Lunch program definition)
- Non-economically disadvantaged

Measuring value-added

$$A^*_{it} = \beta_s X_{it} + f(Z_{jt}; \alpha) + \mu_{jmt} + \mu_k + \mu_t + \theta_{cmt} + \tilde{\epsilon}_{it}$$

- Student i, type m ("adv", "disadv"), school k, classroom c, teacher j, year t
- Z_{jt}: teacher experience
- μ_{jmt}: teacher-student type-year effects

Forecasting:

- Follow Chetty, Friedman, and Rockoff (2014) in allowing drift
- Follow Delgado (2021) in allowing correlation across types, using prior data only
- Shrink estimates

Measuring value-added

$$A^*_{it} = \beta_s X_{it} + f(Z_{jt}; \alpha) + \mu_{jmt} + \mu_k + \mu_t + \theta_{cmt} + \tilde{\epsilon}_{it}$$

- Student i, type m (“adv”, “disadv”), school k, classroom c, teacher j, year t
- Z_{jt}: teacher experience
- μ_{jmt}: teacher-student type-year effects

VA of teacher j with type m students in year t: $\hat{\mu}_{jmt}$
VA of teacher j, in school k and year t is:

$$\widehat{VA}_{jkt} = n_{k0t}\hat{\mu}_{j0t} + n_{k1t}\hat{\mu}_{j1t} + f(Z_{jt}; \hat{\alpha})$$

▸ Details ▸ Other match specifications ▸ Experience ▸ Structural ▸ Drift ▸ Joint distribution ▸ LR test