Who Gets Placed Where and Why?
An Empirical Framework for Foster Care Placement

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Massachusetts Institute of Technology

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Motivation

Foster care

System that provides **temporary care** for children removed from home by child-protective services

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In the U.S.

- Up to **5.91% (1 out of 17)** of children are placed in foster care
- On any given day, nearly **450,000** children are in foster care
- On average, children stay **19 months** in foster care (median = 14 months)
- Exit foster care: **reunification** (55%), **adoption** (35%), **emancipation** (10%)

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Motivating Problem

Many foster **children go through several foster homes** before exiting foster care

- **Prevalent problem:** 56.1% > 1, avg $= 2.56$ (U.S., 2015)
- **Placement disruptions** are **detrimental** for children
- Social workers (say they) try to **minimize disruptions**
  - Do what is best for children, and minimize workload

This paper

1. **How is it done?**
   - **NO** explicit systematic matching algorithm → **Revealed preference** exercise
   - Formulate and estimate **structural model** of **matching in foster care**
   - How do social workers weigh **duration** and **disruptions** when assigning children to foster homes
   - Model accounts for **sample selection** due to **unobservable heterogeneity**

2. **How to improve it?**
   - Use model estimates to study **new policies aimed at improving outcomes**
   - Keep **estimated preferences fixed**
   - Improve placement outcomes by **increasing market thickness** through:
     - **Temporal aggregation** (delaying assignments)
     - **Geographical centralization** (centralizing regional offices)

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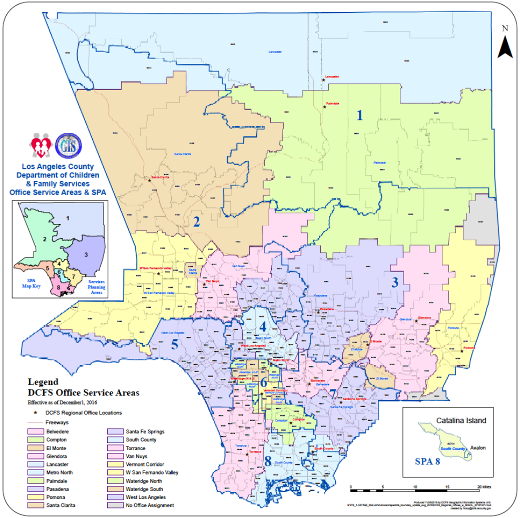
Los Angeles County, CA

Details

- Foster care administered at the **county level**
- County with **most foster children** in the U.S.
  - On any given day, **17,000** children in foster care
  - **40** children assigned to a foster home everyday
  - **19** regional offices (color-coded)
- **Data** Confidential administrative records from LA's child-protective services agency
- **Sample** Every placement assigned in Jan–Feb 2011 (2,087 children; 2,358 placements)
  - Observe outcomes until 2016



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Main Findings

- Within regional offices, social workers do a **“fair job”** assigning children to foster homes
  - Placements **more likely to be disrupted are less likely to be assigned**
  - Social workers **minimize disruptions and the time children stay in foster care**
  - Why “fair”? There might be **room for improvement**

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- $\uparrow$ market thickness by **delaying assignments does not improve outcomes substantially**

- **Decentralization** into regional offices **is sub-optimal**: if system were centralized...
  - Avg. $\mathbb{P}(\textit{disruption}) \downarrow$ 4.2 %-pts $\Longrightarrow$ 8% $\downarrow$ placements per child before exiting foster care
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  - 54% less **distance between foster homes and schools**
- **Moral** *Social workers do a decent job at matching; exogenous institutions cause inefficiencies*
- **Policy Conclusion** *Improve coordination between regional offices, do not delay assignments*

Outline

1. Model Overview
2. Key Estimation Results
3. Counterfactual Policy Analysis

Model Overview

- **Unit of observation**: day within a regional office ("**market**")
- **Empirical model**:

$$\underbrace{(M, \mathbf{T}, \mathbf{R})}_{\text{Endogenous}} \quad \Big| \quad \underbrace{(C, H, \mathbf{X}, \mathbf{Y})}_{\text{Exogenous}}$$

- M = **matching** between children and foster homes
- $\mathbf{T} = (T_{ch})_{(c,h)\in M}$ **duration** of placements
- $\mathbf{R} = (R_{ch})_{(c,h)\in M}$ **termination reason** of placements
  - $R_{ch} \in \{$ disruption, permanency, emancipation $\}$
  - permanency $\equiv$ reunification or adoption

- C = set of **children**
- H = set of **foster homes**
- $\mathbf{X}$ = children **characteristics** (age, school zip-code)
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    - $u(T, R; T_{em}) = \mu_R + \varphi_R \log T + \bar{\varphi}_R \log T_{em}$
    - $\mu_R =$ utility of termination reason R
    - $\varphi_R =$ mg. utility of duration conditional on termination reason R
    - $T_{em,c} =$ time until emancipation (18 – child's age)

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Identification and Estimation

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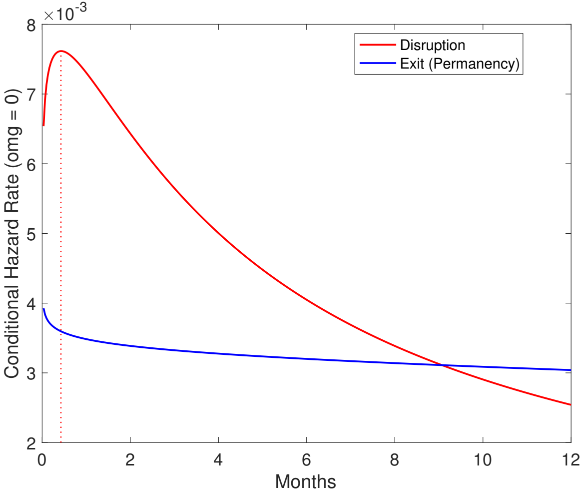
- **Note 2**: $M \,|\, (C, H, \mathbf{X}, \mathbf{Y}) \sim$ Mixed Probit
- **Identification**: exogenous variation in $(C, H, \mathbf{X}, \mathbf{Y})$ across markets

Key Estimation Results

Estimated Hazard Rates



Matching Utility Estimates

Hazard Rates APEs

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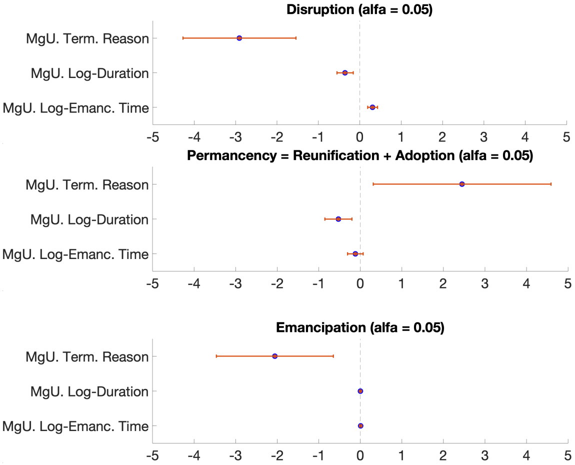

Notes: Estimation via Simulated Maximum Likelihood. Sample size: 1,467 markets; 2,358 placements. Sample period: Jan–Feb 2011.

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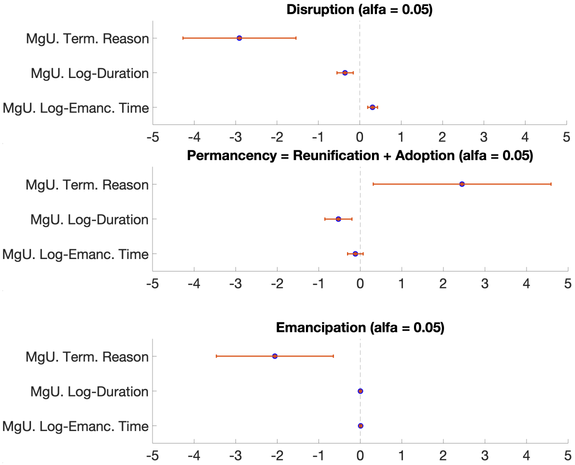

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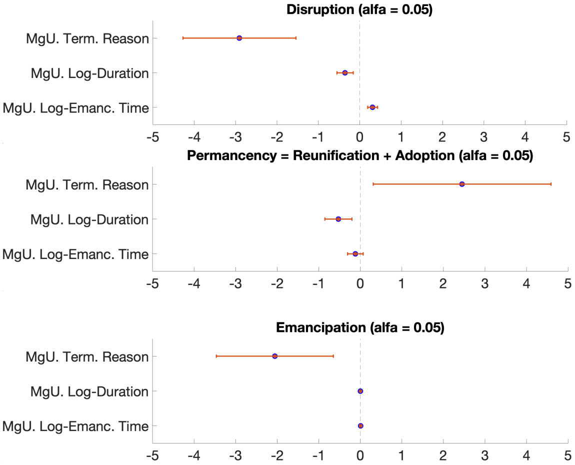

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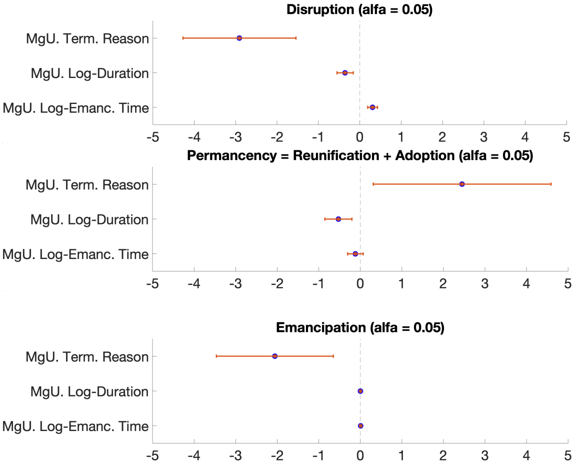
    2. Social workers minimize the time children stay in foster care:
        - $\varphi_{perm} < 0$
    3. "Stronger" preference over termination reason than duration
        - Unwilling to trade off an exit to permanency with long duration, over a disruption with short duration



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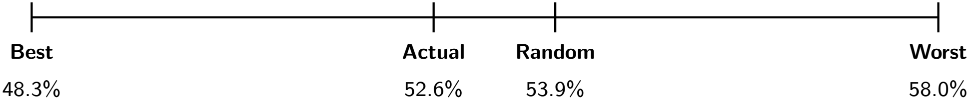
- Simulate assignments under alternative matching policies (change parameters in utility function)
- Average predicted disruption probability across assigned placements:



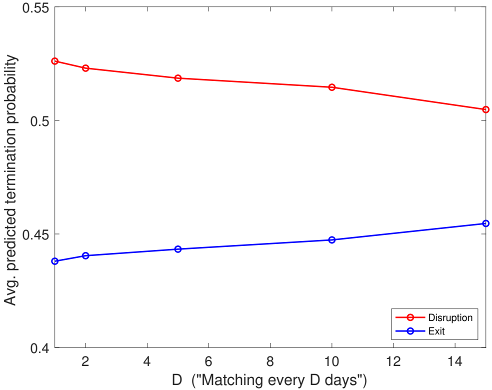
- Social workers
  - could do worse (up to a 10.3% increase)
  - do better than random (2.5% increase)
  - but could also do better (up to 8.2% decrease)

Counterfactual Policy Analysis

Counterfactual Policy Analysis

- Increasing **market thickness** by aggregating markets
  - **Centralization** Pool regional offices together into a single county-wide market
  - **Temporal aggregation** Assign placements within regional offices every $D \geqslant 1$ days
  - **Benchmark** Pool regional offices together and match everyone at once ($D = \infty$)
- Assume zero costs of information aggregation
  - Obtain upper bound of gains from greater market thickness

Temporal and Spatial Aggregation: Expected Outcomes



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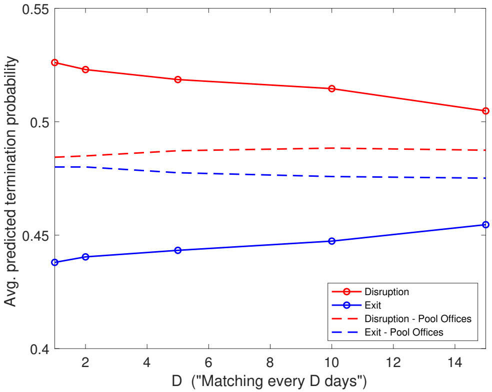

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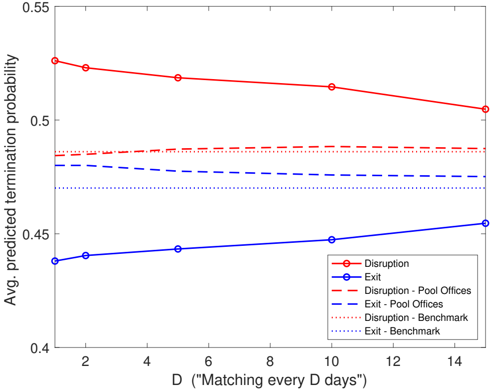

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- dashed lines = spatial aggregation
- dotted lines = maximum market thickness

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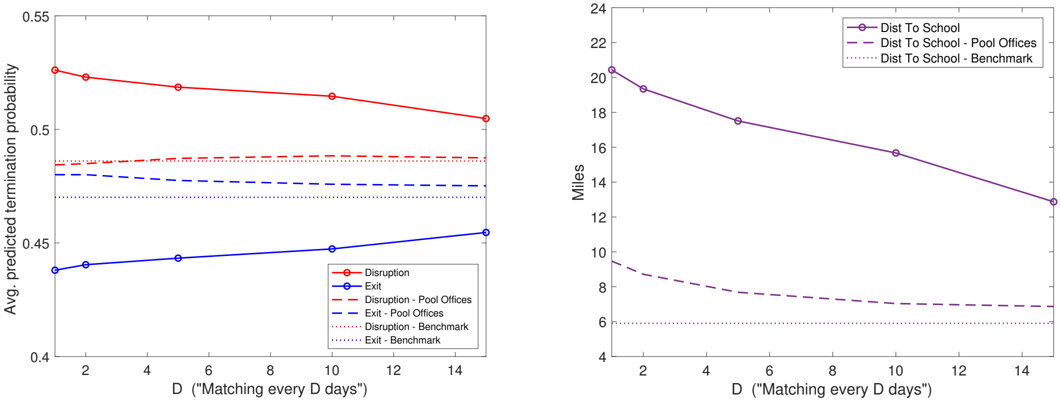


Notes:

- y-axis = avg. termination probability (left), avg. distance to school (right)
- x-axis = temporal aggregation
- dashed lines = spatial aggregation
- dotted lines = maximum market thickness

Conclusion

Related Literature

- **Objective** Formulate and estimate **structural model** of placement assignment and outcomes
- Social workers do a **"fair job"** at minimizing disruptions
  - $\uparrow \mathbb{P}(\textit{disruption}) \implies \downarrow \mathbb{P}(\text{placement})$
  - Better than random, but there is room for improvement
- **However,...**
  - Regional offices **coordinate sub-optimally** with one another.
  - There are **gains from centralizing the assignment of placements** across LA County
    - $\mathbb{P}(\textit{disruption}) \downarrow 4.2$ %-pts $\implies$ 8% $\downarrow$ fewer foster homes per child
    - 54% less distance between foster homes and schools
- **What do we learn?**
  - Social workers do a **fair job** at matching, but **exogenous institutions cause inefficiencies**
  - **Policy recommendation** Improve coordination between regional offices, do not delay assignments

Thank you.

Motivation (Sources) Back

In the U.S.

- **5.91% (1 out of 17)** of children are placed in foster care
  - Estimated share of children from total population who spent at least a day in foster care before their 18th birthday, 2000–2011 (Wildeman and Emanuel 2014)
- Every year, more than **half a million children** go through foster care
  - 2013 (638,041) through 2017 (690,548)
  - Source: U.S. Department of Health and Human Services (AFCARS Report, 2018)
- On any given day, nearly **450,000** children are in foster care
  - 10/30/2013 (400,39) through 10/30/2017 (442,995) (AFCARS Report, 2018)
- On average, children stay **19 months** in foster care (median = 14 months)
  - Average and median length of stay across children who exited during FY 2017 (AFCARS Report, 2018)
- Exit foster care: **reunification** (55%), **adoption** (35%), **emancipation** (10%)
  - Discharge reasons across children who exited during FY 2017 (AFCARS Report, 2018)

Why market design in foster care? (Sources)

Back

- **Prevalent problem:** 56.1% > 1, avg $= 2.56$ (U.S., 2015) (Source: AFCARS)
- **Placement disruptions** detrimental for children's development
  - $\uparrow$ emergency and mental-health services (Rubin et al. 2004; Rubin, Alessandrini, Feudtner, Localio, and Hadley 2004)
  - $\uparrow$ behavioral and attachment problems (Gauthier, Fortin, and Jéliu 2004; Rubin, O'Reilly, Luan, and Localio 2007)
  - affect children's bodily capacity to regulate cortisol (stress hormone) (Fisher, Ryzin, and Gunnar 2011)
- Also, associated with **worse outcomes in adult life**:
  - More and longer placements $\Rightarrow \uparrow$ depression, smoking, drug use, criminal convictions (Dregan and Gulliford 2012)

Why structural model? Back

- **Main Challenge**
  - **Objective:** Recover **preferences over outcomes** from data on which **matchings were chosen**
  - Placement outcomes (duration and disruptions) are **lotteries**

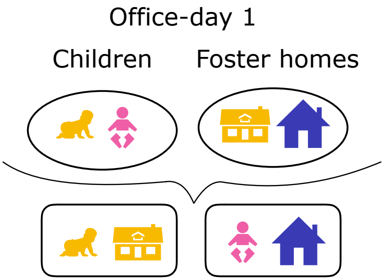
  $\Rightarrow$ Need to estimate **conditional distribution of outcomes**

- **Problem** Possible selection on **unobservables**
  - Unobservables $\rightarrow$ Expected match outcomes $\rightarrow$ Matching $\rightarrow$ Observed outcomes are selected
  - **Endogeneity** when estimating distribution of outcomes conditional on observables

- **Solution**
  - **Structural model** of **matching** and **placement outcomes**, with **unobserved heterogeneity**
  - **Identification** Exogenous variation across dates and regions at which children enter foster care

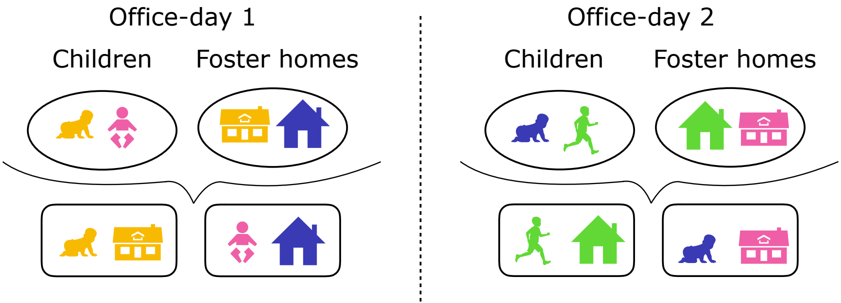
Market Thickness

Back



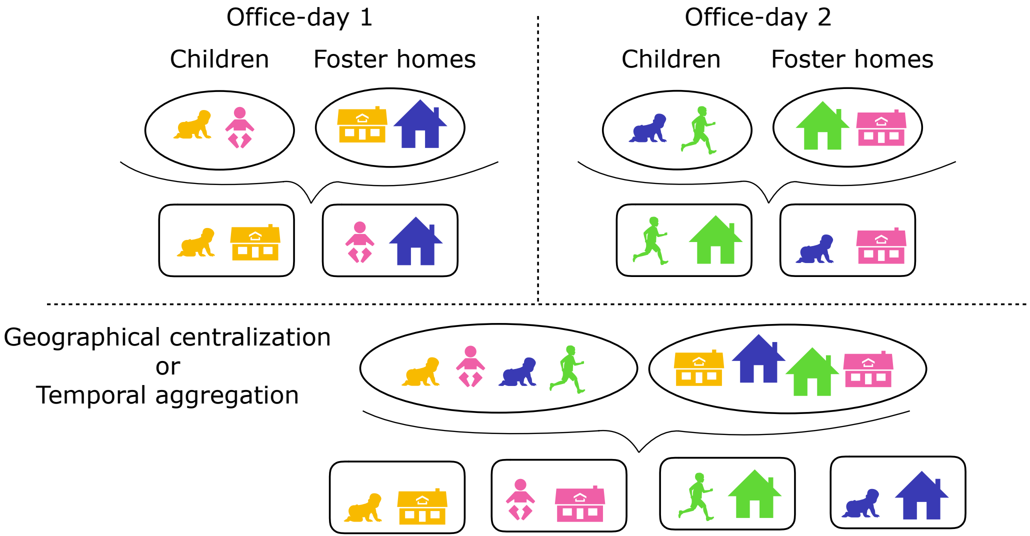
Market Thickness

Back



Market Thickness

Back



Background and Data

Back | Markets and Excess Supply | Summary Statistics

- **Data** Confidential county records (accessed through court order) from the Los Angeles County Department of Children and Family Services (DCFS)
- **Dataset** Records of all children who went through foster care between 2006 and 2016 (FY)
  - 112,755 **children** | 129,084 **foster care episodes** | 266,887 **placements**
  - Avg. episodes per child = 1.14
  - Avg. placements per episode = 2.09
  - Avg. episode duration = 14.02 months (median = 10.32 months)
  - Avg. placement duration = 7.39 months (median = 3.67 months)
- **Sample** Every placement assigned between January 1, 2011, and February 28, 2011
  - 2,087 **children** | 2,358 **placements**
  - **Children characteristics** Age, school zip-code
  - **Foster homes characteristics** Type (county, agency, group-home, relative), zip-code

Description of markets and excess supply

Back

- **Market** $=$ **day** $\times$ **regional office** $\times$ **relatives**
- Foster homes are observed conditional on being matched
  - Excess supply is **not observed, but relatively small**
  - Children are left **unmatched** only if there are **no foster homes available**
- Description of markets
  - **Sample period** $=$ 58 days | **Regional offices** $=$ 19 days | **Office-days** $=$ 1102
  - Office-days with $\geqslant 1$ **child without a relative** $=$ 90.7%
    - At least one **unmatched child** in 88.1% of these office-days
  - 85% children are matched on same day they need a placement
  - Avg. **waiting time** (of those who wait) $=$ 6.5 days
  - **Takeaway** Most children matched right away, but unmatched children in almost all office-days

Summary Statistics

Back | Full Dataset

	n	mean	sd	median
Termination Reasons				
Disruption	2358	0.51	0.5	1
Permanency	2358	0.42	0.49	0
Reunification	2358	0.31	0.46	0
Adoption	2358	0.12	0.32	0
Emancipation	2358	0.052	0.2	0
Censored	2358	0.015	0.12	0
Duration				
Duration (months)	2358	8.37	11.28	4.31
Duration—Disrup	1201	5.4	7.96	2.43
Duration—Perm	999	9.97	9.99	7.31
Duration—Emanc	122	12.94	14.3	7.61
Duration—Cens	36	47.89	27.88	64.56
Placement Characteristics				
Child's Age	2358	8.69	5.97	8.49
County Foster Home	2358	0.086	0.27	0
Agency Foster Home	2358	0.43	0.5	0
Group Home	2358	0.12	0.32	0
Relative Home	2358	0.37	0.48	0
Distance Plac-School (mi.)	1775	18.13	23.77	7.99
No School	2358	0.25	0.43	0

Note: Distance measures at zip-code level, computed using Google Maps API.

Summary Statistics (sample and full dataset)

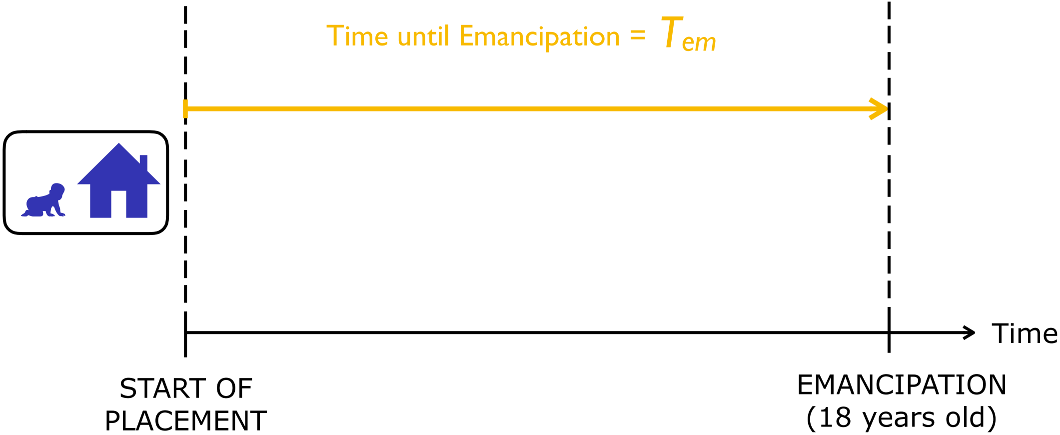
Back

	mean	sd	mean-full	sd-full
Termination Reasons				
Disruption	0.51	0.5	0.49	0.5
Permanency	0.42	0.49	0.37	0.48
Reunification	0.31	0.46	0.26	0.44
Adoption	0.12	0.32	0.11	0.31
Emancipation	0.052	0.2	0.048	0.21
Censored	0.015	0.12	0.090	0.27
Duration				
Duration (months)	8.37	11.28	8.12	10.66
Duration—Disrup	5.4	7.96	4.86	7.38
Duration—Perm	9.97	9.99	10.4	9.90
Duration—Emanc	12.94	14.3	13.23	15.93
Duration—Cens	47.89	27.88	13.99	17.28
Placement Characteristics				
Child's Age	8.69	5.97	8.55	5.91
County Foster Home	0.086	0.27	0.09	0.29
Agency Foster Home	0.43	0.5	0.36	0.48
Group Home	0.12	0.32	0.11	0.32
Relative Home	0.37	0.48	0.43	0.5
Distance Plac-School (mi.)	18.13	23.77	15.75	23.31
No School	0.25	0.43	0.33	0.47

Note: Distance measures at zip-code level, computed using Google Maps API.

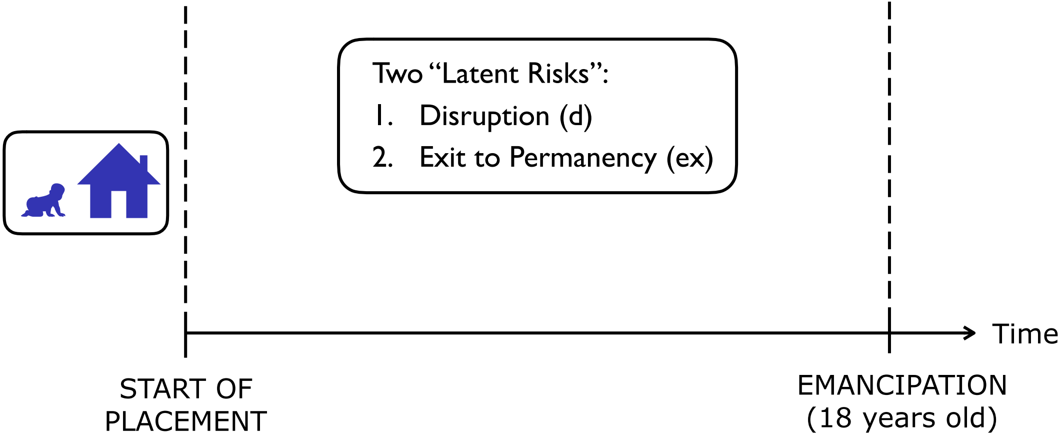
2. Competing Risks Duration Model of Placement Outcomes

Back



2. Competing Risks Duration Model of Placement Outcomes

Back



2. Competing Risks Duration Model of Placement Outcomes

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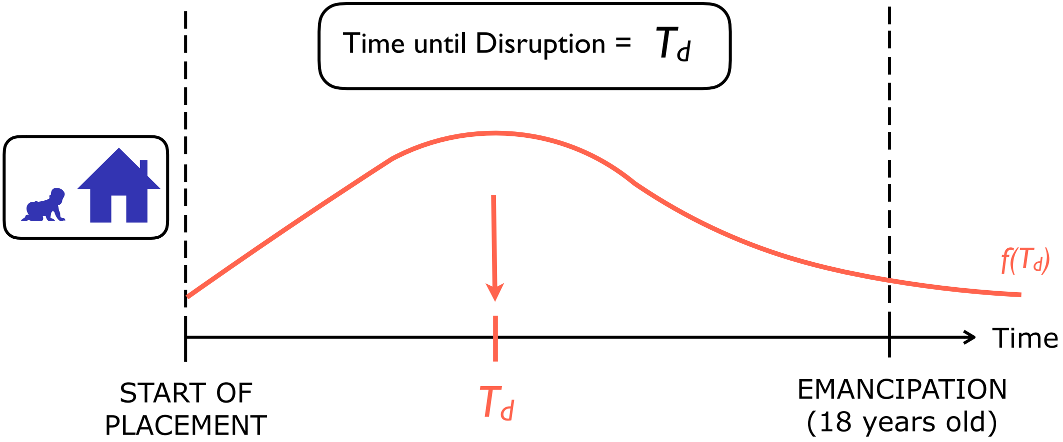

2. Competing Risks Duration Model of Placement Outcomes

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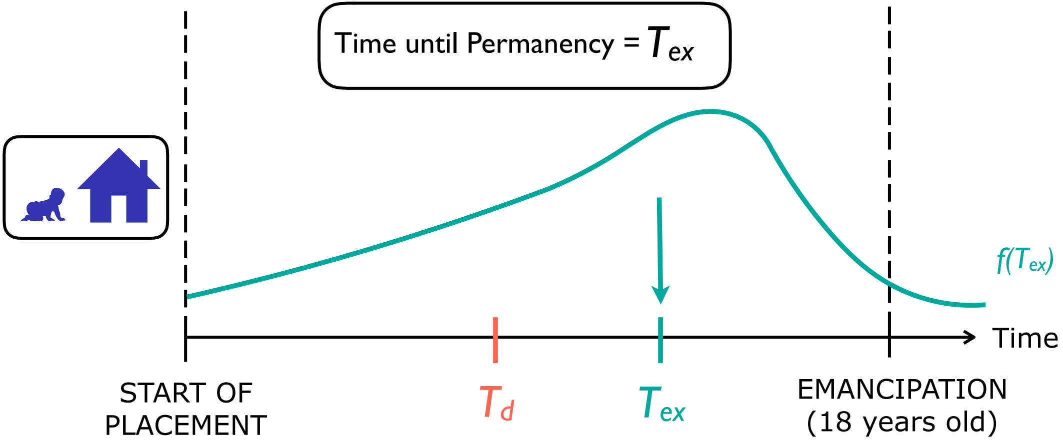

2. Competing Risks Duration Model of Placement Outcomes

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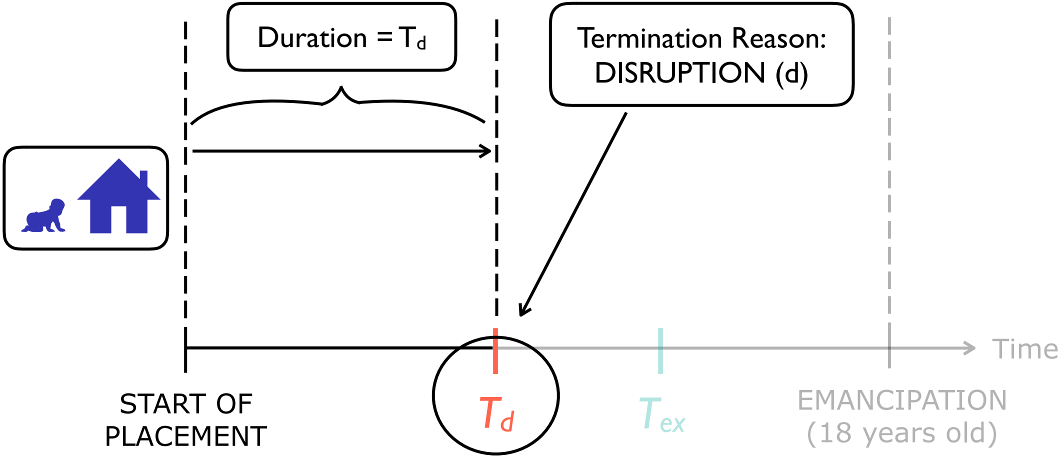

2. Competing Risks Duration Model of Placement Outcomes

Back

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min\{T_R : R \in \mathcal{R}\} \quad \& \quad R = \arg\min\{T_R : R \in \mathcal{R}\}.$$

- Need to specify the **conditional outcome distribution**: $(T, R) \mid \mathcal{I}_{ch}$
  - $\mathcal{I}_{ch}$ = central planner's information about (prospective) placement (c, h)

2. Competing Risks Duration Model of Placement Outcomes

Back

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min\{T_R : R \in \mathcal{R}\} \quad \& \quad R = \arg\min\{T_R : R \in \mathcal{R}\}.$$

Assumption: Normal Mixing Distribution

The **central planner's information** of a placement is $\mathcal{I}_{ch} = (\mathbf{x}_c, \mathbf{y}_h, \boldsymbol{\omega}_{ch})$ where:

$\boldsymbol{\omega}_{ch} = (\omega_d, \omega_{ex})$ are unobservable **frailty terms** (or random effects)

$\boldsymbol{\omega}_{ch} \sim N(0, \boldsymbol{\Sigma}_\omega)$

Note: "Frailty term" means that ω_R shifts the hazard rate of T_R

2. Competing Risks Duration Model of Placement Outcomes

Back

- T_R is the latent duration for $R \in \mathcal{R}$, and

$$T = \min\{T_R : R \in \mathcal{R}\} \quad \& \quad R = \arg\min\{T_R : R \in \mathcal{R}\}.$$

Assumption: Burr Hazard Rates

3a. For $R \in \{d, ex\}$, conditional on $\mathcal{I}_{ch}$, T_R follows a **Burr distribution** with hazard rate:

$$\lambda_R(T \mid \mathcal{I}_{ch}) = \frac{k_R(\mathcal{I}_{ch})\alpha_R T^{\alpha_R - 1}}{1 + \gamma_R^2 k_R(\mathcal{I}_{ch}) T^{\alpha_R}}$$

where $\alpha_R > 0$, $\gamma_R \geqslant 0$, and $k_R(\mathcal{I}_{ch}) = \exp\left(\omega_{R,ch} + g(\mathbf{x}_c, \mathbf{y}_h)\beta_R\right)$.

Note 1: α_R and γ_R determine the shape (duration-dependence) of the hazard rate $\lambda_R(T \mid \mathcal{I}_{ch})$

Note 2: $\lambda_R(T \mid \mathcal{I}_{ch})$ is increasing in $k_R(\mathcal{I}_{ch})$

3b. Latent durations are independent conditional on $\mathcal{I}_{ch}$, $\boldsymbol{\omega}_{ch} \perp \varepsilon_c$, and $\boldsymbol{\omega}_{ch} \perp \eta_h$.

Identification and Estimation Back

- **Identification** Details
  - **Exogenous variation** in $(C, Y, \mathbf{X}, \mathbf{Y})$ across markets identifies distribution of $\boldsymbol{\omega}$ (Ackerberg and Botticini 2002; Sørensen 2007).
    - Intuition akin to traditional **sample selection** models (Heckman 1979)
- **Estimation: Simulated Maximum Likelihood** Details
  - Let $\mathbf{Z}_i \equiv (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i)$. Integrate joint conditional likelihood:

$$\begin{aligned}(M_i, \mathbf{T}_i, \mathbf{R}_i) \,|\, \mathbf{Z}_i &\sim \int (M_i, \mathbf{T}_i, \mathbf{R}_i) \,|\, (\mathbf{Z}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i) \\ &\sim \int (M_i \,|\, \mathbf{Z}_i, \boldsymbol{\Omega}_i)(\mathbf{T}_i, \mathbf{R}_i \,|\, M_i, \mathbf{Z}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i),\end{aligned}$$

where $\boldsymbol{\Omega}_i = (\boldsymbol{\omega}_{ch})_{(c,h)\in C_i \times H_i} \sim G \equiv \bigtimes_{c,h} N(0, \boldsymbol{\Sigma}_\omega)$.

Average Partial Effects on Expected Outcomes

Back

Average Partial Effects

	$\mathbb{P}$(Disrup)	$\mathbb{P}$(Permanency)	$\mathbb{E}(\log T \mid \text{Disrup})$	$\mathbb{E}(\log T \mid \text{Exit})$	$\mathbb{E}(\log T)$
Age At Plac.	0.0139	-0.0115	-0.0406	-0.022	-0.0401
County-FH	0.317	-0.266	-0.969	-0.628	-0.927
Agency-FH	0.320	-0.272	-1.221	-0.874	-1.174
Group Home	0.165	-0.158	0.287	0.450	0.339
Distance To School (zip)	0.00401	-0.00376	-0.007978	-0.00309	-0.00736
No School	0.1136	-0.09686	-0.5244	-0.3653	-0.5212
Number of placements			2358		

Note: Average partial effects of placement characteristics on expected outcomes. Averages taken across the sample of assigned placements in the data. The partial effects with respect to continuous variables is taken by considering a marginal change of one unit.

Average Partial Effects on Expected Outcomes

Back

Average Partial Effects

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Average Partial Effects on Expected Outcomes

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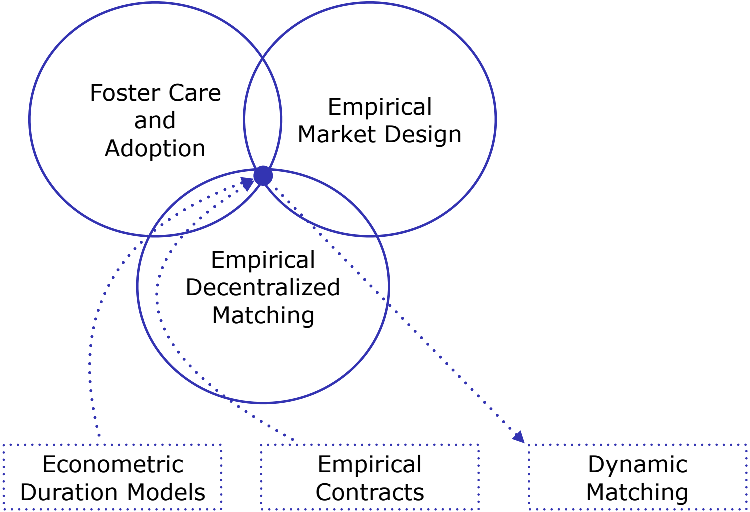
Related Literature

Back

- **Foster Care and Adoption**
  - **Matching** Baccara, Collard-Wexler, Felli, and Yariv (2014); Slaugh, Akan, Kesten, and Ünver (2015); MacDonald (2019); Olberg, Dierks, Seuken, Slaugh, and Ünver (2021)
  - **Foster care outcomes** Doyle Jr. and Peters (2007); Doyle Jr. (2007, 2008, 2013); Doyle Jr. and Aizer (2018); Bald, Doyle Jr., Gross, and Jacob (2022); Gross and Baron (2022); Bald, Chyn, Hastings, and Machelett (2022)
- **Empirical Market Design**
  - **Medical Match** Agarwal (2015)
  - **School Choice** Abdulkadiroğlu, Agarwal, and Pathak (2017); Agarwal and Somaini (2018)
  - **Kidney Exchange** Agarwal, Ashlagi, Azevedo, Featherstone, and Karaduman (2017); Agarwal, Ashlagi, Rees, Somaini, and Waldinger (2019); Agarwal, Hodgson, and Somaini (2022)
- **Empirical Decentralized Matching**
  - **Marriage** Choo and Siow (2006); Galichon and Salanié (2015)

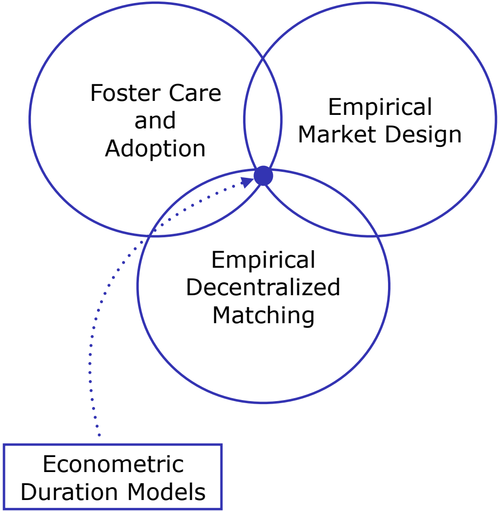
Related Literature (extended)

Back



Related Literature (extended)

Back



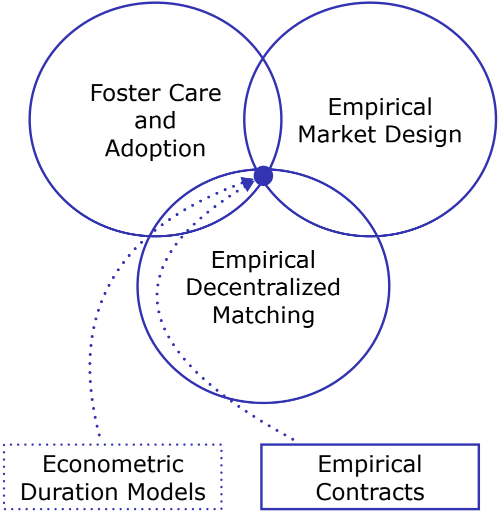
Competing Risks Duration Models

- Heckman and Honoré 1989
- Lancaster 1990
- Kalbfleisch and Prentice 2002

Borrow econometric methods and identification techniques

Related Literature (extended)

Back



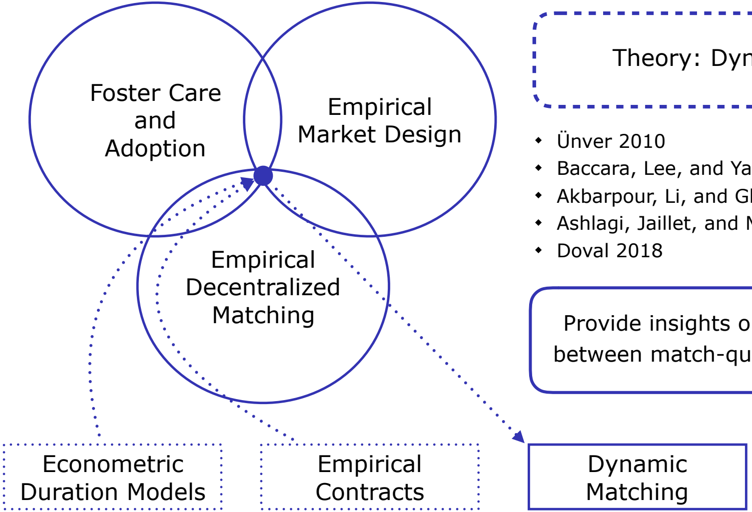
Empirical Contracting Models

- Ackerberg and Botticini 2002
- Srensen 2007
- Ewens, Gorbenko, and Korteweg 2019

Use similar identification strategy for selection on unobservables

Related Literature (extended)

Back



Theory: Dynamic Matching

- Ünver 2010
- Baccara, Lee, and Yariv 2019
- Akbarpour, Li, and Gharan 2019
- Ashlagi, Jaillet, and Manshadi 2013
- Doval 2018

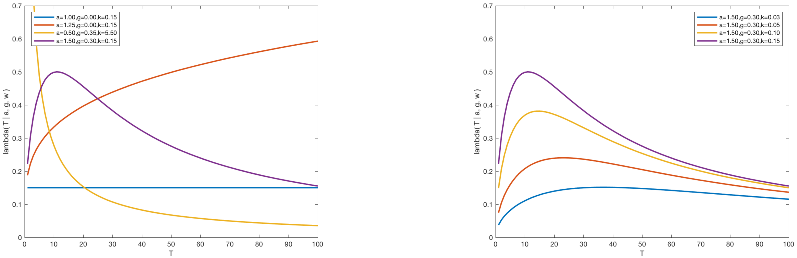
Provide insights on dynamic trade-offs between match-quality and waiting time

Burr Distribution

Back

- The random variable $T \in \mathbb{R}_+$ has a Burr distribution with parameters $\alpha > 0$, $\gamma \geqslant 0$, and $k > 0$, if its hazard function takes the following form:

$$\lambda(T) = \frac{k\alpha T^{\alpha-1}}{1+\gamma^2 k T^{\alpha}}.$$



Left: Examples of Burr hazard functions for different values of α, γ.

Particular cases: Exponential ($\alpha = 1$, $\gamma = 1$), Weibull ($\gamma = 0$), and Log-Logistic ($\gamma = 1$)

Right: Examples of hazard functions for different values of k.

Data Generating Process (DGP)

Back

- Need to identify the distribution of the **endogenous** ("left-hand side") variables

$$(M_i, \mathbf{T}_i, \mathbf{R}_i),$$

conditional on the **exogenous** ("right-hand side") ones

$$(C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i).$$

- Also, need to identify distribution of the **unobserved heterogeneity** ("mixing distribution")

$$(M_i, \mathbf{T}_i, \mathbf{R}_i) \,|\, (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i) \sim \int (M_i, \mathbf{T}_i, \mathbf{R}_i) \,|\, (C_i, H_i, \mathbf{X}_i, \mathbf{Y}_i, \boldsymbol{\Omega}_i) dG(\boldsymbol{\Omega}_i),$$

where $\boldsymbol{\Omega}_i = (\boldsymbol{\omega}_{ch})_{(c,h)\in C_i \times H_i}$.

Identification

Back

1. Duration Distribution (hazard rates and unobserved heterogeneity)
   - **Mixed competing risks** with covariates identified **non-parametrically** (Heckman and Honoré 1989).
   - Distribution of $\boldsymbol{\omega}$ across observed outcomes is **conditional on being matched**: $\boldsymbol{\omega}_{ch} \mid M(c,h) = 1$.
   - **Exogenous variation** in $(C, Y, \mathbf{X}, \mathbf{Y})$ across markets identifies distribution of $\boldsymbol{\omega}$ (Ackerberg and Botticini 2002; Sørensen 2007).
     - Intuition akin to traditional **sample selection** models (Heckman 1979)

2. Matching Distribution (multinomial probit)
   - **Utility index** $\sum_{c,h} M(c,h)\pi(c,h)$ **linear** in utility parameters $(\mu_R, \varphi_R, \bar{\varphi}_R)_{R\in\mathcal{R}}$.
   - Distribution of **individual shocks** ε_c and η_y can be backed out from **composite error** υ_M
   - Exploit variation in $(C, Y, \mathbf{X}, \mathbf{Y})$ across markets, and observing **unmatched children**.

Estimation

Back

- Estimate via **Simulated Maximum Likelihood**.
- Collect all the parameters of the model:

$$\boldsymbol{\theta}_T = (\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\beta}); \quad \boldsymbol{\theta}_M = (\boldsymbol{\mu}, \boldsymbol{\varphi}, \bar{\boldsymbol{\varphi}}, \boldsymbol{\Sigma}_\epsilon, \boldsymbol{\Sigma}_\eta); \quad \boldsymbol{\theta} = [\boldsymbol{\Sigma}_\omega, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M].$$

- The likelihood of observing $(M_i, \mathbf{T}_i, \mathbf{R})$, conditional on $\boldsymbol{\Omega}_i = (\boldsymbol{\omega}_{ch})_{(c,h)\in C_i \times H_i}$, is given by:

$$\mathcal{L}(M_i, \mathbf{T}_i, \mathbf{R}_i \,|\, \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) = \mathcal{L}_M(M_i \,|\, \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) \prod_{(c,h)\in M_i} \mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} \,|\, \boldsymbol{\omega}_{ch}, \boldsymbol{\theta}_T),$$

where:

$$\mathcal{L}_M(M_i \,|\, \boldsymbol{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) = \text{probit choice probability}$$

$$\mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} \,|\, \boldsymbol{\omega}_{ch}, \boldsymbol{\theta}_T) = \text{Burr competing risks conditional likelihood}$$

Estimation

Back

- Let $G = \bigtimes_{c,h} G_{ch}$ denote the distribution of $\mathbf{\Omega}_i$, i.e., $G_{ch} \equiv N(0, \mathbf{\Sigma}_\omega)$. Then,

$$\mathcal{L}(M_i, \mathbf{T}_i, \mathbf{R}_i \,|\, \boldsymbol{\theta}) = \int \mathcal{L}_M(M_i \,|\, \mathbf{\Omega}_i, \boldsymbol{\theta}_T, \boldsymbol{\theta}_M) \prod_{(c,h)\in M_i} \mathcal{L}_{\mathbf{T},\mathbf{R}}(T_{ch}, R_{ch} \,|\, \boldsymbol{\omega}_{ch}, \boldsymbol{\theta}_T) dG(\mathbf{\Omega}_i \,|\, \mathbf{\Sigma}_\omega).$$

- The log-likelihood of the data is $\ell(\boldsymbol{\theta}) = \sum_{i=1}^n \log \mathcal{L}(M_i, \mathbf{T}_i, \mathbf{R}_i \,|\, \boldsymbol{\theta})$.
- Simulated analog of $\mathcal{L}$:

$$\mathcal{L}^{S_\upsilon, S_\omega}(M_i, \mathbf{T}_i, \mathbf{R}_i \,|\, \boldsymbol{\theta}) = \frac{1}{S_\upsilon} \frac{1}{S_\omega} \sum_{s_\upsilon=1}^{S_\upsilon} \sum_{s_\omega=1}^{S_\omega} \mathcal{L}_M^{s_\upsilon}\left(M_i \,|\, \mathbf{\Omega}_i^{s_\omega}, \boldsymbol{\theta}\right) \prod_{(c,h)\in M_i} \mathcal{L}_{\mathbf{T},\mathbf{R}}\left(T_{ch}, R_{ch} \,|\, \boldsymbol{\omega}_{ch}^{s_\omega}, \boldsymbol{\theta}_T, \mathbf{\Sigma}_\omega\right),$$

  where $\mathcal{L}_M^{s_\upsilon}$ is the simulated probit choice probability using a logit-kernel (Train 2009).

- The SMLE of $\boldsymbol{\theta}$ is given by: $\hat{\boldsymbol{\theta}}_{SMLE} = \arg\max_{\boldsymbol{\theta}} \sum_{i=1}^n \log \mathcal{L}^{S_\upsilon, S_\omega}(M_i, \mathbf{T}_i, \mathbf{R}_i \,|\, \boldsymbol{\theta})$
- $\hat{\boldsymbol{\theta}}_{SMLE} \overset{a}{=} \hat{\boldsymbol{\theta}}_{MLE}$ (**consistent**, **asymptotically normal** and **efficient**) if $n, S_\upsilon, S_\omega \to \infty$, and $\sqrt{n}/\min(S_\upsilon, S_\omega) \to 0$ (Gourieroux and Monfort 1997).

Aggregate payoff function

Probit Model Identification

- The aggregate payoff of matching $M \in \mathbb{M}(C, H)$ is a **linear function of the utility function parameters**:

$$\sum_{c,h} M(c,h)\pi(c,h) = \sum_{R \in \mathcal{R}} \Bigg\{ \left[\sum_{c,h} M(c,h)\mathbb{P}(R \,|\, \mathcal{I}_{ch}) \right] \mu_R + \left[\sum_{c,h} M(c,h)\mathbb{P}(R \,|\, \mathcal{I}_{ch})\mathbb{E}\left(\log T \,|\, R, \mathcal{I}_{ch}\right) \right] \varphi_R + \left[\sum_{c,h} M(c,h)\mathbb{P}(R \,|\, \mathcal{I}_{ch}) \log T_{em,c} \right] \bar{\varphi}_R \Bigg\},$$

Expected placement outcomes

Estimation APEs

- **Termination probabilities** and **expected log-duration**:

$$\mathbb{P}(R|\mathcal{I}_{ch}) = \int_0^{T_{em,c}} \bar{F}(T|\mathcal{I}_{ch})\lambda_R(T|\mathcal{I}_{ch})dT$$

$$\mathbb{E}\left(\log T \,|\, R, \mathcal{I}_{ch}\right) = \int_0^{T_{em,c}} \log T \left[\frac{\bar{F}(T|\mathcal{I}_{ch})\lambda_R(T|\mathcal{I}_{ch}}{\mathbb{P}(R|\mathcal{I}_{ch})}\right] dT,$$

  where $\bar{F}(T|\mathcal{I}_{ch})$ denotes the **conditional survival function** of T, given by

$$\bar{F}(T|\mathcal{I}_{ch}) = \exp\left\{-\sum_{R\in\mathcal{R}_0} \gamma_R^{-2} \log\left[1 + \gamma_R^2 k_R(\mathcal{I}_{ch}) T^{\alpha_R}\right]\right\}.$$

- The integrals above have **no closed-form solution**. They need to be computed numerically.

Conditional Hazard Functions

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	Disruption	Exit
Var(ω_R)	0.873***	0.02955
	(0.2912)	(0.02867)
Cov(ω_d, ω_{ex})	0.1573*	0.1573*
	(0.08908)	(0.08908)
Age At Plac.	0.09872***	-0.01615
	(0.01767)	(0.01047)
County-FH	2.217***	-0.02375
	(0.332)	(0.2101)
Agency-FH	2.983***	0.4547***
	(0.2556)	(0.1237)
Group Home	-2.077**	-1.987***
	(0.9188)	(0.5642)
Age At Plac. × *County-FH*	-0.02272	0.01804
	(0.0261)	(0.01636)
Age At Plac. × *Agency-FH*	-0.07878***	-0.01007
	(0.0194)	(0.0124)
Age At Plac. × *Group Home*	0.2569***	0.1419***
	(0.06179)	(0.03894)
Distance To School (zip)	0.02052***	-0.006059***
	(0.002471)	(0.001724)
No School	0.9007***	0.1222
	(0.1603)	(0.08942)
Constant	-8.996***	-6.082***
	(0.5408)	(0.2132)
Alpha (α_R)	1.091***	0.9665***
	(0.07551)	(0.03427)
Gamma (γ_R)	0.9527***	0.2222
	(0.1183)	(0.2361)
Number of placements	2358	

Note: Estimated parameters of unobserved heterogeneity ($\boldsymbol{\Sigma}_\omega$) and conditional hazard rates ($\boldsymbol{\theta}_T$). Standard errors in parenthesis. Significance level of parameters: ***p⩽0.01, **p⩽0.05, *p⩽0.01.

Model Fit

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Goodness of Fit and Estimation Parameters

	Predicted	Sample
$\mathbb{P}$*(Disruption)*	0.514	0.5093
$\mathbb{P}$*(Exit)*	0.4303	0.4237
$\mathbb{P}$*(Emanc/Cens)*	0.05568	0.06701
$\mathbb{E}(\log T \mid \textit{Disruption})$	4.482	4.141
$\mathbb{E}(\log T \mid \textit{Exit})$	4.721	4.994
$\mathbb{E}(\log T \mid \textit{Emanc/Cens})$	7.19	5.534
$\mathbb{E}(\log T)$	4.615	4.596
Number of markets (n)	1467	
Number of assigned placements	2358	
Number of prospective placements	8900	
SMLL	-17005.86	
S_ω	50	
S_υ	50	
$\dim(\boldsymbol{\theta})$	39	

Note: Average predicted outcomes and sample average outcomes. Averages taken across the sample of assigned placements in the data. The number of assigned placements in the data is equal to $\sum_i \sum_{c,h} M_i(c,h)$. The number of prospective placements is equal to $\sum_i \sum_{c,h} |C_i| \times |H_i|$. *SMLL* gives the value of the simulated log-likelihood at the estimated vector of parameters. S_ω, S_υ, and ψ are the parameters of the simulated log-lilkelihood. $\dim(\boldsymbol{\theta})$ refers to the number of parameters estimated.