

Waitlist Mechanisms

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Rationing via Waits

- Many real-world examples of waiting as a rationing tool
- Common reasons
 - ▶ Restrictions (legal or otherwise) on using prices
 - ▶ Arrival of objects over time
 - ▶ In-kind transfers
- How to maximize allocative efficiency with waits?

Outline

- 1 Theory Examples
 - Waiting and “Money Burning”
 - Waitlist Offer Mechanisms
- 2 Deceased Donor Kidney Allocation

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Costs of waiting

- Waiting can be costly in some environments
 - ▶ Literally standing in line is wasteful
 - ▶ Value of object/allocation can decrease
 - Differs from transfers (e.g. auctions) which are costless from a social perspective
- ✓ (When) does it make sense to use waiting time to screen for agent valuations?

- Notation:

- ▶ M objects with quality $x_j > 0$, $x_{j+1} > x_j$
- ▶ N agents indexed by i
- ▶ Valuations $v_i \sim F_i$ with support V_i
- ▶ Agent payoff $v_i x_i - c_i$ where c_i is money burned (waiting)

- Implementable direct mechanism

- ▶ Allocation probabilities $p_{i,j}$ and costs c_i
- ▶ Interim stage expected utility given report s_i

$$U_i(v_i, s_i) = v_i E_{v_{-i}} \left[\sum_m x_{i,j} p_{i,j}(s_i, v_{-i}) \right] - E_{v_{-i}} [c_i(s_i, v_{-i})].$$

- **Lemma 1:** A direct mechanism is IC iff $P_i(v_i) \geq P_i(v'_i)$ for all $v_i \geq v'_i$ and

$$C_i(v_i) = v_i P_i(v_i) - \int_0^{v_i} P_i(x) dx$$

where $P_i(v_i) = E_{v_{-i}} [\sum_j x_{i,j} p_{i,j}(v_i, v_{-i})]$ and $C_i(v_i) = E_{v_{-i}} [c_i(s_i, v_{-i})]$.

✓ Standard result from Vickrey auctions

“Money Burning”

- Objective Function:

$$E_v \left\{ \sum_i^n w_i \left[v_i \sum_j x_j p_{i,j}(v) - c_i(v) \right] \right\}$$

- ▶ w_i are welfare weights
- ▶ Money burning since “payments” are subtracted

✓ Allocative efficiency vs screening costs

- **Corollary 1:** First best is not implementable unless $n - 1$ ($n - m$) agents have zero Pareto weights (and objects are identical)
 - ▶ First-best requires highest value agent to receive the object

When Does Waiting Lists Make Sense?

- Theorem 1 characterizes the optimal mechanism – allocation to agents with highest marginal contribution to social surplus, $\lambda_i(v_i)$, which depends on w_i and F_i
- Corollary 2: If hazard rates are non-decreasing, then $\lambda_i(x) = w_i E[v_i]$ for all x
 - ✓ Full pooling, no private information extracted, allocation based on w_i
- Corollary 3: If hazard rates are decreasing, then priority functions are

$$\lambda_i(x) = w_i \frac{1 - F(x)}{f_i(x)}$$

- ✓ Worth paying the waiting costs only when dispersion in valuations is high enough
 - ▶ Exponential distribution is the cutoff
 - ▶ Screening with Pareto, lottery/priority with uniform
- Slightly different from virtual valuation $v_i + \frac{1 - F_i(v_i)}{f_i(v_i)}$

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Dynamic Assignments via Waitlists

- Many markets in which matching occurs over time
 - ▶ Organs for deceased donors [Agarwal et al, 2021]
 - ▶ Subsidized/public housing [Waldinger, 2021]
 - ▶ Health services (nursing homes)
 - ▶ Adoption/foster care [Robinson-Cortes, 2019]
 - ▶ Ridesharing [Liu et al, 2019]
- ✓ Focus on allocation of objects to agents
 - ▶ Contrast with two-sided dynamic matching [e.g. Doval (2014)]
- Waitlist mechanisms are commonly used
 - ▶ Priority order over potential applicants
 - ▶ Agents can choose (accept/reject) when their turn arrives

First-Come First-Served

- Canonical waitlist procedure
 - ▶ Rationales?
- ✓ Is it efficient when objects and preferences are heterogeneous? [Bloch and Cantala, 2017]
- Model
 - ▶ Time is discrete, t
 - ▶ Agent index $i = 1, \dots, n$ denotes rank on the list
 - ▶ An object arrives each period and must be allocated immediately
 - ▶ Valuations drawn from $\theta \sim F$ on $[\underline{\theta}, \bar{\theta}]$ or $\theta \in \{0, 1\}$
 - ▶ Per-period cost of waiting is c
 - ▶ Constant size waitlist
- Consider offer mechanisms with probabilities over offer sequences $\rho : \mathcal{N} \rightarrow \mathcal{N}$

Queuing disciplines

- Markovian strategy sets thresholds $\theta(i)$
- Value function

$$\begin{aligned}V(i) &= P(\text{accepted by } j < i) V(i-1) \\&= P(i \text{ picks the object } j) \int_{\theta(i)}^{\bar{\theta}} \theta dF(\theta) \\&= P(\text{not accepted by } j \leq i) V(i) - c\end{aligned}$$

- Observations
 - ▶ Rejections by agents above is not a negative signal
 - ▶ In common value case, upper bound changes from $\bar{\theta}$ to $\theta(i-1)$
 - ▶ Waiting is costly

What are good queuing strategies?

- Efficiency criteria on $V(1), \dots, V(n)$
- Focus on monotone queues, i.e. if $i < j$ then for two sequence ρ, ρ' with $\rho(k) = \rho'(k)$ for $k \notin \{i, j\}$, and $\rho'(j) < \rho(j) = \rho(i) = \rho'(j)$, we have that $\rho(\rho) \geq \rho(\rho')$.
- When is FCFS best in this set?
 - ▶ Under private and binary values, all agents prefer FCFS to lotteries
 - ▶ With common and binary values, all agents prefer FCFS
 - ▶ Waste is higher in FCFS under common values [see also Su and Zenios, 2004]
 - ▶ With private values and two agents, FCFS is better than other queuing rules

What are good queuing strategies?

- Two externalities from declining an offer
 1. Allow other agents to accept an offer quickly, save waiting costs
 2. Stay on the list and (potentially) reduce future offers for others
 - ✓ Need to control selectivity using queuing discipline
- Su and Zenios (2004) show that LCFS is best under common values
 - ▶ Minimizes waste, allocative effects are null
 - ▶ Agents internalize their externality on others
- Leshno (2017) shows that batching can be useful
 - ▶ Two object types A and B , two agent types α and β
 - ▶ Would like to get α to decline B and β to decline A
 - ▶ How to get the largest number of mis-matches to be declined?
 - Answer: Lottery amongst the top few
- Incentives to accept/reject are key!
 - ▶ e.g. Arnosti and Shi (2020) show that several pairs of mechanisms are outcome equivalent

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 - Model
 - Estimation Approach
 - Estimates and Counterfactuals

Deceased Donor Organ Allocation

- About 100K patients are waiting for a lifesaving organ transplant
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 - ▶ ~ 20% medically suitable kidneys are discarded
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 - ✓ Similar rationing mechanisms: public housing, long-term care amongst others

Agarwal et.al., 2021: Research Objectives

1. **Empirical Methods:** Estimate “as-if” value of assignments using decisions
 - ▶ **Agent's perspective:** Optimal Stopping Problem [Pakes, 1986; Rust 1987; Hotz and Miller, 1993]
2. **Application:** Kidney Allocation in the New York City area
 - ▶ Administrative data from the OPTN
 - ▶ Detailed donor and patient characteristics
3. **Design Evaluation**
 - ▶ Equilibrium comparison of mechanisms with different priorities
 - ✓ Focus on wait-list offer mechanisms
 - Direct mechanisms are difficult to implement in practice
 - ✓ Equivalent to solving a dynamic game
- ✓ Nature of preferences are important in dynamic mechanism design
 - ▶ Few general theoretical results on optimal designs → motivates empirical work [Agarwal et.al.(AEA P&P)]

Deceased Donor Kidney Allocation

- Waiting lists allocates deceased donor organs
 - ▶ Pre-2014: Coarse priorities and sequential offers
 1. Perfect tissue-type matches
 2. Geography: Local, Regional, National
 3. Points for years waited and some characteristics, e.g, hard-to-match patients, pediatric patients
 - ▶ Post 2014: Attempts to improve match quality
 1. National sharing for extremely difficult to match patients
 2. Top 20% of kidneys → top 20% healthiest patients
- Factors affecting value of an organ transplant
 - ▶ Biological Compatibility
 - ▶ Donor health (age, diabetes etc.)
 - ▶ Similarity of tissue-types

- Administrative data from the OPTN
 - ✓ Formal mechanisms often generate useful data
- This study: Offers to patients in NYRT between 2010 and 2013
 - ▶ Serves NYC, Long Island + neighboring NY counties
 - ▶ Largest Donor Service Area with standard rules
- Detailed donor and patient characteristics
 - ▶ Essentially all donor characteristics known to patient/surgeon
 - ▶ Patient characteristics: demographics and correlates of health

Methodological challenges

1. Ensure Identification of Counterfactuals:

- ▶ Well-known problem in dynamic contexts [Aguirregabiria and Suzuki, 2014; Arcidiacono and Miller, 2020; Kaloupstidi et al., 2021]
- ✓ Solution is wlog for mechanism design counterfactuals

2. Complexity of Beliefs/State-Space:

- ▶ Data and computational curse of dimensionality
- ▶ Complicates both estimation and counterfactuals
- ✓ Assumption on beliefs to simplify beliefs/state-space [c.f. Freshtman and Pakes, 2012]

3. Equilibria in Dynamic Mechanisms:

- ▶ Primarily complicates counterfactual analysis
- ✓ Propose a steady-state concept for equilibrium analysis [c.f. Hopenhayn, 1992; Weintraub et al., 2008; Freshtman and Pakes, 2012]

4. Policy Analysis

- ▶ Compare outcomes under alternative waitlist designs
- ✓ Evaluation of optimal designs

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Payoffs and Discounting

- Agents and objects:
 - ▶ i – agent – patient-surgeon pair
 - ▶ j – objects – organs
 - ▶ Objects arrive at rate λ , with types drawn from F
- Time, t , is continuous
 - ▶ Days since “birth” – joining the kidney list
 - ▶ Finite-horizon, T – 100 years of age
 - ▶ ρ – discount rate
- Primitive Payoffs:
 - ▶ $d_i(t)$ – flow payoff of remaining on the waiting list – dialysis
 - ▶ $D_i(t)$ – NPV of departure without assignment
 - ▶ Exogenous departures at rate $\delta_i(t)$
 - ▶ $\Gamma_{ij}(t)$ – NPV of i assigned j at t
- Primary Restrictions on Payoffs:
 1. Only depends on assignments, no cost of considering offers
 2. Evolve deterministically given agent identity

Mechanism and Beliefs

- Mechanism:
 - ▶ Priority score $s_{ij}(t_i)$
 - ▶ Assignment: highest q_j priority agents that accept ($a_i = 1$)
 - ▶ Technological constraint: Up to n_j offers

Mechanism and Beliefs

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- Beliefs on distribution of s_j^* depend on $\mathcal{F}_{it} = (x_i, t)$

$$\pi_{ij}(t) = H(s_{ij}(t); z_j, \eta_j) \times P(c_{ij} = 1 | x_i, z_j)$$

✓ Idea: Beliefs based on long-run averages/experience [e.g. Freshtman and Pakes, 2012; Weintraub et al., 2009]

- Main Assumption: No inferences based on realization of recent offers

- ▶ Waitlist rules are agent-object specific
- ▶ Cannot reject zero auto-correlation in s_j^* [details](#)
- ▶ Time since last offer is not predictive of acceptance [details](#)

Value Function

- Consider the value from waiting Δt

$$\begin{aligned} V_i(t) = & \frac{1}{1 + \rho \Delta t} [d_i(t) \Delta t + \delta_i(t) \Delta t D_i(t) \\ & + \lambda \Delta t \int \pi_{ij}(t) \int \max \{V_i(t + \Delta t), \Gamma_{ij}(t)\} dG dF \\ & + (1 - (\delta_i(t) + \lambda_i(t)) \Delta t) V_i(t + \Delta t) + o(\Delta t)], \end{aligned}$$

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- Sending $\Delta t \rightarrow 0$ yields the ODE

$$\begin{aligned} (\rho + \delta_i(t)) V_i(t) = & \dot{V}_i(t) + d_i(t) + \delta_i(t) D_i(t) \\ & + \lambda \int \pi_{ij}(t) \max \{0, \Gamma_{ij}(t) - V_i(t)\} dF \end{aligned}$$

✓ See also Arcidiacono et al. (2016) for related derivation of cond. val. func.

- Recall the ODE

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- Estimation requires additional restrictions to address underidentification
 1. Common to set payoff from one action in each state to zero
 2. Setting discount rate [Magnac and Thesmar, 2003]
- “Normalizations” in 1. are not necessarily without loss [Aguirregabiria and Suzuki, 2014; Arcidiacono and Miller, 2020; Kaloupstidi et al., 2021]
 - ▶ Counterfactuals may change transitions to different states
 - ▶ May affect payoffs in these states

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- Normalize the NPV of rejecting every offer to zero
 - ✓ Appropriate for counterfactuals that do not change
 1. Payoffs of remaining on the list – costs/benefits of dialysis
 2. Value and rate of departures – Death and Live-Donor Transplantation

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- Normalize the NPV of rejecting every offer to zero
 - ✓ Appropriate for counterfactuals that do not change
 - Payoffs of remaining on the list – costs/benefits of dialysis
 - Value and rate of departures – Death and Live-Donor Transplantation
- Solution:

$$V_i(t) = \int_t^T \exp(-\rho(\tau - t)) p(\tau|t) \left[\lambda \int \pi_{ij}(\tau) \max\{0, \Gamma_{ij}(\tau) - V_i(\tau)\} dF \right] d\tau$$

where $p(\tau|t)$ is the probability of remaining in the list at τ (conditional on t)

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Estimation Approach

$$V_i(t) = \int_t^T \exp(-\rho(\tau - t)) p_i(\tau|t) \left[\lambda \int \pi_{ij}(\tau) \max\{0, \Gamma_{ij}(\tau) - V_i(\tau)\} dF \right] d\tau$$

- Estimated/set “offline”

- ▶ Donor arrival rate λ = empirical average
- ▶ Discount rate ρ = 5% per year
- ▶ Hazards model for $p_i(\tau|t)$ using (censored) observed departures
 - Gompertz hazards model: $\delta_i(t) = \exp(x_i\beta + pt)$
- ✓ Primary source of discounting is departure – 16% per year

Conditional Choice Probabilities

- Specify binary choice model

$$\Gamma_{ij}(t) - V_i(t) = \chi(z_j, x_i, t)\theta + \eta_j + \varepsilon_{ijt},$$

- ▶ z_j and η_j are observed and unobserved object characteristics
- ▶ x_i are observed agent characteristics
- ▶ $\eta_j \sim N(0, \sigma_\eta)$ and $\varepsilon_{ijt} \sim N(0, 1)$ – scale normalization
- ▶ $\chi(z_j, x_i, t)$ – flexibly piece-wise linear forms with interactions

- Probability of declining an offer:

$$P_{ij}(t) = 1 - \Phi(\chi(z_j, x_i, t)\theta + \eta_j)$$

- ▶ **Estimation:** Gibbs' Sampler using conjugate priors (MCMC)

✓ Identification of σ_η relies on each donor having two kidneys

Estimator for Mechanism/Beliefs

- Hotz-Miller (1987), recover:

$$\mathbb{E} \max \{0, \Gamma_{ij}(t) - V_i(t)\} = \psi(P_{ij}(t))$$

- Use knowledge of the mechanism to estimate inclusive value

$$\begin{aligned} \int \pi_{ij}(t) \max \{0, \Gamma_{ij}(t) - V_i(t)\} dF &= \int \pi_{ij}(t) \psi(P_{ij}(t)) dF \\ &\approx \frac{1}{J} \sum_j 1 \{s_{ij}(t) > s_j^*\} \psi(\hat{P}_{ij}(t)) \end{aligned}$$

- ▶ s_j^* is the score of the last patient that received an offer for j

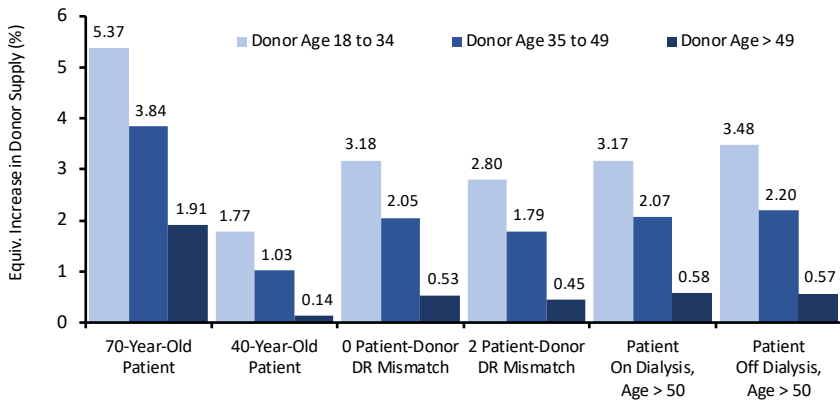
- ✓ Formally justified in the paper

- ▶ $s_{ij}(t)$ independent of other agents
- ▶ Beliefs: $1 \{s_{ij}(t) > s_j^*\}$ does not depend on presence of i'
- ▶ LLN under weak stationarity and continuity assumptions

- ✓ No parametric approximations for the mechanism/beliefs

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NPV of Transplant By Age and Dialysis Status



Steady-State Equilibrium

- Steady state equilibrium of mechanism
 1. Optimality given beliefs π^* – Backwards induction
 2. Consistent beliefs π^* given strategies, queue length, and queue composition
 - ✓ Calculate acceptance rates above each score level
 3. Steady state composition
 - i. Queue Composition: Survival curve calculated using forward simulation
 - ii. Queue length N^* to satisfy detail balance

$$\text{arrival rate of agents} = N^* \times (\text{average departure rate} | m^*)$$

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- ✓ Abstracts away from transition dynamics

Optimal Mechanisms

- Maximize aggregate welfare

$$\max_{V, m, \sigma, \pi} \sum_i \alpha_i \left(\frac{V_i(0)}{\rho} + \int_0^T m_x(x) V_i(\tau) d\tau \right)$$

- Optimal Assignment

- ▶ Full information on payoffs; uncertain object/agent arrivals and departures
- ▶ Steady-state conditions, but no agent optimality
- ✓ Allocate j to i after wait-time t if assignment payoff exceeds a threshold $\underline{\Gamma}_{jit}$

- Optimal Offer Rates

- ▶ Restrict to offer mechanisms independent of the past offers
- ▶ Agents make optimal decisions given π

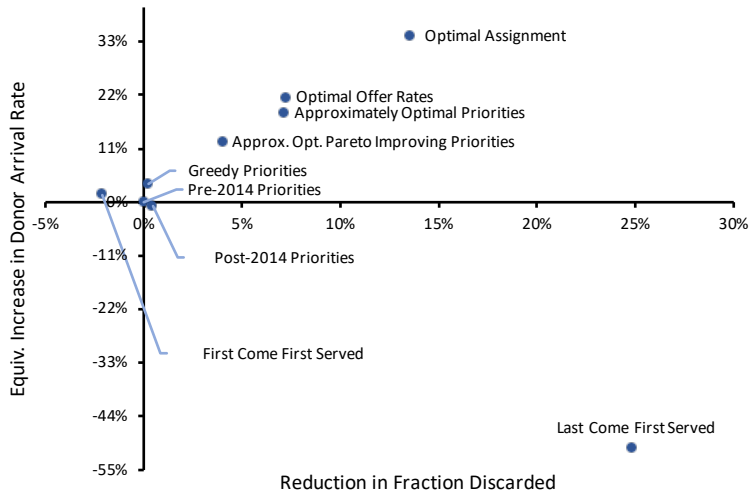
- Approximately Optimal Priorities

- ▶ Implementable version of Optimal Offers Rate

- Approximately Optimal Pareto Improving Priorities

- ▶ Implementable version of a Pareto Constrained Optimal Offers Rate

Welfare and Resource Utilization



Conclusion

- Dynamic assignment design requires an empirical approach
 - ▶ Limited guidance from theory
 - ▶ Simulations commonly used by policy-makers e.g. liver allocation reforms
- Empirical framework for predicting outcomes in dynamic assignment systems
- Many applications and extensions
 - ▶ Rationing through waitlists is empirically under-explored
 - ▶ Interactions with other markets and policies
- Commonly studied mechanisms are far from optimal
- Scope for increasing aggregate outcomes subject to distributional barriers