Interconnected multi-unit auctions: An empirical analysis

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August 20, 2022

Introduction

- Securities/commodities worth trillions of $ are allocated via **multi-unit auctions**
- Often in **parallel**
  - Financial securities
  - International carbon allowances, renewable energy, diamonds
  - Fish, vegetable, wine ...

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- Securities/commodities worth trillions of $ are allocated via **multi-unit auctions**
- Often in **parallel**
  - Financial securities
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  - Fish, vegetable, wine ...
- We leverage this insight to
  1) Develop a method to estimate demand systems for multiple goods
  2) Show how to use these demand systems to achieve higher auction revenue

Part 1: Demand Estimation – Idea

- Parallel auctions
  - Same auction market rules, participants
  - Same time period, economic situation. . .
    - → Can control for unobserved heterogeneity
- Multi-unit auctions
  - Bidders submit full demand schedules
    - → No need for an instrument

Part 1: Demand Estimation – How?

Model of simultaneous multi-unit auctions to identify full demand systems

- Solves challenges
  - Only observe shaded bids
  - Only observe parts of the demand schedules

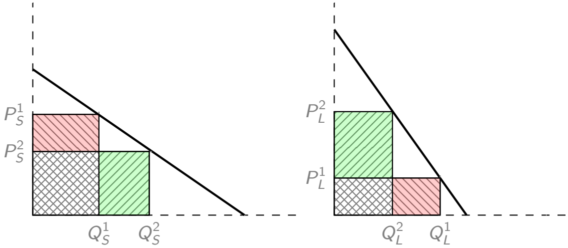
    Because bidders cannot submit multi-dimensional bidding schedules
- Technical contributions
  - Allow demand to depend on multiple goods — Guerre et al. (2000), Hortaçsu (2002)
  - Solve for equilibrium conditions — Wittwer (2021) and Kastl (2011)

Part 2: Increase Revenue

We show

- Auctioneer should behave like a monopolist who price discriminates
- Useful when it is difficult to change the auction format (e.g., Klemperer (2010))

Figure: Uniform price auction



Empirical Application: Canadian Treasury Aucitons

Findings

- 3M, 6M, 12M Treasury bills are weak substitutes despite being cash-like
- Aggregate demand for short bills is less price sensitive than for long bills

- How to increase total revenue:
  - Uniform price auction — issue more of the short bills and less of the long bills
  - Discriminatory price auction — vice versa

So what?

General lesson

(1) Alternative method to estimate demand (e.g., energy, diamonds, fish)

- Identifies substitutes or complements w/o imposing preference correlations

(2) Can achieve higher auction revenues w/o changing the auction format

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Future research

- Alternative objective functions, e.g., reduce CO2 emissions in ETS auctions
- Simultaneous vs. combinatorial auction format vs. sequential auctions

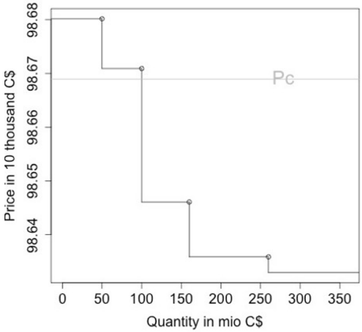
Road Ahead

1. Institutional environment and data
2. Model and identification strategy
3. Estimation findings and counterfactual

Institutional Environment

- Three types of T-bills in Canada: m= 3, 6, 12 months
- Sold every other Thursday in 3 separate discriminatory price auctions, in parallel

Figure: Average demand function in 12M auction



Data Set

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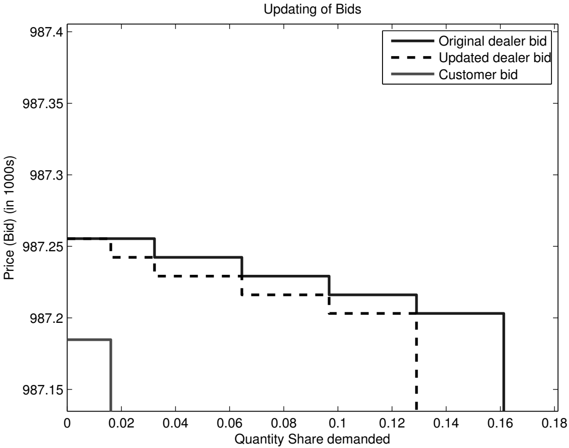
  - Avg: 10.6 bidders participate in one auction
  - Avg: 71 (95) % of active bidders (dealers) go to all 3 auctions
- All individual bids (including updates)
  - Avg: # of steps in bid-function: about 4.5

Summary Stats

Preliminary Evidence: Bid Updating



If observing a customer bid in one maturity, do dealers update other maturities?

$$update_{i,m} = \alpha + \sum_m I_m \left(\beta_m customer_m + \delta_{m,-m} customer_{-m}\right) + \varepsilon_{i,m}$$

Estimates

Micro-Foundation: Dealer Demand

What drives demand in an auction with resale (primary market)? Dealers want goods

1. **For own consumption or to fulfill existing customer orders**
   - Have private info about how much value the good for "personal usage"
   - Heterogeneous business type
2. **To sell them after the auction (secondary market)** where
   - Customers demand different goods
   - It is costly to fail in serving customers

formal details

Micro-Foundation: Dealer Demand

We show

- Goods may be complements or substitutes in the primary market

At time τ when dealer i has type $(s^g_{1i\tau}, s^g_{2i\tau}, s^g_{3i\tau}) \sim F^g$, her willingness to pay for amount q_1 of good 1 can be approximated by

$$v^g_{1i}(q_1, q_2, s^g_{1i\tau}) = f(s^g_{1i\tau}) + \lambda^g_{1i} q_1 + \delta^g_{1i} q_2.$$

- λ: own-price elasticity of demand
- δ: cross-price elasticity of demand

Goal: Measure $\lambda^g_{1i}, \delta^g_{1i}$

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Challenges

1. Dealers may have a latent type (e.g., market maker)

$\Rightarrow$ Generates asymmetries in auction

Goal: Measure $\lambda_{1i}^{g}, \delta_{1i}^{g}$

$$v_{1i}^{g}(q_1, q_2, s_{2i\tau}^{g}) = f(s_{1i\tau}^{g}) + \lambda_{1i}^{g} q_1 + \delta_{1i}^{g} q_2$$

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$=$ Bids for security 1 cannot depend on security 2

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$\Rightarrow$ We observe $b^g_{1i}(q_1, s^g_{1i\tau})$ not $v^g_{1i}(q_1, q_2, s^g_{1i\tau})$ w/o knowing $s^g_{1i\tau}$

Estimation

Stage 1) Estimate dealers' true value v_{mi}^{g} by inversion (Guerre et al (2000)) details

- Assume all play the group-symmetric equilibrium
- Back out which values rationalize the bids we observe

Stage 2) Estimate $\lambda_{mi}^{g}, \vec{\delta}_{mi}^{g}$ in linear regression w/ auction-time-bidder FE details

Results: Demand elasticities

Example: If one dealer wins 1% more of 3M bills

- 3M price $\downarrow$ by C\$ 6.107 $\approx$ 100 %
- 6M price $\downarrow$ by C\$ 1.158 $\approx$ 20 %
- 1Y price $\downarrow$ by C\$ 0.243 $\approx$ 5 %

Table: All maturities

	3M	6M	12M
3M Auction	100%	20%	5%
6M Auction	28%	100%	13%
12M Auction	25%	28%	100%

Take away

- T-Bill demand is rather price-insensitive w.r.t. all maturities
- Bills are imperfect substitutes

Counterfactual: How to split supply across maturities?

Use demand estimates

- To analyze how to split total debt Q_t across different maturities
- To maximize auction revenues$_t$

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- To maximize auction revenues$_t$

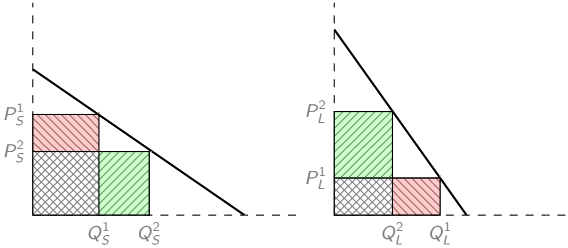
Two maturities: **short (S) and long (L)**

Key factors that determine how to split total supply

- Issuance cost drives a wedge between P_S and P_L → take as given
- Market price elasticities depend on all $\lambda' s$ and $\delta' s$ → focus on
- Auction format → focus on

Counterfactual: How to split supply across maturities?

Uniform price auction

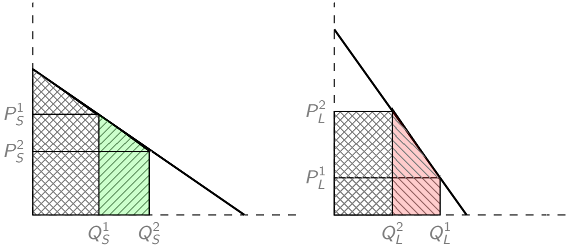


- Issue more S and less L
- Price-quantity trade-off
  - The more debt is issued as S rather than L

  $\rightarrow$ The lower the revenue gain in the long bond auction

Counterfactual: How to split supply across maturities?

Discriminatory price auction



- It can go both ways, since demand changes when supply changes

→ Leverage structural model: Issue more of L than S

CF Back of envelop Trade off

Conclusions

This paper

- Develops a new methodology on how to estimate demand systems for multiple goods that can account for any degree of substitution or complementarity
- Illustrates how to use demand systems to better target the auctioneers' objective

Thank you!

Summary Stats

	Mean			SD			Min			Max		
	3M	6M	12M	3M	6M	12M	3M	6M	12M	3M	6M	12M
Issued amount	5.76	2.12	2.12	1.68	0.52	0.52	3.05	1.22	1.22	10.40	3.80	3.80
Dealers	11.88	11.79	11.03	0.90	0.93	0.83	9	9	9	13	13	12
Global part. (%)	93.67	93.84	98.84	24.34	24.04	10.67	0	0	0	100	100	100
Customers	6.26	5.68	5.35	2.69	2.94	2.54	1	0	0	14	13	15
Global part. (%)	35.66	40.13	39.46	47.90	49.02	48.88	0	0	0	100	100	100
Comp demand as % of announced sup.	16.29	16.91	17.02	7.96	7.61	7.31	0.002	0.019	0.005	25	25	25
Submitted steps	4.83	4.23	4.35	1.86	1.78	1.75	1	1	1	7	7	7
Updates by dealer	2.89	2.18	2.48	3.58	2.87	3.18	0	0	0	31	31	42
Updates by customer	0.12	0.13	0.19	0.40	0.40	0.58	0	0	0	4	3	9
Non-comp dem. as % of announced sup.	0.05	0.15	0.15	0.03	0.10	0.10	$5/10^5$	$4/10^5$	$2/10^3$	0.24	0.58	0.58

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Preliminary Evidence of Interdependency

Table: Probability of Dealer Updating Bids

Coefficient	Verbal description	(1)		(2)	
$\hat{\beta}_{3M}$	update in $3M$ after order for $3M$	0.533	(0.056)	0.711	(0.053)
$\hat{\delta}_{3M,6M}$	update in $3M$ after order for $6M$	0.405	(0.064)	0.531	(0.061)
$\hat{\delta}_{3M,12M}$	update in $3M$ after order for $12M$	0.303	(0.057)	0.446	(0.054)
$\hat{\delta}_{6M,3M}$	update in $6M$ after order for $3M$	0.086	(0.063)	0.248	(0.059)
$\hat{\beta}_{6M}$	update in $6M$ after order in $6M$	0.848	(0.076)	0.929	(0.070)
$\hat{\delta}_{6M,12M}$	update in $6M$ after order in $12M$	0.729	(0.080)	0.762	(0.074)
$\hat{\delta}_{12M,3M}$	update in $12M$ after order for $3M$	0.556	(0.070)	0.664	(0.066)
$\hat{\delta}_{12M,6M}$	update in $12M$ after order for $6M$	0.120	(0.059)	0.244	(0.056)
$\hat{\beta}_{12M}$	update in $12M$ after order for $12M$	0.828	(0.061)	0.934	(0.059)
$\hat{\alpha}$	constant	0.476	(0.007)	0.448	(0.007)

$$update_{i,m} = \alpha + \sum_{m} I_m \left(\beta_m customer_m + \delta_{m,-m} customer_{-m} \right) + \varepsilon_{i,m}$$

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Micro-Foundation: Dealer Demand

- Let there be only 2 auctions, each offering one maturity ($M = 2$)
- Each dealer has a type $\boldsymbol{s}$, which decomposes into ν (known by all bidders) and $\boldsymbol{t}$ (iid private information):

$$\boldsymbol{s} = (\boldsymbol{t}, \nu) \text{ with } \boldsymbol{t} = (\boldsymbol{t_1}, \boldsymbol{t_2}) \text{ and } \nu = (a, b, e, \gamma, \kappa_1, \kappa_2, \rho).$$

- He will use the amount q_m he wins in auction m in two ways

$$\begin{cases} (1-\kappa_m)\% \text{ of } q_m & \text{to fulfill existing customers orders or for personal usage} \\ \kappa_m\% \text{ of } q_m & \text{for future resale in the secondary market} \end{cases}$$

Micro-Foundation: Dealer Demand

- Clients demand in the secondary market: $\{\boldsymbol{x_1}, \boldsymbol{x_2}\} \sim G$ (Vayanos and Vila (2021))
- Aggregate demand for good 1 in the secondary market is

$$p_1(x_1, x_2 | q_1, q_2) = \begin{cases} a - bx_1 - ex_2 & \text{for } x_1 \leq \kappa_1 q_1 \text{ and } x_2 \leq \kappa_2 q_2 \\ a - bx_1 & \text{for } x_1 \leq \kappa_1 q_1 \text{ and } x_2 > \kappa_2 q_2 \\ 0 & \text{for } x_1 > \kappa_1 q_1 \text{ and } x_2 > \kappa_2 q_2 \end{cases}$$

- with $e \geq 0$, $b \geq 0$, i.e., **goods are substitutes in the secondary market**

⇒ Expected benefit from winning $\{q_1, q_2\}$

$$V(q_1, q_2, s) = U(q_1, q_2, s) + \mathbb{E}\Big[\underbrace{p_1 \boldsymbol{x_1} + p_2 \boldsymbol{x_2}}_{\text{resale revenue}} - \underbrace{cost(\boldsymbol{x_1}, \boldsymbol{x_2} | q_1, q_2)}_{\text{cost of turning down clients}} \Big]$$

- where $cost(x_1, x_2 | q_1, q_2)$ increases in x_1 and x_2 & is supermodular
- for **market makers** cost is low

back

Details on Stage 1

Which valuations rationalize the bids we observe if all play the type-symmetric BNE?

- Bidder i of group g draws private type $s^g_{i\tau}$
- Forms beliefs about market clearing price conditional on all info available at τ: $\theta_{i\tau}$ (this might involve knowledge of customer bids etc.), and submit a bid that solves:

One discriminatory price auction

$$v(q_k, s^g_{i\tau}) = b_k + \frac{\Pr\left(b_{k+1} \geq P^c | \theta_{i\tau}\right)}{\Pr\left(b_k > P^c > b_{k+1} | \theta_{i\tau}\right)} \left(b_k - b_{k+1}\right)$$

Details on Stage 1

Which valuations rationalize the bids we observe if all play the type-symmetric BNE?

- Bidder i of group g draws vector of types $s^g_{i\tau} = \left(s^g_{3M}, s^g_{6M}, s^g_{12M}\right)_{i\tau}$
- Forms beliefs about market clearing price conditional on all info available at τ: $\theta_{i\tau}$ (this might involve knowledge of customer bids etc.), and submit a bid that solves:

Simultaneous discriminatory price auctions

$$\mathbb{E}\left[v_m \left(q_{mk}, \vec{Q}^c_{-mi}, s^g_{mi\tau} \right) \middle| \text{win } q_{mk}, \theta^g_{i\tau} \right] = b_{m,k} + \frac{\Pr\left(b_{mk+1} \geq P^c_m | \theta_{i\tau}\right)}{\Pr\left(b_{mk} > P^c_m > b_{mk+1} | \theta_{i\tau}\right)} \left(b_{mk} - b_{mk+1}\right)$$

Details on Stage 1

By resampling we can consistently estimate the joint distribution of

- The market clearing prices $\vec{P}^c_{|\theta_{i\tau}} = (P^c_{3M|\theta_{i\tau}}, P^c_{6M|\theta_{i\tau}}, P^c_{12M|\theta_{i\tau}})$
- How much bidder i wins $\vec{Q}^c_{|\theta_{i\tau}} = (Q^c_{3M|\theta_{i\tau}}, Q^c_{6M|\theta_{i\tau}}, Q^c_{12M|\theta_{i\tau}})$

This allows us to construct the needed (conditional) expectations.

back

Details on Ressampling (simplified)

Assume

- Bidder is from group g
- N_{-g} potential bidders from each $-g$ and $N_g - 1$ from g are ex-ante type-sym and play the type-sym BNE
- Private information is independent across bidders, no updates (just for illustration)
- All $T \times M$ auctions have identical covariates

Details on Ressampling (simplified)

Procedure

1. Fix bidder i and the bidding schedules he submitted in all auctions he participated in. If he did not bid in an auction, replace his bid by 0.

2. Draw a random subsample of $N_g - 1$ bid vector triplets with replacement from the sample of $N_g(T \times M)$ bids in the data set and N_{-g} from $\overline{N_{-g}(T \times M)}$.

3. Construct one realization of bidder i's residual supply $\forall m$ were others to submit these bids to determine
   - realized clearing prices $\vec{p} = \{p_{3M}, p_{6M}, p_{12M}\}$
   - if i would have won $\vec{q}_i = \{q_{i,3M}, q_{i,6M}, q_{i,12M}\}$ for all $(\vec{q}, \vec{p})$.

$\rightarrow$ Repeat many times $\Rightarrow$ Consistent estimate of the joint distr. of $\vec{P}$ and $\vec{Q}_i$

back

Details on Stage 2

Stage 2) Estimate $\lambda^g_{mi}, \vec{\delta}^g_{mi}$

- When bidding for amount $q_{mi\tau k}$ in auction m,
  dealer i guesses how much he wins, $\vec{Q}^c_{-m}$, in other auctions $-m$

$$\hat{\mathbb{E}}[v^g_{mi}(q_{mi\tau k}, \vec{Q}^c_{-m}, s^g_{mi\tau})|\text{win } q_{mi\tau k}]$$

$$= f(s^g_{mi\tau}) + \lambda^g_{mi} * q_{mi\tau k} + \vec{\delta}^g_{mi} \cdot \hat{\mathbb{E}}[\vec{Q}^c_{-m}|\text{win } q_{mi\tau k}] + \epsilon_{mi\tau k}$$

back

Details on Stage 2

Stage 2) Estimate $\lambda^g_{mi}, \vec{\delta}^g_{mi}$

- OLS regressions with bid functions that have > 1 step (88%)

$$\underbrace{\hat{\mathbb{E}}[v^g_{mi}(q_{mi\tau k}, \vec{Q}^c_{-m}, s^g_{mi\tau})|\text{win } q_{mi\tau k}]}_{\textit{estimated}}$$

$$= \underbrace{f(s^g_{mi\tau})}_{\textit{fixed effect}} + \lambda^g_{mi} * \underbrace{q_{mi\tau k}}_{\textit{observed}} + \vec{\delta}^g_{mi} \cdot \underbrace{\hat{\mathbb{E}}[\vec{Q}^c_{-m}|\text{win } q_{mi\tau k}]}_{\textit{estimated}} + \epsilon_{mi\tau k}$$

back

Details on Stage 3

Stage 3)

- Classify dealers into groups based on the estimated $\vec{\delta}_{mi}$
- If classification coincides with the initial guess from Stage 0), terminate, otherwise go to Step 1) using the updated partition

back

Counterfactual: Revenue Gains

Table: Average gain (in bps) per auction when reshuffling 1% of debt

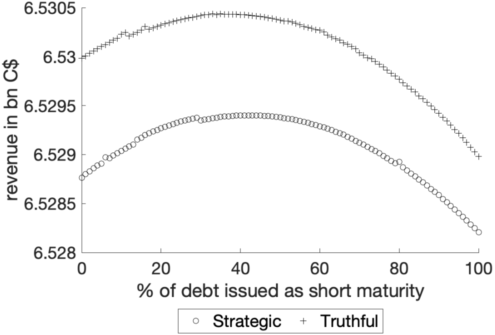
Demand coefficients		$S\uparrow L\downarrow$ Uniform	$S\uparrow L\downarrow$ PayAs	$S\downarrow L\uparrow$ Uniform	$S\downarrow L\uparrow$ PayAs
Independent:	factor$_\lambda$=1, factor$_\delta$=0	+0.020	+0.007	−0.023	−0.010
Weak substitutes:	factor$_\lambda$=1, factor$_\delta$=1	+0.016	−0.002	−0.024	+0.001
Perfect substitutes:	factor$_\lambda$=1, $\delta = \lambda$	+0.011	−0.052	−0.016	+0.048
Independent:	factor$_\lambda$=10, factor$_\delta$=0	+0.234	−0.028	−0.297	+0.007
Weak substitutes:	factor$_\lambda$=10, factor$_\delta$=1	+0.225	−0.036	−0.292	+0.016
Perfect substitutes:	factor$_\lambda$=10, $\delta = \lambda$	+0.119	−0.609	−0.189	+0.590
Independent:	factor$_\lambda$=100, factor$_\delta$=0	+2.344	−0.446	−2.9757	+0.191
Weak substitutes:	factor$_\lambda$=100, factor$_\delta$=1	+2.341	−0.455	−2.970	+0.200
Perfect substitutes:	factor$_\lambda$=100, $\delta = \lambda$	+1.313	−6.720	−1.956	+6.624

Take away: Issue more of the price-insensitive bond and less of the price-sensitive bond in uniform price auction, vice versa in discriminatory price

Back

Counterfactual: Price quantity trade-off

Figure: Illustration of the price-quantity trade-off



On the y-axis is the total revenue earn from issuing both maturities (in billion C$) when issuing x% of the short maturity and (1-x)% of the long maturity. The x-axis scales up x from 0% to 100%.

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Counterfactual: Back-of-the-Envelope Calculation

Canada

- Average price elasticity is below 0.002 → moderate gains from reshuffling
- E.g., 2021
  - Canada issued C$416 billion in bills and C$277 billion in bonds.
  - Cost savings of 0.001 - 0.02 bps per auction (C$595,600 annual) had Canada issued 1% more as long and 1% less as short debt.

Counterfactual: Back-of-the-Envelope Calculation

Other markets

- Higher price elasticities → sizable gains from reshuffling
- E.g., Albuquerque et al. (2022) estimate an average price elasticity of demand of 2.1-2.4 in Portuguese bond auctions between 2014 and 2019.
- Scaling all demand coefficients by a factor of 1,000, our model predicts
  - Similar price elasticity
  - Cost savings of about 40 bps per uniform-price auction if government had issued 1% more as short and 1% less as long debt.

back