Waitlist Mechanisms

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Rationing via Waits

- Many real-world examples of waiting as a rationing tool
- Common reasons
  - Restrictions (legal or otherwise) on using prices
  - Arrival of objects over time
  - In-kind transfers
- How to maximize allocative efficiency with waits?

Outline

Outline

Costs of waiting

- Waiting can be costly in some environments
  - Literally standing in line is wasteful
  - Value of object/allocation can decrease
- Differs from transfers (e.g. auctions) which are costless from a social perspective

✓ (When) does it make sense to use waiting time to screen for agent valuations?

Condorelli (2012)

- Notation:
  - M objects with quality $x_j > 0$, $x_{j+1} > x_j$
  - N agents indexed by i
  - Valuations $v_i \sim F_i$ with support V_i
  - Agent payoff $v_i x_i - c_i$ where c_i is money burned (waiting)

- Implementable direct mechanism
  - Allocation probabilities $p_{i,j}$ and costs c_i
  - Interim stage expected utility given report s_i

$$U_i(v_i, s_i) = v_i E_{v_{-i}}[\sum_m x_{i,j} p_{i,j}(s_i, v_{-i})] - E_{v_{-i}}[c_i(s_i, v_{-i})].$$

- Lemma 1: A direct mechanism is IC iff $P_i(v_i) \geq P_i(v_i')$ for all $v_i \geq v_i'$ and

$$C_i(v_i) = v_i P_i(v_i) - \int_0^{v_i} P_i(x)\mathrm{d}(x)$$

where $P_i(v_i) = E_{v_{-i}}[\sum_j x_{i,m} p_{i,j}(v_i, v_{-i})]$ and $C_i(v_i) = E_{v_{-i}}[c_i(s_i, v_{-i})]$.

✓ Standard result from Vickrey auctions

“Money Burning”

- Objective Function:

$$E_v\left\{\sum_{i}^{n} w_i\left[v_i\sum_{j} x_j p_{i,j}(v) - c_i(v)\right]\right\}$$

  - w_i are welfare weights
  - Money burning since “payments” are subtracted

✓ Allocative efficiency vs screening costs

- Corollary 1: First best is not implementable unless $n-1$ $(n-m)$ agents have zero Pareto weights (and objects are identical)
  - First-best requires highest value agent to receive the object

When Does Waiting Lists Make Sense?

- Theorem 1 characterizes the optimal mechanism – allocation to agents with highest marginal contribution to social surplus, $\lambda_i(v_i)$, which depends on w_i and F_i

- Corollary 2: If hazard rates are non-decreasing, then $\lambda_i(x) = w_i E[v_i]$ for all x
  - ✓ Full pooling, no private information extracted, allocation based on w_i
- Corollary 3: If hazard rates are decreasing, then priority functions are

$$\lambda_i(x) = w_i \frac{1 - F(x)}{f_i(x)}$$

✓ Worth paying the waiting costs only when dispersion in valuations is high enough

- Exponential distribution is the cutoff
- Screening with Pareto, lottery/priority with uniform

- Slightly different from virtual valuation $v_i + \dfrac{1 - F_i(v_i)}{f_i(v_i)}$

Outline

1. Theory Examples
   - Waiting and “Money Burning”
   - Waitlist Offer Mechanisms
2. Deceased Donor Kidney Allocation

Dynamic Assignments via Waitlists

- Many markets in which matching occurs over time
  - Organs for deceased donors [Agarwal et al, 2021]
  - Subsidized/public housing [Waldinger, 2021]
  - Health services (nursing homes)
  - Adoption/foster care [Robinson-Cortes, 2019]
  - Ridesharing [Liu et al, 2019]

✓ Focus on allocation of objects to agents
  - Contrast with two-sided dynamic matching [e.g. Doval (2014)]

- Waitlist mechanisms are commonly used
  - Priority order over potential applicants
  - Agents can choose (accept/reject) when their turn arrives

First-Come First-Seved

- Canonical waitlist procedure
  - Rationales?

✓ Is it efficient when objects and preferences are heterogeneous? [Bloch and Cantala, 2017]

- Model
  - Time is discrete, t
  - Agent index $i = 1, \ldots, n$ denotes rank on the list
  - An object arrives each period and must be allocated immediately
  - Valuations drawn from $\theta \sim F$ on $[\underline{\theta}, \bar{\theta}]$ or $\theta \in \{0, 1\}$
  - Per-period cost of waiting is c
  - Constant size waitlist

- Consider offer mechanisms with probabilities over offer sequences $\rho : \mathcal{N} \to \mathcal{N}$

Queuing disciplines

- Markovian strategy sets thresholds $\theta(i)$
- Value function

$$
\begin{aligned}
V(i) &= P(\text{accepted by } j < i)V(i-1) \\
&+ P(i \text{ picks the object } j)\int_{\theta(i)}^{\bar{\theta}} \theta \, \mathrm{d}F(\theta) \\
&+ P(\text{not accepted by } j \leq i)V(i) - c
\end{aligned}
$$

- Observations
  - Rejections by agents above is not a negative signal
  - In common value case, upper bound changes from $\bar{\theta}$ to $\theta(i-1)$
  - Waiting is costly

What are good queuing strategies?

- Efficiency criteria on $V(1), \ldots, V(n)$
- Focus on monotone queues, i.e. if $i < j$ then for two sequence ρ, ρ' with $\rho(k) = \rho'(k)$ for $k \notin \{i, j\}$, and $\rho'(j) < \rho(j) = \rho(i) = \rho'(j)$, we have that $p(\rho) \geq p(\rho')$.
- When is FCFS best in this set?
  - Under private and binary values, all agents prefer FCFS to lotteries
  - With common and binary values, all agents prefer FCFS
  - Waste is higher in FCFS under common values [see also Su and Zenios, 2004]
  - With private values and two agents, FCFS is better than other queuing rules

What are good queuing strategies?

- Two externalities from declining an offer
  1. Allow other agents to accept an offer quickly, save waiting costs
  2. Stay on the list and (potentially) reduce future offers for others

  ✓ Need to control selectivity using queuing discipline

- Su and Zenios (2004) show that LCFS is best under common values
  - Minimizes waste, allocative effects are null
  - Agents internalize their externality on others

- Leshno (2017) shows that batching can be useful
  - Two object types A and B, two agent types α and β
  - Would like to get α to decline B and β to decline A
  - How to get the largest number of mis-matches to be declined?
    - Answer: Lottery amongst the top few

- Incentives to accept/reject are key!
  - e.g. Arnosti and Shi (2020) show that several pairs of mechanisms are outcome equivalent

Outline

1. Theory Examples
2. Deceased Donor Kidney Allocation
   - Model
   - Estimation Approach
   - Estimates and Counterfactuals

Deceased Donor Organ Allocation

- About 100K patients are waiting for a lifesaving organ transplant
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  - Patients are offered organs in priority order and may accept or decline
  - ✓ Similar rationing mechanisms: public housing, long-term care amongst others

Agarwal et.al., 2021: Research Objectives

1. **Empirical Methods:** Estimate "as-if" value of assignments using decisions
   - Agent's perspective: Optimal Stopping Problem [Pakes, 1986; Rust 1987; Hotz and Miller, 1993]

2. **Application:** Kidney Allocation in the New York City area
   - Administrative data from the OPTN
   - Detailed donor and patient characteristics

3. **Design Evaluation**
   - Equilibrium comparison of mechanisms with different priorities
   - ✓ Focus on wait-list offer mechanisms
     - Direct mechanisms are difficult to implement in practice
   - ✓ Equivalent to solving a dynamic game

✓ Nature of preferences are important in dynamic mechanism design
- Few general theoretical results on optimal designs → motivates empirical work [Agarwal et.al.(AEA P&P)]

Deceased Donor Kidney Allocation

- Waiting lists allocates deceased donor organs
  - Pre-2014: Coarse priorities and sequential offers
    1. Perfect tissue-type matches
    2. Geography: Local, Regional, National
    3. Points for years waited and some characteristics, e.g, hard-to-match patients, pediatric patients
  - Post 2014: Attempts to improve match quality
    1. National sharing for extremely difficult to match patients
    2. Top 20% of kidneys $\rightarrow$ top 20% healthiest patients
- Factors affecting value of an organ transplant
  - Biological Compatibility
  - Donor health (age, diabetes etc.)
  - Similarity of tissue-types

Data

- Administrative data from the OPTN
  - ✓ Formal mechanisms often generate useful data

- This study: Offers to patients in NYRT between 2010 and 2013
  - Serves NYC, Long Island + neighboring NY counties
  - Largest Donor Service Area with standard rules

- Detailed donor and patient characteristics
  - Essentially all donor characteristics known to patient/surgeon
  - Patient characteristics: demographics and correlates of health

Methodological challenges

1. Ensure Identification of Counterfactuals:
   - Well-known problem in dynamic contexts [Aguirregabiria and Suzuki, 2014; Arcidiacono and Miller, 2020; Kaloupstidi et al., 2021]
   - ✓ Solution is wlog for mechanism design counterfactuals
2. Complexity of Beliefs/State-Space:
   - Data and computational curse of dimensionality
   - Complicates both estimation and counterfactuals
   - ✓ Assumption on beliefs to simplify beliefs/state-space [c.f. Freshtman and Pakes, 2012]
3. Equilibria in Dynamic Mechanisms:
   - Primarily complicates counterfactual analysis
   - ✓ Propose a steady-state concept for equilibrium analysis [c.f. Hopenhayn, 1992; Weintraub et al., 2008; Freshtman and Pakes, 2012]
4. Policy Analysis
   - Compare outcomes under alternative waitlist designs
   - ✓ Evaluation of optimal designs

Outline

Payoffs and Discounting

- Agents and objects:
  - i – agent – patient-surgeon pair
  - j – objects – organs
  - Objects arrive at rate λ, with types drawn from F
- Time, t, is continuous
  - Days since "birth" – joining the kidney list
  - Finite-horizon, T – 100 years of age
  - ρ – discount rate
- Primitive Payoffs:
  - $d_i(t)$ – flow payoff of remaining on the waiting list – dialysis
  - $D_i(t)$ – NPV of departure without assignment
  - Exogenous departures at rate $\delta_i(t)$
  - $\Gamma_{ij}(t)$ – NPV of i assigned j at t
- Primary Restrictions on Payoffs:
  1. Only depends on assignments, no cost of considering offers
  2. Evolve deterministically given agent identity

Mechanism and Beliefs

- Mechanism:
  - Priority score $s_{ij}(t_i)$
  - Assignment: highest q_j priority agents that accept $(a_i = 1)$
  - Technological constraint: Up to n_j offers

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- Beliefs on distribution of s_j^* depend on $\mathcal{F}_{it} = (x_i, t)$

$$\pi_{ij}(t) = H(s_{ij}(t); z_j, \eta_j) \times P(c_{ij} = 1 | x_i, z_j)$$

✓ Idea: Beliefs based on long-run averages/experience [e.g. Freshtman and Pakes, 2012; Weintraub et al., 2009]

- Main Assumption: No inferences based on realization of recent offers
  - Waitlist rules are agent-object specific
  - Cannot reject zero auto-correlation in s_j^* details
  - Time since last offer is not predictive of acceptance details

- Consider the value from waiting Δt

$$
\begin{aligned}
V_i(t) = \frac{1}{1+\rho\Delta t}&\Big[d_i(t)\,\Delta t + \delta_i(t)\,\Delta t D_i(t) \\
&+ \lambda\Delta t \int \pi_{ij}(t) \int \max\{V_i(t+\Delta t), \Gamma_{ij}(t)\}\,\mathrm{d}G\mathrm{d}F \\
&+ (1-(\delta_i(t)+\lambda_i(t))\,\Delta t)\,V_i(t+\Delta t) + o(\Delta t)\Big],
\end{aligned}
$$

Value Function

- Consider the value from waiting Δt

$$V_i(t) = \frac{1}{1+\rho\Delta t}[d_i(t)\Delta t + \delta_i(t)\Delta t D_i(t)$$
$$+ \lambda\Delta t \int \pi_{ij}(t) \int \max\{V_i(t+\Delta t), \Gamma_{ij}(t)\}\,\mathrm{d}G\mathrm{d}F$$
$$+ (1-(\delta_i(t)+\lambda_i(t))\Delta t) V_i(t+\Delta t) + o(\Delta t)],$$

- Sending $\Delta t \to 0$ yields the ODE

$$(\rho+\delta_i(t)) V_i(t) = \dot{V}_i(t) + d_i(t) + \delta_i(t) D_i(t)$$
$$+ \lambda \int \pi_{ij}(t) \max\{0, \Gamma_{ij}(t) - V_i(t)\}\,\mathrm{d}F$$

✓ See also Arcidiacono et al. (2016) for related derivation of cond. val. func.

- Recall the ODE

$$\begin{aligned}(\rho + \delta_i(t))\, V_i(t) &= \dot{V}_i(t) + d_i(t) + \delta_i(t)\, D_i(t) \\ &\quad + \lambda \int \pi_{ij}(t) \max\{0, \Gamma_{ij}(t) - V_i(t)\}\, \mathrm{d}F\end{aligned}$$

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- Estimation requires additional restrictions to address underidentification
  1. Common to set payoff from one action in each state to zero
  2. Setting discount rate [Magnac and Thesmar, 2003]

- "Normalizations" in 1. are not necessarily without loss [Aguirregabiria and Suzuki, 2014; Arcidiacono and Miller, 2020; Kaloupstidi et al., 2021]
  - Counterfactuals may change transitions to different states
  - May affect payoffs in these states

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- Normalize the NPV of rejecting every offer to zero
  - ✓ Appropriate for counterfactuals that do not change
    1. Payoffs of remaining on the list – costs/benefits of dialysis
    2. Value and rate of departures – Death and Live-Donor Transplantation

- Recall the ODE

$$\left(\rho+\delta_i\left(t\right)\right)V_i\left(t\right)=\dot{V}_i\left(t\right)+d_i\left(t\right)+\delta_i\left(t\right)D_i\left(t\right)$$
$$+\lambda\int\pi_{ij}\left(t\right)\max\left\{0,\Gamma_{ij}\left(t\right)-V_i\left(t\right)\right\}\mathrm{d}F$$

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- Solution:

$$V_i\left(t\right)=\int_t^T\exp\left(-\rho\left(\tau-t\right)\right)p\left(\tau|t\right)\left[\lambda\int\pi_{ij}\left(\tau\right)\max\left\{0,\Gamma_{ij}\left(\tau\right)-V_i\left(\tau\right)\right\}\mathrm{d}F\right]\mathrm{d}\tau$$

where $p\left(\tau|t\right)$ is the probability of remaining in the list at τ (conditional on t)

Outline

Estimation Approach

$$V_i(t) = \int_t^T \exp(-\rho(\tau - t))\, p_i(\tau|t) \left[\lambda \int \pi_{ij}(\tau) \max\{0, \Gamma_{ij}(\tau) - V_i(\tau)\}\, \mathrm{d}F\right] \mathrm{d}\tau$$

- Estimated/set "offline"
  - Donor arrival rate $\lambda =$ empirical average
  - Discount rate $\rho = 5\%$ per year
  - Hazards model for $p_i(\tau|t)$ using (censored) observed departures
    - Gompertz hazards model: $\delta_i(t) = \exp(x_i\beta + pt)$
  - ✓ Primary source of discounting is departure – 16% per year

Conditional Choice Probabilities

- Specify binary choice model

$$\Gamma_{ij}(t) - V_i(t) = \chi(z_j, x_i, t)\,\theta + \eta_j + \varepsilon_{ijt},$$

  - z_j and η_j are observed and unobserved object characteristics
  - x_i are observed agent characteristics
  - $\eta_j \sim N(0, \sigma_\eta)$ and $\varepsilon_{ijt} \sim N(0, 1)$ – scale normalization
  - $\chi(z_j, x_i, t)$ – flexibly piece-wise linear forms with interactions

- Probability of declining an offer:

$$P_{ij}(t) = 1 - \Phi(\chi(z_j, x_i, t)\,\theta + \eta_j)$$

  - Estimation: Gibbs' Sampler using conjugate priors (MCMC)

✓ Identification of σ_η relies on each donor having two kidneys

Estimator for Mechanism/Beliefs

- Hotz-Miller (1987), recover:

$$\mathbb{E}\max\{0, \Gamma_{ij}(t) - V_i(t)\} = \psi(P_{ij}(t))$$

- Use knowledge of the mechanism to estimate inclusive value

$$\int \pi_{ij}(t)\max\{0, \Gamma_{ij}(t) - V_i(t)\}\,\mathrm{d}F = \int \pi_{ij}(t)\,\psi(P_{ij}(t))\,\mathrm{d}F$$
$$\approx \frac{1}{J}\sum_j 1\left\{s_{ij}(t) > s_j^*\right\}\psi\left(\hat{P}_{ij}(t)\right)$$

  - s_j^* is the score of the last patient that received an offer for j

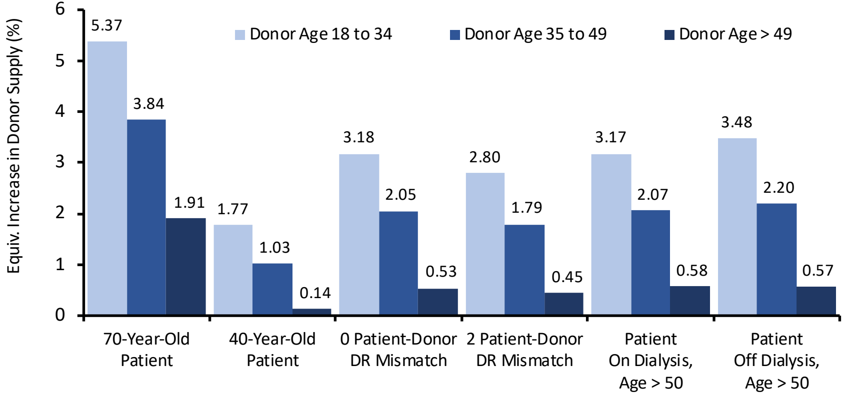
✓ Formally justified in the paper

  - $s_{ij}(t)$ independent of other agents
  - Beliefs: $1\left\{s_{ij}(t) > s_j^*\right\}$ does not depend on presence of i'
  - LLN under weak stationarity and continuity assumptions

✓ No parametric approximations for the mechanism/beliefs

Outline

NPV of Transplant By Age and Dialysis Status



Steady-State Equilibrium

- Steady state equilibrium of mechanism

  1. Optimality given beliefs π^* – Backwards induction
  2. Consistent beliefs π^* given strategies, queue length, and queue composition
     ✓ Calculate acceptance rates above each score level
  3. Steady state composition
     i. Queue Composition: Survival curve calculated using forward simulation
     ii. Queue length N^* to satisfy detail balance

$$\text{arrival rate of agents} = N^* \times (\text{average departure rate } | m^*)$$

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✓ Abstracts away from transition dynamics

Optimal Mechanisms

- Maximize aggregate welfare

$$\max_{V,m,\sigma,\pi} \sum_i \alpha_i \left(\frac{V_i(0)}{\rho} + \int_0^T m_x(x) V_i(\tau) \, d\tau \right)$$

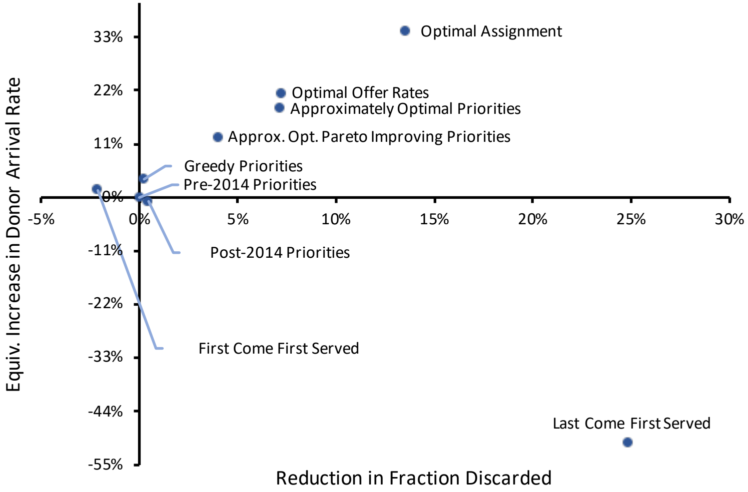
- Optimal Assignment
  - Full information on payoffs; uncertain object/agent arrivals and departures
  - Steady-state conditions, but no agent optimality
  - ✓ Allocate j to i after wait-time t if assignment payoff exceeds a threshold $\underline{\Gamma}_{jit}$

- Optimal Offer Rates
  - Restrict to offer mechanisms independent of the past offers
  - Agents make optimal decisions given π

- Approximately Optimal Priorities
  - Implementable version of Optimal Offers Rate

- Approximately Optimal Pareto Improving Priorities
  - Implementable version of a Pareto Constrained Optimal Offers Rate

Welfare and Resource Utilization



Conclusion

- Dynamic assignment design requires an empirical approach
  - Limited guidance from theory
  - Simulations commonly used by policy-makers e.g. liver allocation reforms
- Empirical framework for predicting outcomes in dynamic assignment systems
- Many applications and extensions
  - Rationing through waitlists is empirically under-explored
  - Interactions with other markets and policies
- Commonly studied mechanisms are far from optimal
- Scope for increasing aggregate outcomes subject to distributional barriers