Welfare effects of dynamic matching: An empirical analysis

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May 19, 2021

Abstract

Allocating resources without monetary payments is expected to yield inefficient allocations. Theory suggests that introducing rationing when resources are allocated repeatedly over time can mitigate this issue, while the magnitude of the resulting efficiency gains is an empirical question in most settings. We study a dynamic assignment mechanism used by the Michigan Department of Natural Resources to allocate bear hunting licenses and find that it yields a more efficient allocation than static mechanisms, allocating participants to types of resources for which they have a higher value without crowding out participants with a high overall value for hunting. Our empirical analysis also highlights the importance of heterogeneity across participants and across allocated resources for determining the efficiency of a dynamic allocation mechanism.

JEL Codes: D47, D61, H42, C51, C33*; Keywords:* Market Design, One-Sided Matching, Dynamic Matching, Dynamic Discrete Choice Models

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We thank Peter Arcidiacono, Gary Biglaiser, Hanming Fang, Luca Flabbi, Alfred Galichon, Philip Haile, Andres Hincapie, Joe Hotz, Fei Li, Arnaud Maurel, Brian McManus, Robert Miller, Tiago Pires, Stan Rabinovitch, Holger Sieg, Christopher Timmins, Jonathan Williams, Kyle Woodward, and Andrew Yates for many helpful comments and suggestions, Scott Knoche for help with the travel cost data, and Frank Lupi for providing access to the lottery data. We also thank Sandeep Sarangi and Jenny Williams for their help with the implementation of this project on the UNC - Chapel Hill computing cluster. Finally, we thank the editor and four anonymous referees who have provided very helpful feedback on several versions of this paper.

1 Introduction

Government agencies are frequently tasked with regulating access to publicly provided resources while facing a mandate that precludes the use of money as a medium of exchange. Examples include amenities such as public housing, which is by definition offered at prices below market clearing prices; transplant organs, for which payment is ruled out for ethical and legal reasons; and access to public schooling, which is considered an essential right that should be tax-funded.

Resource allocation without monetary transfers is generally inefficient, in the sense of failing to maximize total surplus, when individual valuations for the resources being allocated are private information.[1] For instance, when allocating one resource to one individual out of a set of participants, a Vickrey auction is predicted to yield an efficient allocation where the individual with the highest value for the resource receives the allocation, while assigning this resource without monetary transfers would necessarily lead to the same surplus as a simple lottery that assigns the resource at random.

In settings where several types of resources are assigned, or where a resource is allocated repeatedly over time, assignment mechanisms can use opportunity costs instead of monetary transfers to obtain a more efficient allocation. Here we study how successful a particular assignment mechanism has been at reducing the difference between the total surplus of an allocation obtained without relying on monetary transfers and the maximized total surplus that would be obtained if monetary transfers were admissible.

We study an allocation problem in which access to several positions is being allocated.[2] Each position can be assigned to a certain number of individuals, but each individual can be assigned to at most one position. In addition, positions can be reallocated periodically among individuals. The assignment mechanism we study, which we call a dynamic lottery, endows participants with a stock of "preference points" that increases whenever a participant is not assigned to a position but is reset to zero upon receiving an allocation. Every period, applicants choose whether to request access for a position or abstain, and applicants with a

[1] In this paper we call an allocation efficient if it maximizes total surplus, i.e. the sum of valuations across all assignments.

[2] We use the terminology of Hylland and Zeckhauser (1979), where the term positions denotes the different types of resources being allocated.

higher stock of preference points are granted a higher priority by the assignment mechanism.

The Michigan Department of Natural Resources (DNR) has used such a dynamic assignment mechanism to allocate approximately 12,000 bear hunting licenses for 22 different hunting sites and seasons every year since 2000. The DNR offers these licenses at very low cost, and therefore faces significant excess demand, with around 55,000 participants in its lottery every year.

Using data on participants and applications obtained from the DNR and a dynamic model of applicants' choices, we find that the dynamic lottery used by the DNR leads to a total surplus that is significantly higher than the total surplus that would be obtained with static alternatives, while remaining lower than the maximized total surplus obtained by an efficient auction.[3]The increase in average surplus achieved by the dynamic lottery originates both from allocating participants to types of resources for which they have a high relative value (improved match quality), and from allocating access to participants with a high overall value for the allocated resources (improved targeting). The first source of efficiency is achieved by the intertemporal opportunity cost elicited by the dynamic lottery. Since obtaining access today is associated with a loss in priority for access in the future, participants are more selective about their requests for access. The second source of efficiency follows from the heterogeneity that is estimated to exist across types of resources and from the requirement in the dynamic lottery that participants abstain from receiving a license in order to increase their priority in future allocations. Participants with a low overall value for hunting prefer to target highly desirable hunting licenses that require several years of waiting, freeing up less desirable hunting licenses for participants with a high overall value for hunting.

Using a real world application and a centralized assignment mechanism that is already being implemented, our results provide empirical evidence on the magnitude of efficiency gains obtained by a dynamic assignment mechanism that rations applicants' assignments by using a dynamically evolving budget in an artificial currency (preference points). A large body of theoretical results points to the usefulness of leveraging opportunity costs

[3]The static alternatives that we consider are a static lottery for each position (applicants choose one position for which to apply, then a simple lottery allocates access for each position separately) and random serial dictatorship (applicants are randomly assigned to a place in a single queue, then choose which position to receive among positions that are leftover once the mechanism reaches their place in the queue).

when facing allocation problems without monetary transfers and in the presence of private information. Hylland and Zeckhauser (1979) show that introducing an artificial currency can transform the static allocation of many individuals to several positions into a pseudo-market and lead to a Pareto optimal assignment.[4] Jackson and Sonnenschein (2007) show that linking a large number of assignment problems and introducing rationing can lead to an efficient allocation in a simplified setting.[5] Guo and Hörner (2017) consider the repeated allocation of a good by a principal to an agent whose valuation of the good is private information and evolves stochastically over time. They show that an optimal allocation mechanism in their setting provides the agent with a dynamically evolving budget in an artificial currency which decreases when the agent obtains the good and increases if the agent abstains. This feature of their proposed mechanism is remarkably similar to the dynamic lottery that we study, but we observe a rich market where participants and types of resources are estimated to be heterogenous. We show that these two dimensions of heterogeneity have a first order effect on the features of the allocation obtained by the dynamic lottery.

The repeated allocation of several resources to several individuals has been studied as a dynamic one-sided matching problem with private information in Arnosti and Shi (2020), Bloch and Cantala (2017), Leshno (2017), Schummer (2016), and Thakral (2016), who also show that assignments that create intertemporal trade-offs — or, in other words, increase the option value of waiting for a future assignment — can induce applicants to be more selective, leading to an improvement in match quality. On the other hand, this work also shows that allocation mechanisms that elicit intertemporal trade-offs may lead to a crowding out of the participants with a higher overall value for the resources being allocated, leading to a deterioration in targeting. For instance, Arnosti and Shi (2020) show in their setting that there is necessarily a trade-off between match quality and targeting when allocating resources without money. As discussed above, we find in our application that the dynamic lottery leads to an improvement in both match quality and targeting.

To our knowledge, this paper is among the first to provide empirical results on the relative

[4]See also He et al. (2017) for additional results and their review of existing results on no-transfer allocation mechanisms that account for intensity of preferences. In a recent contribution, Arnosti and Randolph (2020) study a static allocation mechanism for hunting licenses in Alaska.

[5]See also Radner (1981), Townsend (1982), Rubinstein and Yaari (1983), and Fang and Norman (2006).

efficiency of an existing dynamic allocation mechanism.[6] Two other examples are found in Agarwal et al. (2020) and Waldinger (2020), who study the problem of the one-time allocation of participants to a resource for which new units arrive continuously (transplant organs and public housing, respectively). In addition to methodological differences, the nature of the allocation also leads to different considerations for welfare.[7] Here, we study the efficiency gains derived from endowing participants with a dynamically evolving budget in an artificial currency (preference points). Agarwal et al. (2020) consider allocation mechanisms with objectives that are specific to the allocation of donor organs (e.g., biological compatibility, organ discard rate, equity for patients with very few possible matches). Waldinger (2020) compares allocation mechanisms along two main dimensions: whether applicants have a choice over housing sites and whether income determines a participant's priority.[8]

In the following section, we provide background on bear hunting in Michigan and on the allocation mechanism used by the DNR to allocate access to hunting. We then provide descriptive evidence on application outcomes and behavior in our data, develop a model of application choices in a dynamic lottery, and discuss our estimation method and results. Finally we compare the dynamic lottery to alternative allocation mechanisms.

[6] A related empirical literature studies one-sided and two-sided static matching mechanisms. Abdulkadiroğlu, Che, and Yasuda (2011), Abdulkadiroğlu, Che, and Yasuda (2015), Abdulkadiroğlu, Agarwal, and Pathak (2017), Agarwal (2015), Agarwal and Somaini (2018), Calsamiglia, Fu, and Güell (2019), Fack, Grenet, and He (2019), Hastings, Kane, and Staiger (2009), He (2017), and Narita (2016) empirically evaluate two-sided matching mechanisms used for school and medical residency assignment. Li (2017) compares the welfare gains under a static lottery and an auction when allocating automobile licenses in the presence of negative externalities.

[7] In terms of methodology, Agarwal et al. (2020) study a binary decision (accepting or rejecting a transplant offer) and use a conditional choice probability estimation approach developed for their specific setting in which new transplant offers arrive continuously over time. We use a full-solution estimation approach that deviates from standard methods because, in each year, participants choose over uncertain alternatives and multiple rounds of allocation. Waldinger (2020) estimates a choice model for a portfolio choice problem (which housing site-specific waitlist to join).

[8] At a more general level, our analysis is also related to any empirical work in which intertemporal tradeoffs play a central role, in particular when dynamic structural choice models are used to capture these tradeoffs. For instance, Marion (2017) studies an affirmative action policy for procurement where a history of participation may exempt firms from participation at a given time. As a result, a firm's decision to participate may be determined by the effect of participation on both current and expected future gains, similarly as in our study in which application choices are modeled as being based on current gains and the intertemporal opportunity cost of receiving an allocation. Reviews of such work in various literatures can be found in Keane and Wolpin (2009), Keane, Todd, and Wolpin (2011), Low and Meghir (2017), and Reiss and Wolak (2007).

2 Allocated Resource and Allocation Mechanism

2.1 Bear Hunting Management in Michigan

In Michigan, the Department of Natural Resources (DNR) is responsible for guaranteeing that the bear population remains at a sustainable level while not being so large that it becomes a nuisance to the public. Hunting is the primary tool available to the DNR to accomplish this goal (Michigan Department of Natural Resources (2009b)). The DNR regulates the intensity of bear hunting in the state of Michigan by issuing hunting licenses. A hunter must have been granted a license by the DNR in order to bear hunt in Michigan, and a hunting license allows for the harvest of at most one bear.[9]

In order to have a finer control over the bear population, the DNR issues hunting licenses that restrict the geographical region and time period in which a hunter can hunt. A hunting license issued by the DNR will only grant access to one of ten geographical regions called bear management units (BMU), which are depicted in Figure 1a. A hunting license will also restrict the dates during which a hunter may hunt. Six of the ten BMU offer three distinct hunting seasons, while the remaining four BMU only offer one hunting season. Table 1 lists the opening and ending dates of each hunting season. This results in a total of twenty-two types of licenses allocated by the DNR.

Table 1 also provides information on BMU characteristics that highlights their heterogeneity. In particular, Table 1 reports the hunter success rate of each BMU, i.e., the proportion of hunters who successfully harvested a bear in each BMU, which ranges from 17% (Gladwin BMU) to 65% (Drummond Island BMU). Receiving a license for an earlier or later hunting season also has an impact on hunters. While the DNR does not provide data on success rate by season, earlier hunting seasons are expected to be more favorable to hunters as the bear population is larger and bears are less alert to hunters.

Bear hunting takes place during the months of September and October. Each spring, the DNR determines license quotas for each type of license (i.e. for each BMU and hunting season). In the summer, hunters can apply to receive one of these licenses. Wildlife in the

[9] A license cannot be transferred to another hunter. The license system is enforced by DNR conservation officers and by reporting from the public. In Michigan, illegally taking a bear carries a mandatory prison sentence of between 5 and 90 days, a fine between $500 and $1000, a $3,500 restitution, and a ban from any hunting for four years.

United States is a public trust resource (see, e.g., The Wildlife Society (2010)), held and managed by the government for the benefit of the public. Therefore, the cost of obtaining a license from the DNR is very low. A hunter can apply for a license by paying an application fee of $4. If allocated a license, a hunter who is a resident of Michigan must pay a license fee of $15, while a non-resident must pay a license fee of $150. Because the monetary cost of obtaining a hunting license is low, the DNR faces a large amount of excess demand (e.g., there were 55,954 lottery participants for 12,993 licenses in 2008). In order to manage this excess demand, the DNR uses a particular dynamic allocation mechanism that we call a dynamic lottery and describe in the next subsection.

2.2 Allocation by Dynamic Lottery

The dynamic lottery used by the DNR to assign bear hunting licenses tracks an applicant's seniority (by using what the DNR calls preference points) and assigns greater priority to hunters who have been participating in the lottery for the longest time without receiving a license. In any given year:

- New participants enter the lottery with zero preference points. Applicants who were present in the lottery in the previous year re-enter the lottery with their stocks of preference points as determined by the previous year's lottery.

- Participants submit first and second round choices simultaneously. For each round, an applicant submits a choice of one out of twenty-three alternatives given by the twenty-two types of licenses and the "preference point-only" option. Applicants who select this last option will not be considered for the allocation of a license.

- After the end of the application submission period, licenses are assigned by the DNR using the applicants' first round choices. Each type of license is assigned to the applicants who applied for this type of license by reverse order of preference points, with random tie-breaking if needed. For each type of license, either all licenses are assigned using first round choices or there are leftover licenses that can be assigned using second round choices.

- Leftover licenses are assigned to applicants who have not received a license in the first round using these applicants' second round choices and the same assignment rule as in the first round.

- Once both rounds have been completed, applicants who have successfully obtained a license have their stock of preference points reset to zero. Applicants who did not receive a license, either because they were unsuccessful in applying for a license or because they chose the preference point-only option, have their stock of preference points increased by one.

Table 2 provides the number of first round applications for each type of license and each possible stock of preference points in 2008, as well as the quotas for each type of license.[10] For example, we see that the DNR allocated 610 licenses for the first season of the Bergland BMU (Bergland 1) in that year. All 507 applicants with two or more preference points who applied for Bergland 1 were successful in obtaining a license. Random tie-breaking was implemented to allocate the remaining 103 licenses to the 349 applicants with one preference point who applied for Bergland 1. Therefore there were no left-over Bergland 1 licenses available in the second round, and none of the participants with zero preference points who applied for Bergland 1 received a license in the first round.[11]

3 Allocation Outcomes and Application Behavior

In this section we describe the allocation outcomes of the dynamic lottery and the behavior of the applicants observed in our data. We use for our analysis the application data collected by the DNR in 2008 and 2009. Applicants are uniquely identified so that their application choices can be linked across time. In each year, we observe each applicant's stock of preference points at the time of application and their choices in both rounds of the lottery. In addition

[10]This information for the year 2009 is provided in the online appendix (section A), as well as information on first round application choices across both 2008 and 2009, and information on second round application choices in both years.

[11]Note that these applicants' stock of preference points will then increase by one, as if they had applied for the preference point-only option. The application website used by the DNR reports detailed application outcomes from previous years, which remain relatively constant over time, as discussed below. Therefore, we will model these applicants as being well-informed about their application chances and applying for a type of license that cannot be obtained (e.g., Bergland 1 with zero preference points) to tie-break between multiple alternatives that lead to the same outcome. In practice, it is possible that some of these applicants are in fact misinformed about their application chances. This could have an effect on our estimates and welfare results — since applicants who we model as opting to abstain from hunting for a year would in fact be under the impression that they may receive a license this year — but we have not explored this possibility here.

to this lottery information, we also observe the gender and age of each applicant as well as their exact addresses. Table 3 provides summary statistics on gender, age, and region of residence.[12] Figure 1b plots the location of applicants' addresses in Michigan and in the surrounding region. We first discuss the heterogeneity in the competitiveness of access across types of licenses in the dynamic lottery allocation.

3.1 Competitiveness of the Allocation for each Type of Resource

Consider a particular participant in the dynamic lottery described in the previous section, taking the rest of the applications as given. This applicant could obtain a license of a particular type with certainty if her stock of preference points is large enough, could obtain a license if successfully drawn by random tie-breaking if her stock of preference points is just below the minimum stock required to obtain this license with certainty, or could have no chance of obtaining a license if her stock of preference points is lower than the stock required to participate in the random tie-breaking.

For each type of license and in each year, this information can be summarized by expected cutoffs that calculate the average minimum stock of preference points needed to obtain a license. For instance, in 2008, random tie-breaking for Bergland 1 licenses is implemented to allocate 103 remaining licenses to 349 applicants with one preference point. For this type of license, 30% of applicants with one preference point receive a license, while 70% are unsuccessful (i.e., would have needed a stock of preference points of two to receive a license), leading to an expected cutoff of 1.70.

Table 2 provides the expected cutoffs for each type of license in 2008 (the cutoffs for 2009 are similar to 2008 and reported in Table 7 of the appendix). These cutoffs measure the competitiveness of the allocation for each type of license in the dynamic lottery.[13] We see that there is a substantial amount of heterogeneity in how competitive access is for different types of licenses. The second and third seasons of Bergland, Baraga, Carney, and Gwinn and the third seasons of Amasa and Newberry have low expected cutoffs (less than one year). Drummond Island and Baldwin have the highest expected cutoffs (around 8 years), while the

[12] Table 3 also provides summary statistics on hourly wage, annual income, and house value, which are imputed and discussed in section 3.4 below and in the appendix (section B).

[13] In a static setting, Azevedo and Leshno (2016) used a similar notion of cutoffs to capture the competitiveness of an allocation across several types of resources.

remaining license types have expected cutoffs ranging from 1.7 to 5.2 years. Overall, license types for earlier seasons or for BMU with higher success rates, larger populations, and more land available tend to be more competitive.[14]

3.2 Application Choices and Preference Points: Local Evidence of Strategic Behavior

As discussed above, an applicant's stock of preference points determines her priority in the allocation of licenses. In section 4 below, we model applicants as strategically submitting application choices in order to maximize their expected utility, taking into account their priority as determined by their stock of preference points. In this section we present local evidence on the strategic behavior of applicants in the dynamic lottery. In particular, we show that applicants tend to apply for more competitive license types as their stock of preference points increases. In doing so, we also uncover compositional differences across applicants with different levels of preference points. This will lead us to include applicant heterogeneity in our structural model in the next section.

Firstly, note from Table 2 that the fraction of applicants applying for the preference point-only option in 2008 increases with preference points, from 21% among applicants with zero preference points to 88% among applicants with eight preference points. This may appear counterintuitive since more license types become available as an applicant accumulates preference points, so that a particular applicant may be expected to be more likely to apply for hunting as her stock of preference points increases. On the other hand, an applicant's stock of preference points keeps a record of her past behavior, and thus is partly guided by preferences. In particular, if applicants vary in their taste for hunting, applicants with a low preference for hunting may apply to hunt less frequently and therefore would exhibit higher levels of preference points, which would explain the pattern above.

In this section, we provide local evidence to disentangle compositional effects from the effect of an increase in preference points on a particular applicant's behavior, using the random tie-breaking implemented by the dynamic lottery. For simplicity, consider applicants with zero preference points who apply for a type of license that will lead them to enter a random tie-breaking procedure (second season of Bergland, Baraga, Carney and Gwinn and

[14] To support these patterns, the online appendix (section F) provides the results from a linear regression of the expected cutoffs of all license types on the observed characteristics of these license types.

third season of Amasa and Gwinn). These applicants will be randomly assigned to either a license, in which case their stock of preference points will be zero again in 2009, or to an increase in their stock of preference points by one. We can therefore estimate, for these applicants, the average causal effect on 2009 outcomes of not receiving a license in 2008 and entering the 2009 lottery with one preference point compared to receiving a license and entering with zero preference points.

We can denote this local average treatment effect by $E(y(1) - y(0)|D)$, where y is some 2009 outcome of interest (e.g., applying for the preference point-only option), $y(1)$ is potential outcome after failing to receive a license in 2008, $y(0)$ is potential outcome after receiving a license in 2008, and D indexes the population of applicants with zero preference points who entered a random tie-breaking procedure in 2008. We estimate this local average treatment effect by inverse probability weighting (see, e.g., Wooldridge (2010)) using the license type-specific probability of being successfully drawn by the random tie-breaking.

Table 4 shows these estimated local average treatment effects on three 2009 outcomes: applying for a license with an expected cutoff less than one, applying for the preference point-only option, and obtaining a license. We show results for all 2008 stocks of preference points at which random tie-breaking is applied for at least one license type. An increase in preference points is estimated to cause a decrease in the likelihood of selecting the preference point-only option among applicants with a 2008 stock of preference points of 2 or greater. This confirms that the descriptive pattern discussed at the beginning of this subsection can indeed be attributed to compositional differences across applicants with different stocks of preference points.[15] In addition, we estimate that an increase in preference points causes a decrease in the likelihood of applying for a license type with an expected cutoff lower than one preference point. This is evidence of strategic behavior, i.e., evidence that applicants tend to target more competitive license types as their priority in the dynamic lottery increases.[16] On the other hand, this evidence is local since it only concerns applicants with application choices

[15]The estimated effect is small but positive for applicants with zero or one preference points in 2008, which is not incompatible with a model of strategic behavior since the option value of waiting also has to be taken into consideration, as will be discussed further in the next section.

[16]Note that our outcome of interest here is applying for a license type with an expected cutoff lower than one rather than the expected cutoff itself because it is impossible for applicants with zero preference points in 2009 (our "control" group) to obtain license types with expected cutoffs greater than one.

and stocks of preference points such that they entered a random tie-breaking in 2008.[17] Our structural model in the next section will allow us to estimate a model of strategic behavior for the entire population of applicants, at the expense of imposing assumptions on this behavior.

3.3 Application Choices and Travel Distance

We see from Figures 1a and 1b that there is significant variation in the geographical location of applicants and of hunting sites. In this section, we document the statistical relationship between application choices and travel distances by estimating a descriptive model of application choices.[18]

As discussed above, applicants with different stocks of preference points have access to a different set of license types. In particular, taking all other application choices as given, applicants with a stock of preference points greater than the expected cut-off of a particular license type will obtain this license with certainty if they choose to apply for it, while other applicants would either not be able to obtain this license or would enter a lottery drawing for it. In addition, participants who do not receive a license obtain an increase in priority for future allocations, while participants who receive a license see their priority decrease.

In order to specify a simple descriptive model without considerations of intertemporal substitutions or of uncertainty in alternatives, we model application choices among license types that can be obtained with certainty. For each possible value of an applicant's stock of preference points $p \in \{0, 1, ..., 10\}$ and each year $t \in \{2008, 2009\}$, we denote the set of license types that can be obtained with certainty by $\mathcal{C}_{p,t}$.[19] For applicant i with a stock of preference points p in year t, we then use a multinomial logit model for the conditional probability of

[17]In a static setting, Agarwal and Somaini (2018) used a regression discontinuity design rather than randomization to provide local evidence on the effect of priority scores on school application choices, for the hypothetical subpopulation of students with characteristics that place them exactly at the threshold required to gain a priority premium.

[18]Driving distances and driving times were calculated using arcGIS as the driving distance and time from each applicant's home to the centroid of each BMU.

[19]For instance, we see from Table 2 that $\mathcal{C}_{0,2008}$ contains the third seasons of the Bergland, Baraga, and Carney hunting sites. Note that an applicant may not necessarily know ex-ante — i.e., at the time of application, before the other applicants' choices are revealed — whether she can obtain these sites with certainty. In section 5.2 below, we show that treating the license types in $\mathcal{C}_{p,t}$ as obtainable with certainty at the time of application is a good approximation in a setting where many participants submit equilibrium application choices.

applying for site $j \in \mathcal{C}_{p,t}$ in the first round:

$$P(c_{1,it} = j | c_{1,it} \in \mathcal{C}_{p,t}) = \frac{e^{\varsigma_{0,j} + g_0(d_{ij})}}{\sum_{j' \in \mathcal{C}_{p,t}} e^{\varsigma_{0,j'} + g_0(d_{ij'})}},$$

where $c_{1,it}$ denotes the first round application choice of applicant i in year t, $\varsigma_{0,j}$ is a license type-specific intercept that accounts for systematic differences in the frequency of applications across types of license, d_{ij} is applicant i's travel distance to the hunting site corresponding to license type j, and $g_0(.)$ is a function in the class of continuous cubic splines with continuous first and second derivatives. If application choices were orthogonal to travel distances, the function $g_0(.)$ would be equal to zero, while it will otherwise capture the predictive effect of travel distances on application choices.

In order to obtain a more compact support for the distance from an applicant's home to each hunting site, we restrict our estimation sample to applicants who reside in Michigan (95% of applicants), so that distance in our estimation sample takes values from 0 to 666 miles. We then estimate the intercepts $\varsigma_{0,j}$ for all license types j (with the normalization $\varsigma_{0,1} = 0$) and the function $g_0(.)$ by pooled maximum likelihood estimation (pooling across both years of data), and obtain standard errors by clustering at the applicant-level. Figure 2 shows the estimated shape of the function $g_0(.)$ for six and eight equidistant knots, with pointwise 95% confidence intervals.[20]

We see that travel distance has a negative predictive effect on application frequencies over the entire range of observed travel distances, but that the magnitude of this effect varies. Application choices seem to be disproportionately affected by distance within a range of approximately 150 miles, while beyond 200 miles the effect of distance on the log-odds ratios of our descriptive model seems to be approximately linear. In the next section, we will interpret these patterns as applicants placing a premium on hunting sites that are very close to their home, perhaps, for instance, because these sites are already familiar to them from hunting other species such as deer, while travel distance beyond a radius of 200 miles will be modeled as affecting application choices through travel costs that are proportional to

[20] The preference point cutoffs for each type of license and both years of our data are reported in Table 7 of the online appendix. There is some variation in the sets $\mathcal{C}_{p,t}$ over time. To check the sensitivity of our results to this variation, we estimated the same model as discussed here but conditional on the set of licenses that could be obtained with certainty in both years of our data. The corresponding results are very similar to the results discussed here.

distance. We will see below that this plays an important role in our ability to report welfare results denominated in monetary terms rather than in arbitrarily normalized utils.

3.4 Imputed annual income, hourly wage, and house value

As will be discussed in the next section, the annual income, hourly wage, and house value of applicants enter our model of preferences. These variables are not observed in our data, and we use imputed values for our analysis. We impute this information using the applicants' home addresses and census data at the block group or census tract level. Our approach relies on residential segregation by income in the United States, so that living in a high income neighborhood is predictive of a high income and vice versa. To obtain an improved measure of income and wage, we also used estimates of house value obtained from the website Zillow as an additional predictor of income and wage within an individual's neighborhood. We provide details in the online appendix (section B), and Table 3 reports information on the imputed household annual income, hourly wage, and house value in our data.

4 Model of Applicants' Preferences and Application Behavior

In this section, we develop a model of application behavior for participants in the dynamic lottery.

4.1 Applicants' Preferences for Hunting

In this subsection, we specify our model for the flow utilities derived by applicants from each license type. The first simplification imposed in our model is a quasilinear specification for the total utility derived by applicants when obtaining a license of type $j \in \mathcal{J} = \{1, ..., 22\}$ or not obtaining a license ($j = 0$): $u(j, \text{income}) = v_j + \mu_0(\text{income} - \text{cost}_j)$, where cost_j is the monetary cost of hunting for $j \in \mathcal{J}$ and $\text{cost}_0 = 0$, and μ_0 is the marginal utility of income.[21]

As discussed above, hunting licenses are provided by the state of Michigan at approximately no cost, so that we take the main monetary cost of hunting to be the cost of traveling from the applicant's home to the hunting site. For applicant $i \in \{1, ..., n\}$ and a license of

[21] The assumption of quasilinearity is important for our ability to extrapolate and calculate welfare under an efficient auction. It is possible that utilities are in fact non-linear in consumption of other goods or that participants face credit constraints, which would invalidate our estimates of welfare under an efficient auction, and would also deviate from the conditions under which the auction considered is efficient (see, e.g., Che, Gale, and Kim (2013)). Since we observe little variation in the monetary cost of hunting, we do not attempt to estimate a model that would include such mechanisms, and this is a potential limitation of our analysis.

type $j \in \mathcal{J} = \{1, 2, ..., 22\}$, we denote this travel cost by TC_{ij}.[22] In addition, since income appears across all alternatives and because of our quasilinear utility specification, it can be excluded from the model without having any impact on our model of application choices or on our counterfactual analysis.[23] We then write the total utility derived in year t by applicant i when not obtaining a license ($j = 0$) or obtaining a license of type $j \in \mathcal{J}$ as:

$$u_{0,it} = v_{0,it}, \qquad\qquad u_{j,it} = v_{j,it} - \mu_0 TC_{ij} \ \forall\, j \in \mathcal{J}.$$

We then assume that the term $v_{j,it}$, $j \in \bar{\mathcal{J}} = \{0\} \cup \mathcal{J}$ can be decomposed into time-constant and time-varying parts:

$$v_{j,it} = v_{j,i} + \epsilon_{j,it} \,\forall\, j \in \bar{\mathcal{J}},$$

where $\epsilon_{j,it}$ is an independent preference shock assumed to be type-1 extreme value distributed and i.i.d. across j, i, and t.[24]

Finally we specify a model for the time-constant utility components $v_{j,i}$. Informed by the empirical patterns highlighted in the previous section, our model accounts for heterogeneity across license types and applicants and for the possibility that applicants derive additional utility from hunting sites that are very close to their home (nearby premium). Since a normalization will be required for estimation, we normalize $v_{0,i} = 0$.[25] The heterogeneity across license types $j \in \mathcal{J}$ is captured by license type-specific parameters $\chi_{0,j}$ that account for attributes that may make some license types more or less attractive to hunters (e.g., hunter success rate or amenities). Applicant heterogeneity is modeled with observed characteristics (x_i) that may have an effect on overall taste for hunting, and discrete types that are unob-

[22] For notational simplicity we omit application fees ($4) and license fees ($15 for in-state applicants and $150 for out-of-state applicants) here, although these costs are included in our final analysis by redefining TC_{ij} to include license fees (application fees are charged even when not obtaining a license, so that they cancel out across all alternatives in our choice model).

[23] Aguirregabiria and Suzuki (2014) and Kalouptsidi, Scott, and Souza-Rodrigues (2017) show that such normalizations may have an effect on counterfactual analyses in general. We show in the online appendix (section D.1) that this is not the case in our application.

[24] Two sources of variation in preferences or choices that are absent from our model are satiation and collusion. With satiation, applicants may derive different utilities from hunting depending on whether they hunted in the recent past or not. We discuss a model with satiation in the appendix (section K) and find that the estimated effect of satiation is minor, with the rest of estimated parameters being virtually unchanged, so that ignoring satiation here seems appropriate. With collusion, applicants may synchronize their application behavior to increase their joint utility. We do not pursue this here but it could be an interesting question for future work.

[25] As for the previous normalization which excluded income from our utility specification, the online appendix (section D.1) shows that this normalization does not have an effect on our analysis.

served (to the econometrician), as in Heckman and Singer (1984) and subsequent papers.[26] An applicant of type $\tau \in \{1, 2, ..., \bar{\tau}\}$ is characterized by an intercept, $\alpha_{0,\tau}$, which determines her overall preference for hunting, and a slope coefficient, $\beta_{0,\tau}$, which determines her sensitivity to the license type-specific quality $\chi_{0,j}$. Finally we use a piecewise-linear function of distance, $h(.,.)$, to account for a nearby premium, so that our model for $v_{j,i}$ is:

$$v_{j,i} = \alpha_{0,\tau_i} + \beta_{0,\tau_i}\chi_{0,j} + x_i\eta_0 + h(d_{ij}, \gamma_0)\ \forall\, j \in \mathcal{J},$$

where τ_i is applicant i's unobserved preference type, d_{ij} is the distance from applicant i's home to the hunting site of a license of type j, and η_0 and γ_0 are vectors of unknown parameters.[27]

We decompose the cost of traveling from applicant i's home to the hunting site corresponding to license type j into the cost of operating a vehicle to drive to the hunting site (mileage cost), and the opportunity cost of time of applicants:

$$TC_{ij} = MC_{ij} + OC_{ij}, \qquad MC_{ij} = 2 \times mc \times d_{ij}, \qquad OC_{ij} = 2 \times \lambda_0 \text{wage}_i \times t_{ij},$$

where MC_{ij} is the cost of operating a vehicle over a round-trip to the hunting site, determined by the round-trip distance, $2 \times d_{ij}$, and the cost of operating a vehicle per mile, mc; and OC_{ij} is the opportunity cost of time corresponding to the round-trip, determined by the driving time from the applicant's address to the hunting site, t_{ij}, and a share λ_0 of the applicant's hourly wage.[28]

As discussed above, the parameter μ_0 represents the marginal utility of income in our model. Our quasilinear utility specification will allow us to calculate compensating variations and report welfare results denominated in monetary terms, even though access to hunting licenses is allocated without money. This is an approach that has been widely used to value

[26]For estimation, x_i will include age, gender, annual income, house value, and indicator variables for regions of residence.

[27]For estimation, $h(.,.)$ will be composed of four equidistant segments from 0 to 200 miles: $h(d, \gamma) = \sum_{k=1}^{4} 1[50 \cdot (k-1) \leq d < 50 \cdot k](\gamma_{1,k} + \gamma_{2,k} d)$, where $\gamma = [\gamma_{1,k}, \gamma_{2,k} : k = 1, ..., 4]$ and $1[.]$ is the indicator function.

[28]The cost of operating a vehicle for one mile, mc, was obtained from American Automobile Association (2008) as 27.5 cents per mile based on the use of an SUV (17 cents per mile in fuel cost, 6.5 cents per mile in maintenance and tire wear, and 4 cents per mile in car depreciation due to additional miles).

environmental and natural resources (see, e.g., Haab and McConnell (2002) for a review).[29] On the other hand, we can note that, in addition to a quasilinear utility specification and the imputation of some variables discussed above, two important restrictions underpin our results.

Firstly, preferences for nearby hunting sites and the effect of travel costs on utilities are not separately identified without imposing a functional form restriction that, for distances greater than 200 miles, an increase in travel distances only has an effect on utilities because of an increase in travel costs TC_{ij}. While section 3.3 provides supporting evidence that the effect of travel distance on utilities appears to be linear beyond 200 miles, one may worry that our estimates of the marginal utility of income, μ_0, conflate both the true marginal utility of income and a separate direct effect of traveling over longer distances on utilities. If that is the case, one can interpret our counterfactual results as pertaining to the relative efficiency of the dynamic lottery rather than its efficiency denominated in monetary terms.

Secondly, our estimates may be biased if preferences are correlated with income or wage beyond the correlation captured by the observed characteristics x_i (which contain annual income and house value), because such correlation would lead to a bias in our estimate of the share of wage that determines the applicants' opportunity cost of time (λ_0), which in turn would lead to a bias in our estimate of the marginal utility of income (μ_0). We do not observe strong evidence that this may be an issue here, since we estimate the effect of income or house value on preferences to be very modest, and our applicants do not seem to drastically vary from the general public in terms of income (for instance, the median imputed household annual income in our data is \$48,125, while for the general public from the same residential location as our applicants it is \$49,000, and for the population of Michigan it is \$49,087).

[29] In a subsequent project, Reeling, Verdier, and Lupi (2020) show that the non-market valuation of bear hunting opportunities based on a dynamic discrete choice model yields markedly different results from using existing methods that rely on ad hoc approaches to the dynamic nature of the allocation mechanism used by the DNR. Following a traditional non-market valuation exercise, Reeling, Verdier, and Lupi (2020) calculate the willingness-to-pay of hunters for a particular hunting site to remain open, conditional on licenses being allocated by a dynamic lottery. Although this is not the central interest of this paper, the results provided here may have additional implications for the non-market valuation of natural resources since the relevant valuation for policy may be the maximized value that would be achieved by an efficient auction rather than the realized value achieved by an allocation through a dynamic lottery, and in section 6 we estimate the former to be significantly greater than the latter.

4.2 Forward-Looking Decisions

In each year, an applicant is endowed with a stock of preference points in the dynamic lottery denoted by $p_{it} \in \{0, 1, 2, ...\}$ and submits an application choice $c_{it} = (c_{1,it}, c_{2,it}) \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}$, where $c_{1,it}$ is her choice for the first round of the allocation, and $c_{2,it}$ is her choice for the second round of the allocation. The choice alternatives $c_{1,it} = 0$ and $c_{2,it} = 0$ correspond to the preference point-only option.

As discussed in section 2.2, the odds of receiving a license depend on an applicant's stock of preference points. We define the success probability $\phi^o_{jp,t}$ to be the probability that an applicant with p preference points applying for a license type $j \in \mathcal{J}$ in the first round receives a license of this type. In principle, we could also introduce success probabilities for second round applications. However, in our data we observe that licenses of any particular type are either not available in the second round or are still in excess-supply at the end of the second round, so that for simplicity we denote $\mathcal{J}_{2,t}$ to be the set of licenses still available in the second round and set the second-round success probability for these licenses to one, while other license types have a second-round success probability of zero.[30]

We assume that, in each time period, applicants choose their first and second round application choices in order to maximize the present discounted value of their expected lifetime utilities, with a discount factor ρ. In addition, we assume that the choice environment provided by the dynamic lottery is stationary, so that $\phi^o_{jp,t}$ is constant over time and we can denote it by ϕ^o_{jp}, and we can write $\mathcal{J}_2$ instead of $\mathcal{J}_{2,t}$. This has two implications for our model of heterogenous forward-looking agents. Firstly, in a stationary environment, agents' expectations for future success probabilities are constant over time and simply given by the past and current success probabilities. Secondly, the assumption of stationarity is used for estimation to address the initial conditions problem. We discuss this last point in Section 5.1 below and present evidence to support our assumption of stationarity in the online appendix (section E).

The expected flow utility of an applicant i with p preference points who submits an

[30]This restriction simplifies notation and also leads to simpler derivations for the form of the Bellman equations and conditional choice probabilities defined below, as discussed in section C of the appendix. Section 5.2 below shows that it is a good approximation in a setting where there are many participants submitting equilibrium application choices.

application $c = (j, j^{'}) \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}$ is:

$$\tilde{u}_{it}(c, p) = \phi^o_{jp} u_{j,it} + (1 - \phi^o_{jp})(1[j^{'} \in \mathcal{J}_2] u_{j^{'},it} + 1[j^{'} \notin \mathcal{J}_2] u_{0,it}), \tag{4.1}$$

where we define $\phi^o_{0p} = 0$ (the probability of obtaining a license is zero when applying for the preference point-only option) and $1[.]$ is the indicator function.

Under the assumption of stationarity, agents only have two sources of uncertainty regarding the future: future realizations of the transitory shocks to their preferences, $\epsilon_{j,it}$, and their future stock of preference points. The former is assumed to be drawn independently over time, while the latter depends on their current stock of preference points and on their application choices. In particular, an applicant with p preference points who submits an application $c = (j, j^{'}) \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}$ will see her stock of preference points reset to zero if she receives a license in the first round (with probability ϕ^o_{jp}) or in the second round (if she chose a site for which leftover licenses are available, i.e., if $j^{'} \in \mathcal{J}_2$). Otherwise, this applicant would see her stock of preference points increase by one. We can therefore define the transition probabilities $\Pi(0, c, p) = P(p_{it+1} = 0 | p_{it} = p, c_{it} = c) = \phi^o_{jp} + (1 - \phi^o_{jp}) 1[j^{'} \in \mathcal{J}_2]$ and $\Pi(p + 1, c, p) = P(p_{it+1} = p + 1 | p_{it} = p, c_{it} = c) = 1 - \Pi(0, c, p)$.

Define $u_{j,i} = v_{j,i} - \mu_0 TC_{ij}$ and $u_i = \{u_{j,i} : j \in \mathcal{J}\}$ to be the time-constant part of an applicant's utilities for all types of licenses. We model participants in the dynamic lottery as solving a dynamic discrete choice problem (see, e.g., Arcidiacono and Ellickson (2011)):

$$max_{c \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}} \{\tilde{u}_{it}(c, p) + \rho \sum_{p^{'} \in \{0, p+1\}} \Pi(p^{'}, c, p) R_{u_i}(p^{'})\}, \tag{4.2}$$

where the integrated value function $R_{u_i}(p)$ is defined recursively by:

$$R_{u_i}(p) = E(max_{c \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}} \{\tilde{u}_{it}(c, p) + \rho \sum_{p^{'} \in \{0, p+1\}} \Pi(p^{'}, c, p) R_{u_i}(p^{'})\}), \tag{4.3}$$

where the expected value operator $E(.)$ integrates over the distribution of the transitory preference shocks $\{\epsilon_{j,it} : j \in \bar{\mathcal{J}}\}$.

This discrete choice problem leads to the conditional choice probabilities

$$P_{u_i}(c, p) = P(c \in argmax_{c \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}} \{\tilde{u}_{it}(c, p) + \rho \sum_{p^{'} \in \{0, p+1\}} \Pi(p^{'}, c, p) R_{u_i}(p^{'})\}), \tag{4.4}$$

where the probability operator $P(.)$ integrates over the distribution of the transitory preference shocks $\{\epsilon_{j,it} : j \in \bar{\mathcal{J}}\}$.

The equations above form our model of application behavior in the dynamic lottery. The

Bellman equation (4.3) and conditional choice probabilities $P_{u_i}(c,p)$ defined in (4.4) do not take the familiar multinomial logit forms, despite our assumption that preference shocks are type-1 extreme value distributed, because applicants make application choices under uncertainty (due to the the random tie-breaking implemented by the dynamic lottery) and submit choices across the same alternatives over two rounds. In the online appendix (section C), we show that analytical derivations can be used to write the Bellman equation and the conditional probabilities in terms of a single integral, so that numerical integration can be used without leading to a prohibitive computational cost. The issue of choosing over uncertain alternatives across several rounds is similar to the issue encountered by Agarwal and Somaini (2018), who study a static allocation problem where applicants (students) choose over uncertain alternatives for the allocation of 16 types of resources (schools) over three rounds (while here each allocation involves 22 types of resources over two rounds). Agarwal and Somaini (2018) rely on simulation and a Gibbs sampler for estimation, while the analytical results shown in section C of the appendix allow us to use a maximum likelihood estimator.[31]

5 Estimation

We take the discount factor, ρ, as known. Brookshire, Eubanks, and Randall (1983) find evidence of discount factors for big game hunting opportunities ranging from $\rho \in [0.95, 0.99]$. We therefore assume $\rho = 0.975$. The remaining unknown parameters that determine the value functions and conditional choice probabilities for each applicant of each type derived in the previous section are then given by the parameters $\theta_0 = [\chi_{0,j} : j \in \mathcal{J}, (\alpha_{0,\tau}, \beta_{0,\tau}) : \tau = 1, ..., \bar{\tau}, \eta_0, \gamma_0, \mu_0, \lambda_0]$. The time-constant part of the utility of applicant i of type τ for site $j \in \mathcal{J}$ evaluated at the candidate vector of parameters θ is:

$$u_{j,i}(\theta,\tau) = \alpha_\tau + \beta_\tau \chi_j + x_i \eta + h(d_{ij}, \gamma) - \mu(MC_{ij} + \lambda \times 2 \times \text{wage}_i \times t_{ij}). \tag{5.1}$$

Before discussing estimation, we first formulate a model for the distribution of unobserved preference types (τ_i) conditional on an applicant's initial stock of preference points.

[31] The analytical simplification of the Bellman equation (4.3) leads to the largest computational gains for estimation since solving for the integrated value function constitutes the main computational burden of our estimation method, but this issue is absent in Agarwal and Somaini (2018) since they consider a static problem.

5.1 *Initial Conditions Problem*

In this section, we derive a model for the probability $P(\tau_i = \tau | p_{i1}, X_i)$ that an applicant is of unobserved preference type $\tau_i = \tau$ given her initial stock of preference points p_{i1} and the characteristics $X_i = \{x_i, \{d_{ij}, MC_{ij}, \text{wage}_i \times t_{ij}\}_{j \in \mathcal{J}}\}$ that are observed to the econometrician. This model is obtained from the model of application choices outlined above, the assumption of stationarity, and two additional assumptions that we discuss here.

Before doing so, note that the choice environment of applicants is identical across all stocks of preference points large enough that every license type can be obtained with certainty. Let $\bar{p} = \min\{p : \phi^o_{jp} = 1 \, \forall \, j \in \mathcal{J}\}$ be the smallest such stock of preference points. We can then censor preference points at $\bar{p}$ without loss of generality.[32] To reflect this censoring, we redefine the transition probabilities $\Pi(p', c, p)$ so that $\Pi(0, c, p)$ is unchanged but $\Pi(p', c, p) = 1 - \Pi(0, c, p)$ for $p' = min(p + 1, \bar{p})$.

We observe some attrition in our data, with some applicants who were observed in the lottery in 2008 not being present in 2009, while some applicants who are observed in 2009 were not present in the lottery in 2008. The number of total applicants in the lottery remains approximately constant across both years (there are 55,463 applicants in 2008 and 56,775 applicants in 2009). We model this phenomenon as attrition and renewal at random, with a rate α that we simply calculate to be 28.9% from the data as the proportion of applicants from the 2008 lottery who do not participate in the 2009 lottery.[33] Note that, since attrition is not included in our model of application choices in section 4, we model applicants as being unaware that they might exit the lottery in the future.

Finally, we assume that applicant unobserved type τ_i is drawn independently of the observed characteristics X_i, i.e. $P(\tau_i = \tau | X_i) = \pi_{0,\tau}$, where $\pi_{0,\tau}$ is the unconditional proportion

[32] In our application, an applicant with a stock of preference points of nine or greater can obtain a license of any type with certainty, so that we can censor preference points at nine.

[33] To evaluate the plausibility of this assumption, we measure the relationship between attrition or renewal and applicant characteristics and find that there are no strong patterns of correlation. A regression of an indicator variable for whether a 2008 applicant is still present in the 2009 data on observed characteristics (x_i) and indicator variables $1[c_{i1} = j, p_{i1} = p]$ for each possible pair of values $(j, p) \in \bar{\mathcal{J}} \times \{0, ..., 9\}$ of 2008 first round application and stock of preference points leads to a low R-squared of 3.5% (logit and probit regressions lead to a pseudo R-squared of 3.2%). Similarly, a regression of an indicator variable for whether a 2009 applicant was present in the 2008 data on observed characteristics x_i leads to a R-squared of 2% (logit and probit regressions lead to a pseudo R-squared of 1.6%; note that we do not have information on 2008 preference points and choices for new entrants in the 2009 data so that only x_i is included in this regression).

of type τ in the population of applicants. Note that we included observed characteristics in our model of preferences (5.1) above, so that the unobserved type τ_i can be seen as the heterogeneity across applicants that remains after conditioning on observed characteristics.

Our assumption of stationarity implies that the dynamic lottery is in its steady state. Therefore we can obtain the probabilities $P(p_{i1} = p|\tau_i = \tau, X_i)$ of all possible initial stocks of preference points for an applicant of type τ with observed characteristics X_i by solving for $\{\zeta_p : p = 0, ..., \bar{p}\}$ in the steady state equations:

$$\zeta_p = \alpha 1[p = 0] + (1 - \alpha) \sum_{p'=0}^{\bar{p}} \sum_{c \in \bar{\mathcal{J}} \times \bar{\mathcal{J}}} \Pi(p, c, p') P_{u_i(\theta_0, \tau)}(c, p') \zeta_{p'}, \ \forall\, p = 1, ..., \bar{p}, \tag{5.2}$$

where the term $\alpha 1[p = 0]$ and the factor $(1 - \alpha)$ account for attrition and renewal at random. Define $\zeta_{u_i(\theta_0,\tau)}(p) = P(p_{i1} = p|\tau_i = \tau, X_i)$ to be the solution to the system of equations (5.2). By Bayes' rule and our assumption that unobserved preference types are independent of observed characteristics, we have $\xi_i(\tau, p; \theta_0, \pi_0) = \frac{\zeta_{u_i(\theta_0,\tau)}(p)\pi_{0,\tau}}{\sum_{\tau'=1}^{\bar{\tau}} \zeta_{u_i(\theta_0,\tau')}(p)\pi_{0,\tau'}}$, where we define $\xi_i(\tau, p; \theta_0, \pi_0) = P(\tau_i = \tau|p_{i1} = p, X_i)$ and $\pi_0 = [\pi_{0,\tau} : \tau = 1, ..., \bar{\tau}]$ collects the proportion of all applicant types.

5.2 Success Probabilities

In the model outlined in section 4.2, application choices depend on first-round success probabilities, ϕ^o_{jp}, and the set of licenses available in the second round, $\mathcal{J}_2$. We assume that applicants in the dynamic lottery participate in a Bayesian Nash equilibrium, so that the beliefs on success probabilities that enter their application choices coincide with the success probabilities determined by the equilibrium distribution of application choices.

To state our equilibrium assumption, we first define the first-round success probabilities of each license type in each time period taking all application choices as given, so that uncertainty only originates from the tie-breaking described in section 2.2. Define $q_{j,t}$ to be the quota of licenses available for type $j \in \mathcal{J}$ and $n_{jp,t} = \sum_{i=1}^{n} 1[c_{1,it} = j, p_{it} = p]$ to be the number of applicants with p preference points choosing to apply for type j in the first round of the dynamic lottery. Taking all applications as given, the success probability of a first round application for a license of type j from an applicant with p preference points in year t

is given by:

$$\phi_{jp,t} = 1[\sum_{p' \geq p} n_{jp',t} \leq q_{j,t}] + \frac{q_{j,t} - \sum_{p'>p} n_{jp',t}}{n_{jp,t}} 1[\sum_{p'>p} n_{jp',t} \leq q_{j,t} < \sum_{p' \geq p} n_{jp',t}]. \qquad (5.3)$$

Our equilibrium assumption, together with our assumption of stationarity, then requires that $\phi^o_{jp} = E(\phi_{jp,t}) \; \forall\, j, p, t$, where the expected value is with respect to the equilibrium distribution of all application choices, i.e., the choices modeled in section 4.2, determined by the equilibrium success probabilities ϕ^o_{jp}, the set of license types $\mathcal{J}_2$, and applicant-specific preferences and stocks of preference points.[34]

In addition, because we have many participants (over 55,000 per year in our data) over few values of preference points (10 values in our data) and choice alternatives (22 hunting sites), the online appendix (section E.2) shows that the ex-post success probabilities, $\phi_{jp,t}$, are consistent estimators of the ex-ante equilibrium success probabilities, ϕ^o_{jp}, under our assumptions. We therefore replace the ex-ante success probabilities ϕ^o_{jp} with the ex-post success probabilities $\phi_{jp,t}$ averaged across the two years of our data in the definition of the likelihood function below. This leads to a large computational gain for estimation since it allows us to use empirical quantities ($\phi_{jp,t}$) rather than solving for equilibrium beliefs (ϕ^o_{jp}) within our estimation procedure.

Similarly, we can define second round success probabilities taking all application choices as given, $\phi^2_{jp,t}$, by using the same definition as (5.3) but replacing quotas $q_{j,t}$ with the number of licenses left-over after the first round of allocation, and replacing first-round application numbers $n_{jp,t}$ with second-round application numbers among applicants who did not receive a license in the first round. If $E(\phi^2_{jp,t}) \in \{0, 1\} \; \forall\, j, p$, our equilibrium assumption then requires that $\mathcal{J}_2 = \{j \in \mathcal{J} : E(\phi^2_{jp,t}) = 1\}$, where the expected value is again with respect to the equilibrium distribution of all application choices. The simplification restricting that $E(\phi^2_{jp,t}) \in \{0, 1\} \; \forall\, j, p$, used here and in section 4.2, is motivated by observing the same restriction being true for ex-post success probabilities, i.e., $\phi^2_{jp,t} \in \{0, 1\} \; \forall\, j, p$, and by the consistency of the observed ex-post success probabilities $\phi^2_{jp,t}$ for the ex-ante success proba-

[34]Note that, since we have many participants (over 55,000 per year in our data) over few values of preference points (10 values in our data) and choice alternatives (22 hunting sites), the expected value operator is taken to be with respect to the equilibrium distribution of all application choices rather than the equilibrium distribution of the other applicants' choices. This is because the difference between the two expected values is negligible.

bilities $E(\phi^2_{jp,t})$ discussed in section E.2 of the appendix. In the likelihood function below, we replace $\mathcal{J}_2$ with the set of license types that were available in the second round of both years of our data (the third seasons of the Bergland, Baraga, and Carney BMU). The third season of the Gwinn BMU is available in the second round in 2009 but not in 2008. We exclude this license type from the second round in the likelihood function below. Including it does not have a significant effect on estimates, as shown in section J of the online appendix.

5.3 Likelihood Function

The results above allow us to define the likelihood function for applicant i submitting choices (c_{i1}, c_{i2}) with stocks of preference points (p_{i1}, p_{i2}) in years $t = 1$ and $t = 2$:

$$\mathcal{L}_i(\theta, \pi) = \sum_{\tau=1}^{\bar{\tau}} P_{u_i(\theta,\tau)}(c_{i2}, p_{i2})\Pi(p_{i2}, c_{i1}, p_{i1})P_{u_i(\theta,\tau)}(c_{i1}, p_{i1})\xi_i(\tau, p_{i1}; \theta, \pi).$$

Our assumption of attrition and renewal at random also allows us to define the likelihood for applicants who are only observed in year $t = 1$:

$$\mathcal{L}_i(\theta, \pi) = \sum_{\tau=1}^{\bar{\tau}} P_{u_i(\theta,\tau)}(c_{i1}, p_{i1})\xi_i(\tau, p_{i1}; \theta, \pi),$$

and for new entrants in year $t = 2$:

$$\mathcal{L}_i(\theta, \pi) = \sum_{\tau=1}^{\bar{\tau}} P_{u_i(\theta,\tau)}(c_{i2}, 0)\pi_\tau.$$

We then estimate the unknown parameters θ_0 and π_0 by solving the maximum likelihood estimation problem:

$$\max_{\theta,\pi:\pi_\tau \geq 0\,\forall\,\tau, \sum_\tau \pi_\tau = 1} \sum_{i=1}^{n} log(\mathcal{L}_i(\theta, \pi)),$$

where each evaluation of the likelihood at candidate parameter vectors θ and π requires (i) solving the Bellman equation (4.3) using a fixed-point algorithm as in Rust (1987), for which numerical integration is required at each iteration (see also Miller (1984), Pakes (1986), and Wolpin (1987)); (ii) obtaining the conditional choice probabilities (4.4) by numerical integration; and (iii) solving the system of linear equations (5.2). We discuss the identification of the parameters of our model in the online appendix (section D).[35]

[35]Intuitively, the conditional choice probabilities (CCP) for each preference type are identified because of the assumption of stationarity, the assumption of serial independence of the preference shocks $\epsilon_{j,it}$, and because application choices are observed over two years. The CCP can then be mapped into the parameters θ_0 because of our distributional assumption on the preference shocks $\epsilon_{j,it}$, because the discount factor ρ is taken to be known, and because the time-constant part of the utility derived when not hunting was normalized to zero ($u_{0,it} = \epsilon_{0,it}$).

5.4 Results

Since our travel costs are based on the cost of driving a vehicle, we restrict our estimation sample to applicants from Michigan and neighboring states (Wisconsin, Illinois, Indiana, and Ohio) who are unlikely to consider traveling by airplane and who represent 99% of all applicants. Our estimation sample is then composed of all observations on these applicants who had no missing information in their observed characteristics, for a total of 108,732 observations and 70,185 applicants over two years.

Table 5 shows the estimation results from the method defined above with three unobserved applicant types.[36] We estimate that the marginal utility of income, μ_0, is positive, as expected, and that the applicants' opportunity cost of time is determined by a share of hourly wage, λ_0, of 0.67.

We estimate a large degree of heterogeneity across license types ($j \in \mathcal{J}$) and across applicant types ($\tau \in \{1, 2, 3\}$). There is an association between the estimated license type-specific intercepts ($\chi_{0,j}$) and the license type characteristics. License types that offer a higher hunting success rate are estimated to be more desirable, as are license types corresponding to earlier hunting seasons, larger populations, and a larger amount of hunting land.[37] In terms of applicant unobserved heterogeneity, we estimate that applicants of type 1 (21% of applicants) and type 2 (56% of applicants) have approximately the same sensitivity to license type quality ($\chi_{0,j}$), but that type 2 applicants have a significantly lower overall taste for hunting, while applicants of type 3 (23% of applicants) have both a lower overall taste for hunting and a higher sensitivity to license type quality.

The observed characteristics x_i are given by applicant age, gender, annual income, house value, and geographical region of residence (out-of-state, Upper Peninsula, and northern, southwestern, and southeastern Lower Peninsula). We find that age, gender, income, and house value have relatively modest estimated effects on preferences for hunting, but that out-of-state applicants have a higher preference for hunting than Michigan applicants, while Upper Peninsula applicants have a lower preference for hunting than Lower Peninsula appli-

[36] We used Knitro for Matlab to solve the maximization problem, using the default interior-point algorithm. For our starting value, we chose $\pi_\tau = \frac{1}{3}$, $\alpha_{0,\tau} = -\tau$, $\beta_{0,\tau} = 1$, $\forall\, \tau$, and all other parameters set at zero.

[37] To support these patterns, the online appendix (section F) provides the results from a linear regression of the estimated license type-specific intercepts ($\chi_{0,j}$) on the observed characteristics of each license type.

cants. Figure 3 depicts the estimated nearby utility premium. We see that this premium is estimated to be high for hunting sites within a radius of 100 miles and fairly low with a small gradient with respect to distance for hunting sites further away than 100 miles, so that the threshold of 200 miles that we selected seems to be large enough to capture the majority of the additional impact that traveling distance has on utilities beyond its effect through travel costs. The online appendix (section G) provides evidence on how our model fits observed patterns in the data, with all results pointing toward a good fit, considering that our model relies on 46 parameters only.

6 Efficiency of the Dynamic Lottery as an Assignment Mechanism

In this section, we evaluate the relative efficiency of the dynamic lottery as an assignment mechanism by comparing its equilibrium allocation with the equilibrium allocation obtained by two alternative static assignment mechanisms without money and with the equilibrium allocation obtained by an efficient auction. We begin by introducing a decomposition of the possible match value of applicant i to license type j which will be useful for us to discuss the relative efficiency of each allocation mechanism.

6.1 Decomposition of Match Values

From our model for preferences in section 4.1, we can write applicant i's value (relative to not obtaining a license) for a license of type j at time t as:[38]

$$W_{j,it} = \frac{1}{\mu_0}(u_{j,it} - u_{0,it}). \tag{6.1}$$

We can decompose an applicant's value for a particular type of license into the applicant's taste for a randomly drawn license, the applicant's persistent preference for this license type, and the applicant's transitory preference shock for this license type. We first define the ex-ante value derived from a randomly drawn license:

$$A_i = \sum_{j \in \mathcal{J}} \frac{q_j}{q} E(W_{j,it}) = \frac{1}{\mu_0} \sum_{j \in \mathcal{J}} \frac{q_j}{q} u_{j,i},$$

where the expected value operator is with respect to the (unconditional) distribution of preference shocks $\epsilon_{j,it} - \epsilon_{0,it}$, q_j is the quota for licenses of type $j \in \mathcal{J}$, and q is the total number of licenses available, $q = \sum_{j \in \mathcal{J}} q_j$, so that $\frac{q_j}{q}$ is the share of type j licenses among

[38]Note that dividing by the marginal utility of income (μ_0) leads to results denominated in monetary terms rather than in utils.

licenses of all types. In our discussion below, the term A_i will capture an applicant's overall taste for hunting.

We define an applicant's persistent preference for a license type as the difference between the ex-ante value derived from this type of license compared to a randomly drawn license:

$$\Delta_{j,i} = E(W_{j,it}) - A_i = \frac{u_{j,i}}{\mu_0} - A_i,$$

where the expected value operator is again with respect to the (unconditional) distribution of preference shock $\epsilon_{j,it} - \epsilon_{0,it}$.

The remaining part of the value of an applicant for a license of type j is then given by the transitory preference shocks $\mathcal{E}_{j,it}$ defined by:

$$\mathcal{E}_{j,it} = \frac{1}{\mu_0}(\epsilon_{j,it} - \epsilon_{0,it}),$$

and we have the decomposition:

$$W_{j,it} = A_i + \Delta_{j,i} + \mathcal{E}_{j,it}. \tag{6.2}$$

Before discussing the allocation mechanisms under consideration in this section and their relative efficiency, we can note that the average overall taste for hunting (A_i) across all applicants is found to be negative (at $-\$1,159$). This is not unexpected because of the large amount of heterogeneity that is estimated to exist across license types (and is also reflected in the large discrepancies in waiting times across license types). Since we define overall taste for hunting to be the ex-ante value of applicants for a randomly drawn license, and the least desirable license types correspond to large quotas, one may expect the average overall taste for hunting to be negative. When considering the total ex-ante value for a license, $A_i + \Delta_{j,i}$, approximately all applicants have a positive ex-ante value for at least one license type except for applicants of type $\tau = 2$. Among applicants of type $\tau = 2$, 45% have a positive ex-ante value for at least one type of license. This subgroup of applicants is mainly composed of applicants who live within 75 miles of a BMU, from which they can derive a fairly large nearby premium (see Figure 3), leading to a positive ex-ante value for the corresponding license types.

Intuitively, we can think of applicants of type $\tau = 2$ as being "local applicants" who mostly seek to hunt near their home, while applicants of type $\tau = 1$ are "high preference applicants" who would be happy to hunt with the majority of license types, and applicants of type $\tau = 3$ are "destination applicants" who mostly seek to hunt with license types of the highest quality.

6.2 Allocation Mechanisms

The appendix (section H.1) discusses how our estimated choice model can be used to calculate characteristics of the equilibrium allocation of the dynamic lottery such as total surplus, or probabilities that applicants of each type obtain a permit. The next three subsections introduce the alternative assignment mechanisms we consider.

6.2.1 Efficient Allocation by Auction

Let $\{a_{j,it}\}_{i\in\mathcal{N},j\in\mathcal{J}}$ denote some sequence of assignments of applicants to permits such that $a_{j,it} = 1$ if applicant i obtains a permit for hunt j and $a_{j,it} = 0$ otherwise. As in Koopmans and Beckmann (1957) and Shapley and Shubik (1971), an assignment maximizes total surplus in any given year t under the constraint that each applicant receives at most one license and that the number of licenses assigned for each type does not exceed its quota if it solves the linear program:[39]

$$\max_{\{a_{j,it}\}_{i\in\mathcal{N},j\in\mathcal{J}}} \sum_{i,j\in\mathcal{N}\times\mathcal{J}} a_{j,it}W_{j,it} \text{ s.t. } \sum_{j\in\mathcal{J}} a_{j,it} \leq 1 \text{ and } \sum_{i\in\mathcal{N}} a_{j,it} \leq q_j \tag{6.3}$$

where q_j is the quota available for licenses of type j.[40]

A social planner cannot solve for this optimal allocation directly since applicants' valuations $W_{j,it}$ are private information. Leonard (1983) shows that an assignment mechanism that is incentive compatible and solves (6.3) can be achieved with an extension of a Vickrey (second bid price) auction. Demange, Gale, and Sotomayor (1986) also show that the efficient allocation can be approximated arbitrarily well through a sequential procedure whereby prices are gradually increased until the market clears. We use simulations to obtain results for the equilibrium allocation of this efficient auction as well as for the other two alternative static mechanisms introduced below. We provide the details of our calculations in the appendix (section H.2).

[39]Koopmans and Beckmann (1957) show that the solution to the constrained maximization problem (6.3) necessarily sets $a_{j,it} = 0$ or $a_{j,it} = 1$ so that we do not need to explicitly impose the constraint that assignment be a binary variable.

[40]Note that this allocation problem is completely unrelated with the allocation problem in different years. Here we maintained the t subscript so that our notation corresponds to the rest of the paper, but the analysis of the auction, or the two other static allocation mechanisms below, can be done for any particular year, ignoring the fact that allocation is repeated in our application.

6.2.2 Allocation by Random Serial Dictatorship

As a benchmark for evaluating allocation mechanisms without monetary transfers, we consider an assignment mechanism that has been widely studied in theoretical work called random serial dictatorship (see, e.g., Abdulkadiroğlu and Sönmez (1998) for a review). Every year, all participants are assigned a place in a centralized queue at random. The first participant chooses any permit among all the permits being allocated. Subsequent participants choose any permit among those that remain after participants ahead of them have made their choice. This process continues until all permits have been allocated. A participant can also choose not to receive a permit, in which case the assignment mechanism simply moves on to the next participant. The appendix (section H.3) provides the details of our calculations for this allocation mechanism.

6.2.3 Allocation by Static Lottery

As another alternative to the dynamic lottery used by the DNR, we consider a static lottery that does not track seniority among applicants. In each year, applicants decide whether to apply for a license of type j or whether to abstain from entering a drawing. Licenses of type j are then allocated to applicants who applied for type j, using a random drawing if needed.[41] As in the dynamic lottery, we model applicants as maximizing their expected utility, i.e., solving the problem $\max_{j \in \bar{\mathcal{J}}} \phi_j^o W_{j,it}$, where ϕ_j^o is the equilibrium success probability for licenses of type $j \in \mathcal{J}$ and $\phi_0^o = 0$. The appendix (section H.4) provides the details of our calculations for this allocation mechanism.

6.3 Comparison of Equilibrium Allocations

In this section we discuss the relative efficiency of the dynamic lottery implemented by the DNR compared to the static alternatives defined above.[42] Table 6 compares equilibrium allocation outcomes for each assignment mechanism. Random serial dictatorship leads to

[41] This assignment mechanism is similar to the mechanism described in Hylland and Zeckhauser (1979) except that here participants are required to choose one site for which to apply instead of being endowed with a budget of application weights that they can divide between several sites.

[42] Note that, across all allocation mechanisms considered in this section, we treat the set of participants as fixed, i.e., we take the extensive margin of participating in the allocation as exogenous. Since participating in the dynamic lottery is approximately free, this is in principle consistent with us observing the universe of individuals interested in hunting, but in practice it is possible that we are ruling out variations by maintaining the set of participants as constant.

the equilibrium allocation with the lowest annual average total surplus (\$207), followed by the static lottery (\$232), and the dynamic lottery (\$266). The maximized average surplus achieved by the efficient auction is \$410. Average surplus per allocated license is \$950 under random serial dictatorship, \$1,069 in the static lottery, \$1,243 in the dynamic lottery, and \$1,886 in the efficient auction.

Using standard terminology in the matching literature, we will say that an allocation yields improvements in match quality if the persistent or transitory average relative preference for the allocated license ($\Delta_{j,i}$ and $\mathcal{E}_{j,it}$ in (6.2)) is higher, and that it yields improvements in targeting if the average overall taste for hunting (A_i in (6.2)) among allocated applicants is higher. Table 6 shows that each part of this decomposition accounts for a significant share of the difference in average surplus per allocated license across allocation mechanisms, which we discuss in the following subsections.

6.3.1 Static Equilibrium Sorting and Match Quality

The efficiency gains of the static lottery (SL) compared to the random serial dictatorship (RSD) originate from an improvement in match quality when allocating licenses by SL, with the average total relative preference, $\Delta_{j,i} + \mathcal{E}_{j,it}$, for the allocated license being \$1,835 under SL compared to \$1,670 under RSD. This improvement in match quality can be explained similarly as in Abdulkadiroğlu, Che, and Yasuda (2011), who compare various static allocation mechanisms of students to schools. Consider for simplicity the case in which only two types of licenses are allocated, $j = 1$ and $j = 2$. Under RSD, applicants for whom both types of licenses are available simply choose the type of license that yields a higher value, i.e. choices are based on ordinal utilities only. For instance, they choose the first type of license, $j = 1$, if:

$$W_{1,it} - W_{2,it} \geq 0, \qquad\qquad W_{1,it} \geq 0.$$

In an allocation by static lottery (SL), the uncertainty associated with each type of license induces applicants to consider the intensity of their preferences in addition to their ordering, resulting in choices based on cardinal rather than ordinal utilities. Without loss of generality, consider the case where oversubscription is greater for licenses of type $j = 1$ than of type $j = 2$, i.e. $\phi_1^o < \phi_2^o$. An applicant chooses $j = 1$ if $\phi_1^o W_{1,it} \geq \phi_2^o W_{2,it}$ and $W_{1,it} \geq 0$, which can be rewritten:

$$W_{1,it} - W_{2,it} \geq \frac{\phi_2^o - \phi_1^o}{\phi_2^o} W_{1,it}, \qquad\qquad W_{1,it} \geq 0,$$

from which we see that the more oversubscribed license type $j = 1$ will be allocated to applicants with a higher relative preference for this type of license, which is a source of efficiency gain for the allocation by SL compared to the allocation by RSD.

6.3.2 Dynamic Equilibrium Sorting: Intertemporal Opportunity Costs as Prices

In an allocation by dynamic lottery (DL), obtaining a license for the current year is associated with a loss of priority in future allocations. This creates an intertemporal opportunity cost of allocation, captured in our model by a loss in continuation value that for applicant i with p preference points is given by $C_i(p) = \frac{1}{\mu_0}\rho(R_{u_i}(p+1) - R_{u_i}(0))$. In an allocation by DL, applicants will only apply to receive a license if, for at least one type of license, the value of receiving a license to the applicant exceeds the opportunity cost of an allocation, i.e. if $W_{j,it} - C_i(p) \geq 0$ for some $j \in \mathcal{J}$ such that $\phi_{jp}^o > 0$. This intertemporal opportunity cost leads applicants to be more selective about applying for a license than in an allocation by RSD or SL.[43] Table 6 reports the fraction of participants who apply for a license under each allocation mechanism — 30% under DL compared to 83% under RSD and SL, reflecting a high increase in selectivity among applicants in an allocation by DL.

Figure 4b shows quantiles across all applicants of the intertemporal opportunity cost of receiving a license for each level of preference points, as well as the average auction price of the licenses allocated by the DL at each level of preference points and the auction price of each license type over its expected preference point-cutoff. We see that license types that would have a higher price in an allocation by auction also require higher stocks of preference points in an allocation by DL. Since the intertemporal opportunity cost $C_i(p)$ is increasing in the level of preference points p, applicants are faced with a higher opportunity cost of obtaining a license when license types with higher auction prices become available, so that the DL mimics the heterogeneity in prices across license types found in an auction. On the other hand, there is substantial heterogeneity across applicants in their opportunity costs, which leads to inefficiencies that we discuss further below.

[43] Note that $C_i(p)$ is necessarily positive and increasing in stock of preference points p since an applicant's choice set increases as her stock of preference points increases.

The intertemporal opportunity cost elicited by the dynamic lottery leads to an increase in match quality both because applicants become more selective about when to apply for a license, leading to an increase in their time varying preference, $\mathcal{E}_{j,it}$, for the allocated license, and because applicants can accumulate preference points in order to receive a license for which they have a persistently high relative preference, leading to an increase in their persistent relative preference, $\Delta_{j,i}$, for the allocated license. Table 6 shows that both of these channels account for a significant share of the increase in average match quality compared to the allocations by RSD or SL.

6.3.3 Targeting

From Table 6, we see that the average overall taste for hunting, A_i, among applicants who receive a license is lower in an allocation by SL than by RSD (\$-766 under SL and \$-720 under RSD). This is explained by the license type-specific uncertainty induced by the static lottery and discussed in section 6.3.1 above, which improves efficiency by increasing match quality, but favors applicants with a relatively low taste for hunting. In an allocation by SL compared to RSD, some applicants will redirect their applications from the most oversubscribed license types ($j = 1$ in our simplified example above) to less competitive license types ($j = 2$ in our example), which will benefit applicants with a low overall taste for hunting who would only consider hunting at the most oversubscribed license types (applicants with $W_{1,it} \geq 0$ but $W_{2,it} \leq 0$ in our example).

This crowding out does not occur to the same extent in an allocation by DL, with the average overall taste for hunting (\$-733) being only slightly lower than the average taste for hunting in an allocation by RSD. This is because, unlike the static lottery, the dynamic lottery reduces the set of participants who apply for a license, and this reduction impacts applicants with high or low taste for hunting differently. Applicants with a low taste for hunting will tend to abstain from applying for a license more frequently in an allocation by DL, preferring to accumulate high levels of preference points to gain access to more desirable license types, while applicants with a high taste for hunting will apply for a license, even if it is of a less desirable type, more frequently.

To illustrate this mechanism, we divide applicants by their type τ_i, which accounts for 91% of the variation in applicants' overall taste for hunting. Table 6 shows that the reduction in the fraction of applicants who apply for a license in an allocation by DL compared to RSD or SL is much greater for applicants with a low overall taste for hunting ($\tau_i = 2$ and $\tau_i = 3$) than for applicants with a high overall taste for hunting ($\tau_i = 1$), which leads to a higher fraction of licenses being allocated to applicants of type $\tau_i = 1$ by DL than by SL. Table 6 also reports the average auction price of licenses allocated to applicants of each type. Supporting the discussion above, we see that applicants of types $\tau_i = 2$ or $\tau_i = 3$ receive licenses with a higher average auction price in an allocation by DL than by SL or RSD. This is accomplished by abstaining from hunting in order to accumulate seniority, which in turn reduces the crowding out of applicants with a high preference for hunting.

6.3.4 Inefficiency of the Dynamic Lottery

Jackson and Sonnenschein (2007) showed that rationing access over time can lead to an efficient allocation, even when the applicants' value for the good is private information, in a setting where a single type of resource is allocated, applicants have identically and independently drawn values for the resource from a distribution that is known to the social planner, and do not discount future utilities. In our application, we find that, although the rationing implemented by the DL yields significant efficiency gains compared to static non-monetary allocation mechanisms, it still leads to an inefficient allocation, with an average surplus of \$266 compared to the maximized average surplus of \$410 achieved by an efficient auction. Three deviations from the setting of Jackson and Sonnenschein (2007) account for this inefficiency: discounting and attrition, heterogeneity across applicants, and heterogeneity across resources.

In the same environment as Jackson and Sonnenschein (2007) but with agents who discount their future utility, Guo and Hörner (2017) show that an optimal non-monetary allocation mechanism still relies on rationing, through the use of a dynamically evolving budget in an artificial currency, but that the total surplus achieved by this optimal mechanism is lower than the maximized total surplus. In our application, discounting and attrition will prevent assignment mechanisms without money from achieving an efficient allocation. This can also be understood by considering the extreme case where there is full discounting (i.e. where agents are myopic), and where the attrition rate is one (i.e. all applicants are replaced every year), since in this case rationing access over time would be impossible.

Additionally, the heterogeneity estimated to exist across participants implies that mechanisms relying on opportunity costs rather than monetary transfers to elicit selectivity will reach over-selectivity for some participants or under-selectivity for others. This occurs because an applicant's opportunity cost for obtaining an allocation today originates from a decrease in the probability of obtaining an allocation in the future, and is therefore determined by her expected value for the allocated resources. Figure 4b shows that both underselectivity among some participants and overselectivity among others are estimated to arise in our application.

Finally, we also estimate that there exists a large degree of heterogeneity across license types, but at a given stock of preference points the dynamic lottery charges a uniform "price" in preference points for hunting across all sites. Therefore, applicants at a given level of preference points incur the same opportunity cost from obtaining a license of any type, so that their choice between types of license will mostly be based on a simple ordering of their preferences across license types. For instance, an applicant i with p preference points will select license type j over license type $j^{'}$ if $W_{j,it} \geq W_{j^{'},it}$ when both license types j and $j^{'}$ can be obtained with certainty with p preference points, i.e. $\phi^{o}_{jp} = \phi^{o}_{j^{'}p} = 1$. In contrast, an applicant's decisions in an auction are influenced by other applicants' preferences through efficient prices, and an applicant only selects license type j over license type $j^{'}$ if the difference in her values for these two types of license overweighs the difference in their prices.[44]

7 Conclusion

Our results show that dynamically evolving budgets in an artificial currency (preference points) can lead to allocations that are more efficient than allocations obtained with static mechanisms. This follows from intertemporal opportunity costs that make participants more selective about requesting an allocation. With heterogenous resources and participants, these opportunity costs also yield a better separation by applicants' types, where applicants with a low value for the resource concentrate on more desirable resources, allowing applicants with a high value for the resource to obtain an assignment more frequently.

[44]In a previous version of this paper, available upon request, we considered allocation via a dynamic allocation mechanism in which applicants are also endowed with a budget in an artificial currency that increases with seniority, but in which prices for licenses in this artificial currency vary across license types. We found that this allocation mechanism improved efficiency compared to the dynamic lottery, but that this efficiency gain was modest.

On the other hand, the heterogeneity among participants and allocated resources leads to the dynamic lottery studied here being significantly less efficient than an efficient auction. The efficiency frontier of allocation mechanisms without monetary transfers and with heterogenous resources and participants is, to our knowledge, unknown. It would be interesting to study whether alternative allocation mechanisms can mitigate the efficiency loss of the dynamic lottery which we studied here.

Data Availability Statement:

The code used to obtain the results discussed in this paper is available at https://doi.org/10.5281/zenodo.4773982. The data used are confidential and subject to a data use agreement with the Michigan Department of Natural Resources (DNR). Access to the data should be obtained from the DNR directly, and we provide contact information in the link above.

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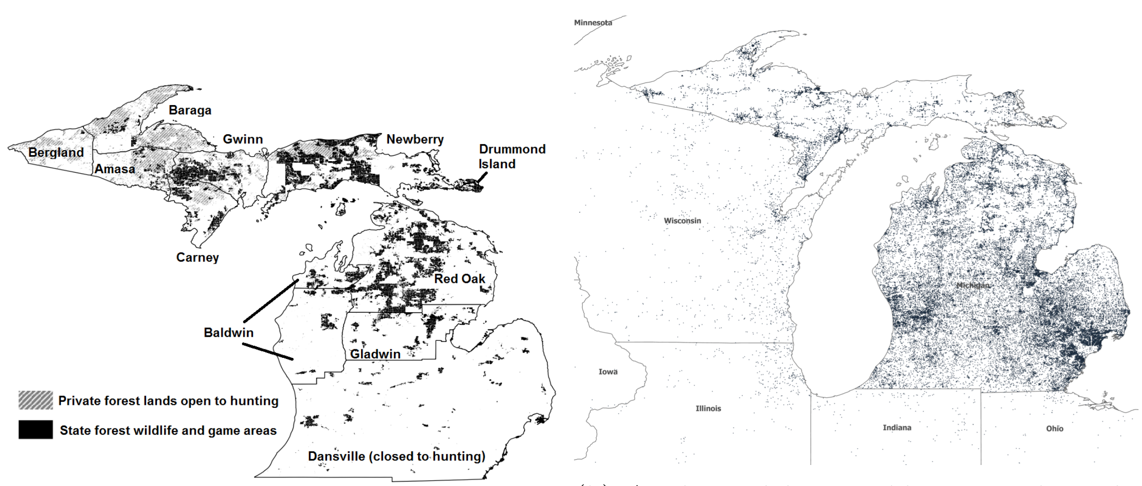
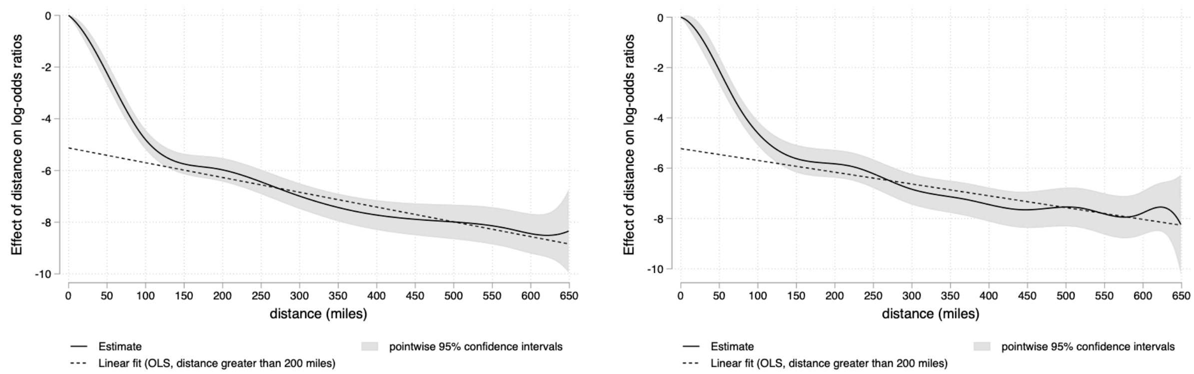


(a) Michigan bear management units.

(b) Applicants' home address. Each applicant is represented by a dot, the region displayed here contains 97.5% of all applicants.

Figure 1: Geographical location of hunting sites and applicants.



Six knots   Eight knots

Figure 2: Relationship between distance and application choices. (Descriptive model of application choices among license types that can be obtained with certainty.)

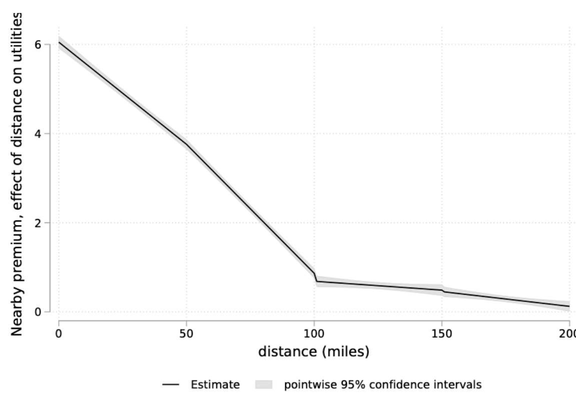
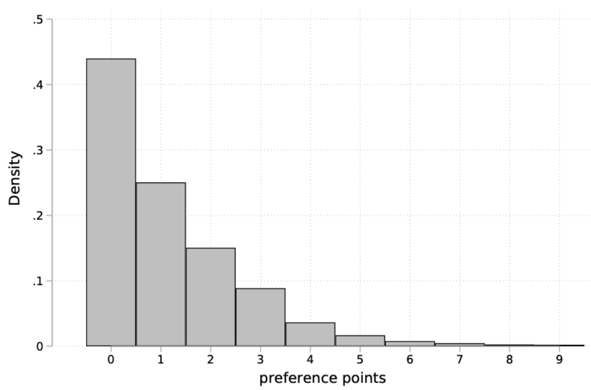
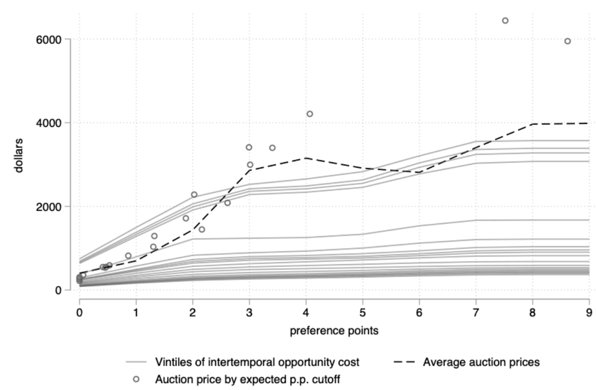


Figure 3: Estimated nearby premium.



(a) Distribution of preference points.



(b) Intertemporal opportunity cost of allocation, average auction price of allocated licenses, and auction price by expected p.p. cutoff of each license type.

Figure 4: Features of the equilibrium allocation by dynamic lottery.

Table 1: Bear Management Unit Characteristics (Source: DNR 2009a)

BMU	Population	Forest land (ac)	Success rate	season 1	season 2	season 3
Bergland	16,452	340,020	0.28	9/10-10/21	9/15-10/26	9/25-10/26
Baraga	78,327	974,399	0.26	9/10-10/21	9/15-10/26	9/25-10/26
Amasa	23,636	500,823	0.33	9/10-10/21	9/15-10/26	9/25-10/26
Gwinn	48,954	574,025	0.24	9/10-10/21	9/15-10/26	9/25-10/26
Carney	57,362	436,796	0.21	9/10-10/21	9/15-10/26	9/25-10/26
Newberry	60,591	1,333,215	0.26	9/10-10/21	9/15-10/26	9/25-10/26
Drummond Isl.	457	22,550	0.65	9/10-10/21		
Red Oak	294,981	1,394,083	0.28	9/18-9/26		
Baldwin	467,081	288,297	0.50	9/18-9/26		
Gladwin	303,693	231,582	0.17	9/18-9/26		

Success rate is the proportion of hunters who succesfully took a bear in the corresponding BMU.

Table 2: Number of first round applicants by type of license and stock of preference points, quotas, and expected preference point cutoffs in 2008.

			applicants by stock of preference points										
BMU and season	quota	cutoff	0	1	2	3	4	5	6	7	8	9	10
Bergland 1	610	1.70	583	349	245	128	83	24	22	5	0	0	0
Bergland 2	620	0.08	464	112	42	21	15	4	1	0	0	0	0
Bergland 3	725	0.00	449	68	28	14	8	1	0	1	0	0	0
Baraga 1	500	2.91	783	478	297	298	107	41	19	6	1	0	0
Baraga 2	850	0.40	692	294	85	37	10	5	2	0	0	0	0
Baraga 3	1,560	0.00	893	186	52	18	5	3	1	1	0	0	0
Amasa 1	205	4.14	349	293	213	166	140	35	21	25	4	0	0
Amasa 2	280	1.75	225	156	146	56	21	8	5	4	1	0	0
Amasa 3	485	0.52	331	184	71	33	17	5	12	3	0	0	0
Carney 1	225	2.94	513	253	240	107	52	28	10	13	0	0	0
Carney 2	340	0.62	379	111	51	16	11	5	1	0	0	0	0
Carney 3	550	0.00	355	46	14	14	2	1	2	1	0	0	0
Gwinn 1	250	3.78	600	414	247	217	149	37	15	2	0	0	0
Gwinn 2	390	0.90	425	233	79	27	7	0	2	0	0	0	0
Gwinn 3	740	0.01	585	106	40	12	2	0	1	0	0	0	0
Newberry 1	375	5.24	1,344	803	590	471	467	359	69	26	6	0	0
Newberry 2	565	2.40	782	455	374	167	71	35	2	2	0	0	0
Newberry 3	1,520	0.30	1,271	422	154	43	18	9	1	0	0	0	0
Drummond Isl.	8	7.83	63	57	52	35	28	31	41	30	3	0	0
Red Oak	2,000	3.70	4,354	2,508	1,975	1,737	1,216	174	57	13	11	0	0
Baldwin	65	7.63	845	355	287	239	256	214	228	141	13	0	0
Gladwin	175	2.38	382	161	118	73	14	8	5	2	0	0	0
pref. pt only			4,356	3,290	2,707	2,156	1,761	1,118	699	475	302	2	0
Total	13,038		21,023	11,334	8,107	6,085	4,460	2,145	1,216	750	341	2	0

Cutoffs correspond to the expected minimum stock of preference points required to obtain a license of each type.

Table 3: Applicant characteristics.

	average	25th perc.	median	75th perc.
age (years)	44.55	34	45	56
male	89.85%			
household annual income ($)	52,757	37,643	48,125	56,161
hourly wage ($)	15.40	11.09	13.62	16.91
house value ($)	137,628	80,964	118,752	170,519
Location of home address				
Outside of Michigan	4.67%			
Upper Peninsula	15.05%			
northern Lower Peninsula	21.36%			
southwestern Lower Peninsula	22.13%			
southeastern Lower Peninsula	36.79%			

Characteristics across 72,789 unique applicants in both years of our data. Annual income, hourly wage, and house value are imputed based on home address using the procedure outlined in the text and appendix.

Table 4: Effect of not being drawn in the 2008 random tie-breaking on 2009 application behavior among applicants who participated in the 2008 random tie-breaking.

p.p. 2008	N_0	N_{k+1}	Cutoff less than one			p.p.-only option			Obtaining a License		
			$E(Y(0))$	$E(Y(k+1)-Y(0))$		$E(Y(0))$	$E(Y(k+1)-Y(0))$		$E(Y(0))$	$E(Y(k+1)-Y(0))$	
0	1,876	660	0.69	-0.11	(0.04)	0.07	0.04	(0.02)	0.40	0.19	(0.04)
1	247	116	0.13	-0.06	(0.03)	0.24	0.02	(0.05)	0.08	0.55	(0.05)
2	330	440	0.11	-0.03	(0.02)	0.26	-0.13	(0.03)	0.06	0.57	(0.03)
3	404	1,143	0.06	-0.05	(0.01)	0.17	-0.07	(0.02)	0.04	0.84	(0.01)
4	81	12	0.10	-0.02	(0.12)	0.46	-0.29	(0.13)	0.02	0.81	(0.11)
5	192	82	0.13	-0.13	(0.02)	0.27	-0.06	(0.06)	0.07	0.69	(0.05)
7	34	101	0.05	-0.05	(0.04)	0.22	-0.07	(0.09)	0.03	0.43	(0.04)

p.p. stands for preference points. N_0 and N_{k+1} report the number of applicants who participated in random tie-breaking in 2008 and have a stock of preference points of zero or $k+1$ in 2009, for each value k of the 2008 stock of preference points. $Y(0)$ denotes potential outcomes with zero preference points, $Y(k+1)$ denotes potential outcomes with $k+1$ preference points, so that $E(Y(k+1)-Y(0))$ denotes (local) average treatment effects. Numbers in parentheses are standard errors (robust to unequal variances across "treatment" and "control" groups) for the estimated average treatment effects. The reported outcomes are choosing a type of license which has an expected cutoff less than one preference point in 2009, choosing the preference point-only option in 2009, and obtaining a license in 2009.

Table 5: Estimates of the preference parameters.

Travel cost		
marginal utility of income, in \$100 (μ_0)	0.28	(0.03)
opp. cost of time, wage share (λ_0)	0.67	(0.16)
License type heterogeneity (χ_j, $j \in \mathcal{J}$)		
Bergland 1	0.83	(0.15)
Bergland 2	−0.58	(0.15)
Bergland 3	−0.88	(0.15)
Baraga 1	1.24	(0.16)
Baraga 2	0.04	(0.15)
Baraga 3	−0.10	(0.15)
Amasa 1	1.51	(0.16)
Amasa 2	0.03	(0.15)
Amasa 3	−0.60	(0.15)
Carney 1	0.88	(0.15)
Carney 2	−1.41	(0.15)
Carney 3	−1.84	(0.15)
Gwinn 1	1.33	(0.16)
Gwinn 2	−0.51	(0.15)
Gwinn 3	−1.16	(0.15)
Newberry 1	1.94	(0.17)
Newberry 2	0.97	(0.15)
Newberry 3	0.11	(0.14)
Drummond Isl.	2.03	(0.18)
Red Oak	1.68	(0.16)
Baldwin	2.25	(0.18)
Gladwin	−2.43	(0.16)

Applicant unobserved heterogeneity		
probability type 1 (π_1)	0.21	(0.01)
probability type 2 (π_2)	0.56	(0.01)
probability type 3 (π_3)	0.23	(0.01)
α_1	0	
α_2	−3.44	(0.04)
α_3	−5.34	(0.99)
β_1	1	
β_2	1.02	(0.03)
β_3	7.19	(0.40)
Applicant demographics		
age (years)	0.02	(0.00)
male	0.21	(0.04)
income (\$10,000)	0.06	(0.02)
house value (\$100,000)	−0.05	(0.02)
Home location (baseline is non-MI)		
Upper Peninsula	−2.25	(0.10)
northern Lower Peninsula	−1.17	(0.09)
southwestern Lower Peninsula	−0.63	(0.08)
southeastern Lower Peninsula	−0.62	(0.08)

Numbers in parentheses are standard errors.

Table 6: Welfare Comparison across Different Assignment Mechanisms

		Random serial dictatorship	Static lottery	Dynamic lottery	Auction
Total surplus ($M)[a]		11.65	13.08	14.98	23.07
Average surplus ($)		207	232	266	410
Average applicant surplus		207	232	266	121
State revenue per applicant		—	—	—	289
Probability of obtaining a license		0.217	0.217	0.214	0.217
Average surplus per license ($)		950	1,069	1,243	1,886
Taste for hunting of allocated applicants (A_i)[b]		-720	-766	-733	-745
Persistent relative preference for allocated license ($\Delta_{j,i}$)		762	794	861	1,405
Time-varying relative preference for allocated license ($\mathcal{E}_{j,it}$)		908	1,041	1,115	1,226
Probability of applying for a license		0.830	0.830	0.302	0.217
by applicant type [c]	1	0.983	0.983	0.636	0.531
	2	0.702	0.702	0.183	0.062
	3	1.000	1.000	0.286	0.306
Fraction of licenses allocated to applicants of type	1	0.517	0.476	0.496	0.516
	2	0.327	0.377	0.361	0.160
	3	0.156	0.147	0.142	0.324
Average auction price of allocated licenses ($)		1,330	1,330	1,372	1,330
by applicant type	1	840	660	662	531
	2	1,372	1,522	1,634	514
	3	2,865	3,008	3,182	3,006

[a] We normalize total surplus absent hunting to $0.
[b] The overall taste for hunting of an applicant is defined as the ex-ante preference for a randomly drawn license.