Dynamic Games in Industrial Organization.

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Background: Static Empirical IO.

• Developed tools that enabled us to better analyze market outcomes conditional on the "state variables" of the problem.

• Common thread: incorporate the institutional background needed to make sense of the data used to analyzes market responses to environmental & policy changes.

• Focus was to incorporate

- **heterogeneity** (in plant productivity, products demanded, bidders and/or consumers) and,
- **equilibrium conditions** when we need to solve for variables that firms could change in response to the environmental change.

• Largely relied on earlier work by our game theory colleagues for the analytic frameworks.

In particular we most often assumed

- Each agent's actions affect all agents' payoffs, and
- At the "equilibrium" or "rest point"
  - agents have correct perceptions, and
  - the system is in some form of "Nash" equilibrium (policies such that no agent has an incentive to deviate).

• Our contribution was the development of an ability to adapt the analysis to the richness of different real world institutions.

To keep the model static we had to rule out circumstance where the static (q or p) choice has an independent effect on

- future costs [l.b.d., adjustment costs, networks,...]
- future demand [durable or experience goods, networks,...]
- future equilibrium choices [collusion, asymmetric information, ...]

• In addition this ruled out analyzing how the state variables might respond to changes in policy or environmental changes.

Dynamic analysis started analogously

• I.e. we took frameworks taken from our theory colleagues. that made assumptions which insured that the

1. state variables evolve as a Markov process
2. and the equilibrium is some form of Markov Perfection (no agent has an incentive to deviate at any value of the state variables).

Star and Ho (1969) and more directly Maskin and Tirole (1988) who use full information environments.

• We started in a similar fashion as before. We tried to adapt the dynamic framework to the richness of different real world institutions for dynamic games was more difficult.

• Ericson and Pakes (1995) develop a framework for doing this,

- **States.**
  - $i \in \mathcal{Z}^+$ (could be multidimensional)
  - s_i will be the number of firms with efficiency level i,
  - $s = [s_i;\ i \in \mathcal{Z}^+]$ is the "industry structure" (a counting measure of the number of firms at each different efficiency level).
  - ζ aggregate state variable that evolves exogenously (perhaps as a Markov process). E.g. outside alternative.

- **Bellman Equation: "Capital Accumulation" Games**

$$V(i,s) = \max\{\phi, \pi(i,s) +$$
$$\sup_{(x \geq 0)} [-cx + \beta \sum V(i',s')pr(i',s'|x,i,s,\zeta)p(\zeta)]\}.$$

- $pr(i',s'|x,i,s,\zeta) = pr(i'|i,x,\zeta)q(s-e(i)|i,x,s,\zeta)$.

- $pr(i'|i,x,\zeta)$. This is a game where my own investment only affects my own state variables.

- $q(s-e(i)|i,x,s,\zeta)$ perception of where my competitors will be.

- $\mathcal{P} = \{p(i'|x,\zeta); x \in \mathcal{R}^+\}$, stochastically increasing in x for every ζ.
- $q[\cdot|i,s,\zeta]$ embodies the incumbent's beliefs about entry and exit.
- Many possible entry models; e.g. Must pay x_e $(> \beta\phi)$ to enter, and enters one period later at state $\omega_e \in \Omega^e \subset \mathcal{Z}^+$ with probability $p^e(\cdot)$. Only enters if the expected discounted value of future net cash flows from entering is greater than the cost of entry.

Dynamic Equilibrium.

1. Every agent chooses optimal policies given its perceptions on likely future industry structures
2. Those perceptions are consistent with the behavior of the agent's competitors.

Doraszelski and Satterwaite (2003) prove existence (to insure this we need random entry fees and exit costs), and E-P show that any equilibria

1. Is "computable", i; never more than $\overline{n}$ firms active & Only observe "i" on $\Omega = \{1, ..., K\} \Rightarrow$ need only compute equilibria for $(i, s) \in \Omega \times S$

$$S \equiv \{s = [s_1, ..., s_k] : \sum s_j \leq \overline{n} < \infty\} \ \Rightarrow \ \#S \leq K^N$$

2. Generates a homogeneous Markov chain for industry structures [for $\{s_t\}$], i.e. $Q(s'|s)$

$$Pr[s_{t+1} = s'|s^t] = Pr[s_{t+1} = s'|s_t] \equiv Q[s'|s_t].$$

3. And provide conditions on the primitives such that insure that any equilibrium $Q[\cdot|\cdot]$ is *ergodic*. [Picture].
   - R is frequently much smaller than S (and the divergence is greatest for large markets with many state variables).
   - In the limit the probabilities of being at the various points in R converges to an invariant measure. This invariant measure is often referred to as a "steady state" of the system, though "steady" seems to be a misnomer (as the state is not constant).

Brute Force Computation

Pakes and McGuire (1994, *RAND*). Important for understanding.

• The first algorithm we consider is a "backward solution" algorithm (the multiple agent analogue of what we do in single agent dynamic problems).

- In memory. Estimates of the value function and policies associated with each $(i, s) \in \Omega \times S$. Assume K and $\overline{n}$ known.
- Updating. *Synchronous*; i.e. it circles through the points in S in some fixed order and updates all estimates associated with *every* $s \in S$ at each iteration (here updating estimates at s involves updating estimates at each (i, s) that has $s_i > 0$).
- Convergence. The values and policies from successive iterations are the same. Converged policies and values satisfy all the properties of equilibrium values and policies (see below).

• Assume $i' = i + \nu - \zeta$ where $\mathcal{P}_v = \{P_\nu(\cdot|x)\}$, ζ exogenous.

Updating 1: Rewrite Bellman Equation.

$$V(i,s) = \max_{\chi \in \{0,1\}} \{[1-\chi]\phi + \tag{1}$$

$$\chi\{\pi(i,s) - \sup_{x \geq 0}[-cx + \beta \sum_{\nu} w(\nu; i, s) p(\nu|x_1)]\}, \quad \&$$

$$w(\nu;\ i,s) \equiv$$

$$\sum_{(\hat{s}_i', \zeta)} V(i+\nu-\zeta, \hat{s}_i' + e(i+\nu-\zeta)|w) q[\hat{s}_i'|i,s,\zeta]\mu(\zeta), \quad (1a)$$

$$q[\hat{s}_i' = s_i^*|i,s,\zeta] \equiv Pr\{\hat{s}_i' = \hat{s}_i^*|i,s,\zeta, \text{equilibrium policies}\ \} \ \ (1b).$$

• $w(\nu; i, s)$ is the EDV of future net cash flow conditional on investment resulting in a particular value of ν, and the current state being (i, s) (it integrates out $\hat{s}_i'$, and ζ). It is all the agent needs to know, and generates a single agent problem.

Updating Rules.

- Calculate $w^{k-1}(\cdot|i,s)$ from the information in memory, i.e. from (x^{k-1}, V^{k-1}) (as in 1a),
- substitutes $w^{k-1}(\cdot)$ for $w(\cdot)$ in (1) and then solve the resultant *single agent* optimization problem for the j^{th} iteration's entry, exit and investment polices at (i,s).
- Incumbents solve for (χ^k, x^k) that

$$\max_{\chi \in \{0,1\}} \{[1-\chi]\phi + \chi \sup_{x \geq 0} [\pi(i,s) - cx + \beta \sum_{\nu} w^{k-1}(\nu; i,s) p(\nu|x)]$$

I.e. we solve the Kuhn-Tucker problem for investment conditional on continuing which if $\nu \in \{0,1\}, \zeta \in \{0,1\}$ is

$$\sum_{\nu} \frac{\partial p(v|x)}{\partial x} w(v; i,s) - c \leq 0,$$

with strict inequality iff $x = 0$. Then substitute the solution in the continuation value and determine whether it is greater than ϕ.

Potential entrants compute

$$V_e^k(s) = \beta \sum_{\zeta} w^{k-1}(\zeta; i_e, s + e(i_e))\mu(\zeta).$$

and set $\chi_e^k = 1 \Leftrightarrow V_e^k(s) > x_e$.

• Substitutes these policies and the w^{k-1} for the w, x and the max operator in (1), and labels the result $V^k(\cdot)$, and put $V^k(\cdot)$ in memory.

• Setting K. Start with the monopoly problem ($\overline{n} = 1$) and an oversized K; $\rightarrow$ a lowest i at which the monopolist remains active and a highest i at which the monopolist invests. $\rightarrow$ 1 and K in Ω.

• Setting $\overline{n}$. Set $\overline{n} = 2$ and do the iterative calculations again starting at $V^0(i_1, i_2) = V^*(i_1)$. Then set $\overline{n} = 3$ and set $V^0(i_1, i_2, i_3) = V^*(i_1, max(i_2, i_3))$. Continue until we reach an $\overline{n}$ that whenever $\overline{n} - 1$ firm's active there is never entry. This is $\overline{n}$.

Convergence & Equilibrium.

- At the end of the iteration calculate $||V^{k-1}(\cdot) - V^{k}(\cdot)||$ and $||x^{k-1}(\cdot) - x^{k}(\cdot)||$. If both are sufficiently small, stop. Else continue.

- **Equilibrium.** At fixed point each incumbent and potential entrant
  - uses, as its perceived distribution of the future states of its competitors, the actual distribution of future states of those competitors, and
  - chooses its policy to maximize its expected discounted value of future net cash flow given this distribution of the future of its competitors.

The computational burden is (essentially) the product of three factors,

- the number of points evaluated at each iteration;
- the time per point evaluated;
- the number of iterations (value &/or policy function iterations).

Number of Points.

Since each of the $\overline{n}$ active firms can only be at K distinct states, the number of points we need to evaluate at each iteration, or

$$\#S \leq K^{\overline{n}}.$$

Exchangeability, of the value and the policy functions in the state variables of a firm's competitors implies that we do not need to differentiate between two vectors of competitors that are permutations of one another notation does not. Pakes (1993) shows that an upper bound for $\#S$ is given by the combinatoric

$$\binom{K+\overline{n}-1}{\overline{n}} << K^{\overline{n}}.$$

but for $\overline{n}$ large enough this bound is tight.

Burden per Point.

Determined by

- the cost of calculating the expected value of future states conditional on outcomes (of obtaining the $w^j(\cdot; i, s)$ from the information in memory).
- The cost of obtaining the optimal polices and the new value function given $w^j(\cdot; i, s)$.

• Think of this as individual firms playing against the rest. Assume that there is positive probability on each of κ points for each of the $m-1$ active competitors of a given firm. Then we need to sum over κ^m possible future states and there are $\kappa \times m$ values of $w^j(\cdot)$ needed at that s. Average m should increase in $\overline{n}$, and κ should be determined by the nature of the state space per firm (it typically goes up exponentially in the number of state variables per firm).

Conclusion

It is clear that the computational burden of the model grows quickly in both the number of firms ever simultaneously active (it grows geometrically in this dimension) and the number of state variables per firm (it grows exponentially in this dimension). This is the problem known as "The Curse of Dimensionality" reappearing in games.

• Consequently pointwise algorithm has been used both as a tool for investigating theoretical issues where analytic solutions were not possible.

• Realistic empirical analysis of most applied problem could not use this. Two possible extensions

- Approximation techniques
- Use a different notion of equilibrium.

Approximation Techniques.

• A number are available. Each has their problems, but they compute equilibria with much less of a computational burden than the standard algorithm. I leave a discussion of them to others at the conference.

In order of appearance

- Deterministic approximation techniques. This starts in economics with the book by Judd(2004). It has now expanded with the use of various AI related tools.
- Stochastic Algorithm (Pakes and Mcguire, 2001)
- Continuous time algorithm. Doraszelski and Judd (2004).
- Oblivious equilibrium (mean field theory). Benkard, Van Roy, Weintraub, 2010. Approximate using moments of the distribution.

Alternative Equilibrium Concept

Premise: the complexity of Markov Perfection both

- limits our ability to do dynamic analysis of market outcomes, but also
- leads to a question of whether some other notion of equilibria will better approximate agents' behavior.

• What assumptions of MP might we relax? The initial frameworks made assumptions which insured that the

1. state variables evolve as a Markov process
2. and the equililbrium is some form of Markov Perfection (no agent has an incentive to deviate at any value of the state variables).

On the Markov Assumption.

Except in situations involving active experimentation to learn (where policies are transient), we are likely to stick with the assumption that states evolve as a time homogenous finite order Markov process. Reasons

- Convenience and fits the data well.
- Realism suggests information access and retention conditions limit the memory used.
- We can bound unilateral deviations (Ifrach and Weintraub, 2014), and have conditions which insure those deviations can be made arbitrarily small by letting the length of the kept history grow (White and Scherer, 1994).

On 2: Perfection.

The fact that Markov Perfect framework becomes unwieldily when confronted by the complexity of real world institutions, not only limits our ability to do empirical analysis of market dynamics

- it also raises the question of whether some other notion of equilibrium will better approximate agents' behavior.

The complexity issue implies that agents

- have access to and can retain a large amount of information (all state variables), and
- can either compute or learn an unrealistic number of strategies (one for each information set).

How demanding is this? Durable goods example.

Decrease the number of state variables by assuming agents only have access to a subset of the state variables.

- Since agents presumably know their own characteristics and these tend to be persistent, we would need to allow for asymmetric information: the "perfectness" notion would then lead us to a "Bayesian" Markov Perfect solution.

Is assuming "Bayesian MP" more realistic? It decreases the information access and retention conditions but increases the burden of computing the policies significantly. The additional burden results from the need to compute posteriors, as well as optimal policies; and the requirement that they be consistent with one another.

• I will come back to the issue of whether learn the policies below, as the equilibrium concept I am going to use has a learning model embedded in it.

Abandon Perfection

Question. If we abandon Markov Perfection can we both

- better approximate agents' behavior and,
- enlarge the set of dynamic questions we are able to analyze.

• I start with strategies that are "rest points" to a dynamical system. This makes my job much easier because

- strategies at the rest point likely satisfy a Nash condition of some sort; else someone has an incentive to deviate.
- However it still leaves opens the question: What is the form of the Nash Condition?

What Conditions Can We Assume for the Rest Point at States that are Visited Repeatedly?

We expect (and I believe should integrate into our modelling) that

1. Agents perceive that they are doing the best they can at each of these points, and
2. These perceptions are at least consistent with what they observe.

Note. It might be reasonable to assume more than this: that agents (i) know and/or (ii) explore, properties of outcomes of states not visited repeatedly. I come back to this below.

Formalization of Assumptions.

• Denote the information set of firm i in period t by $J_{i,t}$. $J_{i,t}$ will contain both public (ξ_t) and private ($\omega_{i,t}$) information, so $J_{i,t} = \{\xi_t, \omega_{i,t}\}$.

• Assume $(J_{1,t}, \ldots J_{n_t,t})$ evolves as a finite state Markov process on $\mathcal{J}$ (or can be adequately approximated by one).

• Policies, say $m_{i,t} \in \mathcal{M}$, will be functions of $J_{i,t}$. For simplicity assume $\#\mathcal{M}$ is finite, and that it is a simple capital accumulation game, i.e. $\forall (m_i, m_{-i}) \in \mathcal{M}^n, \ \& \ \forall\, \omega \in \Omega$

$$P_\omega(\cdot | m_i, m_{-i}, \omega) = P_\omega(\cdot | m_i, \omega),$$

• The public information, ξ, is used to predict competitor behavior and common demand and cost conditions (these evolve as an exogenous Markov process).

• A "state" of the system, is

$$s_t = \{J_{1,t}, \ldots, J_{n_t,t}\} \in \mathcal{S},$$

$\#\mathcal{S}$ is finite. $\Rightarrow$ any set of policies will insure that s_t will wander into a recurrent subset of $\mathcal{S}$, say $\mathcal{R} \subset \mathcal{S}$, in finite time, and after that $s_{t+\tau} \in \mathcal{R}$ w.p.1 forever.

• Note that the agents does not keep track of all of s_t, only $J_{i,t}$; i.e. $J_{i,t}$ is whatever management conditions on when forming dynamic policies.

• Let the agent's perception of the expected discounted value of current and future net cash flow were it to chose m at state J_i, be

$$W(m|J_i), \ \ \forall m \in \mathcal{M} \ \ \& \ \ \forall J_i \in \mathcal{J},$$

• and of expected profits be $\pi^E(m|J_i)$.

Our assumptions imply:

- Each agent choses an action which maximizes its perception of its expected discounted value, and
- For those states that are visited repeatedly (i.e. are in the recurrent class or $\mathcal{R}$), these perceptions are consistent with observed outcomes.

Formally

$$\mathbf{A}. \quad W(m^*|J_i) \geq W(m|J_i), \ \ \forall m \in \mathcal{M} \ \ \& \ \ \forall J_i \in \mathcal{J},$$

$$\mathbf{B}. \ \&, \ \forall J_i \ \text{ which is a component of an } \ s \in \mathcal{R}$$

$$W(m(J_i)|J_i) = \pi^E(m|J_i) + \beta \sum_{J_i'} W(m^*(J_i')|J_i')p^e(J_i'|J_i),$$

where, if $p^e(\cdot)$ provides the empirical probability (the fraction of periods the event occurs)

$$\pi^E(m|J_i) \equiv \sum_{J_{-i}} E[\pi(\cdot)|J_i, J_{-i}]p^e(J_{-i}|J_i),$$

and

$$\left\{ p^e(J_{-i}|J_i) \equiv \frac{p^e(J_{-i}, J_i)}{p^e(J_i)} \right\}_{J_{-i}, J_i},$$

while

$$\left\{ p^e(J_i'|J_i) \equiv \frac{p^e(J_i', J_i)}{p^e(J_i)} \right\}_{J_i', J_i}. \ \spadesuit$$

"Experience Based Equilibrium"

• These are the conditions of a (restricted) EBE (Fershtman and Pakes, 2012). For earlier work based on similar ideas but in a setting with repeated interactions with different draws from an infinite number of potential competitors see Fudenberg and Levine, 1993 ,on self confirming equilibria.

• Bayesian Perfect satisfy them, but so do weaker notions.

• We now turn to its : computation, and overcoming multiplicity issues, and then to an empirical example.

Computational Algorithm.

• Asynchronous reinforcement learning algorithm (first used for dynamic games in Pakes and McGuire, 2001), earlier machine learning literature for single agent problems calls this Q-learning.

• The fact that it is based on learning from realized data makes it a candidate to analyze perturbations to the environment (provided they are not large enough to induce experimentation), as well as to compute equilibrium to dynamic games.

• If there is more than one equilibria, the learning algorithm will pick one out. If algorithm is run many times from the same initial conditions, it will pick out a distribution of equilibria.

• Formally it circumvents the two sources of the curse of dimensionality in computing equilibrium.

Iterations

• The computation problem is now different for management then for the researcher as management just conditions on $J_{i,t}$ but the researcher must compute equilibria for $s_t = (J_{1,t} \dots J_{n_t})$. Here we deal with the computational problem for researchers. We come back to firm's below.

• **Iterations are defined by**

- A location, say $L^k = (J_1^k, \dots J_{n(k)}^k) \in \mathcal{S}$: is the information sets of active agents .
- Objects in memory (i.e. M^k):
  (i) perceived evaluations, W^k,
  (ii) No. of visits to each point, h^k.

Must update (L^k, W^k, h^k)**.** Computational burden determined by; memory constraint, and compute time. I use a simple (not neccesarily optimal) structure to memory.

Update Location.

- Calculate "greedy" policies (policies are now "m") for each agent

$$m^*_{i,k} = arg \max_{m \in \mathcal{M}} W^k(m|J_{i,k})$$

- Take random draws on outcomes conditional on $m^*_{i,k}$:
- E.g.; if we invest in "payoff relevant" $\omega_{i,k} \in J_{i,k}$, draw $\omega_{i,k+1}$ conditional on $(\omega_{i,k}, m^*_{i,k})$.
- Use outcomes to update $L^k \rightarrow L^{k+1}$.

Update W^k.

• "Learning" interpretation: Simple case: assume agent observes the competitors' static controls $b(m_{-i})$ (usually a price or bid) and initially we will assume the agent knows (perhaps through estimation) the primitives; $\pi_i(\cdot), p(\omega_{i,t+1}|\omega_{i,t}, m_{i,t})$. Can generalize and allow them not to be known.

• Its ex poste perception of what its value would have been had it chosen m is

$$V^{k+1}(J_{i,k}, m) =$$

$$\pi(\omega_{i,k}, m, m_{-i,k}, d_k) + \max_{\tilde{m} \in M} \beta W^k(\tilde{m}|J_{i,k+1}(m)),$$

where $J_i^{k+1}(m)$ is what the $k+1$ information would have been given m and *competitors actual play*.

Treat $V^{k+1}(J_{i,k})$ as a random draw from the possible realizations of $W(m|J_{i,k})$, and update W^k as in stochastic integration (Robbins and Monroe,1956)

$$W^{k+1}(m|J_{i,k}) =$$

$$\frac{1}{h^k(J_{i,k})} V^{k+1}(J_{i,k}, m) + \frac{(h^k(J_{i,k}) - 1)}{h^k(J_{i,k})} W^k(m|J_{i,k}),$$

or

$$W^{k+1}(m|J_{i,k}) - W^k(m|J_{i,k}) =$$

$$\frac{1}{h^k(J_{i,k})}[V^{k+1}(J_{i,k}, m) - W^k(m|J_{i,k})].$$

(other weights are more efficient as the early estimates of $V^{k+1}(J_{i,k}, m)$ are noisier than the later estimates, and it would be good to know how to use information on close states to update a given state ....)

Notes.

• If we have equilibrium valuations we tend to stay their, i.e. if * designates equilibrium

$$E[V^*(J_i, m^*)|W^*] = W^*(m^*|J_i).$$

So if it reaches equilibrium it tends to stay at equilibrium.

• As in all computational algorithms for dynamic games there is no guarantee that it will converge, but the learning interpretation provides a rational for using it regardless. The smoothing implicit in the draws tends to make it converge, but it may take many iterations.

• Agents (not only the analyst) could use the algorithm to find equilibrium policies or adjust to perturbations in the environment.

• Algorithm has no curse of dimensionality.
(i) Computing continuation values: integration is replaced by averaging two numbers.
(ii) States: algorithm eventually wanders into $\mathcal{R}$ and stays their, and $\#\mathcal{R} \leq \#\mathcal{J}$.

• Fershtman and Pakes (2012) also provide a test for equilibrium that has no curse of dimensionality.

• Still the number of states can grow large (typically grows linearly in the number of state variables).

• The stochastic approximation literature for single agent problems often augments this with functional form approximations ("TD learning" or Sutton and Barto,1998).

Multiplicity of REBE.

• Recall that $\mathcal{R}$ is the recurrent class (the points that are visited repeatedly).

• $\mathcal{R}$ contains both "interior" and "boundary" points. Points at which there are feasible strategies which can lead outside of $\mathcal{R}$ are boundary points. Interior points are points that can only transit to other points in $\mathcal{R}$ no matter which (feasible) policy is chosen.

• Our conditions only insure that perceptions of outcomes are consistent with the results from actual play at interior points. Perceptions of outcomes for some feasible (but in-optimal) policy at boundary points are not tied down by actual outcomes.

• "MPBE" are a special case of (restricted) EBE and they have multiplicity. Here differing perceptions at boundary points can support a (possibly much) wider range of equilibria.

Narrowing the Set of Equilibria.

• In any empirical appliction the data will rule out equilibria. m^* is observable, at least for states in $\mathcal{R}$, and this implies inequalities on $W(m|\cdot)$. With enough data $W(m^*|\cdot)$ will also be observable up to a mean zero error.

• Use external information to constrain perceptions of the value of outcomes outside of $\mathcal{R}$. If available use it.

• Asker, Fershtman, Jihye, and Pakes, (2020, *RAND*), allow firms to experiment with $m_i \neq m_i^*$ at boundary points. Leads to a stronger notion of, and test for, equilibrium. We insure that perceptions are consistent with the results from **actual play** for each **feasible** action at boundary points (and hence on $\mathcal{R}$).

Boundary Consistency.

• Let $B(J_i|\mathcal{W})$ be the set of actions at $J_i \in s \in \mathcal{R}$ which could generate outcomes which are not in the recurrent class (if it is not empty, J_i is a boundary point) and $B(\mathcal{W}) = \cup_{J_i \in \mathcal{R}} B(J_i|\mathcal{W})$. Then the extra condition needed to insure "Boundary Consistency" is:

• **Extra Condition.** Let τ index future periods, then $\forall (m, J_i) \in B(\mathcal{W})$

$$W(m^*|J_i) \geq$$

$$E\left[\sum_{\tau=0}^{\infty} \delta^{\tau} \pi\Big(m(J_{i,\tau}), m(J_{-i,\tau})\Big)\Big| J_i = J_{i,0}, \mathcal{W}\right],$$

where $E[\cdot|J_i, \mathcal{W}]$ takes expectations over future states starting at J_i using the policies generated by $\mathcal{W}$. ♠

A Note on Estimation.

• Need a candidate for J_i (the firm's information). There might be exogenous information (e.g. participant information) but failing that we would want to empirically analyze the determinants of the dynamic controls (investment, exit,...).

• That is use data to see what management pays attention to when making its decision.

• The problem that is likely to arise is whether the residual from the empirical investigation of controls is serially correlated or not. If it is not serially correlated, we can use standard techniques. CCP techniques discussed in the course by Bob Miller, or Bajari Benkard and Levin (2007), Pakes Ostrovsky and Berry (2007).

• The problem is these estimation techniques do not allow for serially correlated errors, and both the theory and empirical work indicate they are likely to be important. Come back to this below.

E.g.; Dubois Pakes: Institutions for Pharma Advertising

First we use the data to approximate the marginal return to a dollar's advertising, with separate equations for detailing and DTC advertising (i.e. $h \in \{d, D\}$).

$$\mathcal{E}\Big[\Big(\sum_{\tau=1}^{\infty}(\beta\rho)^{\tau}\frac{\partial\pi(\cdot)_{t+\tau}}{\partial\xi_{t+1}}\beta_a\Big)|J_t,\xi_t\Big] \approx \tilde{\theta}_{0,h}\Big(\frac{\widehat{\partial\pi(\cdot)_{t+1}}}{\partial\xi_{t+1}}\beta_a\Big)^{\theta_{1,h}} exp[w_t\beta_{w,h}+\omega_t]$$

where

$$\frac{\widehat{\partial\pi(\cdot)_t}}{\partial\xi_t} = E\Big[\frac{\partial D(\cdot)_{t+1}}{\partial\xi_{t+1}}(p_{t+1}-c)|J_t,\xi_t\Big],$$

- Note c is unknown. In this literature it is either assumed zero, or a static Nash pricing assumption is used to back it out.
- w_t includes time to loss of exclusivity, the advertising expenditures of other firms, whether it is OTC, whether it is generic.

$$\Rightarrow log[a_{h,t}] = \theta_{0,h} + \theta_{1,h} log[\frac{\partial \widehat{\pi(\xi', J')_t}}{\partial \xi'}] + w_t \beta_{w,h} + \omega_{h,t}.$$

• OLS on this equation generates good fits, but there is a worry about serial correlation, endogeneity, and selection biasing parameter estimates, and we want to do counterfactual equilibria.

• Since $\omega \equiv (\omega_{d,t}, \omega_{D,t})$ represents variables that management considers in making its advertising decisions but we do not observed, we expect it to be serially correlated and it is. So

$$\omega_{h,t} = \rho_h \omega_{h,t-1} + \nu_{h,t}$$

where $\{\nu_{h,t}\}$ is i.i.d.

• Table 1. Need for selection correction. It is handled through an inversion technique. Advertising is traditionally a control with much more variance than price, so it should contain a lot of information about incentives. It is often ignored because of these kinds of problems.

• Quasi first difference for serial correlation to produce

$$log[a_{h,t}] - \rho_h log[a_{h,t-1}] =$$

$$\theta_{0,h}(1-\rho_h) \ + \theta_{1,h}\left[log\left(\frac{\partial D(\cdot)_t}{\partial \xi_t}(p_t - c)\right) - \rho_h log\left(\frac{\partial D(\cdot)_{t-1}}{\partial \xi_{t-1}}(p_{t-1} - c)\right)\right]$$

$$+\beta_{w,h}\left(w_t - \rho_h w_{t-1}\right) + M(P_{h,t}) + e_{h,t}.$$

where $M(P_{h,t})$ is a term which corrects for selection generated by zero advertising.

DTC (column 3). About half of the detailing observations are zero, and nearly ninety percent of the DTC observations are also[2]. Ten to twenty percent of the observations with positive advertising are followed by an observation with zero advertising. Not all firms advertise, and management in those that do decide to advertise in some periods but not in others.

Table 1: Properties of the Data.

Market	N	$a_{dt} > 0$	$a_{Dt} > 0$	$a_{Dt} > 0$ $\&a_{Dt-1} > 0$	$a_{dt} > 0$ $\&a_{dt-1} > 0$	$a_{Dt} > 0$ $\&a_{Dt-1} > 0$ $\&a_{dt-1} > 0$	$a_{dt} > 0$ $\&a_{dt-1} > 0$ $\&a_{Dt-1} > 0$
Antiasthma	1389	957	164	148	857	148	160
Antidepressants	1325	664	114	103	564	96	100
Antiulcer	1314	718	83	70	608	70	81

To model management's decision of whether to engage in advertising in a particular period, we allow for a fixed cost of doing advertising, denoted by $exp[f_h + u_{h,t}]$, where here and below $h \in \{d, D\}$ for detailing and DTC respectively. Given the approximation in equation (**??**), and our fixed cost the advertising equations becomes

$$(i) \quad a_{h,t} = 0 \quad \Rightarrow \quad Pr\left\{ u_{h,t} + f_h \geq \theta_{0,h} + \theta_{1,h} log[\frac{\partial \widehat{\pi(\xi', \cdot)}}{\partial \xi'}] + w\beta_{w,h} + \omega_{h,t} \right\}, \text{ and}$$

$$(ii) \quad a_{h,t} > 0 \quad \Rightarrow \quad log[a_{h,t}] = \theta_{0,h} + \theta_{1,h} log[\frac{\partial \widehat{\pi(\xi', J')_t}}{\partial \xi'}] + w_t\beta_{w,h} + \omega_{h,t}. \tag{5}$$

Since $\omega \equiv (\omega_{d,t}, \omega_{D,t})$ represents variables that management considers in mak-

[2] This is among drugs that have some detailing and some DTC during our sample period, we assume that the firms that market drugs that do not have any of either do not have an advertising department and hence do not make advertising decisions.

Something on the (preliminary) estimates.

- Notes on procedure.
  - The parameters $\left(\rho_h, \theta_{0,h}, \theta_{1,h}, \beta_{w,h}\}\right)_{h \in \{d,D\}}$ differ across markets.
  - The c differs both across drugs in a given market, and since it includes rebates, between the branded and generic versions of the drug .
- Notes on Estimation Results.
  - Table 2. Note the importance of the serial correlation parameter and the selection terms.
  - Table 3. Table of dynamic estimates of costs compared to static estimates.
- In process
  - Pointwise computational algorithm with continuous EDV's.
  - Evaluating alternative institutional structures for advertising.

Table 30: GMM estimates of Detailing and DTC equation with cost estimated from Detailing (selection correction with P/(1-P))

	Antiasthma	Anticholesterol	Antiulcer	Antidepressants
	GMM	GMM	GMM	GMM
	b/se	b/se	b/se	b/se
rho				
Constant	0.619***	0.594***	1.000***	0.548***
	(0.035)	(0.038)	(0.107)	(0.040)
theta1d				
Constant	1.253***	0.807***	0.338	1.247***
	(0.086)	(0.051)	(1.476)	(0.125)
betad				
Constant	-0.358*	-0.482***	-0.982***	-0.354*
	(0.158)	(0.102)	(0.203)	(0.157)
betad2				
Constant	0.034***	0.036***	0.041	0.068***
	(0.007)	(0.009)	(0.153)	(0.007)
Md				
Constant	0.488**	0.234***	0.617***	0.409***
	(0.152)	(0.053)	(0.183)	(0.053)
rhoDTC				
Constant	1.000***	0.148*	0.636***	0.999***
	(0.039)	(0.060)	(0.100)	(0.015)
theta1DTC				
Constant	7.824	0.505***	1.493	7.676*
	(5.120)	(0.082)	(1.284)	(3.124)
betaD				
Constant	-1.111	-0.128	-1.887*	0.729
	(0.574)	(0.143)	(0.873)	(0.443)
betaD2				
Constant	0.624	0.051***	0.166	-0.795
	(1.139)	(0.013)	(0.099)	(1.280)
MD				
Constant	0.669***	-0.559***	-0.002*	0.809***
	(0.128)	(0.104)	(0.001)	(0.133)
constantd				
Constant	16.131***	13.736***	-97.386	14.240***
	(0.633)	(0.438)	(297622.844)	(0.770)
constantD				
Constant	711.856	10.524***	12.343***	-1341.365
	(70816.159)	(0.383)	(3.634)	(19507.300)
N	908	514	692	634

Note: Robust standard errors. * for $p < .05$, ** for $p < .01$, and *** for $p < .001$.

Table 31: Statistics on Prices and Marginal Costs Estimates from Detailing First Order Condition

marketgeneric			Mean		
	Price	Marginal cost	Margin	Marginal cost	Margin
Antiasthma: Branded	1.73	0.43	0.72	1.25	0.66
Antiasthma: Generic	0.84	0.17	0.62	0.71	0.67
Anticholesterol: Branded	2.83	0.28	0.89	1.89	0.50
Anticholesterol: Generic	0.75	0.08	0.89	0.38	1.55
Antidepressants: Branded	4.94	0.23	0.93	3.46	0.39
Antidepressants: Generic	0.38	0.08	0.66	0.29	1.67
Antiulcer: Branded	2.87	0.00	1.00	2.07	0.49
Antiulcer: Generic	0.48	0.00	0.99	0.07	2.43

Note: Mean prices, marginal costs and margins relative to price using cost estimates from advertising equation in columns 2 and 3 and cost estimates from price FOC in columns 4 and 5.