Demand Analysis under Latent Choice Constraints

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Introduction

- Textbook models $\rightarrow$ choices are unconstrained at posted prices
  - ✓ Prices do all the "work" in clearing supply and demand

- Models of constraints on choice sets are common

1. Information frictions
   - Consumer brand awareness/advertising
   - Consumer search frictions

2. Supply-side rationing
   - Two-sided matching: student to schools, entry-level labor markets
   - Selective admissions: healthcare providers

- Common Theme: choice sets are unobserved

Research Objectives

1. General model of latent choice sets
    - Random utility model of consumer preferences
    - General model of latent choice sets
    - ✓ Consistent with several common models of choice constraints

      [Two-sided matching; Informative advertising; Fixed-sample search]

2. Identification analysis and estimation using data on observed matches
    - Show conditions under which model is non-parametrically identified
    - ✓ Relies on shifters of choice-sets and preferences excluded from the other side
    - Propose an estimator based on Gibbs' sampling

3. Application to the kidney dialysis market in the US
    - 750,000 patients, 1% of national healthcare expenditure

✓ Approach can be useful in other settings
- e.g. models of school demand when choice data is not available

Related Literature

- Selective admission practices in healthcare
  [Ching et al.,2015; Gandhi, 2020]
- Two-sided matching models with non-transferable utility
  [Agarwal, 2015; Menzel, 2015; Diamond and Agarwal, 2017; He, Sinha and Sun, 2021]
- Models of consideration sets and brand awareness
  [Swait and Ben-Akiva, 1987; Sovinksy Goeree, 2008; Gaynor et al., 2016; Abaluck and Adams-Prassl, 2021; Berseghyan et al., 2021; Berseghyan and Molinari, 2022]
- Incomplete product availability
  [Anupindi et al.,1998; Mausalem et al. 2010; Conlon and Mortimer, 2013; Hickman and Mortimer, 2016]
- Build on identification of demand without constraints
  [Berry and Haile, 2010; Berry, Gandhi and Haile, 2013]
- Study of the US dialysis industry
  [Greico and McDevitt, 2017; Dafny, Cutler and Ody, 2018; Eliason, 2019; Wollmann, 2020; Eliason et al., 2020]

Outline

Outline

Model: Preferences

- Indirect utility of patient i from treatment at j in market t is given by

$$v_{ijt} = u_{jt}(w_i, \omega_i) - g_{jt}(w_i, y_{ij})$$

  - w_i, y_{ij} are observed, ω_i is unobserved
  - Normalize $v_{i0t} = 0$, $g_{jt}(w_i, y_0) = 0$ and $|\partial g_{1t}(w_i, y_0)/\partial y| = 1$ for some y_0

✓ Very general form
  - Product specific observables and unobservables via functions $u_{jt}(\cdot), g_{jt}(\cdot)$
  - Non-linearity in y_{ij} since g_{jt} is general
  - Example – $\omega_i = (\beta_i, \varepsilon_i)$:

$$v_{ijt} = x_{jt}\alpha + \xi_{jt} + \sum_k x_{jtk}(\beta^u_{ik} + \beta^0_k w_{ik}) - \gamma y_{ij} + \varepsilon_{ijt}$$

- y_{ij} is distance in application

- Facility j is in i's choice set if $\sigma_{ijt} = 1$

$$\sigma_{ijt} = \sigma_{jt}(w_i, \omega_i, z_{ij}) \in \{0, 1\}$$

- Assumption 1: $(z_i, y_i) \perp \omega_i | w_i, t$

- Assumption 2: $\sigma_{jt}(w_i, \omega_i, \cdot)$ is non-increasing with image $\{0, 1\}$
  - ✓ Implies that there exists a random variable $\pi_{ijt} = \pi_{jt}(w_i, \omega_i)$ such that

$$\sigma_{ijt} = 1\{\pi_{ijt} > z_{ij}\}$$

✓ Does not commit to a specific model of choice set formation
  - Only require monotonicity and independence

- z_{ij} is excess occupancy in application

Outline

Identification: Primitives and Assumptions

$$\begin{aligned} v_{ijt} &= u_{jt}(w_i, \omega_i) - g_{jt}(w_i, y_{ij}) \\ \sigma_{ijt} &= 1\{\pi_{jt}(w_i, \omega_i) > z_{ij}\} \end{aligned}$$

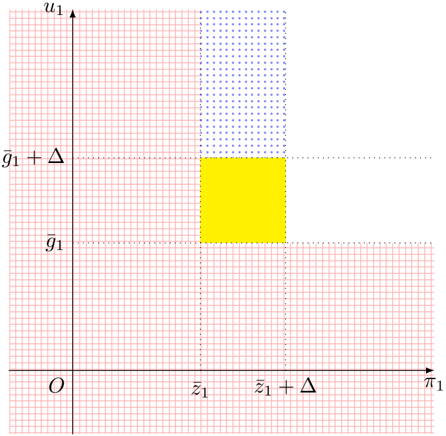
✓ Condition on w_i, t and drop from notation unless necessary

$$\begin{aligned} v_{ij} &= u_{ij} - g_j(y_{ij}) \\ \sigma_{ij} &= 1\{\pi_{ij} > z_{ij}\} \end{aligned}$$

- Identify the distribution of $(\boldsymbol{\pi}_i, \boldsymbol{u}_i)$ using data on $\boldsymbol{s}(\boldsymbol{y}_i, \boldsymbol{z}_i)$

Outline

Identification: $J = 1$, local variation in shifters



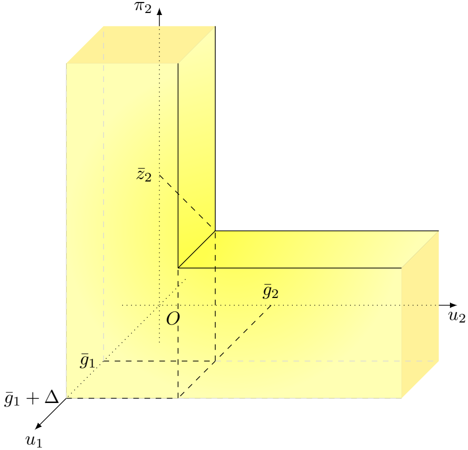
- Combine two slopes to obtain joint distribution at $(\bar{g}_1, \bar{z}_1)$
  - Variation in $\bar{g}_1$ affects consumers on the margin of choosing the outside option
  - Variation in $\bar{z}_1$ affects consumers whose choice set changes

Identification: Induction for $J > 1$

- Previous argument calculated

$$\int_{\bar{g}_1}^{\bar{g}_1+\Delta} \int_{\bar{z}_1}^{\bar{z}_1+\Delta} \int_{u_2<\bar{g}_2 \vee \pi_2<\bar{z}_2} \cdots \int_{u_J<\bar{g}_J \vee \pi_J<\bar{z}_J} f_{U,\Pi}(u,\pi)\, \mathrm{d}g_1 \mathrm{d}z_1 \mathrm{d}u_2 \mathrm{d}\pi_2 \cdots \mathrm{d}u_J \mathrm{d}\pi_J$$

✓ Proof extends to the general case:



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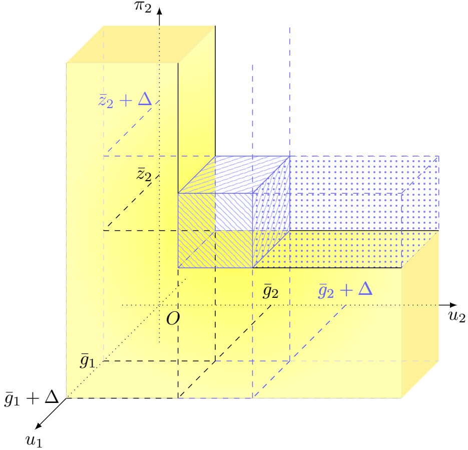

Identification: Result with known $g_j(\cdot)$

- Lemma 1: Suppose Assumptions 1 - 2 hold, $g_j(\cdot)$ is known, and let χ be the support of (g, z). The joint distribution of $(u, \pi)|\chi$ is identified.

- Comments:
  - Local variation sufficient for identification
  - Requires shifters/instruments on both sides

✓ Most prior approaches rely heavily on choice-set formation model
  - Enable analysis under weaker economic and statistical assumptions

Outline

Identifying $g_j(\cdot)$: Two Additional Assumptions

- Definition 1: Goods j and k substitute at y_i if there exist z_i^* with $s_j(y_i, z_i^*)$ and $s_k(y_i, z_i^*)$ strictly increasing and differentiable in y_{ik} and y_{ij} respectively.

- Assumption 3: For all but a finite set of y_i, the substitution network at y_i is connected.

  ✓ Under regularity assumptions, equivalent to increasing $g_j(\cdot)$ and $g_k(\cdot)$

- Similar to the connected substitutes idea in Berry, Gandhi and Haile (2014)

- Assumption 4:
  i. The support Y is rectangular with non-empty interior
  ii. Each $g_j(\cdot)$ has no critical points (differentiable with non zero derivative)

- Lemma 2: Suppose A1, A3 and A4 hold and $|J| > 1$. The function $g_j(\cdot)$ is identified in the support of Y_j

- Define the inclusive value of a consumer given observables

$$V^*\left(g\left(y_i\right)\right) = \sum_{O \in \mathcal{O}} E\left(\max_{j \in O} u_{ij} - g_j\left(y_{ij}\right) \middle| O, g\left(y_i\right)\right) P\left(O\right),$$

Identification of $g_j(\cdot)$

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- By the Envelope Theorem, we get a version of Roy's identity [Inspired by Allen and Rehbeck, 2019]

$$\frac{\partial V^*\left(g\left(y_i\right)\right)}{\partial g_j} = -s_j\left(y_i\right)$$

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- This implies the following

$$\begin{aligned} \frac{\partial s_j\left(y_i\right)}{\partial y_{ik}} &= -\frac{\partial^2 V^*\left(g\left(y_i\right)\right)}{\partial g_j \partial g_k} g_k'\left(y_{ik}\right) \\ \Longrightarrow \quad \frac{\partial s_j\left(y_i\right)}{\partial y_{ik}} \Big/ \frac{\partial s_k\left(y_i\right)}{\partial y_{ij}} &= \frac{g_k'\left(y_{ik}\right)}{g_j'\left(y_{ij}\right)} \end{aligned}$$

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✓ Along with normalizations, implies identification of $g(\cdot)$ when $J > 1$

Outline

Estimation: Specification

- Consider the following specification:

$$\begin{aligned} v_{ijt} &= \delta_j + \beta w_i - y_{ij} + \varepsilon_{i0} + \varepsilon_{ijt} \\ \pi_{ijt} &= \eta_j + \alpha w_i - z_{ij} + \nu_{ijt}, \end{aligned}$$

where

  - y_{ij} is distance
  - z_{ij} is patients per station
  - ε_{i0} is mean-zero normal with unknown variance
  - ε_{ijt} and ν_{ijt} are distributed jointly mean-zero normal (independent today)

✓ Relationship between (δ_j, η_j) and x_j, ξ_j estimated in a second stage

- Approximate the MLE using Gibbs Sampler

- Start with diffuse priors for the parameters and an initial value
  ✓ Normal/Inverse-Wishart to maintain conjugacy

Outline

Kidney Dialysis in the US

- Patients with kidney failure need transplantation or regular dialysis
- Medicare provides near-universal coverage
  - Cost the taxpayer $49.2 billion in 2018
    [1% of Fed budget, 7% of Medicare spending]
- Focus on hemodialysis
  - $\sim$ 90% of dialysis patients
  - Outpatient procedure
  - Facilities not associated with a hospital
  - Between two to three times a week, approx. four hours at a time
- Capacity constraints and selective admission due to capital/labor requirements

Outline

Data

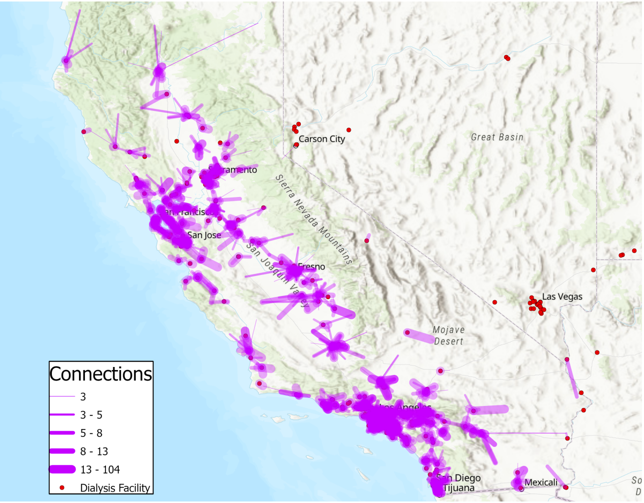
- US Renal Data System (USRDS)

- Comprehensive data on new patient admissions
  - Medicare collects information on all patients due to near-universal eligibility
    - ✓ 90 day wait period, followed by coordination with private insurer for 30 months if applicable
  - Patient zip-code, health conditions, demographics, primary insurer

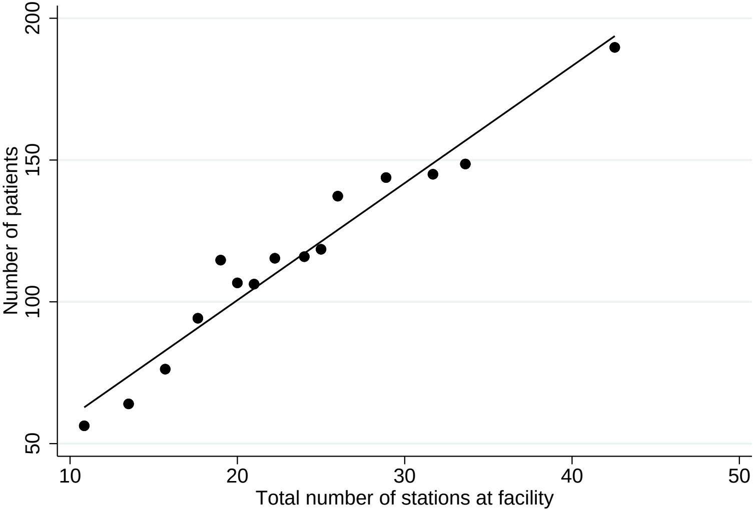
- Data on facilities
  - Location
  - Number of stations, labor inputs
  - Outcome/quality measures

- Focus on CA, between 2015 and 2018
  - Most populous state, isolated

Map of Facilities and Patient Connections



Patients per Station

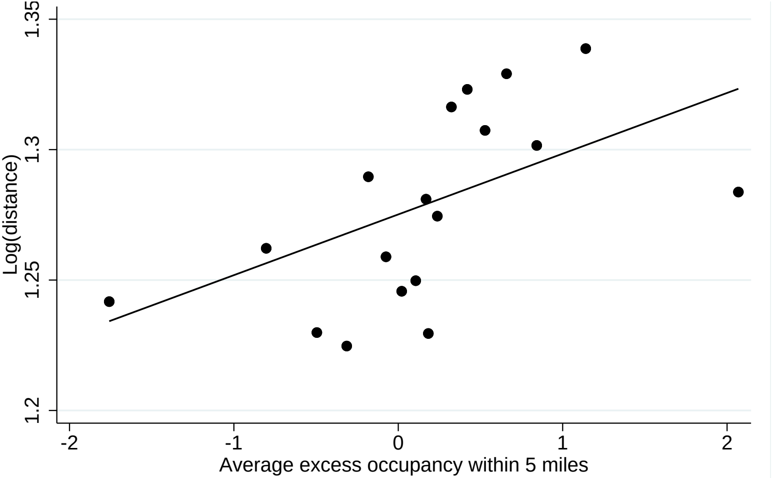


Effects of Capacity Constraints on New Admissions

	Any new patient	Log(days to next patient)	Any new patient	Log(days to next patient)	Any new patient		Log(days to next patient)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Occupancy	0.0001	0.004***	-0.0008***	0.017***				
	(0.0001)	(0.001)	(0.0001)	(0.002)				
Excess occupancy					-0.0003**	-0.0003**	0.017***	0.016***
					(0.0001)	(0.0001)	(0.002)	(0.002)
Occupancy within 5 miles						0.0003***		0.003**
						(0.0001)		(0.001)
Facility FE	X	X			X	X	X	X
Facily-Year FE			X	X				
Observations	708,969	23,666	708,969	23,666	708,969	708,969	23,666	23,666

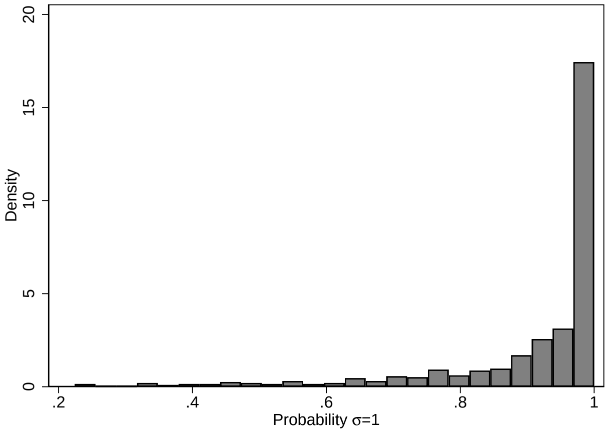
✓ Patients arriving when local facilities are busier, end up traveling further

Effects of Local Capacity Constraints



Outline

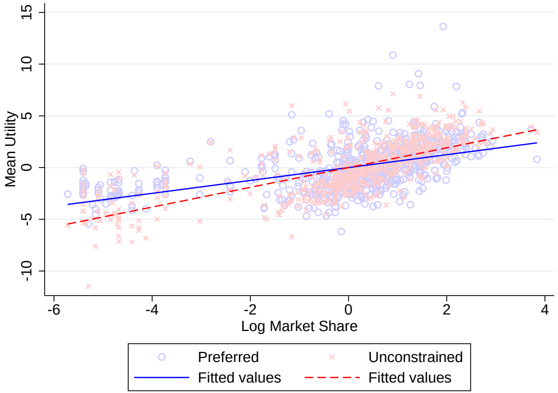
Admission Probabilities



✓ A within-facility 1 s.d. ↑ in occupancy → admission prob. ↓ by 2.28%

Estimates

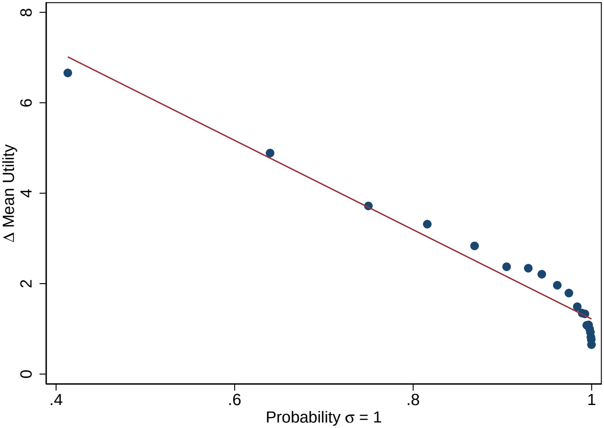
Effects of Constraints on Market Shares



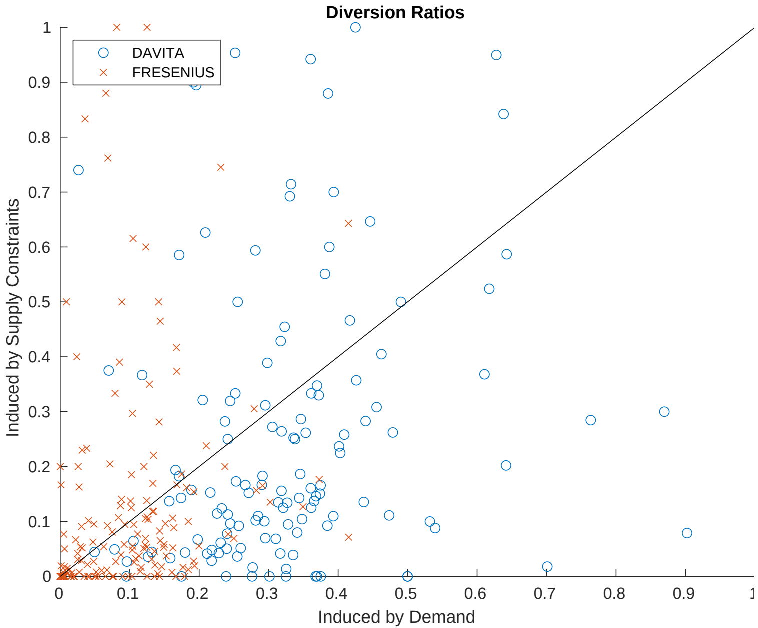
✓ Capacity constraints are important for some patients

- 73.0% of patients get treated at top choice facility
- 5.9% of patients do not get treated at top five choices
- Utility falls by equivalent of 0.72 miles for those facing constraints

Comparing Estimates



Comparing Estimates II



Outline

Conclusion

- Unified approach for identification in several models of choice with constraints
  - Supply-side rationing [e.g. education, healthcare markets]
  - Information frictions [e.g. consumer inattention, brand awareness]
  - ✓ Based on instruments excluded from the other side of the market

✓ Common in a number of markets

- Capacity constraints appear to affect allocations in the dialysis industry

- Open issues left for future work
  - Structural model of constraints
  - ✓ Straightforward in two-sided matching, more complicated in dynamic selection
  - Incorporate endogeneous decisions [e.g. pricing, quality decisions]

Estimates

	Preferred Specification (1)		Unconstrained (2)	Naïve (3)
	Acceptance	Utility	Utility	Utility
Diabetes	7.868	0.654	1.066	1.104
	(1.142)	(0.206)	(0.204)	(0.206)
Hypertension	10.051	-1.488	-0.980	-0.998
	(1.524)	(0.275)	(0.281)	(0.292)
BMI<20	2.605	-0.068	0.057	0.058
	(1.369)	(0.392)	(0.398)	(0.405)
25<=BMI<30	-0.397	-0.109	-0.144	-0.143
	(0.845)	(0.245)	(0.245)	(0.253)
30<=BMI	-1.076	0.477	0.422	0.439
	(0.947)	(0.248)	(0.246)	(0.258)
Age	-0.388	0.000	0.001	0.001
	(0.141)	(0.000)	(0.000)	(0.000)
Age squared	0.004	-2.392	-2.161	-2.224
	(0.001)	(0.284)	(0.279)	(0.297)
Medicare	5.521	0.080	0.059	0.061
	(1.193)	(0.039)	(0.040)	(0.040)
Medicare Advantage	-7.750	-2.145	-2.631	-2.708
	(1.525)	(0.334)	(0.318)	(0.336)
Medicare waiting period	-1.475	3.600	3.610	3.715
	(1.078)	(0.352)	(0.350)	(0.372)
Employed		-6.878	-7.035	-7.212
		(0.386)	(0.363)	(0.428)
Employed x distance		0.002	-0.002	-0.002
		(0.008)	(0.008)	(0.008)
Population density x distance		0.003	0.002	0.002
		(0.001)	(0.001)	(0.001)
Distance squared		0.013	0.013	0.013
		(0.000)	(0.000)	(0.000)
Excess Occupancy				-0.053
				(0.004)
Mean of δ_j		4.062	1.853	1.935
		(1.159)	(1.175)	(1.217)
Standard deviation of δ_j		3.036	3.204	3.186
		(0.127)	(0.118)	(0.119)
Standard deviation of ε_{i0}		11.445	11.772	12.136
		(0.470)	(0.401)	(0.570)
Standard deviation of ε_{ij}		4.274	4.775	4.769
		(0.044)	(0.033)	(0.033)
Mean of η_j	20.437			
	(4.791)			
Standard deviation of η_j	32.142			
	(3.601)			
Standard deviation of ν_{ij}	21.850			
	(2.105)			
Correlation between ε_{ij} and ν_{ij}	-0.138			
	(0.042)			

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Identification with Endogeneity

$$v_{ijt} \quad = \quad u\left(\boldsymbol{x}_{jt}, \xi_{jt}\left(w_i\right), w_i, \omega_i\right) + g\left(\boldsymbol{x}_{jt}, \zeta_{jt}\left(w_i, y_{ij}\right), w_i, y_{ij}\right)$$

- Identification conditions
  i. Additivity: $E\left(\left. u_{ijt}\right| \boldsymbol{x}_{jt}, \xi_{jt}\right) = \tilde{u}\left(\boldsymbol{x}_{jt}\right) + \xi_{jt}$ and $g\left(\boldsymbol{x}_{jt}, \zeta_{jt}\right) = \tilde{g}\left(\boldsymbol{x}_{jt}\right) + \zeta_{jt}$
  ii. Instruments: $E\left[\left.\left(\xi_{jt}, \zeta_{jt}\right)\right| \boldsymbol{r}_{jt}\right] = 0$
  iii. Completeness (rank) condition on instruments

✓ Strategy based on Berry and Haile (2010), Newey and Powell (2003)

- Comments:
  - Extension to non-linear unobservables via Chernozhukov and Hansen (2005)

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