Fold Geometry Toolbox

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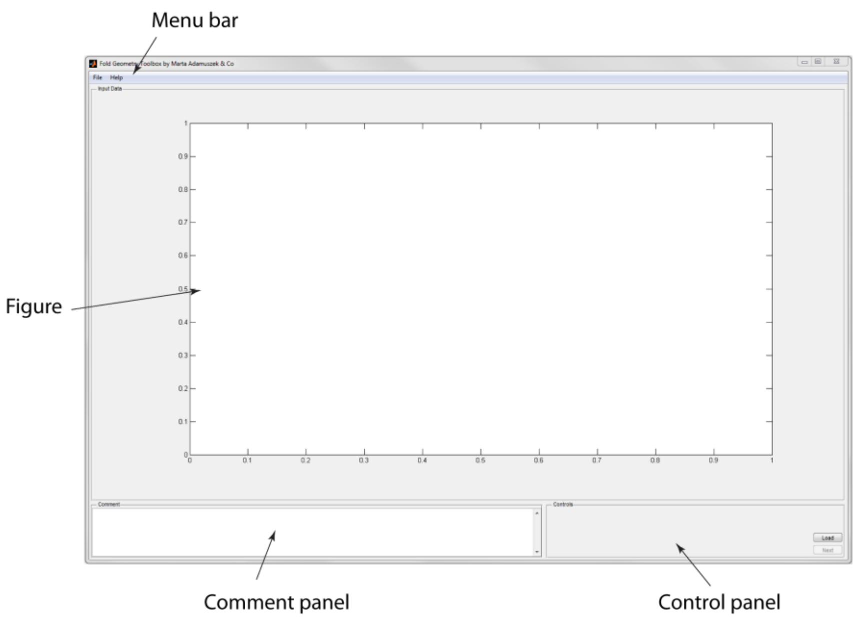
Contents

Workspace overview

If you run the fgt.m file in Matlab, FGT workspace will appear on your screen (as seen in the below screenshot). The workspace contains the following components:

1. Menu bar – provides access to basic commends,
2. Figure window – displays the current work of FGT,
3. Comment panel – allows adding a text comment during the fold analysis,
4. Control panel – displays options and parameters relevant for current analysis. 'Next' and 'Back' buttons enable to move between different steps of the fold analysis.



Menu bar

Menu bar contains choices such as File and Help. The File menu includes load data, save data, and export data options.

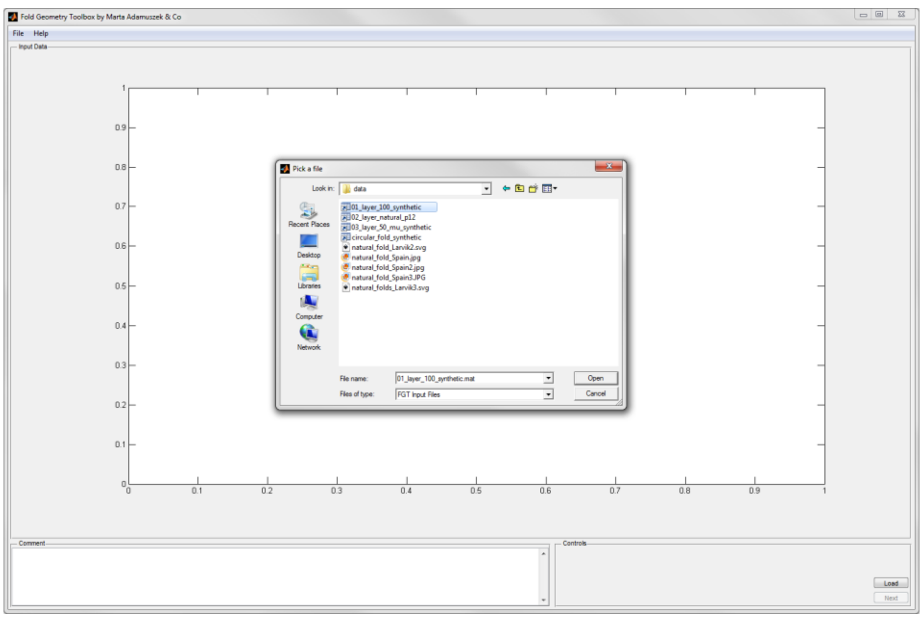
Load data

The data can be loaded in any of the following ways.

1. In the **File** dialog box, click **Load data.**
2. Press Ctrl+L.
3. Click on the figure.
4. Press **Load** in the control panel.

Note, that only the first two possibilities are active throughout the entire fold analysis process.

FGT input files include: Matlab, SVG, JPG, or PNG. Choose a file and click **Open**.



Create a Matlab file

All the fold data is stored in a single MATLAB structure array called *Fold*. Every entry in *Fold* has a field *Face* with two entries, corresponding to the upper and lower interface. Every entry in *Face* has two fields *X* and *Y* that contain the x and y coordinates of a single fold interface, respectively. If the input is specified as a MATLAB file then it must contain the structure *Fold*.

e.g.

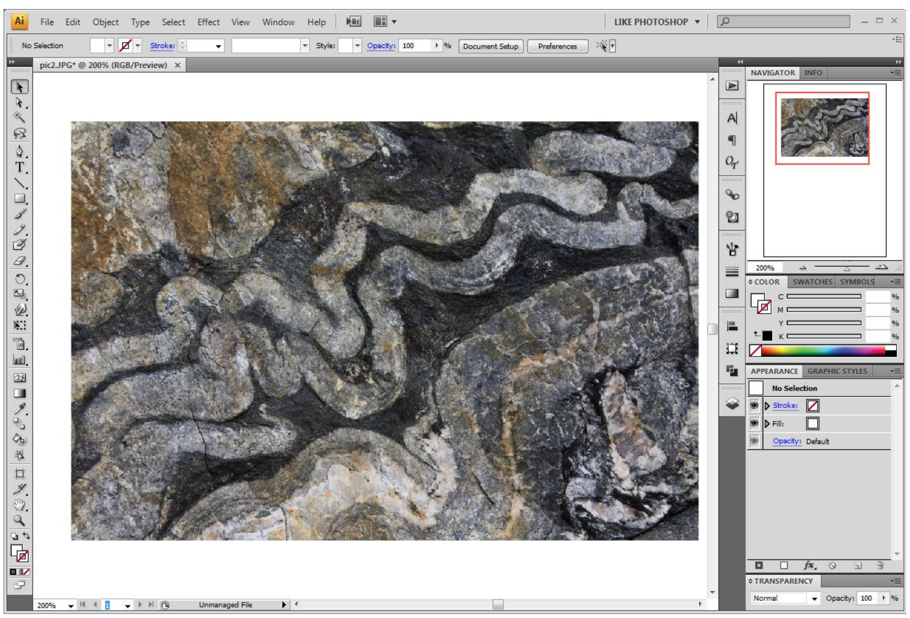
```
>> Fold(1).Face(1).X.Ori = upper_interface_x
>> Fold(1).Face(1).Y.Ori = upper_interface_y
>> Fold(1).Face(2).X.Ori = lower_interface_x
>> Fold(1).Face(2).Y.Ori = lower_interface_y
```

Note, each fold interface must consist of at least 7 points.

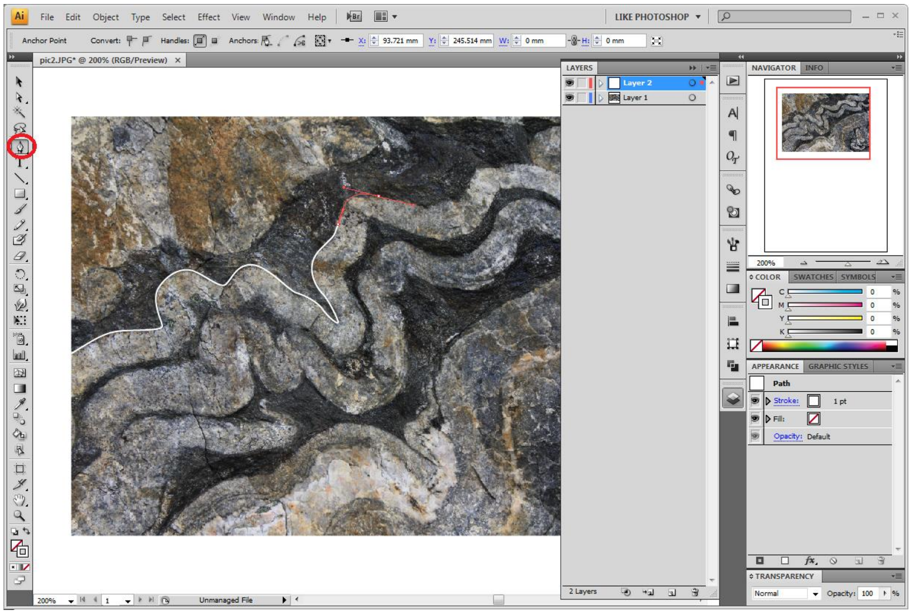
Create an SVG file in e.g. Adobe Illustrator

SVG (Scale Vector Graphics) file contains results of an image digitization in a vector graphics program such as Adobe Illustrator (AI). The following instruction of how to generate SVG file is presented as an example in Adobe Illustrator.

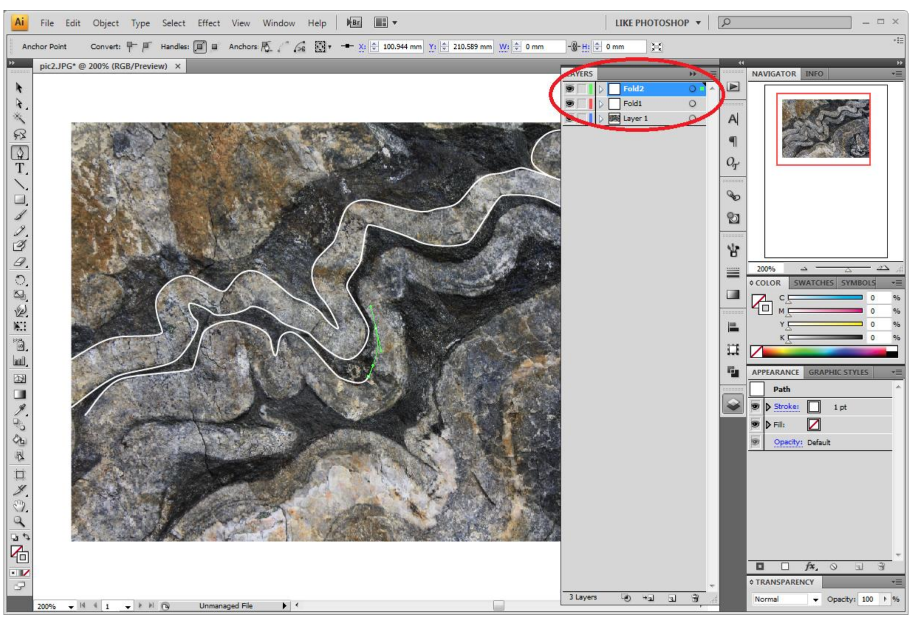
1. Import image.



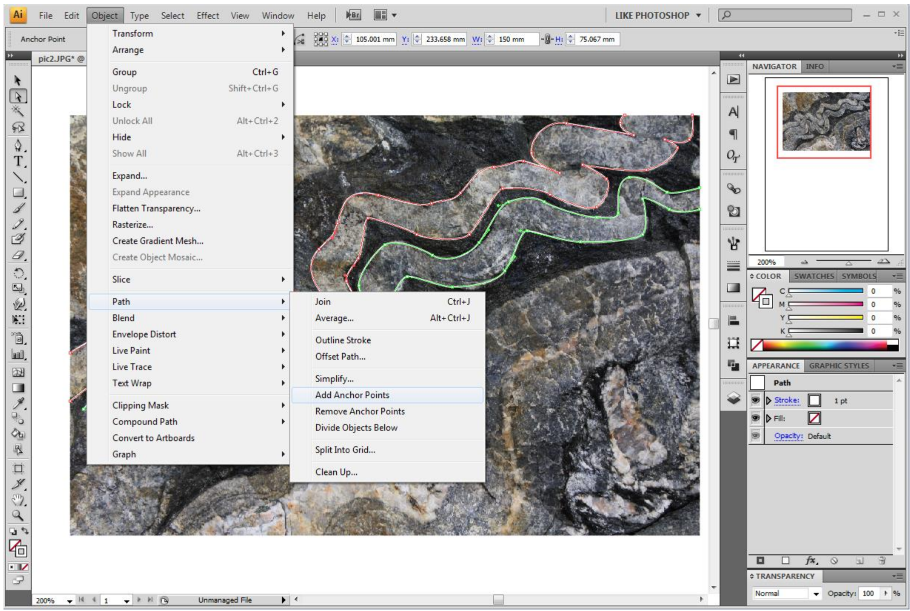
2. Draw separate contours of the upper and lower interface using the *Pen Tool*  (Bezier curve). Note: digitize the fold interfaces separately, preferably in the same direction e.g., here, from left to right



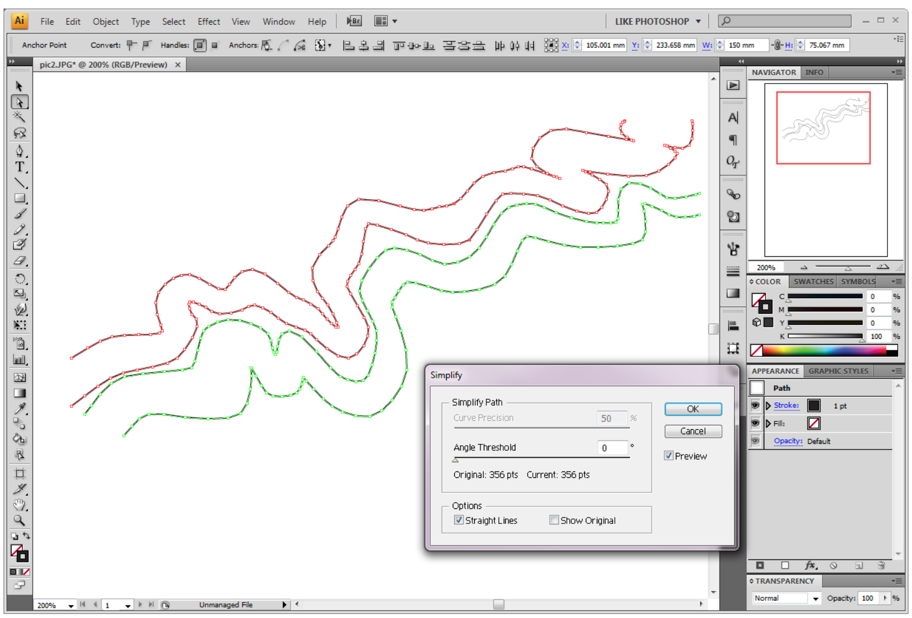
3. In case of multiple layers, draw each fold on the separate layers.



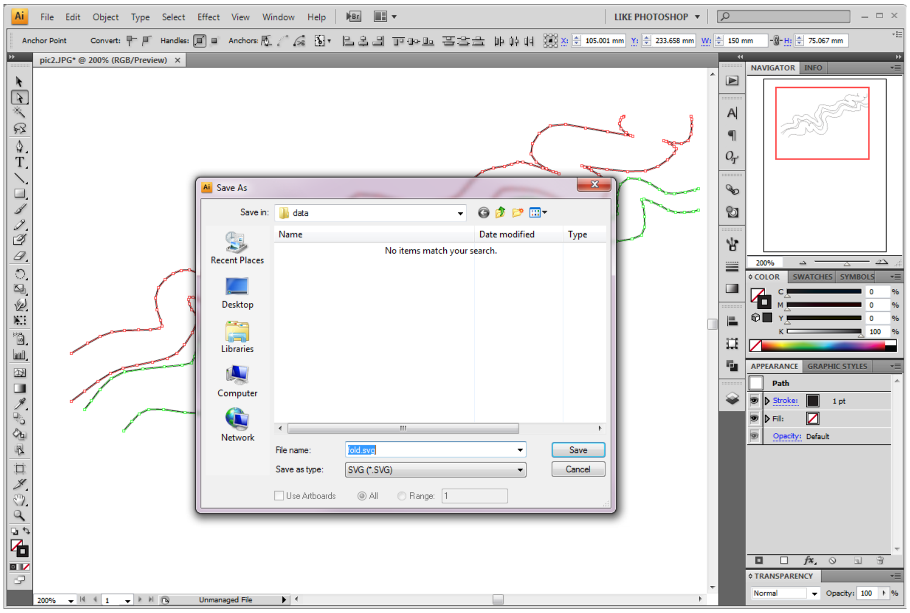
4. Add points on the curve. Mark the lines and select *Object/Path/Add more anchor points* as to obtain around 20-40 points on the fold.



5. Use *Object/Path/Simplify* and select *Straight Lines* to produce straight lines in between the points.

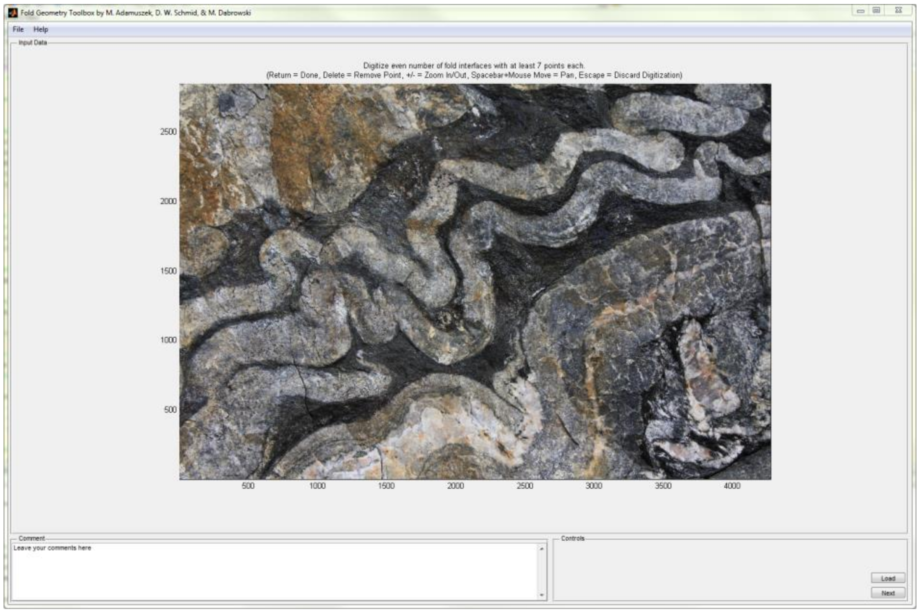


6. Save data as a SVG file.

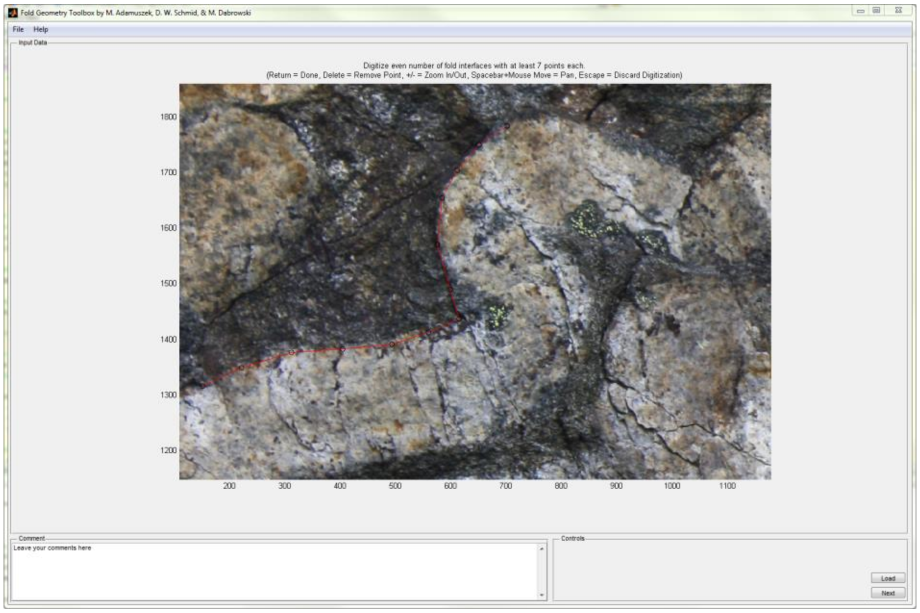


Digitize picture in FGT

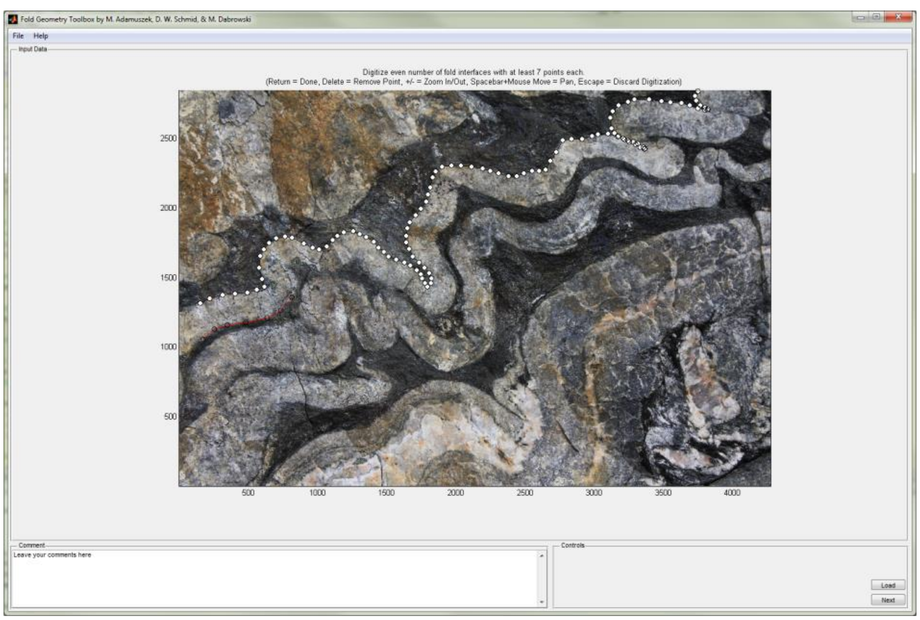
1. Load picture



2. Digitize the first fold interface by clicking with a mouse on the picture. The digitized data points are marked in red. In order to remove a point click **delete**. In order to zoom in or zoom out click + or **–** respectively. To pan use a **space bar** and move the mouse.



3. To finish digitization click enter or click the right mouse button. Then, the digitized interface turns black and the points are indicated with white dots. Start digitizing the second interface. Note, that the digitization of the second interface has to be done in the same direction e.g. here from left to right.

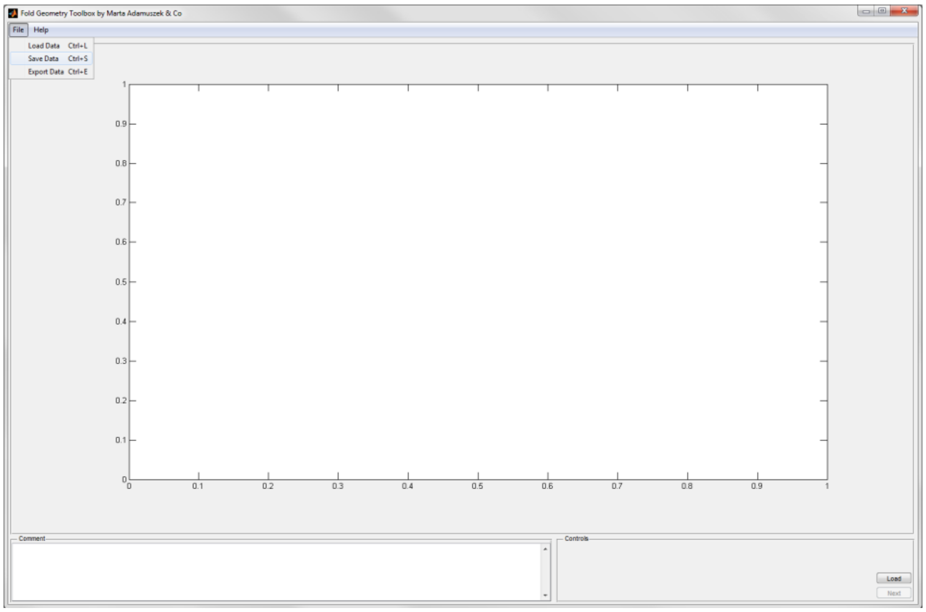


Save data

The fold data can be saved at each step of the fold analysis. The information about the fold is stored in the structure *Fold*.

The data can be saved in either way:

1. Click the **File** tab, and then click **Save data**
2. Click **Ctrl+S**.



Fold geometry parameters saved in Fold structure

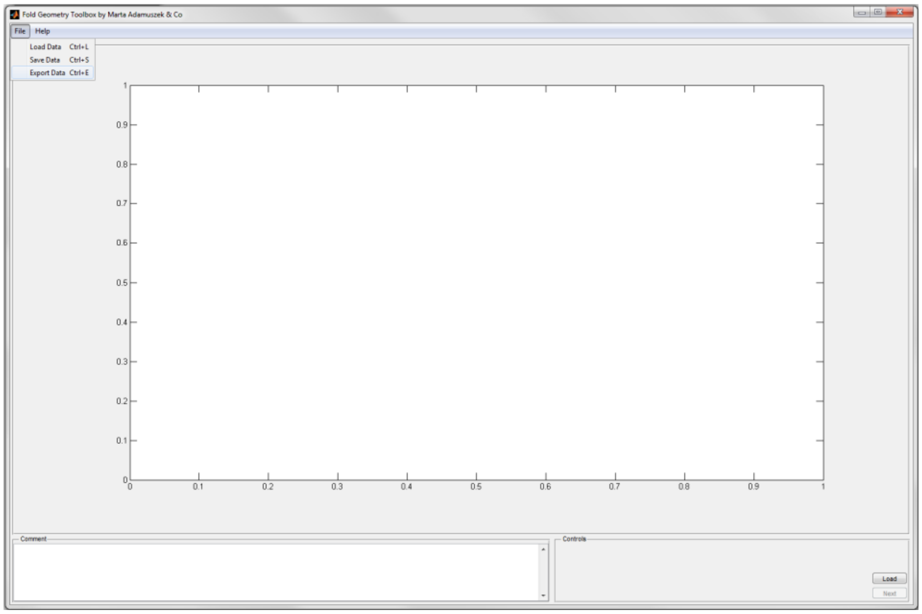
Information about the analysed fold is stored in the following entries in the Fold structure (the presented examples are given for the first interface in the first fold)

Fold coordinates:	Fold(1).Face(1).X.Full, etc.
Arc length:	Fold(1).Face(1).Fold_arclength
Amplitude:	Fold(1).Face(1).Amplitude.Value
Wavelength:	Fold(1).Face(1).Wavelength.Value
Local thickness:	Fold(1).Thickness.Local.Value
Average thickness:	Fold(1).Thickness.Average
Comment:	Fold(1).Comment

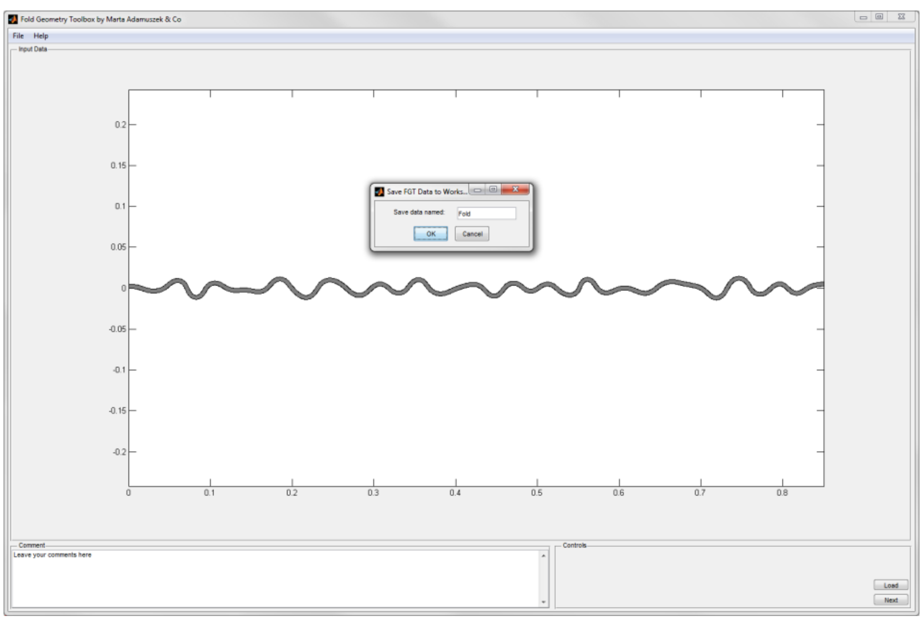
Export data

In order to export the data to the current workspace:

1. Click the **File** tab, and then click **Export data**, or
2. Click **Ctrl+E**.



Then, name the data and click OK.



Comment panel

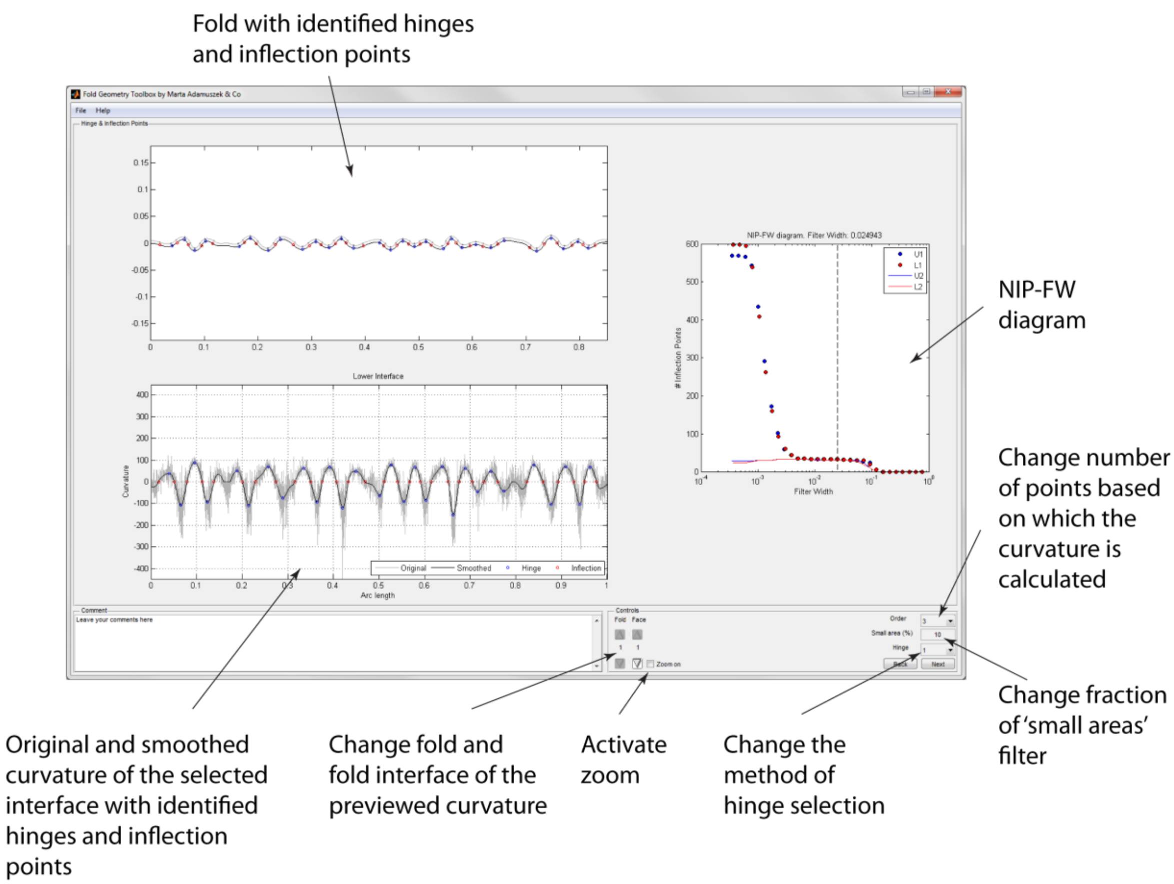
Throughout the work, any comment can be written in the Comment panel. The comment is stored in the structure *Fold* with the other fold data.

Step 1

In step 1, the analysed fold is loaded.

Step 2

In step 2, the curvature is calculated and smoothed and the inflection and hinges points are identified and located on the fold interfaces.



Curvature calculation

Curvature is a measure of how an interface deviates from being a straight line. The parametric form of curvature defines

$$\kappa = \frac{x'y'' - y'x''}{\left(x'^2 + y'^2\right)^{3/2}}$$

Single and double primes denote first and second derivative with respect to arc length, accordingly. The derivatives are approximated by calculating the derivatives of a polynomial that locally interpolates the set of data points. The user can fit a second order polynomial to the parameterized x and y coordinates of sets of three, five, or seven points. The derivatives are evaluated in the central point.

Gaussian filter

The curvature value in each data point is replaced by the weighted average of values within the smoothing window of size Ψ. The smoothed curvature in a point s_j is

$$\tilde{\kappa}\left(s_j\right) = \frac{\sum_{i=1}^{n} w_i \cdot \kappa\left(s_i\right)}{\sum_{i=1}^{n} w_i}$$

The weights w_i are evaluated based on the distance to point s_j :

$$w_i = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(s_i - s_j\right)^2 / 2\sigma^2}$$

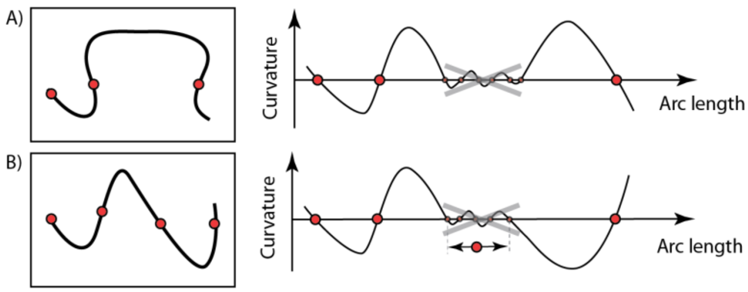
where the standard deviation s is set to Ψ/6.

Inflection point

Inflection points are the points where the interface curvature changes its sign. Two neighbouring inflection points confined a fold.

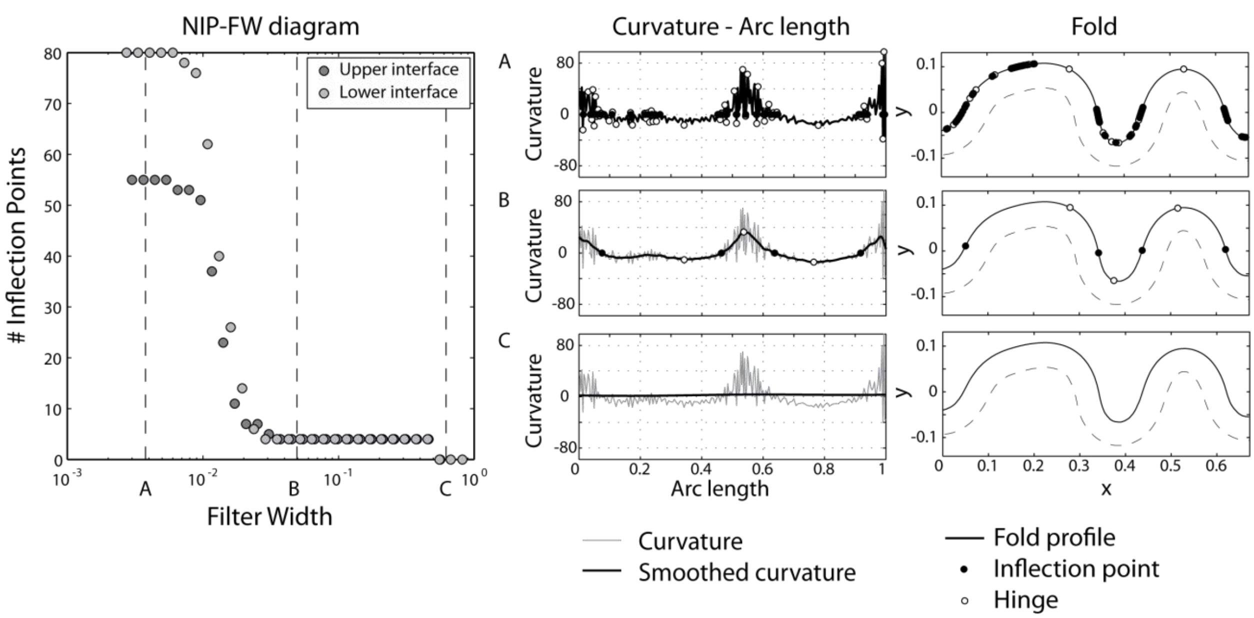
'Small area' filter

The areas bounded by the curvature between inflection points and the horizontal axes are calculated. If an individual value is smaller than a certain percentage, e.g. 10%, of the average of all these values then the curvature between the corresponding inflections is set to zero. If the curvature changes sign around this segment we place an inflection point in the middle of it. Note, the fraction of the 'small area' filter has to be between 0 and 100.



NIP-FW diagram

The amount of smoothing and consequently the number of detected hinge and inflection points is controlled by the width of the smoothing filter. This relation is presented on the NIP-FW (Number of Inflection Points vs. Filter Width) diagram. Plateaus relate to folds on different scales and the filter width should be selected from the range of the plateaus.

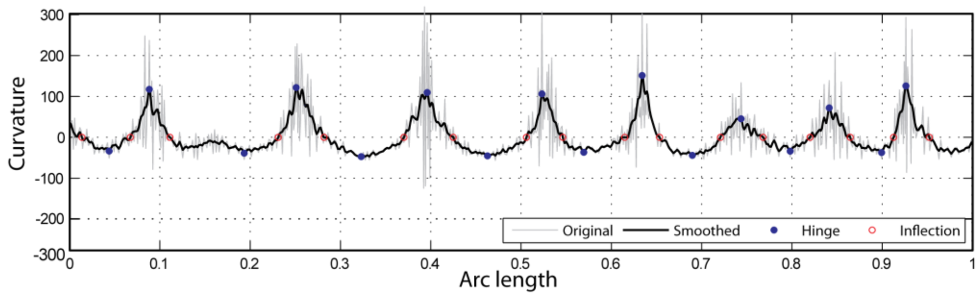


Hinge point

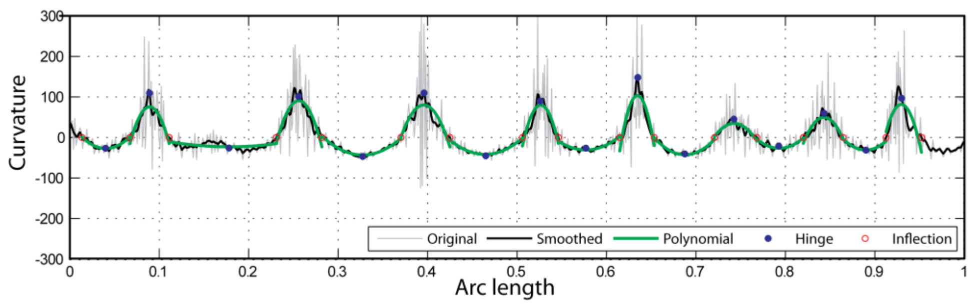
Two methods for identifying hinges are implemented

1. A hinge is the point of local extremum of curvature between two inflection points.
2. A hinge is a point of local extremum of the second order polynomial fitted to all the points in curvature-arc length space between two inflection points. This procedure is recommended for concentric folds.

1



2



Zoom

Zoom can be activated by marking the check box in the control panel.

Zoom in:

1. Press the mouse button.
2. Rotate the mouse scroll wheel upward.

Zoom out:

1. Rotate the mouse scroll wheel upward.
2. Press shift+click.
3. Click right mouse button and select “zoom out”.

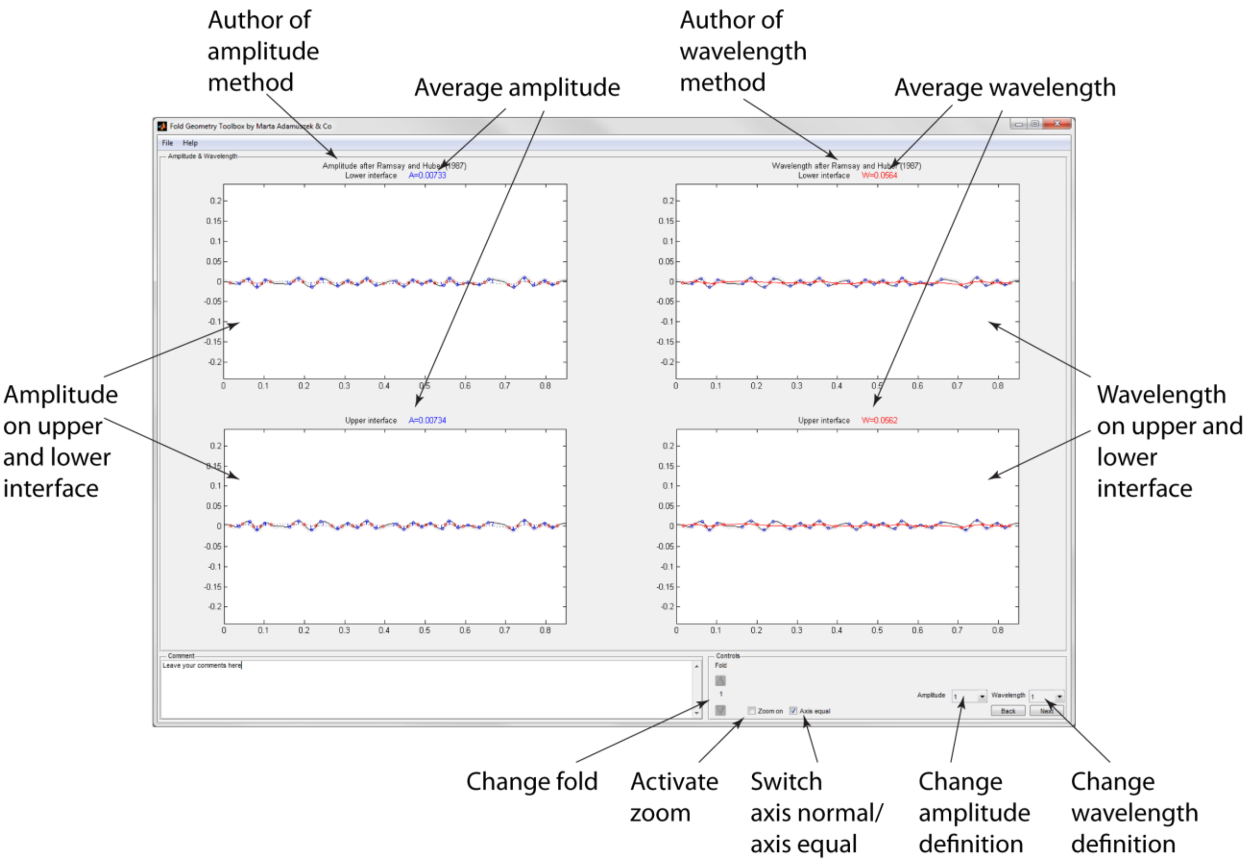
Restore original view:

1. Double click on the picture.
2. Click right mouse button and select “reset to original view”.

Note: zoom options are active only when zoom box is selected.

Step 3

In step 3, arclength, amplitude, and wavelength are calculated.



Arc length (L)

The arc length s is the interface length between two points on it. Using a parametric representation where interface point coordinates x and y are functions of parameter τ

The arc length is given by

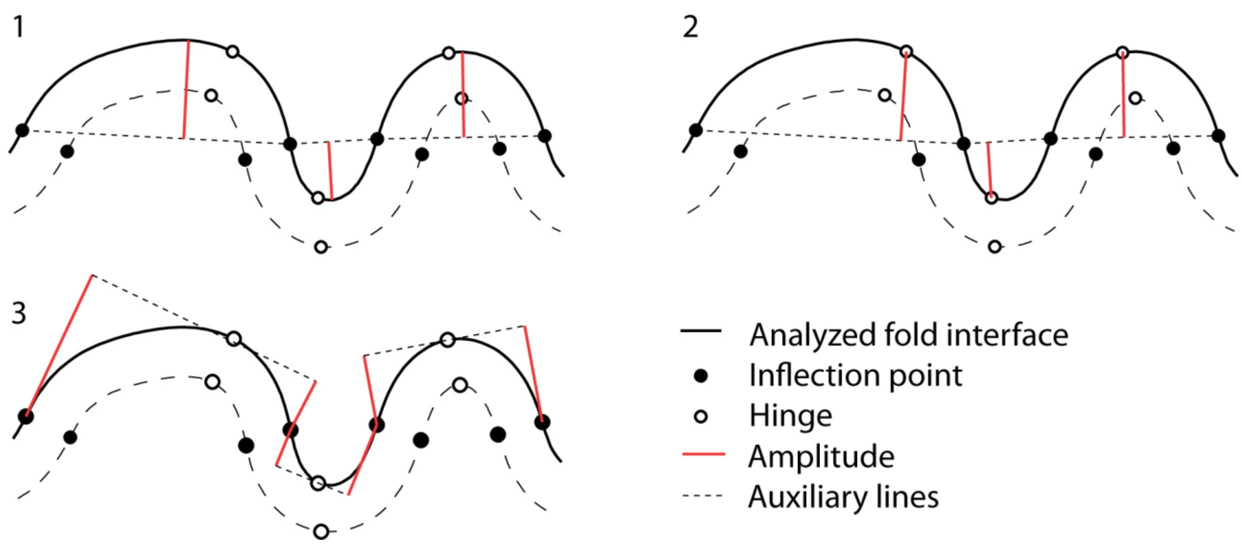
$$s = \int_{t_1}^{t_2} \left(x'(\tau)^2 + y'(\tau)^2 \right)^{1/2} d\tau$$

where the prime denotes the first derivative with respect to τ. The "fold arc length" – L is defined as twice the arc length between two inflection points.

Amplitude (A)

Three definitions of amplitude are employed in FGT:

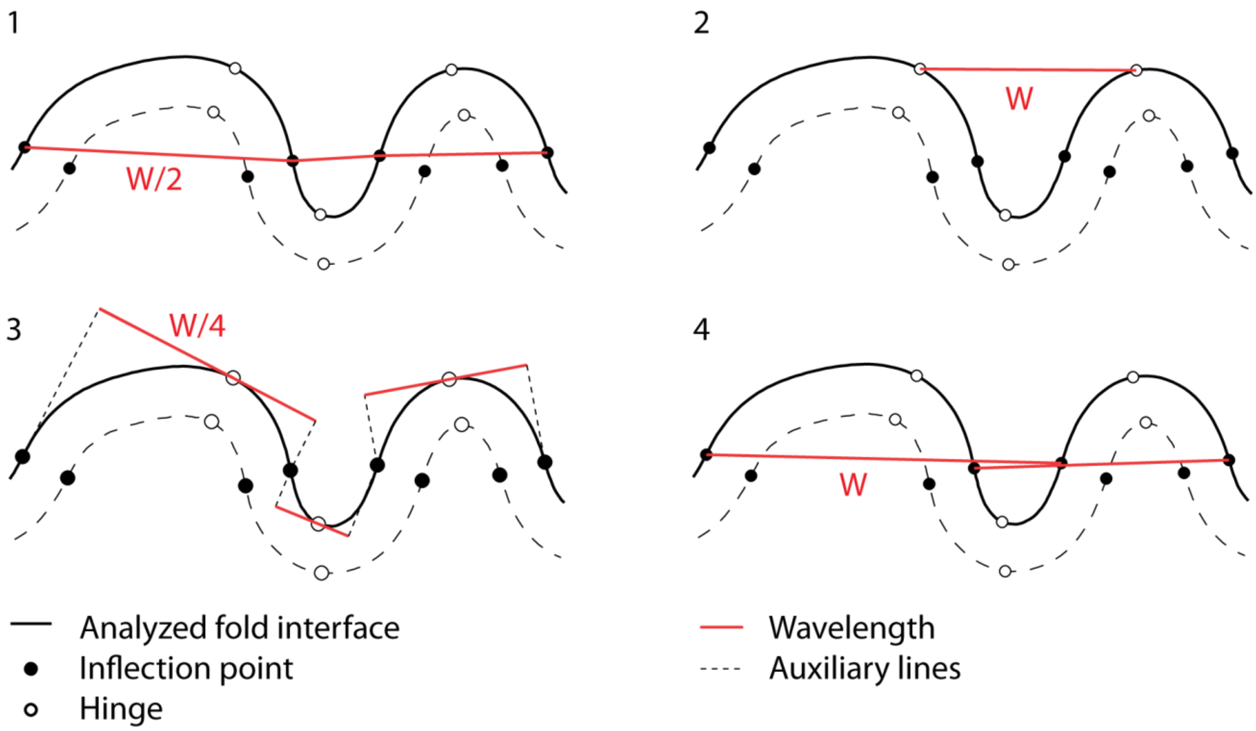
1. The distance between the line joining two inflection points and the extremity of the fold (Ramsay and Huber, 1987).
2. The distance from the hinge to the line joining two inflection points (Park, 1997).
3. The distances between the inflection points and the line tangent at the hinge (Hudleston, 1973). We use the average of these two values in FGT.



Wavelength (λ)

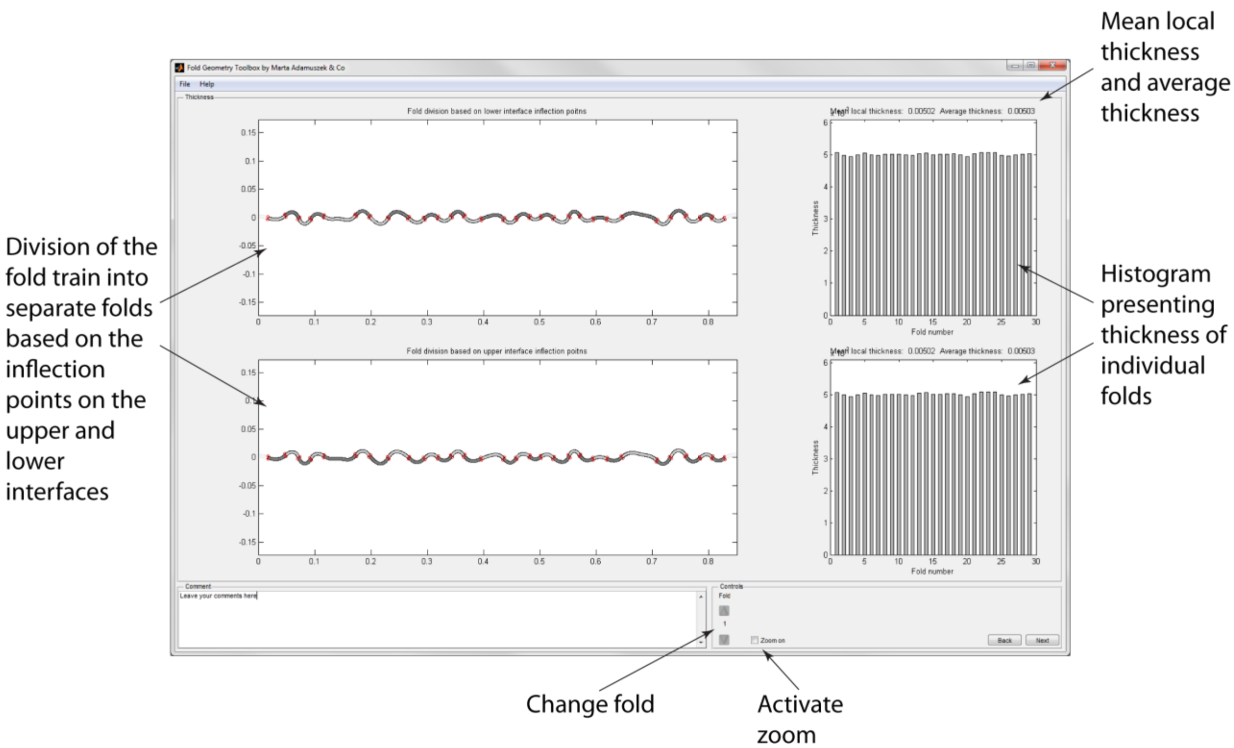
Four definitions of wavelength are employed in FGT:

1. Twice the distance between adjacent inflection points (Price and Cosgrove, 1990; Ramsay and Huber, 1987).
2. The distance between alternating hinges (van der Pluijm and Marshak, 2004). This method is applicable only to a fold train that consists of more than three folds. In FGT we use the average value if more than one wavelength can be constructed for a specific fold hinge.
3. Four times the distance between the hinge and the point on the tangent to the hinge that has the shortest distance to the adjacent inflection points (Hudleston, 1973). For each fold the average value of the two wavelengths is used in FGT.
4. The distance between alternating inflection points (Price and Cosgrove, 1990). We use the average of two values that correspond to one fold.


1
W/2
2
W
3
W/4
4
W
Analyzed fold interface
Inflection point
Hinge
Wavelength
Auxiliary lines

Step 4

In step 4, thickness is calculated.

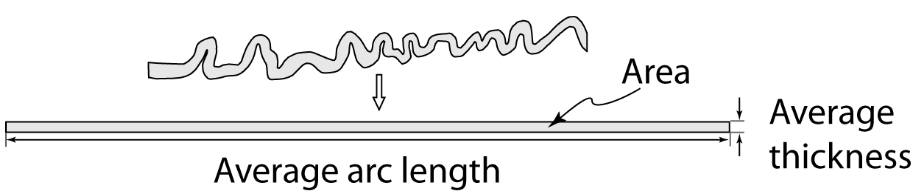


Thickness (h)

Two methods of calculating thickness are identified: average and local thickness:

1. Average thickness

The "average thickness" of an entire fold train is defined as the layer area divided by the average arc length of the two interfaces.



2. Local thickness

The “local thickness” defines the thickness of individual folds. The fold train is divided into discrete folds based on iso-contours of the harmonic function f, which is defined everywhere within the fold domain. Hence,

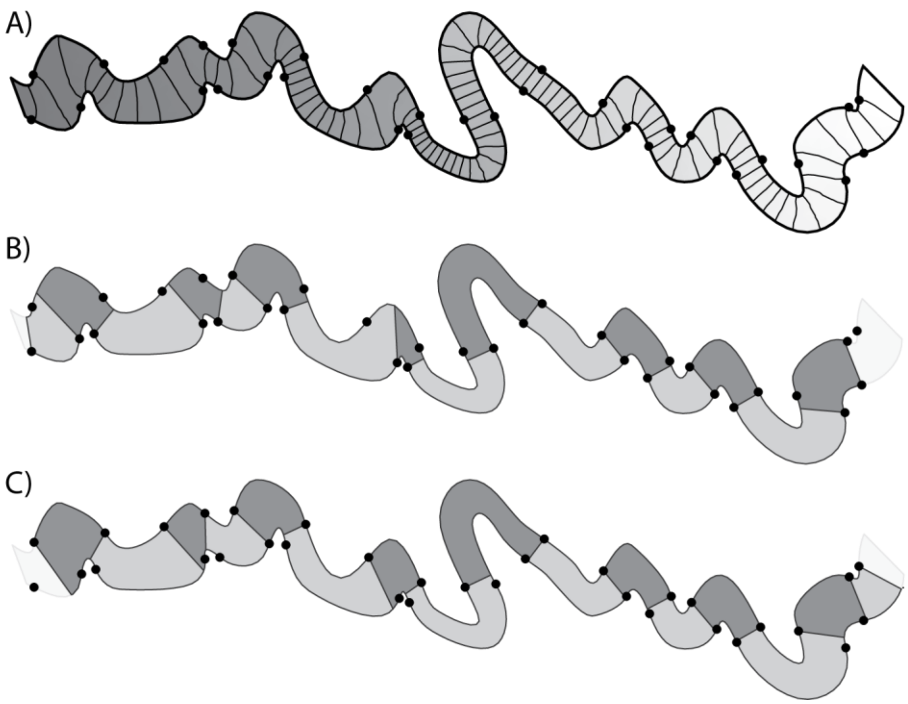
$$\nabla^2\Phi = 0$$

with the following boundary conditions

$$\Phi = 0 \; on \; \partial\Omega_L \qquad \frac{\partial\Phi}{\partial n} = 0 \; on \; \partial\Omega_T$$

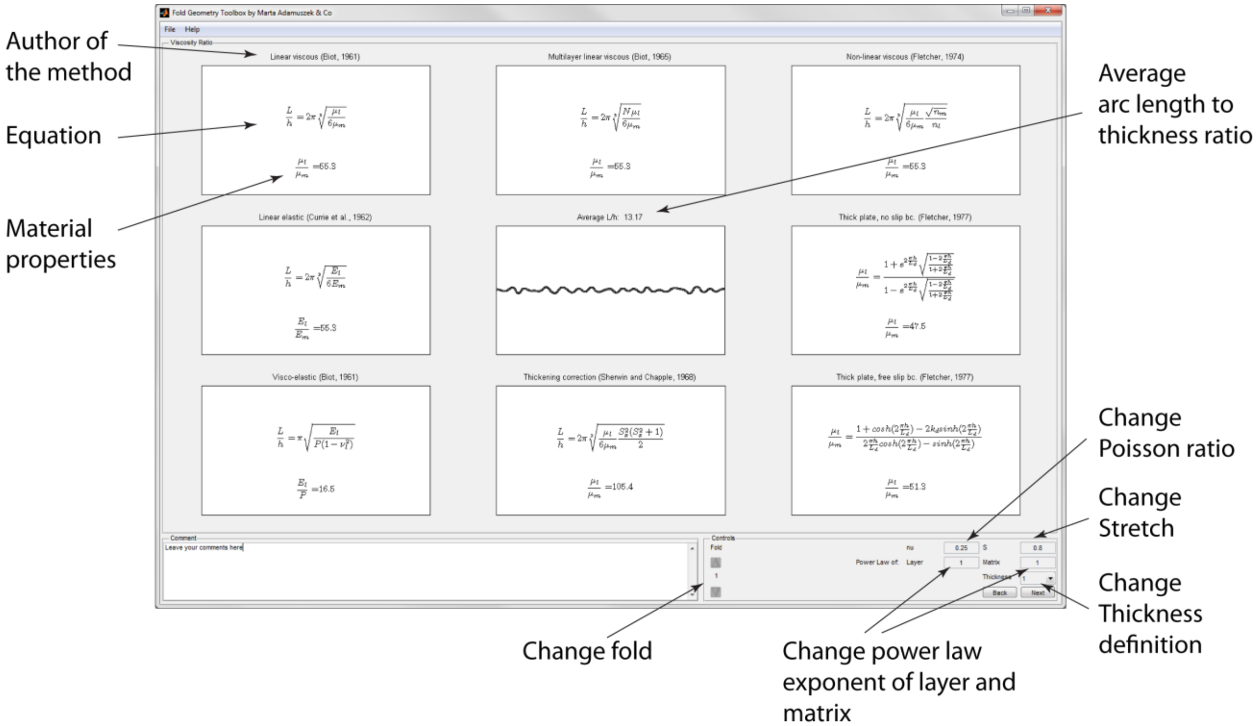
$$\Phi = 1 \; on \; \partial\Omega_R \qquad \frac{\partial\Phi}{\partial n} = 0 \; on \; \partial\Omega_B$$

The iso-contours of the function (A) allow finding a set of analogous points on both interfaces and thus facilitate a division of the fold train into separate folds. The division is made separately based on the position of inflection points on the lower (B) and upper interface (C).



Step 5

In step 5, material properties are estimated based on the



Material properties

These methods require the initial (infinitesimal amplitude) fold wavelength to thickness ratio. As a proxy we use the final fold arc length to thickness ratio (L/h). L/h is calculated per fold train as the ratio of average fold arc length to average thickness. Note that all averages in FGT are calculated as arithmetic means and that ratios are taken on already averaged values.

1. Linear viscous by Biot (1961)

$$\frac{L}{h} = 2\pi \sqrt[3]{\frac{\mu_l}{6\mu_m}} \Rightarrow \frac{\mu_l}{\mu_m} = 6\left(\frac{L}{2\pi h}\right)^3$$

where

L - arc length, h - thickness, μ_l - viscosity of the layer, μ_m - viscosity of the matrix

2. Linear elastic bu Currie (1962)

$$\frac{L}{h} = 2\pi \sqrt[3]{\frac{E_l}{6E_m}} \Rightarrow \frac{E_l}{E_m} = 6\left(\frac{L}{2\pi h}\right)^3$$

where

L - arc length, h - thickness, E_l - elastic moduli of the layer, E_m - elastic moduli of the matrix

3. Visco-elastic by Biot (1961)

$$\frac{L}{h} = 2\pi \sqrt{\frac{E_l}{P\left(1-\nu_l^{\,2}\right)}} \Rightarrow \frac{E_l}{P} = \left(1-\nu_l^{\,2}\right)\left(\frac{L}{2\pi h}\right)^2$$

where

L - arc length, h - thickness, E_l - elastic moduli the layer,
P - layer parallel stress, ν_l - Poisson ratio of the layer

4. Multilayer linear viscous (1965)

$$\frac{L}{h} = 2\pi \sqrt[3]{\frac{N\mu_l}{6\mu_m}} \Rightarrow \frac{\mu_l}{\mu_m} = \frac{6}{N}\left(\frac{L}{2\pi h}\right)^3$$

where

L - arc length, h - thickness, μ_l - viscosity of the layer,
μ_m - viscosity of the matrix, N - number of layers

5. Thickening correction by Sherwin and Chapple (1968)

$$\frac{L}{h} = 2\pi \sqrt[3]{\frac{\mu_l}{6\mu_m}\frac{S_x^2\left(S_x^2+1\right)}{2}} \Rightarrow \frac{\mu_l}{\mu_m} = \frac{12}{S_x^2\left(S_x^2+1\right)}\left(\frac{L}{2\pi h}\right)^3$$

where

L - arc length, h - thickness, μ_l - viscosity of the layer,
μ_m - viscosity of the matrix, S_x - layer stretch in x direction

6. Non-linear viscous Fletcher (1974)

$$\frac{L}{h} = 2\pi \sqrt[3]{\frac{\mu_l}{6\mu_m}\frac{\sqrt{n_m}}{n_l}} \Rightarrow \frac{\mu_l}{\mu_m} = \frac{6n_l}{\sqrt{n_m}}\left(\frac{L}{2\pi h}\right)^3$$

where

L - arc length, h - thickness, μ_l - viscosity of the layer,
μ_m - viscosity of the matrix, n_l - power law exponent of layer,
n_m - power law exponent of matrix

7. Thick plate, no slip boundary conditions Fletcher (1977)

$$\frac{\mu_l}{\mu_m} = \frac{1 + e^{2\frac{\pi h}{L_d}} \sqrt{\left(1 - 2\frac{\pi h}{L_d}\right) \Big/ \left(1 + 2\frac{\pi h}{L_d}\right)}}{1 - e^{\frac{2\pi h}{L_d}} \sqrt{\left(1 - 2\frac{\pi h}{L_d}\right) \Big/ \left(1 + 2\frac{\pi h}{L_d}\right)}}$$

where

L_d - dominant arc length, h - thickness,

μ_l - viscosity of the layer, μ_m - viscosity of matrix

8. Thick plate, free slip boundary conditions Fletcher (1977)

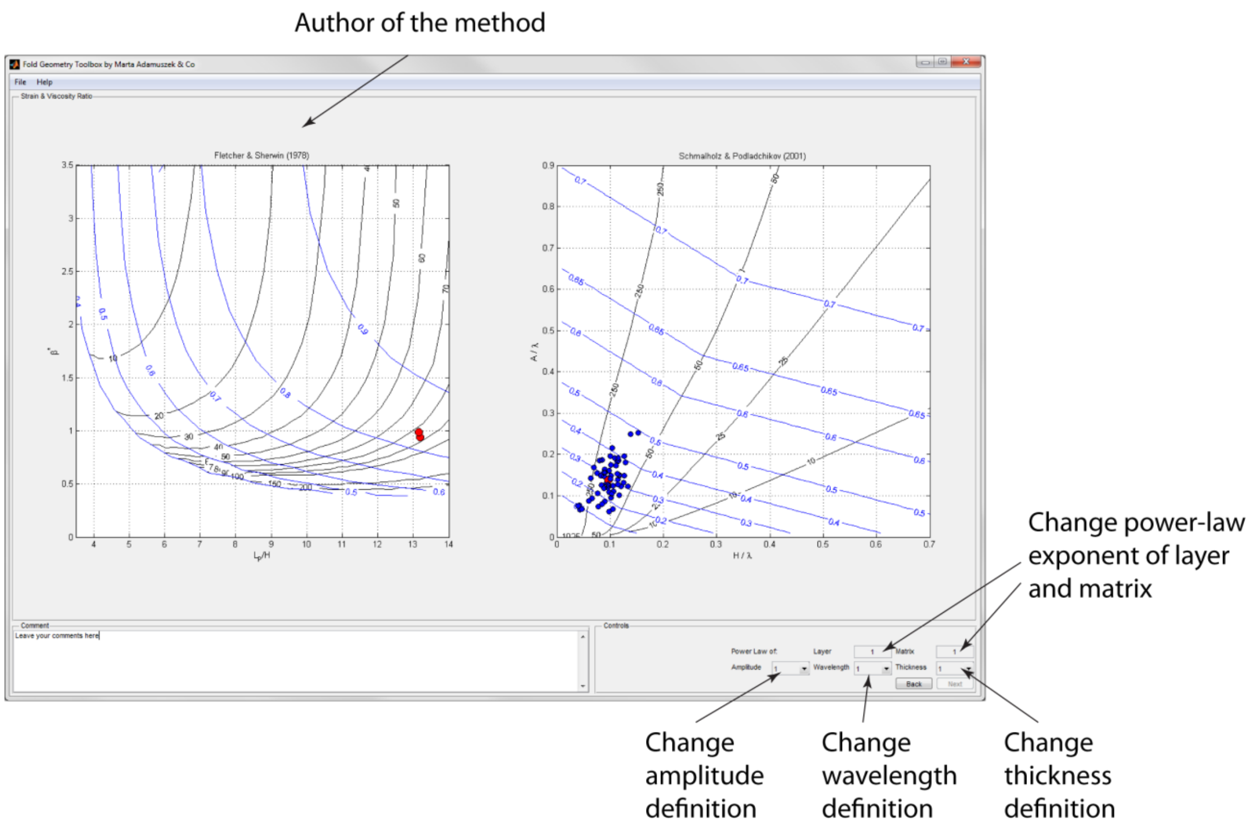
$$\frac{\mu_l}{\mu_m} = \frac{1 + \cosh\left(2\frac{\pi h}{L_d}\right) - 2\frac{\pi h}{L_d}\sinh\left(2\frac{\pi h}{L_d}\right)}{2\frac{\pi h}{L_d}\cosh\left(2\frac{\pi h}{L_d}\right) - \sinh\left(2\frac{\pi h}{L_d}\right)}$$

where

L_d - dominant arc length, h - thickness,

μ_l - viscosity of the layer, μ_m - viscosity of matrix

Step 6



Calculation of viscosity ratio and stretch using Fletcher and Sherwin (1978) method requires estimation of preferred wavelength to thickness ratio and relative bandwidth of the amplification spectrum. Dots on the plot represent values estimated for each interface. The power law exponent for layer and matrix also used in the calculation can be modified in the control panel.

For calculation of the viscosity ratio and bulk shortening using the Schmalholz and Podladchikov (2001) method, amplitude, wavelength, and thickness measures are involved. The amplitude, wavelength, and thickness definitions can be changed in the control panel. Blue dots represent the individual measurement per fold, whereas the red dot represents the average value. All the averages are calculated as arithmetic means and the ratios are taken on previously averaged values.

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