Coding

CHAPTER 9

Every Superman has his kryptonite, but as a Data Scientist, coding can't be yours. Between data munging, pulling in data from APIs, and setting up data processing pipelines, writing code is a near-universal part of a Data Scientist's job. This is especially true at smaller companies, where data scientists tend to wear multiple hats and are responsible for productionizing their analyses and models. Even if you are the rare Data Scientist that never has to write production code, consider the collaborative nature of the field — having strong computer science fundamentals will give you a leg up when working with software and data engineers.

*To test your programming foundation, Data Science interviews often take you on a stroll down memory lane back to your Data Structures and Algorithms class (**you did take one, right?**). These coding questions test your ability to manipulate data structures like lists, trees, and graphs, along with your ability to implement algorithmic concepts such as recursion and dynamic programming. You're also expected to assess your solution's runtime and space efficiency using Big O notation.*

Approaching Coding Questions

Coding interviews typically last 30 to 45 minutes and come in a variety of formats. Early in the interview process, coding interviews are often conducted via remote coding assessment tools like HackerRank, Codility, or CoderPad. During final-round onsite interviews, it's typical to write code on a whiteboard. Regardless of the format, the approach outlined below to solve coding interview problems applies.

After receiving the problem: Don't jump right into coding. It's crucial first to make sure you are solving the correct problem. Due to language barriers, misplaced assumptions, and subtle nuances that are easy to miss, misunderstanding the problem is a frequent occurrence. To prevent this, make sure to repeat the question back to the interviewer so that the two of you are on the same page. Clarify any assumptions made, like the input format and range, and be sure to ask if the input can be assumed to be non-null or well formed. As a final test to see if you've understood the problem, work through an example input and see if you get the expected output. Only after you've done these steps are you ready to begin solving the problem.

When brainstorming a solution: First, explain at a high level how you could tackle the question. This usually means discussing the brute-force solution. Then, try to gain an intuition for why this brute-force solution might be inefficient, and how you could improve upon it. If you're able to land on a more optimal approach, articulate how and why this new solution is better than the first brute-force solution provided. Only after you've settled on a solution is it time to begin coding.

When coding the solution: Explain what you are coding. Don't just sit there typing away, leaving your interviewer in the dark. Because coding interviews often let you pick the language you write code in, you're expected to be proficient in the programming language you chose. As such, avoid pseudocode in favor of proper compilable code. While there is time pressure, don't take many shortcuts when coding. Use clear variable names and follow good code organization principles. Write well-styled code — for example, following PEP 8 guidelines when coding in Python. While you are allowed to cut some corners, like assuming a helper method exists, be explicit about it and offer to fix this later on.

After you're done coding: Make sure there are no mistakes or edge cases you didn't handle. Then write and execute test cases to prove you solved the problem.

At this point, the interviewer should dictate which direction the interview heads. They may ask about the time and space complexity of your code. Sometimes they may ask you to refactor and clean the code, especially if you cut some corners while coding the solution. They may also extend the problem, often with a new constraint. For example, they may ask you not to use recursion and instead tell you to solve the problem recursively. Or, they might ask you to not use surplus memory and instead solve the problem in place. Sometimes, they may pose a tougher variant of the problem as a follow-up, which might require starting the problem-solving process all over again.

Space & Time Complexity

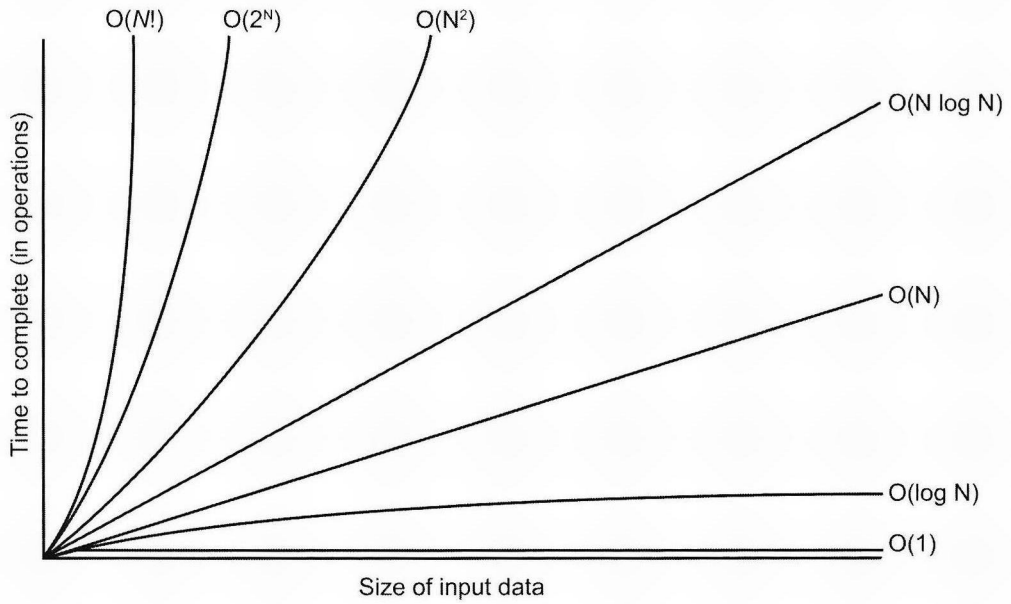
Determining the runtime and space usage (how much memory is utilized) of an algorithm is essential for coding interviews and real-world data science. Because compute and storage resources can be bottlenecks to machine learning model training and deployment, analyzing an algorithm's performance can affect what techniques you choose to implement. Consider OpenAI's GPT-3 language model, which contains over 175 billion parameters and took $12 million in compute resources to train. Much of the work bringing GPT-3 to the world involved optimizing resource usage to efficiently train such a large model.

Computer scientists analyze and classify the behavior of an algorithm's time and space usage via asymptotic complexity analysis. This technique considers how an algorithm performs when the input size goes toward infinity and characterizes the behavior of the runtime and space used as a function of n. In academic settings, we establish tight bounds on performance in terms of n using Big θ (big theta) notation. However, in industry, the technical definitions have been muddled, and we tend to denote these tight bounds on performance using Big O notation.

In the context of companies asking interview questions, we care about not just establishing tight bounds on performance but thinking about the worst-case scenario for this performance. As such, Big O notation often describes the "worst-case upper bound," or the longest an algorithm would run or the maximal amount of space it would need in the worst case.

For instance, consider an array of size N. Here are the following classes of runtime complexities, from fastest to slowest, using Big O notation:

- O(1): Constant time. Example: getting a value at a particular index from an array
- O(log N): Logarithmic time. Example: binary search on a sorted array
- O(N): Linear time. Example: using a for-loop to traverse through an array
- O(N log N): Log-linear time. Example: running mergesort on an array
- O(N^2): Quadratic time. Example: iterating over every pair of elements in an array using a double for-loop
- O(2^N): Exponential time. Example: recursively generating all binary numbers that are N digits long
- O(N!): Factorial time. Example: generating all permutations of an array



The same Big-O runtime analysis concepts apply analogously to space complexity. For example, if we need to store a copy of an input array with N elements, that would be an additional O(N) space. If we wanted to store an adjacency matrix among N nodes, we would need O(N^2) space to keep the N-by-N sized matrix.

For a basic example for both runtime and space complexity analysis, we can look at binary search, where we are searching for a particular value within a sorted array. The code that implements this algorithm is below (with an extra set of conditions that returns the closest if the exact value is not found):

```
def binary_search(a, k):
  lo, hi = 0, len(a) - 1
  best = lo
  while lo <= hi:
    mid = lo + (hi - lo) // 2
    if a[mid] < k:
      lo = mid + 1
    elif a[mid] > k:
      hi = mid - 1
    else:
      best = mid
      break
    if abs(a[mid] - k) < abs(a[best] - k):
      best = mid
    return best
```

If we start the binary search with an input of N elements, then at the next iteration, we would only need to search through $N/2$ elements, and so on. The runtime complexity for binary search is O(log N) since at each iteration we cut the remaining search space in half. The space complexity would simply be O(N) since the array is size N, and we do not need auxiliary space.

Complexity Analysis Applied to ML Algorithms

For an example of complexity analysis for a machine learning technique, consider the Naive Bayes classifier. Recall that the algorithm aims to calculate

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

for each of the n training data points (of dimension d) for each of the classes, and that $P(B|A)$ is the likelihood probability, and $P(A)$ is the prior probability. In simpler terms, Naive Bayes is counting how many times each of the d features co-occurs within each class.

Now, consider the training runtime. For all n training points, Naive Bayes will look at the posterior and prior probabilities over all d features, for all k classes. This will take $O(nkd)$ total runtime since the operations boil down to a series of counts. The space complexity is just $O(kd)$ to store the probabilities needed to compute results for new data points.

As another example, consider logistic regression. Recall that we need to calculate the following:

$$S(x) = \frac{1}{1 + e^{-X\beta}}$$

for any given x. There are n training data points (each with dimension d); hence, β is a d-by-1 vector of weights. Recall that the goal of logistic regression is to find the optimal decision boundary to split the data into two classes. This involves multiplying each of the n training points with β, which is d-by-1 vector, so the training runtime complexity is $O(d)$. The space complexity is just $O(d)$ to store the weights (β) to classify new data points.

Data Structures

Below is a brief overview of the most common data structures used for coding interviews. The best way to become familiar with each data structure is by implementing a basic version of it in your favorite language. Knowing the Big-O for common operations, like inserting an element or finding an element within the structure, is also essential. The table below can be used for reference:

Data Structure	Time Complexity								Space Complexity
	Average				Worst				Worst
	Access	Search	Insertion	Deletion	Access	Search	Insertion	Deletion	
Array	0(1)	0(n)	0(n)	0(n)	0(1)	0(n)	0(n)	0(n)	0(n)
Stack	0(n)	0(n)	0(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Queue	0(n)	0(n)	0(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Linked List	0(n)	0(n)	0(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Hash Map	N/A	0(1)	0(1)	0(1)	N/A	0(n)	0(n)	0(n)	0(n)
Binary Search Tree	0(log (n))	0(log (n))	0(log (n))	0(log (n))	0(n)	0(n)	0(n)	0(n)	0(n)

Arrays

An array is a series of consecutive elements stored sequentially in memory. Arrays are optimal for accessing elements at particular indices, with an O(1) access and index time. However, they are slower for searching and deleting a specific value, with an O(N) runtime, unless sorted. An array's simplicity makes it one of the most commonly used data structures during coding interviews.

Common array interview questions include:

- Moving all the negative elements to one side of an array
- Merging two sorted arrays
- Finding specific sub-sequences of integers within the array, such as the longest consecutive subsequence or the consecutive subsequence with the largest sum

A frequent pattern for array interview questions is the existence of a straightforward brute-force solution that uses O(n) space, and a more clever solution that uses the array itself to lower the space complexity down to O(1). Another pattern we've seen when dealing with arrays is the prevalence of off-by-1 errors — it's easy to crash the program by accidentally reading past the last element of an array.

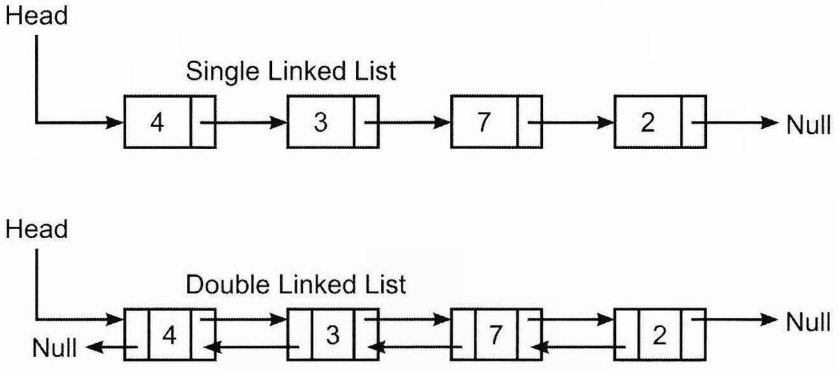
For jobs where Python knowledge is important, interviews may cover list comprehensions, due to their expressiveness and ubiquity in codebases. As an example, below, we use a list comprehension to create a list of the first 10 positive even numbers. Then, we use another list comprehension to find the cumulative sum of the first list:

```
a = [x*2 for x in range(1, 11)] # list creation
c = [sum(a[:x]) for x in range(len(a)+1)] # cumulative sum
```

Arrays are also at the core of linear algebra since vectors are represented as 1-D arrays, and matrices are represented by 2-D arrays. For example, in machine learning, the feature matrix X can be represented by a 2-D array, with one dimension as the number of data points (n) and the other as the number of features (d).

Linked Lists

A linked list is composed of nodes with data that have pointers to other nodes. The first node is called the head, and the last node is called the tail. Linked lists can be circular, where the tail points to the head. They can also be doubly linked, where each node has a reference to both the previous and next nodes. Linked lists are optimal for insertion and deletion, with O(1) insertion time at the head or tail, but are worse for indexing and searching, with a runtime complexity of O(N) for indexing and O(N) for search.



Common linked list questions include:

- Reversing a linked list
- Detecting a cycle in a linked list
- Removing duplicates from a sorted linked list
- Checking if a linked list represents a palindrome

As an example, below we reverse a linked list. Said another way, given the input linked list $4 \rightarrow 1 \rightarrow 3 \rightarrow 2$, we want to write a function which returns $2 \rightarrow 3 \rightarrow 1 \rightarrow 4$. To implement this, we first start with a basic node class:

```
class Node:
  def __init__(self, val):
    self.val = val
    self.next = None
```

Then we create the LinkedList class, along with the method to reverse its elements. The reverse function iterates through each node of the linked list. At each step, it does a series of swaps between the pointers of the current node and its neighbors.

```
class LinkedList:
  def __init__(self):
    self.head = None

  def reverse(self):
    prev = None
```

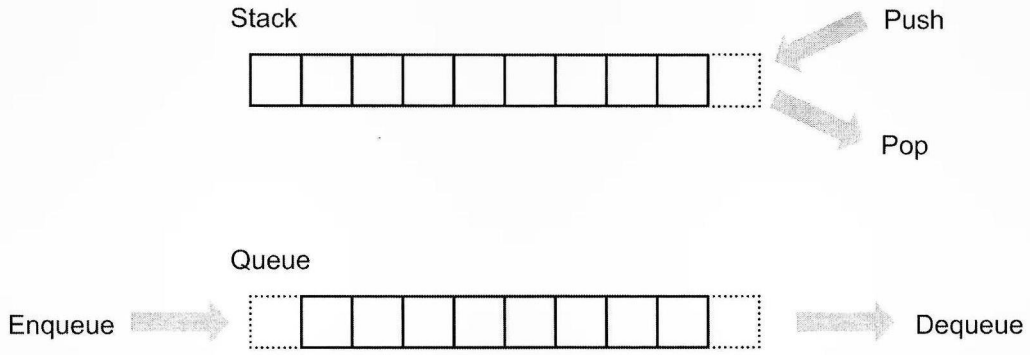
```
curr = self.head
while curr:
  next = curr.next
  curr.next = prev
  prev = curr
  curr = next
self.head = prev
```

Like array interview questions, linked list problems often have an obvious brute-force solution that uses O(n) space, but then also a more clever solution that utilizes the existing list nodes to reduce the memory usage to O(1). Another commonality between array and linked list interview solutions is the prevalence of off-by-one errors. In the linked list case, it's easy to mishandle pointers for the head or tail nodes.

Stacks & Queues

A stack is a data structure that allows adding and removing elements in a last-in, first-out (LIFO) order. This means the element that is added last is the first element to be removed. Another name for adding and removing elements from a stack is pushing and popping. Stacks are often implemented using an array or linked list.

A queue is a data structure that allows adding and removing elements in a first-in, first-out (FIFO) order. Queues are also typically implemented using an array or linked list.



The main difference between a stack and a queue is the removal order: in the stack, there is a LIFO order, whereas in a queue it's a FIFO order. Stacks are generally used in recursive operations, whereas queues are used in more iterative processes.

Common stacks and queues interview questions include:

- Writing a parser to evaluate regular expressions (regex)
- Evaluating a math formula using order of operations rules
- Running a breadth-first or depth-first search through a graph

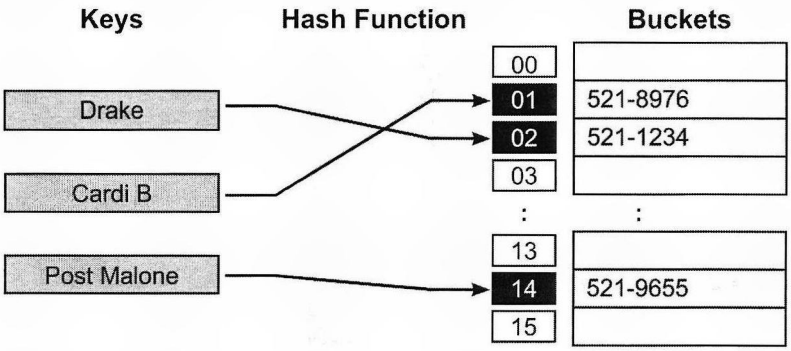
An example interview problem that uses a stack is determining whether a string has balanced parentheses. Balanced, in this case, means every type of left-side parentheses is accompanied by valid right-side parentheses. For instance, the string "({}(){})" is correctly balanced, whereas the string "{}({)" is not balanced, due to the last character, ')'. The algorithm steps are as follows:

1) Starting parentheses (left-sided ones) are pushed onto the stack.
2) Ending parentheses (right-sided ones) are verified to see if they are of the same type as the most recently seen left-side parentheses on the stack.
3) If the parentheses are of the same type, pop from the stack. If they don't match, return false since the parentheses are mismatched.
4) Continue parsing the input until it's completely processed, and the stack is empty (every pair of parentheses was correctly accounted for), in which case, return true. Or, if the stack is not empty, in which case, return false.

```
def check_balance(s):
  left_side = ["(", "{", "["] # left parentheses
  right_side = [")", "}", "]"] # right parentheses
  stack = [] # stack
  for i in s:
    if i in left_side:
      stack.append(i) # push onto stack
    elif i in right_side:
      pos = right_side.index(i) # get right char
      # check match
      if len(stack) == 0 or (left_side[pos] != stack[len(stack)-1]):
        return False
      else:
        stack.pop() # remove and continue
    if len(stack) == 0: # balanced
      return True
    else:
      return False
```

Hash Maps

A hash map stores key-value pairs. For every key, a hash map uses a hash function to compute an index, which locates the bucket where that key's corresponding value is stored. In Python, a dictionary offers support for key-value pairs and has the same functionality as a hash map.



While a hash function aims to map each key to a unique index, there will sometimes be "collisions" where different keys have the same index. In general, when you use a good hash function, expect the elements to be distributed evenly throughout the hash map. Hence, lookups, insertions, or deletions for a key take constant time.

Due to their optimal runtime properties, hash maps make a frequent appearance in coding interview questions.

Common hash map questions center around:

- Finding the unions or intersection of two lists
- Finding the frequency of each word in a piece of text
- Finding four elements a, b, c and d in a list such that $a + b = c + d$

An example interview question that uses a hash map is determining whether an array contains two elements that sum up to some value. For instance, say we have a list [3, 1, 4, 2, 6, 9] and k. In this case, we return true since 2 and 9 sum up to 11.

The brute-force method to solving this problem is to use a double for-loop and sum up every pair of numbers in the array, which provides an O(N^2) solution. But, by using a hash map, we only have to iterate through the array with a single for-loop. For each element in loop, we'd check whether the complement of the number (target - that number) exists in the hash map, achieving an O(N) solution:

```
def check_sum(a, target):
  d = {} # create dictionary
  for i in a:
    if (target - i) in d: # check hashmap
      return True
    else:
      d[i] = i # add to hashmap
  return False
```

Due to a hash function's ability to efficiently index and map data, hashing functions are used in many real-world applications (in particular, with regards to information retrieval and storage). For example, say we need to spread data across many databases to allow for data to be stored and queried efficiently while distributed. Sharding, covered in depth in the databases chapter, is one way to split the data. Sharding is commonly implemented by taking the given input data, and then applying a hash function to determine which specific database shard the data should reside on.

Trees

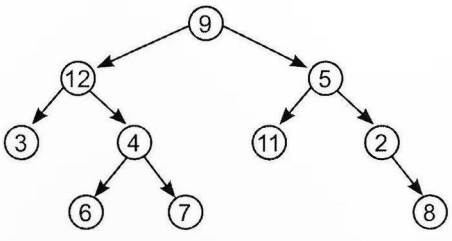
A tree is a basic data structure with a root node and subtrees of children nodes. The most basic type of tree is a binary tree, where each node has at most two children nodes. Binary trees can be implemented with a left and right child node, like below:

```
class TreeNode:
  def __init__(self, val):
    self.val = val
    self.left = None
    self.right = None
```

There are various types of traversals and basic operations that can be performed on trees. For example, in an in-order traversal, we first process the left subtree of a node, then process the current node, and, finally, process the right subtree:

```
def inorder(node):
    if node is None:
        return []
    else:
        return inorder(node.left) + [node.val] + inorder(node.right)
```

The two other closely related traversals are post-order traversal and pre-order traversal. A simple way to remember how these three algorithms work is by remembering that the "post/pre/in" refers to the placement of the processing of the root value. Hence, a post-order traversal processes the left child node first, then the right child node and, in the end, the root node. A pre-order traversal processes the root node first, then the left child node, and then, the right child node.



Level-order Tree Traversal:
9, 12, 5, 3, 4, 11, 2, 6, 7, 8

In-order Tree Traversal:
3, 12, 6, 4, 7, 9, 11, 5, 2, 8

Pre-order Tree Traversal:
9, 12, 3, 4, 6, 7, 5, 11, 2, 8

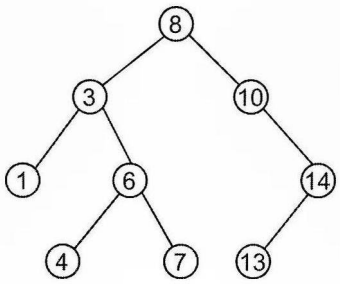
Post-order Tree Traversal:
3, 6,7, 4, 12, 11, 8, 2, 5, 9

For searching, insertion, and deletion, the worst-case runtime for a binary tree is O(N), where N is the number of nodes in the tree.

Common tree questions involve writing functions to get various properties of a tree, like the depth of a tree or the number of leaves in a tree. Oftentimes, tree questions boil down to traversing through the tree and recursively passing some data in a top-down or a bottom-up manner. Coding interview problems also often focus on two specific types of trees: Binary Search Trees and Heaps.

Binary Search Trees

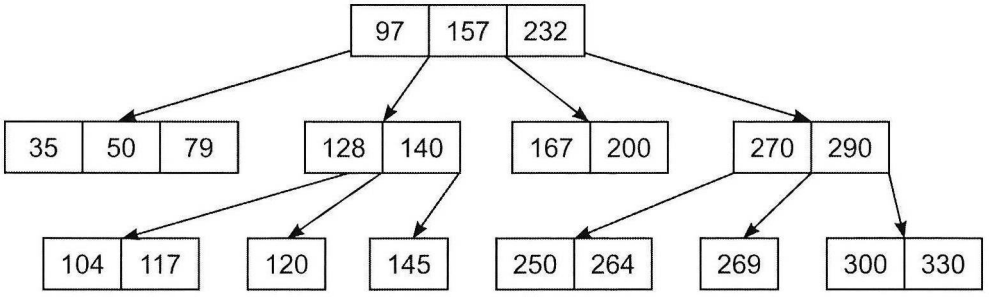
A binary search tree (BST) is composed of a series of nodes, where any node in a left subtree is smaller than or equal to the root, and any node in the right subtree is larger than or equal to the root. When BSTs are height balanced so no one leaf is much deeper than another leaf from the root, searching for elements becomes efficient. To demonstrate, consider searching for the value 9 in the balanced BST below:



Example of a Binary Search Tree

To find 9, we first examine the root value, 8. Since 9 is greater than 8, the node containing 9, if it exists, would have to be on the right side of the tree. Thus, we've cut the search space in half. Next, we compare against the node 10. Since 9 is less than 10, the node, should it exist, has to be on the left of 10. Again, we've cut the search space in half. In conclusion, since 10 doesn't have a left child, we know 9 doesn't occur in the tree. By cutting the search space in half at each iteration, BSTs support search, insertion, and deletion in O(log N) runtime.

Because of their lookup efficiency, BSTs show up frequently not just in coding interviews but in real-life applications. For instance, B-trees, which are used universally in database indexing, are a generalized version of BSTs. That is, they allow for more than 2 nodes (up to M children), but offer a searching and insertion process similar to that of BST. These properties allow B-trees to have O(log N) lookup and insertion runtimes similar to that of BSTs, where N is the total number of nodes in the B-tree. Because of the logarithmic growth of the tree depth, database indexes with millions of records often only have a B-tree depth of four or five layers.



Example of a B-Tree

Common BST questions cover:

- Testing if a binary tree has the BST property
- Finding the k-th largest element in a BST
- Finding the lowest common ancestor between two nodes (the closest common node to two input nodes such that both input nodes are descendants of that node)

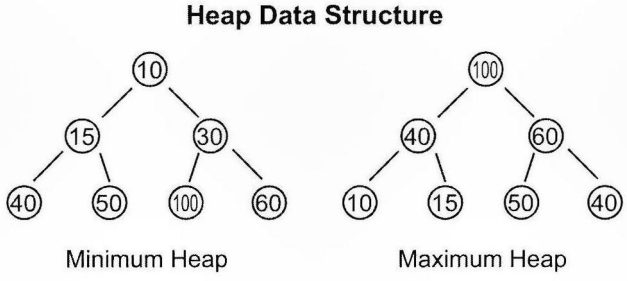
An example implementation of a BST using the TreeNode class, with an insert function, is as follows:

```
class TreeNode:
    def __init__(self, val):
        self.val = val
        self.left = None
        self.right = None

class BST:
    def __init__(self, val):
        self.root = TreeNode(val)

    def insert(self, node, val):
        if node is not None:
            if val < node.val:
                if node.left is None:
```

```
            node.left = TreeNode(val)
        else:
            self.insert(node.left, val)
    else:
        if node.right is None:
            node.right = TreeNode(val)
        else:
            self.insert(node.right, val)
else:
    self.root = TreeNode(val)
return
```

Heaps

Another common tree data structure is a heap. A max-heap is a type of heap where each parent node is greater than or equal to any child node. As such, the largest value in a max-heap is the root value of the tree, which can be looked up in O(1) time. Similarly, for a min-heap, each parent node is smaller than or equal to any child node, and the smallest value lies at the root of the tree and can be accessed in constant time.



To maintain the heap property, there is a sequence of operations known as "heapify," whereby values are "bubbled up/down" within the tree based on what value is being inserted or deleted. For example, say we are inserting a new value into a min-heap. This value starts at the bottom of the heap and then is swapped with its parent node ("bubbled up") until it is no longer smaller than its parent (in the case of a min-heap). The runtime of this heapify operation is the height of the tree, O(log N).

In terms of runtime, inserting or deleting is O(log N), because the heapify operation runs to maintain the heap property. The search runtime is O(N) since every node may need to be checked in the worst-case scenario. As mentioned earlier, heaps are optimal for accessing the min or max value because they are at the root, i.e., O(1) lookup time. Thus, consider using heaps when you care mostly about finding the min or max value and don't need fast lookups or deletes of arbitrary elements.

Commonly asked heap interview questions include:

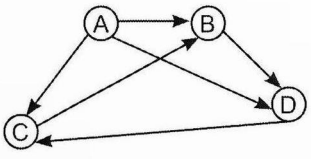
- finding the K largest or smallest elements within an array
- finding the current median value in a stream of numbers
- sorting an almost-sorted array (where elements are just a few places off from their correct spot)

To demonstrate the use of heaps, below we find the k-largest elements in a list, using the heapq package in Python:

```
import heapq
k = 5
a = [13, 5, 2, 6, 10, 9, 7, 4, 3]
heapq.heapify(a) # creates heap
heapq.nlargest(k, a) # finds k-largest
```

Graphs

Graphs are composed of nodes (vertices) and a set of edges that connect the nodes. Edges are often represented in two ways: an adjacency matrix or an adjacency list. In the case of an adjacency matrix, there is a *n*-by-*n* matrix for *n*-nodes where the entries are either Boolean values (denoting an edge is present or not between two nodes) or a weight. In contrast, an adjacency list stores a list of neighbors for each node.



To Graph

	A	B	C	D
A	0	1	1	1
B	0	0	0	1
C	0	1	0	0
D	0	0	1	0

To Adjacency Matrix

Node	Adjacent Node(s)
A	B C D
B	D
C	B
D	C

To Adjacency List

The lookup time to check whether two nodes are neighbors for an adjacency matrix is O(1). For an adjacency list, it could be O(*N*) in the worst case (if a node has edges to every other node). What an adjacency list lacks in time efficiency compared to an adjacency matrix, it makes up for in space efficiency. For a large graph with *n* nodes that are sparse (the *N* nodes don't have many connections to each other), an adjacency list offers a compact way to represent the edges, versus an adjacency matrix which requires you to store an *N*^2 sized matrix in memory. For instance, consider the Facebook friendship graph, which has 2 billion users (nodes), but a maximum of only 5,000 connections per node (the 5,000 friend limit). A full adjacency matrix would need to be 2 billion rows by 2 billion columns, whereas an adjacency list would require considerably less memory.

An example implementation of a graph is below. The Vertex class uses an adjacency list to keep track of its neighbors with weights:

```
class Vertex:
    def __init__(self, val):
        self.val = val
        self.neighbors = {}

    def add_to_neighbors(self, neighbor, w): # add to neighbor dict with weight
        self.neighbors[neighbor] = w

    def get_neighbors(self):
        return self.neighbors.keys()
```

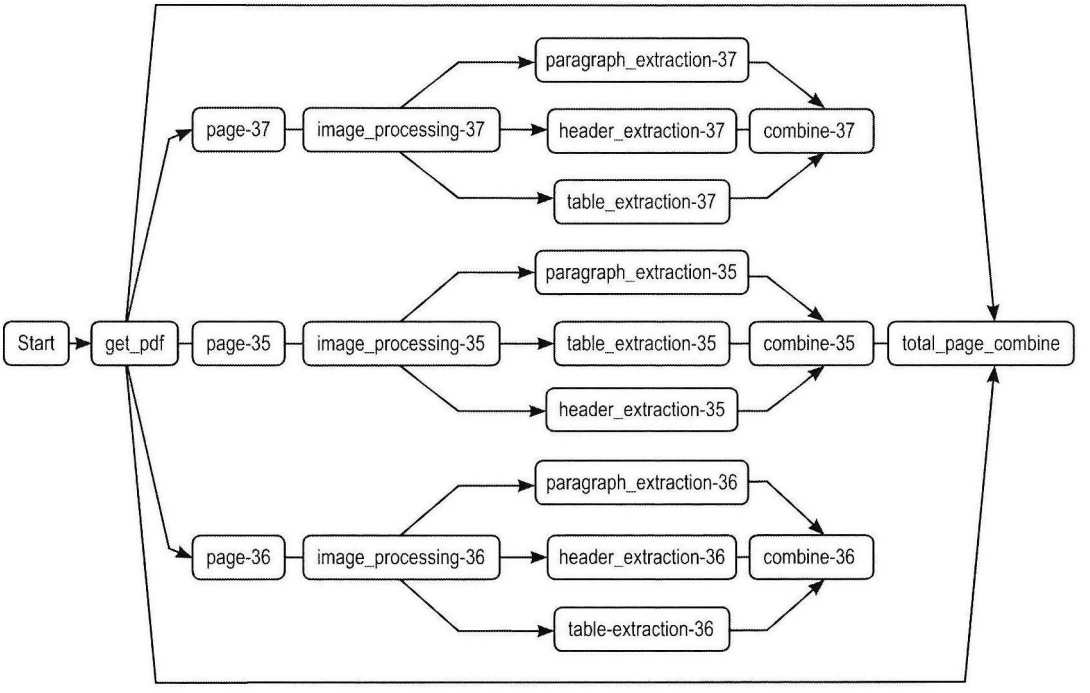
```
class Graph:
    def __init__(self):
        self.vertices = {}

    def add_vertex(self, val):
        new_vertex = Vertex(val)
        self.vertices[val] = new_vertex
        return new_vertex

    def add_edge(self, u, v, weight): # add edge from u to v
        if u not in self.vertices:
            self.add_vertex(Vertex(u))
        if v not in self.vertices:
            self.add_vertex(Vertex(v))
        self.vertices[u].add_to_neighbors(self.vertices[v], weight)

    def get_vertex(self, val):
        return self.vertices[val]

    def get_vertices(self):
        return self.vertices.keys()

    def __iter__(self):
        return iter(self.vertices)
```

Real-World Applications of Graphs

Graphs serve as a prevalent data structure for representing many real-world problems. For example, PageRank, developed by Google co-founders Larry Page and Sergey Brin, is an algorithm that measures how important various web pages are. PageRank represents each web page as a node, and each hyperlink from one page to another represents an edge. The underlying assumption is that important web pages are more likely to have been linked to by other influential pages on the web. In the context of graphs, this means that nodes with a high number of edges from other high-quality nodes are likely to have a better PageRank score.

Another real-world use of graphs is for computational graphs used within scheduling tools like Airflow or large-scale data processing frameworks like Spark or Tensorflow. These frameworks represent the series of computations and data flows that need to be performed as a directed acyclic graph (DAG), which is a directed graph (meaning edges are unidirectional) that has no cycles. Nodes represent operations that create or manipulate data, and edges represent how data (typically multidimensional arrays also known as tensors) must flow from one operation (node) to another.

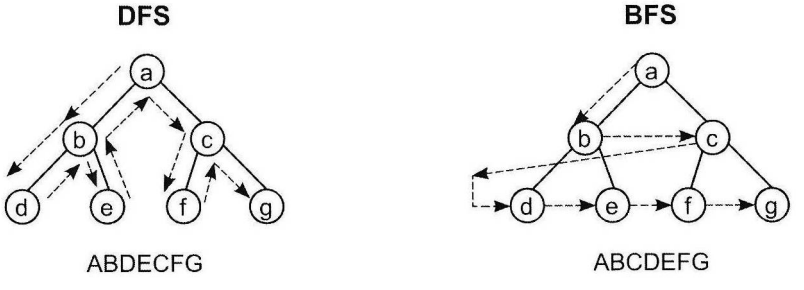
These computational graphs enable parallelism, since it's easy to order the operations in such a way that certain operations that do not depend on one another can be run concurrently. Another advantage is that the computational graph offers a language-agnostic representation of the computations we want, which can easily be ported between underlying frameworks.



An example Airflow execution graph to extract a data table from a large PDF file using image processing

Breadth-First and Depth-First Search

Interview problems related to graphs often involve traversing the graph, either in a breadth-first-search (BFS) or in a depth-first-search (DFS). In BFS, start with one node and add it to a queue. For each node in the queue, process the node and add its neighbors to the queue. In DFS, you also start with one node, but instead of processing all the neighbors next, you recursively process each neighbor one by one.



Below is an example of BFS using a queue for iterative processing:

```
def bfs(graph, v):
    n = len(graph.vertices)
    visited = [False] * (n+1)
    queue = []
```

```
    queue.append(v)
    visited[v] = True

    while queue:
        curr = queue.pop(0)
        for i in graph.get_vertex(curr):
            if visited[i.val] == False:
                queue.append(i.val)
                    visited[i.val] = True
    return visited
```

Below is a primitive example of DFS, where *visited* tracks the set of processed nodes, and *v* is the node currently being processed:

```
def dfs_helper(graph, v, visited):
    visited.add(v)
    for neighbor in graph.get_vertex(v).get_neighbors():
        if neighbor.val not in visited:
            dfs_helper(graph, neighbor.val, visited)
    return visited


def dfs(graph, v):
    visited = set()
    return dfs_helper(graph, v, visited)
```

The runtime for both is $O(E + V)$, where E is the number of edges in the graph and V is the number of vertices. For most basic use cases, DFS and BFS are interchangeable. However, there are some differences in their use cases and advantages.

When interview problems concern optimization, such as finding the shortest path between nodes, consider using BFS. It can find solutions definitively and never gets trapped in recursive sub-calls. Some drawbacks to BFS are that it uses more memory when storing nodes and can take a lot of time if solutions are far away from the root.

When interview problems concern analyzing a graph's structure, such as when finding cycles or orderings in directed graphs (for example, topological sorting), consider using DFS. Because it exhausts paths before trying others, it may be trapped recursively and cannot guarantee a solution. However, DFS has fewer memory requirements and can find long-distance elements in a shorter amount of time compared to BFS.

Algorithms

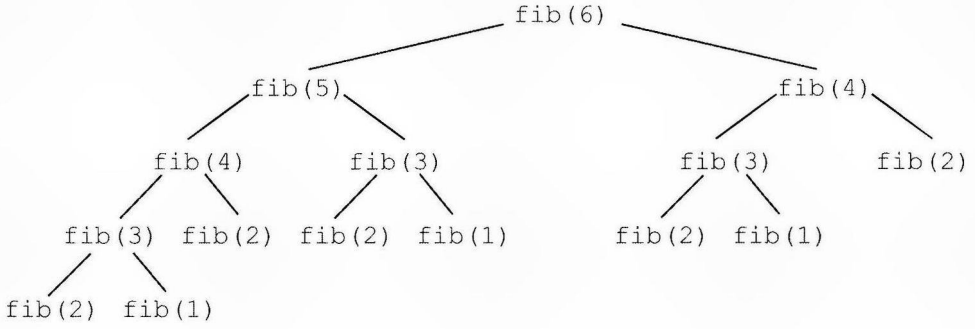
Recursion

A recursive function is one that calls itself directly or indirectly. It can be broken down into two parts: the recursive case (the part that calls itself) and the base case (which terminates the recursion).

For an example of a recursive algorithm, consider the classic problem of producing Fibonacci numbers (0, 1, 1, 2, 3, 5, 8, etc.):

```
def fib(n):
    if n == 0:
        return 0
    if n == 1 or n == 2:
        return 1
    else:
        return fib(n-1)+fib(n-2)
```

If you draw the tree of recursive calls, you'll see that many times fib(n) for a particular n is called repeatedly. This repetition leads to an undesirable exponential runtime and memory usage. Since recursive calls are stored on a stack, and often, there is limited allocated memory for a program, recursive solutions can also lead to stack overflows which crash the program.



In contrast, iterative algorithms do not overwhelm the call stack. Such algorithms rely on using a for-loop or while-loop, rather than calling themselves repeatedly like in recursion. An iterative example of the Fibonacci sequence is as follows: we use a for-loop to iterate up to n, set the current result to be the sum of the previous Fibonacci number at indexes n-2 and n-1, and continue this until we reach the desired n:

```
def fib(n):
    if n == 0:
        return 0
    if n == 1 or n == 2:
        return 1
    else:
        prev2 = 0 # fib(n-2)
        prev = 1 # fib(n-1)
        res = 0 # fi1b(n)
        for i in range(1, n): # process iteratively
            res = prev2 + prev
            prev2 = prev
            prev = res
        return res
```

Compared to the recursive solution, the iterative approach is less expressive code-wise but more memory efficient since there aren't O(N) recursive calls, but instead just two variables to keep track of (the most recent two Fibonacci numbers).

Greedy Algorithms

Greedy algorithms choose at each step what seems to be the best option. In other words, greedy problems reduce problems into smaller ones by making the locally optimal choice at each step.

A classic example where a greedy algorithm works well is when making change with the minimum number of coins. Say, for instance, you wanted to make change for 67 cents, and all you had were pennies, nickels, dimes, and quarters. The greedy approach is to use as many coins of the highest denomination as possible (two quarters), before continuing to the next denomination (one dime). The code is as follows:

```
def minCoins(k):
    coins = [1, 5, 10, 25]
    n = len(coins)
    res = []
    i = n-1
    while(i >= 0 and k >= 0):
        if k >= coins[i]:
            k -= coins[i]
            res.append(coins[i])
        else:
            i -= 1
    return res
```

In the real world, greedy algorithms show up in various areas of machine learning. For example, decision trees are split in a greedy manner to maximize information gain (reduction in entropy based on the feature chosen) at each split. For every feature, we evaluate all possible features, then, in a greedy fashion, choose the feature with the best information gain to split on next. This process takes place recursively until we end up with leaf nodes. The pseudocode for getting the best feature is as follows:

```
info_gains = [getInfoGain(feature) for feature in features]
best_feature_index = np.argmax(info_gains)
best_feature = features[best_feature_index]
```

For interview questions, keep greedy algorithms in mind for optimization problems, where there's an obvious set of choices to select from, and it's easy to know what the appropriate choice is. Keep in mind that it's often easier to reason about a greedy algorithm recursively, but then implement it later iteratively for better memory performance.

Dynamic Programming

Earlier in the recursion section, we saw how an iterative solution to generating the n-th Fibonacci number had its advantage over a recursive version since it used less memory and didn't recompute

work. That's where dynamic programming, the art of storing results of subproblems to speed up recursion, comes in. Although, technically, the iterative solution serves as a form of dynamic programming, to be more explicit, for pedagogical purposes, below we show how an array can be used to cache existing results. Now, instead of making the previous recursive calls all the way down, we get the stored sub-result from the cache to prevent values from being recomputed.

```
dp = [0] * 1000

def fib(n):
    if n == 0 or n == 1:
        return n
    else:
        dp[n] = fib(n-1) + fib(n-2)
        return dp[n]
```

Two properties are needed for dynamic programming (DP) to be applicable:

1) optimal substructure
2) overlapping subproblems

An optimal substructure means the problem can be solved optimally by breaking it down into subproblems and solving those subproblems. Overlapping subproblems indicates the problem can be broken down into subproblems, that are then reusable in other subproblems. If both are present, then calculating and storing the solutions to subproblems in a recursive manner will solve the overall problem. Note that the Fibonacci satisfies the two requirements because:

1) Fib(n) is found by solving the subproblems fib(n-1) and fib(n-2)
2) Subproblems are overlapping: both fib(n) and fib(n-1) require having solved fib(n-2)

Dynamic programming can be implemented with a top-down or bottom-up approach. In a top-down approach, we start with the top and break the problem into smaller chunks (as in the Fibonacci example). In practice, the top-down approaches often are implemented with recursion, and the intermediate results are cached in a hash table.

In contrast, in a bottom-up manner, we start with the smaller problems and continue to the top. These solutions are often implemented iteratively, where an n-dimensional array is used to cache previous results. A bottom-up Fibonacci example would be:

```
def fib(n):
    dp = [0 for _ in range(n+1)]
    dp[1] = 1
    for i in range(2, n+1):
        dp[i] = dp[i-1] + dp[i-2]
    return dp[n]
```

Dynamic programming isn't just an academic exercise or coding interview favorite — it often shows up in the real world, too. For instance, in reinforcement learning, the goal is to understand what

actions to take given a particular state of the universe to maximize an expected eventual payoff. The famous Bellman equations are a core part of the reinforcement learning process and utilize the dynamic programming approach. The Bellman equations break down the total eventual payoff into a series of smaller subproblems that can each be optimized, where the best eventual payoff comes from combining different subproblem payoffs.

Greedy Algorithms Vs. Dynamic Programming

As explained earlier, a greedy algorithm does whatever is locally optimal and hence always chooses the option that leads to the best result for the next step. This contrasts with dynamic programming, which will exhaust the search space and is guaranteed to find the globally optimal solution, not just the locally optimal solution. Therefore, you can think of greedy algorithms as getting the "local maximum/minimum," but not necessarily the "global maximum/minimum" that dynamic programming achieves.

For a concrete example of where greedy algorithms can fall short, consider the classic 0/1 knapsack problem with the values below. In this problem, we are choosing between N items, each with some positive value and positive weight. We want to maximize the amount of value obtained by selecting items with a total weight of no more than W.

Example Knapsack

Aa Item	Value	Weight	Value/Weight
A	3	4	3/4
B	1.2	2	1.2/2
C	2	3	2/3

The greedy implementation is to just sort the elements that maximize value per unit weight and choose the most efficient items.

```
def knapsackGreedy(values, weights, max_weight):
    curr_weight = 0
    value_list, weight_list = ([], []) # values and weights
    total_sorted = sorted(zip(values, weights),
                          key=lambda x: x[0]/x[1], reverse=True)
    values_sorted, weights_sorted = zip(*total_sorted)
    # iterate in sorted fashion
    for value, weight in zip(values_sorted, weights_sorted):
        if weight + curr_weight <= max_weight: # meets weight requirement
            value_list.append(value)
            weight_list.append(value)
            curr_weight += weight
    return sum(value_list)
```

However, this does not maximize the amount of value obtained overall. Take, for example, the table of weights and values provided earlier, along with a total weight limit of 5.

Greedily, we would select item A (the highest value/weight ratio) and end up with a total value of 3, since the weight constraint does not allow for extra items. However, the best possible method would be to take B and C for a total value of 3.2, which is larger than 3.

In contrast, a dynamic programming approach achieves the global maximum. An example of a bottom-up approach is below, where the core logic is in deciding whether to take an item or not. If an item meets the weight requirement, then we either

- Take the item, and get a new optimal maximum value, which now gives us a new weight constraint (current weight minus item's weight), or
- Do not take the item, and continue with the current optimal maximum value and the current weight constraint.

```
def knapsackDP(values, weights, max_weight):
    n = len(values)
    dp = [[0 for x in range(max_weight+1)] for x in range(n+1)] # 2d-array

    for i in range(n+1):
        for w in range(max_weight+1):
            if weights[i-1] <= w: # meets weight requirement
                dp[i][w] = max(values[i-1]+dp[i-1][w-weights[i-1]],
                               dp[i-1][w]) # either take value or not
            else:
                dp[i][w] = dp[i-1][w] # not taking value
    return dp[n][max_weight]
```

The dynamic programming approach coded above leads to the optimal solution of taking items B and C for a total value of 3.2. When solving coding interview problems, it's crucial to not fall for a more apparent greedy approach when the correct answer is found through dynamic programming.

Sorting

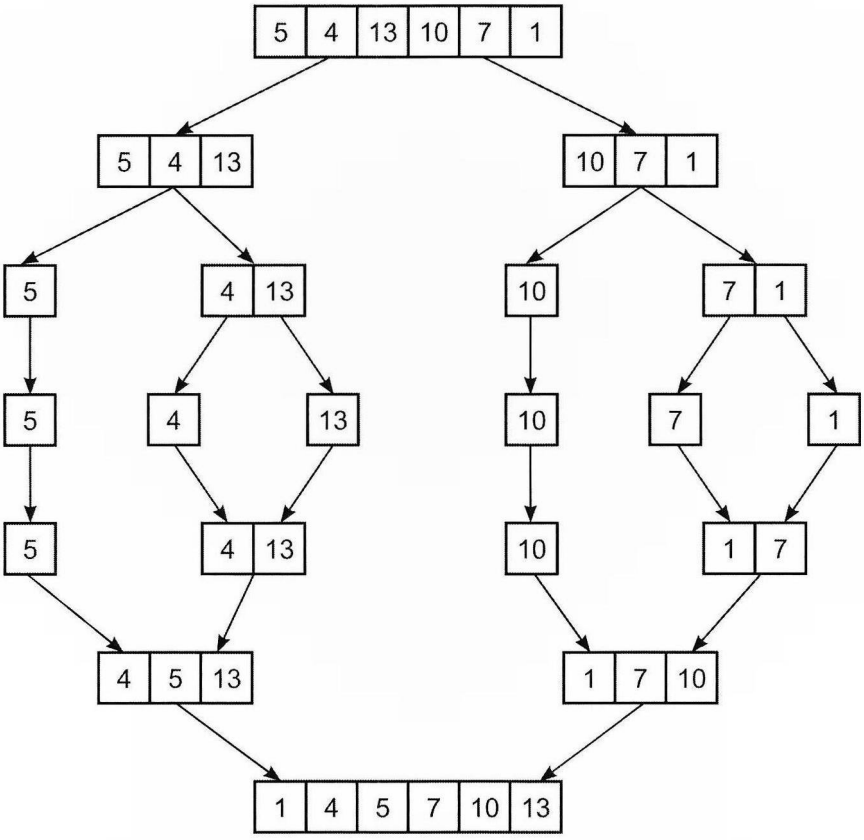
Two unique sorting algorithms — mergesort and quicksort — arise in coding interviews from time to time. While it's rare to be asked to code up these algorithms, both sorts are often modified to solve a problem or used as an intermediate step within a larger coding interview solution. This is because sorting the input can expose some structure, which makes the problem simpler.

Mergesort

Mergesort uses a "divide-and-conquer" approach as follows:

1. Repeatedly divide the input into smaller subarrays, until a base case of a single element is reached (this single element subarray is considered sorted).
2. Repeatedly merge the smaller sorted subarrays into bigger sorted subarrays, until the entire input is merged back together.

Below is an example of mergesort:



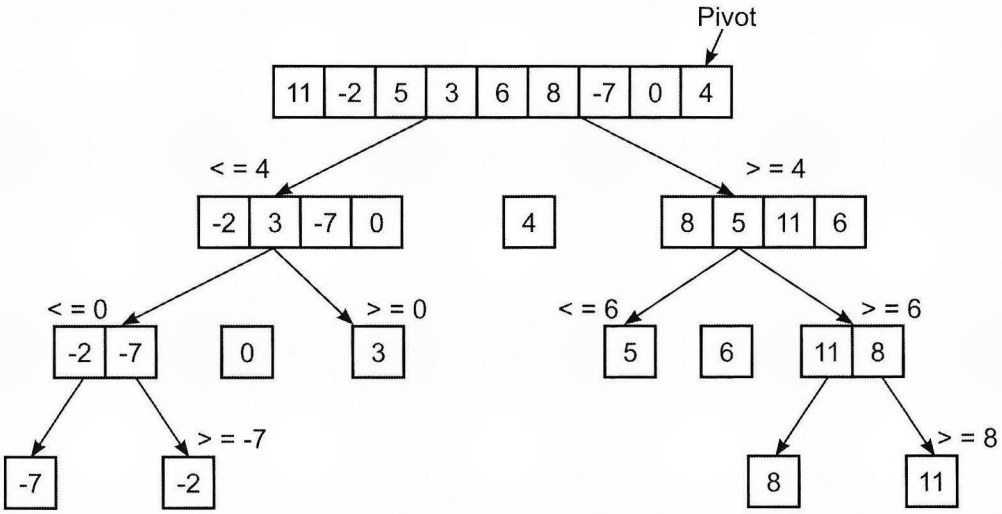
Overall, mergesort has a runtime complexity of O(N log N) and also requires O(N log N) space to support the auxiliary merging steps. An example implementation of mergesort that utilizes a helper function for merging subarrays is as follows:

```
def merge_helper(a, low, high, mid):
    if len(a) == 1:
        return a
    i = j = k = 0 # k is for merged array
    left = a[:mid] # copy of left side
    right = a[mid:] # copy of right side
    while i < len(left) and j < len(right): # compare left and right sides, add
        if left[i] < right[j]:
            a[k] = left[i]
            i += 1
        else:
            a[k] = right[j]
            j += 1
        k += 1
    while i < len(left): # remaining on left
        a[k] = left[i]
```

```
        i += 1
        k += 1
    while j < len(right): # remaining on right
        a[k] = right[j]
        j += 1
        k += 1
    return a

def mergesort(a, low, high):
    if low >= high:
        return a
    mid = (low + high-1) // 2
    mergesort(a, low, mid)
    mergesort(a, mid+1, high)
    merge_helper(a, low, high, len(a) // 2)
    return a
```

Quicksort

Quicksort selects an arbitrary pivot element and puts all elements smaller than the pivot to the left of the pivot, and larger elements to the right of the pivot. The exchanging of elements occurs through swaps. This process of selecting a pivot and swapping the left and right elements ensues recursively, until the base case is hit when just two elements are swapped into their correct relative order.



The worst-case runtime for quicksort is O(N^2), although, in practice the expected runtime is closer to O(N log N) through picking a "smart" pivot whereby the elements are roughly divided into equal halves upon each iteration.

Below is an example implementation of quicksort that utilizes a helper function to partition the array:

```
def helper(a, low, high):
    i = low-1 # smaller element index
    pivot = a[high] # pivot
    for j in range(low, high):
        if a[j] <= pivot:
            i += 1 # increment
            a[i], a[j] = a[j], a[i] # swap
    a[i+1], a[high] = a[high], a[i+1]
    return i+1

def quicksort(a, low, high):
    if len(a) == 1:
        return a
    if low < high:
        pivot = helper(a, low, high) # place pivot
        quicksort(a, low, pivot-1) # recurse left side
        quicksort(a, pivot+1, high) # recurse right side
    return a
```

Matrix Multiplication

Since all machine learning algorithms eventually reduce to a series of matrix multiplications, speeding up these operations is crucial for large-scale machine learning applications. Using the direct mathematical definition of matrix multiplication, an implementation of matrix multiplication gives an O(N^3) runtime, as seen by the three nested for-loops below:

```
def matrix_multiply(A, B):
    m = [[0 for row in range(len(B[0]))] for col in range(len(A))]
    for i in range(len(A)):
        for j in range(len(B[0])):
            for k in range(len(B)):
                m[i][j] = m[i][j] + A[i][k] * B[k][j] # add
    return m
```

Nevertheless, improving upon this runtime is possible. For example, we can break down the multiplication into a series of sub-matrix multiplications. These sub-matrix multiplications can be calculated in parallel, which enables the task to be accomplished in a distributed manner.

Finance companies like to ask candidates about coding up matrix multiplication or implementing other common linear algebra topics such as singular-value decomposition because it allows firms to test your mathematical understanding and programming ability at once. These questions are also picked because optimizing numerical calculations done in linear algebra is a relevant concept when trying to reduce the latency of trading systems.

Interview Questions

Easy Problems

9.1. Amazon: Given two arrays, write a function to get the intersection of the two. For example, if A = [1, 2, 3, 4, 5], and B = [0, 1, 3, 7] then you should return [1, 3].

9.2. D.E. Shaw: Given an integer array, return the maximum product of any three numbers in the array. For example, for A = [1, 3, 4, 5], you should return 60, while for B = [−2, −4, 5, 3] you should return 40.

9.3. Facebook: Given a list of coordinates, write a function to find the k closest points (measured by Euclidean distance) to the origin. For example, if $k = 3$, and the points are: [[2, −1], [3, 2], [4, 1], [−1, −1], [−2, 2]], then return [[−1, −1], [2, −1], [−2, 2]].

9.4. Google: Say you have an n-by-n matrix of elements that are sorted in ascending order both in the columns and rows of the matrix. Return the k-th smallest element of the matrix. For example, consider the matrix below:

$$\begin{bmatrix} 1 & 4 & 7 \\ 3 & 5 & 9 \\ 6 & 8 & 11 \end{bmatrix}$$

If $k = 4$, then return 5.

9.5. Akuna Capital: Given an integer array, find the sum of the largest contiguous subarray within the array. For example, if the input is [−1, −3, 5, −4, 3, −6, 9, 2], then return 11 (because of [9, 2]). Note that if all the elements are negative, you should return 0.

9.6. Facebook: Given a binary tree, write a function to determine whether the tree is a mirror image of itself. Two trees are a mirror image of each other if their root values are the same and the left subtree is a mirror image of the right subtree.

Medium Problems

9.7. Google: Given an array of positive integers, a peak element is greater than its neighbors. Write a function to find the index of any peak elements. For example, for [3, 5, 2, 4, 1], you should return either 1 or 3 because the values at those indexes, 5 and 4, are both peak elements.

9.8. AQR: Given two lists X and Y, return their correlation.

9.9. Amazon: Given a binary tree, write a function to determine the diameter of the tree, which is the longest path between any two nodes.

9.10. D.E. Shaw: Given a target number, generate a random sample of n integers that sum to that target that also are within σ standard deviations of the mean.

9.11. Facebook: You have the entire social graph of Facebook users, with nodes representing users and edges representing friendships between users. Given a social graph and two users as input, write a function to return the smallest number of friendships between the two users. For example, take the graph that consists of 5 users A, B, C, D, E, and the friendship edges are: (A, B), (A, C), (B, D), (D, E). If the two input users are A and E, then the function should return 3 since A is friends with B, B is friends with D, and D is friends with E.

9.12. LinkedIn: Given two strings A and B, write a function to return a list of all the start indices within A where the substring of A is an anagram of B. For example, if A = "abcdcbac" and

B = "abc," then you want to return [0, 4, 5] since those are the starting indices of substrings of A that are anagrams of B.

9.13. Yelp: You are given an array of intervals, where each interval is represented by a start time and an end time, such as [1, 3]. Determine the smallest number of intervals to remove from the list, such that the rest of the intervals do not overlap. Intervals can "touch," such as [1, 3] and [3, 5], but are not allowed to overlap, such as [1, 3] and [2, 5]). For example, if the input interval list given is: [[1, 3], [3, 5], [2, 4], [6, 8]], then return 1, since the interval [2, 4] should be removed.

9.14. Goldman Sachs: Given an array of strings, return a list of lists where each list contains the strings that are anagrams of one another. For example, if the input is ["abc", "abd", "cab", "bad", "bca", "acd"] then return: [["abc", "cab", "bca"], ["abd", "bad"], ["acd"]].

9.15. Two Sigma: Say that there are n people. If person A is friends with person B, and person B is friends with person C, then person A is considered an indirect friend of person C.

Define a friend group to be any group that is either directly or indirectly friends. Given an n-by-n adjacency matrix N, where N[i][j] is one if person i and person j are friends, and zero otherwise, write a function to determine how many friend groups exist.

9.16. Workday: Given a linked list, return the head of the same linked list but with k-th node from the end of a linked list removed. For example, given the linked list $3 \rightarrow 2 \rightarrow 5 \rightarrow 1 \rightarrow 4$ and $k = 3$, remove the 5 node and, thus, return the linked list $3 \rightarrow 2 \rightarrow 1 \rightarrow 4$.

9.17. Goldman Sachs: Estimate π using a Monte Carlo method. Hint: think about throwing darts on a square and seeing where they land within a circle.

9.18. Palantir: Given a string with lowercase letters and left and right parentheses, remove the minimum number of parentheses so the string is valid (every left parenthesis is correctly matched by a corresponding right parenthesis). For example, if the string is ")a(b((cd)e(f)g)" then return "ab((cd)e(f)g)".

9.19. Citadel: Given a list of one or more distinct integers, write a function to generate all permutations of those integers. For example, given the input [2, 3, 4], return the following 6 permutations: [2, 3, 4], [2, 4, 3], [3, 2, 4], [3, 4, 2], [4, 2, 3], [4, 3, 2].

9.20. Two Sigma: Given a list of several categories (for example, the strings A, B, C, and D), sample from the list of categories according to a particular relative weighting scheme. For example, say we give A a relative weight of 5, B a weight of 10, C a weight of 15, and D a weight of 20. How do we construct this sampling? How do you extend the solution to an arbitrarily large number of k categories?

9.21. Amazon: Given two arrays with integers, return the maximum length of a common subarray within both arrays. For example, if the two arrays are [1, 3, 5, 6, 7] and [2, 4, 3, 5, 6] then return 3, since the length of the maximum common subarray, [3, 5, 6], is 3.

9.22. Uber: Given a list of positive integers, return the maximum increasing subsequence sum. In other words, return the sum of the largest increasing subsequence within the input array. For example, if the input is [3, 2, 5, 7, 6], return 15 because it's the sum of 3, 5, 7. If the input is [5, 4, 3, 2, 1], return 5 (since no subsequence is increasing).

9.23. Palantir: Given a positive integer n, find the smallest number of perfect squares that sum up to n. For example, for $n = 7$, you should return 4, since $7 = 4 + 1 + 1 + 1$. For $n = 13$, you should return 2, since $13 = 9 + 4$.

9.24. Facebook: Given an integer n and an integer k, output a list of all of the combinations of k numbers chosen from 1 to n. For example, if $n = 3$ and $k = 2$, return [1, 2], [1, 3], [2, 3].

Hard Problems

9.25. Citadel: Given a string with left and right parentheses, write a function to determine the length of the longest well-formed substring. For example, if the input string is ")(())(," then return 4, since the longest well-formed substring is "(())."

9.26. Bloomberg: Given an m-by-n matrix with positive integers, determine the length of the longest path of increasing integers within the matrix. For example, consider the input matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

In this case, return 5, since one of the longest paths would be 1-2-5-6-9.

9.27. Google: Given a number n, return the number of lists of consecutive positive integers that sum up to n. For example, for $n = 9$, return 3, since the consecutive positive integer lists are: [2, 3, 4], [4, 5], and [9]. Can you solve this in linear time?

9.28. Citadel: Given a continuous stream of integers, write a class with functions to add new integers to the stream, and a function to calculate the median at any time.

9.29. Two Sigma: Given an input string and a regex, write a function that checks whether the regex matches the input string. The input string is composed of the lowercase letters a-z. The regular expression contains lowercase a-z, '?', or '*', where the '?' matches any one character, and the '*' matches an arbitrary number of characters (empty as well). For example, if the input string is "abcdba" and the regex is "a*c?*", return true. However, if the regex was instead "b*c?*" return false.

9.30. Citadel: A fire department wants to build a new fire station in a location where the total distance from the station to all houses in the town (in Euclidean terms) is minimized. Given a list of coordinates for the n houses, return the coordinates of the optimal location for the new fire station.

Solutions

Solution #9.1

The simplest way to check for intersecting elements of two lists is to use a doubly-nested for-loop to iterate over one array and check against every element in the other array. However, this leads to a time complexity of O(N*M) where N is the length of A, and M is the length of B.

A better approach is to use sets (which utilizes a hash map implementation underneath) since the runtime time is O(1) for each lookup operation. Then we can do the series of lookups over the larger set (resulting in a O(min(N, M)) total runtime):

```
def intersection(a, b):
    set_a = set(a)
    set_b = set(b)
```