The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. Cross reference to GARP FRM Part I Foundations of Risk Management, Chapter 5.

READING 5

Modern Portfolio Theory and Capital Asset Pricing Model

EXAM FOCUS

This reading introduces modern portfolio theory, the efficient frontier, and the capital market line. It then continues to discuss the security market line (SML), the calculation of beta, and the capital asset pricing model (CAPM). For the exam, it is important to have a firm grasp of the CAPM calculation. The reading concludes by reviewing some popular risk-adjusted measures of return, such as the Sharpe measure, the Treynor measure, Jensen's alpha, the information ratio, and the Sortino ratio. In general, all of these performance measures evaluate excess return over some form of risk. It would be beneficial to memorize these measures of performance because they are popular concepts on the exam.

MODULE 5.1: MODERN PORTFOLIO THEORY AND THE CAPITAL MARKET LINE

Modern Portfolio Theory

LO 5.a: Explain modern portfolio theory and interpret the Markowitz efficient frontier.

Due to abundance of market data, market risk has attracted significant interest from academics since the 1950s. As a result, numerous market risk models have since been developed. The criterion for a good market model is that it must have acceptable explanatory power without being unnecessarily complex.

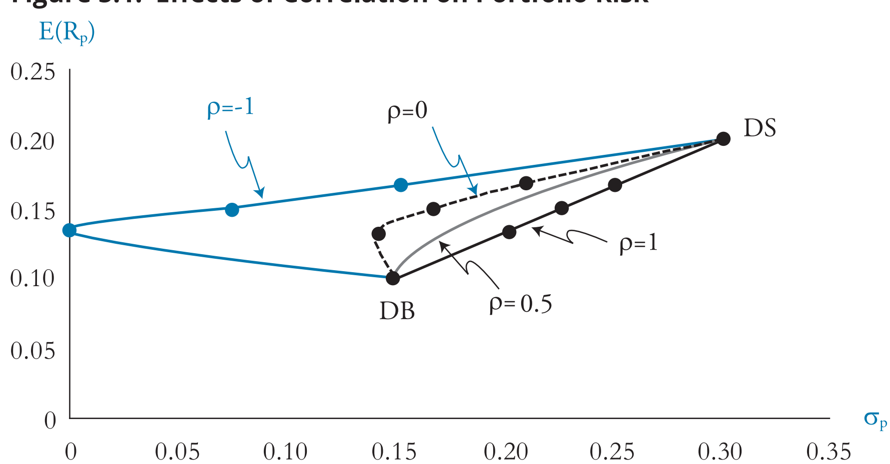
One of the most notable market risk researchers was Harry Markowitz. He laid the foundation for modern portfolio theory in the early 1950s. Markowitz's portfolio theory makes the following assumptions:

- *Returns are normally distributed*. This means that, when evaluating utility, investors only consider the mean and the variance of return distributions. They ignore deviations from normality, such as skewness or kurtosis (we will review those concepts in Book 2).
- *Investors are rational and risk-averse*. Markowitz defines a rational investor as someone who seeks to maximize utility from investments. Furthermore, when presented with two investment opportunities at the same level of expected risk, rational investors always pick the investment opportunity which offers the highest expected return.
- *Capital markets are perfect*. This implies that investors do not pay taxes or commissions. They have unrestricted access to all available information and perfect competition exists among the various market participants.

Because investors are risk-averse, they strive to minimize the risk of their portfolios for a given level of target return. This could be achieved by investing in multiple assets which are not perfectly correlated with each other (i.e., where their correlation coefficients, ρ, are less than 1).

While portfolio returns are calculated as weighted averages of individual asset returns, portfolio variances depend on the correlations among assets. A correlation of +1 offers no diversification benefits and results in portfolio variance being a weighted average of individual variances (solid black line DB-DS in Figure 5.1). When correlation is less than 1, diversification occurs and portfolio variance declines below the weighted average of individual variances. The lower the correlation, the greater the benefit becomes. With perfect negative correlation ($\rho = -1$), it is indeed possible to structure a portfolio with zero variance [i.e., a synthetic risk-free asset (y-intercept of blue curve in Figure 5.1)]. We will further explore the mathematics of covariance and correlation in Book 2.

Figure 5.1: Effects of Correlation on Portfolio Risk

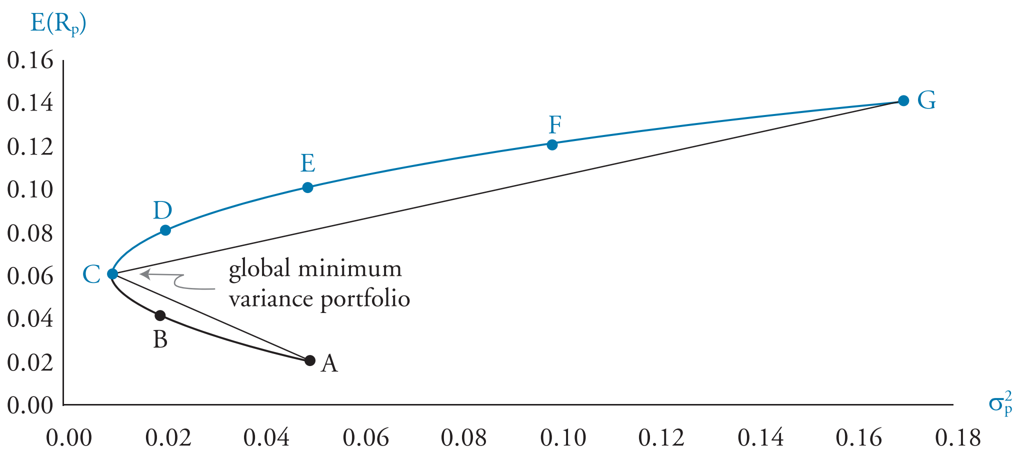


By holding a sufficiently large, diversified portfolio, investors are able to reduce, or even eliminate, the amount of company-specific (i.e., idiosyncratic) risk inherent in each individual security. Examples of company-specific risks include accounting fraud, cyber attacks, loss of key personnel, or any other issue which affects a specific company, without affecting the rest of the market. By holding a well-diversified portfolio, the importance of events affecting individual stocks in the portfolio is diminished, and the portfolio becomes mostly exposed to general market risk. It follows this pattern because when investors can diversify at low- or no-cost, they must not expect to receive compensation for unnecessary exposure to company-specific risk given that it's diversifiable. The compensation they receive must be exclusively determined by their exposure to market risk.

The Efficient Frontier

Rational investors maximize portfolio return per unit of risk. Plotting all those maximum returns for various risk levels produces the **efficient frontier**, which is represented by the blue curve passing through C-D-E-F-G, shown in Figure 5.2.

Figure 5.2: Efficient Frontier



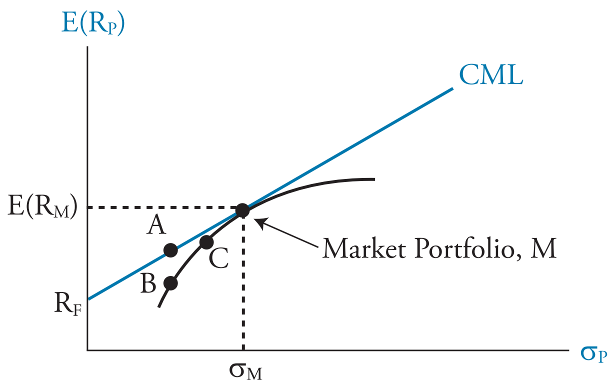
Point C is known as the **global minimum variance portfolio** because it is the efficient portfolio offering the smallest amount of total risk. Point C is, therefore, the leftmost point of the efficient frontier. Points A and B lie on the so-called **portfolio possibilities curve**, which is an extension of the efficient frontier below the global minimum variance portfolio, C. However, points A and B (or any other points below the efficient frontier) are considered inefficient because there is always a portfolio directly above them on the efficient frontier offering a higher return for the same amount of total risk. In general, any portfolio below the efficient frontier is, by definition, inefficient, whereas any portfolio above the efficient frontier is unattainable. In the absence of a risk-free asset, the only efficient portfolios are the portfolios on the efficient frontier. Investors choose their position on the efficient frontier depending on their relative risk aversion. A risk seeker may choose to hold Portfolio G whereas another investor seeking lower risk may choose to hold Portfolio D.

The Capital Market Line (CML)

LO 5.d: Interpret the capital market line.

So far in our analysis, we have only considered risky portfolios. The next step is to introduce a risk-free asset. A common proxy used for the risk-free asset is the U.S. Treasury bill (T-bill). Investors will combine the risk-free asset with a specific efficient portfolio that will maximize their risk-adjusted rate of return. Thus, investors obtain a line tangent to the efficient frontier whose y-intercept is the risk-free rate of return (as shown in Figure 5.3). Assuming investors have identical expectations regarding expected returns, variances/standard deviations, and covariances/correlations (i.e., homogenous expectations), there will only be one tangency line, which is referred to as the **capital market line (CML).**

Figure 5.3: Capital Market Line



Because it is assumed there is only one CML, it follows that there is only one tangency portfolio, which, by definition, becomes the **market portfolio**. We can think of the market portfolio as the portfolio containing all risky asset classes in the world. In practice, a stock market index is often used as a proxy for the market portfolio, such as the S&P 500. All investors hold some combination of the risk-free asset and the market (tangency) portfolio, depending on their desired amount of total risk and return. For example, a more risk-averse investor may invest some of his money in the risk-free asset with the remainder invested in the market (i.e., his investment may be located at point A in Figure 5.3). At any point to the left of M, investors are lending at the risk-free rate because some of their money is invested in Treasuries, whereas at points to the right of M, they are borrowing at the risk-free rate (i.e., using leverage to magnify their investment in the market portfolio).

The equation of the CML is:

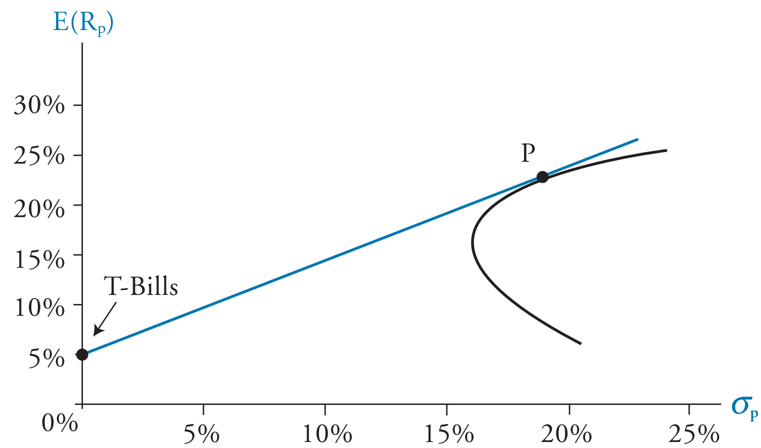
$$E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_P$$

Note that the slope of the CML is equal to the *Sharpe measure*, which we will examine later in this reading.

MODULE QUIZ 5.1

1. At a recent analyst meeting at Invest Forum, analysts Michelle White and Ted Jones discussed the use of the capital market line (CML). White states that the CML assumes that investors hold two portfolios: (1) a risky portfolio of all assets weighted according to their relative market value capitalizations; and (2) the risk-free asset. Jones states that the CML is useful in determining the required rate of return for individual securities. Are White and Jones's statements correct?
   A. Only Jones's statement is correct.
   B. Only White's statement is correct.
   C. Both statements are correct.
   D. Neither statement is correct.

Use the following graph to answer Question 2.



2. In the above mean-variance analysis, a risk analyst has combined the risk-free asset (T-bills) with Portfolio P. Portfolio P is least likely to
   A. be efficient.
   B. have beta of 1.
   C. lie on the security market line.
   D. represent a 100% investment in the market portfolio.

MODULE 5.2: DERIVING AND APPLYING THE CAPITAL ASSET PRICING MODEL

The Capital Asset Pricing Model (CAPM)

LO 5.c: Describe the assumptions underlying the CAPM.

The **capital asset pricing model (CAPM)** was developed by William Sharpe and John Lintner in the 1960s. It builds on the ideas of modern portfolio theory and the CML in that investors are assumed to hold some combination of the risk-free asset and the market portfolio. Its key assumptions are:

- *Information is freely available.*
- *Frictionless markets.* There are no taxes and commissions or transaction costs.

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- *Fractional investments are possible*. Assets are infinitely divisible, meaning investors can take a large position as well as very small positions.
- *Perfect competition*. Individual investors cannot affect market prices through their buying and selling activity and are, therefore, viewed as price takers.
- *Investors make their decisions solely based on expected returns and variances*. This implies that deviations from normality, such as skewness and kurtosis, are ignored from the decision-making process.
- *Market participants can borrow and lend unlimited amounts at the risk-free rate*.
- *Homogenous expectations*. Investors have the same forecasts of expected returns, variances, and covariances over a single period.

Clearly, the CAPM makes a number of unrealistic assumptions. As with any other model, care must be taken when relying solely on the results from the CAPM.

Estimating and Interpreting Systematic Risk

LO 5.f: Interpret beta and calculate the beta of a single asset or portfolio.

The expected returns of risky assets in the market portfolio are assumed to only depend on their relative contributions to the market risk of the portfolio. The systematic risk of each asset represents the sensitivity of asset returns to the market return and is referred to as the asset's **beta**. Beta is computed as follows:

$$\beta_i = \frac{\text{covariance of Asset i's return with the market return}}{\text{variance of the market return}} = \frac{\text{Cov}_{i,M}}{\sigma_M^2} = \rho_{i,M} \times \frac{\sigma_i}{\sigma_M}$$

In the next section, we will demonstrate that the market beta is, by definition, equal to 1. Any security with a beta of 1 moves in a one-to-one relationship with the market. Consequently, any security with a beta greater than 1 moves by a greater amount (has more market risk) and is referred to as cyclical (e.g., luxury goods stock). Any security with a beta below 1 is referred to as defensive (e.g., a utility stock). Cyclical stocks perform better during expansions whereas defensive stocks fare better in recessions.

EXAMPLE: Calculating an asset's beta

The standard deviation of the market return is estimated as 20%.

1. If Asset A's standard deviation is 30% and its correlation of returns with the market index is 0.8, what is Asset A's beta?

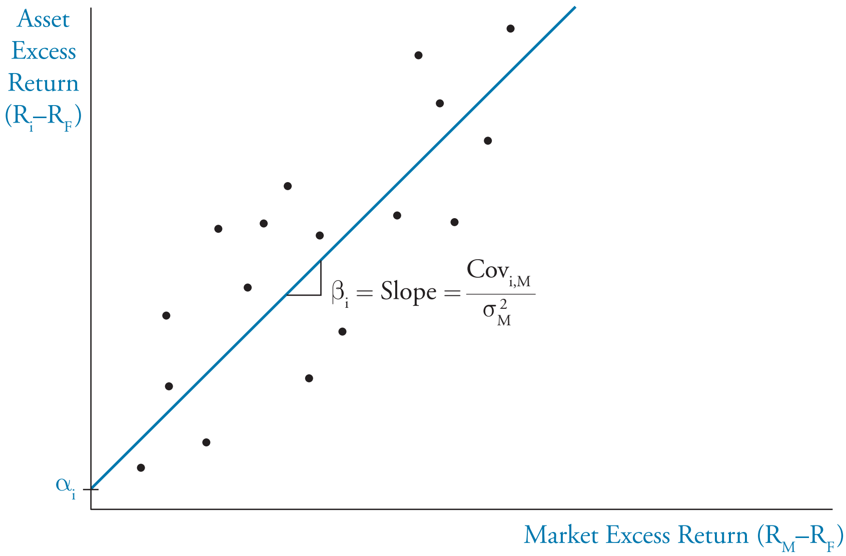
   Using the formula: $\beta_i = \rho_{i,M}\frac{\sigma_i}{\sigma_M}$, we have: $\beta_i = 0.80\frac{0.30}{0.20} = 1.2$.

2. If the covariance of Asset A's returns with the returns on the market index is 0.048, what is the beta of Asset A?

   Using the formula: $\beta_i = \frac{\text{Cov}_{i,M}}{\sigma_M^2}$, we have: $\beta_i = \frac{0.048}{0.2^2} = 1.2$.

In practice, we estimate beta by regressing asset returns against market returns. While regression is a concept discussed in Book 2, for the purposes of this reading, you can think of it as a mathematical estimation procedure that fits a line to a data plot. In Figure 5.4, we represent the excess returns on Asset *i* as the dependent variable and the excess returns on the market index as the independent variable. The **least squares regression line** is the line that minimizes the sum of the squared differences of the points from the line (this is what is meant by the line of *best fit*). The slope of this line is our estimate of beta.

Figure 5.4: Estimating Beta With Regression



Deriving the CAPM

LO 5.b: Understand the derivation and components of the CAPM.

A straightforward CAPM derivation recognizes that expected return

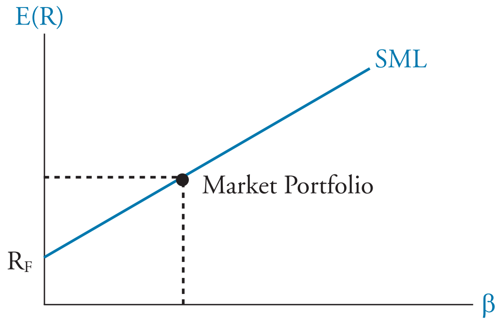
- only depends on beta (company-specific risk can be diversified away) and
- is a linear function of beta.

We therefore obtain the following equation, where expected return is explained as a linear function of beta with an intercept equal to *a* and slope equal to *m*:

$$E(R_P) = a + m \times \beta_P$$

The graphical depiction of the above equation is known as the **security market line (SML)**.

Figure 5.5: The Security Market Line



In Figure 5.5, the intercept occurs when beta is equal to 0 (i.e., when there is no systematic risk). The only asset with zero market risk is the risk-free asset, which is completely uncorrelated with market movements and offers a guaranteed return. Therefore, the intercept of the SML is equal to the risk-free rate of return, R_F.

To calculate the value of the slope we will need to know two points along the line. We already know the coordinates for the risk-free asset, which are $(0, R_F)$. We also know the coordinates for the market portfolio, which must be $(1, R_M)$ [i.e., the market portfolio has a return equal to the market return, by definition, and its systematic (beta) risk is equal to 1]. The latter point can be easily demonstrated, remembering that the covariance of the returns of an asset with itself is equal to the variance (we will further explore the properties of covariance in Book 2):

$$\beta_M = \frac{\text{Cov}_{M,M}}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1$$

We are now ready to calculate the slope of the SML as the *rise over run* of the line. This slope is known as the market risk premium (MRP) because it equals $(R_M - R_F)$:

$$m = \frac{(R_M - R_F)}{(1-0)} = (R_M - R_F) = \text{MRP}$$

Recall that expected return is a linear function of beta:

$$E(R_P) = a + m \times \beta_P$$

Using substitution, we can now obtain the well-known CAPM equation:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

This implies that the expected return of an investment depends on the risk-free rate R_F, the MRP, $[R_M - R_F]$, and the systematic risk of the investment, β. The expected return, $E(R_i)$, can be viewed as the *minimum required return*, or the *hurdle rate*, that investors demand from an investment, given its level of systematic risk. Estimating hurdle rates accurately is very important. If investors use an inflated hurdle rate, they may incorrectly forgo valuable investment opportunities. If, on the other hand, the rate used is too low, investors may purchase overvalued assets.

LO 5.e: Apply the CAPM in calculating the expected return on an asset.

EXAMPLE: Expected return on a stock

Assume you are assigned the task of evaluating the stock of Sky-Air, Inc. To evaluate the stock, you calculate its required return using the CAPM. The following information is available:

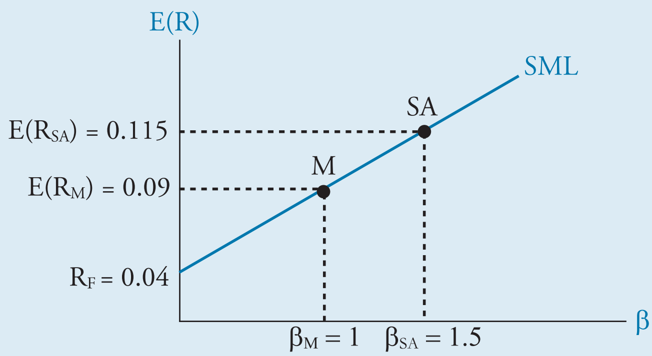
Expected market risk premium	5%
Risk-free rate	4%
Sky-Air beta	1.5

Using CAPM, calculate and interpret the expected return for Sky-Air.

Answer:

The expected return for Sky-Air is:

$$E(R_{SA}) = 0.04 + 1.5(0.05) = 0.115 = 11.5\%$$



In this case, the hurdle rate is 11.5% (i.e., this is the minimum required return given the market risk of Sky-Air). If investors predict that the return will exceed 11.5%, then they should buy Sky-Air stock (stock is undervalued). However, if investors predict that the expected return will be less than 11.5%, then they should either shy away from Sky-Air stock or short the stock, if allowed to do so, because the stock is overvalued.

In the previous example, we calculated the required rate of return, which always lies on the SML. If an analyst determines that the expected return is different from the required rate of return implied by CAPM, then the security may be mispriced according to rational expectations. A mispriced security would not lie on the SML. In general:

- An overvalued security would have a required rate of return (computed by CAPM) that is higher than its expected return (computed by the analyst's valuation). An overvalued security would plot below the SML.
- An undervalued security would have a required rate of return (computed by CAPM) that is lower than its expected return (computed by the analyst's valuation). An undervalued security would plot above the SML.

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MODULE QUIZ 5.2

1. Which of the following statements is most likely an assumption of the capital asset pricing model (CAPM)?
   A. Investors only face capital gains taxes.
   B. Investors' actions affect the prices of assets.
   C. Transaction costs are constant across all assets.
   D. Market participants can lend and borrow unlimited amounts at the risk-free rate.
2. Patricia Franklin makes buy and sell stock recommendations using the capital asset pricing model (CAPM). Franklin has derived the following information for the broad market and for the stock of the CostSave Company (CS):

Expected market risk premium	8%
Risk-free rate	5%
Historical beta for CS	1.50

   Franklin believes that historical betas do not provide good forecasts of future beta, so therefore uses the following formula to forecast beta:

   forecasted beta = 0.80 + 0.20 × historical beta

   After conducting a thorough examination of market trends and the CS financial statements, Franklin predicts that the CS return will equal 10%. Franklin should derive which of the following CS required returns for CS and valuation decisions (undervalued or overvalued)?

	Valuation	CAPM required return
A.	Overvalued	8.3%
B.	Overvalued	13.8%
C.	Undervalue	8.3%
D.	Undervalued	13.8%

3. Albert Dreiden wants to estimate the expected return on the market. He believes that the stock of the Hobart Materials Company is fairly valued, and gathers the following information:

Expected return for Hobart	7.50%
Risk-free rate	4.50%
Beta for Hobart	0.80

   Based on this information, the estimated expected return for the market portfolio is closest to

   A. 3.00%.
   B. 3.75%.
   C. 6.90%.
   D. 8.25%.

MODULE 5.3: PERFORMANCE EVALUATION MEASURES

LO 5.g: Calculate, compare and interpret the following performance measures: the Sharpe performance index, the Treynor performance index, the Jensen performance index, the tracking error, information ratio and Sortino ratio.

It is important for portfolio managers to not only focus on raw returns but to also analyze the risk taken to generate those returns. In other words, portfolio managers must analyze risk-adjusted rates of return to evaluate the true performance of their portfolios given the amount of risk taken. We begin by analyzing three traditional performance measures:

- Sharpe performance index (SPI)
- Treynor performance index (TPI)
- Jensen's performance index (JPI)

In all three cases, for a given portfolio, the higher measure, the better the risk-adjusted return. Note that Sharpe and Treynor are very similar in that they both normalize the risk premium by dividing by a measure of risk.

Sharpe Performance Index

The Sharpe measure computes excess return (portfolio return in excess of the risk-free rate) per unit of total risk (as measured by standard deviation). Investors can apply the Sharpe measure to all portfolios because it uses total risk, and it is more widely used than the other two measures.

$$\text{SPI} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

As previously mentioned, the slope of the CML is the Sharpe measure of the market. A portfolio with a Sharpe measure greater than the Sharpe measure of the market offers better risk-adjusted returns compared to the market. This inevitably assumes that markets are not always efficient, allowing managers to sometimes beat the market.

Treynor Performance Index

The Treynor measure is similar to the Sharpe measure in that both use the same numerator, the portfolio excess return. However, they differ in their calculation of the denominator. While the Sharpe measure uses total risk as measured by standard deviation, the Treynor measure uses systematic risk as measured by beta.

$$\text{TPI} = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

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As previously mentioned, well-diversified portfolios are only exposed to market risk, having diversified away idiosyncratic risk. Beta and TPI should therefore be more relevant metrics for well-diversified portfolios. On the other hand, poorly-diversified portfolios (i.e., portfolios containing few assets) will likely have an unnecessarily high standard deviation due to the presence of excessive company-specific risk.

Recall that the mathematical description of the SML is the CAPM, whose slope is the MRP:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

The slope of the SML can also be viewed as the Treynor measure of the market, or the MRP:

$$TPI_M = \left[\frac{E(R_M) - R_F}{\beta_M}\right] = \left[\frac{E(R_M) - R_F}{1}\right] = MRP$$

Jensen's Performance Index

Jensen's Performance Index, like Treynor, assumes investors are well-diversified and, therefore, uses beta rather than standard deviation as the relevant risk metric. Essentially, it compares the portfolio expected return to the CAPM required return. The difference between the two may be referred to as Jensen's alpha (α_P).

$$JPI = \alpha_P = E(R_P) - \{R_F + [E(R_M) - R_F]\beta_P\}$$

In equilibrium (the absence of mispricing), the portfolio expected return must equal the CAPM required return resulting in zero alpha. If Jensen's alpha is positive, this implies that the portfolio is undervalued and investors would be wise to buy or hold it. Jensen's alpha is most suitable for comparing portfolios that have the same level of systematic risk.

The Treynor measure and Jensen's alpha go hand in hand, in that superior performance implied by the Treynor measure automatically implies superior performance according to Jensen's alpha. However, relative rankings of portfolios may differ according to the two measures.

EXAMPLE: Calculating performance measures

For a portfolio of 10 stocks, assume that the portfolio's expected return is 14% with a standard deviation of 25%. The beta of the portfolio is 1.1. The expected return of the market is 12.5% with a standard deviation of 20.2%. The risk-free rate is 2.6%. Calculate Sharpe, Treynor, and Jensen's alpha for the portfolio of stocks. Compare the above measures to each measure for the market.

Answer:

$$SPI_P = \left[\frac{E(R_P) - R_F}{\sigma_P}\right] = \left[\frac{0.14 - 0.026}{0.25}\right] = 0.456$$

$$TPI_P = \left[\frac{E(R_P) - R_F}{\beta_P}\right] = \left[\frac{0.14 - 0.026}{1.1}\right] = 0.1036$$

$$JPI_P = \alpha_P = 0.14 - [0.026 + (0.125 - 0.026)(1.1)] = 0.0051$$

We can now compare the above measures to Sharpe, Treynor, and Jensen's alpha of the market:

$$SPI_M = \left[\frac{E(R_M) - R_F}{\sigma_M}\right] = \left[\frac{0.125 - 0.026}{0.202}\right] = 0.49$$

$$TPI_M = \left[\frac{E(R_M) - R_F}{\beta_M}\right] = \left[\frac{0.125 - 0.026}{1.0}\right] = 0.099$$

$$JPI_M = \alpha_P = 0.125 - [0.026 + (0.125 - 0.026)(1.0)] = 0.00$$

An alternative approach to evaluating portfolios is to calculate excess return relative to a target return or a benchmark portfolio return. In the following section, we will review three such measures:

- Tracking error
- Information ratio
- Sortino ratio

Tracking Error

If a manager is trying to earn a return higher than the market portfolio or any other reference or benchmark, the difference will have some variability over time. In other words, even if the manager is successful in generating a positive alpha, the alpha will vary over time. **Tracking error** is the term used to describe the standard deviation of the difference between the portfolio return and the benchmark return. This source of variability is another source of risk to use in assessing the manager's success.

$$\text{tracking error} = \sqrt{\frac{\sum(R_P - R_B)^2}{n-1}}$$

PROFESSOR'S NOTE

If you are asked to calculate tracking error on the exam, it would most likely amount to no more than obtaining the standard deviation using the relevant function on your calculator. We will review this computation in detail in Book 2. Also, note that even though the earlier definition of tracking error is typically how it's defined, some practitioners refer to tracking error simply as the difference between portfolio returns and benchmark returns: $R_P - R_B$.

Information Ratio

The **information ratio (IR)** divides the portfolio expected return in excess of the benchmark expected return by the tracking error:

$$IR = \frac{E(R_P - R_B)}{\text{tracking error}} = \frac{\text{active return}}{\text{active risk}}$$

PROFESSOR'S NOTE

Some practitioners refer to the numerator as active return and the denominator as active risk. The definition of tracking error (active risk) for the denominator of the IR is the same as the first definition provided earlier—the standard deviation of the difference between the portfolio return and the benchmark return.

Sortino Ratio

The **Sortino ratio** is reminiscent of the Sharpe measure except for two changes. First, we replace the risk-free rate with a *minimum acceptable return*, denoted R_{MIN}. This return could be determined by the needs of the investor or it can sometimes be set equal to the risk-free rate. Second, we replace standard deviation with *downside deviation*:

$$\text{Sortino} = \frac{R_P - R_{MIN}}{\text{downside deviation}}$$

Downside deviation is a type of semi-standard deviation. It measures the variability of only those returns that fall below the minimum acceptable return. Returns higher than R_{MIN} are ignored from the calculation of downside deviation as they are not considered risky as far as the desired returns of our investor are concerned.

PROFESSOR'S NOTE

It is unlikely that you will be asked to calculate downside deviation, so focus on being able to compute the Sortino ratio given R_P, R_{MIN}, and downside deviation.

EXAMPLE: Calculating the information ratio and the Sortino ratio

An active portfolio manager is trying to beat the FTSE 100. The expected returns of the active portfolio and the FTSE 100 are 15% and 12%, respectively, while the tracking error is 9%. The minimum acceptable return is 4% and the downside deviation is 7%. Compute the information ratio and the Sortino ratio.

Answer:

$$\text{IR} = \frac{E(R_P - R_B)}{\text{tracking error}} = \frac{0.15 - 0.12}{0.09} = 0.33$$

$$\text{Sortino} = \frac{R_P - R_{MIN}}{\text{downside deviation}} = \frac{0.15 - 0.04}{0.07} = 1.57$$

MODULE QUIZ 5.3

1. For a given portfolio, having a Treynor measure greater than the market but a Sharpe measure that is less than the market would most likely indicate the portfolio is
   A. not well-diversified.
   B. generating a negative alpha.
   C. borrowing at the risk-free rate.
   D. not borrowing at the risk-free rate.
2. With respect to performance measures, the use of the standard deviation of portfolio returns is a distinguishing feature of
   A. the beta measure.
   B. the Jensen's alpha.
   C. the Sharpe measure.
   D. the Treynor measure.
3. For a given portfolio, the expected return is 9% with a standard deviation of 16%. The beta of the portfolio is 0.8. The expected return of the market is 12% with a standard deviation of 20%. The risk-free rate is 3%. The portfolio's alpha is
   A. −1.2%.
   B. −0.6%.
   C. +0.6%.
   D. +1.2%.
4. Advanced Quantitative Models global equity fund has averaged a return of 12.5% per year over the last 10 years. The benchmark average return over the same period was 11% per year. The risk-free rate of return during the same period averaged 3.5%. The standard deviation of the fund's return is 16.15%, and the tracking error is 10.5%. What is the information ratio (IR) for the fund?
   A. 0.14
   B. 0.95
   C. 1.05
   D. 1.19

5. Given the following information:

Risk-free rate	4%
Minimum acceptable return	6%
Benchmark return	10%
Expected return on portfolio	12%
Expected return on market	9%
Beta	1.25
Standard deviation (portfolio)	7.3%
Downside deviation (portfolio)	8.2%

What is the Sortino ratio of the portfolio?

A. 0.24
B. 0.73
C. 0.82
D. 0.98

KEY CONCEPTS

LO 5.a

Rational investors seek to maximize return per unit of risk and, therefore, absent a risk-free asset, they will hold a portfolio on the efficient frontier. To reduce total risk, investors diversify across multiple investments. A sufficiently large portfolio will have eliminated company-specific (idiosyncratic) risk and will only be exposed to market risk.

LO 5.b

To derive the capital asset pricing model (CAPM), we must recognize that

- expected return only depends on beta because company-specific risk can be diversified away and
- expected return is a linear function of beta.

The capital asset pricing model (CAPM) equation is:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

The beta of the market is equal to 1, and the slope of the security market line (SML) is equal to the market risk premium (MRP). The SML is the graphical depiction of the CAPM.

LO 5.c

The capital asset pricing model (CAPM) makes the following assumptions:

- Information is freely available.
- There are no taxes and commissions.
- Fractional investments are possible.
- Market participants can borrow and lend at the risk-free rate.
- Individual investors cannot affect market prices.
- Investors have the same forecasts of expected returns, variances, and covariances.

LO 5.d

The capital market line (CML) linearly combines the risk-free asset with the tangency portfolio of the efficient frontier. Given the assumption of homogenous expectations, the tangency portfolio becomes the market portfolio. All investors are assumed to hold some combination of the risk-free asset and the market portfolio. The equation of the CML is:

$$E\left(R_P\right) = R_F + \left[\frac{E\left(R_M\right) - R_F}{\sigma_M}\right]\sigma_P$$

The slope of the CML is the Sharpe performance index.

LO 5.e

The expected return for an asset can be computed using the following formula, given the risk-free rate, the market risk premium (MRP), and an asset's beta:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

The MRP is the return of the market in excess of the risk-free rate.

LO 5.f

Beta can be estimated as the slope from a linear regression of stock returns against market returns. It is the sensitivity of stock returns to market movements. The following formulas can be used to calculate beta:

$$\beta_i = \frac{\text{covariance of Asset i's return with the market return}}{\text{variance of the market return}} = \frac{Cov_{i,M}}{\sigma_M^2} = \rho_{i,M} \times \frac{\sigma_i}{\sigma_M}$$

LO 5.g

Risk-adjusted performance measures include: the Sharpe performance index (SPI), the Treynor performance index (TPI), and Jensen's alpha. Both Treynor and Jensen's alpha are based on beta, whereas Sharpe is based on standard deviation:

$$SPI = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

$$TPI = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

$$JPI = \alpha_P = E(R_P) - \{R_F + [E(R_M) - R_F]\beta_P\}$$

Three relative performance metrics include: tracking error, the information ratio (IR), and the Sortino ratio:

$$\text{tracking error} = \sqrt{\frac{\sum(R_P - R_B)^2}{n - 1}}$$

$$IR = \frac{E(R_P - R_B)}{\text{tracking error}} = \frac{\text{active return}}{\text{active risk}}$$

$$\text{Sortino} = \frac{R_P - R_{MIN}}{\text{downside deviation}}$$

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 5.1

1. **B** The CML assumes all investors have identical expectations and all use mean-variance analysis, implying that they all identify the same risky tangency portfolio (the *market portfolio*) and combine that risky portfolio with the risk-free asset when creating their portfolios. Because all investors hold the same risky portfolio, the weight on each asset must be equal to the proportion of its market value to the market value of the entire portfolio. Therefore, White is correct. The CML is useful for determining the rate of return for efficient portfolios, but it cannot be used to determine the required rate of return for inefficient portfolios or individual securities. The capital asset pricing model (CAPM) is used to determine the required rate of return for inefficient portfolios and individual securities. Therefore, Jones is incorrect. (LO 5.d)

2. **C** The line connecting the risk-free rate with the tangency (market) portfolio is referred to as the capital market line. The market portfolio has a beta of 1, by definition, and lies on the efficient frontier. Had risk been measured on the graph with beta, the graph would represent the security market line. (LO 5.d)

Module Quiz 5.2

1. **D** The CAPM assumes unlimited borrowing and lending at the risk-free rate. Additionally, CAPM assumes no taxes, no transaction costs, and that investor actions do not affect market prices. (LO 5.c)

2. **B** The CAPM equation is:

   $$E(R_i) = R_F + \beta_i[E(R_M - R_F)]$$

   Franklin forecasts the beta for CostSave as follows:

   beta forecast = 0.80 + 0.20 (historical beta)

   beta forecast = 0.80 + 0.20(1.50) = 1.1

   The CAPM required return for CostSave is then:

   0.05 + 1.1(0.08) = 13.8%

   Note that the market premium, $E(R_M) - R_F$, is provided in the question (8%).

   Franklin should decide that the stock is overvalued because she forecasts that the CostSave return will equal only 10%, whereas the required return (minimum acceptable return) is 13.8%. (LO 5.e)

3. **D** The capital asset pricing model (CAPM) equation is:

$$E(R_i) = R_F + \beta_i[E(R_M - R_F)]$$

Using the given information, we can solve for the expected return for the market portfolio as follows:

$$7.50\% = 4.50\% + 0.80[E(R_M) - 4.50\%]$$

$$E(R_M) = [(7.50\% - 4.50\%) / 0.80] + 4.50\% = 8.25\%$$

Based on the information given and using the CAPM, the expected return on the market is 8.25%. (LO 5.e)

Module Quiz 5.3

1. **A** Low diversification can produce a Treynor measure greater than the Sharpe measure because it will likely increase the standard deviation of the portfolio's returns, thus decreasing the Sharpe measure. Using margin is not directly related to the risk-adjusted performance, because adjusting for risk removes the effect of leverage. A Treynor measure greater than the market Treynor would result in a positive alpha (not a negative alpha). (LO 5.g)

2. **C** The Sharpe measure is the portfolio return minus the risk-free rate divided by the standard deviation of the return. The Treynor and Jensen measures use beta as the measure of risk. The answer *beta measure* is a nonsensical choice for this question. (LO 5.g)

3. **A** The alpha is $9\% - [3\% + 0.8 \times (12\% - 3\%)] = -1.2\%$. (LO 5.g)

4. **A** $IR = (12.5 - 11) / 10.5 = 0.14$. (LO 5.g)

5. **B** Sortino ratio = (portfolio return – minimum acceptable return) / downside deviation

$$= (0.12 - 0.06) / 0.082 = 0.7317$$

(LO 5.g)

The following is a review of the Foundations of Risk Management principles designed to address the learning objectives set forth by GARP®. Cross reference to GARP FRM Part I Foundations of Risk Management, Chapter 6.

READING 6

The Arbitrage Pricing Theory and Multifactor Models of Risk and Return

EXAM FOCUS

The relationship between risk and return is one of the most important concepts in finance. The capital asset pricing model (CAPM) asserts that the expected return on any asset is solely determined by its exposure to the market portfolio. Recall from the previous reading that the risk exposure in the CAPM is known as beta. In contrast, arbitrage pricing theory (APT) asserts that expected returns are determined by exposures to multiple factors that are linked to the macroeconomy. The risk exposures in APT are known as factor betas. For the exam, be able to calculate expected returns using single-factor and multifactor models. Also, understand the Fama and French three-factor version of a multifactor model. In addition, be able to describe how to use a multifactor approach to construct a hedged portfolio.

MODULE 6.1: MULTIFACTOR MODEL ASSUMPTIONS AND INPUTS

Arbitrage Pricing Theory

LO 6.a: Explain the arbitrage pricing theory (APT), describe its assumptions and compare the APT to the CAPM.

Investors have historically thought about the expected return for an investment through the filter of the **capital asset pricing model (CAPM)**. This model captures a linear relationship between a financial asset and a single index (e.g., S&P 500 Index). Using CAPM, risk is modeled through the beta (or factor exposure) to this single index. In 1976, economics professor Steven Ross proposed an alternative risk modeling tool called **arbitrage pricing theory (APT)**.[1] This newer approach is a type of multifactor model that measures the linear relationship between a financial asset and multiple risk factors, which includes one or more financial indices (e.g., S&P 500 Index, bond index, or commodity index) and multiple macroeconomic variables (e.g., GDP, interest rate metrics, production measures, employment variables).

In a classic sense, the term *arbitrage* refers to the simultaneous buying and selling of two securities to capture a perceived abnormal price difference between the two assets. In the context of APT, this term simply refers to a model that measures expected return relative to multiple risk factors. In fact, APT assumes that there are no available arbitrage opportunities, and that if one does exist, it will very quickly evaporate due to the trading actions of market participants.

According to arbitrage pricing theory, the expected return for security *i* can be modeled as shown here. The idea is to model systematic risk on a more granular level using a series of risk factors.

$$R_i = E(R_i) + \beta_1 F_1 + \beta_2 F_2 + \ldots + \beta_k F_k + e_i$$

where:
R_i = the actual return on stock *i*
$E(R_i)$ = the expected return on stock *i*
β_1 = the beta (factor sensitivity) for factor 1
F_1 = the first in a series of risk factors that could add return deviation from the expected return
β_k = the beta (factor sensitivity) for factor k
F_k = the last in a series of risk factors that could add return deviation from the expected return
e_i = a random error term that accounts for company-specific (idiosyncratic) risk

Every mathematical model is based on a series of assumptions. Arbitrage pricing theory has very simplistic assumptions, including the following:

1. Market participants are seeking to maximize their profits.
2. Markets are frictionless (i.e., no barriers due to transaction costs, taxes, or lack of access to short selling).
3. There are no arbitrage opportunities, and if any are uncovered, then they will be very quickly exploited by profit-maximizing investors.

One element, which is both good and bad, is that APT does not specify the multiple factors to include in the analysis. This provides analysts with tremendous flexibility. However, if an investor is looking for a clear-cut and direct calculation, then APT might not be the best fit. Factors need to be checked on a periodic basis and factor sensitivities (betas) need to also be updated on a regular basis because financial markets are dynamic. Ultimately, there is no one-size-fits-all approach for

[1] Steven Ross, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory* 13, no. 3 (1976): 341–360.

determining the macroeconomic factors used in an APT model, but Chen, Roll, and Ross propose the following four factors as one way to structure an APT model[2]:

- The spread between short-term and long-term interest rates (i.e., the yield curve)
- Expected versus unexpected inflation
- Industrial production
- The spread between low-risk and high-risk corporate bond yields

The core of the APT model is to find a combination of granular risk factors, such as those presented, that more closely predict the return of a financial asset. In this model, arbitrage is not an expected opportunity because the model is adjusted to account for macroeconomic variables that might explain the current pricing for a given stock. This does not mean that the actual stock returns will not deviate from APT pricing (it very well may). This is the influence of company-specific risk factors. An analyst would be wise to buy a security whose market price drifts lower than APT would suggest (due to unexpected factors) and to potentially short a stock whose price is too much higher than APT's calculated return. This logic introduces model risk and also the need to periodically update model coefficients to ensure robustness.

Multifactor Model Inputs

LO 6.b: Describe the inputs (including factor betas) to a multifactor model.

The inputs into a multifactor model can be best understood by considering its equation, which can be seen as follows for stock *i*:

$$R_i = E(R_i) + \beta_1 F_1 + \beta_2 F_2 + \ldots \beta_k F_k + e_i$$

The first input is the expected return for the stock in question. This type of multifactor model will then offer a series of adjustments that attempt to capture known variables that would influence the returns of a stock (or portfolio). A beta (factor sensitivity) is needed for each variable included in the model, and a value is needed for each factor as well. The error term (e_i) represents firm-specific return that is otherwise unexplained by the model. This idiosyncratic risk could come from factors that are correlated with the stock's return but are excluded from the analysis. It could come from randomness and potentially from irrational market behavior. It could also result from unexpected firm-specific events such as labor strikes, natural disasters, or tariff uncertainty. Because firm-specific events are random, the expected (i.e., default) value for the error term is zero.

A multifactor model could include any number of variables that an analyst desires to consider. They could be macroeconomic variables, or they could be firm attributes (e.g., P/E multiples, revenue trends, historical returns). Consider an example where an analyst tests a stock's sensitivity to deviations from consensus expectations in quarterly GDP releases. The factor for GDP could be expressed as F_{GDP} and the beta (also known as the *factor loading* or the *factor sensitivity*) for GDP might be 2.0. If consensus GDP is 3.2%, but the actual value comes in as 2.2%, then

[2] N. Chen, R. Roll, and S. Ross, "Economic Forces and the Stock Market," *The Journal of Business* 59, no. 3 (1986): 383–403, http://www.jstor.org/stable/2352710.

the deviation is −0.01 (i.e., −1%). With a GDP beta (β_{GDP}) of 2.0, then we would expect the stock to decline by 2% (double the factor's movement due to the beta of 2.0).

MODULE QUIZ 6.1

1. Which of the following statements is correct regarding arbitrage pricing theory (APT)?
   A. APT uses a pre-established series of variables to calculate expected returns.
   B. APT provides more flexibility than traditional CAPM-based models.
   C. APT relies on a strict series of assumptions.
   D. APT is constrained to a five-factor model.
2. Which of the following statements regarding the inputs involved with a multifactor model is correct?
   A. The factors included in a multifactor model are very rigid.
   B. Factor betas describe how much the relationship is amplified between the stock under analysis and the respective factor.
   C. Analysts must include only economic variables as the factors in a multifactor model.
   D. Factor betas must be positive values.

MODULE 6.2: APPLYING MULTIFACTOR MODELS

Calculating Expected Returns

LO 6.c: Calculate the expected return of an asset using a single-factor and a multifactor model.

The number of factors to include in a model should be as small as possible, yet still capture the priced sources of systematic (nondiversifiable) risk. The simplest versions consist of just one macro factor (a **single-factor model**). Consider the differences between a single-factor and a multifactor model using a two-step example.

First, let's examine a single-factor option for the common stock of HealthCare Inc. (HCI). Actual returns are measured using a single-factor model that captures the impact of GDP surprises (unexpected percentage changes denoted by GDP*). The formula for this relationship follows:

$$R_{HCI} = E(R_{HCI}) + \beta_{GDP*}F_{GDP*} + e_{HCI}$$

The following data expands this single-factor example:

- The expected return for HCI is 10%.
- The factor beta for GDP surprises is 2.0.
- The expected GDP growth rate is 3.2%.

Considering the factor beta, we can deduce that the expected returns for HCI are strongly influenced by GDP surprises. This beta suggests a 200% sensitivity. Therefore, the stock price is estimated to change by 2% if the GDP surprise is 1%.

What would this single-factor model prediction be if GDP were actually 2.6% and not the original consensus forecast of 3.2%? The GDP surprise factor is −0.60% (= 2.6% − 3.2%). The formula would suggest that HCI's stock return should be 8.8%:

$$R_{HCI} = E(R_{HCI}) + \beta_{GDP*}F_{GDP*} + e_{HCI}$$
$$R_{HCI} = 0.10 + 2.0(-0.006) + e_{HCI} = 0.088 = 8.8\%$$

Perhaps HCI's actual return was 8.25%. Any deviation from the 8.8% value represents either company-specific risk or systematic risk exposure that is not captured by the single-factor model. A multifactor model enables analysts to include the systematic risk exposure of multiple factors. Maybe surprises in consumer sentiment (CS*) is also a big influencer for HCI's returns. Consider the following multifactor model:

$$R_{HCI} = E(R_{HCI}) + \beta_{GDP*}F_{GDP*} + \beta_{CS*}F_{CS*} + e_{HCI}$$

The information below is added to the single-factor model data:

- The factor beta for CS surprises is 1.5.
- The expected CS growth rate is 1.0%.

If an updated measure of CS presents a growth rate of 0.75%, then the CS surprise factor is −0.25% (= 0.75% − 1.0%) and HCI's stock price should be 8.43%:

$$R_{HCI} = E(R_{HCI}) + \beta_{GDP*}F_{GDP*} + \beta_{CS*}F_{CS*} + e_{HCI}$$
$$R_{HCI} = 0.10 + 2.0(-0.006) + 1.5(-0.0025) + e_{HCI} = 0.0843 = 8.43\%$$

The multifactor model predicts a value of 8.43%, which is much closer to the actual result of 8.25%. This multifactor model is capturing more of the systematic influences. An analyst would likely keep exploring to find a third or fourth factor that would get them even closer to the actual result. Once the proper risk factors have been included, the analyst will be left with company-specific risk (e_i) that cannot be diversified away.

PROFESSOR'S NOTE

Both the factors and the beta exposures will need to be updated and verified on a periodic basis because these elements change dynamically.

Accounting for Correlation

LO 6.d: Explain models that account for correlations between asset returns in a multi-asset portfolio.

Arbitrage pricing theory is an application of a multifactor model that serves as an alternative to CAPM. This theory relies on the use of a **well-diversified portfolio**. A portfolio is well diversified if financial assets are mixed with other assets that have sufficient correlation differences to expel much of the company-specific risk (i.e., nonsystematic risk, idiosyncratic risk). A well-diversified portfolio will then be left with market-linked risk (i.e., systematic risk), which is measured by a beta coefficient.

We understand that diversification is enhanced when correlations between portfolio assets is low. Logic points to higher correlations when constituent assets in a portfolio come from the same asset class and lower correlations when member assets are drawn from different asset classes (e.g., commodities, real estate, industrial firms, utilities). The presence of multiple asset classes will result in a divergent list of factors that might impact the expected returns for a stock. Multifactor models are ideal for this form of analysis.

The main conclusion of APT is that expected returns on well-diversified portfolios are proportional to their factor betas. However, we cannot conclude that the APT relationship will hold for *all* securities. For example, if the APT relationship is violated for one security in the portfolio, then its effect will be too small to produce meaningful arbitrage opportunities for the portfolio. Therefore, we can conclude that the APT relation can hold for well-diversified portfolios even if it does not hold for all securities in the portfolio. But, the APT relationship must hold for nearly all securities in a well-diversified portfolio, or else arbitrage opportunities will become available for the portfolio. Therefore, we can conclude that the APT relationship must hold for nearly all securities.

Hedging Exposure to Multiple Factors

LO 6.e: Explain how to construct a portfolio to hedge exposure to multiple factors.

The granular exposures captured by multifactor models enable a unique hedging opportunity. Using calculated factor sensitivities, an investor can build **factor portfolios**, which retain some exposures and intentionally mitigate others through targeted portfolio allocations. Consider the following example with a series of three well-diversified portfolio as:

	Portfolio 1	**Portfolio 2**	**Portfolio 3**
GDP surprise factor sensitivity (β_{GDP*})	0.50	0.50	
Consumer sentiment surprise factor sensitivity (β_{CS*})	0.30		0.30
Unemployment surprise factor (β_{JOBS*})		0.25	
Manufacturing sector surprise factor (β_{ISM*})			1.25

Suppose that an investor wishes to mitigate all exposure to GDP surprise risk. That investor could find a financial asset (or portfolio) that is correlated with GDP surprise and has an equal factor sensitivity of 0.50. In this example, an investor could take a long position in Portfolio 1 and a short position in Portfolio 2. Doing so would result in a zero beta for GDP surprise, but it would retain a 0.30 beta for consumer sentiment surprise and add a −0.25 beta (because the position is held short) to unemployment surprise. It is possible to find a financial asset that only has an equal factor exposure to the single variable of GDP surprise. In such a circumstance, the investor could neutralize the GDP surprise exposure and not add any other new exposures.

An investor could also decide to be long Portfolio 1 and short Portfolio 3, which would neutralize the consumer sentiment exposure while retaining GDP surprise and adding manufacturing surprise. A third option would be to find derivatives that could hedge the 0.50 beta exposure to GDP surprise and the 0.30 beta exposure to consumer sentiment surprise. In this instance, an investor could form a hedged portfolio (Portfolio H) which has a 50% position in a derivative with exposure to only GDP surprise, a 30% position in a derivative with exposure to only consumer sentiment surprise, and the remaining 20% in the risk-free asset. An investor could take a long position in Portfolio 1 and a short position in Portfolio H. This action would effectively mitigate all exposure to both GDP surprise and consumer sentiment surprise.

An investor might engage in this fully hedged process to exploit a perceived arbitrage opportunity. Perhaps Portfolio 1 has an expected return of 12% and the hedged portfolio has an expected return of 10%. Taking equal long and short positions in these two portfolios will result in a potential 2% arbitrage profit [12% (long) − 10% (short)]. Alternatively, if the hedged portfolio instead had a 14% expected return, then the investor could take a long position in Portfolio H and a short position in Portfolio 1. This action would accomplish the same goal of neutralizing factor sensitivities while isolating the perceived 2% arbitrage opportunity.

One caveat is the potential for error. Because this hedging process is based on the calculated model, there will always be an element of model risk. What if the factor sensitivities have changed? What if different factors are better descriptors for a portfolio? What if the necessary assumptions do not hold during periods of market distress (e.g., the financial crisis of 2007–2009)? Error could also be present if the hedging strategy is either rebalanced too infrequently or too often. Trading costs from frequent rebalancing could erode profits, and infrequent rebalancing could risk undesired exposures as relationships dynamically change in the markets.

The Fama-French Three-Factor Model

LO 6.f: Describe and apply the Fama-French three-factor model in estimating asset returns.

Recall that CAPM is a single-factor model to calculate the expected return of a portfolio. The formula for CAPM is as follows:

$$E(R_i) = R_F + \beta_{i,M} RP_M + e_i$$

where:

$E(R_i)$ = the expected return on stock *i*
R_F = the risk-free rate
$\beta_{i,M}$ = the beta (factor sensitivity) between stock *i* and the market
RP_M = the risk premium for the market
e_i = a random error term which accounts for company-specific (idiosyncratic) risk

As mentioned, because well-diversified portfolios include assets from multiple asset classes, multiple risk factors will influence the systematic risk exposure of the portfolio. Therefore, multifactor APT can be rewritten as follows:

$$E(R_i) = R_F + \beta_1 RP_1 + \beta_2 RP_2 + \beta_3 RP_3 + e_i$$

where:
β_i = the beta (factor sensitivity) between stock *i* and factor exposure *i*
RP_i = risk premium associated with risk factor *i*

As mentioned previously, a major weakness of APT is that it provides no guidance on which other factors to include in a multifactor model. In 1996, economists Eugene Fama and Kenneth French famously specified a multifactor model with three factors: (1) a risk premium for the market, (2) a factor exposure for "small minus big," and (3) a factor exposure for "high minus low".[3] *Small minus big (SMB)* is the difference in returns between small firms and large firms. This factor adjusts for the size of the firm because smaller firms often have higher returns than larger firms. *High minus low (HML)* is the difference between the return on stocks with high book-to-market metrics and ones with low book-to-market values. A high book-to-market value means that the firm has a low price-to-book metric (book-to-market and price-to-book are inverses). This last factor basically means that firms with lower starting valuations are expected to potentially outperform those with higher starting valuations.

PROFESSOR'S NOTE

Notice that SMB is a hedge strategy, which is long small firms and short big firms. Likewise, HML is also a hedge strategy that is long high book-to-market firms and short low book-to-market firms.

The Fama-French three-factor model is as follows:

$$E(R_i) = R_F + \beta_{i,M} RP_M + \beta_{i,SMB} F_{SMB} + \beta_{i,HML} F_{HML} + e_i$$

The SMB and HML factors are chosen because history shows that returns are higher on smaller firms and those with high book-to-market values. Fama and French argue that these differences exist because small firms are inherently riskier than big firms. It is common knowledge that valuation levels when a trade is initiated have an impact on the ultimate outcome.

In 1997, Mark Carhart added a momentum factor to the Fama and French model to yield a four-factor model.[4] In 2015, Fama and French themselves proposed adding factors for "robust minus weak" (RMW) that accounts for the strength of operating profitability and "conservative minus aggressive" (CMA) to adjust for the degree of conservatism in the way a firm invests.[5] The point is that the Fama-French three-factor model is not the only option, but it is a widely known version of a multifactor model.

[3] E. F. Fama and K. R. French, "Multifactor Explanations of Asset Pricing Anomalies," *The Journal of Finance* 51, no. 1 (1996): 55–84.

[4] M. M. Carhart, "On Persistence in Mutual Fund Performance," *The Journal of Finance* 52, no. 1 (1997): 57–82.

[5] E. F. Fama and K. R. French, "A Five-Factor Asset Pricing Model." *Journal of Financial Economics* 116, no. 1 (2015): 1–22.

Reading 6

Consider an example applying the Fama-French three-factor model. A company has a beta relative to the market (β_M) of 0.85, an SMB factor sensitivity (β_{SMB}) of 1.65, and an HML factor sensitivity (β_{HML}) of −0.25. The equity risk premium is 8.5%, the SMB factor is 2.5%, the HML factor is 1.75%, and the risk-free rate is 2.75%. Given this series of inputs, the expected return for this stock is computed as:

$$E(R_i) = R_F + \beta_{i,M}RP_M + \beta_{i,SMB}F_{SMB} + \beta_{i,HML}F_{HML} + e_i$$

$$E(R_i) = 0.0275 + 0.85(0.085) + 1.65(0.025) + -0.25(0.0175) + e_i = 0.1366 = 13.66\%$$

Any return that is different from this calculated 13.66% is considered to be **alpha (α)**. The source of this alpha could be company-specific risk (e_i), or it could be that other factors need to be added to this multifactor model to better predict this stock's future returns.

MODULE QUIZ 6.2

1. What value is derived from adding more factors through a multifactor approach?
   A. All company-specific risk can be mitigated.
   B. The same variables can be added for every stock, which makes the process easy to implement.
   C. Calculations can be derived over multiple time periods because the factor betas remain static.
   D. A richer systematic relationship can be captured.
2. Which of the following statements about correlation and diversification is correct with respect to multifactor models?
   A. Well-diversified portfolios hold constituent assets with high correlations.
   B. The use of well-diversified portfolios removes the need for multifactor models.
   C. The use of multiple assets with lower correlations makes the use of multifactor models more beneficial for analysts to consider.
   D. Well-diversified portfolios typically include assets from the same asset class.
3. Which of the following statements relative to the use of multifactor models and hedging is incorrect?
   A. Multifactor models enable investors to hedge specific factor exposures.
   B. There are still no arbitrage opportunities, even when factoring in the granular exposures captured by multifactor models.
   C. Multifactor models potentially enable investors to eliminate all calculated factor exposures.
   D. The hedging process will most likely contain an element of model risk.
4. Which factors are explicitly considered in the Fama-French three-factor model?
   A. A size factor
   B. A momentum factor
   C. A currency exposure factor
   D. An operational robustness factor

KEY CONCEPTS

LO 6.a

The capital asset pricing model (CAPM) measures the expected return of a financial asset with respect to the broad market only. Arbitrage pricing theory (APT) is a type of multifactor model that expands upon the CAPM to consider any number of macroeconomic factors that may add additional explanatory power to the expected returns of a financial asset. There is not a set series of macroeconomic factors to consider, which presents analysts with a great deal of flexibility. APT also has simplified assumptions relative to the CAPM.

LO 6.b

The inputs in a multifactor model are a series of factors that influence the return on a stock. They include the expected return for the stock, a series of desired factors, and a beta for each factor. The factors are completely customizable by an analyst.

LO 6.c

A single-factor model will only consider the impact of one factor on a dependent variable (a stock's return). This leaves the potential for either company-specific risk or uncaptured systematic risk to influence asset returns. A multifactor model enables analysts to better model the impact of all systematic risk exposures to improve forecasting ability.

LO 6.d

APT relies on well-diversified portfolios. Diversification is based on correlation between constituent assets held in a portfolio. When the assets are all sourced from the same asset class, correlations will be higher than if they are sourced from different asset classes. Therefore, a well-diversified portfolio will hold assets from different categories. This will result in a much broader pool of factors that could influence the systematic risk exposure of a given stock. Multifactor models are ideal for the need to monitor a diverse list of factors.

LO 6.e

Because multifactor models consider factor exposures on a very granular level, investors can use this approach for hedging. A specific factor exposure can be targeted for elimination, or all factor exposures can be targeted. Through the creation of a customized hedged portfolio that is scaled to the factor sensitivities of a specific portfolio, investors can potentially isolate arbitrage opportunities.

LO 6.f

Fama and French specified a three-factor model that includes the equity risk premium plus an adjustment for the size of the firm (SMB) and the firm's valuation (HML). There have been other extensions of both the CAPM and the Fama-French three-factor model (e.g., a momentum factor).

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 6.1

1. **B** Arbitrage pricing theory uses a completely customizable group of variables. It explicitly mixes the return of the market with a collection of macroeconomic variables. As such, it offers more granular flexibility than CAPM. It also uses much fewer restrictive assumptions than CAPM. (LO 6.a)

2. **B** Multifactor models include a series of factors and associated betas for each factor. The selection of factors is completely customizable with no constraints, and a beta factor can be positive or negative. In either instance, the beta factor will measure the relationship between the stock and the factor in question. (LO 6.b)

Module Quiz 6.2

1. **D** Adding multiple risk factors does not eliminate company-specific risk, which is also known as nondiversifiable risk. Each stock will use its own variables, so an analyst will need to source variables for each stock under review and periodically check (and maybe change) the factors deployed because the factors and the factor betas are dynamic over time. Adding multiple risk factors does enhance the discovery of systematic risk influence. (LO 6.c)

2. **C** APT requires a well-diversified portfolio, which means that assets with lower correlations coming from different asset categories need to be included. This requirement will broaden the pool of influential factors and make a multifactor model a more attractive option. Using uncorrelated assets can lessen but not eliminate company-specific risk. (LO 6.d)

3. **B** The use of multifactor models enables investors to focus on granular risk exposures. Investors can hedge a single exposure and retain the others. They can also potentially hedge all calculated risk exposures. This process could produce arbitrage opportunities given the right circumstances. Because this hedging process is based on the calculated model, there will always be an element of model risk. (LO 6.e)

4. **A** The Fama-French three-factor model explicitly adjusts for size (SMB) and valuation (HML). Carhart added a momentum factor one year after Fama and French's original work. Fama and French also added an operating profit measure and an investment conservatism factor in a very recent extension of their own work. (LO 6.f)