1 Equality of $\boldsymbol{c}$ and $\boldsymbol{p}$ Growth with Transitory Shocks

Section 4.1 asserted that in the absence of permanent shocks it is possible to prove that the growth factor for aggregate consumption approaches that for aggregate permanent income. This section establishes that result.

First define $\mathrm{a}(m)$ as the function that yields optimal end-of-period assets as a function of m.

Suppose the population starts in period t with an arbitrary value for $\mathrm{cov}_t(a_{t+1,i}, \boldsymbol{p}_{t+1,i})$. Then if $\hat{m}$ is the invariant mean level of m we can define an 'average marginal propensity to save away from $\hat{m}$' function:

$$\bar{\mathrm{a}}'(\Delta) = \Delta^{-1} \int_{\hat{m}}^{\hat{m}+\Delta} \mathrm{a}'(z) dz$$

where the combination of the bar and the ′ are meant to signify that this is the average value of the derivative over the interval. Since $\psi_{t+1,i} = 1$, $\tilde{R}_{t+1,i}$ is a constant at $\tilde{R}$, so if we define $\hat{a}$ as the value of a corresponding to $m = \hat{m}$, we can write

$$a_{t+1,i} = \hat{a} + (m_{t+1,i} - \hat{m})\bar{\mathrm{a}}'(\overbrace{\tilde{R}a_{t,i} + \xi_{t+1,i}}^{m_{t+1,i}} - \hat{m})$$

so

$$\mathrm{cov}_t(a_{t+1,i}, \boldsymbol{p}_{t+1,i}) = \mathrm{cov}_t\left(\bar{\mathrm{a}}'(\tilde{R}a_{t,i} + \xi_{t+1,i} - \hat{m}), G\boldsymbol{p}_{t,i}\right).$$

But since $R^{-1}(qR\beta)^{1/\gamma} < \bar{\mathrm{a}}'(m) < \frac{\text{Þ}}{R}$,

$$|\mathrm{cov}_t((qR\beta)^{1/\gamma} a_{t+1,i}, \boldsymbol{p}_{t+1,i})| < |\mathrm{cov}_t(a_{t+1,i}, \boldsymbol{p}_{t+1,i})| < |\mathrm{cov}_t(\text{Þ} a_{t+1,i}, \boldsymbol{p}_{t+1,i})|$$

and for the version of the model with no permanent shocks the GIC-Mod says that $\text{Þ} < G$, while the FHWC says that $G < R$; combining these facts we get:

$$|\mathrm{cov}_t(a_{t+1,i}, \boldsymbol{p}_{t+1,i})| < G|\mathrm{cov}_t(a_{t,i}, \boldsymbol{p}_{t,i})|.$$

This means that from any arbitrary starting value, the relative size of the covariance term shrinks to zero over time (compared to the $\check{\boldsymbol{A}}G^n$ term which is growing steadily by the factor G). Thus, $\lim_{n\to\infty} \boldsymbol{A}_{t+n+1}/\boldsymbol{A}_{t+n} = G$.

This logic unfortunately does not go through when there are permanent shocks, because the $\tilde{R}_{t+1,i}$ terms are not independent of the permanent income shocks.

To see the problem clearly, define $\check{\tilde{R}} = \mathbb{M}\left[\tilde{R}_{t+1,i}\right]$ and consider a first order Taylor expansion of $\bar{\mathrm{a}}'(m_{t+1,i})$ around $\hat{m}_{t+1,i} = \check{\tilde{R}}a_{t,i} + 1$,

$$\bar{\mathrm{a}}'_{t+1,i} \approx \bar{\mathrm{a}}'(\hat{m}_{t+1,i}) + \bar{\mathrm{a}}''(\hat{m}_{t+1,i})\left(m_{t+1,i} - \hat{m}_{t+1,i}\right).$$

The problem comes from the $\bar{\mathrm{a}}''$ term (which we implicitly define as the derivative of $\bar{\mathrm{a}}'$). The concavity of the consumption function implies convexity of the a function, so this term is strictly positive but we have no theory to place bounds on its size as we do for its level $\bar{\mathrm{a}}'$. We cannot rule out by theory that a positive shock to permanent income (which has a negative effect on $m_{t+1,i}$) could have a (locally) unboundedly positive effect on $\bar{\mathrm{a}}''$

(as for instance if it pushes the consumer arbitrarily close to the self-imposed liquidity constraint).