Table 1 Microeconomic Model Calibration

Calibrated Parameters			
Description	Parameter	Value	Source
Permanent Income Growth Factor	G	1.03	PSID: Carroll (1992)
Interest Factor	R	1.04	Conventional
Time Preference Factor	β	0.96	Conventional
Coefficient of Relative Risk Aversion	γ	2	Conventional
Probability of Zero Income	q	0.005	PSID: Carroll (1992)
Std Dev of Log Permanent Shock	σ_ψ	0.1	PSID: Carroll (1992)
Std Dev of Log Transitory Shock	σ_θ	0.1	PSID: Carroll (1992)

Table 2 Model Characteristics Calculated from Parameters

Description	Symbol and Formula			Approximate Calculated Value
Finite Human Wealth Factor	$\tilde{R}^{-1}$	$\equiv$	G/R	0.990
PF Value of Autarky Factor	$\beth$	$\equiv$	$\beta G^{1-\gamma}$	0.932
Growth Compensated Permanent Shock	$\underline{\mathbf{\Psi}}$	$\equiv$	$(\mathbb{E}[\mathbf{\Psi}^{-1}])^{-1}$	0.990
Uncertainty-Adjusted Growth	$\underline{G}$	$\equiv$	$G\underline{\mathbf{\Psi}}$	1.020
Utility Compensated Permanent Shock	$\underline{\underline{\mathbf{\Psi}}}$	$\equiv$	$(\mathbb{E}[\psi^{1-\gamma}])^{1/(1-\gamma)}$	0.990
Utility Compensated Growth	$\underline{G}$	$\equiv$	$G\underline{\underline{\mathbf{\Psi}}}$	1.020
Absolute Patience Factor	$\mathbf{Þ}$	$\equiv$	$(R\beta)^{1/\gamma}$	0.999
Return Patience Factor	$\frac{\mathbf{Þ}}{R}$	$\equiv$	$\mathbf{Þ}/R$	0.961
Growth Patience Factor	$\frac{\mathbf{Þ}}{G}$	$\equiv$	$\mathbf{Þ}/G$	0.970
Modified Growth Patience Factor	$\frac{\mathbf{Þ}}{G}\mathbb{E}[\psi^{-1}]$	$\equiv$	$\mathbf{Þ}/\underline{G}$	0.980
Value of Autarky Factor	$\underline{\underline{\beth}}$	$\equiv$	$\beta G^{1-\gamma}\underline{\underline{\mathbf{\Psi}}}^{1-\gamma}$	0.941
Weak Return Impatience Factor	$q^{1/\gamma}\mathbf{Þ}$	$\equiv$	$(q\beta R)^{1/\gamma}$	0.071

Table 3 Definitions and Comparisons of Conditions

Perfect Foresight Versions	Uncertainty Versions
Finite Human Wealth Condition (FHWC)	
$G/R < 1$ The growth factor for permanent income G must be smaller than the discounting factor R for human wealth to be finite.	$G/R < 1$ The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.
Absolute Impatience Condition (AIC)	
$\mathbf{Þ} < 1$ The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time: $\boldsymbol{c}_{t+1} < \boldsymbol{c}_t$	$\mathbf{Þ} < 1$ *If wealth is large enough,* the *expectation* of consumption next period will be smaller than this period's consumption: $\lim_{m_t \to \infty} \mathbb{E}_t[\boldsymbol{c}_{t+1}] < \boldsymbol{c}_t$
Return Impatience Conditions	
Return Impatience Condition (RIC)	Weak RIC (WRIC)
$\mathbf{Þ}/R < 1$ The growth factor for consumption $\mathbf{Þ}$ must be smaller than the discounting factor R, so that the PDV of current and future consumption will be finite: $\mathrm{c}'(m) = 1 - \mathbf{Þ}/R < 1$	$q^{1/\gamma}\mathbf{Þ}/R < 1$ If the probability of the zero-income event is $q = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker. $\mathrm{c}'(m) < 1 - q^{1/\gamma}\mathbf{Þ}/R < 1$
Growth Impatience Conditions	
GIC	GIC-Mod
$\mathbf{Þ}/G < 1$ For an unconstrained PF consumer, the ratio of $\boldsymbol{c}$ to $\boldsymbol{p}$ will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \to \infty} m_{t+1}/m_t = \frac{\mathbf{Þ}}{G}$	$\mathbf{Þ}\,\mathbb{E}[\psi^{-1}]/G < 1$ By Jensen's inequality stronger than GIC. Ensures consumers will not expect to accumulate m unboundedly. $\lim_{m_t \to \infty} \mathbb{E}_t[m_{t+1}/m_t] = \frac{\mathbf{Þ}}{G}\,\mathbb{E}[\psi^{-1}]$
Finite Value of Autarky Conditions	
PF-FVAC	FVAC
$\beta G^{1-\gamma} < 1$ equivalently $\mathbf{Þ} < R^{1/\gamma} G^{1-1/\gamma}$ The discounted utility of constrained consumers who spend their permanent income each period should be finite.	$\beta G^{1-\gamma}\,\mathbb{E}[\psi^{1-\gamma}] < 1$ By Jensen's inequality, stronger than the PF-FVAC because for $\gamma > 1$ and nondegenerate ψ, $\mathbb{E}[\psi^{1-\gamma}] > 1$.

Table 4 Sufficient Conditions for Nondegenerate‡ Solution

Consumption Model(s)	Conditions	Comments
$\bar{c}(m)$: PF Unconstrained	RIC, FHWC°	RIC$\Rightarrow$ $\lvert \mathrm{v}(m)\rvert < \infty$; FHWC$\Rightarrow$ $0 < \lvert \mathrm{v}(m)\rvert$
$\underline{c}(m) = \underline{\kappa} m$		PF model with no human wealth ($h = 0$)
Section 2.3.1:		RIC prevents $\bar{c}(m) = \underline{c}(m) = 0$
Section 2.3.1:		FHWC prevents $\bar{c}(m) = \infty$
Eq (47):		PF-FVAC+FHWC $\Rightarrow$ RIC
Eq (46):		GIC+FHWC $\Rightarrow$ PF-FVAC
$\grave{c}(m)$: PF Constrained	~~GIC~~, RIC	FHWC holds ($G < \text{Þ} < R \Rightarrow G < R$)
Section 5.1.1:		$\grave{c}(m) = \bar{c}(m)$ for $m > m_{\#} < 1$
		(~~RIC~~ would yield $m_{\#} = 0$ so $\grave{c}(m) = 0$)
Appendix C:	GIC,RIC	$\lim_{m\to\infty} \grave{c}(m) = \bar{c}(m)$, $\lim\limits_{m\to\infty} \grave{\kappa}(m) = \underline{\kappa}$
		kinks where horizon to $b = 0$ changes*
Appendix C:	GIC,~~RIC~~	$\lim\limits_{m\to\infty} \grave{\kappa}(m) = 0$
		kinks where horizon to $b = 0$ changes*
$\mathrm{c}(m)$: Friedman/Muth	Section 2.4.1 & 2.4.2 ,	$\underline{\mathrm{c}}(m) < \mathrm{c}(m) < \bar{\mathrm{c}}(m)$
	Section ??	$\underline{\mathrm{v}}(m) < \mathrm{v}(m) < \bar{\mathrm{v}}(m)$
Section ??:	FVAC, WRIC	Sufficient for Contraction
Section ??:		WRIC is weaker than RIC
Figure 7:		FVAC is stronger than PF-FVAC
Section ??: Case 3		~~FHWC~~+RIC $\Rightarrow$GIC, $\lim\limits_{m\to\infty} \kappa(m) = \underline{\kappa}$
Section ??: Case 1		~~RIC~~ $\Rightarrow$~~FHWC~~, $\lim\limits_{m\to\infty} \kappa(m) = 0$
Section 3.1:		"Buffer Stock Saving" Conditions
Theorem 6:		GIC $\Rightarrow \exists\ \check{m}$ s.t. $0 < \check{m} < \infty$
Theorem 7:		GIC-Mod $\Rightarrow \exists\ \hat{m}$ s.t. $0 < \hat{m} < \infty$

‡For feasible m satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines a unique optimal value of c satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < \mathrm{v}(m) < 0$.

°RIC, FHWC are necessary as well as sufficient for the perfect foresight case. *That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the m where the constraint will bind two periods in the future, etc.

**In the Friedman/Muth model, the RIC+FHWC are sufficient, but *not* necessary for nondegeneracy

Table 5 Appendix: Perfect Foresight Liquidity Constrained Taxonomy

For constrained $\grave{c}$ and unconstrained $\bar{c}$ consumption functions

Main Condition Subcondition	Math			Outcome, Comments or Results
~~GIC~~		$1 <$	$Þ/G$	Constraint never binds for $m \geq 1$
and RIC	$Þ/R$	< 1		FHWC holds ($R > G$);
				$\grave{c}(m) = \bar{c}(m)$ for $m \geq 1$
and ~~RIC~~		$1 <$	$Þ/R$	$\grave{c}(m)$ is degenerate: $\grave{c}(m) = 0$
GIC	$Þ/G$	< 1		Constraint binds in finite time $\forall\ m$
and RIC	$Þ/R$	< 1		FHWC may or may not hold
				$\lim_{m \uparrow \infty} \bar{c}(m) - \grave{c}(m) = 0$
				$\lim_{m \uparrow \infty} \grave{\kappa}(m) = \underline{\kappa}$
and ~~RIC~~		$1 <$	$Þ/R$	~~FHWC~~
				$\lim_{m \uparrow \infty} \grave{\kappa}(m) = 0$

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GICholds, the constraint will bind in finite time.