1 Perfect Foresight Liquidity Constrained Solution

{sec:ApndxLiqCons

We briefly interpret FVAC before turning to how all the conditions relate under uncertainty. Analogously to (1), the value for a consumer who spent exactly their permanent income every period would reflect the product of the expectation of the (independent) future shocks to permanent income:

$$= \mathrm{u}(\boldsymbol{p}_t)\left(\frac{1-(\beta\mathcal{G}^{1-\gamma}\mathbb{E}[\psi^{1-\gamma}])^{T-t+1}}{1-\beta\mathcal{G}^{1-\gamma}\mathbb{E}[\psi^{1-\gamma}]}\right),$$

The function $\mathbf{v}_t$ will be finite as T approaches ∞ if the FVAC holds. In the case without uncertainty, Because $\mathrm{u}(xy) = x^{1-\gamma}\mathrm{u}(y)$, the value the consumer would achieve is:

$$\begin{aligned}\mathbf{v}_t^{\text{autarky}} &= \mathrm{u}(\boldsymbol{p}_t) + \beta\mathrm{u}(\boldsymbol{p}_t\mathcal{G}) + \beta^2\mathrm{u}(\boldsymbol{p}_t\mathcal{G}^2) + \ldots \\ &= \mathrm{u}(\boldsymbol{p}_t)\left(\frac{1-(\beta\mathcal{G}^{1-\gamma})^{T-t+1}}{1-\beta\mathcal{G}^{1-\gamma}}\right)\end{aligned}$$

which (for $\mathcal{G} > 0$) asymptotes to a finite number as n, with $n = T - t$, approaches $+\infty$.

1.1 Perfect Foresight Unconstrained Solution

{subsec:ApndxUCP

The first result relates to the perfect foresight case without liquidity constraints.

Proof. (Proof of Proposition 1) Consider a sequence of consumption $\{\boldsymbol{c}_{T-n}\}_{n=t}^{T}$ and a sequence of income $\{\boldsymbol{p}_{T-n}\}_{n=t}^{T}$ and let $\mathrm{PDV}_t^T(\boldsymbol{c})$ and $\mathrm{PDV}_t^T(\boldsymbol{p})$ denote the present discounted value of the consumption sequence and permanent income sequence respectively. The dynamic budget constraint, strictly positive marginal utility, and the can't-die-in-debt condition, Equation (1), imply an exactly-holding intertemporal budget constraint (IBC):

$$\mathrm{PDV}_t^T(\boldsymbol{c}) = \overbrace{\boldsymbol{m}_t - \boldsymbol{p}_t}^{\boldsymbol{b}_t} + \overbrace{\mathrm{PDV}_t^T(\boldsymbol{p})}^{\boldsymbol{h}_t}, \tag{1}$$

where $\boldsymbol{b}$ is beginning-of-period 'market' balances; with $\tilde{\mathfrak{R}} := \mathsf{R}/\mathcal{G}$ 'human wealth' can be written as:

$$\begin{aligned}\boldsymbol{h}_t &= \boldsymbol{p}_t + \tilde{\mathfrak{R}}^{-1}\boldsymbol{p}_t + \tilde{\mathfrak{R}}^{-2}\boldsymbol{p}_t + \cdots + \tilde{\mathfrak{R}}^{t-T}\boldsymbol{p}_t \\ &= \underbrace{\left(\frac{1-\tilde{\mathfrak{R}}^{-(T-t+1)}}{1-\tilde{\mathfrak{R}}^{-1}}\right)}_{\equiv h_t}\boldsymbol{p}_t. \end{aligned}\tag{2}$$

Let h denote the limiting value of normalized human wealth as the planning horizon recedes, we have $h := \lim\limits_{n\to\infty} h_{t_n}$.

Next, since consumption is growing by $\text{\TH}$ but discounted by R:

$$\text{PDV}_t^T(\boldsymbol{c}) = \left(\frac{1 - \frac{\text{\TH}}{\mathsf{R}}^{T-t+1}}{1 - \frac{\text{\TH}}{\mathsf{R}}}\right) \boldsymbol{c}_t$$

from which the IBC (1) implies

$$\boldsymbol{c}_t = \overbrace{\left(\frac{1 - \frac{\text{\TH}}{\mathsf{R}}}{1 - \frac{\text{\TH}}{\mathsf{R}}^{T-t+1}}\right)}^{\equiv \underline{\kappa}_t} (\boldsymbol{b}_t + \boldsymbol{h}_t) \tag{3}$$

defining a normalized finite-horizon perfect foresight consumption function:

$$\bar{\mathrm{c}}_{T-n}(m_{T-n}) = (\overbrace{m_{T-n} - 1}^{\equiv b_{T-n}} + h_{T-n})\underline{\kappa}_{t-n}$$

where $\underline{\kappa}_t$ is the marginal propensity to consume (MPC). (The overbar signifies that $\bar{\mathrm{c}}$ will be an upper bound as we modify the problem to incorporate constraints and uncertainty; analogously, $\underline{\kappa}$ is the MPC's lower bound).

The horizon-exponentiated term in the denominator of (3) is why, for $\underline{\kappa}$ to be strictly positive as n goes to infinity, we must impose the RIC. The RIC thus implies that the consumer cannot be so pathologically patient as to wish, in the limit as the horizon approaches infinity, to spend nothing today out of an increase in current wealth (the RIC rules out the degenerate limiting solution $\bar{\mathrm{c}}(m) = 0$).

Given that the RIC holds, and (as before) defining limiting objects by the absence of a time subscript, the limiting upper bound consumption function will be

$$\bar{\mathrm{c}}(m) = (m + h - 1)\underline{\kappa}, \tag{4}$$

and so in order to rule out the degenerate limiting solution $\bar{\mathrm{c}}(m) = \infty$ we need h to be finite; that is, we must impose the Finite Human Wealth Condition (FHWC), Assumption (I.3).

□

Under perfect foresight in the presence of a liquidity constraint requiring $b \geq 0$, this appendix taxonomizes the varieties of the limiting consumption function $\grave{\mathrm{c}}(m)$ that arise under various parametric conditions.

Results are summarized in table 1.

1.2 If GIC Fails

A consumer is 'growth patient' if the perfect foresight growth impatience condition fails ($\cancel{\text{GIC}}$, $1 < \text{\TH}/\mathcal{G}$). Under $\cancel{\text{GIC}}$ the constraint does not bind at the lowest feasible value of $m_t = 1$ because $1 < (\mathsf{R}\beta)^{1/\gamma}/\mathcal{G}$ implies that spending everything today (setting

$c_t = m_t = 1$) produces lower marginal utility than is obtainable by reallocating a marginal unit of resources to the next period at return R:[1]

$$\begin{aligned} 1 &< (\mathsf{R}\beta)^{1/\gamma}\mathcal{G}^{-1} \\ 1 &< \mathsf{R}\beta\mathcal{G}^{-\gamma} \\ \mathrm{u}'(1) &< \mathsf{R}\beta\mathrm{u}'(\mathcal{G}). \end{aligned}$$

Similar logic shows that under these circumstances the constraint will never bind at $m = 1$ for a constrained consumer with a finite horizon of n periods, so for $m \geq 1$ such a consumer's consumption function will be the same as for the unconstrained case examined in the main text.

RIC fails, FHWC holds. If the RIC fails ($1 < \frac{\text{Þ}}{\mathsf{R}}$) while the finite human wealth condition holds, the limiting value of this consumption function as $n \uparrow \infty$ is the degenerate function

$$\grave{\mathrm{c}}_{T-n}(m) = 0(b_t + h). \tag{5}$$

(that is, consumption is zero for any level of human or nonhuman wealth).

RIC fails, FHWC fails. $\cancel{\text{FHWC}}$ implies that human wealth limits to $h = \infty$ so the consumption function limits to either $\grave{\mathrm{c}}_{T-n}(m) = 0$ or $\grave{\mathrm{c}}_{T-n}(m) = \infty$ depending on the relative speeds with which the MPC approaches zero and human wealth approaches ∞.[2]

Thus, the requirement that the consumption function be nondegenerate implies that for a consumer satisfying $\cancel{\text{GIC}}$ we must impose the RIC (and the FHWC can be shown to be a consequence of $\cancel{\text{GIC}}$ and RIC). In this case, the consumer's optimal behavior is easy to describe. We can calculate the point at which the unconstrained consumer would choose $c = m$ from Equation (14):

$$\begin{aligned} m_{\#} &= (m_{\#} - 1 + h)\underline{\kappa} \\ m_{\#}(1 - \underline{\kappa}) &= (h - 1)\underline{\kappa} \\ m_{\#} &= (h - 1)\left(\frac{\underline{\kappa}}{1 - \underline{\kappa}}\right) \end{aligned} \tag{6}$$

which (under these assumptions) satisfies $0 < m_{\#} < 1$.[3] For $m < m_{\#}$ the unconstrained consumer would choose to consume more than m; for such m, the constrained consumer is obliged to choose $\grave{\mathrm{c}}(m) = m$.[4] For any $m > m_{\#}$ the constraint will never bind and the consumer will choose to spend the same amount as the unconstrained consumer, $\bar{\mathrm{c}}(m)$.

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[1] The point at which the constraint would bind (if that point could be attained) is the $m = c$ for which $\mathrm{u}'(c_{\#}) = \mathsf{R}\beta\mathrm{u}'(\mathcal{G})$ which is $c_{\#} = \mathcal{G}/(\mathsf{R}\beta)^{1/\gamma}$ and the consumption function will be defined by $\grave{\mathrm{c}}(m) = \min[m, c_{\#} + (m - c_{\#})\underline{\kappa}]$.

[2] The knife-edge case is where $\text{Þ} = \mathcal{G}$, in which case the two quantities counterbalance and the limiting function is $\grave{\mathrm{c}}(m) = \min[m, 1]$.

[3] Note that $0 < m_{\#}$ is implied by RIC and $m_{\#} < 1$ is implied by $\cancel{\text{GIC}}$.

[4] As an illustration, consider a consumer for whom $\text{Þ} = 1$, $\mathsf{R} = 1.01$ and $\mathcal{G} = 0.99$. This consumer will save the amount necessary to ensure that growth in market wealth exactly offsets the decline in human wealth represented by $\mathcal{G} < 1$; total wealth (and therefore total consumption) will remain constant, even as market wealth and human wealth trend in opposite directions.

(Stachurski and Toda [2019] obtain a similar lower bound on consumption and use it to study the tail behavior of the wealth distribution.)

1.3 If GIC Holds

Imposition of the GIC reverses the inequality in (5), and thus reverses the conclusion: A consumer who starts with $m_t = 1$ will desire to consume more than 1. Such a consumer will be constrained, not only in period t, but perpetually thereafter.

Now define $b^n_{\#}$ as the b_t such that an unconstrained consumer holding $b_t = b^n_{\#}$ would behave so as to arrive in period $t+n$ with $b_{t+n} = 0$ (with $b^0_{\#}$ trivially equal to 0); for example, a consumer with $b_{t-1} = b^1_{\#}$ was on the 'cusp' of being constrained in period $t-1$: Had b_{t-1} been infinitesimally smaller, the constraint would have been binding (because the consumer would have desired, but been unable, to enter period t with negative, not zero, b). Given the GIC, the constraint certainly binds in period t (and thereafter) with resources of $m_t = m^0_{\#} = 1 + b^0_{\#} = 1$: The consumer cannot spend more (because constrained), and will not choose to spend less (because impatient), than $c_t = c^0_{\#} = 1$.

We can construct the entire 'prehistory' of this consumer leading up to t as follows. Maintaining the assumption that the constraint has never bound in the past, c must have been growing according to $\frac{\text{Þ}}{\mathcal{G}}$, so consumption n periods in the past must have been

$$c^n_{\#} = \frac{\text{Þ}}{\mathcal{G}}^{-n} c_t = \frac{\text{Þ}}{\mathcal{G}}^{-n}. \tag{7}$$

The PDV of consumption from $t-n$ until t can thus be computed as

$$\begin{aligned}
\mathbb{C}^t_{t-n} &= c_{t-n}(1 + \text{Þ}/\mathsf{R} + \cdots + (\text{Þ}/\mathsf{R})^n) \\
&= c^n_{\#}(1 + \frac{\text{Þ}}{\mathsf{R}} + \cdots + \frac{\text{Þ}}{\mathsf{R}}^n) \\
&= \frac{\text{Þ}}{\mathcal{G}}^{-n}\left(\frac{1 - \frac{\text{Þ}}{\mathsf{R}}^{n+1}}{1 - \frac{\text{Þ}}{\mathsf{R}}}\right) \\
&= \left(\frac{\frac{\text{Þ}}{\mathcal{G}}^{-n} - \frac{\text{Þ}}{\mathsf{R}}}{1 - \frac{\text{Þ}}{\mathsf{R}}}\right)
\end{aligned}$$

and note that the consumer's human wealth between $t-n$ and t (the relevant time horizon, because from t onward the consumer will be constrained and unable to access post-t income) is

$$h^n_{\#} = 1 + \cdots + \tilde{\mathcal{R}}^{-n} \tag{8}$$

while the intertemporal budget constraint says

$$\mathbb{C}^t_{t-n} \quad = b^n_{\#} + h^n_{\#}$$

from which we can solve for the $b^n_{\#}$ such that the consumer with $b_{t-n} = b^n_{\#}$ would unconstrainedly plan (in period $t-n$) to arrive in period t with $b_t = 0$:

$$b^n_{\#} = \mathbb{C}^t_{t-n} - \overbrace{\left(\frac{1-\tilde{\mathcal{R}}^{-(n+1)}}{1-\tilde{\mathcal{R}}^{-1}}\right)}^{h^n_{\#}}. \tag{9}$$

Defining $m^n_{\#} = b^n_{\#} + 1$, consider the function $\grave{\mathrm{c}}(m)$ defined by linearly connecting the points $\{m^n_{\#}, c^n_{\#}\}$ for integer values of $n \geq 0$ (and setting $\grave{\mathrm{c}}(m) = m$ for $m < 1$). This function will return, for any value of m, the optimal value of c for a liquidity constrained consumer with an infinite horizon. The function is piecewise linear with 'kink points' where the slope discretely changes; for infinitesimal ϵ the MPC of a consumer with assets $m = m^n_{\#} - \epsilon$ is discretely higher than for a consumer with assets $m = m^n_{\#} + \epsilon$ because the latter consumer will spread a marginal dollar over more periods before exhausting it.

In order for a unique consumption function to be defined by this sequence (9) for the entire domain of positive real values of b, we need $b^n_{\#}$ to become arbitrarily large with n. That is, we need

$$\lim_{n\to\infty} b^n_{\#} = \infty. \tag{10}$$

1.3.1 If FHWC Holds

The FHWC requires $\tilde{\mathcal{R}}^{-1} < 1$, in which case the second term in (9) limits to a constant as $n \uparrow \infty$, and (10) reduces to a requirement that

$$\begin{aligned}
\lim_{n\to\infty} \left(\frac{\frac{\text{Þ}}{\mathcal{G}}^{-n} - \left(\frac{\text{Þ}}{\mathsf{R}}/\frac{\text{Þ}}{\mathcal{G}}\right)^n \frac{\text{Þ}}{\mathsf{R}}}{1-\frac{\text{Þ}}{\mathsf{R}}}\right) &= \infty \\
\lim_{n\to\infty} \left(\frac{\frac{\text{Þ}}{\mathcal{G}}^{-n} - \tilde{\mathcal{R}}^{-n}\frac{\text{Þ}}{\mathsf{R}}}{1-\frac{\text{Þ}}{\mathsf{R}}}\right) &= \infty \\
\lim_{n\to\infty} \left(\frac{\frac{\text{Þ}}{\mathcal{G}}^{-n}}{1-\frac{\text{Þ}}{\mathsf{R}}}\right) &= \infty.
\end{aligned}$$

Given the GIC $\frac{\text{Þ}}{\mathcal{G}}^{-1} > 1$, this will hold iff the RIC holds, $\frac{\text{Þ}}{\mathsf{R}} < 1$. But given that the FHWC $\mathsf{R} > \mathcal{G}$ holds, the GIC is stronger (harder to satisfy) than the RIC; thus, the FHWC and the GIC together imply the RIC, and so a well-defined solution exists. Furthermore, in the limit as n approaches infinity, the difference between the limiting constrained consumption function and the unconstrained consumption function becomes vanishingly small, because the date at which the constraint binds becomes arbitrarily distant, so the effect of that constraint on current behavior shrinks to nothing. That is,

$$\lim_{m\to\infty} \grave{\mathrm{c}}(m) - \bar{\mathrm{c}}(m) = 0. \tag{11}$$

1.3.2 If FHWC Fails

If the FHWC fails, matters are a bit more complex.

Given failure of FHWC, (10) requires

$$\lim_{n\to\infty}\left(\frac{\tilde{\mathfrak{R}}^{-n}\frac{\mathbf{Þ}}{\mathsf{R}}-\frac{\mathbf{Þ}}{\mathcal{G}}^{-n}}{\frac{\mathbf{Þ}}{\mathsf{R}}-1}\right)+\left(\frac{1-\tilde{\mathfrak{R}}^{-(n+1)}}{\tilde{\mathfrak{R}}^{-1}-1}\right)=\infty$$

$$\lim_{n\to\infty}\left(\frac{\frac{\mathbf{Þ}}{\mathsf{R}}}{\frac{\mathbf{Þ}}{\mathsf{R}}-1}-\frac{\tilde{\mathfrak{R}}^{-1}}{\tilde{\mathfrak{R}}^{-1}-1}\right)\tilde{\mathfrak{R}}^{-n}-\left(\frac{\frac{\mathbf{Þ}}{\mathcal{G}}^{-n}}{\frac{\mathbf{Þ}}{\mathsf{R}}-1}\right)=\infty$$

If RIC Holds. When the RIC holds, rearranging (12) gives

$$\lim_{n\to\infty}\left(\frac{\frac{\mathbf{Þ}}{\mathcal{G}}^{-n}}{1-\frac{\mathbf{Þ}}{\mathsf{R}}}\right)-\tilde{\mathfrak{R}}^{-n}\left(\frac{\frac{\mathbf{Þ}}{\mathsf{R}}}{1-\frac{\mathbf{Þ}}{\mathsf{R}}}+\frac{\tilde{\mathfrak{R}}^{-1}}{\tilde{\mathfrak{R}}^{-1}-1}\right)\quad=\infty$$

and for this to be true we need

$$\begin{aligned}\frac{\mathbf{Þ}}{\mathcal{G}}^{-1} &> \tilde{\mathfrak{R}}^{-1}\\ \mathcal{G}/\mathbf{Þ} &> \mathcal{G}/\mathsf{R}\\ 1 &> \mathbf{Þ}/\mathsf{R}\end{aligned}$$

which is merely the RIC again. So the problem has a solution if the RIC holds. Indeed, we can even calculate the limiting MPC from

$$\lim_{n\to\infty}\kappa_{\#}^{n}=\lim_{n\to\infty}\left(\frac{c_{\#}^{n}}{b_{\#}^{n}}\right)\tag{12}$$

which with a bit of algebra[5] can be shown to asymptote to the MPC in the perfect foresight model:[6]

$$\lim_{m\to\infty}\grave{\boldsymbol{\kappa}}(m)=1-\frac{\mathbf{Þ}}{\mathsf{R}}.\tag{14}$$

[5]Calculate the limit of

$$\left(\frac{\frac{\mathbf{Þ}}{\mathcal{G}}^{-n}}{\frac{\mathbf{Þ}}{\mathcal{G}}^{-n}/(1-\frac{\mathbf{Þ}}{\mathsf{R}})-(1-\tilde{\mathfrak{R}}^{-1}\tilde{\mathfrak{R}}^{-n})/(1-\tilde{\mathfrak{R}}^{-1})}\right)=\left(\frac{1}{1/(1-\frac{\mathbf{Þ}}{\mathsf{R}})+\tilde{\mathfrak{R}}^{-n}\tilde{\mathfrak{R}}^{-1}/(1-\tilde{\mathfrak{R}}^{-1})}\right)\tag{13}$$

[6]For an example of this configuration of parameters, see the notebook `doApndxLiqConstr.nb` in the Mathematica software archive.

If RIC Fails. Consider now the ~~RIC~~ case, $\frac{\text{Þ}}{\mathsf{R}} > 1$. We can rearrange (12)as

$$\lim_{n\to\infty}\left(\frac{\frac{\text{Þ}}{\mathsf{R}}(\tilde{\mathcal{R}}^{-1}-1)}{(\tilde{\mathcal{R}}^{-1}-1)(\frac{\text{Þ}}{\mathsf{R}}-1)}-\frac{\tilde{\mathcal{R}}^{-1}(\frac{\text{Þ}}{\mathsf{R}}-1)}{(\tilde{\mathcal{R}}^{-1}-1)(\frac{\text{Þ}}{\mathsf{R}}-1)}\right)\tilde{\mathcal{R}}^{-n}-\left(\frac{\frac{\text{Þ}}{\mathcal{G}}^{-n}}{\frac{\text{Þ}}{\mathsf{R}}-1}\right)=\infty. \tag{15}$$

which makes clear that with ~~FHWC~~ $\Rightarrow \tilde{\mathcal{R}}^{-1} > 1$ and ~~RIC~~ $\Rightarrow \frac{\text{Þ}}{\mathsf{R}} > 1$ the numerators and denominators of both terms multiplying $\tilde{\mathcal{R}}^{-n}$ can be seen transparently to be positive. So, the terms multiplying $\tilde{\mathcal{R}}^{-n}$ in (12) will be positive if

$$\begin{aligned}\frac{\text{Þ}}{\mathsf{R}}\tilde{\mathcal{R}}^{-1}-\frac{\text{Þ}}{\mathsf{R}} &> \tilde{\mathcal{R}}^{-1}\frac{\text{Þ}}{\mathsf{R}}-\tilde{\mathcal{R}}^{-1}\\ \tilde{\mathcal{R}}^{-1} &> \frac{\text{Þ}}{\mathsf{R}}\\ \mathcal{G} &> \text{Þ}\end{aligned}$$

which is merely the GIC which we are maintaining. So the first term's limit is $+\infty$. The combined limit will be $+\infty$ if the term involving $\tilde{\mathcal{R}}^{-n}$ goes to $+\infty$ faster than the term involving $-\frac{\text{Þ}}{\mathcal{G}}^{-n}$ goes to $-\infty$; that is, if

$$\begin{aligned}\tilde{\mathcal{R}}^{-1} &> \frac{\text{Þ}}{\mathcal{G}}^{-1}\\ \mathcal{G}/\mathsf{R} &> \mathcal{G}/\text{Þ}\\ \text{Þ}/\mathsf{R} &> 1\end{aligned}$$

which merely confirms the starting assumption that the RIC fails.

What is happening here is that the $c_{\#}^{n}$ term is increasing backward in time at rate dominated in the limit by $\mathcal{G}/\text{Þ}$ while the $b_{\#}$ term is increasing at a rate dominated by $\mathcal{G}/\mathsf{R}$ term and

$$\mathcal{G}/\mathsf{R} > \mathcal{G}/\text{Þ} \tag{16}$$

because ~~RIC~~ $\Rightarrow \text{Þ} > \mathsf{R}$.

Consequently, while $\lim_{n\uparrow\infty} b_{\#}^{n} = \infty$, the limit of the *ratio* $c_{\#}^{n}/b_{\#}^{n}$ in (12) is zero. Thus, surprisingly, the problem has a well defined solution with infinite human wealth if the RIC fails. It remains true that ~~RIC~~ implies a limiting MPC of zero,

$$\lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = 0, \tag{17}$$

but that limit is approached gradually, starting from a positive value, and consequently the consumption function is *not* the degenerate $\grave{\mathrm{c}}(m) = 0$. (Figure 1 presents an example for $\gamma = 2$, $\mathsf{R} = 0.98$, $\beta = 1.00$, $\mathcal{G} = 0.99$; note that the horizontal axis is bank balances $b = m-1$; the part of the consumption function below the depicted points is uninteresting — $c = m$ — so not worth plotting).

We can summarize as follows. Given that the GIC holds, the interesting question is

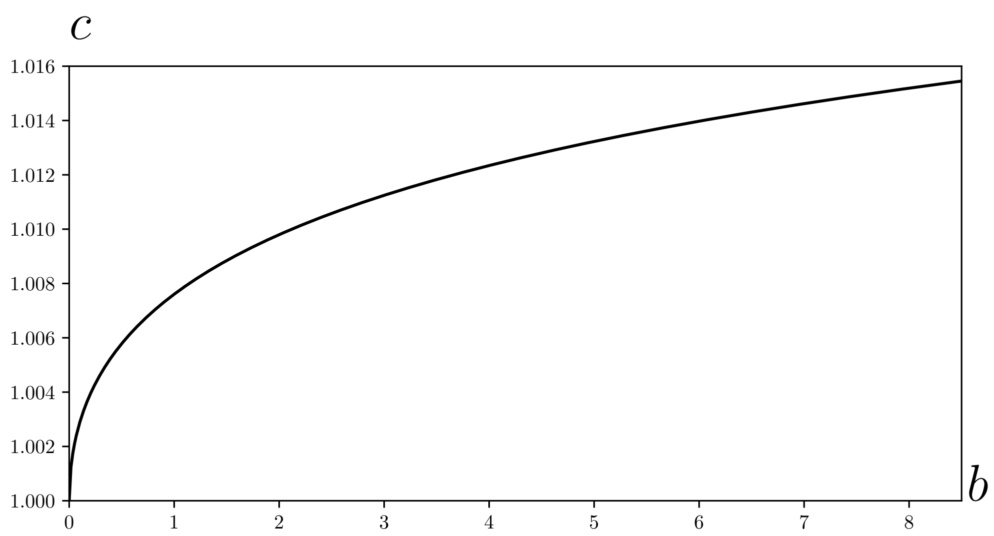

Figure 1 Appendix: Nondegenerate c Function with ~~FHWC~~ and ~~RIC~~ {fig:PFGICHoldsFH

whether the FHWC holds. If so, the RIC automatically holds, and the solution limits into the solution to the unconstrained problem as $m \uparrow \infty$. But even if the FHWC fails, the problem has a well-defined and nondegenerate solution, whether or not the RIC holds.

Although these results were derived for the perfect foresight case, we know from work elsewhere in this paper and in other places that the perfect foresight case is an upper bound for the case with uncertainty. If the upper bound of the MPC in the perfect foresight case is zero, it is not possible for the upper bound in the model with uncertainty to be greater than zero, because for any $\kappa > 0$ the level of consumption in the model with uncertainty would eventually exceed the level of consumption in the absence of uncertainty.

Ma and Toda [2020] characterize the limits of the MPC in a more general framework that allows for capital and labor income risks in a Markovian setting with liquidity constraints, and find that in that much more general framework the limiting MPC is also zero.

References

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John Stachurski and Alexis Akira Toda. An impossibility theorem for wealth in heterogeneous-agent models with limited heterogeneity. Journal of Economic Theory, 182:1–24, July 2019. doi: 10.1016/j.jet.2019.04.001.

Table 1 Appendix: Perfect Foresight Liquidity Constrained Taxonomy

{table:LiqConstrSce

For constrained $\grave{c}$ and unconstrained $\bar{\mathrm{c}}$ consumption functions

Main Condition Subcondition	Math	Outcome, Comments or Results
$\cancel{\text{GIC}}$	$1 < \mathbf{Þ}/\mathcal{G}$	Constraint never binds for $m \geq 1$
and RIC	$\mathbf{Þ}/\mathsf{R} < 1$	FHWC holds ($\mathsf{R} > \mathcal{G}$); $\grave{\mathrm{c}}(m) = \bar{\mathrm{c}}(m)$ for $m \geq 1$
and $\cancel{\text{RIC}}$	$1 < \mathbf{Þ}/\mathsf{R}$	$\grave{\mathrm{c}}(m)$ is degenerate: $\grave{\mathrm{c}}(m) = 0$
GIC	$\mathbf{Þ}/\mathcal{G} < 1$	Constraint binds in finite time $\forall\ m$
and RIC	$\mathbf{Þ}/\mathsf{R} < 1$	FHWC may or may not hold $\lim_{m\uparrow\infty} \bar{\mathrm{c}}(m) - \grave{\mathrm{c}}(m) = 0$ $\lim_{m\uparrow\infty} \grave{\boldsymbol{\kappa}}(m) = \underline{\kappa}$
and $\cancel{\text{RIC}}$	$1 < \mathbf{Þ}/\mathsf{R}$	$\cancel{\text{FHWC}}$ $\lim_{m\uparrow\infty} \grave{\boldsymbol{\kappa}}(m) = 0$

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where $\cancel{\text{GIC}}$ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GICholds, the constraint will bind in finite time.