

Instrumental Variables

EC 425/525, Set 8

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Prologue

Schedule

Last time

Matching and propensity-score methods

- Conditional independence
- Overlap

Today

Instrumental variables (and two-stage least squares)

Upcoming

- Assignment due Sunday
- Proposal due Wednesday 5/22
- Midterm?

Research designs

Research designs

Selection on observables and/or unobservables

We've been focusing on ***selection-on-observables designs***, i.e.,

$$(Y_{0i}, Y_{1i}) \perp\!\!\!\perp D_i | X_i$$

for **observable** variables X_i .

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Selection-on-unobservable designs replace this assumption with two new (but related) assumptions

1. $(Y_{0i}, Y_{1i}) \perp Z_i$
2. $\text{Cov}(Z_i, D_i) \neq 0$

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Our main goal in causal-inference minded (applied) econometrics boils down to isolating **"good" variation** in D_i (exogenous/as-good-as-random) from **"bad" variation** (the part of D_i correlated with Y_{0i} and Y_{1i}).

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Seems more plausible. Possible to validate. May be underpowered.

Instrumental variables

Introduction

Instrumental variables (IV)[†] is the canonical selection-on-unobservables design—isolating *good variation* in D_i via some magical instrument Z_i .

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Consider some model (structural equation)

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i \quad (1)$$

To guarantee consistent OLS estimates for β_1 , want $\text{Cov}(\mathbf{D}_i, \varepsilon_i) = 0$.
In general, this is a heroic assumption.

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Alternative: Estimate β_1 via instrumental variables.

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Instrumental variables

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Let **Lottery**_{*i*} denote an indicator for whether *i* won a lottery scholarship.[†]

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1. $\text{Cov}(\text{Lottery}_i, \text{Grad}_i) \neq 0 (> 0)$ if scholarships increase grad. rates.
2. $\text{Cov}(\text{Lottery}_i, \varepsilon_i) = 0$ since the lottery is randomized.

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Instrument variables

The IV estimator

The IV estimator for our model

$$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i \quad (1)$$

with (valid) instrument Z_i is

$$\hat{\beta}_{IV} = (Z'D)^{-1} (Z'Y)$$

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If you have no covariates, then

$$\hat{\beta}_{IV} = \frac{\text{Cov}(Z_i, Y_i)}{\text{Cov}(Z_i, D_i)}$$

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If you have additional (exogenous) covariates X_i , then

$$Z = [Z_i \quad X_i]$$

$$D = [D_i \quad X_i]$$

Instrumental variables

Proof: Consistency

With a valid instrument Z_i , $\hat{\beta}_{IV}$ is a consistent estimator for β_1 in

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$$\text{plim}(\hat{\beta}_{IV})$$

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$$= \beta \quad \checkmark$$

Two-stage least squares

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First stage Estimate the effect of the instrument \mathbf{Z}_i on our endogenous variable \mathbf{D}_i and (predetermined) covariates \mathbf{X}_i . Save $\widehat{\mathbf{D}}_i$.

$$\mathbf{D}_i = \gamma_1 \mathbf{Z}_i + \gamma_2 \mathbf{X}_i + u_i$$

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Second stage Estimate model we wanted—but only using the variation in \mathbf{D}_i that correlates with \mathbf{Z}_i , i.e., $\widehat{\mathbf{D}}_i$.

$$\mathbf{Y}_i = \beta_1 \widehat{\mathbf{D}}_i + \beta_2 \mathbf{X}_i + \varepsilon_i$$

Note The controls \mathbf{X}_i must match in the first and second stages.

Two-stage least squares

IV estimation

This two-step procedure, with a valid instrument, produces an estimator $\hat{\beta}_1$ that is consistent for β_1 .

$$\hat{\beta}_{2SLS} = (D' P_Z D)^{-1} (D' P_Z Y)$$

$$P_Z = Z(Z'Z)^{-1}Z'$$

where D is a matrix of our treatment and predetermined covariates (X_i) and Z is a matrix of our instrument and our predetermined covariates.

Two-stage least squares

IV estimation

Important notes

- The controls (X_i) must match in the first and second stages.
- If you have exactly **one instrument** and exactly **one endogenous variable**, then 2SLS and IV are identical.
- Your second-stage standard errors are not correct.

Two-stage least squares

The reduced form

In addition to the regressions within the two stages of 2SLS

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there is a third important and related regression: the reduced form.

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The **reduced form** regresses the outcome Y_i (LHS of the second stage) on our instrument Z_i and covariates X_i (RHS of the first stage).

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Thus, the reduced form provides a consistent estimate of the causal effect of our instrument on the outcome.

Two-stage least squares

The reduced form, continued

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$$\hat{\beta}_1^{\text{2SLS}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

when you have exactly one instrument.

Two-stage least squares

The reduced form, intuition

This expression for the 2SLS (and IV) estimator can be very helpful.

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$\hat{\gamma}_1$ estimates the effect of winning the scholarship lottery on graduation—the share of winners who graduated due to winning.

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$\hat{\gamma}_1$ estimates the effect of winning the scholarship lottery on graduation—the share of winners who graduated due to winning. We can scale with $\hat{\gamma}_1$!

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The reduced form, example

To see why this scaling makes sense, imagine that 50% of lottery winners graduate from college due to the lottery, i.e., $\hat{\gamma}_1 = 0.50$.[†]

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However, half of the winners did not graduate, so $\hat{\pi}_1$ "underestimates" the effect of college graduation by combining graduates by nongraduates.

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However, half of the winners did not graduate, so $\hat{\pi}_1$ "underestimates" the effect of college graduation by combining graduates by nongraduates.

Thus, we want to double $\hat{\pi}_1$, i.e., divide by $\hat{\gamma}_1$: $\hat{\pi}_1 / \hat{\gamma}_1 = \$5,000 / 0.5 = \$10,000.$

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$$= \frac{\hat{\pi}_1}{\hat{\gamma}_1} \quad \checkmark$$

Let's push a bit deeper into IV's mechanics and intuition.

IV: Mechanics and intuition

Setup

In this section, we'll use medical trials as a working example.[†]

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$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i$$

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\mathbf{D}_i indicates whether i takes the treatment (medication). ε_i captures all other factors that affect \mathbf{Y}_i . Or in potential-outcomes framework:

$$\mathbf{Y}_i = \mathbf{Y}_{1i} \mathbf{D}_i + \mathbf{Y}_{0i} (1 - \mathbf{D}_i)$$

$$\mathbf{Y}_{0i} = \beta_0 + \varepsilon_i$$

$$\mathbf{Y}_{1i} = \mathbf{Y}_{0i} + \beta_1$$

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IV: Mechanics and intuition

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Goal **Estimate the effect of blood-pressure medication** on blood pressure.

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Analysis 2 **IV!** Instrument medication D_i with intention to treat Z_i .

IV: Mechanics and intuition

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IV: Mechanics and intuition

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 $\therefore Z_i$ is a valid instrument for D_i and IV consistently estimates β_1 .

IV: Mechanics and intuition

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Let's see how IV "solves" this problem.

First, assume noncompliance only affects treated individuals—*i.e.*, treated folks sometimes don't take their pills; control folks never take pills.

IV: Mechanics and intuition

Noncompliance, continued

The **first stage** recovers the share of treatment individuals who take the pill

$$D_i = \gamma_1 Z_i + u_i$$

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which we know IV rescales using the first stage

$$\hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{\hat{\pi}_1}{0.50} = 2 \times \hat{\pi}_1$$

IV: Mechanics and intuition

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If everyone perfectly complies, then $\hat{\gamma}_1 = 1$ and $\hat{\beta}_1^{\text{IV}} = \hat{\pi}_1/1 = \hat{\beta}_1^{\text{ITT}}$.

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Further example $N_{\text{Trt}} = 10$; trt. compliance = 50%; ctrl. compliance = 100%.

$$\bar{\mathbf{Y}}_{\text{Trt}} = \frac{5(\beta_0 + \beta_1) + 5(\beta_0)}{10} = \beta_0 + \frac{\beta_1}{2}$$

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IV consistently estimates β_1 via rescaling the ITT by the rate of compliance

$$\hat{\beta}_1^{\text{IV}} = \frac{\pi}{\gamma} = \frac{\beta_1/2}{1/2} = \beta_1$$

IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of Z_i on Y_i by the causal effect of Z_i on D_i .

IV: Mechanics and intuition

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IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of Z_i on Y_i by the causal effect of Z_i on D_i .
2. IV **does not** compare treated compliers to untreated compliers.
Such a comparison/estimator would re-introduce selection bias.

Thus far, we assumed homogeneous treatment effects.

Q What happens **when treatment effects are heterogeneous?**

A Let's recall what our instruments are doing (with Venn diagrams!).

Credit [Glen Waddell](#) introduced me to IV via Venn.

Figure 1

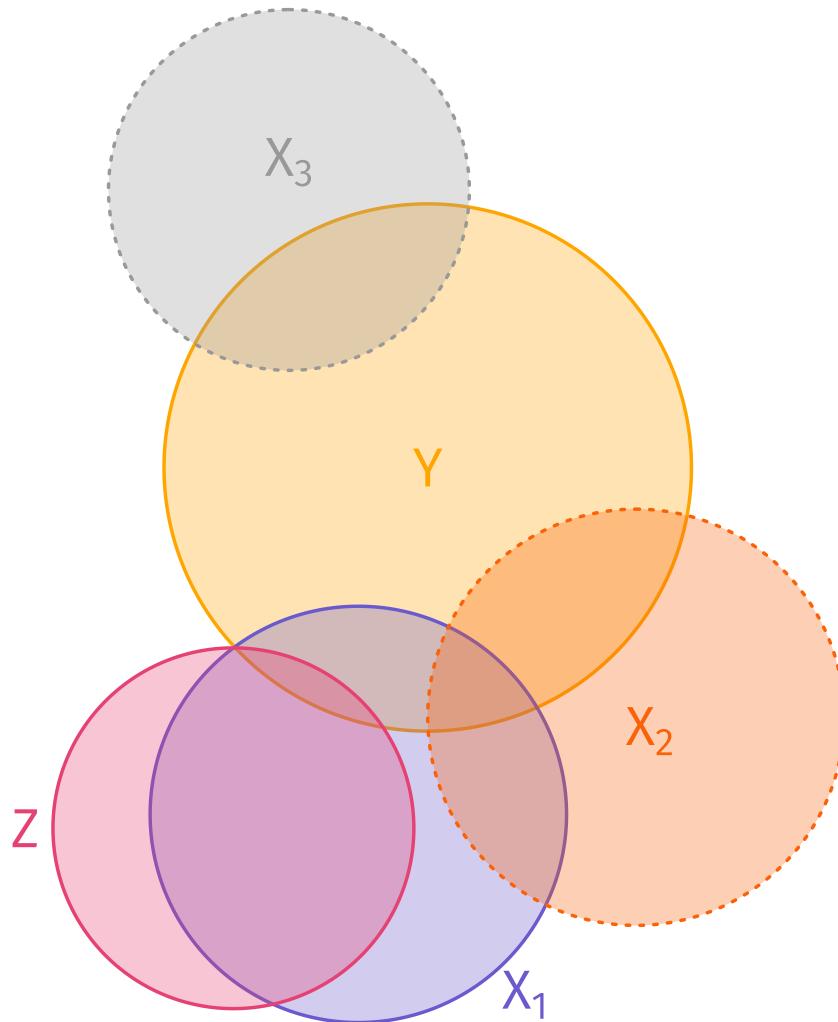


Figure 2

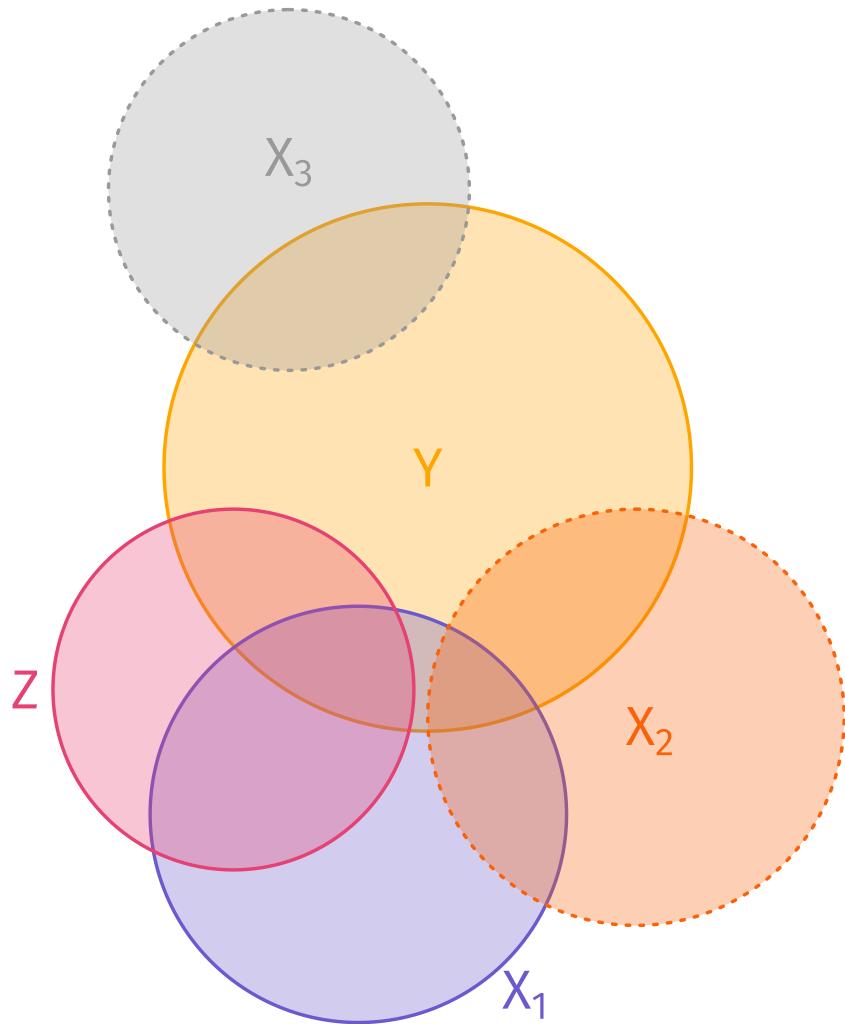


Figure 3

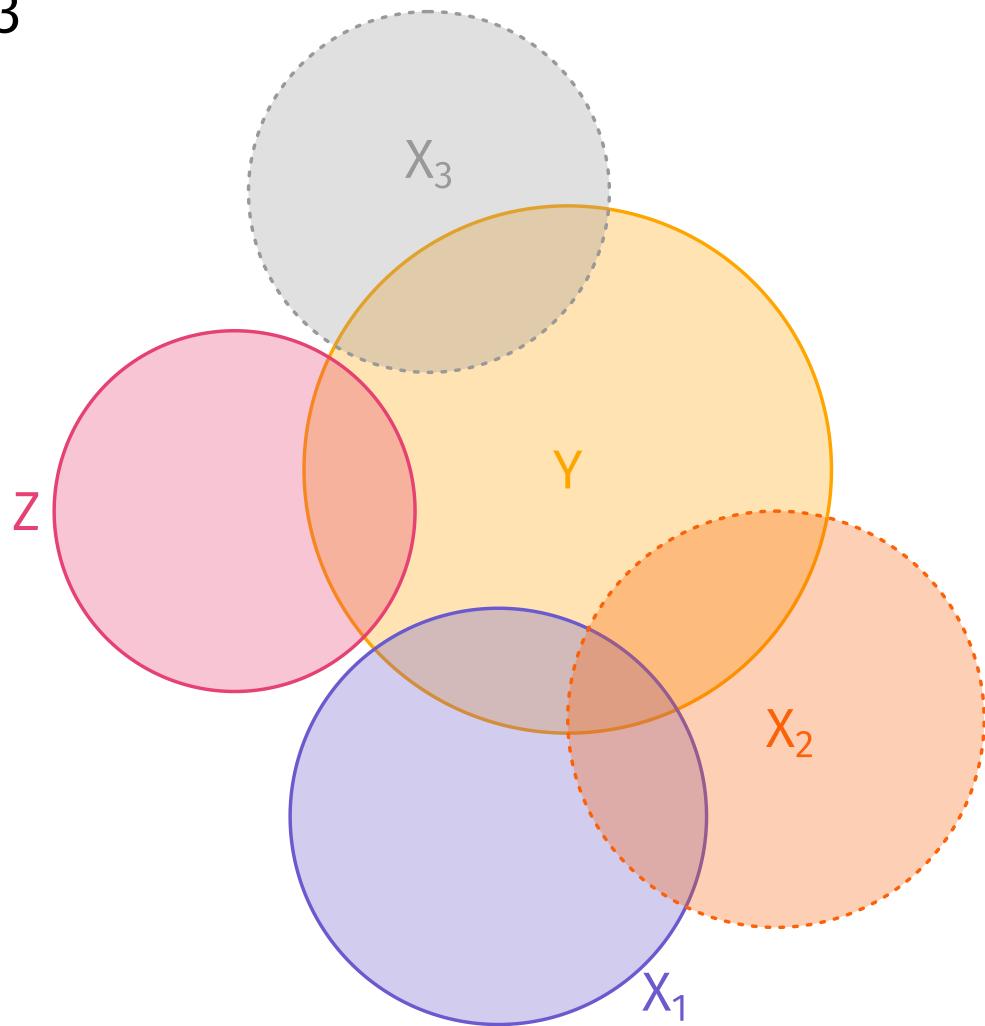


Figure 4

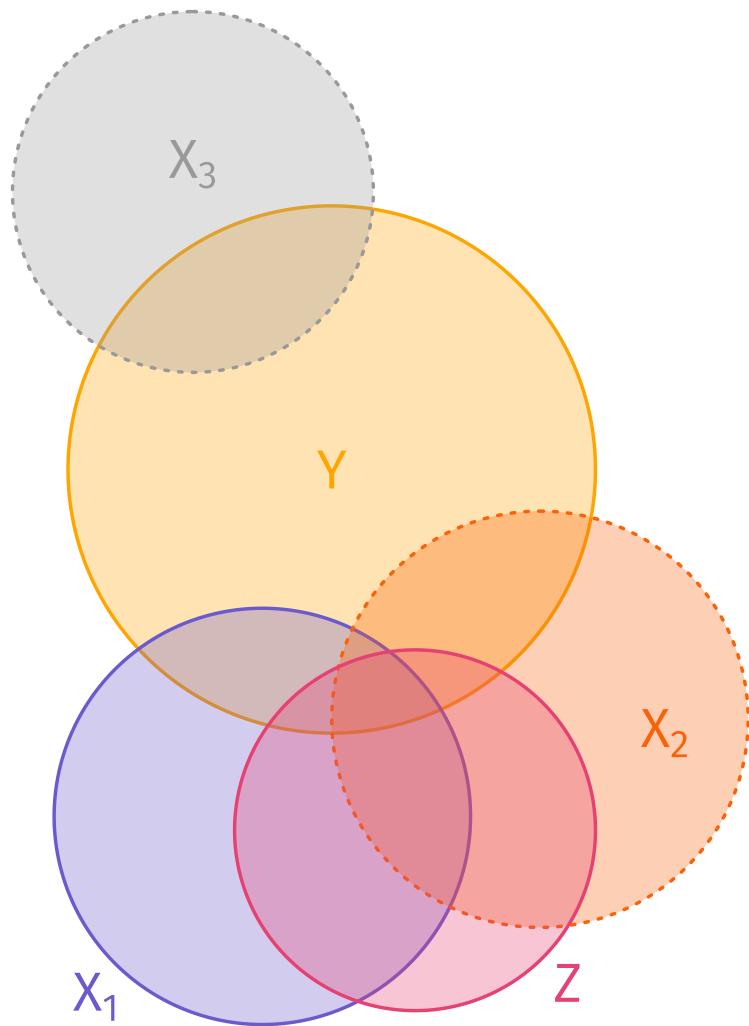
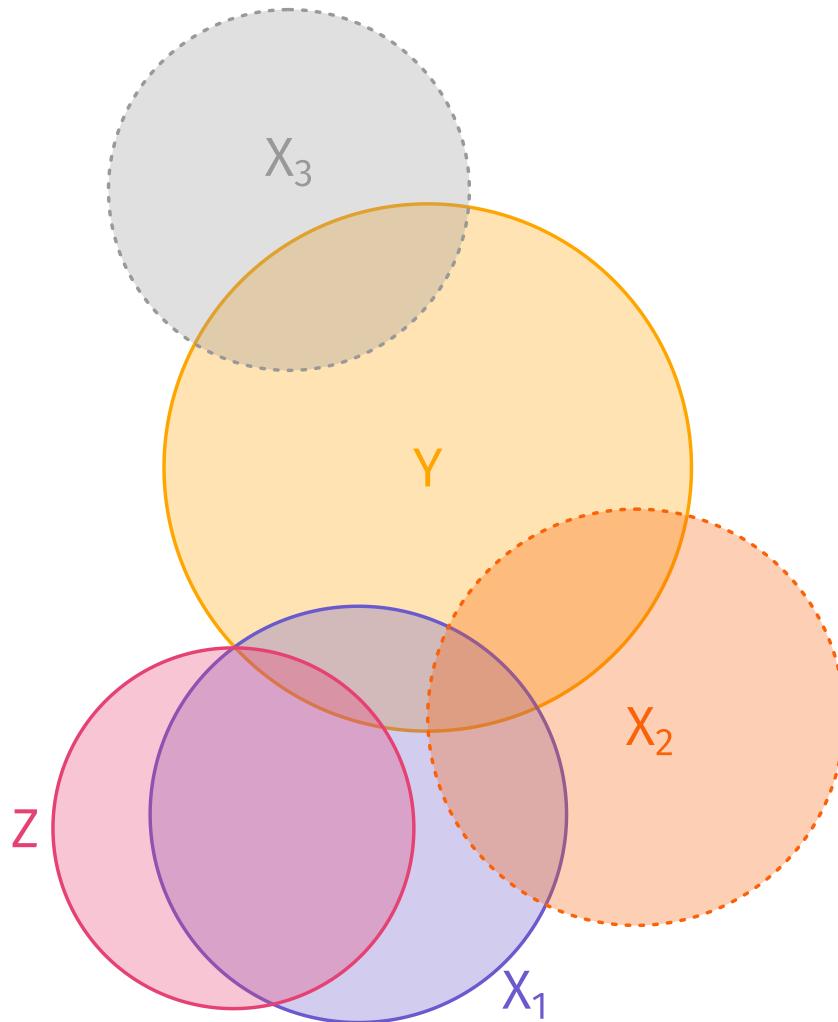


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IV + heterogeneity

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A Not ATE. And not TOT. They estimate the LATE.[†]

[†] See Angrist, Imbens, and Rubin (1996).

IV + heterogeneity

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However, *compliers* are only one of four possible groups.

1. **Compliers** $D_i = 1$ iff $Z_i = 1$.
2. **Always-takers** $D_i = 1 \forall Z_i$.
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4. **Defiers** $D_i = 1$ iff $Z_i = 0$.

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| 1. Compliers $D_i = 1$ iff $Z_i = 1$. | Only take pills when treated . |
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Hence the "local" in *local average treatment effect*.

IV + heterogeneity

The LATE: Medical-trial example

Imagine treatment works for some ($\beta_{1,i} < 0$) and not for others ($\beta_{1,j} = 0$).

Suppose individuals know their response to blood-pressure medication.

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Thus, IV's LATE will indicate no treatment effect $\left(\hat{\beta}_1^{\text{IV}} = 0\right)$.

IV + heterogeneity

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IV + heterogeneity

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IV doesn't estimate the ATE or TOT, so it would be inconsistent for them.[†]

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Takeaway₂ Different instruments have different LATEs.

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IV + heterogeneity

Monotonicity

We've already written down the two classical IV/2SLS assumptions

- *First stage:* $\text{Cov}(\mathbf{Z}_i, \mathbf{D}_i) > 0$
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- **Monotonicity (Uniformity):** $\mathbf{D}_i(z) \geq \mathbf{D}_i(z')$ or $\mathbf{D}_i(z) \leq \mathbf{D}_i(z') \quad \forall i$
Heckman: Uniformity of responses across persons.
Imbens and Angrist (1994): Instrument has monotone effect on \mathbf{D}_i .

IV + heterogeneity

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which is not bound between τ_c and τ_d .

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Example $\tau_c = 1$ and $\tau_d = 2$. $\Pr(\text{complier}) = 2/3$ and $\Pr(\text{defier}) = 1/3$.

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If "defiers" exist, then monotonicity/uniformity is violated.

In this case, the IV estimand is

$$\frac{\tau_c \Pr(\text{complier}) - \tau_d \Pr(\text{defier})}{\Pr(\text{complier}) - \Pr(\text{defier})}$$

which is not bound between τ_c and τ_d .

Example $\tau_c = 1$ and $\tau_d = 2$. $\Pr(\text{complier}) = 2/3$ and $\Pr(\text{defier}) = 1/3$.

Then the "LATE" is 0 .[†]

[†] Some people would instead say that there is no LATE when you violate monotonicity.

Until now, we've focused on using a single instrument.

The 2SLS estimator accommodates multiple instruments.[†]

[†] Whether you can find multiple valid instruments is another question.

Multiple instruments

Multiple instruments

Motivation

Q Why include multiple instruments?

Multiple instruments

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A Multiple instruments can capture more variation in D_i (efficiency).

Multiple instruments

Motivation

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Using terminology from the *system-of-equations* literature,

- one instrument for one endogenous variable: **just identified**
- multiple instruments for one endogenous variable: **over identified**

Multiple instruments

In practice

With (valid) instruments Z_{1i} and Z_{2i} , or first stage becomes

$$D_i = \gamma_0 + \gamma_1 Z_{1i} + \gamma_2 Z_{2i} + \gamma_3 X_i + u_i$$

Multiple instruments

In practice

With (valid) instruments Z_{1i} and Z_{2i} , or first stage becomes

$$D_i = \gamma_0 + \gamma_1 Z_{1i} + \gamma_2 Z_{2i} + \gamma_3 X_i + u_i$$

while our second stage is still

$$Y_i = \beta_0 + \beta_1 \hat{D}_i + \beta_2 X_i + v_i$$

Multiple instruments

Example: Quarter of birth

Back to our quest to estimate the returns to education.

Multiple instruments

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Angrist and Krueger (1991) proposed *quarter of birth* as a set of instruments for years of schooling.

Multiple instruments

Example: Quarter of birth

Back to our quest to estimate the returns to education.

Angrist and Krueger (1991) proposed *quarter of birth* as a set of instruments for years of schooling.

Accordingly, their first stage looks something like[†]

$$\begin{aligned}\text{Schooling}_i = & \gamma_0 + \gamma_1 \mathbb{I}(\text{Born Q1})_i + \gamma_2 \mathbb{I}(\text{Born Q2})_i \\ & + \gamma_3 \mathbb{I}(\text{Born Q3})_i + \gamma_4 \mathbb{I}(\text{Born Q4})_i \\ & + \gamma_5 X_i + u_i\end{aligned}$$

[†] We need to drop one of the quarter-of-birth indicators to avoid perfect collinearity.

Multiple instruments

Example: Quarter of birth

Q Is quarter of birth a valid instrument?

Multiple instruments

Example: Quarter of birth

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Q1 Why would quarter of birth affect schooling? (*First stage*)

Multiple instruments

Example: Quarter of birth

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A1 Students cannot drop out of school until a certain age, and quarter of birth affects your age at the time you begin school.

Multiple instruments

Example: Quarter of birth

Q Is quarter of birth a valid instrument?

Q1 Why would quarter of birth affect schooling? (*First stage*)

A1 Students cannot drop out of school until a certain age, and quarter of birth affects your age at the time you begin school.

Example Some states require students to stay in school until they are 16.

- Students who start school at age **6** drop out after **10** years of schooling.
- Students who start school at age **5** drop out after **11** years of schooling.

Multiple instruments

Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

Multiple instruments

Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

For some group, quarter of birth may affect the number of years in school.

Multiple instruments

Example: Quarter of birth

It turns out that the first stage is also pretty weak in this setting.

Weak instruments can cause several problems for 2SLS/IV:

Multiple instruments

Example: Quarter of birth

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Weak instruments can cause several problems for 2SLS/IV:

1. Our estimator is a ratio of the reduced form and the first stage, so a weak first stage can blow up reduced-form estimates (amplifying reduced-form noise/bias).

Multiple instruments

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Weak instruments can cause several problems for 2SLS/IV:

1. Our estimator is a ratio of the reduced form and the first stage, so a weak first stage can blow up reduced-form estimates (amplifying reduced-form noise/bias).
2. Many weak instruments lead to a finite-sample issue in which 2SLS is biased toward OLS—our first stage is essentially overfitting.

Multiple instruments

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Weak instruments can cause several problems for 2SLS/IV:

1. Our estimator is a ratio of the reduced form and the first stage, so a weak first stage can blow up reduced-form estimates (amplifying reduced-form noise/bias).
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What about our other requirements for a valid instrument?

Multiple instruments

Example: Quarter of birth

Q2 Is quarter of birth uncorrelated with ε_i (*excludable*)?

Multiple instruments

Example: Quarter of birth

Q2 Is quarter of birth uncorrelated with ε_i (*excludable*)?

A2 While quarter of birth may be fairly arbitrary for some families, other families might time births.

If these birth timers differ from other couples along other dimensions (e.g., income or education), then quarter of birth may correlate with ε_i .

Multiple instruments

Example: Quarter of birth

Q3 Is the effect monotone?

Multiple instruments

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A3 Some[†] argue that monotonicity may be violated in this setting.

[†] E.g., Aliprantis (2012)

Multiple instruments

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Consider December births.

- Original idea: December birthdates will start school at age 5.7, inducing more years of education before 16.

[†] E.g., Aliprantis (2012)

Multiple instruments

Example: Quarter of birth

Q3 Is the effect monotone?

A3 Some[†] argue that monotonicity may be violated in this setting.

Consider December births.

- Original idea: December birthdates will start school at age 5.7, inducing more years of education before 16.
- *Redshirting* idea: Parents hold back December kids so they can be older (*i.e.*, 6.7), inducing fewer years of education before 16.

[†] E.g., Aliprantis (2012)

2SLS and R

estimatr

You can implement 2SLS/IV in many ways in R.

Today: `esitmatr` and `iv_robust()`.

2SLS and R

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2SLS and R

estimatr

You can implement 2SLS/IV in many ways in R.

Today: `esitmatr` and `iv_robust()`.

Specifically, we give `iv_robust()` the relationship that we want separated from the instrument by `|`, e.g.,

```
# Estimate 2SLS
iv_robust(Y ~ D | Z, data = sample_df, se_type = "classical") %>%
  tidy() %>% select(1:5)

#>           term estimate std.error statistic      p.value
#> 1 (Intercept) 5.786204  2.9744230  1.945320 0.0546020456
#> 2          D 1.107801  0.3043264  3.640173 0.0004372703
```

2SLS and R

Now in two stages!

Of course, we can estimate 2SLS in two stages.

```
# First stage
stage1 ← lm_robust(D ~ Z, data = sample_df, se_type = "classical")
# First-stage results
stage1 %>% tidy() %>% select(1:5)

#>          term  estimate std.error statistic    p.value
#> 1 (Intercept) 8.8226148  0.3169568 27.835389 2.486413e-48
#> 2           Z  0.3257347  0.1031506  3.157857 2.112927e-03
```

2SLS and R

Second stage

We just need to add \hat{D}_i to our dataset.

```
# Add fitted (first-stage) values to data
sample_df %>% mutate(D_hat = stage1$fitted.values)
# Second stage
stage2 ← lm_robust(Y ~ D_hat, data = sample_df, se_type = "classical")
# Second-stage results
stage2 %>% tidy() %>% select(1:5)
```

```
#>           term estimate std.error statistic    p.value
#> 1 (Intercept) 5.786204  5.4132099  1.068904 0.28773854
#> 2     D_hat   1.107801  0.5538496  2.000184 0.04824759
```

2SLS and R

Standard errors

However, recall that our second-stage standard errors are not correct.

2SLS and R

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Second-stage results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	5.413	1.07	0.2877
D hat	1.108	0.554	2.00	0.0482

2SLS and R

Standard errors

However, recall that our second-stage standard errors are not correct.

Second-stage results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	5.413	1.07	0.2877
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2SLS results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	2.974	1.95	0.0546
D	1.108	0.304	3.64	0.0004

IV and 2SLS

Conclusions

1. IV/2SLS focus on **isolating some "good" variation** in D_i via Z_i .
2. Important **requirements**: strong first stage, excludability, monotonicity.
3. IV and 2SLS **rescale the reduced form** with the first stage.
4. Estimates are **LATE from compliers**.
5. Different instruments can produce **different LATEs**.
6. A **weak first stage** can lead to problems.

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