Instrumental Variables

EC 425/525, Set 8

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Prologue

Schedule

Last time

Matching and propensity-score methods

- Conditional independence
- Overlap

Today

Instrumental variables (and two-stage least squares)

Upcoming

- Assignment due Sunday
- Proposal due Wednesday 5/22
- Midterm?

Research designs

Selection on observables and/or unobservables

We've been focusing on ***selection-on-observables*** **designs**, *i.e.*,

$$(\mathrm{Y}_{0i}, \mathrm{Y}_{1i}) \perp\!\!\!\perp \mathrm{D}_i | \mathrm{X}_i$$

for **observable** variables X_i.

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***Selection-on-unobservable* designs** replace this assumption with two new (but related) assumptions

1. $(\mathrm{Y}_{0i},\, \mathrm{Y}_{1i}) \perp \mathrm{Z}_i$
2. $\mathrm{Cov}(\mathrm{Z}_i,\, \mathrm{D}_i) \neq 0$

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Our main goal in causal-inference minded (applied) econometrics boils down to isolating **"good" variation** in D_i (exogenous/as-good-as-random) from **"bad" variation** (the part of D_i correlated with $\mathbf{Y}_{0i}$ and $\mathbf{Y}_{1i}$).

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   Seems more plausible. Possible to validate. May be underpowered.

Introduction

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$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i \tag{1}$$

To guarantee consistent OLS estimates for β_1, want $\mathbf{Cov}(\mathbf{D}_i,\, \varepsilon_i) = \mathbf{0}$.
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Alternative: Estimate β_1 via instrumental variables.

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For our model

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Back to the returns to a college degree,

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The IV estimator

The IV estimator for our model

$$\mathrm{Y}_i = \beta_0 + \beta_1 \mathrm{D}_i + \varepsilon_i \tag{1}$$

with (valid) instrument Z_i is

$$\hat{\beta}_{\mathrm{IV}} = \left(\mathrm{Z}'\mathrm{D}\right)^{-1} \left(\mathrm{Z}'\mathrm{Y}\right)$$

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If you have no covariates, then

$$\hat{\beta}_{\text{IV}} = \frac{\text{Cov}(\text{Z}_i,\ \text{Y}_i)}{\text{Cov}(\text{Z}_i,\ \text{D}_i)}$$

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If you have additional (exogenous) covariates $\mathbf{X}_i$, then

$$\mathbf{Z} = \begin{bmatrix} \mathrm{Z}_i & \mathbf{X}_i \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} \mathrm{D}_i & \mathbf{X}_i \end{bmatrix}$$

Instrumental variables

Proof: Consistency

With a valid instrument $\mathbf{Z}_i$, $\hat{\beta}_{\text{IV}}$ is a consistent estiamtor for β_1 in

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{X}_i + \varepsilon_i \tag{1}$$

$$\operatorname{plim}\left(\hat{\beta}_{IV}\right)$$

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$$\begin{aligned} &\mathrm{plim}\Big(\hat{\beta}_{IV}\Big) \\ &\quad = \mathrm{plim}\Big(\left(\mathrm{Z'D}\right)^{-1}\left(\mathrm{Z'Y}\right)\Big) \\ &\quad = \mathrm{plim}\Big(\left(\mathrm{Z'D}\right)^{-1}\left(\mathrm{Z'D}\beta + \mathrm{Z'}\varepsilon\right)\Big) \end{aligned}$$

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$$\mathrm{D}_i = \gamma_1 \mathrm{Z}_i + \gamma_2 \mathrm{X}_i + u_i$$

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Second stage Estimate model we wanted—but only using the variation in $\mathbf{D}_i$ that correlates with $\mathbf{Z}_i$, *i.e.*, $\widehat{\mathbf{D}}_i$.

$$\mathbf{Y}_i = \beta_1 \widehat{\mathbf{D}}_i + \beta_2 \mathbf{X}_i + \varepsilon_i$$

Note The controls $\mathbf{X}_i$ must match in the first and second stages.

Two-stage least squares

IV estimation

This two-step procedure, with a valid instrument, produces an estimator $\hat{\beta}_1$ that is consistent for β_1.

$$\hat{\beta}_{2SLS} = \left(\mathrm{D}'\mathrm{P_Z}\mathrm{D}\right)^{-1}\left(\mathrm{D}'\mathrm{P_Z}\mathrm{Y}\right)$$
$$\mathrm{P_Z} = \mathrm{Z}\left(\mathrm{Z}'\mathrm{Z}\right)^{-1}\mathrm{Z}'$$

where $\mathbf{D}$ is a matrix of our treatment and predetermined covariates $(\mathbf{X}_i)$ and Z is a matrix of our instrument and our predetermined covariates.

Two-stage least squares

IV estimation

Important notes

- The controls (X_i) must match in the first and second stages.
- If you have exactly **one instrument** and exactly **one endogenous variable**, then 2SLS and IV are identical.
- Your second-stage standard errors are not correct.

Two-stage least squares

The reduced form

In addition to the regressions within the two stages of 2SLS

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there is a third important and related regression: the reduced form.

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The **reduced form** regresses the outcome Y_i (LHS of the second stage) on our instrument Z_i and covariates X_i (RHS of the first stage).

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Thus, the reduced form provides a consistent estimate of the causal effect of our instrument on the outcome.

Two-stage least squares

The reduced form, continued

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$$\widehat{\beta}_1^{\text{2SLS}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1}$$

when you have exactly one instrument.

The reduced form, intuition

This expression for the 2SLS (and IV) estimator can be very helpful.

$$\widehat{\beta}_1^{\text{2SLS}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1} = \frac{\text{Reduced-form estimate}}{\text{First-stage estimate}}$$

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$\widehat{\gamma}_1$ estimates the effect of winning the scholarship lottery on graduation—the share of winners who graduated due to winning.

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$\widehat{\pi}_1$ gives the estimated causal effect of the scholarship lottery on income, but what share of lottery winners graduate? We need to rescale if $<$ 100%.

$\widehat{\gamma}_1$ estimates the effect of winning the scholarship lottery on graduation—the share of winners who graduated due to winning. We can scale with $\widehat{\gamma}_1$!

The reduced form, example

To see why this scaling makes sense, imagine that 50% of lottery winners graduate from college due to the lottery, *i.e.*, $\widehat{\gamma}_1 = 0.50$.[†]

† Imagine none of the applicants would have graduated otherwise

Two-stage least squares

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Thus, we want to double $\widehat{\pi}_1$, *i.e.*, divide by $\widehat{\gamma}_1$: $\widehat{\pi}_1/\widehat{\gamma}_1$ = $\$5{,}000/0.5$ = \$10,000.

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Two-stage least squares

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Let's push a bit deeper into IV's mechanics and intuition.

Setup

In this section, we'll use medical trials as a working example.†

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IV: Mechanics and intuition

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IV: Mechanics and intuition

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$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i$$

$\mathbf{D}_i$ indicates whether $\boldsymbol{i}$ *takes* the treatment (medication). ε_i captures all other factors that affect $\mathbf{Y}_i$.

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IV: Mechanics and intuition

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D_i indicates whether i *takes* the treatment (medication). ε_i captures all other factors that affect Y_i. Or in potential-outcomes framework:

$$\begin{aligned} \text{Y}_i &= \text{Y}_{1i}\text{D}_i + \text{Y}_{0i}(1 - \text{D}_i) \\ \text{Y}_{0i} &= \beta_0 + \varepsilon_i \\ \text{Y}_{1i} &= \text{Y}_{0i} + \beta_1 \end{aligned}$$

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IV: Mechanics and intuition

Research design

Goal **Estimate the effect of blood-pressure medication** on blood pressure.

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IV: Mechanics and intuition

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Analysis 2 **IV!** Instrument medication $\mathbf{D}_i$ with intention to treat $\mathbf{Z}_i$.

IV: Mechanics and intuition

The IV solution

First question: Is $\mathbf{Z}_i$ a valid instrument for $\mathbf{D}_i$?

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2. $\mathbf{Cov}(\mathbf{Z}_i,\, \mathbf{D}_i) \neq \mathbf{0}$ if assignment to treatment changes the likelihood you take the pills (first stage).

$\therefore$ $\mathbf{Z}_i$ is a valid instrument for $\mathbf{D}_i$ and IV consistently estimates β_1.

Noncompliance

Noncompliant individuals do not abide by their treatment assignment.

IV: Mechanics and intuition

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IV: Mechanics and intuition

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Noncompliant individuals do not abide by their treatment assignment.

Let's see how IV "solves" this problems.

First, assume noncompliance only affects treated individuals—*i.e.*, treated folks sometimes don't take their pills; control folks never take pills.

IV: Mechanics and intuition

Noncompliance, continued

The **first stage** recovers the share of treatment individuals who take the pill

$$\text{D}_i = \gamma_1 \text{Z}_i + u_i$$

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IV: Mechanics and intuition

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which we know IV rescales using the first stage

$$\widehat{\beta}_1^{\mathrm{IV}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1} = \frac{\widehat{\pi}_1}{0.50} = 2 \times \widehat{\pi}_1$$

Noncompliance, continued

IV solves the noncompliance issue by rescaling by the rate of compliance.

IV: Mechanics and intuition

Noncompliance, continued

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If everyone perfectly complies, then $\widehat{\gamma}_1 = 1$ and $\widehat{\beta}_1^{\text{IV}} = \widehat{\pi}_1 / 1 = \widehat{\beta}_1^{\text{ITT}}$.

IV: Mechanics and intuition

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IV solves the noncompliance issue by rescaling by the rate of compliance.

If everyone perfectly complies, then $\widehat{\gamma}_1 = \mathbf{1}$ and $\widehat{\beta}_1^{\text{IV}} = \widehat{\pi}_1/\mathbf{1} = \widehat{\beta}_1^{\text{ITT}}$.

Further example N_{Trt} = 10; trt. compliance = 50%; ctrl. compliance = 100%.

$$\overline{\text{Y}}_{\text{Trt}} = \frac{5(\beta_0 + \beta_1) + 5(\beta_0)}{\mathbf{10}} = \beta_0 + \frac{\beta_1}{\mathbf{2}}$$

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So our reduced-form estimate (the ITT) is $\widehat{\gamma}_1 = \dfrac{\beta_1}{\mathbf{2}}$ (half the true effect).

IV: Mechanics and intuition

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IV consistently estimates β_1 via rescaling the ITT by the rate of compliance

$$\widehat{\beta}_1^{\text{IV}} = \frac{\pi}{\gamma} = \frac{\beta_1/2}{1/2} = \beta_1$$

IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of $\mathbf{Z}_i$ on $\mathbf{Y}_i$ by the causal effect of $\mathbf{Z}_i$ on $\mathbf{D}_i$.

IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of $\mathbf{Z}_i$ on $\mathbf{Y}_i$ by the causal effect of $\mathbf{Z}_i$ on $\mathbf{D}_i$.

2. IV **does not** compare treated compliers to untreated compliers.

IV: Mechanics and intuition

Takeaways

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2. IV **does not** compare treated compliers to untreated compliers.
   Such a comparison/estimator would re-introduce selection bias.

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