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Learn The Limitations Of Low-Pass Sallen-Key Filters

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Fri, 1999-12-17 (All day)

Imperfect Amplifiers, Parasitic Capacitance, And Component Selection All Impact Filter Performance At High Frequencies.

Since professors R.P. Sallen and E.L. Key described it in 1955, the Sallen-Key low-pass filter has become one of the most widely used filters in electronic systems. Perhaps because the mathematics can be somewhat daunting, however, little has been written to help working engineers specify the correct components to achieve their objectives. For example, few realize the limitations of Sallen-Key filters at high frequencies.

The following describes the basic operations of a Sallen-Key low-pass filter and offers a simplified way of working with such circuits. Based on laboratory research, it also demonstrates some of this filter's limitations at high frequencies.

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Sallen-Key basics: The two-stage RC network shown in Figure 1 forms a second-order low-pass filter. This circuit has the limitation that its Q is always less than one-half. With R1 = R2 and C1 = C2, then Q = 1/3. Q approaches the maximum value of one-half when the impedance of the second RC is much larger than the first. But most filters usually require larger Qs than one-half.

Q can be enhanced with an amplifier in positive feedback. With that feedback localized to the filter's cutoff frequency, almost any Q can be realized. Mostly, it's only limited by the physical constraints of the power supply and component tolerances. The Sallen-Key low-pass filter shown in Figure 2 is an example of how an amplifier is used in this manner. C2 is no longer connected to ground, but rather provides a positive feedback path around the amplifier.

The operation can be described qualitatively. At low frequencies, where C1 and C2 appear as open circuits, the signal is simply amplified to the output. R3 and R4 are chosen to give the desired gain. At high frequencies, C1 and C2 appear as short circuits, and the signal is shunted to ground at the amplifier's input. The amplifier amplifies this input to its output, and the signal doesn't appear at V_O. Near the cutoff frequency, where the impedance of C1 and C2 are on the same order as R1 and R2, positive feedback via C2 provides Q enhancement of the signal.

Ideal operation: The standard frequency-domain equation for a second-order low-pass filter is:

$$H_{LP} = \frac{K}{-\left(\frac{f}{f_C}\right)^2 + \frac{jf}{Qf_C} + 1} \qquad (1)$$

where f_C is the corner frequency and Q is the quality factor.

When f<C, Equation 1 reduces to H_{LP} = K, and the circuit passes signals multiplied by gain factor K. When f = f_C, Equation 1 reduces to: $H_{LP} = -jKQ$, and the signals are enhanced by the factor Q. When f>>f_C, Equation 1 reduces to:

$$H_{LP} = -K\left(\frac{f_C}{f}\right)^2$$

and the signals are attenuated by the square of the frequency ratio. With attenuation at higher frequencies increasing by a power of two, the formula describes a second-order low-pass filter.

Deriving the transfer function of the circuit in Figure 2, the Sallen-Key ideal low-pass transfer function is defined by Equation 2.

By letting

$$s = j2\pi f,\ f_C = \frac{1}{2\pi\sqrt{R1R2C1C2}},\ \text{and}$$

$$Q = \frac{\sqrt{R1R2C1C2}}{R1C1 + R2C1 + R1C2(1-K)},$$

Equation 2

$$\frac{V_O}{V_I}(LP) = \frac{K}{s^2(R1R2C1C2) + s(R1C1 + R2C1 + R1C2(1-K)) + 1} \qquad (2)$$

$$\text{where } K = 1 + \frac{R4}{R3}$$

Equation 2 follows the same form as Equation 1. With simplifications, you can deal with the equation more easily.

Simplification 1: Set filter components as ratios. Letting R1 = mR, R2 = R, C1 = C, and C2 = nC, results in:

$$f_C = \frac{1}{2\pi RC\sqrt{mn}},\ \text{and}$$

$$Q = \frac{\sqrt{mn}}{m+1+mn(1-K)}$$

This simplifies things somewhat, but there's interaction between f_C and Q.

The design should start by setting the gain and Q based on m, n, and K, and then selecting C and calculating R to set f_C. It may be observed that K = 1 +\[(m+1)/(mn)\] results in Q = ∞. With larger values, Q becomes negative. In other words, the poles move into the right half of the s-plane and the circuit oscillates. The most frequently designed filters require low Q values, so this should rarely be a design issue.

Simplification 2: Set filter components as ratios and gain = 1. Letting R1 = mR, R2 = R, C1 = C, C2 = nC, and K = 1 results in:

$$f_C = \frac{1}{2\pi RC\sqrt{mn}} \text{ and } Q = \frac{\sqrt{mn}}{m+1}$$

This keeps the gain equal to 1 in the pass band. But again, there's interaction between f_C and Q. Design should start by choosing the ratios m and n to set Q, and then selecting C and calculating R to set f_C.

Simplification 3: Set resistors as ratios and capacitors equal. Letting R1 = mR, R2 = R, and C1 = C2 = C, results in:

$$f_C = \frac{1}{2\pi RC\sqrt{m}} \text{ and } Q = \frac{\sqrt{m}}{1+2m-mK}$$

The main motivation behind setting the capacitors equal is the limited selection of values in comparison with resistors. Interaction exists between setting f_C and Q. Design should start with choosing m and K to set the gain and Q of the circuit before choosing C and calculating R to set f_C.

Simplification 4: Set filter components equal. Letting R1 = R2 = R and C1 = C2 = C results in:

$$f_C = \frac{1}{2\pi RC} \text{ and } Q = \frac{1}{3-K}$$

Now f_C and Q are independent of one another. Design is greatly simplified, although it's simultaneously limited. Q is now determined by the gain of the circuit. The choice of RC sets f_C. The capacitor should be chosen, and the resistor calculated. One minor drawback is that because the gain controls the Q of the circuit, further gain or attenuation may be necessary to achieve the desired signal gain in the passband.

Values of K that are very close to 3 result in high Qs that are sensitive to variations in the component values of R3 and R4. Setting K = 2.9 results in a nominal Q of 10. A worst-case analysis with 1% resistors results in Q = 16. In contrast, if setting K = 2 for a Q of 1, worst-case analysis with the same 1% resistors results in Q = 1.02. Resistor values where K = 3 leads to Q = ∞. And with larger values, Q becomes negative. The poles move into the right half of the s-plane and the circuit will oscillate. The most frequently designed filters require low Q values, so this should rarely become a design issue.

Non-ideal circuit operation: Up to now, we've assumed that the circuit was ideal, but there comes a time (or actually a frequency) when this is no longer valid. Simple logic tells us that the amplifier must be an active component at the frequencies of interest or else we have problems. But what are these problems?

As mentioned previously, there are three basic modes of operation: below cutoff, above cutoff, and in the area of cutoff. Assuming that the amplifier has adequate frequency response beyond cutoff, the filter works as expected. At frequencies well above cutoff, the high-frequency model depicted in Figure 3 is used to show the expected circuit operation. The assumption made here is that C1 and C2 are effective shorts when compared to the impedance of R1 and R2, so the amplifier's input is at ac ground. In response, the amplifier generates an ac ground at its output limited only by its output impedance, Z_O. The formula shows the transfer function of this particular model.

Z_O is the closed-loop output impedance. It depends on the loop transmission and the open-loop output impedance, z_o:

$$Z_O = \frac{z_o}{1 + a(f)b}$$

where a(f) is the open-loop gain of the amplifier and b is the feedback factor. This feedback factor is constant—set by resistors R3 and R4. But the open-loop gain, a(f), depends on frequency.

With dominant-pole compensation, the amplifier's open-loop gain decreases by 20 dB/decade over the usable frequencies of operation. Assuming z_o is mainly resistive (usually a valid assumption up to a few hundred megahertz), Z_O increases at a rate of 20 dB/decade. The transfer function appears to be a first-order high pass.

At frequencies above 100 MHz (or so), the parasitic inductance in the output starts playing a role and the transfer function transitions to a second-order high pass. Plus, at higher frequency, the high-pass transfer function will roll off due to stray capacitance.

Simulation and lab data: To show the effects described above, a Sallen-Key low-pass filter was simulated in Spice and lab tested using a THS3001 operational amplifier from Texas Instruments. The THS3001 is a high-speed, current-feedback amplifier with an advertised bandwidth of 420 MHz. Choosing R1 = R2 = 1 kΩ, C1 = C2 = 1 nF, R3 = open, and R4 = 1 kΩ results in a low-pass filter with f_C = 159 kHz and Q = 1/2.

Figure 4a depicts the simulation circuit with the Spice model modified so that the output impedance of the amplifier is 0 Ω. In Figure 5, curve (a) shows the frequency response as simulated in Spice. It also reveals that with zero output impedance, the attenuation of the signal continues to increase as frequency rises.

Figure 4b depicts the high-frequency model as exemplified in Figure 3, where the input is at ground and the output impedance controls the transfer function. The Spice model used for the THS3001 includes an LRC network for the output impedance. Again, Figure 5 shows the frequency response as simulated in Spice, but this time it's symbolized by curve (b). The magnitude of the signal at the output is seen to cross curve (a) at about 7 MHz. Above this frequency, the output impedance causes the switch in the transfer function, which is described above.

Look to Figure 4c for the simulation circuit using the Spice model with the LCR output impedance. Figure 5's curve (c) shows the frequency response for this model. With the output impedance, the attenuation caused by the circuit follows curve (a) until it crosses curve (b), at which point it follows curve (b). Figure 4d reveals the circuit as tested in the lab, with curve (d) in Figure 5 showing that the measured data agrees with the simulated data.

Comments about component selection: Until now, the choosing of resistor and capacitor values has been left without mention. Theoretically, any values of R and C that satisfy the equations may be used. But practical considerations call for certain guidelines to be followed. Given a specific corner frequency, the values of C and R are inversely proportional to one another. By making C larger, R becomes smaller and vice versa.

In the case of the low-pass Sallen-Key filter, the ratio between the output impedance of the amplifier and the value of filter component R sets the transfer functions seen at frequencies well above cutoff. The larger the resistor's value, the lower the transmission of signals at high frequency. Making R too large may result in C becoming so small that the parasitic capacitors, including the input capacitance of the amplifier, cause errors. The best choice of component values depends on the particulars of your circuit and the tradeoffs you're willing to make.

Here are some general recommendations for capacitors and resistors: Engineers should avoid capacitors with values less than 100 pF. If at all possible, use an NPO type. X7R is okay in a pinch, but avoid Z5U and other low-quality dielectrics. In critical applications, even higher-quality dielectrics, like polyester, polycarbonate, Mylar, etc., may be required. As for resistors, values in the range of a few hundred to a few thousand ohms are the best bet. You also should choose metal-film resistors that possess low temperature coefficients. Finally, use 1%-tolerance capacitors and resistors, preferably those of the surface-mount variety.

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