# Action 'c' and Kashiwara-Nakashima tableaux

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## **ALPHABET MODULE**

Module containing definition of Alphabet class class alphabet.Alphabet (num) Bases: object This class defines the generalized alphabet and its behaviour.  $\underline{\hspace{0.1cm}}$ eq $\underline{\hspace{0.1cm}}$  (other) Function to check equality of the two Alphabet objects, using == Parameters other (Alphabet) – alphabet object that we are comparing with. **Returns** True if both alphabet object are equal, and False otherwise. **\_\_ge**\_\_\_(other) Function to check if invoker is greater than of equal to the other Alphabet object, using >= Parameters other (Alphabet) – alphabet object that we are comparing with. **Returns** True if invoker is greater than or equal to the other, False otherwise. **\_gt\_\_** (other) Function to check invoker object is greater than the other Alphabet object, using > **Parameters** other (Alphabet) – alphabet object that we are comparing with. **Returns** True if invoker is greater than other, False otherwise. init (num) Initialization function for Alphabet **Parameters** num(int) – Number that is to be decorated with a specific type of alphabet **\_le**\_\_(other) Function to check if invoker is less than or equal to the other Alphabet object, using <= Parameters other (Alphabet) – alphabet object that we are comparing with. **Returns** True if invoker is less than or equal to the other, False otherwise. **ne** (other) Function to check inequality of two Alphabet objects, using != **Parameters** other (Alphabet) – alphabet object that we are comparing with. **Returns** True if both alphabet object are not equal, and False otherwise.

This class defines the Barred alphabet, which is a specialization of Alphabet.

class alphabet.Barred(num)
 Bases: alphabet.Alphabet

```
___1t___(other)
           Function to check if invoker is less than the other Alphabet object, using <
               Parameters other (Alphabet) – alphabet object that we are comparing with.
               Returns True if invoker is less than the other, False otherwise.
        str ()
           Function to get the string representation of the Barred object.
               Returns string representation of invoking object.
               Return type str
     clone()
           Function to make a copy of the invoking Barred object.
               Returns Copy of invoking object
               Return type Barred
class alphabet.Ordinary(num)
     Bases: alphabet.Alphabet
     This class defines the Ordinary alphabet, which is a specialization of Alphabet.
     ___lt___(other)
           Function to check if invoker is less than the other Alphabet object, using <
               Parameters other (Alphabet) – alphabet object that we are comparing with.
               Returns True if invoker is less than the other, False otherwise.
     __str__()
           Function to get the string representation of the Ordinary object.
               Returns string representation of invoking object.
               Return type str
     clone()
           Function to make a copy of the invoking Ordinary object.
               Returns Copy of invoking object
               Return type Ordinary
```

# **OPERATION MODULE**

This module contains all the elementary function to apply action c on a kn tableau.

```
operation.action_c (t, m=None, mutable=True)
```

Function to apply action c on tableau.

#### **Parameters**

- t tableau.
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar
- mutable optional. Default set to True, where it directly make changes to the tableau t. If passed in as False, it creates a copy of t and operates on that copy, leaving t unaffected.

**Returns** tableau obtained after applying c on t.

```
operation.e_i(t, i, m, mutable=True)
```

Function to apply e\_i on tableau.

#### **Parameters**

- t tableau.
- i i as in e\_i
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar
- mutable optional. Default set to True, where it directly make changes to the tableau t. If passed in as False, it creates a copy of t and operates on that copy, leaving t unaffected.

**Returns** tableau after applying e i

```
operation.evaluate(poly, q, inp)
```

Function to evaluate polynomial at a particular value.

#### **Parameters**

- poly polynomial.
- **q** variable in polynomial.
- inp input value where we need to evaluate the polynomial.

Returns value obtained after evaluation.

```
\texttt{operation.f\_i} \; (t,i,m,mutable = True)
```

Function to apply f\_i on tableau.

#### **Parameters**

• t – tableau.

- $\mathbf{i} i$  as in f i
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar
- mutable optional. Default set to True, where it directly make changes to the tableau t. If passed in as False, it creates a copy of t and operates on that copy, leaving t unaffected.

**Returns** tableau after applying f i

```
operation.get_crystal_graph(shape, m=None)
```

Function to create crystal graph data.

#### **Parameters**

- **shape** shape of tableaux in graph
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar

**Returns** a list of all k-n tableaux for given shape and value of m, an f\_i dictionary with all the kn tableaux as keys with a list as value, where (i-1)st index in list refer to tableau obtained by applying f\_i to the key tableau. A similar e\_i tableaux dictionary.

#### operation.get\_kn\_schur\_polynomial(shape, m=None)

Function to compute the kn-Schur polynomial in q for a given shape and value of m.

#### **Parameters**

- **shape** shape of tableaux in set.
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar

**Returns** Schur polynomial in variable q. and variable q.

```
operation.get_power_of_q(t, m)
```

Power of q in the monomial that corresponds to tableau.

#### **Parameters**

- t tableau.
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar

**Returns** power of q in the monomial that corresponds to tableau t.

```
operation.get_tableaux_set(shape, m=None)
```

Function to obtain all the kn tableaux of given shape and value of m.

#### **Parameters**

- **shape** shape of tableaux.
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar

**Returns** list of all the tableaux of given shape and value of m.

```
operation.k_of_shape(shape)
```

statistic of shape as defined in OH's and PARK's paper.

Parameters shape - shape.

**Returns** statistic of shape.

```
operation.nth_root_of_unity(n)
```

Function to compute n-th root of unity.

**Parameters** n - n as in n-th root of unity.

**Returns** n-th root of unity in form of sympy complex number expression.

```
operation.order_of_set (shape, m=None)
```

Function to compute order of a set of kn tableaux of a given shape and value of m.

#### **Parameters**

- **shape** shape of tableau.
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar

Returns dictionary with key as order and frequency of that order in set as value of that key.

```
operation.order_of_tableau(t, m=None)
```

Function to compute the order of action c.

#### **Parameters**

- t tableau.
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar

Returns order of tableau t under action c.

```
operation.s_i(t, i, m=None, mutable=True)
```

Function to apply s\_i to the tableau.

#### **Parameters**

- t tableau.
- i i as in  $s_i$
- m m such that allowed entries in t are 1 to m and 1-bar to m-bar
- mutable optional. Default set to True, where it directly make changes to the tableau t. If passed in as False, it creates a copy of t and operates on that copy, leaving t unaffected.

Returns tableau obtained after applying s\_i on t.

## **TABLEAU MODULE**

This module contains definition of Tableau class.

```
class tableau (body)
     Bases: object
     This class defines a young tableau and provide functions to operate on it.
      ___eq___(other)
           Function to check if two Tableau object are same in the sense that they have same entries and same location
           for all location of boxes, using ==
               Parameters other - Tableau object with which invoking Tableau object is being compared.
               Returns True if both Tableau object are same. False otherwise.
        _init___(body)
           Initialization function for Tableau object.
               Parameters body - body of tableau.
      __ne__(other)
           Function to check if two Tableau object are not same.
               Parameters other -
                      param other Tableau object with which invoking Tableau object is being compared.
               Returns False if both Tableau object are same. True otherwise.
        _str___()
           Function to get string representation of the Tableau object.
               Returns string representation of the Tableau object.
               Return type str
     add_new_box(i, element)
           Function to add a new box at the end of a row.
               Parameters
                    • \mathbf{i} – row index.
                    • element – element to be added in new box.
               Returns None
     clone()
           Function to creat a copy of the invoking Tableau object
```

**Returns** copy of invoking tableau object.

## Return type Tableau

#### ${\tt del\_row\_lastbox}\,(i)$

Function to delete the last box in the row.

**Parameters** i – row index

Returns None

## static get\_shape\_transpose(shape)

Function to compute the transpose of a given shape.

**Parameters** shape – shape for which we need to calculate transpose.

**Returns** list representing transpose shape.

Return type list

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