Inference on the mode of weak directional signals: a Le Cam perspective on hypothesis testing near singularities

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In this paper we consider the problem of testing the null hypothesis that the mode of a directional signal is equal to a given value. The problem is tackled under asymptotic scenarios for which the signal strength goes to zero at an arbitrary rate. We study the classical Wald and Watson tests that are known to be asymptotically equivalent when the signal is strong. We show that they exhibit very different (null and non-null) behaviours when the signal becomes arbitrarily weak. The Watson test is shown to be adaptively rate-consistent and essentially adaptively Le Cam optimal. Throughout, our theoretical findings are illustrated via Monte-Carlo simulations. The practical relevance of our results is also shown on a real data example.