Biostatistics
Week V

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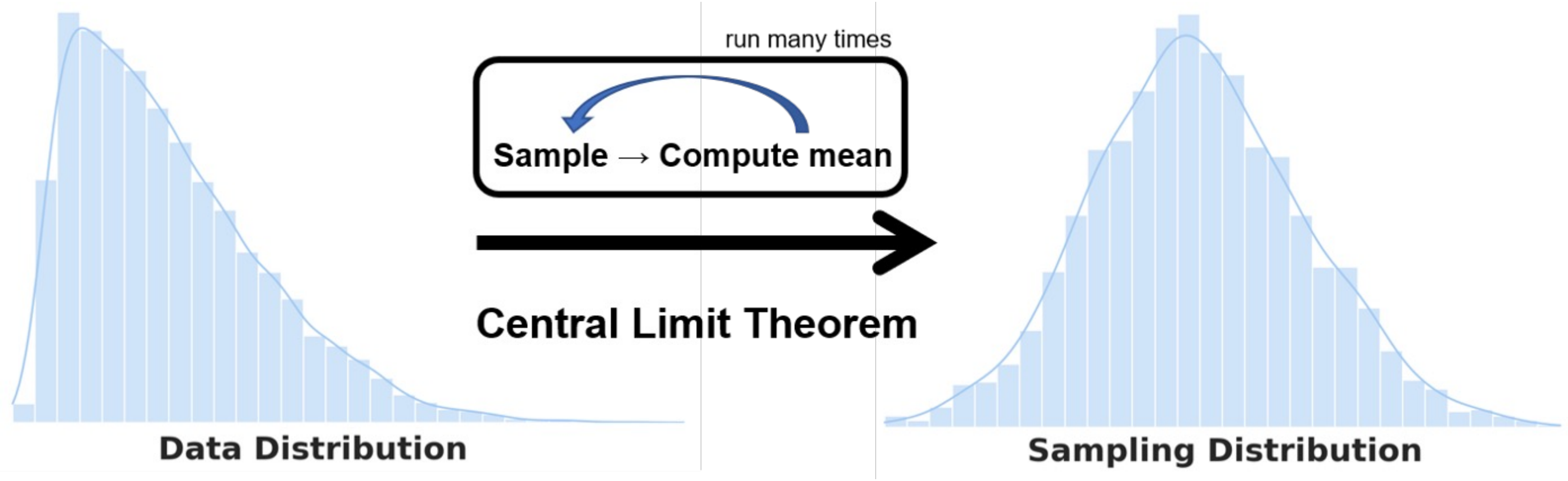
3 November 2022

https://towardsdatascience.com/data-distribution-vs-sampling-distribution-what-you-need-to-know-294819109796

Reminder



Reminder

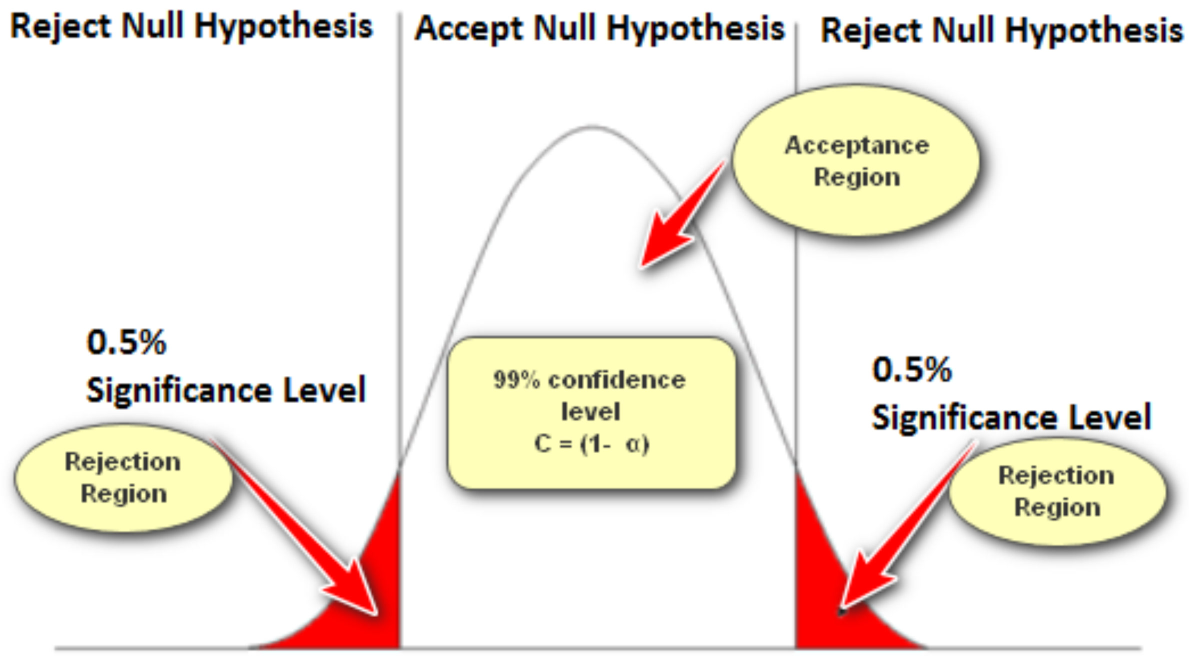
1. **Check assumptions, determine H_0 and H_a, choose α**
    - Assumptions differ based on the test
    - The null hypothesis always contains equality (=)
2. **Calculate the appropriate test statistic**
    - z, t, χ^2, …
3. **Calculate critical values/p value**
    - With the aid of precalculated tables/software
4. **Decide whether to reject/fail to reject H_0**
    - Reject if the statistic is within the critical region/$p \leq \alpha$

Reminder H_0	Decision	
	Fail to reject H_0	Reject H_0
H_0 is True	Correct decision	**Type I Error** α
H_0 is False	**Type II Error** **β**	Correct decision

Barai A. Hypothesis Testing | All a beginner needs to know [Internet]. Medium. 2020 [cited 2021 Oct 4]. Available from: https://medium.com/analytics-vidhya/hypothesis-testing-all-a-beginner-needs-to-know-dd1bdeebdbf1

Reminder

$$\text{test statistic} = \frac{\text{estimator} - \text{null value}}{\text{standard error of estimator}}$$



One-Sample t-Test – Example II

- It is claimed that the post-treatment tumor volume of glioblastoma patients subject to a novel treatment is different than 5 cm^3
- The mean tumor volume of 41 randomly-selected patients is 5.9 cm^3
- Sample standard deviation is 1.74

One-Sample t-Test – Example II (cont.)

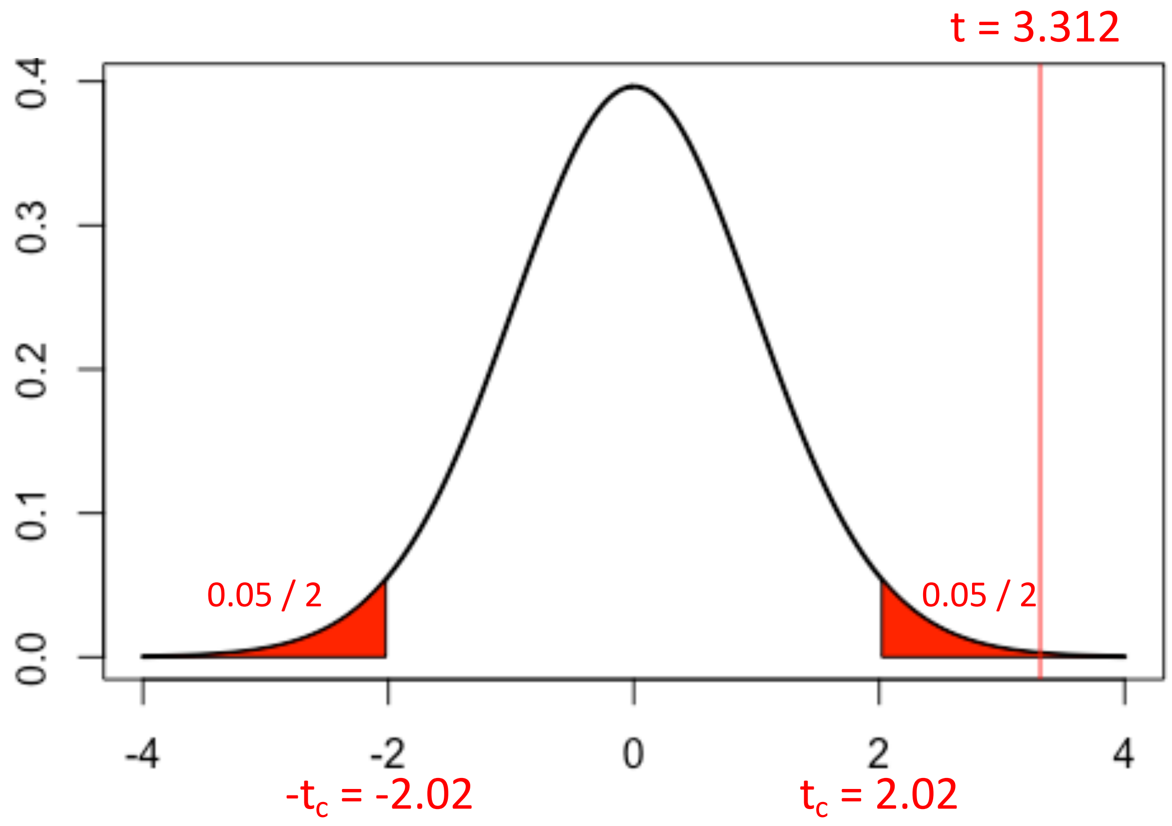
1. Check assumptions, determine H_0 and H_a, choose α
   - Normality of the variable is checked
   - H_0: $\mu = 5$ H_a: $\mu \neq 5$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{5.9 - 5}{1.74/\sqrt{41}} = 3.312 \qquad (\sim t_{n-1} = t_{40})$$

One-Sample t-Test – Example II (cont.)

3. Calculate **critical values**/p value
4. Decide whether to reject/fail to reject H_0



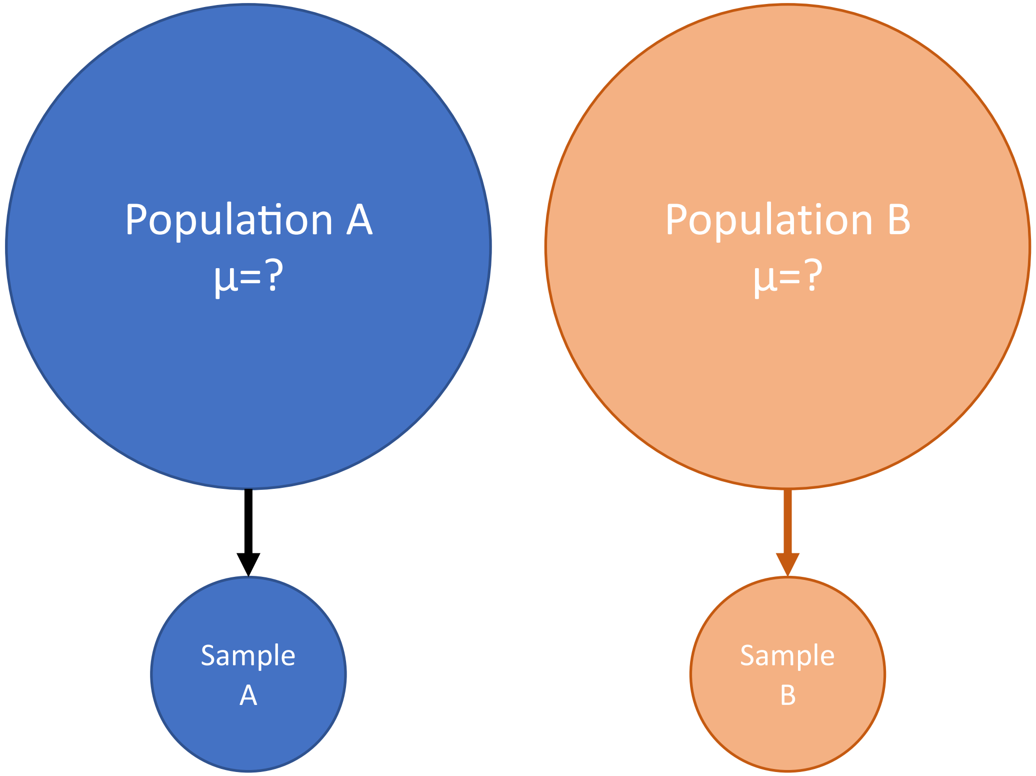
One-Sample t-Test – Example II (cont.)

5. **State a conclusion:**

With 95% confidence, we can conclude that there is enough evidence to say that post-treatment tumor volume of glioblastoma patients subject to a novel treatment is different than 5 cm^3.

Two-Sample t-Test

- The **two-sample t-test** (also known as the **independent samples t-test**) is a method used to test whether the unknown population means of two groups are equal or not



Two-sample t-Test

H_0: $\mu_X = \mu_Y$

H_a: $\mu_X \neq \mu_Y$

or

H_0: $\mu_X - \mu_Y = 0$

H_a: $\mu_X - \mu_Y \neq 0$

Two-sample t-Test

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}} \sim \mathrm{t}(\mathrm{m}),$$

$$m = \frac{(w_X + w_Y)^2}{\left(\frac{w_X^2}{n_X - 1} + \frac{w_Y^2}{n_Y - 1}\right)}$$

$$w_X = s_X^2 / n_X, \quad w_Y = s_Y^2 / n_Y$$

Two-sample t-Test – Example I

id	treatment	perc_benefit	id	treatment	perc_benefit
158	trt1	37.2549020	15	trt2	10.0978368
392	trt1	-4.3864459	143	trt2	0.5048635
457	trt1	-5.1075269	470	trt2	-0.8156940
487	trt1	36.7043369	536	trt2	50.0000000
723	trt1	5.1303099	549	trt2	-3.0303030
832	trt1	3.1806616	750	trt2	-2.8977108
894	trt1	-3.9062500	891	trt2	26.3872135
1104	trt1	5.9443608	997	trt2	4.3651179
1283	trt1	-0.8601855	1000	trt2	2.3582125
1288	trt1	-3.1674208	1209	trt2	8.9702189

- Mean percentage benefit is 7.078674 for group 1, and 9.593976 for group 2
- Is the difference a significant one?

Two-sample t-Test – Example I (cont.)

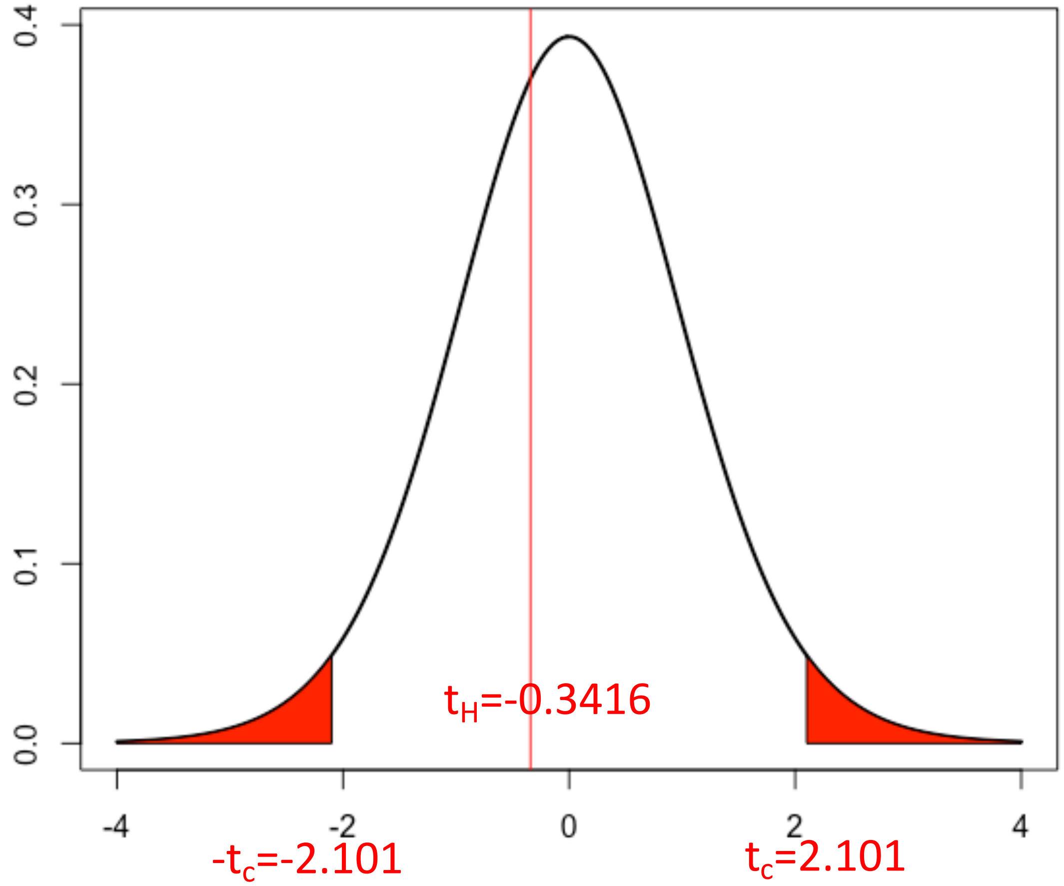
1. Check assumptions, determine H_0 and Ha, choose α
   - We check that the variables are normally distributed
   - H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 \neq \mu_2$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = -0.3416(\sim t_{17.98834})$$

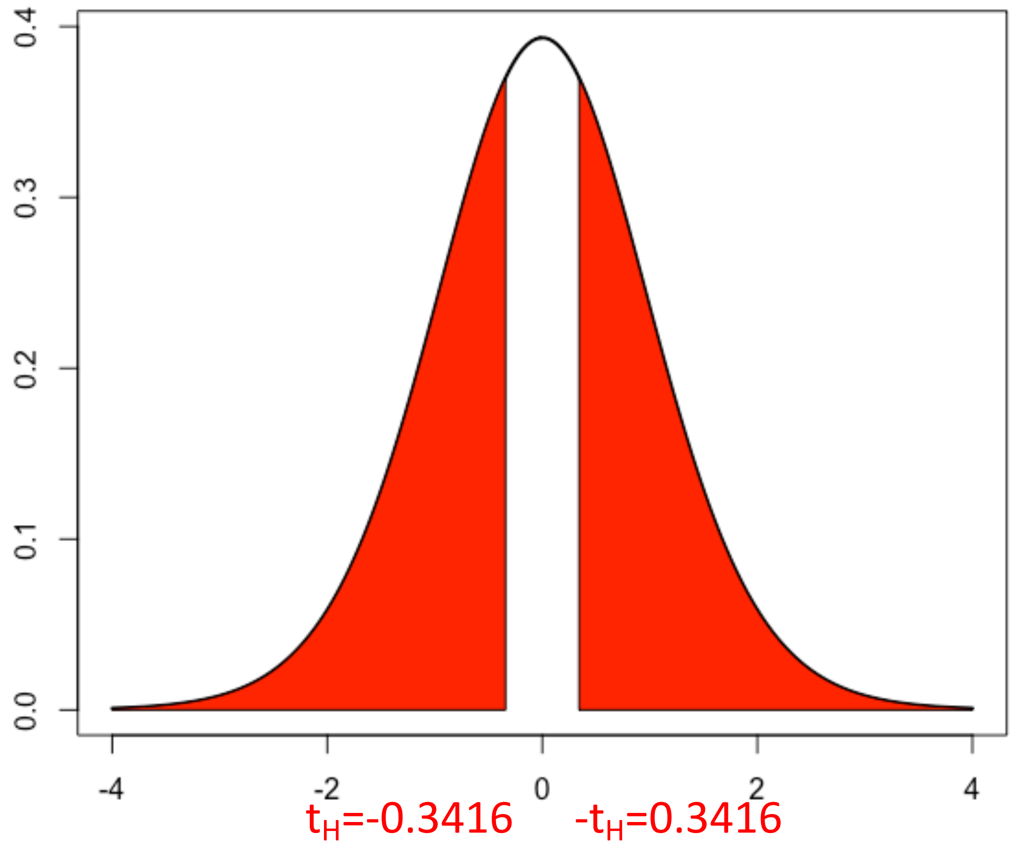
Two-sample t-Test – Example I (cont.)

3. Calculate **critical values**/p value
4. Decide whether to reject/fail to reject H_0

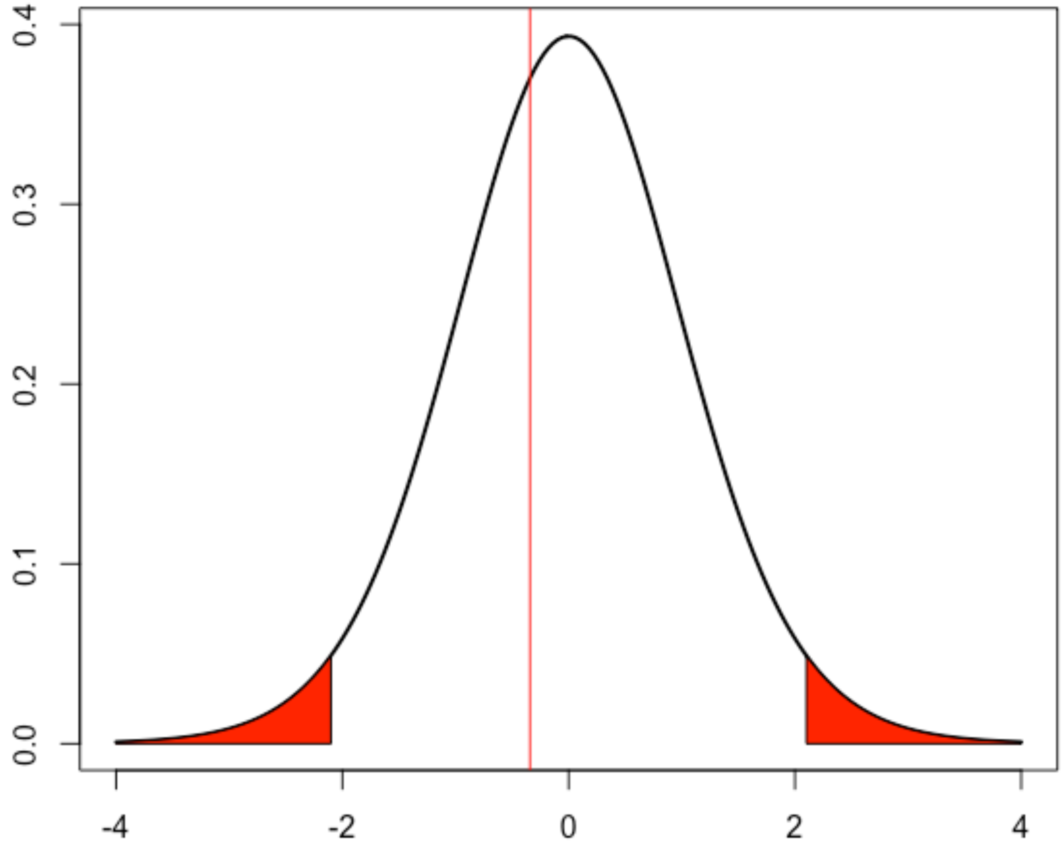


Two-sample t-Test – Example I (cont.)

3. Calculate critical values/**p value**
4. Decide whether to reject/fail to reject H_0



Two-sample t-Test – Example I (cont.)



95% confidence interval for $\mu_1 - \mu_2$ = [-17.98 , 12.95]

Two-sample t-Test – Example I (cont.)

- there is not enough evidence to say mean percentage benefit for treatment 1 and treatment 2 are significantly different

Two-sample t-Test – Example II

- In a study,
  - The sedimentation rate of 12 arthritis patients was measured:

  $\bar{X}_1 = 82.79$ mm and $s_1 = 18.4$ mm
  - The sedimentation rate of 15 healthy controls was measured

  $\bar{X}_2 = 69.03$ mm and $s_2 = 21.4$ mm
- Is there a difference between the mean sedimentation rates of the two groups?

Two-sample t-Test – Example II (cont.)

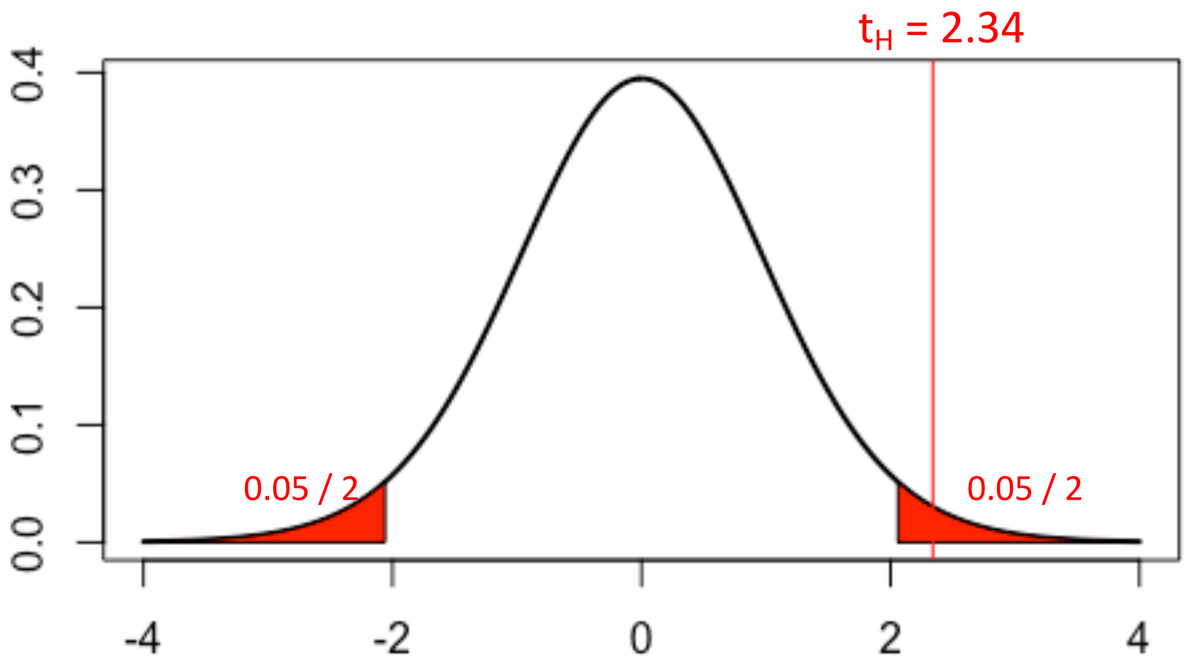
1. Check assumptions, determine H_0 and Ha, choose α
   - We check that the variables are normally distributed
   - H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 \neq \mu_2$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = 2.34 \qquad (\sim t_{25})$$

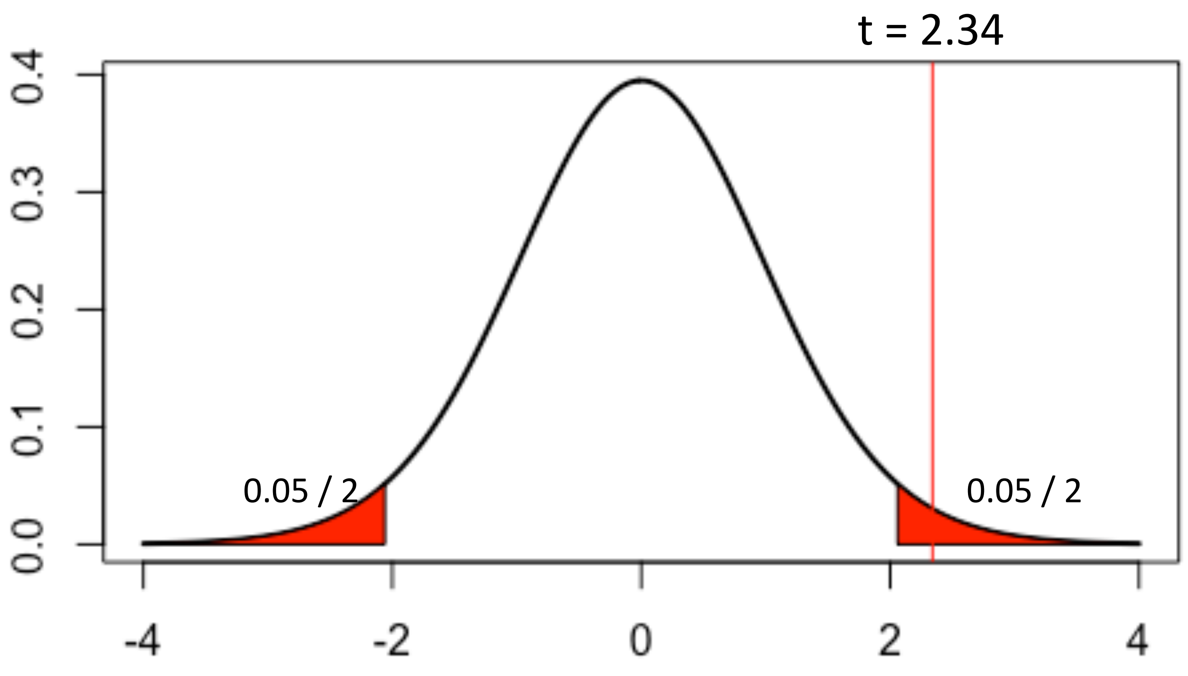
Two-sample t-Test – Example II (cont.)

3. Calculate critical values/p value
4. Decide whether to reject/fail to reject H_0



p = 0.027568

Two-sample t-Test – Example II (cont.)



p = 0.027568

95% confidence interval for $\mu_1 - \mu_2$ = [3.52, 33]

Two-sample t-Test – Example II (cont.)

- With 95% confidence, there is enough evidence to say that there is a difference between the mean sedimentation rates of the two groups

Wong DT, Dallaire A, Singh KP, Madhusudan P, Jackson T, Singh M, et al. High-flow nasal oxygen improves safe apnea time in morbidly obese patients undergoing general anesthesia: a randomized controlled trial. Anesth Analg. 2019 Oct;129(4):1130–6.

Two-sample t-Test – Example III

- "Morbidly obese patients undergoing general anesthesia are at risk of hypoxemia during anesthesia induction"
- A randomized controlled trial investigating:
- Does high-flow nasal oxygenation provide longer safe apnea time compared to conventional facemask oxygenation during anesthesia induction in morbidly obese surgical patients?

Wong DT, Dallaire A, Singh KP, Madhusudan P, Jackson T, Singh M, et al. High-flow nasal oxygen improves safe apnea time in morbidly obese patients undergoing general anesthesia: a randomized controlled trial. Anesth Analg. 2019 Oct;129(4):1130–6.

Two-sample t-Test – Example III (cont.)

- Safe Apnea time in Control Group (n = 20)
  - $\overline{X_c} = 185.5$
  - $s_c = 53$
- Safe Apnea time in High-Flow Nasal Oxygenation Group (n = 20)
  - $\overline{X_T} = 261.4$
  - $s_T = 77.7$

Two-sample t-Test – Example III (cont.)

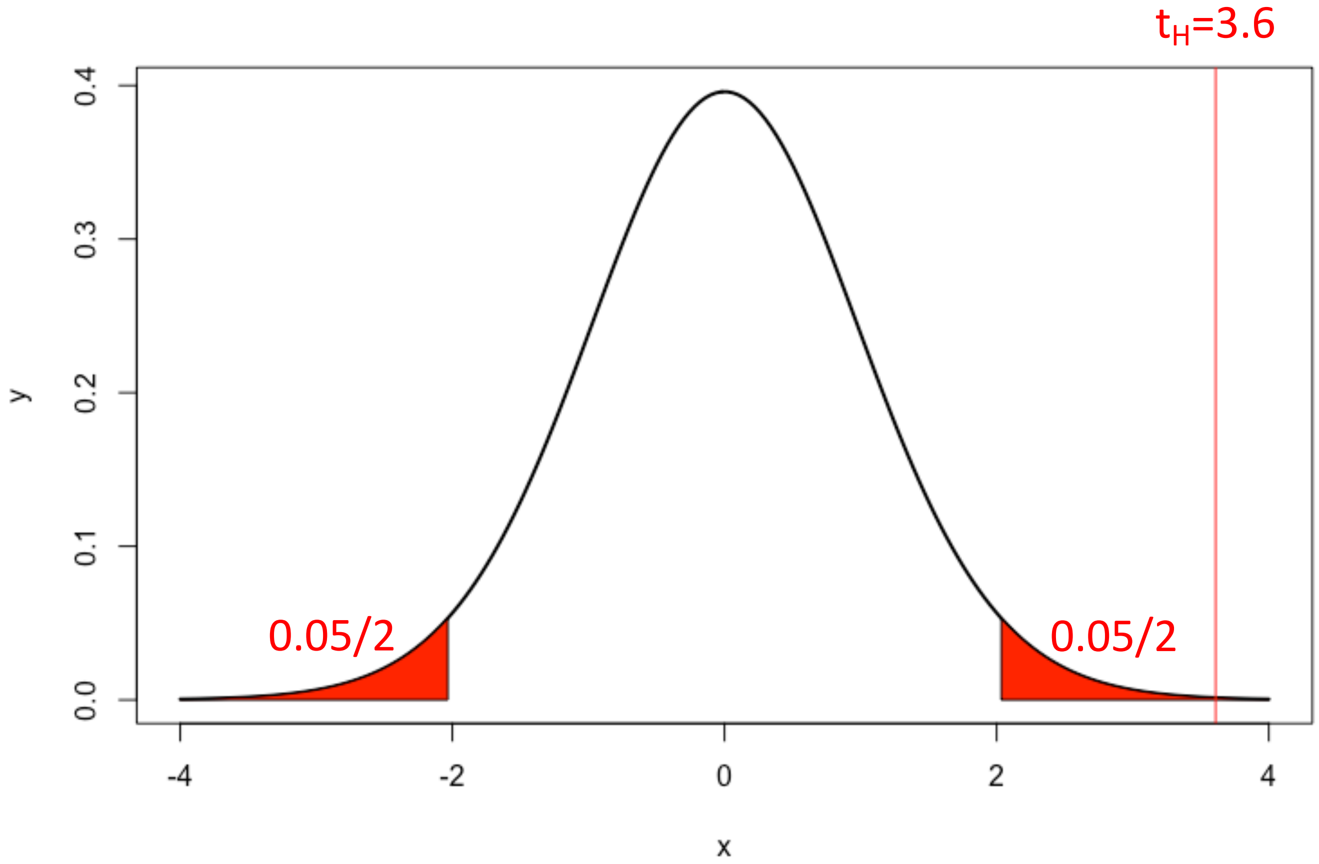
1. Check assumptions, determine H_0 and Ha, choose α
   - We check that the variables are normally distributed
   - H_0: $\mu_c = \mu_T$ H_a: $\mu_c \neq \mu_T$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = 3.6 \qquad (\sim t_{33.53})$$

Two-sample t-Test – Example III (cont.)

3. Calculate **critical values**/p value
4. Decide whether to reject/fail to reject H_0



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Two-sample t-Test – Example III (cont.)

Table 2. Study Outcomes: Safe Apnea Time, Minimum SpO_2, Plateau $ETCO_2$, and Time to Regain Baseline SpO_2

	Control Group (n = 20)	High-Flow Nasal Oxygenation Group (n = 20)	Mean Difference (95% CI)	*P* Value
Safe apnea time (s)	185.5 ± 53.0	261.4 ± 77.7	75.9 (33.3–118.5)	.001
Minimum SpO_2 (%)	87.9 ± 4.7	90.9 ± 3.5	3.1 (0.4–5.7)	.026
Plateau $ETCO_2$ (mm Hg)	38.8 ± 2.5	37.9 ± 3.0	−0.8 (−2.6 to 0.9)	.33
Time to regain baseline SpO_2 (s)	49.6 ± 20.8	37.3 ± 6.8	−12.3 (−22.2 to −2.4)	.016

Values represent mean ± SD.
Control group: facemask oxygenation.
Abbreviations: CI, confidence interval; $ETCO_2$, end-tidal carbon dioxide; SpO_2, oxygen saturation measured by pulse oximetry.

"Safe apnea time was significantly longer (261.4 ± 77.7 vs 185.5 ± 52.9 seconds; mean difference [95% CI], 75.9 [33.3–118.5]; *P* = .001)…"

Barai A. Hypothesis Testing | All a beginner needs to know [Internet]. Medium. 2020 [cited 2021 Oct 4]. Available from: https://medium.com/analytics-vidhya/hypothesis-testing-all-a-beginner-needs-to-know-dd1bdeebdbf1

Brief Summary