Biostatistics
Week VII

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ACIBADEM
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Hypothesis Testing - Steps

1. **Check assumptions, determine H_0 and H_a, choose α**
   - Assumptions differ based on the test
   - The null hypothesis always contains equality (=)
2. **Calculate the appropriate test statistic**
   - z, t, χ^2, …
3. **Calculate critical values/p value**
   - With the aid of precalculated tables/software
4. **Decide whether to reject/fail to reject H_0**
   - Reject if the statistic is within the critical region/$p \leq \alpha$

χ^2 Test of Association

- Used to assess the association between two categorical variables
- More generally, used to investigate the significance of the difference between expected and observed values

- Are the 2 categorical variables **independent**?

χ^2 Test – Test Statistic

$$\chi^2 = \sum \frac{(observed - expected)^2}{expected}$$

Cupples ME, McKnight A. Randomised controlled trial of health promotion in general practice for patients at high cardiovascular risk. BMJ. 1994 Oct 15;309(6960):993–6.

χ^2 Test – Example

TABLE III—*Changes in frequency of physical exercise in patients with angina between baseline and review at two years*

	No (%) of patients	
	Intervention group	Control group
Increased	108 (34)	63 (21)
No change	120 (38)	74 (25)
Decreased	89 (28)	163 (54)

χ^2 Test – Example

1. Determine H_0 and H_a, choose α
   - H_0: there is **no association** between frequency of physical exercise and group
     H_a: there **is association** between frequency of physical exercise and group
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

χ^2 Test – Example

	Intervention Group	Control Group	Total
Increased	108	63	**171**
No change	120	74	**194**
Decreased	89	163	**252**
Total	**317**	**300**	**617**

$$expected_{1,1} = 317 \times \frac{171}{617} \qquad expected_{1,2} = 300 \times \frac{171}{617}$$

$$expected_{2,1} = 317 \times \frac{194}{617} \qquad expected_{2,2} = 300 \times \frac{194}{617}$$

$$expected_{3,1} = 317 \times \frac{252}{617} \qquad expected_{3,2} = 300 \times \frac{252}{617}$$

χ^2 Test – Example

OBSERVED	Intervention Group	Control Group
Increased	108	63
No change	120	74
Decreased	89	163

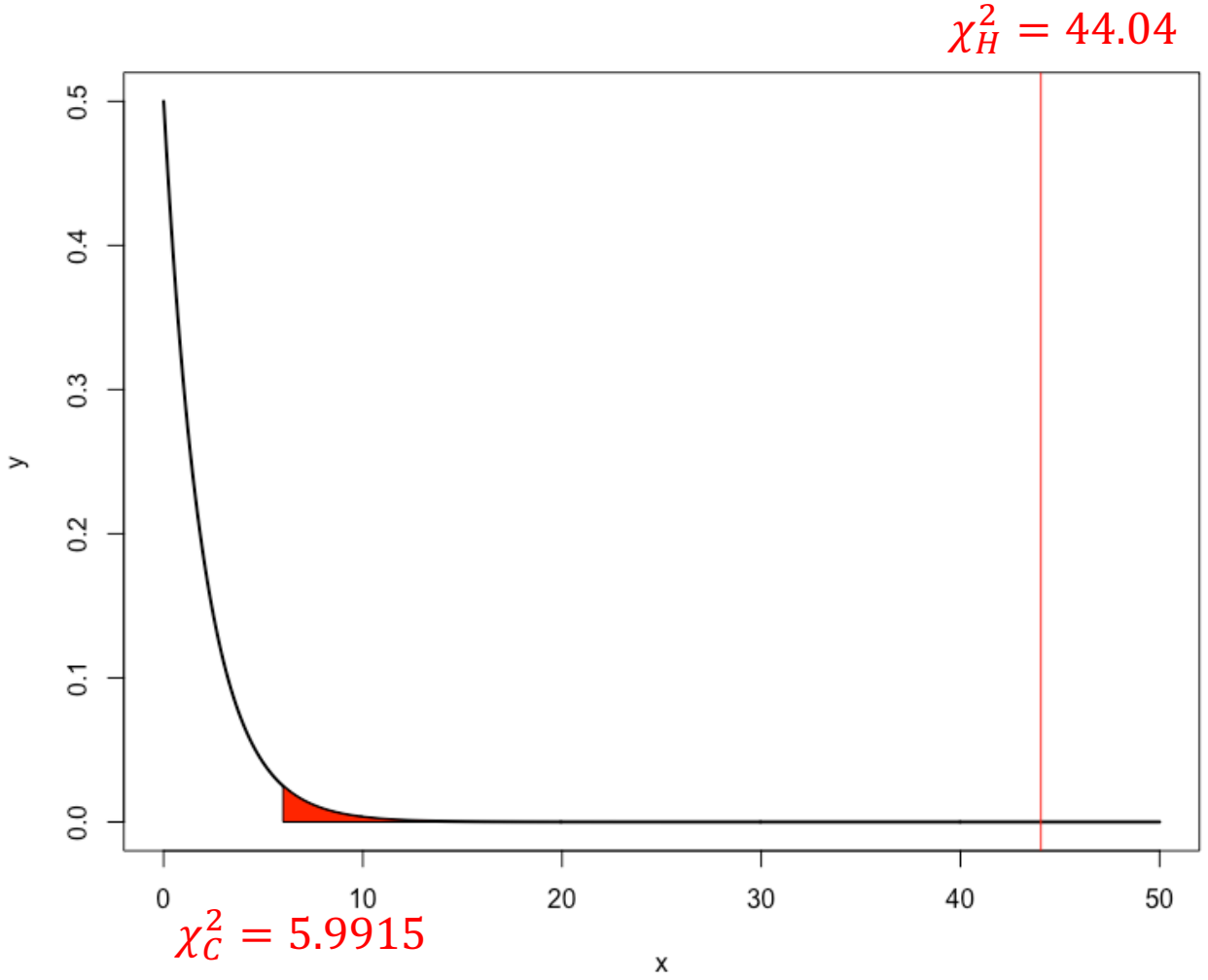
EXPECTED	Intervention Group	Control Group
Increased	87.86	83.14
No change	99.67	94.33
Decreased	139.47	122.53

χ^2 Test – Test Statistic

$$\chi^2 = \sum \frac{(observed\ - expected)^2}{expected}$$

$$\chi^2_H = 44.04 \sim \chi^2_{(3-1)(2-1)=2}$$

χ^2 Test – Test Statistic



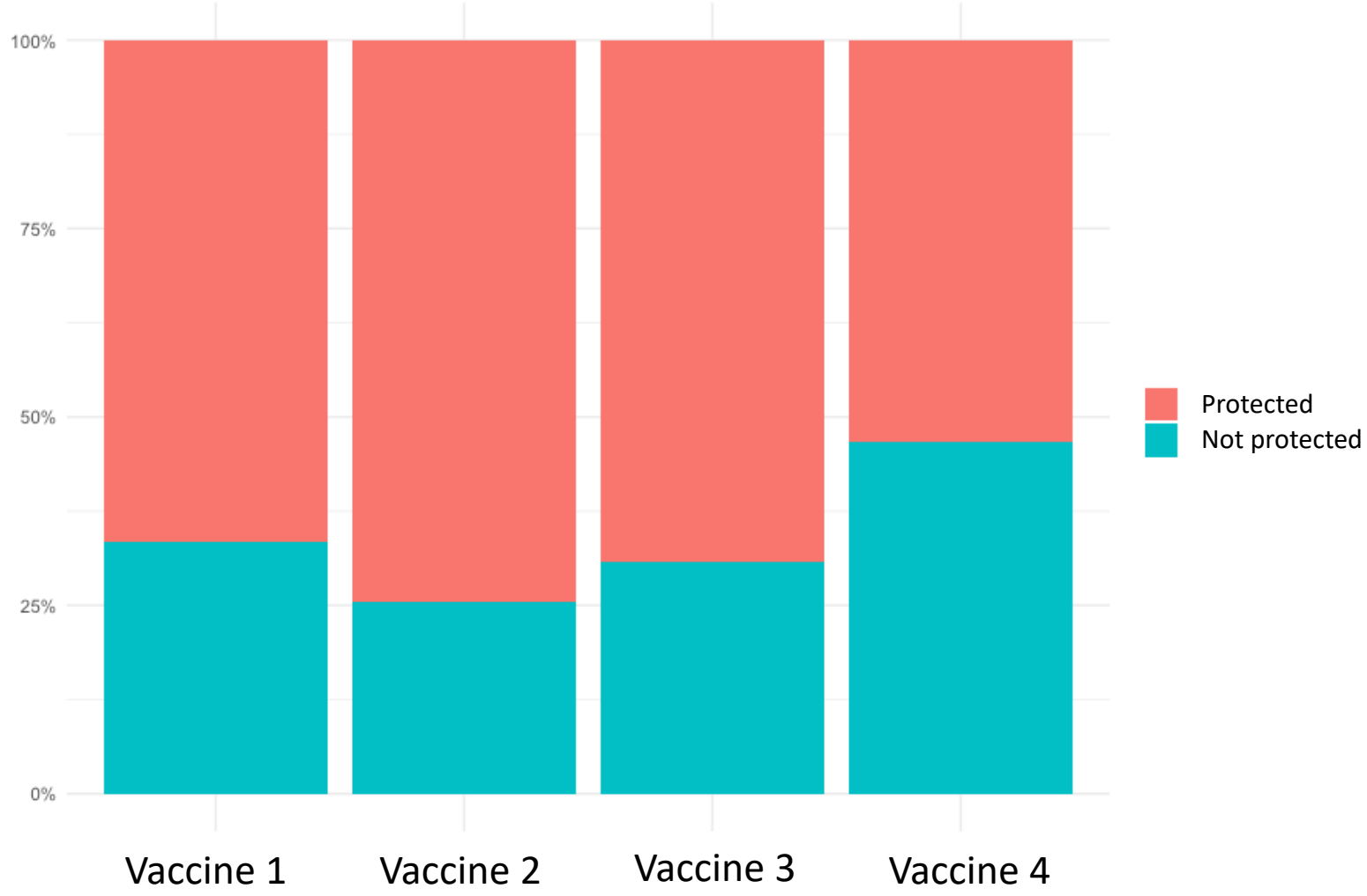
p < 0.001

χ^2 Test – Example

- Is the protection status dependent on different COVID vaccines?

	Protected	Not protected
Vaccine 1	82	41
Vaccine 2	70	24
Vaccine 3	45	20
Vaccine 4	48	42

χ^2 Test – Example



χ^2 Test – Example

1. Check assumptions, determine H_0 and H_a, choose α
   - H_0: there is **no association** between protection status and vaccine type

     H_a: there **is association** between protection status and vaccine type
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

   $\chi_H^2 = 9.297 \sim \chi_3^2$

χ^2 Test – Example

	Proctected	Not protected	Total
Vaccine 1	82	41	**123**
Vaccine 2	70	24	**94**
Vaccine 3	45	20	**65**
Vaccine 4	48	42	**90**
Total	**245**	**127**	**372**

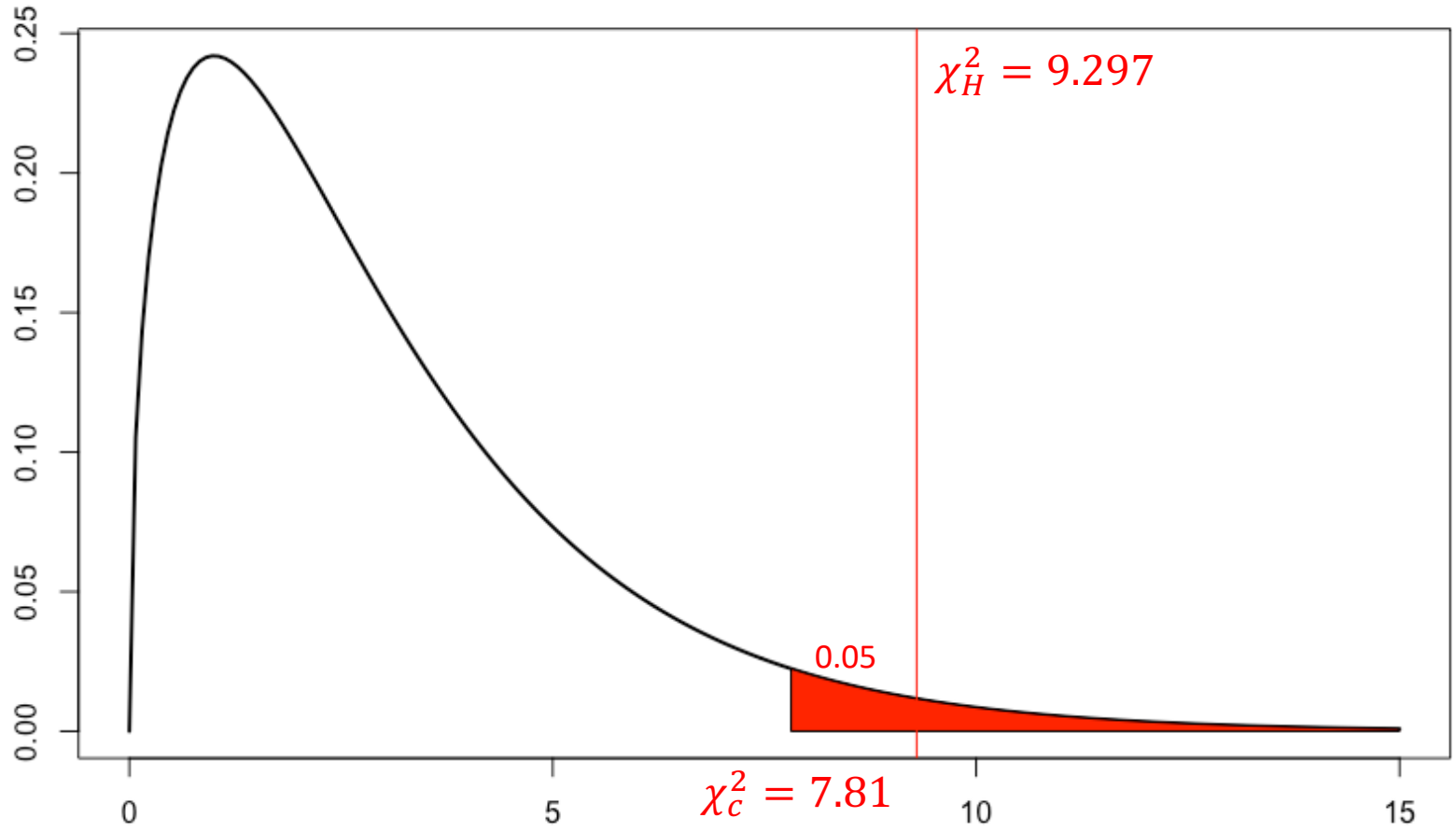
$$expected_{4,1} = 245 \times \frac{90}{372} = 59$$

$$expected_{4,2} = 127 \times \frac{90}{372} = 31$$

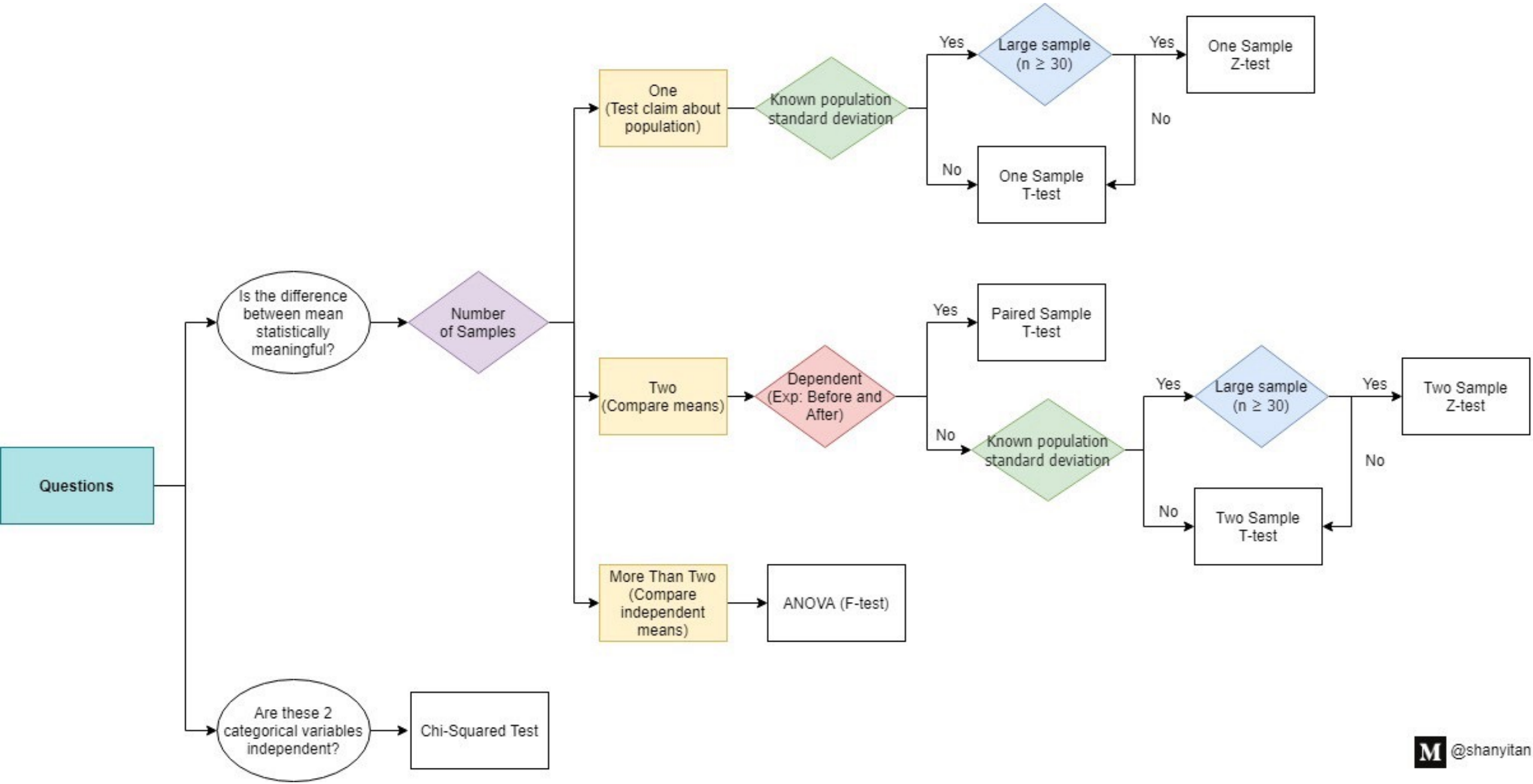
$$\chi^2_H = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{(observed_{ij} - expected_{ij})^2}{expected_{ij}} \sim \chi^2_{(m-1)(n-1)}$$

$$\boldsymbol{\chi^2_H = 9.297 \sim \chi^2_3}$$

χ^2 Test – Example



p = 0.025592


Questions
Is the difference between mean statistically meaningful?
Number of Samples
One (Test claim about population)
Known population standard deviation
Yes
Large sample (n ≥ 30)
Yes
One Sample Z-test
No
No
One Sample T-test
Two (Compare means)
Dependent (Exp: Before and After)
Yes
Paired Sample T-test
No
Known population standard deviation
Yes
Large sample (n ≥ 30)
Yes
Two Sample Z-test
No
No
Two Sample T-test
More Than Two (Compare independent means)
ANOVA (F-test)
Are these 2 categorical variables independent?
Chi-Squared Test
@shanyitan