Biostatistics
Week VI

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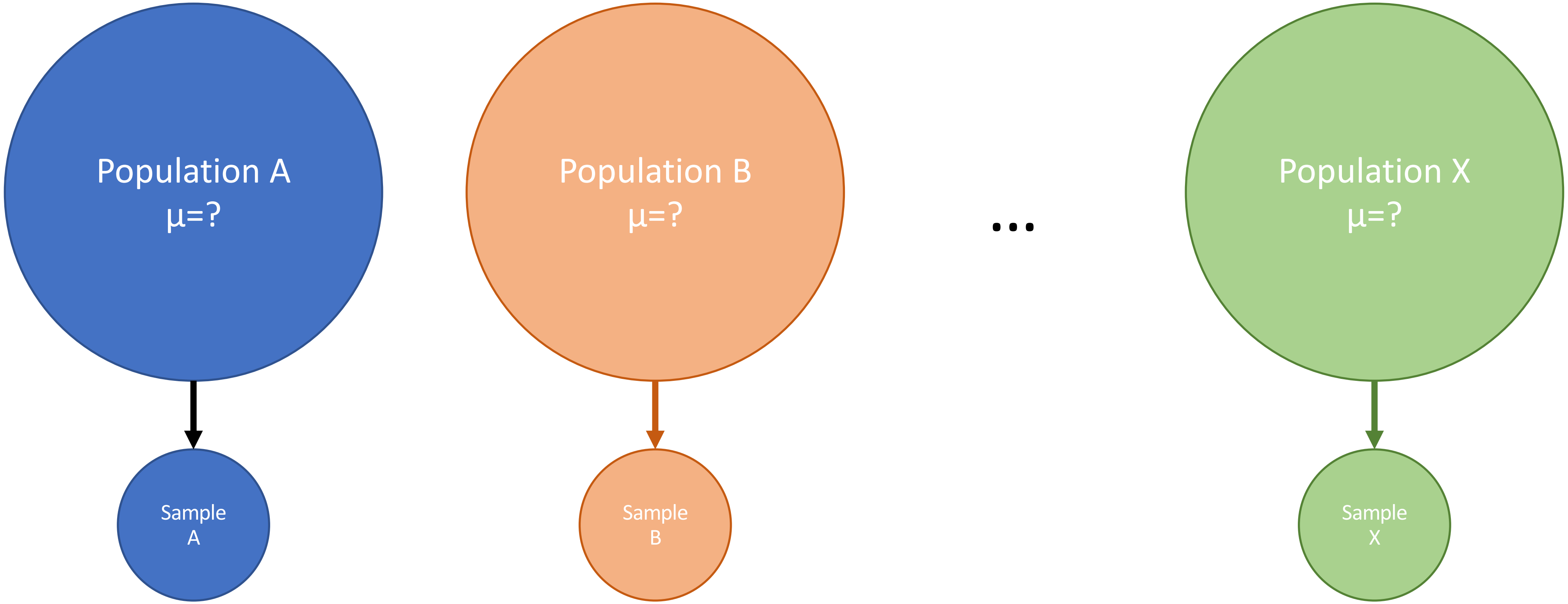
ACIBADEM
MEHMET ALİ AYDINLAR
ÜNİVERSİTESİ

Hypothesis Testing - Steps

1. **Check assumptions, determine H_0 and H_a, choose α**
    - Assumptions differ based on the test
    - The null hypothesis always contains equality (=)
2. **Calculate the appropriate test statistic**
    - z, t, χ^2, …
3. **Calculate critical values/p value**
    - With the aid of precalculated tables/software
4. **Decide whether to reject/fail to reject H_0**
    - Reject if the statistic is within the critical region/$p \leq \alpha$

Analysis of Variance (ANOVA)

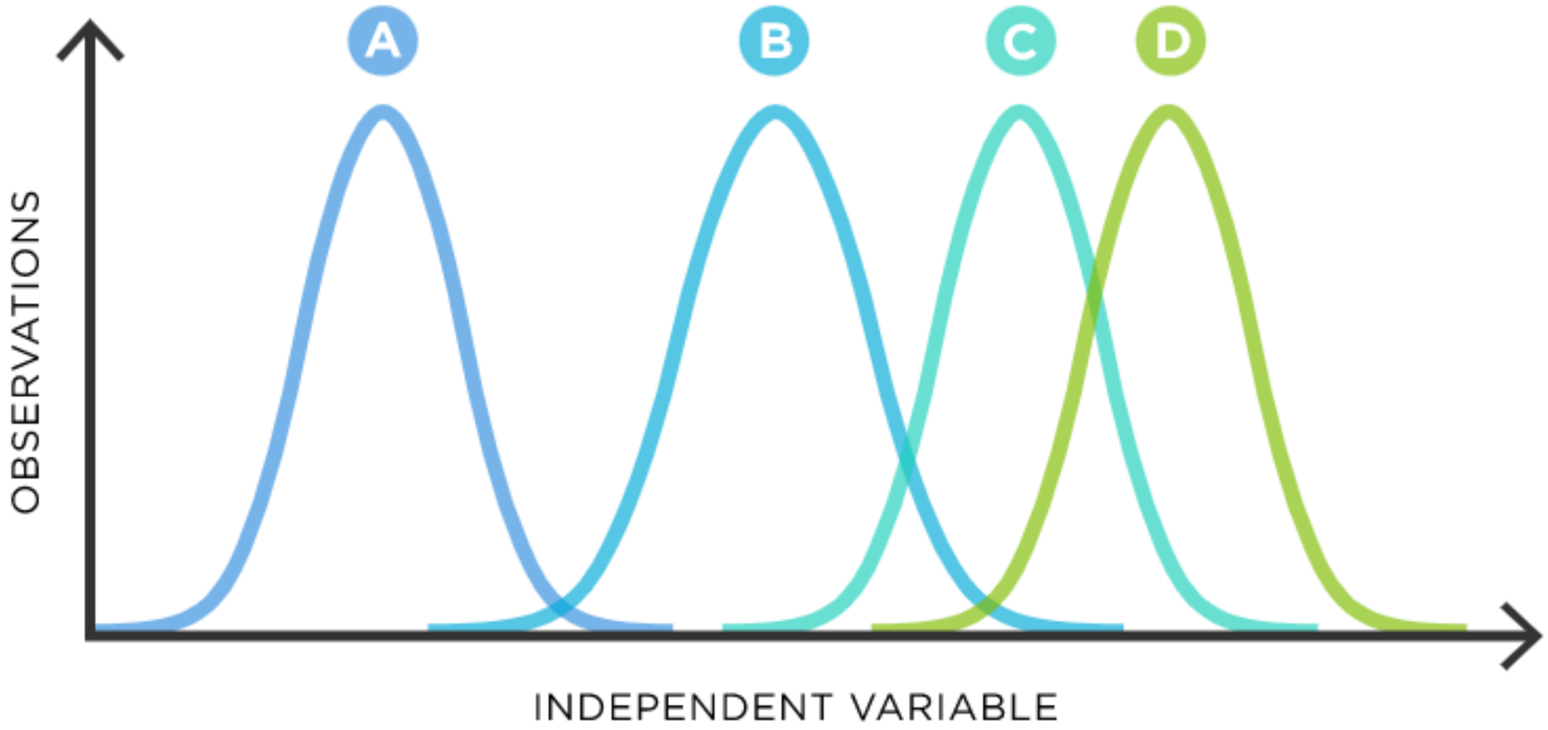
- Analysis of variance (ANOVA) is a statistical technique that is used to check if the means of **two or more groups** are significantly different from each other



ANOVA

$H_0: \mu_1 = \mu_2 = \ldots = \mu_n$

H_a: at least one μ_i is different



One-way ANOVA

k: number of groups
n: total number of samples
n_i: number of samples in group i

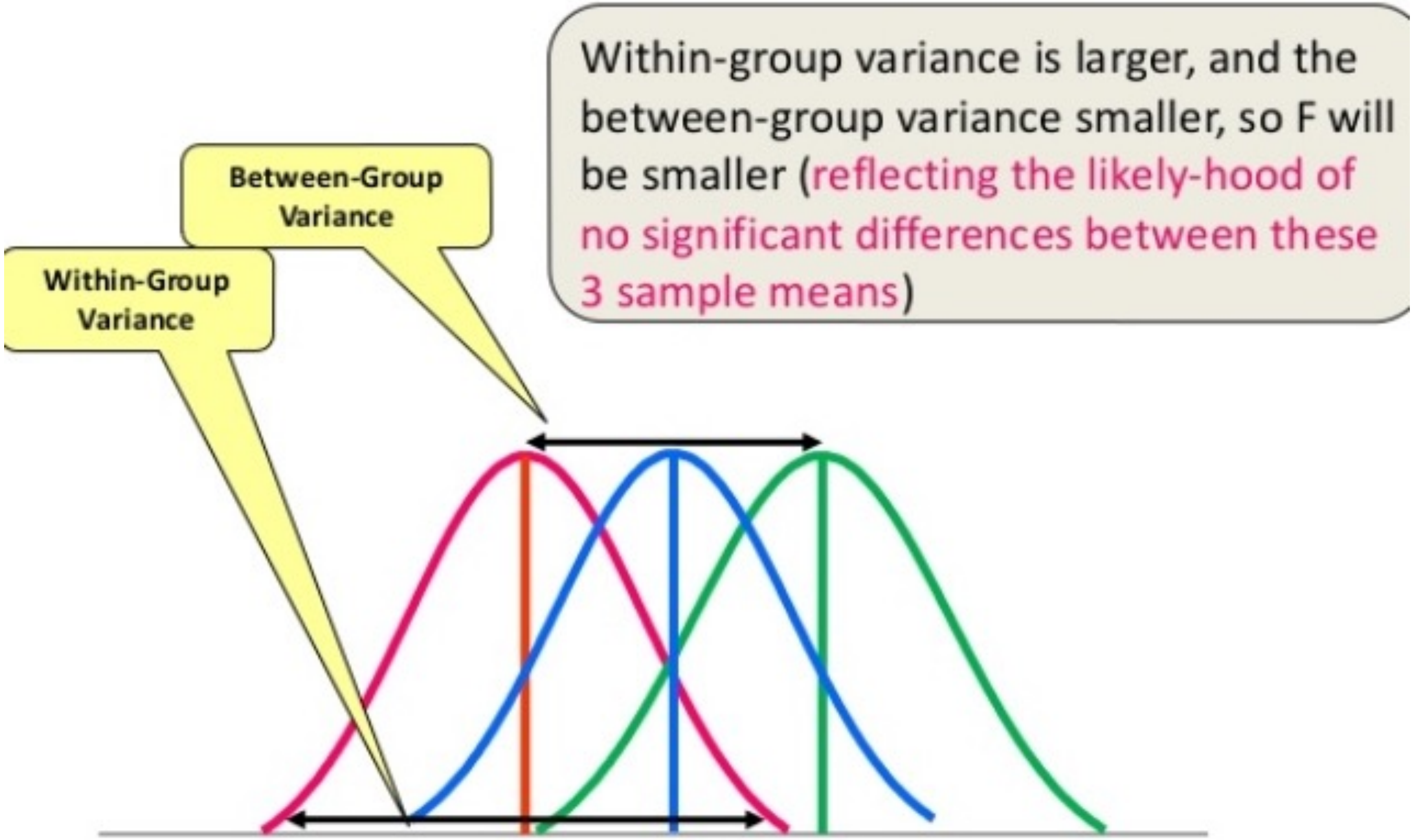
Analysis of Variance(ANOVA)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares (MS)	F
Between	$\sum n_i(\bar{X}_i - \bar{X})^2$	k - 1	SS_b/df_b	$F = \frac{MS_b}{MS_w}$
Within	SS_T - SS_b	n - k	SS_w/df_w	
Total	$\sum (X_j - \bar{X})^2$	n - 1		

How to perform analysis of variance (Anova) - step by step procedure [Internet]. The Genius Blog. 2018 [cited 2021 Oct 6]. Available from: https://kindsonthegenius.com/blog/how-to-perform-analysis-of-variance-anova-step-by-step-procedure/

Analysis of variance (Anova) | introduction, types & techniques [Internet]. Analytics Vidhya. 2018 [cited 2021 Oct 6]. Available from: https://www.analyticsvidhya.com/blog/2018/01/anova-analysis-of-variance/

ANOVA



One-way ANOVA – Example I

Table 1: Percentage benefits for 5 patients from each treatment groups.

Treatment 1	Treatment 2	Treatment 3	Treatment 4
-7.2	-13.0	-3.8	7.0
2.5	-0.4	-2.7	1.5
1.4	-1.6	5.3	9.4
-0.7	4.9	-5.9	9.5
-0.9	-0.7	3.7	9.9

The hypothesis of interest is

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$
$H_1 :$ at least one is different from the others

One-way ANOVA – Example I (cont.)

1. Check assumptions, determine H_0 and H_a, choose α
    - Check that data is normally distributed
    - H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ H_a: at least one mean is different
    - $\alpha = 0.05$

One-way ANOVA – Example I (cont.)

2. Calculate the appropriate test statistic

Sources of variation	Sum of squares	degrees-of-freedom	Mean squared error	F	p-value
Between treatment					
Within treatment					
Total					

One-way ANOVA – Example I (cont.)

2. Calculate the appropriate test statistic

Step 1: Calculate the treatment means and grand mean:

$$\bar{x}_1 = \frac{-7.2+2.5+1.4+(-0.7)+(-0.9)}{5} = -0.98$$
$$\bar{x}_2 = \frac{-13.0+(-0.4)+(-1.6)+4.9+(-0.7)}{5} = -2.16$$
$$\bar{x}_3 = \frac{-3.8+(-2.7)+(5.3)+(-5.9)+3.7}{5} = 0.68$$
$$\bar{x}_4 = \frac{7.0+1.5+9.4+9.5+9.9}{5} = 7.46$$
$$\bar{x} = \frac{-7.2+...+(-0.9)+(-13.0)+...(-0.7)+(-3.8)+...+3.7+7.0+...+9.9}{20} = 0.91$$

One-way ANOVA – Example I (cont.)

2. Calculate the appropriate test statistic

Step 3: Calculate between treatment sum of squared error:

$$5\ (-0.98 - 0.91\ )^2 + 5(-2.16 - 0.91)^2 + 5(0.68 - 0.91)^2 + 5(7.46 - 0.91)^2 = 292.138$$

Step 4: Calculate the total sum of squared error:

$$(-7.2 - 0.91)^2 + \ldots + (-0.9 - 0.91)^2 + (-13.0 - 0.91)^2 + \ldots (-0.7 - 0.91)^2 + (-3.8 - 0.91)^2 + \ldots + (3.7 - 0.91)^2 + (7.0 - 0.91)^2 + \ldots + (9.9 - 0.91)^2 = 667.198$$

Step 5: Calculate the within-group sum of squared error as $667.198 - 292.138 = 375.06$

One-way ANOVA – Example I (cont.)

2. Calculate the appropriate test statistic

Step 6: Total d.o.f.: 20 - 1, 19; between treatment d.o.f: 4-1=3; within treatment d.o.f.: 19-3=16

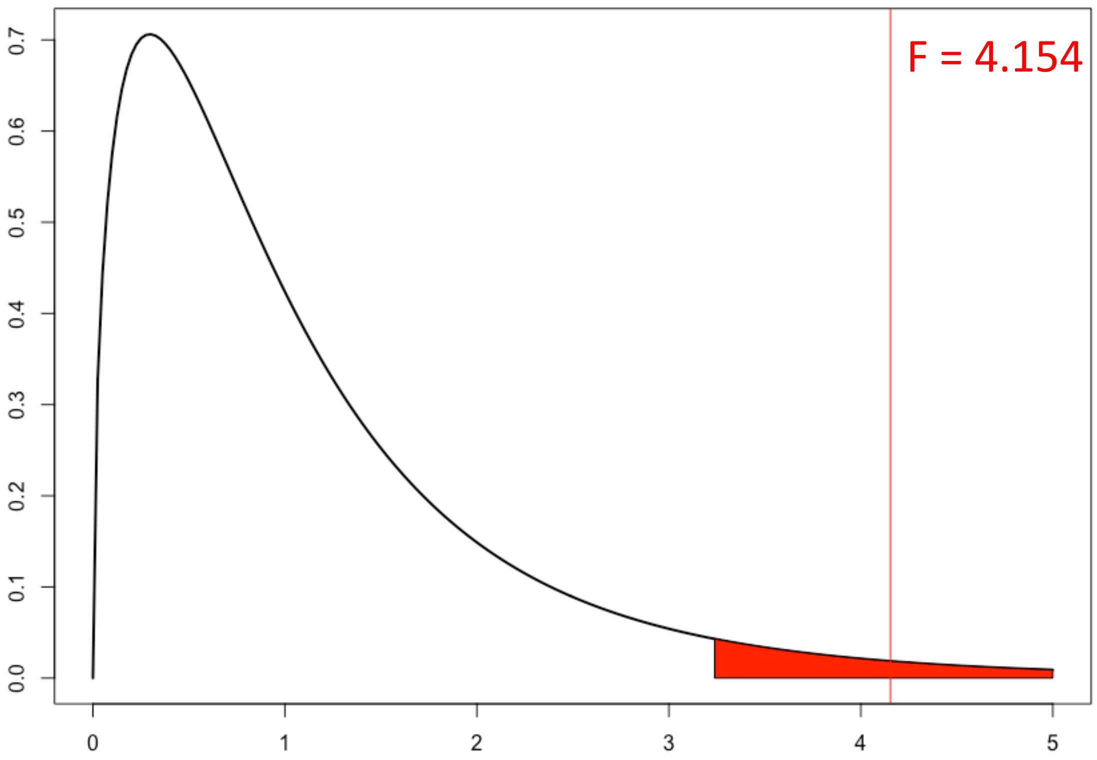
Step 7: Calculate mean suqared error for between treatment as 292.138/3=97.38

Step 8: Calculate mean squared error for within treatment as 375.06.198/16=23.44

Step 9: Calculate F value as 97.38/23.44=4.154

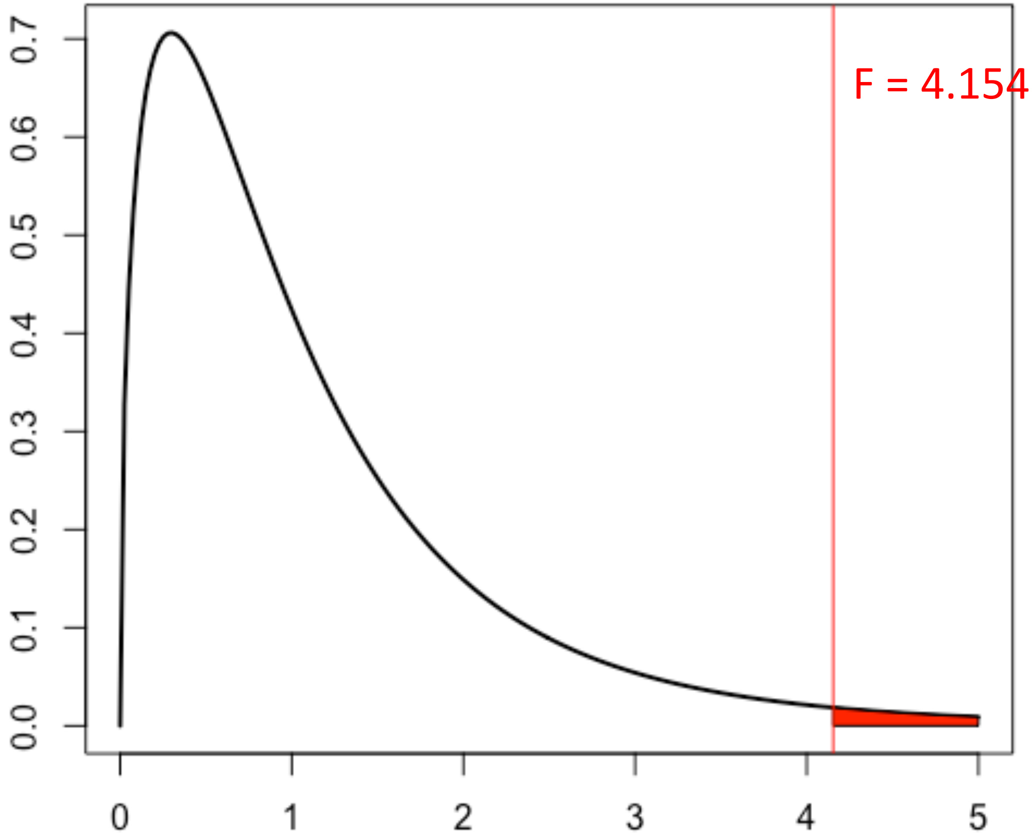
One-way ANOVA – Example II (cont.)

3. Calculate **rejection zone**/p value
4. Decide whether to reject/fail to reject H_0



One-way ANOVA – Example II (cont.)

3. Calculate rejection zone/**p value**
4. Decide whether to reject/fail to reject H_0



p=0.023516

One-way ANOVA – Example II

THE LANCET, AUGUST 12, 1978

MEGALOBLASTIC HÆMOPOIESIS IN PATIENTS RECEIVING NITROUS OXIDE

J. A. L. Amess   J. F. Burman
G. M. Rees   D. G. Nancekievill
D. L. Mollin

Departments of Hæmatology, Cardiothoracic Surgery, and Anæsthetics, St. Bartholomew's Hospital, West Smithfield, London EC1A 7BE

- 22 patients who underwent coronary artery bypass graft surgery (CABG) are separated into 3 different treatment groups (different ventilation strategies)
- Is there a difference in red blood cell folic acid measurements at 24 hours between the 3 treatment groups?

One-way ANOVA – Example II (cont.)

Group I.—8 patients received approximately 50% nitrous oxide and 50% oxygen mixture continuously for 24 h. 1 patient received 2000 μg of hydroxocobalamin intramuscularly immediately before and after the operation.

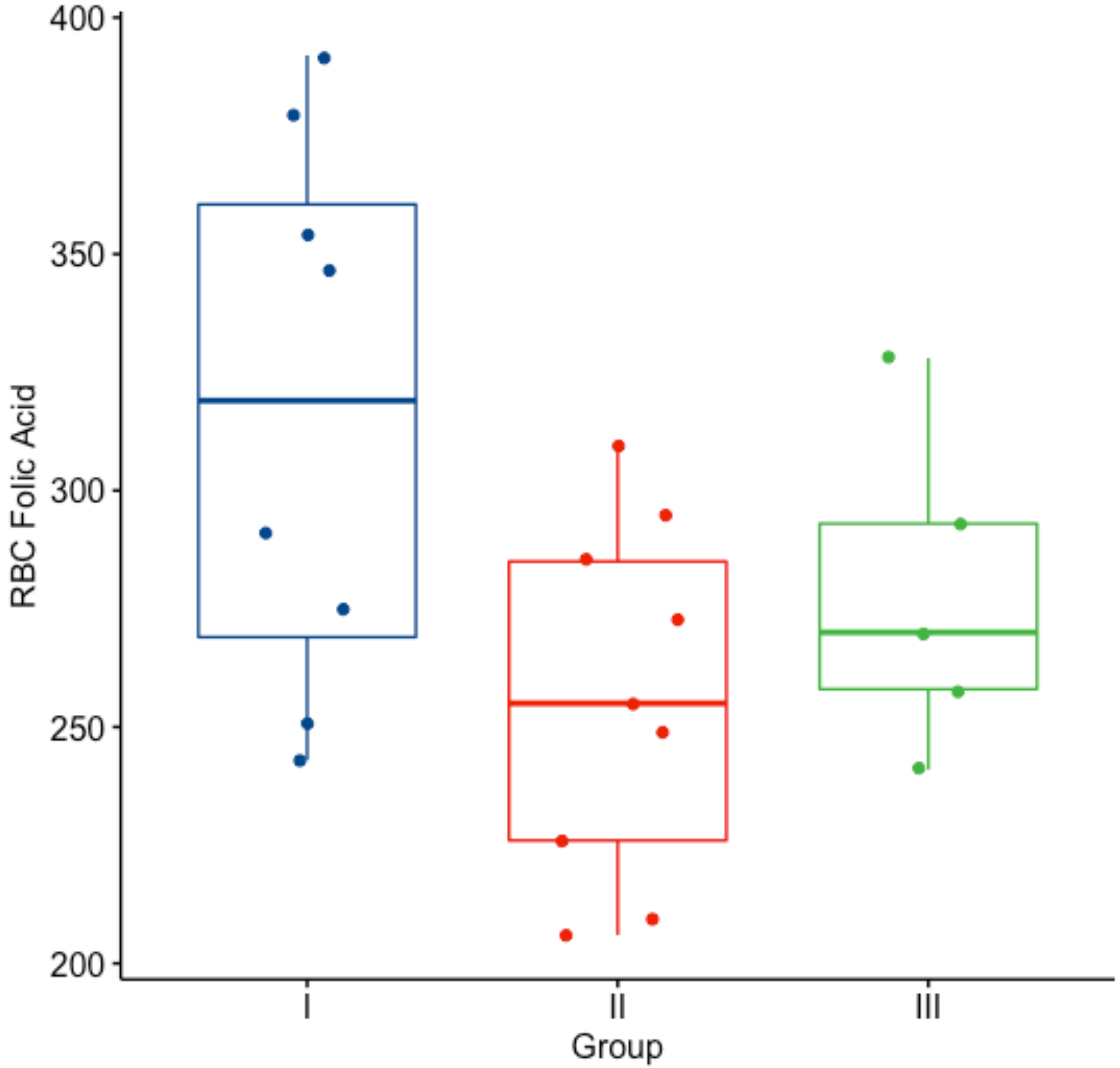
Group II.—9 patients received approximately 50% nitrous oxide and 50% oxygen mixture only during the operation (5–12 h) and thereafter 35–50% oxygen for the remainder of the 24 h period.

Group III.—5 patients received no nitrous oxide but were ventilated with 35–50% oxygen for 24 h.

Group I	Group II	Group III
243	206	241
251	210	258
275	226	270
291	249	293
347	255	328
354	273	
380	285	
392	295	
	309	

Amess JA, Burman JF, Rees GM, Nancekievill DG, Mollin DL. Megaloblastic haemopoiesis in patients receiving nitrous oxide. Lancet. 1978 Aug 12;2(8085):339–42.

One-way ANOVA – Example II (cont.)



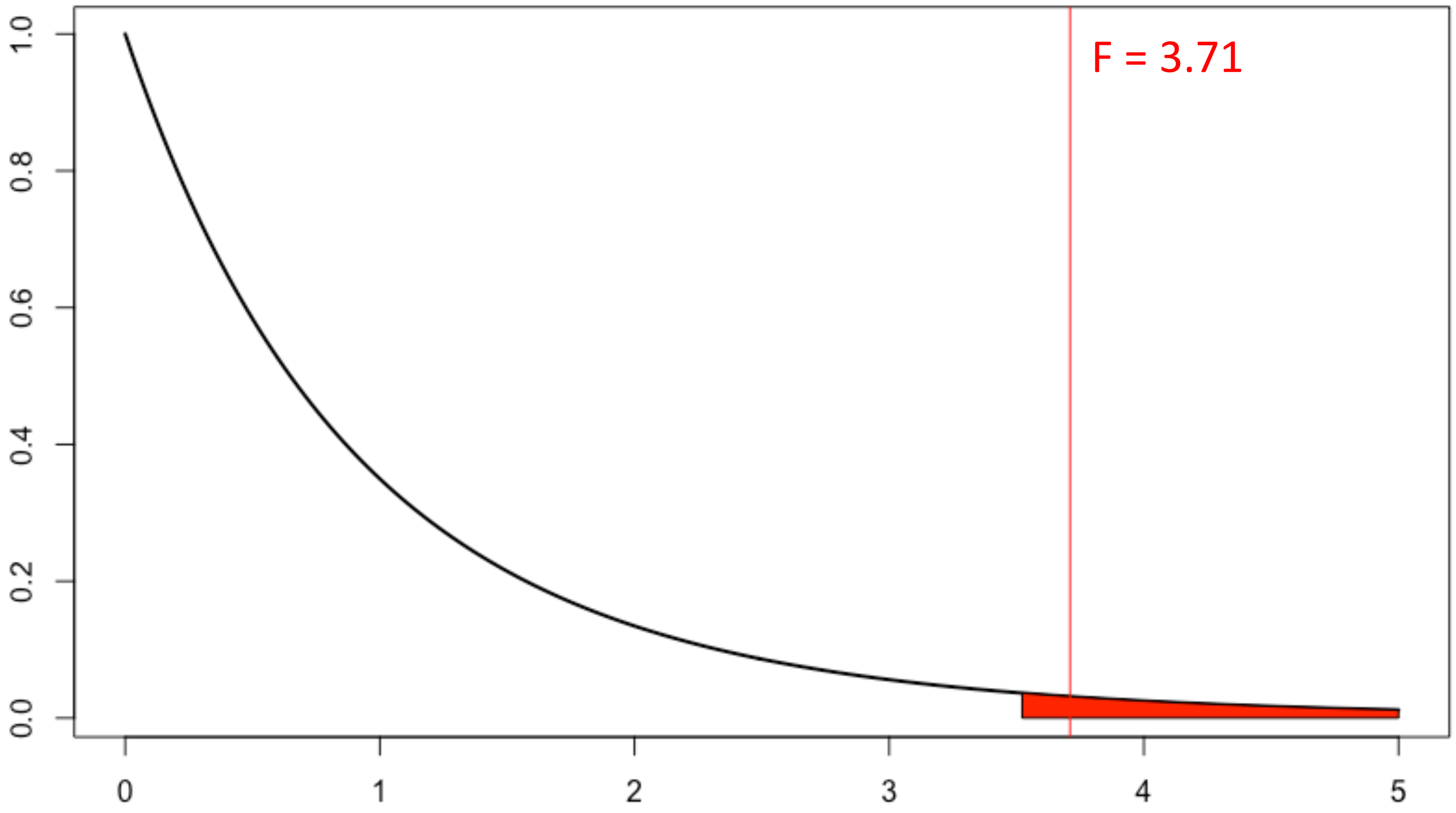
One-way ANOVA – Example II (cont.)

1. Check assumptions, determine H_0 and H_a, choose α
   - Check that data is normally distributed
   - H_0: $\mu_1 = \mu_2 = \mu_3$ H_a: at least one mean is different
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic
   - F = 3.71 $\sim F_{2,19}$

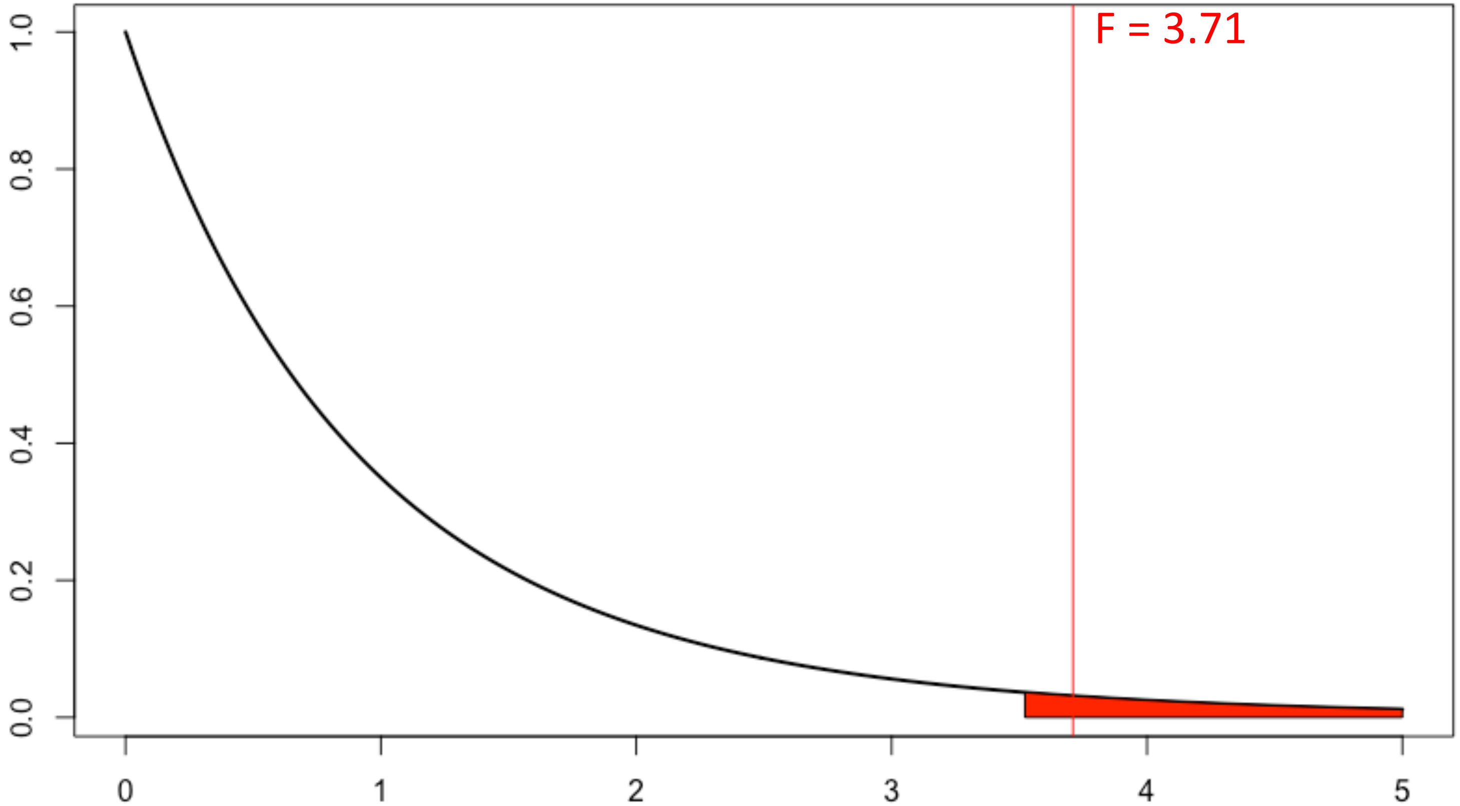
One-way ANOVA – Example II (cont.)

3. Calculate **critical values**/p value
4. Decide whether to reject/fail to reject H_0



One-way ANOVA – Example II (cont.)

3. Calculate critical values/**p value**
4. Decide whether to reject/fail to reject H_0

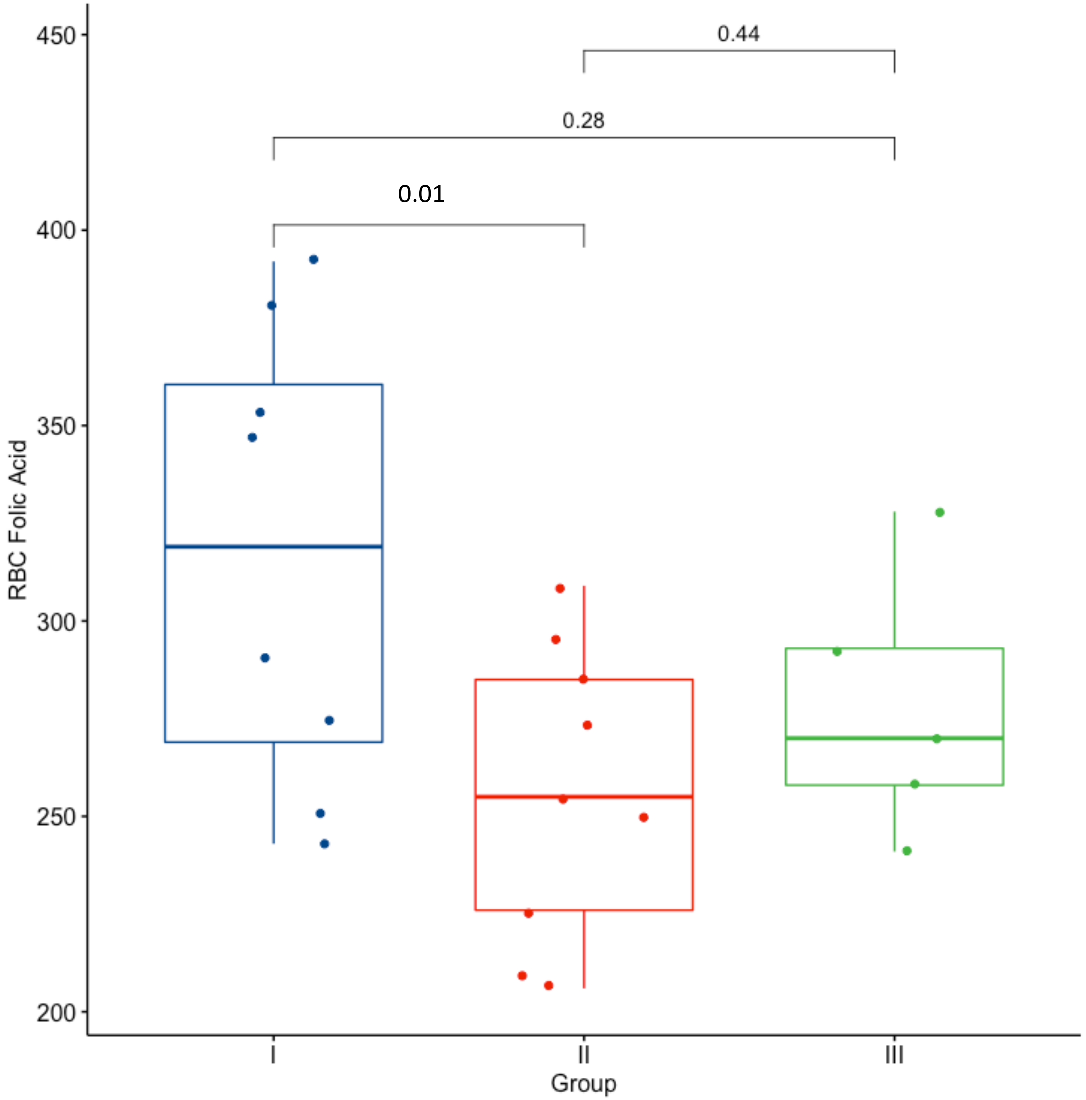


p = 0.043631

One-way ANOVA – Example II (cont.)

- With 95% confidence, we can conclude that the mean RBC folic acid level of at least one group is significantly different than the others
- Next, we perform 2-sample t-tests between all pairs of groups

One-way ANOVA – Example II (cont.)



Brief Summary

- Analysis of variance (ANOVA) is a statistical technique that is used to check if the means of **two or more groups** are significantly different from each other
  - ANOVA checks **the impact** of one or more factors by comparing the means of different samples
  - One-way ANOVA checks the impact of one factor
- Pairwise two-sample t-tests can then be used to determine which group(s) is different