Biostatistics
Week XI

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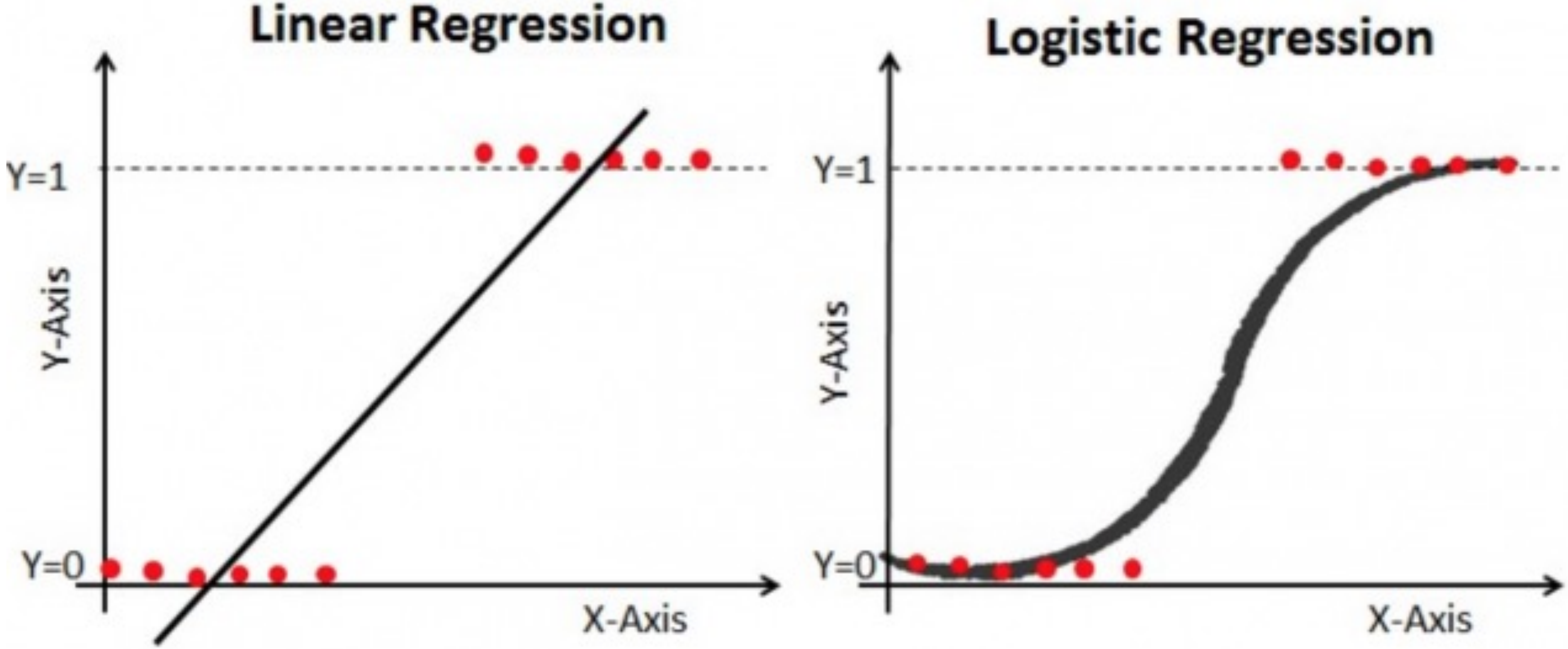
Regression Analysis

- The variable to be predicted is called the **dependent variable**
  - Also called the **response variable**
- The value of this variable depends on the value of the **independent variable(s)**
  - Also called the **explanatory** or **predictor variable(s)**

Dependent variable = f(Independent variable , Independent variable , … , Independent variable)

R demo for linear regression



Science O-OD. Logistic regression with python [Internet]. Medium. 2019 [cited 2021 Oct 9].
Available from: https://medium.com/@ODSC/logistic-regression-with-python-ede39f8573c7

Logistic Regression

- Logistic regression is a specialized form of regression used when the dependent variable is **binary outcome**
  - Having a binary outcome (dependent variable) violates the assumption of linearity in linear regression

Logistic Regression

- The goal of logistic regression is to find the best fitting model to describe the relationship between the binary outcome and a set of independent variables
    - e.g., predicting whether the treatment will be successful or not, the presence/absence of a disease, etc.

Logistic Regression

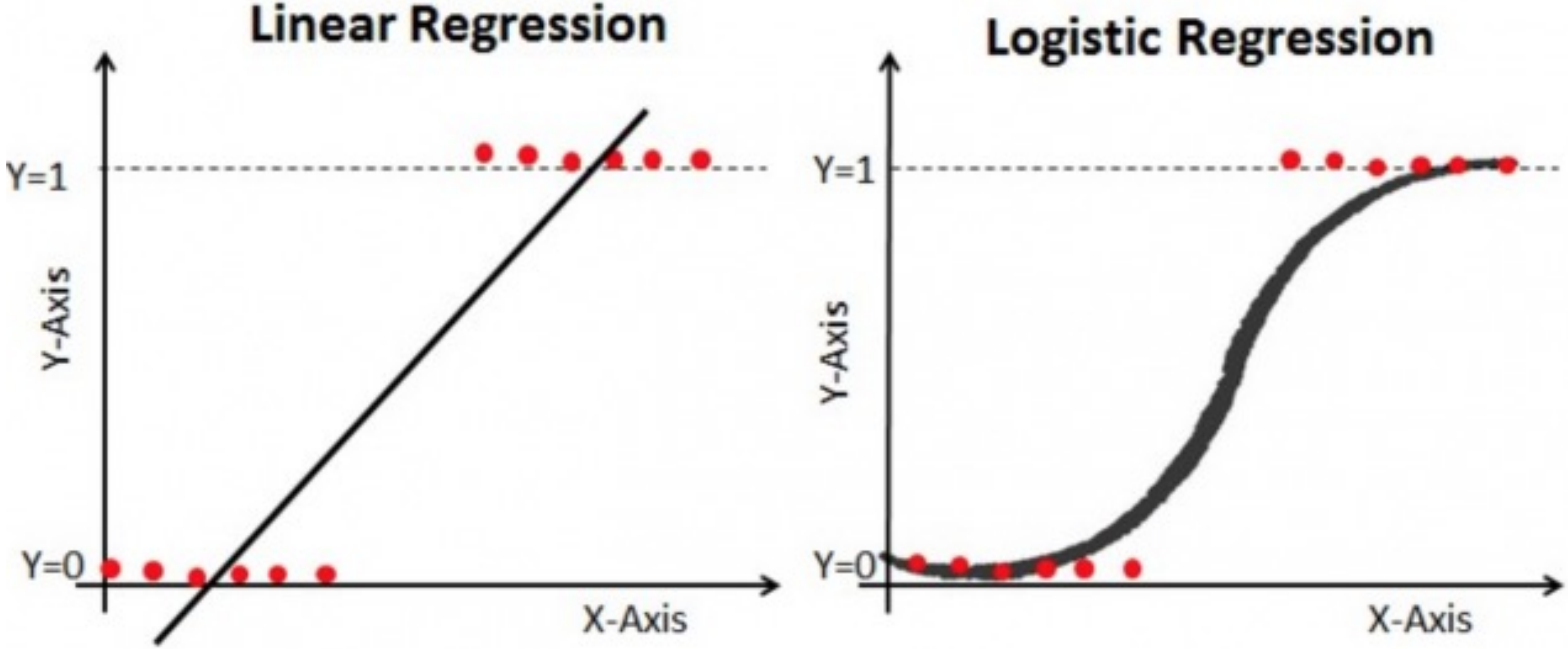
- Logistic regression generates the coefficients of the following formula to predict a **logit transformation** of the probability of presence of the outcome:

$$logit(P(Y=1)) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p$$

where *P(Y = 1)* indicates the probability that the outcome is 1 (where the binary outcome variable is encoded as 0 and 1)

- *logit* is in fact the log of odds:

$$logit(p) = ln\left(\frac{p}{1-p}\right)$$



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Logistic Regression – Example

- Identification of risk factors for metastasis with prostate cancer
- n = 52 patients
- y = metastasis status (0 = none, 1 = metastasis)
- x = phosphatase, X-ray result (binary), tumor size

https://www.ebpi.uzh.ch/dam/jcr:ffffffff-c1f2-5119-ffff-ffff9de0876f/slides-logistic.pdf

Metastasis – Logistic Regression Model

	Estimate	Std. Error	z value	Pr(>|z|)	OR
(Intercept)	-0.5418	0.8298	-0.65	0.5138	
$\log_2$(phosph)	2.3645	1.0267	2.30	0.0213	10.6
X-ray	1.9704	0.8207	2.40	0.0163	7.2
Size	1.6175	0.7534	2.15	0.0318	5.0

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Interpretation

	Estimate	Std. Error	z value	Pr(>|z|)	OR
(Intercept)	-0.5418	0.8298	-0.65	0.5138	
$\log_2$(phosph)	2.3645	1.0267	2.30	0.0213	10.6
X-ray	1.9704	0.8207	2.40	0.0163	7.2
Size	1.6175	0.7534	2.15	0.0318	5.0

- With 95% confidence, it could be said that a patient with $\log_2$(phosphatase) = 0, negative X-ray result, size = 0 was equally-likely in terms of having nodal metastases (p = 0.5138)

- With 95% confidence, it could be said that $\log_2$(phosphatase) and having nodal metastases are associated (p = 0.0213)
  - A one unit increase in $\log_2$(phosphatase) was associated with approximately 963.87% increase in the odds of having nodal metastases
  - (exp(2.3645) – 1) * 100 = 963.87

Interpretation (cont.)

	Estimate	Std. Error	z value	Pr(>|z|)	OR
(Intercept)	-0.5418	0.8298	-0.65	0.5138	
$\log_2$(phosph)	2.3645	1.0267	2.30	0.0213	10.6
X-ray	1.9704	0.8207	2.40	0.0163	7.2
Size	1.6175	0.7534	2.15	0.0318	5.0

- With 95% confidence, it could be said that a positive X-ray result and having nodal metastases are associated (p = 0.0163)
  - Presence of positive X-ray result was associated with approximately 617.35% increase in the odds of having nodal metastases
  - (exp(1.9704) – 1) * 100 = 617.35

- With 95% confidence, it could be said that Size and having nodal metastases are associated (p = 0.0318)
  - Presence of a one unit increase in Size was associated with approximately 404.05% increase in the odds of having nodal metastases
  - (exp(1.6175) – 1) * 100 = 404.05

Poisson Regression

- Linear regression was for continuous outcome, whereas logistic regression for binary outcome
- For **count** outcome, Poisson regression can be used

Poisson Regression - Example

- For 59 epilepsy patients the following data were collected:
  - **treatment:** the **treatment group**, a factor with levels placebo and Progabide
  - **base**: the **number of seizures** collected during 8-week period **before** the trial started
  - **age**: the **age of the patient**
  - **seizure rate:** the **number of seizures** occurred during the 2-week period **after** the trial was started

Poisson Regression – Example (cont.)

- First 10 patients:

treatment	base	age	seizure.rate	subject
placebo	11	31	5	1
placebo	11	30	3	2
placebo	6	25	2	3
placebo	8	36	4	4
placebo	66	22	7	5
placebo	27	29	5	6
placebo	12	31	6	7
placebo	52	42	40	8
placebo	23	37	5	9
placebo	10	28	14	10

Poisson Regression – Example (cont.)

- A Poisson regression with treatment group, previous seizures and age are related to the mean number of of seizure for patient i, λ_i, is given by:

$$log(\lambda_i) = \beta_0 + \beta_1 * I(treatment = Progabide) + \beta_2 * (base - 6) + \beta_3(age - 18)$$

Poisson Regression – Example (cont.)

$$log(\lambda_i) = \beta_0 + \beta_1 * I(treatment = Progabide) + \beta_2 * (base - 6) + \beta_3(age - 18)$$

	Estimate	Std. Error	z value	p
(Intercept)	0.75	0.14	5.33	<0.001
treament = Progabide	-0.12	0.09	-1.28	0.20
base	0.03	0.00	26.37	<0.001
age	0.05	0.01	5.95	<0.001

Poisson Regression – Example (cont.)

	Estimate	Std. Error	z value	p
(Intercept)	0.75	0.14	5.33	<0.001
treament = Progabide	-0.12	0.09	-1.28	0.20
base	0.03	0.00	26.37	<0.001
age	0.05	0.01	5.95	<0.001

- A patient in placebo group, with 6 previous seizures, and aged 18 had approximately 2 seizures on average in the first two weeks after the trial was started
  - exp(0.75)
- With 95% confidence, it could be said that there was no difference between placebo and progabide (p-value = 0.199)
  - Negative estimate for β_1 indicates lowered mean number of seizures for progabide, but the difference from placebo was not significant

Poisson Regression – Example (cont.)

	Estimate	Std. Error	z value	p
(Intercept)	0.75	0.14	5.33	<0.001
treament = Progabide	-0.12	0.09	-1.28	0.20
base	0.03	0.00	26.37	<0.001
age	0.05	0.01	5.95	<0.001

- With 95% confidence, it could be said that previous number of seizures occurred in the 8-week interval prior to the study start and mean seizure rate was significantly associated (p-value < 0.001)
- One unit increase in previous seizure is associated with approximately 2.6% increase in the mean number of seizures in the first two weeks of the trial
  - (exp(0.03) - 1) * 100

Poisson Regression – Example (cont.)

	Estimate	Std. Error	z value	p
(Intercept)	0.75	0.14	5.33	<0.001
treament = Progabide	-0.12	0.09	-1.28	0.20
base	0.03	0.00	26.37	<0.001
age	0.05	0.01	5.95	<0.001

- With 95% confidence, it could be said that age sand mean seizure rate was significantly associated (p-value < 0.001)
- One unit increase in age is associated with approximately 4.8% increase in the mean number of seizures in the first two weeks of the trial
    - (exp(0.05) - 1) * 100

Brief Summary

Dependent Variable	Regression Model
Continuous	Linear Regression
Binary	Logistic Regression
Count	Poisson Regression