

Q&A session on Gödel's Incompleteness Theorems

MAT100 Infinity room
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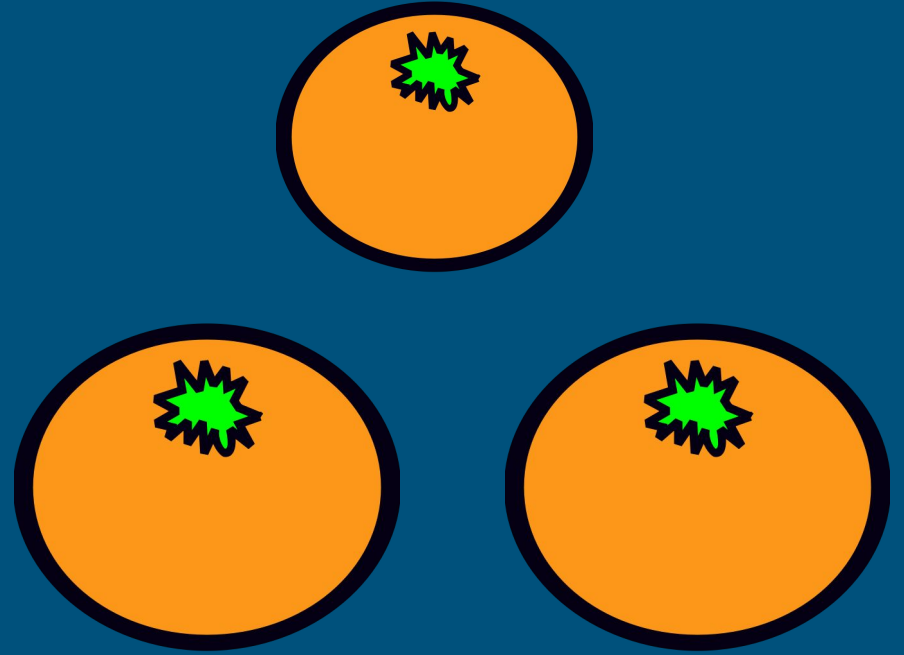
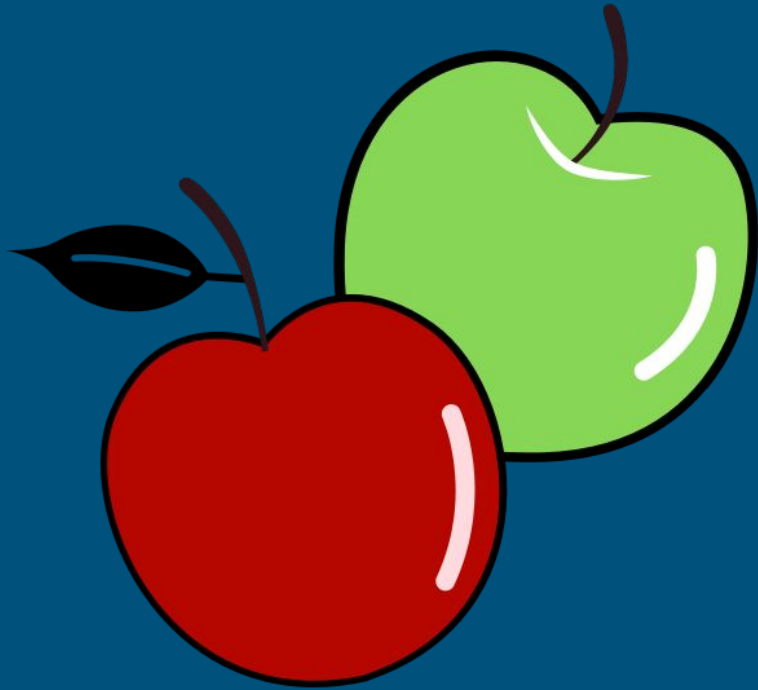
Before we start...

Agenda

- Russel's paradox
- Godel's incompleteness theorems

The class was over...

Math works or not?

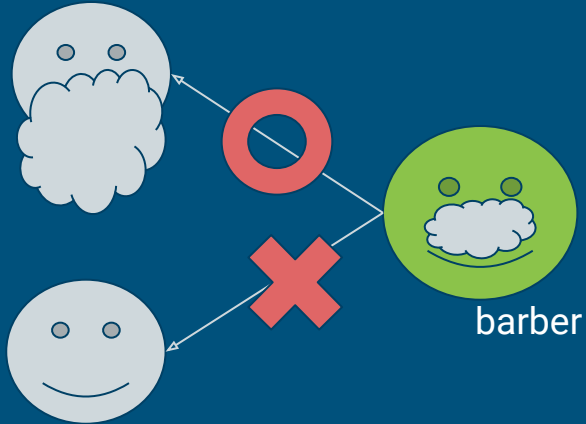


Russell's Paradox

R = the set of all sets
that are not members
of themselves

Let $R = \{x \mid x \notin x\}$,
then $R \in R \iff R \notin R$

Barber's Paradox



Barber's Paradox

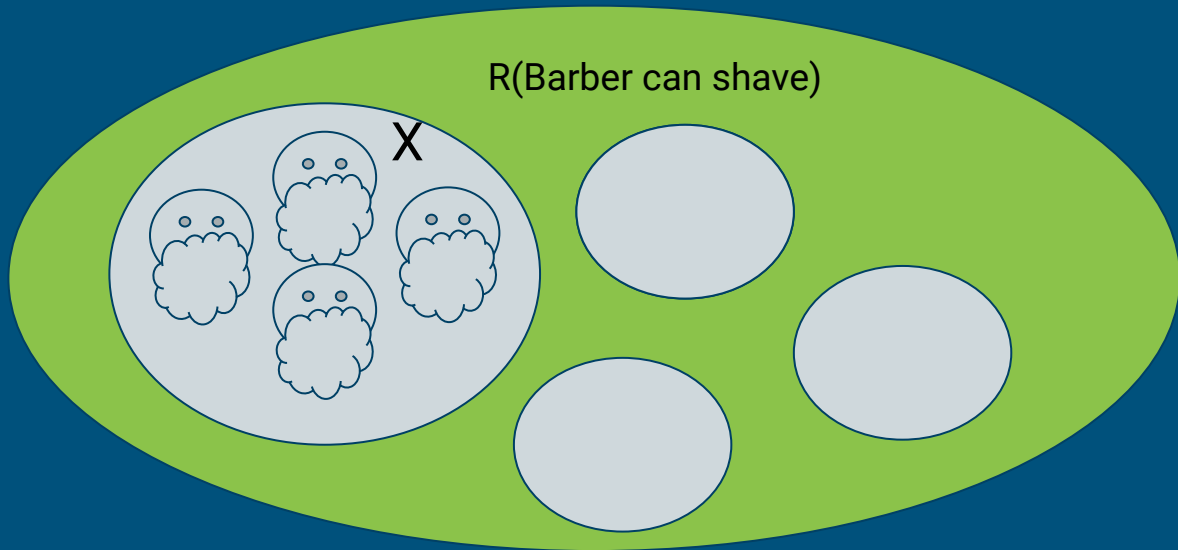


Does not shave by
himself $= R$

Did shave by
himself $\neq R$

R = the set of all sets
that are not members
of themselves

Let $R = \{x \mid x \notin x\}$,
then $R \in R \iff R \notin R$





Any questions?

LIAR or not?

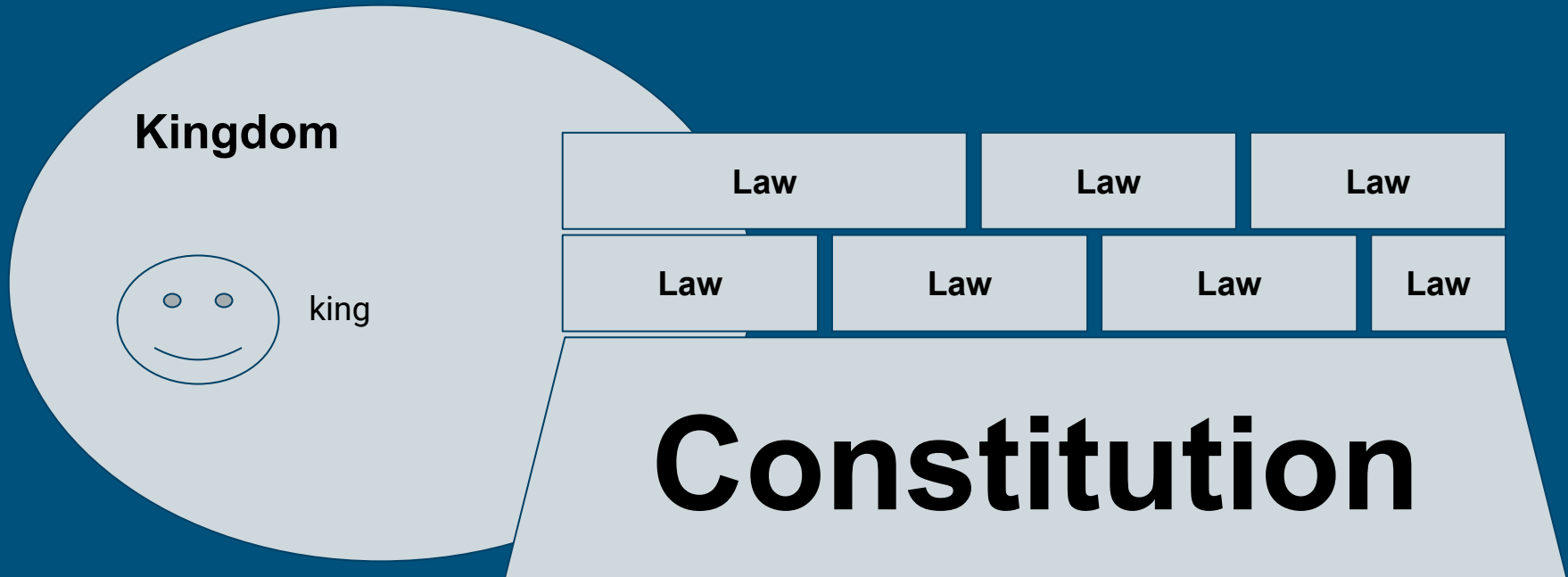
I am a liar!!!



If he is a liar..
His statement
is true
→ NOT a liar

If he is NOT a
liar..
His statement
is false
→ A liar

Gödel's Incompleteness Theorems



Gödel's Incompleteness Theorems

Law A

?

Law $\neg A$

?



There does not exist the automatic proof of the consistency of the system of the kingdom by using the system.

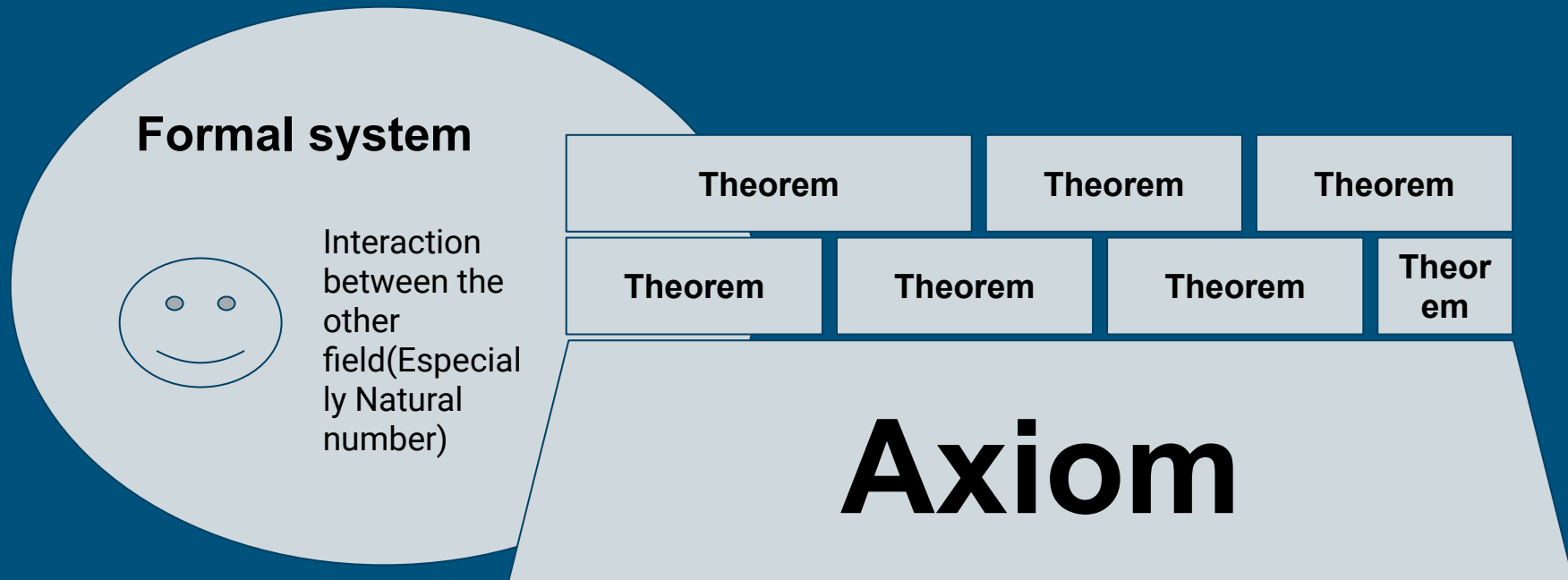
Gödel's Incompleteness Theorems

1. the King is not involved in the process of judging the new law. It is completely **automatic**.
2. the system in the Kingdom must be **consistent**.
3. the Kingdom has a good relationship with **another country**.



Any questions?

Gödel's Incompleteness Theorems



Gödel's Incompleteness Theorems

1. The sequence of logical expression can be **automatically** judged whether it is formal or not.
2. The formal system is **Consistent** (without contradiction)
3. In the formal system, **natural number** (**another country**) can be used.

Gödel's Incompleteness Theorems

If the formal system (= kingdom) meets the strict rule, There may exist

Theorem A

Theorem

$\neg A$

Both cannot
be proved nor
declined



There does not exist the automatic proof of the consistency of the formal system by using the system.

Last question...

We used 'I am a liar' as an example in the slide,

Now, I would like to ask you questions here,,,,

If One say 'I tell you always truth' then can you tell his statement is true or not?

Thank you for listening..

Today's explanation is just an brief idea of Godel's incompleteness theorem. The real proof and other stories are very interesting and we can feel deep deep inside the mathematics. If you are interested in, please contact me i am very happy to share this

Gödel's Incompleteness Theorems

Summarize

- ① In mathematics or formal systems that meet the requirement, mathematics or formal system has the problems which I can neither prove nor decline.
- ② The mathematics or formal system which meets the requirement cannot prove that the formal system itself has no contradiction.