

Numerical Sequences

Nanako Akiyama

Do you know what numerical sequences are?

- A list of numbers that are linked by a rule.
- If you work out the rule, you can work out the next numbers in the sequence.

Various types of numerical sequences

Let's be familiar with sequences!

Arithmetic Sequence

- Made by **adding** the same value each time

We are adding 1 each time!

1 2 3 4 5

Can you guess what comes next?

6

We are adding 5
each time!

8 13 18 23 28 33 ☐

Can you guess what comes next?

38

Geometric Sequence

- Made by **multiplying** by the same number each time.

We are
multiplying by 1
each time!

1 1 1 1 1 □

Can you guess what comes next?

1

We are
multiplying by
3 each time!

2 6 18 54 162 ☐

Can you guess what comes next?

486

Square Numbers

$0^2, 1^2, 2^2, 3^2, 4^2, \dots$

- The **squares** of whole numbers

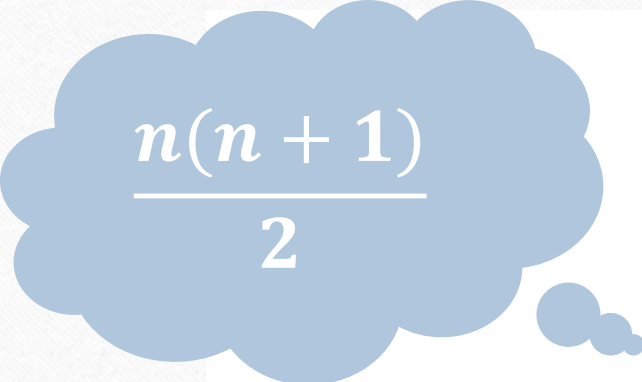
0 1 4 9 16 25 36 49 64 ☐

Can you guess what comes next?

81

Triangular Numbers

- Generated from a pattern of dots that form a **triangle**
- By adding another row of dots and counting all the dots, we can find the next number of the sequence


$$\frac{n(n+1)}{2}$$



$$= 1$$



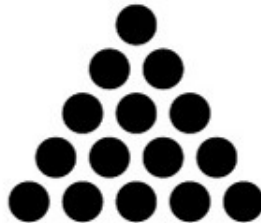
$$= 3$$



$$= 6$$



$$= 10$$



$$= 15$$

What about these sequences?

0 1 1 2 3 5 8 13 21 34...

Can you find a rule here?

Hint: Try calculating the difference
between numbers so far.

Fibonacci Numbers

- Found by adding the two numbers before it together.
- The Rule is $x_n = x_{n-1} + x_{n-2}$
(x_n – nth element)

0 1 1 2 3 5 8 13 21 34

1+1 1+2 2+3 3+5 5+8 8+13 13+21

\square \square \square

55 89 144

Closed formula

$$x_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$$

$$\frac{1}{\sqrt{5}} \approx 0.44721359$$

$$\frac{1+\sqrt{5}}{2} \approx 1.618$$

$$\frac{1-\sqrt{5}}{2} \approx -0.618$$

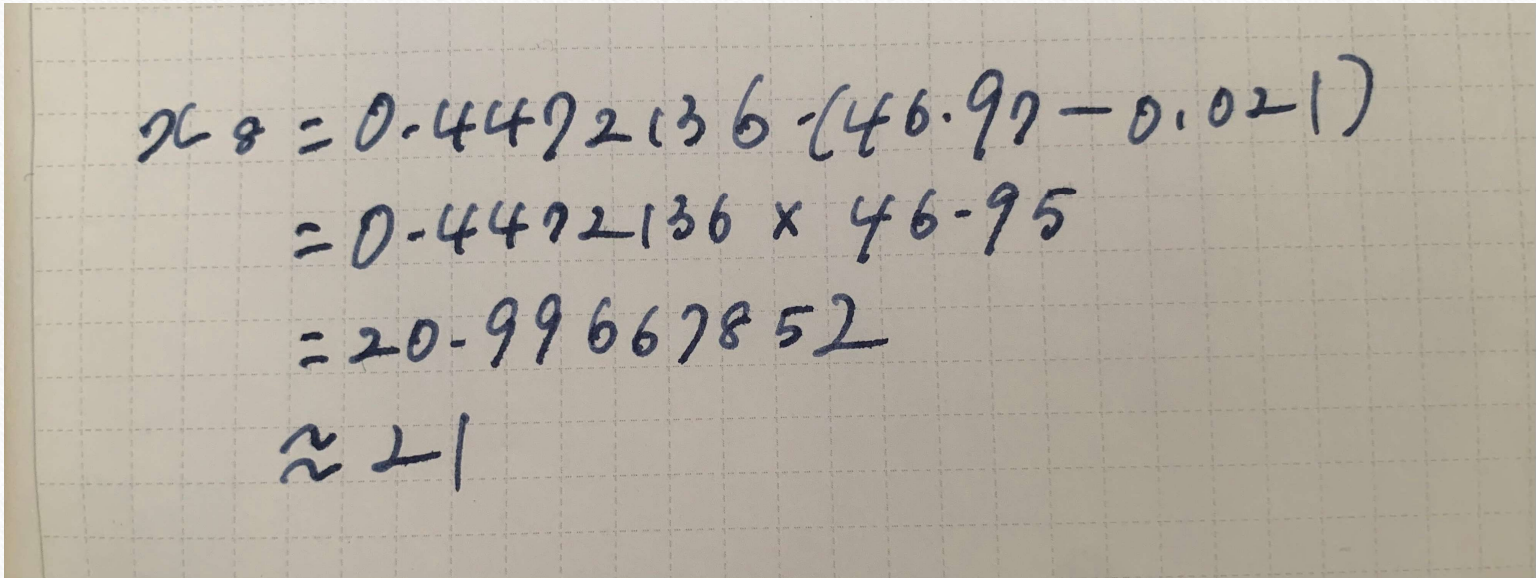
$$x_n = 0.4472136 \cdot (1.618^n - (-0.618)^n)$$

Let's calculate the 7th number!

$$\begin{aligned}x_7 &= 0.4472(36 - (29.03 - 0.03)) \\&= 12.9691944 \\&\approx 13\end{aligned}$$

Can you calculate the 8th number?

- You can use the computer!


$$\begin{aligned}x_8 &= 0.4472136 \cdot (46.97 - 0.021) \\&= 0.4472136 \times 46.95 \\&= 20.99667852 \\&\approx 21\end{aligned}$$

Let me introduce an interesting site!

-
- The On-Line Encyclopedia of Integer Sequences®
(OEIS®)

Reference

- Number Sequences - Square, Cube and Fibonacci (mathsisfun.com)
- What is a number sequence? - BBC Bitesize
- The magic of Fibonacci numbers | Arthur Benjamin
- The On-Line Encyclopedia of Integer Sequences® (OEIS®)
- Closed-Form Expression to Calculate n-th Fibonacci Number



Thank you!