# Chapter 3 Embarrassingly Parallel Computations

A computation that can be divided into a number of completely independent parts, each of which can be executed by a separate processor.

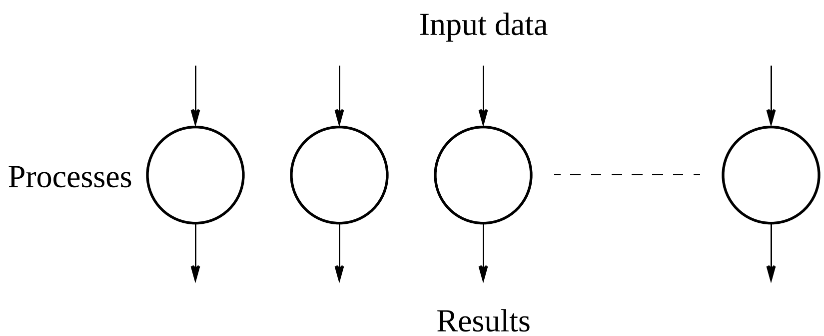



Figure 3.1 Disconnected computational graph (embarrassingly parallel problem).

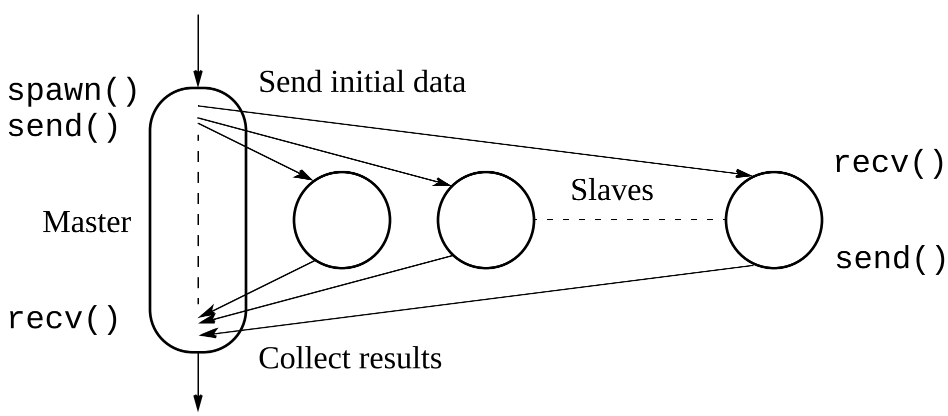



Figure 3.2 Practical embarrassingly parallel computational graph with dynamic process creation and the master-slave approach.

# Embarrassingly Parallel Examples

**Low level image operations:**

(a) Shifting

Object shifted by $\Delta x$ in the $x$-dimension and $\Delta y$ in the $y$-dimension:

$$x' = x + \Delta x$$

$$y' = y + \Delta y$$

where $x$ and $y$ are the original and $x'$ and $y'$ are the new coordinates.

(b) Scaling

Object scaled by a factor $S_x$ in the $x$-direction and $S_y$ in the $y$-direction:

$$x' = xS_x$$

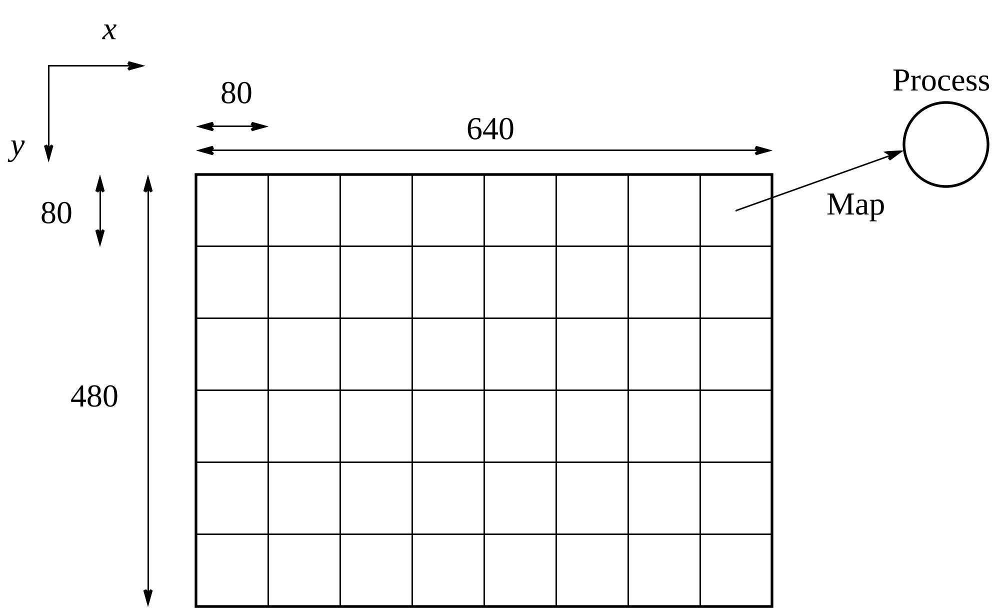
$$y' = yS_y$$

(c) Rotation

Object rotated through an angle $\theta$ about the origin of the coordinate system:

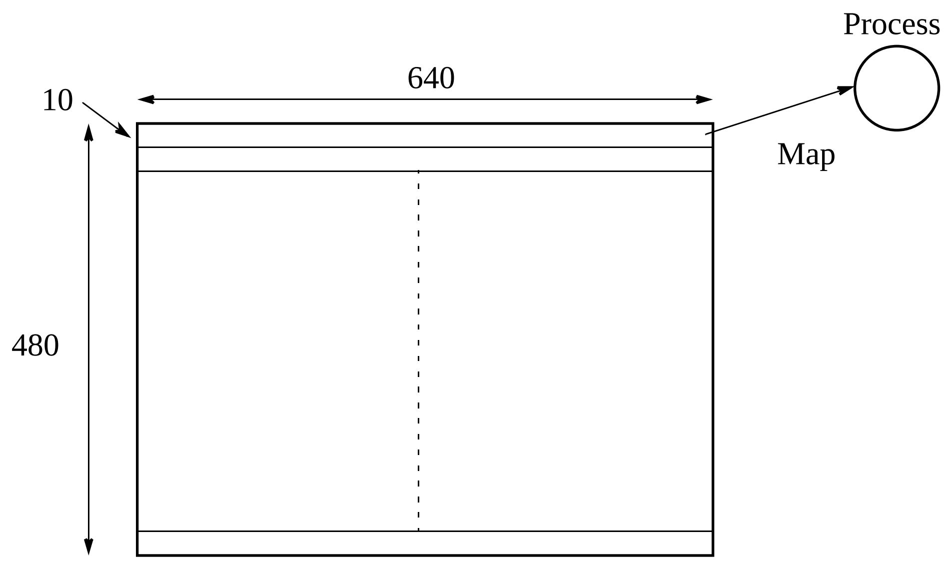
$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$




(a) Square region for each process

Figure 3.3 Partitioning into regions for individual processes.



(b) Row region for each process

# Pseudocode to Perform Image Shift

Master

```
for (i = 0, row = 0; i < 48; i++, row = row + 10)/* for each process*/
    send(row, Pi);                         /* send row no.*/

for (i = 0; i < 480; i++)                  /* initialize temp */
    for (j = 0; j < 640; j++)
      temp_map[i][j] = 0;

for (i = 0; i < (640 * 480); i++) {        /* for each pixel */
    recv(oldrow,oldcol,newrow,newcol, PANY);/* accept new coords */
    if !((newrow < 0)||(newrow >= 480)||(newcol < 0)||(newcol >= 640))
      temp_map[newrow][newcol]=map[oldrow][oldcol];
}
for (i = 0; i < 480; i++)                  /* update bitmap */
    for (j = 0; j < 640; j++)
      map[i][j] = temp_map[i][j];
```

Slave

```
recv(row, Pmaster);                          /* receive row no. */
for (oldrow = row; oldrow < (row + 10); oldrow++)
    for (oldcol = 0; oldcol < 640; oldcol++) {/* transform coords */
      newrow = oldrow + delta_x;      /* shift in x direction */
      newcol = oldcol + delta_y;      /* shift in y direction */
      send(oldrow,oldcol,newrow,newcol, Pmaster);/* coords to master */
    }
```

# Analysis

## Sequential

$$t_s = n^2 = \mathrm{O}(n^2)$$

## Parallel

### Communication

$$t_{\text{comm}} = t_{\text{startup}} + mt_{\text{data}}$$

$$t_{\text{comm}} = p(t_{\text{startup}} + 2t_{\text{data}}) + 4n^2(t_{\text{startup}} + t_{\text{data}}) = \mathrm{O}(p + n^2)$$

### Computation

$$t_{\text{comp}} = 2\left(\frac{n^2}{p}\right) = \mathrm{O}(n^2/p)$$

### Overall Execution Time

$$t_p = t_{\text{comp}} + t_{\text{comm}}$$

For constant $p$, this is $\mathrm{O}(n^2)$. However, the constant hidden in the communication part far exceeds those constants in the computation in most practical situations.

# Mandelbrot Set

Set of points in a complex plane that are quasi-stable (will increase and decrease, but not exceed some limit) when computed by iterating the function

$$z_{k+1} = z_k^2 + c$$

where $z_{k+1}$ is the $(k + 1)$th iteration of the complex number $z = a + bi$ and $c$ is a complex number giving the position of the point in the complex plane.
The initial value for $z$ is zero.

Iterations continued until magnitude of $z$ is greater than 2 or number of iterations reaches arbitrary limit. Magnitude of $z$ is the length of the vector given by

$$z_{\text{length}} = \sqrt{a^2 + b^2}$$

Computing the complex function, $z_{k+1} = z_k^2 + c$, is simplified by recognizing that

$$z^2 = a^2 + 2abi + bi^2 = a^2 - b^2 + 2abi$$

or a real part that is $a^2 - b^2$ and an imaginary part that is $2ab$.

The next iteration values can be produced by computing:

$$z_{\text{real}} = z_{\text{real}}^2 - z_{\text{imag}}^2 + c_{\text{real}}$$

$$z_{\text{imag}} = 2z_{\text{real}}z_{\text{imag}} + c_{\text{imag}}$$

## Seq. Routine computing value of one pt, returning no of iterations

```
structure complex {
  float real;
  float imag;
};

int cal_pixel(complex c)
{
int count, max;
complex z;
float temp, lengthsq;
max = 256;
z.real = 0; z.imag = 0;
count = 0;                      /* number of iterations */
do {
  temp = z.real * z.real - z.imag * z.imag + c.real;
  z.imag = 2 * z.real * z.imag + c.imag;
  z.real = temp;
  lengthsq = z.real * z.real + z.imag * z.imag;
  count++;
} while ((lengthsq < 4.0) && (count < max));
return count;
}
```

## Scaling Coordinate System

For computational efficiency, let

```
scale_real = (real_max - real_min)/disp_width;
scale_imag = (imag_max - imag_min)/disp_height;
```

Including scaling, the code could be of the form

```
for (x = 0; x < disp_width; x++) /* screen coordinates x and y */
  for (y = 0; y < disp_height; y++) {
    c.real = real_min + ((float) x * scale_real);
    c.imag = imag_min + ((float) y * scale_imag);
    color = cal_pixel(c);
    display(x, y, color);
  }
```

where `display()` is a routine to display the pixel (x, y) at the computed color.

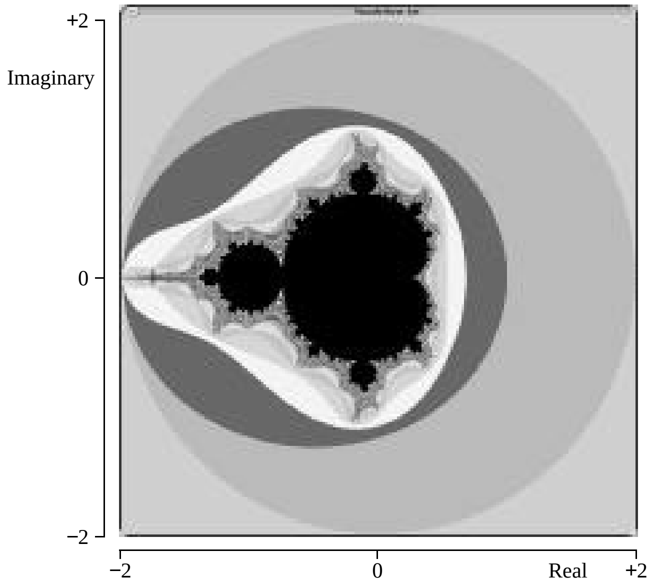



Figure 3.4 Mandelbrot set.

# Parallelizing Mandelbrot Set Computation

## Static Task Assignment

Master

```
for (i = 0, row = 0; i < 48; i++, row = row + 10)/* for each process*/
    send(&row, P_i);                    /* send row no.*/
for (i = 0; i < (480 * 640); i++) {/* from processes, any order */
    recv(&c, &color, P_ANY);    /* receive coordinates/colors */
    display(c, color);          /* display pixel on screen */
}
```

Slave (process i)

```
recv(&row, P_master);           /* receive row no. */
for (x = 0; x < disp_width; x++)/* screen coordinates x and y */
    for (y = row; y < (row + 10); y++) {
      c.real = min_real + ((float) x * scale_real);
      c.imag = min_imag + ((float) y * scale_imag);
      color = cal_pixel(c);
      send(&c, &color, P_master);/* send coords, color to master */
    }
```

# Dynamic Task Assignment

## Work Pool/Processor Farms

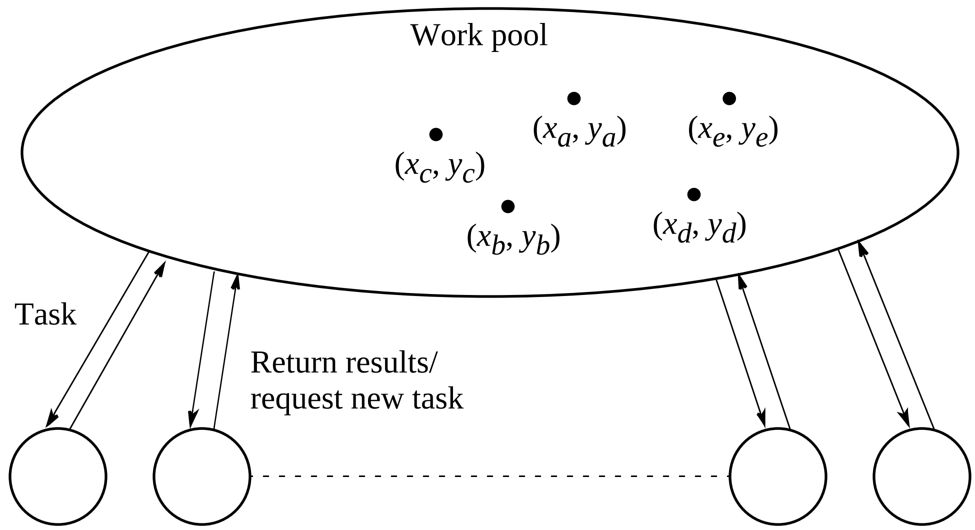



Figure 3.5 Work pool approach.

# Coding for Work Pool Approach

Master

```
count = 0;                      /* counter for termination*/
row = 0;                        /* row being sent */
for (k = 0; k < procno; k++) {  /* assuming procno<disp_height */
    send(&row, P_k, data_tag);  /* send initial row to process */
    count++;                    /* count rows sent */
    row++;                      /* next row */
}

do {
    recv (&slave, &r, color, P_ANY, result_tag);
    count--;                  /* reduce count as rows received */
    if (row < disp_height) {
      send (&row, P_slave, data_tag);       /* send next row */
      row++;                                /* next row */
      count++;
    } else
      send (&row, P_slave, terminator_tag); /* terminate */
    rows_recv++;
    display (r, color);                     /* display row */
} while (count > 0);
```

Slave

```
recv(y, Pmaster, ANYTAG, source_tag);/* receive 1st row to compute */
while (source_tag == data_tag) {
    c.imag = imag_min + ((float) y * scale_imag);
    for (x = 0; x < disp_width; x++) {/* compute row colors */
      c.real = real_min + ((float) x * scale_real);
      color[x] = cal_pixel(c);
    }
    send(&i, &y, color, Pmaster, result_tag);/* row colors to master */
    recv(y, Pmaster, source_tag);      /* receive next row */
};
```

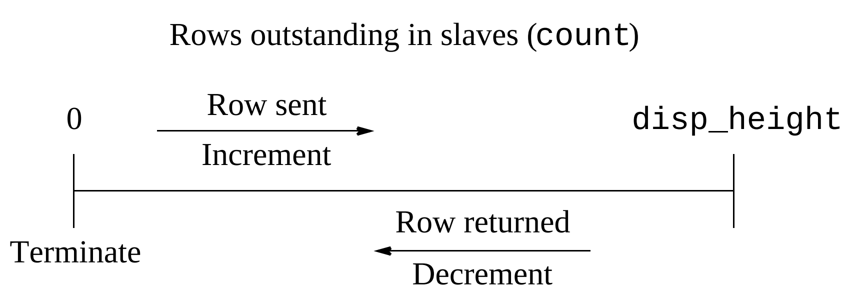



Figure 3.6 Counter termination.

# Analysis

## Sequential

$$t_s \leq \max \times n = O(n)$$

## Parallel program

**Phase 1: Communication - Row number is sent to each slave**

$$t_{comm1} = s(t_{startup} + t_{data})$$

**Phase 2: Computation - Slaves perform their Mandelbrot computation in parallel**

$$t_{comp} \leq \frac{\max \times n}{s}$$

**Phase 3: Communication - Results passed back to master using individual sends**

$$t_{comm2} = \frac{n}{s}(t_{startup} + t_{data})$$

**Overall**

$$t_p \leq \frac{\max \times n}{s} + \left(\frac{n}{s} + s\right)(t_{startup} + t_{data})$$

# Monte Carlo Methods

Basis of Monte Carlo methods is the use of random selections in calculations.

## Example - To calculate π

A circle is formed within a square. Circle has unit radius so that square has sides 2 × 2.

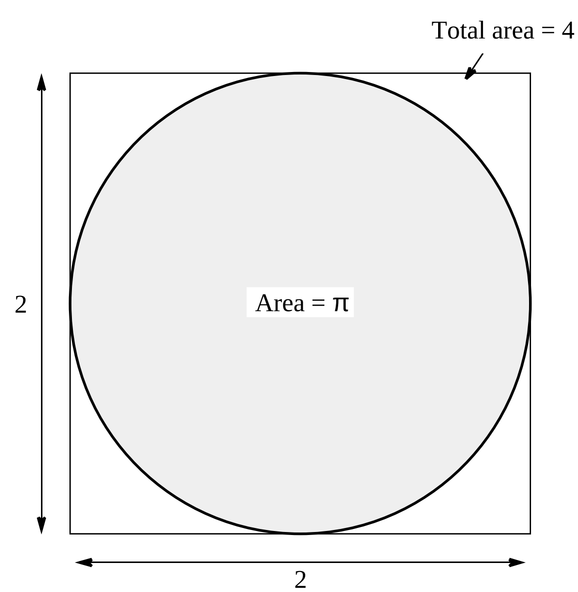



Figure 3.7 Computing π by a Monte Carlo method.

The ratio of the area of the circle to the square is given by

$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi(1)^2}{2 \times 2} = \frac{\pi}{4}$$

Points within the square are chosen randomly and a score is kept of how many points happen to lie within the circle.

The fraction of points within the circle will be $\pi/4$, given a sufficient number of randomly selected samples.

# Computing an Integral

One quadrant of the construction in Figure 3.7 can be described by the integral

$$\int_0^1 \sqrt{1 - x^2}\, dx = \frac{\pi}{4}$$

A random pair of numbers, $(x_r, y_r)$ would be generated, each between 0 and 1, and then counted as in circle if $y_r \le \sqrt{1 - x_r^2}$; that is, $y_r^2 + x_r^2 \le 1$.

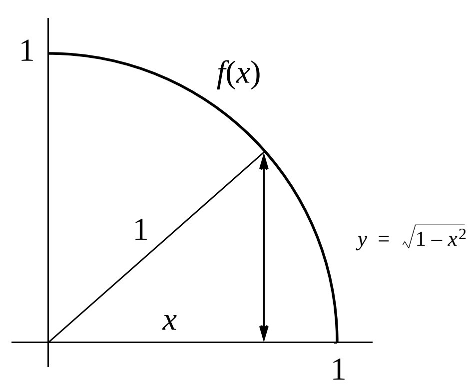



Figure 3.8 Function being integrated in computing π by a Monte Carlo method.

## Alternative (better) Method

Use the random values of $x$ to compute $f(x)$ and sum the values of $f(x)$:

$$\text{Area} = \int_{x_1}^{x_2} f(x)\,dx = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} f(x_r)(\mathrm{x}_2 - \mathrm{x}_1)$$

where $x_r$ are randomly generated values of $x$ between $x_1$ and $x_2$.

## Example

Computing the integral

$$I = \int_{x_1}^{x_2} (x^2 - 3x)\,dx$$

### Sequential Code

```
sum = 0;
for (i = 0; i < N; i++) {          /* N random samples */
    xr = rand_v(x1, x2);           /* generate next random value */
    sum = sum + xr * xr - 3 * xr; /* compute f(xr) */
}
area = (sum / N) * (x2 - x1);
```

The routine randv(x1, x2) returns a pseudorandom number between x1 and x2.

# Parallel Implementation

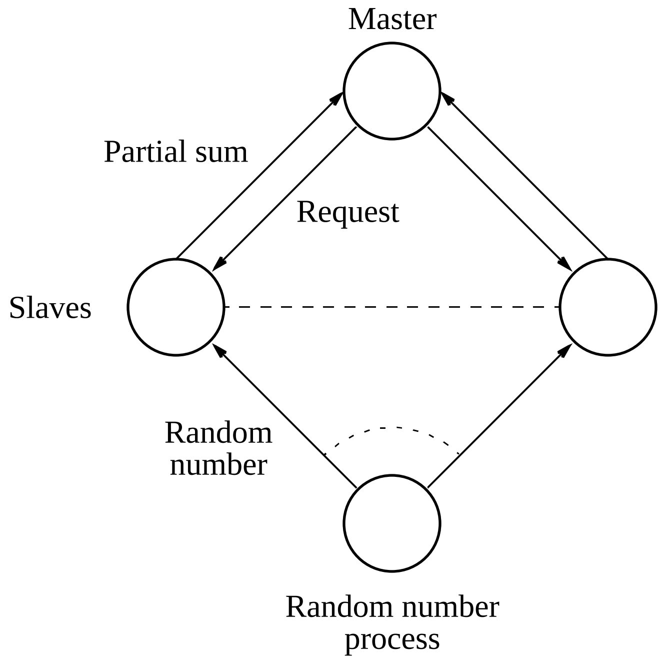



Figure 3.9 Parallel Monte Carlo integration.

# Pseudocode

Master

```
for(i = 0; i < N/n; i++) {
    for (j = 0; j < n; j++)     /* n=no of random numbers for slave */
      xr[j] = rand();           /* load numbers to be sent */
    recv(P_ANY, req_tag, P_source); /* wait for a slave to make request */
    send(xr, &n, P_source, compute_tag);
}
for(i = 0; i < slave_no; i++) { /* terminate computation */
    recv(P_i, req_tag);
    send(P_i, stop_tag);
}
sum = 0;
reduce_add(&sum, P_group);
```

Slave

```
sum = 0;
send(Pmaster, req_tag);
recv(xr, &n, Pmaster, source_tag);
while (source_tag == compute_tag) {
    for (i = 0; i < n; i++)
      sum = sum + xr[i] * xr[i] - 3 * xr[i];
    send(Pmaster, req_tag);
    recv(xr, &n, Pmaster, source_tag);
};
reduce_add(&sum, Pgroup);
```

# Random Number Generation

The most popular way of creating a pueudorandom number sequence:

$$x_1, x_2, x_3, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_{n-1}, x_n,$$

is by evaluating $x_{i+1}$ from a carefully chosen function of $x_i$, often of the form

$$x_{i+1} = (ax_i + c) \bmod m$$

where $a$, $c$, and $m$ are constants chosen to create a sequence that has similar properties to truly random sequences.

# Parallel Random Number Generation

It turns out that

$$x_{i+1} = (ax_i + c) \bmod m$$

$$x_{i+k} = (Ax_i + C) \bmod m$$

where $A = a^k \bmod m$, $C = c(a^{k-1} + a^{n-2} + \ldots + a^1 + a^0) \bmod m$, and $k$ is a selected "jump" constant.

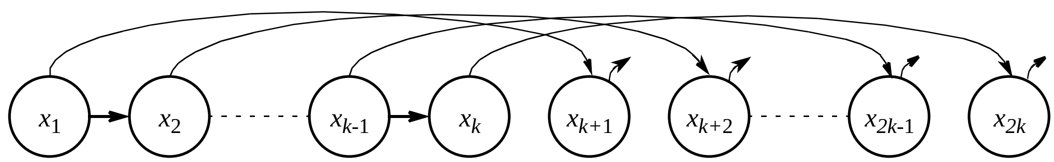



Figure 3.10 Parallel computation of a sequence.