0.1 Terms

Clock and Health Parameters

term	bits	scale fctr	eff. range	units	explanation
WN	10	1		weeks	GPS week modulo 1024
URA	4				User Range Accuracy
SH	6	1			Satellite health
T_{GD}	8	2^{-31}	-	-	Estimated group delay differential
IODC	10	-	-	-	Issue of Data, Clock[1]
t_{oc}	16	2^4	604,784	seconds	Clock data reference time
a_{f2}	8	2^{-55}	-	sec/sec^2	-
a_{f1}	16	2^{-43}	-	sec/sec	-
a_{f0}	22	2^{-31}	-	seconds	-

Ephemeris Parameters

term	bits	scale fctr	eff. range	units	explanation
M_0	32	2^{-31}	-	semi-circles	Mean anomaly at reference time
Δn	16	2^{-43}	-	semi-circles/sec	Mean motion difference from computed value
e	32	2^{-33}	0.03	d.less	Eccentricity
$\sqrt{A}$	32	2^{-19}	-	$\sqrt{\text{meters}}$	Square root of the semi major axis
Ω_0	32	2^{-31}	-	semi-circles	Longitude of ascending node of orbit plane at weekly epoch
i_0	32	2^{-31}	-	semi-circles	Inclination angle at reference time
ω	32	2^{-31}	-	semi-circles	Argument of perigee
$\dot{\Omega}$	24	2^{-43}	-	semi-cirlces/sec	Rate of right ascension
IDOT	14	2^{-43}	-	semi-circles/sec	Rate of inclination angle
C_{uc}	16	2^{-29}	-	radians	Amplitude of the cosine harmonic correction term to the argument of latitude
C_{us}	16	2^{-29}	-	radians	Amplitude of the sine harmonic correction term to the argument of latitude
C_{rc}	16	2^{-5}	-	meters	Amplitude of the cosine harmonic correction term to the orbit radius
C_{rs}	16	2^{-5}	-	meters	Amplitude of the sine harmonic correction term to the orbital radius
C_{ic}	16	2^{-29}	-	radians	Amplitude of the cosine harmonic correction term to the angle of inclination

[1] Guaranteed unique in any 7-day period

C_{is}	16	2^{-29}	-	radians	Amplitude of the sine harmonic correction term to the angle of inclination
t_{oe}	16	2^{4}	604,784	seconds	Reference time ephemeris
IODE	8	-	-	-	Issue of Data (Ephemeris)

0.2 Almanac

Block II and IIA SVs transmit three sets of almanac data spanning at least 60 days. Sets 1 and 2 will transmit for up to six days each, set 3 will be transmitted for the remainder of the 60 day minimum.

Block IIR/IIR-M, IIF, and Block III SVs transmit five sets of almanac data spanning at least 60 days. Sets 1, 2, and 3 will transmit for up to six days each; sets 4 and 5 will be transmitted for up to 32 days; the 5th set will be transmitted for the remainder of the 60 day minimum.

Almanac Parameters

term	bits	scale fctr	eff. range	units	explanation
e	16	2^{-21}	-	d.less	Eccentricity
t_{oa}	8	2^{12}	602,112	seconds	Reference time almanac
δi	16	2^{-19}	-	semi-circles	correction to inclination (relative to i_0 = 0.30 semi-circles)
$\dot{\Omega}$	16	2^{-38}	-	semi-cirlces/sec	Rate of right ascension
$\sqrt{A}$	16	2^{-11}	-	$\sqrt{\text{meters}}$	Square root of the semi major axis
Ω_0	24	2^{-23}	-	semi-circles	Longitude of ascending node of orbit plane at weekly epoch
ω	24	2^{-23}	-	semi-circles	Argument of perigee
M_0	24	2^{-23}	-	semi-circles	Mean anomaly at reference time
a_{f0}	11	2^{-20}	-	seconds	-
a_{f1}	11	2^{-38}	-	sec/sec	-
WN_a	8	-	-	weeks	Week number to which t_{oa} is referenced (mod 256)

UTC Parameters

term	bits	scale fctr	eff. range	units	explanation
A_0	32	2^{-30}	-	seconds	-
A_1	24	2^{-50}	-	sec/sec	-
$\Delta\mathrm{t}_{LS}$	8	1	-	seconds	Current delta time
t_{ot}	8	2^{12}	602,112	seconds	-
WN_t	8	1	-	weeks	Current week number (??)
WN_{LSF}	8	1	-	weeks	Week number leap seconds becomes effective

DN	8	1	7	days	Day number leap seconds becomes effective
Δt_{LSF}	8	1	-	seconds	Delta time due to leap seconds (soon or recent)

Ionospheric Parameters

term	bits	scale fctr	eff. range	units	explanation
α_0	8	2^{-30}		seconds	-
α_1	8	2^{-27}		sec/semi-circle	-
α_2	8	2^{-24}		sec/semi-circles^2	-
α_3	8	2^{-24}		sec/semi-circles^3	-
β_0	8	2^{11}		seconds	-
β_1	8	2^{14}		sec/semi-circle	-
β_2	8	2^{16}		sec/semi-circles^2	-
β_3	8	2^{16}		sec/semi-circles^3	-

Constants

term	def	units	explanation
c	2.99792458×10^8	meters/sec	speed of light in vacuum
μ	3.986005×10^{14}	meters^3/sec^2	Earth's universal gravitational parameter
$\dot{\Omega}_e$	$7.2921151467 \times 10^{-5}$	rad/sec	Earth's rotation rate
π	3.1415926535898		Pi

0.3 Equations

Geometric Range Correction: When computing geometric range, account for the effects due to earth rotation rate during the time of signal propagation so as to evaluate the path delay in an inertially stable coordinate system. If working in Earth-fixed coordinates, add the following to position estimate (x, y, z):

$$(-\dot{\Omega}_e \mathrm{y} \Delta t, \dot{\Omega}_e \mathrm{x} \Delta t, 0)$$

Group Delay Application: The user who utilizes the L1 frequency will modify the code phase offset with this equation, where T_{GD} is provided to the user as subframe 1 data:

$$(\Delta t_{SV})_{L1} = \Delta t_{SV} - T_{GD}$$

Satellite Clock Correction: The polynomial defined in the following allows the user to determine the effective satellite PRN code phase offset referenced to the phase center of the satellite antennas (Δt_{SV}) with respect to the GPS system time (t) at the time of data transmission.

The coefficients transmitted in subframe 1 describe the offset apparent to the control segment two-frequency receivers for the interval of time in which the parameters are transmitted. This estimated correction accounts for the deterministic satellite clock error characteristics of bias, drift, and aging, as well as for the satellite implementation characteristics of group delay bias and mean differential group delay. Since these coefficients do not include corrections for relativistic effects, the user's equipment must determine the requisite relativistic correction. Accordingly, the offset given below includes a term to perform this function.
The user will correct the time received from the satellite with the equation (in seconds):

$$t = t_{SV} - (\Delta t_{SV})_{L1}$$

where

$$\begin{aligned} t &= \text{GPS system time (seconds)} \\ t_{SV} &= \text{effective SV PRN code phase time at message transmission time (seconds)} \\ (\Delta t_{SV})_{L1} &= \text{SV PRN code phase time offset (seconds)} \end{aligned}$$

SV PRN code phase offset: Using a_{f0}, a_{f1}, a_{f2} from subframe 1:

$$(\Delta t_{SV})_{L1} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_r - T_{GD}$$

Relativistic correction term: Using e, A, and E_k from subframes 2 and 3:

$$\Delta t_r = Fe(A)^{\frac{1}{2}} \sin E_k$$

Definition of F:

$$F = \frac{-2(\mu)^{\frac{1}{2}}}{c^2} = -4.442807633(10)^{-10} d \text{ sec/(meter)}^{\frac{1}{2}}$$

Relativistic correction term used by control segment:

$$\Delta t_r = -\frac{2\vec{R} \times \vec{V}}{c^2}$$

where

$$\begin{aligned} \vec{R} &= \text{Instantaneous position vector of SV} \\ \vec{V} &= \text{Instantaneous velocity vector of SV} \end{aligned}$$

Ionospheric Model: Utilizing parameters from page 18 of subframe 4.

$$T_{iono} = \begin{cases} F \times \left[5.0 \times 10^{-9} + (\mathrm{AMP})\left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\right] & |x| < 1.57 \\ F \times (5.0 \times 10^{-9}) & |x| \geq 1.57 \end{cases} \text{(sec)}$$

$$\mathrm{AMP} = \begin{cases} \sum_{n=0}^{3} \alpha_n \phi_m^n & \mathrm{AMP} \geq 0 \\ 0 & \mathrm{AMP} < 0 \end{cases} \text{(sec)}$$

$$x = \frac{2\pi(t - 50400)}{\mathrm{PER}} \text{(radians)}$$

$$\mathrm{PER} = \begin{cases} \sum_{n=0}^{3} \beta_n \phi_m^n, & \text{if } \mathrm{PER} \geq 72,000 \\ 72,000, & \text{if } \mathrm{PER} < 72,000 \end{cases} \text{(sec)}$$

$$F = 1.0 + 16.0[0.53 - E]^3$$

α_n and β_n are transmitted data words with$n = 0, 1, 2,$ and 3

Other ionospheric equations:

$$\phi_m = \phi_i + 0.064 \cos(\lambda_i - 1.617) \text{ (semi-circles)}$$

$$\lambda_i = \lambda_u + \frac{\psi \sin A}{\cos \phi_i} \text{ (semi-circles)}$$

$$\phi_i = \begin{cases} \phi_u + \psi \cos A, & \text{if } |\phi_i| \leq 0.416 \\ \phi_i = +0.416, & \text{if } \phi_i > 0.416 \\ \phi_i = -0.416, & \text{if } \phi_i < -0.416 \end{cases} \text{(semi-circles)}$$

$$\psi = \frac{0.00137}{E + 0.11} - 0.022 \text{ (semi-circles)}$$

$$t = \begin{cases} 4.32 \times 10^4 \lambda_i + \text{GPS time (sec)} \\ t - 86400, & \text{if } t \geq 86400 \\ t + 86400, & \text{if } t < 0 \end{cases}$$

Terms Used in Computation of Ionospheric Delay:

Satellite Transmitted:

α_n	Coefficients of cubic equation representing amplitude of vertical delay (4 coefficients = 8 bits each)
β_n	Coefficients of cubic equation representing period of model (4 coefficients = 8 bits each)

Receiver Generated:

E	Elevation angle between the user and satellite (semi-circles)
A	Azimuth angle between the user and satellite, measured clockwise positive from true north (semi-circles)
ϕ_u	User geodetic latitude in WGS-84 (semi-circles)
λ_u	User geodetic longitude in WGS-84 (semi-circles)
GPS time	Receiver computed system time[2]

Computed:

x	Phase (radians)
F	Obliquity factor (dimensionless)
t	Local time (sec)
ϕ_m	Geomagnetic latitude of the earth projection of the ionospheric intersection point (mean ionospheric height assumed 350km) (semi-circles)
λ_i	Geomagnetic longitude of the earth projection of the ionospheric intersection point (semi-circles)
ϕ_i	Geomagnetic latitude of the earth projection of the ionospheric intersection point (semi-circles)
ψ	Earth's central angle between user position and earth projection of ionospheric intersection point

[2]Referenced to UTC midnight January 5 (morning of January 6), 1980

Universal Coordinated Time (UTC) Depending on the relationship of the effectivity date to the user's current GPS time, the following three different UTC/GPS-time relationships exist:
a. When the effectivity time indicated by the WN_{LSF} and the DN values is not in the past (relative to the user's present time), *and* the user's present time does not fall in the timespan which starts at DN + 3/4 and ends at DN + 5/4, the UTC/GPS-time relationship is given by:

$$t_{UTC} = (t_E - \Delta t_{UTC}) \mod 86,400 \text{ (secs)}$$

where

$$\begin{aligned}
\Delta t_{UTC} &= \Delta t_{LS} + A_0 + A_1(t_E - t_{ot} + 604800(\mathrm{WN} - \mathrm{WN}_t)) \text{ (secs)} \\
t_E &= \text{GPS time as estimated by the user on the basis of correcting } t_{SV} \text{ for factors described in 2.5.5.2 a} \\
\Delta t_{LS} &= \text{Delta time due to leap seconds} \\
A_0 \text{ and } A_1 &= \text{Constant and first order terms of polynomial} \\
t_{ot} &= \text{reference time for UTC data; referenced to start of week number } (\mathrm{WN}_t) \text{ in page 18 subframe 4 wo} \\
\mathrm{WN} &= \text{Current week number (derived from subframe 1)} \\
\mathrm{WN}_t &= \text{UTC reference week number; eight LSBs of full week number}
\end{aligned}$$

WN, WN_t, and WN_{LSF} can all be truncated – see 2.3.5(b). $|\mathrm{WN} - \mathrm{WN}_t| \leq 127$

b. When the user's current time falls within the timespan of DN + 3/4 to DN + 5/4, proper accommodation of the leap second event with a possible week number transition is provided by the following expression for UTC:

$$t_{UTC} = W[\mathrm{mod}(86400 + \Delta t_{LSF} - \Delta t_{LS})] \text{ (secs)}$$

where

$$\begin{aligned}
W &= t_E - \Delta t_{UTC} - 43200)[\mathrm{mod}86400] + 43200 \text{ secs} \\
\Delta t_{UTC} &= \text{as above}
\end{aligned}$$

c. When the effectivity time of the leap second event (per WN_{LFS} and DN) is in the past relative to the user's current time, handle identical to "a" except:

$$\Delta t_{UTC} = \Delta t_{LSF} + A_0 + A_1(t_E - t_{ot} + 604800(\mathrm{WN} - \mathrm{WN}_t)) \text{ (secs)}$$

Almanac Data: Calculate per precise ephemeris. Nominal inclination angle of 0.30 semi-circles implicit, δi (correction to inclination) is transmitted. Other parameters not included in almanac are set to 0 for position determination.

$$t = t_{SV} - \Delta t_{SV} \text{ (accurate to ~2 microseconds)}$$

where

$$\begin{aligned}
t &= \text{GPS system time (secs)} \\
t_{SV} &= \text{Effective satellite PRN code phase at time of transmission (secs)} \\
\Delta t_{UTC} &= \text{as above} \\
\Delta t_{SV} &= a_{f0} + a_{f1} t_k \text{ (Satellite PRN code phase time offset (secs))} \\
t_k &= t - t_{oa} \text{ (Time from epoch)}
\end{aligned}$$

Elements of Coordinate Systems

$A = (\sqrt{A})^2$	Semi-major axis
$n_0 = \sqrt{\frac{\mu}{A^3}}$	Computed mean motion - rad/sec
$t_k = t - t_{oe}$	Time from ephemeris reference epoch
$n = n_0 + \Delta n$	Corrected mean motion
$M_k = E_k - e \sin E_k$	Kepler's equation for eccentric anomaly (may be solved by iteration) - radians
$v_k = \tan^{-1}\left\{\frac{\sin v_k}{\cos v_k}\right\} = \tan^{-1}\left\{\frac{\sqrt{1-e^2}\sin E_k \ / \ (1-e\cos E_k)}{(\cos E_k - e) \ / \ (1-e\cos E_k)}\right\}$	True anomaly
$E_k = \cos^{-1}\left\{\frac{e+\cos v_k}{1+e\cos v_k}\right\}$	Eccentric anomaly
$\Phi_k = v_k + \omega$	Argument of latitude
$\left.\begin{array}{ll} \delta u_k = C_{us}\sin 2\Phi + C_{uc}\cos 2\Phi_k & \text{argument of latitude correction} \\ \delta r_k = C_{rc}\cos 2\Phi + C_{rs}\sin 2\Phi_k & \text{radius correction} \\ \delta i_k = C_{ic}\cos 2\Phi_k + C_{is} sin 2\Phi_k & \text{inclination correction} \end{array}\right\}$	Second Harmonic Perturbations
$u_k = \Phi_k + \delta u_k$	Corrected argument of latitude
$r_k = A(1 - e\cos E_k) + \delta r_k$	Corrected radius
$i_k = i_0 + \delta i_k + (\text{IDOT})t_k$	Corrected inclination
$\left.\begin{array}{l} x'_k = r_k \cos u_k \\ y'_k = r_k \sin u_k \end{array}\right\}$	Positions in orbital plane
$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t_k - \dot{\Omega}_e t_{oe}$	Corrected longitude of ascending node
$\left.\begin{array}{l} x_k = x'_k \cos\Omega_k - y'_k \cos i_k \sin\Omega_k \\ y_k = \sin\Omega_k + y'_k \cos i_k \cos\Omega_k \\ z_k = y'_k \sin i_k \end{array}\right\}$	Earth-Centered, Earth-Fixed coordinates

User Range Accuracy:

$$\text{URA (meters)} = \begin{cases} 2^{(1+\frac{N}{2})} & \text{if } N \leq 6 \\ 2^{(N-2)} & \text{if } 6 < N < 15 \\ \infty & \text{if } N = 15 \\ & \\ 2.8 & \text{if } N = 1 \\ 5.7 & \text{if } N = 3 \\ 11.3 & \text{if } N = 5 \end{cases}$$

0.4 Coordinate Systems

Earth-Centered Earth-Fixed (ECEF): Equations from IS-GPS-200H Table 20-IV give SV's antenna phase center in WGS-84 ECEF system defined as:

Origin	=	Earth's center of mass
Z-Axis	=	Direction of IERS Reference Pole (IRP) (Rotational axis of the WGS-84 ellipsoid)
X-Axis	=	Intersection of IERS Reference Meridian (IRM) and the plane passing through the origin and normal to the Z-axis
Y-Axis	=	Completes a right-handed, Earth-Centered, Earth-Fixed orthogonal coordinate system

Earth-Centered, Inertial (ECI): In an ECI coordinate system, GPS signals propagate in straight lines at constant speed c. A stable ECI coordinate system of convenience may be defined as being coincident with the ECEF coordinate system at a given time t_0. The x, y, z coordinates in the ECEF coordinate system at some other time can be transformed to the x', y', z' coordinates in the selected ECI coordinate system of convenience by simple rotation (neglecting polar motion, nutation, and precession, which can be neglected for small values of $(t - t_0)$):

$$\begin{aligned} x' &= x\cos(\theta) - y\sin(\theta) \\ y' &= x\sin(\theta) + y\cos(\theta) \\ z' &= z \end{aligned}$$

where

$$\theta = \dot{\Omega}_e(t - t_0)$$

0.5 Other

Geometric Range: The user must account for the geometric range (D) from the satellite to receiver in an ECI coordinate system:

$$D = \left| \overrightarrow{r}(t_R) - \overrightarrow{R}(t_T) \right|$$

where

$$\begin{aligned} t_T &= \text{GPS system time of transmission} \\ t_R &= \text{GPS system time of reception} \\ \overrightarrow{R}(t_T) &= \text{position vector of SV in selected ECI coordinate system at time } t_T \\ \overrightarrow{r}(t_R) &= \text{position vector of receiver in selected ECI coordinate system at time } t_R \end{aligned}$$

NMCT Validity Time: To use NMCT data (page 13 of subframe 4) examine AODO term provided in subframe 2. If AODO is 27900 seconds (binary 11111), NMCT is invalid. Otherwise, validity time (t_{nmct}) is:

$$\text{OFFSET} = t_{oe} \mod 7200$$

$$t_{\text{nmct}} = \begin{cases} t_{oe} - \text{AODO} & \text{if OFFSET} = 0 \\ t_{oe} - \text{OFFSET} + 7200 - \text{AODO} & \text{if OFFSET} > 0 \end{cases}$$

Calculation of t_{nmct} must account for beginning and end of week crossovers:

$$t_{\text{nmct}} = \begin{cases} t_{\text{nmct}} + 604,800 & \text{if } t - t_{\text{nmct}} > 302,400 \\ t_{\text{nmct}} - 604,800 & \text{if } t - t_{\text{nmct}} < -302,400 \end{cases}$$

Different SVs will transmit NMCT with different t_{nmct}; best performance is usually obtained by applying data from the NMCT with the latest (largest) t_{nmct}. If the same largest t_{nmct} is provided by two or more visible SVs, the NMCT from any SV (with largest t_{nmct}) may be used; The estimated range deviation (ERD) value provided by the selected NMCT for the other SVs with the same t_{nmct} should be set to zero if those SVs are used in the positioning solution. Do not apply data from multiple NMCTs and do not apply the data from one NMCT to only a subset of SVs.

IODE Datasets: Cutovers to new datasets will occur on hour boundaries except the first dataset of a new upload. Cutover to 4-hour datasets will occur modulo 4 hours relative to start of week. Cutover from 4-hour to 6-hour datasets will occur modulo 12 hours. Cutover from 12-hour to 24-hour datasets will occur modulo 24 hours.

Start of transmission interval for each dataset corresponds to beginning of curve fit interval for dataset.

IODC Values and Data Set Lengths (Block II/IIA)

days spanned	transmission int. (h)	curve fit int. (h)	IODC Range
1	2	4	(see IS-GPS-200H table 20-XL)
2–14	4	6	(see IS-GPS-200H table 20-XL)
15–16	6	8	240–247
17–20	12	14	248–255, 496
21–27	24	26	497–503
28–41	48	50	504–510
42–59	72	74	511, 752–756
60–63	96	98	757

IODC Values and Data Set Lengths (Block IIR/IIR-M/IIF & III)

days spanned	transmission int. (h)	curve fit int. (h)	IODC Range
1	2	4	(see IS-GPS-200H table 20-XIL)
2–14	4	6	(see IS-GPS-200H table 20-XIL)
15–16	6	8	240–247
17–20	12	14	248–255, 496
21–62	24	26	497–503, 1021–1023