Pipe flow experiment

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Scheduled submission date: 30.04.2018
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Abstract

The purpose of the experiment was to examine the difference between laminar and turbulent flows by studying a classical fluid flow. We found out how the skin friction coefficient varied with Reynolds number and deduced that the transition occurred between Re =7000 and Re=3800. In the laboratory we measured the static pressure drop, as this allowed us to measure shear stress and skin friction coefficient.
Understanding the features of a fully developed pipe flow helps engineers in prediction of a flow around a wing and also gives engineers a deep insight of behaviour of the flow at high Reynold's numbers.

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1. Introduction

In the laboratory we examined the significant features of laminar and turbulent flow on a fully developed pipe flow. In 1883, Osborne Reynolds was the first to study the transition in a pipe flow from a laminar to turbulent state. He varied the Reynolds number by increasing velocity from some low value to some high value. He observed a change in path of flow, from ordered streaks to chaotic random motion with increasing Reynolds numbers. During the experiment, students altered the speed of the flow by altering the power of the fan and took the readings of the pressure from a manometer. This in turn allowed students to estimate pressure gradient and subsequently the skin friction coefficient. Clearly, a study on pipe flow has its own benefits and the results obtained in this laboratory are relevant to the calculation of losses in pipe systems. The practical importance of this subject has led to several studies, such as Chézy (1775), Taylor-Couette flow (1890)[i] and Floris Takens (1981)[ii]. Engineers often use the idea of Reynolds number in construction of wind tunnels and the idea of drag coefficient when building a profile of a plane wing.

2. Theoretical background

To describe relations between pressure and shear stress a thin slice across the flow has to be considered.

Then,

$$\frac{\pi * D^2}{4} * \frac{dp}{dx} * dx = \pi * D * \tau_w * dx \qquad (1)$$

$$\tau_w = \frac{D}{4} * \frac{dp}{dx} \qquad (2)$$

It can be easily seen that the shear stress can be obtained simply by measuring the pressure gradient, which is relatively easy.

The variation of the skin friction will be presented as a dimensionless function of the Reynolds number.

$$c_f = \frac{\tau_w}{\frac{1}{2} * \rho * V^2} = f(Re_D) \qquad (3)$$

$$Re_D = \frac{\rho * V * D}{\mu} \qquad (4)$$

Here, τ_w is the wall shear stress, ρ is the density, V is the bulk mean velocity, D is the pipe diameter and μ is the fluid viscosity. The actual measurements made are of the pressure gradient along the pipe, so that c_f will be found from a combination of (1) and (2) as

$$c_f = \frac{\Delta p}{\frac{1}{2} * \rho * V^2} * \frac{D}{4 * L} = f(Re_D) \qquad (5)$$

where Δp is the pressure drop recorded over a length L.

Equation (3) is a typical non-dimensional relation showing the relationship between the shear stress, scaled on the dynamic pressure, and the Reynolds number. This indicates the relative importance of stresses related to momentum changes to those resulting from the action of viscosity.

Thus: $$\frac{Momentum\ related\ stress\ (\frac{1}{2} * \rho * V^2)}{Viscous\ shear\ stress\ (\mu * \frac{V}{D})} = \frac{\rho * V * D}{\mu} \qquad (6)$$

The factor ½ is dropped as a matter of convention.[iii]

3. Apparatus and procedure

The test rig consists of a circular section Perspex pipe (Figure 1) with a variable speed fan (Figure 3) mounted downstream and controlled by a transformer. Pressure tappings (Figure 4) are allocated at seven positions along the pipe. Relative to tapping 1, the positions of other tappings are listed below:

Tapping	1	2	3	4	5	6	7
Position(m)	0	0.71	1.32	1.93	2.54	3.15	3.77

Table 1: Tappings with their relative position.

The mean velocity in the pipe flow can be determined from the pressure difference across an orifice plate. This consists of a plate across the pipe with a hole in it whose diameter is less than that of the pipe. In general, such a device needs to be calibrated, through for this experiment the calibration factor has been 'adjusted' in order to give good results.

$$\frac{1}{2} * \rho_a * V^2 = k\Delta p \qquad (7)$$

where, $k = 0.125 \; and \; 0.100$ for laminar and turbulent flows, respectively.

The calibration factor (k) is a function of geometry and Reynolds number, and, because the velocity profiles are very different for laminar and turbulent flow, there are substantial differences between the factors in the two regimes. In principle the calibration factor remained a function of Re_D in both regimes, but the variation within each regime was small enough to be ignored.

The pressure tappings were connected to a multi-tube manometer (Figure 2) that was filled with alcohol (density,$\rho_m = 800Kg/m^3$). The range of pressure to be measured in this experiment was very wide. For low-pressure differences (that would occur at low Reynolds numbers), students tilted the manometer to a shallow angle (as low as $20°$ to the horizontal). There was a scale that was fixed to the spindle that stops at $20°$. The errors become magnified at smaller angles as small pressure differences manifest as large displacements in the alcohol column.

Before starting, students leveled the manometer (graduations in the manometer are in cm), equalizing the alcohol level across the tubes with no flow. The pressures measured were below atmospheric, so the alcohol level rose in the tubes. Students made sure that the manometer was vertically set and that the fluid is 10 cm above the bottom of the scale.

During the experiment the position of the manometer had been constantly altered in order to obtain a full-range reading as far as possible (this in turn minimized random error).

The next step would be to plot a graph of $(\Delta h_{1-7} * sin\theta)/\sqrt{\Delta h_0 * sin\theta}$ against $\sqrt{\Delta h_0 * sin\theta}$ as this graph would give a horizontal line in the laminar regime and a nearly straight line in the turbulent regime, with a very clear indication of transition as seen in Figure 5.

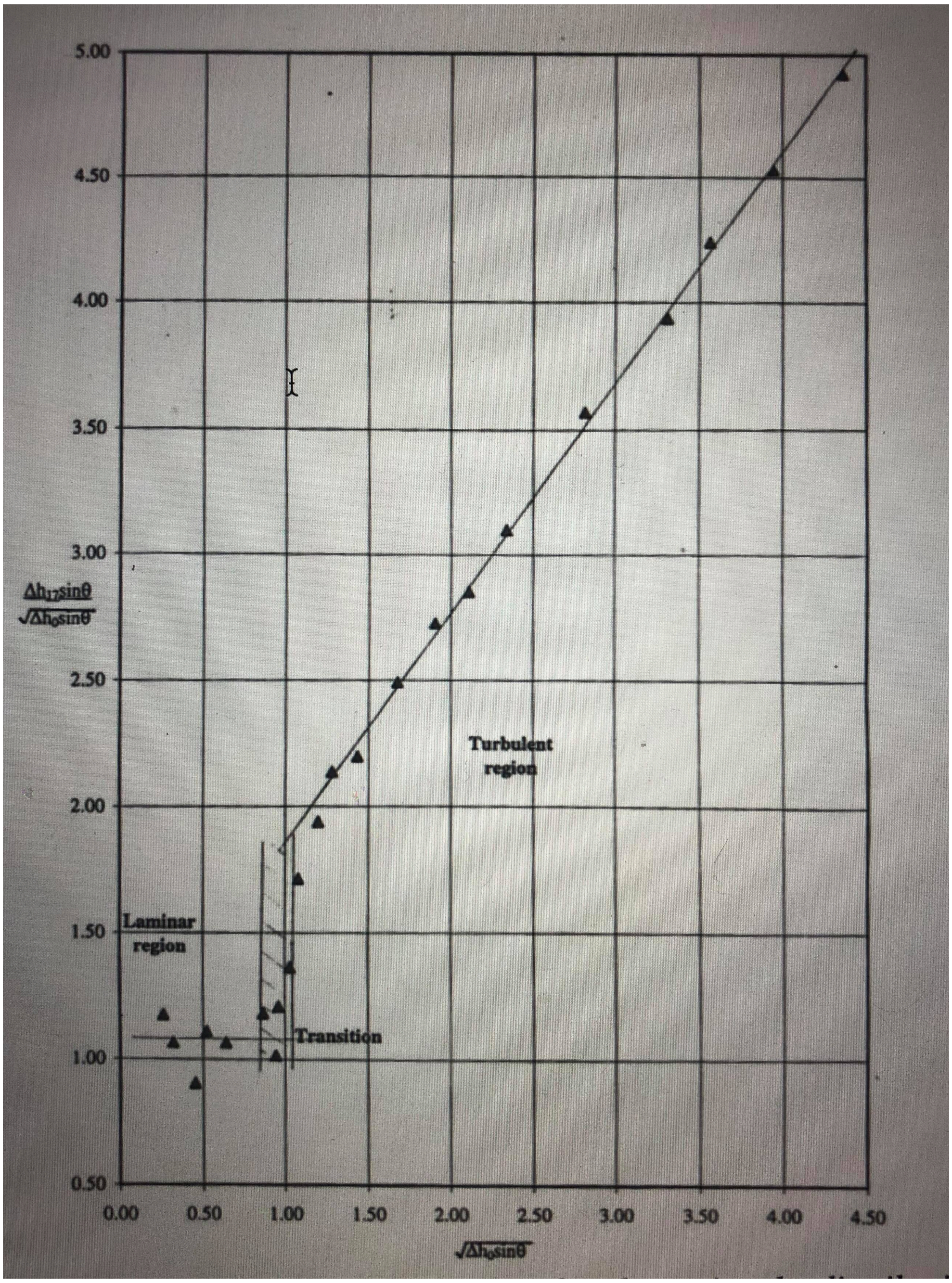

Figure 5: A Graphical representation of different flow states.[iv]

There were seven static holes and the tappings on either side of the orifice plate corresponding to manometer readings. The only readings that needed to be converted to dimensional form were the pressure difference across the orifice plate. $\Delta p_0 = \Delta h_0 * \rho_m * g * sin\theta$. These were required for calculating the Reynolds numbers.[v]
The skin friction coefficients can be found directly from the manometer readings without working out any of the pressures as such:

$$c_f = \frac{\Delta p}{\frac{1}{2} * \rho_a * V^2} * \frac{D}{4L} = \frac{\Delta h * \rho_m * g * sin\theta}{k * \Delta h_0 * \rho_m * g * sin\theta} * \frac{D}{4L} \qquad (7)$$

$$c_f = \frac{\Delta h * D}{k * \Delta h_0 * 4L} = \frac{1 * d(h/h_0)}{4k * d(x/D)} \qquad (8)$$

Figure 1: The full view of the Perspex pipe and the first pressure tapping.

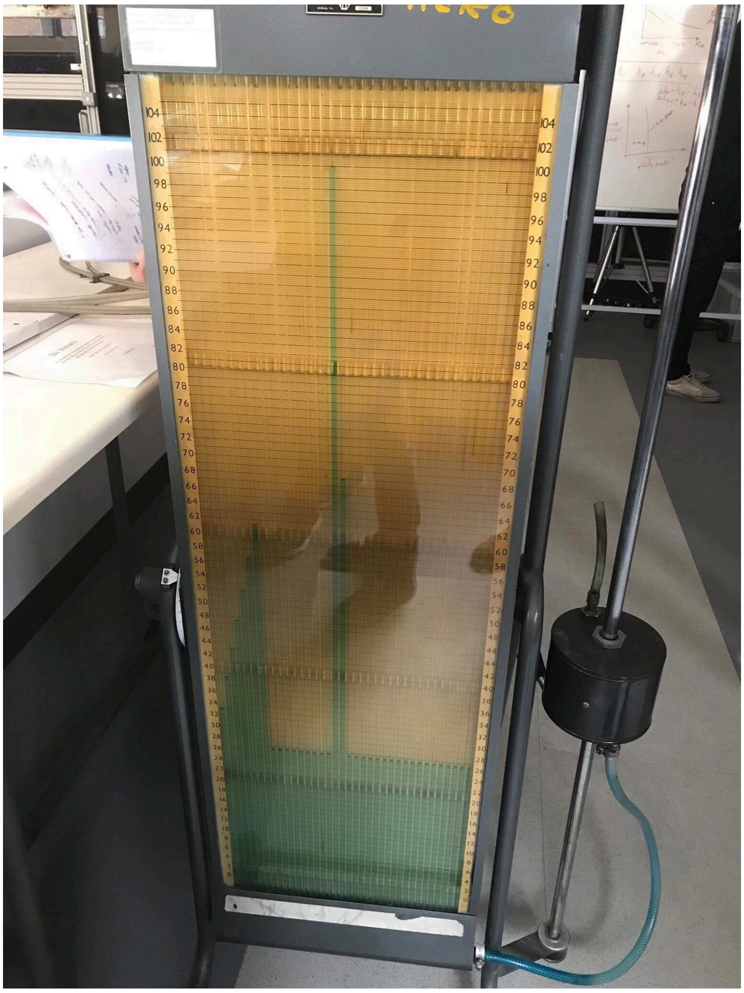

Figure 2: Manometer filled with alcohol.

Figure 3: Fan and orifice plate.

Figure 4: The detailed view of tapping.

The only instruments that gave the students some values were the manometer and the inclination scale. The precision on these instruments is the smallest reading possible, i.e. +/- 0.2 cm and +/- 1° respectively.

4. Results and Discussion

Height/Trial	1	2	3	4	5	6	7	8
h1	32.7	31.8	24.9	15.0	12.6	5.2	4.7	4.2
h2	38.2	36.6	31.0	21.3	18.0	6.2	5.4	4.7
h3	42.7	41.5	36.0	26.5	22.8	7.0	6.0	5.2
h4	47.2	46.2	40.2	31.9	27.3	7.7	6.7	5.7
h5	51.8	51.2	46.5	37.2	31.9	8.4	7.4	6.2
h6	56.5	56.2	51.8	42.8	36.6	9.2	8.0	6.7
h7	61.2	61.2	56.7	48.5	41.3	9.9	8.6	7.2
h17	100.0	100.2	96.6	89.3	75.3	17.8	14.3	10.5
h18	67.8	67.8	63.4	55.9	47.5	11.8	9.4	7.6
angle	90°	45°	35°	25°	24°	21.5°	21.5°	21.5°
href	26	23.9	17.8	7.2	5.8	2.8	2.7	2.8

Table 2: Each trial and its respective height of liquid in manometer.

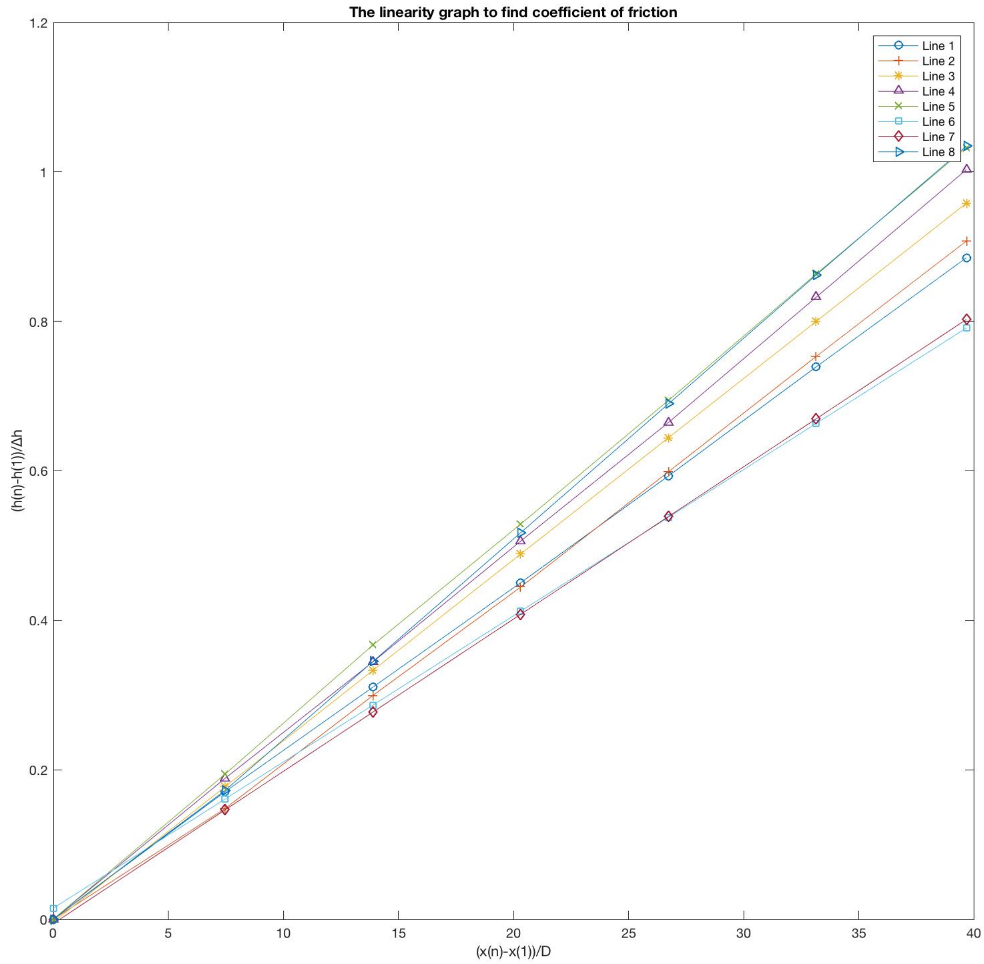

Figure 6: The linear graphs from which the skin friction can be determined.

The following skin friction coefficients have been found using equation 8:

Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8
0.00444	0.00454	0.00464	0.00474	0.00482	0.00585	0.00575	0.00585

Table 3: Coefficients of friction found from Figure 6

Overall, the right pattern can be observed. If the Reynolds number is large, the viscosity effect is small. In the laboratory, the students initially set the power of fan to maximum and with each subsequent trial the power of the fan was lower.
Using this simple equation $\Delta p_0 = \Delta h_0 * \rho_m * g * sin\theta$, Reynolds number can be found.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8
$\Delta p_0(Pa)$	2527.1	1798	1494.5	1107.8	887.4	172.6	140.9	83.40
$V(m/s)$	20.3	17.1	15.6	13.4	12.0	5.94	5.36	4.13
Re_D	13060	11000	10030	8620	7715	3820	3446	2655

Table 4: Results obtained from the manometer readings. Viscosity of air is treated as $1.81 * 10^{-5} \frac{kg}{m*s}$.

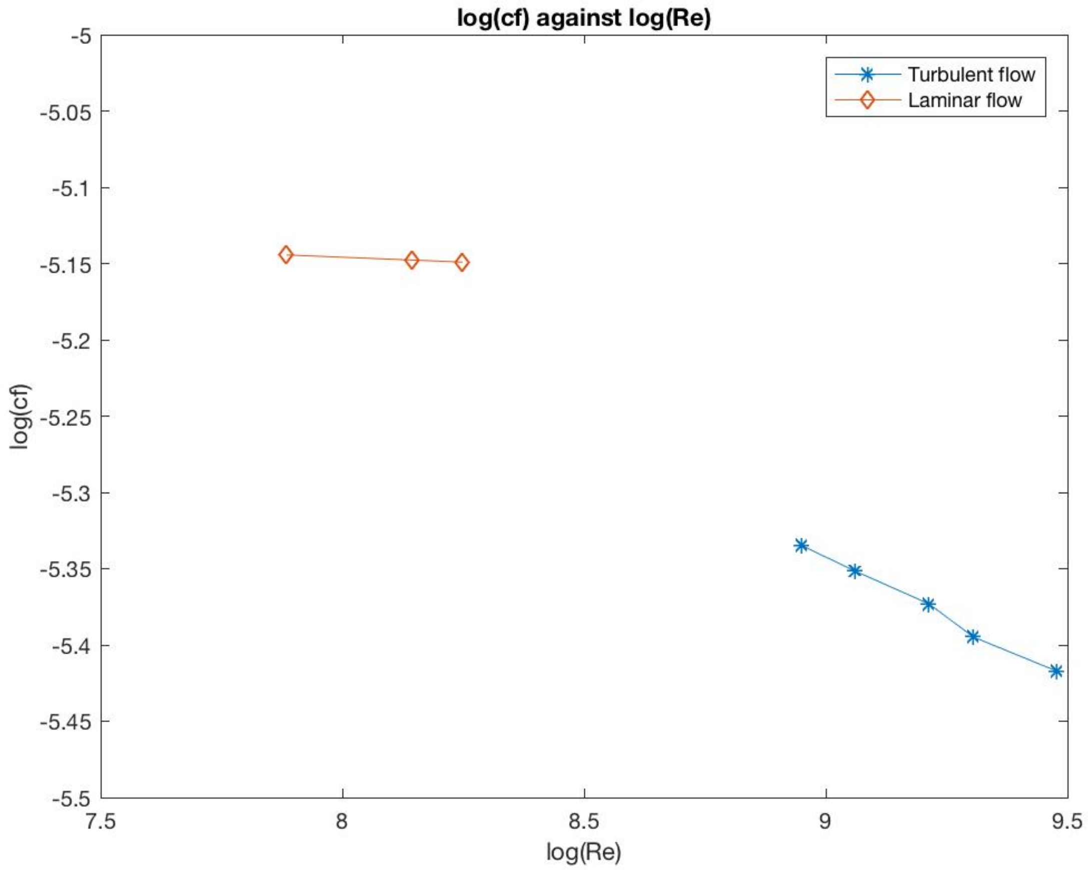

Figure 7. Log(cf) against log(Re) graph.

Here is the list of answers on the questions from the handout:

1. The difference between accuracy and precision is that accuracy shows how results are close to the actual or 'theoretically predicted' value, whereas precision shows how results are close to each other. The results can be stated to be precise but not very accurate. First, the last value obtained from the gradient of 'Line 8' does not follow the theoretical pattern as its value is higher than the preceding value (the gradient of 'Line 7'). This could have happened due to the noise in the data collected. Second, the uncertainties mentioned in Section 4 ('Apparatus and procedure') were magnified as these uncertainties were carried through a number of formulae (the results of the overall uncertainty of the experiment is listed below).
2. $c_f = \frac{16}{Re_D} = \frac{16}{2655} = 0.00603$ which is roughly equals to 0.00585. This result is the most accurate as it lies within 3% uncertainty. The established formula is therefore proven by the experimental data.
3. Using the formula $c_f{}^{-0.5} = -0.4 + 4 * \log\left(c_f{}^{0.5} * Re_d\right)$, the best estimate can be obtained using the gradient of the first line and the respective Reynolds number, i.e. $c_f = 0.00444$ and $Re_d = 13060$. Although the number on the left-hand side and the right-hand side are not quite equal, they lie within 25% uncertainty. It is hard to prove the established formula using the experimental data listed above.

4. The transition occurred between 7715 and 3820 Re_D. The critical value is different to the one provided in the handout, where the critical value is said to be between 2500 and 2300. The highest Reynolds number also exceeds the highest theoretical value (12000).
5. Most of the graphs are linear and the gradients of such graphs were found using the equation of straight line. There are however some non-linear graphs as well, 'Line 3' is non-linear probably due to some random error (N.B. the rest 4 lines obtained from the turbulent data points are all quite straight), but 'Line 6' and 'Line 7' demonstrate that the flow was not fully developed in the pipe.
6. The uncertainty on the evaluation of coefficient of friction is the uncertainty on the gradient of each line. The uncertainty on the gradient is the difference between the gradient of the steepest best-fit line and the gradient of the shallowest best-fit line all over the gradient of the best-fit line that passes through the most data points. This though can be simplified to maximum gradient minus minimum gradient all over average gradient. Uncertainty on Reynolds number is equal to uncertainty on pressure difference. To avoid fiddly calculations, the average differences are calculated. Thus

$$c_f(\%uncertainty) = \frac{0.00241 - 0.00222}{0.00232} * 100\% = 8.2\%$$

$$Re_D(\%\ uncertainty) = \Delta p(\%uncertainty) =$$

$$= \theta(\%uncertainty) + (\Delta h_0)(\%uncertainty) = \frac{0.2 + 0.2}{avg(\Delta h_0)} * 100\% + \sin(1°) * 100\%$$

$$= 1.88\%$$

5. Conclusion

Overall, the results are quite precise and relatively accurate. The total uncertainty of the experiment is about 10%, which is a small error. Most of the theoretically established formulae are proven using the relations above. However, the data for turbulent state does not coincide with the theoretical values. The transition occurred between Re=7715 and Re=3820, this range lies outside the theoretical range mentioned in the handout. Also, one of the problems was that the tappings were not ideally insulated and the momentum of the flow was partially lost and this lead to erratic laminar results. However, some sources state that transition would normally occur between Re=2000 and Re=13000.[vi]

References

[i] H.W Broer, B Krauskopf, Gert Vegter .*Global Analysis of Dynamical Systems.*

[ii] Taylor Geoffrey I. *Stability of a viscous liquid contained between two rotating cylinders.*

[iii] Jonathan Morrison. *Pipe flow.* Available from: https://bb.imperial.ac.uk/bbcswebdav/pid-1233469-dt-content-rid-3999125_1/courses/DSS-AE1_113-17_18/AE1-113%20Pipe%20Flow%20Handout.pdf/ [Accessed 20th of March 2018]

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[vi] Unknown Author. *Pipe flow.* Available from https://en.wikipedia.org/wiki/Laminar–turbulent_transition [Accessed 29th of March 2018]