Strain measurements
with resistance strain gauges

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1. Abstract

The purpose of the experiment was to determine Young's Modulus of Brass, Copper and Aluminum specimens. The values of Young's Modulus obtained during the experiments are 114 GPa, 133 GPa and 77.1 GPa respectively. The experimental measurements and theoretical values are relatively close to each other.
Stress and strain measurements are key elements of material testing. The knowledge of the stress-strain curve allows engineers to compare and select different materials, and predict the behavior of a structure made from a particular material. The experiment also gave a deep insight of the use of strain gauges.

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2. Introduction

The objective of the experiment is to become familiar with the use of resistance strain gauges and to determine Young's Modulus of three specimens. There are several types of strain gauges, such as metal strain gauges, semiconductor strain gauges, photomechanical strain gauges and many others. Among strain gauges, the electrical strain gauge has the advantages of lower cost and being the established product and it is, therefore, the most commonly used type of device. The gauges can be used in aerospace industry where the gauges measure the amount of stress an airplane can handle until it fails. Strain gauges are also widely applied in long-term monitoring of structures, for example, engineers put the sensors on bridges to monitor weak points of a bridge, where excessive stress and vibrations may lead to failure. The Tsina Ma Bridge in Hong Kong has more than 300 sensors.[i]

3. Theoretical Background

The experiment is based on the phenomena discovered by Lord Kelvin that the resistance of the electrical conduct varies if this is deformed.

The gauge is a conducting element which is bonded to the surface on which the strain is to be measured so that any strain on the surface is equal to the strain in the strain gauge conductor. Strain gauges measure strain in the direction they are oriented.

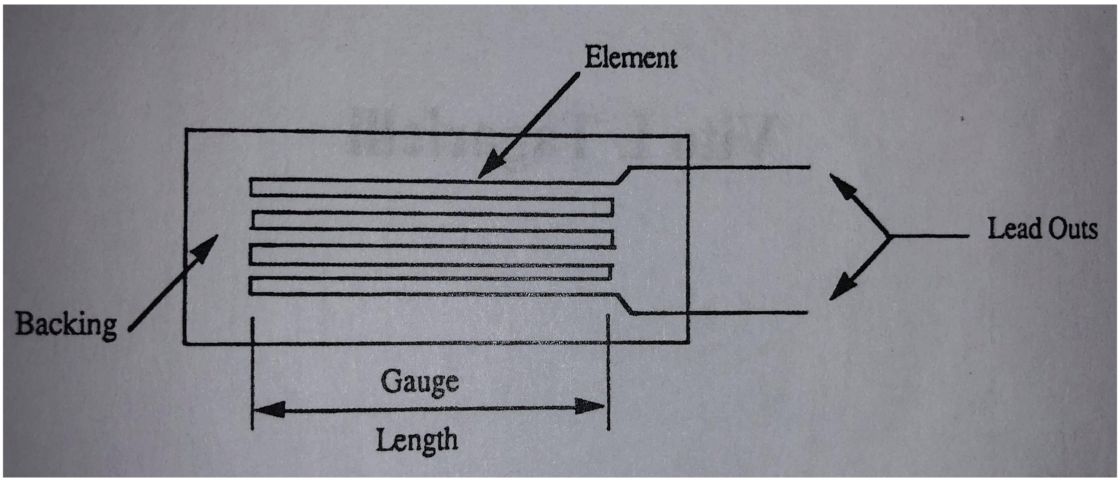


Figure 1. Schematic drawing of a gauge.

Recall that the electrical resistance R of a wire can be written as:

$$R = \rho * \frac{l}{A}, \qquad (1)$$

Where ρ is the resistivity of the material, l is the length of the 'dogbone' specimen and A is its cross-sectional area.

From the formula above we can deduce that the change in resistance is approximately proportional to the imposed strain along the direction of the wire.

Therefore:

$$\frac{\Delta R}{R} = S * \varepsilon, \qquad (2)$$

Where S is called the gauge factor and **ε** is the strain. The gauge factor is constant and is equal to 2.1.

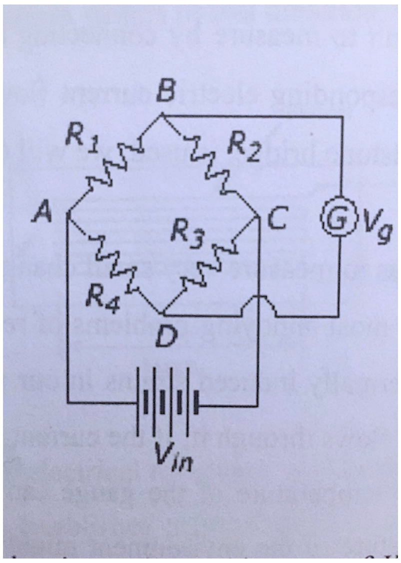


Figure 2. The circuit used in the experiment.

The configuration of the electric circuit used is so-called 'quarter-bridge' Wheatstone bridge where one of the resistors is replaced by the electrical strain gauge. The Wheatstone bridge allows us to measure small changes in resistance and to fix the problem of fictitious, thermally induced strains in our readings. Due to Joule effect, the relatively small gauge heats up if the current flowing through the gauge is large. The gauge's resistivity will change and both the length and area of the conductor will increase, as a result, the gauge will measure a change in resistance given by

$$\Delta R = \Delta R(thermal) + \Delta R(actual)$$

For the sake of simplicity, all resistors in the experiment have the same resistance R.

$$V(g) = \frac{V(in)}{4} * [\frac{\Delta R(1)}{R} - \frac{\Delta R(2)}{R} + \frac{\Delta R(3)}{R} - \frac{\Delta R(4)}{R}], \qquad (3)$$

$$\Delta R(2) = \Delta R(3) = \Delta R(4) = 0, \qquad (4)$$

$$V(g) = \frac{V(in) * \Delta R(1)}{4R}, \qquad (5)$$

$$\varepsilon = \frac{\Delta R}{R * S} = \frac{4V(g)}{V(in) * S}, \qquad (6)$$

V(g) is small and therefore it is difficult to measure accurately. To amplify the output voltage we use a voltage amplifier, which multiplies the output voltage by a constant factor 'GAIN'.

$$\overline{V}(g) = V(G) * GAIN \qquad (7)$$

Hence, the strain is given by

$$\varepsilon = \frac{4\overline{V}(g)}{V(in) * S * GAIN} \qquad (8)$$

In order to eliminate thermal effects from the reading, a dummy gauge must be used. The dummy gauge is connected to the second arm of the Wheatstone bridge. However, the dummy gauge is not attached to the strained component, this gauge can be attached to the same material.[ii]

$$\Delta R(1) = \Delta R(thermal) + \Delta R(actual), \qquad (9)$$

$$\Delta R(2) = \Delta R(thermal), \qquad (10)$$

$$\begin{aligned} V(g) &= V(in) * \frac{\Delta R(1) - \Delta R(2)}{4R} \\ &= V(in) * \left(\frac{\Delta R(thermal) + \Delta R(actual) - \Delta R(thermal)}{4R}\right), \qquad (11) \end{aligned}$$

$$V(g) = V(in) * \frac{\Delta R(actual)}{4R}, \qquad (12)$$

4. Experimental Apparatus and Procedures

A range of various apparatuses were used throughout the experiment. Figure 3-6 demonstrate the apparatuses used and Figure 7 shows the Wheatstone bridge configuration. Figure 8 represents the general setup of the experiment.

Figure 3. Brass, Aluminum and Copper dogbone specimens.

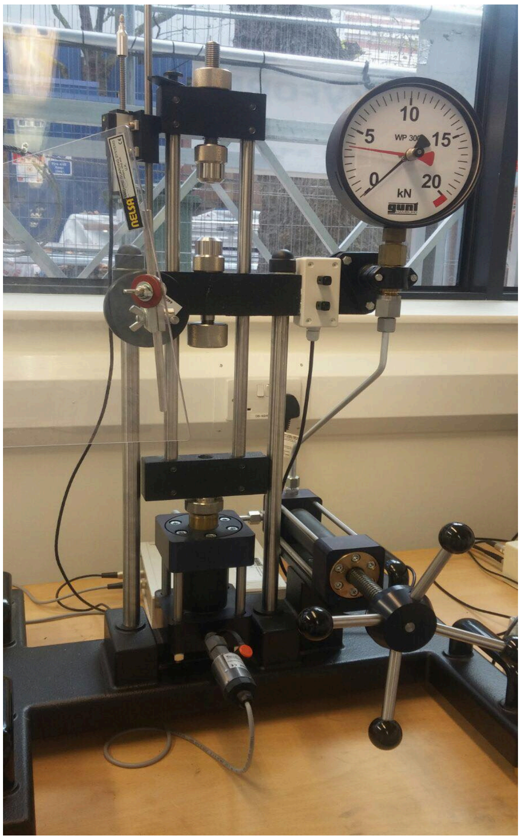


Figure 4. Hand-operated tensile machine.

Figure 5. Voltmeter.

Figure 6. Voltage amplifier.

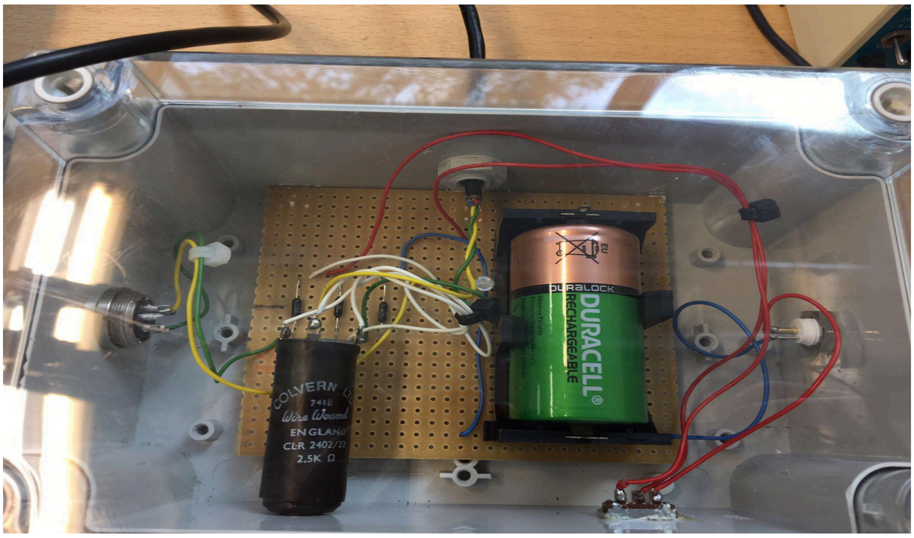


Figure 7. Wheatstone bridge.

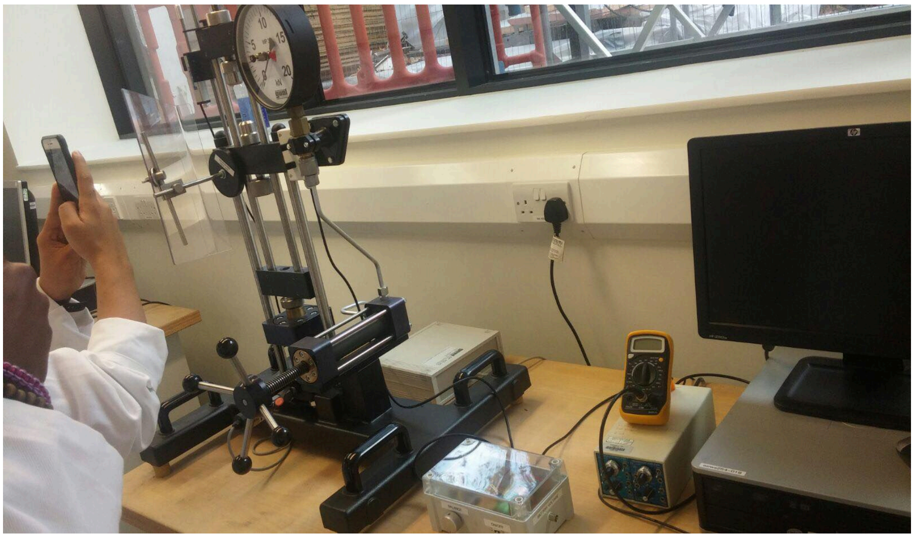


Figure 8. General view of the circuit.

The following procedure was followed for each experiment:

• Measure the specimen's diameter D using a digital caliper.

• Calculate the cross-section area A.

• Take a note of the GAIN of the voltage amplifier.

• Fasten the specimen's ends to the grips of the tensile machine, making sure not to induce any load while doing so.

• Connect the strain gauge wires to the Wheatstone bridge.

• Switch off the bridge and take a note of the voltage provided by the battery via the appropriate port, using a voltmeter set on DC voltage; use a full scale of 2 Volts and take a note if the reading; this is the V(in) to be used in the strain calculations.

• Switch-on the bridge and read the output voltage of the amplifier $\overline{V}(g)$; this will be initially a non-zero value (the bridge is initially unbalanced due to slight differences between the resistance of the fixed resistors and strain gauge).

• Observing $\overline{V}(g)$, turn the dial of the potentiometer on the bridge until it $\overline{V}(g) = 0$. Now the bridge is balanced and you are ready to start the test.

• Slowly operate the wheel of the tensile machine until you read a value of force 0.5kN; stop the loading phase and take a note of $\overline{V}(g)$; this will allow you to calculate the tensile strain in the specimen at an applied force of 0.5kN.

• Increase the load in steps of 0.5kN, stopping after each loading step and annotating the value of $\overline{V}(g)$. Terminate the procedure when you reach a load of 3 to 3.5kN.

• Unload the specimen slowly until the load drops to zero (additional readings can be taken).

• Mount the next specimen on the machine and repeat the entire procedure.[iii]

5. Results

There are three constants in the equations (1) – (12) that are required to determine Young's Modulus of the specimens.
The constants are represented in the table below:

Table 1: Constants and their values.

Constant	Value
Gauge Factor	2.1
GAIN	500
V(in)	1.315V

The uncertainty on Voltmeter is 0.001V, i.e. the smallest possible reading a student could take during the experiment. The uncertainty on Force applied to the specimens is 50 N, i.e. the smallest possible reading the student could take during the experiment. The uncertainty on the micrometer is 0.005 mm, i.e. the smallest reading the student could take divided by two. The further uncertainty calculations are based on the formula $\frac{a}{b} * 100\%$, where 'a' is the uncertainty of a device and 'b' is the value measured by the device.

- Brass

Diameter Average = $\frac{5.90+5.93+5.95}{3} = 5.92\ mm$

Cross-sectional Area = $\pi * r^2 = 27.53\ mm^2$

The uncertainty on the Cross-sectional Area is $2 * \frac{0.005}{5.92} * 100\% = 0.17\%$
The uncertainty on the Stress is therefore: $\frac{50}{2000} * 100\% + 0.17\% = 2.67\%$, where 2000 N is the average force applied to the specimen.
The uncertainty on the Strain is $\left(\frac{0.001}{1.315} + \frac{0.001}{0.217}\right) * 100\% = 0.54\%$, where 0.217 V is the average voltage across the specimen.

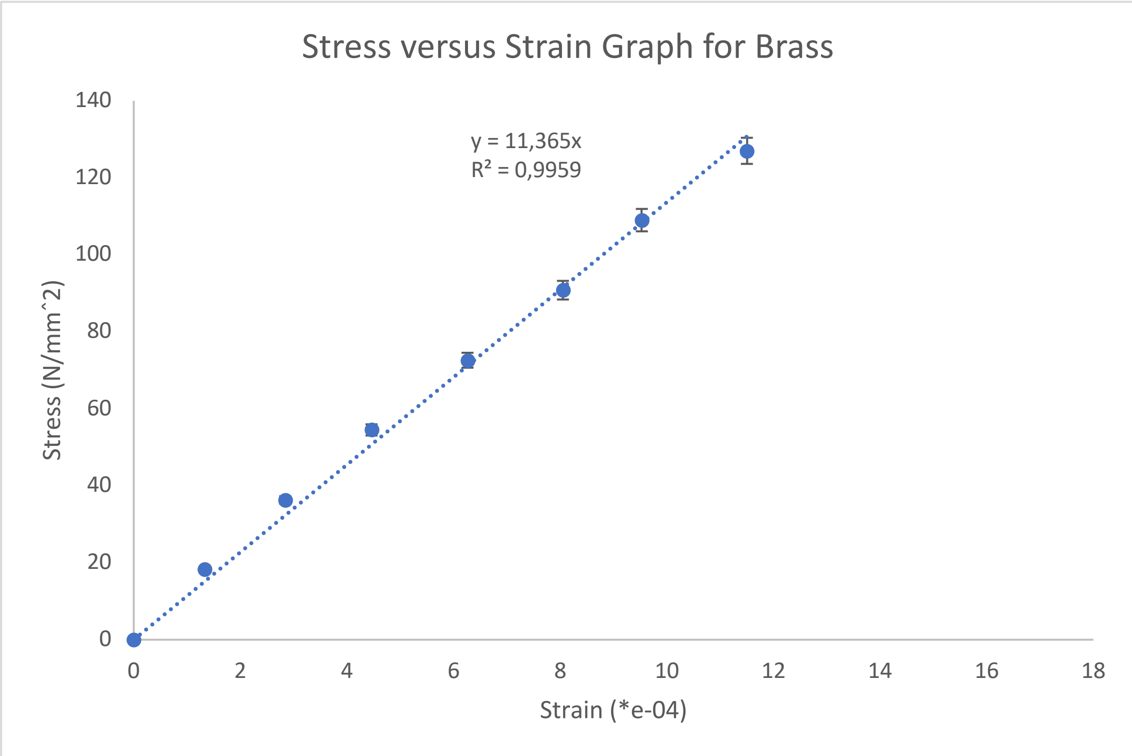
Table 2: Force versus Voltage measurements for Brass.

Force / kN (+/- 0.05kN)	$\overline{V}(g)$/ Volts Loading (+/- 0.001 V)	$\overline{V}(g)$/ Volts Unloading (+/- 0.001 V)
0	0	0.014
0.5	0.046	0.060
1.0	0.098	0.124
1.5	0.157	0.178
2.0	0.216	0.235
2.5	0.278	0.295
3.0	0.329	0.350
3.5	0.398	0.398

Table 3: Stress versus Strain measurements for Brass.

σ Stress/ $N * mm^{-2}$	ε Strain / (*e-04)
0	0
18.2	1.33
36.3	2.84
54.5	4.46
72.6	6.26
90.8	8.05
109.0	9.53
127.0	11.5

Graph 1: Stress versus Strain Graph for Brass.



From the Graph 1 the gradient is 11.365. Therefore, Young's Modulus of Copper is

$$11.365 * 10^{10} = 114GPa$$

The total uncertainty on Young's Modulus is 2.67%+0.54%=3.21%

• Copper

Diameter Average = $\frac{5.89+5.96+5.95}{3} = 5.93\ mm^2$

Cross-sectional Area = $27.65\ mm^2$

The uncertainty on the Cross-sectional Area is $2 * \frac{0.005}{5.93} * 100\% = 0.17\%$

The uncertainty on the Stress is therefore: $\frac{50}{2000} * 100\% + 0.17\% = 2.67\%$, where 2000 N is the average force applied to the specimen.

The uncertainty on the Strain is $\left(\frac{0.001}{1.315} + \frac{0.001}{0.188}\right) * 100\% = 0.61\%$, where 0.188 V is the average voltage across the specimen.

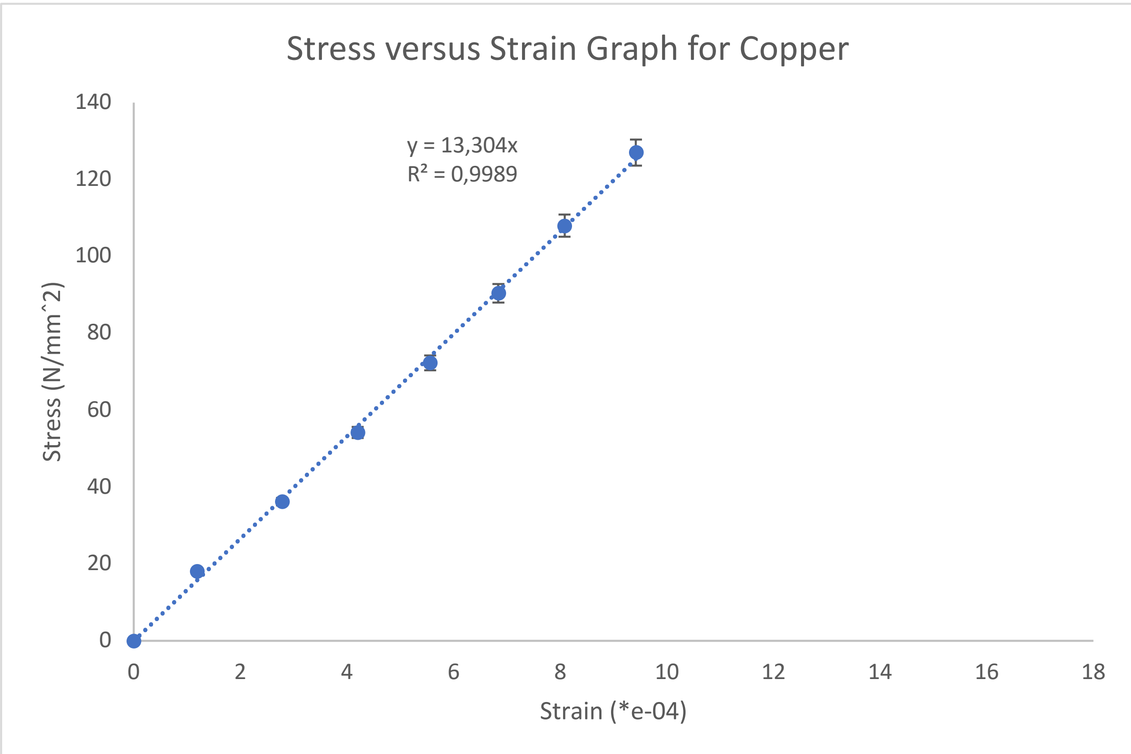
Table 4: Force versus Voltage measurements for Copper.

Force/ kN (+/- 0.05kN)	$\overline{V}(g)$/ Volts Loading (+/- 0.001 V)	$\overline{V}(g)$/ Volts Unloading (+/- 0.001 V)
0	0	0.024
0.5	0.041	0.084
1.0	0.096	0.130
1.5	0.145	0.168
2.0	0.192	0.216
2.5	0.236	0.259
3.0	0.279	0.300
3.5	0.325	0.325

Table 5: Stress versus Strain measurements for Copper.

Stress/ $N * mm^{-2}$ σ	Strain / (*e-04) ε
0	0
18.1	1.19
36.2	2.78
54.2	4.20
72.3	5.56
90.4	6.84
108.0	8.08
127.0	9.42

Graph 2: Stress versus Strain Graph for Copper.



From the Graph 2 the gradient is 13.304. Therefore, Young's Modulus of Copper is

$$13.304 * 10^{10} = 133GPa$$

The total uncertainty on Young's Modulus is $2.67\% + 0.61\% = 3.28\%$

• Aluminum

Diameter Average = $\frac{5.91+5.94+5.97}{3} = 5.94\ mm^2$

Cross-sectional Area = $27.71\ mm^2$

The uncertainty on the Cross-sectional Area is $2 * \frac{0.005}{5.94} * 100\% = 0.17\%$

The uncertainty on the Stress is therefore: $\frac{50}{2000} * 100\% + 0.17\% = 2.67\%$, where 2000 N is the average force applied to the specimen.

The uncertainty on the Strain is $\left(\frac{0.001}{1.315} + \frac{0.001}{0.363}\right) * 100\% = 0.35\%$, where 0.188 V is the average voltage across the specimen.

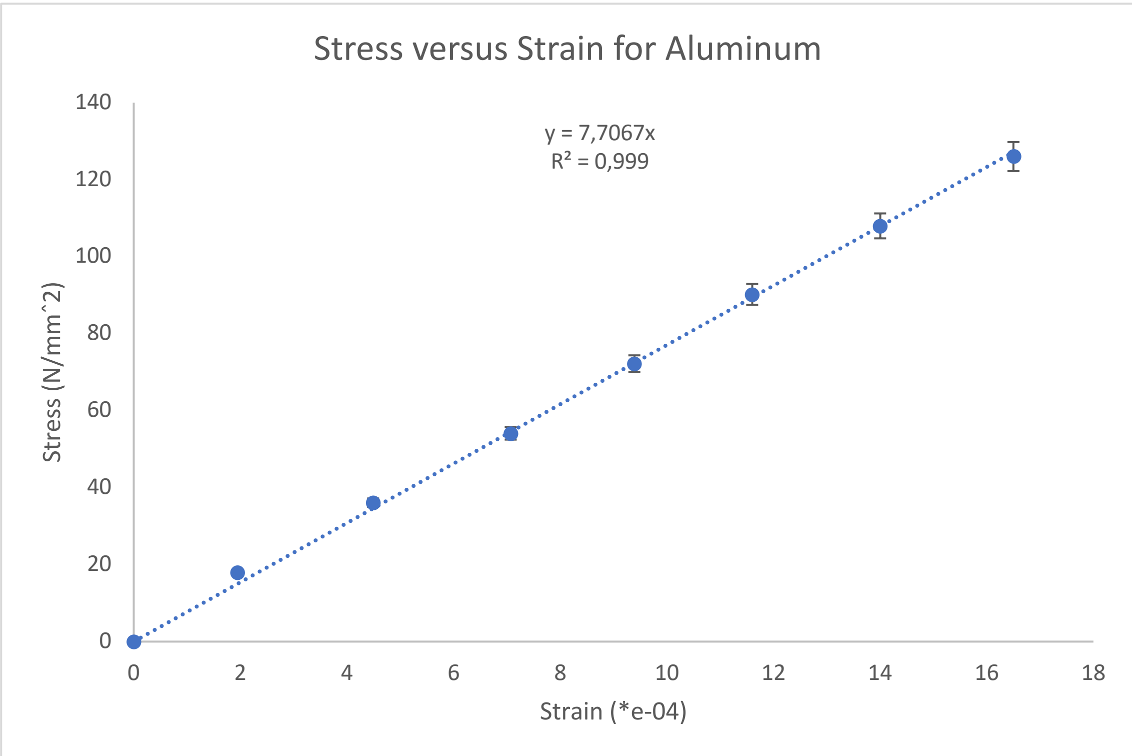
Table 6: Force versus Voltage measurements for Aluminum.

Force/ kN (+/- 0.05kN)	$\overline{V}(g)$/ Volts Loading (+/- 0.001 V)	$\overline{V}(g)$/ Volts Unloading (+/- 0.001 V)
0	0	0.029
0.5	0.067	0.106
1.0	0.155	0.194
1.5	0.244	0.274
2.0	0.324	0.358
2.5	0.401	0.433
3.0	0.482	0.510
3.5	0.569	0.569

Table 7: Stress versus Strain measurements for Aluminum.

σ Stress/ $N * mm^{-2}$	ε Strain / (*e-04)
0	0
18.0	1.94
36.1	4.49
54.1	7.07
72.2	9.39
90.2	11.6
108.0	14.0
126.0	16.5

Graph 3: Stress versus Strain Graph for Aluminum.



From the Graph 3 the gradient is 7.7067. Therefore, Young's Modulus of Aluminum is

$$7.7067 * 10^{10} = 77.1GPa$$

The total uncertainty on Young's Modulus is $2.67\% + 0.35\% = 3.02\%$

6. Discussion

The goal of the experiment has been met and the Young's Modulus of three specimens have been determined. From the experimental data, it can be deduced that the stiffest specimen is Copper and the most flexible is Aluminum. The results obtained from the experiment are relatively accurate but the theoretical Young's Modulus values of the metals listed above are still different to the experimental ones. The difference might have occurred due to various reasons, such as human error, thermal expansion of resistors and the surface of the metals which is not ideally smooth and polished. Moreover, the results would have tended to be more accurate if the experiment had been carried out several times but due to time limits this was not possible.

The room temperature varied as well during the experiment which also affected the obtained results. The extension in one direction caused the material to shrink in another direction, thus the diameter of the specimens was not constant throughout the experiment. This neglect affected the 'stress' values.

The differences between real and experimental values are listed below

• Young's Modulus of Brass is 102-125 GPa[iv]. 114 GPa lies within this range.

•Young's Modulus of Copper is 133 GPa[v]. The experimental value corresponds to the theoretical value.

•Young's Modulus of Aluminum is 69 GPa[vi]. The difference is $\frac{77.1-69}{69} * 100\% = 11.8\%$.

7. Conclusion

Overall, the experiment was done successfully and the ultimate targets were achieved. The average uncertainty of the experiment is 3.93%. If the certain factors, mentioned in the previous section, had been taken into account the uncertainty of the experiment would have lowered. Students became familiar with the use of electrical strain gauges. The experiment also showed the right pattern in stiffness of metals. The students were able to prove experimentally that Aluminum was the most flexible specimen (i.e. YM is the lowest) and Copper was the stiffest specimen (i.e. YM is the highest).

8. Appendix

Problem solution.

Problem Solution.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu \sigma_{yy}}{E}$$

$$\frac{4\overline{V_g}}{V_{in} \times S \times GAIN} = \frac{1}{E}\left(\frac{\Delta p R}{t} - \nu \frac{\Delta p R}{2t}\right)$$

$$\Delta p \left(\frac{R}{Et} - \frac{\nu R}{2tE}\right) = \frac{4\overline{V_g}}{V_{in} \times S \times GAIN}$$

$$\Delta p = \frac{4 \times \overline{V_g} \times 2Et}{(2R - \nu R) \times V_{in} \times S \times GAIN}$$

$$= \frac{4 \times (-1.11) \times 2 \times 70 \times 10^{9} \times 0.11 \times 10^{-3}}{(2 \times 65 \times 10^{-3} - 0.33 \times 65 \times 10^{-3}) \times 1 \times 1000 \times 2.1}$$

$$= -0.3 \text{ MPa.}$$

9. References

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