CSCI 3104 Spring 2014 Problem Set 1
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1. Determine whether the following claims are true or false.

a) $n + 3 = O(n^3)$

True

$$\lim_{nto\infty} \frac{n+3}{n^3} = 0 \tag{1}$$

b) $3^{2n} = O(3^n)$

False

$$\lim_{nto\infty} \frac{3^{2n}}{3^n} = \infty \tag{2}$$

c) $n^n = o(n!\,)$

False

$$\lim_{nto\infty} \frac{n^n}{n!} = \infty \tag{3}$$

d) $\frac{1}{3n} = o(1)$

True

$$\lim_{nto\infty} \frac{\frac{1}{3n}}{1} = 0 \tag{4}$$

e) $ln^3 n = \theta(lg^3 n)$

True

$$\lim_{nto\infty} \frac{ln^3 n}{lg^3 n} = lg^3(10) \tag{5}$$

2. Simplify the following expressions.

a) $\frac{d}{dt}(3t^4 + \frac{1}{3}t^3 - 7)$

$$12t^3 + 3t^2 \tag{6}$$

b) $\sum_{i=0}^{k} 2^i$

$$\begin{aligned}\sum_{i\ =0}^{k-1} 2^i &= \frac{1-2^i}{1-2} \\ &= 2^i - 1 \qquad i > 0\end{aligned}$$

$$\sum_{i\ =0}^{k} 2^i = 2^{i+1} - 1 \qquad i >= 0 \tag{7}$$

c) $\theta(\sum_{k=1}^{n} \frac{1}{k})$

I use the integral test to compare the equations

$$\begin{aligned}\sum_{k\ =1}^{n} \frac{1}{k} &> \int_1^n 1/n \\ &= ln(n)\Big|_1^n \\ &= ln(n)\end{aligned}$$

We can see this integral does not converge and implies the series in divergent

$$\theta(\infty) \tag{8}$$

3. Describe an O(n) time algorithm that takes input T and returns an array containing the same values in ascending order.

Merge sort is an O(n) sorting algorithm it insures that the correct result is always given by first spliting the list down to each element then comparing two elements at a time creating new sorted lists of length two it then continues to compare the elements of each list next to each other creating half the number of correctly sorted lists until only one correctly sorted list remains

4. Should Acme develop a faster algorithm or stick with the current algorithm?

a) Let $n = 41, f(n) = 1.99^n, g(n) = n^3$ and $t = 17$ days

If the integral of f(n) is less than the [integral of g(n)] + t(in microseconds) then Acme should stick with the current algorithm

$$\begin{aligned}\int_0^{41} 1.99^n &= \frac{1.99^n}{ln(1.99)}|_0^{41} \\ &= 1.99^{40} - 1 \\ &= 8.99752x10^{11}\end{aligned}$$

$$\begin{aligned}\int_0^{41} n^3 \mathrm{d}n &= \frac{n^4}{4}\Big|_0^{41} \\ &= \frac{41^4}{4} - 0 \\ &= 706440 + 1.4688x10^{12} \\ &\approx 1.4688x10^{12}\end{aligned}$$

$$8.99752x10^{11} < 1.4688x10^{12} \tag{9}$$

They should continue to use the current algorithm

b) Let $n = 10^6, f(n) = n^{2.00}, g(n) = n^{1.99}$ and $t = 2$ days

$$\begin{aligned}\int_0^{10^6} n^{2.00} &= \frac{n^3}{3}\Big|_0^{10^6} \\ &= 3.33x10^{17}\end{aligned}$$

$$\begin{aligned}\int_0^{10^6} n^{1.99} &= \frac{n^{2.99}}{2.99}\Big|_0^{10^6} \\ &= 2.912x10^{17} + 1.728x10^{11} \\ &\approx 2.912x10^{17}\end{aligned}$$

$$3.33x10^{17} > 2.912x10^{17} \tag{10}$$

They should develop the new algorithm

5. Using the mathematical definition of Big-O, answer the following.
   a) Is $2^{nk} = O(2^n)$ for $k > 1$?

$$\begin{aligned}\lim_{nto\infty} \frac{2^{nk}}{2^n} &= \lim_{nto\infty} 2^k \\ &= \infty\end{aligned} \tag{11}$$

Since the limit diverges to infinity we can conclude that $2^{nk}$ is not $O(2^n)$

b) Is $2^{n+k} = O(2^n)$, for $k = O(1)$?

$$\lim_{nto\infty} \frac{2^{n+k}}{2^n} = \lim_{nto\infty} 2^k \tag{12}$$

Since k has a constant run time we can conclude that $2^{n+k}$ is not $O(2^n)$ when the run time of k is not 0

6. Is an array that is in sorted order also a min-heap?
   Yes given that an algorithm exists to correctly index into the sorted array given the position in the min-heap.