HW 5
Due 28.04.2006.
You may leave your homeworks in the Homework Box # 1.

1. For $n=0,1,2,3,4,5$, fit a polynomial of degree n by least squares to the following data.

t	0	1	2	3	4	5
y	1	2,7	5,8	6,6	7,5	9,9

   Write your own MATLAB function. Do not use MATLAB's polyfit function. You may use polyfit function for comparison purposes. Plot the data and show the fitted polynomials for each n. Also, compute error for each n.

2. Consider the following system:

$$\begin{bmatrix} 0,16 & 0,10 \\ 0,17 & 0,11 \\ 2,02 & 1,29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cong \begin{bmatrix} 0,26 \\ 0,28 \\ 3,31 \end{bmatrix}$$

   a. Solve this system by using normal equations method. Calculate sensitivity and conditioning.
   b. Solve the same system by using either Householder's or Gram-Schmidt's method. Calculate sensitivity and conditioning.
   c. Now replace the **b** vector with this one:

$$\begin{bmatrix} 0,27 \\ 0,25 \\ 3,33 \end{bmatrix}$$

   and solve for **x** with any method you like. Compare your results with the previous steps. Can you explain the difference.

3. Show that if $v \neq 0$, then the Householder matrix, i.e.

$$H = I - 2\frac{vv^T}{v^T v}$$

   is orthogonal and symmetric.

4. Consider the following matrix:

$$\begin{bmatrix} 1 & 2 & -4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

   a. Apply Gershgorin's theorem to define the existence region of eigenvalues.
   b. Find eigenvalues of this system and show that they are in the region found in the previous part.
   c. Find the eigenvectors.
   d. Can you diagonalize this matrix? If yes, show. If no, explain why.