CMPE 597: Spec. Topics Graph Algorithms
Final, Fall 2006, due Jan. 22nd, 4:00pm
Good Luck !!!

"I, hereby, declare that the answers that I am submitting are my own answers that I have worked out alone by myself without consultation to any other person or the Internet resources during the exam period. If the aforementioned exam-taking criteria are violated, I understand that I will be subject to YÖK's academic dishonesty rules."

`Name: ________________________ Signature: _______________________`

Note: There are 6 problems. Each is worth 17 points.

Problem 1

Show that the following variation of 3SAT, called 1-in-3SAT, is NP-complete. The input is the same as the one for 3SAT. The problem is to determine whether there exists a satisfying assignment such that in every clause *exactly* one of the variables is true.

Problem 2

Prove that the clique problem remains NP-complete for regular graphs. Note that a regular graph is an undirected graph in which all vertices have the same degree.

Problem 3

Suppose you are given connected undirected graph $G(V, E)$ and its corresponding Laplacian matrix L. In the class, we have shown that, given a partition vector p consisting of 1's and -1's the following holds:

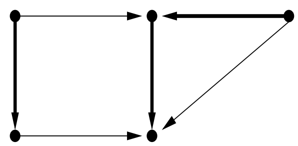
$$p^T Lp = 4 \cdot (no.\ \ of\ \ \ edges\ \ \ cut)$$

What two integers i_1 and i_2 can you use so that given a partition vector p consisting of i_1's and i_2's the following holds ?

$$p^T Lp = (no.\ \ of\ \ \ edges\ \ \ cut)$$

Problem 4

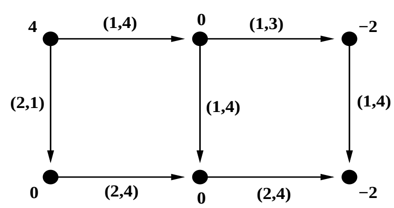
Suppose you are given a directed acyclic graph. Give an algorithm that finds a set of simple vertex disjoint path(s) such that the total path-length(s) (i.e. summation of path-length(s)) is maximum. As an example, for the graph in the following figure, a solution is given by bold paths with total path-length of 3.



Note that the solution may not be unique. (The above example graph also has another solution).

Problem 5

Find the minimum cost flow in the following network by (a) Cycle canceling and (b) Successive shortest path algorithms. Show the steps of your algorithms by drawing appropriate residual networks at each step.



Note that in the figure, the values on the nodes are the source/demand values and the values in parantheses are the (cost,capacity) values. Take all lower bounds to be 0 in the network.

Problem 6

Design an efficient algorithm to find a minimum-size vertex cover for a given tree.