Computer Science 373 – Analysis of Algorithms
Prof. Steven Skiena
Spring 2006

Homework 5 – Intractability
Due Thursday May 4, 2006

Each of the problems should be solved on a separate sheet of paper to facilitate grading. Limit the solution of each problem to one sheet of paper. Please don't wait until the last minute to look at the problems.

The numbered problems all come from *The Algorithm Design Manual*, by Skiena.

1. Give the 3-SAT formula that results from applying the reduction of SAT to 3-SAT for the formula:

$$(x + y + \overline{z} + w + u + \overline{v}) \cdot (\overline{x} + \overline{y} + z + \overline{w} + u + v) \cdot (x + \overline{y} + \overline{z} + w + u + \overline{v}) \cdot (x + \overline{y})$$

2. Draw the graph that results from the reduction of 3-SAT to vertex cover for the expression

$$(x + \overline{y} + z) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + y + z) \cdot (x + \overline{y} + \overline{x})$$

3. Exercises 6-1 and 6-3.

4. Exercises 6-4 and 6-5.

5. Assume that the Hamiltonian cycle problem for undirected graphs is NP-complete. Prove that the traveling salesman problem for undirected graphs is also NP-complete.

6. A graph G is *regular* if every vertex in G has the same degree. Prove that the clique problem remains NP-complete for regular graphs.

7. Given an undirected graph G with n vertices and m edges, and an integer k, give an $O(m+n)$ algorithm that finds the maximum induced subgraph H of G such that each vertex in H has degree $\geq k$, or prove that no such graph exists. An induced subgraph $F = (U, R)$ of a graph $G = (V, E)$ is a subset of U of the vertices V of G, and all edges R of G such that both vertices of each edge are in U.