17.1 Electric Current

Charges move in a direction perpendicular to a surface of area (cross-sectional area of a wire) A. The current is the rate at which charge flows through this surface. The direction of current is the same as the flow of positive charge. Negative charge flowing to the left is equivalent to an equal amount of positive charge flowing to the right. In a conductor, negative electrons moving actively to the left are equivalent to positive holes migrating to the right.

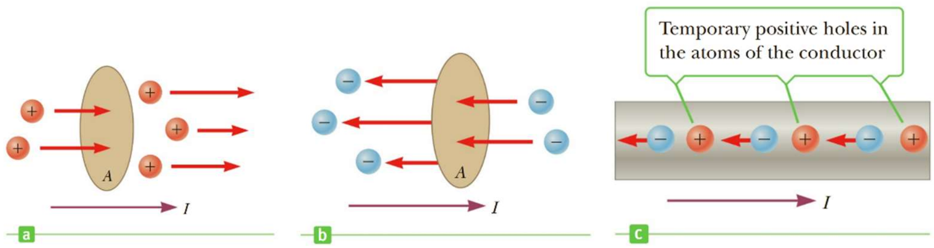


Figure1. The time rate of flow of charge through area A is the current I

Average Current: The average current I_{av} is equal to the amount of charge ΔQ divided by the time interval Δt:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

where the SI unit of current is coulomb per second or the ampere (A).

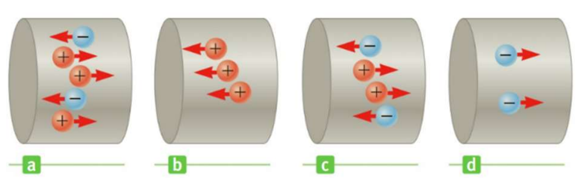
The total number of electrons passing a unit cross-sectional area is expressed by

$$N = \frac{I_{av}\Delta t}{q_e}$$

.The instantaneous current is the limit of the average current as the time interval goes to zero, which can be expressed by

$$\lim_{\Delta t \to 0} I_{av} = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

Quick Quiz 17.1 Consider positive and negative charges all moving horizontally with the same speed through the four regions in Figure 17.2. Rank the magnitudes of the currents in these four regions from lowest to highest. (I_a is the current in Figure 17.2a, I_b the current in Figure 17.2b, etc.) (a) I_d, I_a, I_c, I_b (b) I_a, I_c, I_b, I_d (c) I_c, I_a, I_d, I_b (d) I_d, I_b, I_c, I_a (e) I_a, I_b, I_c, I_d (f) none of these.



Solution- The correct answer is (d). Negative charges moving in one direction are equivalent to positive charges moving in the opposite direction. Thus, I_a, I_b, I_c, I_d are equivalent to the movement of 5,3,4 and 2 charges respectively, giving $I_d < I_b < I_c < I_a$.

Example 17.1 The amount of charge that passes through the filament of a certain lightbulb in 2.00 s is 1.67 C. Find (a) the average current in the light bulb and (b) the number of electrons that pass through the filament in 5.00 s. (c) If the current is supplied by a 12.0-V battery, what total energy is delivered to the light bulb filament during 2.00 s? What is the average power?

Solution-

(a) The average current, $I_{av} = \frac{\Delta Q}{\Delta t} = \frac{16.7}{2} = 0.835$ A

(b) The total number of electrons, $N = \frac{I_{av}\Delta t}{q_e} = \frac{0.835\times 5}{1.6\times 10^{-1}} = 2.61 \times 10^{-1}$ electrons

(c) The average power, $P_{av} = \frac{\Delta U}{\Delta t} = \frac{q\Delta V}{\Delta t} = \frac{1.67\times 12}{2} = 10$ W

Problems

Problem 5 If a current of 80.0 mA exists in a metal wire, (a) how many electrons flow past a given cross section of the wire in 10.0 min? (b) In what direction do the electrons travel with respect to the current?

17.2 A Microscopic View: Current and Drift Speed

Macroscopic currents: The motion of the microscopic charge carriers making up the current.

The mobile charge ΔQ in this element:

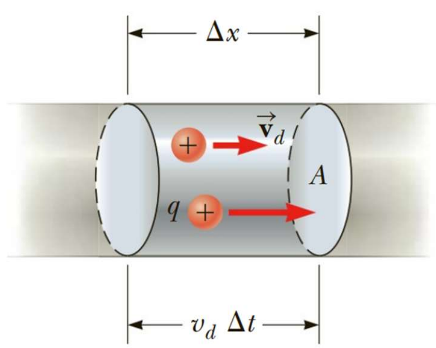
$$\Delta Q = number\ of\ carriers \times charge\ per\ carrier = (nA\Delta x)q$$

The current in a conductor is related to the motion of the charge carriers by

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = nqv_d A$$

Where, n is the number of mobile charge carriers per unit volume, q is the charge on each carrier, v_d the drift speed of the charges, A the cross-sectional area of the conductor. If the carriers move with a constant average speed called the drift speed v_d, the distance they move in the time interval Δt,

$$\Delta x = v_d \Delta t$$



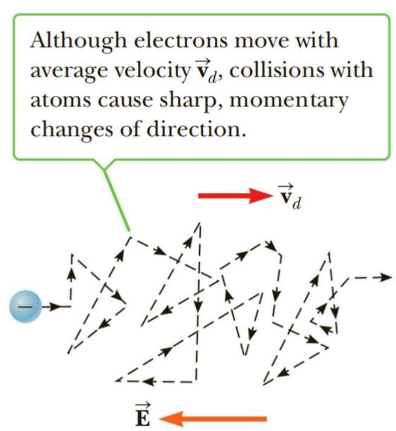


Figure 3. A section of a uniform conductor of cross-sectional area A. Figure 4. The zigzag motion of a charge carrierin a conductor. The drift velocity v_d is opposite the direction of the electric field.

Quick Quiz 17.2 Suppose a current-carrying wire has a cross-sectional area that gradually becomes smaller along the wire so that the wire has the shape of a very long, truncated cone. How does the drift speed vary along the wire? (a) It slows down as the cross section becomes smaller. (b) It speeds up as the cross section becomes smaller. (c) It doesn't change. (d) More information is needed.

Solution- The correct answer is (b). Under steady-state conditions, the current is the same in all parts of the wire. Thus, the drift velocity, $v_d = \frac{1}{nqA}$ is inversely proportional to the cross-sectional area A.

Problems

Problem 6 A copper wire has a circular cross section with a radius of 1.25 mm. (a) If the wire carries a current of 3.70 A, find the drift speed of electrons in the wire. (Take the density of mobile charge carriers in copper to be $n = 1.10 \times 10^{29}$ electrons/m3.) (b) For the same wire size and current, find the drift speed of electrons if the wire is made of aluminum with n = 2.11×10^{29} electrons/m3.

17.3 Current and Voltage Measurements in Circuits

The circuit shown in the following figure is a drawing of the actual circuit necessary for measuring the current. The most important quantities that characterize how the bulb works in different situations are the current I and the potential difference ΔV across the bulb. To measure the current in the bulb, we place an ammeter in line with the bulb. The voltmeter measures the potential difference between the two ends of the bulb's filament.

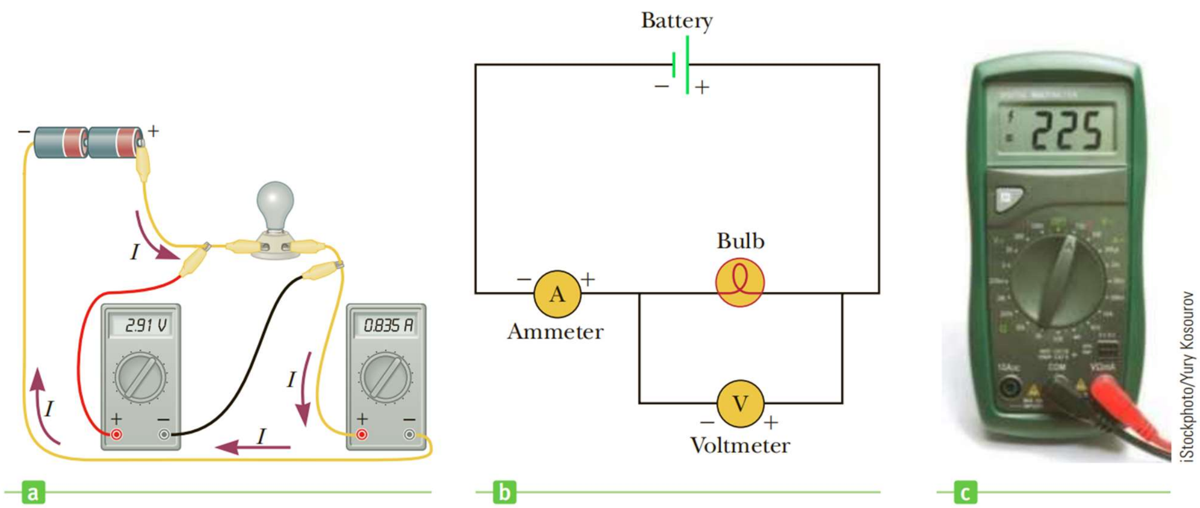
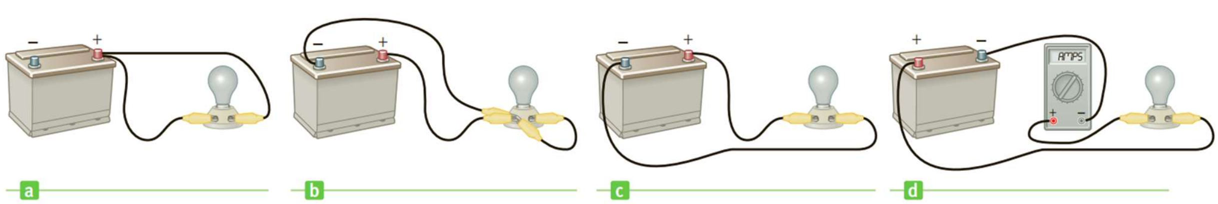


Figure 5. (a) A sketch of an actual circuit (b) A schematic diagram of the circuit shown in (a) (c) A

Quick Quiz 17.3 Look at the four "circuits" shown in the following figure and select those that will light the bulb.



Solution – The answer is (c) and (d). Neither circuit (a) nor circuit (b) applies a difference in potential across the bulb. Circuit (a) has both lead wires connected to the same battery terminal. Circuit (b) has a low resistance path (a "short") between the two battery terminals as well as between the bulb terminals.

17.4 Resistance, Resistivity and Ohm's Law

Resistance and Ohm's Law

The resistance R of a conductor is defined as the ratio of the potential difference ΔV across the conductor to the current I in it,

$$R = \frac{\Delta V}{I}$$

and the SI unit is volt per ampere or ohms (Ω)

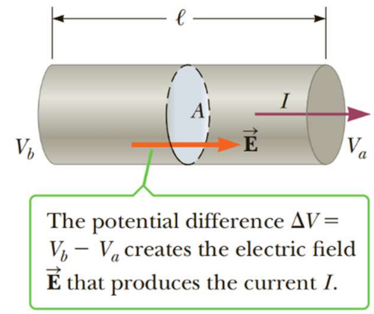


Figure 1. The potential difference ΔV is proportional to the current, I.

Ohm's law states that in many conductors, the applied voltage ΔV is directly proportional to the current I it causes. The proportionality constant is the resistance:

$$\Delta V = IR$$

Materials that obey Ohm's law are said to be ohmic. Materials having resistance that changes with voltage or current are nonohmic. Ohmic materials have linear current-voltage relationship over a large range of applied voltages. Nonohmic materials have a nonlinear current-voltage relationship. Two common nonohmic semiconductors are the diode and transistor.

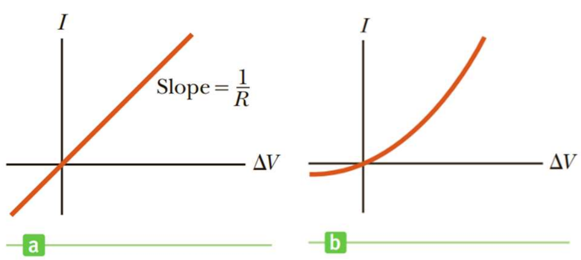


Figure: I-V curves for ohmic (a) and nonohmic (b) materials

Quick Quiz 17.4 In the figure (b), does the resistance of the diode increase or decrease as the positive voltage ΔV increases? (a) increases (b) decreases

Solution- The correct answer is (b). The slope of the line tangent to the curve at a point is the reciprocal of the resistance at that point. As ΔV increases, the slope (and hence 1/R) increases. Thus, the resistance decreases.

Quick Quiz 17.6 Suppose an electrical wire is replaced with one having every linear dimension doubled (i.e., the length and radius have twice their original values). The wire now has (a) more resistance than before (b) less resistance (c) the same resistance

Solution- The correct answer is (b). Since $R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$, doubling all linear dimensions increases the numerator of this expression by a factor of 2, but increases the denominator by a factor of 4. Thus, the result is that the resistance will be reduced to one-half of its original value.

Resistivity

The resistance R of an ohmic conductor increases with length l, because the electrons going through it must undergo more collisions in a longer conductor. A smaller cross-sectional area A also increases the resistance of a conductor, just as a smaller pipe slows the fluid moving through it. The resistance R is proportional to the conductor's length l, and inversely proportional to its cross-sectional area A,

$$R = \rho \frac{l}{A}$$

where ρ is resistivity with its SI unit being ohm meter (Ωm). The following table shows resistivities and temperature coefficients of resistivity for various materials (at 20°C).

Problems

Problem 15 Nichrome wire of cross-sectional radius 0.791 mm is to be used in winding a heating coil. If the coil must carry a current of 9.25 A when a voltage of 1.20×10^2 V is applied across its ends, find (a) the required resistance of the coil and (b) the length of wire you must use to wind the coil. ($\rho = 150 \times 10^{-8}\, \Omega\, m$)

Problem 18 A rectangular block of copper has sides of length 10 cm, 20 cm and 40 cm. If the block is connected to a 6 V source across two of its opposite faces, what are (a) the maximum current and (b) the minimum current the block can carry? ($\rho = 1.7 \times 10^{-8}\, \Omega\, m$)

17.5 Temperature Variation of Resistance

In an old-fashioned carbon filament incandescent lamp, the electrical resistance is typically 10 V, but changes with temperature.

As the temperature of the material increases, its constituent atoms vibrate with greater amplitudes. As a result, the electrons find it more difficult to get by those atoms. The resistance of a carbon filament incandescent light bulb increases with temperature.

For most metals and other conducting materials, resistivity increases with increasing temperature. Over a limited temperature range, the resistivity of a conductor varies with temperature

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where α is the temperature coefficient of resistivity and ρ_0 the resistivity at reference temperature (T_0). The SI unit of α is $°C^{-1}$.

Because the resistance of a conductor with a uniform cross section is proportional to the resistivity, $R = \rho \frac{l}{A}$, the resistance of a conductor varies with temperature, which can be expressed as

$$R = R_0[1 + \alpha(T - T_0)]$$

Quick Quiz 17.9 Two resistors, A and B, are connected in a series circuit with a battery. The resistance of A is twice that of B. Which resistor dissipates more power? (a) resistor A does (b) resistor B does (c) more information is needed

Solution – The correct answer is (a). The power dissipated by a resistor may be expressed as $P = I^2R$, where I is the current carried by the resistor of resistance R. Since resistors connected in series carry the same current, the resistor having the largest resistance will dissipate the most power.

Example 17.4 A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made of platinum and has a resistance of 50.0 Ω at 20.0°C. (a) When the device is immersed in a vessel containing indium at its melting point, its resistance increases to 76.8 Ω. From this information, find the melting point of indium. (b) The indium is heated further until it reaches a temperature of 235°C. Estimate the ratio of the new current in the platinum to the current I_{mp} at the melting point, assuming the coefficient of resistivity for platinum doesn't change significantly with temperature. ($\alpha = 3.92 \times 10^{-3} {}^\circ C^{-1}$)

Solution-

For platinum, $\alpha = 3.92 \times 10^{-3} {}^\circ C^{-1}, R_0 = 50\ \Omega, T_0 = 20\ ^\circ\mathrm{C}$

(a) $R = 76.8\ ^\circ\mathrm{C}, T = ?\ (The\ melting\ point\ of\ indium)$

$$R = R_0[1 + \alpha(T - T_0)]$$

$$T - T_0 = \frac{R - R_0}{R_0\alpha} = \frac{76.8 - 50}{50 \times 3.92 \times 10^{-3}} = 137\ ^\circ\mathrm{C}$$

$$T = 137 + T_0 = 137 + 50 = 157\ ^\circ\mathrm{C}$$

(b) $\frac{I}{I_{mp}} = ?$

$$R = R_0 = 76.8\ ^\circ\mathrm{C}, T_0 = T_{mp} = 157\ ^\circ\mathrm{C}, T = 235\ ^\circ\mathrm{C}$$

$$R = R_0[1 + \alpha(T - T_0)]$$

At the melting point, $R = R_{mp}\left[1 + \alpha\left(T - T_{mp}\right)\right]$

By Ohm's law, $\Delta V = IR, R = \frac{\Delta V}{I}$

$$\frac{\Delta V}{I} = \frac{\Delta V}{I_{mp}}\left[1 + \alpha\left(T - T_{mp}\right)\right]$$

$$\frac{I}{I_{mp}} = \frac{1}{1 + \alpha\left(T - T_{mp}\right)} = \frac{1}{1 + 3.92 \times 10^{-3} \times (235 - 157)} = 0.766$$

Problems

Problem 25 At 20.0°C, the carbon resistor in an electric circuit connected to a 5.0 V battery has a resistance of $2.0 \times 10^2\ \Omega$. What is the current in the circuit when the temperature of the carbon rises to 80.0°C? ($\alpha = -0.5 \times 10^{-3} {}^\circ\mathrm{C}^{-1}$)

Problem 34 A length of aluminium wire has a resistance of 30.0 Ω at 20.0°C. When the wire is warmed in an oven and reaches thermal equilibrium, the resistance of the wire increases to 46.2 Ω. (a) Neglecting thermal expansion, find the temperature of the oven. (b) Qualitatively, how would thermal expansion be expected to affect the answer? ($\alpha = 3.9 \times 10^{-3} {}^\circ C^{-1}$)

17.6 Electrical Energy and Power

As the positive charge ΔQ moves from A to B through the battery, electrical potential energy of the system increases by the amount $\Delta Q \Delta V$ and the chemical potential energy in the battery decreases by the same amount

As the charge moves from C to D through the resistor, it loses this electrical potential energy during collisions with atoms in the resistor. In the process, the energy is transformed into internal energy in the resistor.

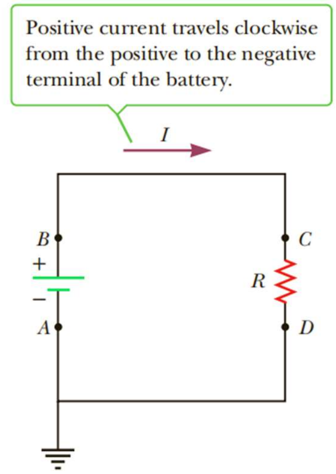


Figure 17.11 A circuit consisting of a battery and a resistance R. Point A is grounded.

Power: The rate at which energy is delivered to the resistor.

$$P = I\Delta V$$

$$P = I^2 R = \frac{\Delta V^2}{R}$$

The SI unit of power is watt (W). A kilowatt - hour is the amount of energy converted or consumed in one hour by a device supplied with power at the rate of 1 kW.

$$1kWh = 10^3 W \times 3600\ s = 3.6 \times 10^6\ J$$

Example 17.5 A circuit provides a maximum current of 20.0 A at an operating voltage of $1.20 \times 10^2\ V$. (a) How many 75.0 W bulbs can operate with this voltage source? (b) At \$0.120 per kilowatt - hour, how much does it cost to operate these bulbs for 8.00 h?

Solution-

(a) The total power, $P_{total} = I\Delta V = 20 \times 1.2 \times 10^2 = 2.4 \times 10^3\ W$
Number of bulbs, $\frac{P_{total}}{P_{bulb}} = \frac{2.4 \times 10^3}{75} = 32$

(b) Energy = $Pt = 2.4 \times 10^3\ W \times 8\ h = 19.2\ kWh$
Cost = $19.2\ kWh \times \$0.12 = \$2.3$

Problems

Problem 38 If electrical energy costs \$0.12 per kilowatt - hour, how much does it cost to (a) burn a 100 - W lightbulb for 24 h? (b) Operate an electric oven for 5.0 h if it carries a current of 20.0 A at 220 V?