ELECTRICITY AND MAGNETISM

Topic 17

Current and Resistance (Part II)

Lesson Outlines

17.4 Resistance, Resistivity and Ohm's Law

17.5 Temperature Variation of Resistance

17.6 Electrical Energy and Power

Objectives

- To understand the relationship between *resistance and resistivity* and to calculate *Ohm's Law* by using current, voltage and resistance
- To get the concept about correlation between resistivity and resistance due to temperature variation of the material
- To know the process of energy transfer in a simple circuit.

17.4 Resistance, Resistivity and Ohm's Law

17.4.1 Resistance and Ohm's Law

The resistance R of a conductor is defined as the *ratio of the potential difference ΔV across the conductor to the current I* in it

$$R \equiv \frac{\Delta V}{I}$$

SI units of volts per ampere, called **ohms** (Ω)

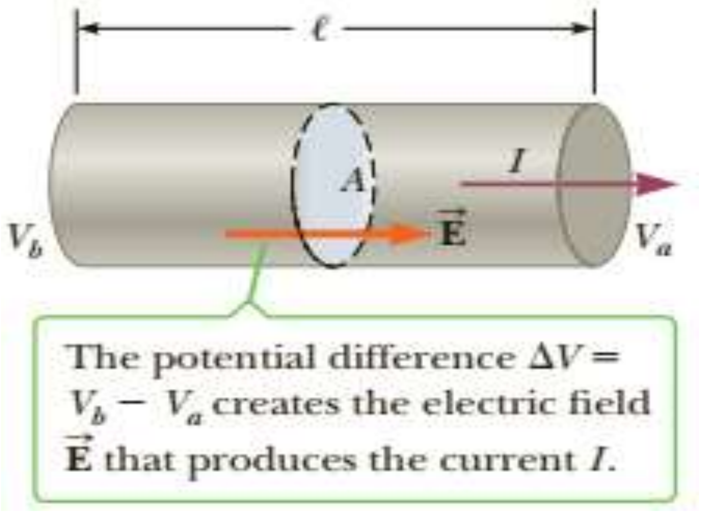


Figure 1. The potential difference ΔV is proportional to the current, I.

Ohm's Law

In many conductors, the applied voltage ΔV is directly proportional to the current I it causes. The proportionality constant is the resistance:

$$\Delta V = IR$$

Ohmic

- Materials that obey Ohm's law
- Ohmic materials have a *linear current-voltage* relationship over a large range of applied voltages
- Ohmic conductors: metals such as aluminium, copper and etc.

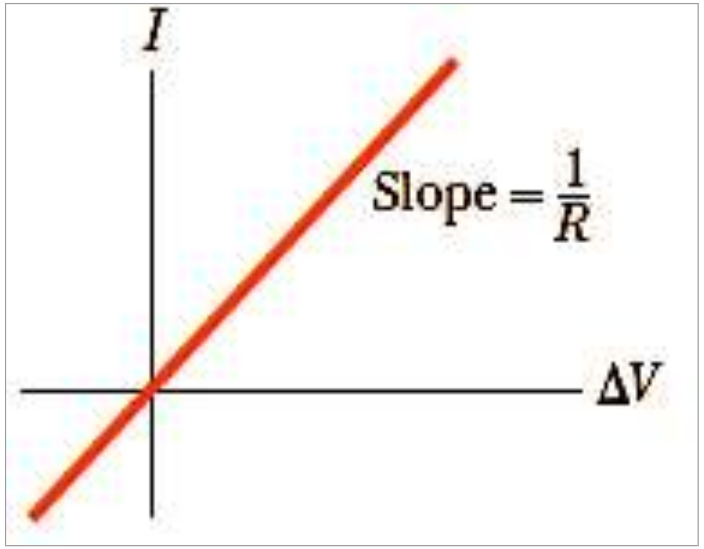


Figure 2. I-V curve for ohmic material

Nonohmic

- Materials *changes with voltage or current* are called nonohmic: *nonlinear current–voltage relationship.*
- Nonohmic conductors: diodes, semiconductors, transistors, etc.

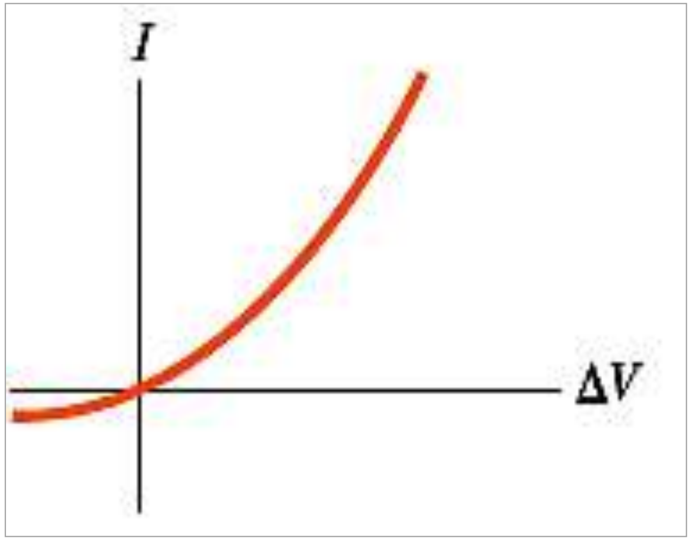


Figure 3. I-V curve for non-ohmic material

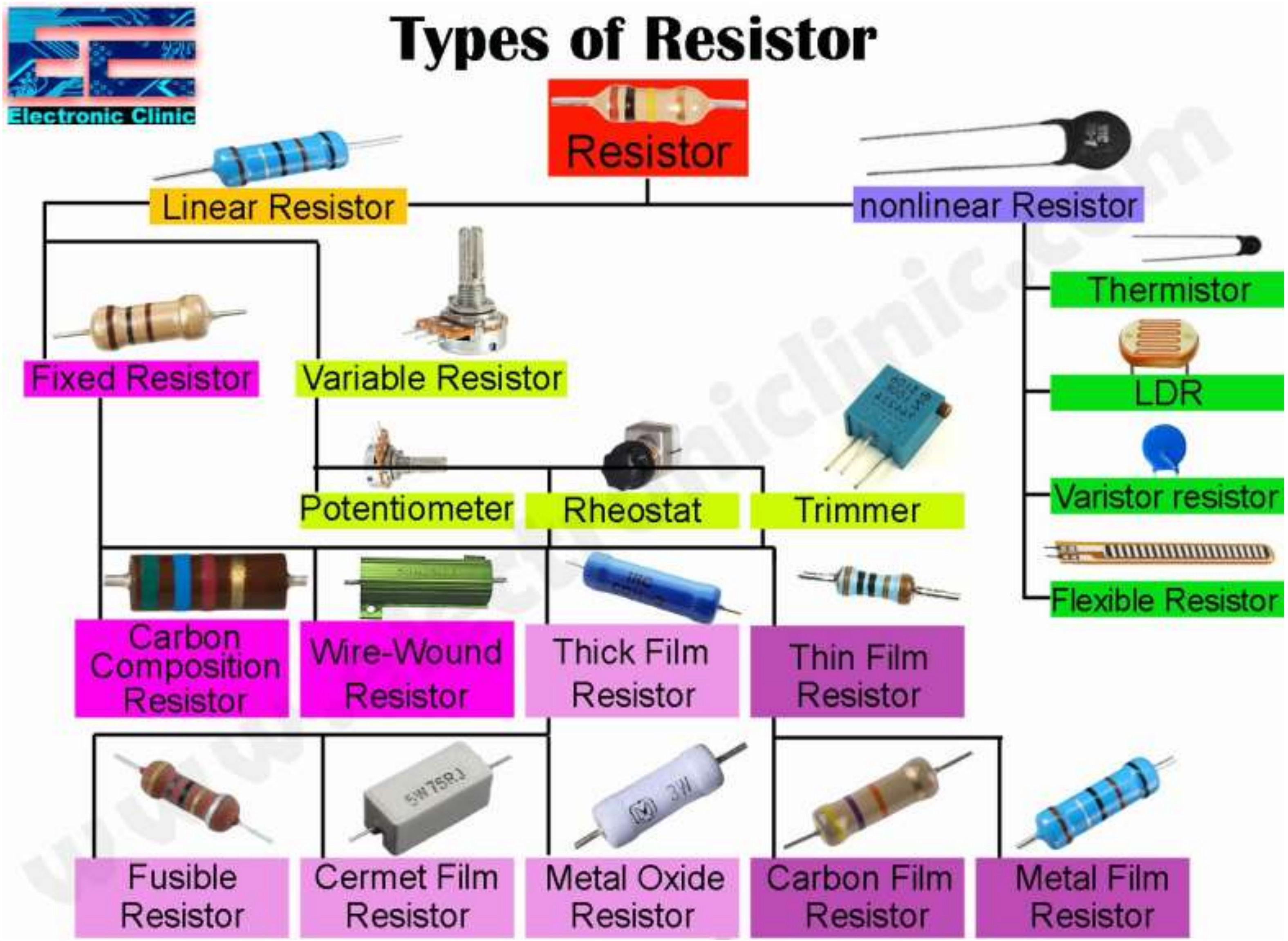


Figure 4. https://www.electronicclinic.com/

17.4.2 Resistivity

- The resistance *R of an ohmic conductor increases with length l ,* because the electrons going through it must undergo more collisions in a longer conductor.
- A smaller *cross-sectional area A also increases the resistance of a conductor*, just as a smaller pipe slows the fluid moving through it.

Resistivity: The resistance R *is proportional to the conductor's length* l *, and inversely proportional to its cross-sectional area A,*

$$R = \frac{\rho l}{A}$$

where, ρ = proportionality constant = resistivity

The SI unit of resistivity is the ohm meter (Ω m)

Table 17.1 Resistivities and temperature coefficients of resistivity for various Materials (at 20°C)

Material	Resistivity ($\Omega \cdot m$)	Temperature Coefficient of Resistivity [$(°C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10.0×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[a]	150×10^{-8}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10}–10^{14}	
Hard rubber	$\approx 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] A nickel-chromium alloy commonly used in heating elements.

Problem 17.15 (Pg 586): Nichrome wire of cross-sectional radius 0.791 mm is to be used in winding a heating coil. If the coil must carry a current of 9.25 A when a voltage of 1.20 x 10^2 V is applied across its ends, find (a) the required resistance of the coil and (b) the length of wire you must use to wind the coil. ($\rho = 150 \times 10^{-8}\ \Omega$ m)

Solution:

(a) The required resistance of the coil, $R = \frac{\Delta V}{I} = \frac{1.20 \times 10^2}{9.25} = 13\ \Omega$

(b) The length of wire, $R = \rho l / A$

$$l = \frac{RA}{\rho} = \frac{\mathrm{R} \times \pi r^2}{\rho}$$

$$l = \frac{RA}{\rho} = \frac{13 \times \pi (0.791 \times 10^{-3})^2}{150 \times 10^{-8}} = 17\ \text{m}$$

Problem 17.18 (Pg 586): A rectangular block of copper has sides of length 10 cm, 20 cm and 40 cm. If the block is connected to a 6 V source across two of its opposite faces, what are (a) the maximum current and (b) the minimum current the block can carry? ($\rho = 1.7 \times 10^{-8}\ \Omega\ \mathrm{m}$)

Solution:

With different orientations of the block, the ratios L/A are :

$$\left(\frac{L}{A}\right)_1 = \left(\frac{10\ cm}{20\ cm \times 40cm}\right) = \frac{1}{80\ cm} = \frac{1}{0.8\ m}$$

$$\left(\frac{L}{A}\right)_2 = \left(\frac{20\ cm}{10\ cm \times 40cm}\right) = \frac{1}{20\ cm} = \frac{1}{0.2\ m}$$

$$\left(\frac{L}{A}\right)_3 = \left(\frac{40\ cm}{10\ cm \times 20cm}\right) = \frac{1}{5\ cm} = \frac{1}{0.05\ m}$$

$$I_{max} = \left(\frac{\Delta V}{R_{min}}\right) = \left(\frac{\Delta V}{\rho\left(\frac{l}{A}\right)_{min}}\right)$$

$$= \left(\frac{60 \times 0.8}{1.7 \times 10^{-8}}\right)$$

$$= 2.8 \times 10^{8}\ \text{A}$$

$$I_{min} = \left(\frac{\Delta V}{R_{max}}\right) = \left(\frac{\Delta V}{\rho\left(\frac{l}{A}\right)_{max}}\right)$$

$$= \left(\frac{60 \times 0.05}{1.7 \times 10^{-8}}\right)$$

$$= 1.8 \times 10^{7}\ \text{A}$$

17.5 Temperature Variation of Resistance

Figure 5. In an old-fashioned carbon filament incandescent lamp, the electrical resistance is typically 10 V, but changes with temperature.

- As the temperature of the material increases, its constituent atoms vibrate with greater amplitudes. As a result, the electrons find it more difficult to get by those atoms. The resistance of a carbon filament incandescent light bulb increases with temperature.

- For most metals and other conducting materials, *resistivity* ***increases*** *with increasing temperature.*
- Over a limited temperature range, the ***resistivity*** *of a conductor* ***varies*** *with* ***temperature***

$$\boldsymbol{\rho = \rho_0[1 + \alpha(T - T_0)]}$$

Where, $\alpha = temperature\ coefficient\ of\ resistivity$

$\rho_0 =$ *resistivity* at reference temperature $\boldsymbol{T_0}$ **(20°C)**

- ➢ SI units of α are $\boldsymbol{°C^{-1}}$

- Because the resistance of a conductor with a uniform cross section is proportional to the resistivity, $R = \frac{\rho l}{A}$
- The **resistance** of a conductor **varies** with **temperature,**

$$\boldsymbol{R = R_0[1 + \alpha(T - T_0)]}$$

Example 17.4(Pg 576):A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made of platinum and has a resistance of 50.0 Ω at 20.0°C. **(a)** When the device is immersed in a vessel containing indium at its melting point, its resistance increases to 76.8 Ω. From this information, find the ***melting point*** of indium. **(b)** The indium is heated further until it reaches a temperature of 235°C. Estimate the ***ratio of the new current*** in the platinum to the current I_{mp} at the melting point, assuming the coefficient of resistivity for platinum doesn't change significantly with temperature. ($\alpha = 3.92 \times 10^{-3} \circ C^{-1}$)

Solution:

For platinum, $\alpha = 3.92 \times 10^{-3} {}^{\circ}C^{-1}$

$$R_o = 50\ \Omega,\ T_o = 20.0\ ^{\circ}\mathrm{C}$$

(a) $R = 76.8\ \Omega$, T = ? (The melting point of indium)

$$R = R_o\ [1 + \alpha\ (T - T_o)]$$

$$R = R_o + R_o\ \alpha\ (T - T_o)$$

$$T - T_o = \frac{R - R_o}{R_o\ \alpha} = \frac{(76.8 - 50)\Omega}{50\Omega \times (3.92 \times 10^{-3} {}^{\circ}C^{-1})} = 137\ ^{\circ}C$$

$$\boldsymbol{T} = T_o + 137\ ^{\circ}C \boldsymbol{= 157\ ^{\circ}C}$$

(b) $\frac{I}{I_{mp}}$ = ? (when the temperature rises from 156.7 °C to 235°C)

$R_o = R_{mp} = 76.8\ \Omega$, $T_o = T_{mp} = 157$ °C, $T = 235$°C

$$R = R_o\,[1 + \alpha\,(T - T_o)]$$

At the melting point, $R = R_{mp}\,[1 + \alpha\,(T - T_{mp})]$

Ohm's law, $\Delta V = IR$

$$R = \frac{\Delta V}{I}$$

$$\frac{\Delta V}{I} = \frac{\Delta V}{I_{mp}}\,[1 + \alpha\,(T - T_{mp})]$$

$$\boldsymbol{\frac{I}{I_{mp}}} = \frac{1}{1+\alpha\,(T - T_{mp})} = \frac{1}{1+3.92\times10^{-3}{}^{\circ}C^{-1}(235-157)^{\circ}\mathrm{C}} = 0.766$$

Problem.25 (Pg 587): At 20.0°C, the carbon resistor in an electric circuit connected to a 5.0 V battery has a resistance of 2.0 × 10^2 Ω. What is ***the current*** in the circuit when the temperature of the carbon rises to 80.0°C? $(\alpha = -0.5 \times 10^{-3}{}^{\circ}\mathrm{C}^{-1})$

Solution:

$$\alpha = -0.5 \times 10^{-3}{}^{\circ}\mathrm{C}^{-1}, \Delta V = 5\ \mathrm{V}, R_0 = 2 \times\ 10^2\Omega, T_0 = 20\ {}^{\circ}\mathrm{C}, \mathrm{T} = 80\ {}^{\circ}\mathrm{C}, \mathrm{I} = ?$$

$$I = \frac{\Delta V}{R} = \frac{\Delta \mathrm{V}}{R_0[1+\alpha(T-T_0)]}$$

$$= \frac{5\ \mathrm{V}}{(2\times 10^2\Omega)\left[1+\left(-0.5\times10^{-3}\ {}^{\circ}\mathrm{C}^{-1}\right)(80-20){}^{\circ}\mathrm{C}\right]}$$

$$I = 2.6 \times 10^{-2} A = \mathbf{26\ mA}$$

Problem.34 (Pg 587): A length of aluminium wire has a resistance of 30.0 Ω at 20.0°C. When the wire is warmed in an oven and reaches thermal equilibrium, the resistance of the wire increases to 46.2 Ω. (a) Neglecting thermal expansion, find the temperature of the oven. (b) Qualitatively, how would thermal expansion be expected to affect the answer? $(\alpha = 3.9 \times 10^{-3}°\mathrm{C})$

Solution:

(a) The aluminium wire $\alpha = 3.9 \times 10^{-3}°\mathrm{C}, R_0 = 30\ \Omega$ at $T_0 = 20°\mathrm{C}$.

If $R = 46.2\Omega$ at temperature $T = ?$

$$\mathrm{R} = \mathrm{R}_0[1 + \alpha(\mathrm{T} - \mathrm{T}_{\mathrm{mp}})]$$

$$T = T_0 + \frac{\left({}^{R}/_{R_0}\right) - 1}{\alpha} = 20 + \frac{\left({}^{46.2}/_{30}\right) - 1}{3.9 \times 10^{-3}}$$

$$T = 1.6 \times 10^{2}\,°C$$

(b) The expansion of the cross-sectional area contributes slightly more than the expansion of the length of the wire, so the answer would be slightly reduced.

17.6 Electrical Energy and Power

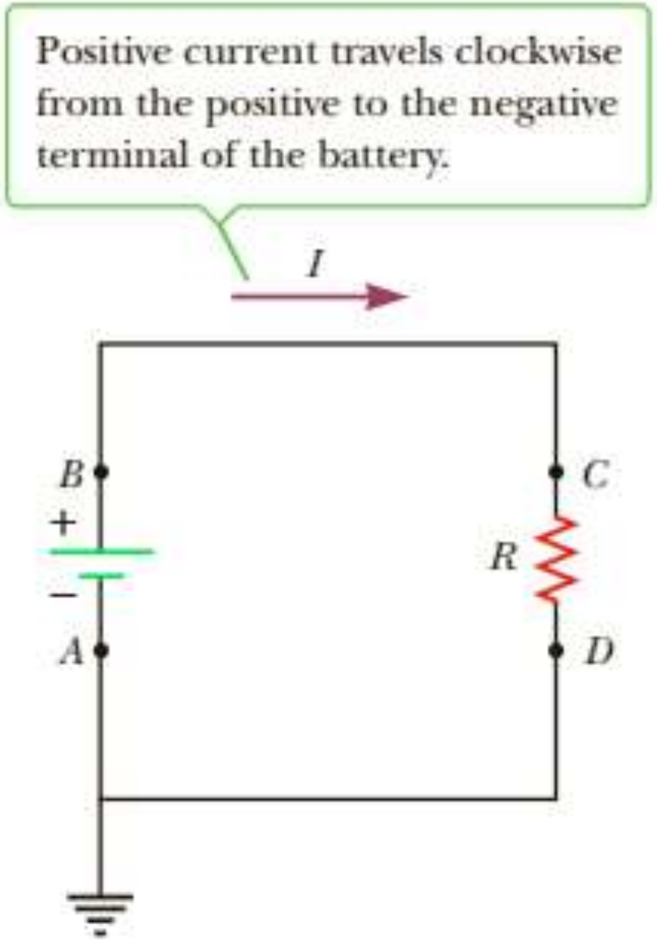


Figure 6. A circuit consisting of a battery and a resistance ***R***. Point ***A*** is grounded.

As the positive charge *ΔQ moves from A to B through the battery,*

- ***electrical potential energy of the system increases*** by the amount $\boldsymbol{\Delta Q \Delta V}$
- ***chemical potential energy in the battery decreases*** by the same amount

As the *charge moves from C to D through the resistor,*

- It ***loses this electrical potential energy*** during collisions with atoms in the resistor.
- In the process, the ***energy is transformed into internal energy in the resistor.***

Power: The rate at which energy is delivered to the resistor.

$$\lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} \Delta V = I \Delta V \quad \Rightarrow \quad \boldsymbol{P = I\Delta V}$$

$$P = I \Delta V \quad \Rightarrow \quad \boldsymbol{P = I^2 R = \frac{\Delta V^2}{R}}$$

SI unit of power is the watt (W).

The **unit of energy** - the **kilowatt - hour**

A **kilowatt - hour** is the amount of energy converted or consumed in one hour by a device supplied with power at the rate of 1 kW.

$$1\ \text{kWh} = \left(10^3\ \text{W}\right)\left(3600\ \text{s}\right) = 3.60 \times 10^6\ \text{J}$$

Example 17.5 (Pg 579): A circuit provides a maximum current of 20.0 A at an operating voltage of 1.20 x 10^2 V. (a) How many 75.0 W bulbs can operate with this voltage source? (b) At $0.120 per kilowatt - hour, how much does it cost to operate these bulbs for 8.00 h?

Solution:

(a) The number of bulbs that can be lighted

The total power $P_{total} = I\Delta V \ = (20 \ \times 1.2 \ \times 10^2) = 2.4 \ \times 10^3 \ W$

Number of bulbs $= \frac{P_{total}}{P_{bulb}} = \frac{2.4 \times 10^3}{75} = 32$

(b) The cost of electricity for 8 h, the energy in kWh is

$$energy = Pt = (2.4 \times 10^3 W)(8h) = 19.2 \text{ kWh}$$

$$cost = 19.2 \text{ kWh} \times \$\, 0.12 = \$\, 2.30$$

Problem.38 (Pg 588): If electrical energy costs \$0.12 per kilowatt - hour, how much does it cost to (a) burn a 100 - W lightbulb for 24 h? (b) Operate an electric oven for 5.0 h if it carries a current of 20.0 A at 220 V?

Solution:

(a) The energy used by 100W bulb in 24 h is

$$energy = Pt = \frac{100 \times 24}{1000} = 2.4 \text{ kWh}$$

Cost of this energy, $cost = 2.4\text{kWh} \times \$\, 0.12 = \$\, 0.29$

(b) The energy used by the oven in 5 h is

$$energy = Pt = (I\Delta V)t \; = \frac{20 \times 220 \; \times 5}{1000} = 22 \text{ kWh}$$

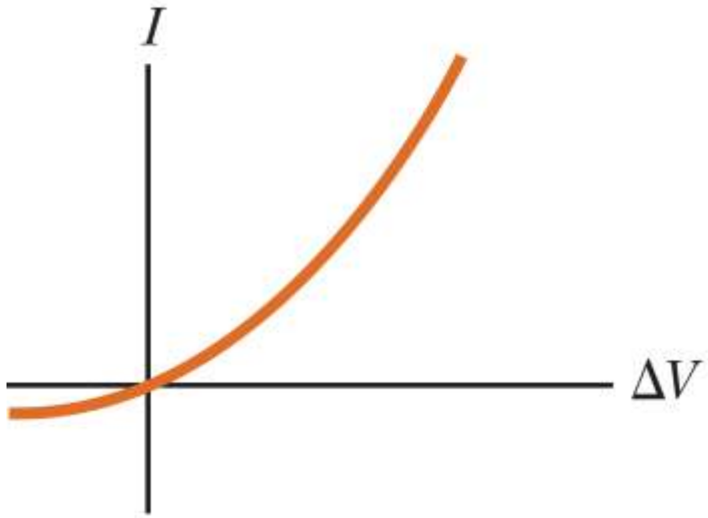
Cost of this energy, $cost = 22 \text{ kWh} \times \$\, 0.12 = \$\, 2.30$

Quiz 17.4

In the figure, does the resistance of the diode increase or decrease as the positive voltage ΔV increases?

(a) increases

(b) decreases



Answer: (b)

The slope of the line tangent to the curve at a point is the reciprocal of the resistance at that point. As ΔV increases, the slope (and hence $1/R$) increases. Thus, the resistance decreases.

Quiz 17.6

Suppose an electrical wire is replaced with one having every linear dimension doubled (i.e., the length and radius have twice their original values). The wire now has

a. More resistance than before
b. Less resistance
c. The same resistance.

Answer: b

$R = \rho L/A = \rho L/\pi r^2$. Doubling all linear dimensions increases the numerator of this expression by a factor of 2, but increases the denominator by a factor of 4. Thus, the result is that the resistance will be reduced to one-half of its original value.

Quiz 17.9

Two resistors, A and B, are connected in a series circuit with a battery. The resistance of A is twice that of B. Which resistor dissipates more power?

a. Resistor A does.

b. Resistor B does.

c. More information is needed.

Answer: a

The power dissipated by a resistor may be expressed as $P = I^2R$, where I is the current carried by the resistor of resistance R. Since resistors connected in series carry the same current, the resistor having the largest resistance will dissipate the most power.

Quiz 17.10

The diameter of wire A is greater than the diameter of wire B, but their lengths and resistivities are identical. For a given voltage difference across the ends, what is the relationship between P_A and P_B, the dissipated power for wires A and B, respectively?

a. $P_A = P_B$

b. $P_A < P_B$

c. $P_A > P_B$

Answer: ***c***

Increasing the diameter of a wire increases the cross-sectional area. Thus, the cross-sectional area of A is greater than that of B, and from $R = \rho L/A$, we see that $R_A < R_B$. Since $P = (\Delta V)^2/R$, the wire having the smallest resistance dissipates the most power for a given potential difference.

Summary

17.4 Resistance, Resistivity, and Ohm's Law

- **Resistance**

$$\boxed{R \equiv \frac{\Delta V}{I}}$$

SI units: ohms (Ω) (1 Ω =1 V/A)

- **Ohm's law**

$$\boxed{\Delta V = IR}$$

.

- **Resistivity (ρ)**

$$\boxed{R = \rho \frac{l}{A}}$$

SI unit of resistivity : ohm-meter (Ω m)

17.5 Temperature Variation of Resistance

- Resistivity of a conductor varies with temperature

$$\boldsymbol{\rho = \rho_0[1 + \alpha(T - T_0)]}$$

- Resistance of a conductor varies with temperature

$$R = R_0[1 + \alpha(T - T_0)]$$

17.6 Electrical Energy and Power

- Power P

$$\boldsymbol{P = I\Delta V}$$

- Apply Ohm's Law,

$$\boldsymbol{P = I\Delta V = I^2R = \frac{\Delta V^2}{R}}$$

SI unit of power -watt (W)

$$\boldsymbol{1\ kWh = 3.60 \times 10^6\ J}$$

THANK YOU!