15.2 Coulomb's Law

An electric force has the following properties:

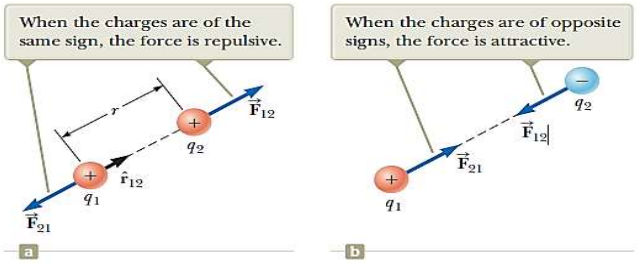
1. It is directed along a line joining the two particles and is inversely proportional to the square of the separation distance r, between them.
2. It is proportional to the product of the magnitudes of the charges, $|q_1|$ and $|q_2|$ of the two particles.
3. It is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

Coulomb proposed the following mathematical form for the electric force between two charges –

The magnitude of the electric force, **F,** between charges $\mathbf{q_1}$ and $\mathbf{q_2}$ separated by a distance **r** is given by,

$$F = k_e \frac{(|q_1||q_2|)}{r^2}$$

Where Coulomb constant, $k_e = 8.9876 \times 10^9 \mathrm{Nm^2/C^2}$ and the SI unit of electric force is Newton (N).



Two point-charges separated by a distance r exert a force on each other given by Coulomb's law. The force on q_1 is equal in magnitude and opposite in direction to the force on q_2.

Quick Quiz 15.2 Object A has a charge of $+2\mu C$ and object B has a charge of $+6\mu C$. Which statement is true? (a) $\vec{F}_{AB} = -3\vec{F}_{BA}$ (b) $\vec{F}_{AB} = -\vec{F}_{BA}$ (c) $3\vec{F}_{AB} = -\vec{F}_{BA}$

Solution- The correct answer is (b). By Newton's third law, the two objects will exert forces having equal magnitudes but opposite directions on each other.

Example 15.1 The electron and proton of a hydrogen atom are separated (on the average) by a distance of about 5.3×10^{-11} m. (a) Find the magnitudes of the electric force and the gravitational force that each particle exerts on the other, and the ratio of the electric force F_e to the gravitational force F_g . (b) Compute the acceleration caused by the electric force of the proton on the electron. Repeat for the gravitational acceleration.

Solution-

$(a) r = 5.3 \times 10^{-11} m, F_e = ?, F_g = ?$

$$F_e = k\frac{|q_1||q_2|}{r^2} = 8.99\times10^9 \times \frac{1.6\times10^{-19}\times1.6\times10^{-19}}{(5.3\times10^{-11})^2} = 8.2\times10^{-8}N$$

$$F_g = g\frac{m_e m_p}{r^2} = 6.67\times10^{-11} \times \frac{9.1\times10^{-3}\ \times1.67\times10^{-27}}{(5.3\times10^{-11})^2} = 3.6\times10^{-47}N$$

$$\frac{F_e}{F_g} = \frac{8.2\times10^{-8}}{3.6\times10^{-4}} = 2.3\times10^{39}$$

(b) a_e =?, a_g =?

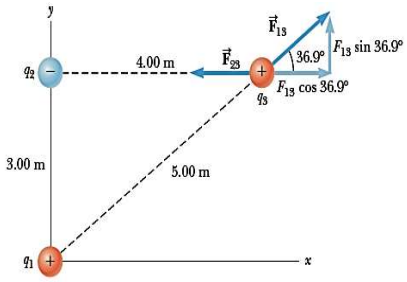
$$F_e = m_e a_e$$

$$a_e = \frac{F_e}{m_e} = \frac{8.2\times10^{-8}}{9.1\times10^{31}} = 9\times10^{22}\ ms^{-2}$$

$$F_g = m_e a_g$$

$$a_g = \frac{F_g}{m_e} = \frac{3.6\times10^{-47}}{9.1\times10^{-31}} = 4\times10^{-17}\ ms^{-2}$$

Example 15.3 Consider three point-charges at the corners of a triangle, as shown in Figure, where $q_1 = 6.00\times10^{-9}C$,$q_2 = -2.00\times10^{-9}C$, and $q_3 = 5.00\times10^{-9}C$. (a) Find the components of the force $\vec{F}_{23}$ exerted by q_2 on q_3. (b) Find the components of the force $\vec{F}_{13}$ exerted by q_1 on q_3. (c) Find the resultant force on q_3 in terms of components and also in terms of magnitude and direction.



Solution

(a) $q_1 = 6\times10^{-9}C, q_2 = -2\times10^{-9}C, q_3 = 5\times10^{-9}C,$

$\vec{F}_{23x}$ =?, $\vec{F}_{23y}$ =?

$$F_{23} = k_e\frac{|q_2||q_3|}{r^2} = (8.99\times10^9)\frac{(2.00\times10^{-9})(5.00\times10^{-9})}{(4.00)^2} = 5.62\times10^{-9}N$$

$$F_{23x} = -5.62\times10^{-9}N$$

$$F_{23} = 0$$

(b) F_{13x} =? , F_{13y} =?

$$F_{13} = k_e\frac{|q_1||q_3|}{r^2} = (8.99\times10^9)\frac{(6.00\times10^{-9})(5.00\times10^{-9})}{(5.00)^2} = 1.08\times10^{-8}N$$

$$F_{13x} = F_{13}cos\theta = (1.08\times10^{-8})\,cos(36.9°) = 8.64\times10^{-9}N$$

$$F_{13y} = F_{13}sin\theta = (1.08\times10^{-8})\,sin(36.9°) = 6.48\times10^{-9}N$$

(c) The resultant force, $\vec{F}$ =? , θ =?

$F_x = F_{13x} - F_{23x} = 8.64 \times 10^{-9} - 5.62 \times 10^{-9} = 3.02 \times 10^{-9} N$

$F_y = F_{13} + F_{23y} = 6.48 \times 10^{-9} + 0 = 6.48 \times 10^{-9} N$

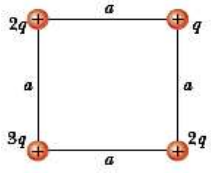
$$\vec{F} = \sqrt{{F_x}^2 + {F_y}^2} = \sqrt{(3.02 \times 10^{-9})^2 + (6.48 \times 10^{-9})^2} = 7.15 \times 10^{-9} N$$

Qick Quiz 15.2 Object A has a charge of $+2\mu C$ and object B has a charge of $+6\mu C$. Which statement is true? (a) $\vec{F}_{AB} = -3\vec{F}_{BA}$ (b) $\vec{F}_{AB} = -\vec{F}_{BA}$ (c) $\overrightarrow{3F}_{AB} = -\vec{F}_{BA}$

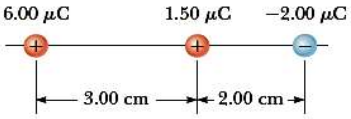
Solution :The correct answer is (b). By Newton's third law, the two objects will exert forces having equal magnitudes but opposite directions on each other.

Problems

Problem 6 Four point charges are at the corners of a square of side a as shown in Figure. Determine the magnitude and direction of the resultant electric force on q, with k_e , q, and a left in symbolic form.



Problem 8 Calculate the magnitude and direction of the Coulomb force on each of the three charges shown in Figure.



Problem 11 Three point charges are located at the corners of an equilateral triangle as in Figure. Find the magnitude and direction of the net electric force on the 2.00 μC charge.