Mechanical Systems Dynamics
Fundamentals of experimental modal analysis
I Experimental setup
II Post-processing

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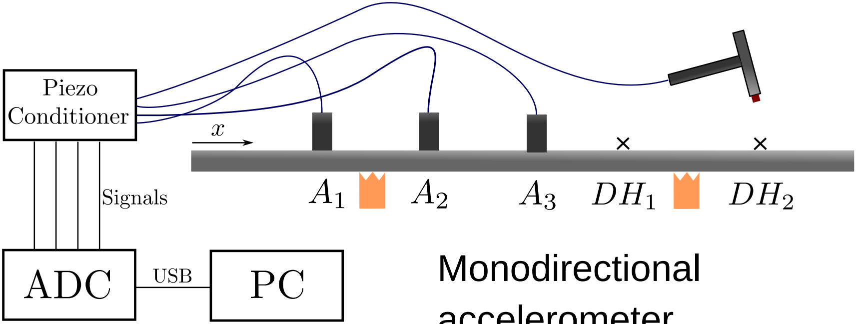
Free-free beam: to realize a perfet pin or a perfect incastr is almost impossible.



Monodirectional accelerometer

Beam's structural properties

Property	Symbol	Value	Unit
Length	L	1.2	[m]
Thickness	h	0.008	[m]
Width	b	0.04	[m]
Density	ρ	2700	[kg/m^3]
Young's Modulus	E	0.68E11	[N/m^2]

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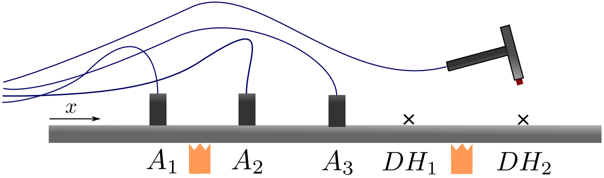
Sensors

Sensor	Symbol	x [mm]	Transducer	Sensitivity
Accelerometer	A_1	105	Piezo	10.2 $[mV/(m/s^2)]$
Accelerometer	A_2	415		
Accelerometer	A_3	600		
Dynamometric Hammer	DH_1	815		2.17 $[mV/N]$
Dynamometric Hammer	DH_2	1065		

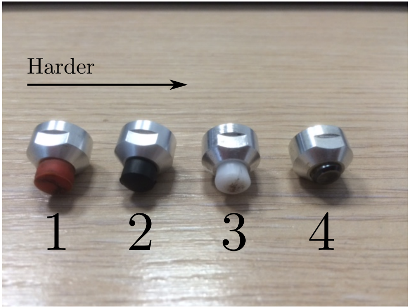


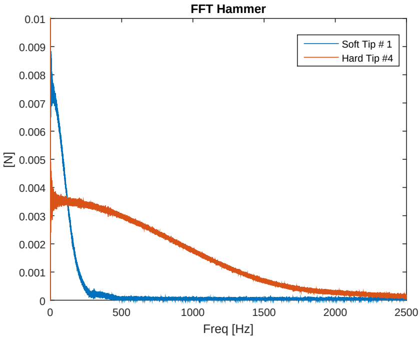
Supports: flexible modes are coincident with those of a free-free beam

Some preliminary considerations

- **Constraints**: theory Vs reality, why free-free beam for this lab.?
- Sensors **Positioning**
- **SIMO** Vs **MIMO** test configurations, **Reciprocity**
- Hammer's **tip**.

What's the influence of different hammer's tips?





...in these tests we use the hammer's tip # 2 (intermediate)

Signal processing for deriving the experimental FRF

1. Measurements are performed so as to collect a data set of N pairs of sampled time histories for the input force F_k and the output vibration x_j (the length of all the 2N time histories is indicated with T_0)
2. If needed, a Hanning (or other) window, is used to minimize spectral leakage
3. Discrete Fourier Transform is applied to all the signals, thus obtaining 2N discrete spectra F_{k_i} and X_{j_i} with fundamental frequency $\omega_0 = 2\pi/T_0$
4. PSD (Power Spectral Density - real) $G_{XX}(n\omega_0)$ and $G_{FF}(n\omega_0)$, as well as CSD (Cross-Spectral Density - complex) functions $G_{XF}(n\omega_0)$ are computed.
5. Finally the experimental FRF G_{jk}^{EXP} and the coherence function γ_{jk}^2 are estimated.

Analytical solution

$$w(\xi,t) = [Acos(\gamma\xi) + Bsin(\gamma\xi) + CCosh(\gamma\xi) + DSinh(\gamma\xi)]e^{j\omega t}$$

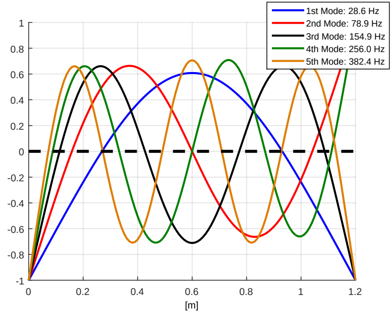


Figure: Available in **modes.mat** (modes.x, modes.y), normalized to 1

Available Experimental Data

- **DH1.mat** hammer in DH1 position
- **DH2.mat** hammer in DH2 position
- **RDH1.mat** Acc1 and DH1 position interchanged (**reciprocity** against **DH1.mat**)

In these *.mat file

- **freq**: frequency vector (resolution 0.02 Hz)
- **frf**: frequency response functions (complex), collected by columns (A_1,A_2,A_3)
- **cohe**: coherence functions, collected by columns (A_1,A_2,A_3)

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Assignment

Single-mode identification (up to 5th mode)

1. Identification of the natural **frequencies**
2. Identification of the damping ratio ξ by the **"half-power points"** method
3. Identification of the damping ratio ξ by the **"slope of the phase diagram"** method
4. Comparison Analytical Vs Experimental **mode shapes**

For the oral examination...

...report, for each mode, the identification results (items 1 to 4), for at least one test configuration among DH1, DH2 and RDH1. Collect the results in table form (for each sensor, items 1 to 3) and plot a diagram for the comparison (item 4)

How to...

- **Half-Power points method:**

8.2.5.3 on Advanced Dynamics of Mechanical Systems

$$|G(\omega_1)| = \frac{\sqrt{2}}{2}|G_i| \quad ; \quad |G(\omega_2)| = \frac{\sqrt{2}}{2}|G_i| \quad ; \quad \omega_1 < \omega_i < \omega_2$$

$$\xi_i = \frac{\omega_2^2 - \omega_1^2}{4\omega_i^2}$$

- **Slope of the phase:**

$$\xi_i = -\frac{1}{\omega_i \cdot \frac{\partial \angle G(\Omega)}{\partial \Omega}\Big|_{\Omega=\omega_i}}$$

$i = peak$

Ogni modo di vibrare ha un proprio smorzamento che dipende solo dal modo di vibrare stesso

- **Analytical Vs Experimental visualization:**

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How to...

- For each frequency (mode shape), the ratio among the peaks $|G^k(\omega_i)|$ along with the relative phase $\angle(G^k(\omega_i)$ of different accelerometers k give the shape of the mode.
- The analytical modes (modes.mat) are given normalized to 1 (maximum displacement).
- A common normalization is required for analytical/experimental visualization (without distorting the experimental modes shapes!!)
- In order to find the damping ratio ξ with a higher precision, increasing the local frequency resolution around the peak, could be required (i.e. interpolation).