Exercises

Read Carefully

These exercises are provided only to give an example of possible exams questions. Any topic object of the lessons can be the object of the written and of the oral exam.

1 Sampling

1.1 Example 1

Your acquisition system is configured as follows:

- Sampling rate: 250 Hz
- Number of samples: 2500
- Antialiasing Filter Cut-off frequency: 100 Hz
- Range of the A/D converter: ±5 V

You have to acquire these four signals

- Sine wave with frequency 8.84 Hz, Amplitude 2 V
- Sine wave with frequency 175 Hz, Amplitude 1 V
- Sine wave with frequency 10 Hz, Amplitude 6 V.

For each signal:

- Identify the best window (Hanning – rectangular) for reducing or avoiding the leakage
- Plot the spectrum modulus.

1.2 Example 2

Your acquisition system is configured as follows:

- Sampling rate: 500 Hz
- Number of samples: 2500
- Antialiasing Filter Cut-off frequency: 150 Hz
- Range of the A/D converter: ±5 V

You have to acquire these four signals

- Sine wave with frequency 1408.84 Hz, Amplitude 2 V
- Sine wave with frequency 187.5 Hz, Amplitude 1 V

For each signal:

- Identify the presence of leakage, aliasing or clipping errors
- Identify the best window (Hanning – rectangular) for reducing or avoiding the leakage
- Plot the spectrum modulus.

1.3 Example 3

Your acquisition system is configured as follows:

- Sampling rate: 5000 Hz
- Number of samples: 10000
- Antialiasing Filter Cut-off frequency: 2150 Hz
- Range of the A/D converter: ±5 V

You have to acquire these four signals

- Sine wave with frequency 237.199 Hz, Amplitude 2 V
- Sine wave with frequency 280.5 Hz, Amplitude 1.2 V
- Sine wave with frequency 12287 Hz, Amplitude 1 V
- Sine wave with frequency 4 Hz, Amplitude 52 V.

For each signal:

- Identify the presence of leakage, aliasing or other measurement errors;
- Identify the best window (Hanning – rectangular) for reducing or avoiding the leakage
- Plot the spectrum modulus.

1.4 Example 4

You have to acquire a quasi-periodic signal that is the sum of these two sine waves

Amplitude: 1 V	Frequency: Log(700) Hz	Phase: 90°
Amplitude: 0.1 V	frequency: Log(1100) Hz	Phase: -45°

Choose the optimal sampling frequency, the number of samples, the acquisition time and the windows to be used among the one listed below. Explain all your choices

- Sampling frequencies: 10, 50, 100, 1000 Hz
- Number of samples: 100, 200, 500, 1000, 2000 Samples
- Window: Rectangular, Hanning or High order window.

Answer

The frequencies are not rational so it is impossible to avoid the leakage. Consequently, it is better to choose the maximum number of samples and the minimum sampling frequencies that avoids aliasing. This grants the largest possible T with the lowest possible frequency resolution

1.5 Exercise 5

Sketch the spectra of a sine wave with a frequency of 10 Hz, sampled at 1000 Hz, for T equal to:

a. 10.00 seconds
b. 10.05 seconds

Plot the spectra of the same signal as before sampled with a sampling frequency of 225 Hz for T:

c. 10.00 seconds
d. 10.05 seconds

1.6 Exercise 6

You have to acquire a single signal that is the sum of the two sine waves (a and b) and the broadband noise (c)

a. Sine wave with frequency 278 Hz, Amplitude 2 V
b. Sine wave with angular velocity 1750 rad/s, amplitude 2 V
c. Broadband noise up to 10 MHz

The acquisition system can be configured as follows:

- Available sampling frequencies: 50, 100, 200, 500, 1000 or 2000 Hz
- Number of samples: 1024, 2048, 8096, 16384 or 32768
- Antialiasing filter cut-off frequency: 40, 80, 150, 250, 320, 640 Hz
- Windows: Rectangular or Hanning
- 12 bits A/D converter range: ±1 V, ±2.5 V, ±5 V or ±10 V

1. Discuss the necessity of the antialiasing filter.
2. Choose the sampling frequency, the number of samples and the window to correctly distinguish between the two spectral components.
3. Choose the A/D converter range, compute the LSB and the converter standard uncertainty.
4. Plot the signal spectrum.

1.7 Exercise 7

You have to acquire a single signal that is the sum of the two sine waves (a and b) and the broadband noise (c)

a) Sine wave with frequency 9.12315 Hz, Amplitude 2 V
b) Sine wave with frequency 203 Hz, amplitude 2 V
c) Broadband noise up to 10 MHz

The acquisition system can be configured as follows:

Available sampling frequencies: 50, 100, 200, 500, 1000 or 2000 Hz

Number of samples: 1024, 2048, 8096, 16384 or 32768

Antialiasing filter cut-off frequency: 40, 80, 150, 250, 320, 640 Hz

Windows: Rectangular or Hanning

12 bits A/D converter range: ±1 V, ±2.5 V, ±5 V or ±10 V

- Discuss the necessity of the antialiasing filter;
- Choose the sampling frequency, the MINIMUM number of samples and the window to correctly distinguish between the two spectral components.
- Choose the A/D converter range, compute the LSB and the converter standard uncertainty
- Plot the signal spectrum

1.8 Exercise 8

An acceleration signal is the sum of two harmonic components: the first has a pulsation of 157 rad/s, the second has a frequency of 25.1 Hz. Both the signals have a peak-to-peak value of 7 m/s^2 and are acquired with accelerometers with sensitivity of 980.655 mV/g.

Configure the acquisition system in order to reduce the measurement errors (sampling frequency, number of samples/buffer, acquisition time, window)

Plot the spectrum modulus in terms of 0-Peak amplitude with the chosen acquisition parameter

Solution:

The two frequencies are 25.1 Hz and 24. 98…….. Hz. The second frequency is not rational so it is impossible to avoid the leakage.

The sampling frequency larger than 50.2 Hz to avoid aliasing; a decent choice could be 100 Hz. The number of samples should be chosen to distinguish between the two spectral components, i.e. should be larger than 50 s.

Compute the crest factor of purely harmonic signal (sine wave) not affected by noise, given that its RMS value is 1 Pa and the signal duration is 13.138s.

Solution: CF is equal to sqrt(2) for any harmonic signal

2 Questions

2.1 Sampling

Describe what is the aliasing problem and how can we avoid it.

Describe what is the leakage problem and how can we avoid it.

Digitalization of analog signals: describe the meaning of A/D conversion and sampling

Compute the LSB of a 12-bit converter with full range 0-10 V

You have to acquire a square wave with a frequency of 127.34 Hz. Please determine the sampling frequency, the number of samples and the windows to be used. Is an antialiasing filter necessary or not? Justify your answers

2.2 Time Domain

Definition of auto- and cross- correlation. Describe a practical usage of one of these two functions in a real case.

Definitions of the biased and unbiased auto- and cross- correlation and implications for time big time delays

Describe one practical use of the autocorrelation and one for the cross-correlation

Plot the autocorrelation of the signal: [0;0;0;0;0;1;1;1;1;0;0;0;0;0]

Plot the cross-correlation between the signals X= [0;0;0;1;1;1;1;0;0;0], Y=[0;0;0;1;2;2;1;0;0;0] <u>for time delays between -3 and +3 samples</u>

Plot the biased auto-correlation of the signals X= [0;0;0;1;2;2;1;0;0;0] <u>for time delays between -3 and +3 samples</u>

Compute and plot the auto-correlation for positive and negative time lags of a square wave with amplitude 7 pW/m^2 and period 1 s

Is there any link between the autocorrelation and the RMS?

Definition of the probability density function. Sketch qualitatively the probability density function of a square wave, of a sine wave, of a triangle wave, of the Gaussian noise and of the uniform noise.

Which are the differences between the RMS and the standard deviation?

Definition and of Crest Factor and typical uses in experimental mechanics.

Definition and of Skewness and Kurtosis and their uses in experimental mechanics.

How can we check the periodicity of a signal in time domain?

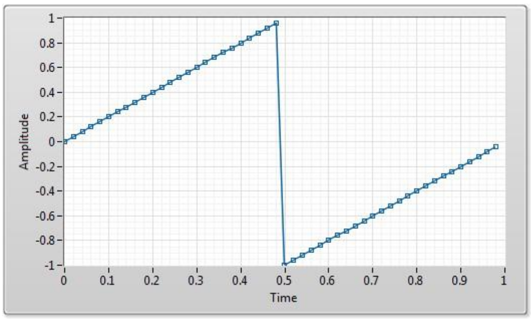
When a certain phenomenon is said to be stationary?

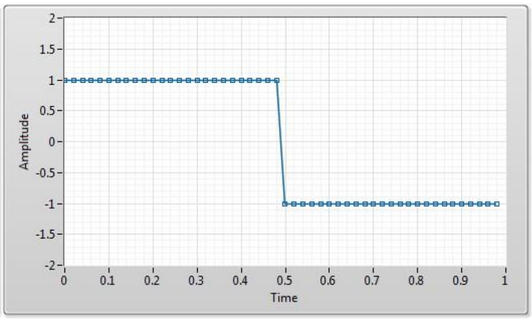
Which is the RMS of a sine wave whose zero-to-peak amplitude is 10 V?

Which is the RMS of a square wave whose amplitude is 3.2 V?

Which is the RMS of a DC signal whose constant amplitude is 1.32 V?

Plot the probability density function of the following signals (samples are indicated by dots)





Compute the Crest factor of the following signal

2.3 Frequency Domain

Define, if there is any, the relationship between autocorrelation and autospectrum of the same signal.

Demonstrate that the Fourier transform of the autocorrelation is a real function

Demonstrate that the Fourier transform of an odd signal is an imaginary even function

Demonstrate that the elements at negative frequencies of the FT are the complex conjugate of the positive ones.

Compare the two approaches of the counter-rotating vectors and of the single vector in the Fourier analysis

2.4 Linear Systems, Dirac and Convolution

Definition of linearity, homogeneity and causality of a system.

Describe system response through convolution, by a step by step calculation (scaling ad delayed impulse response)

Describe leakage through convolution.

Describe aliasing through convolution.

Demonstrate the sampling property of the Dirac function

Compute the Fourier Transform of the Dirac function

Graphically describe the response of a system to a ramp using the convolution

2.5 Windows

Are there any situations in which the rectangular window to observe a signal is the best solution? And which are the main differences between the rectangular and the Hanning windows?

Explain advantages and drawbacks of the Hanning window if compared to the rectangular one

Which is the main difference between the Fourier transform of a rectangular window and of an hanning window?

2.6 Frequency Response, Coherence and Estimators

Which are the stimuli that are commonly used to identify the frequency response function of a system?

Describe a practical use of the frequency response function on (a car, a motorbike, a bike, a kite surfer, your stereo system, the calibration of a sensor)

Which are the actuators that can be used to identify the vibration response of mechanical systems? In which force range they are usually employed?

Compare the advantages and the drawbacks of the sweep versus the white noise stimuli for the estimation of the FRF of a mechanical system

Explain the difference between random and pseudo-random signals

What is a Bode diagram?

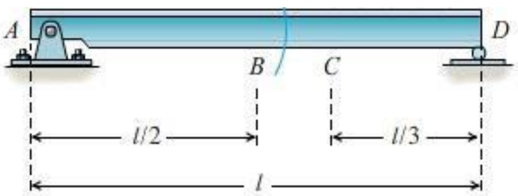
Plot the Frequency Response Function (Bode Diagram, modulus and phase) of a 1 degree of freedom mechanical system (natural frequency: 1000 Hz) in terms of:

- Absolute mass displacement / absolute ground displacement
- Relative displacement foundation-mass / absolute ground displacement

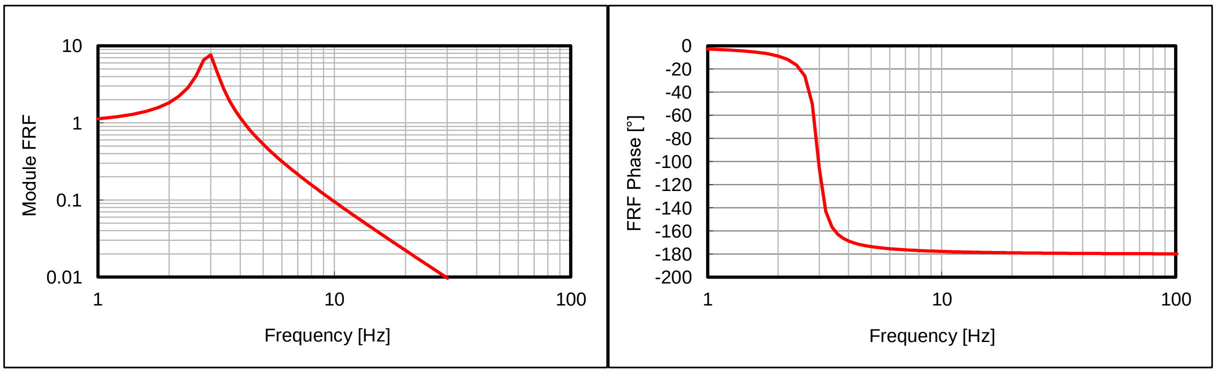
Supposing that a 1DOF mechanical system has a natural frequency of 732 Hz and has a lumped mass of 3.5 kg:

- Outline two excitation sources that can be used to derive the FRF
- <u>Plot</u> for each of the selected excitation sources a <u>time history</u> and its <u>spectrum</u> of the stimulus that you would use to identify the frequency response function.

Describe the procedure (position of the accelerometers, impact point, sampling frequency, window, ecc) that you would adopt to identify the first two bending vibration modes of the structure in the figure below with two accelerometers and a dynamometric hammer.



The experimental identification of the Frequency Response Function of a mechanical system on a shaker lead to these plots. The FRF is expressed as a ratio between the mechanical system response (m/s^2) and the shaker vibration stimulus (m/s^2); mod and phase are plotted in log scales.



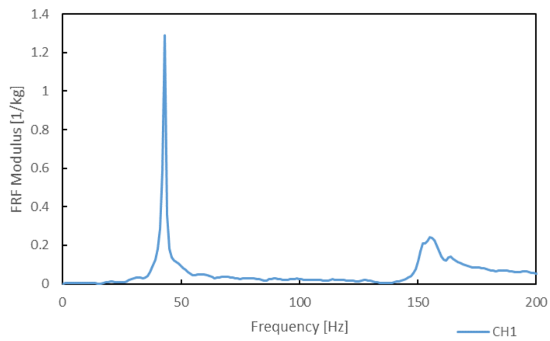
Said A the stimulus acceleration amplitude (m/s^2), □ the stimulus angular velocity (rad/s) and □ the stimulus phase (radians), estimate the system responses to input X= A*sin($\omega\tau+\phi$)

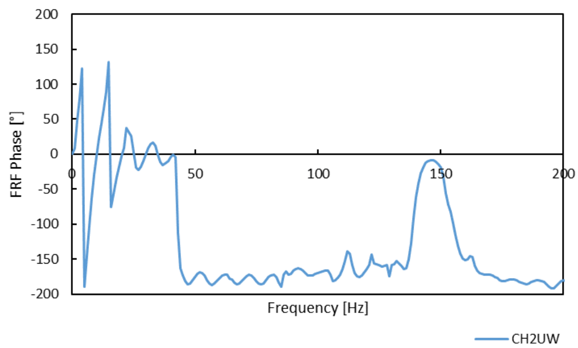
- A=2.5m/s^2, $\omega=6.3$ rad/s and $\phi=0.7$ radians
- A=65m/s^2, $\omega=31.5$ rad/s and $\phi=0$ radians
- The stimulus is the sum of the two components above

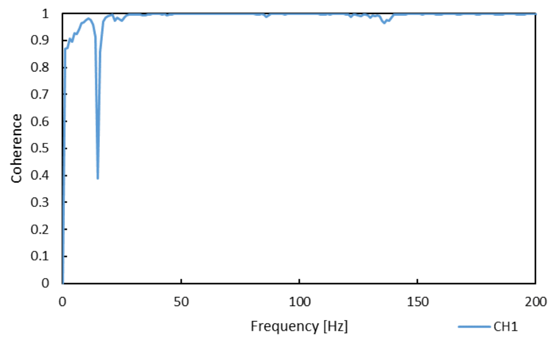
Coherence function and correlation coefficient: definitions, analogies and examples of practical use.

Definition of H1 and H2 estimators of the frequency response function. Which is the best choice if the system has a dominant noise on the input?

You have estimated the FRF of a mechanical system using an impact hammer and an accelerometer with a plastic tip. The FRF modulus, phase and coherence were computed using the H1 estimator.







Qualitatively plot the FRF modulus using the H2 estimator and list the reasons for which the coherence could be lower than 1 between 0 and 50 Hz.

Solution: The coherence is the ratio between H1 and H2, so H2 is H1/coherence. Consequently, H2 is equal to H1 except at low frequencies (where it tends to infinite at frequency=0) and at approximately 20 Hz where the low value increases

You have estimated the FRF of a mechanical system using a piezoelectric shaker. The input force was measured by a piezoelectric load cell and the output with a triaxial MEMS accelerometer. You have acquired a single buffer of 1024 samples with a 12-bit data acquisition board. The sampling rate was 1.024 MHz. The input spectrum was a sweep sine (linear sweep rate during the acquisition time) between the frequencies of 1 and 100 Hz. The system is supposed to be nonlinear. Plot the spectrum of the input and the coherence between the input and the output. Justify your answer.

Solution: The time duration of the buffer is 1ms, i.e. the frequency resolution of the spectrum is 1 kHz. The spectrum of the signal is therefore composed by one line at 0 Hz and one line at 1000 Hz (with lower amplitude). Depending on the window there will be spectral leakage at the higher frequencies. Given that only one buffer was acquired the coherence is one everywhere. The computation of averages using the Welch technique is useless, given that, in any case, the frequency resolution of the spectrum will be larger than 1000 Hz.

2.7 Time-Frequency Analysis

Explain the procedure to obtain the short-time Fourier transform of a signal and the function of the overlap

Explain the main advantage of the wavelet transform with respect to the Fourier transform

Why are the time and the frequency strictly correlated in STFT?

Is there any drawback in the use of Hanning window in STFT?

2.8 Rotating machineries

Describe what is the full-spectrum

Predict the full-spectrum in case the orbit is a circle/line/ellipse

Describe the measurement chain necessary to obtain the orbit plot

Full spectrum: definition and experimental setup. Said H the last digit of your day of birth, plot the full spectrum and the orbit plot if the signals X and Y are

X= 0.7*H*cos(13πt+(H°+27°))

Y= 0.7*H*cos(13πt+(H°-63°))

Suppose a possible defect of the rotor rotating at 13πt rad/s.

[The two signals have the same amplitude and are 90° out of phase, so the orbit is a circle. The full spectrum is therefore a single bin at 6.5 Hz, with amplitude FS+ 1.4H. The circular orbit is given by a rotor imbalance]