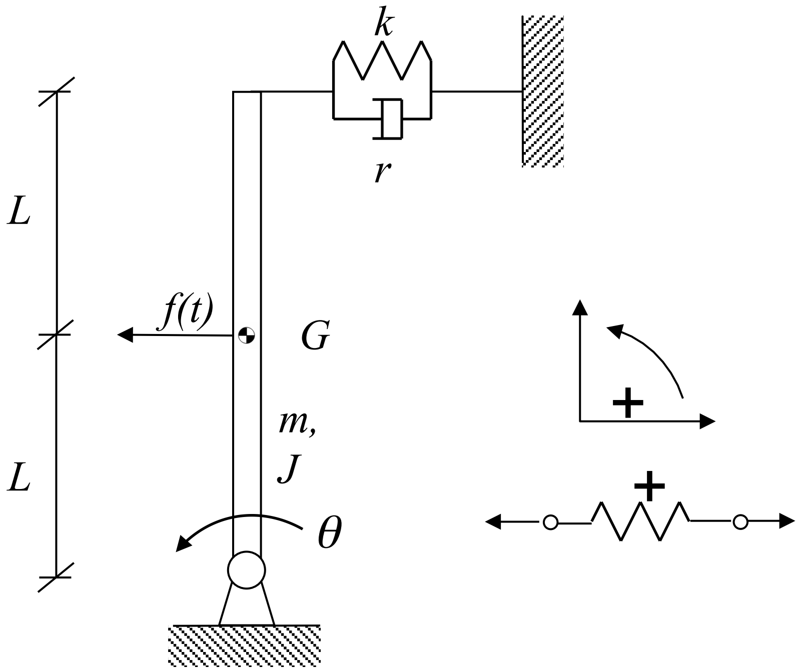
POLITECNICO DI MILANO

POLITECNICO
MILANO 1863

PROPORTIONAL-DERIVATIVE (PD) CONTROLLER

PROPORTIONAL-INTEGRAL (PI) CONTROLLER

Mechanical system



L = 1 m
m = 80 kg
J = 20 kgm²
k = 250 N/m (or 125 N/m)
r = 25 Ns/m

$$\left(mL^2 + J\right)\ddot{\vartheta} + 4rL^2\dot{\vartheta} + \left(4kL^2 - mgL\right)\vartheta = f\left(t\right)L = F\left(t\right)$$

$$m^*\ddot{\vartheta} + r^*\dot{\vartheta} + k^*\vartheta = F\left(t\right)$$

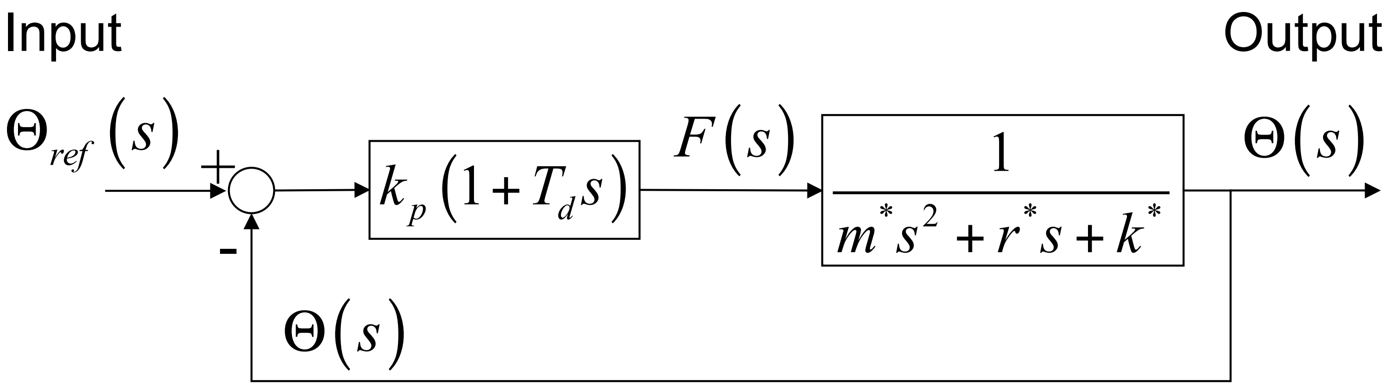
$$F\left(t\right) = k_p\left(\vartheta_{ref} - \vartheta\right) + k_d\left(\dot{\vartheta}_{ref} - \dot{\vartheta}\right)$$

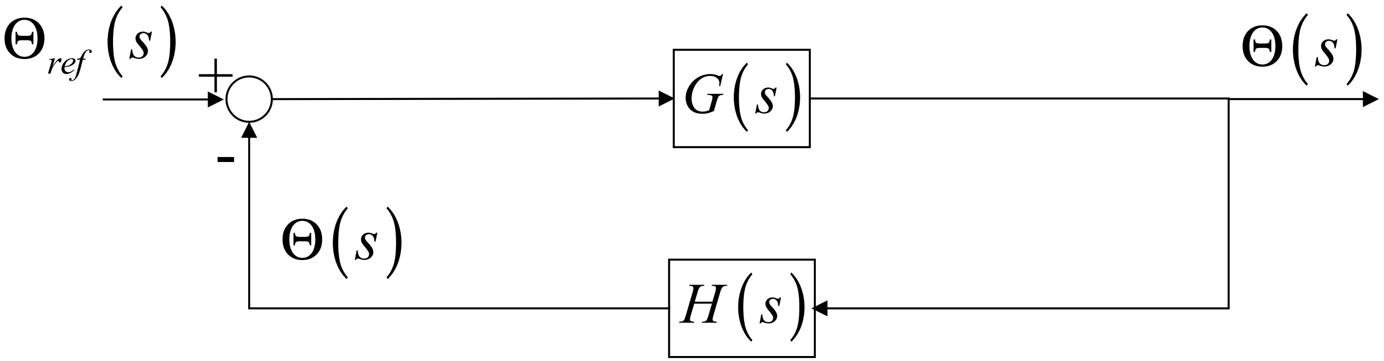
$$m^*\ddot{\vartheta} + r^*\dot{\vartheta} + k^*\vartheta = k_p\left(\vartheta_{ref} - \vartheta\right) + k_d\left(\dot{\vartheta}_{ref} - \dot{\vartheta}\right)$$

Laplace domain

$$m^*\ddot{\vartheta} + r^*\dot{\vartheta} + k^*\vartheta = k_p\left(\vartheta_{ref} - \vartheta\right) + k_d\left(\dot{\vartheta}_{ref} - \dot{\vartheta}\right)$$

$$\left(m^*s^2 + r^*s + k^*\right)\Theta(s) = k_p\left(1 + T_d s\right)\left(\Theta_{ref}(s) - \Theta(s)\right)$$





Laplace domain

$$GH(s) = G(s)H(s) = \frac{k_p(1+T_d s)}{m^* s^2 + r^* s + k^*}$$ open loop transfer function

$$L(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{k_p(1+T_d s)}{m^* s^2 + (r^* + k_p T_d)s + (k^* + k_p)}$$ closed loop transfer function

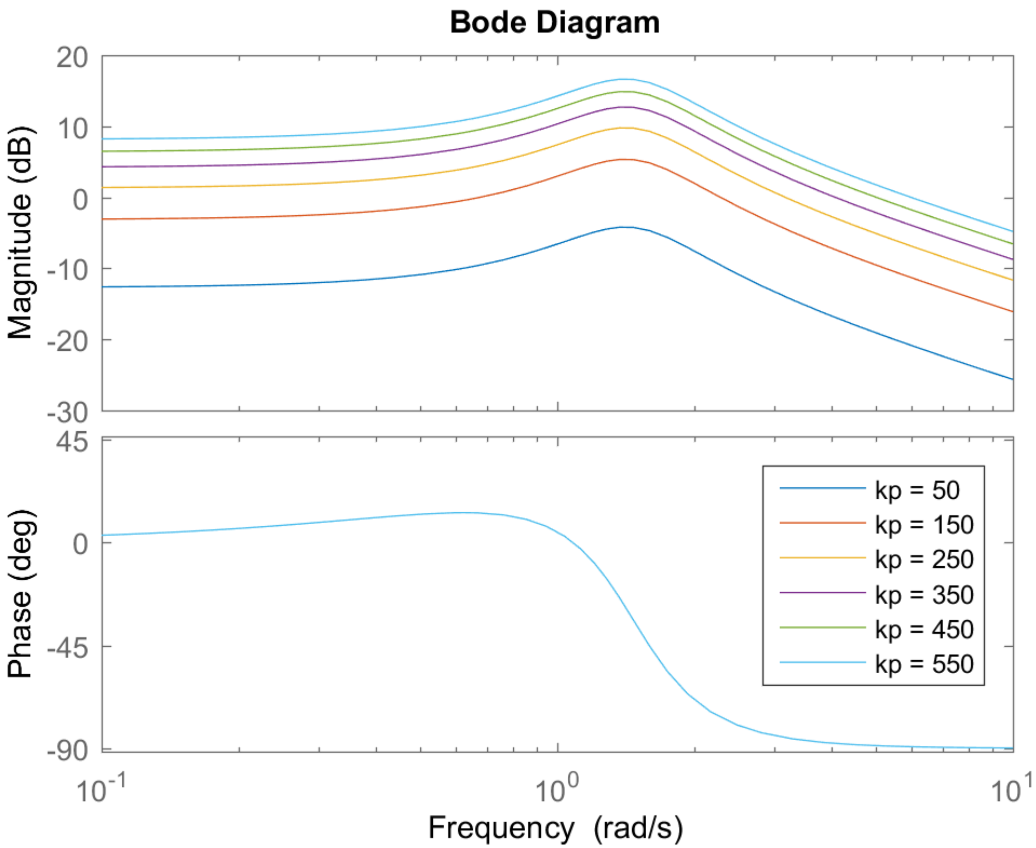
The stability analysis can be performed:

1) studying the poles of the closed loop transfer function *L(s)*;

2) using Nyquist criterion (we must draw the Nyquist diagram of *GH*(*s*));

3) using Bode criterion, if and only if the uncontrolled system is stable (we must draw the Bode diagram of *GH*(*s*)).

k = 250 N/m (zero before the c.c. poles)
Bode diagram of the open loop transfer function GH(s)



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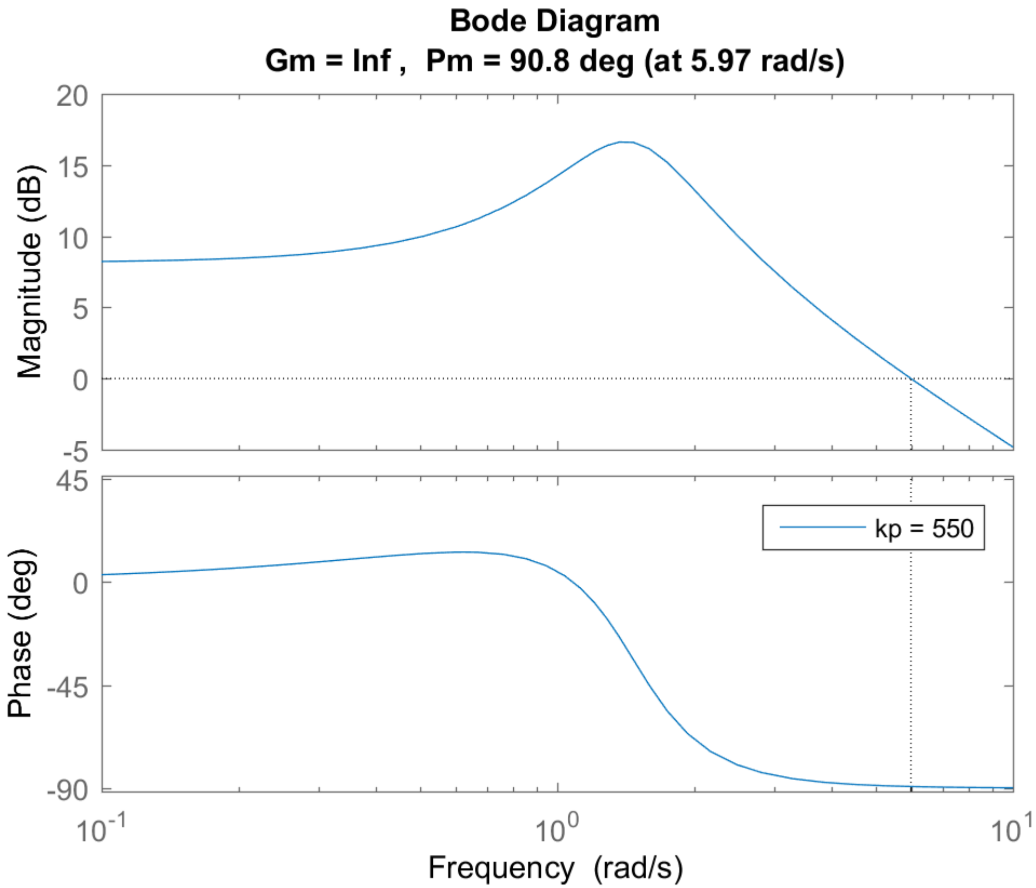
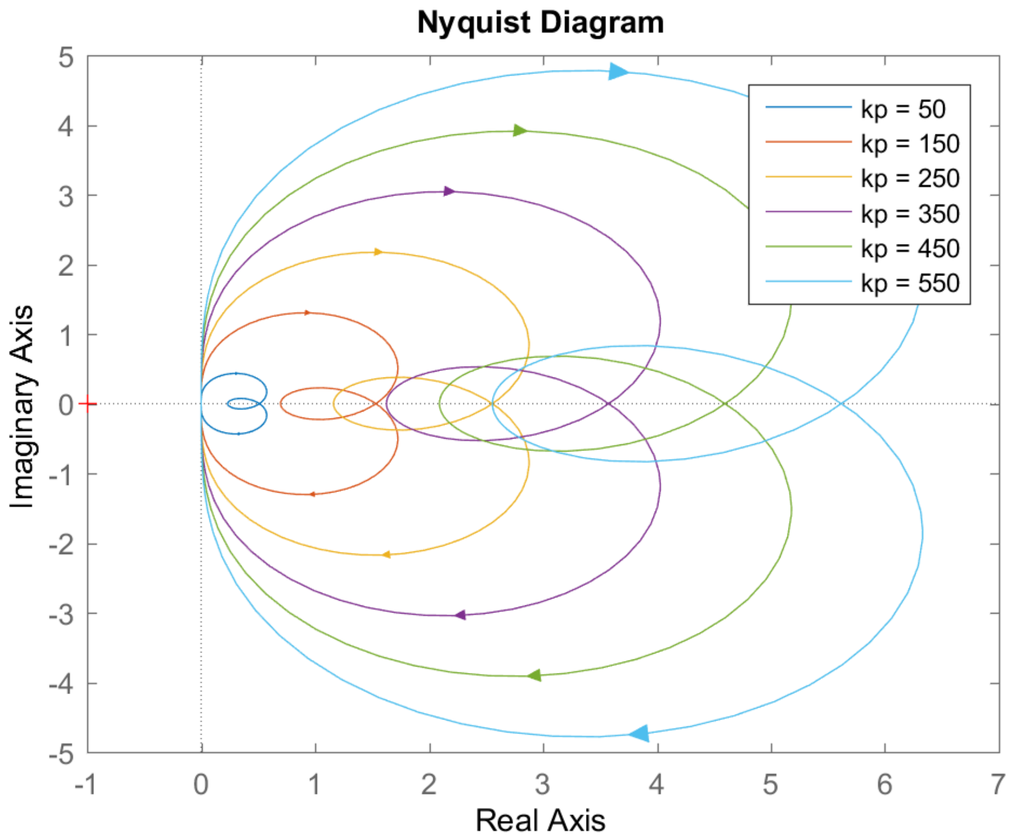
Bode diagram of the open loop transfer function GH(s)



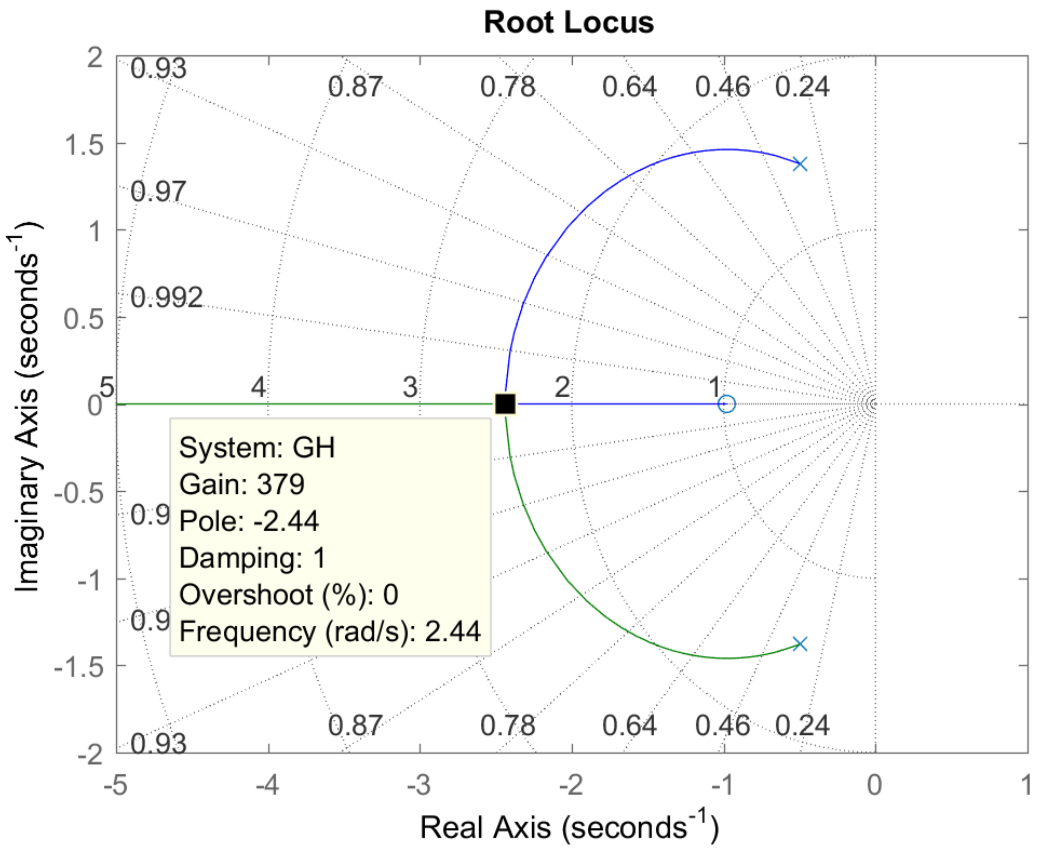
Using Bode criterion, being Pm>0 and Gm>0 we can say that the controlled system is stable.

k = 250 N/m (zero before the c.c. poles)
Nyquist diagram of the open loop transfer function GH(s)



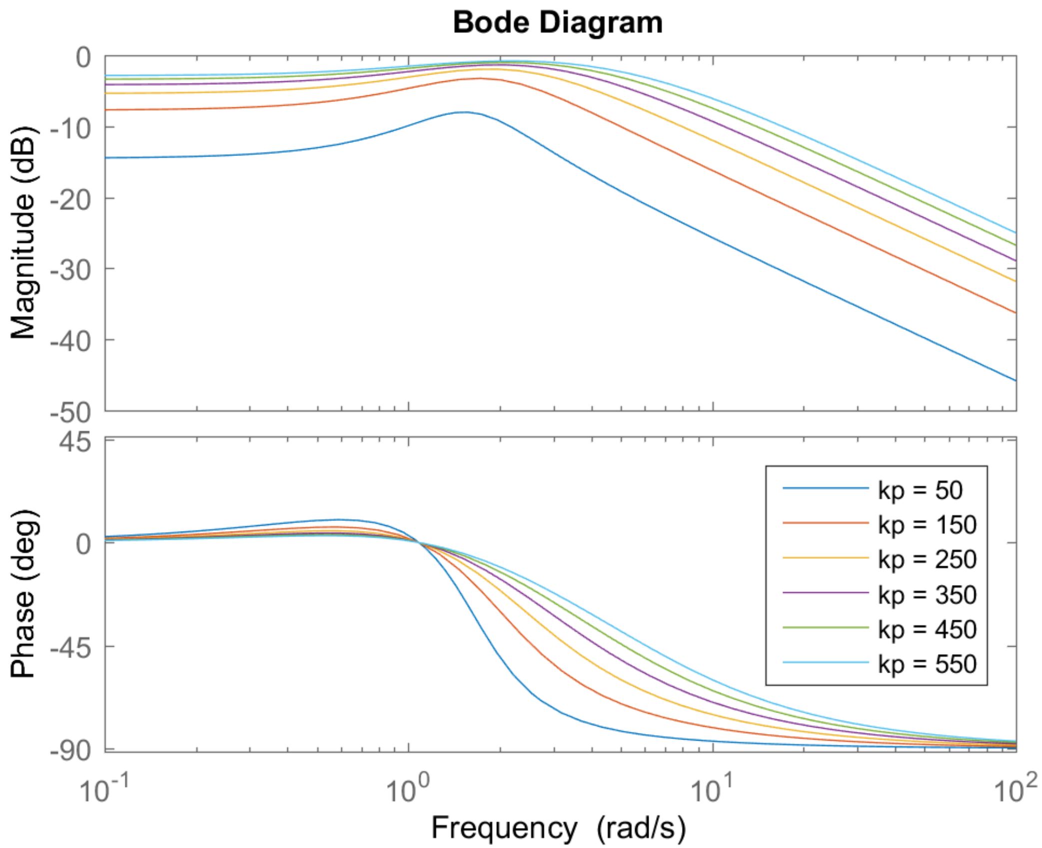
No encirclment of the point (-1,0) is found. Using Nyquist criterion we can say that the controlled system is stable.

k = 250 N/m (zero before the c.c. poles)
Root locus

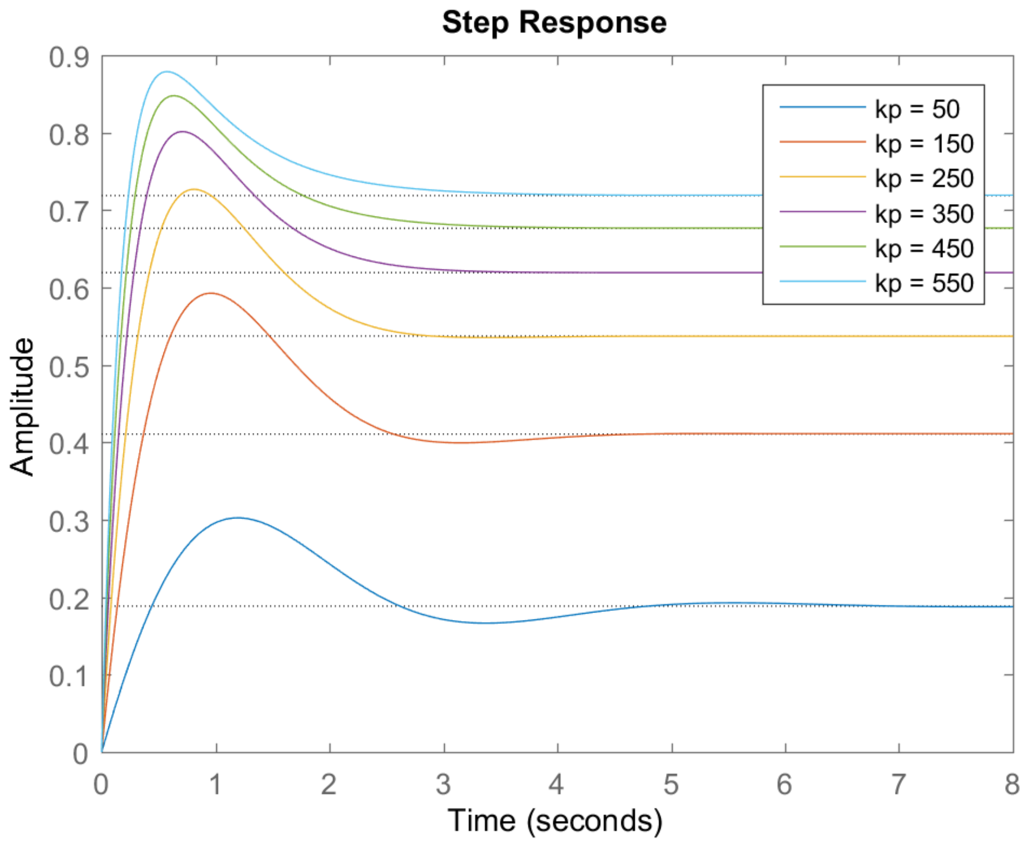


The poles of the system have negative real parts for all the possible values of the proportional gain k_p, however they may become real if k_p is larger than 379.

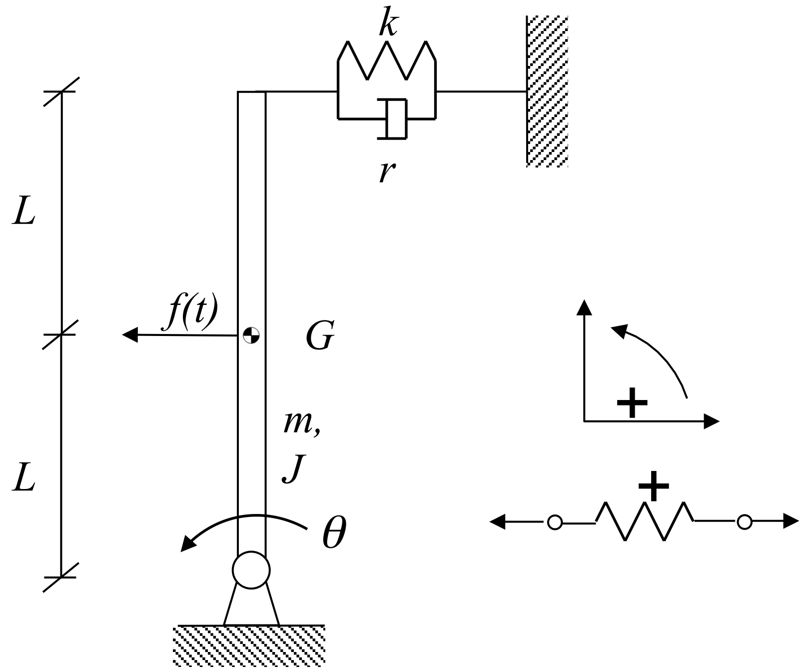
k = 250 N/m (zero before the c.c. poles)
Bode diagram of the closed loop transfer function L(s)



k = 250 N/m (zero before the c.c. poles)
Step response



Mechanical system



L = 1 m
m = 80 kg
J = 20 kgm^2
k = 250 N/m (or 125 N/m)
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$$\left(mL^2+J\right)\ddot{\vartheta}+4rL^2\dot{\vartheta}+\left(4kL^2-mgL\right)\vartheta=f\left(t\right)L=F\left(t\right)$$

$$m^*\ddot{\vartheta}+r^*\dot{\vartheta}+k^*\vartheta=F\left(t\right)$$

$$F\left(t\right)=k_p\left(\vartheta_{ref}-\vartheta\right)+k_i\int_0^t\left(\vartheta_{ref}-\vartheta\right)dt$$

$$m^*\ddot{\vartheta}+r^*\dot{\vartheta}+k^*\vartheta=k_p\left(\vartheta_{ref}-\vartheta\right)+k_i\int_0^t\left(\vartheta_{ref}-\vartheta\right)dt$$

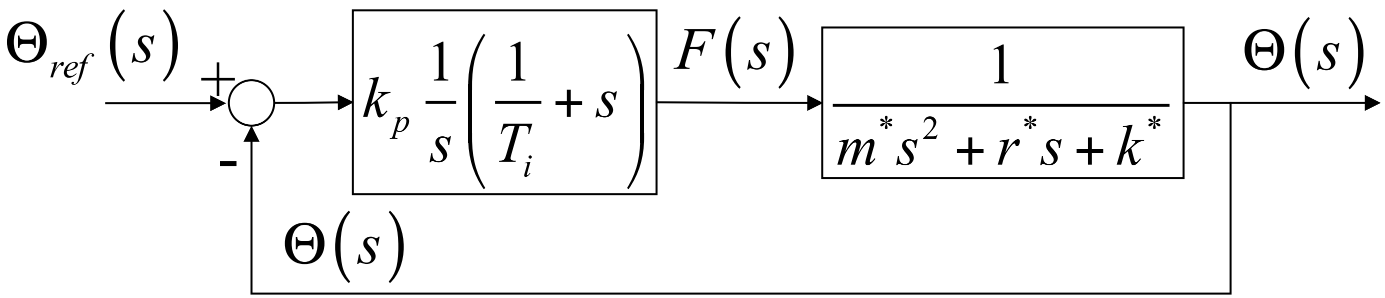
Laplace domain

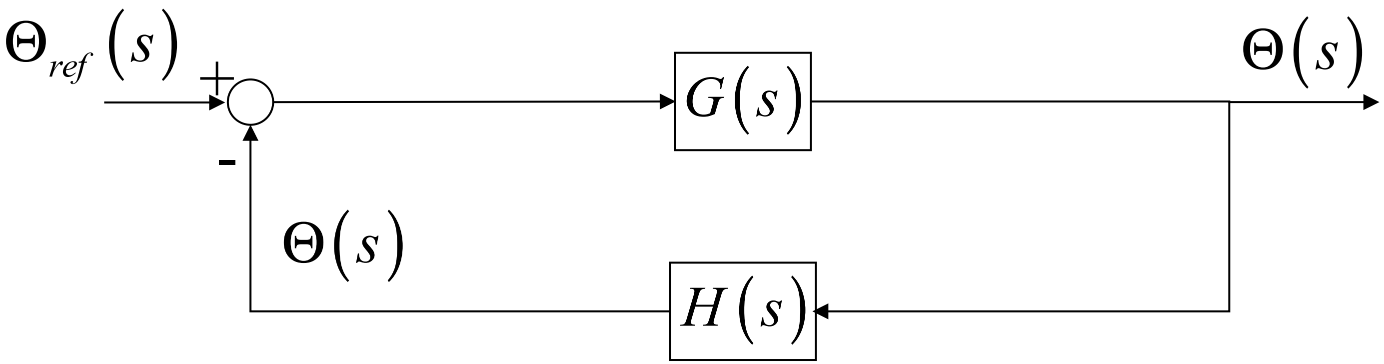
$$m^*\ddot{\vartheta} + r^*\dot{\vartheta} + k^*\vartheta = k_p\left(\vartheta_{ref} - \vartheta\right) + k_i\int_0^t\left(\vartheta_{ref} - \vartheta\right)dt$$

$$\left(m^*s^2 + r^*s + k^*\right)\Theta(s) = k_p\frac{1}{s}\left(\frac{1}{T_i} + s\right)\left(\Theta_{ref}(s) - \Theta(s)\right)$$

Input

Output





Laplace domain

$$GH(s) = G(s)H(s) = \frac{k_p\left(\frac{1}{T_i} + s\right)}{s\left(m^* s^2 + r^* s + k^*\right)}$$

open loop transfer function

$$L(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{k_p\left(\frac{1}{T_i} + s\right)}{s\left(m^* s^2 + r^* s + k^*\right) + k_p\left(\frac{1}{T_i} + s\right)}$$

closed loop transfer function

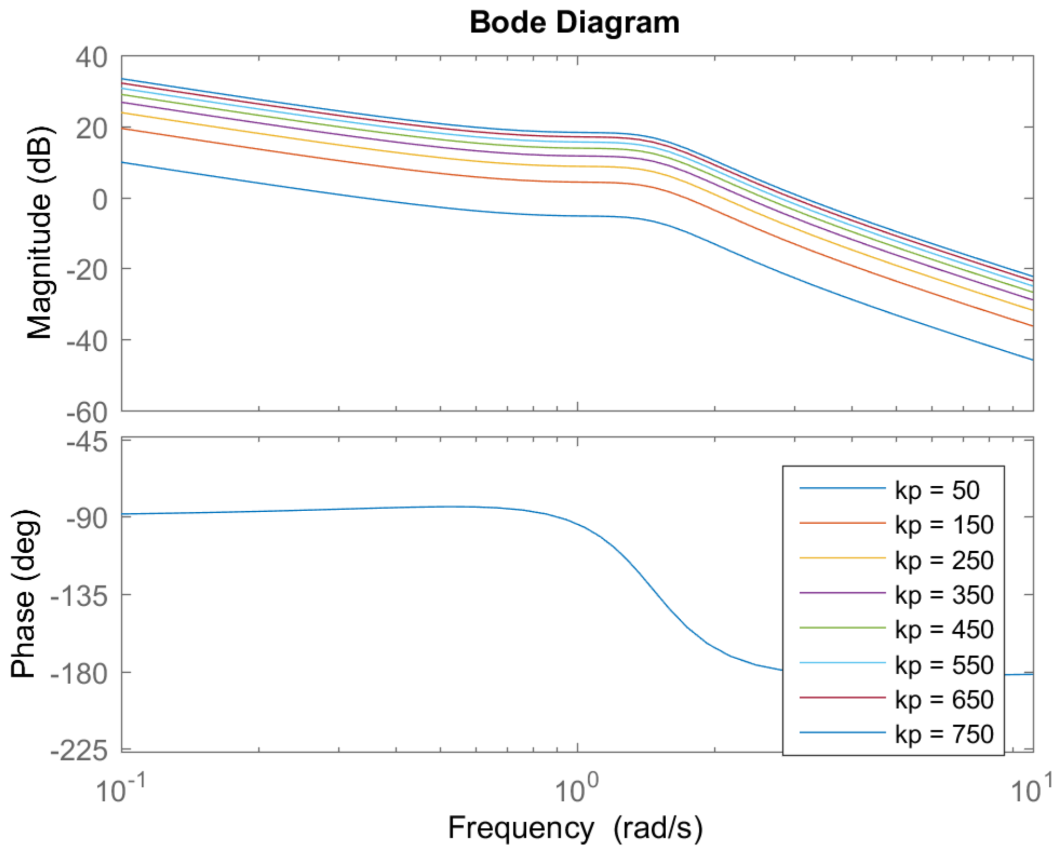
The stability analysis can be performed:

1) studying the poles of the closed loop transfer function *L(s)*;

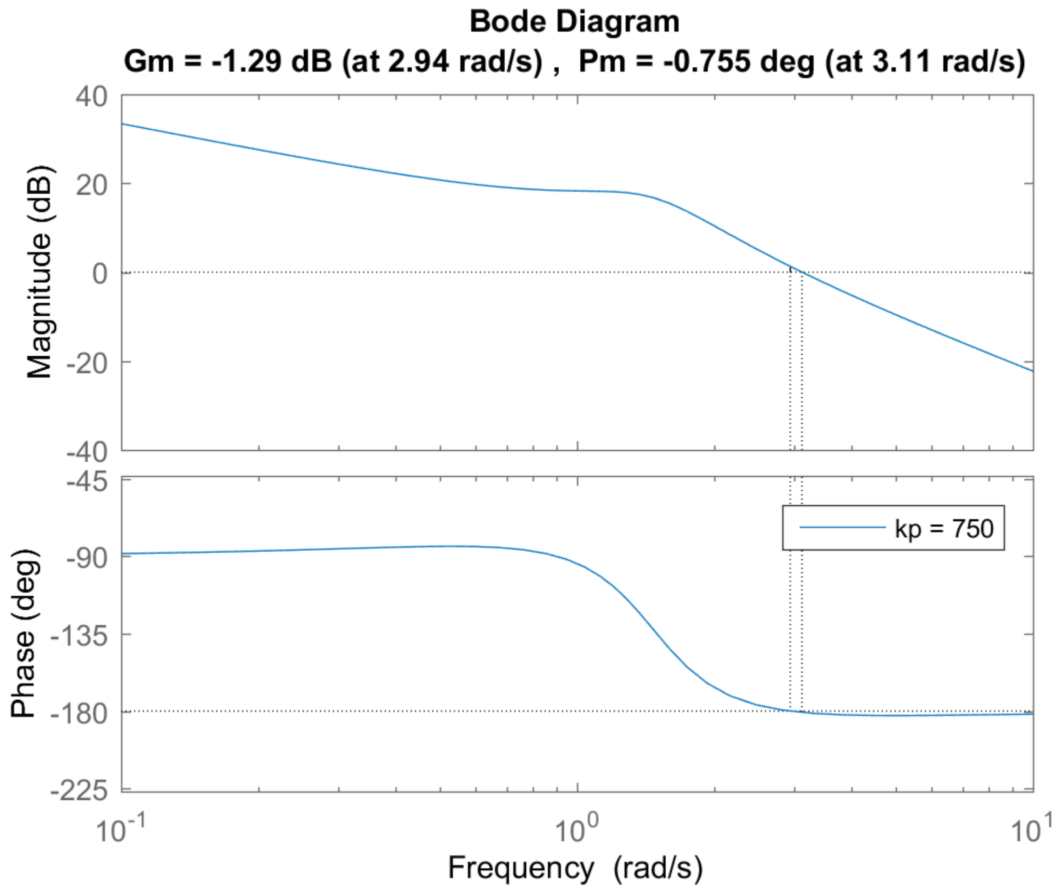
2) using Nyquist criterion (we must draw the Nyquist diagram of *GH*(*s*));

3) using Bode criterion, if and only if the uncontrolled system is stable (we must draw the Bode diagram of *GH*(*s*)).

k = 250 N/m (zero just before the c.c. poles) Bode diagram of the open loop transfer function GH(s)

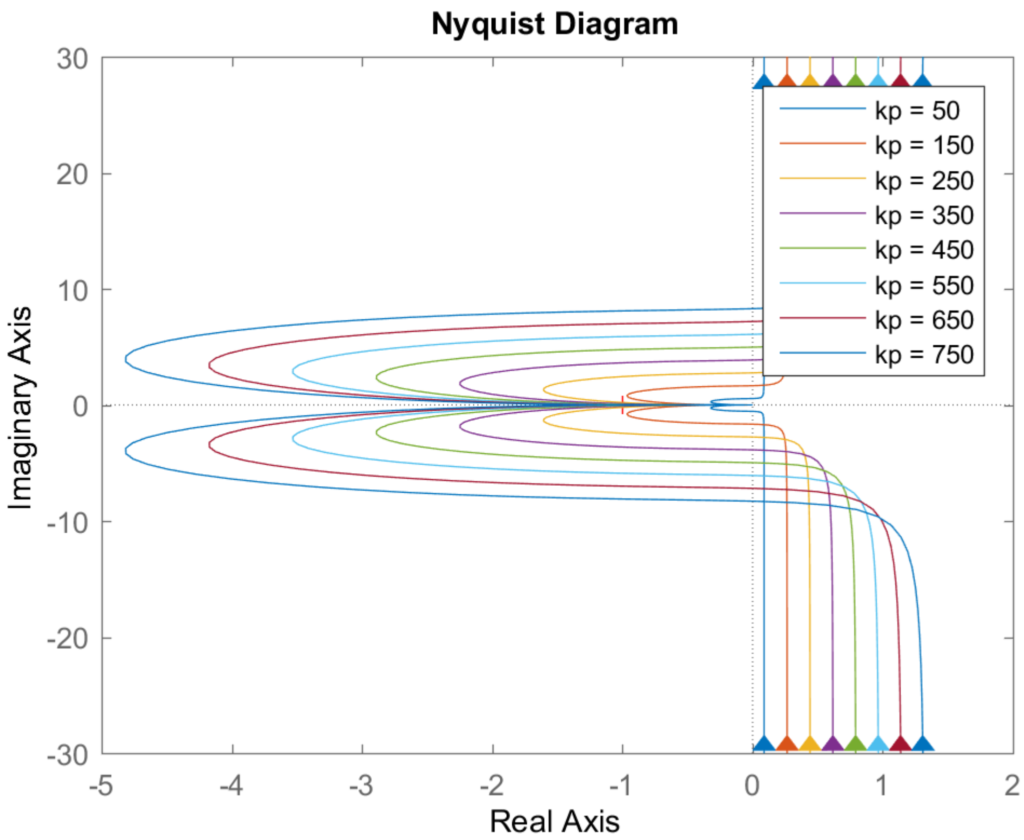


k = 250 N/m (zero just before the c.c. poles)
Bode diagram of the open loop transfer function GH(s)

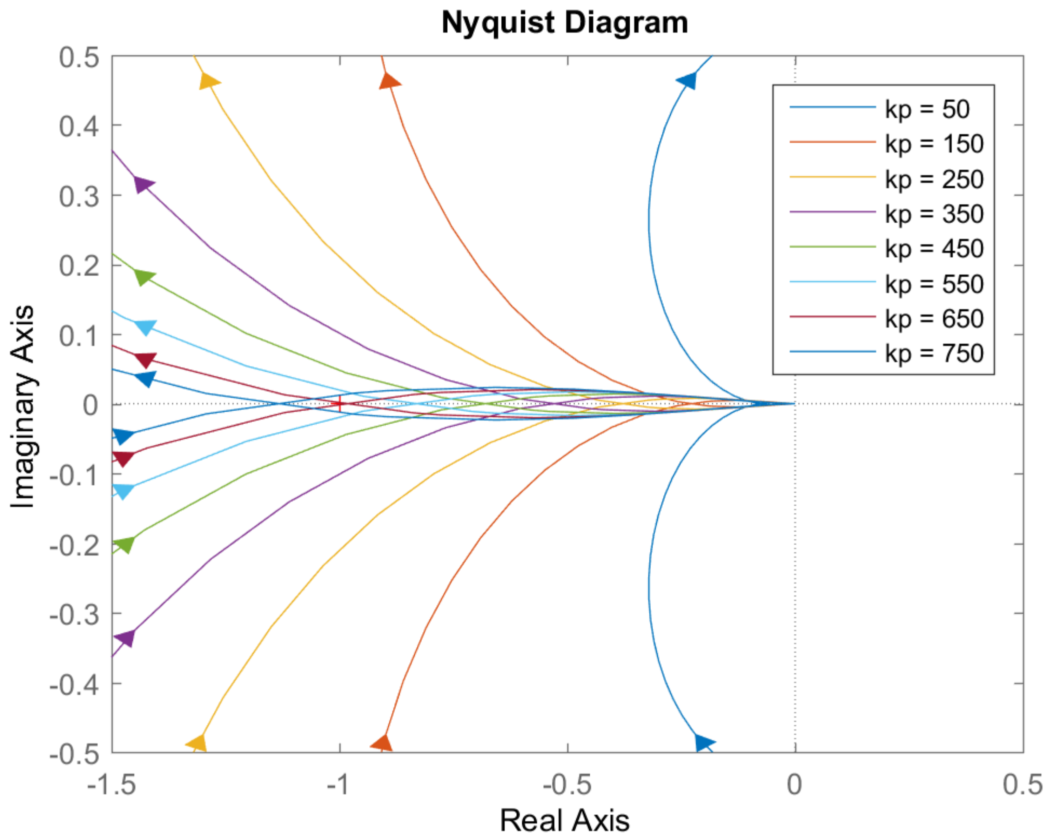


Using Bode criterion, being Pm<0 and Gm<0 we can say that the controlled system is unstable when k_p is equal to 750.

k = 250 N/m (zero just before the c.c. poles)
Nyquist diagram of the open loop transfer function GH(s)

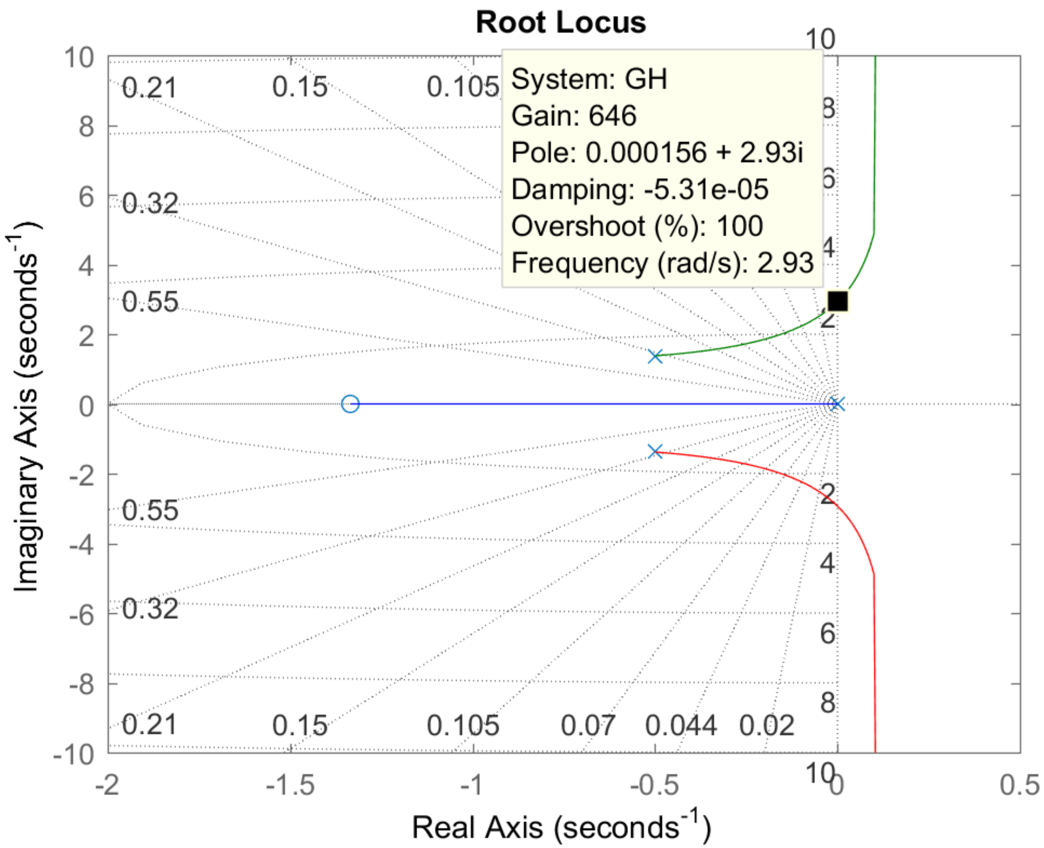


k = 250 N/m (zero just before the c.c. poles) Nyquist diagram of the open loop transfer function GH(s)



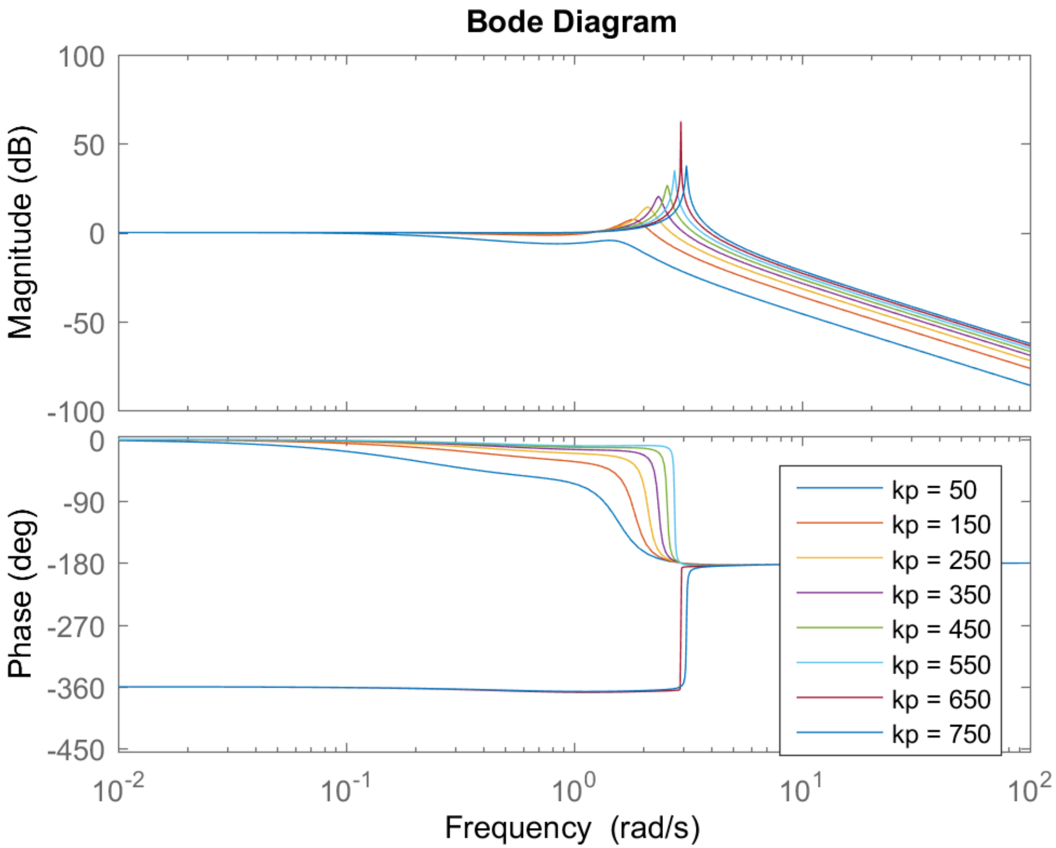
No encirclment of the point (-1,0) is found when k_p is smaller than 650. If k_p is equal to 650 and 750 one encirclement is found. For these two cases, using Nyquist criterion we can say that the controlled system is unstable.

k = 250 N/m (zero just before the c.c. poles)
Root locus

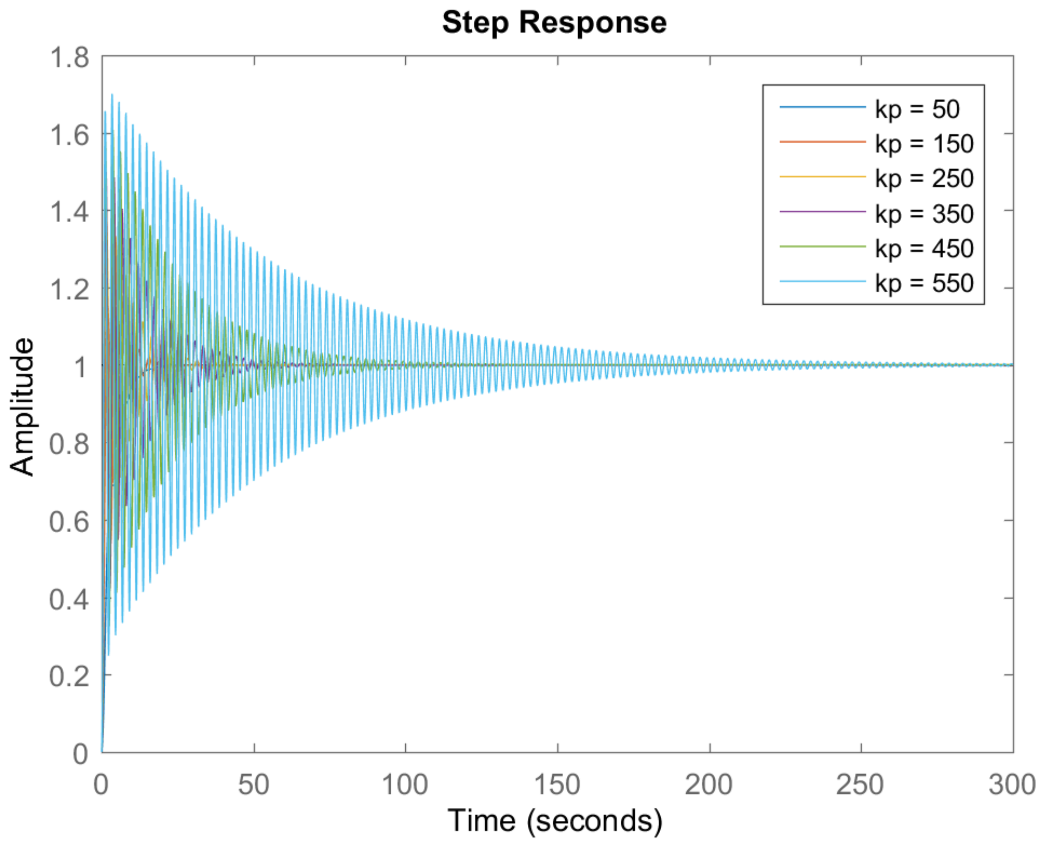


The poles of the system have negative real parts if the proportional gain k_p is smaller than 646.

k = 250 N/m (zero just before the c.c. poles) Bode diagram of the closed loop transfer function L(s)



k = 250 N/m (zero just before the c.c. poles)
Step response



Increasing the proportional gain k_p the damping ratio of the poles of the controlled system decreases thus incresasing the maximum overshoot. The steady state error is null for any value of the gain.