POLITECNICO DI MILANO

THE FINITE ELEMENT METHOD (1)

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Exercise

DATA:

- Mass per unit length **$m = 9.75$ kg/m**
- Flexural rigidity **$EJ = 1.34 \cdot 10^4$ Nm^2**
- Axial stiffness **$EA = 2.57 \cdot 10^7$ N**
- Length **$L = 2$ m**
- Structural Damping Matrix $[C_s] = \alpha[M]+\beta[K]$ (α **$= 0.2$ s^{-1}; $\beta = 1 \cdot 10^{-4}$ s**)

We want to calculate:

- **Natural frequencies;**
- **Mode shapes.**

The length of the beam finite element influences the maximum frequency of the structure analysis.

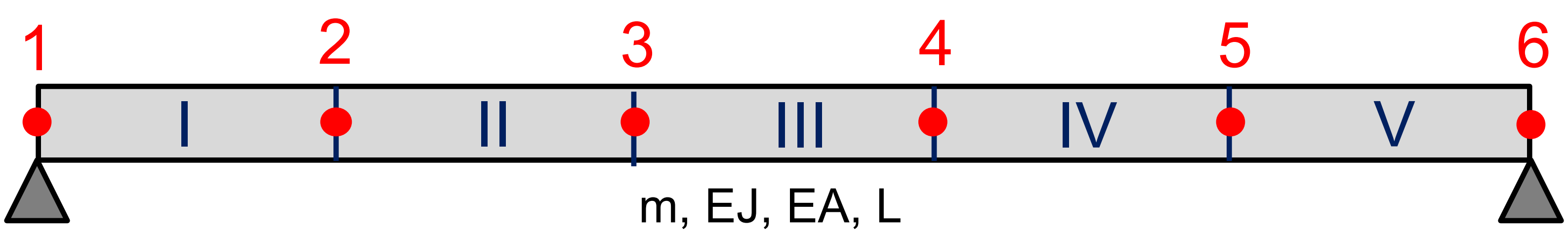
The beam finite element must have a quasi-static behaviour (the considered shape functions are the static ones) during vibration of the structure.

$$\Downarrow$$

$$\omega_{fe,i} = \left(\frac{\pi}{l_{fe,i}}\right)^2 \sqrt{\frac{EJ}{m}} > \Omega_{max} \Rightarrow l_{fe,i,max} < \sqrt{\frac{\pi^2}{\Omega_{max}} \cdot \sqrt{\frac{EJ}{m}}}$$

For the considered structure:

f_{max} [Hz]	Ω_{max} [rad/s]	$l_{fe,i,max}$ [m]
5	31.4	3.41
10	62.8	2.41
20	125.7	1.71
50	314.2	1.08
100	628.3	0.76
200	1256.6	0.54
300	**1885.0**	**0.44**



```
! FEM(1)
! 1st Exercise
!                    ----------------------------------------
! list of nodes :
*NODES
! n. of node - constraint code (x,y,theta) - x coordinate- y coordinate.
1                  1 1 0            0.0       0.0
2                  0 0 0            0.4       0.0
3                  0 0 0            0.8       0.0
4                  0 0 0            1.2       0.0
5                  0 0 0            1.6       0.0
6                  1 1 0            2.0       0.0
! end card *NODES
*ENDNODES
!                    ----------------------------------------
! list of elements :
*BEAMS
! n. of elem. - n. of input node - n. of output node - m, EA, EJ
1       1      2    9.75    2.57e7   1.34e4
2       2      3    9.75    2.57e7   1.34e4
3       3      4    9.75    2.57e7   1.34e4
4       4      5    9.75    2.57e7   1.34e4
5       5      6    9.75    2.57e7   1.34e4
*ENDBEAMS
```

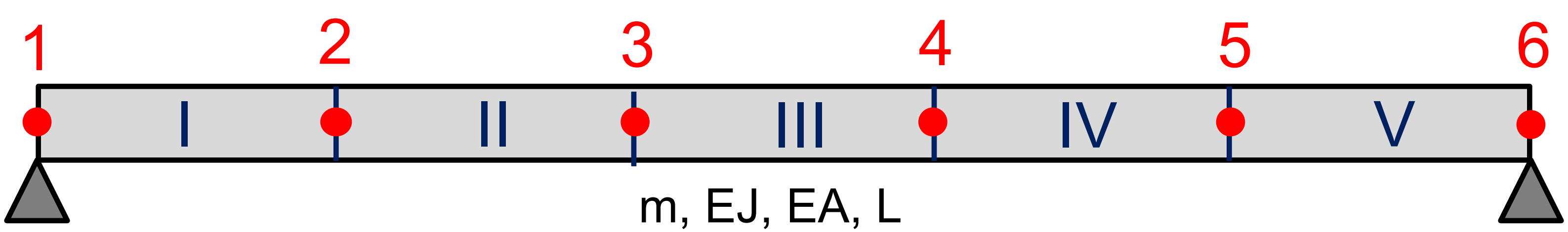
! = symbol for comment

*NODES = start definition of nodes

*ENDNODES = end definition of nodes

*BEAMS = start definition of beam finite element

*ENDBEAMS = end definition of beam finite element



Definition of input file (*.inp)

```
! FEM(1)
! 1st Exercise
!           ---------------------------------------------
! list of nodes :
*NODES
! n. of node - constraint code (x,y,theta) - x coordinate- y coordinate.
1                  1 1 0                 0.0       0.0
2                  0 0 0                 0.4       0.0
3                  0 0 0                 0.8       0.0
4                  0 0 0                 1.2       0.0
5                  0 0 0                 1.6       0.0
6                  1 1 0                 2.0       0.0
! end card *NODES
*ENDNODES
!           ---------------------------------------------
! list of elements :
*BEAMS
! n. of elem. - n. of input node - n. of output node - m, EA, EJ
1       1      2     9.75    2.57e7    1.34e4
2       2      3     9.75    2.57e7    1.34e4
3       3      4     9.75    2.57e7    1.34e4
4       4      5     9.75    2.57e7    1.34e4
5       5      6     9.75    2.57e7    1.34e4
*ENDBEAMS
```

Constraint code:

0 = free d.o.f.

1 = constrained d.o.f.

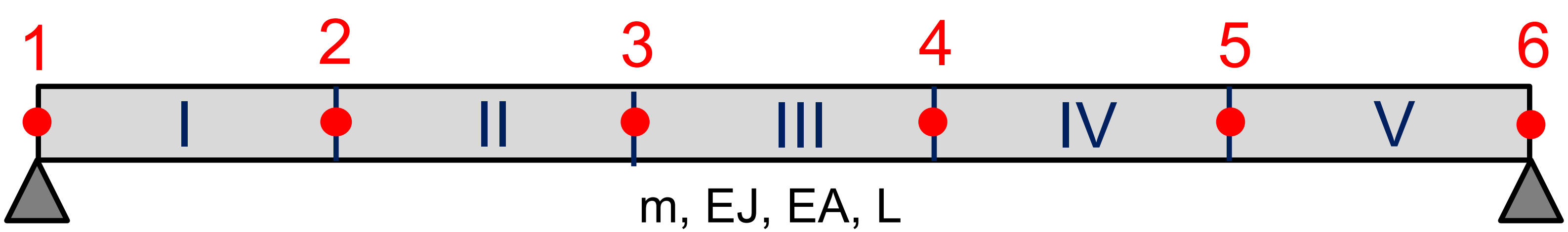
Coordinates of node:

x [m], y [m]

Left and right nodes of the beam finite element

Properties of the beam finite element:

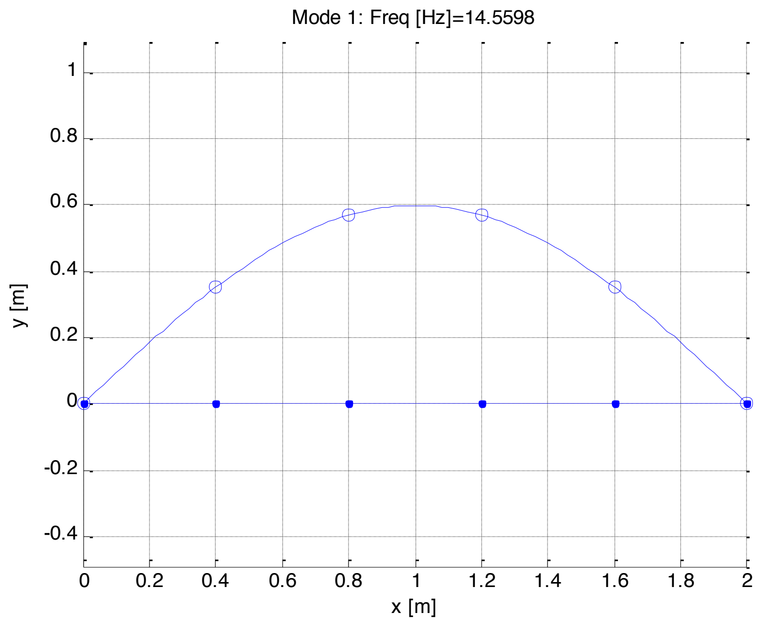
m [kg/m], EA [N], EJ [Nm^2]



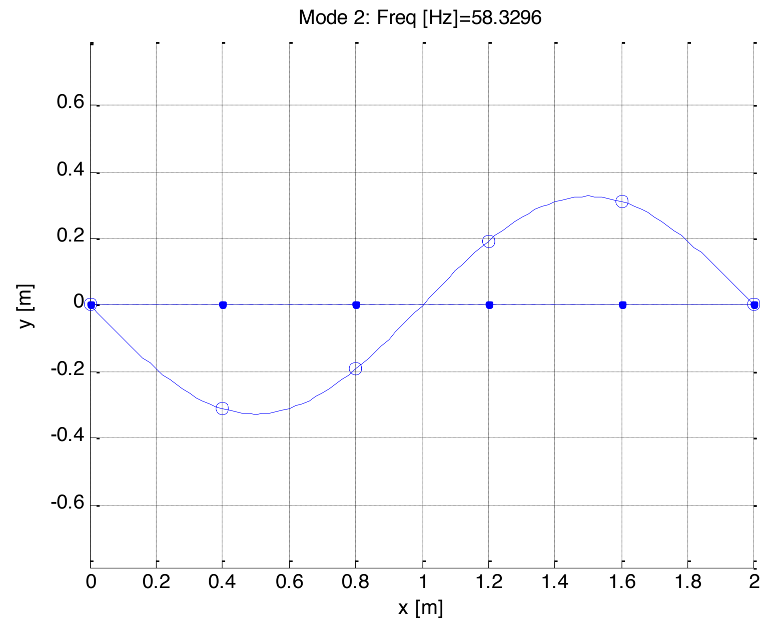
If we divide the structure in 5 beam finite elements (6 nodes – 3 d.o.f. for each node = 18 d.o.f.):

$$[\mathbf{M}]_{18\times 18} = \begin{bmatrix} [\mathbf{M_{FF}}]_{14\times 14} & [\mathbf{M_{FC}}]_{14\times 4} \\ [\mathbf{M_{CF}}]_{4\times 14} & [\mathbf{M_{CC}}]_{4\times 4} \end{bmatrix}$$

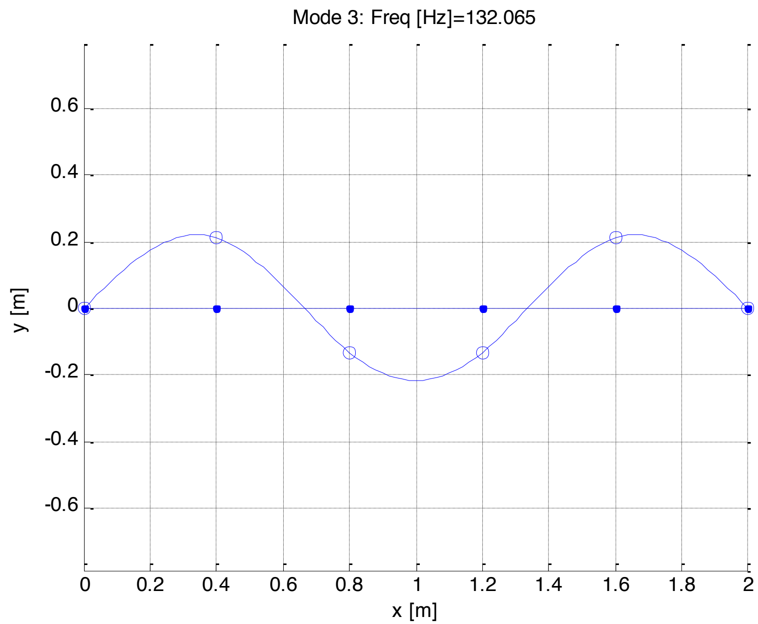
$$[\mathbf{K}]_{18\times 18} = \begin{bmatrix} [\mathbf{K_{FF}}]_{14\times 14} & [\mathbf{K_{FC}}]_{14\times 4} \\ [\mathbf{K_{CF}}]_{4\times 14} & [\mathbf{K_{CC}}]_{4\times 4} \end{bmatrix}$$



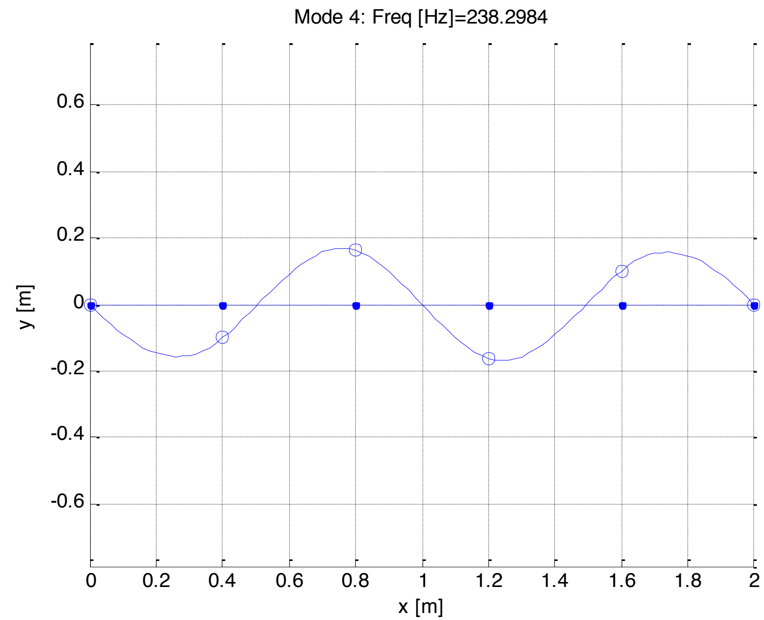
Mode 1: Freq [Hz]=14.5598
y [m]
x [m]



Mode 2: Freq [Hz]=58.3296
y [m]
x [m]



Mode 3: Freq [Hz]=132.065
y [m]
x [m]



Mode 4: Freq [Hz]=238.2984
y [m]
x [m]

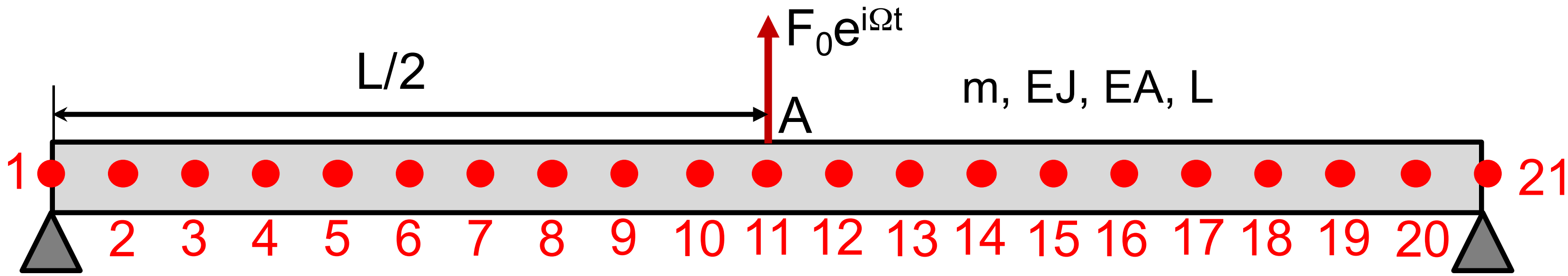
Effect of beam finite element length

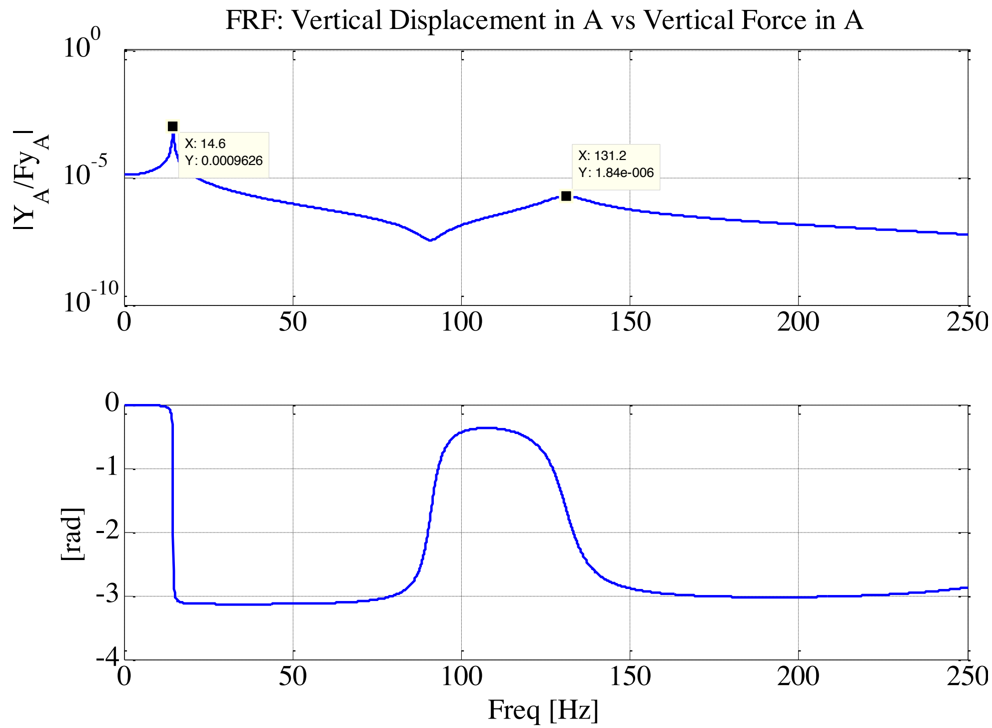
	$l_{fe} = 1m$	$l_{fe} = 0.5m$	$l_{fe} = 0.4m$	$l_{fe} = 0.1m$
f_{real} [Hz]	f_{fem} [Hz]	f_{fem} [Hz]	f_{fem} [Hz]	f_{fem} [Hz]
14.56	14.62	14.56	14.56	14.56
58.23	64.63	58.46	58.33	58.23
131.02	162.46	133.42	132.06	131.03
232.93	296.19	258.54	238.3	232.96

$$f_{real,k} = \frac{1}{2\pi}\left(\frac{k\pi}{L}\right)^2 \sqrt{\frac{EJ}{m}}$$

Frequency Response Function

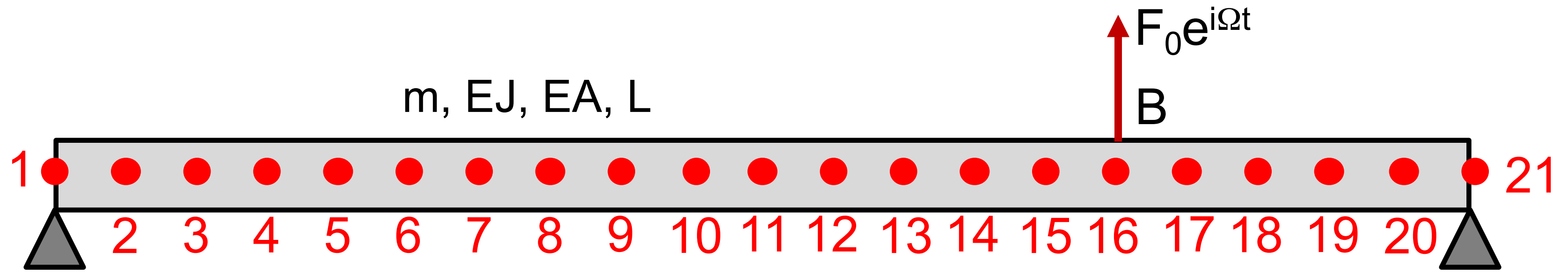
A harmonic force $F=F_0e^{i\Omega t}$ is applied in A: we want to calculate the FRF of the system (l_{fe} = 0.1m).

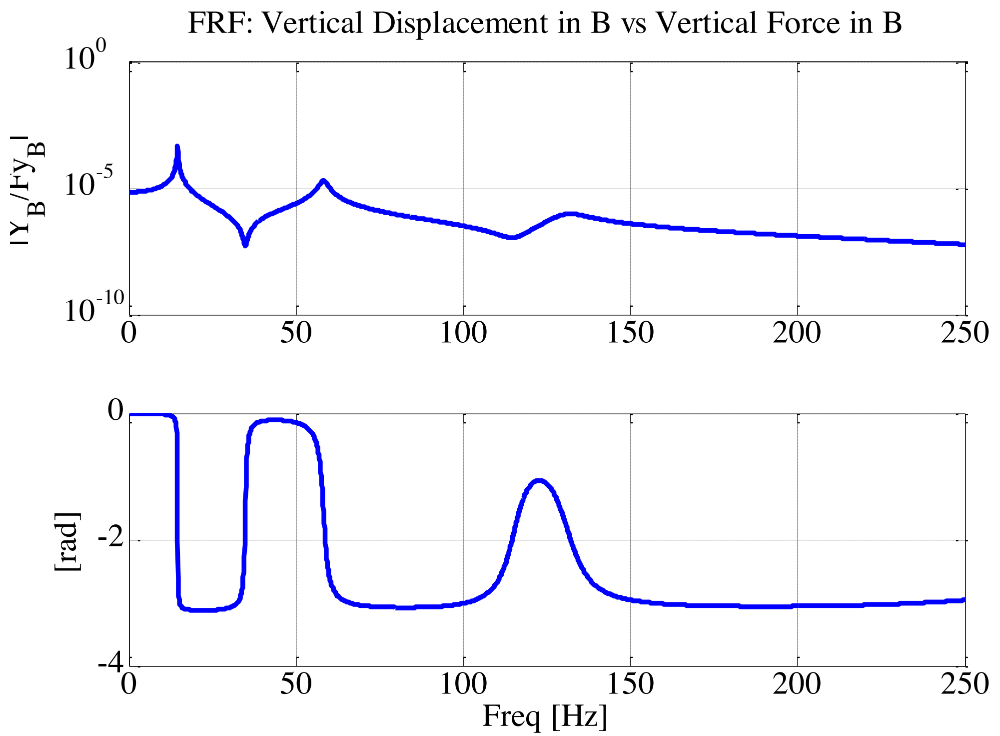




Now it's your turn…

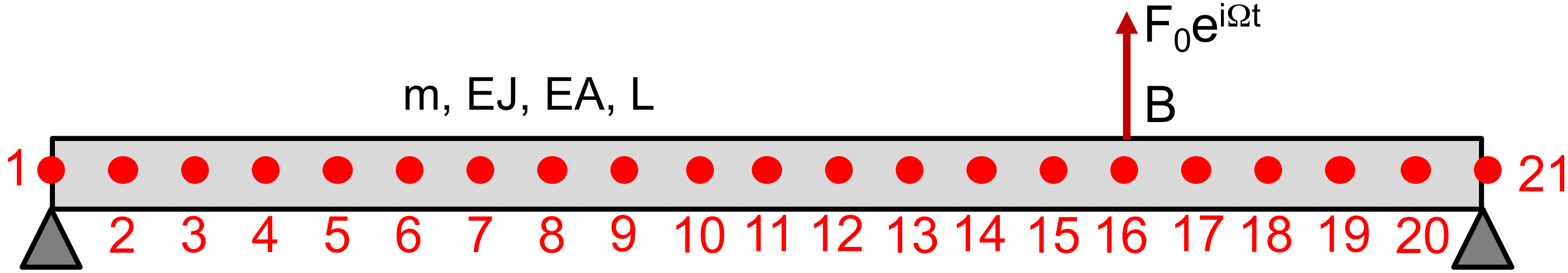
A harmonic force $F=F_0e^{i\Omega t}$ is applied in B: we want to calculate the FRF of the system for the displacement in B (l_{fe} = 0.1m).

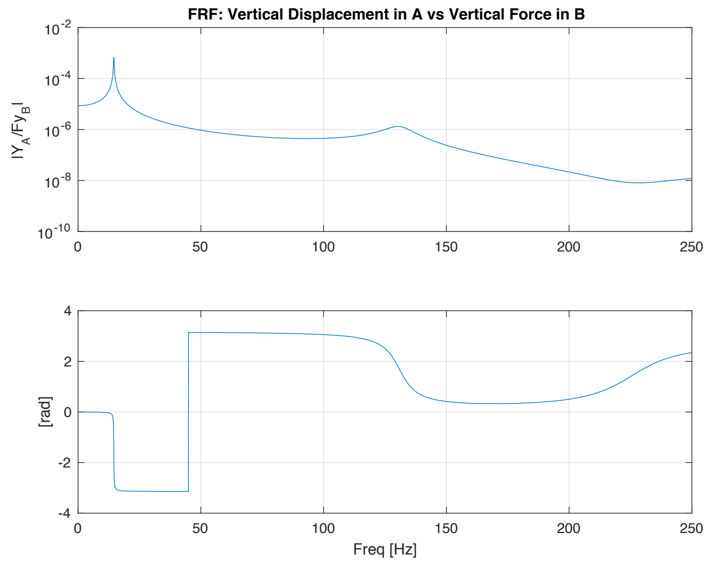




Frequency Response Function

A harmonic force $F=F_0e^{i\Omega t}$ is applied in B: we want to calculate the FRF of the system for the displacement <u>in A</u> (i.e. l_{fe} = 0.1m).





Modal shapes and Frequency Response Function

A harmonic force $F=F_0e^{i\Omega t}$ is applied in A: we want to calculate the modal shapes and the FRF of the system for the displacement in A (i.e. l_{fe} = 0.1m).