Fermi Problems: Cheat Sheet

SI prefixes

Name	*Order*
giga (G)	10^9
mega (M)	10^6
kilo (k)	10^3
hecto (h)	10^2
deca (da)	10^1
(base)	10^0
deci (d)	10^{-1}
centi (c)	10^{-2}
milli (m)	10^{-3}
micro (μ)	10^{-6}
nano (n)	10^{-9}

SI base units

Name	*Unit*
length	meter (m)
mass	kilogram (kg)
time	second (s)
temperature	kelvin (K)

SI derived units

Name	*Unit*	*Conversion*
volume	liter (L)	dm^3
force	newton (N)	$\mathrm{kg{\cdot}m{\cdot}s^{-2}}$
energy	joule (J)	$\mathrm{N{\cdot}m}$
power	watt (W)	$\mathrm{J{\cdot}s^{-1}}$
metric ton	ton	1000 kg

Scientific Notation

A number in the form $a \times 10^n$, where $1 \leq a < 10$. The answer we want is just the value of $\boldsymbol{n}$.

- The restrictions on a are important! Notably, $720 = 7.2 \times 10^2 \rightarrow n = 2$; correct, not anything else (such as 0.72×10^3; wrong).
- Pay particular attention to single digits! $5 = 5 \times 10^{\mathbf{0}} \rightarrow n = 0$, **NOT** 5×10^1, which is 50(!!)
- For the purposes of this class, we will **not** take rounding into effect. Namely, 9.999×10^3 will still be answered as $n = 3$, even though this number is essentially 1×10^4.

Useful tips and tricks

- General problem solving technique: **Break it down!**
- The speed of light is approximately 3×10^8 m/s.
- The mass of an electron is approximately 1×10^{-30} kg.
- 1 quart is approximately 1 liter. There are 4 quarts in 1 gallon.
- 1 gram of TNT contains approximately 4×10^3 J of energy.
- Various distance conversions (approximately): $\begin{cases} 1 \text{ mile} & \approx 1.6\,\text{km} \approx 5000 \text{ feet} \\ 1 \text{ foot} & = 12 \text{ inches} \approx 30\,\text{cm} \end{cases}$

Have fun!!!

Problems

Team Name:

You will have approximately 20 minutes to solve 5 problems by applying some of the techniques we just learned. The problems are arranged roughly in order of difficulty, and it's OK if you do not finish all 5 (all questions have the same weight). Express each answer in proper scientific notation and remember that *we are only looking for the correct integer value of n.* The team with the **lowest** total score wins, which is computed as

$$\text{Score} = \sum |\text{guess} - \text{actual}|.$$

1) ________ How many letters are in the average English word?

2) ________ What is the sum of the ages of everyone in this room?

3) ________ How many seconds are in a year?

4) ________ If a faucet leaked one drop of water each second, then how many gallons of water are wasted each year?

5) ________ What is the diameter of the Earth in kilometers?

Fermi Problems Lesson Plan

1 Introduction

1.1 Background

Fermi problems, or back-of-the-envelope calculations, involve making approximations and estimates that would otherwise burden a series of calculations. Their namesake, Italian physicist Enrico Fermi, was known for his ability to make good approximations with little data, such as at the Trinity test for the first atomic bomb.

1.2 Practicality

- Fast
- Double check figures, so you don't get duped.
- Fun!
- Examples:
  - Receipts
  - Utility bills
  - Project resources

2 Mathematical Background

2.1 Order of Magnitude

The whole idea of Fermi problems is to estimate based on order of magnitude, or commonly known as the power of 10. For this, we turn to scientific notation, which is a number in the form $a \times 10^n$, where $1 \leq a < 10$.

- Example: $2017 = 2.017 \times 10^3$, whereas $33 = 3.3 \times 10^1$, so we say 2017 is of order 3, 33 is of order 1, and 2017 is 2 orders of magnitude larger than 33.
- The restrictions on a are important! Notably, $720 = 7.2 \times 10^2 \to n = 2$; correct, not anything else (such as 0.72×10^3; wrong).
- Pay particular attention to single digits! $5 = 5 \times 10^{\mathbf{0}} \to n = 0$, **NOT** 5×10^1, which is 50(!!)

2.2 Dimensional Analysis

The second component of Fermi problems is dimensional analysis, which is the task of identifying and keeping track of units during calculations. With any scientific or engineering discipline, you will have to learn how to convert between quantities using different units and their relationships.

- Example: If I can run 1 mile in 10 minutes, then how many feet do I run in 3000 seconds?

$$\frac{3000\text{ s}}{1}\frac{1\text{ min}}{60\text{ s}}\frac{1\text{ mi}}{10\text{ min}}\frac{5280\text{ ft}}{1\text{ mi}} = 26,400\text{ ft}.$$

- If at any time in science class you're unsure of how to start a problem, dimensional analysis is a *great* way to figure out how the constants you're given mix and match to obtain the right answer.

3 Strategies

3.1 Break it down

Let's try to tackle a very classic Fermi problem: How many piano tuners are in New York City? If we try to guess an answer straight up, we would likely be very far off. Let's break the problem down and tackle it one step at a time.

1. How many people live in NYC? 8,000,000.
2. Does every person own a piano? No.
3. Can we assume that families own pianos, not individuals? Yes.
4. How large is the average family? Five people.
5. How many families in NYC then? 1,600,000.
6. Does every family own a piano? No... maybe 1 in 5 does.
7. So how many pianos are there in NYC? 320,000.
8. How often are pianos tuned? Once per year.
9. How many pianos can one piano tuner do? 4 per day = 800 per year.
10. Therefore, NYC can support $320,000/800 = \boxed{400}$ piano tuners.

3.2 Judicious grouping/rounding

The second trick, one that is often handy for multiplying a lot of little numbers, is to group numbers together that cancel out completely or multiply to get a power of 2 or 10.
Example: The electric potential energy between two neon ($Z = 10$) nuclei spaced 1 m apart.

3.3 Set some bounds—then guess

Lastly, as a good first step to build intuition, or when you're completely lost, a good strategy is to set lower and upper bounds for what the answer *could* be, then guess something in between.
Example: How many students are here this weekend?