# Econometrics I - TA section

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TEXAS
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- Define the 2SLS estimator of $\beta$ using $z_i$ as an instrument for $x_i$. How does this differ from OLS? $X'X^{-1}Z'y$

# Chapter 13

Method of moments.

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and consider the GMM estimator $\hat{\beta}$ of $\beta$. Let

$$J = n\bar{g}_n(\hat{\beta})'\hat{\Omega}^{-1}\bar{g}_n(\hat{\beta})$$

denote the test of over-identifying restrictions. Show that $J \to d\chi^2_{\ell-k}$ as $n \to \infty$.

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  - $J = n\bar{g}_n(\hat{\beta})'C(II')C'\bar{g}_n(\hat{\beta})$
  - $J = n\bar{g}_n(\hat{\beta})'C(C^{-1}CC'C'^{-1})C'\bar{g}_n(\hat{\beta})$

Z' X)$^{-1}X'Z\hat{\Omega}Z'y$

Thus $\hat{e} = (X'Z\hat{\Omega}^{[} - 1Z'X)^{-1}X'Z\hat{\Omega}Z'e$

Plugging into $C'\bar{g}_n(\hat{\beta})$ we get the result.

- $D_n \to_p I_\ell R(R'R)^{-1}R'$ where $R = C'E(z_i x_i')$ Apply LLN
- $\sqrt{n} C' \bar{g}_n$