Econometrics I - TA section

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Cheat sheet- Sigma algebra

Sigma algebras are "possible probability events". Ω is all events. A subset $S \subset \Omega$ is a sigma algebra if

1. $\Omega \in S$
2. $A \in S$ implies $A^c \in S$
3. $\cup_{n=1}^{\infty} A_n \in S$ i.e. $A_1 \cup A_2 \cup A_3 ... \in S$

Cheat sheet- Measures

Sigma algebra's are useful because you can measure them (just like you can measure distances)...

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You measure events A in a sigma algebra using a probability measure μ.

1. $\mu(A) \geq 0$
2. $\mu(\emptyset) = 0$
3. $\mu(\cup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mu(A_n)$ when A_n are disjoint

1.1 HIE- "Warm up" $A = \{a, b, c, d\}, B = \{a, c, e, f\}$ mechanics of unions and intersections...

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- $B \setminus A = \{e, f\}$

Probability/Measure Theory- “Warm-up”

1.2 HIE Describe the sample space for...

- A coin flip -

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- A coin flip - 2 outcomes $\{heads, tails\}$
- A 6-sided die roll -

Probability/Measure Theory- "Warm-up"

1.2 HIE Describe the sample space for...

- A coin flip - 2 outcomes $\{heads, tails\}$
- A 6-sided die roll - 6 outcomes $\{1, 2, 3, 4, 5, 6\}$

1.2 HIE

Describe the sample space for...

- 2, 6 sided die rolls -

1.2 HIE

Describe the sample space for...

- 2, 6 sided die rolls - $6 \times 6 = 36$ outcomes

$$\{(1,1),(1,2),(1,3),(1,4),(1,5),\\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)...\}$$

- Shoot 6 free-throws -

1.2 HIE

Describe the sample space for...

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- Shoot 6 free-throws - $2^6 = 64$ outcomes

$$\{000000,\\ 100000, 110000, 111000, 111100, 111110,\\ 111111, 010000, 011000, ...\}$$

1.6 HIE

Prove $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

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- One of the properties of probability measures involves disjoint events...
- For any sets A and B, unions consist of 3 disjoint events

$$A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A).$$

$$P(A \cup B) = P(A \setminus B) + P(A \cap B) + P(B \setminus A)$$

1.6 HIE

Prove $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

- One of the properties of probability measures involves disjoint events...
- For any sets A and B, unions consist of 3 disjoint events

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$$P(A \cup B) = P(A \setminus B) + P(A \cap B) + P(B \setminus A)$$

- Can further break up $A \setminus B$ into disjoint events

$$P(A \setminus B) = P(A) - P(A \cap B) \; P(B \setminus A) = P(B) - P(A \cap B)$$

The result follows.

Bayes rule/conditional probability

Cheat sheet- Bayes' rule/Independence

- Conditional distribution (discrete RV) - $P(A|B) = P(A \cap B)/P(B)$
- Bayes' rule - $P(A|B)P(B) = P(B|A)P(A)$
- Independence - $P(A \cap B) = P(A)P(B)$

Definition of conditional probabilities is a little deeper i.e. continuous outcomes...

1.8 HIE

$P(A \cap B) = P(A)$. Can A and B be independent?

Bayes rule/conditional probability

1.8 HIE

$P(A \cap B) = P(A)$. Can A and B be independent?

- Yes, $B = \Omega$
- $P(A \cap B) = P(A)P(B) = P(A)$

Bayes rule/conditional probability

1.10 HIE

Is $P(A|B) \leq P(A)$ is $P(A|B) \geq P(A)$ or $P(A|B) \geq P(A)$ or are both possible?

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- Both are possible. Depends how "related" A and B are
- Ex: $A = B$ Then $P(A|B) = 1 \geq P(A)$

Bayes rule/conditional probability

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- Both are possible. Depends how "related" A and B are
- Ex: $A = B$ Then $P(A|B) = 1 \geq P(A)$
- Ex: $A = B^c$, then $P(A|B) = 0 \leq P(A)$

Bayes rule/conditional probability

1.11 HIE $P(A) > 0$ and $P(A|B) = 0$. A and B are mutually exclusive i.e. $P(A \cap B) = 0$

1.17 HIE

Suppose 1 % of athletes use banned steroids. Suppose a drug test has a detection rate of 40 % and a false positive rate of 1 %. If an athlete tests positive what is the conditional probability that the athlete has taken banned steroids?

Bayes rule/conditional probability

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Suppose 1 % of athletes use banned steroids. Suppose a drug test has a detection rate of 40 % and a false positive rate of 1 %. If an athlete tests positive what is the conditional probability that the athlete has taken banned steroids?

- $P(\text{steriods}) = .01$
- $P(\text{positive}|\text{steriods}) = .4$
- $P(\text{positive}|\text{no steriods}) = .01$
- $P(\text{steriods}|\text{positive}) =?$

Bayes rule/conditional probability

1.17 HIE

- $P(\text{steriods}|\text{positive}) =$

Bayes rule/conditional probability

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- $P(\text{steriods}|\text{positive}) = \dfrac{P(\text{positive}|\text{steriods})P(\text{steriods})}{P(\text{positive})}$

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- and...
  $P(\text{positive}) = P(\text{positive}|\text{no steriods})P(\text{no steriods}) + P(\text{positive}|\text{steriods})P(\text{steriods})$

Bayes rule/conditional probability

1.17 HIE

- $P(\text{steriods}|\text{positive}) = \dfrac{P(\text{positive}|\text{steriods})P(\text{steriods})}{P(\text{positive})}$
- and...
  $P(\text{positive}) = P(\text{positive}|\text{no steriods})P(\text{no steriods}) +$
  $P(\text{positive}|\text{steriods})P(\text{steriods})$
- So, we get $= \dfrac{.4 \times .01}{.4 \times .01 + .01 \times .99}$

Bayes rule/conditional probability

1.22 HIE: In the poker game "Five Card Draw" a player first receives five cards drawn at random. The player decides to discard some of their cards and then receives replacement cards.

- Assume a player is dealt a hand with one pair and three unrelated cards
- and decides to discard the three unrelated cards to obtain replacements.
- Calculate the following conditional probabilities for the resulting hand after the replacements are made.

1.22 HIE

four of a kind

1.22 HIE

four of a kind

- How many possible outcomes are there?
- 2 pair, 1 irrelevant. 1 irrelevant, 2 pair. 1 pair, irrelevant card, 1 pair.
- What is the probability one of these events occurs?

Bayes rule/conditional probability

1.22 HIE

four of a kind

- 5/52 cards in the hand, so 47/52 cards left in the deck
- 2 pair, 1 irrelevant card

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- 5/52 cards in the hand, so 47/52 cards left in the deck
- 2 pair, 1 irrelevant card
- $2/47 \times 1/46 \times 45/45$ is the probability
- multiply by 3 for the answer

1.22 HIE

three of a kind

- This time 3 outcomes are:
- 2 irrelevant, 1 pair. 1 pair, 2 irrelevant. 1 irrelevant, 1 pair, 1 irrelevant
- Order doesn't matter among remaining pair

1.22 HIE

three of a kind

- This time 3 outcomes are:
- 2 irrelevant, 1 pair. 1 pair, 2 irrelevant. 1 irrelevant, 1 pair, 1 irrelevant
- Order doesn't matter among remaining pair
- Probability of 2 irrelevant, 1 pair is
- $45/47 \times 44/46 \times 2/45$
- multiply by 3 for the answer

Bayes rule/conditional probability

1.22 HIE

two pair

Bayes rule/conditional probability

1.22 HIE

two pair

- Outcomes?

1.22 HIE

two pair

- Outcomes?
- Like before... 2 pair, 1 irrelevant. 1 irrelevant, 2 pair. 1 pair, irrelevant card, 1 pair.

Bayes rule/conditional probability

1.22 HIE

two pair

- Outcomes?
- Like before... 2 pair, 1 irrelevant. 1 irrelevant, 2 pair. 1 pair, irrelevant card, 1 pair.
- The 3 cards you discarded matter! Actually 6 possible events.
- 3/13 of the types only have 3 in the deck. 9/13 have 4 in the deck. This doubles the number of events.
- 1/13 have 2 in the deck

1.22 HIE

two pair

- types with 3 cards 3 cards left $= 45/47 \times 2/46 \times 42/45$
- first card could be anything... second card is it's pair (2 left)... last card is not one of the remaining pairs (45 - 2- 1).
- types with 4 cards 4 cards left $= 45/47 \times 3/46 \times 41/45$
- Final answer,
  $3 \times (9/13 \times (3 \text{ cards left}) + \times 9/13 \times (4 \text{ cards left}))$