Econometrics I - TA section

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2.4

Suppose that the random variables Y and X only take the values 0 and 1, and have the following joint probability distribution

	$x = 0$	$x = 1$
$y = 0$	.1	.2
$y = 1$	.4	.3

Find $E(y|x)$, $E(y^2|x)$ and $var(y|x)$ for $x = 0$ and $x = 1$

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- $Pr(y = 0|x = 0) = \dfrac{Pr(y = 0, x = 0)}{P(x = 0)} = \dfrac{.1}{.1 + .4} = .2$
- $Pr(y = 1|x = 0) = \dfrac{.1}{.1 + .4} = .8$
- $Pr(y = 0|x = 1) = \dfrac{.2}{.2 + .3} = .4$
- $Pr(y = 1|x = 1) = \dfrac{.2}{.2 + .3} = .6$

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- $E(y|x=1) = .6$
- $var(y^2|x=0) = E(y^2|x=0) - (E(y|x=0))^2 = .8 - .8^2 = .16$
- $var(y^2|x=1) = E(y^2|x=1) - (E(y|x=1))^2 = .24$

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$$\text{Minimize } E((e^2 - h(x))^2)$$

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$$E((e^2 - h(x))^2) =$$
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$$E((e^2 - \sigma^2(x))^2) + E((\sigma^2(x) - h(x))^2) + 2E((\sigma^2(x) - h(x))(e^2 - \sigma^2(x)))$$

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$$E((\sigma^2(x) - h(x))(e^2 - \sigma^2(x)))$$

- Note $\sigma^2(x) = E(e^2|x)$ so using LIE

$$= E(E((\sigma^2(x) - h(x))(e^2 - \sigma^2(x))|x)) = 0$$

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- Thus

$$E((e^2 - g(x))^2) = E((e^2 - \sigma^2(x))^2) + E((\sigma^2(x) - h(x))^2)$$

$$\geq E((e^2 - \sigma^2(x))^2)$$

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What if,

$$E(e|x) = 0$$
$$E(e^2|x) = \sigma^2$$
$$E(e^3|x) = x^3$$

3.10

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- What about P_1 and P_2. Both are $n \times n$

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$$P = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} X_1'X_1 & X_1'X_2 \\ X_2'X_1 & X_2'X_2 \end{bmatrix}^{-1} \begin{bmatrix} X_1 & X_2 \end{bmatrix}' =$$

$$\begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} (X_1'X_1)^{-1} & 0 \\ 0 & (X_2'X_2)^{-1} \end{bmatrix}^{-1} \begin{bmatrix} X_1 & X_2 \end{bmatrix}' =$$

$$X_1(X_1'X_1)^{-1}X_1' + X_2(X_2'X_2)^{-1}X_2' =$$

$$P_1 + P_2$$

3.22

You estimate a least-squares regression

$$y_i = x_{1i}'\tilde{\beta}_1 + \tilde{u}_i$$

and then regress the residuals on another set of regressors

$$\tilde{u}_i = x_{2i}'\tilde{\beta}_2 + \tilde{e}_i$$

Does this second regression give you the same estimated coefficients as from estimation of a least-squares regression on both set of regressors?

$$y_i = x_{1i}'\hat{\beta}_1 + x_{2i}'\hat{\beta}_2 + \hat{e}_i$$

Explain your reasoning. In other words, is it true that $\tilde{\beta}_2 = \hat{\beta}_2$

3.22

- $\tilde{U} = Y - X_1(X_1'X_1)^{-1}X_1'Y$
- $\tilde{\beta} = (X_2'X_2)^{-1}X_2'\tilde{U}$

Meanwhile

- $$\hat{\beta} = \begin{bmatrix} X_1'X_1 & X_1'X_2 \\ X_2'X_1 & X_2'X_2 \end{bmatrix}^{-1} \begin{bmatrix} X_1 & X_2 \end{bmatrix}' Y$$
- Can use the partition inverse formula to show they are different
- $A^{-1} = \begin{bmatrix} (A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1} & -A_{11}^{-1}A_{12}(A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1} \\ -A_{22}^{-1}A_{21}(A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1} & (A_{22} - A_{12}A_{11}^{-1}A_{12})^{-1} \end{bmatrix}$

So, no these are different.

Computer questions

- Question 3.22
- Homework Question 1
- Homework Question 2