Econometrics I - TA section

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12.7

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• Define the 2SLS estimator of β using z_i as an instrument for x_i . How does this differ from OLS? $X'X^{-1}Z'y$

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$$y_i = x_i' \beta + e_i$$

and consider the GMM estimator $\hat{\beta}$ of β . Let

$$J = n\bar{g}_n(\hat{\beta})'\hat{\Omega}^{-1}\bar{g}_n(\hat{\beta})$$

denote the test of over-identifying restrictions. Show that $J \to d\chi^2_{\ell-k}$ as $n \to \infty$.

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 - Do an eigenvalue decomposition of $\Omega = PDP^{-1}$
 - Ω is symmetric so $P' = P^{-1}$
 - $\Omega > 0$ i.e. PSD means $D^{1/2} > 0$ (See appendix A.10)
 - $C = (PD^{1/2})^{-1}$

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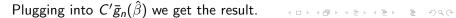
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 - $J = n\bar{g}_n(\hat{\beta})'C(II')C'\bar{g}_n(\hat{\beta})$
 - $J = n\bar{g}_n(\hat{\beta})'C(C^{-1}CC'C'^{-1})C'\bar{g}_n(\hat{\beta})$

$$(Z' X)^{-1}X'Z\hat{\Omega}Z'y$$

Thus
$$\hat{e} = (X'Z\hat{\Omega}^{[} - 1Z'X)^{-1}X'Z\hat{\Omega}Z'e$$



- $D_n \rightarrow_p I_\ell R(R'R)^{-1}R'$ where $R = C'E(z_ix_i')$ Apply LLN
- $\sqrt{n}C'\bar{g}_n$