Econometrics I - TA section

Eric Schulman

The University of Texas at Austin

September 19, 2020

1.5 HIE

If $P(A) = 1/2$ and $P(B) = 2/3$ can A and B be disjoint? Explain.

1.5 HIE

If $P(A) = 1/2$ and $P(B) = 2/3$ can A and B be disjoint? Explain.

- No, $P(A) + P(B) > 1$
- Disjoint events sum to at most 1.

Homework 1

1.12 HIE

1. Drawing a King with one card.

Homework 1

1.12 HIE

1. Drawing a King with one card. $4/52$

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card.

Homework 1

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card. $3/51$
3. Drawing two Kings with two cards.

Homework 1

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card. $3/51$
3. Drawing two Kings with two cards. $4/52 \times 3/51$

Homework 1

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card. $3/51$
3. Drawing two Kings with two cards. $4/52 \times 3/51$
4. Drawing a King on the second card, conditional on the first card is not a King.

Homework 1

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card. $3/51$
3. Drawing two Kings with two cards. $4/52 \times 3/51$
4. Drawing a King on the second card, conditional on the first card is not a King. $4/51$
5. Drawing a King on the second card, when the first card is placed face down (so is unknown).

Homework 1

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card. $3/51$
3. Drawing two Kings with two cards. $4/52 \times 3/51$
4. Drawing a King on the second card, conditional on the first card is not a King. $4/51$
5. Drawing a King on the second card, when the first card is placed face down (so is unknown).
   $3/51 \times 4/52 + 4/51 \times 48/52$
   - Does it make a difference?

1.12 HIE

1. Drawing a King with one card. $4/52$
2. Drawing a King on the second card, conditional on a King on the first card. $3/51$
3. Drawing two Kings with two cards. $4/52 \times 3/51$
4. Drawing a King on the second card, conditional on the first card is not a King. $4/51$
5. Drawing a King on the second card, when the first card is placed face down (so is unknown).
   $3/51 \times 4/52 + 4/51 \times 48/52$
   - Does it make a difference?
   - As expected, drawing a card face down doesn't effect the probability of drawing a king...

1.14 HIE

1. Getting three heads in a row from three coin flips.

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.
   $1/3$ i.e. 4 possible outcomes, 1 is “ruled out” b/c only tails

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.
   $1/3$ i.e. 4 possible outcomes, 1 is "ruled out" b/c only tails
4. Rolling a six from a pair of dice.

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.
   $1/3$ i.e. 4 possible outcomes, 1 is "ruled out" b/c only tails
4. Rolling a six from a pair of dice.
   $11/36$ i.e. 36 outcomes, don't double count (6,6)

Homework 1

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.
   $1/3$ i.e. 4 possible outcomes, 1 is "ruled out" b/c only tails
4. Rolling a six from a pair of dice.
   $11/36$ i.e. 36 outcomes, don't double count (6,6)
5. Rolling "snakes eyes" from a pair of dice. (Getting a pair of ones.)

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.
   $1/3$ i.e. 4 possible outcomes, 1 is "ruled out" b/c only tails
4. Rolling a six from a pair of dice.
   $11/36$ i.e. 36 outcomes, don't double count (6,6)
5. Rolling "snakes eyes" from a pair of dice. (Getting a pair of ones.)
   $1/36$

1.14 HIE

1. Getting three heads in a row from three coin flips.
   $(1/2)^3$
2. Getting a heads given that the previous coin was a tails.
   $1/2$
3. From two coin flips getting two heads given that at least one coin is a heads.
   $1/3$ i.e. 4 possible outcomes, 1 is "ruled out" b/c only tails
4. Rolling a six from a pair of dice.
   $11/36$ i.e. 36 outcomes, don't double count (6,6)
5. Rolling "snakes eyes" from a pair of dice. (Getting a pair of ones.)
   $1/36$

1.19 HIE

1. "When I selected door A the probability that it has the prize was 1/3. No information was revealed. So the probability that Door A has the prize remains 1/3."

1.19 HIE

1. "When I selected door A the probability that it has the prize was $1/3$. No information was revealed. So the probability that Door A has the prize remains $1/3$." False

1.19 HIE

1. "When I selected door A the probability that it has the prize was $1/3$. No information was revealed. So the probability that Door A has the prize remains $1/3$." False
2. "The original probability was $1/3$ on each door. Now that door B is eliminated, doors A and C each have each probability of $1/2$. It does not matter if I stay with A or switch to C."

Homework 1

1.19 HIE

1. "When I selected door A the probability that it has the prize was $1/3$. No information was revealed. So the probability that Door A has the prize remains $1/3$." False
2. "The original probability was $1/3$ on each door. Now that door B is eliminated, doors A and C each have each probability of $1/2$. It does not matter if I stay with A or switch to C." False

Homework 1

1.19 HIE

1. "When I selected door A the probability that it has the prize was $1/3$. No information was revealed. So the probability that Door A has the prize remains $1/3$." False
2. "The original probability was $1/3$ on each door. Now that door B is eliminated, doors A and C each have each probability of $1/2$. It does not matter if I stay with A or switch to C." False

1.19 HIE

1. "The host inadvertently revealed information. If door C had the prize, he was forced to open door B. If door B had the prize he would have been forced to open door C. Thus it is quite likely that door C has the prize."

1.19 HIE

1. "The host inadvertently revealed information. If door C had the prize, he was forced to open door B. If door B had the prize he would have been forced to open door C. Thus it is quite likely that door C has the prize." True

1.19 HIE

1. "The host inadvertently revealed information. If door C had the prize, he was forced to open door B. If door B had the prize he would have been forced to open door C. Thus it is quite likely that door C has the prize." True
2. Assume a prior probability for each door of 1/3. Calculate the posterior probability that door A and door C have the prize. What choice do you recommend for the contestant?

1.19 HIE

1. "The host inadvertently revealed information. If door C had the prize, he was forced to open door B. If door B had the prize he would have been forced to open door C. Thus it is quite likely that door C has the prize." True
2. Assume a prior probability for each door of $1/3$. Calculate the posterior probability that door A and door C have the prize. What choice do you recommend for the contestant?
   $1/3$ chance A. $2/3$ chance B or C. We know C does not have the prize. Thus $2/3$ chance B has the prize.

1.21 HIE

1. A straight

1.21 HIE

1. A straight
   This is five cards in a sequence (e.g., 4,5,6,7,8), with aces allowed to be either 1 or 13 (low or high) and with the cards allowed to be of the same suit (e.g., all hearts) or from some different suits. The number of such hands is $10 \times (4 choose 1)^5$
2. A flush

Homework 1

1.21 HIE

1. A straight
   This is five cards in a sequence (e.g., 4,5,6,7,8), with aces allowed to be either 1 or 13 (low or high) and with the cards allowed to be of the same suit (e.g., all hearts) or from some different suits. The number of such hands is $10 \times (4 choose 1)^5$
2. A flush
   Here all 5 cards are from the same suit (they may also be a straight). The number of such hands is (4-choose-1)* (13-choose-5).
3. A full house

1.21 HIE

1. A straight
   This is five cards in a sequence (e.g., 4,5,6,7,8), with aces allowed to be either 1 or 13 (low or high) and with the cards allowed to be of the same suit (e.g., all hearts) or from some different suits. The number of such hands is $10 \times (4 choose 1)^5$
2. A flush
   Here all 5 cards are from the same suit (they may also be a straight). The number of such hands is (4-choose-1)* (13-choose-5).
3. A full house
   This hand has the pattern AAABB where A and B are from distinct kinds. The number of such hands is $(13 - choose - 1)(4 - choose - 3)(12 - choose - 1)(4 - choose - 2)$

1.21 HIE

1. A straight

Homework 1

1.21 HIE

1. A straight
   - This is five cards in a sequence (e.g., 4,5,6,7,8)
   - with aces allowed to be either 1 or 13 (low or high) i.e. 10 such sequences, can't start on J,Q,K
   - the cards allowed to be of the same suit (e.g., all hearts) or from some different suits i.e. $(4 - \textit{choose} - 1)$
   - number of such hands is $10 \times (4 - \textit{choose} - 1)^5$
   - Denominator is $(52 - \textit{choose} - 5)$
2. A flush

Homework 1

1.21 HIE

1. A straight
   - This is five cards in a sequence (e.g., 4,5,6,7,8)
   - with aces allowed to be either 1 or 13 (low or high) i.e. 10 such sequences, can't start on J,Q,K
   - the cards allowed to be of the same suit (e.g., all hearts) or from some different suits i.e. $(4 - \mathit{choose} - 1)$
   - number of such hands is $10 \times (4 - \mathit{choose} - 1)^5$
   - Denominator is $(52 - \mathit{choose} - 5)$
2. A flush
   - Here all 5 cards are from the same suit (they may also be a straight). The number of such hands is $(4 - \mathit{choose} - 1) \times (13 - \mathit{choose} - 5)$
   - Denominator is $(52 - \mathit{choose} - 5)$

1.21 HIE

3. A full house
   - This hand has the pattern AAABB where A and B are from distinct kinds.
   - The number of such hands is $(13-choose-1)(4-choose-3)(12-choose-1)(4-choose-2)$
   - Denominator is $(52-choose-5)$

Homework 2 Hints

2.1 HIE

Let $X \sim U[0,1]$. Find the PDF of $Y = X^2$

2.1 HIE

Let $X \sim U[0,1]$. Find the PDF of $Y = X^2$

- $Y = g(X)$ so, $X = g^{-1}(Y) = Y^{1/2}$

Homework 2 Hints

2.1 HIE

Let $X \sim U[0,1]$. Find the PDF of $Y = X^2$

- $Y = g(X)$ so, $X = g^{-1}(Y) = Y^{1/2}$
- $Pr(Y < y) = F_y(y) = F_x(g^{-1}(y)) = y^{1/2}$

Homework 2 Hints

2.1 HIE

Let $X \sim U[0,1]$. Find the PDF of $Y = X^2$

- $Y = g(X)$ so, $X = g^{-1}(Y) = Y^{1/2}$
- $Pr(Y < y) = F_y(y) = F_x(g^{-1}(y)) = y^{1/2}$
- If CDF is continuous and differentiable, density is first derivative.
- $dF_y(y)/dy = f_y(y) = 1/2y^{-1/2}$

Homework 2 Hints

2.1 HIE

Let $X \sim U[0,1]$. Find the PDF of $Y = X^2$

- $Y = g(X)$ so, $X = g^{-1}(Y) = Y^{1/2}$
- $Pr(Y < y) = F_y(y) = F_x(g^{-1}(y)) = y^{1/2}$
- If CDF is continuous and differentiable, density is first derivative.
- $dF_y(y)/dy = f_y(y) = 1/2y^{-1/2}$ where $0 \leq Y \leq 1$
- $f_y(y) = 0$ otherwise
- Note the bounds are important. This is a PDF, density must integrate to 1.

2.8 HIE

Show that if the density satisfies $f(x) = f(-x)$ for $x \in \mathcal{R}$ then the distribution function satisfies $F(-x) = 1 - F(x)$

Homework 2 Hints

2.8 HIE

Show that if the density satisfies $f(x) = f(-x)$ for $x \in \mathcal{R}$ then the distribution function satisfies $F(-x) = 1 - F(x)$

- We know: $f(t) = f(-t)$
- Also we know $F(x) = \int_{-\infty}^{x} f(t)dt$ for a continuous RV
- $F(-x) = \int_{-\infty}^{-x} f(t)dt$
- $= \int_{-\infty}^{-x} f(-t)dt$ from what we are given
- letting $s = -t$ i.e. change of variable
- $= \int_{x}^{\infty} f(s)ds$
- $= \int_{\infty}^{-\infty} f(s)ds - \int_{x}^{-\infty} f(s)ds$
- $= 1 - F(x)$

2.13 HIE

Find a which minimizes $E[(X-a)^2]$. Your answer should be an expectation or moment of X

Homework 2 Hints

2.13 HIE

Find a which minimizes $E[(X-a)^2]$. Your answer should be an expectation or moment of X

- $E((X-a)^2) = a^2 - 2E(X)a + E(X^2) = (a - E(X))^2 + E(X^2) - E(X)^2$.
- Considering this as a quadratic function on a
- it is clear that the minimum is acquired when $a = E(X)$.

Homework 2 Hints

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$

Homework 2 Hints

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$
   - $(1 - p) + p = 1$

Homework 2 Hints

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$
    - $(1 - p) + p = 1$
    - This should be clear from the definition of the density function for discrete RV
2. $E(x) = p$

Homework 2 Hints

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$
   - $(1 - p) + p = 1$
   - This should be clear from the definition of the density function for discrete RV
2. $E(x) = p$
   - $E(x) = \sum_{x=0}^{1} x \times \pi(x; p)$
   - $0 \times (1 - p) + 1 \times p = p$

Homework 2 Hints

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$
   - $(1 - p) + p = 1$
   - This should be clear from the definition of the density function for discrete RV
2. $E(x) = p$
   - $E(x) = \sum_{x=0}^{1} x \times \pi(x; p)$
   - $0 \times (1 - p) + 1 \times p = p$
3. $Var(x) = p(1 - p)$

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$
    - $(1 - p) + p = 1$
    - This should be clear from the definition of the density function for discrete RV
2. $E(x) = p$
    - $E(x) = \sum_{x=0}^{1} x \times \pi(x; p)$
    - $0 \times (1 - p) + 1 \times p = p$
3. $Var(x) = p(1 - p)$
    - $Var(x) = E(x^2) - E(x)^2$
    - $[(0)^2(1 - p) + (1)^2 p] - p^2 = p - p^2 = p(1 - p)$

Homework 2 Hints

3.1 HIE

For the Bernoulli distribution show...

1. $\sum_{x=0}^{1} \pi(x; p) = 1$
   - $(1-p) + p = 1$
   - This should be clear from the definition of the density function for discrete RV
2. $E(x) = p$
   - $E(x) = \sum_{x=0}^{1} x \times \pi(x; p)$
   - $0 \times (1-p) + 1 \times p = p$
3. $Var(x) = p(1-p)$
   - $Var(x) = E(x^2) - E(x)^2$
   - $[(0)^2(1-p) + (1)^2 p] - p^2 = p - p^2 = p(1-p)$