Econometrics I - TA section

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8.19

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  $\beta_2 + \beta_3 exp/100 \geq 0$ and $exp \leq 50$ i.e.
  $\beta_2 + \beta_3 \geq 0$
- To estimate with the inequality constraint use KKT i.e. find solution to OLS with and without the constraint.
  - first estimate $y_i = ... + \beta_2 exp + \beta_3 exp^2/100 + ... + e_i$
  - Are $\hat{\beta}_2 + \hat{\beta}_3 \geq 0$? If so, need to run constrained least squares i.e. $y_i = .. + \beta_2(exp + exp^2/100)... + e_i$

8.22 Take the linear model

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$$E(x_i e_i) = 0$$

with n observations. Consider the restriction:
$\dfrac{\beta_1}{\beta_2} = 2$

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- Find an explicit expression for the constrained least squares estimator under the restriction.
- The restricted model is $y_i = \beta_2(2x_{i1} + x_{i2}) + e_i$
- The CLS estimator is
  $\tilde{\beta}_2 = \left(\sum_{i=1}^n (2x_{i1} + x_{i2})(2x_{i1} + x_{i2})'\right)^{-1} \left(\sum_{i=1}^n (2x_{i1} + x_{i2}) y_i\right)$

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- It's just OLS, $\tilde{\beta}_2 \to \mathcal{N}(0, V_\beta)$
- $V_\beta = \sigma^2 E((2x_{i1} + x_{i2})(2x_{i1} + x_{i2})')^{-1}$

9.7 Take the model $y_i = x_i\beta_1 + x_i^2\beta_2 + e_i$
$E(e_i|x_i) = 0$ where y_i is wages and x_i is age. Describe how you test the hypothesis that the expected wage for a 40-year old worker is 20 dollars an hour.

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- $E(y_i|x_i = 20) = 40\beta_1 + 1600\beta_2$, as a result:
- $H_0 : 20\beta_1 + 400\beta_2 = 20$ $H_1 : 20\beta_1 + 400\beta_2 \neq 20$
- Use a Wald test with the restrictions given in H_0
- Use a χ_q^2 with $q = 1$

9.8

You want to test $H_0 : \beta_2 = 0$ against $H_1 : \beta_2 \neq 0$ in the model
$y_i = x_{1i}'\beta_1 + x_{2i}'\beta_2 + e_i \ E(x_i e_i) = 0$
You read a paper which estimates the model
$y_i = x_{1i}'\hat{\gamma}_1 + (x_{2i} - x_{1i})'\hat{\gamma}_2 + e_i$
and reports $\gamma_2 = 0$ against $\gamma_2 \neq 0$. Is this related to the test you wanted to conduct?

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- Rearranging, the model in the paper is
  $y_i = x'_{1i}(\gamma_1 - \gamma_2) + x'_{2i}\gamma_2 + e_i$
- So, $\gamma_2 = 0$ is the same as testing $\beta_2 = 0$ i.e. $\beta_1 = \gamma_1 - \gamma_2$

9.17

You have two regressors x_1 and x_2 and estimate a regression with all quadratic terms.

$y_i = \alpha + \beta_1 x_{2i} + \beta_3 x_{1i}^2 + \beta_4 x_{2i}^2 + \beta_5 x_{1i} x_{2i} + e_i$

One of your advisors asks: Can we exclude the variable x_{2i} from this regression?

9.17

- What is the relevant null and alternative hypotheses?
  $H_0 : \beta_2 = 0, \beta_4 = 0, \beta_5 = 0$
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- What is an appropriate test statistic? Be specific
  Run a Wald-test i.e. $W = (r\hat{\beta} - \theta_0)' \hat{V}_{\theta}^{-1} (r\hat{\beta} - \theta_0)'$
  In this case, $W = \begin{bmatrix} \hat{\beta}_2 \\ \hat{\beta}_4 \\ \hat{\beta}_5 \end{bmatrix} \hat{V}^{-1}_{\beta_2,\beta_4,\beta_5} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_4 \\ \hat{\beta}_5 \end{bmatrix}'$

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- What is the rule for acceptance/rejection for the null hypothesis? For 95 percent confidence, reject if W is greater than 5.991.

9.20

You are reading a paper and it reports the result of two nested OLS regressions:

Short Regression	Long Regression
$R^2 = .20$	$R^2 = .26$
$\sum_{i=1}^{n} \bar{e}_i^2 = 106$	$\sum_{i=1}^{n} \hat{e}_i^2 = 100$
coefficients=5	coefficients=8
$n = 50$	$n = 50$

You are curious if the estimate of $\hat{\beta}_2$ is statistically different from 0. If there a away to determine an answer from this information?

9.20

- Yup use (9.12)

$$F = \frac{(\bar{\sigma}^2 - \hat{\sigma}^2)(n-k)}{\hat{\sigma}^2 q}$$

- Note (1) $1 - R^2_{short} = \bar{\sigma}^2 \times SST$, $1 - R^2 = \hat{\sigma}^2 \times SST$
- and (2) $SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$ in the denominator and numerator cancel.

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- $q = 3,\ n = 50,\ k = 8,\ R_{short} = .2,\ R^2 = .26$
- Use an F stat with $q, n-k$ degrees of freedom for the test.

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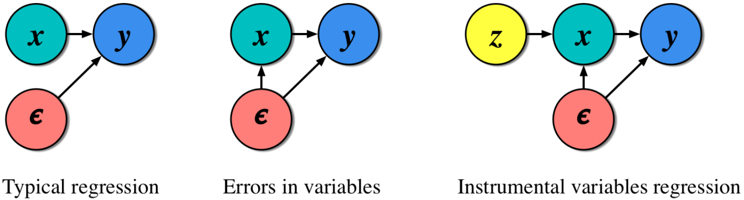

Figure: DAG for IV, OLS, simultaneous equations

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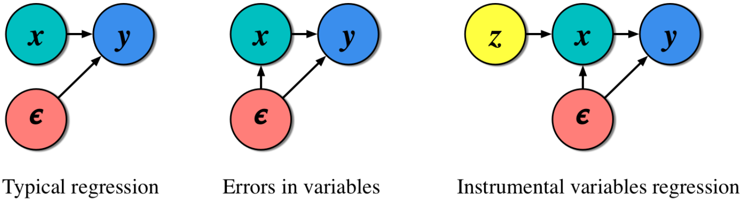

Figure: DAG for IV, OLS

- In this situation, we (1) project x_i onto z_i.
- (2) project the result onto y_i i.e. follow the arrows.

$$\hat{\beta}_{2sls} = (X'P_zX)^{-1}X'P_zy$$

- P_z is the projection matrix $P_z = Z(Z'Z)^{-1}Z'$

Chapter 12

12.3 Take the linear model $y = X\beta + e$ Let the OLS estimator for β be $\hat{\beta}$ and the OLS residual be $\hat{e} = y - X\hat{\beta}$.
Let the IV estimator for β using some instrument Z be β and the IV residual be $\bar{e} = y - X\bar{\beta}$. If X is indeed endogenous.

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Chapter 12

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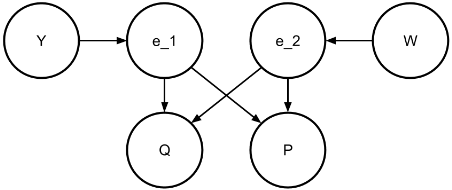
$$Q = a_0 + a_1 P + a_2 Y + e_1$$

$$Q = b_0 + b_1 P + b_2 W + e_2$$



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- W and Y go on the RHS because $Y, W \perp e_1, e_2$.

$$P = c_0 + c_1 Y + c_2 W + u_1$$

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- Is there a unique mapping between $c_0, c_1, c_2, d_0, d_1, d_2, u_1, u_2$ and $a_0, a_1, a_2, b_0, b_1, b_2, e_1, e_2$?

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- Note the distribution of e_1 and e_2 are actually parameters.

12.8 Is there a unique mapping between the reduced form (1) (2) and the structural form (3) (4)?

$$P = c_0 + c_1 Y + c_2 W + u_1 \tag{1}$$

$$Q = d_0 + d_1 Y + d_2 W + u_2 \tag{2}$$

$$Q = a_0 + a_1 P + a_2 Y + e_1 \tag{3}$$

$$Q = b_0 + b_1 P + b_2 W + e_2 \tag{4}$$

- From (1)

$$W = \frac{P - c_0 - c_1 Y - u_1}{c_2} \tag{5}$$

$$Y = \frac{P - c_0 - c_2 W - u_1}{c_1} \tag{6}$$

- Just plug (5) into (2) to get (3).
- Just plug (6) into (2) to get (4).

12.8

$$a_0 = d_0 - \frac{c_0}{c_2} \qquad b_0 = c_0 - \frac{c_0}{c_1}$$

$$a_1 = \frac{d_2}{c_1} \qquad b_1 = \frac{d_1}{c_2}$$

$$a_2 = d_1 - \frac{c_1}{c_2} \qquad b_2 = d_2 - \frac{c_2}{c_1}$$

$$e_1 = u_1 - \frac{u_2}{c_2} \qquad e_2 = u_1 - \frac{u_2}{c_1}$$