Econometrics I - TA section

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4.7

Let $\tilde{\beta}$ be the GLS estimator. Assume that $\Omega = c^2\Sigma$ with Σ known and c^2 unknown. Define the residual vector $\tilde{e} = y - X\tilde{\beta}$ and

$$\tilde{c}^2 = \frac{1}{n-k}\tilde{e}'\Sigma^{-1}\tilde{e}$$

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- Show (4.18) $E(\tilde{\beta}|X) = \beta$

$$\begin{aligned} E(\tilde{\beta}|X) &= E((X'\Sigma^{-1}X)^{-1}(X'\Sigma^{-1}y)|X) \\ &= E((X'\Sigma^{-1}X)^{-1}(X'\Sigma^{-1}(X\beta + e))|X) \\ &= \beta + E((X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}e|X) = \beta \end{aligned}$$

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\begin{aligned}
var(\tilde{\beta}|X) &= var((X'\Sigma^{-1}X)^{-1}(X'\Sigma^{-1}y)|X) \\
&= var((X'\Sigma^{-1}X)^{-1}(X'\Sigma^{-1}(X\beta + e))|X) \\
&= var((X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}e|X) \\
&= (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}\Omega(X'\Sigma^{-1})'((X'\Sigma^{-1}X)^{-1})'
\end{aligned}
$$

Plugging in $\Omega^{-1} = c^2\Sigma$ things cancel out and we get

$$= (X'\Omega^{-1}X)^{-1}$$

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$$\tilde{e} = y - X\tilde{\beta} = X\beta + e - X\tilde{\beta} = e - X(\tilde{\beta} - \beta)$$

Note

$$\tilde{\beta} - \beta = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}(\beta X + e) - \beta = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}e$$

Thus we get

$$\tilde{e} = e - X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}e = M_1 e$$

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  Start with,

$$(I - X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})'\Sigma^{-1}(I - X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})$$

$$= (I - X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})'(\Sigma^{-1} - \Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})$$

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Note that the second term,

$$(X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})'(\Sigma^{-1} - \Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}) =$$

$$(X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})'\Sigma^{-1}M_1 = 0$$

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$$E\left(\frac{1}{n-k}\tilde{e}'\Sigma^{-1}\tilde{e}|X\right) = \frac{1}{n-k}E((M_1 e)'\Sigma^{-1}M_1 e|X) =$$

$$= \frac{1}{n-k}E(e'(\Sigma^{-1} - \Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1})e|X)$$

$$= \frac{1}{n-k}E(e'\Sigma^{-1}M_1 e|X) = \frac{1}{n-k}tr(E(e'\Sigma^{-1}M_1 e|X))$$

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- Is $\tilde{c}^2$ a reasonable estimator for c^2? Yes, it's unbiased anyway.

4.20

$$y = X\beta + e$$

$$E(e|x) = 0$$

$$E(ee'|X) = \Omega$$

Assume Ω is known. Consider $\hat{\beta} = (X'X)^{-1}(X'y)$ and $\tilde{\beta} = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}y)$. Compute the conditional covariance between $\hat{\beta}$ and $\tilde{\beta}$, $E\left((\hat{\beta} - \beta)(\tilde{\beta} - \beta)' \mid X\right)$

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$$= E(\hat{\beta}\tilde{\beta} - \beta\tilde{\beta}' - \hat{\beta}\beta' + \beta\beta'|X) = E(\hat{\beta}\tilde{\beta}'|X) - \beta\beta'$$

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$$= E(\hat{\beta}\tilde{\beta} - \beta\tilde{\beta}' - \hat{\beta}\beta' + \beta\beta'|X) = E(\hat{\beta}\tilde{\beta}'|X) - \beta\beta'$$

Note that

$$E(\hat{\beta}\tilde{\beta}'|X) = E((X'X)^{-1}(X'y)((X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}y))'|X) =$$

$$E((X'X)^{-1}(X'(X\beta + e))((X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}(X\beta + e)))'|X) =$$

$$E((X'X)^{-1}ee'((X'\Omega X)^{-1})'|X) = \beta'\beta + (X'X)^{-1}\Omega((X'\Omega X)^{-1})'$$

4.20 Find the conditional covariance matrix for $\hat{\beta} - \tilde{\beta}$

$$E((\hat{\beta} - \tilde{\beta})(\hat{\beta} - \beta)'|X) =$$

$$E(\hat{\beta}\hat{\beta}' - \tilde{\beta}\hat{\beta}' - \hat{\beta}\tilde{\beta}' + \tilde{\beta}\tilde{\beta}'|X) =$$

$$(X'X)^{-1}\Omega((X'\Omega X)^{-1})' + (X'\Omega X)^{-1}\Omega((X'X)^{-1})'$$

7.11 Take a regression model with i.i.d. observations (y_i, x_i) and scalar x_i

$$y_i = x_i\beta + e_i$$

$$E(e_i|x_i) = 0$$

$$\Omega = E(x_i^2 e_i^2)$$

Let $\hat{\beta}$ the OLS estimator of β with residuals $\hat{e}_i = y_i - x_i\hat{\beta}$. Consider the estimators of Ω

$$\tilde{\Omega} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 e_i^2$$

$$\hat{\Omega} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \hat{e}_i^2$$

7.11 Find the asymptotic distribution of $\sqrt{n}(\tilde{\Omega} - \Omega)$ as $n \to \infty$

- General approach (1) we are given $x_i e_i$ are i.i.d. (2) compute mean of $x_i e_i$ (3) compute variance of $x_i e_i$ show it is bounded (4) apply CLT

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- First compute the mean. From the question $E(x_i^2 e_i^2) = \Omega$, thus $E(\tilde{\Omega}) = \Omega$.

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- Next compute variance. $var(x_i^2 e_i^2) = E(x_i^4 e_i^4) - \Omega^2$

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- Next compute variance. $var(x_i^2 e_i^2) = E(x_i^4 e_i^4) - \Omega^2$
- Apply the Lindeberg-Levy central limit theorem assuming $var(x_i^2 e_i^2) < \infty$. The limiting distribution is normal with variance given above.

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  First note that

$$\hat{\Omega} = \frac{1}{n} \sum_{i=1}^{n} x_i^2 (-(\hat{\beta} - \beta) x_i + e_i)^2$$

$$\hat{\Omega} = \tilde{\Omega} - \frac{2}{n} x_i^3 e_i (\hat{\beta} - \beta) + \frac{1}{n} x^4 (\beta - \hat{\beta})^2$$

  Thus,

$$\sqrt{n}(\hat{\Omega} - \Omega) = \sqrt{n}(\tilde{\Omega} - \Omega) - \frac{2}{n} \sum_{i=1}^{n} x_i^3 e_i \sqrt{n}(\hat{\beta} - \beta) + \frac{1}{n} \sum_{i} x_i^4 (\hat{\beta} - \beta) \sqrt{n}(\hat{\beta} - \beta)$$

7.11

- First, note that $\sqrt{n}(\tilde{\Omega} - \Omega) \xrightarrow{d} N(0, E(x_i^4 e_i^4))$ and $\sqrt{n}(\beta - \hat{\beta}) \xrightarrow{d} N(0, V_\beta)$
- Also note, $\frac{1}{n}\sum_{i=1}^{n} x_i^3 e_i \xrightarrow{p} E(x_i^3 e_i) = 0$
- And, $\frac{1}{n}\sum_i x_i^4(\hat{\beta} - \beta) \xrightarrow{p} 0$
- Apply Slutsky's theorem. We have a normally distributed random variable, which is the sum of 3 other normally distributed random variables (2 of them are multiplied by 0).
- It's variance is $E(x_i^4 e_i^4)$.

7.20

The data $\{y_i, x_i, w_i\}$ is from a random sample $i = 1, ..., n$. The parameter β is estimated by minimizing the criterion function

$$S(\beta) = \sum_{i=1}^{n} w_i (y_i - x_i' \beta)^2$$

That is $\hat{\beta} = \text{argmin} S(\beta)$

7.20

- Find an explicit expression for $\hat{\beta}$

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  Just OLS, however x_i and y_i are multiplied by $\sqrt{w_i}$

$$\hat{\beta} = \left(\sum_{i=1}^{n} w_i x_i x_i' \right)^{-1} \left(\sum_{i=1}^{n} w_i x_i y_i \right)$$

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  Let $y_i = x_i'\beta + e_i$ and $E(e_i | x_i, w_i) = 0$

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$$E\left(\hat{\beta}\right) = \beta$$

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- Find the probability limit for $\hat{\beta}$ as $n \to \infty$
  Apply the weak LLN, consistency follows from the previous part.
- Find the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta)$ as $n \to \infty$
  Apply CLT. Assuming that $var(e_i|w_i, x_i) < \infty$ and $var(w_i) < \infty$ and $var(x_i) < \infty$

7.28

$$y_i = x_i'\beta + e_i$$

$$E(e_i|x_i) = 0$$

An econometrician is worried about the impact of some unusually large values of the regressors. The model is thus estimated on the subsample for which $|x_i| \leq c$ for some fixed c. Let $\tilde{\beta}$ denote the OLS estimator on this subsample. It equals

$$\tilde{\beta} = \left(\sum_{i=1}^{n} x_i x_i' \mathbf{1}(|x_i| \leq c) \right)^{-1} \left(\sum_{i=1}^{n} x_i y_i \mathbf{1}(|x_i| \leq c) \right)$$

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- $\mathbf{1}(|x_i| \leq c)$ is basically a weight, can use generalized least squares techniques.
- This question is basically a special cause of 4.7

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$$E\left(\tilde{\beta}\right) = E\left(E(\beta|X)\right)$$

$$E\left(\left(\sum_{i=1}^{n} x_i x_i' \mathbf{1}(|x_i| \leq c)\right)^{-1} \left(\sum_{i=1}^{n} x_i y_i \mathbf{1}(|x_i| \leq c)\right) \mid X\right) =$$

$$E\left(\left(\sum_{i=1}^{n} x_i x_i' \mathbf{1}(|x_i| \leq c)\right)^{-1} \left(\sum_{i=1}^{n} x_i (x_i'\beta + e_i) \mathbf{1}(|x_i| \leq c)\right) \mid X\right) =$$

$$\beta + E\left(\left(\sum_{i=1}^{n} x_i x_i' \mathbf{1}(|x_i| \leq c)\right)^{-1} \sum_{i=1}^{n} x_i e_i \mathbf{1}(|x_i| \leq c) \mid X\right) = \beta$$

This establishes consistency. Now apply weak LLN.

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- $$avar\left(\left(\sum_{i=1}^{n} x_i x_i' \mathbf{1}(|x_i| \leq c)\right)^{-1} \sum_{i=1}^{n} x_i e_i \mathbf{1}(|x_i| \leq c)\right) =$$

  $$E(x_i x_i')^{-1} var(x_i e_i)(E(x_i x_i') Pr(|x_i| \leq c))^{-1})'$$

- Thus we can apply the central limit theorem.

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$$E(x_i x_i')^{-1} var(x_i e_i)(E(x_i x_i') Pr(|x_i| \le c))^{-1})'$$

- Thus we can apply the central limit theorem.
- Note this is inefficient assuming homoskedasticity i.e. the variance is

$$\sigma^2 (E(x_i x_i') Pr(|x_i| \le c))^{-1} > \sigma^2 E(x_i x_i')^{-1}$$