Econometrics I Homework 1

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1 Part I

2.2

$$E(E(xy|x))$$
$$= E(xE(y|x))$$
$$= E(a(x + bE(x))|x)$$
$$a(x + bE(x))$$

2.4

$$E(y|x = 0) = E(y^2|x = 0) = .8$$
$$E(y|x = 1) = E(y^2|x = 1) = .6$$
$$E(y^2|x = 0) - (E(y|x = 0))^2 = .16$$
$$E(y^2|x = 1) - (E(y|x = 1))^2 = .24$$

2.5 a)

$$E((e^2 - g(x))^2)$$

b)

$$\text{Minimize } E((e^2 - h(x))^2)$$

c)

$$E((e^2 - g(x))^2)$$
$$= E((e^2 - \sigma^2(x) + \sigma^2(x) - h(x))^2)$$
$$= E((e^2 - \sigma^2(x))^2) + E((\sigma^2(x) - h(x))^2) + 2E((\sigma^2(x) - h(x))(e^2 - \sigma^2(x)))$$

Since

$$E((\sigma^2(x) - h(x))(e^2 - \sigma^2(x)))$$
$$= E(E((\sigma^2(x) - h(x))(e^2 - \sigma^2(x))|x))$$
$$= E(E(e^2\sigma^2(x) - e^2g(x) + g(x)\sigma^2(x) + \sigma^2(x)\sigma^2(x)|x))$$
$$= E(\sigma^2(x)\sigma^2(x) - \sigma^2(x)g(x) + g(x)\sigma^2(x) + \sigma^2(x)\sigma^2(x)) = 0$$

We have

$$E((e^2 - g(x))^2) = E((e^2 - \sigma^2(x))^2) + E((\sigma^2(x) - h(x))^2) \geq E((e^2 - \sigma^2(x))^2)$$

2.7
$$E(y^2|x) - (E(y|x))^2 = V(y|x) = V(\beta x + e|x) = V(e|x) = \sigma^2(x)$$

2.10 True

$$E(x^2 e) = E(x^2 E(e|x)) = 0$$

2.11 False

Suppose e has a degenerate distribution and $p(x = 1) = .5$ and $p(x = -1) = .5$

$$E(ex) = 0$$
$$E(ex^2) = \bar{e}$$

2.12 False.

Suppose e, x are uniformly distributed across the unit circle around $(0, 0)$. The conditional distribution of e given x is uniform over the interval $(-\sqrt{1 - x^2}, \sqrt{1 - x^2})$, So its expectation, found by integrating the conditional density, is zero.

However,they share a distribution so they are not independent.

$$E(e|x) = 0$$

2.13 Use the same counter example as 2.11

$$E(ex) = 0$$
$$E(e|x) = \bar{e}$$

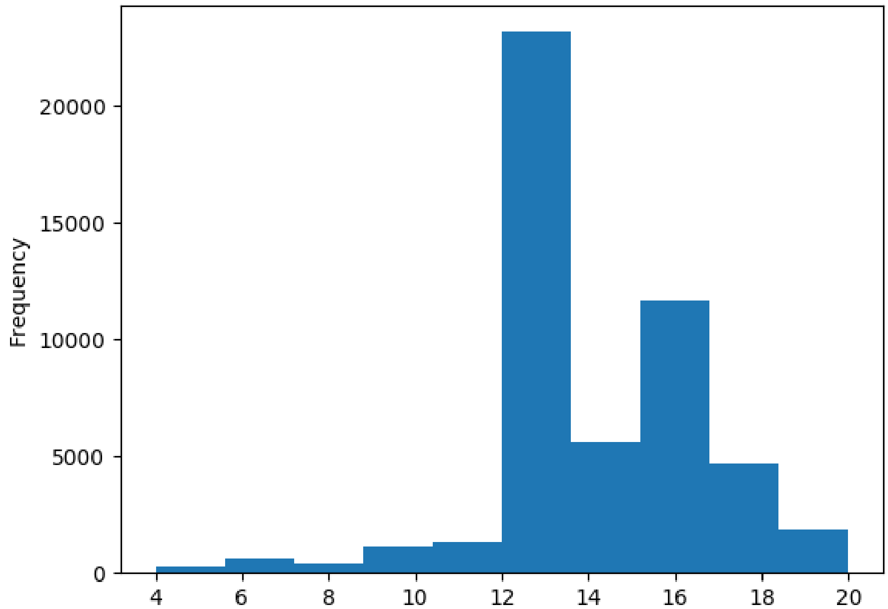
2.14 False

Suppose e, x are uniformly distributed across the unit square around $(0, 0)$. The conditional distribution of e given x is uniform over the interval $(-1, 1)$, So its expectation, found by integrating the conditional density, is zero. Similarly, the variance along this interval is 1.

Again,they share a distribution so they are not independent.

2 Part II

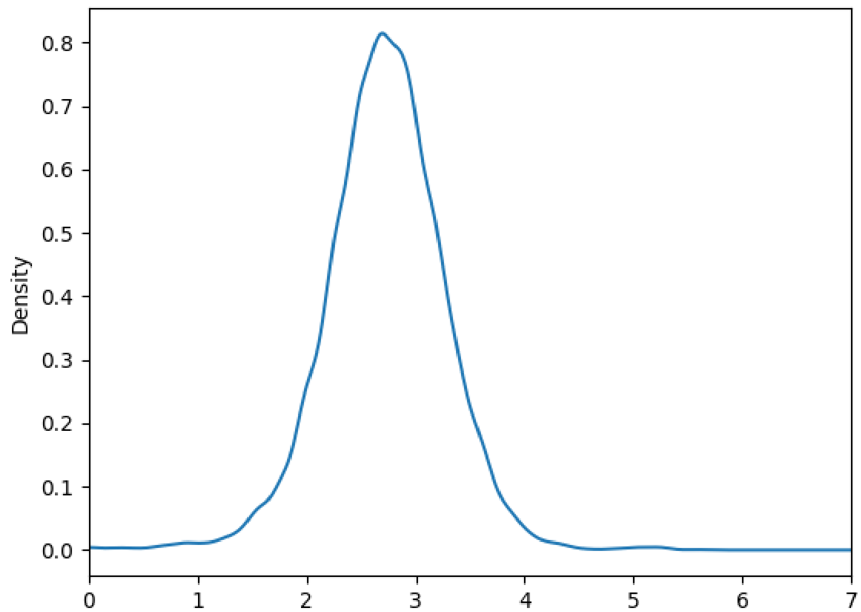
a) Below is the histogram.

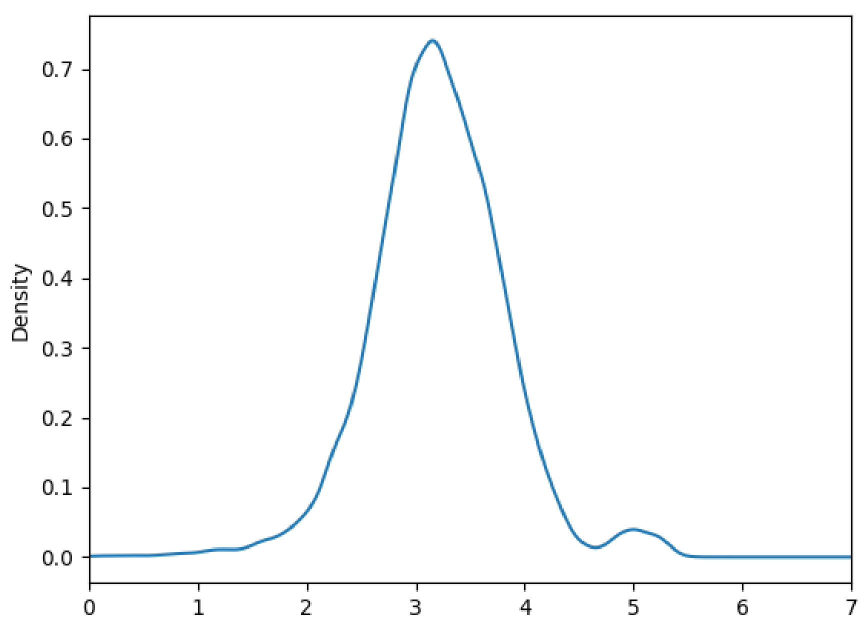


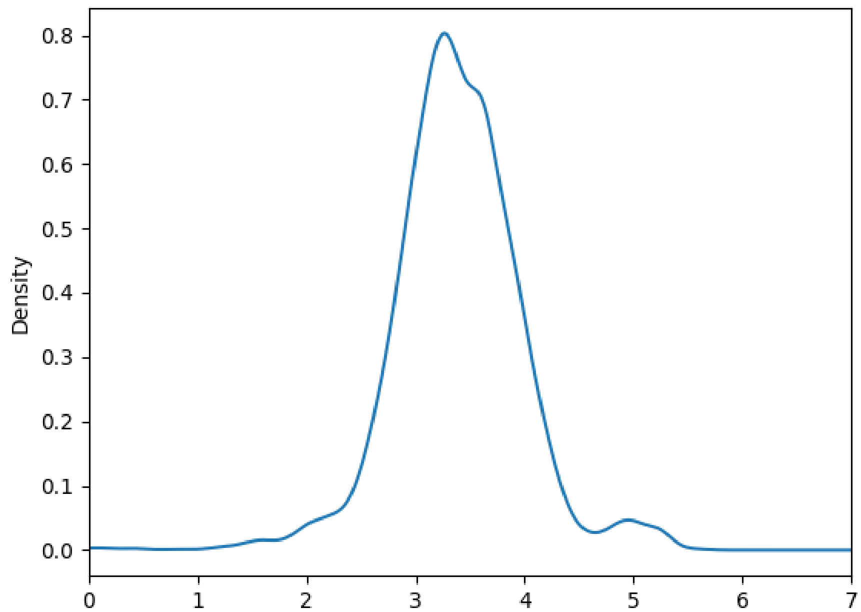
b)

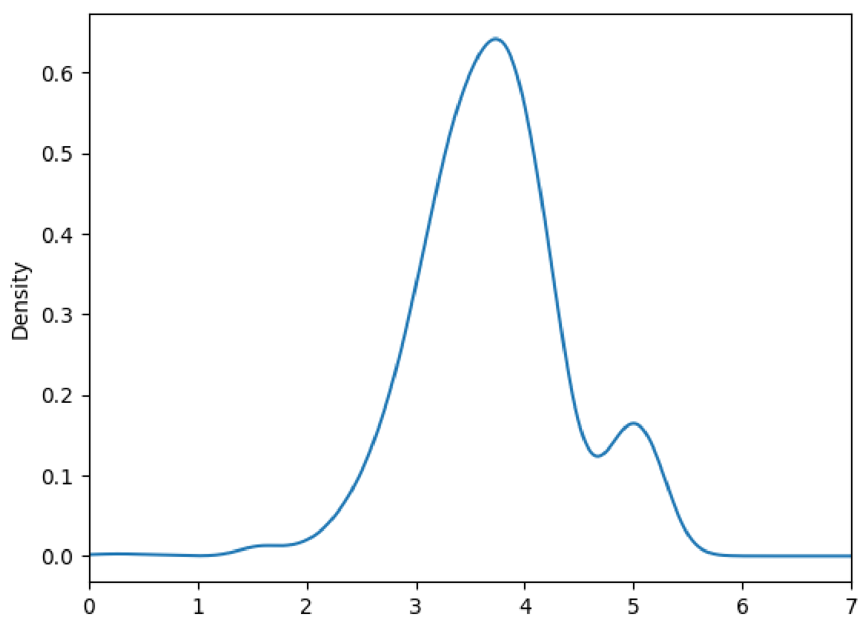
Years	Observations
12	13896
16	11640
18	4670
20	1875

c) Below are the kernel density plots.


Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
1
2
3
4
5
6
7


Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
1
2
3
4
5
6
7





d) The table below show the results.

Years	Mean	Variance
12	2.712	0.329
16	3.201	0.416
18	3.381	0.363
20	3.689	0.619

By the table, we can see 20 years of education has the highest variance.

e) The table below show the results.

Year	25th	50th	75th
12	2.403	2.733	3.055
16	2.839	3.180	3.585
18	3.062	3.362	3.710
20	3.275	3.690	4.089

f) The table below show the results.

Years	Difference
12	0
16	0.489
18	0.669
20	0.977

g) The table below show the results.

Year 1	Year 2	Diff	SE	T-Value	Reject
12	12	0.0	0.00486691158299	0.0	False
12	16	-0.488918	0.00486691158299	-100.457462777	True
12	18	-0.669176	0.00486691158299	-137.494970241	True
12	20	-0.976912	0.00486691158299	-200.725245359	True
16	12	0.488918	0.00597543294539	81.821282852	True
16	16	0.0	0.00597543294539	0.0	False
16	18	-0.180258	0.00597543294539	-30.1665629463	True
16	20	-0.487994	0.00597543294539	-81.6667908266	True
18	12	0.669176	0.00882675326792	75.8122316274	True
18	16	0.180258	0.00882675326792	20.4218095382	True
18	18	0.0	0.00882675326792	0.0	False
18	20	-0.307736	0.00882675326792	-34.8640263334	True
20	12	0.976912	0.0181678405989	53.7714989472	True
20	16	0.487994	0.0181678405989	26.8603431317	True
20	18	0.307736	0.0181678405989	16.9385104793	True
20	20	0.0	0.0181678405989	0.0	False

h) The table below show the results.

Dep. Variable:	lwage	**R-squared:**	0.203
Model:	OLS	**Adj. R-squared:**	0.203
Method:	Least Squares	**F-statistic:**	2727.
Date:	Sat, 03 Feb 2018	**Prob (F-statistic):**	0.00
Time:	13:45:04	**Log-Likelihood:**	-30104.
No. Observations:	32081	**AIC:**	6.022e+04
Df Residuals:	32077	**BIC:**	6.025e+04
Df Model:	3		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	2.7121	0.005	516.939	0.000	2.702	2.722
educ_16	0.4889	0.008	62.916	0.000	0.474	0.504
educ_18	0.6692	0.010	63.969	0.000	0.649	0.690
educ_20	0.9769	0.015	64.203	0.000	0.947	1.007

Omnibus:	10460.946	**Durbin-Watson:**	1.767
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	213631.163
Skew:	-1.068	**Prob(JB):**	0.00
Kurtosis:	15.460	**Cond. No.**	4.98

3 Python Code