IIR elliptic BPF model

Table of Contents

Configure input parameters

```
clear all;
```

Sampling frequency

```
Fo =8096; %  [Hz]
T = 1/Fo;
```

Bits per sample

```
NumOfBits = 16;
```

Filter passband

```
fp = [1250 ,2500];
Wp=fp/(Fo/2); % normalized passband
```

Filter stopband

```
fs = fp + [-250,250];
Ws=fs/(Fo/2); % normalized stopband
```

Filter ripple tolerances

Max ripple in passband [dB]

```
Rp = 2.5;
```

Min ripple in stopband [dB]

```
Rs =40;
```

Input test signal

```
t = T*(0:1:Fo); % time axis [s] Num of Samples is Fo for 1 second
% Dirac impulse
x_in = zeros(size(t));
x_in(1) = 2^(NumOfBits-1);
% scale the input signal to base 2 and save in memory (quantized)
scale_2 = 2^ceil(log2(max(abs(x_in))));
x_q = quantization(x_in/scale_2, NumOfBits, 1);
```

Direct realization - MATLAB reference

```
[n,Wn] = ellipord(Wp,Ws,Rp,Rs);
[b,a] = ellip(n,Rp,Rs,Wn);
N = 2*n; % for BPF
[H,meta_res] = freqz(b,a,Fo);
figure
subplot(2,1,1)
hold on
f = (meta_res/pi)*Fo/2;
Loss = -20*log10(abs(H));
rectangle('Position',[0, (Rs), fs(1)/1000, 2*Rs],'FaceColor',[.8 .8 .8],'Linestyle','none')
rectangle('Position',[fs(2)/1000, (Rs), (Fo/2-fs(2))/1000, 2*Rs],'FaceColor',[.8 .8 .8],'Linest
rectangle('Position',[fp(1)/1000, 0, (fp(2)-fp(1))/1000, Rp],'FaceColor',[.8 .8 .8],'Linestyle
plot(f/1000, Loss)
title('Elliptic BPF - Direct realization - MATLAB ref')
xlabel('{\it f} [kHz]'), ylabel('Loss [dB]')
axis([-0.1,(Fo/2)/1000, -3,max(Loss)])
```

Direct realization - DSP Simulation

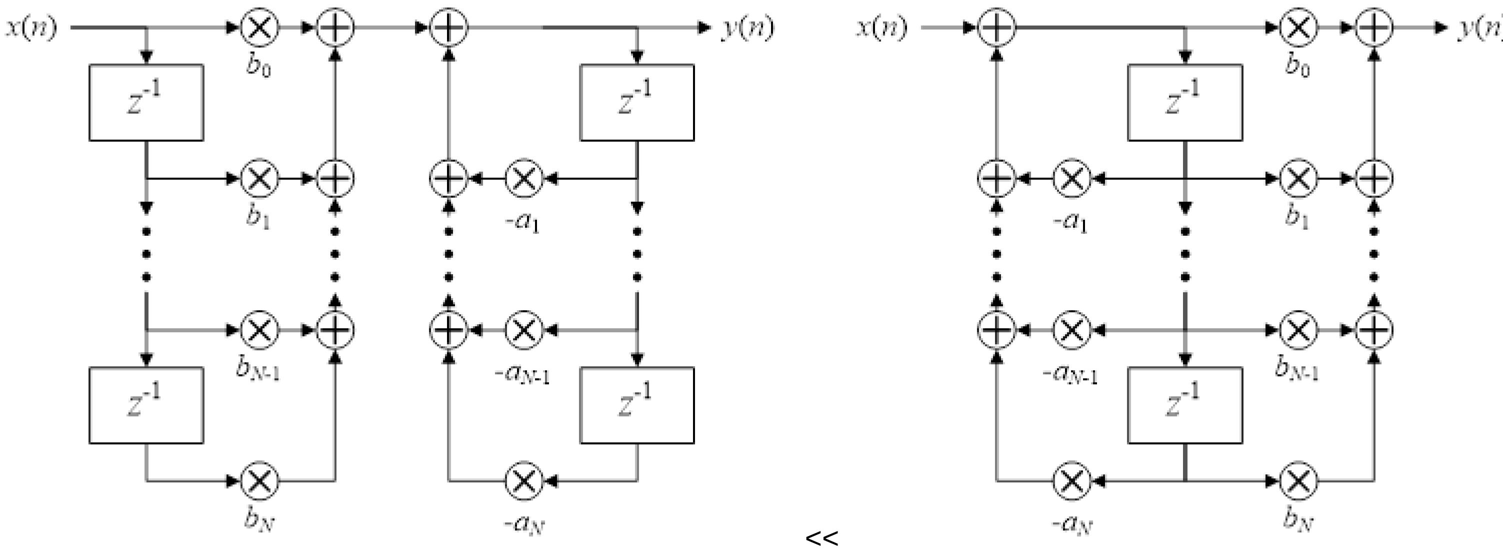
$$H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{k=0}^{N} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

$$y(n)+\sum_{k=1}^{N} a_k y(n-k)=\sum_{k=0}^{N} b_k x(n-k)$$

$$y(n) = s\left(\sum_{k=0}^{N} b'_k x(n-k) - \sum_{k=1}^{N} a'_k y(n-k)\right),\quad b'_k = \frac{b_k}{s}, k = 0,\ldots,N,\quad a'_k = \frac{a_k}{s}, k = 1,\ldots,N$$

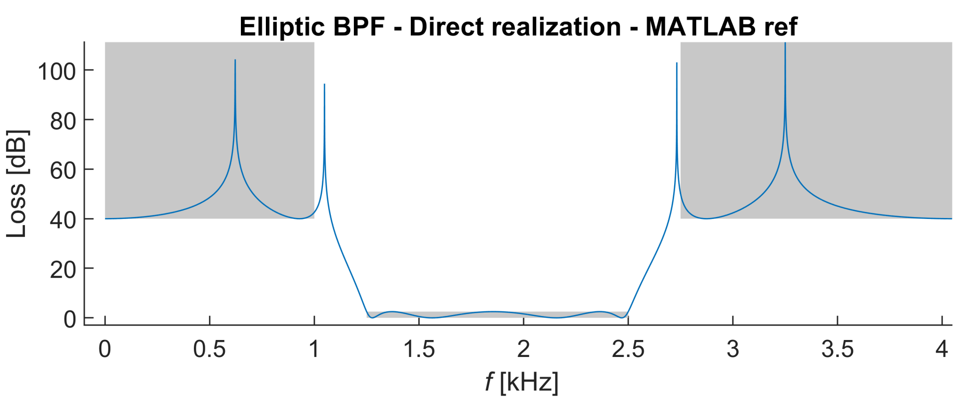
s – scaling coefficient

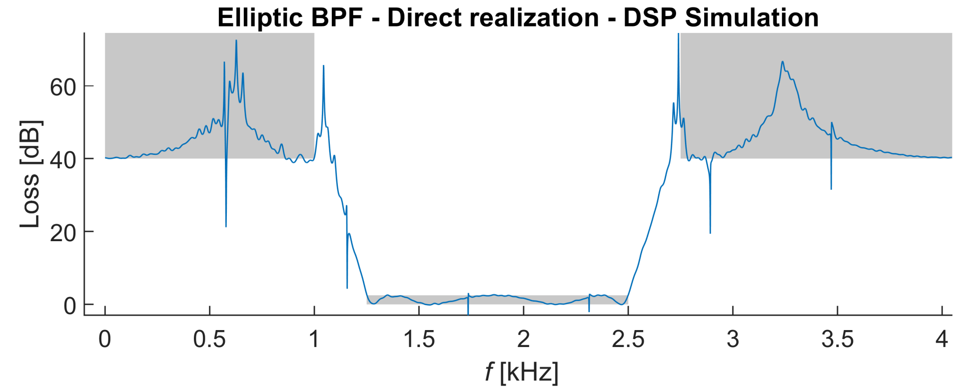
```
% scale the filter coeffs to base 2 and save in memory (quantized)
sc = 2^ceil(log2(max(abs([b a]))));
b_q = quantization(b/sc, NumOfBits,1);
a_q = quantization(a/sc, NumOfBits,1);
```




```
% filtering and save in memory (quantized)
x_z = zeros(1,N);
sum_b = zeros(1,N);
meta_res = zeros(1, length(x_q));
% b coeffs branch
for n = 1:length(x_q)
    sum_b(N) = quantization(b_q(N+1)*x_z(N), NumOfBits,1);
    % prev sums
    for k = N:-1:2
        sum_b(k-1) = quantization(sum_b(k)+ b_q(k)*x_z(k-1), NumOfBits,1);
    end
    meta_res(n) = quantization(sum_b(1)+ b_q(1)*x_q(n), NumOfBits,1);
    % shift delayed samples
    for z = N:-1:2
        x_z(z) = x_z(z-1);
    end
    % load next sample
    x_z(1) = x_q(n);
end
y_z = zeros(1,N);
sum_a = zeros(1,N);
y = zeros(1,length(x_q));
% a coeffs branch
```

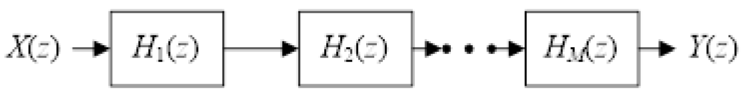
```
for n = 1:length(x_q)
    sum_a(N) = quantization(-a_q(N+1)*y_z(N), NumOfBits,1);
    for k = N:-1:2
        sum_a(k-1)=  quantization(sum_a(k) - a_q(k)*y_z(k-1), NumOfBits,1);
    end
    y(n) = quantization((meta_res(n) + sum_a(1)), NumOfBits,1) *sc;
    for z = N:-1:2
        y_z(z) = y_z(z-1);
    end
    y_z(1) = y(n);
end

%figure
subplot(2,1,2)
hold on
Y = fft(y);
M = abs(Y);
Loss = -20*log10(M);
Faxis = 0:Fo;
rectangle('Position',[0, (Rs), fs(1)/1000-0, 2*Rs],'FaceColor',[.8 .8 .8],'Linestyle','none')
rectangle('Position',[fs(2)/1000, (Rs), (Fo/2)/1000-fs(2)/1000, 2*Rs],'FaceColor',[.8 .8 .8],'L
rectangle('Position',[fp(1)/1000, 0, fp(2)/1000-fp(1)/1000, Rp],'FaceColor',[.8 .8 .8],'Linesty
plot(Faxis(1:Fo/2)/1000, Loss(1:Fo/2))
xlabel('{\it f} [kHz]'), ylabel('Loss [dB]')
axis([-0.1,(Fo/2)/1000, -3,max(Loss)])
title('Elliptic BPF - Direct realization - DSP Simulation')
```





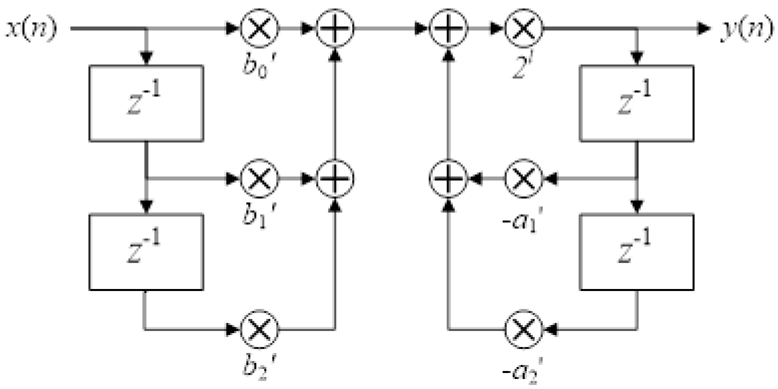
Cascade realization - MATLAB reference

```
[sos,G] = tf2sos(b,a);
%Transfer Function to Second Order Section conversion
%SOS is an L by 6 matrix with the following structure:
% SOS = [ b01 b11 b21  1 a11 a21
%          ...
%          b0L b1L b2L  1 a1L a2L ]

%Each row of the SOS matrix describes a 2nd order transfer function:
%                  b0k +  b1k z^-1 +  b2k  z^-2
%      Hk(z) =  -------------------------------
%                  1 +  a1k z^-1 +  a2k  z^-2
%where k is the row index.

%G is a scalar which accounts for the overall gain of the system. If
%G is not specified, the gain is embedded in the first section.
% The second order structure thus describes the system H(z) as:
%        H(z) = G*H1(z)*H2(z)*...*HL(z)

v = size(sos);
NumOfSections = v(1);
```




```
Hcas = G;
for i=  1:NumOfSections
      b_sec = sos(i, 1:3); %SOS is an L by 6 matrix
      a_sec = sos(i, 4:6); %SOS is an L by 6 matrix
      [H_sec,meta_res] = freqz(b_sec,a_sec,Fo);
      Hcas = Hcas.*H_sec;
end

figure
subplot(2,1,1)
f = (meta_res/pi)*Fo/2;
Loss = -20*log10(abs(Hcas));
hold on
rectangle('Position',[0, (Rs), fs(1)/1000-0, 2*Rs],'FaceColor',[.8 .8 .8],'Linestyle','none')
rectangle('Position',[fs(2)/1000, (Rs), (Fo/2)/1000-fs(2)/1000, 2*Rs],'FaceColor',[.8 .8 .8],'
rectangle('Position',[fp(1)/1000, 0, fp(2)/1000-fp(1)/1000, Rp],'FaceColor',[.8 .8 .8],'Linesty
plot(f/1000, Loss)
title('Elliptic BPF - Cascade realization - MATLAB ref')
xlabel('{\it f} [kHz]'), ylabel('Loss [dB]')
axis([-0.1,(Fo/2)/1000, -3,max(Loss)])
```

Cascade realization - DSP Simulation



$$b_i' = \frac{b_i}{2^l}, \quad i = 0,1,2 \qquad\qquad a_i' = \frac{a_i}{2^l}, \quad i = 1,2$$

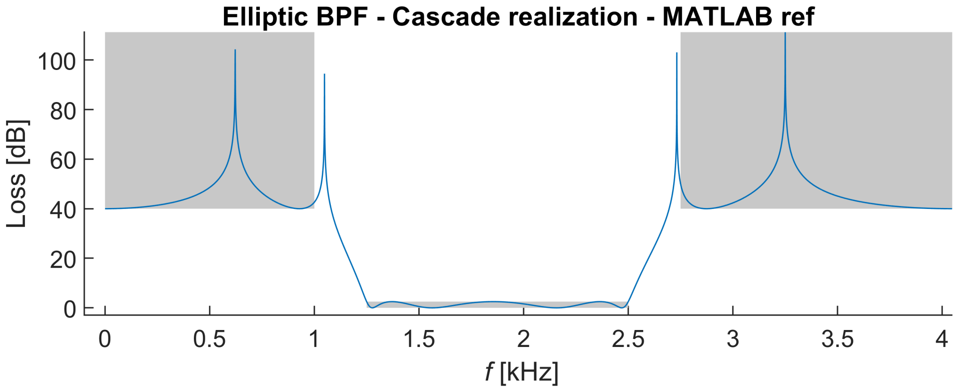
$$w(n) = x(n) \cdot b_0' + w_1(n) \qquad y(n) = 2^l \cdot \left(w(n) + y_1(n)\right)$$

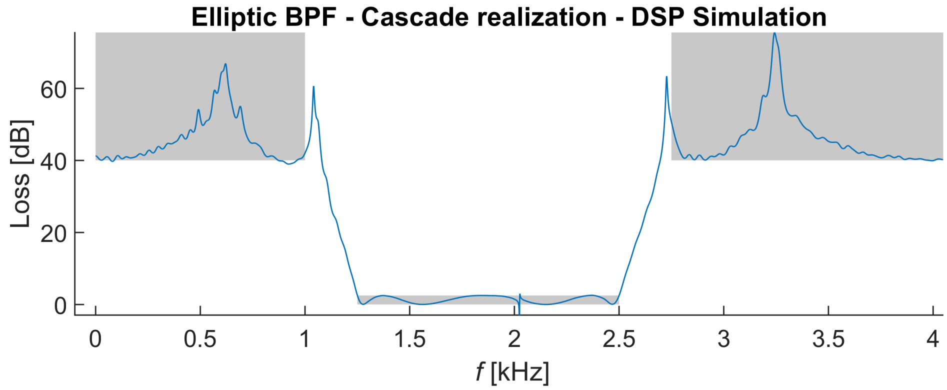
$$w_1(n) = x(n-1) \cdot b_1' + w_2(n) \qquad y_1(n) = 2^l \cdot \left(y(n-1) \cdot (-a_1') + y_2(n)\right)$$

$$w_2(n) = x(n-2) \cdot b_2' \qquad y_2(n) = 2^l \cdot y(n-2) \cdot (-a_2')$$

```
% scale the filter coeffs to base 2, for each section
sc_sec = zeros(1,3);
for i = 1:NumOfSections
      b_sec = sos(i, 1:3);
      a_sec = sos(i, 4:6);
      sc_sec(i)=2^ceil(log2(max(abs([b_sec a_sec]))));
end

% filtering and save in memory (quantized)
G_q = quantization(G, NumOfBits,1);
ws=zeros(size(x_q));
ys=zeros(size(x_q));
xs = G_q*x_q;
for i = 1:NumOfSections
    b_p0 = sos(i, 1)/sc_sec(i);
    b_p0q = quantization(b_p0, NumOfBits,1);

    b_p1 = sos(i, 2)/sc_sec(i);
    b_p1q = quantization(b_p1, NumOfBits,1);

    b_p2 = sos(i, 3)/sc_sec(i);
    b_p2q = quantization(b_p2, NumOfBits,1);

    a_p1 = sos(i, 5)/sc_sec(i);
    a_p1q = quantization(a_p1, NumOfBits,1);

    a_p2 = sos(i, 6)/sc_sec(i);
    a_p2q = quantization(a_p2, NumOfBits,1);

    x2=0; x1=0;
    for n = 1:length(xs)
```

```
        z2 = quantization(b_p2q*x2, NumOfBits,1);
        z1 = quantization(z2 + b_p1q*x1, NumOfBits,1);
        ws(n) = quantization(z1 + b_p0q*xs(n), NumOfBits,1);
        x2 = x1;
        x1 = xs(n);
    end
    y2=0; y1=0;
    for n = 1:length(xs)
        v2 = quantization(-a_p2q*y2, NumOfBits,1);
        v1 = quantization(v2 - a_p1q*y1, NumOfBits,1);
        ys(n) = quantization(ws(n) + v1, NumOfBits,1)*sc_sec(i);
        y2 = y1;
        y1 = ys(n);
    end
    xs = ys;
end
y_cas = xs;

%figure
subplot(2,1,2)
hold on
Ykas=fft(y_cas);
Mkas=abs(Ykas);
Loss = -20*log10(Mkas);
Faxis = 0:Fo;
rectangle('Position',[0, (Rs), fs(1)/1000-0, 2*Rs],'FaceColor',[.8 .8 .8],'Linestyle','none')
rectangle('Position',[fs(2)/1000, (Rs), (Fo/2)/1000-fs(2)/1000, 2*Rs],'FaceColor',[.8 .8 .8],'
rectangle('Position',[fp(1)/1000, 0, fp(2)/1000-fp(1)/1000, Rp],'FaceColor',[.8 .8 .8],'Linesty
plot(Faxis(1:Fo/2)/1000, Loss(1:Fo/2))
xlabel('{\it f} [kHz]'), ylabel('Loss [dB]')
axis([-0.1,(Fo/2)/1000, -3,max(Loss)])
title('Elliptic BPF - Cascade realization - DSP Simulation')
```


Elliptic BPF - Cascade realization - MATLAB ref
Loss [dB]
100
80
60
40
20
0
0
0.5
1
1.5
2
2.5
3
3.5
4
f [kHz]


Elliptic BPF - Cascade realization - DSP Simulation
Loss [dB]
60
40
20
0
0
0.5
1
1.5
2
2.5
3
3.5
4
f [kHz]