# Aggregate Dynamics in Lumpy Economies

**Isaac Baley**
UPF, CREi, Barcelona GSE

**Andrés Blanco**
U Michigan

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## Overview

- How do aggregate shocks propagate in economies with ...
  - Heterogeneous agents
  - Lumpiness = Inaction + Large adjustment
- We develop a sufficient statistic approach:
  1. Propagation = $f$(ergodic moments)
  2. Ergodic moments = $g$(microdata on adjustments)

# Overview

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  - Heterogeneous agents
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- We develop a sufficient statistic approach:
  1. Propagation = $f$(ergodic moments)

     (1) $\mathbb{V}ar$[capital/productivity]

     (2) $\mathbb{C}ov$[capital/productivity, time elapsed since last adjustment]
  2. Ergodic moments = $g$(microdata on adjustments)

     Plant-level investment data recovers $\mathbb{V}ar[\cdot]$ and $\mathbb{C}ov[\cdot]$
- Investment dynamics following an aggregate productivity shock

## Simplified Environment in Investment

- Continuum of firms, steady state
- Firm's state: capital gap (unobservable)

$$x_t \equiv \log\left(\text{capital}_t/\text{tfp}_t\right) - \underbrace{\mathbb{E}\left[\log\left(\text{capital}/\text{tfp}\right)\right]}_{\text{cross-sectional mean}}$$

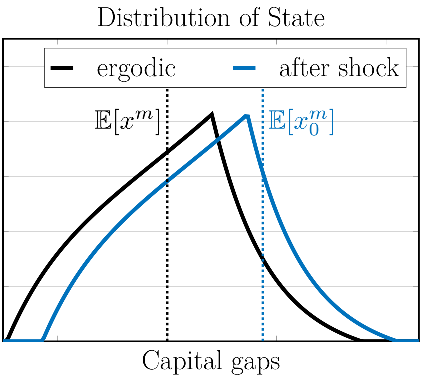
- Between adjustments, capital gap follows:

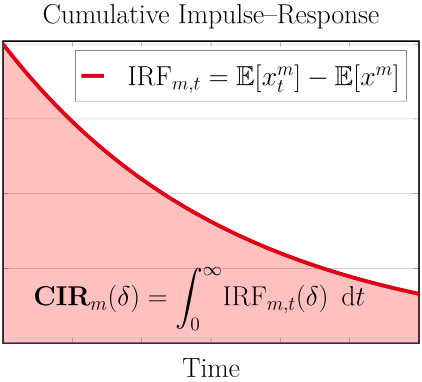
$$\mathrm{d}x_t = -\nu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$$

- Firm policy: reset capital gap to $\hat{x}$ at dates $\{T_k\}_k$ if
  - Capital gap is too large (Ss policy)
  - Or/and at random $i.i.d.$ dates (Calvo, sticky info)
- Observable actions in panel data: $(\Delta x, \tau, a)$
  - Adjustment size (investment): $\Delta x_k = \hat{x} - x_{T_k^-}$
  - Duration of completed spells: $\tau_k = T_k - T_{k-1}$
  - Duration of uncompleted spells (age): $a_{tk} = t - T_{k-1}$

# Propagation of Aggregate Shocks

- Unanticipated decrease in productivity of size $\delta$ (small) for all firms




- Cumulative Impulse–Response: area under IRF
  - Real output ($m = 1$), capital misallocation ($m = 2$)
  - Measure of impact & persistence in one scalar

$$\textbf{Propagation} = f(\textbf{ergodic moments})$$

# Propagation = $f$(ergodic moments)

- **Input:** Stochastic process between adjustments + firm policy
  - Assume firms use steady-state policy
- **Output**: CIR as linear combination of two ergodic moments

$$\boxed{\frac{\text{CIR}_m(\delta)}{\delta} = \frac{\mathbb{E}[x^{m+1}] + \nu \mathbb{C}ov[x^m, a]}{\sigma^2} + o(\delta)}$$

- Ergodic moments encode agents' **responsiveness** to shocks

  $\Rightarrow$ Information about speed of convergence

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- Real output response ($m = 1$):

$$\frac{\text{CIR}_1(\delta)}{\delta} = \frac{\mathbb{V}ar[x] + \nu \mathbb{C}ov[x, a]}{\sigma^2} + o(\delta)$$

$\Rightarrow$ $\nu = 0$ : $\text{CIR}_1(\delta)/\delta \approx \mathbb{V}ar[x]/\sigma^2$

$\Rightarrow$ $\nu \neq 0$: $\mathbb{C}ov[x, a]$ corrects dispersion due to drift

$$\textbf{Ergodic moments} = g(\textbf{microdata})$$

## Ergodic moments = $g$(microdata)

- **Input:** Stochastic process between adjustment + $(\Delta x, \tau, a)$
  - Assume constant reset state $(\hat{x})$
- **Result**: Ergodic moments and parameters $(\nu, \sigma^2)$

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**i. Drift:** frequency $\times$ size of investment

$$\nu = \frac{\mathbb{E}[\Delta x]}{\mathbb{E}[\tau]}$$

**ii. Volatility:** frequency $\times$ dispersion of investment (adjusted for drift)

$$\sigma^2 = \frac{\mathbb{E}[\Delta x^2]}{\mathbb{E}[\tau]} - 2\nu\hat{x}$$

**iii. Reset capital gap:** $\hat{x} = \frac{\mathbb{E}[\Delta x]}{2}\left(1 - \mathbb{CV}^2[\tau]\right) + \frac{\mathbb{C}ov[\tau, \Delta x]}{\mathbb{E}[\tau]}$

- Set $\hat{x} = 0$ for today

## Ergodic moments = $g$(microdata)

- Variance of capital gap

$$\mathbb{V}ar[x] = \frac{1}{3}\frac{\mathbb{E}\left[\Delta x^3\right]}{\mathbb{E}\left[\Delta x\right]}$$

- Covariance between capital gap and age

$$\mathbb{C}ov[x, a] \;=\frac{1}{2\nu}\left\{\mathbb{V}ar[x] + \sigma^2\mathbb{E}[a] \;-\; \frac{\mathbb{E}[\tau\Delta x^2]}{\mathbb{E}[\tau]}\right\}$$

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  - Large investment rate $\implies$ Large capital misallocation
  - Dispersed investment rate $\implies$ Indicative of Calvo adjustments

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- Covariance between capital gap and age

$$\mathbb{C}ov[x,a] \;=\frac{1}{2\nu}\left\{\underbrace{\mathbb{V}ar[x]+\sigma^2\mathbb{E}[a]}_{>0}\;\underbrace{-\;\frac{\mathbb{E}[\tau\Delta x^2]}{\mathbb{E}[\tau]}}_{<0}\right\}$$

  - If depreciation $\nu$ is large $\Longrightarrow \mathbb{C}ov[x,a]<0$
  - If volatility $\sigma/\nu$ is large + irreversibility $\Longrightarrow \mathbb{C}ov[x,a]>0$

$$\textbf{Propagation} = f(g(\textbf{microdata}))$$

# Main Contribution

- Alvarez, Le Bihan and Lippi (2016)'s environment:
  - Large set of price-setting models
  - Zero drift + Symmetric policies + $m = 1$
  - $x =$ markup gap, $\Delta x =$ price change

**Output effect of a money shock**

$$\underbrace{\frac{\text{CIR}_1(\delta)}{\delta}}_{\text{output dynamics}} = \qquad\qquad = \underbrace{\frac{\mathbb{E}[\tau]\mathbb{K}ur[\Delta x]}{6}}_{\frac{\text{kurtosis of price changes}}{\text{freq. of price changes}}}$$

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- By connecting to ergodic moments, we allow for:
    - Non-zero drift, asymmetries, $m \geq 1$, mean-preserving spreads...
- Sufficient statistics across fields in economics

# Bringing the Theory to the Data

# Investment fluctuations

- **Input:** Plant-level investment data from Chile (structures)
- **Output:** Parameters and ergodic moments

| **Parameters** | **Value** | **Interpretation** |
|---|---|---|
| $\nu$ | 0.11 | 6% depreciation + 5% (price + growth) |
| $\sigma^2$ | 0.08 | In line w/Bachmann, Caballero, Engel (2013) |
| **Ergodic moments** | | |
| $\mathbb{V}[x]$ | 0.23 | Calvo (standard Ss model 0.002) |
| $\mathbb{C}ov[x,a]$ | 0.90 | Fixed Costs + Irreversibility |
| **Propagation ($\delta = -1\%$)** | | |
| $\text{CIR}_1$ | 4.3 | Cumulative capital drop of 4.3% |
| Half-life | 3 | 3 years |

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Our sufficient statistics are powerful devices for **discriminating between models** and **retrieving primitives** in lumpy economies