Lumpy Investment, Uncertainty, and Monetary Policy *

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Abstract

I study the impact of fluctuations in firm-level uncertainty on the effectiveness of monetary policy when investment is lumpy. I first document empirically that high uncertainty hinders firms' investment responses to monetary stimulus, especially at the extensive margin (changes in the number of firms choosing between (dis)investing or staying inactive). I then develop a heterogeneous firm New Keynesian model with random fixed costs and partial irreversibility of capital adjustment. These adjustment costs create a sizable extensive margin of investment which is more sensitive to changes in the interest rate and firm-level uncertainty than the intensive margin. Hence, monetary policy works primarily through the extensive margin of investment. Upon an uncertainty shock, firms tend to stay inactive at the extensive margin, so monetary stimulus can hardly motivate investment at the extensive margin. As a result, the effectiveness of monetary policy is reduced. I then parameterize the model. I find that its quantitative implications for both monetary policy and uncertainty are primarily shaped by the specifications of capital adjustment costs. Unlike much of the prior literature, I use the dynamic moments of investment to identify this key model element. Based on this parameterization, I show that an aggregate shock to firm-level uncertainty estimated by Bloom et al. (2018) reduces the effectiveness of monetary stimulus on investment by half. This reduction is about 85% of what I find in the data. Therefore, the effect of monetary policy depends on lumpy investment and time-varying uncertainty.

Keywords: lumpy investment; uncertainty; firm heterogeneity; monetary policy;
JEL Codes: E52, E32, E22

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1. Introduction

Fluctuations in aggregate investment is primarily driven by changes in the number of firms implementing new investment projects (the extensive margin) rather than changes in the size of ongoing investment projects (the intensive margin)[1]. The extensive margin, as documented in Bloom (2009), is heavily influenced by time-varying firm-level uncertainty. Aggregate shocks to firm-level uncertainty significantly changes firms' tendency at the extensive margin from (dis)investing to staying inactive. However, most New Keynesian models abstract away from both the extensive margin of investment and the time-varying firm-level uncertainty. As a result, all responses in aggregate investment to monetary shocks are from firms operating along the uncertainty-irrelevant intensive margin. Therefore, a key question for New Keynesian modeling is: Do these abstractions matter for the investment channel of monetary policy transmission?

In this paper, I argue that incorporating both the extensive margin of investment and time-varying firm-level uncertainty is essential for the investment channel of monetary policy transmission. First, I document that extensive-margin investment is significantly less responsive to monetary shocks during high uncertainty periods. Second, I build a heterogeneous firm New Keynesian model consistent with both the existence of the extensive margin of investment and the aggregate investment sensitivity to aggregate shocks. In the model, the responses of aggregate investment to monetary shocks are primarily driven by the extensive margin rather than the intensive margin. Along with an uncertainty shock, fewer firms are close to making an extensive margin investment, so a monetary shock generates less aggregate investment response than it would otherwise. These results illustrate how time-varying firm-level uncertainty interacting with extensive-margin investment plays an essential role in determining the investment channel of monetary policy.

I first document how firm-level uncertainty affects the investment channel of monetary policy using both quarterly firm-level data from Compustat and quarterly aggregate data from NIPA. At firm-level, by extending Jordà (2005)'s method to Tobit Local Projection and Probit Local Projection, I show that Compustat firms' investment is significantly and persistently less responsive to a monetary stimulus at both intensive and extensive margin. More importantly, the reduction of responsiveness at the extensive margin is substantial. I then run the aggregate-level Local Projection for the economy-wide results.[2] At aggregate-level, I show that the aggregate investment rate is 60%

[1] Check the evidence in Doms et al. (1998) or Gourio and Kashyap (2007) as well as the theory in Winberry (2018a).

[2] Compustat firms are all large firms that could potentially overcome lumpy adjustment costs. As a result, the magnitude implied from Compustat firms is potential downward biased. Therefore, I run the aggregate-level Local Projection for the economy-wide result. I show that aggregate investment rate responses are 0.71% lower at the peak when the

less responsive to monetary policy along with an uncertainty shock estimated by Bloom et al. (2018).

Motivated by this evidence, I embed a model of heterogeneous firms with extensive margin investment decisions into the benchmark New Keynesian framework. In the model, there are three types of firms: production firms, retailers, and a final good producer. Production firms use both capital and labor to produce an identical intermediate good sold competitively to retailers. Retailers buy the intermediate good and set prices subject to price adjustment costs as in Rotemberg (1982)[3]. The zero-profit final good producer buys the retailer goods as the inputs and produces one final good using a CES production technology. The critical model ingredient is that production firms which adjust their capital stock incur random fixed cost and partial irreversibility. Production firms are subject to an exogenous idiosyncratic AR(1) productivity process with potential shocks to the variance of the idiosyncratic productivity shocks. The specification of adjustment costs and the shock to the variance of the idiosyncratic productivity allows the analysis of monetary policy with different firm-level uncertainty environments.

To illustrate the mechanism intuitively, I present a simplified two-period model to show that the effects of uncertainty shocks and monetary policy shocks are primarily determined by the concavity of the distribution of firm value functions, shaped by the specification of capital adjustment costs. Firm value functions become more concave when capital adjustment cost specifications force a wedge between the marginal product and the cost of capital, due to the resulting losses to the firm. An uncertainty shock increases the odds of a firm facing more concave productivity-capital mismatched value functions. Therefore, an uncertainty shock lowers the slope of the expected value function and leads firms to decrease investment. Monetary policy works similarly by changing the slope of the expected value function. As an uncertainty shock lowers the slope of the expected value function, many firms become less responsive to monetary stimulus. For some firms, the slope of the expected value function is pushed even lower than that of the investment cost function. Conventional monetary policy is 100% ineffective at generating investment from these firms. Therefore, monetary policy is overall much less effective with the uncertainty shock.

After showing the mechanism intuitively, I calibrate the model. The first critical aspect of this calibration is that, as recommended by Nakamura and Steinsson (2018), I pin down the level of the

central bank conducted a conventional monetary stimulus at the state with an aggregate shock to firm-level uncertainty estimated by Bloom et al. (2018). Given the peak response of investment to the same monetary policy stimulus is 1.2% in the absence of the uncertainty shock, it reduces the effectiveness of monetary policy by about 60%.

[3]Since I am not studying the interaction of investment and price setting, I introduce a New Keynesian Block to separate rigidity price setting from firms' production decisions to keep the model tractable. This follows the setting as in Ottonello and Winberry (2018).

random fixed costs using Koby and Wolf (2019)'s estimate of the partial equilibrium elasticity of aggregate investment to real interest rates. I find that this calibration approach gives rise to much larger random fixed cost than those in the calibrations of Khan and Thomas (2008), Reiter et al. (2013), and Bachmann and Bayer (2013).[4] The second critical aspect of this calibration is that, I pin down the level of partial irreversibility so that the partial equilibrium aggregate investment declines sharply in response to an uncertainty shock. Therefore, the magnitudes of random fixed cost and partial irreversibility generate empirically consistent sensitivity of aggregate investment in response to interest rate shocks and uncertainty shocks, respectively.

Solving the quantitative model is difficult because the distribution of firms is an infinite dimension state. I solve the model following the MIT shock strategy, as documented in Boppart et al. (2018). My algorithm is to first solve the steady-state, and then solve transition paths given different shocks or different combinations of shocks, all of which eventually converge back to steady-state. Since this yields a global solution, the algorithm captures all the non-linear dynamics, which is very important for understanding uncertainty shocks. Since the algorithm is also fast, I can solve thousands of models to demonstrate the identification.

Equipped by the technique, I then quantify the mechanism addressed in the simplified two-period example. During times of average uncertainty, for a conventional monetary policy shock, the peak response of aggregate investment is a raise of 1.8%.[5] In contrast, during times of elevated uncertainty (as estimated by Bloom et al. (2018)), the peak response of aggregate investment to the same monetary shock is 0.86%. This reduction of 52% compared to when facing normal levels of uncertainty is only slightly smaller than the 60% reduction that I estimated in the data. I further validate the mechanism through several alternative models. I show that without a reasonable specification of capital adjustment costs, the model cannot match all the empirical moments and fails to generate uncertainty-dependent responses as in the data. Combining all three types of adjustment costs is essential to generate both empirically consistent impulse responses and the differential responses.

Related Literature. This paper primarily contributes to three strands of literature. First, this paper contributes to the literature that studies how time-varying economic uncertainty affects the business cycle and monetary policy outcomes. Since the seminal paper Bloom (2009), uncertainty shocks

[4]In Khan and Thomas (2008), Reiter et al. (2013), and Bachmann and Bayer (2013), the implied partial equilibrium elasticity of aggregate investment to real interest rate is always much larger than 5. However, as suggested by Koby and Wolf (2019), this elasticity should be around 5.

[5]This resolves the excessive response puzzle of lumpy investment responses to monetary policy as in Reiter et al. (2013). In their version of calibration, the same monetary policy shock generates more than a 5% increase in aggregate investment which only lasts for one period.

have received substantial attention. Bloom et al. (2018) argued that an uncertainty shock shapes the bust-boom cycles with sharp recessions using a heterogeneous firm general equilibrium model with partial irreversibility[6]. In terms of monetary policy, most literature features the interaction of uncertainty shocks and monetary policy through menu costs and firm pricing decisions. Vavra (2013) provides evidence and builds a menu cost model arguing that higher volatility leads to an increase in aggregate flexibility so that a nominal stimulus mostly generates inflation rather than output growth.[7] Baley and Blanco (2019b) also builds a price-setting model featuring imperfect information, markup Poisson shocks, and learning. In their model, an aggregate uncertainty shock increases monetary policy neutrality so that the real effect of monetary policy is reduced. There is also a strand of empirical literature using either VAR or Local Projection on aggregate time series data studying the relationship between high uncertainty and the real effects of monetary policy. This includes Aastveit et al. (2017), Castelnuovo and Pellegrino (2018), Caggiano et al. (2017), and Paccagnini and Colombo (2020). I provide direct evidence of this interaction on the extensive margin of investment as well as building a lumpy investment model arguing that higher uncertainty leads to a substantial drop in firm-level investment via the extensive margin reducing the effectiveness of nominal stimulus on aggregate investment.

Second, this paper contributes to the literature that studies the transmission of monetary policy to the aggregate economy featuring endogenous capital accumulation. Popularized by Christiano et al. (2005) and Smets and Wouters (2007), New Keynesian models assume convex capital adjustment costs or investment adjustment costs in order to generate empirically consistent investment responses to monetary policy shocks. These assumptions work well in Representative Agent New Keynesian (RANK) models but fail to capture the observed lumpiness in plant-level investment[8]. Reiter et al. (2013) is the first paper that attempted to fill this gap. However, by introducing standard fixed capital adjustment costs as in Khan and Thomas (2008), they argued that monetary policy shocks lead to large but very short-lived dynamic consequences that are not consistent with empirical evidence or the consensus view in the literature. In this paper, I incorporate the standard fixed capital adjustment costs along with partial irreversibility and convex adjustment costs. The comprehensive adjustment cost structure helps to generate consistent lumpiness in plant-level investment

[6]There is also another important strand of literature featuring financial frictions. Arellano et al. (2019) and Gilchrist et al. (2014) show that with financial frictions, firms reduce investment to avoid default after a surprise increase in risk.

[7]Li (2019) revisits these results and shows that monetary policy is still very effective under uncertainty shocks in the menu cost models. Unlike that work, this paper is the first to explore how the interaction between an uncertainty shock and capital adjustment costs at firm-level affects monetary policy.

[8]Although the literature such as Ottonello and Winberry (2018) and Jeenas (2018) has started to explore New Keynesian models with heterogeneous firms, they essentially forgo the micro-level lumpy investment.

while maintaining reasonable aggregate responses to monetary policy shocks.

Third, this paper contributes to the literature which studies whether micro-level lumpy investment has aggregate implications. Since Caballero et al. (1995) and Caballero and Engel (1999), who find that firm level extensive margin investment behavior generates procyclical responsiveness to shocks, there has been ongoing debate whether micro-level lumpy investment has aggregate implications. During the 2000s, Thomas (2002), Khan and Thomas (2003), and Khan and Thomas (2008) show that in an otherwise standard RBC framework that extensive margin investment is irrelevant for aggregate dynamics. However, Bachmann et al. (2013) shows that the results of these models are very sensitive to the calibration. Meanwhile, House (2014) suggests that the extreme sensitivity of aggregate investment to the relative price of investment goods drives these irrelevance results in a stylized partial equilibrium model. Following that train of thought, Winberry (2018b) and Koby and Wolf (2019) show that matching the empirically consistent interest rate sensitivity of aggregate investment is the key to breaking the irrelevance results. More recently, Baley and Blanco (2019a) shows that the steady-state misallocation and irreversibility are sufficient statistics for aggregate dynamics. I extend this literature by showing the sensitivity of the dynamic moments to both real interest and firm-level uncertainty and how matching them matters for the dynamics of aggregate investment.[9]

Layout. This paper is organized as follows. Section 2 provides the empirical evidence that both the aggregate-level and firm-level response to monetary policy vary with uncertainty. Section 3 develops a New Keynesian model with heterogeneous firms to interpret this evidence. Section 4 shows the mechanism in a minimalist framework. Section 5 then calibrates the full model and verifies that it is consistent with the distribution of investment during periods of high/low uncertainty. Section 6 uses the model to study the monetary transmission mechanism in the presence of uncertainty. Section 7 discusses robustness of the mechanism and several correlated extensions. Section 8 concludes.

[9]The lumpy investment literature gets a lot of attention recently. Chen et al. (2019) incorporates the lumpy nature of firm-level investment into the study of how tax policy affects investment behavior. Zorzi (2020) shows the lumpy nature of durable adjustment of residential investment accounts for the non-linear effect which amplifies the aggregate response of durable spending during booms and dampens it during recessions. Also, in the asset pricing literature, Wu (2020) shows that incorporating lumpy investment could potentially resolve asset pricing puzzles in both time-series and cross-sectional. These studies overturn the aggregate irrelevant results in the previous literature and show interesting empirical findings and theoretical applications.

2. Empirical Motivation

This section provides empirical findings on the relationship between uncertainty and monetary policy using both U.S. aggregate-level data and firm-level data. I show how the lumpy investment distribution varies over high and low uncertainty periods in Section 2.1. I then run firm-level local projections for both the intensive and extensive margins in Section 2.2 and 2.3, respectively, to show how the effectiveness of monetary stimulus is affected by uncertainty through both margins. Finally, I show in Section 2.4 the aggregate differential impulse responses. The main finding is that high uncertainty over firm-level idiosyncratic productivity reduces the responsiveness of investment to monetary policy along both the intensive and extensive margins, especially the extensive margin, and the aggregate magnitude is significantly substantial.

2.1 Lumpy Investment and Uncertainty

Data and Variables. The sample includes all Compustat firms except regulated utilities (SICs 4900-4999), the financial sector (SICs 6000-6999), and non-profit organizations and governmental enterprises (SICs 8000s & 9000s)[10]. **Investment** is defined as the ratio of quarterly capital expenditures (Item *capxy*) to the lag of quarterly property, plant, and equipment (Item *ppentq*) as per the expression: $i_{jt}^{phy} = \frac{capxy_{jt}}{ppentq_{jt-1}}$[11]. **Uncertainty** is measured as the interquartile range (IQR) of the cross-sectional spread of stock returns from Compustat publicly-traded Compustat firms as in Bloom et al. (2018)[12]. The details of the Compustat firm-level data are in Appendix B.2.

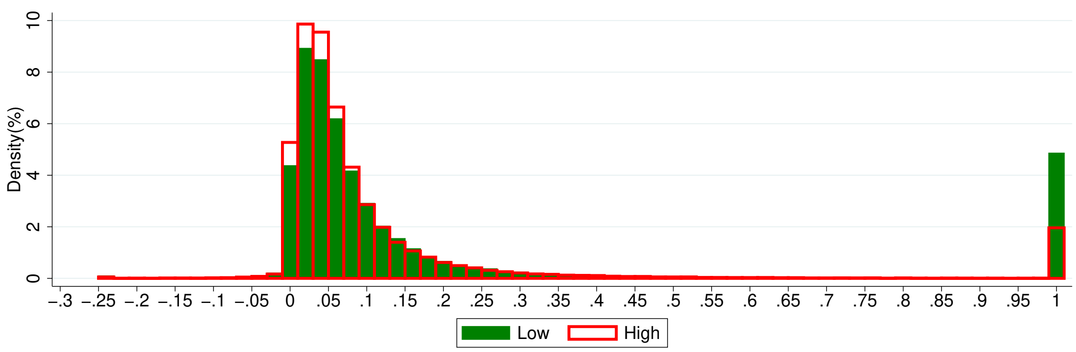
In Figure 1, I plot the distributions of investment during high and low uncertainty periods (with historical top 10% and bottom 10% of the IQR stock market return dispersion measure), respectively.

[10] I drop firms with missing or non-positive book values of assets or sales, firms with cash holdings, capital expenditures, or property, plant, and equipment lines larger than total assets, and firms with debt larger than total assets as in the standard corporate finance literature.

[11] This definition follows Erickson and Whited (2012) and many others. As the capital expenditures (Item *capxy*) is a cumulative value within a fiscal year, I make a difference between quarters except for the first fiscal quarter which suffers less from mismeasurement problems or firm-specific depreciation rates. Other measures usually use the difference between quarterly property, plant, and equipment numbers, adjusted by the price index, to back out capital, and then take the log-difference of this capital capital. This requires assuming the same depreciation rate for all firms and the same price index for all firms, which may introduce measurement errors. However, capital expenditures (Item *capxy*) is a direct measure of a firm's "monetary expense" on forming property, plant, and equipment (Item *ppentq*), hence no adjustments are required for either inflation or depreciation.

[12] The uncertainty index is calculated from all Compustat firms with 25+ years of data in the spirit of Bloom et al. (2018) over 1960Q1 to 2018Q4. It reflects the volatility of news about firm performance. I use the whole feasible Compustat panel, which dates back to the first quarter in 1960.

Figure 1: Distributions of Investment Rate by Uncertainty
(Top 10% vs. Bottom 10% of Uncertainty)



Notes: The investment rate is calculated as capital expense over capital stock for the corresponding fiscal quarter of all firms. Low and High Uncertainty means the stock market return dispersion is within the historical bottom 10% and top 10%, respectively. I truncate the investment rate at 100%. Any observed investment rate larger than 100% is accumulated at 100% in the graph.

First, the distribution of firm-level investment shows lumpy characteristics: skewed to the right, significant mass around zero, and very few negative rates. Second, during the high uncertainty periods, the distribution of investment shifts to the left with significant increments at lower investment rates. This pattern indicates the roles played by uncertainty interacting with lumpy adjustments.

2.2 IRFs for a Monetary Policy Shock at Firm-level (Intensive Margin)

From the previous Section 2.1, it is clear that the distribution of investment rate is significantly left-censored around zero, even though the sample includes only the largest firms. To explore how uncertainty affects the effectiveness of monetary policy while at the same time avoiding estimation biases created by this left-censored property, I employ the *Local Projection* (LP) method of Òscar Jordà (2005) which I extend to properly deal with both the panel and left-censored nature of the data. I refer to this as *Tobit Local Projection*, defined as follows:

$$I_{j,t+h} = \begin{cases} I^*_{j,t+h} = \beta_h^m r_t^m + \boldsymbol{\gamma_h r_t^m u_t} \\ \qquad\qquad + \sum_{l=0}^{L} \Gamma'_{h,t-l} X_{j,t-l} + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{j,h,t}, \ \text{if } |I_{j,t+h}| > 1\% \\ 0, \ \text{otherwise} \end{cases} \tag{1}$$

where j indicates firm j, h indicates quarters in the future and l indexes lags. $I^*_{j,t+h}$ is all non-inaction observations of the h quarter ahead investment rate[13], $u_t = IQR^{stock}_t$ is the uncertainty measure at time t, r^m_t is the sign-flipped real interest rate, $X_{j,t-l}$ is a vector containing the firm-level controls including Tobin's Q, sales, leverage, and size for $t-l$, and Z_{t-l} is a vector of aggregate controls including uncertainty u_t, CPI, output gap, investment, and private consumption from NIPA. For uncertainty, investment, and firm-level controls, I use the same Compustat Quarterly data as constructed in Section 2.1[14]. The details of the time series aggregate variables are in Appendix B.1. The data period is from 1985Q1 to 2009Q2 just before the ZLB is reached. I control the lag up to $L = 4$. The coefficient of interest here is the set of γ_h which indicates the **differential intensive margin impulse responses** to the monetary policy shock interacting with uncertainty.

Figure 2: Firm-level Differential Responses to Monetary Shocks
(Intensive Margin)

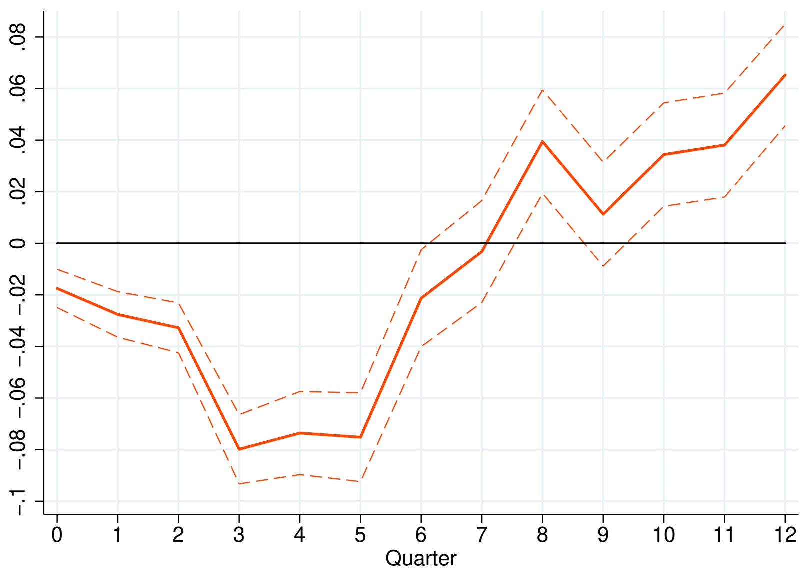


Figure 2 displays the estimates of γ_h from estimating the firm-level specification *Tobit Local Projection* (1). The dashed lines indicate the 95% confidence interval. I find that, during higher uncertainty periods, firm-level intensive margin investment is less responsive to monetary shocks. In

[13]Follow literature, I define inaction as when the absolute value of the investment rate is less than 1%. I also vary this cutoff from 0.05% to 3%, the regressions results hold. For space limit, the results are available upon request.

[14]The firm-level control variables follows the literature: A size measure using total assets (Item *atq*); financial condition measures using leverage measured by total debt (Item *dlcq* + *dlttq*) over total assets (Item *atq*), and cash holding (Item *cheq*); operational status measures using revenue (Item *revtq*), sales (Item *saleq*), and sales growth which is Δ*saleq* divided by sales; and finally, investment opportunity measured using Tobin's Q.

comparison to a lower uncertainty state, the effects of a one percentage point decrease in the real interest rate will reduce investment rate by 0.18% $\left(\{\gamma_3 = 0.08\} \times \{SD_{IQR} = 0.022\}\right)$ buttoned at quarter 3 if the stock market volatility $u_t = IQR_t^{stock}$ increases by one standard deviation ($SD_{IQR} = 0.022$). However, all these Compustat firms are large firms. Consequently, even though the results are significant, the magnitudes are potentially much smaller than the true economy-wide responses. If there was a quarterly census of firms I would likely obtain much bigger estimates in such a sample. For the results of a corresponding *OLS Local Projection*, please refer to Appendix B.4.1.

2.3 IRFs for a Monetary Policy Shock at Firm-level (Extensive Margin)

Given that the distribution of investment rates is significantly left-censored around zero, it is both interesting and important to know how this mass of inactive firms would respond to monetary policy interacting with uncertainty. To explore how uncertainty affects the effectiveness of monetary policy while at the extensive margin of lumpy investment, I employ a similar *Local Projection* (LP) method of Òscar Jordà (2005) which I extend to properly incorporate both the panel structure and the possibility of regime switch between action and inaction. This dynamic regression for extensive margin adjustment is entirely new. I refer to this as *Probit Local Projection*:

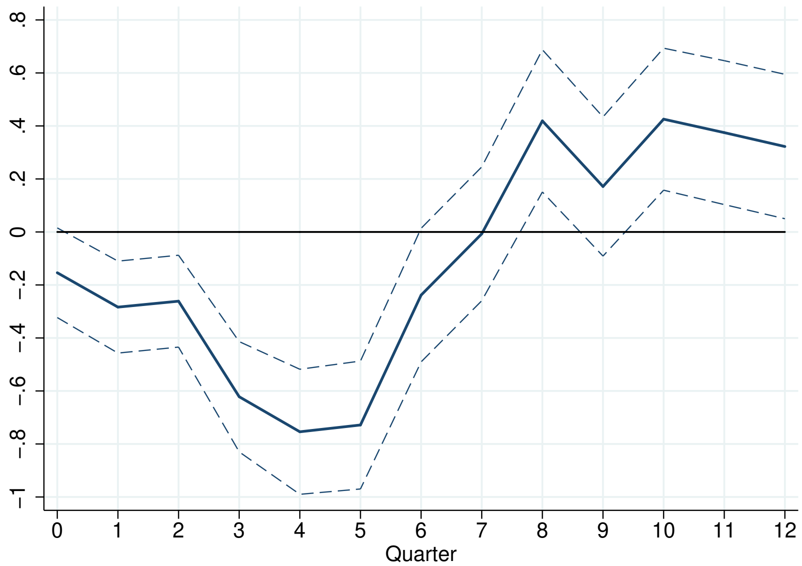
$$A^*_{j,t+h} = \beta^m_h r^m_t + \boldsymbol{\gamma_h r^m_t u_t} + \sum_{l=0}^{L} \Gamma'_{h,t-l} X_{j,t-l} + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{j,h,t} \tag{2}$$

$$A_{j,t+h} = \begin{cases} 1, & \text{if } |I_{j,t+h}| > 1\% \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where j indicates firm j, h indicates quarters in the future and l indexes lags. $I_{j,t+h}$ is the h quarter ahead investment rate measure, $A_{j,t+h}$ is the observed choice of either inaction (=0) or action (=1), $u_t = IQR_t^{stock}$ is the uncertainty measure at time t, r_t^m is the sign-flipped real interest rate, $X_{j,t-l}$ and Z_{t-l} are same controls as in the *Tobit Local Projection*. For uncertainty, investment, and firm-level controls, I use the same Compustat quarterly data as constructed in previous specification (1). The data period is from 1985Q1 to 2009Q2 just before the ZLB is reached. I include all lags up to $L = 4$ as well. The coefficient of interest here is the set of γ_h which indicates the **extensive margin differential impulse responses** to monetary policy shocks interacting with uncertainty.

Figure 3 displays the estimates of γ_h from estimating the firm-level specification *Probit Local Projection* (2). The dashed lines indicate the 95% confidence interval. This shows that with higher

Figure 3: Firm-level Differential Responses to Monetary Shocks (Extensive Margin)



uncertainty, firm-level investment at the extensive margin is less responsive to the monetary shocks. The estimated coefficients of this probit model are harder to interpret. Suppose we fix all other regressors at $X^*_{j,t}\beta_h$, the probability of a firm making engaging in investment is $P(A_{j,h} = 1|X^*_{j,t}) = \Phi(X^*_{j,t}\beta_h + \boldsymbol{\gamma_h r^m_t u_t})$. In comparison to a lower uncertainty state, the effects of a one percentage point decrease in the real interest rate will generate roughly -1.6% $\left(\{\gamma_5 = 0.73\} \times \{SD_{IQR} = 0.022\}\right)$ in the term of $\boldsymbol{\gamma_h r^m_t u_t}$ buttoned at quarter 5 if the uncertainty $u_t = IQR^{stock}_t$ increases by one standard deviation ($SD_{IQR} = 0.022$). The difference of probability of a firm making active investment is therefore lowered by $\Delta P_{j,h} = \Phi(X^*_{j,t}\beta_h - 0.016) - \Phi(X^*_{j,t}\beta_h) < 0$ which depends on the value of $X^*_{j,t}\beta_h$ but is certainly significantly negative. Given the normalization of control vector X around 0, to an average firm when $X_h = 0$, the magnitude is $\Delta P_h = \Phi(-0.016) - \Phi(0) \simeq 0.6\%$.[15] This result shows that the extensive margin is significantly important for the reduction of effectiveness. Again, recall that all the Compustat firms are large firms. As a result, even though the results are significant, the magnitudes likely bias the true economy-wide responses. In order to try and estimate economy-wide responses directly, I now turn to aggregate-level regressions.

[15]This is only an illustration which is not a correct number for almost every firm in the sample. That's because the firm-level controls $X_{j,t}$ and aggregate-level controls Z_t both vary over time. Even though their mean is normalized to 0, for each specific firm j at any specific time $t+h$, the combination of $\beta^m_h r^m_t + \sum_{l=0}^{L} \Gamma'_{h,t-l} X_{j,t-l} + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l}$ is almost never exactly 0.

2.4 IRFs for a Monetary Policy Shock at Aggregate-level

To examine the economy-wide responses directly, we still need to rely on the impulse response estimates for aggregate investment for empirical moments. I again employ the Local Projection method, but now at the aggregate level, using data from NIPA and the same firm-level uncertainty measure constructed in Section 2.1:

$$\Delta_h I_{t+h} = \alpha_h + \beta_h^m r_t^m + \boldsymbol{\gamma_h r_t^m u_t} + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \tag{4}$$

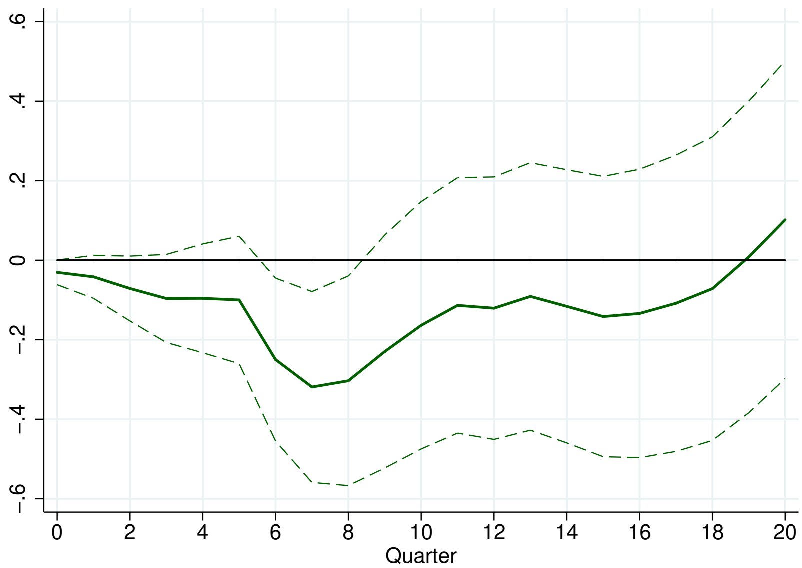
where h indicates quarters in the future, and l indexes lags. $\Delta_h I_{t+h}$ is the h quarter ahead investment measure, $u_t = IQR_t^{stock}$ is the uncertainty measure at time t, r_t^m is the sign-flipped real interest rate, and Z_{t-l} is a vector containing uncertainty u_t, CPI, output gap, investment, and private consumption. I choose the horizon $H = 20$ and the lag $L = 4$ as suggested by Jordà (2005)[16]. We are interested in γ_h, which measures how the semi-elasticity of investment $\Delta_h I_{t+h}$ with respect to monetary shocks r_t^m depends on the state of uncertainty u_t. I use quarterly data series from 1960Q1 to 2009Q2[17].

Figure 4 displays the estimates of the coefficient of the sign-flipped real interest rate interacted with uncertainty, γ_h from estimating the first aggregate-level specification (4). The dashed lines indicates the 95% confidence interval. This shows that during the times of higher uncertainty, measured by micro-level stock market volatility, real non-residential private fixed investment is less responsive to the monetary shocks. In comparison to a lower uncertainty state, the effects of a one percentage point unanticipated decrease in the real interest rate will generate a 0.6% $\left(\{\gamma_7 = 0.30\} \times \{SD_{IQR} = 0.022\}\right)$ lower investment rate bottomed at quarter 7 if the stock market volatility measure $u_t = IQR_t^{stock}$ increases by one standard deviation ($SD_{IQR} = 0.022$). As hypothesized following the firm-level regressions, the economy-wide differential response is much larger than the Compustat sample firm-level differential response, further indicating that the fixed costs

[16] The economy-wide responses are not as timely as Compustat firms.

[17] The main sample ends as the federal fund rate hits zero lower bound in 2009. However, In the appendix, I replace the federal fund rate with the shadow rate constructed by Wu and Xia (2016) from 2009Q3 to 2015Q4. In this way, we could extend the whole series through 2018Q2. The output gap is constructed as the gap between real GDP from the BEA and the real potential GDP forecasts of the Congress Budget Office. The short-term interest rate is the quarterly averaged daily effective federal fund rate. I also transformed the output gap, investment, and private consumption series using natural logarithms. For the monetary policy shock, I use the real interest rate (r_t^m) constructed as the short-term interest rate minus the CPI. I flip the sign, so now a positive increment in the shock indicates monetary stimulus. The investment series (ΔI_t) is the real non-residential private fixed investment, which is most relevant for the firm's investment choice as per real-option theory. I also use alternative aggregate investment measures such as the real private fixed investment, which includes residential investment, the real gross private investment, and the real gross fixed capital formation cost as calculated by the OECD. The results are slightly different but generally consistent.

Figure 4: Aggregate-level Differential Response to Monetary Shocks



matter.

2.5 Robustness Checks

In the Empirical Appendix B, I show various robustness checks using different monetary policy indicators, different investment measures, and different Local Projection sample periods. I briefly summarize the results of these robustness checks here. The results in the main text hold in almost all of these exercises.

Alternative Monetary Policy Shock Measures: I use the high frequency identified monetary policy shocks as in Gertler and Karadi (2015). The idea is to use high frequency identified financial variable movements from the 30mins window around FOMC announcements as an external instrument for the one-year government bond rate in a monthly IV-VAR with log industrial production, the employment rate, the log of the consumer prices index and a measure of corporate interest spreads. I take the predicted residual of the instrumented one-year government bond rate and then aggregate them to a quarterly series. As these shocks are exogenous disturbances, there is no need to include further macro controls in the local projection regressions. The results hold.

Alternative Investment Measures: I choose alternative measures of investment and also components of investment measures for the aggregate local projection regressions. These alternatives includes real gross fixed capital formation, real gross private investment, and real private fixed investment. Components includes equipment, structures, and intellectual property. I also include the output gap as an external validation. My results hold for most measures excluding intellectual property.

Alternative Time Series Periods: I choose alternative periods for both the firm-level and the aggregate local projection regressions. There are three major trade-offs: 1985, 1994, and 2009. There are natural concerns about potential structural monetary policy changes at these time points. My results hold for various combinations of sample period choices.

2.6 Inferring the Reduction of the Effectiveness of Monetary Policy

The previous sections showed a negative correlation between uncertainty and the effectiveness of monetary policy using both firm-level data and aggregate-level data. This finding suggests that besides the well-known fact that increasing uncertainty lowers investment, as in Section 2.1, uncertainty also plays a crucial role in determining the effectiveness of monetary policy. The model in 3 will explain why and how uncertainty affects monetary policy effectiveness, and I will estimate the model parameters to match micro-level investment data and quantify the effect of uncertainty on monetary policy effectiveness in Section 6.

To make the comparison between the empirical and the quantitative results in Figure 4 easier, I carry out a back of the envelope calculation to map my estimated coefficient to a monetary policy shock in Christiano et al. (2005) and a level of uncertainty shock in Bloom et al. (2018). In Figure 4, the effects of a -100 base points cut in real interest rate will generate 0.6% lower investment rate if the stock market volatility measure $u_t = IQR_t^{stock}$ increases by one standard deviation ($SD_{IQR} = 0.022$).

First, I show the re-scaled marginal effect of a **doubling of uncertainty** on the peak impulse response of investment to a standard **-25bps Taylor Rule residual shock**. In a standard SVAR estimation such as Christiano et al. (2005), around half of an -25bps monetary policy shock would goes into the changes in the real interest rate (-12.5bps). I re-scale the -100bps changes in the real interest rate within the aggregate local projection to -12.5bps for an -25bps monetary policy shock. I also re-scale the one standard deviation ($SD_{IQR} = 0.022$) changes in the uncertainty level to the mean uncertainty level of the normal state ($IQR = 0.07$). Therefore, the gap of peak response of investment to an -25bps monetary policy shock with a 100% higher uncertainty level is re-scaled as:

$$\frac{dI}{drdu} = -0.6\% \times \frac{12.5}{100} \times \frac{0.07}{0.022} = -0.24\%$$

Second, I can compare the effectiveness of a monetary policy shock with an uncertainty shock as estimated in Bloom et al. (2018) to a monetary policy shock with no uncertainty shock.[18] Their estimated aggregate shock to firm-level uncertainty is $\Delta u = 3$ and the well-established peak response of investment to an identified -25bps monetary policy shock is $\frac{dI}{dr} = 1.2\%$ as in Christiano et al. (2005). Therefore, the reduction in the effectiveness of monetary policy to the uncertainty shock is:

$$\frac{dI/drdu}{dI/dr} \times \Delta u = \frac{-0.24\%}{1.2\%} \times 3 = -60\% \tag{5}$$

As a result, the effectiveness of monetary policy following an estimated aggregate shock to firm-level uncertainty as in Bloom et al. (2018) is only 40% relative to the effectiveness of the same monetary policy shock with average uncertainty. In the following quantitative analysis, I will implement exactly the same uncertainty shock within a model.

3. The Model

The model builds on the class of Real Business Cycle models with capital adjustment costs, including random adjustment costs as in Khan and Thomas (2008), partially irreversibility as in Abel and Eberly (1996), and quadratic adjustment costs. I then extend the Real Business Cycle framework to the New Keynesian framework by introducing price rigidity and a monetary authority. In the model, firms use both capital and labor to produce an identical intermediate good, which is sold to retailers at the same wholesale price P_t^W. Firms that adjust their capital stock incur adjustment costs. Firms are also subject to an exogenous idiosyncratic process for productivity. To keep the model tractable, I then introduce a New Keynesian Block to separate rigidity in price setting from firms' production decisions. Finally, a representative household closes the model.

[18] Since I do not have an economy-wide representative firm sample and my quantitative model is for the whole economy, it is better to rely on Bloom et al. (2018) rather than using the Compustat sample.

3.1 Production Block

There is a fixed unit mass of firms $j \in [0,1]$ which produce output y_{jt} according to a decreasing returns to scale production function. For each firm, its output is then sold to a corresponding retailer at an economy-wide wholesale price P_t^W.

Technology: The production function is as follows:

$$y_{jt} = z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1 \tag{6}$$

where k_{jt} and n_{jt} denote the idiosyncratic capital and labor employed by the firm j. The technology is decreasing returns to scale so $\alpha + \nu < 1$. For each firm, the idiosyncratic productivity is z_{jt}. I assume the shocks follow the following log-normal AR(1) processes:

$$log(z_{jt}) = \mu_t + \rho^z log(z_{jt-1}) + \sigma_{t-1}^z \epsilon_{jt}, \ \epsilon_{jt} \sim N(0,1) \tag{7}$$

where the variance of the idiosyncratic innovation, σ_{t-1}^z and the corresponding adjustment in the mean $\mu_t = -(1-\rho^z) \times \frac{{\sigma_{t-1}^z}^2}{2(1-\rho^{z2})}$ is fixed during normal times. An aggregate shock to firm-level uncertainty is an unexpected sharp rise in the variance of productivity shock σ_{t-1}^z and a corresponding adjustment in μ_t so that the mean of z_{jt} is unchanged.

Adjustment Costs: The deterministic investment cost function includes three components: a direct cost i_{jt}, a partially irreversible cost governed by S, and a quadratic cost governed by ϕ_k. In addition, firms who actively adjust their capital stock also pay a random fixed cost ξ_{jt} in units of labor if they adjust more than a small proportion of their current capital stock ($|ak|$). When an investment adjustment is large enough, $i_{jt} \notin [-ak_{jt}, ak_{jt}]$, firms have to pay the random adjustment costs. The random fixed costs ξ_{jt} is uniformly distributed with support $U[0,\bar{\xi}]$ independently across firms and time.

$$c(i_{jt}) = i_{jt} + |i_j| \left(\mathbf{1}_{(i_{jt}<0)} \cdot S + \frac{\phi_k}{2} |\frac{i_{jt}}{k_{jt}}| \right) + \mathbf{1}_{(|i_{jt}|>ak_{jt})} \cdot \xi_{jt} \cdot w_t \tag{8}$$

This specification of capital adjustment costs is comprehensive enough to nest previous literature. The existence of partial irreversibility generates real options with respect to investment, as articulated in Dixit et al. (1994), which is the critical component for the wait-and-see effects of uncertainty shocks. The quadratic adjustment cost smooths out the investment behavior, as documented in Winberry (2018a) and Koby and Wolf (2019), which is essential to match the cross-section of the investment distribution when the random fixed cost is relatively large. Finally, the random fixed costs make firms pay the fixed cost infrequently, which is the key in generating lumpy investment patterns as in the microdata, as addressed in Cooper and Haltiwanger (2006). I also allow a region $i_{jt} \in [-ak_{jt}, ak_{jt}]$ within where firms do not have to pay the random adjustment cost in order to generate empirically plausible small investment adjustment behaviors around zero investment following Khan and Thomas (2008).

Firm Optimization: I denote by $\tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t)$ the original value function of a firm, $V^A(k_{jt}, z_{jt}; \Omega_t)$ the value function of a firm with an active investment choice, $V^{NA}(k_{jt}, z_{jt}; \Omega_t)$ the value function of a firm without an active investment choice, and $V(k_{jt}, z_{jt}; \Omega_t) = E_{\xi_{jt}} \tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t)$ the dimension-reduced value function of a firm with expected draw of ξ_{jt}. The state variables are given in two parts: (i) individual state of capital stock k_{jt}, individual state of productivity z_{jt}, and individual state of the random fixed cost draw ξ_{jt}; (ii) aggregate state $\Omega_t = (U_{t-1}, \Theta_t, \mu_t(k, z, \xi))$ where U_{t-1} indicates the current degree of uncertainty, Θ_t is the vector of all the aggregate variables including aggregate productivity, inflation, interest rate, wholesale price, stochastic discount factor, and wage at time t, and $\mu_t(k, z, \xi)$ is the current distribution of firms. The original dynamic problem of the firm consists of choosing investment and hours to maximize its recursive value function:

$$\begin{aligned}\tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t) = \max_{n} & \left\{ p_t^w y_{jt} - w_t n_{jt} \right\} \\ & + \max_{i \notin [-ak, ak]} \left\{ -c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^*_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \\ & + \max_{i \in [-ak, ak]} \left\{ -c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^C_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\}\end{aligned} \tag{9}$$

where the real wholesale price $p_t^w = \frac{P_t^W}{P_t}$ is from retailers, the real wage $w_t = \frac{W_t}{P_t}$ is from households, and the stochastic discount factor $\Lambda_{t,t+1} = \frac{\pi_{t+1}}{R_t^n}$ is derived from the household problem as households own all the firms. R_t^n is the nominal interest rate of one-period bonds and π_{t+1} is the inflation. All the aggregate prices are components of the aggregate state $\Theta_t \in \Omega_t$, however, for simplicity, I will only denote them by time t. The choice of k^C_{jt+1} is constrained by the no random fixed cost region $k^C_{jt+1} \in [(1-\delta-a)k_{jt}, (1-\delta+a)k_{jt}]$ while k^*_{jt+1} is not constrained. I can then separate the firm's original recursive value function depending on its investment choice as:

$$V^A(k_{jt}, z_{jt}; \Omega_t) = \max_{i,n} \left\{ p_t^w y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^*_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{10}$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_t) = \max_{i \in [-ak, ak], n} \left\{ p_t^w y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^C_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{11}$$

The firm will choose to pay the fixed cost if and only if the value from doing so is higher than not paying the fixed cost, that is, if and only if $V^A(k_{jt}, z_{jt}; \Omega_t) - w_t \xi_{jt} > V^{NA}(k_{jt}, z_{jt}; \Omega_t)$. For each tuple of $(k_{jt}, z_{jt}; \Omega_t)$, there is a unique threshold $\xi^*(k_{jt}, z_{jt}; \Omega_t)$ which makes the firm indifferent between these two options. The threshold is:

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t} \tag{12}$$

If a firm in state of $(k_{jt}, z_{jt}; \Omega_t)$ draws a random fixed cost ξ_{jt} below $\xi_t^*(k_{jt}, z_{jt}; \Omega_t)$, the firm pays the fixed cost and then actively adjusts its capital; otherwise, it does not. The firms' optimal choice to only pay the fixed cost infrequently is part of what generates lumpy investment patterns as in the microdata.

Given the distribution of the random fixed cost and the optimal thresholds over the space of $(k_{jt}, z_{jt}; \Omega_t)$, the value function is eventually determined as:

$$\begin{aligned} V(k_{jt}, z_{jt}; \Omega_t) = -\frac{w_t \xi^*(k_{jt}, z_{jt}; \Omega_t)}{2} + \frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}} V^A(k_{jt}, z_{jt}; \Omega_t) \\ + \left(1 - \frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t) \end{aligned} \tag{13}$$

where the firm expects to pay the random fixed cost in units of labor when the draw is lower than $\xi^*(k_{jt}, z_{jt}; \Omega_t)$. If so, with probability $\frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}}$, the value would be the active value function $V^A(k_{jt}, z_{jt}; \Omega_t)$; otherwise, its value would be the non-active value function $V^{NA}(k_{jt}, z_{jt}; \Omega_t)$. Therefore, the capital stock evolves by the law of motion:

$$k_{jt+1} = \begin{cases} (1-\delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1-\delta)k_{jt} + i_{jt}^C & otherwise \end{cases} \tag{14}$$

3.2 New Keynesian Block

I design the New Keynesian block of the model to generate a New Keynesian Phillips curve relating nominal variables to the real economy[19]. I separate the nominal rigidities from the firm problems to

[19] I follow Ottonello and Winberry (2018). Similarly to their problem, studying the joint dynamic decision of investment and price setting with nominal rigidities is outside this paper's scope. Nor do I have any micro moments that would provide insight on this joint problem. Therefore, a New Keynesian block is a parsimonious way to model price rigidity. This is a possibly exciting direction for further research on monetary policy.

keep the model more tractable. The New Keynesian block consists of retailers who make the pricing decisions, a final good producer who produces final goods, and a monetary authority who sets the interest rate rule. The whole New Keynesian block is adding essentially two equations to the general equilibrium model: i) a New Keynesian Phillips curve which links wholesale prices to inflation, and ii) a Taylor Rule which links the monetary policy shock and inflation to the nominal interest rate. Without the New Keynesian block, the economy is reduced to a standard RBC model with lumpy investment.

Retailers: For each production firm j, there is a corresponding retailer j who produces a differentiated variety $Y_t(j)$ using good y_{jt} from production firm j' as its only input. The production function is simply a one-to-one transformation:

$$Y_t(j) = y_{jt}$$

where the retailers are monopolistic competitors who set their prices $P_t(j)$ subject to the demand curve generated by the final good producer and the wholesale price of the input P_t. Retailers pay a quadratic menu cost in term of final good, $\frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$, to adjust their prices as in Rotemberg (1982), where Y_t is the final good.

Final Good Producer: There is a representative final good producer who produces the final good Y_t using intermediate goods from all retailers with the production function:

$$Y_t = \left(\int Y_t(j)^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}}$$

where γ is the elasticity of substitution between intermediate goods. The final good producer's profit maximization problem gives the demand curve $\left(\frac{P_t(j)}{P_t}\right)^{-\gamma} Y_t$ where the price index is $P_t = (\int P_t(j)^{1-\gamma} dj)^{\frac{1}{1-\gamma}}$. The final good is taken as the numeraire in the model.

Price Setting by Retailers: The resulting price stickiness would come from the price-setting decisions made by retailers maximizing profits. I follow Rotemberg (1982) except the marginal cost is now the wholesale price P^W from production firms:

$$\Pi_t(j) = (P_t(j) - P_t^W)\left(\frac{P_t(j)}{P_t}\right)^{-\gamma} Y_t - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$$

Through a standard derivation via retailers' profit maximization process (see Appendix A.1), we have the ***New Keynesian Phillips curve***. This paper will directly focus on the linearized version for computational simplicity[20],

$$log\pi_t = \frac{\gamma - 1}{\psi} log \frac{p_t^w}{p^{w*}} + \beta E_t log\pi_{t+1} \tag{15}$$

where $p_t^{w*} = \frac{\gamma-1}{\gamma}$ is the steady state wholesale price, or in other words the marginal cost for retailer firms. The Phillips Curve links the New Keynesian block to the production block through the relative the real wholesale price p_t^w for production firms. If the expectation of future inflation unchanged, when aggregate demand for the final good Y_t increases, retailers must increase production of their differentiated goods because of the nominal rigidity. This in turn increases demand for the production goods y_{jt}, which increases the real wholesale price p_t^w and generates inflation through the Phillips curve.

Monetary Authority: The monetary authority sets the nominal risk-free interest rate R_t^n according to the log version of a ***Taylor rule***:

$$logR_t^n = log(\frac{1}{\beta}) + \phi_\pi log\pi_t + \epsilon_t^m, \text{ where } \epsilon_t^m \sim N(0, \sigma_m^2) \tag{16}$$

where π_t is gross inflation in the final good price, ϕ_π is the weight on inflation in the reaction function, and ϵ_t^m is the monetary policy shock.

3.3 Household Block

The general equilibrium model is completed by introducing the household block. There is a unit measure continuum of identical households with preferences over consumption C_t and labor supply N_t whose expected utility is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \theta N_t \right)$$

subject to the budget constraint:

$$P_t C_t + \frac{1}{R_t^n} B_t \leq B_{t-1} + W_t N_t + \Pi_t^F$$

[20]For robustness when I also solve the quantitative model using the non-linearized version, the results are almost identical. Therefore, in order to save computational time, I use the linearized version throughout the whole paper.

where β is the discount factor of households, θ is the disutility of working, P_t is the price index, R_t^n is the nominal interest rate, B_t is one period bonds, W_t is the nominal wage, and Π_t^F is the nominal profits from all the firms.

Households choose over consumption, labor, and bonds, which supplies two Euler equations that determines both the real wage and the stochastic discount factor for the firms' problem in terms of aggregate consumption and aggregate labor supply:

$$w_t = \frac{W_t}{P_t} = -\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = \theta C_t^{\eta} \tag{17}$$

$$\Lambda_{t,t+1} = \beta \frac{U_c(C_{t+1}, N_{t+1})}{U_c(C_t, N_t)} = \beta \Big(\frac{C_t}{C_{t+1}} \Big)^{\eta} \tag{18}$$

where the ***Stochastic Discount Factor*** is linked through the Euler equation for bonds

$$\Lambda_{t,t+1} = \frac{1}{R_t^n} \frac{P_{t+1}}{P_t} = \frac{\pi_{t+1}}{R_t^n} \tag{19}$$

3.4 Equilibrium Definition

I now characterize and define the equilibrium of the model. The aggregate state vector is $\Omega_t = (U_{t-1}, \Theta_t)$, where U_{-1} is the uncertainty state today, which is determined in the prior period, and Θ_t is the collection of aggregate prices. I also define $\mu(k, z, \xi)$ as the distribution of firms over their state vector (k, z, ξ).

A ***Recursive Competitive Equilibrium*** *for this economy is defined by a set of value functions and policy functions* $\{V(k, z; \Omega)$, $V^A(k, z; \Omega)$, $V^{NA}(k, z; \Omega)$, $\xi^*(k, z; \Omega)$, $k^*(k, z; \Omega)$, $k^C(k, z; \Omega)\}$, *a set of quantity functions* $\{C(\Omega)$, $N^s(\Omega)$, $N^d(\Omega)$, $Y(\Omega)$, $K(\Omega)\}$, *a set of price functions* $\{w(\Omega)$, $\Lambda(\Omega)$, $p^w(\Omega)$, $R^n(\Omega)$, $\pi(\Omega)\}$, *and a distribution* $\mu'(\Omega)$ *that solves the firm's problem, retailer's problem, household's problem, and market clearing such that:*

*(i).**[Firm Optimization]** Taking the aggregate prices* $\{w(\Omega)$, $\Lambda(\Omega)$, $p^w(\Omega)\}$ *as given,* $V(k, z; \Omega)$, $V^A(k, z; \Omega)$, $V^{NA}(k, z; \Omega)$, *and* $\xi^*(k, z; \Omega)$ *solve the firms' Bellman Equations (10) — (13) with associated decision rules* $k^*(k, z; \Omega)$ *and* $k^C(k, z; \Omega)\}$.

*(ii).**[Household Optimization]** Taking the aggregate prices* $\{w(\Omega)$, $R^n(\Omega)$, $\pi(\Omega)\}$ *as given,* $C(\Omega)$, $N^s(\Omega)$, *and* $\Lambda(\Omega)\}$ *solve the household's utility maximization (17) — (19).*

*(iii).**[New Keynesian Block]** Retailers optimization leads to the NKPC (15) and monetary au-*

thority operation leads to the Taylor rule (16). For all Ω, both equations hold.

*(iv).**[Market Clearing]** For all Ω, labor supply $N^s(\Omega)$ equals labor demand $N^d(\Omega) = \int (n(k,z) + \xi(k,z))\, d\mu(k,z;\Omega)$, and the final goods market clears $Y(\Omega) = C(\Omega) + I(\Omega) + \Theta_p(\Omega) + \Theta_k(\Omega)$, where $\Theta_p(\Omega)$ is the price adjustment cost and $\Theta_k(\Omega)$ is the aggregate capital adjustment cost.*

3.5 Solution method

The critical challenge in solving the model is that the aggregate state vector Ω contains an infinite-dimensional object μ, which is the cross-sectional distribution of firms. I follow the MIT shock literature to overcome this challenge. As documented by Boppart et al. (2018), a reasonable MIT shock around the steady-state of the model provides a reasonably accurate approximation and preserves the non-linearity of the transition path very well[21].

The solution method involves two parts. First, I solve the *Stationary Equilibrium* at the steady-state, which delivers the value functions, the policy functions, and the steady-state aggregate variables. The *Stationary Equilibrium* also provides the cross-section moments for the calibration. Second, I solve the *Transitional Equilibrium* starting at the *Stationary Equilibrium* given a path of MIT shocks and a long enough period for the model to transit back to the same *Stationary Equilibrium*. The *Transitional Equilibrium* then provides the dynamic moments for the calibration and the impulse response functions. The advantage here is: i).non-linearity from non-convex adjustments and uncertainty shocks are fully captured; ii).adding multiple MIT shocks will not increase the solution time; iii). the interactions between different shocks are fully captured during the transition, which is essential for the quantitative results. The details of the solution methods are in Appendix A.8.

4. Illustrating the Mechanism

Before the quantitative analysis, this section introduces a minimum framework to illustrate aggregate investment responses to uncertainty shocks under various model specifications and shows how they affect the effectiveness of monetary policy. I give the basic intuition in the context of a simple two-period model.

[21] Compared to the a classical global method such as Krusell and Smith (1998) on the RBC models. There are too many aggregate prices and quantities to predict over the transition path because of the New Keynesian block. And finally, to capture the full dynamics of uncertainty shocks, linearization techniques, i.e., Reiter (2009) and Winberry (2018b) may need more adjustments.

4.1 A two-period model

I provide a graphical illustration of the centrality of value function concavity through a simple two-period model. A unit continuum of firms populates the model. All firms $j \in [0,1]$ have a common production function $y = z(k^{\alpha} n^{1-\alpha})^{\gamma}$ and begin period 1 with a common initial capital stock of k_0 and initial productivity z_1. The single choice variable of firms is their investment i_j. Denoting the investment choice set of firm j as I_j, we have its problem as follows:

$$\max_{i_j \in I_j, n_j} p * (y_j - c(i_j)) - w * n_j + \frac{1}{1+r} E_1\{V_2(k_1, z_2)\}$$

subject to:

$$\begin{aligned} k_1 &= (1-\delta)k_0 + i_j \\ y_j &= z_1(k_0^{\alpha} n_1^{1-\alpha})^{\gamma} \\ c(i_j) &= i_j + |i_j|\left(\mathbf{1}_{(i_j<0)} \cdot S + \frac{\phi_k}{2}\left|\frac{i_j}{k_0}\right|\right) + \xi_j \end{aligned}$$

The aggregate prices $\{p, w, r\}$ are taken as given by all firms, which will be further determined in general equilibrium, and labor is freely adjusted by all firms. I assume $\delta = 0$ for simplicity in this example. Under these conditions, we could reformulate the question as follows:

$$z_1^{\eta} k_0^{\mu} + \max_{i_j \in I_j} \left\{ -c(i_j)) + \frac{1}{R} E_1 \left\{ V_2(k_0 + i_j, z_2) \right\} \right\} \tag{20}$$

subject to:

$$R = h\left(\frac{p}{w^{(1-\alpha)\gamma}}\right)^{-\eta} \cdot (1+r)$$

$$c(i_j) = i_j + |i_j|\left(\mathbf{1}_{(i_j<0)} \cdot S + \frac{\phi_k}{2}\left|\frac{i_j}{k_0}\right|\right) + \xi_j$$

where $\{\eta, \mu, h\}$ are parameters[22] and R is the intertemporal price function which is taken as given by all firms. Since k_0 and z_1 are predetermined, the investment decision at period 1 is reduced to a trade off between the cost function $c(i_j)$ and the expected future value $\frac{1}{R}E_1\{V_2(k_0 + i_j, z_2)\}$.

[22]Parameters are: $\eta = \frac{1}{1-(1-\alpha)\gamma} > 1$, $\mu = \frac{\alpha\gamma}{1-(1-\alpha)\gamma} < 1$, $h = [(1-\alpha)\gamma]^{\frac{(1-\alpha)\gamma}{1-(1-\alpha)\gamma}}[1-(1-\alpha)\gamma]$

4.2 Value Functions and the Investment Decision

For the firm's maximization problem to be well-defined in this model, the value function must be concave in the capital; otherwise, the optimal investment will be infinite whenever the firm invests. Concavity is accomplished by assuming decreasing returns to scale in production ($\gamma < 1$) and the combination of convex and non-convex adjustment costs in this simple model and the full model. Therefore, the expected future value function is also concave in the capital. To demonstrate the shape of the value function, I solve the steady-state as in the full models where the idiosyncratic productivity shock follows an AR(1) process with a *three-state Tauchen* discretization. These steady states include various specifications of adjustment costs.

Adjustment Costs and the Concavity of the Value Function: I first show how adjustment costs affect the concavity of the value function in this three-state Tauchen discretization example. I solve for the steady-state of four models with with the same calibration as the full model in Section 5. The *Baseline* model includes all three types of moderate adjustment costs, and the *type-x only* models retain only type-x adjustment cost but set all other adjustment costs to zero.

Before the numerical graphs, recall that it is the slope of value function over the capital, not the level of the value function, that matters for the investment choice.[23] Therefore, to make the results more intuitive, I normalize the level of value functions around the *optimal capital stock* (k^*) for each productivity[24]. At the *optimal capital stock* (k^*), firms have no incentive at all to adjust their capital.

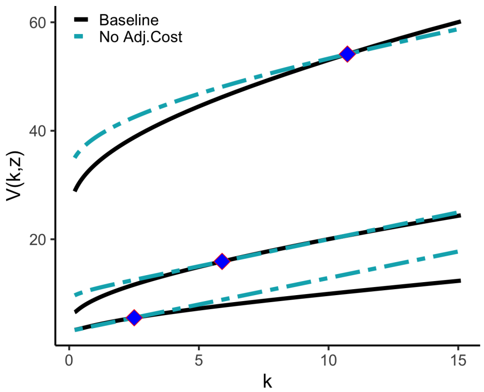
$$\frac{\partial y}{\partial k} = \mu z^{\eta} k^{\mu-1} = 1$$
$$k^*_{2j} = \mu^{1-\mu} z_{2j}^{\frac{\eta}{1-\mu}}, \; z_{2j} = z_l, z_m, z_h$$

The example results are in Figure 5. Start with the sub-figure (a), I emphasize three observations. First, the blue diamonds on each value function are the optimal capital stocks for the baseline model. Second, the value function of a given productivity is concave in the capital stock. Third, capital adjustment costs lower the value for a given productivity — the further current capital is from the optimum, the more substantial the loss in value. These three observations give us a sense of distribution of value function concavity over firms' investment decision space, shaped by the specification of adjustment costs. The key for the *Baseline* model is that it generates significantly more

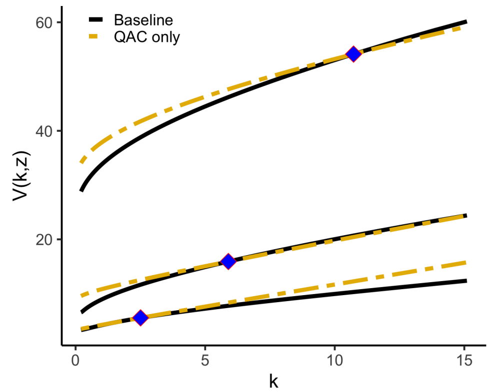
[23]The level of value function across different models is not comparable: the intertemporal price function R is all different across models.

[24]I also show the level of non-adjusted value function plots in the Appendix A.2.

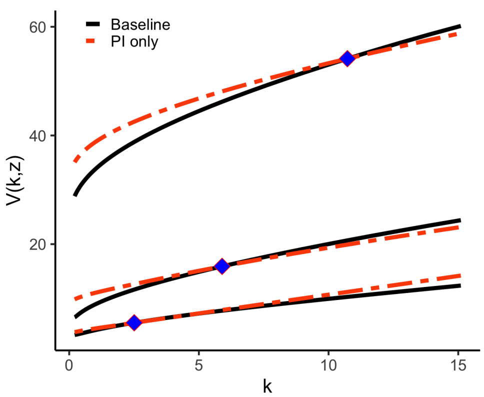
Figure 5: Adjustment Costs and Value Function Concavity



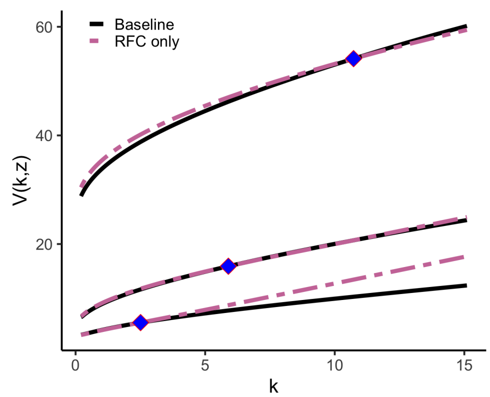
(a) No Adj. Cost



(b) QAC only



(c) PI only



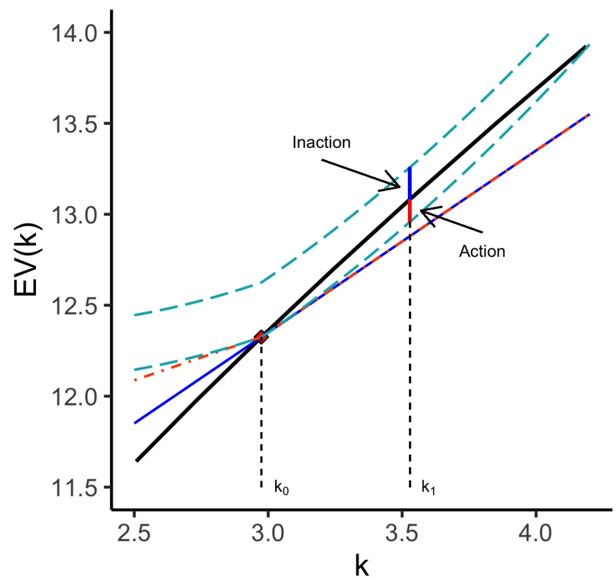
(d) RFC only

Notes: For each model, there are three value functions for three idiosyncratic productivity states. The highest is corresponding to high z, the middle one is corresponding to median z, and the lowest is corresponding to low z. The calibrations of the models are in Section 5. To specify the differences across models: *Baseline* — $\{S = 0.30, \bar{\xi} = 0.40, \phi_k = 2.40\}$, *QAC only* — $\{S = 0.00, \bar{\xi} = 0.00, \phi_k = 2.40\}$, *PI only* — $\{S = 0.30, \bar{\xi} = 0.00, \phi_k = 0.01\}$, *RFC only* — $\{S = 0.00, \bar{\xi} = 0.40, \phi_k = 0.00\}$.

concavity within the productivity-capital mismatch regions: the bottom right of the value function with low productivity and the top left of the value function with low productivity. Then in sub-figures (b), (c), and (d), I show other specifications. These specifications cannot generate enough concavity compared the baseline model.

Expected Value Function and Investment: The curvature of the value function over the decision space affects investment through the expected value function which is a combination of the three

Figure 6: Expected Value Function and Investment



Notes: The black line is the expected value function, a probability-weighted function of the three state value functions. The blue down-sloped solid line is the investment cost function with no adjustment costs. The red down-sloped dashed line is the investment cost function with partial irreversibility. The lower green down-sloped dashed convex line is the investment cost function with partial irreversibility and quadratic adjustment costs. Finally, the higher green down-sloped dashed convex line is the investment cost function with partial irreversibility, quadratic adjustment costs, and the maximum random fixed adjustment cost. Firms with $\{k_0\}$ who draw a random adjustment cost in the *Action* region would choose $k' = k_1$, otherwise $i = 0$.

state value functions with transition probability weights. Consider firms in the two-period example with median productivity $z_1 = z_m$ and a capital stock k_0, and optimal next period capital choice k_1. Examine the firm's decision space in Figure 6. First, the expected value function (the black solid line) is a probability weighted curve $EV(k) = \sum_{i\in\{l,m,h\}}\{\pi_{m,i} * V(k, z_i)\}$ where $\pi_{m,i}$ is the transition probability from z_m to z_i. Therefore, the *slope of* $EV(k)$ depends on the slopes of $V(k, z_i)$ and $\pi_{m,i}$. Here, I demonstrate the case that *slope of* $EV(k) > 1$ for (k_0, z_m).

If there is no random fixed cost, the firm chooses k_1, which delivers the max value between the expected value function (the solid black line) and the investment cost function (the lower green up-sloped dashed convex line). However, since firms have to pay a random fixed cost, the true investment cost function is a random draw between the bounds. Therefore, firms who draw a fixed cost within the *Action* region would choose $i = k_1 - k_0$, otherwise they choose $i = 0$. Given the initial capital stock fixed at k_0 and the cost function bends fixed, the *slope of* $EV(k)$ determined both the intensive margin ($k_1 - k_0$) and the extensive margin *(Action/Inaction)* of investment.

4.3 Effect of Uncertainty Shocks and Monetary Policy Shocks

Effect of Uncertainty Shocks: Suppose we fix the intertemporal price R. An uncertainty shock changes the transition probability $\pi_{m,i}$, but not the value function $V(k, z_i)$. Therefore, how an uncertainty shock affects investment depends on the curvature of $V(k, z_i)$ and how much the shock shapes the transition probability $\pi_{m,i}$.

The most common volatility shock yields transition probabilities $\{\pi'_{m,i}\} = \{\pi_{m,l}+\epsilon, \pi_{m,m}-2\epsilon, \pi_{m,h}+\epsilon\}$. The uncertainty shock would change the slope of the expected value function by:

$$\Delta s(k, z_m) = \epsilon \left(\frac{\partial V(k, z_h)}{\partial k} + \frac{\partial V(k, z_l)}{\partial k} - 2\frac{\partial V(k, z_m)}{\partial k} \right) \tag{21}$$

If $\Delta s(k_1, z_m) < 0$, the shock lowers the slope of expected value function and the *Action* region shrinking or even vanish if expected value function moves below the cost function for any $k > k_0$, as well as next period optimal choice of capital k_1' moving closer to k_0. If $\Delta s(k_1, z_m) > 0$, we would observe the opposite. There is a huge literature discussing this investment-uncertainty relationship dating back to the 1990s. This literature was inconclusive because various assumptions that were made led to different curvatures for value functions. Literature featuring constant returns to scale and/or low adjustment costs ends with a positive investment-uncertainty relationship. In contrast, literature featuring decreasing returns to scale and/or partial irreversibility ends with a negative investment-uncertainty relationship. The reason is that returns to scale shapes the concavity of the value function even without adjustment costs. Adjustment costs further increase concavity by worsening productivity-capital mismatch. With a reasonable calibration, $\Delta s(k_1, z_m) < 0$ is possible because $\frac{\partial V(k,z_l)}{\partial k}$ is sufficiently small for any $k > k_0$, and therefore an uncertainty shock generates investment drops even in a partial equilibrium model.

Effectiveness of Monetary Policy Shocks: In such an environment, monetary policy works through the intertemporal price R. The intertemporal price R enters into firm's investment decision as a multiplier to the expected value function. A monetary stimulus would lower R, therefore, increases the slope of the price adjusted expected value function ($\frac{1}{R}E_1\{V_2(k_1, z_2)\}$). As a result, the *Inaction* region shrinks (extensive margin), the next period capital choice k_1' increases (intensive margin), and therefore, the aggregate investment increases.

However, when an uncertainty shock hits, for a substantial mass of firms (especially the high productivity high capital stock firms), the value function slope in the low productivity state is very flat. As a result, their expected value functions are much flatter and may locate entirely below the investment cost function for any $k_1' > k_0$. A conventional monetary stimulus increases that slope only slightly, which cannot enlarge the *Action* region much or move it back above the investment cost function. Therefore, for a substantial mass of firms, monetary stimulus is less effective or even completely ineffective. In this case, the effectiveness of monetary policy on aggregate investment is reduced by uncertainty shocks.

5. Parameterization

Having highlighted the primary mechanism of this paper, I now take the full model to the data. I first describe the fixed parameters taken from the literature. Then I estimate the capital adjustment cost parameters to match U.S. micro-level investment data and a deep dynamic moment from the newly developed lumpy investment literature. I show that in order to generate reasonable aggregate dynamics, it is essential to match the dynamic moments rather than just the cross-section moments.

5.1 Calibration

My calibration proceeds in two steps. First, I fix a set of parameters to match standard macroeconomic targets in the steady-state. Second, given these fixed parameters' values, I choose the remaining capital adjustment cost parameters to match moments in the data.

Fixed Parameters Table 1 lists the parameters that I fix. The frequency of the model is a quarter, so I set the discount factor $\beta = 0.99$ to match an annual interest rate of 4%. I choose unit elasticity of intertemporal substitution $\eta = 1$ for log utility. Leisure preference $\theta = 2$ matches the fact that households spend a third of their time working.

On the firm side, I choose the capital coefficient $\alpha = 0.25$ and the labor coefficient $\nu = 0.60$ to match a labor share of two-thirds and a decreasing returns to scale imply of 85%. Capital depreciates at a rate of $\delta = 0.026$ quarterly, which generates the average aggregate nonresidential fixed investment rate in Bachmann et al. (2013). I choose the idiosyncratic productivity persistence and volatility following the estimation in Bloom et al. (2018) using U.S. manufacturing firms during

Table 1: **Fixed Parameters**

Parameter	Description	Value
Household Block		
β	Discount factor	0.99
η	Elasticity of intertemporal substitution	1
θ	Leisure preference	2
Production Block		
α	Capital coefficient	0.25
ν	Labor coefficient	0.60
δ	Capital depreciation	0.026
ρ_z	Persistence of TFP shock	0.95
σ_z	Volatility of TFP shock	0.05
New Keynesian Block		
γ	Demand elasticity	10
ψ	Price adjustment cost	90
ϕ_π	Taylor rule coefficient	1.5

normal uncertainty regimes.

For the New Keynesian block, I choose the elasticity of substitution in final goods production $\gamma = 10$, matching a steady-state markup of 11% as in Ottonello and Winberry (2018). The coefficient on inflation in the Taylor rule $\phi_\pi = 1.5$ is chosen within the literature's reasonable range. Finally, I set the price adjustment cost parameter $\psi = 90$, consistent with Kaplan et al. (2018)'s calibration: a 0.1 slope of the Phillips curve on the marginal cost which is also within the range of the NKPC slope estimation.[25]

Fitted Parameters I then choose the remaining adjustment costs parameters, listed in Table 2, in order to match the moments in Table 3. The *Annualized Cross-section Moments* are taken from Zwick and Mahon (2017) computed using the annual IRS corporate income tax returns. The *Annualized Dynamic Moments* are taken from both Zwick and Mahon (2017) and Koby and Wolf (2019).

[25]The slope of NKPC is undetermined though there is a vast literature. Schorfheide (2008) provides a very comprehensive summary of various strands of literature. The estimation ranges from almost zero (0.0004) to almost half (0.437). In this paper, I match to Kaplan et al. (2018).

Table 2: **Fitted Parameters**

Parameter	Description	Value
Adjustment Costs		
ϕ_k	Quadratic adjustment cost	2.40
$\bar{\xi}$	Upper bound of fixed cost	0.40
a	Bounds for no fixed cost	0.01
S	Resale loss in capital	0.33

To generate *Annualized Cross-section Moments* and *Auto-correlation of investment rates*, I first use Monte Carlo stochastic simulation to solve the steady-state with a very large number of firms for hundreds of quarters, and then I aggregate the quarterly results to annual moments. To generate *Elasticity to real interest rate shock*, I solve the *Partial Equilibrium* responses to a 0.25% increment in the real interest rate to match a 1.25% aggregate response in aggregate investment.

Table 3: **Target Moments**

Moments	Data	Model
Annualized Cross-section Moments		
Average investment rate (%)	10.4%	10.1%
Standard deviation of investment rates	0.16	0.12
Spike rate (%)	14.4%	15.3%
Positive rate (%)	85.6%	84.7%
Annualized Dynamic Moments		
Autocorrelation of investment rates	0.40	0.41
Real interest rate elasticity of aggregate investment	5	4.85

Notes: All the moments are from The Zwick and Mahon (2017) Appendix Table B.1. Statistics drawn from the distribution of investment rates pooled over firms and time for U.S. firms from 1998 to 2010. *Annualized Cross-section Moments* were already calculated in their appendix. The spike rate is the fraction of observations with an investment rate greater than 20%. The inaction rate is a fraction of observations with an absolute value of investment rate smaller than 1%. *Annualized Dynamic Moments* are also from Zwick and Mahon (2017). auto-correlation is reported in Appendix Table B.1. as well. However, the elasticity to the real interest rate shock is inferred and recalculated by Winberry (2018a) and Koby and Wolf (2019). An elasticity of 5 means that a 1% unexpected one-time change in real interest rate generates a 5% response in partial equilibrium investment.

I first target four *Annualized Cross-section Moments* related to the distribution of investment rates. These moments feature significant lumpiness and asymmetry in investment behaviors. The average investment rate is 10.4%, with a standard deviation of 0.16. About one-fourth of the observations feature an absolute value of investment rate $< 1\%$ and 14.4% observations featuring investment rate spikes $> 20\%$. In most of the literature, these cross-section moments usually pin down all the capital adjustment cost parameters. However, some of them failed to replicate reasonable aggregate dynamics as in the data. The essential reason is that they did not target dynamic moments. In the appendix A.4, I show that only matching the cross-section moments cannot guarantee plausible dynamics as in the data, and it is essential also to match certain dynamic moments.

I then target two *Annualized Dynamic Moments* related to the dynamics of investment rates. The autocorrelation of annualized investment rates ties closely to the random fixed cost, while the investment interest rate elasticity is a function of the overall magnitude of adjustment costs. The former is essential for the persistence of firms' behaviors, which matters for the consequences of an uncertainty shock, and the latter is the key for the aggregate responses of aggregate investment to the real interest rate and therefore also the response to a conventional monetary policy shock. Also, even though I do not have a dynamic moment to target for a partial equilibrium elasticity to uncertainty shocks, choosing a sizable partial irreversibility is important to generate substantial aggregate investment drops when the economy is hit by an uncertainty shock. Having sizable adjustment costs of all three types helps us fixes the puzzling dynamics addressed in the previous literature, for instance, Bachmann and Bayer (2013) for the non-significant response to an uncertainty shock, and Reiter et al. (2013) for the excessive response to a monetary policy shock, respectively.

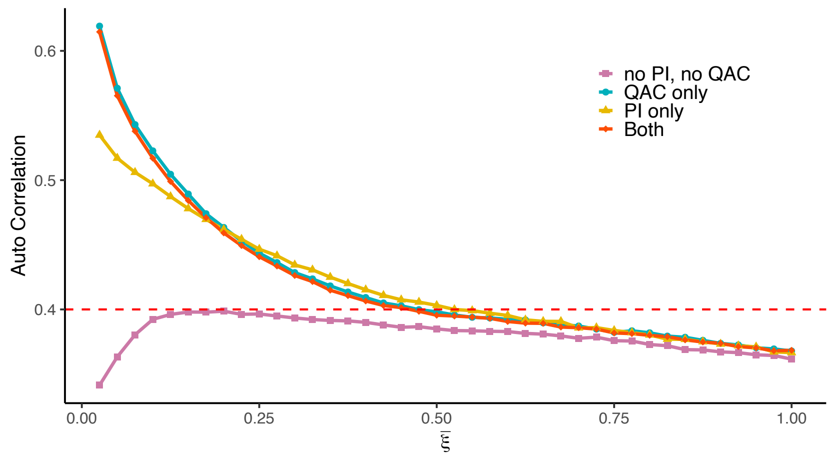
5.2 Identification

In this section, I discuss the key roles played by the random fixed costs and partial reversibility on the sensitivity of the aggregate investment in responses to interest rate and uncertainty shocks. Although the moments are jointly determined by all the adjustment cost parameters, identifying these parameters can be understood by checking the variations of these moments in two steps. First, the *Annualized Dynamic Moments* pin down the overall strength of the adjustment costs. Second, given the overall strength of adjustment costs, the *Annualized Cross-section Moments* pin down the allocation of the adjustment costs. To formally understand the identification of the key capital adjustment cost parameters, I plot the variation of critical moments under various calibration combinations.

5.2.1 The role of random fixed costs

Both dynamics moments are closely related to the identification of the random fixed costs. The increment in the random fixed costs enlarges the inaction region, decreases autocorrelation of investment rates, and decreases investment sensitivity to changes in interest rate.

Figure 7: Autocorrelation of Investment Rates



Notes: The moment *Autocorrelation of Investment Rates* is the average across all firms at the steady-state. To show how other adjustment costs affect the moment, I show four steady states with different combinations of Partial Irreversibility (PI) and Quadratic Adjustment Cost (QAC). I solve 40 steady states for each group, starting from a very low level of $\bar{\xi} = 0.025$ to a very high level of $\bar{\xi} = 1.0$. Group *no PI, no QAC* means that there is no Partial Irreversibility or Quadratic Adjustment Cost; Group *QAC only* means that there is only a moderate Quadratic Adjustment Cost $\phi_k = 2.4$; Group *PI only* means that there is only a moderate Partial Irreversibility $S = 0.33$; and Group *Both* means that there is a combination of moderate Quadratic Adjustment Cost $\phi_k = 2.4$ and moderate Partial Irreversibility $S = 0.33$.

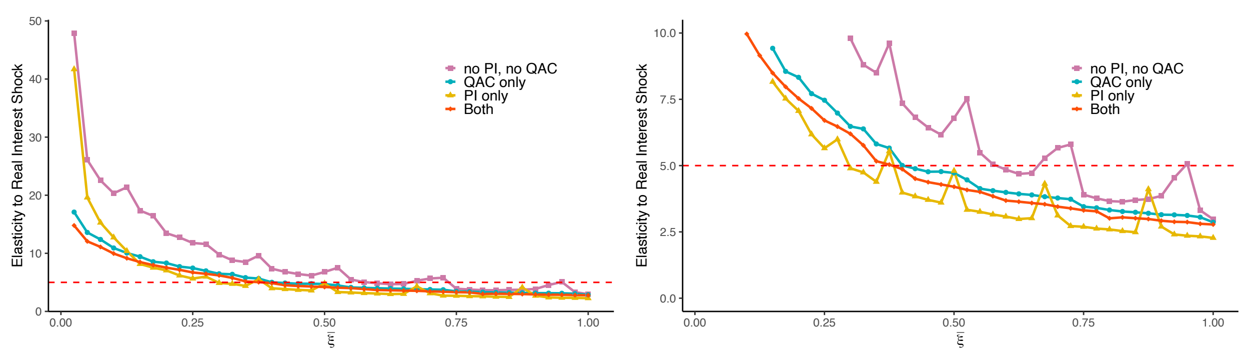
Autocorrelation of investment rates: Firstly, I show the identification of the random adjustment cost $\bar{\xi}$ exploring the variation of the *autocorrelation of investment rates* moment in Figure 7. To show how other adjustment costs affect the moment, I show four groups of steady states with different combinations of Partial Irreversibility (PI) and Quadratic Adjustment Cost (QAC). Except for the *no PI, no QAC* case,[26] the autocorrelation of investment rates monotonically decreases with the size of the random adjustment cost $\bar{\xi}$. Since the random adjustment cost is independent across time, a larger

[26]When $\bar{\xi}$ is very low, and there are no other adjustment costs, an increase in the size of the random adjustment cost would increase the average autocorrelation. This is because, compared to the no cost scenario, some firms would persistently remain inactive.

random adjustment cost would create more randomness in the cost of investment, which lowers firm incentives to engage in persistent investment across periods. This mechanism is in contrast to partial irreversibility and the quadratic adjustment cost, both of which determine adjustment costs based on firm investment decisions. Therefore, the other three simulated groups show a much higher autocorrelation when either PI or QAC is present. However, the size of the random adjustment cost still dominates this moment. Without sufficiently large random adjustment costs, it is impossible to match the autocorrelation of investment rates.

Real interest rate elasticity of aggregate investment: Secondly, I show how the *real interest rate elasticity of aggregate investment* identifies the overall strength of adjustment costs, especially the random fixed costs. Since all three types of adjustment costs govern the overall strength of adjustment costs, it is hard to show them graphically. Here, I show the variation as a function of the random adjustment cost and the quadratic adjustment cost with the four groups as in the identification above, respectively.

Figure 8: PE Real Interest Rate Elasticity of Agg. Inv.
(against RFC $\bar{\xi}$)



(a) Whole Domain

(b) Zoomed In

Notes: The moment is the elasticity of partial equilibrium aggregate investment responding to a one-time real interest shock. In this specification, I use a -25bps real interest shock; therefore, an elasticity of one means aggregate investment increases by 0.25%. To show how other adjustment costs affect the moment, I show four groups of steady states with different combinations of Partial Irreversibility (PI) and Quadratic Adjustment Cost (QAC). The details of the groups are same as in the previous Figure 7.

The results for the random fixed cost are in Figure 8 (a). First, in the *no PI, no QAC* case, we can see that when the random adjustment cost is very small, aggregate investment is strongly responsive to real interest shocks. I truncated the left tail at $\bar{\xi} = 0.025$, which gives us an elasticity of 47. This is already a tremendous response: a 1% real interest rate reduction leads to 47% more aggregate investment increment. Also, partial irreversibility cannot substantially help to resolve this over-response issue when the random adjustment cost is tiny. Moreover, in the cases with moderate quadratic adjustment costs, this elasticity is brought down some even when the random adjustment cost is tiny, but not enough unless the random adjustment cost is large.[27] Inspecting the target as in Figure 8 (b), we can observe that without any quadratic adjustment costs, the moment is not monotone over the random fixed cost $\bar{\xi}$. This is because, at a certain $\bar{\xi}$, the existence of the free adjustment zone triggers a higher elasticity. If we have moderate quadratic adjustment costs, this reversal vanishes. Therefore, to hit the target of interest rate elasticity, we need moderate random fixed costs.[28]

5.2.2 The role of partial irreversibility

Since there is no feasible moment to target for the investment sensitivity to changes in uncertainty in the partial equilibrium, I am not able to choose it to exactly target any empirically moment. In this subsection, I emphasize the key role of partial reversibility for the investment sensitivity to changes in uncertainty. The reduction of the partial reversibility (increment in S) creates higher real option value which increases the investment sensitivity to changes in uncertainty.

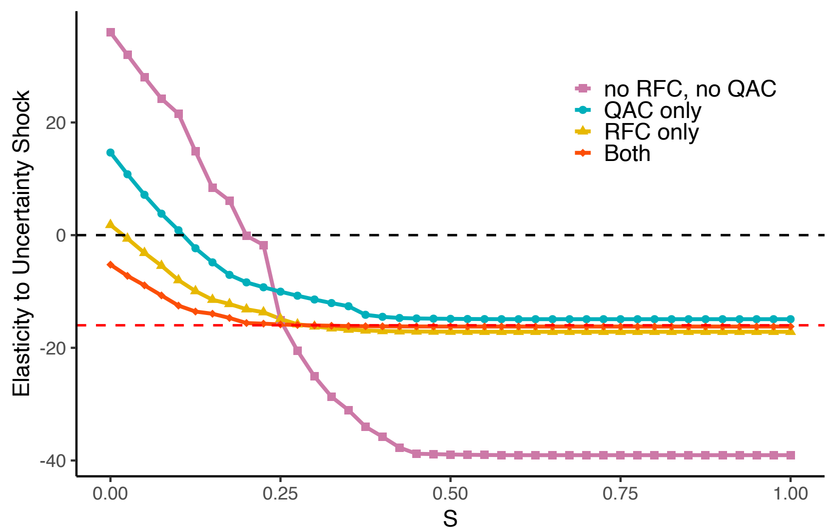
Uncertainty elasticity of aggregate investment: The results for the partial irreversibility are in Figure 9. As emphasized in literature, partial irreversibility creates real option which is the key of generating decreasing investment upon uncertainty shocks. Two empirical observations are essential: first, an uncertainty shock generates negative investment responses in the partial equilibrium; second, the negative responses are large enough. We can observe that we do need large enough partial irreversibility to generate negative responses in most groups, and the elasticity is monotonously decreasing. Eventually, it converges to around -18 in 3 cases with a moderate level of other adjustment costs and to -40 in the case without any adjustment costs. Since there is almost no variation of this moment after the irreversibility is "large enough" ($S > 0.3$), I choose $S = 0.33$ as estimated by Bloom (2009)[29].

[27] This mechanism holds in Winberry (2018a) when matching the relative variation of aggregate investment to aggregate output.

[28] I also show the results against QAC ϕ_k in Appendix A.3.

[29] I solve the quantitative results under various of choices of partial irreversibility $S > 0.3$ from the baseline choice of 33% to almost 100%, there is no significant differences in the results. The results are available upon requests.

Figure 9: PE Uncertainty Elasticity of Agg. Inv.
(against PI S)



Notes: The moment is the elasticity of partial equilibrium aggregate investment responding to a one-time uncertainty shock. In this specification, I use a $\Delta\sigma = 3$ uncertainty shock; therefore, an elasticity equals one means aggregate investment increases by 1%. To show how other adjustment costs affect the moment, I show four groups of steady states with different combinations of Partial Irreversibility (PI) and Quadratic Adjustment Cost (QAC). The details of the groups are same as in the previous Figure 7.

6. Quantitative Analysis

I now quantitatively analyze the effects of an uncertainty shock, a conventional monetary policy shock, and their interactions. Section 6.1 begins the analysis by detailing the investment policy at the steady-state and how it changes with an uncertainty shock. Section 6.2 studies the impulse response of aggregate investment to a conventional monetary stimulus. Section 6.3 studies the different effects of monetary policy under uncertainty shocks and shows that is consistent with the empirical results from Section 2, high uncertainty lowers the effectiveness of monetary policy. Section 6.4 validates the mechanism by showing that the differential aggregate effect of monetary policy depends on the specification of lumpy investment adjustment costs.

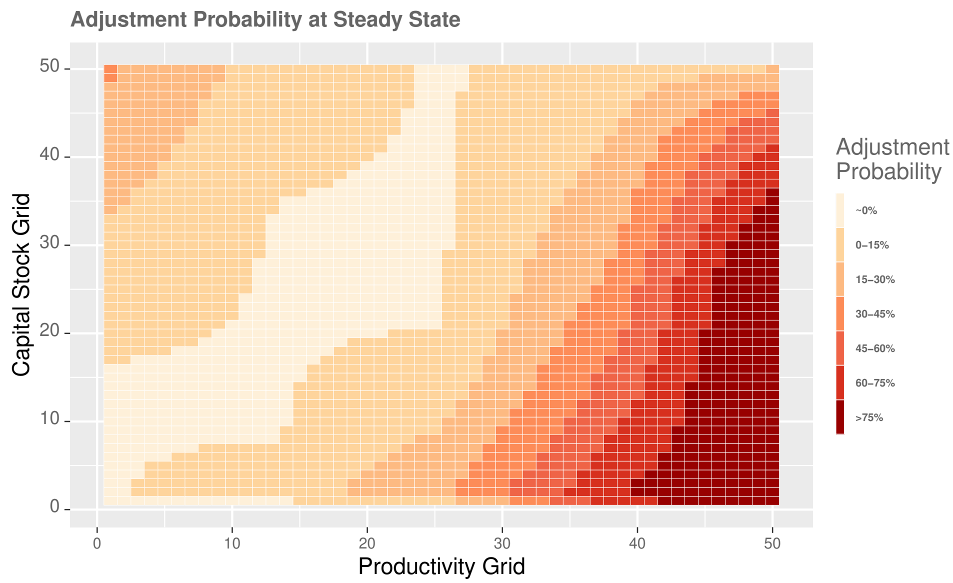
6.1 Uncertainty and Lumpy Investment

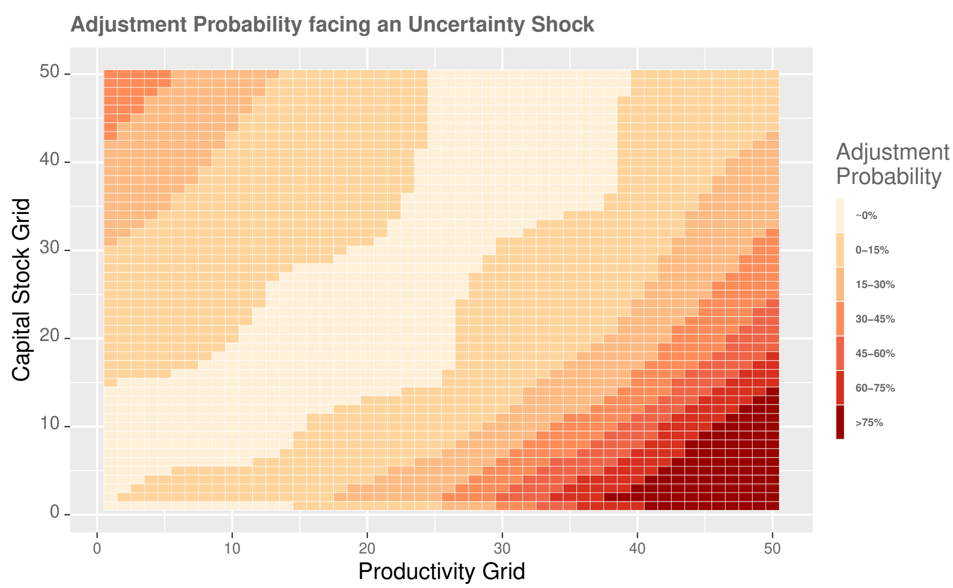
In this section, I present investment behavior in the steady-state and subject to an uncertainty shock as estimated in Bloom et al. (2018), respectively. Here, I illustrate the changes in both the extensive margin and the intensive margin when firms are facing an uncertainty shock. The economy is initially in steady-state and unexpectedly receives a change of variance $\Delta\sigma_z = 3.0$ innovation, which reverts to 0 in the next quarter. I compute the perfect foresight transition path of the economy, which converges back to steady-state.

The investment policy of firms consists of two parts: i). Should the random fixed cost be paid to adjust (the extensive margin); ii). How much investment should be done given the fixed adjustment cost is paid (the intensive margin). I show both investment policies using two-dimensional contour map. First, I show the adjustment probability over the capital-productivity grids in Figure 10. The range of adjustment probability ranges from 0% to 100% with warmer colors indicating a higher rate. The pale section in the middle is the complete inaction region (almost zero probability of adjustment), the section with high productivity and low capital (bottom right) is the positive investment region, and section with low productivity and high capital (top left) is the negative investment region. When an uncertainty shock hits, the inaction region grows, and the positive investment region shrinks. This results in a smaller mass of firms at the extensive margin of positive investment, and, more importantly, a smaller mass of high productivity and high capital firms (top right) who account for most of the aggregate investment response.

Second, I show the investment rate conditional on adjusting over the capital-productivity grids in Figure 11. The range of investment rate ranges from -75% to +75% with warmer colors indicating a higher rate. Similarly to the extensive margin adjustments, the pale section in the middle is the inaction region (almost zero investment rate even though allowed to adjust), the section with high productivity and low capital (bottom right) is the positive investment region, and the section with low productivity and high capital (top left) is the negative investment region. When an uncertainty shock hits, the inaction region grows and most positive investment regions shrink. The exception is that the most productive but low capital stock firms seize the opportunity and increase their investment. However, their mass is too small to make any sizeable aggregate impact. As a result, an uncertainty shock also works at the intensive margin: firms at the intensive margin lower their positive investment rate, and, more importantly, the mass of high productivity and high capital firms (top right) which account for most of the aggregate investment responses lower their positive investment rate the most.

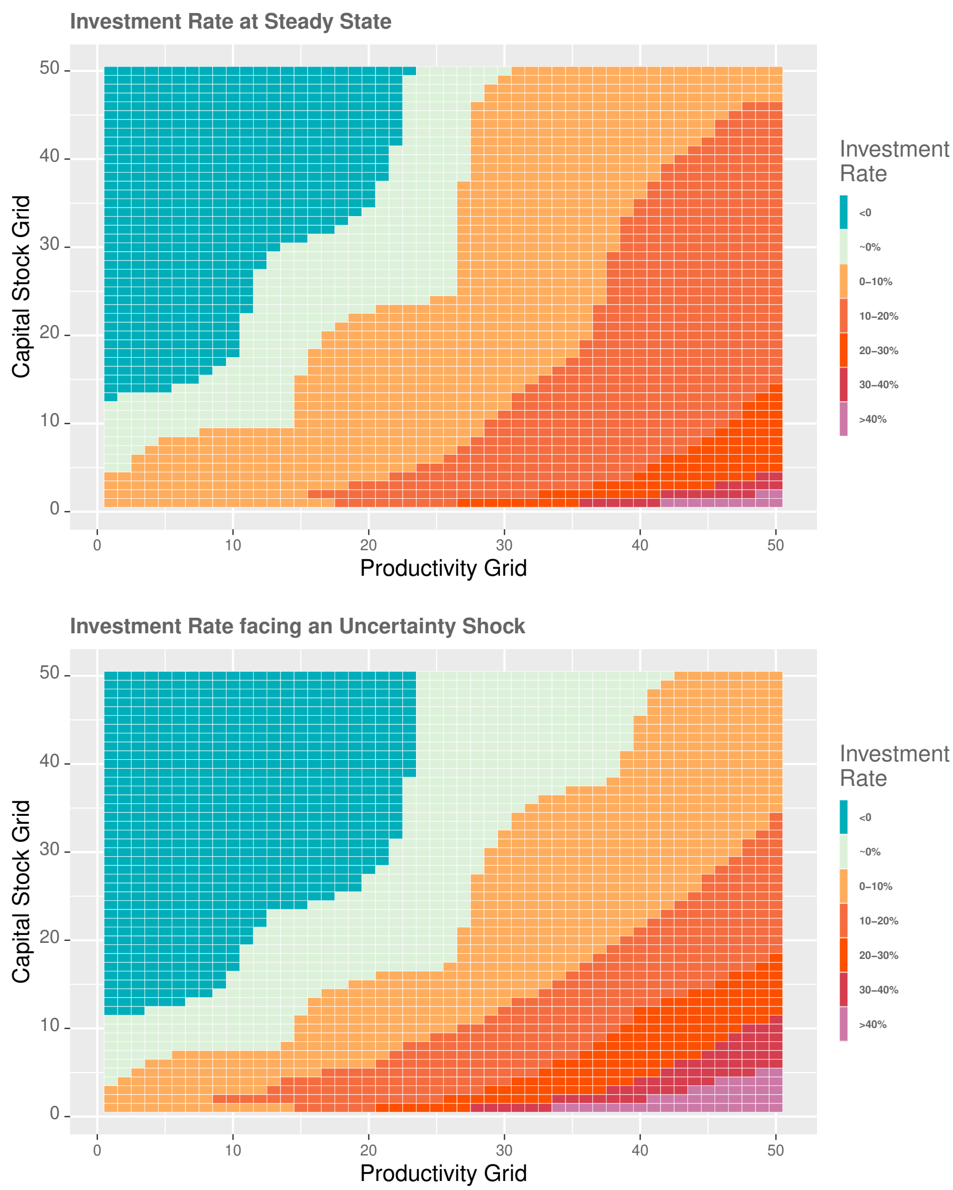
Figure 10: Extensive Margin of Investment





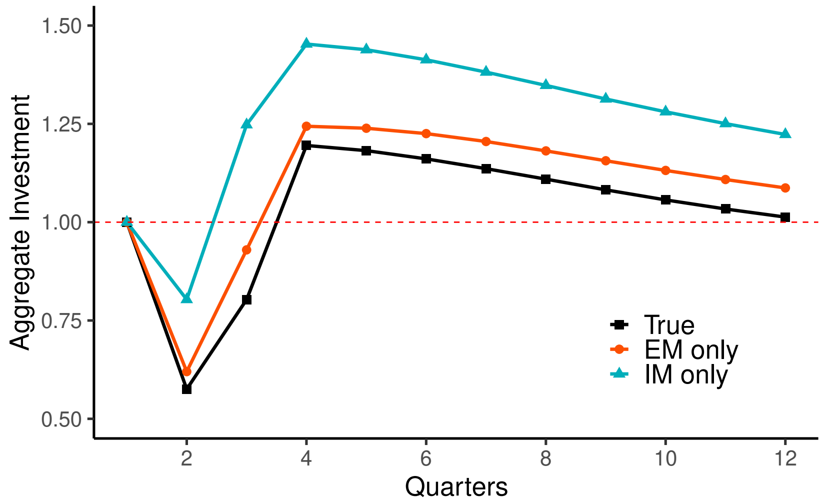
Notes: I solve the steady-state of the model, extract the steady-state cutoff for the random adjustment cost as a function of capital stock and productivity $\xi^*(k,z)[ss]$. Then I interpolate $\xi^*(k,z)$ over a denser grid of productivity and calculate the adjustment probability $Prob(k,z) = \xi^*(k,z)/\bar{\xi}$. For the case with an uncertainty shock, I solve the transition path and extract the first period of $\xi^*(k,z)[t=1]$. The interpolation part is identical. The behavior at the bottom row of the graph may look distorted because that is the lowest capital stock grid point in the quantitative analysis. The measure of firms there is almost zero.

Figure 11: Intensive Margin of Investment



Notes: I solve the steady-state of the model, extract the steady-state active investment rate policy as a function of capital stock and productivity $i(k,z)[ss]$. Then I interpolate $i^*(k,z)$ over a denser grid of productivity and calculate the investment rate $i\%(k,z) = \frac{i(k,z)}{k(k,z)}$. For the case with an uncertainty shock, I solve the transition path and extract the first period of $i(k,z)[t=1]$. The interpolation part is identical. The behavior at the bottom row of the graph may look distorted because that is the lowest capital stock grid point in the quantitative analysis. The measure of firms there is almost zero.

Figure 12: Aggregate Investment Responses to an Uncertainty Shock



Notes: I solve the steady-state of the model, extract the steady-state active investment policy as a function of capital stock and productivity $i(k,z)[ss]$ and the steady-state adjustment probability rate $Prob(k,z) = \xi^*(k,z)/\bar{\xi}$. Then I solve the transition paths for the distribution of investment, active investment policy, and adjustment probability. For the *True* path, I just plot the percentage changes in aggregate investment. For the *EM only* (extensive margin only) case, I hold the active investment policy $i(k,z)[ss]$ at steady state, but allow the adjustment probability $\{Prob(k,z)\}_{t=1}^{t=T}$ to change overtime. For the *IM only* (intensive margin only) case, I hold the adjustment probability $Prob(k,z)[ss]$ at steady state, but allow the active investment policy $\{i(k,z)\}_{t=1}^{t=T}$ to change overtime.

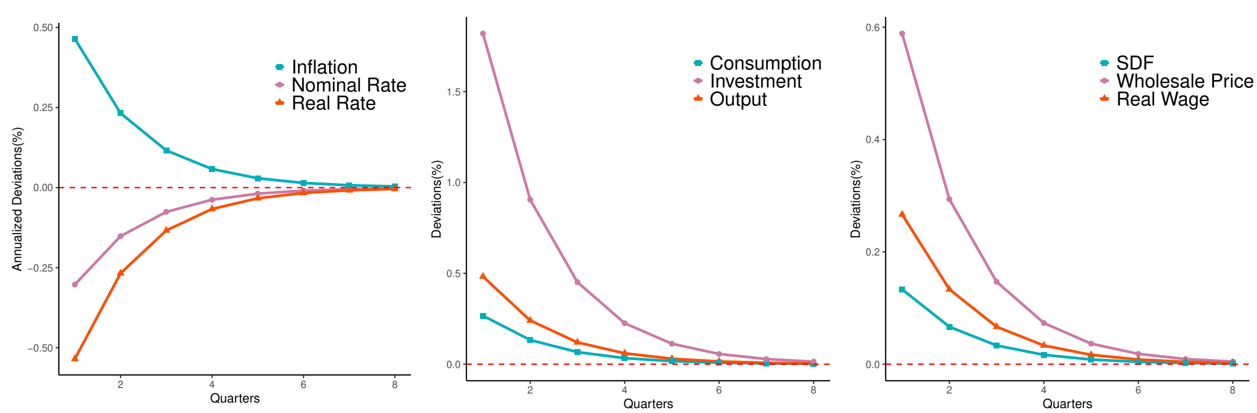
Finally, I present the aggregate investment responses to the same uncertainty shock in Figure 12. Beyond the true transition path, I also calculate two counterfactual cases holding either extensive margin or the intensive margin behavior constant. In the *True* case, upon the uncertainty shock, aggregate investment drops initially by 43%, then gradually recovers after four quarters. It then overshoots for six quarters and goes below the steady-state level. I truncated at quarter 12, but it eventually comes back to the steady-state level. Which margin generates this aggregate behavior? In the extensive margin only case, I hold the active investment policy at the steady-state level, but use the actual adjustment probability to calculate the *EM only* case. In the intensive margin only case, I hold the adjustment probability at the steady-state level, but use the actual active investment policy to calculate the *IM only* case. The enormous initial drop in aggregate investment almost equals the case with only the extensive margin; the two paths almost overlap. The intensive margin also responds with a 20% drop initially, but quickly recovers and then overshoots for a while. These patterns indicate that a certain level of lumpiness is essential for an uncertainty shock to generate large enough aggregate investment responses.

6.2 Monetary Policy and Investment

I now quantitatively analyze the effect of a conventional monetary policy shock. The economy is initially in the steady-state and unexpectedly receives an $\epsilon_1^m = -25bps$ innovation to the Taylor rule which reverts to 0 according to $\epsilon_{t+1}^m = \rho_m \epsilon_t^m$ with $\rho_m = 0.5$. I compute the perfect foresight transition path of the economy as it converges back to the steady-state.

Figure 13 plots the responses of the key aggregate variables to this expansionary monetary policy shock. The shock cuts down the nominal interest rate and lowers the real interest rate due to sticky prices. The lowered real interest rate stimulates investment demand by increasing the stochastic discount factor, so firms put more weight on future values. It also increases household consumption demand due to standard intertemporal substitution reasons. The wholesale price increases more than the real wage, which incentivizes firms to produce more output. Overall, investment increases by approximately 1.8%, consumption increases by 0.35%, and output increases by 0.5%. These magnitudes are broadly in line with the peak effects of monetary policy shocks estimated in Christiano et al. (2005) and the quantitative results from the most recent heterogeneous firm New Keynesian model in Ottonello and Winberry (2018).

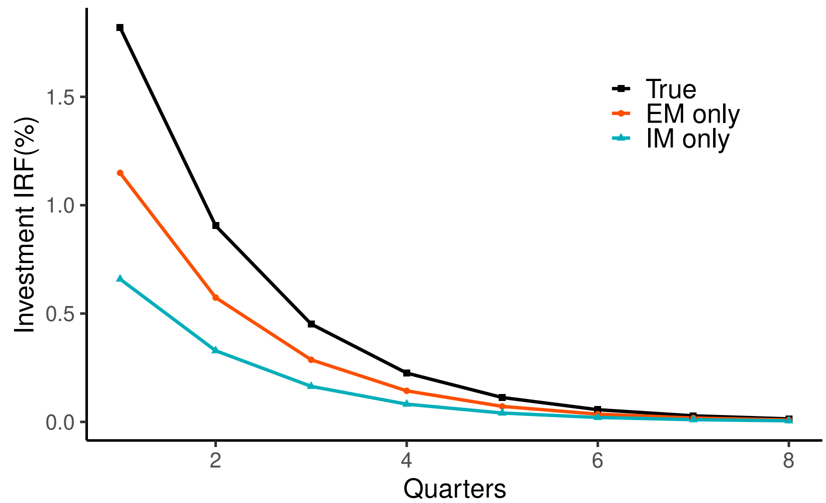
Figure 13: Aggregate Responses to a Monetary Policy Shock



(a) Monetary Rates (b) Aggregate Quantities (c) Aggregate Prices

Figure 14 plots the responses of aggregate investment to this expansionary shock with two counterfactuals. The shock stimulates investment by 1.8%, which narrows the gap between Reiter et al.

Figure 14: Aggregate Investment Responses to a Monetary Policy Shock



Notes: The construction of two counterfactuals follows the same process as in 12.

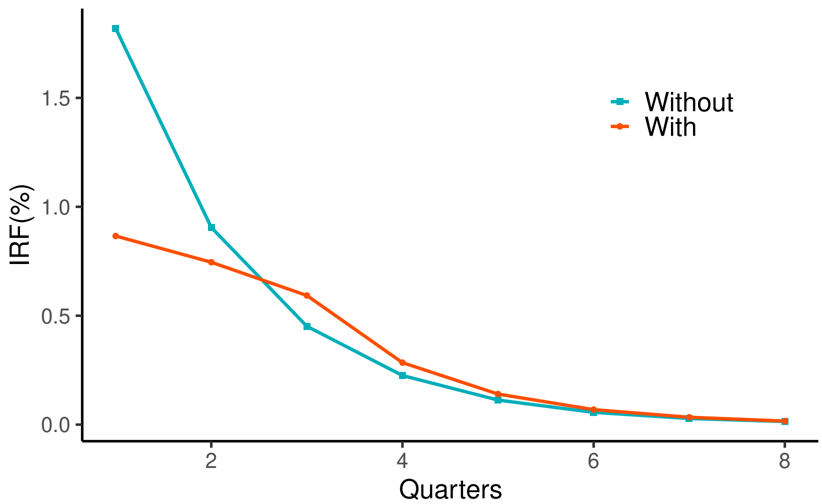
(2013) and the aggregate investment responses in the RANK literature. The same monetary policy shock in Reiter et al. (2013) generates more than a 5% peak response and only lasts for one quarter. However, by matching the real interest rate elasticity of aggregate investment, this model can generate a much more realistic peak response and persistence. I then discuss which margin matters for the monetary policy stimulus as in Section 6.1 . I conduct exactly the same counterfactuals as in Section 6.1 to construct the *EM only* case and *IM only* case. Unlike in the pure uncertainty shock scenario, both margins will have quantitative relevance, but the extensive margin matters more by a factor of two. These patterns indicate that with a reasonable degree of lumpiness, monetary policy mainly works through the extensive margin of aggregate investment.

6.3 Uncertainty and the Effectiveness of Monetary Policy

Section 6.1 and Section 6.2 show how micro-level lumpy investment matters for an uncertainty shock and a monetary policy shock, respectively. I now show how these two shocks interact, and therefore, how an uncertainty shock affects the effectiveness of a monetary policy shock.

The economy is initially in the steady-state and unexpectedly receives potential shocks. There are four cases: *Case One* the economy always stays at steady state; *Case Two* the economy unexpectedly receives an $\epsilon_1^m = 0.0025$ innovation to the Taylor rule which reverts to 0 according to $\epsilon_{t+1}^m = \rho_m \epsilon_t^m$ with $\rho_m = 0.5$ as in Section 6.2; *Case Three* the economy unexpectedly receives a $\Delta\sigma_z = 3.0$ innovation to the variances of the AR(1) process which reverts to 0 in the next quarter as in Section 6.1; *Case Four* the economy unexpectedly receives both an $\epsilon_1^m = 0.0025$ innovation to the Taylor rule which reverts to 0 according to $\epsilon_{t+1}^m = \rho_m \epsilon_t^m$ with $\rho_m = 0.5$ and a $\Delta\sigma_z = 3.0$ innovation to the AR(1) process which reverts to 0 in the next quarter. I compute the perfect foresight transition paths of the economy as it converges back to steady state.

Figure 15: Differential Responses to a Monetary Policy Shock



Notes: I solve the transition paths of the *Case Two* monetary policy shock only, *Case Three* uncertainty shock only, and *Case Four* with both shocks. I then extract the whole paths of aggregate investment. For the *Without* path, I plot the percentage changes in aggregate investment from the *Case Two* monetary policy shock only. For the *With* path, I calculate the gap between the aggregate investment paths of *Case Three* and *Case Four* to isolate the effect of monetary policy under uncertainty. The Without path, calculated as the gap between Case Two and Case One, shows the effect of monetary policy without uncertainty.

To compare the effectiveness of monetary policy, I calculate two differential paths. *Case Two* minus *Case One* (Without) — measuring the difference in aggregate investment between a monetary policy shock without any uncertainty shocks and the steady-state economy - is as in Section 6.2, while *Case Four* minus *Case Three* (With) captures the difference in aggregate investment between a joint monetary policy and uncertainty shock and a one-time uncertainty shock. These measures net out the effect of the uncertainty shock.

The results are in Figure 15. Compared to the impulse response of aggregate investment with average uncertainty, when the monetary stimulus is conducted simultaneously with an uncertainty shock, the initial responses is much weaker: 0.86% v.s. 1.81%.This is a reduction of 52%. This drop is almost the same magnitude as the drop in aggregate investment along the extensive margin caused by the uncertainty shock. Monetary stimulus primarily affects the intensive margin only when the extensive margin is frozen. Once the uncertainty is resolved investment returns to its prior path.

The different responses to monetary policy illustrate a new understanding of the investment channel of monetary policy: when investment is lumpy, the ability of monetary policy to stimulate investment is state-dependent. This effect is not only state-dependent on the first order moment as documented in the literature on productivity differences but also on the second order moment in terms of uncertainty differences.

6.4 Mechanism Validation

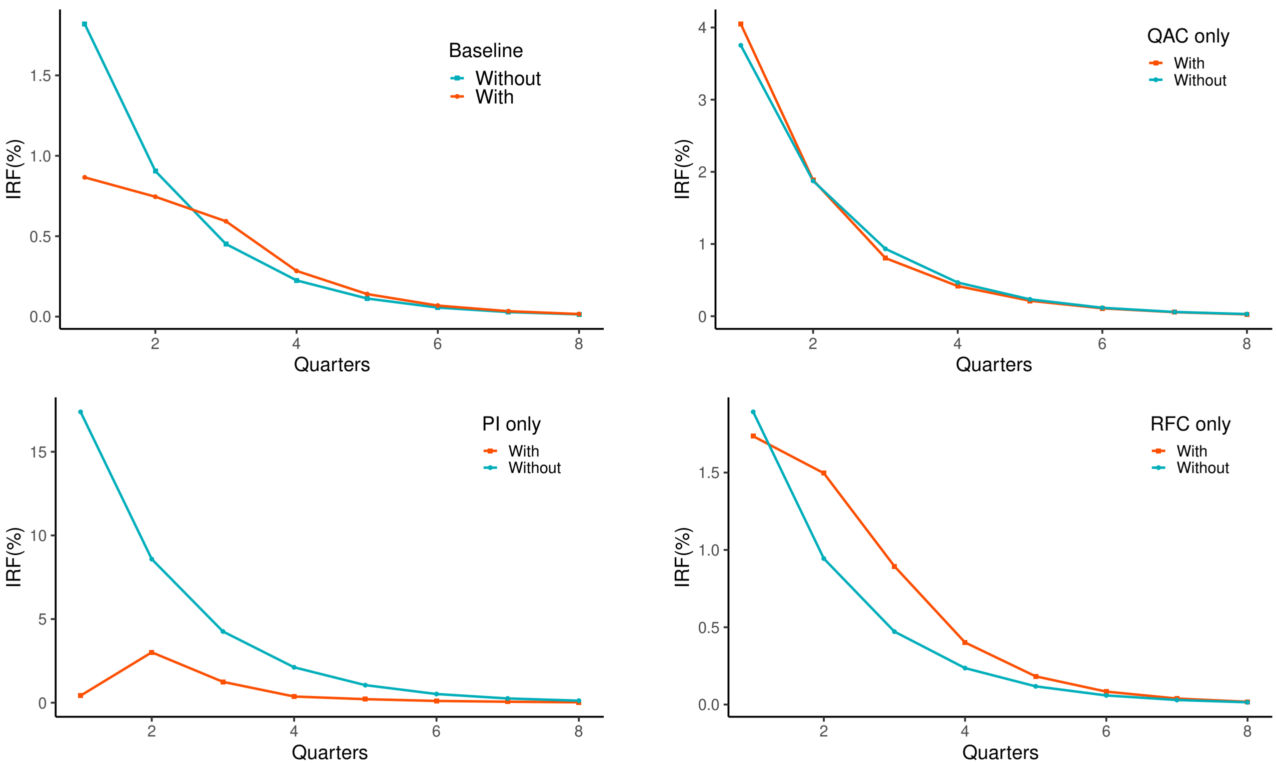
I further inspect the primary mechanism by solving the same experiments under alternative model specifications with different levels of investment lumpiness. I solve three alternative models with only *Quadratic Adjustment Costs (QAC)*, with only *Partial Irreversibility (PI)*, and with only *Random Fixed Costs (RFC)*. I then conduct the same four experiments as in Section 6.3 with the same magnitudes of shocks, and plot the impulse responses to monetary plot the impulse responses to monetary policy with and without an uncertainty shock. The alternative models are not re-calibrated because none of them could match all the target moments.

The results are in Figure 16. I first address two general understandings across all the four models. Compared to the full model (baseline), 1). *QAC* and *PI* models generate excessive responses in investment, and 2). models without partial irreversibility cannot generate impulse response functions that vary with uncertainty. Why is this so?

The *QAC* model is the closest to a representative firm New Keynesian model, which features no lumpiness in investment. Still, firms that make dramatic investment adjustments have to pay a sizable cost, which is completely irreversible if they plan to disinvest when uncertainty is resolved. Also, from the illustration of the distribution of concavity in Section 4, quadratic adjustment costs do not by themselves generate losses in value because of capital productivity mismatch. Therefore, when uncertainty is high, the effectiveness of monetary policy is almost unchanged.

The *PI only* model is the most significant for the gap between the differential effectiveness of monetary policy caused by the uncertainty shock, but it fails to generate a reasonable response in

Figure 16: Differential Investment IRFs in Alternative Models



Notes: The alternative models are chosen by fixing each specific adjustment cost at the calibration level but setting other adjustment costs to almost zero. Therefore, some of them fail to match both the cross-section investment moments and the dynamic investment moments.

levels following the monetary policy shock. Since the *PI* and *QAC* models both feature deterministic adjustment costs, the autocorrelation of idiosyncratic investment will be very high. Though the magnitude is completely wrong, *PI* is the only adjustment cost that creates a sizable gap. Recall the discussion on concavity in Section 4; partial irreversibility brings significant losses in value because of capital productivity mismatch. This concavity drives the ”wait-and-see” effect, which significantly expands the mass of inactive firms and compromises the effectiveness of monetary policy.

Finally, the *RFC* model is the most popular model specification in the lumpy investment literature. However, it still cannot generate the correct prediction of differential monetary stimulus effectiveness. Monetary stimulus conducted along with the uncertainty shock generates a similar impulse response in aggregate investment. The random adjustment cost is not deterministic; upon an uncertainty shock, firms hesitate to adjust their investment. However, there is no deterministic "punishment" in the future. If a firm encounters a bad productivity draw, they will not lose much value because of capital-productivity mismatch. Therefore, even though investment drops a lot, monetary policy is almost identically effective. These three alternative models help clarify how the components of adjustment costs matter and how they are all necessary to match the model moments and generate empirically consistent aggregate dynamics.

7. Extensions

I provide several extensions to further inspect this mechanism. First, I show that when uncertainty is high, the monetary policy shock to the Taylor rule residual should be twice as large (-50bps vs -25bps) to generate the same magnitude of aggregate investment response. The results are in Appendix A.5. Second, I show that other types of first moment policy which also work through changing the relative price of investment cannot alleviate the reduction in the effectiveness of the policy. A natural inference is that first moment policies which cannot directly target the firms who are constrained by the lumpy costs are very likely to be heavily affected by the uncertainty shocks. The results are in Appendix A.6. Third, I show that other types of uncertainty shock such as tail risk and ambiguity also lower the effectiveness of monetary policy. The results are in Appendix A.7.

8. Conclusion

Lumpy investment and uncertainty play essential roles in determining the responsiveness of aggregate investment to monetary policy. Abstracting them away from the model could lead to misunderstanding of how monetary policy affects aggregate investment. In a heterogeneous firm New Keynesian model with a comprehensive capital adjustment cost structure, this paper carefully examines the transmission mechanism of the investment channel of monetary policy. Since firms pursue asymmetric generalized (S,s) investment rules while only more productive firms are making positive investments, monetary policy primarily works through the extensive margin. Following high uncertainty shocks, the (S,s) inaction region is enlarged, and firms at the extensive margin choose to stay inactive. A conventional monetary policy shock is not large enough to motivate firms on the extensive margin to pay the fixed costs and bear the risk of the potential loss from disinvestment because of partial irreversibility. Therefore, the aggregate effect of monetary policy depends on lumpy investment and time-varying uncertainty.

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Appendices

A. Theoretical Appendix

A.1 Standard Rotemberg (1982) Price Setting of Retailers

We can rewrite the profit in real dollars as follows.

$$\Pi_t(j) = (P_t(j) - P_t^W)\left(\frac{P_t(j)}{P_t}\right)^{-\gamma}\frac{Y_t}{P_t} - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t$$

Each period, retailers choose a price to maximize the expected present discounted value of flow profit, which is discounted by the household's stochastic discount factor $\Lambda_{t,t+1}$, since households also own the retailers. The optimization is:

$$\max_{P_t(j)}\left\{\sum_{t=0}^{\infty}\Lambda_{t-1,t}\Pi_t(j)\right\}$$

Through the first order condition, the optimal price-setting rule can be written as follows:

$$\begin{aligned}(\gamma - 1)\left(\frac{P_t(j)}{P_t}\right)^{-\gamma}\frac{Y_t}{P_t} =& \gamma p_t^w\left(\frac{P_t(j)}{P_t}\right)^{-\gamma-1}\frac{Y_t}{P_t} - \psi\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)\frac{Y_t}{P_{t-1}(j)} \\ &+ E_t\psi\Lambda_{t,t+1}\left[\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)\left(\frac{P_{t+1}(j)}{P_t(j)}\right)\left(\frac{Y_{t+1}}{P_t(j)}\right)\right]\end{aligned}$$

where $p_t^w = P_t^w/P_t$ is the real wholesale price. In equilibrium all retailers behave identically. This means they all charge the same price and produce the same output in each period. The optimal condition for price can be written in terms of the inflation rate as:

$$(\gamma - 1) = \gamma p_t^w - \psi(\pi_t - 1)\pi_t + E_t\Lambda_{t,t+1}\psi(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}$$

Reorganizing terms, I obtain the ***New Keynesian Phillips curve***:

$$(\pi_t - \bar{\pi})\pi_t = \frac{\gamma}{\psi}(p_t^w - p^{w*}) + E_t\Lambda_{t,t+1}(\pi_{t+1} - \bar{\pi})\pi_{t+1}\frac{Y_{t+1}}{Y_t}$$

where $p_t^{w*} = \frac{\gamma-1}{\gamma}$ is the steady state whole sale price, or in other words, the marginal cost of retailer firms, and $\bar{\pi} = 1$ is the steady state inflation rate. In the paper, I directly focus on the linearized version for computational simplicity:

$$log\pi_t = \frac{\gamma - 1}{\psi} log\frac{p_t^w}{p^{w*}} + \beta E_t log\pi_{t+1}$$

For robustness, I also solve the quantitative model using the non-linearized version; the results are almost identical. Therefore, in order to save computational time, I use the linearized version through out the whole paper.

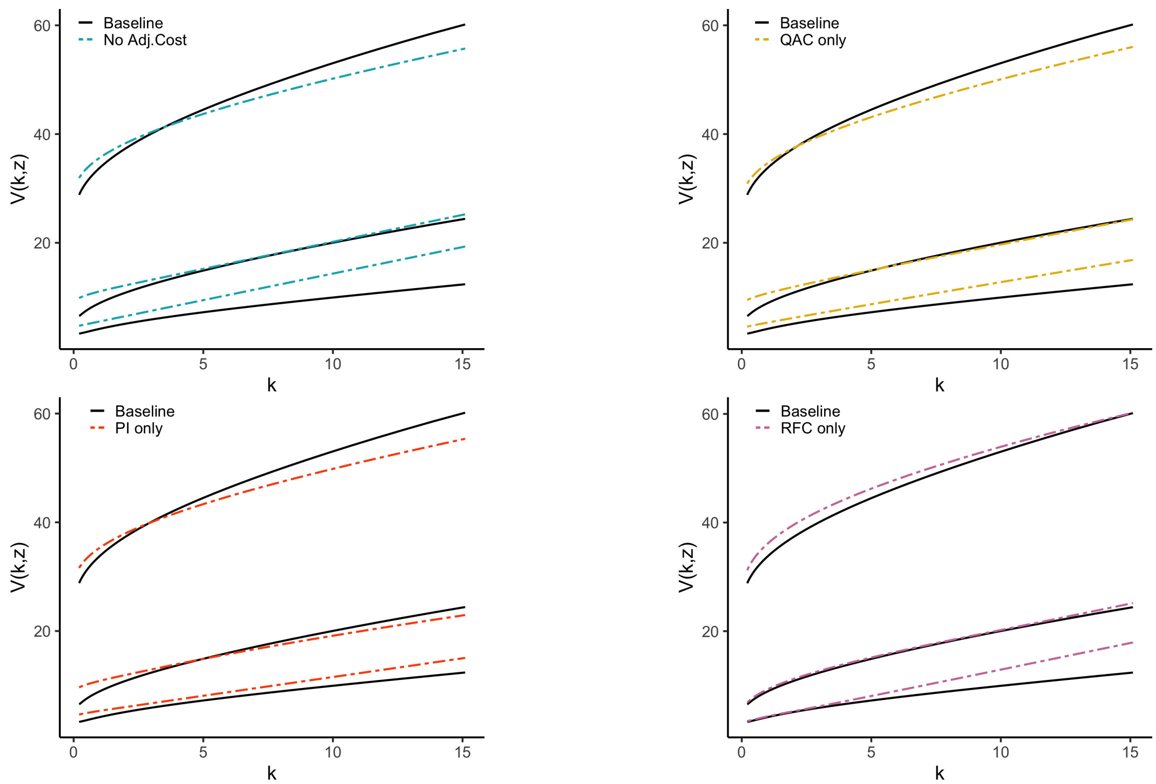
A.2 Original value functions in the two-period example

I present the original value functions as in Section 4.2 in Figure 17. The only difference is that I move the value functions vertically in the paper to make it easier to compare the concavity.

A.3 Real interest rate elasticity of aggregate investment (against QAC ϕ_k)

The results for the quadratic adjustment cost are in Figure 18 (a). First, in the *no PI, no RFC* case, we could see that when the size of quadratic adjustment cost is very low, aggregate investment is very responsive to real interest shocks. I truncated the left tail at $\phi_k = 0.13$, which gives us an elasticity of 195. This is already a huge response: a 1% real interest rate reduction leads to almost 200% more aggregate investment increment. Partial irreversibility cannot really help to resolve this over-response issue without a random fixed cost. In the cases with a moderate random fixed cost, this elasticity is brought down even when the quadratic adjustment cost is tiny, but the elasticity overshoots and drops to 8. Figure 18 (b) gives additional details around the moment target. First, the two cases without random fixed costs never hit the target. The *RFC only* case roughly hits the target for any quadratic adjustment cost ϕ_k larger than 0.9. However, when with both a moderate random fixed cost and moderate partial reversibility, the quadratic adjustment cost must increase to hit the target. Therefore, to match the target moment of the interest rate elasticity of aggregate investment, we need both moderate random fixed adjustment costs and quadratic adjustment costs.

Figure 17: Adjustment Costs and Value Function Concavity



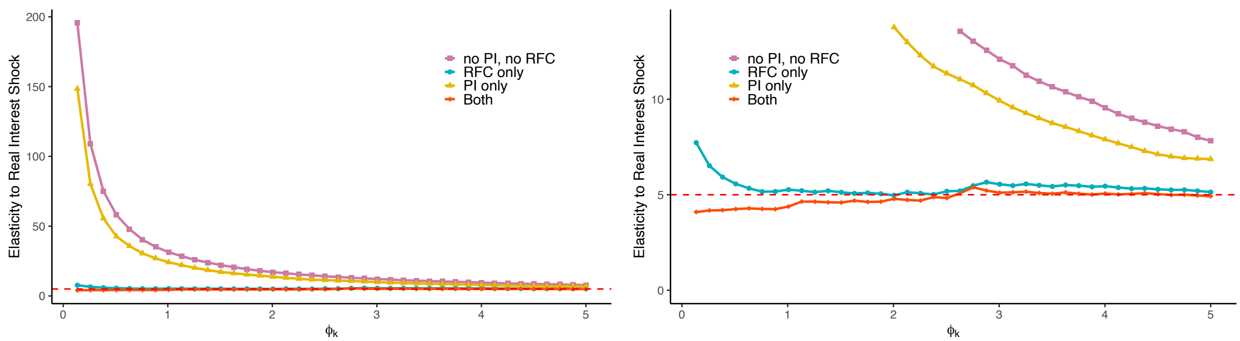
Notes: the details are same as in Figure 5.

A.4 Discussion on Dynamic Moments

The identification strategy in Section 5.2 provides a powerful tool to organize various results in the existing literature which studies whether the micro-level lumpy investment has aggregate implications. Winberry (2018a) pioneered the research showing that matching the empirically plausible interest rate dynamics, a heterogeneous firm RBC model could generate both cross-sectional lumpy investment distribution and state-dependent dynamics of aggregate investment. Koby and Wolf (2019) further generalized that idea by showing that in order to generate state-dependent dynamics of aggregate investment the parameterization should feature data-consistent small price elasticities of partial equilibrium investment[30]. Both of their papers inspire me to show the property of

[30] Koby and Wolf (2019), in one of their applications, embed a heterogeneous firm block with lumpy investment into a standard medium-scale New Keynesian model. They calibrate their lumpy investment parameters to match the cross-section moments and self imputed bonus depreciation response as in Zwick and Mahon (2017). They show that the response of aggregate investment to expansionary monetary policy shocks is affect by the micro-level lumpiness of in-

Figure 18: Real Interest Rate Elasticity of Agg. Inv. (against QAC ϕ_k)



(a) Whole Domain

(b) Zoomed In

The details of the groups are similar as in the previous Figure 7.

Table 4: **Why we need *Dynamic Moments***

Parameters	**Symbol**	Cali.1	Cali.2	Cali.3	Cali.4	Cali.5
Quad. Adj. Cost	ϕ_k	0.45	0	0	1.82	2.27
Random Fixed Cost	$\bar{\xi}$	0	0.16	0	0.50	0.33
Partial Irreversibility	S	0	0	0.08	0.17	0.17
Moments	**Data**					
Cross-section Moments						
Average inv. rate (%)	10.4%	10.3%	10.6%	10.9%	10.3%	10.4%
S.D. of inv. rates	0.16	0.15	0.14	0.15	0.12	0.13
Spike rate (%)	14.4%	13.7%	16.1%	14.7%	13.4%	16.8%
Positive rate (%)	85.6%	86.3%	83.9%	85.3%	86.6%	83.2%
Dynamic Moments						
Auto corr. of inv. rates	0.40	0.70	0.40	0.48	0.39	0.42
Elasticity to RIR shock	5	65	18	182	4	8

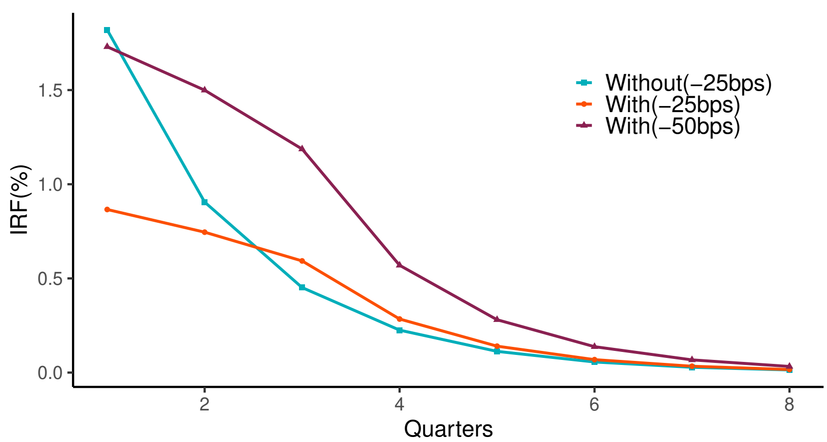
vestment. In a classical TFP recessions, the effectiveness of monetary policy is dampened. Therefore, lumpy investment generate state-dependent effectiveness of monetary policy which differs at a TFP boom or a TFP recession. I became aware of Koby and Wolf (2019) after I finished the first version of the draft. In the first version, I matched to a partial equilibrium investment response to real interest rate shock equals to 7 implied from Winberry (2018a). Re-calibrating to

”dynamic moments” over the adjustment cost parameters spaces. Matching dynamic moments is essential to generate empirically plausible aggregate dynamics.

In Table 4, I show some counter examples of various calibrations which could all target on the cross-section moments, but have very different dynamic moments which are essential whether the model will generate state-dependent dynamics of aggregate investment.

A.5 How large should the monetary stimulus be?

Figure 19: Differential Responses to Monetary Policy Shocks



Notes: The construction of two counterfactuals follows the same process as in Figure 15.

A natural subsequent question would be: How much stronger should monetary stimulus be in the presence of uncertainty to generate the peak impulse response expected absent an uncertainty shock? To answer this question, I repeat the experiments in this section with a sequence of increasing monetary policy shocks. I then plot the results against the benchmark, as in Figure 15. I double the magnitudes (-50bps) with the persistence as well 0.5 as in the previous section. The results are in Figure 19. These differential impulse responses indicate that in order to reach the same peak effect of monetary policy without an uncertainty shock, an enhanced monetary policy shock larger than -50bps is necessary. This is more than twice as large monetary policy shock, which when flipped

5 in Koby and Wolf (2019) further improved the model’s performance to resolve the excessive response puzzle of lumpy investment to monetary policy shocks as in Reiter et al. (2013)

indicates that monetary policy in the presence of an uncertainty shock is at most only 50% effective compared to the same monetary policy during normal times.

A.6 Would Investment Subsidy Policy Perform Better?

In this section, I briefly explore if fiscal stimulus, specifically an investment stimulus policy, is less affected by high uncertainty compared to monetary policy. There are two common investment stimulus policies in the U.S., the investment tax credit and the bonus depreciation allowance.As documented by Winberry (2018a), the investment stimulus shock is isomorphic to an investment-specific technological shock. The key shock transmission channel is through the relative price of investment $q(\Omega)$. To avoid introducing corporate taxes into the main model which increases complexity, I introduce an investment stimulus policy shock ϵ_t^{ω} in a very reduced form way following Winberry (2018a) which does not require much modification of the model:

$$\omega_t = \rho_\omega \omega_{t-1} + \epsilon_{t-1}^{\omega}$$

where ω_t is the stimulus policy at time $t+1$, ρ_ω is the persistence of the stimulus policy, and ϵ_t^{ω} is the shock at time t. The stimulus policy ω_t enters directly into the relative price of investment and the investment cost function equation (8):

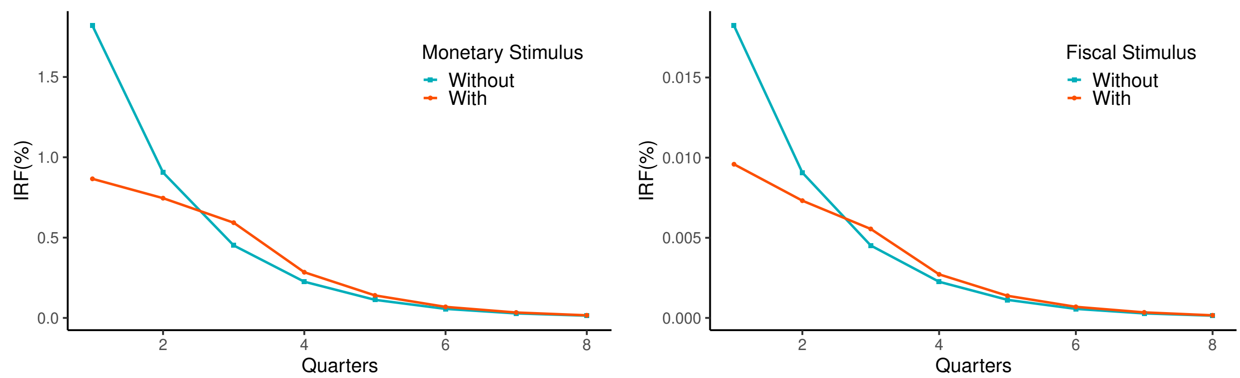
$$q_t(\Omega) = 1 - \omega_t$$

$$c(i_{jt}) = \mathbf{q_t}(\Omega) \times i_{jt} + |i_j| \Big({}_{(i_{jt}<0)} \cdot S + {}_{(i_{jt}\neq 0)} \cdot \frac{\phi_k}{2} |\frac{i_{jt}}{k_{jt}}| \Big) + {}_{(|i_{jt}|>ak_{jt})} \cdot \xi_{jt}$$

I choose the persistence $\rho_\omega = 0.5$ and the shock $\epsilon_t^{\omega} = 39bps$ in the experiments so that the aggregate investment stimulus is similar to the $\epsilon_1^m = -25bps$ monetary policy shock with persistence $\rho_m = 0.5$ to the Taylor rule residual. I then conduct the investment stimulus policy shock with/without the same uncertainty shock as in the main body.

The results are in Figure 20. On the left panel (a) is the same baseline monetary stimulus as in Section 6.3. Compared to the impulse response of aggregate investment with baseline uncertainty, when the monetary stimulus is conducted along with an uncertainty shock, the initial responses is much weaker: 0.86% v.s. 1.81%. This is a reduction of effectiveness of 52%. On the left panel (b) is the fiscal stimulus whose magnitude is matched to the left panel under baseline uncertainty (1.82%). When the same fiscal stimulus is conducted along with the same uncertainty shock, the initial responses is also much weaker: 0.96% v.s. 1.82%. The reduction is similar across monetary and fiscal stimulus with the same uncertainty shock.

Figure 20: Monetary Stimulus vs Investment Subsidy Stimulus



(a) Monetary Stimulus

(b) Investment Subsidy Stimulus

Notes: I choose the magnitudes of the fiscal stimulus to match the 1.8% aggregate investment increment as created by the -25bps Taylor rule shock as in the monetary stimulus. This provides better comparison of the gaps of with/without an uncertainty shock between both types of stimulus.

This result is not surprising. Revisiting the example in Section 4.1, notice that the two shocks are almost isomorphic to each other. If I restate the firms' problem here including the relative price of investment channel of investment subsidy stimulus:

$$\max_{i_j \in I_j} h * R * (z_1^{\eta} k_0^{\mu} - c(i_j)) + E_1\{V_2(k_0 + i_j, z_2)\}$$

where $R = \left(\frac{p}{w^{(1-\alpha)\gamma}}\right)^{\eta}(1+r)$ and $c(i_j) = q \times i_j + |i_j|\left({}_{(i_j<0)} \cdot S + {}_{(i_j \neq 0)} \cdot \frac{\phi_k}{2}|\frac{i_j}{k_0}|\right) + \xi_j$.

The relative cost of investment is $\mathbf{R} \times \mathbf{q}$. Monetary stimulus works through lowering $\mathbf{R}$, or in other words, increasing the rate interest rate $1/\mathbf{R}$, while investment subsidy stimulus works through lowering the relative cost of investment $\mathbf{q}$. Changes in either price cannot chance the lumpy adjustment cost structure $\left(\mathbf{1}_{(i_j<0)} \cdot S + \mathbf{1}_{(i_j \neq 0)} \cdot \frac{\phi_k}{2}|\frac{i_j}{k_0}|\right) + \xi_j$ which is highly affected by the uncertainty shock. Therefore, investment subsidy stimulus would not perform better than monetary stimulus. More generally, other stimulus policies (first moment policies) which cannot directly target the firms who are constrained by the lumpy costs are very likely to be heavily affected by uncertainty shocks.

A.7 How About Other Uncertainty Shocks?

In this section, I explore the model mechanism for other uncertainty shocks. As documented by a large literature, i.e., Jurado et al. (2015), Orlik and Veldkamp (2014), and Ilut and Schneider (2014), there are several measures and types of uncertainty shocks which are not the same, conceptually or statistically. Therefore, I introduce two additional types of uncertainty shocks at the firm-level to test the model mechanism: tail risk and ambiguity through a generalized AR(1) process in equation (7) of firms' idiosyncratic productivity:

$$log(z_{jt}) = (1 - \gamma_t^z(z_{jt-1}))\left(\mu_t + \rho^z log(z_{jt-1}) + \sigma_{t-1}^z \epsilon_{jt}\right) + \gamma_t^z(z_{jt-1})\underline{z} \tag{22}$$

$$\epsilon_{jt} \sim N(0,1)(\alpha_t)$$

Volatility Shock: An aggregate shock to firm-level volatility is an unexpected positive jump in the variance of the productivity shock σ_{t-1}^z and a corresponding adjustment in μ_t. This is the formulation in the main text. Without a volatility shock, σ_{t-1}^z and μ_t are constant.

Tail Risk Shock: I replace the volatility shock in the main body by a one-time aggregate shock to firm-level tail risk. In the numerical iterations, it is an unexpected up-jump in the probability of drawing a low productivity $\underline{z}$ at the left tail of the distribution $\gamma_t^z(z_{jt-1})$. This probability is state-dependent on a firm's previous period productivity, $\gamma_t^z(z_{jt-1})$ is a decreasing function of z_{jt-1}. Without a tail risk shock, $\gamma_t^z(z_{jt-1})$ equals zero.

$$log(z_{jt}) = (1 - \gamma_t^z(z_{jt-1}))\left(\mu_t + \rho^z log(z_{jt-1}) + \sigma^{z*} \epsilon_{jt}\right) + \gamma_t^z(z_{jt-1})\underline{z}$$

$$\epsilon_{jt} \sim N(0,1)$$

$$\gamma_t^z(z_{jt-1}) = \tau\left(\frac{\bar{z} - z_{jt-1}}{\bar{z} - \underline{z}}\right)^{\lambda_\tau}$$

where τ governs the absolute size of the firm-level tail risk, and λ_τ governs the state-dependent (on z_{jt-1}) size of the tail risk. In the exercise, I put $\tau = 0.2$ and $\lambda_\tau = 0$ to generate almost 40% drop of aggregate investment when tail risk shock hits. The difference between IRFs of investment to monetary policy with/without tail risk shocks is robust to different magnitudes of tail risk shocks with different combinations of τ and λ_τ parameters. Of course, the gap is also of different magnitudes depending on the fraction of firms on the extensive margin. This exercise could be understood as a very deduced firm-level version of the tail risk shock to aggregate TFP as in Orlik and Veldkamp

(2014).

Ambiguity Shock: I replace the volatility shock in the main body by a one-time aggregate shock to firm-level ambiguity. In the numerical iterations, it is an unexpected up-jump in firm's subjective uncertainty of which distribution to draw the next period productivity when their objective fundamentals are the same as steady-state. I denote the dispersion of distribution by $\epsilon_{jt} \sim N(0,1)(\alpha_t)$ where α_t indicates that there are a continuum of distributions from $\epsilon_{jt} \sim N(-\alpha_t, 1)$ to $\epsilon_{jt} \sim N(+\alpha_t, 1)$. Without an ambiguity shock, α_t equals zero.

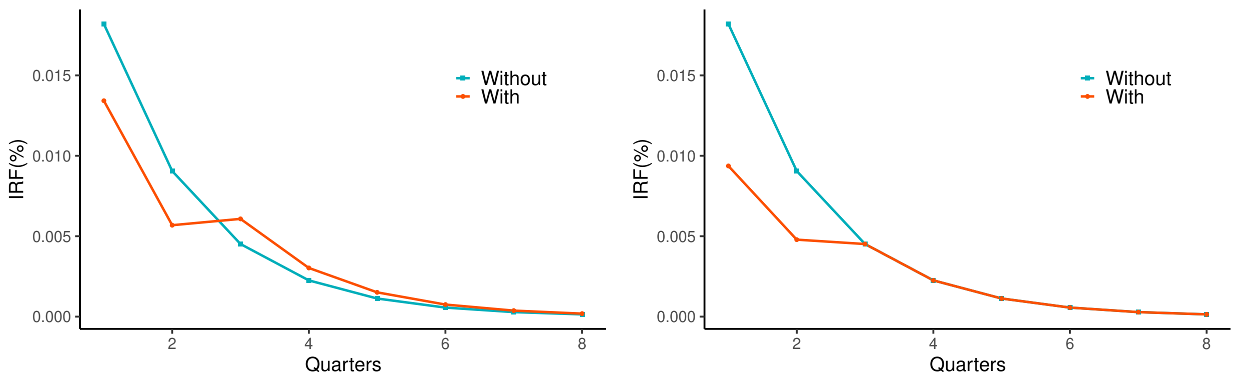
$$log(z_{jt}) = \mu_t + \rho^z log(z_{jt-1}) + \sigma^z_{t-1}\epsilon_{jt}$$

$$\epsilon_{jt} \sim N(0,1)(\alpha_t)$$

And meanwhile, firms are now maxmin agents who are always afraid of the worst scenarios. So when the firm-level ambiguity shock hits, firms maximize their Bellman equation follows the worst scenarios of their potential draw $-\alpha_t$ of the distribution: $log(z_{jt}) = \mu_t + \rho^z log(z_{jt-1}) + \sigma^z_{t-1}\epsilon_{jt}, \epsilon_{jt} \sim N(-\alpha_t, 1)$. However, their underlying fundamental productivity processes are still the steady-state AR(1) process: $log(z_{jt}) = \mu_t + \rho^z log(z_{jt-1}) + \sigma^z_{t-1}\epsilon_{jt}, \epsilon_{jt} \sim N(0,1)$. Therefore, in the computation, I solve the forward process of decision rules following the one-time shock $\epsilon_{j1} \sim N(-\alpha_1, 1)$ at period 1 for all firms, but solve the backward process of the distribution following the steady-state productivity process $\epsilon_{jt} \sim N(0,1)$ for any t from T to 0. Then I iterate the process until all the aggregate prices and distribution in each period are converged. I generate 40% drop of aggregate investment in period 2 by choosing an $\alpha_1 = 0.6$. Also, the gap between differential IRFs exists for various choices of α_1 and the magnitudes depends on the fraction of firms on the extensive margin. This exercise could be understood as a very deduced firm-level version of the ambiguity shock to aggregate TFP as in Ilut and Schneider (2014).

To compare the differences between these different uncertainty shocks, I solve the same experiments under alternative uncertainty shock specifications as in Sections 6.1 and 6.2. To make the results more comparable, I choose the magnitudes of alternative uncertainty shocks to match the 40% peak aggregate investment drop as in the volatility shock scenario. The results are in Figure 21. In both scenarios, monetary policy is less effective compared to the case without an uncertainty shock. However, neither the tail risk shock nor the ambiguity shock performs as well as the volatility shock. Nevertheless, the IRF gaps with/without an uncertainty shock are robust to different magnitudes of all types of uncertainty shock.

Figure 21: Differential Investment IRFs with Alternative Uncertainty Shocks



(a) Tail Risk Shock (b) Ambiguity Shock

Notes: I choose the magnitudes of the tail risk shock and the ambiguity shock to match the 40% aggregate investment drop as created by the volatility shock in the previous sections. This provides a better comparison of the gaps of with/without an uncertainty shock between all these three types of uncertainty shocks.

A.8 Details of the Computation Methods

Part I: Solving the Stationary Equilibrium

I first assume the economy is at steady-state with normal uncertainty. This part is very similar as solving an Aiyagari model. The only two differences are: (1).I have a New Keynesian block incorporates nominal rigidity; and (2).Firms own capital which is subject to adjustment costs. At the stationary equilibrium, there are no monetary policy shocks, so I solve $\{\pi^* = 1, \Lambda^* = \beta, p^{w*} = \frac{\gamma-1}{\gamma}, R^{n*} = 1/\beta\}$. I now search for equilibrium wage to clear the labor market. The algorithm is as following:

Step.1. Guess an equilibrium wage;

Step.2. Solve the firm's problem using Value Function Iteration;

Step.3. Calculate aggregate variables from the firm distribution using Young (2010);

Step.4. Update the wage with a given weight and return to Step 2 until convergence.

After the convergence, I have the stationary equilibrium aggregate prices $\Theta_t^* = \{\pi^* = 1, \Lambda^* = \beta, p^{w*} = \frac{\gamma-1}{\gamma}, R^{n*} = 1/\beta, w^* = w^*\}$, uncertainty state $U_{-1}^* =$ normal, aggregate state $\Omega_t^* = (U_{-1}^*, \Theta_t^*)$, aggregate quantities $\{C^*(\Omega^*), N^*(\Omega^*), Y^*(\Omega^*), K^*(\Omega^*)\}$, firm value functions $\{V^*(k,z;\Omega^*), V^{A^*}(k,z;\Omega^*), V^{NA^*}(k,z;\Omega^*)$, policy functions $\xi^{**}(k,z;\Omega^*)$, $k'^*(k,z;\Omega^*)$, $l'^*(k,z;\Omega^*)\}$, and distribution $\mu(k,z;\Omega^*)$ at the stationary equilibrium state.

Part II: Solving the Transitional Equilibrium

With the stationary equilibrium solutions in hand, I now move to the solution of the transitional equilibrium using a shooting algorithm. The key assumption here is that after a sufficiently long enough time, the economy will always converge back to its initial stationary equilibrium after any temporary and unexpected (MIT) shocks. The following steps outline the shooting algorithm:

Step.1. Fix a sufficient long transition period t = 1 to t = T (say 200);

Step.2. Guess a sequence of aggregate price $\{p_t^w, w_t, \Lambda_t, \pi_t\}$ of length T such that the initial prices $\{p_1^w = p^{w*}, w_1 = w^*, \Lambda_1 = \Lambda^*, \pi_1 = \bar{\pi}\}$ (just simply assuming all the prices stay at steady state works well) and terminal prices $\{p_T^w = p^{w*}, w_T = w^*, \Lambda_T = \Lambda^*, \pi_T = \bar{\pi}\}$. Provide a predetermined shock process of interest, i.e., $\{\epsilon_t^m\}$ and $\{U_{t-1}\}$. This implies a time series for the aggregate state $\{\Omega_t\}_{t=1}^T$. The aggregate state is just time t.

Step.3. I know that at time T, the economy is back to its steady state. I have the steady state value function $V(k,z;\Omega_T) = V^*(k,z;\Omega^*)$ in hand for time T. I solve for the firms' problem by ***backward induction*** given $V(k,z;\Omega_T)$ and $\{p_{T-1}^w, w_{T-1}, \Lambda_{T-1}\}$. This yields the firm value function $V(k,z;\Omega_{T-1})$ and associated policy functions for capital $k'(k,z;\Omega_{T-1})$ and labor $l(k,z;\Omega_{T-1})$. By iterating backward, I solve the whole series of both policy functions $\{k'(k,z;\Omega_t)\}_{t=1}^T$ and $\{l'(k,z;\Omega_t)\}_{t=1}^T$.

Step.4. Given the policy functions and the steady state distribution as the initial distribution $\mu(k,z;\Omega_1) = \mu(k,z;\Omega^*)$, I use ***forward simulation*** with the non-stochastic simulation in Young (2010) to recover the whole path $\{\mu(k,z;\Omega_t)\}_{t=1}^T$.

Step.5. Using the distribution $\{\mu(k,z)\}_1^T$, I obtain all the ***aggregate quantities***: aggregate output $\{Y\}_{t=1}^T$, aggregate investment $\{I\}_{t=1}^T$, aggregate labor demand $\{N\}_{t=1}^T$, and aggregate capital adjustment costs $\{\Theta_k\}_{t=1}^T$, the latter of which follows from the guessed inflation $\{\pi\}_{t=1}^T$, we could calculate aggregate adjustment costs $\{\Theta_p\}_{t=1}^T$. I then use the goods market clearing condition to calculate aggregate consumption $\{C\}_{t=1}^T$. I then calculate the *Excessive Demand* $\{\Delta C\}_{t=1}^T$ by taking the differences between currently iterated $\{C\}_{t=1}^T$ and the previous iteration $\{C_{old}\}_{t=1}^T$.

Step.6. Given all the aggregate quantities in the previous step and the *Excessive Demand* $\{\Delta C\}_{t=1}^T$, I update all the ***aggregate prices***. I update all equilibrium prices with a line search: $X_t^{new} = speed \cdot f_X(\{\Delta C\}_{t=1}^T) + (1 - speed) \cdot X_t^{old}$. Repeat Steps 2-7 until X_t^{new} and X_t^{old} are close enough. I only update

$\{p_t^w, w_t, \Lambda_t, \pi_t\}$ because $\{R_t^n\}$ can be calculated accurately from Taylor rule. The $f_X(\{\Delta C\}_{t=1}^T)$ is chosen by the connections of the New Keynesian prices with the *Excessive Demand* $\{\Delta C\}_{t=1}^T$ through the equations of the New Keynesian prices. Updating all prices in all periods simultaneously reduces the computation burden dramatically.[31] This updating rule allows me to solve the transitional equilibrium in seconds on a dual-core macbook without any parallel computation.

In all the experiments with both the Taylor rule shock and an uncertainty shock, I set T = 200, and a step size of 0.01 to ensure convergence, with the necessary distance between X_t^{new} and X_t^{old} smaller than 1e-7. I also tested with various T from 50 to 400 to ensure that the choice of T = 200 does not affect the accuracy of the solution.

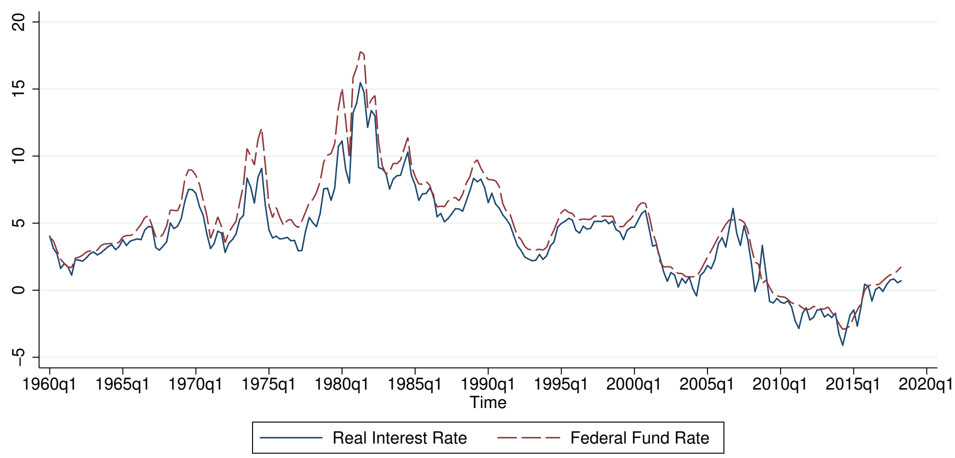
[31]There is an alternative updating rule which is more stable but much more time consuming. In put it here: Step.6'. Using the household first order condition for consumption $\{C\}_{t=1}^T$, I obtain a new $\{\Lambda\}_{t=1}^T$; using the household first order condition for labor, $\{C\}_{t=1}^T$, and $\{N\}_{t=1}^T$, I obtain a new $\{w\}_{t=1}^T$; using the definitions of the stochastic discount factor and Taylor rule simultaneously, I update π_{t+1} with Λ_t, R_t^n, then I update R_{t+1}^n with the updated π_{t+1}, and repeat until I have a new $\{R^n\}_{t=0}^T$ and $\{\pi\}_{t=1}^T$. Finally, I get a new $\{p^w\}_{t=1}^T$ through the New Keynesian Phillips curve.

B. Empirical Appendix

B.1 Description of the Indicators

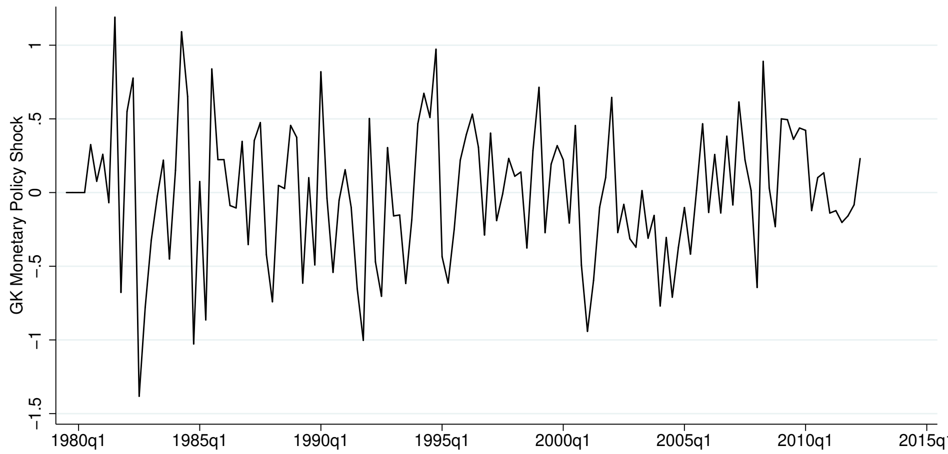
Real Interest Rate as Monetary Policy Indicator: The first possible indicator for monetary policy is to directly use the real interest rate. While this makes it easy to interpret the magnitudes of the local projection results, it may suffer from endogeneity issues. In Figure 22, I show the real interest rate and federal funds rate from 1960Q1 to 2018Q2. They follows each other very closely, peaking around 1982Q1, and bottomed around 2014Q2. For the data series between 2009Q3 and 2015Q4, I use the shadow rate calculated by Wu and Xia (2015) to replace the federal funds rate.

Figure 22: Real Interest Rate & Federal Funds Rate



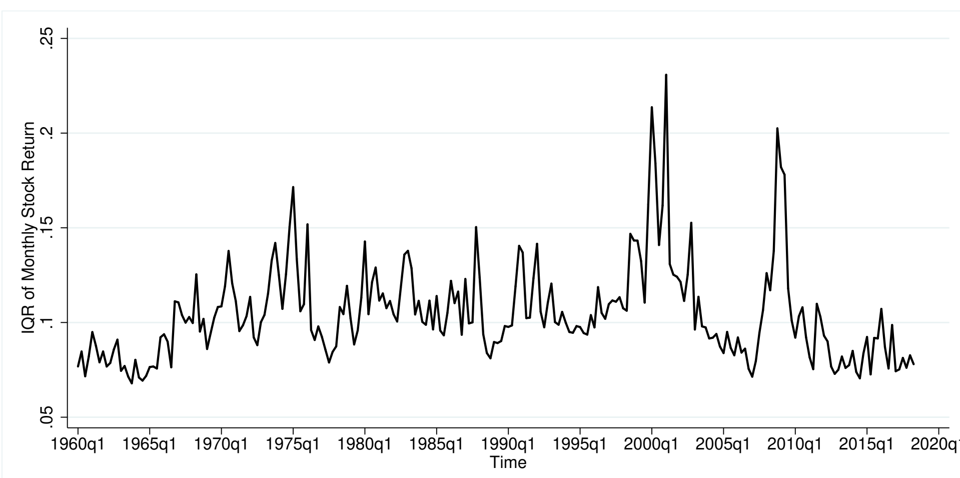
High Frequency Identified Residual as Monetary Policy Indicator: The second indicator for monetary policy is to use the residual from a VAR in which a monetary policy indicator is instrumented for with high-frequency identified shocks following Gertler and Karadi (2015). This avoids endogeneity issues but makes it more difficult to interpret the magnitudes of the local projection results. The idea in Gertler and Karadi (2015) to isolate interest rate surprises using the movements in financial markets data within a short window around central bank policy announcements. They use financial market surprises from Fed Funds Futures during the 30 minutes interval around the FOMC policy announcements as proxies for the one-year government bond rate in a vector autoregression. The structural residual is then the estimated monetary policy shock. I plot the shock series in Figure 23.

Figure 23: GK Monetary Policy Shock



Uncertainty Indicator: In Figure 24, I show the firm-level uncertainty measured using the IQR of monthly returns for all Compustat firms with more than 25 years of observable data. This measure is initially used by Bloom et al. (2018). I extended it to 2018Q2 to accommodate other data series after the federal funds rate reached the zero lower bound at 2009Q2. There are several obvious peaks around 1975Q1, 2001Q1, and 2008Q4, respectively. The uncertainty series is plotted in Figure 24.

Figure 24: IQR of Stock Market Returns



B.2 Description of the Compustat Data

Lumpy Investment in Compustat: In Table 5, I present the summary statistics of physical investment divided into 5-year periods. I also show the standard deviations of each variable and their correlations with the aggregate investment measures.

Table 5: Summary Statistics of Physical Investment over Time

Variable	1975s	1980s	1985s	1990s	1995s	2000s
Average investment rate (%)	N/A	9.8%	8.6%	8.4%	10.0%	8.3%
Inaction rate (%)	N/A	9.2%	10.6%	10.1%	7.0%	10.7%
Spike rate (%)	N/A	15.3%	12.5%	12.6%	16.7%	11.8%
Disinvestment rate (%)	N/A	2.4%	3.6%	2.9%	2.7%	2.3%
Aggregate investment rate (%)	N/A	4.5%	4.2%	4.2%	5.1%	4.2%
Observations	0	15,932	76,905	80,090	95,810	86,928
	2005s	2010s	2015s		σ	β_{Agg}
Average investment rate (%)	8.5%	8.6%	8.4%		0.4%	0.80
Inaction rate (%)	8.2%	8.8%	9.7%		1.3%	-0.84
Spike rate (%)	11.8%	10.8%	9.4%		2.4%	0.83
Disinvestment rate (%)	1.3%	0.9%	0.9%		1.0%	0.19
Aggregate investment rate (%)	4.5%	4.4%	3.9%		0.7%	1.0
Observations	68,483	57,429	32,996			

The average statistic excludes extreme value outliers (0.5% top and bottom). Inaction is defined by investment below 10% of the sample average. Spikes are defined by investment above twice the sample average. Aggregate investment is total investment divided by total lagged capital. σ is the standard deviation of a statistic over time at a quarterly frequency. β_{Agg} is the correlation of a statistic with the aggregate investment rate. Data Source: Compustat Fundamental Quarterly from 1975Q1 to 2019Q2.

Determinants of Investment: In Table 6, I show the determinants of investment at both the firm-level and in aggregate. The results are from an OLS regression of investment on all the determinant variables. The results indicate that Tobin's Q, leverage, and cash holding are important firm-level determinants, and all aggregate-level determinants are important.

Table 6: Determinants of Investment

Variable	Investment
Firm-Level	
Tobin's Q	0.003***
Total Asset	-0.000
Leverage	-0.090***
Cash Holding	-0.000***
Revenue	0.000
Sales Growth	0.001
Aggregate-Level	
GDP Growth	0.001***
Uncertainty	-0.052
CPI	0.007***
Fixed Effects	
Firm FE	Yes
One Digit Sector FE	Yes
Quarter FE	Yes
Observations	227206
adj. R^2	0.036

Standard errors in parentheses: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Extreme value outliers excluded.

B.3 Additional Results: Using GK HFI Residuals as the Policy Indicator

B.3.1 IRFs to Monetary Policy Shock at Firm-level (OLS)

To explore how uncertainty affects the effectiveness of monetary policy, I employ the OLS *Local Projection* (LP) method from *Òscar Jordà* (2005) with adjustments applying to the panel nature of the data:

$$\Delta_h I_{j,t+h} = f_{j,h} + \alpha_{n,h,t+h} + \beta_h^m r_t^m + \boldsymbol{\gamma_h r_t^m u_t} + \sum_{l=0}^{L} \Gamma'_{h,t-l} X_{j,t-l} + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{j,h,t} \tag{23}$$

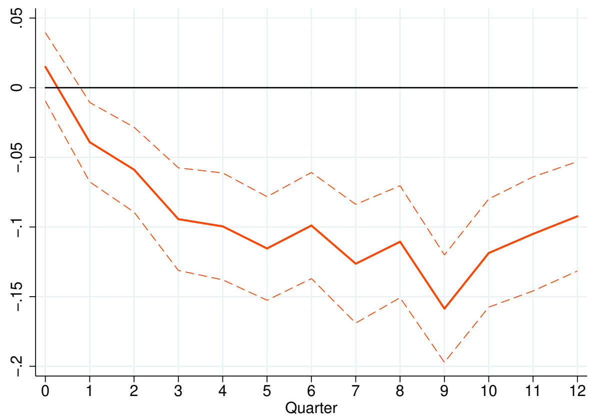
where j indicates firm j, h indicates quarters in the future and l indexes lags. $\Delta_h I_{j,t+h}$ is the h quarter investment rate measure, $f_{j,h}$ is the firm fixed effect, $\alpha_{n,h,t+h}$ is the quarter sector dummy to control for seasonality and the industry fixed effect, $u_t = IQR_t^{stock}$ is the uncertainty measure at time t,. r_t^m is the sign-flipped GK HFI residual, $X_{j,t-l}$ is a vector containing the firm-level controls including Tobin's Q, sales, leverage, and size for $t-l$, and Z_{t-l} is a vector of aggregate controls including uncertainty u_t, CPI, output gap, investment, and private consumption from NIPA. For uncertainty, investment, and firm-level controls, I use the same Compustat Quarterly data as constructed in Section 2.1[32]. Figure 25 displays the results for the coefficients of the sign-flipped GK HFI residual interacted with uncertainty γ_h from estimating the firm-level specification (23) using Compustat public firms.

B.3.2 IRFs to Monetary Policy Shock at Firm-level (Censored, Alternative Periods)

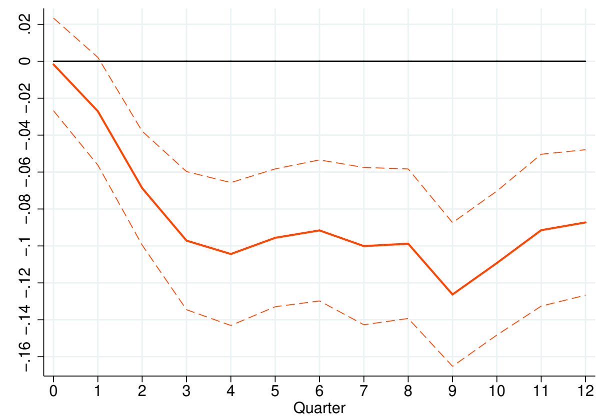
Figure 26 displays the results of the coefficient of the sign-flipped GK HFI residual interacted with uncertainty γ_h from estimating the firm-level Tobit and Probit specifications (1) and (2) with alternative choices of periods. The regressions including pre-85 periods are almost identical to the results in the main text because I have very few firm-level observations before 1985.

[32] The firm-level control variables follows classic literature: A size measure using total asset (Item *atq*); financial condition measures using leverage measured as total debt (Item *dlcq* + *dlttq*) over total asset (Item *atq*), and cash holding (Item *cheq*); operational status measures using revenue (Item *revtq*), sales (Item *saleq*), and sales growth which is $\Delta saleq$ divided by sales, and finally, investment opportunity measured using Tobin's Q.

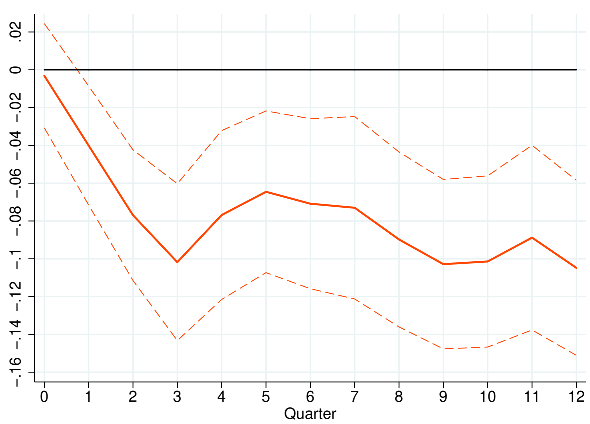
Figure 25: Firm-level Differential Response to Monetary Shocks (OLS)



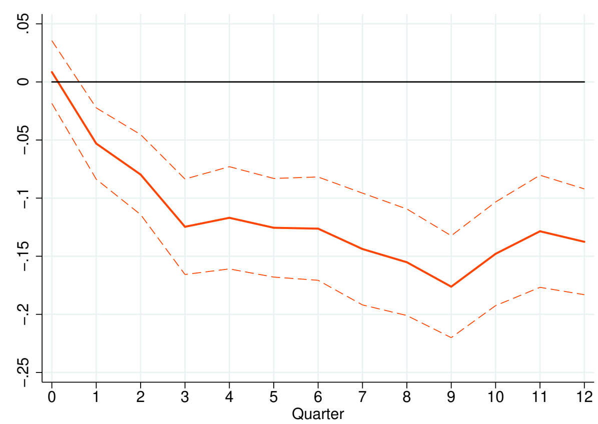
(a) OLS FE



(b) OLS Non-FE

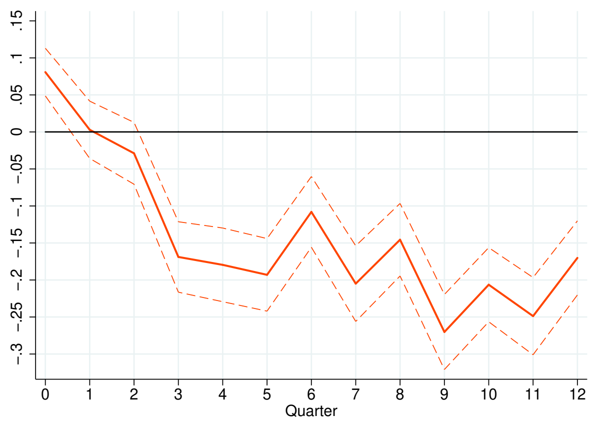


(c) OLS FE Post-94

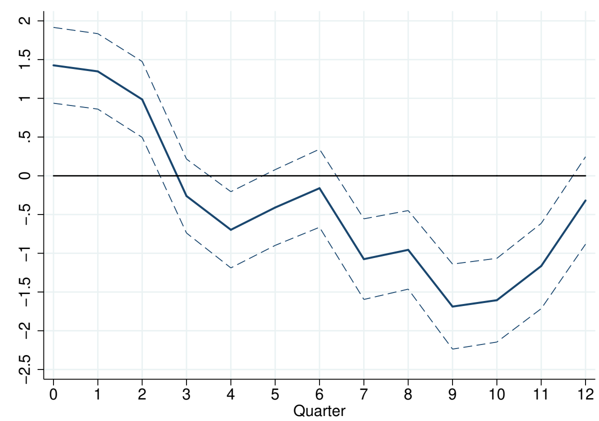


(d) OLS Non-FE Post-94

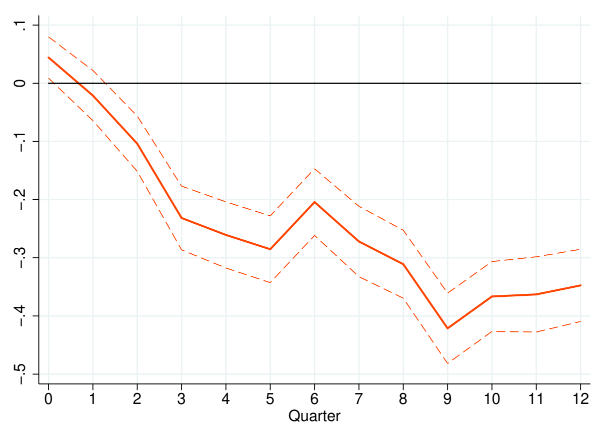
Figure 26: Firm-level Differential Response to Monetary Shocks
(Censored, Alternative Periods)



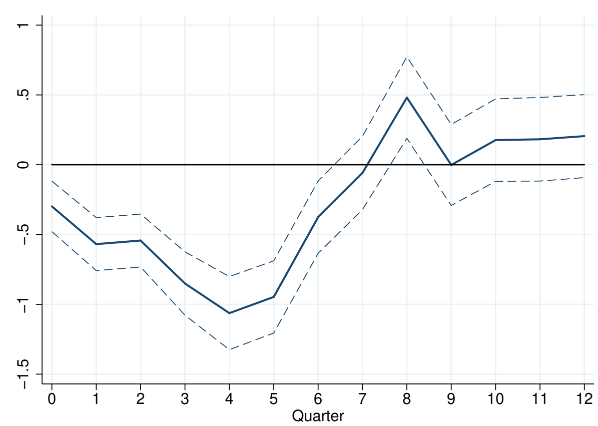
(a) Tobit including Pre-85



(b) Probit including Pre-85



(c) Tobit Post-94



(d) Probit Post-94

B.3.3 IRFs to Monetary Policy Shock at the Aggregate-level

Figure 27 and 28 displays the results of the coefficient of the sign-flipped GK HFI residual interacted with uncertainty γ_h from estimating the aggregate-level Local Projection with non-residential real private fixed investment and different investment components and output gap. Only the results for intellectual property are different. There could be a potentially different mechanism for intangible investment, however, which is beyond the scope of this paper.

Figure 27: Aggregate-level Differential Response to Monetary Shocks

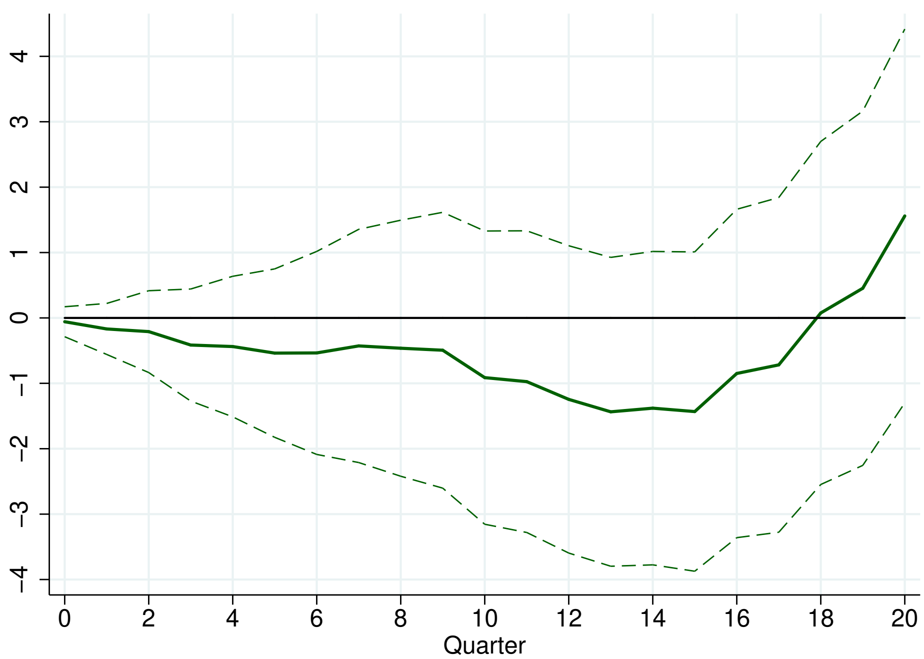
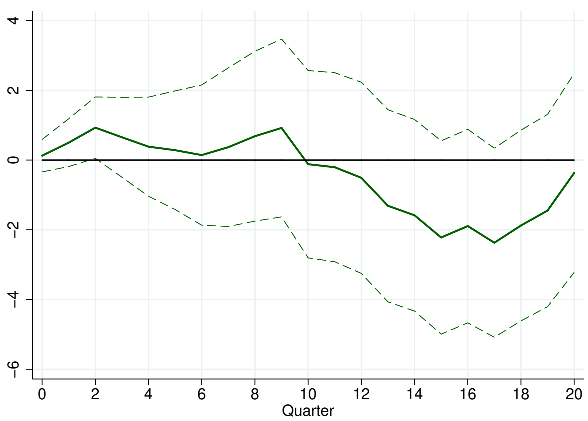
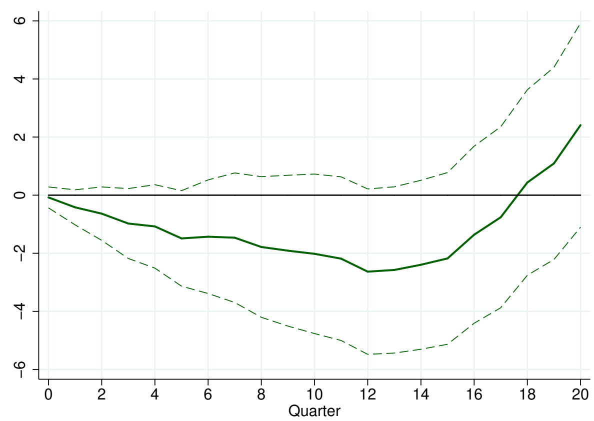


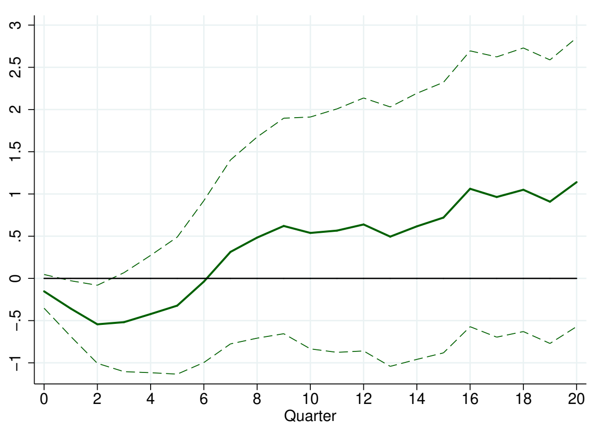
Figure 28: Aggregate-level Differential Response to Monetary Shocks
(Investment Components)



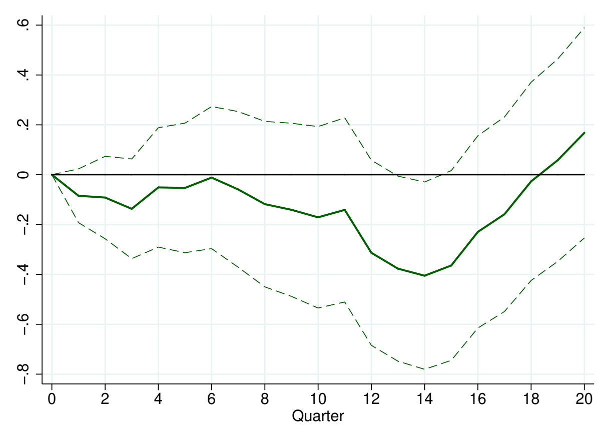
(a) Structure



(b) Equipment



(c) Intellectual Property



(d) Output Gap

B.4 Additional Results: Real Interest Rate as the Policy Indicator

B.4.1 IRFs to Monetary Policy Shock at Firm-level (OLS)

To explore how uncertainty affects the effectiveness of monetary policy, I employ the OLS *Local Projection* (LP) method from Òscar Jordà (2005) with adjustments applying to the panel nature of the data:

$$\Delta_h I_{j,t+h} = f_{j,h} + \alpha_{n,h,t+h} + \beta_h^m r_t^m + \boldsymbol{\gamma_h r_t^m u_t} + \sum_{l=0}^{L} \Gamma'_{h,t-l} X_{j,t-l} + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{j,h,t} \tag{24}$$

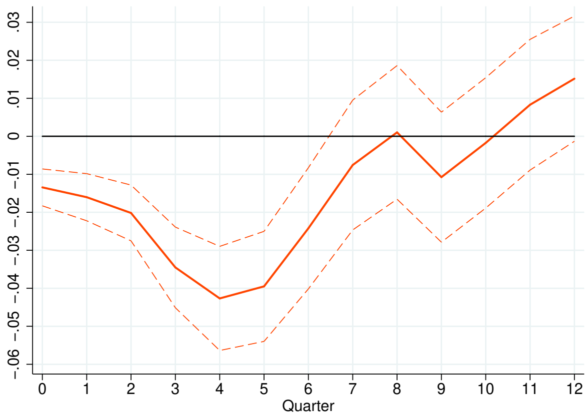
where j indicates firm j, h indicates quarters in the future and l indexes lags. $\Delta_h I_{j,t+h}$ is the h quarter investment rate measure, $f_{j,h}$ is the firm fixed effect, $\alpha_{n,h,t+h}$ is the quarter sector dummy to control for seasonality and the industry fixed effect, $u_t = IQR_t^{stock}$ is the uncertainty measure at time t,. r_t^m is the sign-flipped real interest rate, $X_{j,t-l}$ is a vector containing the firm-level controls including Tobin's Q, sales, leverage, and size for $t-l$, and Z_{t-l} is a vector of aggregate controls including uncertainty u_t, CPI, output gap, investment, and private consumption from NIPA. For uncertainty, investment, and firm-level controls, I use the same Compustat Quarterly data as constructed in Section 2.1[33]. Figure 25 displays the results for the coefficients of the sign-flipped real interest rate interacted with uncertainty γ_h from estimating the firm-level specification (24) using Compustat public firms.

B.4.2 IRFs to Monetary Policy Shock at Firm-level (Censored, Alternative Periods)

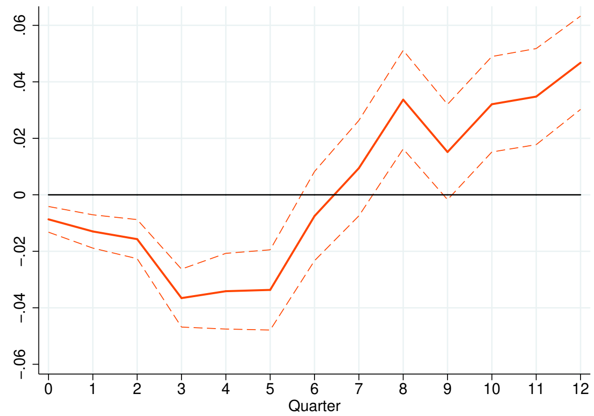
Figure 30 displays the results of the coefficient of the real interest rate interacted with uncertainty γ_h from estimating the firm-level Tobit and Probit with alternative choices of periods. The regressions including pre-85 periods are almost identical to the results in the main text because I have very few observations before 1985.

[33]The firm-level control variables follows classic literature: A size measure using total asset (Item *atq*); financial condition measures using leverage measured as total debt (Item *dlcq* + *dlttq*) over total asset (Item *atq*), and cash holding (Item *cheq*); operational status measures using revenue (Item *revtq*), sales (Item *saleq*), and sales growth which is Δ*saleq* divided by sales, and finally, investment opportunity measured using Tobin's Q.

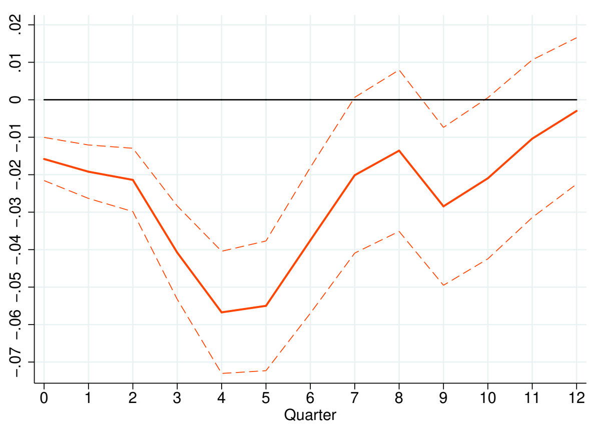
Figure 29: Firm-level Differential Response to Monetary Shocks (OLS)



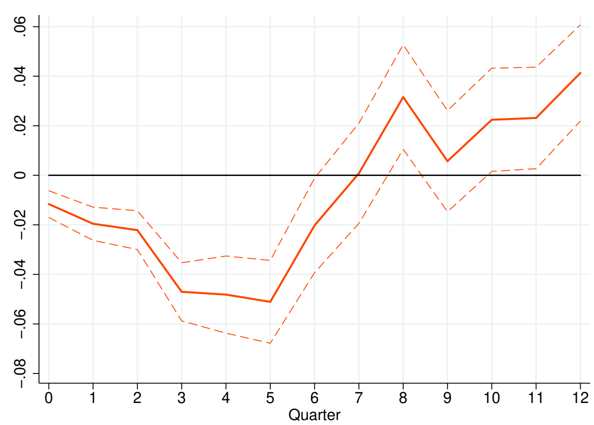
(a) OLS FE



(b) OLS Non-FE

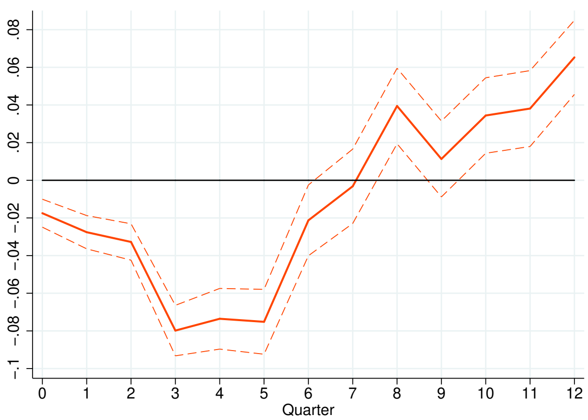


(c) OLS FE Post-94

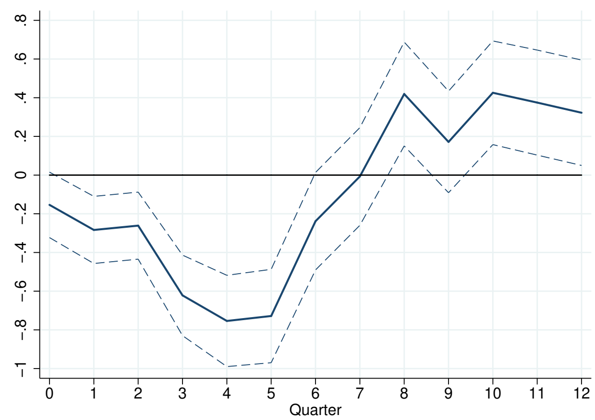


(d) OLS Non-FE Post-94

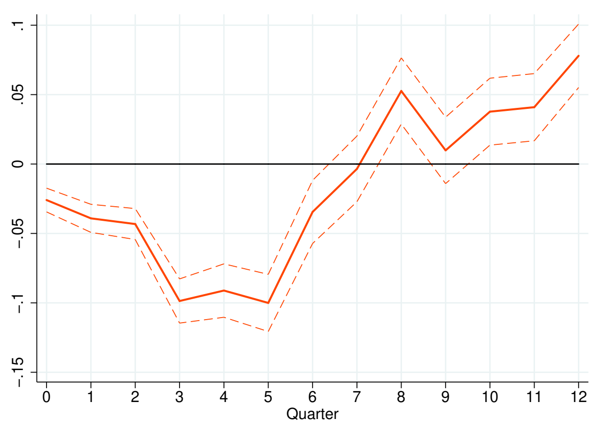
Figure 30: Firm-level Differential Response to Monetary Shocks
(Censored, Alternative Periods)



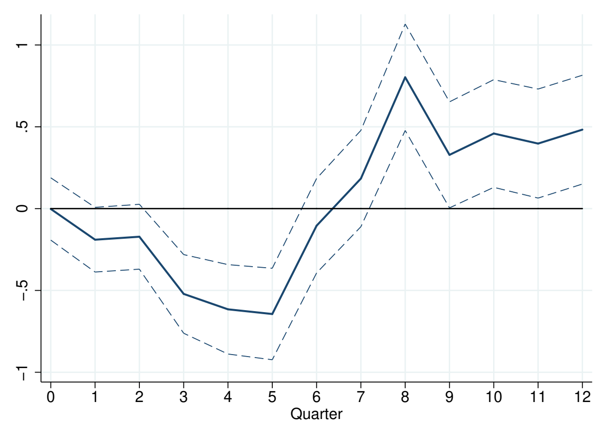
(a) Tobit including Pre-85



(b) Probit including Pre-85



(c) Tobit Post-94

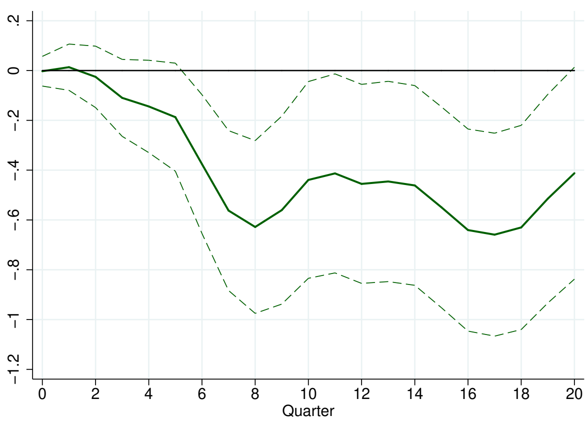


(d) Probit Post-94

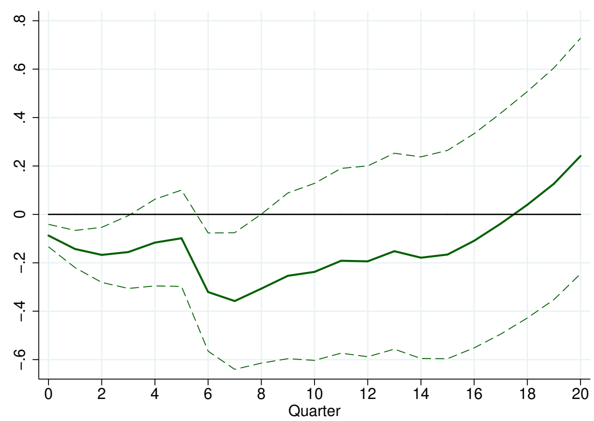
B.4.3 IRFs to Monetary Policy Shock at Aggregate-level (Investment Components)

Figure 31 displays the coefficient of the real interest rate interacted with uncertainty γ_h from estimating the aggregate-level Local Projection with different investment components and output gap. Only the result of intellectual property is different. There could be a potential different mechanism for intangible investment, however, which is beyond the scope of this paper.

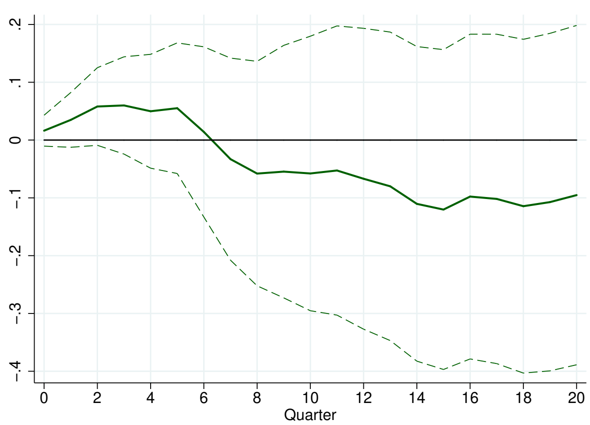
Figure 31: Aggregate-level Differential Response to Monetary Shocks (Investment Components)



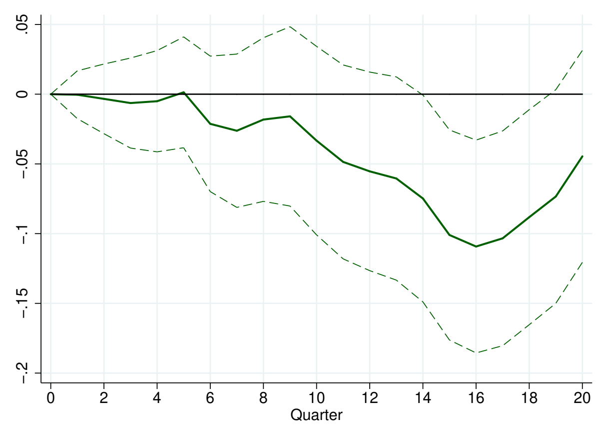
(a) Structure



(b) Equipment



(c) Intellectual Property



(d) Output Gap

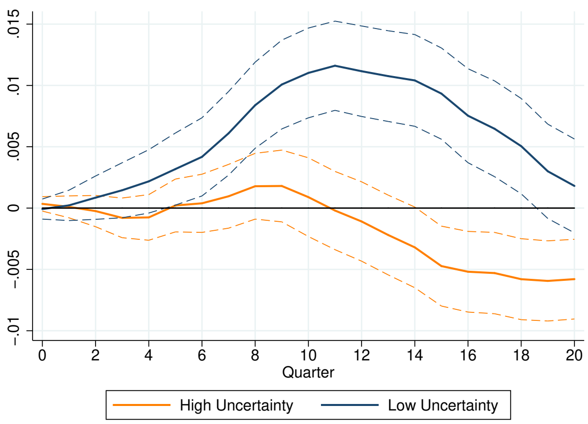
B.4.4 Results of the grouped Local Projection at Aggregate-level

I present more Local Projection results of aggregate-level impulse responses using grouped dummy of uncertainty. To make the results more intuitive, I present the IRFs of the Top 20% of IQR (High Uncertainty) against the Bottom 20% of IQR (Low Uncertainty) using the same Local Projection method. In the alternative regression below, $\mathbf{1}(u_t) = \{h, m, l\}$ indicates high, middle, and low uncertainty, respectively.

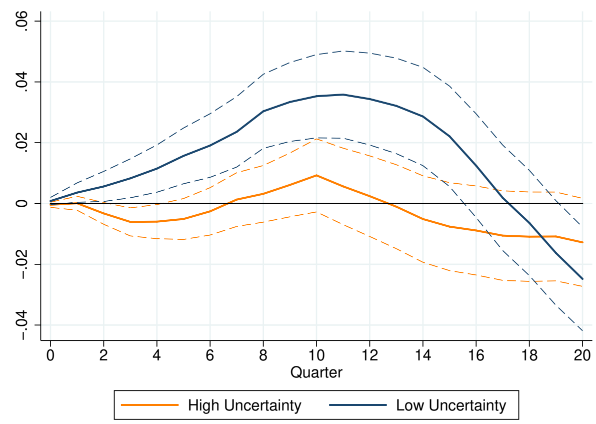
$$\Delta_h I_{t+h} = \alpha_h + \beta_h^m r_t^m + \boldsymbol{\gamma_h r_t^m} \times \mathbf{1}(\boldsymbol{u_t}) + \sum_{l=0}^{L} \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \tag{25}$$

Other Investment Measures: I first show the IRFs of other investment measures. Figure 32 Panel (a) shows the responses of the output gap. Panel (b) shows the responses of the real private fixed investment which also includes the residential investment. Panel (c) shows the responses of the real gross fixed investment which also includes the investments in inventories upon real private fixed investment. Panel (d) shows the responses of the real gross fixed capital formation which is only the value of net additions to fixed assets but excluding stocks of inventories and other operating costs measured by OECD. All three types of investment are less responsive to monetary stimulus when uncertainty is high. Figure 33 shows the responses of non-residential real private fixed investment and its components.

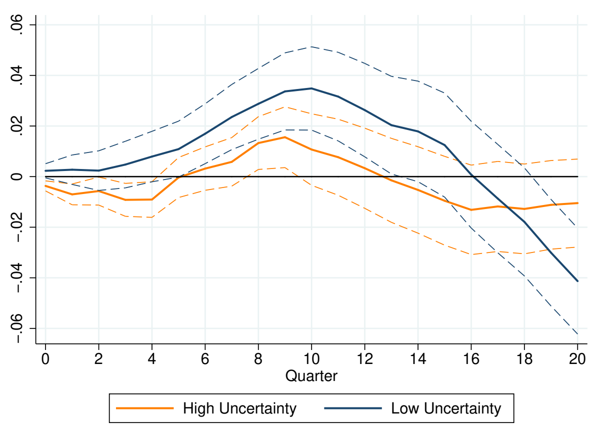
Figure 32: IRFs of Output Gap and Aggregate Investment in Other Measures



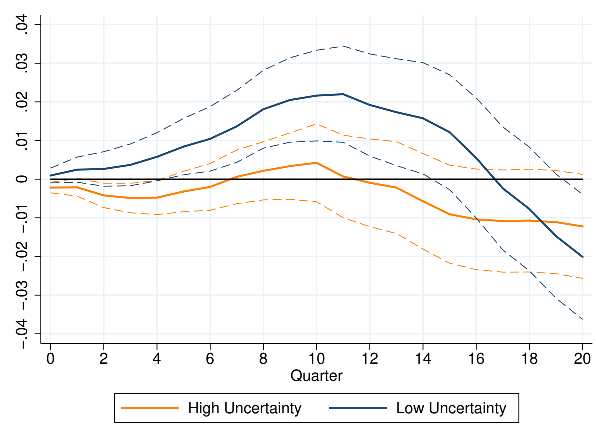
(a) Output Gap



(b) Real Private Fixed Investment

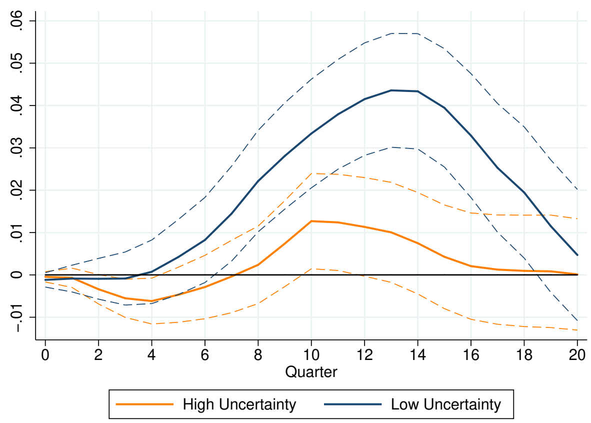


(c) Real Gross Private Investment

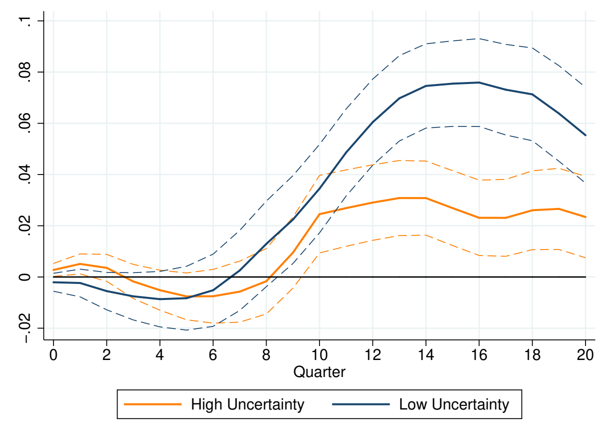


(d) Real Gross Fixed Capital Formation

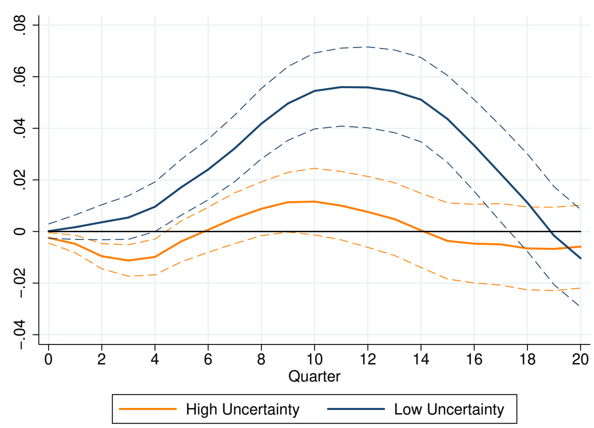
Figure 33: IRFs of Non-residential Real Private Fixed Investment
(All Components)



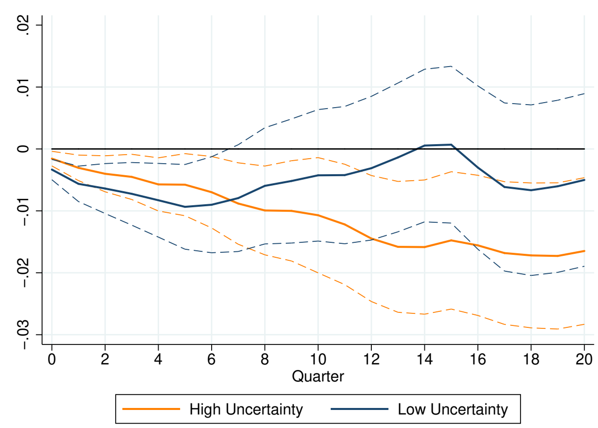
(a) All Non-residential



(b) Structure



(c) Equipment



(d) Intellectual Property