

Lumpy Investment, Fluctuations in Volatility and Monetary Policy

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This version: March 8, 2021

First version: October 16, 2019

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Abstract

I argue that monetary policy is less effective at stimulating investment during periods of elevated volatility in firm-level TFP than during normal times. Empirically, I document that high volatility weakens investment responses to monetary stimulus. I then develop a heterogeneous firm New Keynesian model with lumpy investment to interpret these findings. In the model, non-convex capital adjustment costs create a sizable extensive margin of investment which is more sensitive to changes in both interest rate and volatility than the intensive margin. When volatility is high, firms tend to stay inactive at the extensive margin, so monetary stimulus motivates less investment at the extensive margin. I find that the quantitative implications of the model are primarily shaped by the specifications of the capital adjustment costs. Unlike much of the prior literature, I use the dynamic moments of investment to identify this key model element. Based on this parameterization, high volatility reduces the effectiveness of monetary stimulus for investment by 30%. This reduction is about half of what I find in the data. Therefore, the effect of monetary policy depends on both the lumpy nature of firm-level investment and fluctuations in volatility.

Keywords: Lumpy investment; Ss model; irreversibility; volatility; uncertainty; firm heterogeneity; monetary policy;

JEL Codes: E52, E32, E22

*Email: min.fang.ur@gmail.com. Department of Economics, 238 Harkness Hall, Rochester, NY 14620. I am greatly indebted to my advisors Narayana Kocherlakota and Yan Bai for their continuous advice and encouragement. I thank Zirou Chen for her support and discussions. I am also very grateful to George Alessandria, Issac Bailey, Mark Bils, Macro Brianti, Gaston Chaumont, Mariacristina De Nardi, Jonathan Garita (discussant), Zibin Huang, Fernando Martin, Roman Merga, Andrew Owens, Albert Queralto, Claudia Sahm, Tianchen Song, Aleh Tsyvinski, Christian Wolf, Yajie Wang, Daniel Yi Xu, and Zhiwei Xu (discussant) for their inspiring comments and suggestions. This paper also benefits from the comments and discussions in the seminars and conferences at the University of Rochester, Boston College, EGSC (WUSTL), ES World Congress (Milan), EEA (Rotterdam), YES (UPenn), Chinese Young Economist Society, and ES European Winter Meeting (Nottingham). I thank the Dean's Post-Field Dissertation Fellowship from AS&E at University of Rochester for financial support. All errors are my own.

1 Introduction

How effective is monetary policy at stimulating aggregate investment when volatility is high? Starting from [Bloom \(2009\)](#), much literature has brought renewed attention that *elevated volatility* (when all firms experience an increase in the variance of their productivity shocks) may cause recessions ([Bloom et al., 2018](#); [Arellano, Bai, and Kehoe, 2019](#); [Gilchrist, Sim, and Zakrajšek, 2014](#)). Since monetary policy is often used as the primary stabilization mechanism for the economy, especially during recessions, it is important to quantify the effects of monetary policy on aggregate investment in times of elevated volatility.

In this paper, I take two approaches to answer this question. First, I document empirically that aggregate investment is significantly less responsive to monetary shocks during high volatility periods. Second, I build a heterogeneous firm New Keynesian model which is consistent with both the micro distribution of firm-level extensive-margin investment and the macro sensitivity of aggregate investment to aggregate shocks. In the model, the response of aggregate investment to monetary shocks is primarily driven by the extensive margin rather than the intensive margin. When volatility increases, fewer firms are close to making an extensive margin investment, so monetary stimulus generates less aggregate investment response than it would otherwise. Based on a parameterization consistent with dynamic micro investment moments, the model explains about half of the reduction that I find in the data.

My baseline empirical specification estimates how the semi-elasticity of aggregate investment with respect to a monetary policy shock depends on the level of economic volatility. I employ the local projections in the spirit of [Jordà \(2005\)](#) and estimate differences in non-residential private investment dynamics in response to monetary policy shocks. I find that when moving from the 20th percentile to the 80th percentile of volatility in firm-level performance, the same conventional monetary stimulus generates 63% less aggregate investment.

Motivated by this evidence, I embed a model of heterogeneous firms with lumpy extensive-margin investment decisions into the benchmark New Keynesian framework. These firms make investment decisions subject to both random fixed costs and partial irreversibility. The presence of these costs generates the extensive margin choice of whether to invest or not which is more sensitive to changes in both the interest rate and volatility than the intensive margin of investment. Volatility shocks are introduced as potential shocks to the variance of firms' idiosyncratic productivity shocks. I also employ a group of retailer firms with sticky prices to link nominal variables to real outcomes through a New Keynesian Phillips curve,¹ and a family of representative

¹Since I am not studying the interaction of investment and price setting, I introduce a New Keynesian block to separate rigidity in price setting from firms' production decisions to keep the model tractable.

households to supply labor and consume final goods as in [Ottonello and Winberry \(2020\)](#).

The key mechanism in the model is that the monetary policy shock and the volatility shock interact non-linearly at the extensive margin of investment. With data consistent specifications of capital adjustment costs, there exists a sizable extensive margin of investment.² Since the extensive margin is more sensitive to changes in both the interest rate and volatility than the intensive margin, there are two important observations: first, monetary policy works primarily through the extensive margin of investment; second, a volatility shock significantly decreases the investment incentive at the extensive margin of investment. As a result, upon a volatility shock, firms tend to stay inactive at the extensive margin, so the same conventional monetary stimulus motivates much less investment at the extensive margin.

Solving the quantitative model is difficult because the distribution of firms is an infinite dimension state. I solve the model following the MIT shock strategy, as documented in [Boppart, Krusell, and Mitman \(2018\)](#). My algorithm is to first solve the steady-state, and then solve transition paths given different shocks or different combinations of shocks, all of which eventually converge back to the steady-state. Since this yields a global solution, the algorithm captures all the non-linear dynamics, which is very important for the non-linear interaction between volatility shocks and monetary policy shocks. Since the algorithm is also fast, I can solve thousands of models to demonstrate the identification of the non-convex capital adjustment costs.

Unlike much of the prior literature, I use the dynamic moments of investment to parameterize the model. The first critical aspect of this parameterization is that I pin down the level of the random fixed costs using the autocorrelation of firm-level investment rate. This parameterization gives rise to much larger random fixed costs than those in the parameterizations of [Khan and Thomas \(2008\)](#), [Reiter, Sveen, and Weinke \(2013\)](#), and [Bachmann and Bayer \(2013\)](#).³ The second critical aspect of this parameterization is that I pin down the level of partial irreversibility using the covariance of the capital gap with the duration since last capital adjustment. The magnitudes of the random fixed costs and partial irreversibility generate data-consistent sensitivities of aggregate investment in response to changes in interest rates and volatility. These are both essential elements to generate the volatility-dependent effectiveness of monetary policy in the model.

Equipped by the technique and the parameterization, I then quantify the model results. With normal volatility, the peak response of aggregate investment to a conventional expansionary

²Fluctuations in aggregate investment are primarily driven by changes in the number of firms implementing new investment projects (the extensive margin) rather than changes in the size of ongoing investment projects (the intensive margin). Check the evidence in [Doms, Dunne et al. \(1998\)](#) or [Gourio and Kashyap \(2007\)](#).

³In these papers, the implied partial equilibrium elasticity of aggregate investment to the real interest rate is always much larger than 5. However, as suggested by [Koby and Wolf \(2019\)](#), this elasticity should be around 5.

monetary policy shock is +2.0%.⁴ In contrast, during times of elevated volatility, the peak response of aggregate investment to the same monetary shock is +1.4%. This reduction of 30% compared to normal times is roughly half of the 63% reduction that I estimated in the data.

To further inspect the mechanism, I decompose the investment channel of monetary policy into both the extensive and intensive margins. The extensive margin accounts for 60% of the total impulse response of aggregate investment with respect to monetary policy shocks. With elevated volatility, the extensive margin of the investment channel of monetary policy is substantially less responsive while the intensive margin responsiveness remains almost unchanged. I then show several alternative parameterizations without a reasonable specification of lumpy investment mechanism. In each case, the models cannot match all the empirical moments and fails to generate volatility-dependent effectiveness of monetary policy as in the data.

Related literature. This paper primarily contributes to three strands of literature. First, this paper contributes to the literature that studies how time-varying volatility affects the business cycle and monetary policy outcomes. Since the seminal paper [Bloom \(2009\)](#), volatility shocks have received substantial attention. [Bloom et al. \(2018\)](#) argued that a volatility shock shapes the bust-boom cycles with sharp recessions using a heterogeneous firm general equilibrium model with partial irreversibility⁵. In terms of monetary policy, most literature features the interaction of volatility shocks and monetary policy through menu costs and firm pricing decisions. [Vavra \(2013\)](#) provides evidence and builds a menu cost model arguing that higher volatility leads to an increase in aggregate flexibility so that a nominal stimulus mostly generates inflation rather than output growth.⁶ [Baley and Blanco \(2019b\)](#) also builds a price-setting model featuring imperfect information, markup Poisson shocks, and learning. In their model, volatility shocks increase monetary policy neutrality so that the real effect of monetary policy is reduced. There is also a strand of empirical literature using either VAR or Local Projection on aggregate time series data studying the relationship between high volatility and the real effects of monetary policy. This includes [Aastveit, Natvik, and Sola \(2017\)](#), [Castelnuovo and Pellegrino \(2018\)](#), [Caggiano, Castelnuovo, and Nodari \(2017\)](#), and [Paccagnini and Colombo \(2020\)](#). I provide direct evidence of this interaction on the extensive margin of investment as well as building a lumpy investment

⁴This resolves the excessive response puzzle of lumpy investment responses to monetary policy as in [Reiter, Sveen, and Weinke \(2013\)](#). In their version of calibration, the same monetary policy shock generates more than a 5% increase in aggregate investment which only lasts for one period.

⁵There is also another important strand of literature featuring financial frictions. [Arellano, Bai, and Kehoe \(2019\)](#) and [Gilchrist, Sim, and Zakrajšek \(2014\)](#) show that with financial frictions, firms reduce investment to avoid default after a surprise increase in risk.

⁶[Li \(2019\)](#) revisits these results and shows that monetary policy is still very effective under volatility shocks in the menu cost models. Unlike that work, this paper is the first to explore how the interaction between a volatility shock and capital adjustment costs at firm-level affects monetary policy.

model arguing that higher volatility leads to a substantial drop in firm-level investment via the extensive margin reducing the effectiveness of nominal stimulus on aggregate investment.

Second, this paper contributes to the literature that studies the transmission of monetary policy to the aggregate economy featuring endogenous capital accumulation. Popularized by [Christiano, Eichenbaum, and Evans \(2005\)](#) and [Smets and Wouters \(2007\)](#), New Keynesian models assume convex capital adjustment costs or investment adjustment costs in order to generate empirically consistent investment responses to monetary policy shocks. These assumptions work well in Representative Agent New Keynesian (RANK) models but fail to capture the observed lumpiness in plant-level investment⁷. [Reiter, Sveen, and Weinke \(2013\)](#) is the first paper that attempted to fill this gap. However, by introducing standard fixed capital adjustment costs as in [Khan and Thomas \(2008\)](#), they argued that monetary policy shocks lead to large but very short-lived dynamic consequences that are not consistent with empirical evidence or the consensus view in the literature. [Koby and Wolf \(2019\)](#) shows that with lumpy investment, the effect of monetary policy depends on the level of aggregate TFP. In this paper, I incorporate the standard fixed capital adjustment costs along with partial irreversibility and convex adjustment costs. The comprehensive adjustment cost structure helps to generate consistent lumpiness in plant-level investment while maintaining reasonable aggregate responses to monetary policy shocks.

Third, this paper contributes to the literature which studies whether micro-level lumpy investment has aggregate implications. Since [Caballero et al. \(1995\)](#) and [Caballero and Engel \(1999\)](#), who find that firm level extensive margin investment behavior generates procyclical responsiveness to shocks, there has been ongoing debate whether micro-level lumpy investment has aggregate implications. During the 2000s, [Thomas \(2002\)](#), [Khan and Thomas \(2003\)](#), and [Khan and Thomas \(2008\)](#) show that in an otherwise standard RBC framework that extensive margin investment is irrelevant for aggregate dynamics. However, [Bachmann, Caballero, and Engel \(2013\)](#) shows that the results of these models are very sensitive to the calibration. Meanwhile, [House \(2014\)](#) suggests that the extreme sensitivity of aggregate investment to the relative price of investment goods drives these irrelevance results in a stylized partial equilibrium model. [Winberry \(2018b\)](#) shows that a lumpy investment model matching the business cycle dynamics of real interest rate generates state-dependent aggregate implications. Following that train of thought, [Koby and Wolf \(2019\)](#) shows that matching the empirically consistent interest rate sensitivity of aggregate investment is the key to breaking the irrelevance results. More recently, [Baley and Blanco \(2019a\)](#) shows that the steady-state misallocation and irreversibility are sufficient statistics for aggregate dynamics. I extend this literature by showing the sensitivity of the dynamic moments to both real

⁷Although the literature such as [Ottonello and Winberry \(2020\)](#), [Jeenas \(2018\)](#), and [Deng and Fang \(2020\)](#) has started to explore New Keynesian models with heterogeneous firms, they essentially forgo the micro-level lumpy investment.

interest and firm-level volatility and how matching them matters for the dynamics of aggregate investment.⁸

Layout. This paper is organized as follows. Section 2 provides the empirical evidence that the aggregate investment is less responsive to monetary policy when volatility is high. Section 3 develops a New Keynesian model with heterogeneous firms to interpret this evidence. Section 4 shows the mechanism in a minimalist framework. Section 5 then calibrates the full model and verifies that it is consistent with the distribution of investment during periods of high/low volatility. Section 6 validates the model mechanism and examines firm behavior in the model. Section 7 concludes.

2 Empirical Motivation

In this section, I empirically analyze the relationship between volatility in firm-level performance and the effect of monetary policy using U.S. aggregate-level data. Section 2.1 provides the details of my data and measures. Section 2.2 estimates volatility-dependent impulse responses of aggregate investment to identified monetary policy shocks. Section 2.3 provides a summary of further robustness checks and additional results for the baseline estimation.

2.1 Data and measures

Data and variables: I use both aggregate-level time series data and firm-level data from Compustat. The aggregate-level data includes various investment measures, output gap, inflation, consumption from NIPA and time series financial variables from Bloomberg.

Fluctuations in volatility (σ_t): The main measure I use for volatility (σ_t) is the Interquantile Range (IQR) of sales growth, more specifically, the IQR of sales growth ($IQR_{sg,t}$) of firms appearing for at least 25 years or more in Compustat between 1960 and 2010 as in Bloom et al. (2018). Figure 1 plots the changes in volatility over the business cycle. There are significant fluctuations in volatility measured by IQR sales growth: a peak of 0.32 in 2008Q2 and a bottom of 0.17 in 2006Q4. I group the level of the IQR sales growth $IQR_{sg,t}$ (time series) into three groups: $\{h, m, l\}$

⁸The lumpy investment literature has received a lot of attention recently. Chen et al. (2019) incorporates the lumpy nature of firm-level investment into the study of how tax policy affects investment behavior. Zorzi (2020) shows that the nature of the adjustment of residential investment generates a non-linear effect which amplifies the aggregate response of durable spending during booms and dampens it during recessions. Also, in the asset pricing literature, Wu (2020) shows that incorporating lumpy investment could potentially resolve asset pricing puzzles in both time series and the cross section. These studies overturn the aggregate irrelevance results from previous literature and present interesting empirical findings and theoretical applications.

which are Top 20%, Middle 60%, and Bottom 20%, respectively. The Top 20% is shown as the black shadowed area and the Bottom 20% is denoted as the blue-gray shadowed area. This is also the measure I am using for the following regression in section 2.2.

Monetary policy shocks (ϵ_t^m): The proxy for the monetary policy shock is the quarterly accumulation of the residual from a monthly VAR in which the one year government bond rate is instrumented for with high-frequency identified shocks following Gertler and Karadi (2015). The idea is to isolate interest rate surprises using the movements in financial markets data within a short window around central bank policy announcements. I follow Gertler and Karadi (2015) exactly, using financial market surprises from Fed Funds Futures during the 30 minutes interval around the FOMC policy announcements as proxies for the one-year government bond rate in a vector autoregression. The structural residual is then the estimated monetary policy shock.⁹ The full sample is from 1980Q1 to 2012Q2. I flip the sign and re-scale it dividing by 25bps to match a conventional monetary policy expansion. This provides a more intuitive comparison of the empirical results.

2.2 Different investment impulse responses to monetary shocks

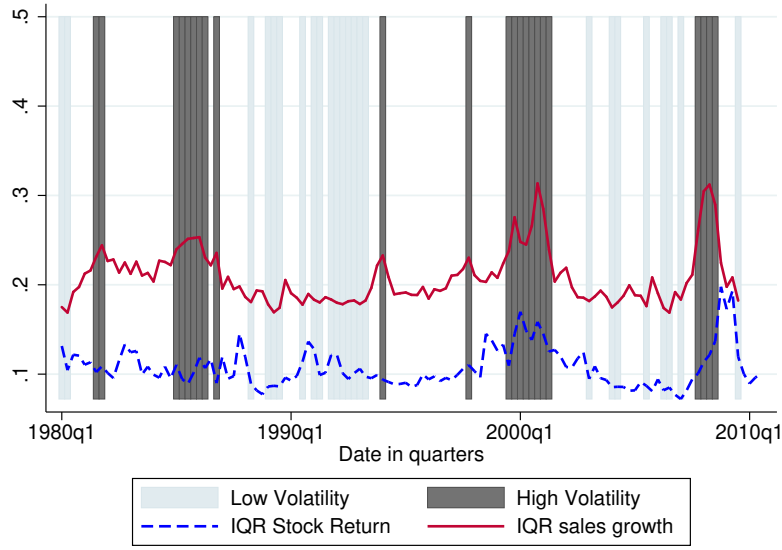
To explore the correlation between volatility and the effect of monetary shocks, I employ the following *Local Projection (LP)* empirical specification of Òscar Jordà (2005):

$$\Delta_h I_{t+h} = \alpha_h + (\beta_{j,h} + \gamma_{j,h} \epsilon_t^m) \times \mathbf{1}_{\sigma_t \in J^\sigma} + \sum_{l=0}^L \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \quad (1)$$

where h indicates quarters in the future and l indicates lags. $\Delta_h I_{t+h} = I_{t+h} - I_t$ is the change of the log investment measure, and I_t is log real non-residential private fixed investment which includes only investment from firms rather than households and government in this benchmark regression. Hence, $\Delta_h I_{t+h}$ measures the change of investment in period $t + h$ relative to period t . $\sigma_t = IQR_{sg,t}$ is the volatility measure at time t , $\mathbf{1}_{\sigma_t \in J^\sigma}$ indicates that σ_t belongs to one of the $J^\sigma = \{h, m, l\}$ groups as defined in Section 2.1, ϵ_t^m is the high-frequency-identified monetary policy shock. I then control for a period h fixed effect, and a vector Z_{t-l} of aggregate variables including volatility, CPI, output gap, investment, and consumption. I choose the horizon $H = 20$ and the lag $l = 4$ as suggested by Jordà (2005). The sample period is from 1980Q3 to 2010Q3. Varying the sample period by excluding pre-1985 quarters or post-2008 quarters do not change

⁹See Cloyne et al. (2018) for detailed explanations of the advantages of using this monetary policy shock series. I choose the measure in Gertler and Karadi (2015) because it covers a longer sample period. I also employ the real interest rate in the robustness checks. I also use the direct measure as in Ottonello and Winberry (2020) which the results also hold qualitatively. For the sake of space, the results are only available upon request.

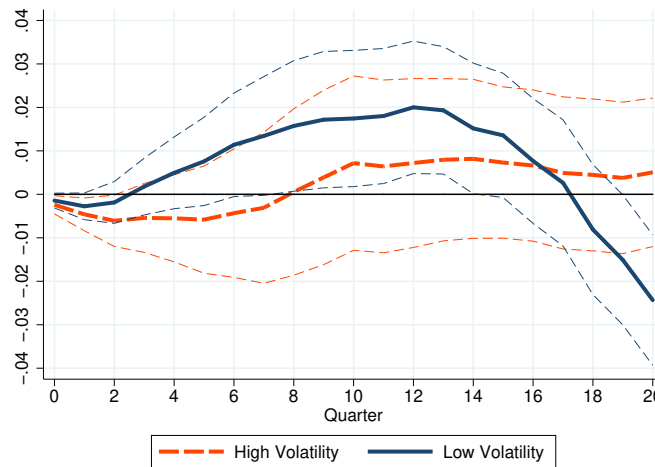
Figure 1: Volatility over Business Cycle



Notes: The Interquartile Range (IQR) of sales growth by quarter is calculated using the sample of Compustat firms with 25 years (100 quarters) or more in Compustat between 1962 and 2010. The Interquartile Range (IQR) of stock returns by quarter using the sample of Compustat firms with 25 years (100 quarters) or more in Compustat between 1960 and 2010. The *Low Volatility* is defined as the IQR sales growth of the Bottom 20% while *High Volatility* is defined as the IQR sales growth of the Top 20%. The correlation between IQR sales growth and IQR stock return is 0.47.

the main results. These supplemental results are in Empirical Appendix B.3.

Figure 2: Volatility-dependent Effectiveness of Monetary Policy



Notes: The dashed lines indicates the 90% confidence interval. The *Low Volatility* is defined as IQR sales growth at its Bottom 20% while *High Volatility* is defined as IQR sales growth at its Top 20%.

The coefficients of interest are $\gamma_{h,h}$ and $\gamma_{l,h}$, which measures how the semi-elasticity of the aggregate private investment rate $\Delta_h I_{t+h}$ with respect to monetary shocks depends on the state of volatility: low (Bottom 20%) or high (Top 20%).

Figure 2 displays the estimation of the coefficients $\gamma_{h,h}$ and $\gamma_{l,h}$ of the sign-flipped real interest rate interacted with volatility. The dashed lines indicates the 90% confidence interval. This shows that during the times of higher volatility, measured by the IQR of sales growth, real non-residential private fixed investment is less responsive to the monetary shocks. When volatility is low, the effects of a 25bps unanticipated monetary stimulus will generate a peak increase of 2.0% in the investment rate in quarter 12. When volatility is high, the same 25bps unanticipated monetary stimulus generates a much smaller peak increase of 0.75% in the investment rate at quarter 12.

2.3 Robustness checks

In the empirical appendix B.3, I show various robustness checks using alternative specifications: investment measures, volatility measures, local projection specifications, sample periods, monetary policy indicators, and all possible combinations. The results in the main text hold qualitatively in almost all of these exercises. I briefly summarize the results of these robustness checks here. For more details, please refer to the empirical appendix B.3.

Alternative investment measures: I check robustness using both alternative measures of investment and components of investment measures for the aggregate local projection regressions. These alternatives includes real gross fixed capital formation, real gross private investment, and real private fixed investment. Components includes equipment, structures, and intellectual property. I also include the output gap as an external validation. I check all these measures interacting with all the other robustness checks. My results hold for all the measures excluding intellectual property.

Alternative volatility measures: I check robustness using the other quarterly measure: the IQR of monthly stock returns for all Compustat firms with more than 25 years of observable data. This alternative measure has a very high correlation (0.47) with my primary IQR of sales growth measure. I also check this measure interacting with all the other robustness checks. My results hold for all the robustness checks when I replace the IQR of sales growth with the IQR of stock returns.

Alternative local projection specifications: I check robustness using an alternative local projection specification which directly interacts the volatility measure with the monetary policy

shocks. The direct interaction specification is as follows:

$$\Delta_h I_{t+h} = \alpha_h + \beta_h^m r_t^m + \gamma_h r_t^m \sigma_t + \sum_{l=0}^L \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \quad (2)$$

The main difference of this direct interaction specification (2) is that now I insert $\beta_h^m r_t^m + \gamma_h r_t^m \sigma_t$ instead of $\gamma_{j,h} \epsilon_t^m \times \mathbf{1}_{\sigma_t \in J^\sigma}$ as in the baseline grouped specification (1). Now γ_h will be measuring the semi-elasticity of investment with respect to monetary shocks r_t^m conditional on a continuous measure of volatility σ_t . Comparing investment impulse responses in the low volatility state to the high volatility state will be less intuitive in this specification, but a significantly negative γ_h is still strong evidence that the main result holds. I also apply this specification to all the other robustness checks and all the results hold.

Alternative sample periods: I choose alternative sample periods for all the aggregate local projection regressions in the robustness checks. There are two major cutoffs: 1985 and 2008. There are natural concerns about potential structural monetary policy changes at these time points. My results hold for various combinations of sample period choices.

Alternative monetary policy indicator: The high frequency identified monetary policy shock series in [Gertler and Karadi \(2015\)](#) only covers 1980-2012. Therefore, to obtain longer sample periods, I employ the real interest rate as the policy indicator. I also redo all the robustness checks using the real interest rate as the monetary policy indicator. Most results hold.

2.4 Remarks

I have shown a negative impact of heightened volatility on the effects of monetary policy on investment. This finding suggests that besides well-known factors such as the level of aggregate TFP that would affect the effects of monetary policy, volatility also plays a crucial role in determining the effectiveness of monetary policy. Table 5 provides a summary. The effectiveness of monetary policy on investment is reduced by 62% when measured volatility increased by 44%. The model in Section 3 explains why and how volatility affects the effect of monetary policy on investment, and in Section 5 I estimate the model parameters to match micro-level investment data and quantify the effect of elevated volatility in comparison to this data exercise.

Table 1: Volatility-dependent Effectiveness of Monetary Policy

	Low Volatility		High Volatility		Δ Effectiveness	
Source	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%

Notes: *Low Volatility* is defined as the IQR sales growth of the Bottom 20% while *High Volatility* is defined as the IQR sales growth of the Top 20%. $\frac{dI}{d\epsilon^m}$ denotes the peak impulse response of investment with respect to an expansionary 25bps monetary shock.

3 The Model

The model builds on the class of Real Business Cycle models with capital adjustment costs, including random adjustment costs as in [Khan and Thomas \(2008\)](#), partially irreversibility as in [Abel and Eberly \(1996\)](#), and quadratic adjustment costs. I then extend the Real Business Cycle framework to the New Keynesian framework by introducing price rigidity and a monetary authority. In the model, firms use both capital and labor to produce an identical intermediate good, which is sold to retailers at the same real wholesale price p_t^w . Firms that adjust their capital stock incur adjustment costs. Firms are also subject to an exogenous idiosyncratic process for productivity. To keep the model tractable, I then introduce a New Keynesian block to separate rigidity in price setting from firms' production decisions. Finally, a representative household closes the model.

3.1 Production Firms

There is a fixed unit mass of firms $j \in [0, 1]$ which produce output y_{jt} according to a decreasing returns to scale production function. For each firm, its output is then sold to a corresponding retailer at an economy-wide wholesale price P_t^W .

Technology: The production function is as follows:

$$y_{jt} = z_{jt} k_{jt}^\alpha n_{jt}^\nu, \quad \alpha + \nu < 1 \quad (3)$$

where k_{jt} and n_{jt} denote the idiosyncratic capital and labor employed by the firm j . The technology is decreasing returns to scale so $\alpha + \nu < 1$. For each firm, the idiosyncratic productivity is z_{jt} . I assume the shocks follow the following log-normal AR(1) process:

$$\log(z_{jt}) = \mu_t + \rho^z \log(z_{jt-1}) + \sigma_t^z \epsilon_{jt}, \quad \epsilon_{jt} \sim N(0, 1) \quad (4)$$

where the variance of the idiosyncratic innovation, σ_t^z , and the corresponding adjustment in the

mean, $\mu_t = -(1 - \rho^z) \times \frac{\sigma_t^{z2}}{2(1-\rho^{z2})}$, is fixed during times of normal volatility. An aggregate shock to volatility of firm-level TFP is an unexpected sharp rise in the variance of productivity shocks σ_t^z and a corresponding adjustment in μ_t so that the mean of z_{jt} is unchanged. The timing of the productivity process is such that at the beginning of the period t , σ_t^z is realized by firms, then firms make investment decisions. After the investment decision is made, z_{jt} is realized and firms make production decisions.

Adjustment costs: The deterministic investment cost function includes three components: a direct cost i_{jt} , a partially irreversible cost governed by S , and a quadratic cost governed by ϕ_k . In addition, firms who actively adjust their capital stock also pay a random fixed cost ξ_{jt} in units of labor if they adjust more than a small proportion of their current capital stock ($|ak|$). When an investment adjustment is large enough, $i_{jt} \notin [-ak_{jt}, ak_{jt}]$, firms have to pay the random adjustment cost. The random fixed costs ξ_{jt} is uniformly distributed with support $U[0, \bar{\xi}]$ independently across firms and time.¹⁰

$$c(i_{jt}) = i_{jt} + |i_{jt}| \left(\mathbf{1}_{(i_{jt} < 0)} \cdot S + \frac{\phi_k}{2} \left| \frac{i_{jt}}{k_{jt}} \right| \right) + \mathbf{1}_{(|i_{jt}| > ak_{jt})} \cdot \xi_{jt} \cdot E_t[w_t] \quad (5)$$

This specification of capital adjustment costs is comprehensive enough to nest previous literature. The existence of partial irreversibility generates real options with respect to investment, as articulated in [Dixit, Dixit, and Pindyck \(1994\)](#), which is the critical component for the wait-and-see effects of volatility shocks. The quadratic adjustment cost smooths out the investment behavior, as documented in [Winberry \(2018a\)](#) and [Koby and Wolf \(2019\)](#), which is essential to match the cross-section of the investment distribution when the random fixed cost is relatively large. Finally, the random fixed costs make firms pay the fixed cost infrequently, which is the key to generating lumpy investment patterns as in the microdata, as addressed in [Cooper and Haltiwanger \(2006\)](#). I also allow a region $i_{jt} \in [-ak_{jt}, ak_{jt}]$ within where firms do not have to pay the random adjustment cost in order to generate empirically plausible small investment adjustment behaviors around zero investment following [Khan and Thomas \(2008\)](#).

Firm optimization: I denote by $\tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t)$ the original value function of a firm, $V^A(k_{jt}, z_{jt}; \Omega_t)$ the value function of a firm with an active investment choice, $V^{NA}(k_{jt}, z_{jt}; \Omega_t)$ the value function of a firm without an active investment choice, and $V(k_{jt}, z_{jt}; \Omega_t) = E_{\xi_{jt}} \tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t)$ the dimension-reduced value function of a firm with expected draw of ξ_{jt} . The state variables are given in two parts: (i) individual state of capital stock k_{jt} , individual state of productivity z_{jt} , and

¹⁰This assumption of uniform distribution from 0 to $\bar{\xi}$ is not distinguishing the mean and the variable of the random fixed cost, see [Fang \(2021\)](#). However, the calibration of a relative large $\bar{\xi}$ is sufficient to deliver a reasonably sizable mean and variance, which generates reasonable dynamics of aggregate investment.

individual state of the random fixed cost draw ξ_{jt} ; (ii) aggregate state $\Omega_t = (\sigma_t, \Theta_t, \mu_t(k, z, \xi))$ where σ_t indicates the current degree of volatility, Θ_t is the vector of all aggregate variables including aggregate productivity, inflation, interest rate, wholesale price, stochastic discount factor, and wage at time t , and $\mu_t(k, z, \xi)$ is the current distribution of firms. The original dynamic problem of the firm consists of choosing investment and hours to maximize its recursive value function:

$$\begin{aligned} \tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t) = & \max_{i \in [-ak, ak]} \left\{ -c(i_{jt}) + E[\max_n (p_t^w y_{jt} - w_t n_{jt}) + \Lambda_{t,t+1} V(k_{jt+1}^*, z_{jt+1}; \Omega_{t+1})] \right\} \\ & + \max_{i \in [-ak, ak]} \left\{ -c(i_{jt}) + E[\max_n (p_t^w y_{jt} - w_t n_{jt}) + \Lambda_{t,t+1} V(k_{jt+1}^C, z_{jt+1}; \Omega_{t+1})] \right\} \end{aligned} \quad (6)$$

where the real wholesale price $p_t^w = \frac{P_t^w}{P_t}$ is from retailers, the real wage $w_t = \frac{W_t}{P_t}$ is from households, and the stochastic discount factor $\Lambda_{t,t+1} = \frac{\pi_{t+1}}{R_t^n}$ is derived from the household problem as households own all the firms. R_t^n is the nominal interest rate of one-period bonds and π_{t+1} is inflation. All the aggregate prices are components of the aggregate state $\Theta_t \in \Omega_t$, however, for simplicity, I will only index them by time t . The choice of k_{jt+1}^C is constrained by the no random fixed cost region $k_{jt+1}^C \in [(1 - \delta - a)k_{jt}, (1 - \delta + a)k_{jt}]$ while k_{jt+1}^* is not constrained. I can then separate the firm's original recursive value function depending on its investment choice as:

$$V^A(k_{jt}, z_{jt}; \Omega_t) = \max_{i,n} \left\{ -c(i_{jt}) + E[p_t^w y_{jt} - w_t n_{jt} + \Lambda_{t,t+1} V(k_{jt+1}^*, z_{jt+1}; \Omega_{t+1})] \right\} \quad (7)$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_t) = \max_{i \in [-ak, ak], n} \left\{ -c(i_{jt}) + E[p_t^w y_{jt} - w_t n_{jt} + \Lambda_{t,t+1} V(k_{jt+1}^C, z_{jt+1}; \Omega_{t+1})] \right\} \quad (8)$$

The firm will choose to pay the fixed cost if and only if the value from doing so is higher than not paying the fixed cost, that is, if and only if $V^A(k_{jt}, z_{jt}; \Omega_t) - w_t \xi_{jt} > V^{NA}(k_{jt}, z_{jt}; \Omega_t)$. For each tuple of $(k_{jt}, z_{jt}; \Omega_t)$, there is a unique threshold $\xi_t^*(k_{jt}, z_{jt}; \Omega_t)$ which makes the firm indifferent between these two options. The threshold is:

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t} \quad (9)$$

If a firm in state $(k_{jt}, z_{jt}; \Omega_t)$ draws a random fixed cost ξ_{jt} below $\xi_t^*(k_{jt}, z_{jt}; \Omega_t)$, the firm pays the fixed cost and then actively adjusts its capital; otherwise, it does not. The firms' optimal choice to only pay the fixed cost infrequently is part of what generates lumpy investment patterns as in the microdata.

Given the distribution of the random fixed cost and the optimal thresholds over the space of

$(k_{jt}, z_{jt}; \Omega_t)$, the value function is eventually determined as:

$$V(k_{jt}, z_{jt}; \Omega_t) = -\frac{w_t \xi^*(k_{jt}, z_{jt}; \Omega_t)}{2} + \frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}} V^A(k_{jt}, z_{jt}; \Omega_t) + \left(1 - \frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t) \quad (10)$$

where the firm expects to pay the random fixed cost in units of labor when the draw is lower than $\xi^*(k_{jt}, z_{jt}; \Omega_t)$. If so, with probability $\frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}}$, the value would be the active value function $V^A(k_{jt}, z_{jt}; \Omega_t)$; otherwise, its value would be the non-active value function $V^{NA}(k_{jt}, z_{jt}; \Omega_t)$. Therefore, the capital stock evolves by the law of motion:

$$k_{jt+1} = \begin{cases} (1 - \delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1 - \delta)k_{jt} + i_{jt}^C & \text{otherwise} \end{cases} \quad (11)$$

3.2 New Keynesian block

I design the New Keynesian block of the model to generate a New Keynesian Phillips curve relating nominal variables to the real economy¹¹. I separate the nominal rigidities from the firm problems to keep the model more tractable. The New Keynesian block consists of retailers who make the pricing decisions, a final good producer who produces final goods, and a monetary authority who sets the interest rate rule. The whole New Keynesian block is adding essentially two equations to the general equilibrium model: i) a New Keynesian Phillips curve which links wholesale prices to inflation, and ii) a Taylor Rule which links the monetary policy shock and inflation to the nominal interest rate. Without the New Keynesian block, the economy is reduced to a standard RBC model with lumpy investment.

Retailers: For each production firm j , there is a corresponding retailer j who produces a differentiated variety $Y_t(j)$ using good y_{jt} from production firm j' as its only input. The production function is simply a one-to-one transformation:

$$Y_t(j) = y_{jt}$$

where the retailers are monopolistic competitors who set their prices $P_t(j)$ subject to the demand curve generated by the final good producer and the wholesale price of the input P_t . Retailers

¹¹I follow [Ottonello and Winberry \(2020\)](#). Similarly to their problem, studying the joint dynamic decision of investment and price setting with nominal rigidities is outside this paper's scope. Nor do I have any micro moments that would provide insight on this joint problem. Therefore, a New Keynesian block is a parsimonious way to model price rigidity. This is a possibly exciting direction for further research on monetary policy.

pay a quadratic menu cost in term of final good, $\frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 P_t Y_t$, to adjust their prices as in [Rotemberg \(1982\)](#), where Y_t is the final good.

Final good producer: There is a representative final good producer who produces the final good Y_t using intermediate goods from all retailers with the production function:

$$Y_t = \left(\int Y_t(j)^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}$$

where γ is the elasticity of substitution between intermediate goods. The final good producer's profit maximization problem gives the demand curve $\left(\frac{P_t(j)}{P_t} \right)^{-\gamma} Y_t$ where the price index is $P_t = \left(\int P_t(j)^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}$. The final good is taken as the numeraire in the model.

Price setting by retailers: The resulting price stickiness comes from the price-setting decisions made by retailers maximizing profits. I follow [Rotemberg \(1982\)](#) except the marginal cost is now the wholesale price P^W from production firms:

$$\Pi_t(j) = (P_t(j) - P_t^W) \left(\frac{P_t(j)}{P_t} \right)^{-\gamma} Y_t - \frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 P_t Y_t$$

Through a standard derivation via retailers' profit maximization process (see Appendix [A.1](#)), we have the **New Keynesian Phillips curve**. This paper will directly focus on the linearized version for computational simplicity¹²,

$$\log \pi_t = \frac{\gamma-1}{\psi} \log \frac{p_t^w}{p^{w*}} + \beta E_t \log \pi_{t+1} \quad (12)$$

where $p_t^{w*} = \frac{\gamma-1}{\gamma}$ is the steady state wholesale price, or in other words the marginal cost for retailer firms. The Phillips Curve links the New Keynesian block to the production block through the relative the real wholesale price p_t^w for production firms. If the expectation of future inflation is unchanged, when aggregate demand for the final good Y_t increases, retailers must increase production of their differentiated goods because of the nominal rigidity. This in turn increases demand for the production goods y_{jt} , which increases the real wholesale price p_t^w and generates inflation through the Phillips curve.

Monetary authority: The monetary authority sets the nominal risk-free interest rate R_t^n accord-

¹²For robustness when I also solve the quantitative model using the non-linearized version, the results are almost identical. Therefore, in order to save computational time, I use the linearized version throughout the whole paper.

ing to the log version of a **Taylor rule**:

$$\log R_t^n = \log\left(\frac{1}{\beta}\right) + \phi_\pi \log \pi_t + \epsilon_t^m, \text{ where } \epsilon_t^m \sim N(0, \sigma_m^2) \quad (13)$$

where π_t is gross inflation in the final good price, ϕ_π is the weight on inflation in the reaction function, and ϵ_t^m is the monetary policy shock.

3.3 Households

The general equilibrium model is completed by introducing the household block. There is a unit measure continuum of identical households with preferences over consumption C_t and labor supply N_t whose expected utility is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \theta N_t \right)$$

subject to the budget constraint:

$$P_t C_t + \frac{1}{R_t^n} B_t \leq B_{t-1} + W_t N_t + \Pi_t^F$$

where β is the discount factor of households, θ is the disutility of working, P_t is the price index, R_t^n is the nominal interest rate, B_t is one period bonds, W_t is the nominal wage, and Π_t^F is the nominal profits from all firms.

Households choose over consumption, labor, and bonds, which supplies two Euler equations that determine both the real wage and the stochastic discount factor for the firms' problem in terms of aggregate consumption and aggregate labor supply:

$$w_t = \frac{W_t}{P_t} = - \frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = \theta C_t^\eta \quad (14)$$

$$\Lambda_{t,t+1} = \beta \frac{U_c(C_{t+1}, N_{t+1})}{U_c(C_t, N_t)} = \beta \left(\frac{C_t}{C_{t+1}} \right)^\eta \quad (15)$$

where the stochastic discount factor is linked through the Euler equation for bonds

$$\Lambda_{t,t+1} = \frac{1}{R_t^n} \frac{P_{t+1}}{P_t} = \frac{\pi_{t+1}}{R_t^n} \quad (16)$$

3.4 Equilibrium definition

I now characterize and define the equilibrium of the model. The aggregate state vector is $\Omega_t = (\sigma_t, \Theta_t)$, where σ_t is the volatility state today, which is determined in the prior period, and Θ_t is the collection of aggregate prices. I also define $\mu(k, z, \xi)$ as the distribution of firms over their state vector (k, z, ξ) .

*A **Recursive Competitive Equilibrium** for this economy is defined by a set of value functions and policy functions $\{V(k, z; \Omega), V^A(k, z; \Omega), V^{NA}(k, z; \Omega), \xi^*(k, z; \Omega), k^*(k, z; \Omega), k^C(k, z; \Omega)\}$, a set of quantity functions $\{C(\Omega), N^s(\Omega), N^d(\Omega), Y(\Omega), K(\Omega)\}$, a set of price functions $\{w(\Omega), \Lambda(\Omega), p^w(\Omega), R^n(\Omega), \pi(\Omega)\}$, and a distribution $\mu'(\Omega)$ that solves the firm's problem, retailer's problem, household's problem, and market clearing such that:*

- (i) [**Firm Optimization**] Taking the aggregate prices $\{w(\Omega), \Lambda(\Omega), p^w(\Omega)\}$ as given, $V(k, z; \Omega)$, $V^A(k, z; \Omega)$, $V^{NA}(k, z; \Omega)$, and $\xi^*(k, z; \Omega)$ solve the firms' Bellman Equations (7) – (10) with associated decision rules $k^*(k, z; \Omega)$ and $k^C(k, z; \Omega)$.
- (ii) [**Household Optimization**] Taking the aggregate prices $\{w(\Omega), R^n(\Omega), \pi(\Omega)\}$ as given, $C(\Omega)$, $N^s(\Omega)$, and $\Lambda(\Omega)$ solve the household's utility maximization (14) – (16).
- (iii) [**New Keynesian Block**] Retailers optimization leads to the NKPC (12) and monetary authority operation leads to the Taylor rule (13). For all Ω , both equations hold.
- (iv) [**Market Clearing**] For all Ω , labor supply $N^s(\Omega)$ equals labor demand $N^d(\Omega) = \int (n(k, z) + \xi(k, z)) d\mu(k, z; \Omega)$, and the final goods market clears $Y(\Omega) = C(\Omega) + I(\Omega) + \Theta_p(\Omega) + \Theta_k(\Omega)$, where $\Theta_p(\Omega)$ is the price adjustment cost and $\Theta_k(\Omega)$ is the aggregate capital adjustment cost.

3.5 Solution method

The critical challenge in solving the model is that the aggregate state vector Ω contains an infinite-dimensional object μ , which is the cross-sectional distribution of firms. I follow the MIT shock literature to overcome this challenge. As documented by Boppart, Krusell, and Mitman (2018), a reasonable MIT shock around the steady-state of the model provides a reasonably accurate approximation and preserves the non-linearity of the transition path very well¹³.

The solution method involves two parts. First, I solve the *Stationary Equilibrium* at the steady-state, which delivers the value functions, the policy functions, and the steady-state aggregate

¹³Compared to the a classical global method such as Krusell and Smith (1998) applied to RBC models. There are too many aggregate prices and quantities to predict over the transition path because of the New Keynesian block. And finally, to capture the full dynamics of volatility shocks, linearization techniques are usually ill-suited

variables. The *Stationary Equilibrium* also provides the cross-section moments for the calibration. Second, I solve the *Transitional Equilibrium* starting at the *Stationary Equilibrium* given a path of MIT shocks and a long enough period for the model to transit back to the same *Stationary Equilibrium*. The *Transitional Equilibrium* then provides the dynamic moments for the calibration and the impulse response functions. The advantage here is: i) non-linearity from non-convex adjustments and volatility shocks are fully captured; ii) adding multiple MIT shocks will not increase the solution time; iii) the interactions between different shocks are fully captured during the transition, which is essential for the quantitative results. The details of the solution methods are in the Appendix [A.2](#).

4 Illustrating the Mechanism

Before the quantitative analysis, this section introduces a minimalist framework to graphically illustrate aggregate investment responses to volatility shocks under various model specifications and shows how they affect the effectiveness of monetary policy. I give the basic intuition in the context of a simple two-period model.

4.1 A two-period model

I provide a graphical illustration of the centrality of the interaction between lumpy capital adjustment costs and heightened volatility through a simple two-period model. A unit continuum of firms populates the model. All firms $j \in [0, 1]$ have a common production function $y = z(k^\alpha n^{1-\alpha})^\gamma$ and begin period 1 with a common initial capital stock of k_0 and initial productivity z_1 . The single choice variable of firms is their investment i_j . Denoting the investment choice set of firm j as I_j , we have its problem as follows:

$$\max_{i_j \in I_j, n_j} p * (y_j - c(i_j)) - w * n_j + \frac{1}{1+r} E_1 \{ V_2(k_1, z_2) \}$$

subject to:

$$\begin{aligned} k_1 &= (1 - \delta)k_0 + i_j \\ y_j &= z_1(k_0^\alpha n_1^{1-\alpha})^\gamma \\ c(i_j) &= i_j + |i_j| \left(\mathbf{1}_{(i_j < 0)} \cdot S + \frac{\phi_k}{2} \left| \frac{i_j}{k_0} \right| \right) + \mathbf{1}_{(|i_j| > ak_j)} \cdot \xi_j \end{aligned}$$

The aggregate prices $\{p, w, r\}$ are taken as given by all firms, which will be further determined in general equilibrium, and labor is freely adjustable by all firms. I assume $\delta = 0$ for simplicity in this example. Under these conditions, I reformulate the question as follows:

$$z_1^\eta k_0^\mu + \max_{i_j \in I_j} \left\{ -c(i_j) + \frac{1}{R} E_1 \left\{ V_2(k_0 + i_j, z_2) \right\} \right\} \quad (17)$$

subject to:

$$R = h \left(\frac{p}{w^{(1-\alpha)\gamma}} \right)^{-\eta} \cdot (1 + r)$$

$$c(i_j) = i_j + |i_j| \left(\mathbf{1}_{(i_j < 0)} \cdot S + \frac{\phi_k}{2} \left| \frac{i_j}{k_0} \right| \right) + \xi_j$$

where $\{\eta, \mu, h\}$ are parameters¹⁴ and R is the intertemporal price function which is taken as given by all firms. Since k_0 and z_1 are predetermined, the investment decision at period 1 is reduced to a tradeoff between the cost function $c(i_j)$ and the expected future value $\frac{1}{R} E_1 \{ V_2(k_0 + i_j, z_2) \}$.

4.2 Value functions and investment decisions

For the firm's maximization problem to be well-defined in this model, the value function must be concave in capital; otherwise, the optimal investment will be infinite whenever the firm invests. Concavity is accomplished by assuming decreasing returns to scale in production ($\gamma < 1$) and the combination of convex and non-convex adjustment costs in this simple model and the full model. Therefore, the expected future value function is also concave in the capital. To demonstrate the shape of the value function, I solve the steady-state as in the full models where the idiosyncratic productivity shock follows an AR(1) process with a *three-state Tauchen* discretization. These steady states include various specifications of adjustment costs.

Adjustment costs and the concavity of the value function: I first show how adjustment costs affect the concavity of the value function in this three-state Tauchen discretization example. I solve for the steady-state of four models with the same calibration as the full model in Section 5. The *Baseline* model includes all three types of moderate adjustment costs, and the *type-x only* models retain only the type-x adjustment cost but set all other adjustment costs to zero.

Before the results, recall that it is the slope of value function with respect to capital, not the level of the value function, that matters for the investment choice. Therefore, to make the results more intuitive, I normalize the level of all value functions around the *optimal capital stock* (k^*) for

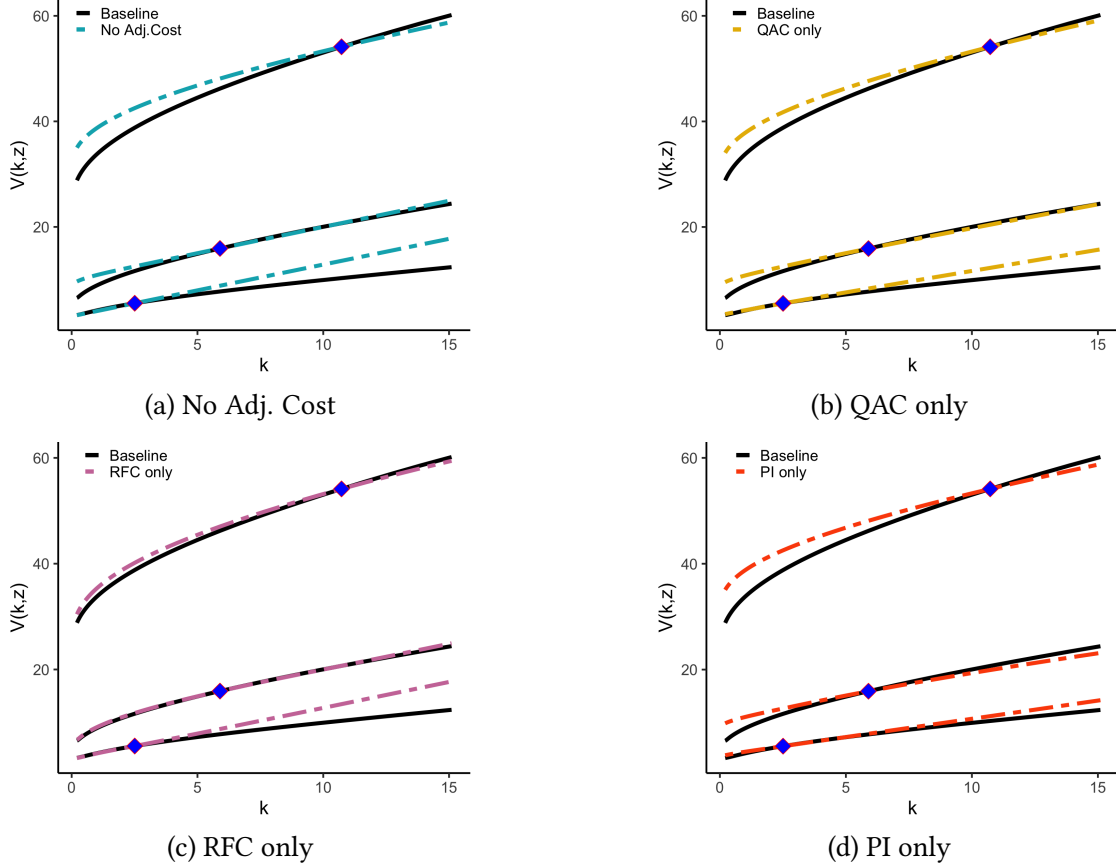
¹⁴Parameters are: $\eta = \frac{1}{1-(1-\alpha)\gamma} > 1$, $\mu = \frac{\alpha\gamma}{1-(1-\alpha)\gamma} < 1$, $h = [(1-\alpha)\gamma]^{\frac{(1-\alpha)\gamma}{1-(1-\alpha)\gamma}} [1 - (1-\alpha)\gamma]$

each productivity. At the *optimal capital stock* (k^*), firms have no incentive at all to adjust their capital.

$$\frac{\partial y}{\partial k} = \mu z^\eta k^{\mu-1} = 1$$

$$k_{2j}^* = \mu^{1-\mu} z_{2j}^{\frac{\eta}{1-\mu}}, \quad z_{2j} = z_l, z_m, z_h$$

Figure 3: Adjustment Costs and Value Function Concavity



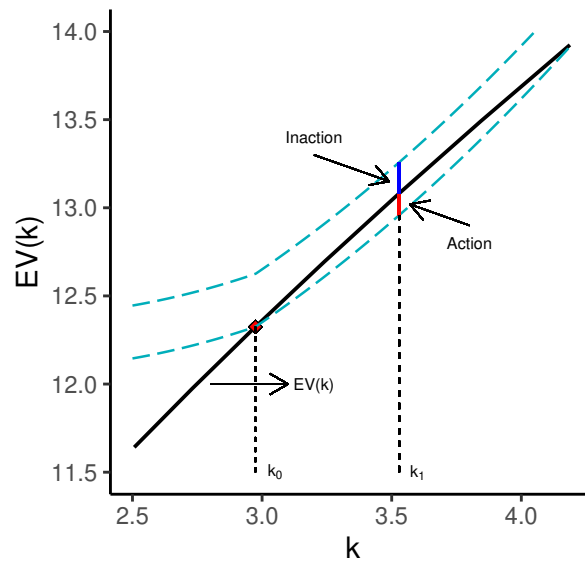
Notes: For each model, there are three value functions for three idiosyncratic productivity states. The highest corresponds to high z , the middle one corresponds to median z , and the lowest corresponds to low z . The calibrations of the models are in Section 5. To specify, the *Baseline* is $\{S = 0.30, \bar{\xi} = 0.70, \phi_k = 4.0\}$ in the cost of capital adjustment function $c(i_j) = i_j + |i_j| \left(\mathbf{1}_{(i_j < 0)} \cdot S + \frac{\phi_k}{2} \left| \frac{i_j}{k_0} \right| \right) + \mathbf{1}_{(|i_j| > ak_j)} \cdot \xi_j$. Additionally, *QAC only* means that other adjustment costs are set to zero but the quadratic adjustment costs $\phi_k = 4.0$ is retained, while *PI only* only preserves partial irreversibility at $S = 0.30$, and *RFC only* keeps only the random fixed cost at $\bar{\xi} = 0.70$.

The example results are in Figure 3.¹⁵ Starting with the sub-figure (a), I emphasize three

¹⁵The level of the value function across different models is not comparable: the intertemporal price function R is

observations. First, the blue diamonds on each value function are the optimal capital stocks for the baseline model. Second, the value function of a given productivity is concave in the capital stock. Third, capital adjustment costs lower the value for a given productivity — the further current capital is from the optimum, the more substantial the loss in value. These three observations give us a sense of the distribution of value function concavity over firms' investment decision space, shaped by the specification of adjustment costs. The key for the *Baseline* model is that it generates significantly more concavity within the productivity-capital mismatch regions: the bottom right of the value function with low productivity and the top left of the value function with low productivity. Then in sub-figures (b), (c), and (d), I show other specifications. These specifications cannot generate enough concavity compared to the baseline model.

Figure 4: Expected Value Function and Investment



Notes: The black line is the expected value function, a probability-weighted function of the three state value functions. The lower green down-sloped dashed convex line is the investment cost function with partial irreversibility and quadratic adjustment costs. The higher green down-sloped dashed convex line is the investment cost function with partial irreversibility, quadratic adjustment costs, and the maximum random fixed adjustment cost. Firms with $\{k_0\}$ who draw a random adjustment cost in the *Action* region would choose $k' = k_1$, otherwise $i = 0$.

Expected value function and investment: The curvature of the value function over the decision space affects investment through the expected value function which is a combination of the three state value functions with transition probability weights. Consider firms in the two-period example with median productivity $z_1 = z_m$, and a capital stock k_0 , and optimal next period capital choice k_1 . Examine the firm's decision space in Figure 4. First, the expected value function (the all different across models. I also show the level of non-adjusted value function plots in the Appendix A.4 Figure 13.

black solid line) is a probability weighted curve $EV(k) = \sum_{i \in \{l, m, h\}} \{\pi_{m,i} * V(k, z_i)\}$ where $\pi_{m,i}$ is the transition probability from z_m to z_i . Therefore, the *slope of $EV(k)$* depends on the slopes of $V(k, z_i)$ and $\pi_{m,i}$. Here, I demonstrate the case that the *slope of $EV(k)$* > 1 for (k_0, z_m) .

If there is no random fixed cost, the firm chooses k_1 , which delivers the max value between the expected value function (the solid black line) and the investment cost function (the lower green up-sloped dashed convex line). However, since firms have to pay a random fixed cost, the true investment cost function is a random draw between the bounds. Therefore, firms who draw a fixed cost within the *Action* region would choose $i = k_1 - k_0$, otherwise they choose $i = 0$. Given the initial capital stock fixed at k_0 and upper and lower bounds of the cost function, the *slope of $EV(k)$* determined both the intensive margin ($k_1 - k_0$) and the extensive margin (*Action/Inaction*) of investment.

4.3 Effect of volatility shocks and monetary shocks

Effect of volatility shocks: Suppose I fix the intertemporal price R . A volatility shock changes the transition probability $\pi_{m,i}$, but not the value function $V(k, z_i)$. Therefore, how a volatility shock affects investment depends on the curvature of $V(k, z_i)$ and how much the shock shapes the transition probability $\pi_{m,i}$.

The most common volatility shock yields transition probabilities $\{\pi'_{m,i}\} = \{\pi_{m,l} + \epsilon, \pi_{m,m} - 2\epsilon, \pi_{m,h} + \epsilon\}$. The volatility shock would change the slope of the expected value function by:

$$\Delta s(k, z_m) = \epsilon \left(\frac{\partial V(k, z_h)}{\partial k} + \frac{\partial V(k, z_l)}{\partial k} - 2 \frac{\partial V(k, z_m)}{\partial k} \right) \quad (18)$$

If $\Delta s(k_1, z_m) < 0$, the shock lowers the slope of the expected value function and the *Action* region shrinks or even vanishes as next period's optimal choice of capital k'_1 moves closer to k_0 and the expected value function moves towards and possibly falls below the cost function for any $k > k_0$. If $\Delta s(k_1, z_m) > 0$, we would observe the opposite. There is a huge literature discussing this investment-volatility relationship dating back to the 1990s. This literature was inconclusive because various assumptions that were made that led to different curvatures for value functions. Literature featuring constant returns to scale and/or low adjustment costs ends with a positive investment-volatility relationship. In contrast, literature featuring decreasing returns to scale and/or partial irreversibility ends with a negative investment-volatility relationship. The reason is that returns to scale shapes the concavity of the value function even without adjustment costs. Adjustment costs further increase concavity by worsening productivity-capital mismatch. With a reasonable calibration, $\Delta s(k_1, z_m) < 0$ is possible because $\frac{\partial V(k, z_l)}{\partial k}$ is sufficiently small for any

$k > k_0$, and therefore a volatility shock generates investment drops even in a partial equilibrium model.

Effectiveness of monetary policy: In such an environment, monetary policy works through the intertemporal price R . The intertemporal price R enters into a firm's investment decision as a multiplier to the expected value function. A monetary stimulus would lower R , increasing the slope of the price adjusted expected value function ($\frac{1}{R}E_1\{V_2(k_1, z_2)\}$). As a result, the *Inaction* region shrinks (extensive margin), the next period capital choice k'_1 increases (intensive margin), and therefore aggregate investment increases.

However, when a volatility shock hits, for a substantial mass of firms (especially the high productivity high capital stock firms), the value function slope in the low productivity state is very flat. As a result, their expected value functions are much flatter and may locate entirely below the investment cost function for any $k'_1 > k_0$. A conventional monetary stimulus increases that slope only slightly, which cannot enlarge the *Action* region much or move it back above the investment cost function. Therefore, for a substantial mass of firms, monetary stimulus is less effective or even completely ineffective. In this case, the effectiveness of monetary policy on aggregate investment is reduced by heightened volatility.

5 Quantitative Analysis

Having highlighted the primary mechanism of this paper, I now take the full model to the data and quantify the mechanism in the model. I first parameterize the model and show that in order to generate reasonable aggregate dynamics, it is essential to match the dynamic moments rather than just the cross-section moments. I then show the quantitative result of heightened volatility on the effectiveness of monetary policy to compare to the data in Section 2.

5.1 Parameterization

My parameterization proceeds in three steps. First, I fix a set of parameters to match standard macroeconomic targets in the steady-state. Second, given these fixed parameter values, I choose the remaining parameters to match moments in the data. Finally, I show the identification of my key capital adjustment cost parameters which is essential for the model mechanism.

Table 2: Fixed Parameters

Parameter	Description	Value
<i>Household Block</i>		
β	Discount factor	0.99
η	Elasticity of intertemporal substitution	1
θ	Leisure preference	2
<i>Production Block</i>		
α	Capital coefficient	0.25
ν	Labor coefficient	0.60
δ	Capital depreciation	0.026
a	Freely adjustment region (no fixed costs)	0.001
<i>New Keynesian Block</i>		
γ	Demand elasticity	10
ψ	Price adjustment cost	90
ϕ_π	Taylor rule coefficient	1.5

5.1.1 Calibration

Fixed Parameters Table 2 lists the parameters that I fix. The frequency of the model is a quarter, so I set the discount factor $\beta = 0.99$ to match an annual interest rate of 4%. I choose unit elasticity of intertemporal substitution $\eta = 1$ for log utility. Leisure preference $\theta = 2$ matches the fact that households spend a third of their time working. On the firm side, I choose the capital coefficient $\alpha = 0.25$ and the labor coefficient $\nu = 0.60$ to match a labor share of two-thirds and decreasing returns to scale imply of 85%. Capital depreciates at a rate of $\delta = 0.026$ quarterly, which generates the average aggregate nonresidential fixed investment rate in [Bachmann, Caballero, and Engel \(2013\)](#). Freely adjustment region parameter $a = 0.001$ so the model could generate empirically consistent tiny investment rates around zero as in [Khan and Thomas \(2008\)](#).

For the New Keynesian block, I choose the elasticity of substitution in final goods production $\gamma = 10$, matching a steady-state markup of 11% as in [Ottonello and Winberry \(2020\)](#). The coefficient on inflation in the Taylor rule $\phi_\pi = 1.5$ is chosen within the literature’s reasonable range. Finally, I set the price adjustment cost parameter $\psi = 90$, consistent with [Kaplan, Moll, and Violante \(2018\)](#)’s calibration, which implies a 0.1 slope of the Phillips curve in terms of marginal cost, broadly consistent with other literature.¹⁶

Fitted Parameters I then choose the remaining adjustment costs parameters and volatility parameters, listed in Table 3, in order to match the moments in Table 4. The volatility moments are from my own calculation in Section 2. The *Annualized Cross-section Moments* are taken from the calculations of [Zwick and Mahon \(2017\)](#) using annual IRS corporate income tax returns. The *An-*

¹⁶The slope of the NKPC is undetermined though there is a vast literature. [Schorfheide \(2008\)](#) provides a very comprehensive summary of various strands of literature. Different estimates range from almost zero (0.0004) to almost half (0.437). In this paper, I match to [Kaplan, Moll, and Violante \(2018\)](#).

nualized Dynamic Moments are taken from both Zwick and Mahon (2017) and Bailey and Blanco (2019a) to identify the key adjustment parameters $\bar{\xi}$ and S .

To generate these moments, I first use Monte Carlo stochastic simulation to simulate the steady-state with a large enough number N of firms for T_{ss} quarters, and then I choose a magnitude of the volatility shock σ_z^h at the quarter $T_{ss} + 1$ and keep simulating the economy until quarter T_{total} which is sufficient length to assure that the economy has converged back to the steady-state. For annual moments, I aggregate the quarterly results to an annual frequency. More details of the simulation process are in the theoretical appendix A.3.

Though the fitted parameters are jointly determined, they are closely tied to specific moments. I first choose the idiosyncratic productivity persistence and volatility following my own calculations in Section 2 of the Interquantile Range of sales growth of firms appearing in Compustat for at least 25 years. Among the N simulated firms, I take the largest firms that account for 45% of total output as representing Compustat firms. Among these firms, those who meet this Compustat threshold for at least 100 quarters – 25 years – represent the simulation analogue of my empirical sample. I then calculate the IQR of sales growth IQR_{sg} for both periods of normal and elevated volatility to match the data moments. This pins down the normal volatility level, $\sigma_z^l = 0.05$, and the elevated volatility level, $\sigma_z^h = 0.13$.

Table 3: **Fitted Parameters**

Parameter	Description	Value
<i>Productivity and Volatility</i>		
ρ_z	Persistence of TFP shock (fixed)	0.95
σ_z^l	Volatility of TFP shock (normal)	0.05
σ_z^h	Volatility of TFP shock (elevated)	0.13
<i>Adjustment Costs</i>		
ϕ_k	Quadratic adjustment cost	2.40
$\bar{\xi}$	Upper bound of fixed cost	0.70
S	Resale loss in capital	0.33

I then target four *Annualized Cross-section Moments* related to the distribution of investment rates. I choose the level of quadratic adjustment costs $\phi_k = 4.0$ to match these moments. These moments generate significant lumpiness and asymmetry in investment behaviors. The average investment rate is 10.4%, with a standard deviation of 0.16. About one-fourth of the observations feature an absolute value of the investment rate $< 1\%$ and 14.4% observations show investment rate spikes $> 20\%$. In most of the literature, these cross-section moments usually pin down all the capital adjustment cost parameters. However, matching these moments has often failed to replicate reasonable aggregate dynamics as in the data, primarily because the consideration of dynamic moments is essential.

Table 4: Target Moments

Moments	Data	Model
Volatility Moments		
IQR sales growth IQR_{sg} (normal volatility)	0.18	0.18
IQR sales growth IQR_{sg} (elevated volatility)	0.26	0.26
Annualized Cross-section Moments		
Average investment rate (%)	10.4%	10.1%
Standard deviation of investment rates	0.16	0.12
Spike rate (%)	14.4%	15.3%
Positive rate (%)	85.6%	84.7%
Annualized Dynamic Moments		
Autocorrelation of investment rates	0.40	0.40
Covariance of capital gap and age since last adj.	0.20	0.29

*capital gap: $x = \log(\frac{k_t}{z_t}) - E[\log(\frac{k_t}{z_t})]$, without frictions, capital gap= 0.

Notes: All the cross-section moments are from The [Zwick and Mahon \(2017\)](#) Appendix Table B.1. Statistics are drawn from the distribution of investment rates pooled over firms and time for U.S. firms from 1998 to 2010. *Annualized Cross-section Moments* are drawn directly from the appendix. The spike rate is the fraction of observations with an investment rate greater than 20%. The inaction rate is the fraction of observations with an absolute value of the investment rate smaller than 1%. *Annualized Dynamic Moments* are also from [Zwick and Mahon \(2017\)](#). While the auto-correlation is reported in their Appendix Table B.1, the covariance between the capital gap and age since last adjustment is inferred and recalculated by [Baley and Blanco \(2019a\)](#). A covariance of 0.29 means that a non-optimal k/z ratio lasts for considerably long periods until firms readjust the capital stock.

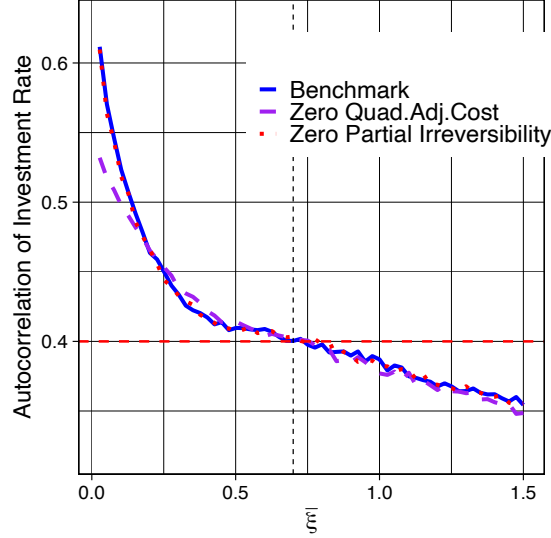
Finally, I target two *Annualized Dynamic Moments* related to the dynamics of investment rates. The autocorrelation of annualized investment rates depends largely on the random fixed cost $\bar{\xi} = 0.70$. This is essential for the persistence of firms' investment behaviors, which is the key for the aggregate responses of aggregate investment to the real interest rate and therefore also the response to a conventional monetary policy shock. The covariance between the capital gap and age since last adjustment measures how likely a firm would stay at a non-optimally high k/z ratio for a long time. This helps to pin down the level of irreversibility $S = 0.30$ which constrains firms' downward adjustment ability. Having sizable adjustment costs of both types fixes the puzzling dynamics addressed in the previous literature, for instance, [Bachmann and Bayer \(2013\)](#) for the non-significant investment responses to a volatility shock, and [Reiter, Sveen, and Weinke \(2013\)](#) for the excessive and non-persistent investment responses to a monetary policy shock.

5.1.2 Identification

Although the moments are jointly determined by all the adjustment cost parameters, identifying the lumpy adjustment cost parameters can be understood by checking the variations of these moments in two steps. To formally understand the identification of the key capital adjustment

cost parameters, I plot the variation of critical moments under various calibration combinations. I pin down both lumpy adjustment cost parameters in a sequential way.

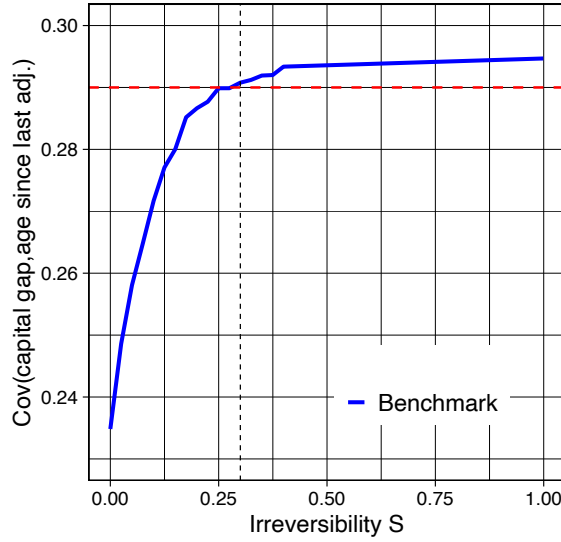
Figure 5: Autocorrelation of Investment Rates



Autocorrelation of investment rates: Firstly, I show the identification of the random adjustment cost $\bar{\xi}$ exploring the variation of the *autocorrelation of investment rates* moment in Figure 5. To show how other adjustment costs affect the moment, I show two alternative calibrations with either zero quadratic adjustment costs or zero partial irreversibility. The autocorrelation of investment rates monotonically decreases with the size of the random adjustment cost $\bar{\xi}$. Since the random adjustment cost is independent across time, a larger random adjustment cost would create more randomness in the cost of investment, which lowers firm incentives to engage in persistent investment across periods. This mechanism is in contrast to partial irreversibility and the quadratic adjustment cost, both of which determine adjustment costs based on firm investment decisions. Therefore, the other two simulated alternatives show no changes in this autocorrelation when either quadratic adjustment costs or partial irreversibility is present. Without sufficiently large random adjustment costs, it is impossible to match the autocorrelation of investment rates.

Covariance between capital gap and age since last adjustment: Secondly, conditional on the choices of $\bar{\xi}$ and ϕ_k , I show the identification of the partial irreversibility S exploring the variation of the *covariance between the capital gap and age since last adjustment* moment in Figure 6. Conditional on my choice of $\bar{\xi}$, this moment has a lower value of 0.235 when $S = 0$. It hits the target 0.29 when $S = 0.30$. This moment is from [Baley and Blanco \(2019a\)](#) which is essentially

Figure 6: Covariance between the capital gap and age since last adjustment



tied to the asymmetric adjustments of capital stocks¹⁷. A larger covariance means that a non-optimal k/z ratio lasts for considerably longer periods until firms readjust the capital stock, which essentially implies more costly disinvestment.

5.2 The role of lumpy adjustment costs

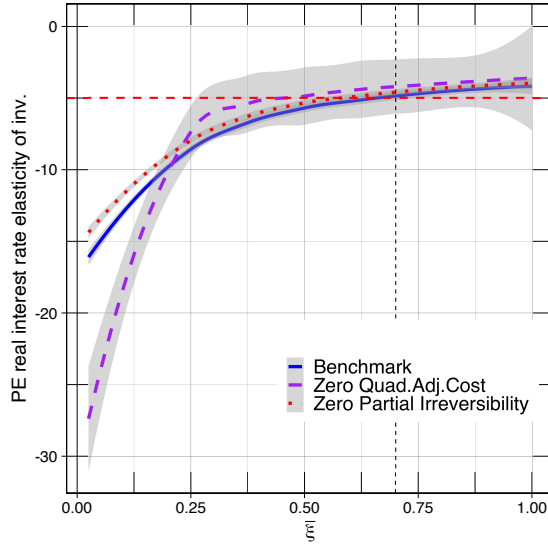
In this section, I discuss the key roles played by the random fixed costs and partial irreversibility with respect to the sensitivity of the aggregate investment in responses to interest rate and volatility shocks in a partial equilibrium fashion. The former governs the responses of investment with respect to monetary policy shocks and the latter governs the responses of investment to volatility shocks.

5.2.1 The role of random fixed costs

The size of the random fixed costs governs how much the aggregate investment responds to real interest rate changes. Since the investment channel of monetary policy works mainly through changing the real interest rate on the firm side, this real interest sensitivity actually determines the responses of investment with respect to monetary policy shocks. A reasonable sensitivity of aggregate investment with respect to the real interest rate helps to fix the puzzling excessive and non-persistent impulse responses of lumpy investment to monetary policy as in [Reiter, Sveen,](#)

¹⁷[Lanteri, Medina, and Tan \(2020\)](#) uses the asymmetric mobility of firms moving across MRPK groups which implies exactly a relatively high irreversibility of capital adjustment.

Figure 7: PE Real Interest Rate Elasticity of Agg. Inv.



Notes: The moment is the elasticity of partial equilibrium aggregate investment responding to a one-time real interest shock. In this specification, I use a -25bps real interest shock; therefore, an elasticity of one means aggregate investment increases by 0.25%. To show how other adjustment costs affect the moment, I show two alternative of steady states with either zero partial irreversibility or zero quadratic adjustment costs.

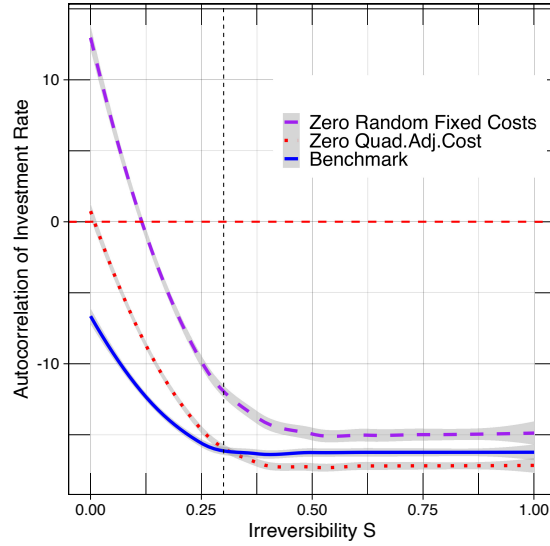
and Weinke (2013).

In Figure 7, I plot how sensitive aggregate investment is with respect to a one-time change in the real interest rate in a partial equilibrium over a set of the random fixed cost parameter choices. In a Khan and Thomas (2008)-type of model with only the random fixed costs, I truncated the lowest choice of $\bar{\xi} = 0.025$ which gives an elasticity of -28. This is already a tremendous response: a 1% real interest rate reduction will generate 28% more aggregate investment. However, according to the estimates of Koby and Wolf (2019)'s estimation, this partial equilibrium elasticity should be around 5. My parameter choice of $\bar{\xi} = 0.70$ yields a similar elasticity. This will create reasonable peaks in and persistence of the impulse responses of investment with respect to monetary policy shocks.

5.2.2 The role of partial irreversibility

The degree of partial irreversibility governs how much aggregate investment responds to changes in volatility. A reasonable investment-volatility sensitivity helps to generate large investment drops in response to a volatility shock, as empirical studies have suggested. Reduced ability to reverse investment (increment in S) creates higher real option values which increases the sensi-

Figure 8: PE Volatility Elasticity of Agg. Inv.



Notes: The moment is the elasticity of partial equilibrium aggregate investment responding to a one-time volatility shock. In this specification, I use a $\Delta\sigma = 3$ volatility shock as in [Bloom et al. \(2018\)](#); therefore, an elasticity of one means aggregate investment increases by 1%. To show how other adjustment costs affect the moment, I show two alternative of steady states with either zero random fixed cost or zero quadratic adjustment costs.

tivity of investment to volatility.

In Figure 8, I plot how aggregate investment responds to a one-time change in volatility in partial equilibrium as I vary the degree of irreversibility. Without irreversibility, aggregate investment is not very responsive to changes in volatility; more specifically, in a conventional investment model even without random fixed costs, the response is positive. My parameter choice of $S = 0.30$ yields a reasonably large and negative elasticity. This will create steep declines in investment following volatility shocks.

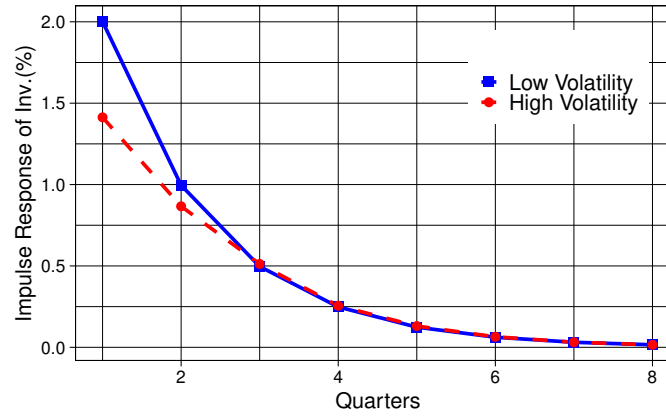
5.2.3 Remarks

Both lumpy capital adjustment cost parameters are essential for the quantitative analysis of the volatility-dependent effectiveness of monetary policy. They jointly govern the responses of firm-level investment at the extensive margin to both monetary shocks and volatility shocks. Without the lumpy adjustment costs generating sizeable responses in aggregate investment following changes in the interest rate or the volatility regime, the model is unable to replicate the volatility-dependent effectiveness of monetary policy as in the data.

5.3 Volatility-dependent effectiveness of monetary policy

I now quantitatively analyze if the model can explain the observed reduction in the effectiveness of monetary policy during periods of high volatility observed in Section 2. The economy is initially in the steady-state and unexpectedly receives shocks. There are two cases: for *Low Volatility* the economy always stays at the low volatility steady state and unexpectedly receives only monetary policy shocks;¹⁸ for *High Volatility* the economy unexpectedly receives the same monetary policy shock along with a one-time volatility shock.

Figure 9: Differential Responses to a Monetary Policy Shock



Notes: I solve the transition paths of each case separately, *Low Volatility* with a monetary policy shock only and *High Volatility* with both the same monetary policy shock and an elevated volatility shock. I then extract the paths of aggregate investment. For the *Low Volatility* path, I plot the percentage changes in aggregate investment with respect to the monetary policy shock. For the *High Volatility* path, I calculate the gap between the aggregate investment paths of having both the monetary policy shock and the elevated volatility shock and the elevated volatility shock only to isolate the changes in aggregate investment with respect to the monetary policy shock only.

More specifically, the monetary policy shock is an $\epsilon_1^m = 0.0025$ innovation to the Taylor rule residual which reverts to 0 according to $\epsilon_{t+1}^m = \rho_m \epsilon_t^m$ with $\rho_m = 0.5$. The volatility shock is a one-time change of the volatility level of the AR(1) productivity process from normal volatility $\sigma_z^l = 0.05$ to an elevated volatility $\sigma_z^h = 0.13$. I compute the perfect foresight transition paths of the economy as it converges back to steady state. To compare the effectiveness of monetary policy, I calculate two differential paths of aggregate investment with respect to the monetary policy shock only. For the *Low Volatility* case, I calculate the percentage changes of aggregate investment relative to its steady-state level. For the *High Volatility* case, I first net out the effects of the volatility shock by taking the difference between this case and a transition path with only

¹⁸Too see how other aggregate variables respond to monetary policy shocks, please refer to Appendix A.5. The impulse responses are generally consistent with the literature.

the volatility shock, and then calculate the percentage changes of aggregate investment relative to the same steady-state level.

Table 5: Volatility-dependent Effectiveness of Monetary Policy

	Low Volatility		High Volatility		Δ Effectiveness	
Source	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%
Model	2.0%	0.18	1.4%	0.26	30%	44%

Notes: The *Low Volatility* case is defined as IQR sales growth at its Bottom 20% while the *High Volatility* case is defined as IQR sales growth at its Top 20%, both in the data. $\frac{dI}{d\epsilon^m}$ denotes the peak impulse responses of investment with respect to an expansionary 25bps monetary shock.

The impulse responses of investment are plotted in Figure 9. Compared to the impulse responses of aggregate investment under normal volatility, when the monetary stimulus is conducted simultaneously with heightened volatility shock, the initial responses is much weaker: 1.40% versus 2.00%. This is a reduction of 30% in the effectiveness of monetary policy at the peak. In Table 5, I provide a direct comparison between this quantitative result and the empirical result in Section 2. The reduction of effectiveness in the model is $\frac{1.4\%}{2.0\%} = 30\%$. Therefore, my quantitative result could explain $\frac{30\%}{62\%} = 48\%$ of the reduction of monetary policy effectiveness as I find in the data.

This volatility-dependent effectiveness of monetary policy illustrate a new understanding of the investment channel of monetary policy: when investment is lumpy, the ability of monetary policy to stimulate investment is state-dependent. This effect is not only state-dependent on the first order moment as documented in the literature on productivity differences but also on the second order moment in terms of volatility differences. In times of elevated volatility, policy makers should be more aware of this reduction in effectiveness and potentially implement more aggressive monetary stimulus to achieve their policy goals.

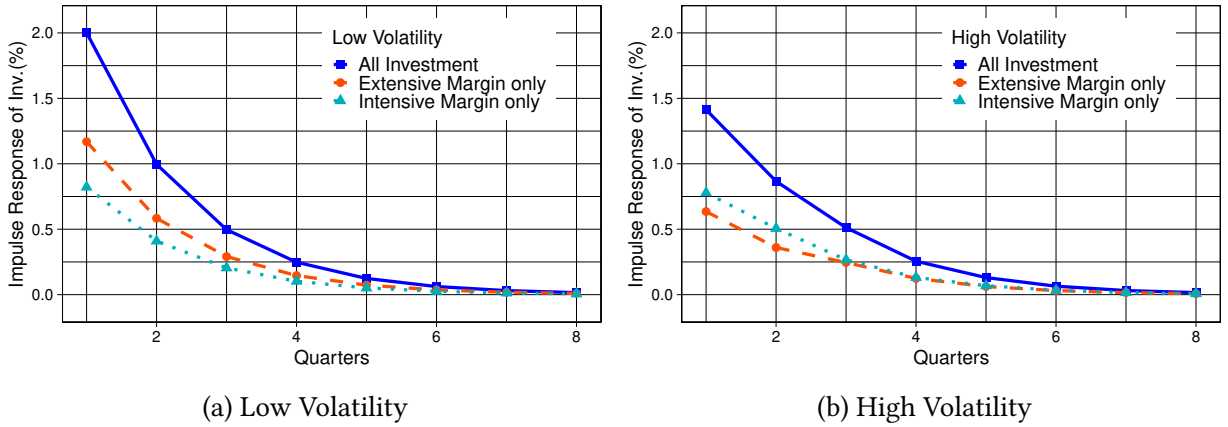
6 Inspecting the mechanism in the model

In this section, I explore the mechanism by which the model generates volatility-dependent monetary policy effectiveness. I first show the essential role of the extensive margin in the investment channel of monetary policy in Section 6.1. I then show how the heightened volatility affects the extensive margin of investment in Section 6.2. Finally, I show that alternative parameterizations of the baseline model without the extensive margin cannot generate the volatility-dependent monetary policy effectiveness.

6.1 Investment channel of monetary policy and Decomposition

I now decompose the investment channel of monetary policy into the extensive margin and the intensive margin. Since the extensive margin (whether to adjust the capital stock) and the intensive margin (how much to adjust conditional on adjusting) are simultaneous decisions, I cannot precisely separate the total investment response into two parts. Instead, I show two counterfactuals with two *x-margin only* scenarios. The *x-margin only* response is constructed by forcing the other margin's investment policy to act as if there is no monetary policy shock, while allowing the *x-margin* investment policy to be conducted optimally following the monetary policy shock.

Figure 10: Volatility and the Investment Channel of Monetary Policy



Notes: I solve the steady-state of the model and extract the steady-state active investment policy as a function of capital stock and productivity $i(k, z)[ss]$ and the steady-state adjustment probability rate $Prob(k, z) = \xi^*(k, z)/\bar{\xi}$. Then I solve the transition paths for the distribution of investment, active investment policy, and adjustment probability. For the *All Investment*, I just plot the percentage changes in aggregate investment. For the *Extensive Margin only*, I hold the active investment policy $i(k, z)[ss]$ at steady state, but allow the adjustment probability $\{Prob(k, z)\}_{t=1}^{t=T}$ to change overtime. For the *Intensive Margin only*, I hold the adjustment probability $Prob(k, z)[ss]$ at steady state, but allow the active investment policy $\{i(k, z)\}_{t=1}^{t=T}$ to change overtime.

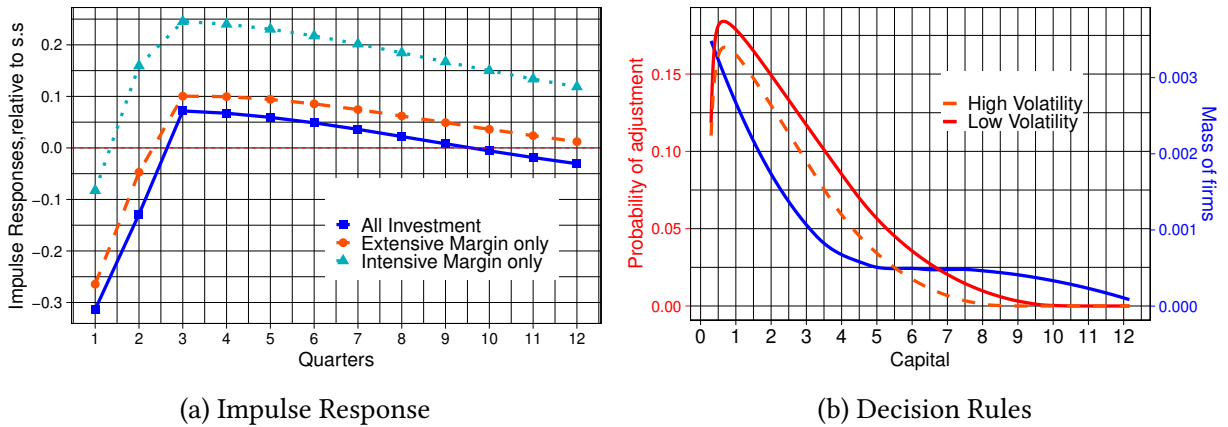
Figure 10 panel (a) the results of these two extensive margin only and intensive margin only counterfactual cases. First, the peak response of aggregate investment with respect to a conventional monetary stimulus is 2.0%, which narrows the puzzling gap between [Reiter, Sveen, and Weinke \(2013\)](#) and the aggregate investment responses in the RANK literature. The same monetary policy shock in [Reiter, Sveen, and Weinke \(2013\)](#) generates more than a 5% peak response which only lasts for 1 quarter. However, this model can generate a much more realistic peak response and persistence. This helps to resolve the overreaction and non-persistence puzzle of lumpy investment in New Keynesian models and show that lumpy investment can co-exist with reasonable impulse responses of investment to monetary policy.

Second, I show that the extensive margin accounts for most of the investment responses to monetary policy, and it accounts for most of the weakening of the investment channel of monetary policy when volatility is high. In the low volatility state in panel (a), both margins of investment are quantitatively relevant, but the extensive margin matters more by a factor of 40% (1.16%/0.82%). These patterns indicate that with a reasonable degree of lumpiness, monetary policy mainly works through the extensive margin of aggregate investment. Following an elevated volatility shock in panel (b), the aggregate investment is 30% (1.4%/2.0%) less responsive, as showed in Section 5.3. With a further decomposition, the extensive margin accounts for most of the decline (1.16% \rightarrow 0.65%) while the intensive margin accounts for only a little of the decline (0.82% \rightarrow 0.78%). This dramatic decrease in the responsiveness of investment at the extensive margin is the key to why aggregate investment is less responsive to monetary stimulus.

6.2 Volatility and investment policy at the extensive margin

I now further inspect why there is a dramatic decrease in the responsiveness of investment at the extensive margin to monetary policy. In particular, I explore how the volatility shock changes investment and investment policy at the extensive margin.

Figure 11: Volatility and Investment



Notes: The left panel (a) shows how aggregate investment responds to the high volatility shock. The decomposition counterfactuals in panel (a) are constructed in exactly the same way as in the decomposition of the investment channel of monetary policy. The right panel (b) shows the probabilities of adjusting the capital stock (the extensive margin) in the low volatility steady state and how this distribution changes when the high volatility shock hits. I show more comprehensive 2D heat maps of both extensive margin and intensive margin investment policies in the Appendix A.6

The results are in Figure 11. In panel (a), I show how aggregate investment responds to the

volatility shock. The volatility shock significantly decreases aggregate investment by almost 30%. The extensive margin accounts for almost all the decline in aggregate investment. Conversely, considering only the intensive margin, the initial decrease is much smaller and the overshoot is very substantial and persistent.

In panel (b), I show the steady state distribution of firms over capital stock and the probability of capital stock adjustment for firms of average productivity. These firms are mostly small and are quite likely to adjust capital stocks. However, when volatility is high, the probability of adjustment falls across the state space. In other words, the marginal propensity of conducting investment at the extensive margin is much smaller when volatility is high. This is why there is a dramatic decrease in the responsiveness of investment to monetary policy.

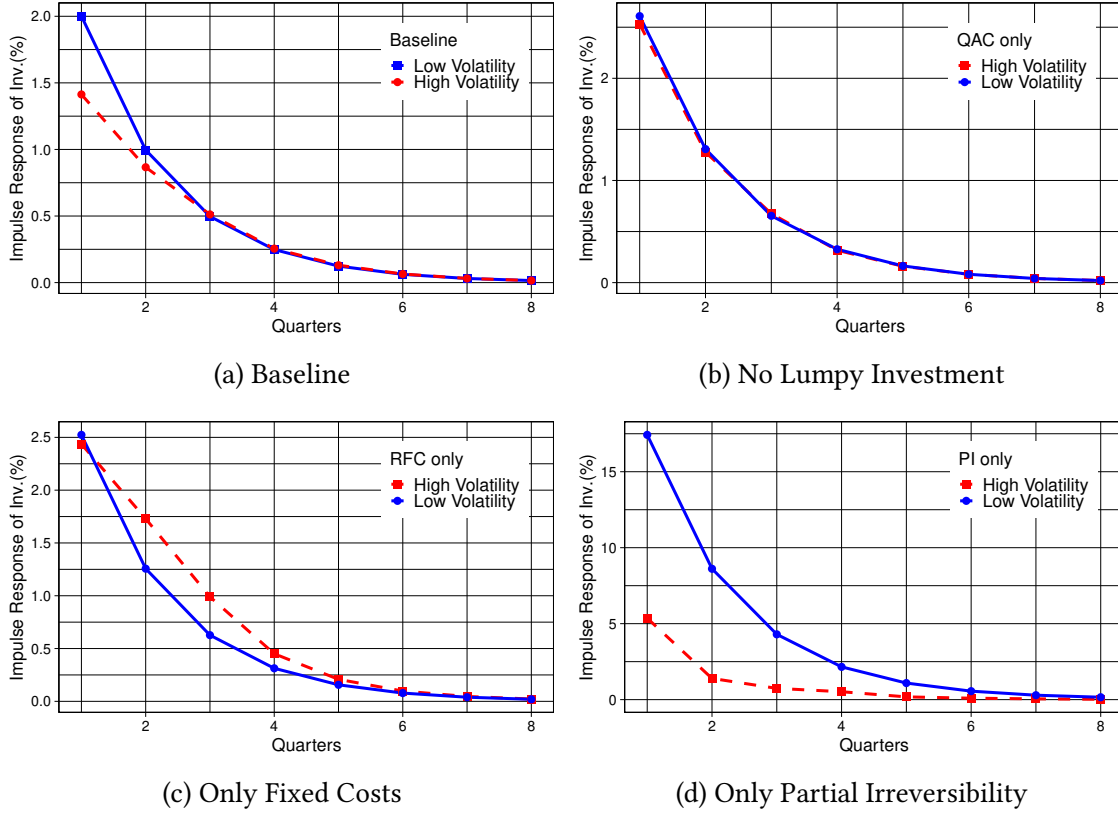
6.3 Insufficiency of alternative parameterizations

I further inspect the model mechanism by solving the same experiments under alternative model specifications with/without lumpy investment and with alternative causes of lumpy investment. I solve three alternative models: *Quadratic Adjustment Costs (QAC) only* in which there is no lumpiness to investment, *Random Fixed Costs (RFC) only* in which lumpy investment is only caused by the random fixed costs, and *Partial Irreversibility (PI) only* in which lumpy investment is only caused by the asymmetrically downward adjustment costs. I then conduct the same four experiments as in Section 5.3 with the same magnitudes of shocks, and plot the impulse responses to monetary policy with and without the volatility shock. The alternative models are not recalibrated because none of them could match all the target moments.

The results are in Figure 12. First, I show that lumpy investment is a necessary component of the model. In a model without lumpy investment like the *QAC only* model in panel (b), the response of aggregate investment to monetary policy is not volatility-dependent at all. This parameterization is the closest to a representative firm New Keynesian model. Even though firms that make dramatic investment adjustments still have to pay a sizable cost, quadratic adjustment costs do not by themselves generate inaction when volatility is high. Here, the investment channel of monetary policy operates exclusively through the volatility-irrelevant intensive margin. Therefore, lumpy investment, or in other words, extensive-margin investment, is necessary to generate volatility-dependent effectiveness of monetary policy.

Second, I show that both causes of lumpy investment: fixed costs and irreversibility, are important to generate reasonable differential impulse responses. In panel (c), the only fixed costs model *RFC only* is the most popular model specification in the lumpy investment literature. However, it still cannot generate the volatility-dependent monetary policy effectiveness. As I discussed

Figure 12: Differential Investment IRFs in Alternative Parameterizations



Notes: The alternative models are chosen by fixing each specific adjustment cost at the calibration level but setting other adjustment costs to almost zero. Therefore, some of them fail to match both the cross-section investment moments and the dynamic investment moments. I choose not to re-calibrate because they will fail to match key moments regardless.

in Section 5.2, random fixed costs control the sensitivity of investment to interest rate changes, so a large enough fixed cost helps to roughly match the peak responses but not the differential responses. In panel (d), the only partial irreversibility model *PI only* manages to generate differential effectiveness of monetary policy following the volatility shock, but it totally fails to generate a reasonable response in levels following the monetary policy shock.

7 Conclusion

In this paper, I have argued that monetary policy is less effective at stimulating investment during periods of high volatility than during normal times. My argument had two main components. First, I showed empirically that aggregate investment is less responsive to identified monetary policy shocks during high volatility periods. Second, I built a heterogeneous firm New Keynes-

sian model with lumpy investment and volatility shocks that is quantitatively consistent with my empirical results. In the model, lumpy capital adjustment costs create a sizable extensive margin of investment which is more sensitive to both changes in interest rates and volatility than the intensive margin. Monetary policy stimulates investment through a combination of firm-level investment at both the extensive margin and the intensive margin. High volatility significantly weakens firms' investment incentives at the extensive margin. As a result, a conventional monetary stimulus is not large enough to motivate firms on the extensive margin to pay the fixed costs and bear the risk of the potential disinvestment loss. Therefore, monetary policy is less effective at stimulating aggregate investment when volatility is high.

My results in this paper may be of independent interest to policymakers who are concerned about volatility during economic recessions. Since monetary policy is used as one of the primary stimulus mechanisms during economic downturns, it is crucial for policymakers to understand that conventional monetary stimulus may not be as powerful as they expected in high volatility recessions such as the Great Recession and the current COVID-19 recession.

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Appendices

A Theoretical Appendix

A.1 Standard **Rotemberg (1982)** Price Setting of Retailers

We can rewrite the profit in real dollars as follows.

$$\Pi_t(j) = (P_t(j) - P_t^w) \left(\frac{P_t(j)}{P_t} \right)^{-\gamma} \frac{Y_t}{P_t} - \frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t$$

Each period, retailers choose a price to maximize the expected present discounted value of flow profit, which is discounted by the household's stochastic discount factor $\Lambda_{t,t+1}$, since households also own the retailers. The optimization is:

$$\max_{P_t(j)} \left\{ \sum_{t=0}^{\infty} \Lambda_{t-1,t} \Pi_t(j) \right\}$$

Through the first order condition, the optimal price-setting rule can be written as follows:

$$\begin{aligned} (\gamma - 1) \left(\frac{P_t(j)}{P_t} \right)^{-\gamma} \frac{Y_t}{P_t} &= \gamma p_t^w \left(\frac{P_t(j)}{P_t} \right)^{-\gamma-1} \frac{Y_t}{P_t} - \psi \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right) \frac{Y_t}{P_{t-1}(j)} \\ &\quad + E_t \psi \Lambda_{t,t+1} \left[\left(\frac{P_{t+1}(j)}{P_t(j)} - 1 \right) \left(\frac{P_{t+1}(j)}{P_t(j)} \right) \left(\frac{Y_{t+1}}{P_t(j)} \right) \right] \end{aligned}$$

where $p_t^w = P_t^w/P_t$ is the real wholesale price. In equilibrium all retailers behave identically. This means they all charge the same price and produce the same output in each period. The optimal condition for price can be written in terms of the inflation rate as:

$$(\gamma - 1) = \gamma p_t^w - \psi(\pi_t - 1)\pi_t + E_t \Lambda_{t,t+1} \psi(\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t}$$

Reorganizing terms, I obtain the **New Keynesian Phillips curve**:

$$(\pi_t - \bar{\pi})\pi_t = \frac{\gamma}{\psi} (p_t^w - p^{w*}) + E_t \Lambda_{t,t+1} (\pi_{t+1} - \bar{\pi})\pi_{t+1} \frac{Y_{t+1}}{Y_t}$$

where $p_t^{w*} = \frac{\gamma-1}{\gamma}$ is the steady state whole sale price, or in other words, the marginal cost of retailer firms, and $\bar{\pi} = 1$ is the steady state inflation rate. In the paper, I directly focus on the linearized

version for computational simplicity:

$$\log \pi_t = \frac{\gamma - 1}{\psi} \log \frac{p_t^w}{p^{w*}} + \beta E_t \log \pi_{t+1}$$

For robustness, I also solve the quantitative model using the non-linearized version; the results are almost identical. Therefore, in order to save computational time, I use the linearized version through out the whole paper.

A.2 Details of the Computation Methods

Part I: Solving the Stationary Equilibrium

I first assume the economy is at steady-state with normal volatility. This part is very similar as solving an Aiyagari model. The only two differences are: (1).I have a New Keynesian block incorporates nominal rigidity; and (2).Firms own capital which is subject to adjustment costs. At the stationary equilibrium, there are no monetary policy shocks, so I solve $\{\pi^* = 1, \Lambda^* = \beta, p^{w*} = \frac{\gamma-1}{\gamma}, R^{n*} = 1/\beta\}$. I now search for equilibrium wage to clear the labor market. The algorithm is as following:

- Step.1. Guess an equilibrium wage;
- Step.2. Solve the firm's problem using Value Function Iteration;
- Step.3. Calculate aggregate variables from the firm distribution using Young (2010);
- Step.4. Update the wage with a given weight and return to Step 2 until convergence.

After the convergence, I have the stationary equilibrium aggregate prices $\Theta_t^* = \{\pi^* = 1, \Lambda^* = \beta, p^{w*} = \frac{\gamma-1}{\gamma}, R^{n*} = 1/\beta, w^* = w^*\}$, volatility state $U_{-1}^* = \text{normal}$, aggregate state $\Omega_t^* = (U_{-1}^*, \Theta_t^*)$, aggregate quantities $\{C^*(\Omega^*), N^*(\Omega^*), Y^*(\Omega^*), K^*(\Omega^*)\}$, firm value functions $\{V^*(k, z; \Omega^*), V^{A^*}(k, z; \Omega^*), V^{NA^*}(k, z; \Omega^*)\}$, policy functions $\{\xi^{**}(k, z; \Omega^*), k'^*(k, z; \Omega^*), l'^*(k, z; \Omega^*)\}$, and distribution $\mu(k, z; \Omega^*)$ at the stationary equilibrium state.

Part II: Solving the Transitional Equilibrium

With the stationary equilibrium solutions in hand, I now move to the solution of the transitional equilibrium using a shooting algorithm. The key assumption here is that after a sufficiently long enough time, the economy will always converge back to its initial stationary equilibrium after any temporary and unexpected (MIT) shocks. The following steps outline the shooting algorithm:

- Step.1. Fix a sufficient long transition period $t = 1$ to $t = T$ (say 200);
- Step.2. Guess a sequence of aggregate price $\{p_t^w, w_t, \Lambda_t, \pi_t\}$ of length T such that the initial

prices $\{p_1^w = p^{w*}, w_1 = w^*, \Lambda_1 = \Lambda^*, \pi_1 = \bar{\pi}\}$ (just simply assuming all the prices stay at steady state works well) and terminal prices $\{p_T^w = p^{w*}, w_T = w^*, \Lambda_T = \Lambda^*, \pi_T = \bar{\pi}\}$. Provide a predetermined shock process of interest, i.e., $\{\epsilon_t^m\}$ and $\{U_{t-1}\}$. This implies a time series for the aggregate state $\{\Omega_t\}_{t=1}^T$. The aggregate state is just time t .

Step.3. I know that at time T , the economy is back to its steady state. I have the steady state value function $V(k, z; \Omega_T) = V^*(k, z; \Omega^*)$ in hand for time T . I solve for the firms' problem by **backward induction** given $V(k, z; \Omega_T)$ and $\{p_{T-1}^w, w_{T-1}, \Lambda_{T-1}\}$. This yields the firm value function $V(k, z; \Omega_{T-1})$ and associated policy functions for capital $k'(k, z; \Omega_{T-1})$ and labor $l(k, z; \Omega_{T-1})$. By iterating backward, I solve the whole series of both policy functions $\{k'(k, z; \Omega_t)\}_{t=1}^T$ and $\{l'(k, z; \Omega_t)\}_{t=1}^T$.

Step.4. Given the policy functions and the steady state distribution as the initial distribution $\mu(k, z; \Omega_1) = \mu(k, z; \Omega^*)$, I use **forward simulation** with the non-stochastic simulation in Young (2010) to recover the whole path $\{\mu(k, z; \Omega_t)\}_{t=1}^T$.

Step.5. Using the distribution $\{\mu(k, z)\}_1^T$, I obtain all the **aggregate quantities**: aggregate output $\{Y\}_{t=1}^T$, aggregate investment $\{I\}_{t=1}^T$, aggregate labor demand $\{N\}_{t=1}^T$, and aggregate capital adjustment costs $\{\Theta_k\}_{t=1}^T$, the latter of which follows from the guessed inflation $\{\pi\}_{t=1}^T$, we could calculate aggregate adjustment costs $\{\Theta_p\}_{t=1}^T$. I then use the goods market clearing condition to calculate aggregate consumption $\{C\}_{t=1}^T$. I then calculate the *Excessive Demand* $\{\Delta C\}_{t=1}^T$ by taking the differences between currently iterated $\{C\}_{t=1}^T$ and the previous iteration $\{C_{old}\}_{t=1}^T$.

Step.6. Given all the aggregate quantities in the previous step and the *Excessive Demand* $\{\Delta C\}_{t=1}^T$, I update all the **aggregate prices**. I update all equilibrium prices with a line search: $X_t^{new} = speed \cdot f_X(\{\Delta C\}_{t=1}^T) + (1 - speed) \cdot X_t^{old}$. Repeat Steps 2-7 until X_t^{new} and X_t^{old} are close enough. I only update $\{p_t^w, w_t, \Lambda_t, \pi_t\}$ because $\{R_t^n\}$ can be calculated accurately from Taylor rule. The $f_X(\{\Delta C\}_{t=1}^T)$ is chosen by the connections of the New Keynesian prices with the *Excessive Demand* $\{\Delta C\}_{t=1}^T$ through the equations of the New Keynesian prices. Updating all prices in all periods simultaneously reduces the computation burden dramatically.¹⁹ This updating rule allows me to solve the transitional equilibrium in seconds on a dual-core macbook without any parallel computation.

In all the experiments with both the Taylor rule shock and a volatility shock, I set $T = 200$, and a step size of 0.01 to ensure convergence, with the necessary distance between X_t^{new} and X_t^{old} smaller than $1e-7$. I also tested with various T from 50 to 400 to ensure that the choice of $T = 200$

¹⁹There is an alternative updating rule which is more stable but much more time consuming. In put it here: Step.6'. Using the household first order condition for consumption $\{C\}_{t=1}^T$, I obtain a new $\{\Lambda\}_{t=1}^T$; using the household first order condition for labor, $\{C\}_{t=1}^T$, and $\{N\}_{t=1}^T$, I obtain a new $\{w\}_{t=1}^T$; using the definitions of the stochastic discount factor and Taylor rule simultaneously, I update π_{t+1} with Λ_t, R_t^n , then I update R_{t+1}^n with the updated π_{t+1} , and repeat until I have a new $\{R^n\}_{t=0}^T$ and $\{\pi\}_{t=1}^T$. Finally, I get a new $\{p^w\}_{t=1}^T$ through the New Keynesian Phillips curve.

does not affect the accuracy of the solution.

A.3 Details of the Simulation Methods

Part I: Simulating the sample

I first assume the economy is at steady-state with normal volatility. There are $N = 100,000$ firms starting with an average capital stock $k_{j0} = E[k_{jt}]$ and random draws of productivity z_{j0} . These firms then draw next period productivity z_{jt} from the AR(1) process of idiosyncratic productivity process as well as the idiosyncratic random fixed costs ξ_{jt} . Since the economy is at the steady state, they take all the aggregate prices as given for all the periods. I simulate this economy for 200 quarters until they converges to the steady-state distribution. Then I keep simulating this economy for an additional 300 quarters which is used for calculation of moments. Finally, I keep simulating the economy starting from the quarter 500 forwards with the transitional investment policy functions and aggregate prices until the economy converges back to the steady state at the quarter 700.

Part II: Calculation in the sample

I calculate the moments to match in this sample. For all moments which could be calculated at steady state, I use the stationary equilibrium simulated sample to calculate. To ensure the moments are not affected by the number of firms and the sample period choices, I also simulate the stationary equilibrium for more than 4000 quarters to ensure the moments are stable. I then aggregate the 4000 quarters into 1000 years to calculate the annualized moments.

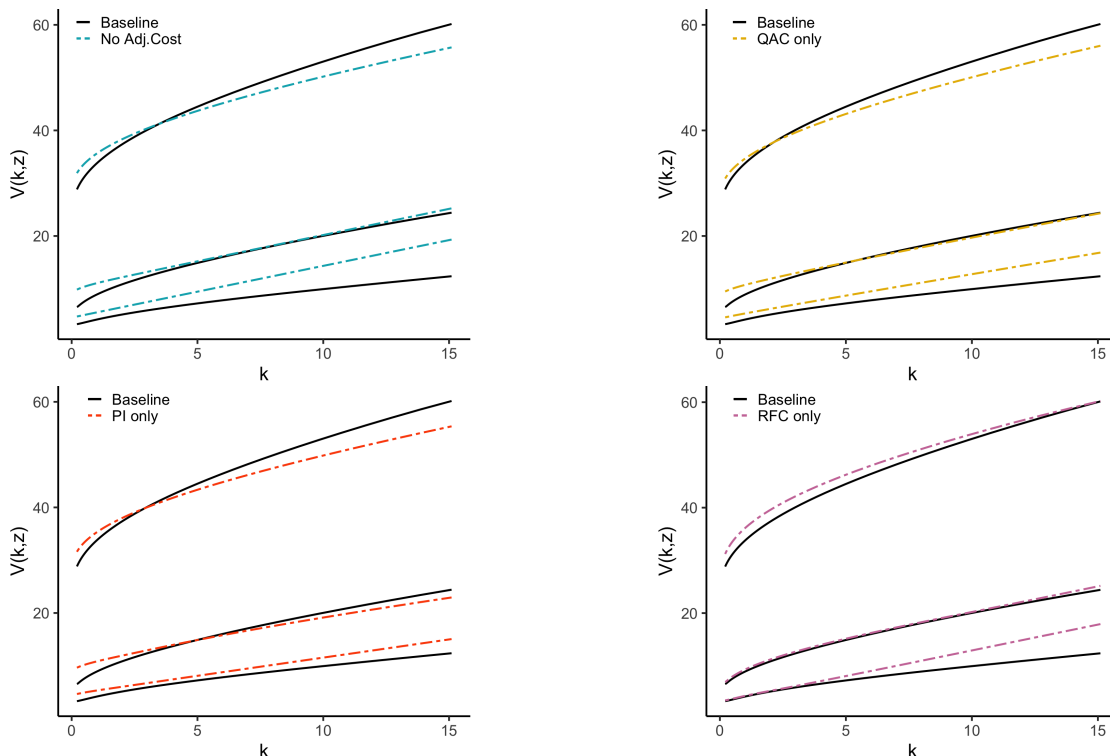
For the volatility levels, I first select "Compustat firms" from all the firms. "Compustat firms" are the largest firms in the whole economy which accounts for 45% of total output (data in 2019). Roughly 10% of all the firms are "Compustat firms" in the whole economy. I then keep counting the quarters since the first quarter a firm became a "Compustat firm" and further select "25 year+ Compustat firms" from the "Compustat firms". Roughly 1% of all the firms are "25 year+ Compustat firms". I calculate the IQR_{sg} for these firms at the quarter 500 and the quarter 501, respectively, as the measured volatility to parameterize σ_z^l and σ_z^h . I also report the volatility of other samples in the model which are larger than the "25 year+ Compustat firms". The calculation of sales growth is exactly the same in the data: $sales\ growth_{jt} = 2 * (sales_{jt} - sales_{jt-4}) / (sales_{jt} + sales_{jt-4})$.

Sources	Low Volatility		High Volatility	
	σ_z^l	IQR_{sg}	σ_z^h	IQR_{sg}
All firms	0.05	0.24	0.13	0.48
Compustat	0.05	0.21	0.13	0.38

A.4 Original value functions in the two-period example

I present the original value functions as in Section 4.2 in Figure 13. The only difference is that I move the value functions vertically in the paper to make it easier to compare the concavity.

Figure 13: Adjustment Costs and Value Function Concavity

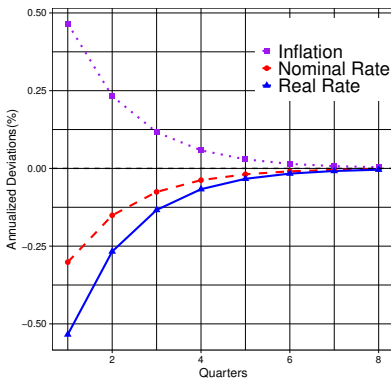


Notes: the details are same as in Figure 3.

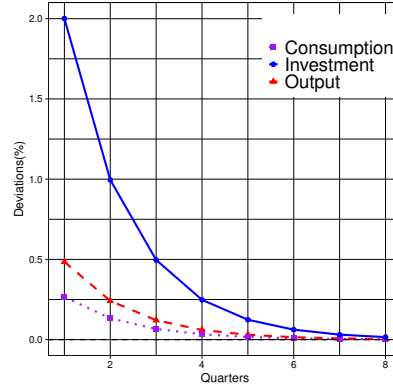
A.5 Impulse responses to monetary policy shocks

Figure 14 plots the responses of the key aggregate variables to this expansionary monetary policy shock. The shock cuts down the nominal interest rate and lowers the real interest rate due to sticky prices. The lowered real interest rate stimulates investment demand by increasing the stochastic discount factor, so firms put more weight on future values. It also increases household consumption demand due to standard intertemporal substitution reasons. The wholesale price increases more than the real wage, which incentivizes firms to produce more output. Overall, investment increases by approximately 1.8%, consumption increases by 0.35%, and output increases by 0.5%. These magnitudes are broadly in line with the peak effects of monetary policy shocks estimated in [Christiano, Eichenbaum, and Evans \(2005\)](#) and the quantitative results from the most recent heterogeneous firm New Keynesian model in [Ottonello and Winberry \(2020\)](#).

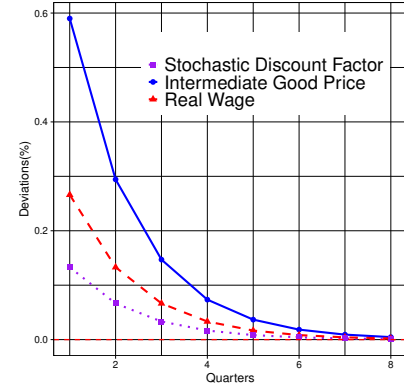
Figure 14: Aggregate Responses to a Monetary Policy Shock



(a) Monetary Rates



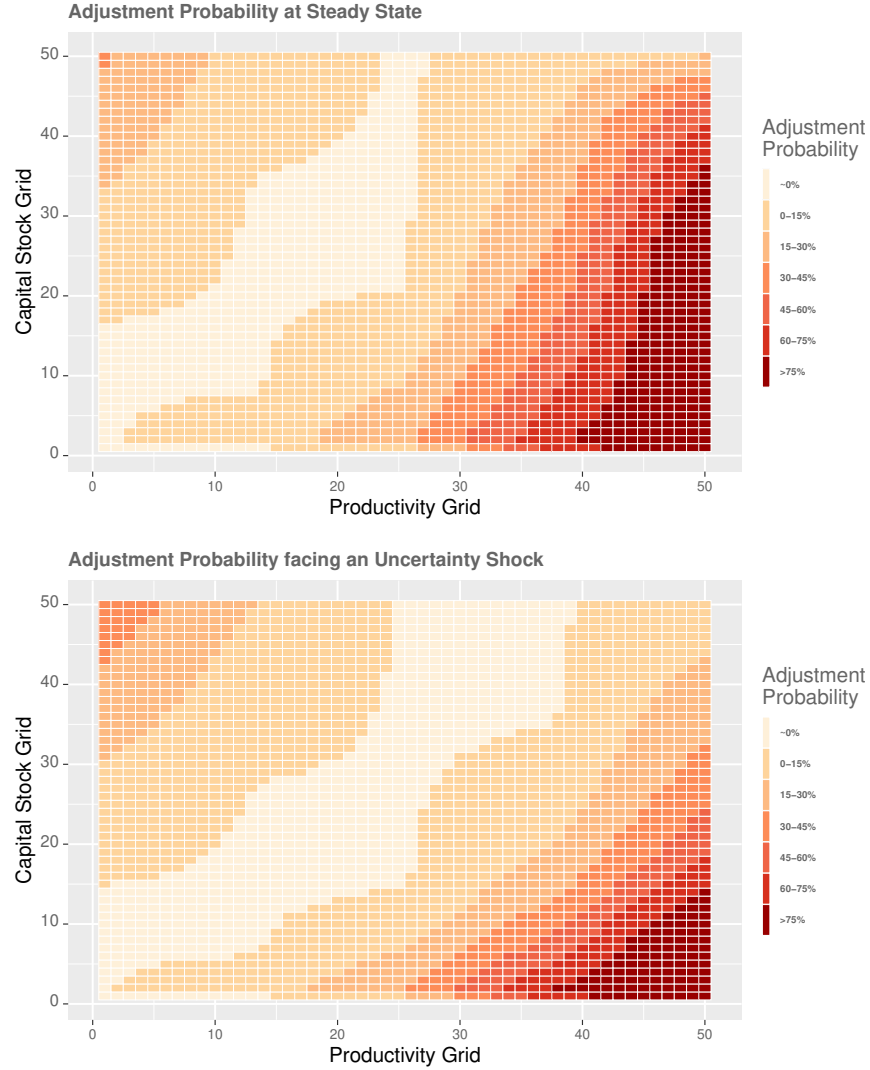
(b) Aggregate Quantities



(c) Aggregate Prices

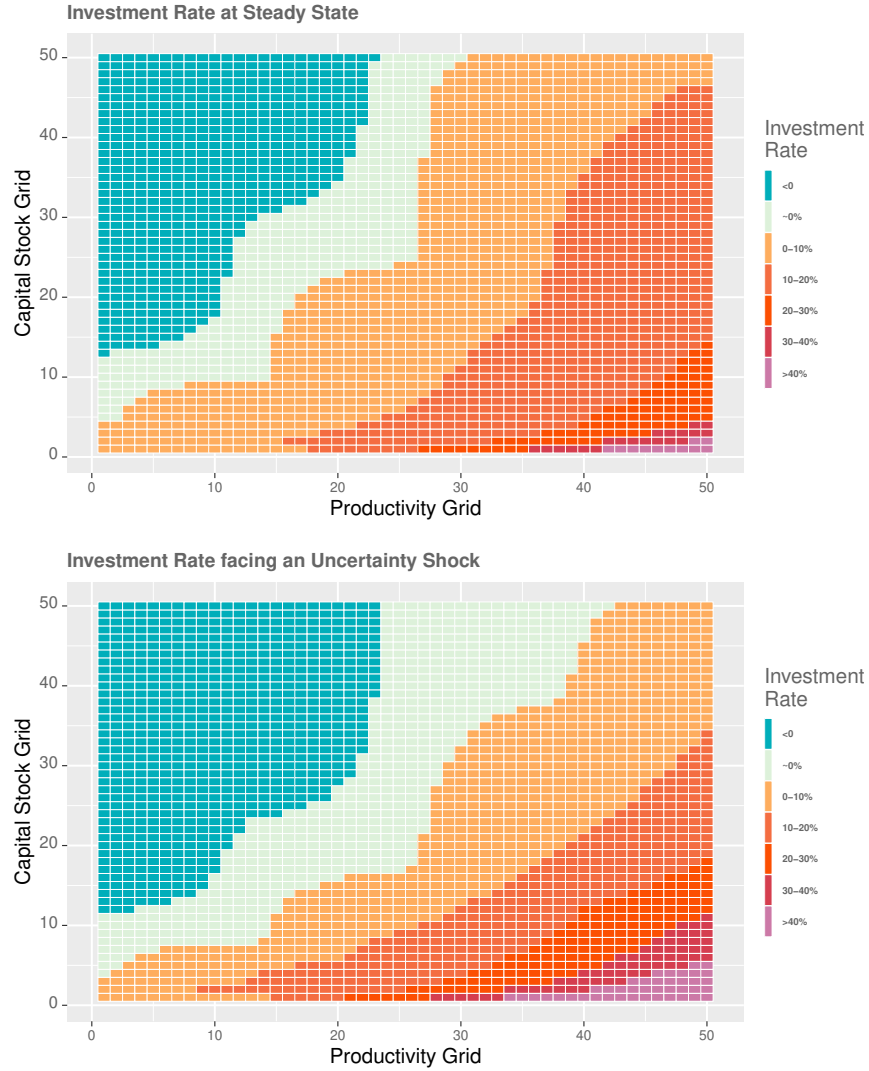
A.6 Volatility and investment policy at both margins

Figure 15: Extensive Margin of Investment



Notes: I solve the steady-state of the model, extract the steady-state cutoff for the random adjustment cost as a function of capital stock and productivity $\xi^*(k, z)[ss]$. Then I interpolate $\xi^*(k, z)$ over a denser grid of productivity and calculate the adjustment probability $Prob(k, z) = \xi^*(k, z)/\bar{\xi}$. For the case with a volatility shock, I solve the transition path and extract the first period of $\xi^*(k, z)[t = 1]$. The interpolation part is identical. The behavior at the bottom row of the graph may look distorted because that is the lowest capital stock grid point in the quantitative analysis. The measure of firms there is almost zero.

Figure 16: Intensive Margin of Investment



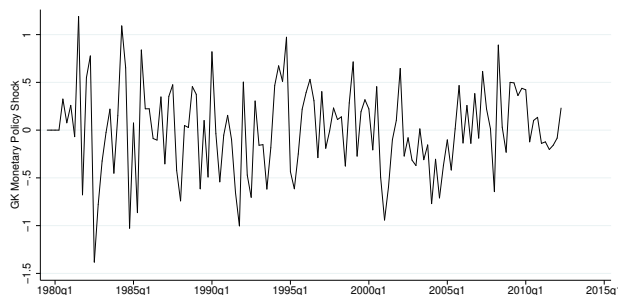
Notes: I solve the steady-state of the model, extract the steady-state active investment rate policy as a function of capital stock and productivity $i(k, z)[ss]$. Then I interpolate $i^*(k, z)$ over a denser grid of productivity and calculate the investment rate $i\%(k, z) = \frac{i(k, z)}{k(k, z)}$. For the case with a volatility shock, I solve the transition path and extract the first period of $i(k, z)[t = 1]$. The interpolation part is identical. The behavior at the bottom row of the graph may look distorted because that is the lowest capital stock grid point in the quantitative analysis. The measure of firms there is almost zero.

B Empirical Appendix

B.1 Description of the Indicators

High Frequency Identified Residual as Monetary Policy Indicator: The main indicator for monetary policy is to use the residual from a VAR in which a monetary policy indicator is instrumented for with high-frequency identified shocks following [Gertler and Karadi \(2015\)](#). This avoids endogeneity issues but makes it more difficult to interpret the magnitudes of the local projection results. The idea in [Gertler and Karadi \(2015\)](#) to isolate interest rate surprises using the movements in financial markets data within a short window around central bank policy announcements. They use financial market surprises from Fed Funds Futures during the 30 minutes interval around the FOMC policy announcements as proxies for the one-year government bond rate in a vector autoregression. The structural residual is then the estimated monetary policy shock. I plot the whole shock series in Figure 17. The whole period is from 1980Q3 to 2012Q3.

Figure 17: GK Monetary Policy Shock



Real Interest Rate as Monetary Policy Indicator: The second possible indicator for monetary policy is to directly use the real interest rate. Since the high-frequency identified shock series is constrained by the availability of the financial measures, directly using real interest rate provides a much longer period. In Figure 18, I show the real interest rate and federal funds rate from 1960Q1 to 2018Q2. They follow each other very closely, peaking around 1982Q1, and bottomed around 2014Q2. For the data series between 2009Q3 and 2015Q4, I use the shadow rate calculated by Wu and Xia (2015) to replace the federal funds rate.

Volatility Indicator: In Figure 19, I show the firm-level volatility measured using the IQR stock return of monthly stock returns and IQR sales growth for all Compustat firms with more than 25 years of observable data. This measure is initially used by [Bloom et al. \(2018\)](#). There are several obvious peaks around 1975Q1, 2001Q1, and 2008Q4 in both measures, respectively. Both measures are plotted in Figure 19. The main study uses the IQR sales growth.

Figure 18: Real Interest Rate & Federal Funds Rate

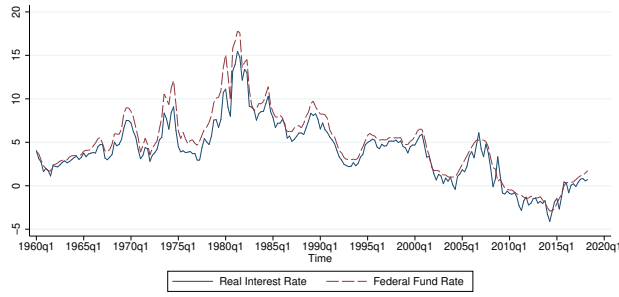
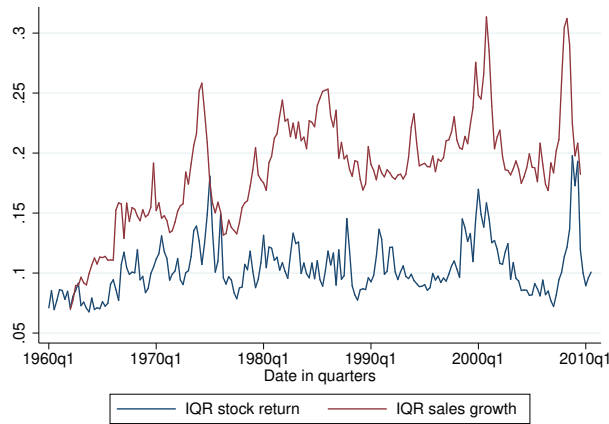


Figure 19: Volatility Indicators



B.2 Description of firm-level investment data

Lumpy Investment in Compustat: In Table 6, I present the summary statistics of physical investment divided into 5-year periods. I also show the standard deviations of each variable and their correlations with the aggregate investment measures.

Table 6: Summary Statistics of Physical Investment over Time

Variable	1975s	1980s	1985s	1990s	1995s	2000s
Average investment rate (%)	N/A	9.8%	8.6%	8.4%	10.0%	8.3%
Inaction rate (%)	N/A	9.2%	10.6%	10.1%	7.0%	10.7%
Spike rate (%)	N/A	15.3%	12.5%	12.6%	16.7%	11.8%
Disinvestment rate (%)	N/A	2.4%	3.6%	2.9%	2.7%	2.3%
Aggregate investment rate (%)	N/A	4.5%	4.2%	4.2%	5.1%	4.2%
Observations	0	15,932	76,905	80,090	95,810	86,928
	2005s	2010s	2015s	σ		β_{Agg}
Average investment rate (%)	8.5%	8.6%	8.4%	0.4%		0.80
Inaction rate (%)	8.2%	8.8%	9.7%	1.3%		-0.84
Spike rate (%)	11.8%	10.8%	9.4%	2.4%		0.83
Disinvestment rate (%)	1.3%	0.9%	0.9%	1.0%		0.19
Aggregate investment rate (%)	4.5%	4.4%	3.9%	0.7%		1.0
Observations	68,483	57,429	32,996			

Note: The average statistic excludes extreme value outliers (0.5% top and bottom). Inaction is defined by investment below 10% of the sample average. Spikes are defined by investment above twice the sample average. Aggregate investment is total investment divided by total lagged capital. σ is the standard deviation of a statistic over time at a quarterly frequency. β_{Agg} is the correlation of a statistic with the aggregate investment rate. Data Source: Compustat Fundamental Quarterly from 1975Q1 to 2019Q2.

Determinants of Investment: In Table 7, I show the determinants of investment at both the firm-level and in aggregate. The results are from an OLS regression of investment on all the determinant variables. The results indicate that Tobin's Q, leverage, and cash holding are important firm-level determinants, and all aggregate-level determinants are important.

B.3 Robustness checks of the main result at the aggregate-level

This empirical appendix section shows various robustness checks using different investment measures, different volatility measures, different local projection specifications, different sample periods, different monetary policy indicators, and different combinations of the above robustness checks. The results in the main text hold in almost all of these exercises. For space, I am not able to show all the different combinations of the robustness checks, results for not shown combinations are available upon request. To preserve readability of this appendix, I restate both specifications here:

Table 7: Determinants of Investment

Variable	Investment
<i>Firm-Level</i>	
Tobin's Q	0.003***
Total Asset	-0.000
Leverage	-0.090***
Cash Holding	-0.000***
Revenue	0.000
Sales Growth	0.001
<i>Aggregate-Level</i>	
GDP Growth	0.001***
Volatility	-0.052
CPI	0.007***
<i>Fixed Effects</i>	
Firm FE	Yes
One Digit Sector FE	Yes
Quarter FE	Yes
Observations	227206
adj. R^2	0.036

Standard errors in parentheses: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Extreme value outliers excluded.

Baseline specification (Grouped):

$$\Delta_h I_{t+h} = \alpha_h + (\beta_{j,h} + \gamma_{j,h} \epsilon_t^m) \times \mathbf{1}_{\sigma_t \in J^\sigma} + \sum_{l=0}^L \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \quad (19)$$

Alternative specification (Interacted):

$$\Delta_h I_{t+h} = \alpha_h + \beta_h^m r_t^m + \gamma_h r_t^m \sigma_t + \sum_{l=0}^L \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \quad (20)$$

where h indicates quarters in the future and l indicates lags. $\Delta_h I_{t+h} = I_{t+h} - I_t$ is the change of log investment measure, I_t is log investment. Hence, $\Delta_h I_{t+h}$ measures the changes of investment of period $t+h$ relative to period t . $\sigma_t = IQR_{sg,t}$ is the volatility measure at time t , ϵ_t^m is the monetary policy indicator. I control for period h fixed effect, and a vector Z_{t-l} of aggregate variables including volatility, CPI, output gap, investment, and consumption. I choose the horizon $H = 20$ and the lag $l = 4$ as suggested by Jordà (2005). In **Grouped** specifications, $\mathbf{1}_{\sigma_t \in J^\sigma}$ indicates σ_t belongs to one of $J^\sigma = \{h, m, l\}$ groups as defined in Section 2.1. The coefficients of interest are $\gamma_{h,h}$ and $\gamma_{l,h}$, which measures how the semi-elasticity of aggregate private investment rate $\Delta_h I_{t+h}$ with respect to monetary shocks depending on the state of low volatility (Bottom 20%) and high volatility

(Top 20%), respectively. In **Interacted** specifications, γ_h will be the coefficient of interest which measures the semi-elasticity of investment with respect to monetary shocks r_t^m conditional on a continuous measure of volatility σ_t . A significantly negative γ_h is strong evidence that the results hold.

I provide the following Table 8 to show all the alternatives in each dimension. The baseline in the paper is {Grouped, GK-HFI, IQR sales growth, RPFI-NR, 80-12}. I show other important alternatives in this section. For closer connected ones with the baseline specification, I will show more robustness checks; otherwise, less are listed below in the subsections. More results are available upon requests.

Table 8: Alternatives of Robustness Checks

Choices	LP Form	MP shock ϵ_t^m	Volatility σ_t	Investment	Periods
1	Grouped	GK-HFI	IQR sales growth	RGFCF	80-10
2	Interacted	RIR	IQR stock return	RGPI	85-10
3				RPFI	85-07
4				RPFI-NR	80-07
5				RPFI-NR-EQMT	60-10
6				RPFI-NR-Struct	60-07
7				RPFI-NR-IP	60-18

Notes: There are $2 \times 2 \times 2 = 8$ main and $2 \times 2 \times 2 \times 7 \times 7 = 392$ total specifications. I cannot show all of them in this appendix. So I mainly show the ones with significant differences between specifications. RGFCF stands for real gross fixed capital formation, RGPI stands for real gross private investment, RPFI stands for real private fixed investment, RPFI-NR, stands for the real non-residential private fixed investment, RPFI-NR-EQMT/Struct/IP stands for the equipment/structure/intellectual property components of RPFI-NR. GK-HFI stands for [Gertler and Karadi \(2015\)](#) high frequency identified monetary policy shocks and RIR stands for real interest rate. 80-12 stands for sample period from 1980Q3 to 2012Q3. I ignore the notation of quarter Q just for simplicity. The choices of specific quarters only depend on availability of combinations of all measures.

B.3.1 Main Specification 1: {Grouped, GK-HFI, IQR sales growth}

Figure 20: {Grouped, GK-HFI, IQR sales growth}
(Alternative investment measures and Output Gap)

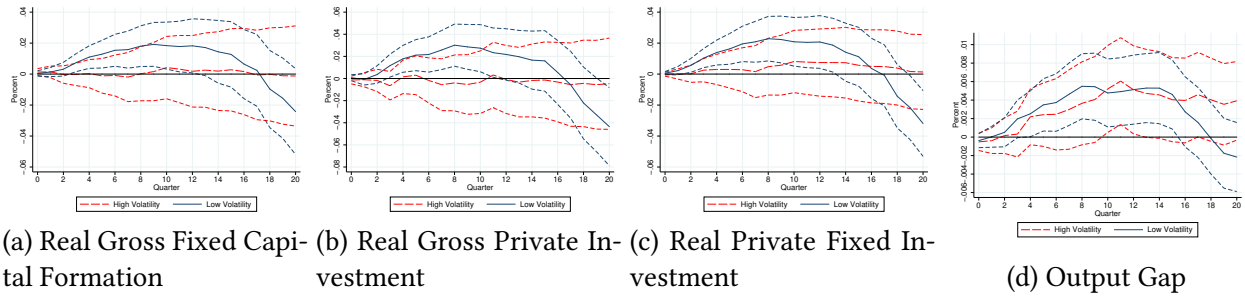


Figure 21: {Grouped, GK-HFI, IQR sales growth}
(Investment components)

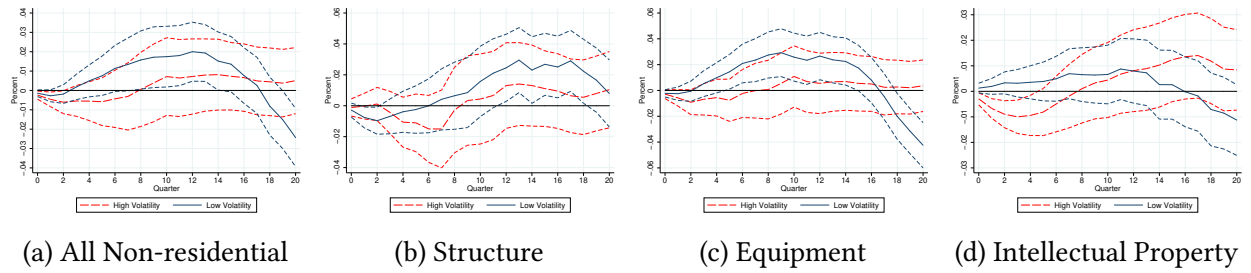


Figure 22: {Grouped, GK-HFI, IQR sales growth}
(Alternative investment measures and Output Gap, Post-1985)

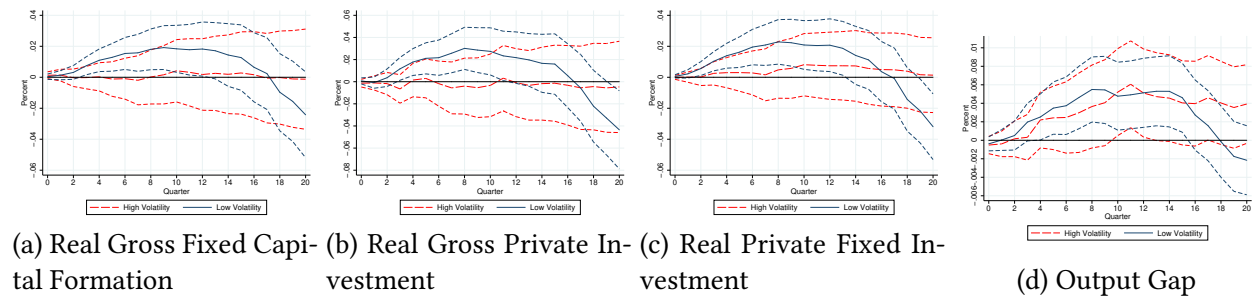


Figure 23: {Grouped, GK-HFI, IQR sales growth}
(Investment components, Post-1985)

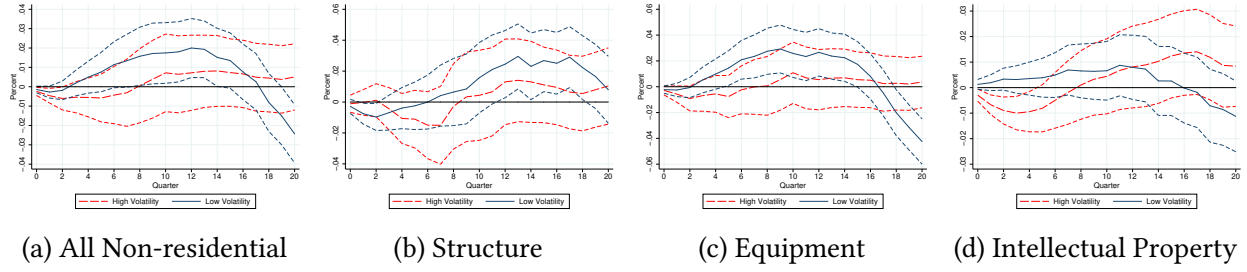


Figure 24: {Grouped, GK-HFI, IQR sales growth}
(Alternative investment measures and Output Gap, Pre-ZLB)

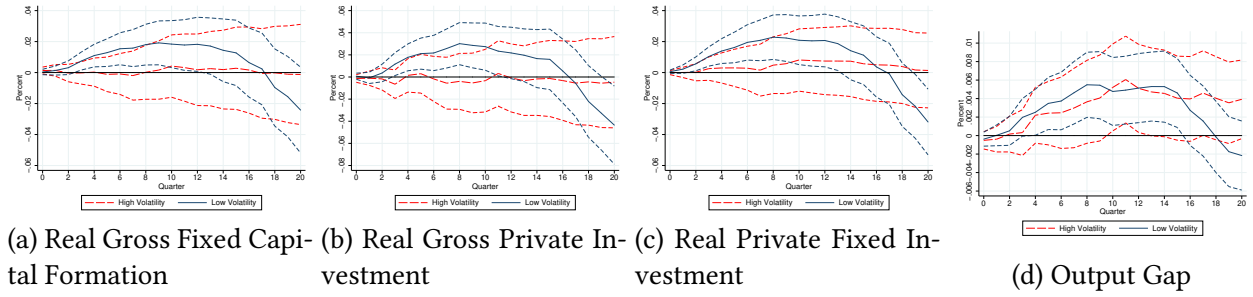
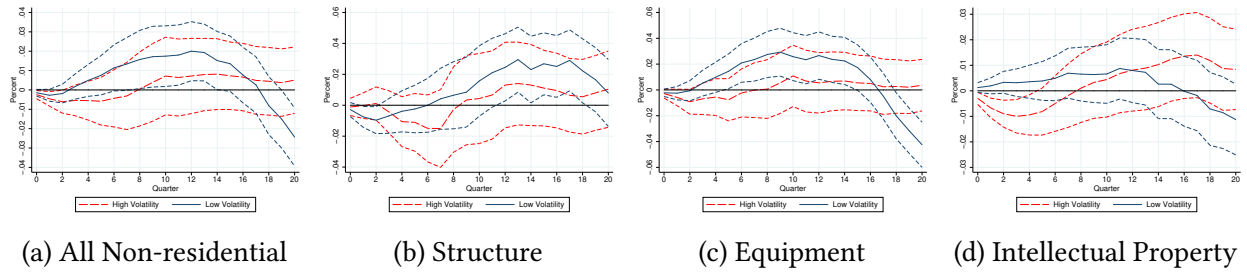


Figure 25: {Grouped, GK-HFI, IQR sales growth}
(Investment components, Pre-ZLB)



B.3.2 Main Specification 2: {Grouped, GK-HFI, IQR stock return}

Figure 26: {Grouped, GK-HFI, IQR stock return}
(Alternative investment measures and Output Gap)

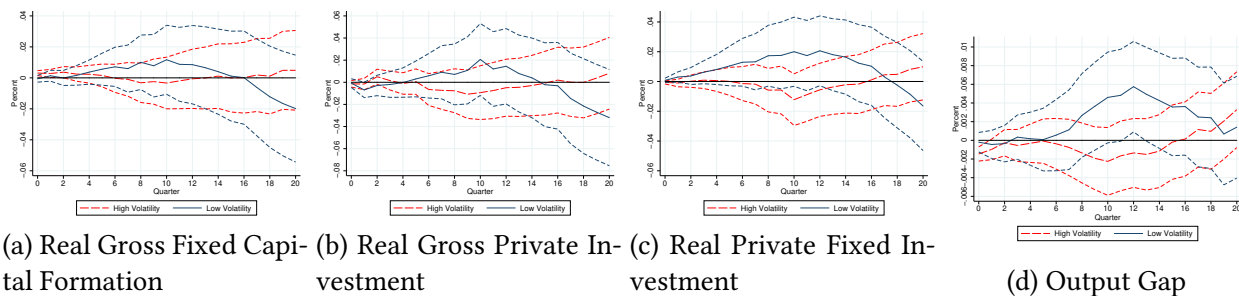
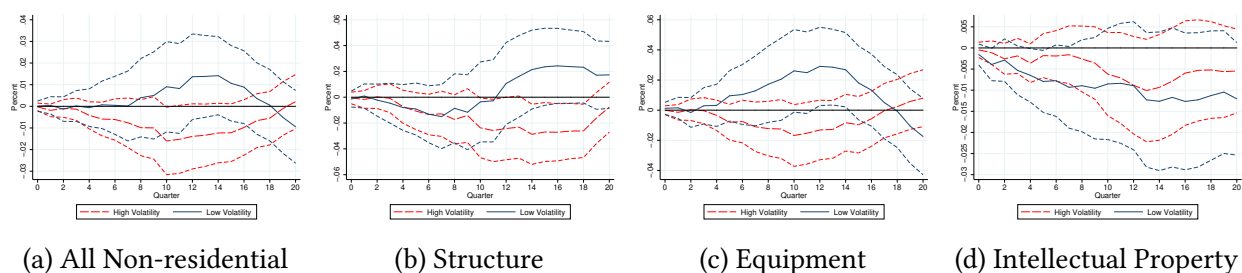


Figure 27: {Grouped, GK-HFI, IQR stock return}
(Investment components)



B.3.3 Main Specification 3: {Grouped, RIR, IQR sales growth}

Figure 28: {Grouped, RIR, IQR stock return}
(Alternative investment measures and Output Gap)

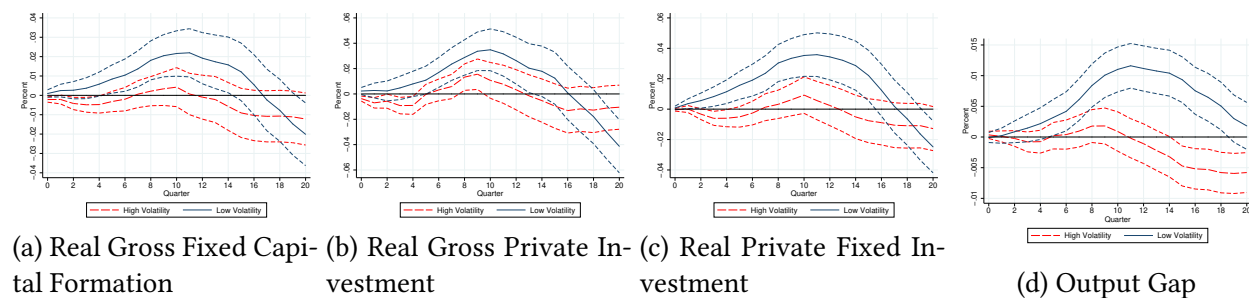
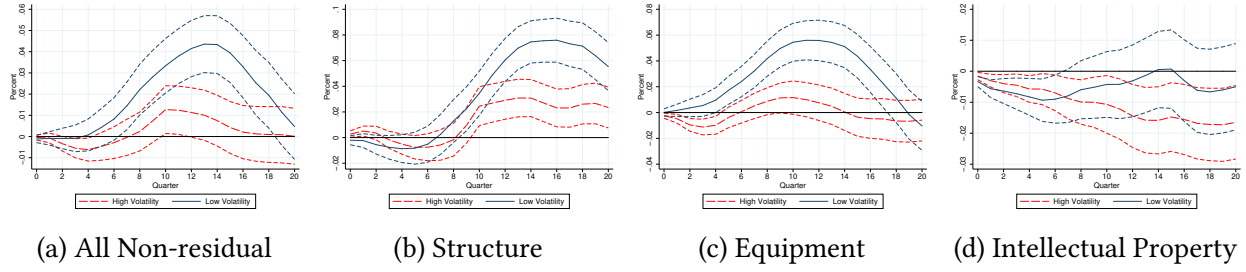
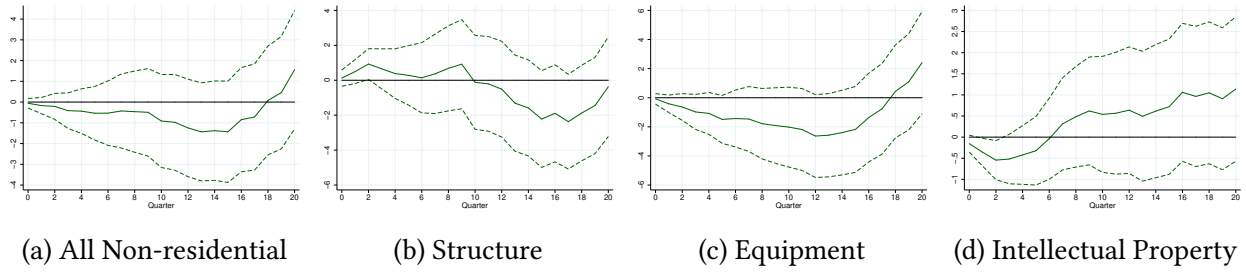


Figure 29: {Grouped, RIR, IQR stock return}
(Investment components)



B.3.4 Main Specification 4: {Interacted, GK-HFI, IQR sales growth}

Figure 30: {Interacted, GK-HFI, IQR sales growth}
(Investment components)



B.3.5 Main Specification 5: {Interacted, RIR, IQR sales growth}

Figure 31: {Interacted, RIR, IQR sales growth}
(Investment components)

