Firm Uncertainty Cycles and the Propagation of Nominal Shocks

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Motivation

- **Firms operate in constantly changing environments**
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- **Questions...**
  - How does uncertainty affect firms' decisions?
  - Does firm-level uncertainty matter in the aggregate?

Our contributions

- **We answer these questions in a general framework**
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  - Uncertainty and price flexibility move in cycles
  - Identify uncertainty's moments from micro-price data

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- **In the context of price-setting...**
  - Positive relationship between uncertainty and price flexibility
  - Uncertainty and price flexibility move in cycles
  - Identify uncertainty's moments from micro-price data
- **Aggregate effects are quantitatively important**
  - *Heterogeneous* uncertainty *amplifies* real effects of nominal shocks
    - Real effects up to 9 $\times$ Golosov and Lucas (2007)
  - *Average* uncertainty *dampens* real effects of nominal shocks
    - Monetary policy less effective in more uncertain times

Roadmap

1. Price-setting with uncertainty cycles (one firm)

2. Aggregate effects of heterogeneous uncertainty

Price-setting with uncertainty cycles

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- **Stochastic process for marginal costs**

$\Longrightarrow$ stochastic process for markup-gaps

Markup-gap and Information

- **Unobserved markup-gap:** $d\mu_t = \sigma_f dW_t + \sigma_u u_t dQ_t$
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- **Learning technology** Filter $\mu_t | \mathcal{I}_t$, with $\mathcal{I}_t = \sigma\{(s_r, Q_r)_{r \leq t}\}$
  - Bayesian firms solve filtering problem

Filtering with Jumps ⇒ Uncertainty Cycles

Filtering equations

Markup-gap's posterior distribution is Normal $\mu_t|\mathcal{I}_t \sim \mathcal{N}(\hat{\mu}_t, \gamma\Omega_t)$

$$\begin{aligned} &\textbf{(estimate)} & d\hat{\mu}_t &= \Omega_t \, d\hat{Z}_t \\ &\textbf{(uncertainty)} & d\Omega_t &= \frac{\sigma_f^2 - {\Omega_t}^2}{\gamma} dt + \frac{\sigma_u^2}{\gamma} dQ_t \end{aligned}$$

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- **Higher uncertainty $\Omega_t \implies$ More volatile estimates**

$$\hat{\mu}_{t+\Delta} = \underbrace{\frac{\gamma}{\Omega_t \Delta + \gamma}}_{\text{weight on prior}} \hat{\mu}_t + \underbrace{\left(1 - \frac{\gamma}{\Omega_t \Delta + \gamma}\right)}_{\text{weight on signal}} \left(\frac{s_t - s_{t-\Delta}}{\Delta}\right)$$

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- **Uncertainty cycles**
  - If $\lambda = 0$ (no jumps), then Ω_t converges to σ_f
  - If $\lambda > 0$ (jumps), then Ω_t features cycles
    - "Long-run" uncertainty $\Omega^* = \sqrt{\sigma_f^2 + \lambda \sigma_u^2}$ $\quad (\mathbb{E}[d\Omega_t] = 0)$

Pricing policy

- **Stopping Time Problem**

$$V(\hat{\mu}_0, \Omega_0) = \max_{\tau} \; \mathbb{E}\Big[\underbrace{\int_0^{\tau} e^{-rt} \left(-\hat{\mu}_t^2\right) dt}_{\text{payoff from inaction}} + \underbrace{e^{-r\tau} \left(-\bar{\theta} + \max_{x} \; V(x, \Omega_\tau)\right)}_{\text{payoff from action}} \Big]$$

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- **Policy: Inaction region that depends on uncertainty**

$$\text{change price if } \ (\hat{\mu}_t, \Omega_t) \notin [-\bar{\mu}(\Omega_t), \bar{\mu}(\Omega_t)]$$

$$\text{and reset markup-gap estimate to } x = 0$$

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Key: Elasticity of inaction region $\mathcal{E}(\Omega) < 1/2 < 1$.

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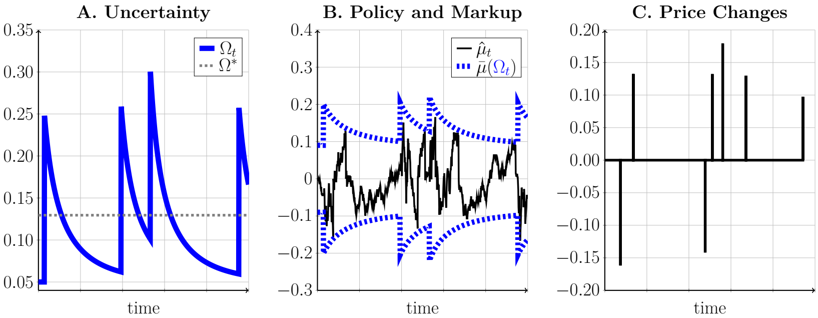
3. Uncertainty decreases expected time to adjustment

$$\mathbb{E}[\tau \big| 0, \Omega] = \left(\frac{\bar{\mu}(\Omega)}{\Omega}\right)^2 (1 + \mathcal{L}^{\tau}(\Omega)) \quad \text{with} \quad \mathcal{L}^{\tau}(\Omega) \propto \left(\frac{\Omega}{\Omega^*} - 1\right)(1 - \mathcal{E}(\Omega^*))$$

Key: Expected time is decreasing and convex in uncertainty.

Uncertainty Cycles ⇒ Adjustment Cycles



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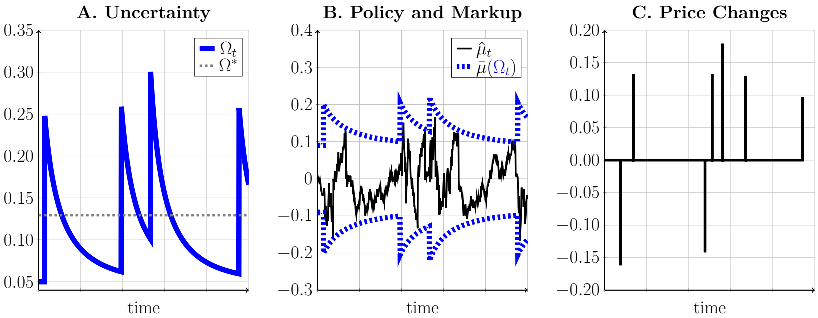
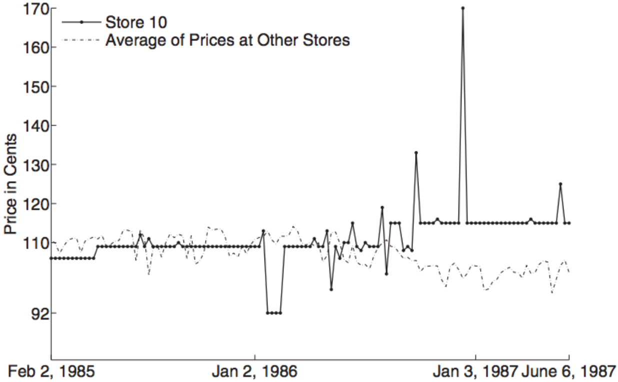


- **Low uncertainty**: small price changes, unlikely to be changed
- **High uncertainty**: large price changes, likely to be changed
- **Suggestive evidence**: Bachmann, et.al. ('13), Vavra ('14)
  - IFO Firm Survey: $corr(freq_i, std(forecast\ errors_i)) > 0$

More suggestive evidence

- At product level, recurrent episodes of very frequent price changes. (Campbell & Eden, '14)



NOTE: Weekly observations of the price of Fleischmann's Margarine at a store in Sioux Falls, South Dakota, and the average of all other stores' prices for the identical product. Dates are the final days of the given week.

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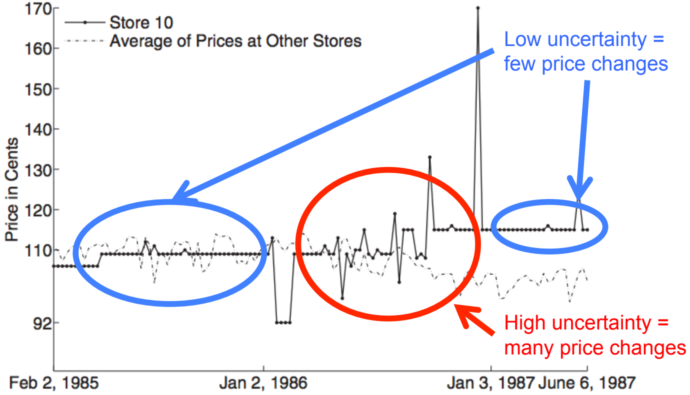

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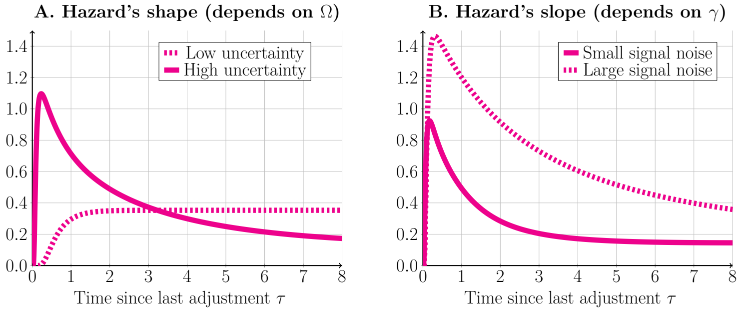

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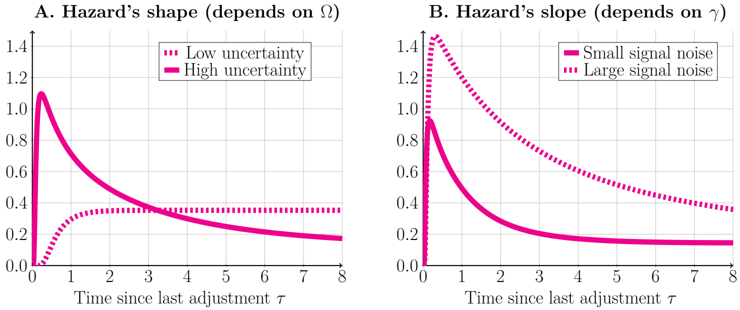

- **Hazard rate:** $h(\tau|\Omega) = Prob(\text{adjust } \tau \mid \text{no adjustment until } \tau)$
- **Shape:** driven by uncertainty Ω
  - **Low uncertainty**: increasing hazard (standard menu cost model)
  - **High uncertainty:** non-monotonic hazard (learning)
- **Slope:** driven by information friction γ

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- Price statistics reflect behavior of high uncertainty firms
  - i.e. aggregate decreasing hazard rate Evidence

Aggregate effects of *heterogenous* uncertainty

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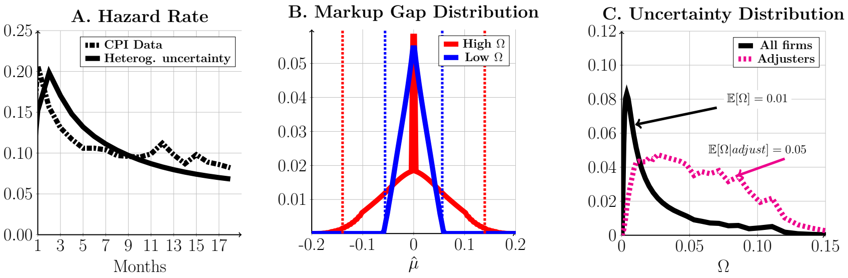
3. **Equilibrium with constant money supply**
   - Nominal wage = Money supply
   - Steady state distribution of markup gaps and uncertainty

Calibration to match micro-price data



	US Data	No heterogeneity (baseline)	Heterogeneous uncertainty
Moments			
$\mathbb{E}[\tau]$ in months	10	10	10
std$[\lvert\Delta p\rvert]$	0.08	0.007	0.05
hazard rate slope	-0.007	0.007	-0.005

Details

Propagation of nominal shocks

- **Unanticipated increase in money supply** $\delta = 1\%$
  - True markup-gaps fall in 1%
- **Output effects = inaction errors + forecast errors**
  - Deviation from steady state (IRF):

$$\tilde{Y}_t = -\int_0^1 \mu_t(z)dz = \underbrace{-\int_0^1 \hat{\mu}_t(z)dz}_{\text{inaction error}} + \underbrace{\int_0^1 \varphi_t(z)dz}_{\text{forecast error}}$$

  - Total effect (area under IRF):

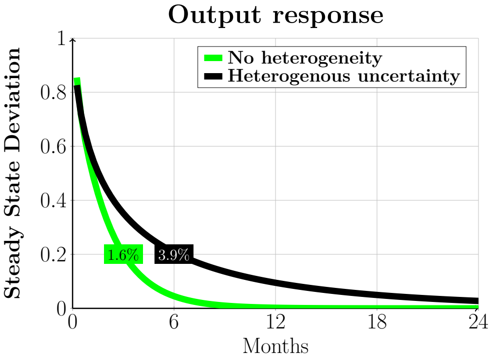
$$\mathcal{M}(\delta) = \int_0^\infty \tilde{Y}_t \; dt \;= \mathcal{I} + \mathcal{F}$$

- **Three exercises:**

A) Disclosed money shock (fully observed)
B) Undisclosed money shock (partially observed)
C) Aggregate uncertainty shock

A) Effects of *disclosed* monetary shock

- Fully observed shock $\Rightarrow$ Only inaction errors
- Only *first* price change matters
- Persistence driven by low uncertainty firms



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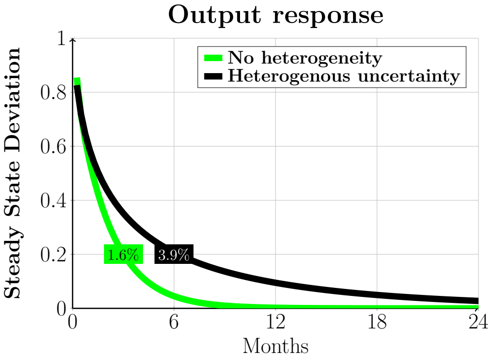

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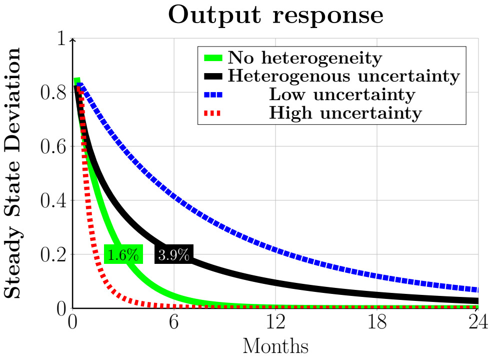
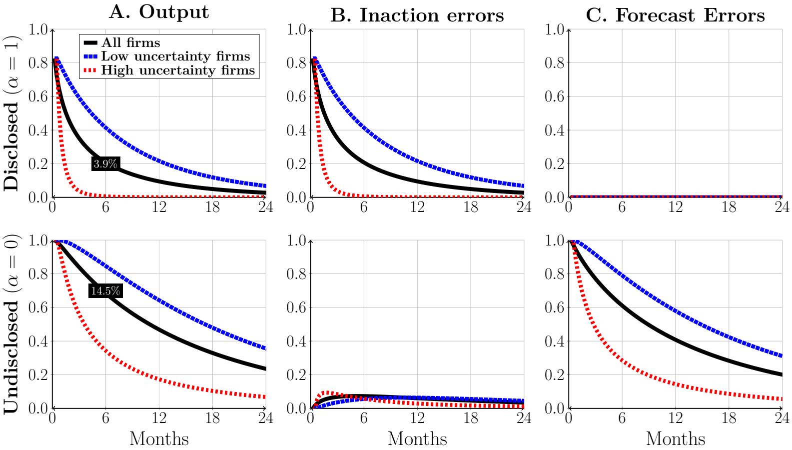


B) Effects of *undisclosed* monetary shock

- Fraction α is observed
- Forecast errors arise
- Incomplete pass-through



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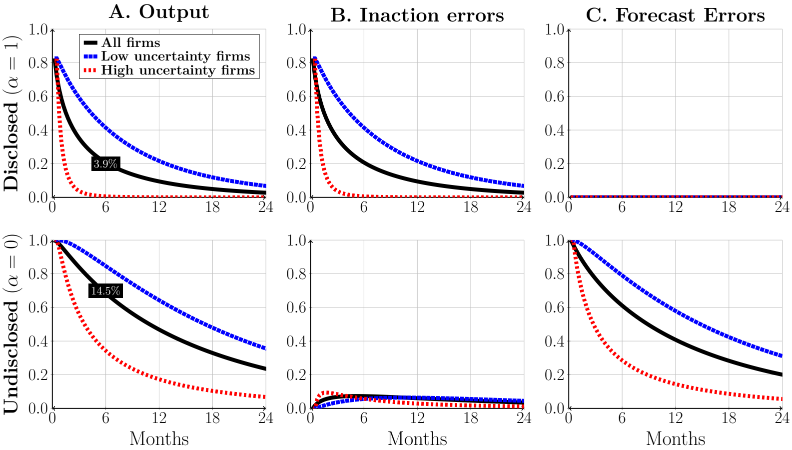
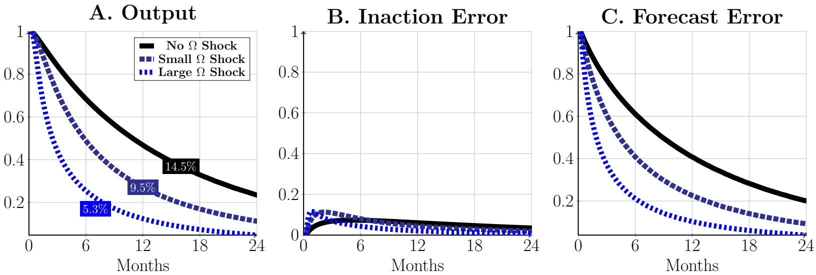
$$\mathcal{M}(\delta,\alpha) \geq \delta \underbrace{\left(\alpha \frac{\mathbb{E}[\tau]}{6} + (1-\alpha)\sqrt{\frac{\gamma^2 \mathbb{E}[\tau]}{\mathbb{V}[\Delta p]}}\right)}_{\text{no heterog} \;=\; 1.6\alpha \;+\; 10(1-\alpha)}$$



C) Effect of *aggregate uncertainty* shock

- Uncertainty shock $\kappa\mathbb{E}[\Omega]$
- Forecast errors fall with κ
- Faster learning



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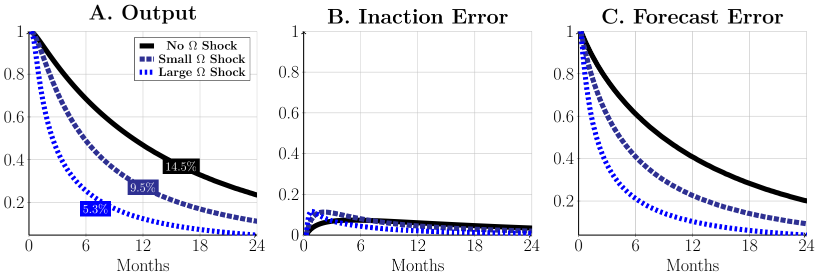

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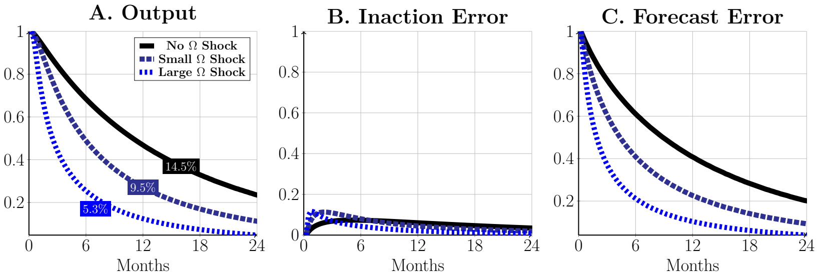


- **Monetary policy is less effective in uncertain times**
  - Castelnuovo, et al (2015), Aastveit, et al (2013)
- **Forecast errors are smaller in uncertain times**
  - Gorodnichenko et al (2016)

Conclusions

- Pricing theory with menu costs and idiosyncratic uncertainty cycles
- Macro implications:
  - Uncertainty heterogeneity amplifies effects of money shocks
  - Average uncertainty dampens effects of money shocks
- Information friction identified with hazard rate
- General framework, potential applications...
  - Portfolio choice s.t. adjustment fees and uncertain returns
  - Occupational choice s.t. mobility costs and uncertain skills.

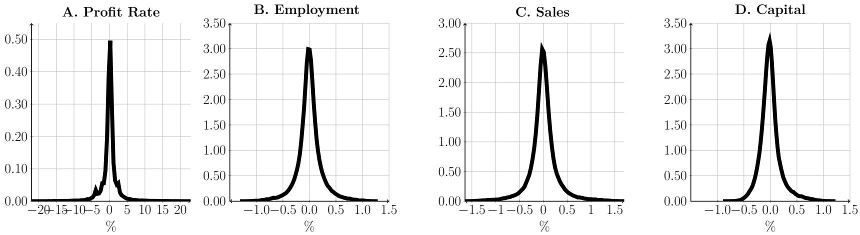
APPENDIX

Characteristics of *idiosyncratic* shocks Return

- **Evidence of fat-tailed (leptokurtic) risk**
  - **Price change distribution**
    - For US CPI: Klenow and Malin (2011)
    - For French CPI: Alvarez, Le Bihan and Lippi (2016)
  - **Employment growth distribution**
    - For US Census data: Davis and Haltinwanger (1992)
  - **Profit rate, employment, sales and capital growth**
    - Own computations using COMPUSTAT 1980-2015 annual
- **Evidence of idiosyncratic uncertainty**
  - **Heterogeneity and time-variation in firm-level uncertainty**
    - German firms: Bachmann, Elstner and Hristov (2016) w/IFO Survey
    - US firms: Senga (2016) using I/B/E/S

Evidence of Leptokurtic Shocks Return

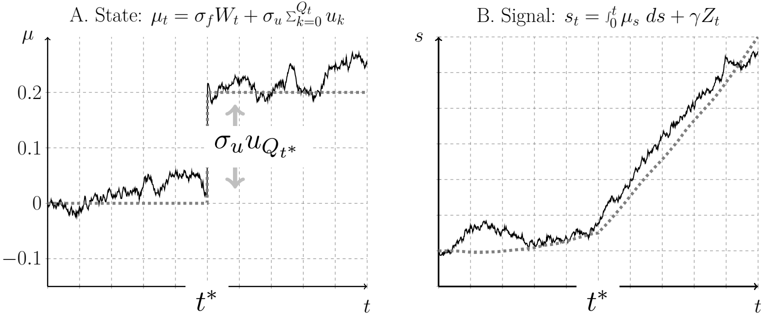
- COMPUSTAT 1980-2015 annual data, growth rates



Moment	Profits	Employment	Sales	Capital
Mean	-0.035	0.002	0.004	-0.002
Median	0.078	-0.002	-0.004	-0.005
Standard Deviation	1.337	0.150	0.179	0.101
Skewness	-0.170	0.483	0.539	0.699
Kurtosis	30.324	11.174	11.097	10.488

Markup gap and signal Return

Figure: Illustration of the Markup Gap and the Signal Processes



Sufficient Conditions for Optimal ST Return

Proposition

Let $\phi : R \times R^+ \to R$ be a function and let ϕ_x. Assume ϕ satisfies the following conditions:

1. ***Hamilton-Jacobi-Bellman** (HJB) equation:*

$$r\phi(\hat{\mu}, \Omega) = -\hat{\mu}^2 + \left(\frac{\sigma_f^2 - \Omega^2}{\gamma}\right)\phi_\Omega(\hat{\mu}, \Omega) + \frac{\Omega^2}{2}\phi_{\hat{\mu}^2}(\hat{\mu}, \Omega) + \tag{1}$$

$$+ \lambda\left[\phi\left(\hat{\mu}, \Omega + \frac{\sigma_u^2}{\gamma}\right) - \phi(\hat{\mu}, \Omega)\right] \tag{2}$$

2. *value matching condition*

$$\phi(0, \Omega) - \bar{\theta} = \phi(\bar{\mu}(\Omega), \Omega) \tag{3}$$

3. *Two smooth pasting conditions*

$$\phi_{\hat{\mu}}(\bar{\mu}(\Omega), \Omega) = 0, \quad \phi_\Omega(\bar{\mu}(\Omega), \Omega) = \phi_\Omega(0, \Omega) \tag{4}$$

Then ϕ is the value function $\phi = V$ and
$\tau = \inf\{t > 0 : \phi(0, \Omega_t) - \theta > \phi(\hat{\mu}_t, \Omega_t)\}$ *is the optimal stopping time.*

Markup process in discrete time

- Stochastic process with permanent and transitory shocks:

$$
\begin{array}{rrcl}
(Total) & \mu_t & = & \mu_{t-1} + \mu_t^P + \mu_t^T \\
(Permanent) & \mu_t^P & = & \mu_{t-1}^P + \sigma_F \varepsilon_t^F + \sigma_U \varepsilon_t^U J_t \\
(Transitory) & \mu_t^T & = & \gamma \varepsilon_t^T \\
 & J_t & = & \begin{cases} 1 & \text{w.p. } 1 - e^{-\lambda} \\ 0 & \text{w.p. } e^{-\lambda} \end{cases} \\
 & \epsilon_t^F, \epsilon_t^U, \epsilon_t^T & \sim & \mathcal{N}(0,1)
\end{array}
$$

- Firm observes total markup μ_t, but not its components separately.
- Firm knows realization of binomial J_t, but not the size of the shock.
- Timing assumption: Choose price before observing productivity.

Return

Young prices: more flexible and dispersed

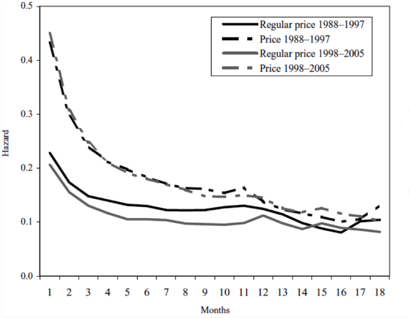
- Price age is current date minus the last stopping time: $a = t - \tau_{t-1}$
- **Age thresholds:**
  - Young price if $a < 7$, (20% age percentile)
  - Old price if $a > 66$, (80% age percentile)
- **Frequency and dispersion for young and old prices**

Statistic	**Data***				**Model**			
	All	Young	Old	Ratio	All	Young	Old	Ratio
Frequency %	15	36	13	2.8	11.4	16.4	8.3	2.0
std(Price gap)	11.4	15.1	6.9	2.2	3.4	3.8	3.0	1.3
Uncertainty*100					0.3	0.4	0.2	2.5

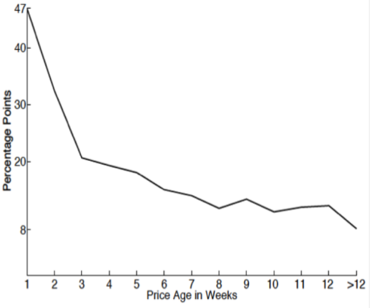
*Campbell and Eden (2014), average all products without discounts, with thresholds $\underline{a} < 3$ and $\overline{a} > 4$ weeks.

- **Frequency ratio informs about underlying uncertainty.**

Hazard Rate of Price Adjustment Return



Nakamura and Steinsson ('08)



Campbell and Eden ('14)

- Include controls for observed and unobserved heterogeneity

Representative Household

$$\max_{\{C_t, c_t(z), l_t, M_t\}} \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\log C_t - l_t + \log\frac{M_t}{P_t}\right)dt\right] \quad s.t$$

$$M_0 \geq \mathbb{E}\left[\int_0^\infty Q_t\left(\int_0^1 p_t(z)c_t(z)dz + R_t M_t - E_t l_t - \Pi_t\right)dt\right]$$

$$C_t = \left(\int_0^1 \left(A_t(z)c_t(z)\right)^{\frac{\eta-1}{\eta}} dz\right)^{\frac{\eta}{\eta-1}}$$

- Q_t : time zero nominal Arrow-Debreu price
- C_t : aggregate consumption with price P_t
- l_t : labor with price E_t
- $R_t M_t$: opportunity cost of money (R_t nominal i-rate)
- Π_t : firms' profits
- $A_t(z)$: quality shocks

Return

Environment: Firms

- Continuum of monopolistic firms, indexed with $z \in [0, 1]$.
- Firms choose price to maximize expected profits, discounted at Q_t.
- Period profits are

$$\Pi\left(p_t(z), A_t(z)\right) = c_t\left(p_t(z), A_t(z)\right)\left(p_t(z) - A_t(z)W_t\right)$$

  where quality $a_t(z) = \log A_t(z)$ is *iid* across firms:

$$da_t(z) = \sigma_f dW_t(z) + \sigma_u u_t(z) dq_t(z)$$

- Firms observe noisy signals about quality:

$$ds_t(z) = a_t(z)dt + \gamma dZ_t(z)$$

- Pay menu cost θ.
- Assumption: firms cannot invert the demand function

Return

Equilibrium Definition

An equilibrium with constant money growth is a set of stochastic processes for

i) consumption strategies $c_t(z)$, labor supply l_t and money holdings M_t for the representative consumer

ii) labor demand $l_t(z)$ and pricing policy $p_t(z)$ for firms

iii) prices W_t, R_t, Q_t

iv) measure of firms that reprice N_t

such that:

- Given prices, $c_t(z)$, l_t and M_t solve the consumer's problem with initial $M_0 = M$.
- Given the prices and demands, firms' policies $l_t(z)$ and $p_t(z)$ solve her problem.
- Markets clear at each date.

Return

Steady State with Constant Money Supply

- **Steady state equilibrium with zero money growth**
- Constant money supply $M \Rightarrow$
  - Constant wage $W = M$
  - Constant nominal interest rate $R = r$ and discount $Q_t = e^{-rt}$
- Fixed distribution $f(\hat{\mu}, \Omega)$

Related literature

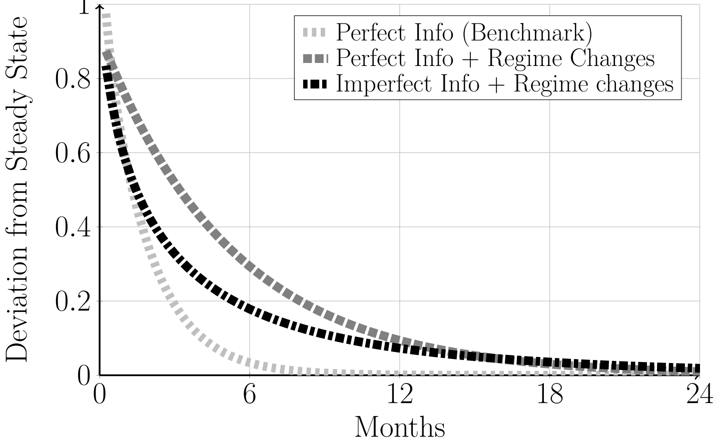
- **Price-setting with menu costs**
  Barro ('72), Caplin & Spulber ('87), Caplin & Leahy ('91), Danziger ('99), Dotsey, King & Wolman ('99), Golosov & Lucas ('07), Gertler & Leahy ('08), Nakamura & Steinsson ('10), Midrigan ('11), Alvarez & Lippi ('14).

- **Price-setting with idiosyncratic information frictions**
  Bachmann & Moscarini ('12), Alvarez, Lippi & Paciello ('11,'13), Bonomo, Carvalho, Garcia & Malta ('14), Argente and Yeh (2016).

- **Uncertainty and real option effects**
  Bernanke ('82), Dixit ('91), Bloom ('09), Vavra ('14), Senga ('15).

- **Price micro-data** Bils & Klenow ('04, '10), Nakamura & Steinsson ('08, '13), Campbell and Eden ('14), Baley, Kochen, Sámano (2016).

Disclosed Money Shock: 3 calibrations



Calibration details Return

	US Data	No uncertainty (Baseline)	Heterogenous Uncertainty
Parameters			
σ_f		0.016	0
σ_u			0.219
λ			0.011
γ			0.467
Moments			
$\mathbb{E}[\tau]$ in months	10	10^*	10^*
std$[\lvert\Delta p\rvert]$	0.08	0.007	0.05^*
hazard rate slope	-0.007	0.007	-0.005^*
kurtosis$[\Delta p]$	3.95	1.027	1.84

- θ such that $\frac{\text{Average menu costs}}{\text{revenue}} = 0.5\%$
- B such that Average markup $= 20\%$
- $r = 4\%$ year