Nonconvex Adjustment Cost in Lumpy Investment Models: Mean versus Variance

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Abstract

This paper revisits the canonical assumption of the nonconvex capital adjustment in lumpy investment models, which is assumed to follow a uniform distribution starting from zero to an upper bound without distinguishing the *Mean* and the *Variance*. I show that in order to generate an empirical-consistent interest-elasticity of aggregate investment, both a sizable *Mean* and a sizable *Variance* are necessary. The *Mean* governs how much the extensive margin accounts for aggregate investment dynamics. In contrast, the *Variance* governs how sensitive the extensive margin is to interest rate changes. As a result, both *Mean* and *Variance* are quantitatively important in a reasonably calibrated lumpy investment model.

Keywords: Lumpy investment; Ss model; firm heterogeneity;
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1 Introduction

In all current lumpy investment models such as Khan and Thomas (2008), the nonconvex capital adjustment cost ξ_{jt} is uniformly distributed with support $U[0, \bar{\xi}]$ independently across firms and time. A conventional calibration of a small upper bound $\bar{\xi}$ which matches the firm-level lumpy investment moments claims that lumpy investment is irrelevant for aggregate dynamics.

However, recent literature (Bachmann et al., 2013; Winberry, 2018; Koby and Wolf, 2019; Fang, 2020) reverse the claim that microeconomic investment lumpiness is inconsequential for macroeconomic analysis. The key argument is that the upper bound $\bar{\xi}$ of the distribution of nonconvex capital adjustment cost should be calibrated much larger.

The upper bound $\bar{\xi}$ determines how sensitive the aggregate investment is to the real interest rate. The calibration of a small (large) upper bound $\bar{\xi}$ delivers a huge (small) interest-elasticity of aggregate investment. Quasi-experimental evidence on firm-level investment responses to tax changes (Zwick and Mahon, 2017; Koby and Wolf, 2019) suggests a small interest-elasticity of aggregate investment, consequently, the calibration of a large upper bound $\bar{\xi}$.

What is the intuitive economics meaning of the calibration of a large upper bound $\bar{\xi}$? In all current models, the setup implicitly assumes that the mean $\mu_\xi = \bar{\xi}/2$ and the standard deviation $\sigma_\xi = \bar{\xi}/\sqrt{12}$ are isometric ($\mu_\xi = \sqrt{3}\sigma_\xi$). This assumption means when calibrating $\bar{\xi}$, we choose the expected size (*Mean*) and the uncertainty (*Variance*) of the nonconvex adjustment cost faced by the firms jointly. Then, which one determines the interest-elasticity of aggregate investment?

In this paper, I answer this question through disciplining the economics meanings of the expected size (*Mean*) and the uncertainty (*Variance*) of the nonconvex adjustment cost. I assume the nonconvex adjustment cost follows a uniform distribution with with mean and variance $\{\mu_\xi, \sigma_\xi\}$:

$$\xi_{jt} \sim U[\mu_\xi - \sqrt{3}\sigma_\xi, \mu_\xi + \sqrt{3}\sigma_\xi] \tag{1}$$

I first compare the interest-elasticity of aggregate investment over both dimensions of $\{\mu_\xi, \sigma_\xi\}$ departing from a conventional calibration lumpy investment model. The model would generate unrealistically large interest-elasticity of aggregate investment when either of the *Mean* or the *Variance* approaching zero. It shows both a sizable *Mean* and a sizable *Variance* are necessary to generate the empirical-consistent interest-elasticity of aggregate investment.

However, further inspection of the mechanism shows that the *Mean* and the *Variance* play different roles. A decomposition of the interest-elasticity into both the extensive margin and the intensive margin shows different roles of the *Mean* and the *Variance*. Without a sizable *Mean*, the unrealistically large interest-elasticity is mainly from the unconstrained intensive margin.[1] Without a sizable *Variable*, the unrealistically large interest-elasticity is mainly from the oversensitive extensive margin. The distribution of the extensive margin adjustment probability confirms

[1]This unconstrained intensive margin is usually constrained by a quadratic adjustment cost in recent literature. To keep this note dedicated on disciplining the *Mean* and the *Variance*, I leave out the quadratic adjustment cost.

these patterns from a micro perspective. Therefore, a sizable *Mean* and a sizable *Variance* are both necessary quantitatively in a reasonably calibrated lumpy investment model.

This paper is organized as follows. Section 2 presents the model and the solution method. Section 3 shows the interest-elasticity of aggregate investment with respect to the *Mean* and the *Variance*, respectively. Section 4 further inspects the mechanism. Finally, section 5 concludes.

2 The Model

The economy consists of a fixed unit mass of firms $j \in [0, 1]$ which produce homogeneous output y_{jt} and a unit measure continuum of identical households who consume output and supply labor.

Technology: The production function is as follows:

$$y_{jt} = A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1 \tag{2}$$

where k_{jt} and n_{jt} indicates the idiosyncratic capital and labor employed by the firm j, and A_t is the aggregate TFP shock. For each firm, the idiosyncratic TFP z_{jt} follows a log-normal AR(1):

$$log(z_{jt}) = -(1-\rho^z)\frac{\sigma^{z2}}{2(1-\rho^{z2})} + \rho^z log(z_{jt-1}) + \epsilon_{jt}, \quad \epsilon_{jt} \sim N(0, \sigma^z) \tag{3}$$

Adjustment Costs: The investment cost function includes three components: a direct cost i_{jt} and a fixed nonconvex capital adjustment cost ξ_{jt} in units of labor if they adjust more than a small proportion of their current capital stock ($|ak|$):

$$c(i_{jt}) = i_{jt} + \mathbf{1}_{(|i_{jt}|>ak_{jt})} \cdot w_t \cdot \xi_{jt}, \quad \xi_{jt} \sim \text{eq.}(1) \tag{4}$$

Firm Optimization: I denote by $V^A(k_{jt}, z_{jt}; \Omega_t)$, $V^{NA}(k_{jt}, z_{jt}; \Omega_t)$, and $V(k_{jt}, z_{jt}; \Omega_t) \equiv E_{\xi_{jt}} \tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t)$ the value functions of a firm with an active investment choice, without an active investment choice, and with expected draw of ξ_{jt}. The aggregate state $\Omega_t = (\Theta_t, \mu_t(k, z, \xi))$ where Θ_t is the vector of stochastic discount factor and wage at time t, and $\mu_t(k, z, \xi)$ is the distribution of firms. The value functions are as follows:

$$V^A(k_{jt}, z_{jt}; \Omega_t) = \max_{i,n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^*_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{5}$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_t) = \max_{i \in [-ak, ak], n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^C_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{6}$$

where the stochastic discount factor $\Lambda_{t,t+1}$ is derived from the household problem since households own all the firms. k^C_{jt+1} and k^*_{jt+1} are constrained and non-constrained capital choices.

The firm will choose to pay the fixed cost if and only if $V^A(k_{jt}, z_{jt}; \Omega_t) - w_t \xi_{jt} > V^{NA}(k_{jt}, z_{jt}; \Omega_t)$.

There is a unique threshold $\xi^*(k_{jt}, z_{jt}; \Omega_t)$ which the firm breaks even:

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t} \tag{7}$$

If a firm draws a fixed cost ξ_{jt} below $\xi^*(k_{jt}, z_{jt}; \Omega_t)$ (ξ^* *for short*), the firm pays the fixed cost and then actively adjusts its capital; otherwise, it does not. The value function is:

$$V(k_{jt}, z_{jt}; \Omega_t) = -\frac{w_t(\xi^* + \underline{\xi})}{2} + \frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi} V^A(k_{jt}, z_{jt}; \Omega_t) + \left(1 - \frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t) \tag{8}$$

where $\underline{\xi} = \mu_\xi - \sqrt{3}\sigma_\xi$ is the lower bound of fixed cost. The firm expects to pay the fixed cost drawing ξ_{jt} lower than $\xi^*(k_{jt}, z_{jt}; \Omega_t)$. With probability $\frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi}$, the firm chooses active investment, otherwise, it stays inactive. Therefore, the capital stock evolves by the law of motion:

$$k_{jt+1} = \begin{cases} (1-\delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1-\delta)k_{jt} + i_{jt}^C & otherwise \end{cases} \tag{9}$$

Household Optimization: Households' expected utility is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \theta N_t \right)$$

subject to the budget constraint: $C_t + \frac{1}{R_t} B_t \leq B_{t-1} + w_t N_t + \Pi_t^F$. Where β is the discount factor of households, θ is the disutility of working, R_t is the interest rate, B_t is one period bonds, w_t is the nominal wage, and Π_t^F is the nominal profits from all the firms. The first order conditions of consumption, labor, and bonds deliver:

$$w_t = -\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = \theta C_t^\eta \tag{10}$$

$$\Lambda_{t,t+1} = \frac{1}{R_t} = \beta \frac{U_c(C_{t+1}, N_{t+1})}{U_c(C_t, N_t)} = \beta \left(\frac{C_t}{C_{t+1}} \right)^\eta \tag{11}$$

Equilibrium Definition: *A Recursive Equilibrium for this economy is defined by a set of value functions and policy functions* $\{V(k, z; \Omega), V^A(k, z; \Omega), V^{NA}(k, z; \Omega), \xi^*(k, z; \Omega), k^*(k, z; \Omega), k^C(k, z; \Omega)\}$, *a set of quantity functions* $\{C(\Omega), N(\Omega), Y(\Omega), K(\Omega)\}$, *a set of price functions* $\{w(\Omega), \Lambda(\Omega), R(\Omega)\}$, *and a distribution* $\mu'(\Omega)$ *that solves firms' and households' problems and market clearing such that:*

(i). Taking price functions as given, the policy functions solve firms' optimization.

(ii). Taking price functions as given, the quantity functions solve households' optimization.

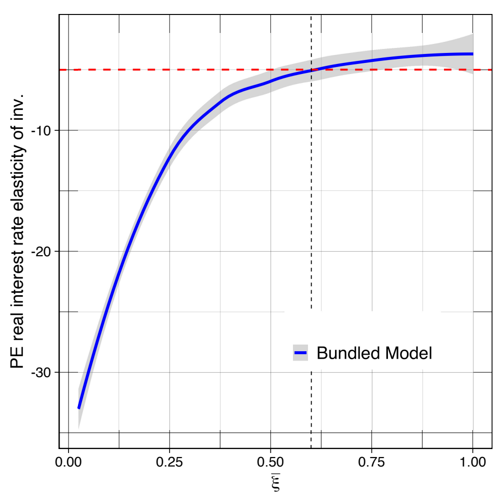
(iii). Market clears $Y(\Omega) = C(\Omega) + I(\Omega) + \Theta_k(\Omega)$, *where* $\Theta_k(\Omega)$ *is the total adjustment cost.*

Solution Method: I follow the sequence space solution strategy as in Boppart et al. (2018) to solve the model which involves two parts. First, I solve the *Stationary Equilibrium* at the steady-state, which delivers all the steady-state equilibrium objects and provides the cross-sectional moments for the calibration. Second, I solve the *Transitional Equilibrium* starting from the *Stationary Equilibrium* and transit back to the same *Stationary Equilibrium*. The *Transitional Equilibrium* then provides the dynamic moments for the calibration and the impulse response functions.

3 Mean versus Variance for the Interest-Elasticity

Benchmark Calibration: I calibrate the benchmark model with bundled mean and variance ($\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$) to hit the target investment moments. For fixed parameters, I choose discount factor $\beta = 0.99$ to match an annual interest rate of 4%., elasticity of intertemporal substitution $\eta = 1$ for log utility, leisure preference $\theta = 2$ to match a third of working time, the capital coefficient $\alpha = 0.25$ and the labor coefficient $\nu = 0.60$ to match a labor share of two-thirds and decreasing returns to scale imply of 85%, quarterly capital depreciation $\delta = 0.026$, freely adjustment region $a = 0.001$, and persistence of idiosyncratic TFP shock $^z = 0.95$. For fitted parameters, I choose $\sigma^z = 0.05$ and $\bar{\xi} = 0.6$ to match average investment rate (10.5%), a standard deviation of investment rates (0.13), spike rate (17%), and partial equilibrium interest-elasticity of aggregate investment (-5) as close to data moments as in Zwick and Mahon (2017) and Koby and Wolf (2019).[2]

Figure 1: Interest-Elasticity over $\bar{\xi}$



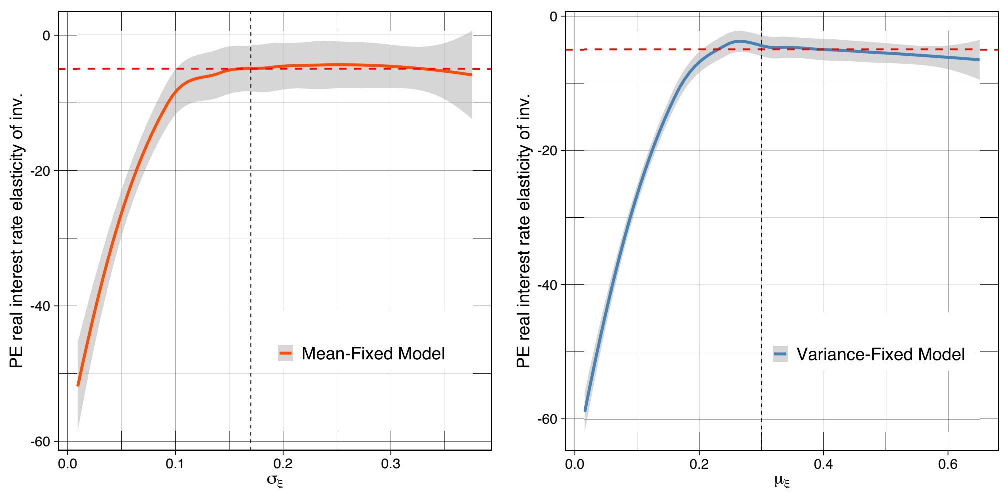
Note: The bundled model bundled mean and variance by $\bar{\xi}$: $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$. Therefore, increasing $\bar{\xi}$ increases both μ_ξ and σ_ξ simultaneously.

Upper Bound $\bar{\xi}$ and Interest-Elasticity of Aggregate Investment: The partial equilibrium

[2]To make the results more intuitive, I only include the nonconvex fixed cost and did not include the quadratic adjustment cost to constrain extreme investment behaviors. As a result, the model cannot exactly match all the micro-investment moments as in Zwick and Mahon (2017).

interest-elasticity of aggregate investment is defined by how the aggregate investment of all heterogeneous firms as a whole responds to an unexpected real interest rate shock. For instance, -5 means when firms face an unexpected real interest rate cut of 1%, the aggregate investment will increase by 5%. Quasi-experimental evidence in Koby and Wolf (2019) suggests this interest-elasticity should be about -5. In Figure 1, I plot this interest-elasticity against the choice of $\bar{\xi}$ from 0.025 to 1. A large calibration of $\bar{\xi}$ would give us an interest-elasticity equals -5. Otherwise, the aggregate investment will be oversensitive to interest rate changes, claiming that lumpy investment is irrelevant for aggregate dynamics.

Figure 2: Interest-Elasticity over μ_ξ and σ_ξ



(a) $\mu_\xi^* = \bar{\xi}/\sqrt{12}$, change σ_ξ

(b) $\sigma_\xi^* = \bar{\xi}/2$, change μ_ξ

Note: The variance-fixed model fixes the variance by choosing $\sigma_\xi^* = \bar{\xi}/2$ and the mean-fixed model fix the mean by choosing $\mu_\xi^* = \bar{\xi}/\sqrt{12}$. The two model are identical at both vertical dotted line when $\mu_x i = 0.3$ and $\sigma_x i \simeq 0.17$.

Mean/Variance and Interest-Elasticity of Aggregate Investment: Now I depart from the benchmark calibration of $\bar{\xi} = 0.6$. Instead, I study two groups of calibrations: 1).fix $\sigma_\xi^* = \bar{\xi}/\sqrt{12} = 0.6/\sqrt{12}$, variate μ_ξ from 0 to $2\mu_\xi^* = \bar{\xi}$ to show how the interest-elasticity changes; and 2).fix $\mu_\xi^* = \bar{\xi}/2 = 0.6/2$, variate σ_ξ from 0 to $2\sigma_\xi^* = \bar{\xi}/\sqrt{3}$ to show how the interest-elasticity changes. I conduct the same quasi-experimental real interest rate shock as the one in Figure 1.

The findings are plotted in Figure 2 which are quite impressive. First, the interest-elasticity is not solely determined by the expected size (*Mean*) of the nonconvex cost. Unlike common claims that the interest-elasticity is controlled by the expected size of the nonconvex cost, the uncertainty (*Variance*) plays a role. In panel (a), even though the *Mean* is fixed to be relatively large when the *Variance* approaches zero, the interest-elasticity is still huge. Given the *Mean* fixed, the model hits the targeted interest-elasticity when the *Variance* is equal or larger compared to the bundled model. Second, the interest-elasticity is not solely determined by the uncertainty (*Variance*) of the nonconvex cost. In panel (b), even though the *Variance* is fixed to be relatively large when the *Mean* approaches zero, the interest-elasticity is still huge. Given the *Variance* fixed, the model hits the targeted interest-elasticity when the *Mean* is equal or larger than the bundled model.

The calibration of a large upper bound $\bar{\xi}$ coincidentally matches the sizeable requirement of both the *Mean* and the *Variance* which delivers reasonable interest-elasticity of aggregate investment. What is the mechanism behind choosing the *Mean* and the *Variance*, respectively?

4 The Mechanism

To further inspect the mechanism behind the differences between the *Mean* and the *Variance*, I demonstrate results from three models with three alternative calibrations: 1).the carefully calibrated bundled model (*Bundled*); 2).a mean-fixed model ($\mu_\xi^* = \bar{\xi}/\sqrt{12}$) with zero variance (*Zero-*σ_ξ); and a variance-fixed model ($\sigma_\xi^* = \bar{\xi}/2$) with zero mean (*Zero-*μ_ξ).

A Decomposition of the Interest-Elasticity: I first show the decomposition of the interest-elasticity in all three models into both extensive margin and intensive margin investment in Table 1. The carefully calibrated *Bundled* model has an interest-elasticity of -5.1, consists of 96% in the extensive margin and 4% in the extensive margin. The *Zero-*σ_ξ model has an interest-elasticity smaller than -600, which is almost all coming from the extensive margin. The *Zero-*μ_ξ model has an interest-elasticity of -80, but which is mainly coming from the intensive margin.

Table 1: A Decomposition of the Interest-Elasticity

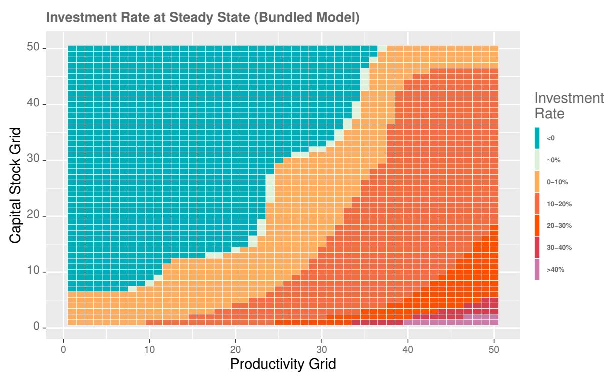
	Bundled			Zero-σ_ξ			Zero-μ_ξ		
Decomp.	Total	EM	IM	Total	EM	IM	Total	EM	IM
Elasticity	-5.1	-4.9	-0.2	-601.8	-601.7	-0.1	-80.0	-4.4	-75.6
Percentage	—	96%	4%	—	100%	0%	—	5.5%	94.5%

Note: The *Bundled* model has the calibration that matches the micro investment moments. The $Zero-\sigma_\xi$ deviates by setting σ_ξ to zero while all other parameters are unchanged. The $Zero-\mu_\xi$ deviates by setting μ_ξ to zero while all other parameters are unchanged.

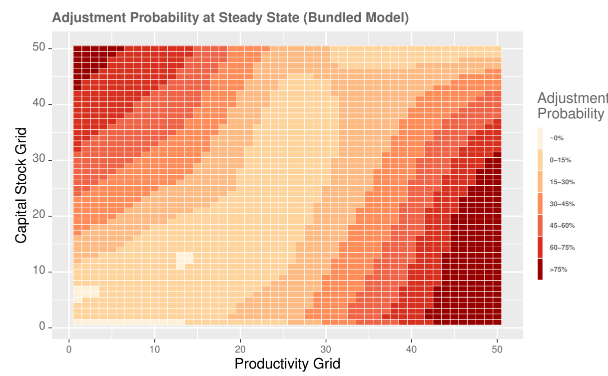
This decomposition shows that the *Mean* and the *Variance* play different roles. Without a sizable *Mean*, the response of the aggregate investment to the interest rate is mainly from the intensive margin. When the quadratic adjustment cost is absent, the intensive margin would be much too sensitive to real interest rate changes, which delivers a falsely large interest-elasticity of aggregate investment. Without a sizable *Variance*, the response of the aggregate investment to the interest rate is mainly from the extensive margin. The extensive margin is super sensitive to changes in real interest rates. A firm either choose to pay μ_ξ^* and invest a lot or stay total inactive when the real interest rate changes. Firms on the extensive margin choose to "all-in" which creates the unrealistically huge interest-elasticity.

Distributions of the Extensive Margin and the Intensive Margin: In figure 3, I plot the distributions of the extensive margin (adjustment probability) and the intensive margin (investment rate) at a steady state on two-dimensional plots with respect to productivity and capital stock. Since the intensive margin (investment rate) distribution is not much different across models, I only plot for the *Bundled* model. Warmer and darker color indicates larger investment rate and higher adjustment probability. In subplot (a), we could see that productive and low capital firms invest more when they have the chance to. In subplot (b), we could see the distribution of extensive margin is layered from 0 in the diagonal to higher probabilities away from the diagonal.

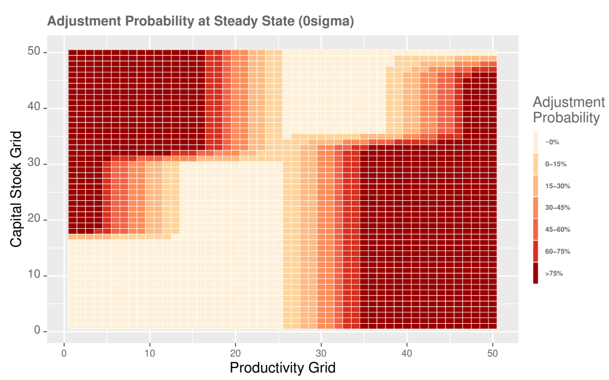
Figure 3: Distributions of the Extensive Margin and the Intensive Margin



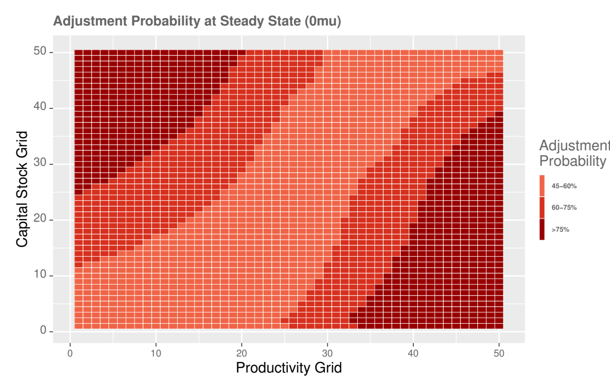
(a) Intensive Margin (Bundled)



(b) Extensive Margin (Bundled)



(c) Extensive Margin (Zero-σ_ξ)



(d) Extensive Margin (Zero-μ_ξ)

However, for the $Zero-\sigma_\xi$ model and the $Zero-\mu_\xi$ model, the extensive margin distributions are entirely different. The extensive margin in $Zero-\sigma_\xi$ model shows a vertical sorting pattern that is sharply moving from 0 probability of adjusting to almost 100% possibility of adjusting. That is why the extensive margin is super interest-rate sensitive. However, the extensive margin adjusting probability in $Zero-\mu_\xi$ model is always higher than 45% and has much smaller variations. As a result, the extensive margin is not that sensitive to interest rate changes.

5 Conclusion

Nonconvex capital adjustment cost plays an essential role in generating the data-consistent lumpy investment behaviors. Literature usually assumes a uniform distribution of the nonconvex adjustment cost starting from 0 to an upper bound, which did not distinguish the separate roles played by the *Mean* and the *Variance* of the distribution. In this paper, I show that both a sizable *Mean* and a sizable *Variance* are necessary for the lumpy investment models to generate empirical-consistent interest-elasticity of aggregate investment. The *Mean* governs how much the extensive margin accounts for aggregate investment dynamics. In contrast, the *Variance* controls how sensitive the extensive margin is to interest rate changes. Therefore, both are necessary quantitatively in a reasonably calibrated lumpy investment model. It will be interesting to further research to distinguish more realistic calibrations of *Mean* and the *Variance* using microdata on firm-level investment.

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