Lumpy Investment, Fluctuations in Volatility, and Monetary Policy

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 - 2. Why does it matter?

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 - 2. Why does it matter?

[One of the reasons]: It affects the amount of monetary policy stimulus required in high volatility times like now or in the Great Recession.

The detailed questions

Q1: What are the estimates of $\frac{dI}{de^m}(\sigma)$ for different σ in the data?

In words: How does an increase in volatility of firm-level TFP affect the impact of monetary policy on aggregate investment?

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In words: How does an increase in volatility of firm-level TFP affect the impact of monetary policy on aggregate investment?

Q2: Could micro-founded macro models explain the estimates? And how?

In words: What key micro-foundations of the model could replicate the observations in the data?

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▶ Empirical: Monetary stimulus is less effective in times of high volatility

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 - ▶ Aggregate transmission depends on the level of volatility:
 - Monetary policy is less effective when volatility is elevated

► Aggregate investment = extensive margin + intensive margin:

$$I = \sum_{j \in EM} i_j + \sum_{j \in IM} i_j$$

▶ (1) Extensive margin is less responsive to MP with elevated volatility

$$rac{d\sum_{j\in EM}i_j}{de^m}\left(\sigma_t
ight)\downarrow$$
 , when $\sigma_t\uparrow$

▶ (2) Reasonable sensitivity of investment to interest rate and volatility

Both data consistent
$$\frac{d\sum_{j\in EM}i_j}{dr}$$
 & $\frac{d\sum_{j\in EM}i_j}{d\sigma}$

▶ The inv. channel of monetary policy works through both margins:

$$\frac{dI}{d\epsilon_t^m} = \frac{d\sum_{j \in EM} i_j}{d\epsilon_t^m} + \frac{d\sum_{j \in IM} i_j}{d\epsilon_t^m}$$

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Both data consistent $\frac{a \sum_{j \in EM} i_j}{a}$ & $\frac{a \sum_{j \in EM} i_j}{a}$

Monetary policy is less effective stimulating aggregate investment:

$$\underbrace{\frac{dI}{d\varepsilon_{t}^{m}}\left(\sigma_{t}\right)}_{\downarrow\downarrow} = \underbrace{\frac{d\sum_{j\in\textit{EM}}i_{j}}{d\varepsilon_{t}^{m}}\left(\sigma_{t}\right)}_{\downarrow\downarrow} + \underbrace{\frac{d\sum_{j\in\textit{IM}}i_{j}}{d\varepsilon_{t}^{m}}\left(\sigma_{t}\right)}_{\approxeq}, \text{ when } \sigma_{t}\uparrow$$

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1 Extensive margin is less responsive to MP with elevated volatility

$$\frac{d\sum_{j\in EM}i_j}{d\epsilon_t^m}\left(\sigma_t\right)\downarrow \text{ , when }\sigma_t\uparrow$$

- ► Could we take the extensive margin mechanism 1 as granted?
 - ▶ Is any extensive margin (lumpy inv.) model sufficient to generate this result?

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Extensive margin is less responsive to MP with elevated volatility

$$\frac{d\sum_{j\in EM}i_j}{d\epsilon_t^m}\left(\sigma_t\right)\downarrow \text{ , when } \sigma_t\uparrow$$

- No. We still need two dynamic properties from the micro-foundation:
 - ▶ Data consistent interest rate sensitivity of $I \Rightarrow$ a reasonable $\frac{d \sum_{j \in EM} i_j}{dr}$
 - \blacktriangleright Data consistent volatility sensitivity of $I\Rightarrow$ a reasonable $\frac{d\sum_{j\in EM}i_j}{d\sigma}$

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Extensive margin is less responsive to MP with elevated volatility

$$\frac{d\sum_{j\in EM}i_j}{d\epsilon_i^m}\left(\sigma_t\right)\downarrow$$
 , when $\sigma_t\uparrow$

▶ (2) Reasonable sensitivity of investment to interest rate and volatility

Both data consistent
$$\frac{d\sum_{j\in EM}i_j}{dr}$$
 & $\frac{d\sum_{j\in EM}i_j}{d\sigma}$

Literature review

1. Volatility/State-dependent Effects of Monetary Policy

Vavra (2013), Koby&Wolf (2019), Baley&Blanco (2019), Li (2020), McKay&Wieland (2020), Castelnuovo&Pellegrino (2018), Eickmeier et al. (2016); I show that the inv. channel of monetary policy is also volatility-dependent.

2. New Keynesian Models with Capital Accumulation

Christiano et al. (2005), Smets&Wouters (2003,2007), Reiter et al. (2013), Ottonello&Winberry (2018), Jeenas (2018);

I show that the lumpy inv. could co-exist with reasonable inv. IRFs. w.r.t. MP.

3. Volatility in RBC and/or for Stimulus Policy

Abel et al. (1996), Dixit et al. (1994), Bloom (2009), Bloom et al. (2018), Bachmann&Bayer (2013), Gilchrist et al. (2014), Arellano,Bai,&Kehoe (2019); I show that second-moment shocks reduce the effects of first-moment policy.

4. Aggregate Implications of Lumpy Investment

Caballero et al. (1995), Caballero&Engel (1999), Thomas (2002), Khan&Thomas (2008), Bachmann et al. (2013), House (2014), Winberry (2018b), Koby&Wolf (2019), Baley&Blanco (2020);

I show that lumpy investment matters for monetary policy as well.

Roadmap

- Q1: What are the estimates of $\frac{dI}{d\epsilon^m}(\sigma)$ for different σ in the data?
 - 0. Local projection of investment responses to identified monetary shocks
- Q2: Could micro-founded macro models explain the estimates? And how?
 - 1. A heterogeneous firm New Keynesian model with lumpy investment
 - 2. Volatility shock and the solution method
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[Empirical Motivation] Q1: What is $\frac{dI}{d\epsilon^m}(\sigma)$ for different σ in the data?

Data Details

Quarterly National Income and Product Account + Monetary Shocks + Volatility

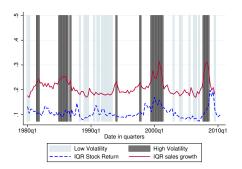
- ▶ Investment Indicator: Real non-residential private fixed investment
- ▶ Monetary Shocks: High-frequency-identified from Gertler-Karadi-2015
- ▶ Volatility Indicator: Interquantile Range (IQR) of sales growth

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Figure: Top20% vs Bottom 20%: 0.18 vs 0.26



Empirical strategy

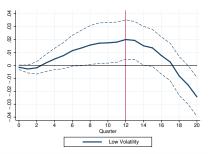
Baseline Local Projection Specification following Jorda (2005)

$$\Delta_{h}I_{t+h} = \alpha_{h} + \gamma_{j,h} \epsilon_{t}^{m} \times \mathbf{1}_{\sigma_{t} \in J^{\sigma}} + \sum_{l=0}^{L} \Gamma_{h,t-l}^{\prime} Z_{t-l} + \epsilon_{h,t}$$
 (1)

- $\sigma_t \in J^{\sigma} \equiv \{h, m, l\}$ indicates which group level of volatility at time t belongs to
- $ightharpoonup \sigma_t = IQR_{se,t}$ is the sales growth interquantile range of 25yr+ Compustat firms
- $ightharpoonup \epsilon_t^m$ is sign-flipped and standardized monetary policy shock (/-25bps)
- Z_{t-1}: conditional on volatility group, consumer price index (CPI), output gap, and consumption up to four quarters L = 4; α_h: h-period ahead fixed effect
- \triangleright Coefficient $\gamma_{i,h}$ measures slope of investment semi-elasticity w.r.t. volatility

Inv. response to monetary stimulus with low volatility

► Impulse Response to monetary policy shock:

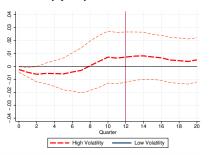


► Effectiveness of monetary policy:

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dI}{de^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg}\uparrow$
Data	2.0%	0.18				

Inv. response to monetary stimulus with high volatility

► Impulse Response to monetary policy shock:

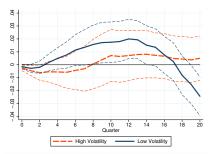


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Data			0.75%	0.26		

High volatility lowers inv. responses to monetary stimulus

► Impulse Response to monetary policy shock:



► Reduction in the effectiveness of monetary policy: 62%

	Low V	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg}\uparrow$	
Data	2.0%	0.18	0.75%	0.26	62%	44%	

Robustness of main result & additional facts

Robustness Checks:

Choices	LP Form	MP shock ϵ_t^m	Volatility σ_t	Investment	Periods
1	Grouped	GK-HFI	IQR sales growth	RGFCF	80-10
2	Interacted	RIR	IQR stock return	RGPI	85-10
3				RPFI	85-07
4				RPFI-NR	80-07
5				RPFI-NR-EQMT	60-10
6				RPFI-NR-Struct	60-07
7				RPFI-NR-IP	60-18

- ► Interacted: replacing $\gamma_{j,h} \epsilon_t^m \times \mathbf{1}_{\sigma_t \in J^{\sigma}}$ with $(\beta_h + \gamma_h) \sigma_t \times \epsilon_t^m$
- Almost all hold in all $2 \times 2 \times 2 \times 7 \times 7 2 \times 2 \times 7 \times 3 = 308$ alternatives
- Additional Facts
 - ► Firm-level regressions using Compustat Quarterly
 - ► Tobit and Probit Local Projections to inspect mechanism

[Quantitative Theory] Q2: Could micro-founded macro models explain the estimates?

And how?

Roadmap of the Quantitative Theory

- 1. A heterogeneous firm New Keynesian model with lumpy investment
- 2. Volatility shock and the solution method
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Model overview

Heterogeneous Production Firms:

- Produce and invest subject to capital adj. costs
- ► Face idiosyncratic productivity shocks

A New Keynesian Block

- Retailers differentiate production firms' output + Rotemberg sticky price
- ► Monetary authority follows Taylor Rule

A Family of Representative Households

Owns firms + choose consumption, hours of working, and saving.

Production firms

Enter period with state variables (z_{jt}, k_{jt})

1. Production:

$$y_{jt} = z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1$$
 (2)

- ▶ Sell at relative price p_t^w
- 2. Idiosyncratic TFP shock:

$$log(z_{jt}) = -\frac{\sigma_z^2}{2(1+\rho)} + \rho_z log(z_{jt-1}) + \sigma_z \epsilon_{jt}$$
(3)

Production Firms

Enter period with state variables (z_{jt}, k_{jt})

Cost of Investment:

$$c(i_j) = i_j + \frac{\Phi_k}{2} \left| \frac{i_j}{k_j} \right|^2 k_j + \mathbf{1}_{(i_j < 0)} \cdot S \cdot |i_j| + \mathbf{1}_{(i_j \notin [-ak,ak])} \cdot \xi_j \cdot w_t$$

$$\xi_j \sim U[0, \bar{\xi}]$$

$$(4)$$

- 1. Quadratic Adj. Costs ϕ_k :
 - Extremely costly to make huge changes in capital stock
- 2. Partial Irreversibility *S*: Disinvestment will cost *S* proportional loss in inv.
 - Caution of investment today because of potential disinvest costs tomorrow
- 3. Random Fixed Costs ξ_i : Randomly occurred cost paid in unit of labor
 - "Lucky" or "unlucky" draws determine inaction or action

Optimal Investment Decisions

Extensive Margin:
$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t}$$
 (5)

Intensive Margin:
$$k_{jt+1} = \begin{cases} (1-\delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1-\delta)k_{jt} + i_{jt}^C & otherwise \end{cases}$$
 (6)

- Both irreversibility and fixed cost create inactions at the extensive margin:
 - ► Irreversibility governs $\xi_t^*(k_{it}, z_{it}; \Omega_t)$ sensitivity to volatility
 - Fixed Cost governs $\xi_t^*(k_{jt}, z_{jt}; \Omega_t)$ sensitivity to interest rate

Retailers and final good producer

- Monopolistically competitive retailers
 - ► Technology: $\tilde{y}_{jt} = y_{jt} \Rightarrow \text{marginal cost} = p_t^w$
 - Subject to price adj. costs: $AC_p = \frac{\psi_p}{2} \left(\frac{p_{jt}}{p_{jt-1}} 1 \right)^2 P_t Y_t$
- ▶ Perfectly competitive final good producer

► Technology:
$$Y_t = \left(\int \tilde{y}_{jt}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_t = \left(\int p_{jt}^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}}$$

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- ▶ Perfectly competitive final good producer
 - ► Technology: $Y_t = \left(\int \tilde{y}_{jt}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_t = \left(\int p_{jt}^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}}$
- ► Implies the New Keynesian Phillips curve

$$log\Pi_{t} = \frac{\gamma - 1}{\Psi_{p}}log\frac{p_{t}^{w}}{p^{w*}} + \beta E_{t}log\Pi_{t+1}$$

Monetary authority and household

► Monetary authority follows the **Taylor rule**

$$log R_t^n = log \frac{1}{\beta} + \phi_{\Pi} log \pi_t + \epsilon_t^m$$

Monetary authority and household

► Monetary authority follows the **Taylor rule**

$$log R_t^n = log \frac{1}{\beta} + \phi_{\Pi} log \pi_t + \mathbf{e}_t^m$$

► A family of representative household with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \Theta N_t \right)$$

- Labor-leisure choice $\Rightarrow w_t = \theta C_t^{\eta}$
- Consumption-saving choice $\Rightarrow \Lambda_{t,t+1} = \beta \left(\frac{C_t}{C_{t+1}}\right)^{\eta}$

Stationary Equilibrium

- ► An equilibrium of this model satisfies
 - 1. Production firms choose investment policies $k'_t(z,k)$ and $\xi^*_t(z,k)$
 - 2. Retailers and final good producers generate NK Phillips curve
 - 3. Monetary authority follows Taylor rule $log R_t^n = log \frac{1}{\beta} + \varphi_\Pi log \pi_t + \varepsilon_t^m$
 - 4. Households choose labor supply N_t and generate SDF $\Lambda_{t,t+1}$

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Solve the stationary equilibrium

- ▶ Solve the stationary equilibrium (policy/distribution) with no aggregate risk
 - Non-stochastic simulation (Young, 2010) for value/policy functions
 - Stochastic simulation for parameterization sample
- ► Compute the stationary equilibrium moments
 - Steady state investment distribution moments
 - Use for identification of lumpy investment parameters

Solve the transitional equilibrium

▶ Volatility shock: a MIT shock (unexpected increase) to the variance σ_z • timing

$$log(z_{jt}) = -\frac{\sigma_z^2}{2(1+\rho)} + \rho_z log(z_{jt-1}) + \sigma_z \epsilon_{jt}$$

Solve the transitional equilibrium

▶ Volatility shock: a MIT shock (unexpected increase) to the variance σ_z \circ timing

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- Compute perfect foresight transition path following aggregate shocks
 - Case One: MP shock only; Case Two: MP shock + Vol. shock

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- Compute perfect foresight transition path following aggregate shocks
 - Case One: MP shock only; Case Two: MP shock + Vol. shock
 - ▶ I update all aggregate price paths all at once using *excessive demand* which is super fast (seconds for 200 periods even without parallel computing)
 - Captures all non-linear dynamics following a volatility shock (Global sol.)
 - Captures all non-linear dynamics of interactions between shocks which is the key that the results in this model is achieved

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Fixed parameters

Parameter	Description	Value
Households		
β	Discount factor	0.99
η	Elasticity of intertemporal substitution	1
θ	Leisure preference	2
Production Firms	-	
α	Capital coefficient	0.25
ν	Labor coefficient	0.60
δ	Capital depreciation	0.026
ρ_z	Persistence of TFP shock	0.95
New Keynesian		
γ	Demand elasticity	10
ψ_p	Price adjustment cost	90
φπ	Taylor rule coefficient	1.5

Parameters to be computed

► How large should the volatility be, respectively?

Parameter	Description	Value
Volatility Level		
σ_z^l	Volatility of TFP shock (normal time)	0.05
σ_z^h	Volatility of TFP shock (elevated)	0.13

▶ Moments to match

Moment	Data	Model
IQR sales growth IQR_{sg} (normal time)	0.18	0.18
IQR sales growth IQR_{sg} (elevated)	0.26	0.26



Parameters to be computed

▶ Recap of the Cost Function of Investment:

$$c(i_j) = i_j + \frac{\Phi_k}{2} \left| \frac{i_j}{k_j} \right|^2 k_j + \mathbf{1}_{(i_j < 0)} \cdot S \cdot |i_j| + \mathbf{1}_{(i_j \notin [-ak,ak])} \cdot \xi_j \cdot w_t$$
$$\xi_j \sim U[0,\bar{\xi}]$$

► How large should the adjustment costs be, respectively?

Parameter	Description	Value
Adjustment Costs		
ξ	Upper bound of fixed cost	
S	Partial Irreversibility	
Φ_k	Quadratic adjustment cost	

Targets

► Cross-section Moments of Investment: (Zwick and Mohan 2017)

Moment	Description (annual)	Data	Model
$\mathbf{E}\left[i/k\right]$	Mean investment rate	10.4%	
$\sigma(i/k)$	Standard dev. of investment rates	0.16	
$P(i/k \geqslant 20\%)$	Spike rate of investment	14.4%	
P(i/k < 20%)	Positive rate of investment	85.6%	

▶ Dynamic Moments of Investment: (Zwick and Mohan 2017, Baley and Blanco 2020)

$Cor(\frac{i}{k}, \frac{i+1}{k+1})$ Autocorrelation of investment rates 0.40 $Cov(x, age)$ Covariance of capital gap and age since last adj. 0.29	Moment	Description (annual)	Data	Model
	$Cor(\frac{i}{k}, \frac{i+1}{k+1})$	Autocorrelation of investment rates	0.40	
	Cov(x, age)	Covariance of capital gap and age since last adj.	0.29	

^{*}capital gap: $x = log(\frac{k_t}{z_t}) - E\left[log(\frac{k_t}{z_t})\right]$, without frictions, capital gap= 0.

▶ I pin down these parameters using both cross-section and dynamic moments

1.The choice of ϕ_k

► The Choice of ϕ_k : (the conventional cost in the literature)

I choose quad. adj. costs to match the cross-section moments $\Rightarrow \phi_k = 4.00$

Cross-section Moments of Investment: (Zwick and Mohan 2017)

Moment	Description	Data	Model
$\mathbf{E}\left[i/k\right]$	Mean investment rate (annual)	10.4%	10.1%
$\sigma(i/k)$	Standard dev. of investment rates (annual)	0.16	0.12
$P(i/k \geqslant 20\%)$	Spike rate of investment (annual)	14.4%	15.3%
P(i/k < 20%)	Positive rate of investment (annual)	85.6%	84.7%

- Next, I pin down the lumpy adj. parameters using both dynamic moments
- ▶ Dynamic Moments of Investment: (Zwick and Mohan 2017, Baley and Blanco 2020)

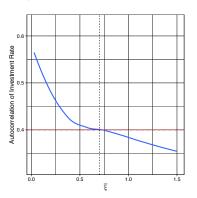
$Cor(\frac{i}{k}, \frac{i+1}{k+1})$ Autocorrelation of investment rates 0.40 $Cov(x, age)$ Covariance of capital gap and age since last adj. 0.29	Moment	Description (annual)	Data	Model
	$Cor(\frac{i}{k}, \frac{i+1}{k+1})$	Autocorrelation of investment rates	0.40	
		Covariance of capital gap and age since last adj.	0.29	

^{*}capital gap: $x = log(\frac{k_t}{z_t}) - E\left[log(\frac{k_t}{z_t})\right]$, without frictions, capital gap = 0.

1.The choice of ξ

First, the autocorrelation of investment rates almost uniquely pins down the upper bound of random fixed costs $\bar{\xi} = 0.70$

Figure: Autocorrelation of Investment Rates

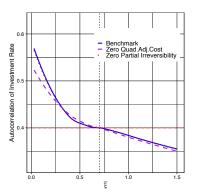


▶ The reason is that this cost is the only "random" cost, which will decrease the autocorrelation monotonically with the existence of other costs

1. The choice of $\bar{\xi}$

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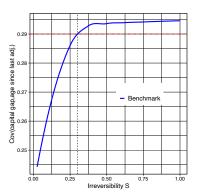


► The reason is that this cost is the only "random" cost, which will decrease the autocorrelation monotonically with the existence of other costs

2.The choice of *S*

Second, conditional on $\xi = 0.7$, the covariance of capital gap and no-adjustment age since last adjustment suggests a large partial irreversibility S = 0.3

Figure: Covariance of capital gap and no-adjustment age since last adjustment



► The reason is that larger irreversibility constraints firms to disinvest so positive capital gap $x = log(\frac{k_t}{z_t}) - E\left[log(\frac{k_t}{z_t})\right] > 0$ lasts longer age (natural depreciation)

How does the choice of $\bar{\xi}$ and S matter for the story?

- ightharpoonup $\bar{\xi}$ governs how sensitive lumpy investment is w.r.t monetary policy shocks
- S governs how sensitive lumpy investment is w.r.t volatility shocks
- Only empirically consistent ξ & S could generate data consistent IRFs to monetary policy shocks and volatility shocks, and eventually volatilitydependent IRFs to monetary policy

Fly to sensitivity

Parameters to be computed

► How large should the adjustment costs be, respectively?

Parameter	Description	Value	
Adjustment Costs			
ξ	Upper bound of fixed cost	0.70	
S	Partial Irreversibility	0.30	
Φ_k	Quadratic adjustment cost	4.00	

Targets

► Cross-section Moments of Investment: (Zwick and Mohan 2017)

Moment	Description	Data	Model
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Dynamic Moments of Investment: (Zwick and Mohan 2017, Baley and Blanco 2020)

Moment	Description (annual)	Data	Model
$Cor(\frac{i}{k}, \frac{i_{+1}}{k_{+1}})$	Autocorrelation of investment rates	0.40	0.40
Cov(x, age)	Covariance of capital gap and age since last adj.	0.29	0.29

^{*}capital gap: $x = log(\frac{k_t}{z_t}) - E\left[log(\frac{k_t}{z_t})\right]$, without frictions, capital gap= 0.

► The dynamic moments are essential so that investment is of empirically consistent sensitivity to monetary shocks and volatility shocks

Roadmap of the Quantitative Theory

- 1. A heterogeneous firm New Keynesian model with lumpy investment
- 2. Volatility shock and the solution method
- 3. Parameterization and identification of lumpy investment
- 4. Volatility-dependent effectiveness of monetary policy
- 5. Inspecting the mechanism in the model

Two experiments:

- ▶ Low Vol.: a conventional MP shock to TR residual $\epsilon_1^m = -25 bps$ with $\rho^m = 0.5$
- ▶ High Vol.: the same MP shock when a volatility shock hits as well

A Fair Comparison:

- ▶ Impulse Responses of Low Volatility vs. High Volatility w.r.t the MP shock
- Compute the peak responses in both cases when the MP shock hits

	Low V	olatility/	High V	olatility	Δ Effe	ctiveness
Sources	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg}\uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%
Model		0.18		0.26		44%

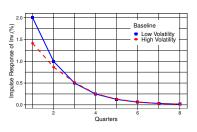
► The model explains ???% of the reduction in the effectiveness of monetary policy

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg}\uparrow$
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	Low Volatility High Volatility		Δ Effectiveness			
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Data	2.0%	0.18	0.75%	0.26	62%	44%
Model		0.18		0.26		44%

▶ Monetary policy generates less IRFs of investment when volatility is high

Figure: Differential IRFs w.r.t. a monetary shock



Compare the peak impulse response in both cases when the MP shock hits

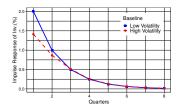
	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d \in m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg}\uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%
Model	2.0%	0.18	1.4%	0.26	30%	44%

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	Low Volatility		High Volatility		Δ Effectiveness	
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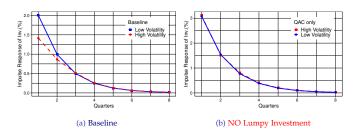
- ▶ The model explains $\frac{30}{62} = 48\%$ of the reduction in the effectiveness of MP
- ▶ The result is within the confidence interval of my estimates.

Figure: Differential Investment IRFs in Alternative Models

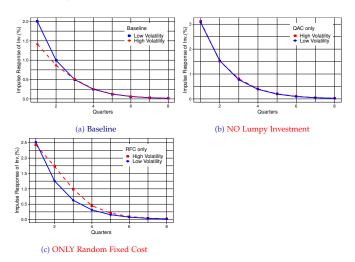


(a) Baseline

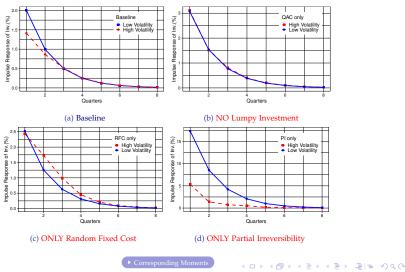












- ▶ The model successfully replicates the reduction in the effectiveness of MP
- ▶ The result is within the confidence interval of my estimates.
- Specification of lumpy investment parameters is the key.

Let's fly to the conclusion if we do not have enough time Conclusion

Roadmap of the Quantitative Theory

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Key mechanism in the model

Monetary policy is less effective stimulating aggregate investment:

$$\underbrace{\frac{dI}{d\varepsilon_{t}^{m}}\left(\sigma_{t}\right)}_{\downarrow\downarrow} = \underbrace{\frac{d\sum_{j\in\textit{EM}}i_{j}}{d\varepsilon_{t}^{m}}\left(\sigma_{t}\right)}_{\downarrow\downarrow} + \underbrace{\frac{d\sum_{j\in\textit{IM}}i_{j}}{d\varepsilon_{t}^{m}}\left(\sigma_{t}\right)}_{\approxeq}, \text{ when } \sigma_{t}\uparrow$$

Extensive margin is less responsive to MP with elevated volatility

$$\frac{d\sum_{j\in EM}i_j}{d\epsilon_i^m}(\sigma_t)\downarrow$$
, when $\sigma_t\uparrow$

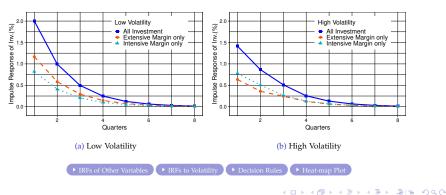
Reasonable sensitivity of investment to interest rate and volatility

Both data consistent
$$\frac{d\sum_{j\in EM}i_j}{dr}$$
 & $\frac{d\sum_{j\in EM}i_j}{d\sigma}$

Key mechanism in the model: Inspection (1)

▶ The decrease of the extensive margin accounts for most of the drops (90%)

	Low Volatility			High Volatility		
IRFs	Total	EM	IM	Total	EM	IM
Number	2.0%	1.17%	0.82%	1.4%	0.63%	0.78%

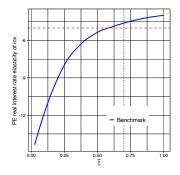


Key mechanism in the model: Inspection (2)

- Elasticity of investment to real interest rate should be -5 (Koby-Wolf-2020)
- ► Elasticity of investment to volatility should be negatively large (Bloom-2009)
- \blacktriangleright My choices of $\bar{\xi}$ and S match both sensitivities in considerable ranges

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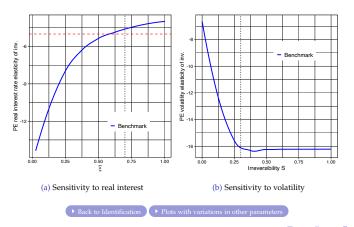


(a) Sensitivity to real interest

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Summary of the mechanism

Lumpy investment and volatility play essential roles:

- 1. The inv. channel of monetary policy works mainly through the extensive margin
- 2. Extensive margin adjustment probability is lowered when volatility is elevated
- 3. The decrease of the extensive margin accounts for most of the drops (90%).

Which one of the lumpy capital adjustment costs plays the central role? Both

- 4. Random fixed costs governs the sensitivity of investment to interest rate
- 5. Irreversibility governs the sensitivity of investment to volatility
- 6. Jointly, they determine the volatility-dependent effectiveness of monetary policy

▶ I estimated the volatility-dependent effectiveness of monetary policy in the data.

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- I show that this estimate is consistent with the implications of a macro model of investment with a plausible parameterization of firm-level adjustment costs.
- This implies that fluctuations in volatility interacting with lumpy investment plays an essential role in monetary policy transmission to aggregate investment
- ► Further work: using the model to do normative analysis (ZLB, other types of monetary policy, second moment policy, ...)

Backup Slides

Data Details Dack

- 1. Monetary policy shocks ϵ_t^m : high-frequency identified as in Gertler-Karadi-2015
 - use HFI FFR30 within 30mins window around FOMC announcements as an IV for the one-year government bond rate in the following VAR
 - run a monthly IV-VAR with log industrial production, employment rate, log CPI and a measure of corporate interest spreads
 - predict the residual of the instrumented one-year government bond rate and then accumulate them to a quarterly series.
 - ▶ sign-flipped and standardized (dividing by -25bps)

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 - sign-flipped and standardized (dividing by -25bps)
- 1. High vs. Low Volatility $\sigma_{z,t} \colon Top~20\%$ vs. Bottom 20% in IQR sales growth
 - measures including IQR sales growth, IQR stock return, ...
 - compare the impulse responses of inv. during High vs. Low Volatility times

Recursive Production Firms' Problem Back

Value Function

$$V^{A}(k_{jt}, z_{jt}; \Omega_{t}) = \max_{i,n} \left\{ -c(i_{jt}) + \mathbb{E}[p_{t}^{w}y_{jt} - w_{t}n_{jt} + \Lambda_{t,t+1}V(k_{jt+1}^{*}, z_{jt+1}; \Omega_{t+1})] \right\}$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_{t}) = \max_{i \in [-ak, ak], n} \left\{ -c(i_{jt}) + \mathbb{E}[p_{t}^{w}y_{jt} - w_{t}n_{jt} + \Lambda_{t,t+1}V((k_{jt+1}^{C}, z_{jt+1}; \Omega_{t+1})] \right\}$$

$$V(k_{jt}, z_{jt}; \Omega_{t}) = -\frac{w_{t}\xi^{*}(k_{jt}, z_{jt}; \Omega_{t})}{2} + \frac{\xi^{*}(k_{jt}, z_{jt}; \Omega_{t})}{\bar{\xi}} V^{A}(k_{jt}, z_{jt}; \Omega_{t}) + \left(1 - \frac{\xi^{*}(k_{jt}, z_{jt}; \Omega_{t})}{\bar{\xi}}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_{t})$$

$$(7)$$

Optimal Investment Decisions

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t}$$
(8)

$$k_{jt+1} = \begin{cases} (1-\delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1-\delta)k_{jt} + i_{jt}^C & otherwise \end{cases}$$
(9)

Enter period with state variables (z_{it}, k_{it})

1. Idiosyncratic TFP shock:

$$log(z_{jt}) = -\frac{\sigma_z^2}{2(1+\rho)} + \rho_z log(z_{jt-1}) + \sigma_z \epsilon_{jt}$$
 (10)

- 2. Volatility shock: (timing)
 - ► t^- : A heightened change in the standard deviation of TFP innovation ($\sigma_z \uparrow$)
 - t: Firms making investment decisions under uncertainty
 - \triangleright t^+ : Productivity z_{it} arrives and firms making production decisions

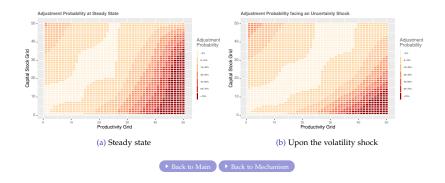
Details of the simulation

- ▶ I simulate 100k firms starting from a steady-state for 500 quarters
- ▶ Both volatility shock and monetary shock hit at the quarter 501
- ▶ The economy convergences back to steady-state in the quarter 700
- ▶ Largest firms who account for 45% of output are "Compustat" firms (~ 10%)
- ▶ "Compustat firms" older than 100+ quarters are used to calculate IQR_{sg} (~ 1%)
- Additional IQRs: (I choose $\sigma^l = 0.05$ and $\sigma^h = 0.13$ to match IQR_{sq} in the data)

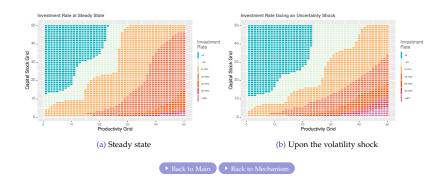
	Low Volatility		High Volatility		
Sources	σ_z^l	IQR_{sg}	σ_z^h	IQR_{sg}	
All firms	0.05	0.24	0.13	0.48	
Compustat	0.05	0.21	0.13	0.38	

▶ Back

Volatility and inv. decision rules at extensive margin

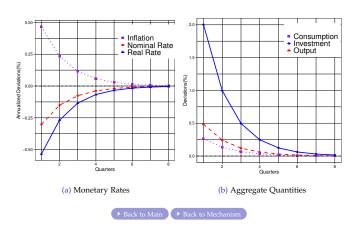


Volatility and inv. decision rules at intensive margin



Impulse responses to a monetary shock

Figure: Impulse responses to a monetary shock

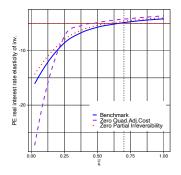


Sensitivity of investment w.r.t. lumpy parameter choices

- ► Elasticity of investment to real interest rate should be -5 (Koby-Wolf-2020)
- ▶ Elasticity of investment to volatility should be negatively large (Bloom et al.-2018)

Sensitivity of investment w.r.t. lumpy parameter choices

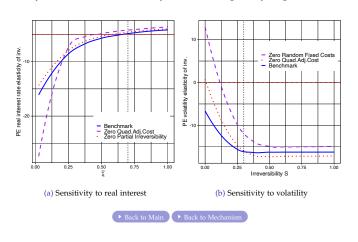
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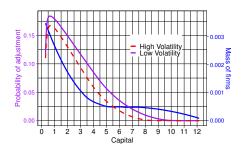
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How do volatility shocks change the investment policy?

Volatility shocks significantly lowered adjustment probability

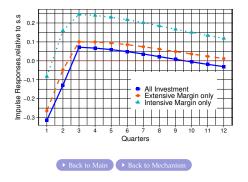
Figure: How does high volatility chance inv. incentive



Firms have much weaker incentive to invest in the extensive margin

The effect of volatility shock

Figure: A Decomposition of the inv. channel of monetary policy



Corresponding Moments of Alternative Models

Table: Moments in Alternative Parameterizations

Adjustment Costs	Benchmark	QAC Only	RFC Only	PI Only
ϕ_k (Quadratic adjustment cost)	4.00	3.20	0.0001	0.001
ξ̄ (Upper bound of fixed cost)	0.70	0.001	0.70	0.001
S (Resale loss in capital)	0.30	0.0001	0.0001	0.30
Annualized Cross-section Moments				
Average investment rate (%)	10.1%	10.1%	10.5%	10.3%
Standard deviation of investment rates	0.12	0.11	0.13	0.12
Spike rate (%)	15.3%	11.9%	14.3%	12.5%
Positive rate (%)	84.7%	88.1%	85.7%	87.5%
Annualized Dynamic Moments				
Autocorrelation of investment rates	0.40	0.78	0.39	0.62
Covariance of capital gap and age since last adj.	0.29	-0.10	0.07	-0.49

^{*}capital gap: $x = log(\frac{k_t}{z_t}) - E\left[log(\frac{k_t}{z_t})\right]$, without frictions, capital gap = 0.

