A Note on Nonconvex Adjustment Costs in Lumpy Investment Models: Mean versus Variance

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Abstract

This paper revisits the canonical assumption of nonconvex capital adjustment costs in lumpy investment models as in Khan and Thomas (2008), which are assumed to follow a uniform distribution from zero to an upper bound, without distinguishing between the *mean* and the *variance* of the distribution. Unlike the usual claim that the upper bound stands for the size (represented by the *mean*) of a nonconvex cost, I show that in order to generate an empirically consistent interest elasticity of aggregate investment, both a sizable *mean* and a sizable *variance* are necessary. The *mean* governs the importance of the extensive margin in aggregate investment dynamics, while the *variance* governs how sensitive the extensive margin is to changes in the real interest rate. As a result, both the *mean* and the *variance* are quantitatively important for aggregate investment dynamics.

Keywords: Lumpy investment; Ss model; firm heterogeneity;
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1 Introduction

In all current lumpy investment models such as Khan and Thomas (2008), the nonconvex capital adjustment cost ξ_{jt} is uniformly distributed with support $U[0, \bar{\xi}]$ independently across firms and time. A conventional calibration of a small upper bound $\bar{\xi}$ which matches the firm-level lumpy investment moments claims that lumpy investment is irrelevant for aggregate dynamics.

However, recent literature (Bachmann et al., 2013; Winberry, 2018; Koby and Wolf, 2019; Fang, 2020) reverses the claim that microeconomic investment lumpiness is inconsequential for macroeconomic analysis. The key argument is that the upper bound $\bar{\xi}$ of the distribution of nonconvex capital adjustment costs should be calibrated as much larger.

The upper bound $\bar{\xi}$ determines how sensitive the aggregate investment is to changes in the real interest rate. The calibration of a small (large) upper bound $\bar{\xi}$ generates a large (small) interest elasticity of aggregate investment. Quasi-experimental evidence on firm-level investment responses to tax changes (Zwick and Mahon, 2017; Koby and Wolf, 2019) suggests a small interest-elasticity of aggregate investment. Consequently, the upper bound $\bar{\xi}$ should be large.

What is the economics meaning of such a calibration of a large upper bound $\bar{\xi}$? In all current models, the setup implicitly assumes that the mean $\mu_\xi = \bar{\xi}/2$ and the standard deviation $\sigma_\xi = \bar{\xi}/\sqrt{12}$ are isometric ($\mu_\xi = \sqrt{3}\sigma_\xi$). This assumption means that when calibrating $\bar{\xi}$, we choose the expected size (*mean*) and the uncertainty (*variance*) of the nonconvex adjustment cost faced by the firms jointly. ***Then, which one determines the interest elasticity of aggregate investment?***

In this paper, I answer this question through disciplining the economic meanings of the expected size (*mean*) and the uncertainty (*variance*) of the nonconvex adjustment cost. I assume the nonconvex adjustment cost follows a uniform distribution with mean and variance $\{\mu_\xi, \sigma_\xi\}$:

$$\xi_{jt} \sim U[\mu_\xi - \sqrt{3}\sigma_\xi, \mu_\xi + \sqrt{3}\sigma_\xi] \tag{1}$$

I first compare the interest elasticity of aggregate investment over both dimensions of $\{\mu_\xi, \sigma_\xi\}$, departing from a conventionally calibrated lumpy investment model. The model generates unrealistically large interest elasticities of aggregate investment when either of the *mean* or the *variance* approach zero. I find that both a sizable *mean* and a sizable *variance* are necessary to generate an empirically consistent interest elasticity of aggregate investment.

Further inspection of the mechanism shows that the *mean* and the *variance* play different roles. A decomposition of the interest elasticity between the extensive margin and the intensive margin indicates the different roles of the *mean* and the *variance*. Without a sizable *mean*, the unrealistically large interest elasticity is mainly from the unconstrained intensive margin.[1] Without a sizable *Variable*, the unrealistically large interest elasticity is mainly from the oversensitive extensive margin. The underlying distribution of the extensive margin adjustment probability confirms these patterns. Therefore, a sizable *mean* and a sizable *variance* are both quantitatively necessary for aggregate investment dynamics.

This paper is organized as follows. Section 2 presents the model and the solution method. Section 3 shows the interest elasticity of aggregate investment with respect to the *mean* and the *variance*, respectively. Section 4 further inspects the mechanism. Finally, section 5 concludes.

2 The Model

The economy consists of a fixed unit mass of firms $j \in [0, 1]$ which produce homogeneous output y_{jt} and a unit measure continuum of identical households who consume output and supply labor.

Technology: The production function is as follows:

$$y_{jt} = z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1 \tag{2}$$

where k_{jt} and n_{jt} indicates the idiosyncratic capital and labor employed by the firm j. For each firm, the idiosyncratic TFP z_{jt} follows a log-normal AR(1):

$$log(z_{jt}) = -(1-\rho^z)\frac{\sigma^{z2}}{2(1-\rho^{z2})} + \rho^z log(z_{jt-1}) + \epsilon_{jt}, \quad \epsilon_{jt} \sim N(0, \sigma^z) \tag{3}$$

Adjustment Costs: The investment cost function includes two components: a direct cost i_{jt} and a fixed nonconvex capital adjustment cost ξ_{jt} paid in units of labor if the firm adjusts by more than a small proportion of their current capital stock ($|ak|$):

$$c(i_{jt}) = i_{jt} + \mathbf{1}_{(|i_{jt}|>ak_{jt})} \cdot w_t \cdot \xi_{jt}, \quad \xi_{jt} \sim \text{eq.}(1) \tag{4}$$

Firm Optimization: I denote by $V^A(k_{jt}, z_{jt}; \Omega_t)$, $V^{NA}(k_{jt}, z_{jt}; \Omega_t)$, and $V(k_{jt}, z_{jt}; \Omega_t) \equiv E_{\xi_{jt}} \tilde{V}(k_{jt}, z_{jt}, \xi_{jt}; \Omega_t)$ the value functions of a firm with an active investment choice, without an active investment choice, and with expected draw of ξ_{jt}. The aggregate state $\Omega_t = (\Theta_t, \mu_t(k, z, \xi))$ where Θ_t is a vector comprising the stochastic discount factor and wage at time t, and $\mu_t(k, z, \xi)$ is the

[1] This unconstrained intensive margin is usually constrained by a quadratic adjustment cost in recent literature. To keep this note dedicated on disciplining the *Mean* and the *Variance*, I leave out the quadratic adjustment cost.

distribution of firms. The value functions are as follows:

$$V^A(k_{jt}, z_{jt}; \Omega_t) = \max_{i,n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^*_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{5}$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_t) = \max_{i \in [-ak, ak], n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^C_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{6}$$

where the stochastic discount factor $\Lambda_{t,t+1}$ is derived from the household problem since households own all the firms. k^C_{jt+1} and k^*_{jt+1} are the constrained and non-constrained capital choices.

The firm will choose to pay the fixed cost if and only if $V^A(k_{jt}, z_{jt}; \Omega_t) - w_t \xi_{jt} > V^{NA}(k_{jt}, z_{jt}; \Omega_t)$. There is a unique threshold $\xi^*(k_{jt}, z_{jt}; \Omega_t)$ at which the firm breaks even:

$$\xi^*_t(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t} \tag{7}$$

If a firm draws a fixed cost ξ_{jt} below $\xi^*(k_{jt}, z_{jt}; \Omega_t)$ (which I denote as ξ^* *for short*), the firm pays the fixed cost and then actively adjusts its capital, otherwise it does not. The value function is:

$$V(k_{jt}, z_{jt}; \Omega_t) = -\frac{w_t(\xi^* + \underline{\xi})}{2} + \frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi} V^A(k_{jt}, z_{jt}; \Omega_t) + \left(1 - \frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t) \tag{8}$$

where $\underline{\xi} = \mu_\xi - \sqrt{3}\sigma_\xi$ is the lower bound of the fixed cost. The firm expects to pay the fixed cost when drawing ξ_{jt} lower than $\xi^*(k_{jt}, z_{jt}; \Omega_t)$. With probability $\frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi}$, the firm chooses to actively invest, otherwise it stays inactive. Therefore, the capital stock evolves by the law of motion:

$$k_{jt+1} = \begin{cases} (1-\delta)k_{jt} + i^*_{jt} & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1-\delta)k_{jt} + i^C_{jt} & otherwise \end{cases} \tag{9}$$

Household Optimization: Households' expected utility is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \theta N_t \right)$$

subject to the budget constraint: $C_t + \frac{1}{R_t} B_t \leq B_{t-1} + w_t N_t + \Pi^F_t$. Here β is the discount factor of households, θ is the disutility of working, R_t is the real interest rate, B_t is one period bonds, w_t is the nominal wage, and Π^F_t is the nominal profits from all the firms. The first order conditions of consumption, labor, and bonds deliver:

$$w_t = -\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = \theta C_t^\eta \tag{10}$$

$$\Lambda_{t,t+1} = \frac{1}{R_t} = \beta \frac{U_c(C_{t+1}, N_{t+1})}{U_c(C_t, N_t)} = \beta \Big(\frac{C_t}{C_{t+1}} \Big)^{\eta} \tag{11}$$

Equilibrium Definition: *A Recursive Equilibrium for this economy is defined by a set of value functions and policy functions* $\{V(k,z;\Omega), V^A(k,z;\Omega), V^{NA}(k,z;\Omega), \xi^*(k,z;\Omega), k^*(k,z;\Omega), k^C(k,z;\Omega)\}$, *a set of quantity functions* $\{C(\Omega), N(\Omega), Y(\Omega), K(\Omega)\}$, *a set of price functions* $\{w(\Omega), \Lambda(\Omega), R(\Omega)\}$, *and a distribution* $\mu'(\Omega)$ *that solves the firms' and households' problems and satisfies market clearing such that:*

(i). Taking the price functions as given, the policy functions solve firms' optimization.

(ii). Taking the price functions as given, the quantity functions solve households' optimization.

(iii). Goods market clears: $Y(\Omega) = C(\Omega) + I(\Omega) + \Theta_k(\Omega)$, *where* $\Theta_k(\Omega)$ *is the total adjustment cost.*

Solution Method: I follow the sequence space solution strategy as in Boppart et al. (2018) to solve the model which involves two parts. First, I solve the *Stationary Equilibrium* at the steady-state, which delivers all the steady-state equilibrium objects and provides the cross-sectional moments for the calibration. Second, I solve the *Transitional Equilibrium* starting from the *Stationary Equilibrium* and transit back to the same *Stationary Equilibrium.* The *Transitional Equilibrium* then provides the dynamic moments for the calibration and the impulse response functions.

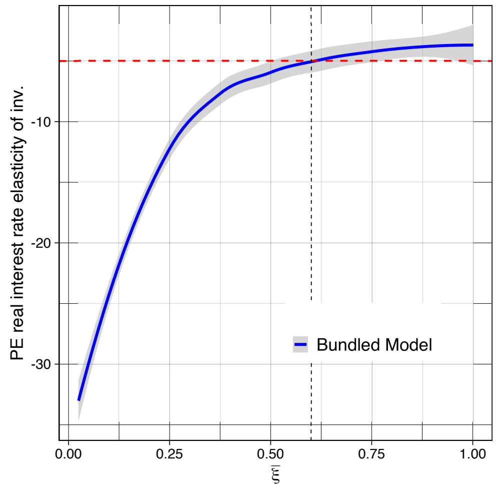
3 Mean, Variance, and the Elasticity of Investment

Benchmark Calibration: I calibrate the benchmark model with mean and variance bundled (*henceforth, bundled model*) as in Khan and Thomas (2008) (the uniform distribution starts from 0 to an upper bound: $\xi_{jt} \sim U[0, \bar{\xi}]$, so $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$) to hit the target investment moments. For fixed parameters, I choose discount factor $\beta = 0.99$ to match an annual interest rate of 4%., elasticity of intertemporal substitution $\eta = 1$ for log utility, leisure preference $\theta = 2$ to match a one-third of working time share, capital exponent $\alpha = 0.25$ and the labor exponent $\nu = 0.60$ to match a labor share of two-thirds and decreasing returns to scale imply of 85%, quarterly capital depreciation $\delta = 0.026$, freely capital adjustment region $a = 0.001$, and persistence of idiosyncratic TFP shock $^z = 0.95$. For fitted parameters, I choose $\sigma^z = 0.05$ and $\bar{\xi} = 0.6$ to match the average investment rate (10.5%), the standard deviation of investment rates (0.13), the spike rate[2] (17%), and the partial equilibrium interest elasticity of aggregate investment (-5), reflecting the empirical moments as measured in Zwick and Mahon (2017) and Koby and Wolf (2019).[3]

[2] Spike rate is defined as the proportion of investment rate larger than 20% in a quarter.

[3] To make the results more intuitive, I only include the nonconvex fixed cost and did not include the quadratic adjustment cost which usually serves to constrain extreme investment behaviors. As a result, the model cannot exactly match all the micro-investment moments as in Zwick and Mahon (2017).

Figure 1: Interest Elasticity over $\bar{\xi}$



Note: In the benchmark model, the uniform distribution starts from 0 to an upper bound: $\xi_{jt} \sim U[0, \bar{\xi}]$. It bundles the mean and the variance by $\bar{\xi}$: $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$. Therefore, increasing $\bar{\xi}$ increases both μ_ξ and σ_ξ simultaneously.

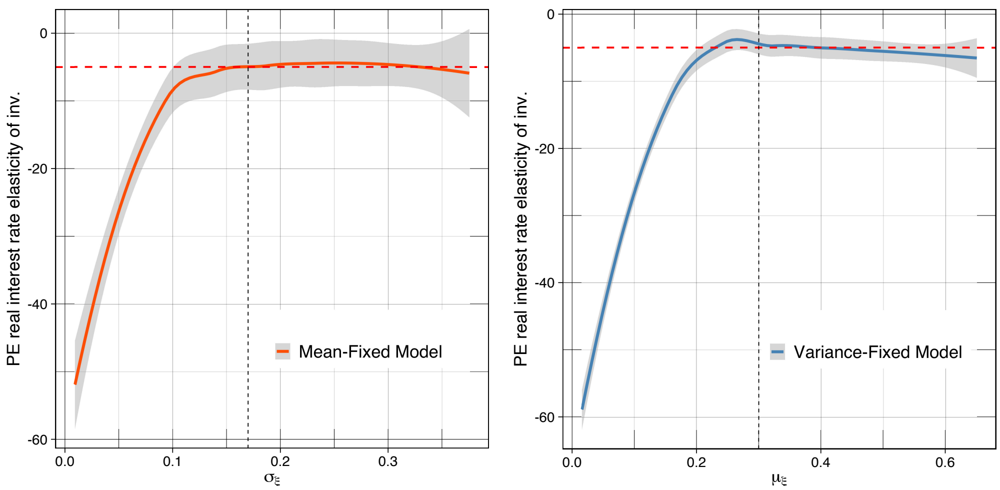
Upper Bound $\bar{\xi}$ and Interest Elasticity of Aggregate Investment: The partial equilibrium interest-elasticity of aggregate investment is defined by how aggregate investment, as yielded by the collective decisions of all heterogeneous firms, responds to an unexpected real interest rate shock[4]. For instance, -5 means when firms face an unexpected real interest rate cut of 1%, the partial equilibrium aggregate investment increases by 5%. Quasi-experimental evidence in Koby and Wolf (2019) suggests this interest-elasticity should be about -5. In Figure 1, I plot the model's interest elasticity against the choice of $\bar{\xi}$ from 0.025 to 1. A large value of $\bar{\xi}$ gives an interest elasticity of -5. Otherwise, the aggregate investment will be oversensitive to interest rate changes, incorrectly implying that lumpy investment is irrelevant for aggregate dynamics as in Khan and Thomas (2008)[5].

Mean, Variance, and Interest Elasticity of Aggregate Investment: Now I depart from the benchmark calibration of $\bar{\xi} = 0.6$. Instead, I study two alternative groups of calibrations: One, fixing the variance $\sigma^*_\xi = \bar{\xi}/\sqrt{12} = 0.6/\sqrt{12}$ and varying the mean μ_ξ from 0 to $2\mu^*_\xi = \bar{\xi}$ to show how the interest-elasticity changes, and two, fixing the mean $\mu^*_\xi = \bar{\xi}/2 = 0.6/2$ and varying the variance σ_ξ from 0 to $2\sigma^*_\xi = \bar{\xi}/\sqrt{3}$, to show how the interest elasticity changes. I use an identical quasi-experimental real interest rate shock as the one in the bundled model experiment above.

[4]By partial equilibrium, I assume the firms not taking consideration of wage changes as a feedback loop from household decisions. This is consistent with the reduced form estimation from the partial equilibrium perspective.

[5]However, this irrelevant result is not consistent with the joint dynamics of aggregate investment and the real interest rate over the business cycle as show in Winberry (2018).

Figure 2: Interest-Elasticity over μ_ξ and σ_ξ



(a) $\mu_\xi^* = \bar{\xi}/2$, change σ_ξ (b) $\sigma_\xi^* = \bar{\xi}/\sqrt{12}$, change μ_ξ

Note: The variance-fixed model fixes the variance by choosing $\sigma_\xi^* = \bar{\xi}/\sqrt{12}$ and the mean-fixed model fixes the mean by choosing $\mu_\xi^* = \bar{\xi}/2$. The two models are identical along both vertical dotted lines when $\mu_\xi = 0.3$ and $\sigma_\xi \simeq 0.17$.

The findings plotted in Figure 2 are quite interesting. First, the interest elasticity is not solely determined by the expected size (*mean*) of the nonconvex cost. Unlike common claims that the interest elasticity is controlled by the expected size of the nonconvex cost, the uncertainty (*variance*) plays a role. In panel (a), even though the *mean* is set to be relatively large, when the *variance* approaches zero, the interest elasticity is massive. Given the fixed *mean*, the model hits the targeted interest elasticity when the *variance* is equal or larger to that of the bundled model. Second, the interest elasticity is not solely determined by the uncertainty (*variance*) of the nonconvex cost. In panel (b), even though the *variance* is fixed to be relatively large when the *mean* approaches zero, the interest elasticity is again massive. Given the fixed *Variance*, the model hits the targeted interest elasticity when the *mean* is equal or larger to that of the bundled model. What is the mechanism behind choosing the *mean* and the *variance*, respectively?

4 The Mechanism

To further inspect the mechanism behind the differences between the *mean* and the *variance*, I demonstrate results from three models with three alternative calibrations: 1) the carefully calibrated bundled model (*Bundled*); 2).a mean-fixed model ($\mu_\xi^* = \bar{\xi}/\sqrt{12}$) with zero variance (*Zero*-σ_ξ); and 3).a variance-fixed model ($\sigma_\xi^* = \bar{\xi}/2$) with zero mean (*Zero*-μ_ξ).

A Decomposition of the Interest Elasticity: I first show the decomposition of the interest elasticity in all three models in terms of both extensive margin and intensive margin investment in Table 1 following the equation below:

$$\frac{d\sum I_j}{dr} = \frac{d\sum_{EM} I_j}{dr} + \frac{d\sum_{IM} I_j}{dr} \tag{12}$$

where $d\sum I_j$ is aggregate investment, $d\sum_{EM} I_j$ is all the extensive margin investment, and $d\sum_{IM} I_j$ is all the intensive margin investment. The carefully calibrated *Bundled* model has an interest elasticity of -5.1, 96% of the investment response is from the extensive margin, and 4% is from the intensive margin. The *Zero-σ_ξ* model has an interest elasticity of about -600, which is almost entirely from the extensive margin. The *Zero-μ_ξ* model has an interest elasticity of -80, but which is mainly from the intensive margin.

Table 1: A Decomposition of the Interest Elasticity

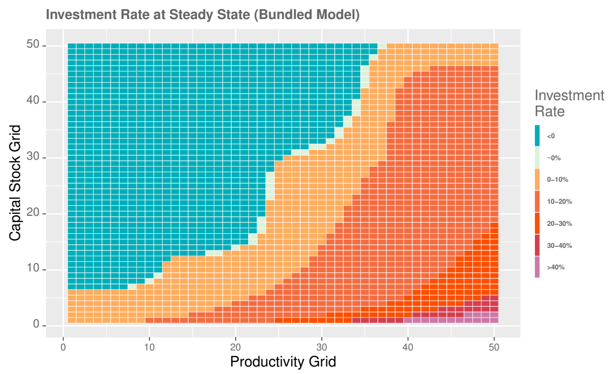
	Bundled			Zero-σ_ξ			Zero-μ_ξ		
Decomp.	Total	EM	IM	Total	EM	IM	Total	EM	IM
Elasticity	-5.1	-4.9	-0.2	-601.8	-601.7	-0.1	-80.0	-4.4	-75.6
Percentage	—	96%	4%	—	100%	0%	—	5.5%	94.5%

Note: The *Bundled* model has the calibration that matches the micro investment moments. The Zero-σ_ξ deviates by setting σ_ξ to zero while all other parameters are unchanged. The Zero-μ_ξ deviates by setting μ_ξ to zero while all other parameters are unchanged. EM stands for the extensive margin and IM stands for the intensive margin.

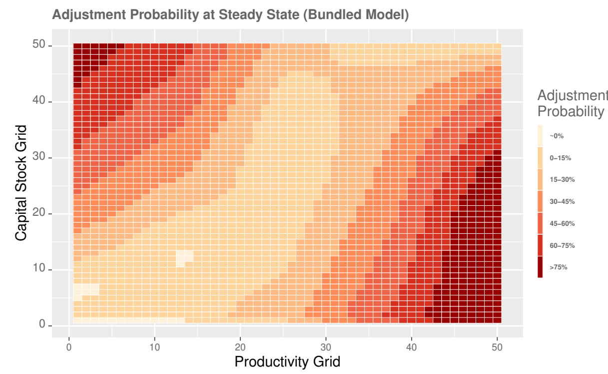
This decomposition shows that the *mean* and the *variance* play different roles. Without a sizable *mean*, the response of the aggregate investment to the interest rate is mainly from the intensive margin. The intensive margin is much too sensitive to real interest rate changes, which delivers a falsely large interest elasticity of aggregate investment. Without a sizable *variance*, the response of the aggregate investment to the interest rate is mainly from the extensive margin. The extensive margin is extremely sensitive to changes in real interest rates. A firm either chooses to pay μ_ξ^* and invest a lot or stay total inactive when the real interest rate changes. Firms on the extensive margin choose to "all-in" which creates the unrealistically large interest elasticity.

Distributions of the Extensive Margin and the Intensive Margin: In Figure 3, I plot the interpolated distributions of the extensive margin (adjustment probability) and the intensive margin (investment rate conditional on adjustment) at the steady states using two-dimensional interpolation with respect to productivity and capital stock. Since the intensive margin distributions are not much changed across models, I only plot these for the *Bundled* model. Warmer and darker colors indicate larger investment rates and higher adjustment probabilities. In subplot (a), we see that productive and low capital firms invest more in response to the opportunity presented by the interest rate shock. In subplot (b), we see the extensive margin distribution is layered from 0 along diagonal to higher probabilities away from the diagonal.

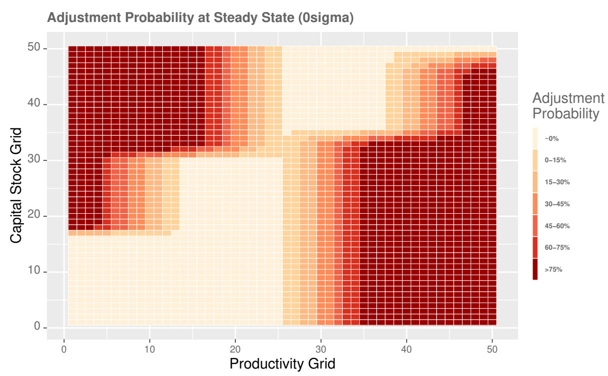
Figure 3: Distributions of the Extensive Margin and the Intensive Margin



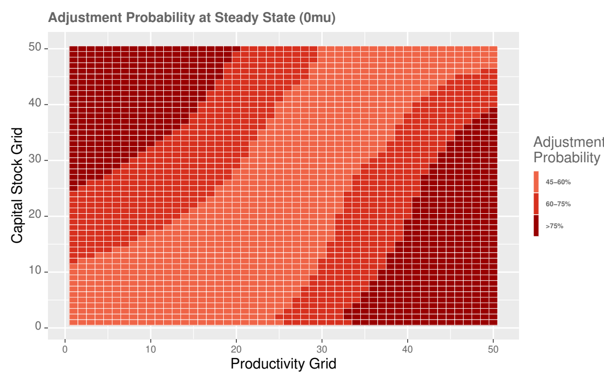
(a) Intensive Margin (Bundled)



(b) Extensive Margin (Bundled)



(c) Extensive Margin (Zero-σ_ξ)



(d) Extensive Margin (Zero-μ_ξ)

However, for the *Zero-*σ_ξ model and the *Zero-*μ_ξ model, the extensive margin distributions are entirely different. The extensive margin in the *Zero-*σ_ξ model shows a vertical sorting pattern that is sharply moving from a 0% probability of adjusting to almost a 100% possibility of adjusting. That is why the extensive margin is extremely interest rate sensitive. However, the extensive margin adjustment probability in the *Zero-*μ_ξ model is always higher than 45% and has much smaller variations. As a result, the extensive margin is not that sensitive to interest rate changes. But the intensive margin is unconstrained when the adjustment probability is high everywhere over the distribution.

5 Conclusion

Nonconvex capital adjustment costs play an essential role in generating the data-consistent lumpy investment behaviors. The literature usually assumes a uniform distribution for the nonconvex adjustment cost, starting from 0 to an upper bound, which does not distinguish the separate roles played by the *mean* and the *variance* of the distribution. In this paper, I show that both a sizable *mean* and a sizable *variance* are necessary for the lumpy investment models to generate an empirically consistent interest elasticity of aggregate investment. The *mean* governs the degree to which the extensive margin accounts for aggregate investment dynamics. In contrast, the *variance* controls how sensitive the extensive margin is to interest rate changes. Therefore, both of them are quantitatively necessary in a reasonably calibrated lumpy investment model.

There are two potential directions of future research. First, more realistic estimations of the *mean* and the *variance* using microdata on firm-level investment to better represent the expected size and the uncertainty of the nonconvex capital adjustment cost faced by firms. Second, the separate roles of the *mean* and the *variance* potentially matter for other dynamic models with nonconvex adjustment costs such as firm entry and exit, worker hire and fire, trade entry and exit, inventory dynamics, and many others.

References

Rüdiger Bachmann, Ricardo J Caballero, and Eduardo MRA Engel. Aggregate implications of lumpy investment: new evidence and a dsge model. *American Economic Journal: Macroeconomics*, 5(4):29–67, 2013.

Timo Boppart, Per Krusell, and Kurt Mitman. Exploiting mit shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control*, 89:68–92, 2018.

Min Fang. Lumpy investment, fluctuations in volatility, and monetary policy. *Available at SSRN 3543513*, 2020.

Aubhik Khan and Julia K Thomas. Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica*, 76(2):395–436, 2008.

Yann Koby and Christian K Wolf. Aggregation in heterogeneous-firm economies: A sufficient statistics approach. Technical report, Working Paper, 2019.

Thomas Winberry. Lumpy investment, business cycles, and stimulus policy. 2018.

Eric Zwick and James Mahon. Tax policy and heterogeneous investment behavior. *American Economic Review*, 107(1):217–48, 2017.