Nonconvex Adjustment Costs in Lumpy Investment Models: Mean versus Variance

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- Unknown: Expected Size (*Mean*) vs Uncertainty (*Variance*),

  which one determines the interest elasticity of aggregate investment?

This paper

- Theory: Separate the roles of mean and variance in lumpy investment models:

$$\xi_{jt} \sim U[\mu_\xi - \sqrt{3}\sigma_\xi, \mu_\xi + \sqrt{3}\sigma_\xi] \tag{1}$$

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$$\xi_{jt} \sim U[\mu_\xi - \sqrt{3}\sigma_\xi, \mu_\xi + \sqrt{3}\sigma_\xi] \quad (1)$$

- Results: Both mean and variance matter for the interest elasticity of agg. inv.

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$$\frac{d\sum I_j}{dr} = \frac{d\sum_{EM} I_j}{dr} + \frac{d\sum_{IM} I_j}{dr}$$

1. Expected Size (*Mean*) governs the importance of the extensive margin:
2. Uncertainty (*Variance*) governs governs the sensitivity of the extensive margin:

larger $\mu_\xi \Rightarrow$ larger $\frac{d\sum_{EM} I_j}{dr} / \frac{d\sum I_j}{dr}$	larger $\sigma_\xi \Rightarrow$ smaller $\frac{d\sum_{EM} I_j}{dr}$

Outline of today's talk

- A quick revisit of the lumpy investment models
- Show interest elasticity of agg. inv. along calibrations of the bundled model
- Show interest elasticity of agg. inv. along calibrations of the separate models
- Show mechanism of mean and variance in decomposition of EM and IM
- Show mechanism of mean and variance in distributions of EM and IM
- Conclusion and future research

The model: setup

Technology:

$$y_{jt} = z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1 \tag{2}$$

$$log(z_{jt}) = -(1-\rho^z)\frac{\sigma^{z2}}{2(1-\rho^{z2})} + \rho^z log(z_{jt-1}) + \epsilon_{jt}, \quad \epsilon_{jt} \sim N(0, \sigma^z) \tag{3}$$

Adjustment Costs:

$$c(i_{jt}) = i_{jt} + \mathbf{1}_{(|i_{jt}|>ak_{jt})} \cdot w_t \cdot \xi_{jt}, \quad \xi_{jt} \sim \text{eq.(1)} \tag{4}$$

Value Functions:

$$V^A(k_{jt}, z_{jt}; \Omega_t) = \max_{i,n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^*_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{5}$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_t) = \max_{i\in[-ak,ak],n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k^C_{jt+1}, z_{jt+1}; \Omega_{t+1})] \right\} \tag{6}$$

The model: optimization

Value Function:

$$V(k_{jt}, z_{jt}; \Omega_t) = -\frac{w_t(\xi^* + \underline{\xi})}{2} + \frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi} V^A(k_{jt}, z_{jt}; \Omega_t) + \left(1 - \frac{\xi^* - \underline{\xi}}{2\sqrt{3}\sigma_\xi}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t) \tag{7}$$

Extensive Margin:

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t} \tag{8}$$

Intensive Margin:

$$k_{jt+1} = \begin{cases} (1-\delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1-\delta)k_{jt} + i_{jt}^C & \textit{otherwise} \end{cases} \tag{9}$$

There is also a representative household to complete the GE model.

The calibration of parameters is very standard as in the literature.

Interest elasiticity of the bundled model

- The benchmark is calibrated $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$, a $\bar{\xi} = 0.6 \Rightarrow \frac{d\sum I_j}{dr} = -5$

Figure: Interest Elasticity over $\bar{\xi}$

Graph_Param/Identification_RIR_sensitivity_xibar-eps-conv

Note: In the benchmark model, the uniform distribution starts from 0 to an upper bound: $\xi_{jt} \sim U[0, \bar{\xi}]$. It bundles the mean and the variance by $\bar{\xi}$: $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$. Therefore, increasing $\bar{\xi}$ increases both μ_ξ and σ_ξ simultaneously.

Interest elasiticity of the separate models

- The Mean-Fixed Model fixed $\mu_\xi = \bar{\xi}/2$, varying σ_ξ
- The Variance-Fixed Model fixed $\sigma_\xi = \bar{\xi}/\sqrt{12}$, varying μ_ξ

Figure: Interest-Elasticity over μ_ξ and σ_ξ

Graph_Param/Identification_RI

(a) $\mu^*_\xi = \bar{\xi}/2$, change σ_ξ

(b) $\sigma^*_\xi = \bar{\xi}/\sqrt{12}$, change μ_ξ

Note: The variance-fixed model fixes the variance by choosing $\sigma^*_\xi = \bar{\xi}/\sqrt{12}$ and the mean-fixed model fixes the mean by choosing $\mu^*_\xi = \bar{\xi}/2$. The two models are identical along both vertical dotted lines when $\mu_\xi = 0.3$ and $\sigma_\xi \simeq 0.17$.

A decomposition of the extreme calibrations

- 1).the carefully calibrated bundled model (*Bundled*);
- 2).a mean-fixed model ($\mu_\xi^* = \bar{\xi}/\sqrt{12}$) with zero variance (*Zero*-σ_ξ);
- 3).a variance-fixed model ($\sigma_\xi^* = \bar{\xi}/2$) with zero mean (*Zero*-μ_ξ).

Table: A Decomposition of the Interest Elasticity

	Bundled			Zero-σ_ξ			Zero-μ_ξ		
Decomp.	Total	EM	IM	Total	EM	IM	Total	EM	IM
Elasticity	-5.1	-4.9	-0.2	-601.8	-601.7	-0.1	-80.0	-4.4	-75.6
Percentage	—	96%	4%	—	100%	0%	—	5.5%	94.5%

Note: The *Bundled* model has the calibration that matches the micro investment moments. The Zero-σ_ξ deviates by setting σ_ξ to zero while all other parameters are unchanged. The Zero-μ_ξ deviates by setting μ_ξ to zero while all other parameters are unchanged. EM stands for the extensive margin and IM stands for the intensive margin.

Distributions of EM & IM in the extreme calibrations

Figure: Distributions of the Extensive Margin and the Intensive Margin

Graph_Param/SS_Investment_Pol

Graph_Param/SS

(a) Intensive Margin (Bundled)

(b) Extensive Margin (Bundled)

Graph_Param/SS_Adjustment_Pol

Graph_Param/SS

Conclusion

Both mean and variance matter for the interest elasticity of agg. inv.:

- The mean governs the importance of the extensive margin.
- The variance governs governs the sensitivity of the extensive margin.

Future research:

- more realistic estimations of the mean and the variance using microdata on firm-level investment.
- the separate roles of the mean and the variance potentially matter for other dynamic models with such as firm entry and exit, worker hire and fire, trade entry and exit, inventory dynamics, and many others.