

LUMPY INVESTMENT, FLUCTUATIONS IN VOLATILITY, AND MONETARY POLICY

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 2. Why does it matter?

[One of the reasons]: It affects the amount of monetary policy stimulus required in high volatility times like now or in the Great Recession.

The detailed questions

Q1: What are the estimates of $\frac{dI}{d\epsilon^m}(\sigma)$ for different σ in the data?

In words: How does an increase in volatility of firm-level TFP affect the impact of monetary policy on aggregate investment?

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In words: How does an increase in volatility of firm-level TFP affect the impact of monetary policy on aggregate investment?

Q2: Could micro-founded macro models explain the estimates? And how?

In words: What key micro-foundations of the model could replicate the observations in the data?

Answers in this paper

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- **Empirical:** Monetary stimulus is less effective in times of high volatility

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► Model consistent with both micro & macro evidence of investment:

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 - (1) **Lumpy Investment:** Inaction/Spikes (E.M.) of firm-level investment

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- ▶ Model consistent with both micro & macro evidence of investment:
 - (1) **Lumpy Investment:** Inaction/Spikes (E.M.) of firm-level investment
 - (2) **Sensitivity of Investment:** Responsiveness to interest rate and volatility
- ▶ Aggregate transmission depends on the level of volatility:

Monetary policy is less effective when volatility is elevated

Key mechanism in the model

- ▶ Aggregate investment = **extensive margin** + **intensive margin**:

$$I = \sum_{j \in EM} i_j + \sum_{j \in IM} i_j$$

- ▶ ① Extensive margin is less responsive to MP with elevated volatility

$$\frac{d \sum_{j \in EM} i_j}{d \sigma_j^m} (\sigma_i) \downarrow, \text{ when } \sigma_i \uparrow$$

- ▶ ② Reasonable sensitivity of investment to interest rate and volatility

$$\text{Both data consistent } \frac{d \sum_{j \in EM} i_j}{dr} \quad \& \quad \frac{d \sum_{j \in EM} i_j}{d \sigma}$$

Key mechanism in the model

- ▶ The inv. channel of monetary policy works through both margins:

$$\frac{dI}{d\epsilon_t^m} = \frac{d\sum_{j \in EM} i_j}{d\epsilon_t^m} + \frac{d\sum_{j \in IM} i_j}{d\epsilon_t^m}$$

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Both data consistent $\frac{d\sum_{j \in EM} i_j}{dr}$ & $\frac{d\sum_{j \in EM} i_j}{d\sigma}$

Key mechanism in the model

- ▶ Monetary policy is less effective stimulating aggregate investment:

$$\underbrace{\frac{dI}{d\epsilon_t^m}(\sigma_t)}_{\downarrow\downarrow} = \underbrace{\frac{d\sum_{j \in EM} i_j}{d\epsilon_t^m}(\sigma_t)}_{\downarrow\downarrow} + \underbrace{\frac{d\sum_{j \in IM} i_j}{d\epsilon_t^m}(\sigma_t)}_{\approx}, \text{ when } \sigma_t \uparrow$$

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- ▶ Could we take the extensive margin mechanism **①** as granted?
 - ▶ Is any extensive margin (lumpy inv.) model sufficient to generate this result?

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- ▶ No. We still need two dynamic properties from the micro-foundation:

- ▶ Data consistent interest rate sensitivity of $I \Rightarrow$ a reasonable $\frac{d\sum_{j \in EM} i_j}{dr}$
- ▶ Data consistent volatility sensitivity of $I \Rightarrow$ a reasonable $\frac{d\sum_{j \in EM} i_j}{d\sigma}$

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$$\text{Both data consistent } \frac{d\sum_{j \in EM} i_j}{dr} \quad \& \quad \frac{d\sum_{j \in EM} i_j}{d\sigma}$$

Literature review

1. Volatility/State-dependent Effects of Monetary Policy

Vavra (2013), Baley&Blanco (2019), Li (2020), McKay&Wieland (2020), Castelnuovo&Pellegrino (2018), Eickmeier et al. (2016);

I show that the inv. channel of monetary policy is also volatility-dependent.

2. New Keynesian Models with Capital Accumulation

Christiano et al. (2005), Smets&Wouters (2003,2007), Reiter et al. (2013), Ottonello&Winberry (2018), Jeenas (2018);

I show that the lumpy inv. could co-exist with reasonable inv. IRFs. w.r.t. MP.

3. Volatility in RBC and/or for Stimulus Policy

Abel et al. (1996), Dixit et al. (1994), Bloom (2009), Bloom et al. (2018), Bachmann&Bayer (2013), Gilchrist et al. (2014), Arellano,Bai,&Kehoe (2019);

I show that second-moment shocks reduces the effects of first-moment policy.

4. Aggregate Implications of Lumpy Investment

Caballero et al. (1995), Caballero&Engel (1999), Thomas (2002), Khan&Thomas (2008), Bachmann et al. (2013), House (2014), Winberry (2018b), Koby&Wolf (2019), Baley&Blanco (2020);

I show that lumpy investment matters for monetary policy as well.

Roadmap

Q1: What are the estimates of $\frac{dI}{d\epsilon^m}(\sigma)$ for different σ in the data?

0. Local projection of investment responses to identified monetary shocks

Q2: Could micro-founded macro models explain the estimates? And how?

1. A heterogeneous firm New Keynesian model with lumpy investment
2. Volatility shock and the solution method
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[Empirical Motivation]

Q1: What is $\frac{dI}{d\epsilon^m}(\sigma)$ for different σ in the data?

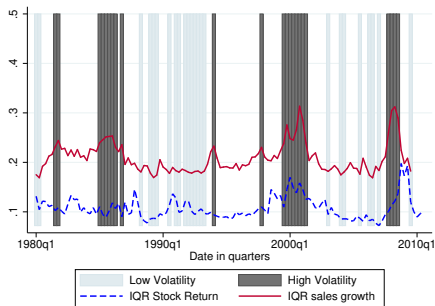
Quarterly National Income and Product Account + Monetary Shocks + Volatility

- ▶ Investment Indicator: Real non-residential private fixed investment
- ▶ Monetary Shocks: High-frequency-identified from Gertler-Karadi-2015
- ▶ Volatility Indicator: Interquantile Range (IQR) of sales growth

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Figure: Top20% vs Bottom 20%: 0.18 vs 0.26



Empirical strategy

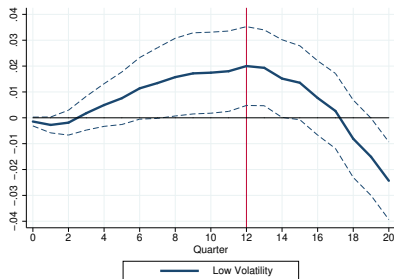
Baseline Local Projection Specification following Jorda (2005)

$$\Delta_h I_{t+h} = \alpha_h + \gamma_{j,h} \epsilon_t^m \times \mathbf{1}_{\sigma_t \in J^\sigma} + \sum_{l=0}^L \Gamma'_{h,t-l} Z_{t-l} + \epsilon_{h,t} \quad (1)$$

- ▶ $\sigma_t \in J^\sigma \equiv \{h, m, l\}$ indicates which group level of volatility at time t belongs to
- ▶ $\sigma_t = IQR_{sg,t}$ is the sales growth interquantile range of 25yr+ Compustat firms
- ▶ ϵ_t^m is sign-flipped and standardized monetary policy shock (/ -25bps)
- ▶ Z_{t-l} : conditional on volatility group, consumer price index (CPI), output gap, and consumption up to four quarters $L = 4$; α_h : h-period ahead fixed effect
- ▶ Coefficient $\gamma_{j,h}$ measures slope of investment semi-elasticity w.r.t. volatility

Inv. response to monetary stimulus with low volatility

- Impulse Response to monetary policy shock:

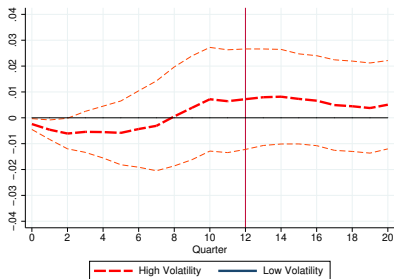


- Effectiveness of monetary policy:

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18				

Inv. response to monetary stimulus with high volatility

- Impulse Response to monetary policy shock:

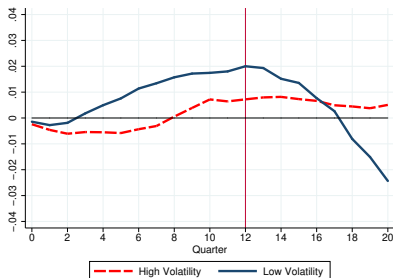


- Effectiveness of monetary policy:

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data			0.75%	0.26		

High volatility lowers inv. responses to monetary stimulus

- Impulse Response to monetary policy shock:



- Reduction in the effectiveness of monetary policy: **62%**

Sources	Low Volatility		High Volatility		Δ Effectiveness	
	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%

► With Confidence Intervals

Robustness of main result & additional facts

► Robustness Checks:

Choices	LP Form	MP shock ϵ_t^m	Volatility σ_t	Investment	Periods
1	Grouped	GK-HFI	IQR sales growth	RGFCF	80-10
2	Interacted	RIR	IQR stock return	RGPI	85-10
3				RPFI	85-07
4				RPFI-NR	80-07
5				RPFI-NR-EQMT	60-10
6				RPFI-NR-Struct	60-07
7				RPFI-NR-IP	60-18

► Interacted: replacing $\gamma_{j,h} \epsilon_t^m \times \mathbf{1}_{\sigma_t \in J^\sigma}$ with $(\beta_h + \gamma_h) \sigma_t \times \epsilon_t^m$

► Almost all hold in all $2 \times 2 \times 2 \times 7 \times 7 - 2 \times 2 \times 7 \times 3 = 308$ alternatives

► Additional Facts

► Firm-level regressions using Compustat Quarterly

► Tobit and Probit Local Projections to inspect mechanism

[Quantitative Theory]

Q2: Could micro-founded macro
models explain the estimates?
And how?

Roadmap of the Quantitative Theory

1. **A heterogeneous firm New Keynesian model with lumpy investment**
2. Volatility shock and the solution method
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Model overview

Heterogeneous Production Firms:

- ▶ Produce and invest subject to capital adj. costs
- ▶ Face idiosyncratic productivity shocks

A New Keynesian Block

- ▶ Retailers differentiate production firms' output + Rotemberg sticky price
- ▶ Monetary authority follows Taylor Rule

A Family of Representative Households

- ▶ Owns firms + choose consumption, hours of working, and saving.

Production firms

Enter period with state variables (z_{jt}, k_{jt})

1. Production:

$$y_{jt} = z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1 \quad (2)$$

► Sell at relative price p_t^w

2. Idiosyncratic TFP shock:

$$\log(z_{jt}) = -\frac{\sigma_z^2}{2(1 + \rho)} + \rho_z \log(z_{jt-1}) + \sigma_z \epsilon_{jt} \quad (3)$$

Production Firms

Enter period with state variables (z_{jt}, k_{jt})

Cost of Investment:

$$c(i_j) = i_j + \frac{\phi_k}{2} \left| \frac{i_j}{k_j} \right|^2 k_j + \mathbf{1}_{(i_j < 0)} \cdot S \cdot |i_j| + \mathbf{1}_{(i_j \notin [-ak, ak])} \cdot \xi_j \cdot w_t \quad (4)$$

$$\xi_j \sim U[0, \bar{\xi}]$$

1. Quadratic Adj. Costs ϕ_k :

- ▶ Extremely costly to make huge changes in capital stock

2. Partial Irreversibility S : Disinvestment will cost S proportional loss in inv.

- ▶ Caution of investment today because of potential disinvest costs tomorrow

3. Random Fixed Costs ξ_j : Randomly occurred cost paid in unit of labor

- ▶ "Lucky" or "unlucky" draws determine inaction or action

Retailers and final good producer

- ▶ Monopolistically competitive **retailers**

- ▶ Technology: $\tilde{y}_{jt} = y_{jt} \Rightarrow \text{marginal cost} = p_t^w$
- ▶ Subject to price adj. costs: $AC_p = \frac{\psi_p}{2} \left(\frac{p_{jt}}{p_{jt-1}} - 1 \right)^2 P_t Y_t$

- ▶ Perfectly competitive **final good producer**

- ▶ Technology: $Y_t = \left(\int \tilde{y}_{jt}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_t = \left(\int p_{jt}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}$

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- ▶ Implies the **New Keynesian Phillips curve**

$$\log \Pi_t = \frac{\gamma-1}{\psi_p} \log \frac{p_t^w}{p^{w*}} + \beta E_t \log \Pi_{t+1}$$

Monetary authority and household

- Monetary authority follows the **Taylor rule**

$$\log R_t^n = \log \frac{1}{\beta} + \phi_{\pi} \log \pi_t + \epsilon_t^m$$

Monetary authority and household

- ▶ Monetary authority follows the **Taylor rule**

$$\log R_t^n = \log \frac{1}{\beta} + \phi_{\Pi} \log \pi_t + \epsilon_t^m$$

- ▶ A family of representative household with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \theta N_t \right)$$

- ▶ Labor-leisure choice $\Rightarrow w_t = \theta C_t^{\eta}$
- ▶ Consumption-saving choice $\Rightarrow \Lambda_{t,t+1} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\eta}$

Stationary Equilibrium

► An **equilibrium of this model satisfies**

1. Production firms choose investment policies $k'_t(z, k)$ and $\xi_t^*(z, k)$
2. Retailers and final good producers generate NK Phillips curve
3. Monetary authority follows Taylor rule $\log R_t^n = \log \frac{1}{\beta} + \Phi_\Pi \log \pi_t + \epsilon_t^m$
4. Households choose labor supply N_t and generate SDF $\Lambda_{t,t+1}$

Roadmap of the Quantitative Theory

1. A heterogeneous firm New Keynesian model with lumpy investment
2. **Volatility shock and the solution method**
3. Parameterization and identification of lumpy investment
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Solve the stationary equilibrium

- ▶ Solve the **stationary equilibrium** (policy/distribution) with no aggregate risk
 - ▶ Non-stochastic simulation (Young, 2010) for value/policy functions
 - ▶ Stochastic simulation for parameterization sample
- ▶ Compute the **stationary equilibrium** moments
 - ▶ Steady state investment distribution moments
 - ▶ Use for identification of lumpy investment parameters

Solve the transitional equilibrium

- Volatility shock: a MIT shock (unexpected increase) to the variance σ_z ► timing

$$\log(z_{jt}) = -\frac{\sigma_z^2}{2(1+\rho)} + \rho_z \log(z_{jt-1}) + \sigma_z \epsilon_{jt}$$

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- ▶ Compute **perfect foresight transition path** following aggregate shocks
 - ▶ Case One: MP shock only; Case Two: MP shock + Vol. shock

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- ▶ Compute **perfect foresight transition path** following aggregate shocks
 - ▶ Case One: MP shock only; Case Two: MP shock + Vol. shock
 - ▶ I update all aggregate price paths all at once using **excessive demand** which is super fast (seconds for 200 periods even without parallel computing)
 - ▶ Captures all non-linear dynamics following a volatility shock (Global sol.)
 - ▶ Captures all non-linear dynamics of interactions between shocks **which is the key that the results in this model is achieved**

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Fixed parameters

Parameter	Description	Value
<i>Households</i>		
β	Discount factor	0.99
η	Elasticity of intertemporal substitution	1
θ	Leisure preference	2
<i>Production Firms</i>		
α	Capital coefficient	0.25
ν	Labor coefficient	0.60
δ	Capital depreciation	0.026
ρ_z	Persistence of TFP shock	0.95
<i>New Keynesian</i>		
γ	Demand elasticity	10
ψ_p	Price adjustment cost	90
ϕ_π	Taylor rule coefficient	1.5

Parameters to be computed

- How large should the volatility be, respectively?

Parameter	Description	Value
<i>Volatility Level</i>		
σ_z^l	Volatility of TFP shock (normal time)	0.05
σ_z^h	Volatility of TFP shock (elevated)	0.13

- Moments to match

Moment	Data	Model
IQR sales growth IQR_{sg} (normal time)	0.18	0.18
IQR sales growth IQR_{sg} (elevated)	0.26	0.26

► Details of the simulation

Parameters to be computed

- Recap of the Cost Function of Investment:

$$c(i_j) = i_j + \frac{\phi_k}{2} \left| \frac{i_j}{k_j} \right|^2 k_j + \mathbf{1}_{(i_j < 0)} \cdot S \cdot |i_j| + \mathbf{1}_{(i_j \notin [-ak, ak])} \cdot \xi_j \cdot w_t$$

$$\xi_j \sim U[0, \bar{\xi}]$$

- How large should the adjustment costs be, respectively?

Parameter	Description	Value
<i>Adjustment Costs</i>		
$\bar{\xi}$	Upper bound of fixed cost	
S	Partial Irreversibility	
ϕ_k	Quadratic adjustment cost	

Targets

► Cross-section Moments of Investment: (Zwick and Mohan 2017)

Moment	Description (annual)	Data	Model
$E[i/k]$	Mean investment rate	10.4%	
$\sigma(i/k)$	Standard dev. of investment rates	0.16	
$P(i/k \geq 20\%)$	Spike rate of investment	14.4%	
$P(i/k < 20\%)$	Positive rate of investment	85.6%	

► Dynamic Moments of Investment: (Zwick and Mohan 2017, Baley and Blanco 2020)

Moment	Description (annual)	Data	Model
$Cor(\frac{i}{k}, \frac{i+1}{k+1})$	Autocorrelation of investment rates	0.40	
$Cov(x, age)$	Covariance of capital gap and age since last adj.	0.29	

*capital gap: $x = \log(\frac{k_t}{z_t}) - E\left[\log(\frac{k_t}{z_t})\right]$, without frictions, capital gap= 0.

► I pin down these parameters using both cross-section and dynamic moments

1.The choice of ϕ_k

- The Choice of ϕ_k : (the conventional cost in the literature)

I choose quad. adj. costs to match the cross-section moments $\Rightarrow \phi_k = 4.00$

- Cross-section Moments of Investment: (Zwick and Mohan 2017)

Moment	Description	Data	Model
$E[i/k]$	Mean investment rate (annual)	10.4%	10.1%
$\sigma(i/k)$	Standard dev. of investment rates (annual)	0.16	0.12
$P(i/k \geq 20\%)$	Spike rate of investment (annual)	14.4%	15.3%
$P(i/k < 20\%)$	Positive rate of investment (annual)	85.6%	84.7%

- Next, I pin down the lumpy adj. parameters using both dynamic moments

- Dynamic Moments of Investment: (Zwick and Mohan 2017, Baley and Blanco 2020)

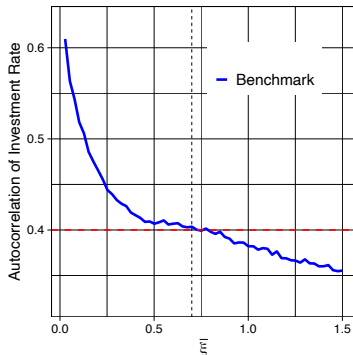
Moment	Description (annual)	Data	Model
$Cor(\frac{i}{k}, \frac{i}{k+1})$	Autocorrelation of investment rates	0.40	
$Cov(x, age)$	Covariance of capital gap and age since last adj.	0.29	

*capital gap: $x = \log(\frac{k_t}{z_t}) - E\left[\log(\frac{k_t}{z_t})\right]$, without frictions, capital gap= 0.

1.The choice of $\bar{\xi}$

- First, the autocorrelation of investment rates almost uniquely pins down the upper bound of random fixed costs $\bar{\xi} = 0.70$

Figure: Autocorrelation of Investment Rates

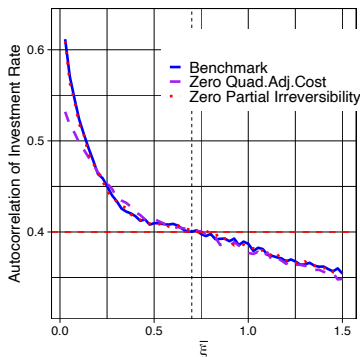


- The reason is that this cost is the only "random" cost, which will decrease the autocorrelation monotonically with the existence of other costs

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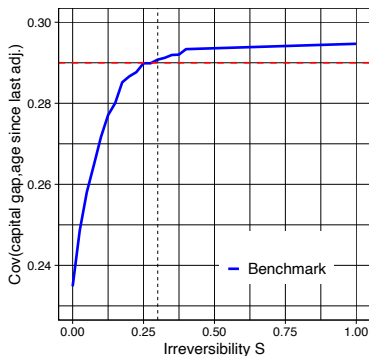


- The reason is that this cost is the only "random" cost, which will decrease the autocorrelation monotonically with the existence of other costs

2.The choice of S

- Second, conditional on $\bar{\xi} = 0.7$, the covariance of capital gap and no-adjustment age since last adjustment suggests a large partial irreversibility $S = 0.3$

Figure: Covariance of capital gap and no-adjustment age since last adjustment



- The reason is that larger irreversibility constraints firms to disinvest so positive capital gap $x = \log(\frac{k_t}{z_t}) - E \left[\log(\frac{k_t}{z_t}) \right] > 0$ lasts longer *age* (natural depreciation)

How does the choice of $\bar{\xi}$ and S matter for the story?

- ▶ $\bar{\xi}$ governs how sensitive lumpy investment is w.r.t monetary policy shocks
- ▶ S governs how sensitive lumpy investment is w.r.t volatility shocks
- ▶ Only empirically consistent $\bar{\xi}$ & S could generate data consistent IRFs to monetary policy shocks and volatility shocks, and eventually volatility-dependent IRFs to monetary policy

▶ Fly to sensitivity

Parameters to be computed

- How large should the adjustment costs be, respectively?

Parameter	Description	Value
<i>Adjustment Costs</i>		
$\bar{\xi}$	Upper bound of fixed cost	0.70
S	Partial Irreversibility	0.30
ϕ_k	Quadratic adjustment cost	4.00

Targets

► Cross-section Moments of Investment: (Zwick and Mohan 2017)

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► Dynamic Moments of Investment: (Zwick and Mohan 2017, Baley and Blanco 2020)

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$Cov(x, age)$	Covariance of capital gap and age since last adj.	0.29	0.29

*capital gap: $x = \log(\frac{k_t}{z_t}) - E\left[\log(\frac{k_t}{z_t})\right]$, without frictions, capital gap= 0.

► The dynamic moments are essential so that investment is of empirically consistent sensitivity to monetary shocks and volatility shocks

Roadmap of the Quantitative Theory

1. A heterogeneous firm New Keynesian model with lumpy investment
2. Volatility shock and the solution method
3. Parameterization and identification of lumpy investment
4. **Volatility-dependent effectiveness of monetary policy**
5. Inspecting the mechanism in the model

The volatility-dependent effectiveness of monetary policy

Two experiments:

- ▶ **Low Vol.:** a conventional MP shock to TR residual $\epsilon_1^m = -25bps$ with $\rho^m = 0.5$
- ▶ **High Vol.:** the same MP shock when a volatility shock hits as well

A Fair Comparison:

- ▶ Impulse Responses of **Low Volatility** vs. **High Volatility** w.r.t the MP shock
- ▶ Compute the peak responses in both cases when the MP shock hits

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%
Model		0.18		0.26		44%

- ▶ The model explains ???% of the reduction in the effectiveness of monetary policy

The volatility-dependent effectiveness of monetary policy

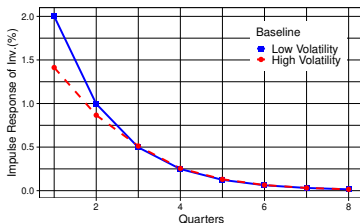
	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
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The volatility-dependent effectiveness of monetary policy

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Data	2.0%	0.18	0.75%	0.26	62%	44%
Model		0.18		0.26		44%

- Monetary policy generates less IRFs of investment when volatility is high

Figure: Differential IRFs w.r.t. a monetary shock



► IRFs of Other Variables

► IRFs to Volatility

► Decision Rules

► Heat-map Plot

The volatility-dependent effectiveness of monetary policy

- Compare the peak impulse response in both cases when the MP shock hits

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
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Model	2.0%	0.18	1.4%	0.26	30%	44%

The volatility-dependent effectiveness of monetary policy

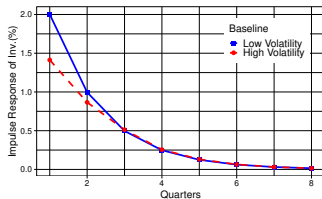
- ▶ Compare the peak impulse response in both cases when the MP shock hits

	Low Volatility		High Volatility		Δ Effectiveness	
Sources	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m}$	IQR_{sg}	$\frac{dl}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%
Model	2.0%	0.18	1.4%	0.26	30%	44%

- ▶ The model explains $\frac{30}{62} = 48\%$ of the reduction in the effectiveness of MP
- ▶ The result is within the confidence interval of my estimates.

Differential impulse responses with alternative parameters

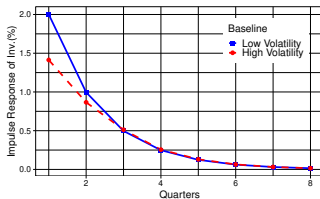
Figure: Differential Investment IRFs in Alternative Models



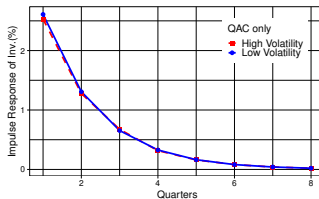
(a) Baseline

Differential impulse responses with alternative parameters

Figure: Differential Investment IRFs in Alternative Models



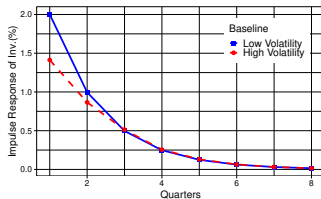
(a) Baseline



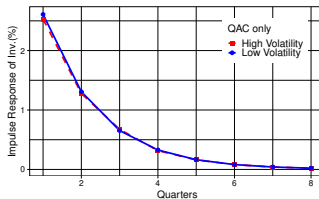
(b) NO Lumpy Investment

Differential impulse responses with alternative parameters

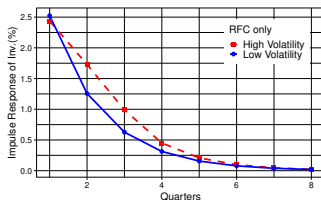
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(a) Baseline



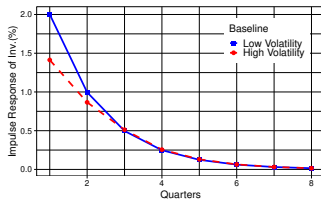
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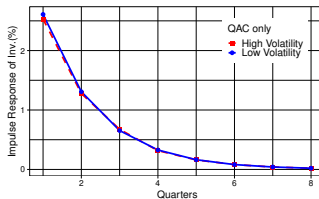
(c) ONLY Random Fixed Cost

Differential impulse responses with alternative parameters

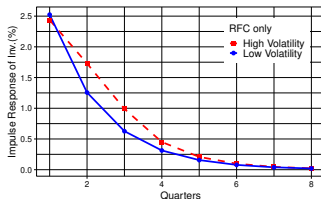
Figure: Differential Investment IRFs in Alternative Models



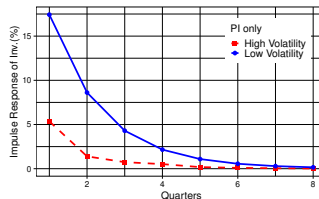
(a) Baseline



(b) NO Lumpy Investment



(c) ONLY Random Fixed Cost



(d) ONLY Partial Irreversibility

The volatility-dependent effectiveness of monetary policy

- ▶ The model successfully replicates the reduction in the effectiveness of MP
- ▶ The result is within the confidence interval of my estimates.
- ▶ Specification of lumpy investment parameters is the key.

Let's fly to the conclusion if we do not have enough time

▶ Conclusion

Roadmap of the Quantitative Theory

1. A heterogeneous firm New Keynesian model with lumpy investment
2. Volatility shock and the solution method
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4. Volatility-dependent effectiveness of monetary policy
5. **Inspecting the mechanism in the model**

Key mechanism in the model

- ▶ Monetary policy is less effective stimulating aggregate investment:

$$\underbrace{\frac{dI}{d\epsilon_t^m}(\sigma_t)}_{\downarrow\downarrow} = \underbrace{\frac{d\sum_{j \in EM} i_j}{d\epsilon_t^m}(\sigma_t)}_{\downarrow\downarrow} + \underbrace{\frac{d\sum_{j \in IM} i_j}{d\epsilon_t^m}(\sigma_t)}_{\approx}, \text{ when } \sigma_t \uparrow$$

- ▶ ① Extensive margin is less responsive to MP with elevated volatility

$$\frac{d\sum_{j \in EM} i_j}{d\epsilon_t^m}(\sigma_t) \downarrow, \text{ when } \sigma_t \uparrow$$

- ▶ ② Reasonable sensitivity of investment to interest rate and volatility

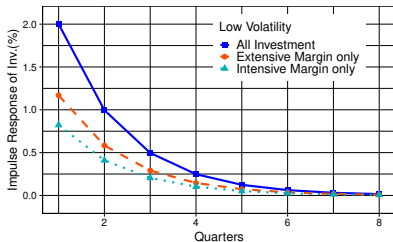
$$\text{Both data consistent } \frac{d\sum_{j \in EM} i_j}{dr} \quad \& \quad \frac{d\sum_{j \in EM} i_j}{d\sigma}$$

Key mechanism in the model: Inspection

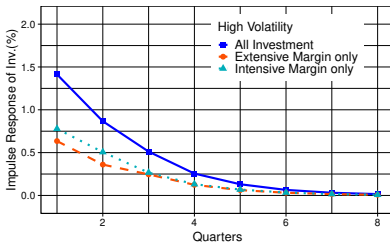
1

- The decrease of the extensive margin accounts for most of the drops (90%)

IRFs	Low Volatility			High Volatility		
	Total	EM	IM	Total	EM	IM
Number	2.0%	1.17%	0.82%	1.4%	0.63%	0.78%



(a) Low Volatility



(b) High Volatility

► IRFs of Other Variables

► IRFs to Volatility

► Decision Rules

► Heat-map Plot

Key mechanism in the model: Inspection

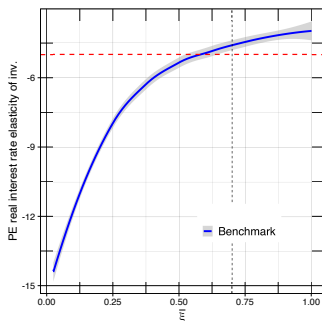
2

- ▶ Elasticity of investment to real interest rate should be -5 (Koby-Wolf-2020)
- ▶ Elasticity of investment to volatility should be negatively large (Bloom-2009)
- ▶ My choices of $\bar{\xi}$ and S match both sensitivities in considerable ranges

Key mechanism in the model: Inspection

2

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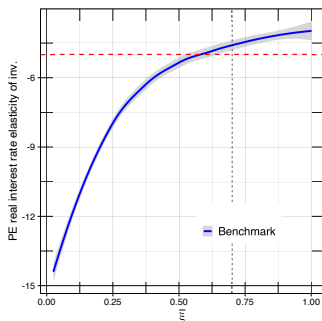


(a) Sensitivity to real interest

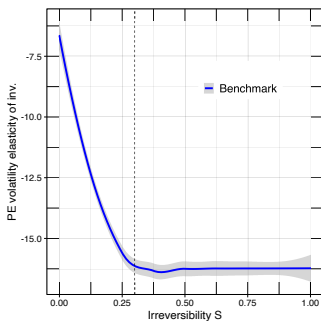
Key mechanism in the model: Inspection

2

- ▶ Elasticity of investment to real interest rate should be -5 (Koby-Wolf-2020)
- ▶ Elasticity of investment to volatility should be negatively large (Bloom-2009)
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(a) Sensitivity to real interest



(b) Sensitivity to volatility

▶ Back to Identification

▶ Plots with variations in other parameters

Summary of the mechanism

Lumpy investment and volatility play essential roles:

1. The inv. channel of monetary policy works mainly through the extensive margin
2. Extensive margin adjustment probability is lowered when volatility is elevated
3. The decrease of the extensive margin accounts for most of the drops (90%).

Which one of the lumpy capital adjustment costs plays the central role? Both

4. Random fixed costs governs the sensitivity of investment to interest rate
5. Irreversibility governs the sensitivity of investment to volatility
6. Jointly, they determine the volatility-dependent effectiveness of monetary policy

Conclusion

- ▶ I estimated the volatility-dependent effectiveness of monetary policy in the data.

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- ▶ This implies that fluctuations in volatility interacting with lumpy investment plays an essential role in monetary policy transmission to aggregate investment

Conclusion

- ▶ I estimated the volatility-dependent effectiveness of monetary policy in the data.
- ▶ I show that this estimate is consistent with the implications of a macro model of investment with a plausible parameterization of firm-level adjustment costs.
- ▶ This implies that fluctuations in volatility interacting with lumpy investment plays an essential role in monetary policy transmission to aggregate investment
- ▶ Further work: using the model to do normative analysis (ZLB, other types of monetary policy, second moment policy, ...)

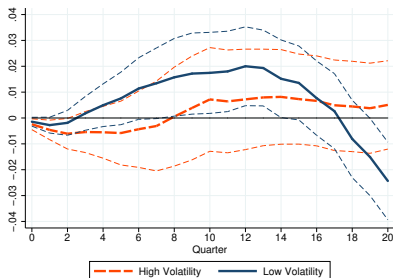
Backup Slides

1. Monetary policy shocks ϵ_t^m : **high-frequency identified** as in Gertler-Karadi-2015
 - ▶ use HFI FFR30 within 30mins window around FOMC announcements as an IV for the one-year government bond rate in the following VAR
 - ▶ run a monthly IV-VAR with log industrial production, employment rate, log CPI and a measure of corporate interest spreads
 - ▶ predict the residual of the instrumented one-year government bond rate and then accumulate them to a quarterly series.
 - ▶ sign-flipped and standardized (dividing by -25bps)

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1. High vs. Low Volatility $\sigma_{z,t}$: **Top 20% vs. Bottom 20%** in IQR sales growth
 - ▶ measures including IQR sales growth, IQR stock return, ...
 - ▶ compare the impulse responses of inv. during High vs. Low Volatility times

High volatility lowers inv. responses to monetary stimulus

- Impulse Response to monetary policy shock:



- Reduction in the effectiveness of monetary policy: **62%**

Sources	Low Volatility		High Volatility		Δ Effectiveness	
	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m}$	IQR_{sg}	$\frac{dI}{d\epsilon^m} \downarrow$	$IQR_{sg} \uparrow$
Data	2.0%	0.18	0.75%	0.26	62%	44%

► Back to Main

Recursive Production Firms' Problem ▶ Back

▶ Value Function

$$\begin{aligned}V^A(k_{jt}, z_{jt}; \Omega_t) &= \max_{i,n} \left\{ -c(i_{jt}) + \mathbb{E}[p_t^w y_{jt} - w_t n_{jt} + \Lambda_{t,t+1} V(k_{jt+1}^*, z_{jt+1}; \Omega_{t+1})] \right\} \\V^{NA}(k_{jt}, z_{jt}; \Omega_t) &= \max_{i \in [-ak, ak], n} \left\{ -c(i_{jt}) + \mathbb{E}[p_t^w y_{jt} - w_t n_{jt} + \Lambda_{t,t+1} V((k_{jt+1}^C, z_{jt+1}; \Omega_{t+1})] \right\} \\V(k_{jt}, z_{jt}; \Omega_t) &= -\frac{w_t \xi^*(k_{jt}, z_{jt}; \Omega_t)}{2} + \frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}} V^A(k_{jt}, z_{jt}; \Omega_t) \\&\quad + \left(1 - \frac{\xi^*(k_{jt}, z_{jt}; \Omega_t)}{\bar{\xi}}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t)\end{aligned}\tag{7}$$

▶ Optimal Investment Decisions

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t}\tag{8}$$

$$k_{jt+1} = \begin{cases} (1 - \delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1 - \delta)k_{jt} + i_{jt}^C & \text{otherwise} \end{cases}\tag{9}$$

Enter period with state variables (z_{jt}, k_{jt})

1. Idiosyncratic TFP shock:

$$\log(z_{jt}) = -\frac{\sigma_z^2}{2(1+\rho)} + \rho_z \log(z_{jt-1}) + \sigma_z \epsilon_{jt} \quad (10)$$

2. Volatility shock: (timing)

- ▶ t^- : A heightened change in the standard deviation of TFP innovation ($\sigma_z \uparrow$)
- ▶ t : Firms making investment decisions under uncertainty
- ▶ t^+ : Productivity z_{jt} arrives and firms making production decisions

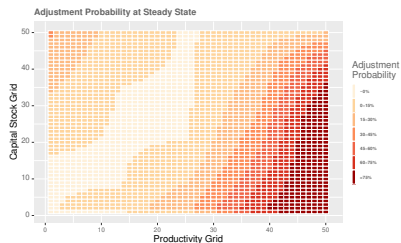
Details of the simulation

- ▶ I simulate 100k firms starting from a steady-state for 500 quarters
- ▶ Both volatility shock and monetary shock hit at the quarter 501
- ▶ The economy convergences back to steady-state in the quarter 700
- ▶ Largest firms who account for 45% of output are "Compustat" firms ($\sim 10\%$)
- ▶ "Compustat firms" older than 100+ quarters are used to calculate IQR_{sg} ($\sim 1\%$)
- ▶ Additional IQRs: (I choose $\sigma^l = 0.05$ and $\sigma^h = 0.13$ to match IQR_{sg} in the data)

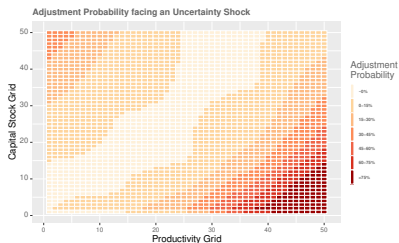
	Low Volatility		High Volatility	
Sources	σ_z^l	IQR_{sg}	σ_z^h	IQR_{sg}
All firms	0.05	0.24	0.13	0.48
Compustat	0.05	0.21	0.13	0.38

▶ Back

Volatility and inv. decision rules at extensive margin



(a) Steady state

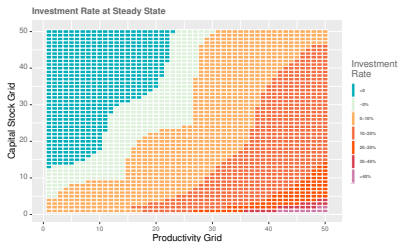


(b) Upon the volatility shock

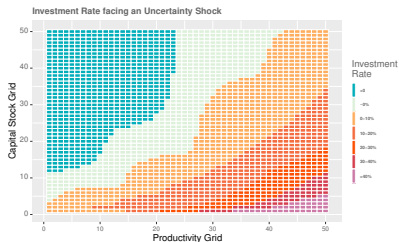
► Back to Main

► Back to Mechanism

Volatility and inv. decision rules at intensive margin



(a) Steady state



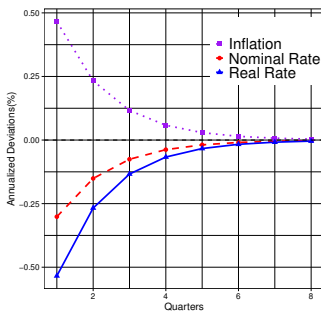
(b) Upon the volatility shock

► Back to Main

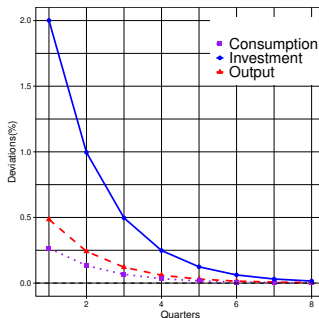
► Back to Mechanism

Impulse responses to a monetary shock

Figure: Impulse responses to a monetary shock



(a) Monetary Rates



(b) Aggregate Quantities

► Back to Main

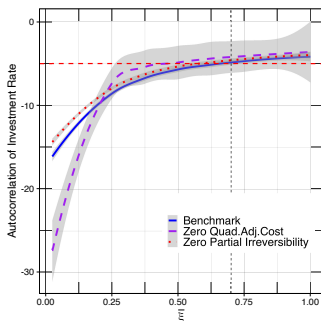
► Back to Mechanism

Sensitivity of investment w.r.t. lumpy parameter choices

- ▶ Elasticity of investment to real interest rate should be -5 (Koby-Wolf-2020)
- ▶ Elasticity of investment to volatility should be negatively large (Bloom et al.-2018)

Sensitivity of investment w.r.t. lumpy parameter choices

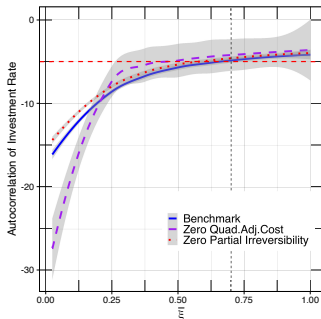
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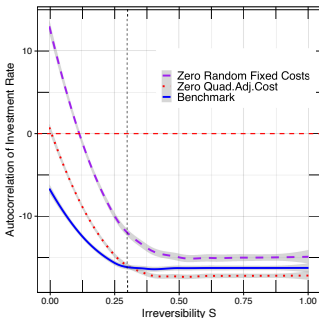
(a) Sensitivity to real interest

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(a) Sensitivity to real interest



(b) Sensitivity to volatility

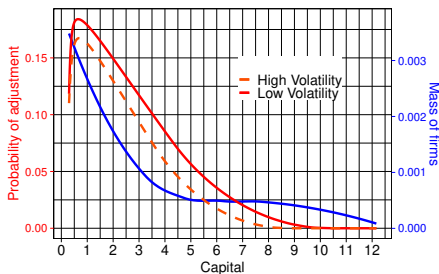
▶ Back to Main

▶ Back to Mechanism

How do volatility shocks change the investment policy?

- ▶ Volatility shocks significantly lowered adjustment probability

Figure: How does high volatility change inv. incentive



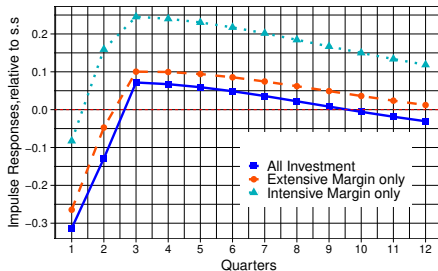
- ▶ Firms have much weaker incentive to invest in the extensive margin

▶ Back to Main

▶ Back to Mechanism

The effect of volatility shock

Figure: A Decomposition of the inv. channel of monetary policy



► Back to Main

► Back to Mechanism