Uniswap price bot calculations

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1 Introduction

Here, we consider how to make a swap on a Uniswap v2 pool (or v3 pool with a max-width position) in order to move the price of an asset to some target price.

1.1 General mechanics

Consider a Uniswap pool with two assets, labelled X and Y. Let the global reserves in the pool at time $t = 0$ be denoted by x_0 and y_0, respectively.

There are two types of events that can alter a Uniswap pool: swap events and liquidity provision events. Both events alter the individual reserves, x_0 and y_0, but swaps alter the price and leave liquidity unchanged, whilst liquidity provision events leave the price unchanged and alter liquidity.

Liquidity in the pool k_0 is defined by the constant-product formula

$$k_0 = x_0 \cdot y_0 \tag{1}$$

The price of asset Y in terms of asset X at time $t = 0$ is denoted p_0^y and simply given by

$$p_0^y = \frac{y_0}{x_0}. \tag{2}$$

1.2 Swaps

Swaps involve a user trading an amount Δx of asset X for an amount Δy of asset Y, or vice versa. Consider a trade $(\Delta x, \Delta y)$. If the user wants to swap Δx amount of X, what is the amount Δy they get of asset Y in return, or vice versa?

To answer these questions, first note that the reserves after the swap must obey the constant-product rule, meaning that

$$(x_0 + \Delta x)(y_0 + \Delta y) = k_0. \tag{3}$$

Rearranging, we find that the change Δy induced by a swap size of Δx is therefore

$$\Delta y = \frac{k}{x_0 + \Delta x} - y_0 = y_0 \cdot \frac{x_0}{x_0 + \Delta x} - y_0 = y_0 \left(\frac{x_0}{x_0 + \Delta x} - 1 \right) = -y_0 \left(\frac{\Delta x}{x_0 + \Delta x} \right). \tag{4}$$

By symmetry, the change Δx induced by a swap of size Δy is

$$\Delta x = -x_0 \left(\frac{\Delta y}{y_0 + \Delta y} \right). \tag{5}$$

The price at time $t = 1$ after a swap is $p_1^y = (y_0 + \Delta y)/(x_0 + \Delta x)$.

1.3 Targeting a price

Now suppose we want to make a swap to target a particular price, p_*^y, so that the price after the swap is given by

$$p_*^y = \frac{y_0 + \Delta y}{x_0 + \Delta x}. \tag{6}$$

If the target price is greater than the current price at time $p_*^y > p_0^y$, then we want to increase p_0^y. We can do this by swapping in Δy for an amount Δx. Substituting in to equation (6) for Δx from equation (5), we have

$$p_*^y = \frac{y_0 + \Delta y}{x_0 - x_0 \left(\frac{\Delta y}{y_0 + \Delta y} \right)} = \frac{y_0 + \Delta y}{x_0 \left(\frac{y_0}{y_0 + \Delta y} \right)} = \frac{(y_0 + \Delta y)^2}{x_0 y_0}. \tag{7}$$

Solving for Δy, the amount we need to swap in to drive the price to the target, we obtain

$$\Delta y = \sqrt{x_0 y_0 p_*^y} - y_0. \tag{8}$$

If the target price is less than the current price at time $p_*^y < p_0^y$, then we want to decrease p_0^y. We can do this by swapping in Δx for an amount Δy. Substituting in to equation (6) for Δy from equation (4), we have

$$p_*^y = \frac{y_0 + \Delta y}{x_0 + \Delta x} = \frac{y_0 \left(\frac{x_0}{x_0 + \Delta x} \right)}{x_0 + \Delta x} = \frac{x_0 y_0}{(x_0 + \Delta x)^2} \tag{9}$$

Solving for Δx, the amount we need to swap in to drive the price to the target, we obtain

$$\Delta x = \sqrt{\frac{x_0 y_0}{p_*^y}} - x_0. \tag{10}$$