

lec 9

Camera model, Calibration

① Why we should Calibrate Camera?

extrinsic, intrinsic parameters of Camera

Location orientation

focal length
size of Pixels

CV

Look for 3rd D

Pose estimation

- Given 3D model of object, its image (2D projection)
- Determine (location, orientation) of object such that when project on the image plane, it will match (translation, rotation)

② translation Matrix

$$\begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

dx, dy, dz, 1

Scaling Matrix

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

diagonal = $S_x, S_y, S_z, 1$

rotation around z-axis

$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation matrices are orthonormal matrices

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(R^z)^{-1} = (R^z)^T$$

$$(R^z)(R^z)^T = I$$

③ Euler Angle

rotation around arbitrary axis

if angles are small

$$\cos \theta \approx 1 \quad \sin \theta \approx \theta$$

$$R = \begin{bmatrix} 1 - \alpha & \beta \\ \alpha & 1 - \gamma \\ -\beta & \gamma & 1 \end{bmatrix}$$

g) in front of lens $\rightarrow \frac{y}{Y} = \frac{f}{Z} \Rightarrow y = \frac{fy}{Z} \quad X = \frac{fX}{Z}$

Perspective Projection

at lens: $-\frac{y}{Y} = \frac{f}{Z} \Rightarrow y = -\frac{fY}{Z}, \quad X = -\frac{fX}{Z}$

original image center: $-\frac{y}{Y} = \frac{f}{Z-f} \Rightarrow y = \frac{fY}{Z-f} \quad X = \frac{fX}{Z-f}$

$X = \frac{Ch_1}{Ch_4} \quad y = \frac{Ch_2}{Ch_4} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix}$

Camera Model

- Camera is at the origin of the world Co-ordinate
- translated by some amount (G)
- rotated around (Z) axis in Counter Clockwise
- rotated again around (X) axis u u u
- translated by (C)

\Rightarrow Since we are moving Camera instead of object
we need to use inverse transformation

$$Ch = P C R_x^X R_z^Z G W_h$$

Camera matrix

- ① Select known 3D Points (x, y, z) and their corresponding image point (x, y)
- ② Solve Camera matrix elements using Least Square Fit

homogenous system n points
 \Rightarrow no-unique solution n equations
 $2n$ unknowns

matrix of rotation & translation / \rightarrow extrinsic

Camera Parameters

① Extrinsic Parameters

\rightarrow Location, orientation of Camera reference frame with respect to known world reference frame

- a) 3D-translation vector
- b) 3X3 rotation matrix

② Intrinsic Parameters

Parameters necessary to link pixel Co-ordinates of an image point with the corresponding Co-ordinates in the Camera reference frame

- a) Perspective projection (focal length)
- b) transformation between Camera Plane Co-ordinates, pixel Co-ordinates

Image, Camera Co-ordinate

$$X = -(x_{im} - o_x) S_x$$

$$y = -(y_{im} - o_y) S_y$$

$$x_{im} = \frac{-X}{S_x} + o_x$$

$$y_{im} = \frac{-y}{S_y} + o_y$$

$x_{im}, y_{im} \rightarrow$ image Co-ordi

$o_x, o_y \rightarrow$ image center

S_x, S_y

\rightarrow effective size of pixels in horizontal, vertical directions

using known 3D points, img Co-ordinates

↓
estimate Camera matrix (pseudo inverse method)

Camera Parameters

Extrinsic → translation, rotation

intrinsic → horizontal fd, vertical fd
focal length and
translation of o_x, o_y

