

lec 3

Image Pyramids Frequency Domain

- Any univariate function can be re-written as
- ① a weighted sum of sines / cosines of different frequencies

Amplitude $\leftarrow (A) \sin(\omega x + \theta)$

Fourier transform stores:

- ② ① magnitude \rightarrow how much signal at Particular Freq.
② Phase \rightarrow spatial information (indirectly)

Amplitude $= \pm \sqrt{R(\omega)^2 + I(\omega)^2}$ Phase $= \theta = \tan^{-1} \frac{I(\omega)}{R(\omega)}$

- ③ In mathematical way, they are represented as Real, Complex numbers

- ④ (FFT) \Rightarrow Fast Fourier transform $= N \log N$

- ⑤ Fourier transform of Convolution of two functions is the product of their Fourier transform

$$F[g * h] = F[g] F[h]$$

- ⑥ Inverse Fourier transform of product of 2 Fourier transform is the Convolution of 2 inverse Ft

$$F^{-1}[gh] = F^{-1}[g] * F^{-1}[h]$$

Convolution in spatial \equiv Multiplication in Frequency

Properties of Fourier transform

- ① linearity ② Symmetric about origin
③ energy of signal = energy of its Fourier transform

⑧ Gaussian filter give more smooth image than Box filter

⑨ lower resolution \rightarrow Sampling
Subsampling by factor of 2 \rightarrow row $\frac{1}{2}$ \Rightarrow $\frac{\delta_{\text{row}}}{2}$
Col $\frac{1}{2}$ \Rightarrow $\frac{\delta_{\text{col}}}{2}$
 $= \frac{1}{4}$ \Rightarrow $\frac{\delta_{\text{area}}}{4}$

Aliasing Problem

- ① Wagon wheel ② Checkboards disintegrate
③ Striped shirts looks funny on Color TV

⑩

Nyquist sampling theorem
Sampling Frequency must be \geq
 $2 \times F_{\text{max}}$

إذا كان Sample
Frequency \geq

\downarrow
re Construct the original perfectly from
Sampled version

Anti-Aliasing \rightarrow Solution: Sample more often
Get rid of all frequencies that are greater than half
the new Sampling frequency

Template Matching

- ① Correlation
- ② Zero mean Correlation
- ③ Sum Square difference
- ④ Normalized cross Correlation

① Correlation

Problem : response is stronger in higher intensity

② Zero mean

Bright pixel above average / dark pixels below

Problem : ① response is sensitive to gain / Contrast

② pixels in filter (near) have little effect

③ doesn't require pixels in image to be near or proportional to values in filter

③ Sum Square difference (SSD)

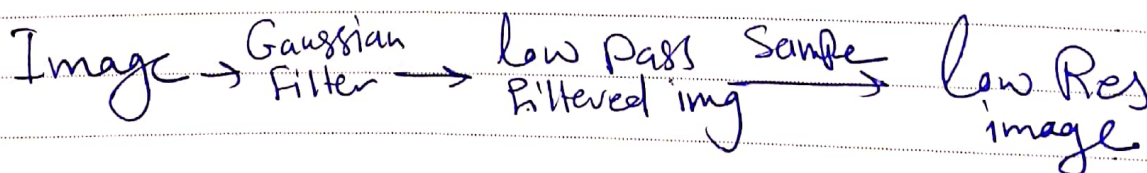
Fast Problem : response is sensitive to overall average intensity

④ Normalized Cross Correlation

✓ invariant to mean and Scale of intensity and Contrast

Problem : Slow

Sampling



mexican hat
Laplacian Filter \rightarrow (2nd) derivative of Gaussian
 \downarrow
2D edge detection $(\nabla^2) \rightarrow$ operator

how many 2d derivative filters?
 $\rightarrow 4 \begin{cases} \rightarrow 2 \text{ in } x \\ \rightarrow 2 \text{ in } y \end{cases}$

we can
re Construct
original image
from Laplacian
Pyramid

Image Pyramids

Pixels \rightarrow

great
poor

 spatial resolution
across Frequency

Fourier transform \rightarrow

Poor
great

 in spatial
in Frequency

Pyramids

 \rightarrow Balance between spatial, frequency

applications

- ① Compression
- ② object detection (scale Search Features)
- ③ detection stable points
- ④ Registration \rightarrow Coarse-to-Fine

Denoising

Smoothing with larger standard deviation Suppress noise
But also Blur Image

Salt, Pepper noise (by) Gaussian Smoothing

(31) → median filter → non linear

↓
Select median intensity
Not Convolution
Robust to outliers

Non Linear Filters

- ① weighted median
pixels further from center count less
- ② Clipped mean
average, ignoring low bright, dark pixels
- ③ Bilateral filter
weigh by spatial distance, intensity distance

