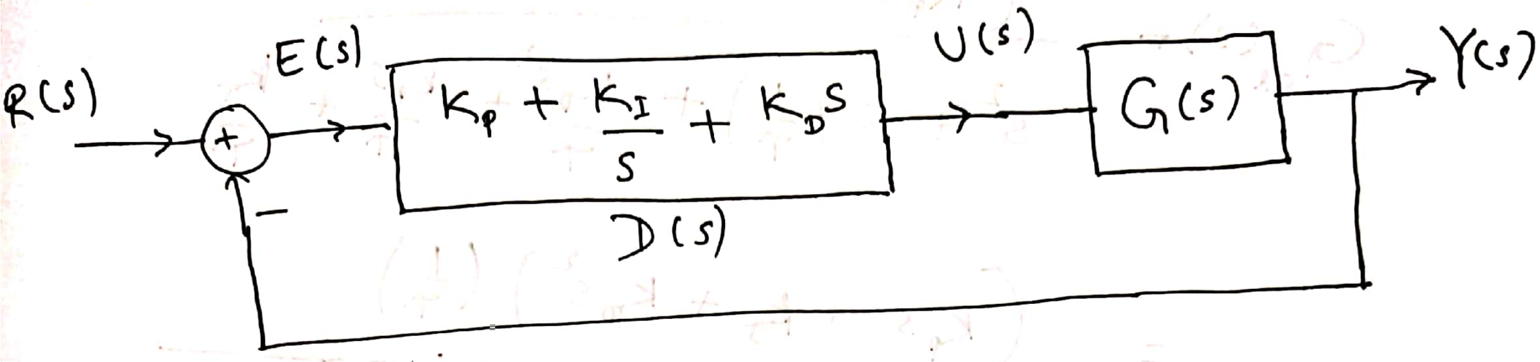
PID Controller



$$U(s) = \left[K_p + \frac{K_I}{s} + K_D s \right] E(s)$$

Control Law:

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau) \cdot d\tau + K_D \frac{d}{dt} e(t)$$

When $G(s) = \dfrac{1/\tau}{s + \frac{1}{\tau}}$ std. first order

$$L(s) = D(s)\,G(s) = \frac{(K_p s + K_I + K_D s^2)\, 1/\tau}{s\left(s + \frac{1}{\tau}\right)}$$

Type-1 system

Transfer function of closed-loop system

$$G_{cl}(s) = \frac{L(s)}{1 + L(s)}$$

$$G_{cl}(s) = \frac{(K_p s + K_I + K_D s^2)\, 1/\tau}{s^2 + s\left(\frac{1}{\tau}\right) + \frac{K_D s^2}{\tau} + \frac{K_p s}{\tau} + \frac{K_I}{\tau}}$$

$$= \frac{(K_p s + K_I + K_D s^2)\left(\frac{1}{\tau}\right)}{s^2\left(\frac{K_D + \tau}{\tau}\right) + \left(\frac{K_p + 1}{\tau}\right) s + \frac{K_I}{\tau}}$$

To compare the characteristic equation of closed-loop system with that of standard second order system divide by $\left(\frac{K_D + \tau}{\tau}\right)$

$$\alpha(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$= s^2 + \frac{K_p + 1}{K_D + \tau} s + \frac{K_I}{K_D + \tau}.$$

$$\omega_n^2 = \frac{K_I}{K_D + \tau} \Rightarrow \omega_n = \sqrt{\frac{K_I}{K_D + \tau}}$$

$$\zeta = \frac{1}{2\omega_n} \frac{K_p + 1}{K_D + \tau} = \frac{K_p + 1}{2\sqrt{K_I(K_D + \tau)}}$$

Given the plant model

$$G(s) = \frac{4}{s+4}$$

Design PID Controller such that poles of closed-loop system satisfy $\zeta = 0.6$ $\omega_n = 10$ rad/sec.

Soln:

$$\frac{1}{\tau} = 4 \Rightarrow \tau = \frac{1}{4} = 0.25$$

$$\omega_n = 10 \text{ rad/sec.}$$

$$\omega_n^2 = \frac{K_I}{K_D + \tau} = 100$$

Let $K_D = 3.75$

Then $K_I = 400$

$$\zeta = 0.6 = \frac{1}{20} \frac{K_p + 1}{K_D + \tau}.$$

$$K_p + 1 = 48 \Rightarrow K_p = 47.$$