Radix - 2 8 - point DIT FFT.

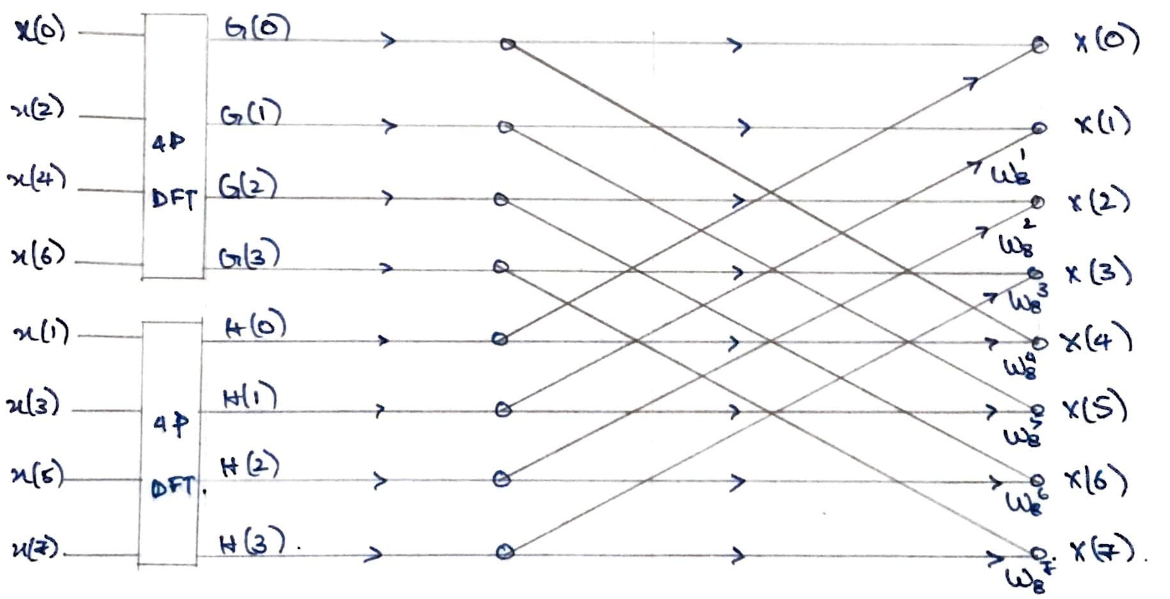
$\underline{N=8} . = 2\times2\times2.$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{7} x(n) W_8^{nk} = \sum_{n \to even} x(n) W_8^{nk} + \sum_{n \to odd} x(n) W_8^{nk}.$$

$$X(k) = \sum_{n=0}^{3} x(2n) W_8^{2nk} + \sum_{n=0}^{3} x(2n+1) W_8^{(2n+1)k}.$$

$$= \sum_{n=0}^{3} x(2n) \cdot W_4^{nk} + \sum_{n=0}^{3} x(2n+1) W_4^{nk} . W_8^{k}$$

$$= G(k) + H.(k). W_8^{k}$$



$X(0) = G(0) + H(0).$

$X(1) = G(1) + W_8^1 H(1).$

$X(2) = G(2) + W_8^2 H(2).$

$X(3) = G(3) + W_8^3 H(3).$

$X(4) = G(0) + W_8^4 H(4)$

$X(5) = G(1) + W_8^5 H(1)$

$X(6) = G(2) + W_8^6 H(2).$

$X(7) = G(3) + W_8^7 H(3).$

$$G(k) = \sum_{n=0}^{3} x(2n)\, W_4^{nk} \qquad H(k) = \sum_{n=0}^{3} x(2n+1)\, W_4^{kn}$$

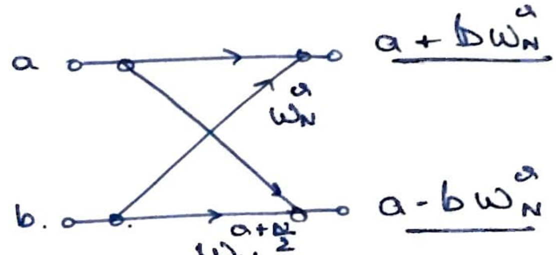
Σ odd + Σ even.

$$\sum_{n=0}^{1} x(2(2n))\, W_4^{2nk} + \sum_{n=0}^{1} x(2(2n+1))\, W_4^{nk(2n+1)}.$$

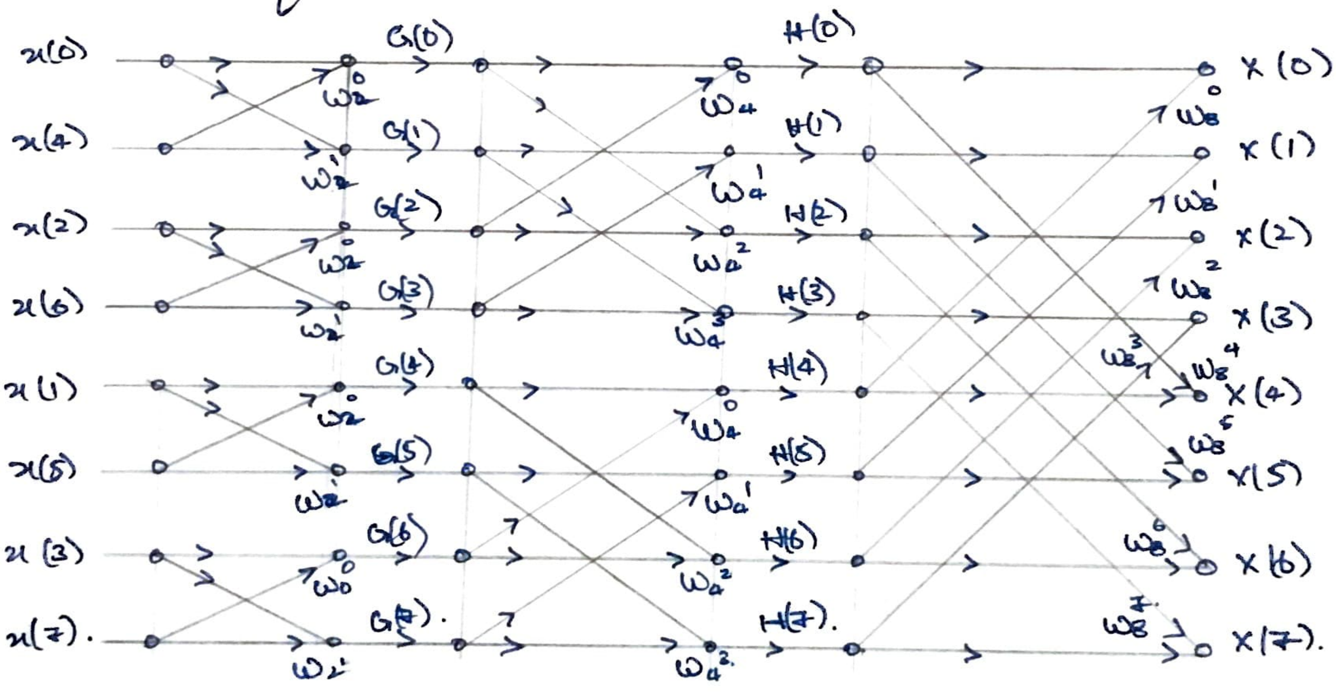
$$\sum_{n=0}^{1} x(4n)\, W_2^{nk} + \sum_{n=0}^{1} x(4n+2)\, W_2^{nk} \cdot W_4^{k}.$$

$$P(k) + Q(k)\, W_4^{k}.$$

$$\text{S}^{\text{lly}} \quad R(k) + S(k)\, W_4^{k}.$$



DFT of. $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
0 1 2 3 4 5 6 7



$G(0) = x(0) + x(4) = 6.$

$G(1) = x(0) - x(4) = 1 - 5 = -4.$

$G(2) = x(2) + x(6) = 3 + 7 = 10$

$G(3) = x(2) - x(6) = 3 - 7 = -4.$

$G(4) = x(1) + x(5). = 2 + 6 = 8.$

$G(5) = x(1) - x(5) = 2 - 6 = -4.$

$G(6) = x(3) + x(7). = 4 + 8 = 12.$

$G(7) = x(3) - x(7) = 4 - 8 = -4.$

$H(0) = G(0) + G(2) = 6 + 10 = 16.$ $\boxed{W_4^1 = -j.}$

$H(1) = G(1) + W_4^1 G(3) = -4 - j(-4) = -4 + 4j.$

$H(2) = G(0) - G(2) = 6 - 10 = -4.$

$H(3) = G(1) - W_4^1 G(3) .= -4 + j(-4) = -4 - 4j.$

$H(4) = G(4) + G(6). = 8 + 12 = 20$

$H(5) = G(5) + W_4^1 G(7) = -4 - j(-4) = -4 + 4j.$

$H(6) = G(4) - G(6) = 8 - 12 = -4.$

$H(7) = G(5) - W_4^1 G(7) = -4 + j(-4) = -4 - 4j.$

$X(0) = H(0) + H(4) = 16 + 20 = 36.$

$X(1) = H(1) + W_8^1 H(5) = -4 + 4j + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right)(-4 + 4j) = -4 + 9.65j.$

$X(2) = H(2) + W_8^2 H(6) = -4 + (-j)(-4) = -4 + 4j.$

$X(3) = H(3) + W_8^3 H(7) = -4 - 4j + \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right)(-4 - 4j) = -4 + 1.65j$

$X(4) = H(0) - H(4). = -4$

$X(5) = H(1) - W_8^1 H(5) = -4 - 1.657j$

$X(6) = H(2) - W_8^2 H(6). = -4 - 4j$

$X(7) = H(3) - W_8^3 H(7) = -4 - 9.657j.$

Radix 2 - DIF FFT

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn}. \qquad \underline{N=8.}$$

$$X(K) = \sum_{n=0}^{7} x(n) W_N^{Kn} = \sum_{n=0}^{3} x(n) W_8^{Kn} + \sum_{4}^{7} x(n) W_8^{Kn}.$$

$$= \sum_{n=0}^{3} x(n) W_8^{Kn} + \sum_{n=0}^{3} x\left(n+\frac{N}{2}\right) W_N^{K\left(n+\frac{N}{2}\right)}.$$

$$\sum_{n=0}^{3} x(n) W_8^{Kn} + \sum_{n=0}^{3} x\left(n+\frac{N}{2}\right) W_8^{Kn} \cdot W_8^{K\frac{N}{2}}.$$

$$X(K) = X_1(K) + X_2(K) W_8^{K\frac{N}{2}} = X_1(K) + X_2(K)(-1)^K.$$

For even sequence. $= \sum_{n=0}^{3} \left[x(n) + x\left(n+\frac{N}{2}\right)(-1)^K\right] W_8^{Kn}.$

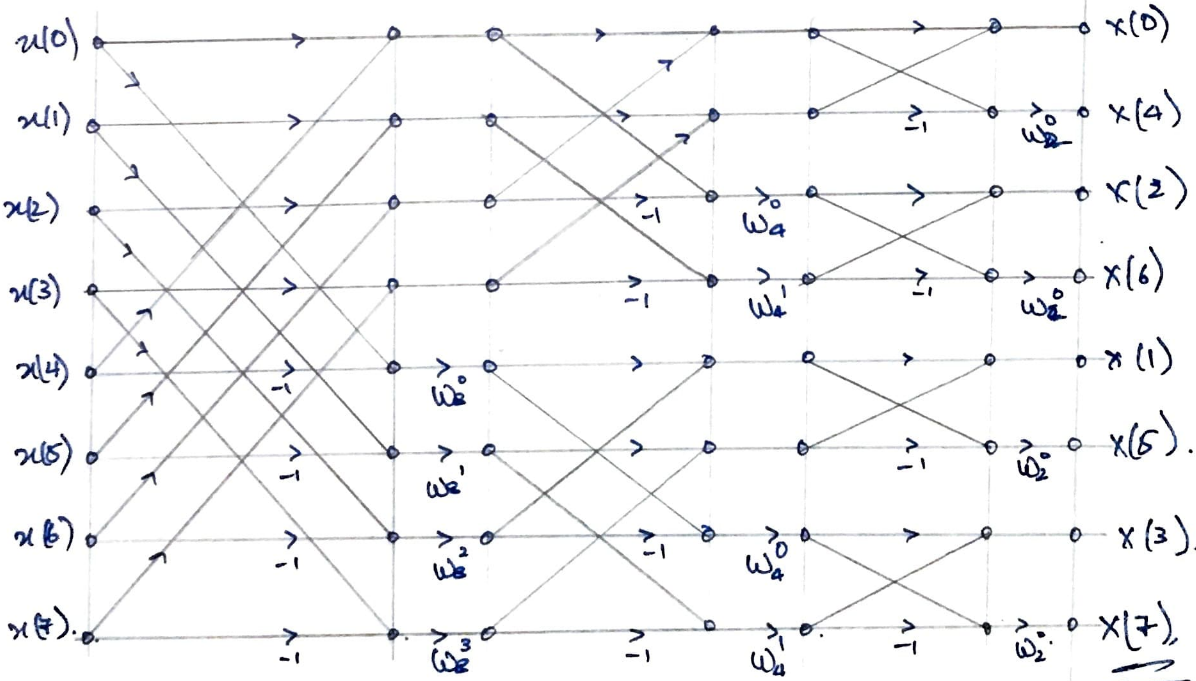
$K = 2K.$

$$X(2K) = X_1(2K) + X_2(2K)(-1)^{2K} = X_1(2K) + X_2(2K).$$

For odd.

$K = 2K+1.$

$$X(2K+1) = X_1(2K+1) + X_2(2K+1)(-1)^{2K+1} = X_1(2K+1) - X_2(2K+1).$$



$X_1(0) = x(0) + x(4) = 1 + 5 = 6$ $\boxed{x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}}$

$X_1(1) = x(1) + x(5) = 8$

$X_1(2) = x(2) + x(6) = 10$

$X_1(3) = x(3) + x(7) = 12$

$X_1(4) = x(0) - W_8^0 x(4) = -4$

$X_1(5) = x(1) - W_8^1 x(5) = -2.82 + 2.82j$

$X_1(6) = x(2) - W_8^2 x(6) = 4j$

$X_1(7) = x(3) - W_8^3 x(7) = 2.82 + 2.82j$.

$\longrightarrow$

$X_2(0) = X_1(0) + X_1(2) = 16$

$X_2(1) = X_1(1) + X_1(3) = 20$

$X_2(2) = X_1(0) - W_4^0 X_1(2) = -4$

$X_2(3) = X_1(1) - W_4^1 X_1(3) = 4j$

$X_2(4) = X_1(4) + X_1(6) = -4 + 4j$

$X_2(5) = X_1(5) + X_1(7) = 5.65j$

$X_2(6) = X_1(4) - W_4^0 X_1(6) = -4 - 4j$

$X_2(7) = X_1(5) - W_4^1 X_1(7) = 5.65j$.

$\longrightarrow$

$X(0) = X_2(0) + X_2(1) = 36$

$X(4) = X_2(0) - X_2(1) W_2^0 = -4$

$X(2) = X_2(2) + X_2(3) = -4 + 4j$

$X(6) = X_2(2) - X_2(3) = -4 - 4j$

$X(1) = X_2(4) + X_2(5) = -4 + 9.65j$

$X(5) = X_2(4) - X_2(5) = -4 - 1.65j$

$X(3) = X_2(6) + X_2(7) = -4 + 1.65j$

$X(7) = X_2(6) - X_2(7) = -4 - 9.65j$.

$x(k) = \{ 36$ ← 0th place.

$-4 + 9.65j$

$-4 + 4j$

$-4 + 1.65j$.

-4

$-4 - 1.65j$

$-4 - 4j$

$-4 - 9.65j \}$.

Complex Radix $\underline{N=6} = \underline{3 \times 2}$. DIT FFT.

$x(n) = \{7, 8, 9, 8, 7, 6\}$

2 - Stages.

Radix 2 in 1st stage.

— " - 3 — " - 2nd " .

Sol: $X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK}$.

$$X(K) = \sum_{n=0}^{1} x(3n) W_N^{3nK} + \sum_{n=0}^{1} x(3n+1) W_N^{(3n+1)K} + \sum_{n=0}^{1} x(3n+2) W_N^{(3n+2)K}.$$

$\underline{N=6}$.

$$= \sum_{n=0}^{1} x(3n) W_2^{nK} + \sum_{n=0}^{1} x(3n+1) W_2^{nK} \cdot W_6^{K} + \sum_{n=0}^{1} x(3n+2) W_2^{nK} \cdot W_6^{2K}.$$

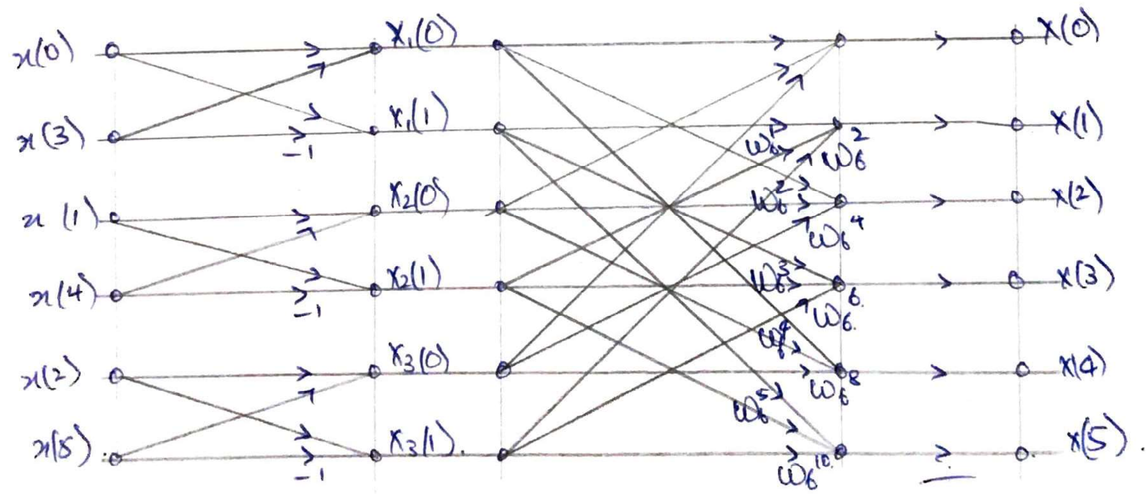
$$= X_1(K) + X_2(K) W_6^{K} + X_3(K) W_6^{2K}.$$

$$X_1(K) = \sum_{n=0}^{1} x(3n) W_2^{nK} = x(0) + x(3) W_2^{K}.$$

$$X_2(K) = \sum_{n=0}^{1} x(3n+1) W_2^{nK} = x(1) + x(4) W_2^{K}$$

$$X_3(K) = \sum_{n=0}^{1} x(3n+2) W_2^{nK} = x(2) + x(5) W_2^{K}.$$

$$X(K) = X_1(K) + X_2(K) W_6^{K} + X_3(K) W_6^{2K}.$$



$X_1(0) = x(0) + x(3) = 15$

$X_1(1) = x(0) - x(3) = -1$

$X_2(0) = x(4) + x(1). = 8+7 = 15$

$X_2(1) = x(1) - x(4). = 1$

$X_3(0) = x(2) + x(5). = 9+6 = 15$

$X_3(1) = x(2) - x(5). = 9-6 = \underline{3.}$

$X(0) = X_1(0) + X_2(0) + X_3(0) = 15 + 15 + 15 = \underline{45}$

$X(1) = X_1(1) + X_2(1) W_6^1 + X_3(1) W_6^2 = -2 - 3.46j.$

$X(2) = X_1(0) + X_2(0) W_6^2 + X_3(0) W_6^4 = 0$

$X(3) = X_1(1) + X_2(1) W_6^3 + X_3(1) W_6^6 = 1$

$X(4) = X_1(0) + X_2(0) W_6^4 + X_3(0) W_6^8 = 0$

$X(5) = X_1(1) + X_2(1) W_6^5 + X_3(1) W_6^{10} = -2 + 3.46j.$

<u>Direct cal:</u>

$W_6^0 = 1 \quad W_6^1 = 0.5 - 0.866j. \quad W_6^2 = -0.5 - 0.866j.$

$W_6^3 = -1 \quad W_6^4 = -0.5 + 0.866j \quad W_6^5 = 0.5 + 0.866j.$

$X(k) = \sum_{n=0}^{5} x(n) W_6^{nk}.$

$X(0) = 7 + 8 + 9 + 8 + 7 + 6 = 45.$

$X(1) = 7 + 8W_6^1 + 9W_6^2 + 8W_6^3 + 7W_6^4 + 6W_6^5 = -2 - 3.46j.$

$X(2) = 7 + 8W_6^2 + 9W_6^4 + 8W_6^6 + 7W_6^8 + 6W_6^{10} = 0$

$X(3) = 7 + 8(-1) + 9 - 8 + 7 - 6 = 1$

$X(4) = 7 + 8W_6^4 + 9W_6^8 + 8W_6^{12} + 7W_6^{16} + 6W_6^{20} = 0.$

$X(5) = 7 + 8W_6^5 + 9W_6^{10} + 8W_6^{15} + 7W_6^{20} + 6W_6^{25} = \underline{\underline{-2 + 3.46j}}.$

(b) $N = 6 = \underline{2 \times 3}$. $X(k) = \sum_{n=0}^{5} x(n) W_6^{kn}$.

$$X(k) = \sum_{n=0}^{2} x(2n) W_6^{2nk} + \sum_{n=0}^{2} x(2n+1) W_6^{(2n+1)k}$$

$$\underline{X_1(k) + W_6^{k} X_2(k)}.$$

$$X_1(k) = \sum_{n=0}^{2} x(2n) W_6^{2nk} = x(0) + x(2) \cdot W_6^{2k} + x(4) W_6^{4k}.$$

$X_1(0) = x(0) + x(2) + x(4)$. $X_1(1) = x(0) + x(2) W_6^{2} + x(4) W_6^{4}$

$X_2(0) = x(1) + x(3) + x(5)$. $X_2(1) = x(1) + x(3) W_6^{2} + x(5) W_6^{4}$

$X_1(2) = x(0) + x(2) W_6^{4} + x(4) W_6^{8}$ $X_2(2) = x(1) + x(3) W_6^{4} + x(5) W_6^{8}$

$$X(k) = X_1(k) + W_6^k X_2(k).$$

$$X(0) = X_1(0) + X_2(0).$$

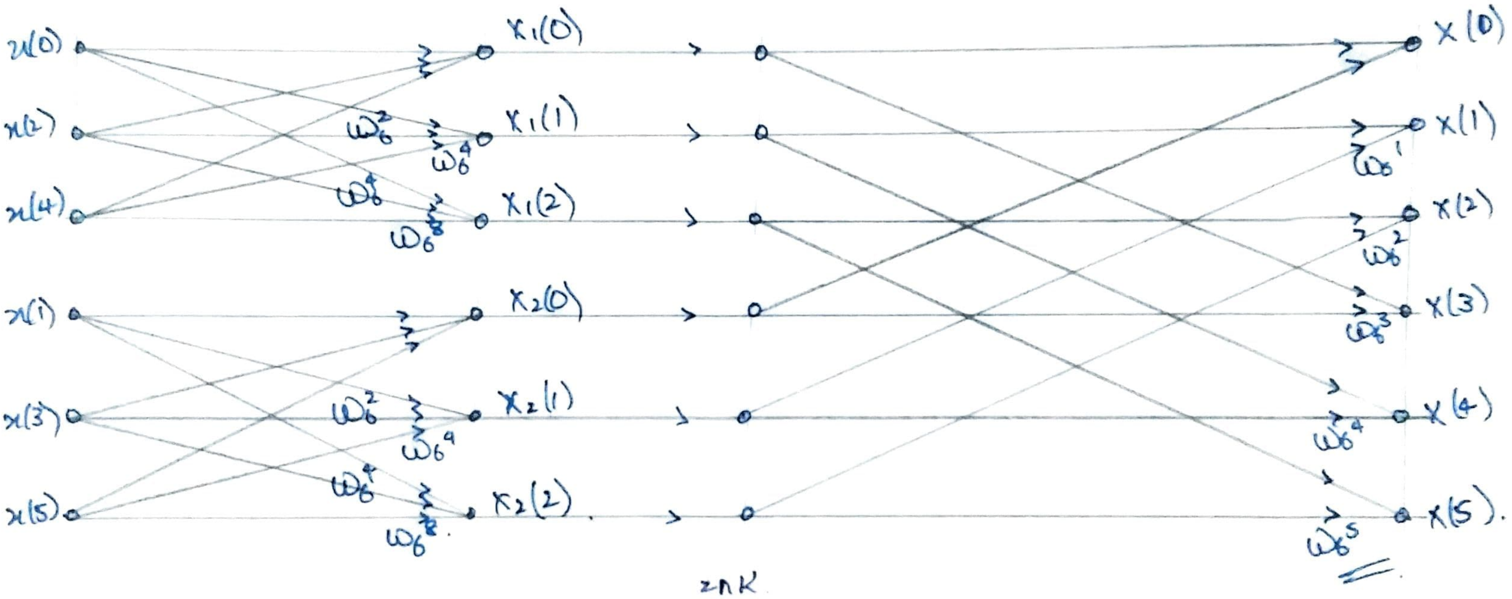
$$X(1) = X_1(1) + W_6^1 X_2(1)$$

$$X(2) = X_1(2) + W_6^2 X_2(2).$$

$$X(3) = X_1(0) + W_6^3 X_2(0).$$

$$X(4) = X_1(1) + W_6^4 X_2(1).$$

$$X(5) = X_1(2) + W_6^5 X_2(2).$$



$$X_1(k) = \sum_{n=0}^{2} x(2n)\, W_6^{2nk}$$

$X_1(0) = 9$

$X_1(1) = -3 + 1.73j$

$X_1(2) = -3 - 1.73j$

$X_2(0) = 12.$

$X_2(1) = -3 + 1.7j$

$X_2(2) = -3 - 1.73j.$

$X(0) = 9 + 12 = 21.$

$X(1) = -3 + 5j + W_6^1(-3 + 1.7j) = -3 + 5.19j.$

$X(2) = -3 - 1.73j + W_6^2(-3 - 1.7j) = -3 + 1.7j.$

$X(3) = 9 - 12 = -3.$

$X(4) = (-3 + 1.73j) + W_6^4(-3 + 1.7j) = -3 - 1.73j$

$X(5) = -3 + 5.196j.$

(b) $N = 6 = 2 \times 3$. $\quad X(K) = \sum_{n=0}^{5} x(n) W_6^{nk}$ $\quad x(n) = [1, -1, 2, -2, 3, -3]$

$$X(K) = \sum_{n=0}^{2} x(2n) W_6^{2nk} + \sum_{n=0}^{2} x(2n+1) W_6^{(2n+1)k}$$

$$X_1(K) + W_6^{K} X_2(K).$$

$$X_1(K) = \sum_{n=0}^{2} x(2n) W_6^{2nk} = x(0) + x(2) \cdot W_6^{2k} + x(4) W_6^{4k}.$$

$X_1(0) = x(0) + x(2) + x(4)$. $\quad X_1(1) = x(0) + x(2) W_6^{2} + x(4) W_6^{4}$

$X_2(0) = x(1) + x(3) + x(5)$. $\quad X_2(1) = x(1) + x(3) W_6^{2} + x(5) W_6^{4}$

$X_1(2) = x(0) + x(2) W_6^{4} + x(4) W_6^{8}$ $\quad X_2(2) = x(1) + x(3) W_6^{4} + x(5) W_6^{8}$

$$X(k) = X_1(k) + W_6^k X_2(k).$$

$$X(0) = X_1(0) + X_2(0).$$

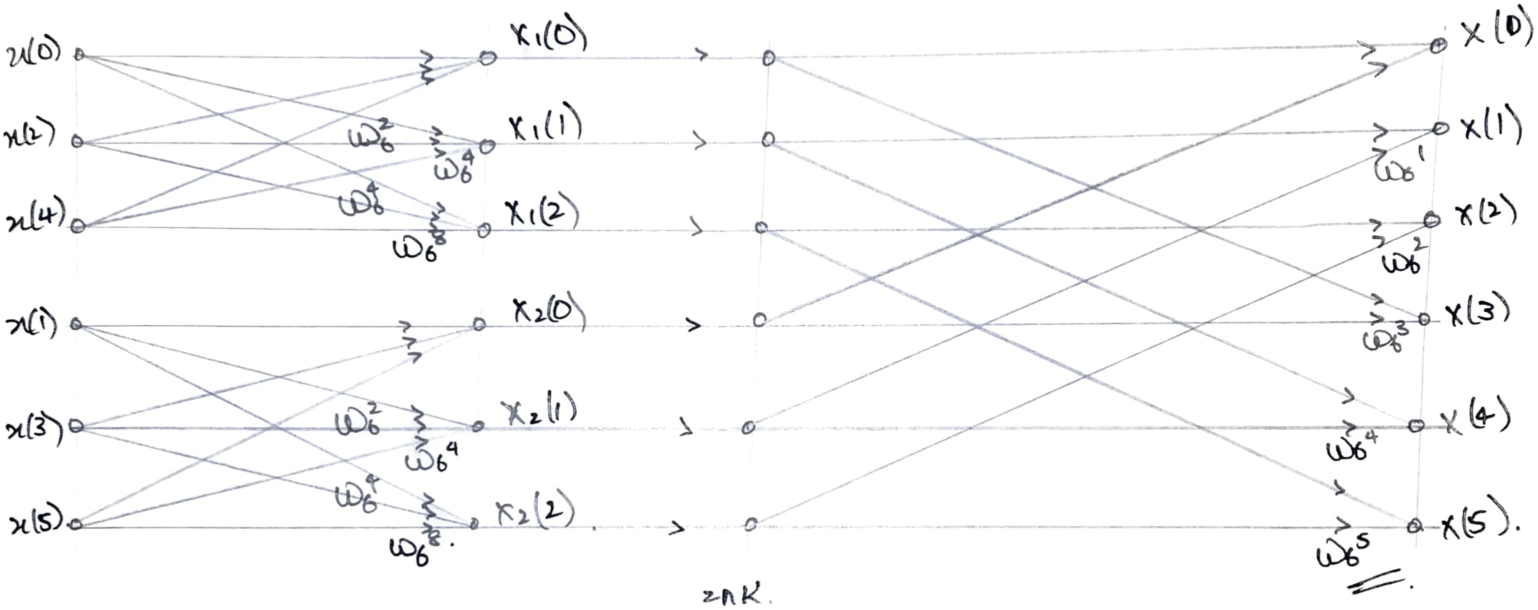
$$X(1) = X_1(1) + W_6^1 X_2(1)$$

$$X(2) = X_1(2) + W_6^2 X_2(2).$$

$$X(3) = X_1(0) + W_6^3 X_2(0).$$

$$X(4) = X_1(1) + W_6^4 X_2(1).$$

$$X(5) = X_1(2) + W_6^5 X_2(2).$$



$$X_1(k) = \sum_{n=0}^{2} x(2n)\, W_6^{2nk}$$

$X_1(0) = 9$

$X_1(1) = -3 + 1.73j$

$X_1(2) = -3 - 1.73j$

$X_2(0) = 12.$

$X_2(1) = -3 + 1.7j$

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$X(0) = 9 + 12 = 21.$

$X(1) = -3 + 5j + W_6^1(-3 + 1.7j) = -3 + 5.19j.$

$X(2) = -3 - 1.73j + W_6^2(-3 - 1.7j) = -3 + 1.7j.$

$X(3) = 9 - 12 = -3.$

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$X(5) = -3 + 5.196j.$