Radix-2 8-point DIT FFT.

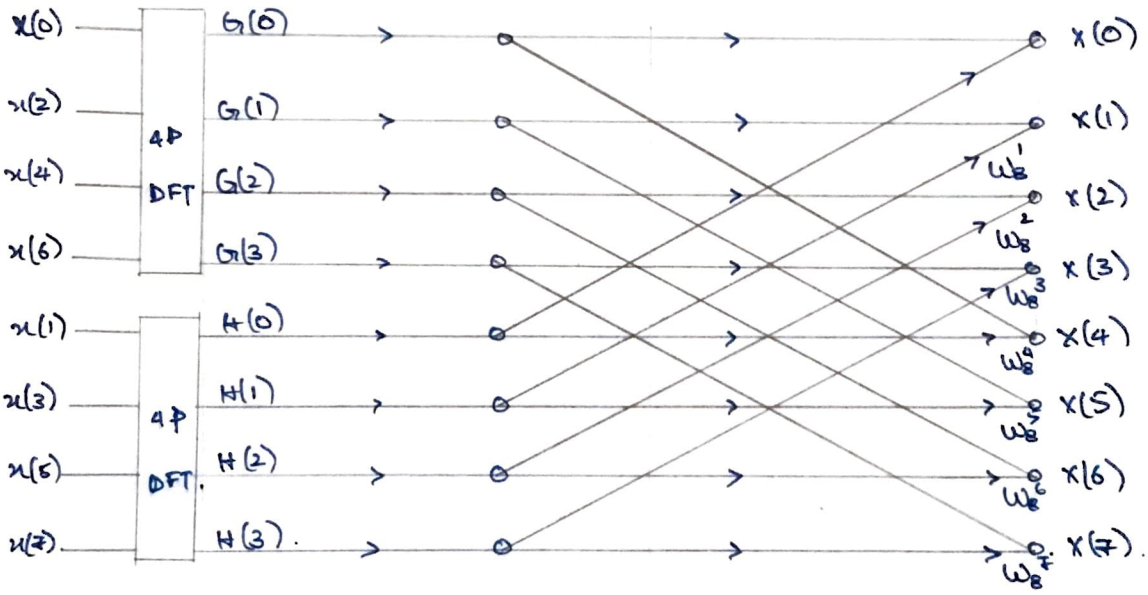
$N = 8 = 2 \times 2 \times 2$.

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{7} x(n) W_8^{nk} = \sum_{n \to \text{even}} x(n) W_8^{nk} + \sum_{n \to \text{odd}} x(n) W_8^{nk}$$

$$X(k) = \sum_{n=0}^{3} x(2n) W_8^{2nk} + \sum_{n=0}^{3} x(2n+1) W_8^{(2n+1)k}$$

$$= \sum_{n=0}^{3} x(2n) \cdot W_4^{nk} + \sum_{n=0}^{3} x(2n+1) W_4^{nk} \cdot W_8^{k}$$

$$= G_1(k) + H_1(k) \cdot W_8^{k}$$



$X(0) = G_1(0) + H(0)$.

$X(1) = G_1(1) + W_8^1 H(1)$.

$X(2) = G_1(2) + W_8^2 H(2)$.

$X(3) = G_1(3) + W_8^3 H(3)$.

$X(4) = G_1(0) + W_8^4 H(4)$

$X(5) = G_1(1) + W_8^5 H(1)$

$X(6) = G_1(2) + W_8^6 H(2)$.

$X(7) = G_1(3) + W_8^7 H(3)$.

$$G(k) = \sum_{n=0}^{3} x(2n)\, W_4^{nk} \qquad H(k) = \sum_{n=0}^{3} x(2n+1)\, W_4^{kn}$$

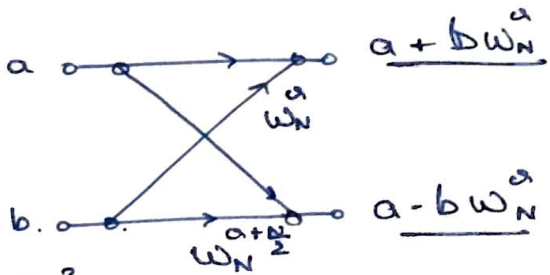
Σ odd + Σ even.

$$\sum_{n=0}^{1} x(2(2n))\, W_4^{2nk} + \sum_{n=0}^{1} x(2(2n+1))\, W_4^{nk(2n+1)}$$

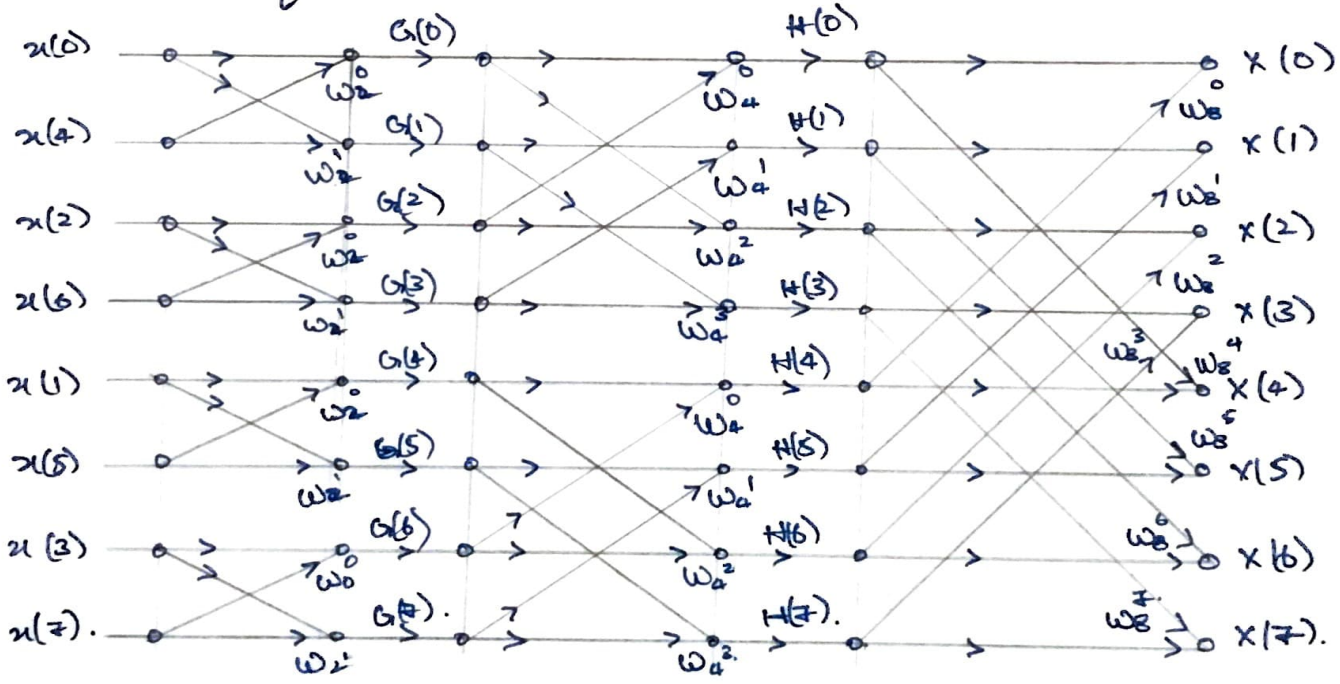
$$\sum_{n=0}^{1} x(4n)\, W_2^{nk} + \sum_{n=0}^{1} \{x(4n+2)\, W_2^{nk} \cdot W_4^{k}$$

$$P(k) + Q(k)\, W_4^{k}$$

Silly $R(k) + S(k)\, W_4^{k}$



DFT of $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
0 1 2 3 4 5 6 7



$G(0) = x(0) + x(4) = 6.$

$G(1) = x(0) - x(4) = 1 - 5 = -4.$

$G(2) = x(2) + x(6) = 3 + 7 = 10$

$G(3) = x(2) - x(6) = 3 - 7 = -4.$

$G(4) = x(1) + x(5) = 2 + 6 = 8.$

$G(5) = x(1) - x(5) = 2 - 6 = -4.$

$G(6) = x(3) + x(7) = 4 + 8 = 12.$

$G(7) = x(3) - x(7) = 4 - 8 = -4.$

$\boxed{W_4' = -j.}$

$H(0) = G(0) + G(2) = 6 + 10 = 16.$

$H(1) = G(1) + W_4' G(3) = -4 - j(-4) = -4 + 4j.$

$H(2) = G(0) - G(2) = 6 - 10 = -4.$

$H(3) = G(1) - W_4' G(3) = -4 + j(-4) = -4 - 4j.$

$H(4) = G(4) + G(6) = 8 + 12 = 20$

$H(5) = G(5) + W_4' G(7) = -4 - j(-4) = -4 + 4j.$

$H(6) = G(4) - G(6) = 8 - 12 = -4.$

$H(7) = G(5) - W_4' G(7) = -4 + j(-4) = -4 - 4j.$

$x(0) = H(0) + H(4) = 16 + 20 = 36.$

$x(1) = H(1) + W_8' H(5) = -4 + 4j + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)(-4 + 4j) = -4 + 9.65j.$

$x(2) = H(2) + W_8^2 H(6) = -4 + (-j)(-4) = -4 + 4j.$

$x(3) = H(3) + W_8^3 H(7) = -4 - 4j + \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)(-4 - 4j) = -4 + 1.65j$

$x(4) = H(0) - H(4) = -4$

$x(5) = H(1) - W_8' H(5) = -4 - 1.657j$

$x(6) = H(2) - W_8^2 H(6) = -4 - 4j$

$x(7) = H(3) - W_8^3 H(7) = -4 - 9.654j.$

Radix 2 - DIF FFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}. \qquad N = 8.$$

$$x(K) = \sum_{n=0}^{7} x(n)\, W_N^{Kn} = \sum_{n=0}^{3} x(n)\, W_8^{Kn} + \sum_{4}^{7} x(n)\, W_8^{Kn}.$$

$$= \sum_{n=0}^{3} x(n)\, W_8^{Kn} + \sum_{n=0}^{3} x\left(n+\frac{N}{2}\right) W_N^{K\left(n+\frac{N}{2}\right)}.$$

$$\sum_{n=0}^{3} x(n)\, W_8^{Kn} + \sum_{n=0}^{3} x\left(n+\frac{N}{2}\right) W_8^{Kn} \cdot W_8^{K\frac{N}{2}}.$$

$$X(K) = X_1(K) + X_2(K)\, W_8^{K\frac{N}{2}} = X_1(K) + X_2(K)(-1)^K.$$

For even sequence. $= \sum_{n=0}^{3} \left[x(n) + x\left(n+\frac{N}{2}\right)(-1)^K\right] W_8^{K(n)}.$

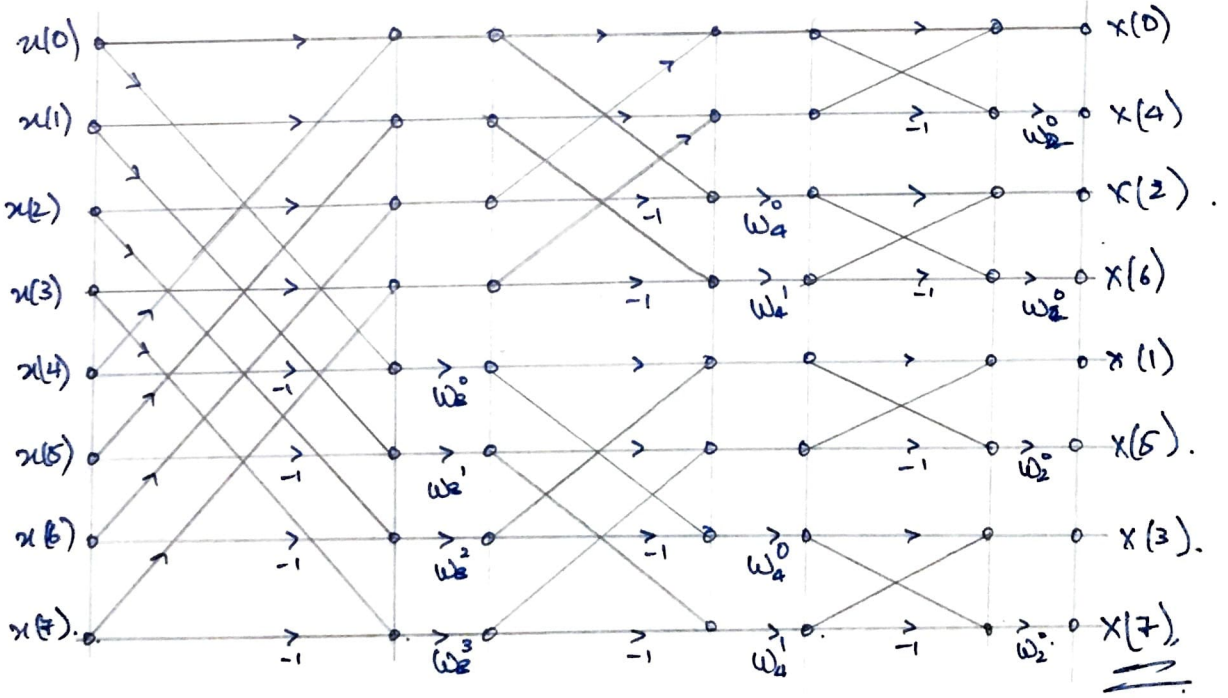
$K = 2R.$

$$X(2K) = X_1(2K) + X_2(2K)(-1)^{2K} = X_1(2K) + X_2(2K).$$

For odd.

$K = 2K+1.$

$$X(2K+1) = X_1(2K+1) + X_2(2K+1)(-1)^{2K+1} = X_1(2K+1) - X_2(2K+1).$$



$x_1(0) = x(0) + x(4) = 1 + 5 = 6$ $\quad x(n) = \{1, 2, 3, 4, 8, 6, 7, 8\}$

$x_1(1) = x(1) + x(5) = 8$

$x_1(2) = x(2) + x(6) = 10$

$x_1(3) = x(3) + x(7) = 12$

$x_1(4) = x(0) - W_8^0 x(4) = -4$

$x_1(5) = x(1) - W_8^1 x(5) = -2.82 + 2.82j$

$x_1(6) = x(2) - W_8^2 x(6) = 4j$

$x_1(7) = x(3) - W_8^3 x(7) = 2.82 + 2.82j$.

$\longrightarrow$

$x_2(0) = x_1(0) + x_1(2) = 16$

$x_2(1) = x_1(1) + x_1(3) = 20$

$x_2(2) = x_1(0) - W_4^0 x_1(2) = -4$

$x_2(3) = x_1(1) - W_4^1 x_1(3) = 4j$

$x_2(4) = x_1(4) + x_1(6) = -4 + 4j$

$x_2(5) = x_1(5) + x_1(7) = 5.65j$

$x_2(6) = x_1(4) - W_4^0 x_1(6) = -4 - 4j$

$x_2(7) = x_1(5) - W_4^1 x_1(7) = 5.65j$.

$\longrightarrow$

$x(0) = x_2(0) + x_2(1) = 36$

$x(4) = x_2(0) - x_2(1) W_2^0 = -4$

$x(2) = x_2(2) + x_2(3) = -4 + 4j$

$x(6) = x_2(2) - x_2(3) = -4 - 4j$

$x(1) = x_2(4) + x_2(5) = -4 + 9.65j$

$x(5) = x_2(4) - x_2(5) = -4 - 1.65j$

$x(3) = x_2(6) + x_2(7) = -4 + 1.65j$

$x(7) = x_2(6) - x_2(7) = -4 - 9.65j$.

$X(k) = \{$ 36 ← 0th place.

$-4 + 9.65j$

$-4 + 4j$

$-4 + 1.65j$

-4

$-4 - 1.65j$

$-4 - 4j$

$-4 - 9.65j$ $\}$.