① Digital butterworth low pass Filter. Using IIT at $\underline{T=1}$.

$A_P = -1.932$ dB. $\omega_P = 0.2\pi$ rad $= 0.6285$

$A_S =$ ~~133~~ -13.9794 dB. $\omega_S = 0.6\pi$ rad $= 1.8857$.

Solution: For IIT.

$$\Omega_P = \frac{\omega_P}{T} = 0.6285 \text{ rad/s}.$$

$$\Omega_S = \frac{\omega_S}{T} = 1.8857 \text{ rad/s}.$$

Step 1: Order — N.

$$N \geq \frac{\log_{10}\left[\frac{10^{0.1A_S}-1}{10^{0.1A_P}-1}\right]}{2\log_{10}\frac{\Omega_S}{\Omega_P}} = \frac{\log_{10}\left[\frac{10^{0.1\times 13.9794}-1}{10^{0.1\times 1.932}-1}\right]}{2\log_{10}\left[\frac{1.8857}{0.6285}\right]}$$

$N \geq 1.7099$ $\boxed{\therefore N = 2.}$

Step 2:

Calculating Cutoff Ω_c.

$$\Omega_{cP} = \frac{\Omega_P}{\left(10^{0.1A_P}-1\right)^{1/2N}} = \frac{0.6285}{\left(10^{0.1\times1.932}-1\right)^{1/4}} = 0.726.$$

$$\Omega_{cS} = \frac{\Omega_S}{\left(10^{0.1A_S}-1\right)^{1/2N}} = \frac{1.8857}{\left(10^{0.1\times13.9794}-1\right)^{1/4}} = 0.852.$$

$$\therefore \Omega_c = \frac{\Omega_{cP}+\Omega_{cS}}{2} = 0.7889 \text{ rad/s}.$$

Step 3: Designing Normalised Filter.

$$p_k = \pm e^{j\frac{(2k+1+N)\pi}{2N}}$$

$$p_0 = \pm(0.707 + 0.707j).$$

$$p_1 = \pm(-0.707 - 0.707j).$$

$$H(s) = \frac{1}{(s+0.707-0.707j)(s+0.707+0.707j)}.$$

$$\frac{1}{(s+a+jb)(s+a-jb)} = \frac{1}{s^2+2as+a^2+b^2}.$$

$$= \frac{1}{(s^2+1.414s+1)}$$

Step 4: De normalising:

$$s \to \frac{s}{\Omega_c} = \frac{s}{0.789}.$$

$$\therefore H(s) = \frac{1}{s^2+1.414s+1} \quad s \to \frac{s}{0.789}.$$

$$H(s) = \frac{0.623}{s^2+1.11s+0.623}.$$

Step 5: $s \to z$ transformation.

$$\frac{b}{(s+a)^2+b^2} = \frac{1.78 \times 0.561}{(s+0.555)^2+(0.561)^2} \rightarrow \frac{e^{-at}\sin bt\, z^{-1}}{1-2e^{-at}\cos bt\, z^{-1}+e^{-2aT}z^{-2}}$$

$$H(z) = \frac{e^{-0.555}\sin(0.561)z^{-1}}{1-2e^{-0.555}\cos(0.561)z^{-1}+e^{-2(0.555)}z^{-2}}.$$

$$H(z) = \frac{0.305\, z^{-1}}{1 - 0.9721 z^{-1} + 0.3295 z^{-2}}$$

(2) Design a chebyschev Filter.

$0.75 \le H(\omega) \le 1$ $\quad 0 \le \omega \le 0.25\pi$

$|H(\omega)| \le 0.23$ $\quad 0.63\pi \le \omega \le \pi$

BLT : $T = 2$

Sol:

$A_p = 0.75$ $\quad \omega_p = 0.25\pi$ $\quad \Omega_p = \frac{2}{2}\tan\frac{\omega_p}{2} = 0.414$ rad/s.

$A_s = 0.23$ $\quad \omega_s = 0.63\pi$. $\quad \Omega_s = \frac{2}{2}\tan\frac{\omega_s}{2} = 1.522$ rad/s.

$A_p = -20\log(0.75) = 2.49$ dB. $\quad \Omega_p = 0.414$ rad/s.

$A_s = -20\log(0.23) = 12.76$ dB. $\quad \Omega_s = 1.522$ rad/s.

Normallizing:

$$\Omega_p' = \frac{\Omega_p}{\Omega_p} = 1 \qquad \Omega_s' = \frac{\Omega_s}{\Omega_p} = 3.676.$$

Finding ε.

$$\varepsilon = \sqrt{10^{0.1A_p} - 1} = 0.8798.$$

Order N

$$N \ge \frac{A_s - 20\log_{10}(\varepsilon) + 6}{6 + 20\log_{10}(\Omega_s')} \ge \cancel{2.657}\ 1.148 = \cancel{3} = 2.$$

$N = 2$.

$$H_1(s) = \frac{K}{(s-s_1)(s-s_2)}.$$

$$s_1 = -0.3583 + j\,0.7927.$$

$$s_2 = -0.3583 - j\,0.7927.$$

$$H_1(s) = \frac{K}{s^2 + 0.7166s + 0.7567}.$$

Finding K.

$$K = \frac{b_0}{\sqrt{1-\varepsilon^2}} \quad \Big\} \quad \because N \text{ is even.} \qquad b_0 = 0.7567.$$

$$\therefore K = 0.56.$$

Frequency transformation.

$$LP - LP \qquad s \to \frac{s}{\omega_{2P}} = \frac{s}{0.414}.$$

$$H(s) = \frac{0.56(0.414)^2}{s^2 + 0.7166 \times 0.414s + 0.7567 \times 0.414^2}$$

$$H(s) = \frac{0.0959}{s^2 + 0.2966s + 0.129}.$$

Finding $H(z)$.

$$s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \qquad \underline{T=2.}$$

$$s \to \left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$H(z) = \frac{0.0959}{\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 0.3\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.129.}$$

$$\cancel{0.995}\ \frac{0.0959\left[1+2z^{-1}+z^{-2}\right]}{1.429+(-1.742)z^{-1}+0.829z^{-2}}$$