

“Event 4 Report”

Submitted for the fulfillment of the CIE (Event-4) for the course

CONTROL SYSTEMS

(EC540)

Submitted by

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## Problem Statement

For the plant  $G(s) = 1/s(s+6)$  design a **phase-lead** controller for damping ratio  $\zeta = 0.4$  and natural frequency **15 rad/sec**.

What is the phase margin and gain margin of the compensated system?

## Solution

### Clearing Workspace

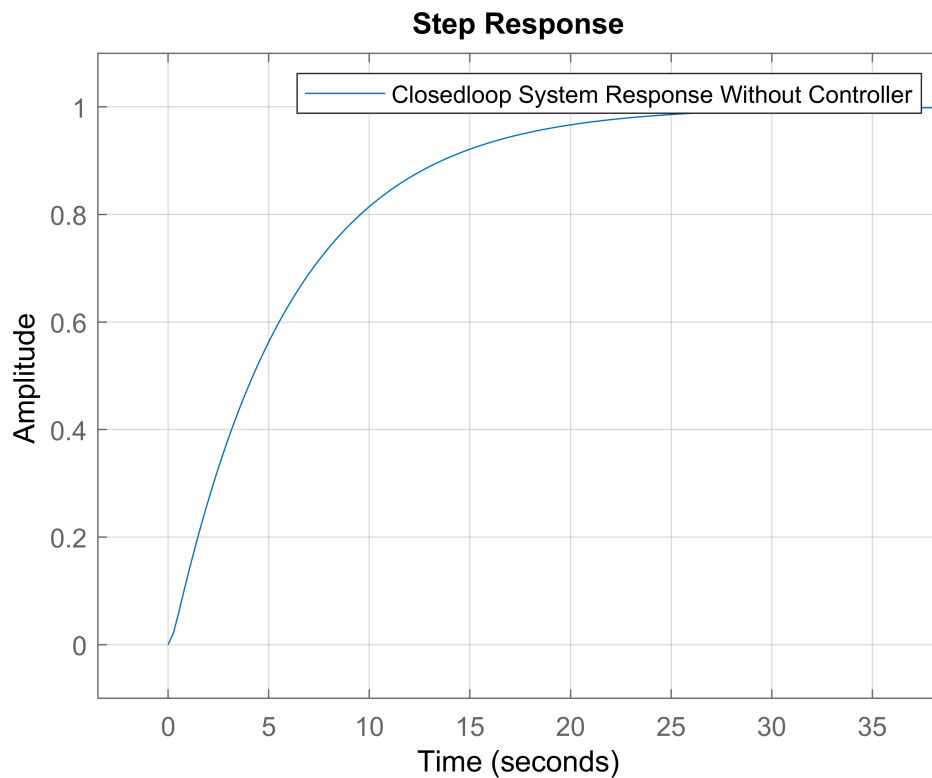
```
close all;  
clear;  
clc;
```

### System without a controller

```
s=tf('s');  
G=1/(s*(s+6));
```

Gcl = Closedloop Transfer Function

```
Gcl = G/(1+G); % system without controller  
step(Gcl); % plotting step response  
grid on;  
setAxisLimits(axis);  
legend('Closedloop System Response Without Controller');
```



Poles for the system without controller

```
disp(pole(Gc1));  
  
0  
-6.0000  
-5.8284  
-0.1716
```

One of the pole is on the imaginary axis, and therefore, the system without controller is marginally stable.

Time domain parameters of system

```
stepinfo(Gc1)  
  
ans = struct with fields:  
    RiseTime: 12.8096  
    SettlingTime: 22.9766  
    SettlingMin: 0.9016  
    SettlingMax: 0.9993  
    Overshoot: 0  
    Undershoot: 0  
    Peak: 0.9993  
    PeakTime: 42.6770
```

## Designing a Phase Lead Controller

$\zeta = 0.4$

$\omega_n = 15$

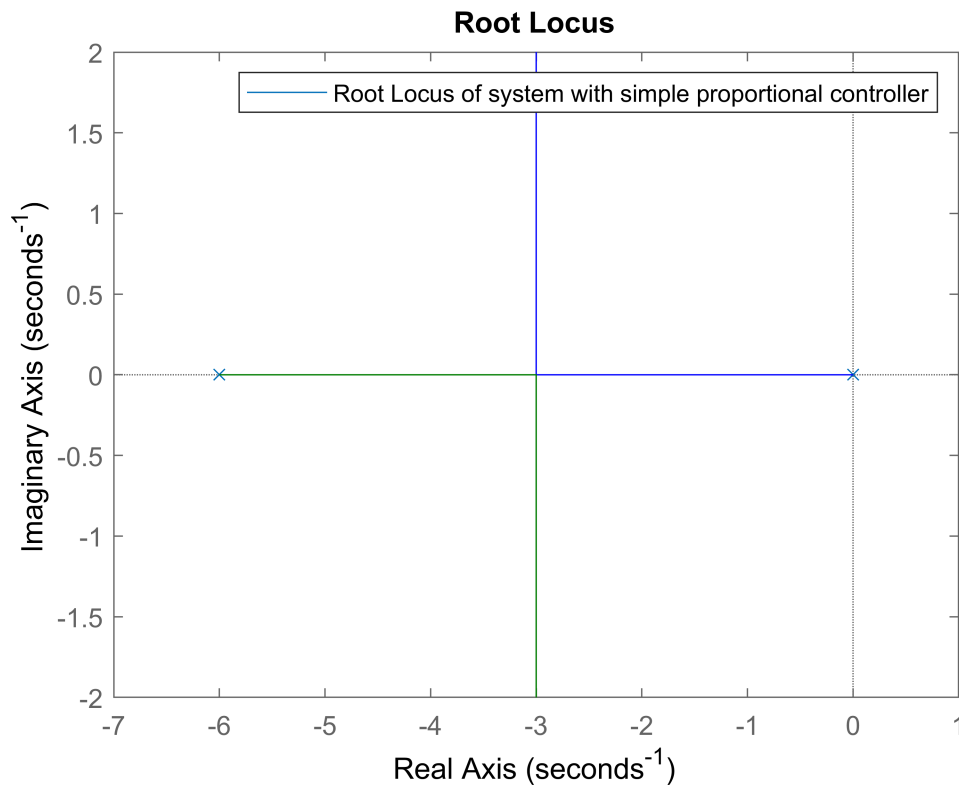
```
zita = 0.4;  
wn = 15;  
desiredPoles = roots([1 2*zita*wn wn^2]);
```

Root Locus must pass through desired poles.

```
disp(desiredPoles);  
  
-6.0000 +13.7477i  
-6.0000 -13.7477i
```

Root Locus of a system with a simple proportional controller

```
figure;  
rlocus(G);  
legend('Root Locus of system with simple proportional controller');
```



We can see that no matter what, the root locus doesn't pass through **desired poles**.

```
syms s1
G1=1/(s1*(s1+6));
phi=double(angle(subs(G1,s1,-6+13.74i)))*180/pi;
sphi=180-phi;
```

The zero of the controller is usually taken just below the desired poles, but as in this system, a pole already exists at  $S = -6$ .

∴ We take the zero of the controller slightly towards left of -6. i.e  $S = -7$  or  $Z = 7$ .

```
z=-7;
p=z-13.7477/tand(90-sphi);
disp(p);
```

```
-13.0034
```

And thus we the pole of the controller as **-13** or **P = 13**.

Then we find out **k** using magnitude criteria.

```
Ds=(s1-z)/(s1-p);
k=1/(double(abs(subs(Ds*G1,-6+13.7477i))))
```

```
k = 230.8210
```

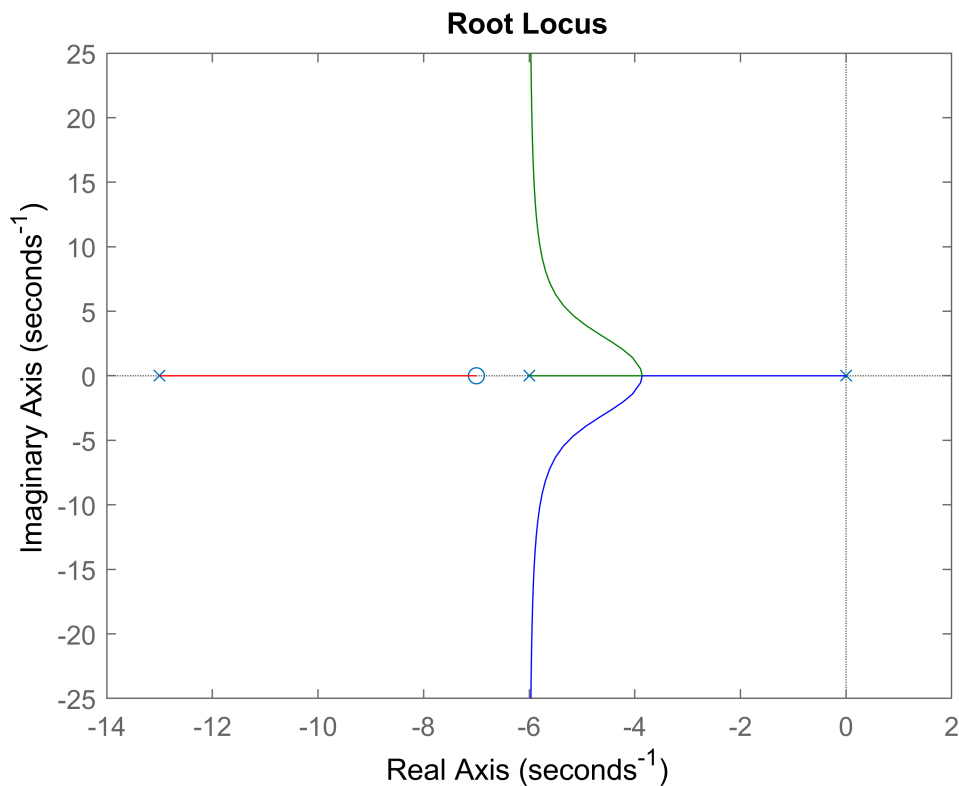
Thus at  $k = 230.5863$ , the RL passes through the desired pole location.

## Verification of design

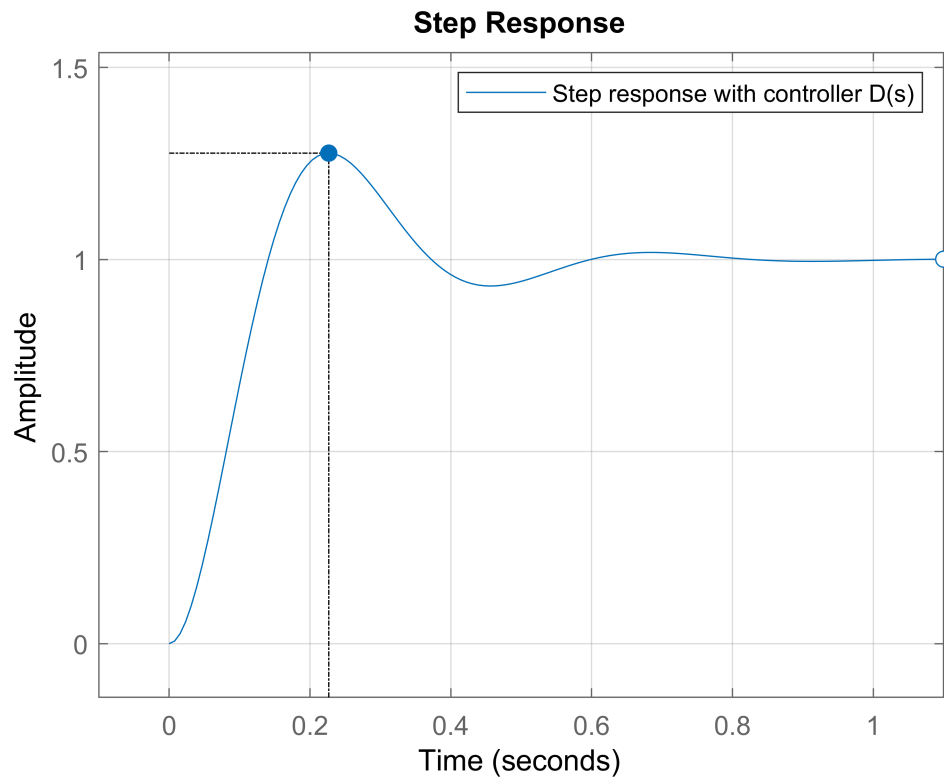
Ds = Controller Transfer Function

Ls = Closedloop Transfer Function with Controller

```
Ds = (s-z)/(s-p);  
Ls = k*Ds*G/(1+k*Ds*G);  
figure;  
rlocus(Ds*G);
```



```
figure;  
response = stepplot(Ls);  
grid on;  
response.showCharacteristic('PeakResponse');  
response.showCharacteristic('SettlingTime');  
response.showCharacteristic('RiseTime');  
response.showCharacteristic('SteadyState');  
setAxisLimits(axis);  
legend('Step response with controller D(s)');
```



Time Domain parameters of system with controller.

```
stepinfo(Ls)
```

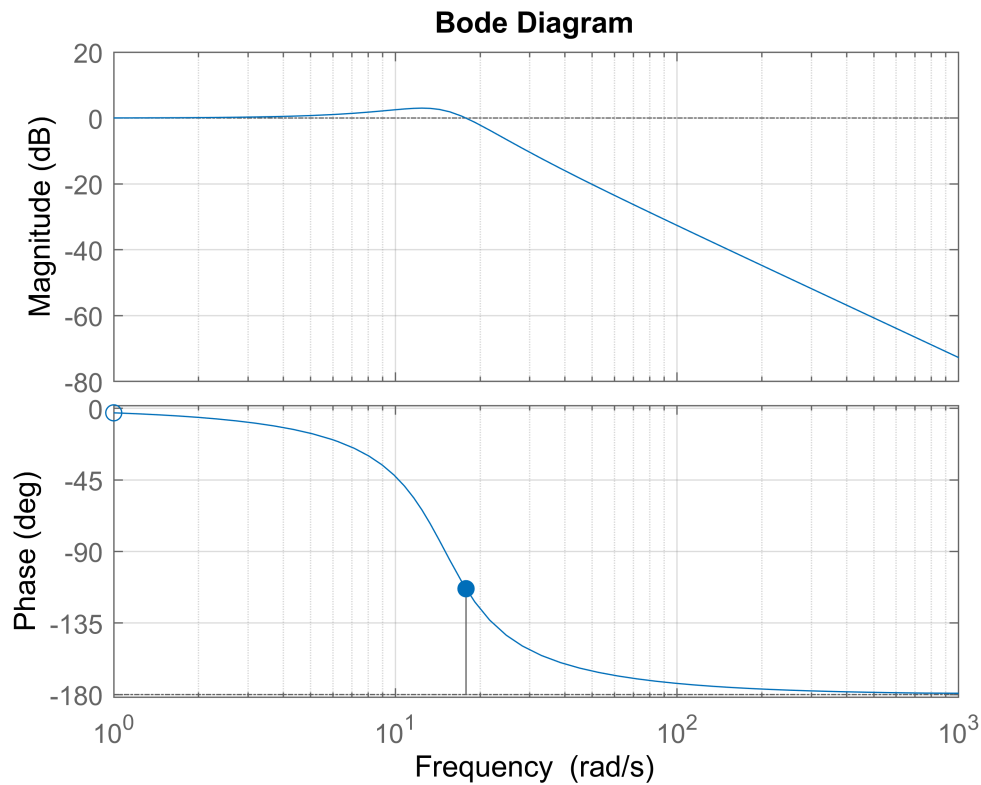
```
ans = struct with fields:
    RiseTime: 0.0952
    SettlingTime: 0.5612
    SettlingMin: 0.9312
    SettlingMax: 1.2771
    Overshoot: 27.7119
    Undershoot: 0
    Peak: 1.2771
    PeakTime: 0.2267
```

```
[gainMargin, phaseMargin, wcg, wcp] = margin(Ls)
```

```
Warning: The closed-loop system is unstable.
gainMargin = Inf
phaseMargin = 66.5761
wcp = Inf
wcp = 17.8151
```

**Bode Plot of the closed loop transfer function.**

```
response = bodeplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```

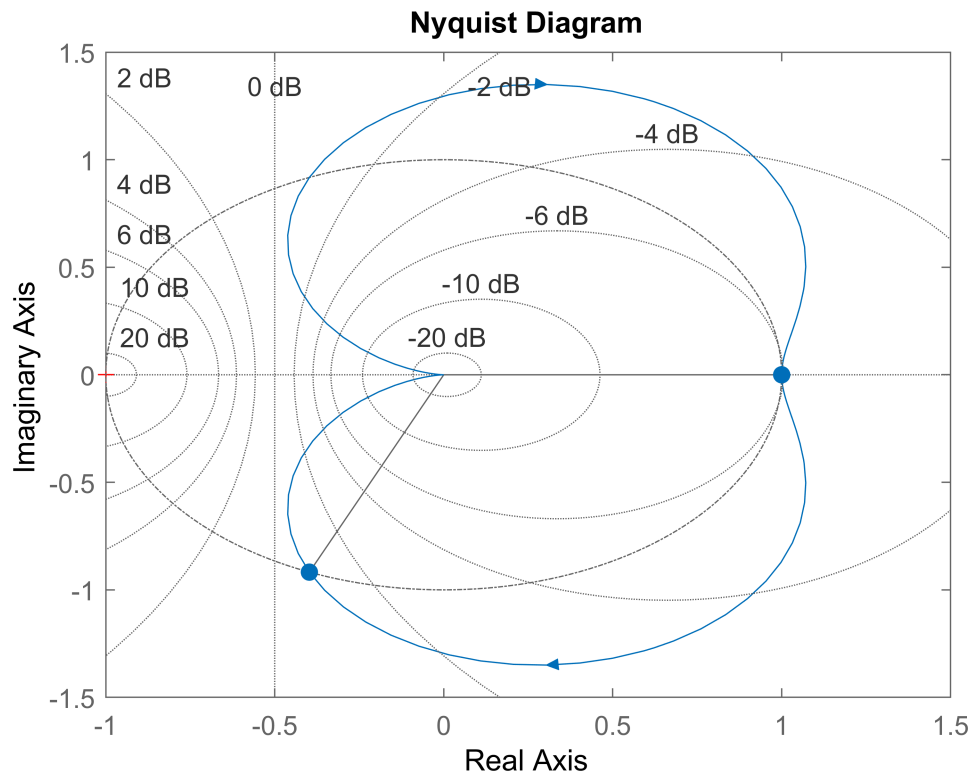


We can see from the **bode plot** that once the gain crossed the **0db** point, it never crosses it back again, hence no matter what the gain is, the system is going to remain stable.

Where as the phase when the gain crosses the **0db** is the **phase margin** and it's angle is  $66.57^\circ$

### Nyquist Plot of the closed loop transfer function

```
response = nyquistplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```



### Inference

Since the **Gain Margin is infinity**, any amount of gain will not result in system getting unstable, this can be shown in the Nyquist plot.

**Phase Margin is 66.5761**, this means that the system is stable for any value on phase but not 66.5761. As seen on the Nyquist plot, At a gain of **0db**, the plot touches the -1 point at an angle of 66.5761 degrees.



Given:  $G(s) = \frac{1}{s(s+6)}$

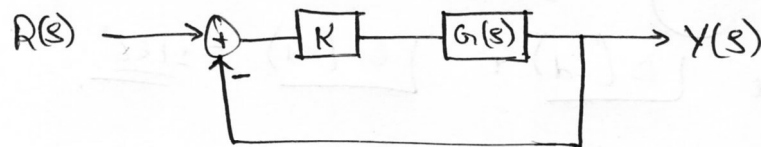
$Z = 0.4$

$\omega_n = 15 \text{ rad/s}$

To Do: Design a phase lead controller.

$S_d = -Z\omega_n \pm \omega_n \sqrt{1-Z^2} = -6 \pm 3\sqrt{2}j$

Step 1: We check if a simple proportional controller can solve.



$\alpha(s) = 1 + K G(s) = 1 + \frac{K}{s(s+6)} = \frac{s^2 + 6s + K}{s^2 + 6s}$

Finding roots of  $s^2 + 6s + K$ .

Poles are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$   
 $b = 6$   
 $c = K$

$= \frac{-6 \pm \sqrt{36 - 4K}}{2} = -3 \pm \sqrt{9 - K}$

Since the real part is always  $-3$ , it never passes through  $S_d$  i.e.  $-6 \pm 3\sqrt{2}j$ . So we use a phase lead controller.

Step 2: Phase lead Controller.

$$D(s) = \frac{K(s+z)}{s+p}$$

'Zero' of the controller is chosen below  $s_d$ , but in this system, a pole already exists at  $s = -6$ .

$\therefore$  We chose the pole location to be  $-7$ .

ie  $\underline{z = 7}$ .

$\therefore \boxed{z = 7}$

$\rightarrow$  Finding  $P$  using angle criteria.

ie  $\angle D(s_d) + \angle G(s_d) = \pm 180^\circ$

ie  $\angle s_d + z - \angle s_d + p - \angle s_d - \angle s_d + 6 = \pm 180^\circ$

$\angle -6 + 3\sqrt{2}j + 7 - \angle s_d + p - \angle -6 + 3\sqrt{2}j - \angle -6 + 3\sqrt{2}j + 6 = \pm 180^\circ$

$\pm 180^\circ = 85.839^\circ - \angle s_d + p - 113.57^\circ - 90 = \pm 180$

$\angle s_d + p = 62.269^\circ = \tan^{-1}\left(\frac{3\sqrt{2}}{p-6}\right)$

$p = \frac{3\sqrt{2}}{\tan(62.269)} + 6 = 13.22$   $\therefore \boxed{p = 13.22}$

Finding  $K$  using magnitude criteria.

$L(s) = \frac{K(s+7)}{s(s+6)(s+13.22)}$

$K = \frac{1}{|L(s)|} = \frac{|s_d| |s_d + 6| |s_d + 13.22|}{|s_d + 7|}$

$K = \frac{|-6 + 3\sqrt{2}j| |3\sqrt{2}j| |-6 + 3\sqrt{2}j + 13.22|}{|-6 + 3\sqrt{2}j + 7|} = \boxed{232.31}$

$$\therefore L(s) = \frac{232.31(s+7)}{s(s+6)(s+13.22)}$$

Sketching the RL of  $L(s)$  above, we can see that the point  $-6 \pm 3\sqrt{2}j$  passes through.