

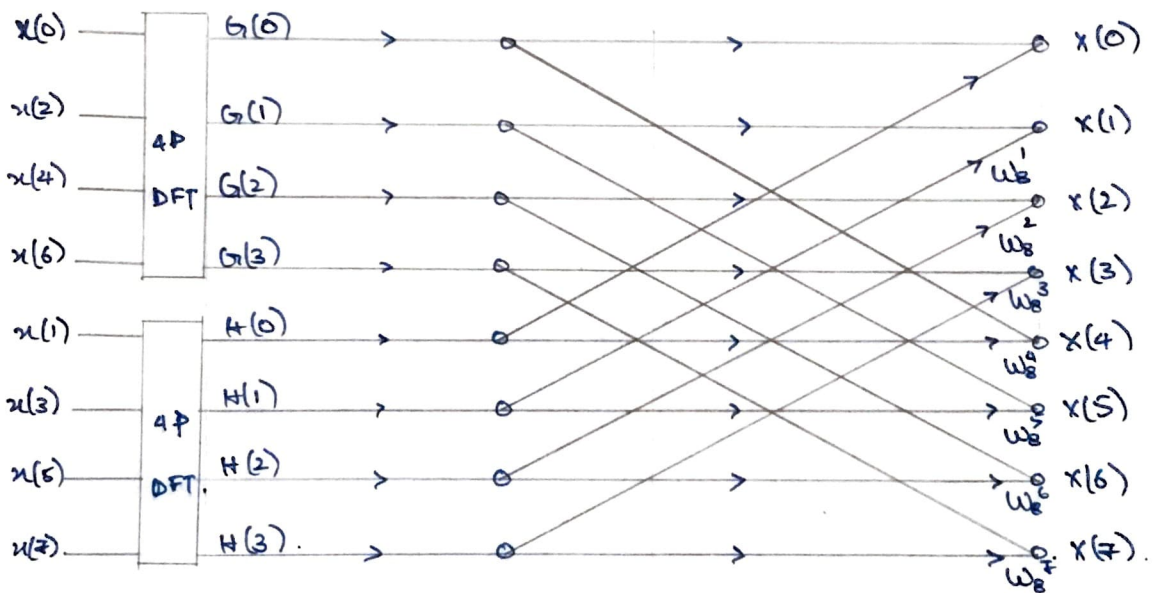
# Radix-2 8-point DIT FFT.

$$X(K) = \sum_{n=0}^{N-1} x(n) \omega_N^{nK} = \sum_{n=0}^7 x(n) \omega_8^{nK} \quad \text{where } N=8 = 2 \times 2 \times 2.$$

$$X(K) = \sum_{n=0}^3 x(2n) \omega_8^{2nK} + \sum_{n=0}^3 x(2n+1) \omega_8^{(2n+1)K} = \sum_{n \rightarrow \text{even}} x(n) \omega_8^{nK} + \sum_{n \rightarrow \text{odd}} x(n) \omega_8^{nK}$$

$$= \sum_{n=0}^3 x(2n) \omega_4^{nK} + \sum_{n=0}^3 x(2n+1) \omega_4^{nK} \cdot \omega_8^K$$

$$= G(K) + H(K) \cdot \omega_8^K$$



$$X(0) = G(0) + H(0)$$

$$X(1) = G(1) + \omega_8^1 H(1)$$

$$X(2) = G(2) + \omega_8^2 H(2)$$

$$X(3) = G(3) + \omega_8^3 H(3)$$

$$X(4) = G(0) + \omega_8^4 H(4)$$

$$X(5) = G(1) + \omega_8^5 H(1)$$

$$X(6) = G(2) + \omega_8^6 H(2)$$

$$X(7) = G(3) + \omega_8^7 H(3)$$

$$G(k) = \sum_{n=0}^3 x(2n) \omega_4^{nk}$$

$$H(k) = \sum_{n=0}^3 x(2n+1) \omega_4^{nk}$$

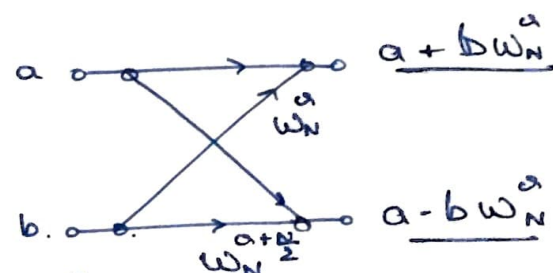
$$\sum_{\text{odd}} + \sum_{\text{even}}$$

$$\sum_{n=0}^1 x(2(2n)) \omega_4^{2nR} + \sum_{n=0}^1 x(2(2n+1)) \omega_4^{nR(2n+1)}$$

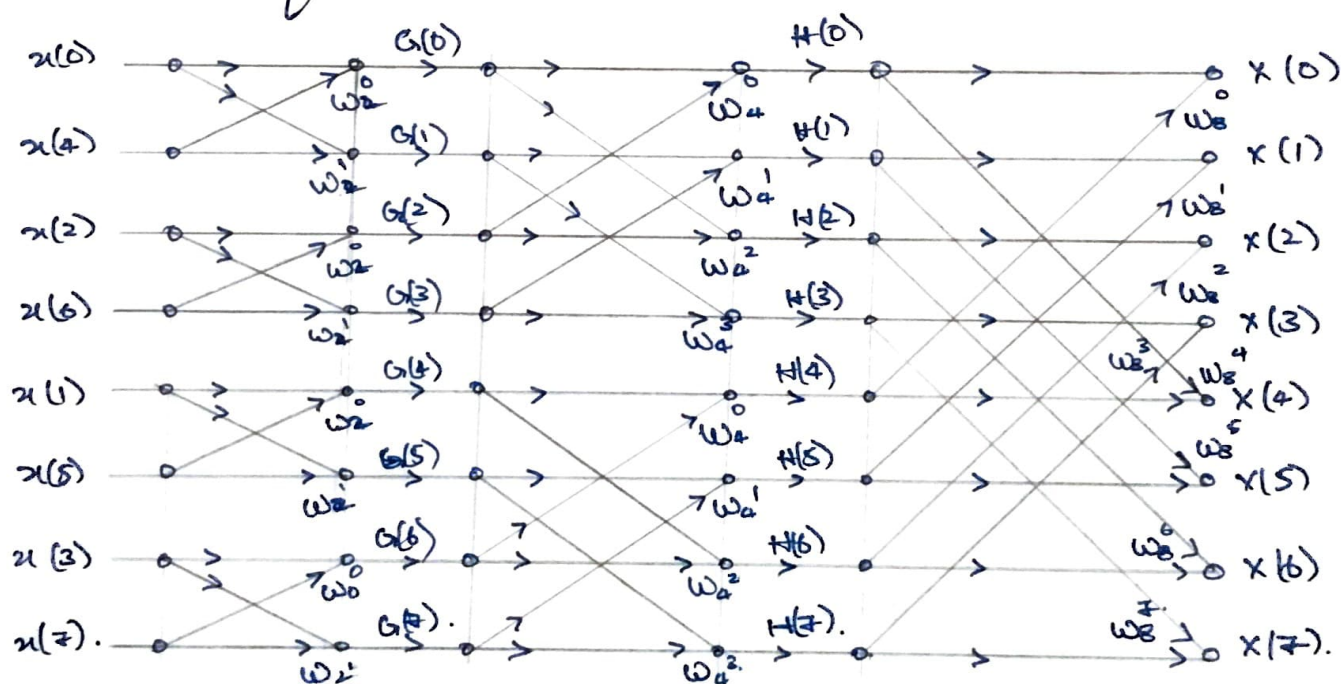
$$\sum_{n=0}^1 x(4n) \omega_2^{nR} + \sum_{n=0}^1 x(4n+2) \omega_2^{nR} \cdot \omega_4^R$$

$$P(k) + Q(k) \omega_4^k$$

$$\text{Silly } R(k) + S(k) \omega_4^k$$



$$\text{DFT } x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$$



$$G(0) = u(0) + u(4) = 6.$$

$$G(1) = u(0) - u(4) = 1 - 5 = \underline{\underline{-4.}}$$

$$G(2) = u(2) + u(6) = 3 + 7 = 10$$

$$G(3) = u(2) - u(6) = 3 - 7 = -4.$$

$$G(4) = u(1) + u(5) = 2 + 6 = 8.$$

$$G(5) = u(1) - u(5) = 2 - 6 = -4.$$

$$G(6) = u(3) + u(7) = 4 + 8 = 12.$$

$$G(7) = u(3) - u(7) = 4 - 8 = \underline{\underline{-4.}}$$

$$H(0) = G(0) + G(2) = 6 + 10 = 16.$$

$$\boxed{\omega_a' = -\dot{y}}$$

$$H(1) = G(1) + \omega_a' G(3) = -4 - \dot{y}(-4) = -4 + 4\dot{y}.$$

$$H(2) = G(0) - G(2) = 6 - 10 = -4.$$

$$H(3) = G(1) - \omega_a' G(3) = -4 + \dot{y}(-4) = \underline{\underline{-4 - 4\dot{y}}}.$$

$$H(4) = G(4) + G(6) = 8 + 12 = 20$$

$$H(5) = G(5) + \omega_a' G(7) = -4 - \dot{y}(-4) = -4 + 4\dot{y}.$$

$$H(6) = G(4) - G(6) = 8 - 12 = -4.$$

$$H(7) = G(5) - \omega_a' G(7) = -4 + \dot{y}(-4) = \underline{\underline{-4 - 4\dot{y}}}.$$

$$X(0) = H(0) + H(4) = 16 + 20 = \underline{\underline{36.}}$$

$$X(1) = H(1) + \omega_8' H(5) = -4 + 4\dot{y} + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\dot{y}\right)(-4 + 4\dot{y}) = -4 + \underline{\underline{9.65\dot{y}}}.$$

$$X(2) = H(2) + \omega_8^2 H(6) = -4 + (-\dot{y})(-4) = \underline{\underline{-4 + 4\dot{y}}}.$$

$$X(3) = H(3) + \omega_8^3 H(7) = -4 - 4\dot{y} + \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\dot{y}\right)(-4 - 4\dot{y}) = -4 + 1.65\dot{y}$$

$$X(4) = H(0) - H(4) = -4$$

$$X(5) = H(1) - \omega_8' H(5) = -4 - 1.65\dot{y}$$

$$X(6) = H(2) - \omega_8^2 H(6) = -4 - 4\dot{y}$$

$$X(7) = H(3) - \omega_8^3 H(7) = -4 - 9.65\dot{y}.$$

## Radin 2 - DIF FFT

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn}$$

$$N=8$$

$$X(K) = \sum_{n=0}^7 x(n) W_N^{Kn} = \sum_{n=0}^3 x(n) W_8^{Kn} + \sum_{n=4}^7 x(n) W_8^{Kn}$$

$$= \sum_{n=0}^3 x(n) W_8^{Kn} + \sum_{n=0}^3 x\left(n + \frac{N}{2}\right) W_N^{K\left(n + \frac{N}{2}\right)}$$

$$\sum_{n=0}^3 x(n) W_8^{Kn} + \sum_{n=0}^3 x\left(n + \frac{N}{2}\right) W_8^{Kn} \cdot W_8^{KN/2}$$

$$X(K) = X_1(K) + X_2(K) W_8^{KN/2} = X_1(K) + X_2(K) (-1)^K$$

$$\text{For even sequence.} = \sum_{n=0}^3 \left[ x(n) + x\left(n + \frac{N}{2}\right) (-1)^K \right] W_8^{Kn}$$

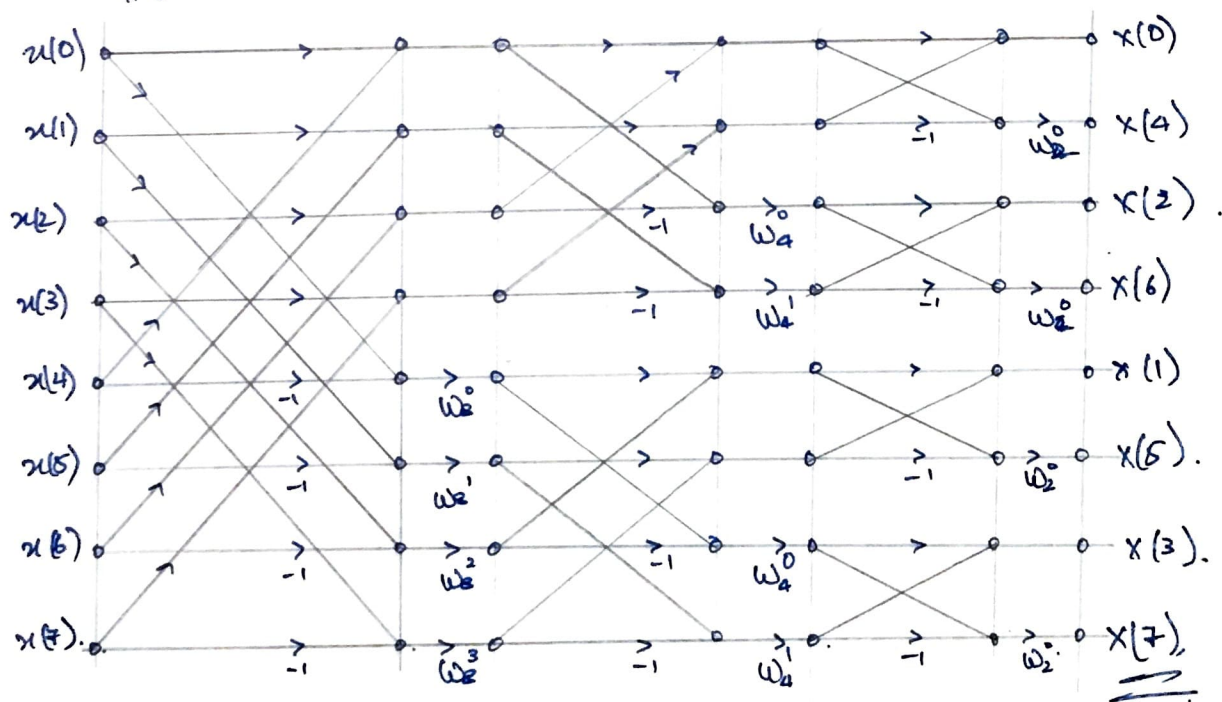
$$K=2R$$

$$X(2R) = X_1(2R) + X_2(2R) (-1)^{2R} = X_1(2R) + X_2(2R)$$

For odd.

$$K=2R+1$$

$$X(2R+1) = X_1(2R+1) + X_2(2R+1) (-1)^{2R+1} = X_1(2R+1) - X_2(2R+1)$$





$$X_1(0) = x(0) + x(4) = 1 + 5 = 6 \quad \underline{x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}}$$

$$X_1(1) = x(1) + x(5) = 8$$

$$X_1(2) = x(2) + x(6) = 10$$

$$X_1(3) = x(3) + x(7) = 12$$

$$X_1(4) = x(0) - \omega_8^0 x(4) = -4$$

$$X_1(5) = x(1) - \omega_8^1 x(5) = -2.82 + 2.82j$$

$$X_1(6) = x(2) - \omega_8^2 x(6) = 4j$$

$$X_1(7) = x(3) - \omega_8^3 x(7) = 2.82 + 2.82j$$

→

$$X_2(0) = X_1(0) + \cancel{x(4)} X_1(2) = 16$$

$$X_2(1) = X_1(1) + X_1(3) = 20$$

$$X_2(2) = X_1(0) - \omega_4^0 X_1(2) = -4$$

$$X_2(3) = X_1(1) - \omega_4^1 X_1(3) = 4j$$

$$X_2(4) = X_1(4) + X_1(6) = -4 + 4j$$

$$X_2(5) = X_1(5) + X_1(7) = 5.65j$$

$$X_2(6) = X_1(4) - \omega_4^0 X_1(6) = -4 - 4j$$

$$X_2(7) = X_1(5) - \omega_4^1 X_1(7) = 5.65j$$

→

$$X(0) = X_2(0) + X_2(1) = 36$$

$$X(4) = X_2(0) - X_2(1) \omega_2^0 = -4$$

$$X(2) = X_2(2) + X_2(3) = -4 + 4j$$

$$X(6) = X_2(2) - X_2(3) = -4 - 4j$$

$$X(1) = X_2(4) + X_2(5) = -4 + 9.65j$$

$$X(5) = X_2(4) - X_2(5) = -4 - 1.65j$$

$$X(3) = X_2(6) + X_2(7) = -4 + 1.65j$$

$$X(7) = X_2(6) - X_2(7) = -4 - 9.65j$$

$$\begin{aligned}
 x(k) = \{ & 36 \quad \leftarrow \text{oth place.} \\
 & -4 + 9.65\dot{y} \\
 & -4 + 4\ddot{y} \\
 & -4 + 1.65\ddot{y} \\
 & -4 \\
 & -4 - 1.65\ddot{y} \\
 & -4 - 4\ddot{y} \\
 & -4 - 9.65\dot{y} \} .
 \end{aligned}$$

Complex Radix  $N=6 = 3 \times 2$  DIT FFT.

$$x(n) = \{0, 7, 8, 9, 8, 7, 6\}$$

Sol:

$$X(K) = \sum_{n=0}^{N-1} x(n) \omega_N^{nK}$$

2 - 8 stages.

Radix 2 in 1st stage.

- " - 3 - " - 2nd " .

$$X(K) = \sum_{n=0}^1 x(3n) \omega_N^{3nK} + \sum_{n=0}^1 x(3n+1) \omega_N^{(3n+1)K} \\ + \sum_{n=0}^1 x(3n+2) \omega_N^{(3n+2)K}$$

$$N=6$$

$$= \sum_{n=0}^1 x(3n) \omega_2^{nK} + \sum_{n=0}^1 x(3n+1) \omega_2^{nK} \cdot \omega_6^K + \sum_{n=0}^1 x(3n+2) \omega_2^{nK} \cdot \omega_6^{2K}$$

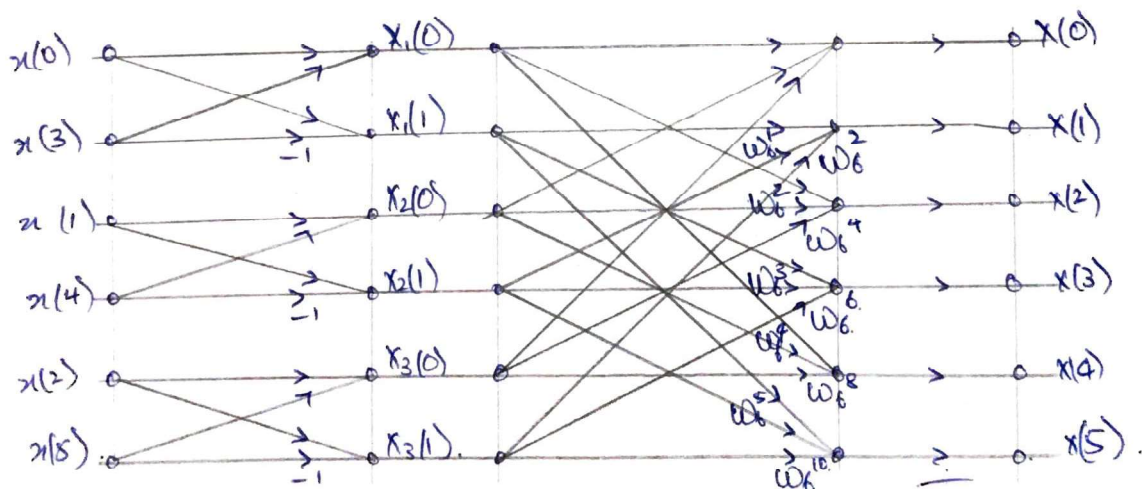
$$= X_1(K) + X_2(K) \omega_6^K + X_3(K) \omega_6^{2K}$$

$$X_1(K) = \sum_{n=0}^1 x(3n) \omega_2^{nK} = x(0) + x(3) \omega_2^{3K}$$

$$X_2(K) = \sum_{n=0}^1 x(3n+1) \omega_2^{nK} = x(1) + x(4) \omega_2^K$$

$$X_3(K) = \sum_{n=0}^1 x(3n+2) \omega_2^{nK} = x(2) + x(5) \omega_2^K$$

$$X(K) = X_1(K) + X_2(K) \omega_6^K + X_3(K) \omega_6^{2K}$$



$$x_1(0) = x(0) + x(3) = 15$$

$$x_1(1) = x(0) - x(3) = -1$$

$$x_2(0) = x(4) + x(1) = 8 + 7 = 15$$

$$x_2(1) = x(1) - x(4) = 1$$

$$x_3(0) = x(2) + x(5) = 9 + 6 = 15$$

$$x_3(1) = x(2) - x(5) = 9 - 6 = 3$$

$$x(0) = x_1(0) + x_2(0) + x_3(0) = 15 + 15 + 15 = 45$$

$$x(1) = x_1(1) + x_2(1) \omega_6^1 + x_3(1) \omega_6^2 = -2 + 3 \cdot 46j$$

$$x(2) = x_1(0) + x_2(0) \omega_6^2 + x_3(0) \omega_6^4 = 0$$

$$x(3) = x_1(1) + x_2(1) \omega_6^3 + x_3(1) \omega_6^6 = 1$$

$$x(4) = x_1(0) + x_2(0) \omega_6^4 + x_3(0) \omega_6^8 = 0$$

$$x(5) = x_1(1) + x_2(1) \omega_6^5 + x_3(1) \omega_6^{10} = -2 + 3 \cdot 46j$$

Direct cal:

$$\omega_6^0 = 1 \quad \omega_6^1 = 0.5 - 0.866j \quad \omega_6^2 = -0.5 - 0.866j$$

$$\omega_6^3 = -1 \quad \omega_6^4 = -0.5 + 0.866j \quad \omega_6^5 = 0.5 + 0.866j$$

$$x(k) = \sum_{n=0}^5 x(n) \omega_6^{nk}$$

$$x(0) = 7 + 8 + 9 + 8 + 7 + 6 = 45$$

$$x(1) = 7 + 8\omega_6^1 + 9\omega_6^2 + 8\omega_6^3 + 7\omega_6^4 + 6\omega_6^5 = -2 + 3 \cdot 46j$$

$$x(2) = 7 + 8\omega_6^2 + 9\omega_6^4 + 8\omega_6^6 + 7\omega_6^8 + 6\omega_6^{10} = 0$$

$$x(3) = 7 + 8(-1) + 9 - 8 + 7 - 6 = 1$$

$$x(4) = 7 + 8\omega_6^4 + 9\omega_6^8 + 8\omega_6^{12} + 7\omega_6^{16} + 6\omega_6^{20} = 0$$

$$x(5) = 7 + 8\omega_6^5 + 9\omega_6^{10} + 8\omega_6^{15} + 7\omega_6^{20} + 6\omega_6^{25} = \underline{\underline{-2 + 3 \cdot 46j}}$$



6

$$N=6 = \underline{2 \times 3}$$

$$X(K) = \sum_{n=0}^5 x(n) \omega_6^{Kn}$$

$$X(K) = \sum_{n=0}^2 x(2n) \omega_6^{2nK} + \sum_{n=0}^2 x(2n+1) \omega_6^{(2n+1)K}$$

$$X_1(K) = \sum_{n=0}^2 x(2n) \omega_6^{2nK} = x(0) + \omega_6^K x(2) + \omega_6^{4K} x(4)$$

$$X_1(0) = x(0) + x(2) + x(4) \quad X_1(1) = x(0) + x(2) \omega_6^2 + x(4) \omega_6^4$$

$$X_2(0) = x(1) + x(3) + x(5) \quad X_2(1) = x(1) + x(3) \omega_6^2 + x(5) \omega_6^4$$

$$X_1(2) = x(0) + x(2) \omega_6^4 + x(4) \omega_6^8 \quad X_2(2) = x(1) + x(3) \omega_6^4 + x(5) \omega_6^8$$

$$X(k) = X_1(k) + \omega_6^k X_2(k).$$

$$X(0) = X_1(0) + X_2(0).$$

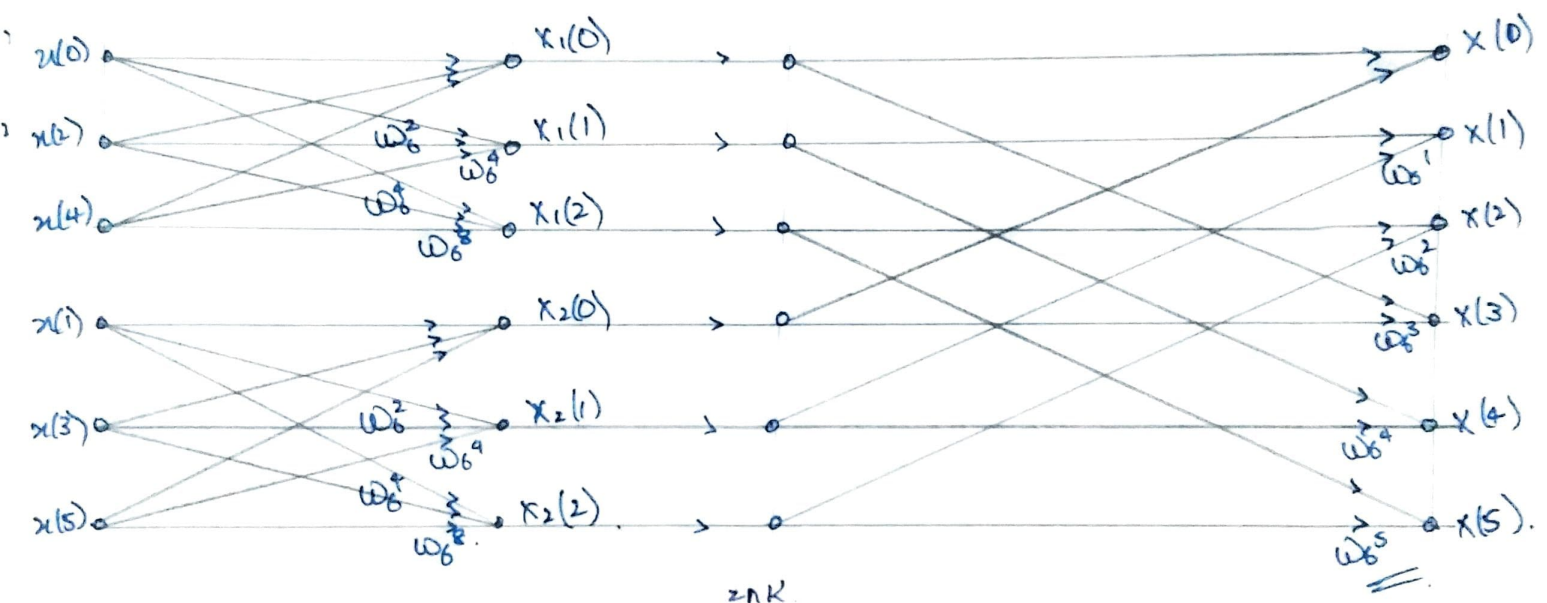
$$X(1) = X_1(1) + \omega_6^1 X_2(1)$$

$$X(2) = X_1(2) + \omega_6^2 X_2(2).$$

$$X(3) = X_1(0) + \omega_6^3 X_2(0).$$

$$X(4) = X_1(1) + \omega_6^4 X_2(1).$$

$$X(5) = X_1(2) + \omega_6^5 X_2(2).$$



$$X_1(k) = \sum_{n=0}^{2} x(2n) \omega_6^k$$

$$X_1(0) = 9$$

$$X_1(1) = -3 + 1.73j$$

$$X_1(2) = -3 - 1.73j$$

$$X_2(0) = 12$$

$$X_2(1) = -3 + 1.73j$$

$$X_2(2) = -3 - 1.73j$$

$$X(0) = 9 + 12 = 21$$

$$X(1) = -3 + 5.196j + \omega_6^1(-3 + 1.73j) = -3 + 5.196j$$

$$X(2) = -3 - 1.73j + \omega_6^2(-3 - 1.73j) = -3 + 1.73j$$

$$X(3) = 9 - 12 = -3$$

$$X(4) = (-3 + 1.73j) + \omega_6^4(-3 + 1.73j) = -3 - 1.73j$$

$$X(5) = -3 + 5.196j$$