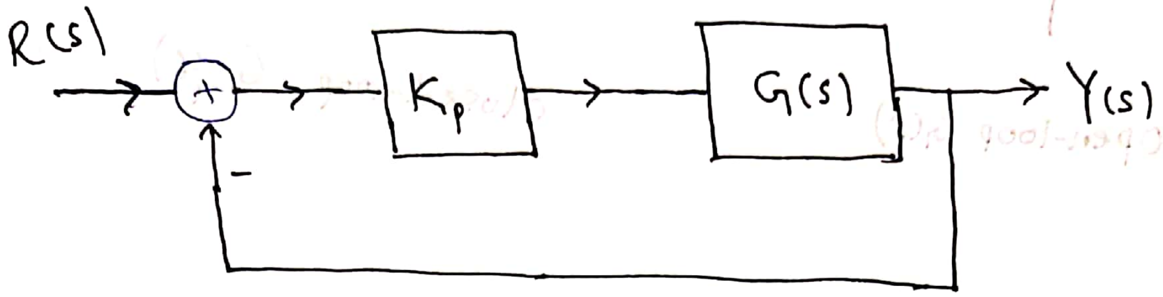
Proportional Controller



$$L(s) = D(s)G(s) = KG(s)$$

Proportional Controller does not alter the Type of system as it does not contain any poles.

When $G(s) = \dfrac{1/\tau}{s + 1/\tau}$ (Standard First Order System)

$$L(s) = \frac{K_p/\tau}{s + 1/\tau}$$ Type-0 system.

Steady state Error for step reference input $e_{ss} = \dfrac{1}{1+K_p}$

Transfer function of Closed loop System

$$G_{cl}(s) = \frac{K_p/\tau}{s + \underbrace{\frac{K_p}{\tau}}_{} + \frac{1}{\tau}}$$

shifts the pole by $\frac{K_p}{\tau}$.

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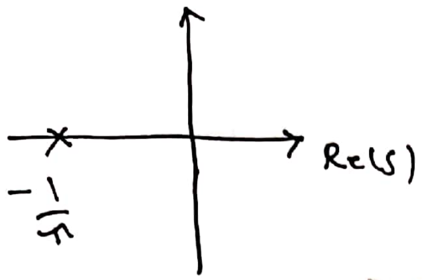
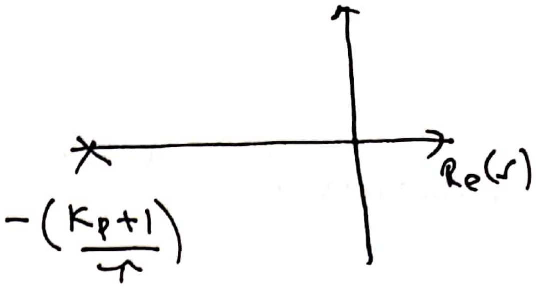


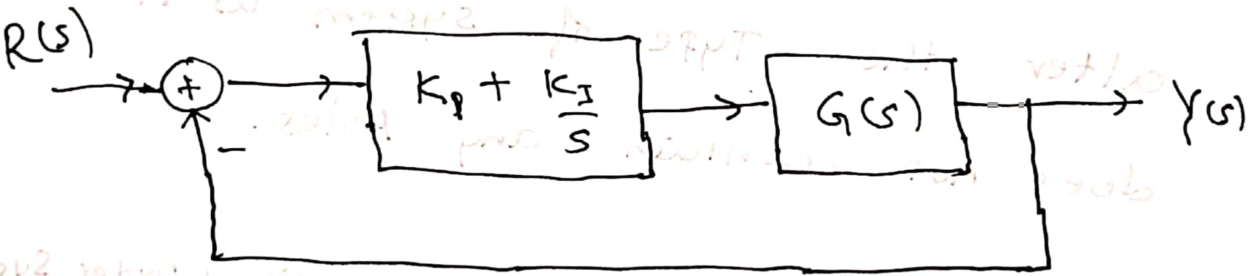
Open-loop $G(s)$



closed-loop $G_{cl}(s)$

PI Controller: $D(s) = K_p + \frac{K_I}{s} = \frac{U(s)}{E(s)}$

Control Law $\quad u(t) = K_p e(t) + K_I \int_0^t e(\tau) \cdot d\tau$



When $\quad G(s) = \frac{1/\tau}{s + \frac{1}{\tau}}$ standard first order

$$L(s) = D(s)G(s)$$

$$= \frac{(K_p s + K_I)(1/\tau)}{s\,(s + \frac{1}{\tau})}$$

Type-1 system.

Transfer function of Closed-loop System

$$G_{cl}(s) = \frac{L(s)}{1 + L(s)}$$

$$= \frac{\dfrac{(K_p s + K_I)\left(\frac{1}{\tau}\right)}{s\left(s + \frac{1}{\tau}\right)}}{1 + \dfrac{(K_p s + K_I)\frac{1}{\tau}}{s\left(s + \frac{1}{\tau}\right)}} = \frac{(K_p s + K_I)\left(\frac{1}{\tau}\right)}{s\left(s + \frac{1}{\tau}\right) + \frac{K_p s}{\tau} + \cdots}$$

Comparing the characteristic equation with that of a standard second system

$$\alpha(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$= s^2 + \left(\frac{K_p + 1}{\tau}\right)s + \frac{K_I}{\tau}$$

Natural frequency of CLS

$$\omega_n = \sqrt{\frac{K_I}{\tau}}$$

$$2\zeta\omega_n = \frac{K_p + 1}{\tau}$$

$$\zeta = \frac{1}{2\sqrt{\frac{K_I}{\tau}}} \cdot \frac{K_p + 1}{\tau}$$

Ex: Given the plant model

$$G(s) = \frac{10}{s+10}$$

Design PI controller such that poles of closed loop system satisfy $\omega_n = 10$ rad/sec.

$$\zeta = 0.6$$

sol: $\tau = 0.1$ $\quad 1/\tau = 10$.

$$\omega_n = \sqrt{\frac{K_I}{\tau}} = 10$$

$$= \sqrt{10 K_I} \Rightarrow K_I = 10$$

$$\zeta = 0.6 = \frac{1}{2\omega_n}\left(\frac{K_p+1}{\tau}\right)$$

$$= \frac{1}{2\cancel{0}} \cdot (1\cancel{0})(K_p+1)$$

$$1.2 = K_p + 1$$

$$\Rightarrow K_p = 0.2.$$