Standard Second Order System

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ξ = Damping Factor

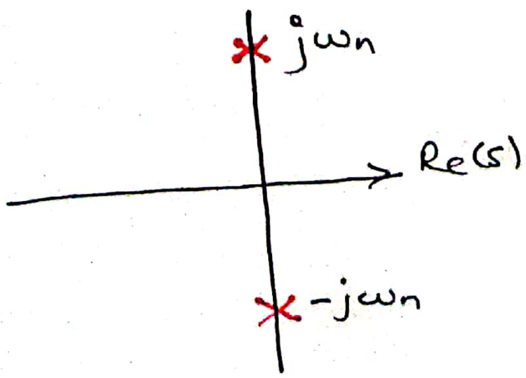
ω_n = Natural Frequency rad/sec

Case I $(\xi = 0)$ Undamped system

$$G(s) = \frac{\omega_n^2}{s^2 + \omega_n^2} = \frac{b(s)}{a(s)}$$

Poles of $G(s) \Rightarrow$ Roots of $a(s) = s^2 + \omega_n^2 = 0$

$= \pm j\omega_n$



Impulse Response of the System (SSOS)
when $\xi = 0$

$$g(t) = \mathcal{L}^{-1}\left\{\frac{\omega_n^2}{s^2+\omega_n^2}\right\} = \omega_n \sin \omega_n t \quad \text{for } t \geq 0.$$

Step Response of the SSOS
when $\xi = 0$

$$Y(s) = G(s)\,U(s) = \frac{\omega_n^2}{s\,(s^2+\omega_n^2)}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + \omega_n^2}$$

$A = 1 \qquad B = -1 \qquad C = 0$

$$\therefore \quad y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega_n^2}\right\}$$

$$= u_s(t) - \cos \omega_n t \quad \text{for } t \geq 0.$$

Case II: $0 < \xi < 1$ Underdamped system

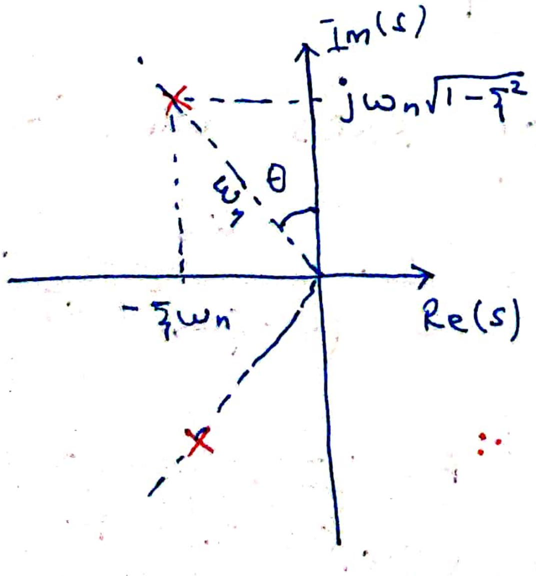
$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{b(s)}{a(s)}$$

Poles of $G(s) \Rightarrow$ Roots of

$$a(s) = s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$P_{1,2} = -\frac{2\xi\omega_n}{2} \pm \frac{\sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



$$\sin|\theta| = \frac{\xi\omega_n}{\omega_n} = \xi$$

When $\xi = 0$, $\theta = 0$

Poles on Imaginary axis

When $\xi = 1$ $\theta = 90°$

Poles on Real axis.

Impulse Response of SSOS when $0 < \xi < 1$

$$G(s) = \frac{\omega_n^2}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}} \frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

Using $\mathcal{L}\{e^{-at}\sin\omega t\} = \frac{\omega}{(s+a)^2+\omega^2}$

$$g(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \sin \omega_n\sqrt{1-\xi^2}\, t \quad \text{for } t \geq 0$$

Step Response of SSOS when $0 < \xi < 1$

$$Y(s) = G(s)U(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{A}{s} + \frac{Bs+C}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$A = 1 \quad B = -1 \quad C = -2\xi\omega_n$

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$$Y(s) = \frac{1}{s} - \frac{(s + 2\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$- \frac{\xi\omega_n\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}\left\{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)\right\}}$$

$$\therefore\ y_s(t) = u_s(t) - e^{-\xi\omega_n t}\cos\omega_n\sqrt{1-\xi^2}\,t$$

$$- e^{-\xi\omega_n t}\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_n\sqrt{1-\xi^2}\,t$$

for $t \geq 0$.

Ex:

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

$\omega_n = 5$ rad/sec $\quad \xi = \frac{2\xi\omega_n}{2\omega_n} = \frac{4}{10} = 0.4$