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JSS Mahavidyapeetha

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“Event 4 Report”

Submitted for the fulfillment of the CIE (Event-4) for the course

CONTROL SYSTEMS

(EC540)

Submitted by

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# Probelm Statement

For the plant **G(s) = 1/s(s+6)** design a **phase-lead** controller for damping ratio **ζ = 0.4** and natural frequency **15 rad/sec**.

What is the phase margin and gain margin of the compensated system?

# Solution

## Clearing Workspace

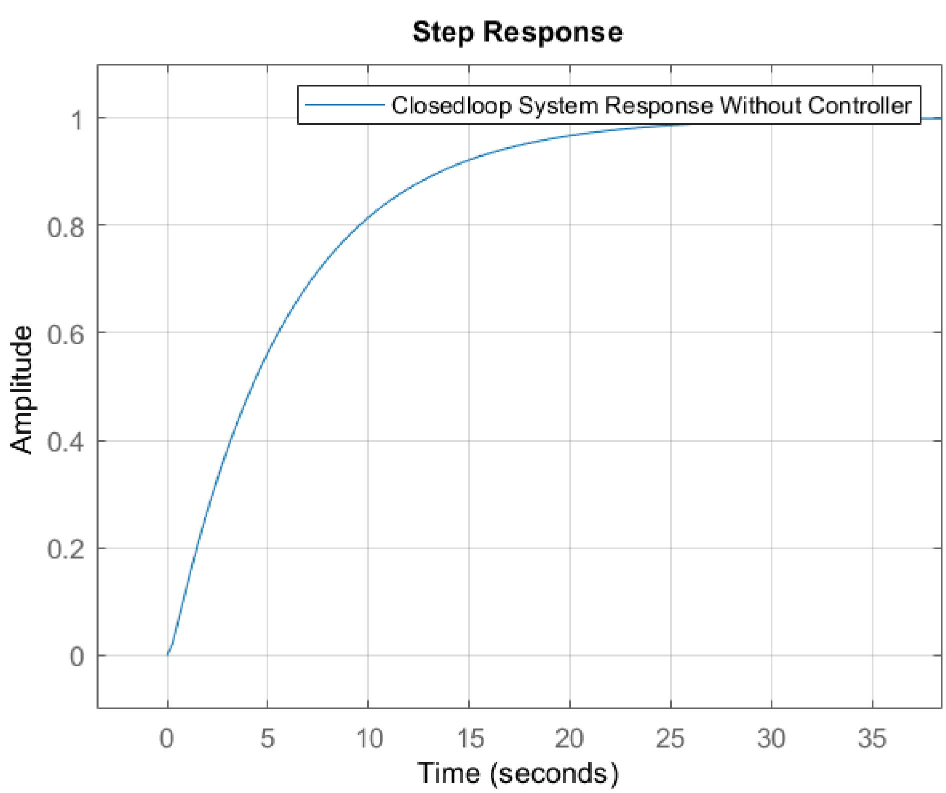
```
close all;
clear;
clc;
```

## System without a controller

```
s=tf('s');
G=1/(s*(s+6));
```

Gcl = Closedloop Transfer Function

```
Gcl = G/(1+G); % system without controller
step(Gcl); % plotting step response
grid on;
setAxisLimits(axis);
legend('Closedloop System Response Without Controller');
```



Poles fo the system without controller

```
disp(pole(Gcl));
```

```
         0
   -6.0000
   -5.8284
   -0.1716
```

One of the pole is on the imaginary axis, and therefore, the system without controller is marginally stable.

Time domain parameters of system

```
stepinfo(Gcl)
```

```
ans = struct with fields:
        RiseTime: 12.8096
    SettlingTime: 22.9766
     SettlingMin: 0.9016
     SettlingMax: 0.9993
       Overshoot: 0
      Undershoot: 0
            Peak: 0.9993
        PeakTime: 42.6770
```

# Designing a Phase Lead Controller

**ζ = 0.4**

**ωn = 15**

```
zita = 0.4;
wn = 15;
desiredPoles = roots([1 2*zita*wn wn^2]);
```

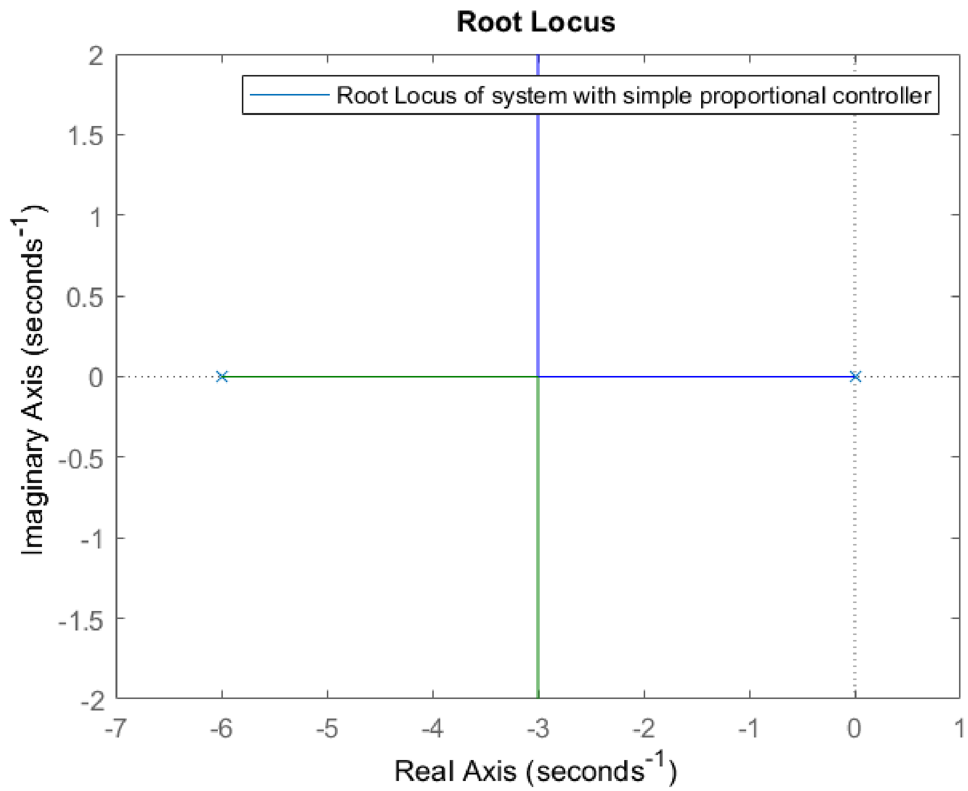
Root Locus must pass throught desired poles.

```
disp(desiredPoles);
```

```
  -6.0000 +13.7477i
  -6.0000 -13.7477i
```

Root Locus of a system with a simple proportional controller

```
figure;
rlocus(G);
legend('Root Locus of system with simple proportional controller');
```




We can see that no matter what, the root locus doesn't pass through **desired poles.**

```
syms s1
G1=1/(s1*(s1+6));
phi=double(angle(subs(G1,s1,-6+13.74i)))*180/pi;
sphi=180-phi;
```

The zero of the controller is usually taken just below the desired poles, but as in this system, a pole already exists at S = -6.

∴ We take the zero of the controller slightly towards left of -6. i.e S = -7 or **Z = 7**.

```
z=-7;
p=z-13.7477/tand(90-sphi);
disp(p);
```

```
-13.0034
```

And thus we the pole of the controller as **-13** or **P = 13**.

Then we find out **k** using magnitude criteria.

```
Ds=(s1-z)/(s1-p);
k=1/(double(abs(subs(Ds*G1,-6+13.7477i))))
```

```
k = 230.8210
```

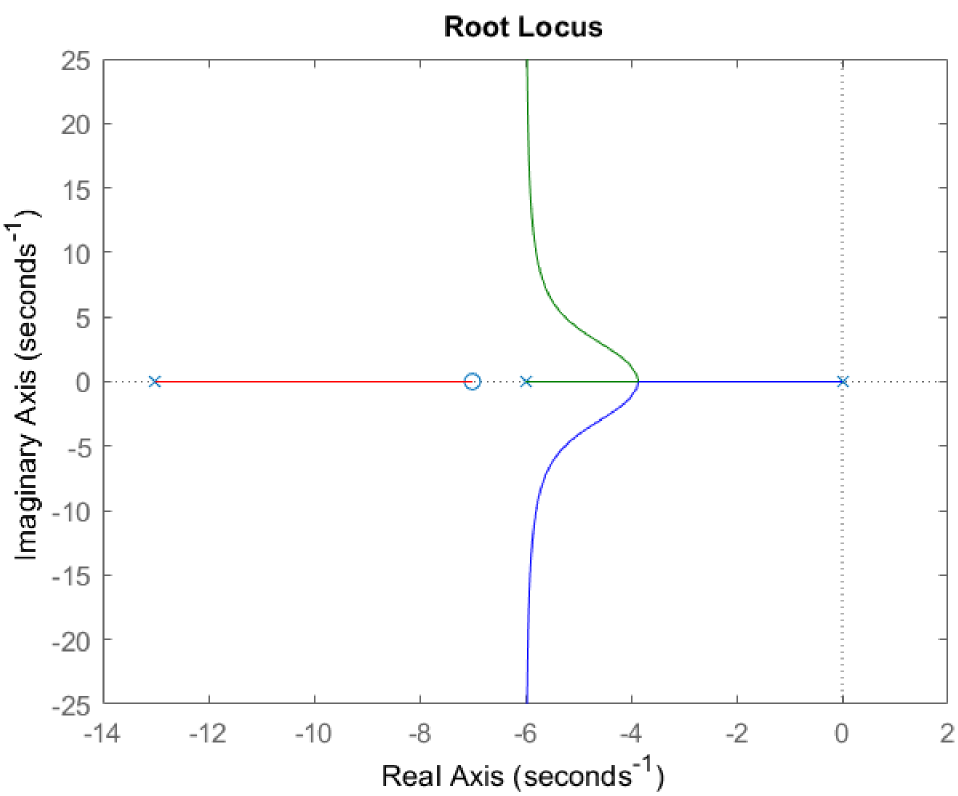
Thus at k = 230.5863, the RL passes through the desired pole location.

## Verification of design

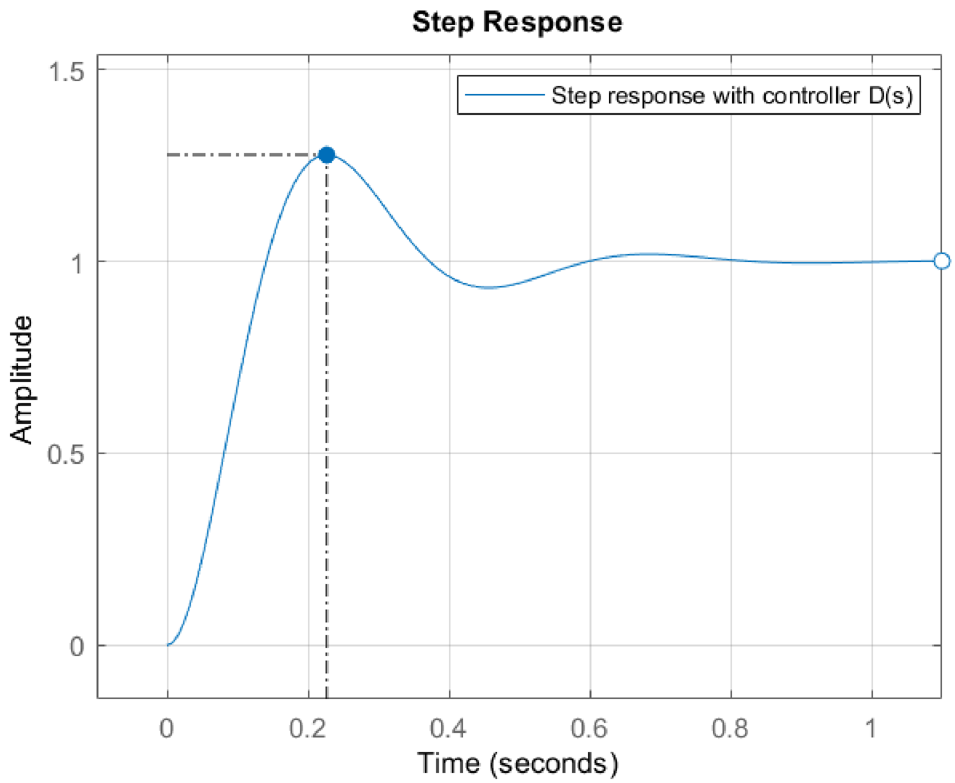
Ds = Controller Transfer Function

Ls = Closedloop Transfer Function with Controller

```
Ds = (s-z)/(s-p);
Ls = k*Ds*G/(1+k*Ds*G);
figure;
rlocus(Ds*G);
```




```
figure;
response = stepplot(Ls);
grid on;
response.showCharacteristic('PeakResponse');
response.showCharacteristic('SettlingTime');
response.showCharacteristic('RiseTime');
response.showCharacteristic('SteadyState');
setAxisLimits(axis);
legend('Step response with controller D(s)');
```




Time Domain parameters of system with controller.

```
stepinfo(Ls)
```

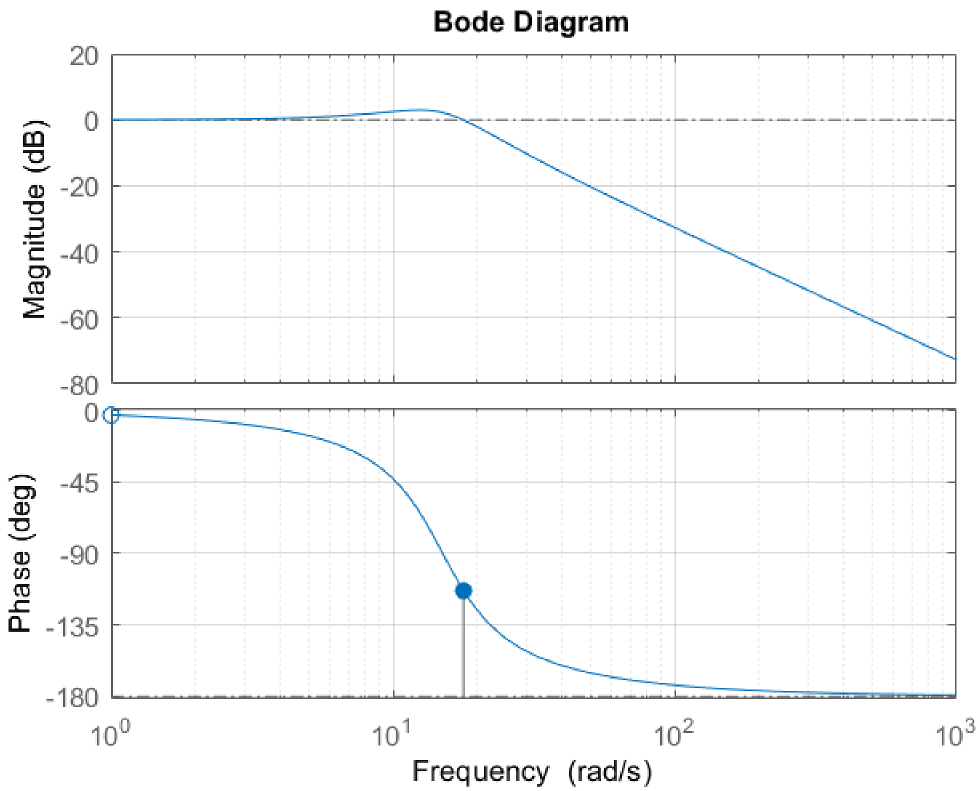
```
ans = struct with fields:
        RiseTime: 0.0952
    SettlingTime: 0.5612
     SettlingMin: 0.9312
     SettlingMax: 1.2771
       Overshoot: 27.7119
      Undershoot: 0
            Peak: 1.2771
        PeakTime: 0.2267
```

```
[gainMargin, phaseMargin, wcg, wcp] = margin(Ls)
```

```
Warning: The closed-loop system is unstable.
gainMargin = Inf
phaseMargin = 66.5761
wcg = Inf
wcp = 17.8151
```

## Bode Plot of the closed loop transfer function.

```
response = bodeplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```

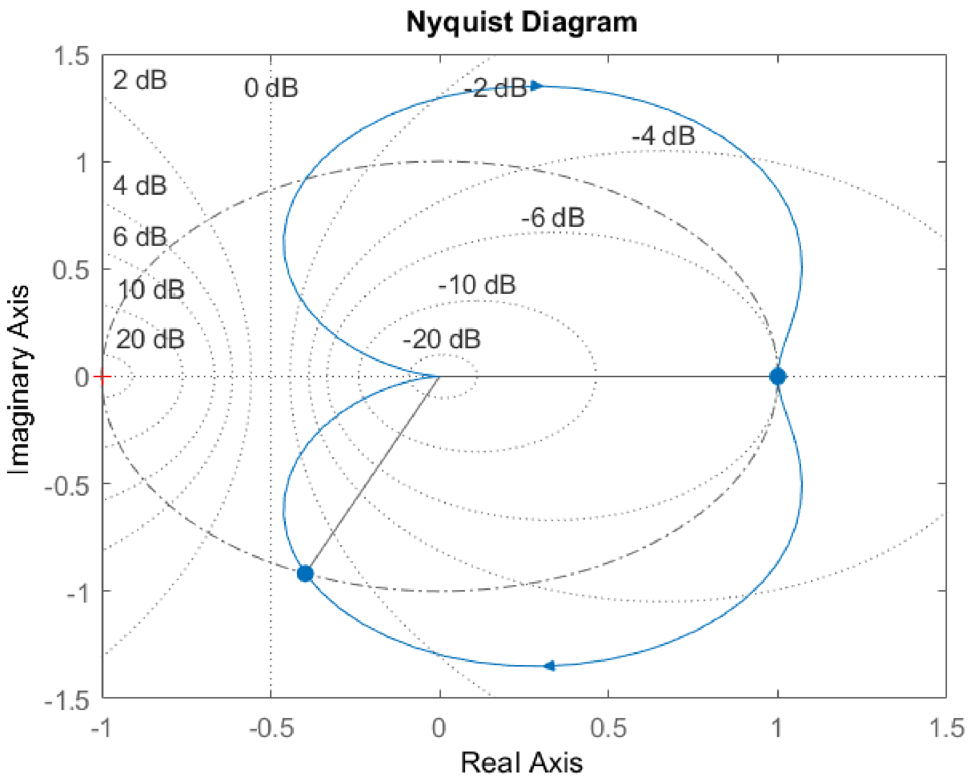



We can see from the **bode plot** that once the gain crossed the **0db** point, it never crosses it back again, hence no matter what the gain is, the system is going to remain stable.

Where as the phase when the gain crosses the **0db** is the **phase margin** and it's angle is 66.57°

### Nyquist Plot of the closed loop transfer function

```
response = nyquistplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```




Since the **Gain Margin is infinity**, and the **Phase Margin is 66.5761**, the system is unstable at a phase of 66.5761 else it's stable.

Given: $G(s) = \dfrac{1}{s(s+6)}$

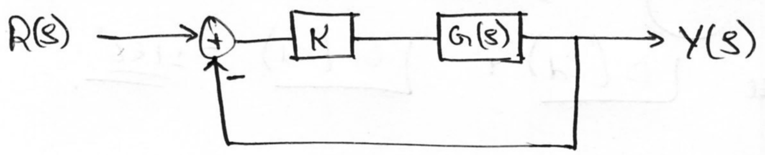
To Do: Design a phase lead controller.

$\zeta = 0.4$

$\omega_n = 15$ rad/s.

$$S_d = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2} = -6 \pm 3\sqrt{21}j.$$

Step 1: We check if a simple proportional controller can solve.




$$\alpha(s) = 1 + K\,G(s) = 1 + \frac{K}{s(s+6)} = \frac{s^2+6s+K}{s^2+6s}.$$

Finding roots of $s^2+6s+K$.  $a=1$, $b=6$, $c=K$.

Poles are $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$= \frac{-6 \pm \sqrt{36-4K}}{2} = -3 \pm \sqrt{9-K}.$$

Since the real part is always $-3$, it never passes through $S_d$ i.e. $-6 \pm 3\sqrt{21}j$. So we use a phase lead controller.

Step 2: Phase lead Controller.

$$D(s) = \frac{K(s+z)}{s+P}.$$

'Zero' of the controller is chosen below Sd, but in this system, a pole already exists at $s = -6$.

∴ We chose the pole location to be $-7$.

ie $z = 7$.

$\therefore \boxed{z = 7.}$

→ Finding P using angle criteria.

ie $\angle D(s_d) + \angle G(s_d) = \pm 180°$.

ie $\angle s_d + z - \angle s_d + P - \angle s_d - \angle s_d + 6 = \pm 180°$

$\angle -6 + 3\sqrt{21}j + 7 - \angle s_d + P - \angle -6 + 3\sqrt{21}j - \angle -6 + 3\sqrt{21}j + 6 = \pm 180°$

$\pm 180° = 85.839° - \angle s_d + P - 113.57° - 90 = \pm 180$

$\angle s_d + P = 62.269° = \tan^{-1}\left(\frac{3\sqrt{21}}{P-6}\right).$

$$P = \frac{3\sqrt{21}}{\tan(62.269)} + 6 = 13.22. \qquad \therefore \boxed{P = 13.22.}$$

Finding K using magnitude criteria.

$$L(s) = \frac{K(s+7)}{s(s+6)(s+13.22)}$$

$$K = \frac{1}{|L(s)|} = \frac{|s_d|\,|s_d+6|\,|s_d+13.22|}{|s_d+7|}$$

$$K = \frac{|-6+3\sqrt{21}j|\,|3\sqrt{21}j|\,|-6+3\sqrt{21}j+13.22|}{|-6+3\sqrt{21}j+7|} = \boxed{232.31.}$$

$$\therefore \quad L(s) = \frac{232.31\,(s+7)}{s(s+6)(s+13.22)}$$

Sketching the RL of L(s) above, we can see that the point $-6 \pm 3\sqrt{21}\,j$ passes through.