MASON'S GAIN FORMULA: LEC 24

Transfer function

$$T = \sum_{n=1}^{K} \frac{P_n \Delta_n}{\Delta}$$

$P_n = n^{th}$ path

K = total number of path in given SFG

Δ = determinant / characteristic function of given SFG.

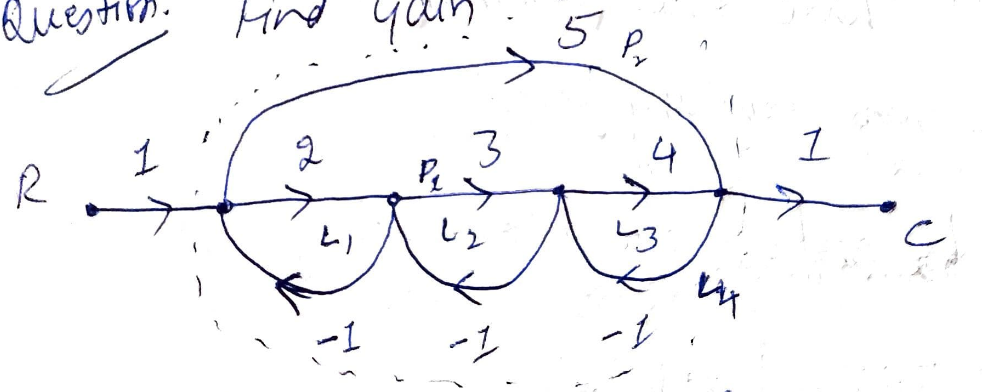
$\Delta = 1 -$ {sum of loop gains of individual loops}

$+$ {sum of product of loop gain of two non-touching loops}

$-$ {sum of product of loop gain of three non-touching loops}

Δ_n = system determinant w.r.t n^{th} path reference.

Question: Find Gain



(a) $\frac{11}{9}$ (b) $\frac{22}{15}$ (c) $\frac{24}{23}$ (d) $\frac{44}{23}$ ✓

$$T = \sum_{n=1}^{K} \frac{P_n \Delta_n}{\Delta}$$

$P_n = P_1, P_2, P_3 \ldots$

$\Delta_n = \Delta_1, \Delta_2, \Delta_3 \ldots$

Path Gain:

$$\left.\begin{aligned} P_1 &= 1 \times 2 \times 3 \times 4 \times 1 = 24 \\ P_2 &= 1 \times 5 \times 1 = 5 \end{aligned}\right\} \text{Gain}$$

Loop gain: $\boxed{\text{Individual Loop gain}}$

$L_1 = 2 \times -1 = -2$

$L_2 = 3 \times -1 = -3$

$L_3 = 4 \times -1 = -4$

$L_4 = 5 \times -1 \times -1 \times -1 = -5$

Non-touching Loops:

two	three	Four
$L_1 \& L_3 = 2 \times 4 = 8$	NIL	NIL

$\Delta = 1 - \{-2-3-4-5\} + \{8\} + \{0\}$

$\Delta = 1 + 14 + 8 = 23$

$\boxed{\Delta = 23}$

$\Delta_1 = 1 - \{$ sum of Individual loop gains which are not common with reference path $\}$ → Here reference path is Path P_1

$\therefore \Delta_1 = 1 - \{0\}$ [∵ since all loops are connected to path 'P_1']

$\therefore \Delta_1 = 1$

$\Delta_2 = 1 - \{-3\}$ → Here reference path is Path 'P_2'

[∵ since only loop L_2 is not connected with path P_2]

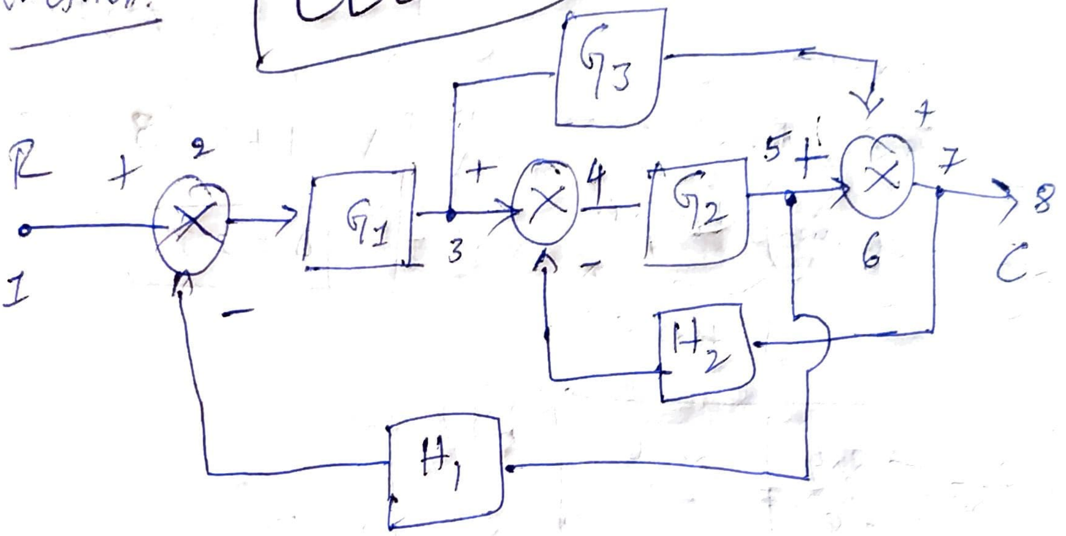
$\Delta_2 = 1 + 3 = 4$

$$\therefore T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$T = \frac{(24 \times 1) + (5 \times 4)}{23}$$

$$\therefore T = \frac{44}{23} \quad \text{(option D))}$$

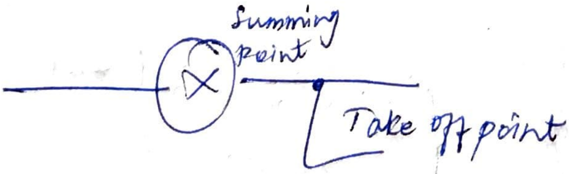
Question: $\boxed{\text{LEC 25}}$



Draw SFG and findout C/R

Note: ① Assign Nodes to all take off / Summing points, Input / output

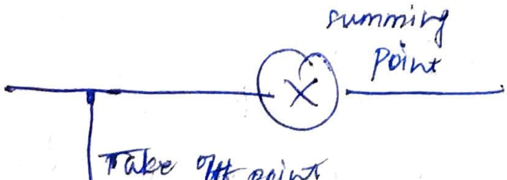
*** ② ***



First summing point, afterwards take off point

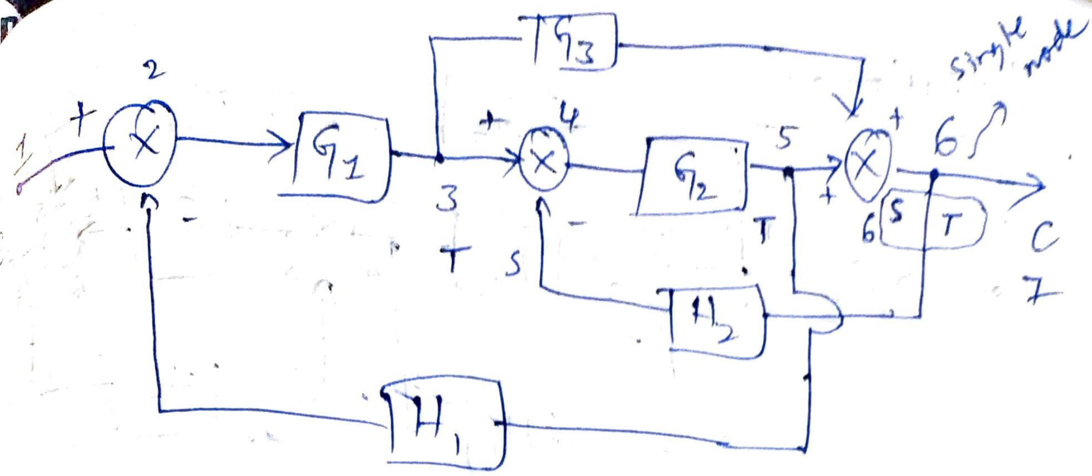
In this condition assign single node

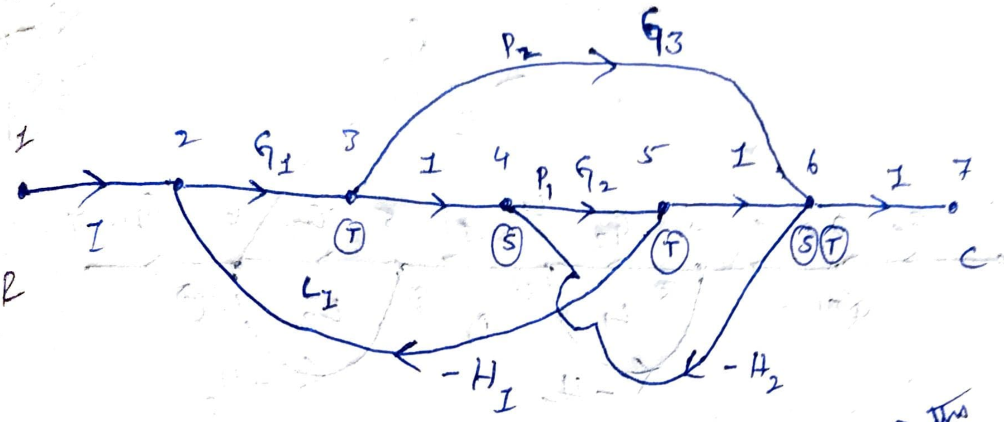
③



First take off point, afterwards summing point

Assign individual node for both.





$P_1 = G_1 G_2$ | $\Delta_1 = 1 - 0 = 1$

$P_2 = G_1 G_3$ | $\Delta_2 = 1 - 0 = 1$

{ paths all have common loops }

$L_1 = -G_1 G_2 H_1$ (2-3-4-5-2)

$L_2 = -G_2 H_2$ (4-5-6-4)

$L_3 = G_1 G_2 G_3 H_1 H_2$ (2-3-6-4-5-2)

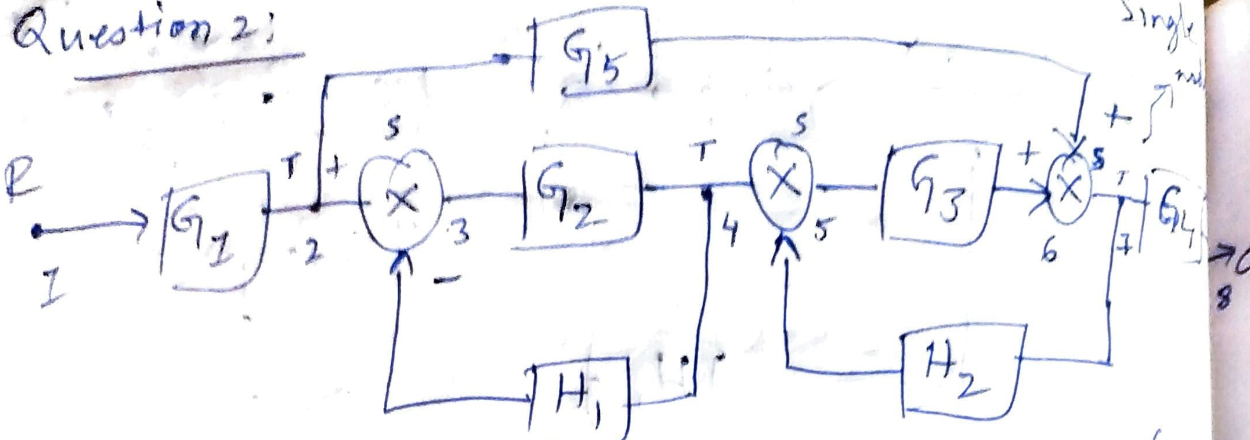
Non-touching loops are not existing here ∴ it is zero

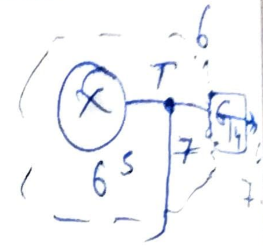
$\therefore \Delta = 1 - (-G_1G_2H_1 - G_2H_2 + G_1G_2G_3H_1H_2) + 0$

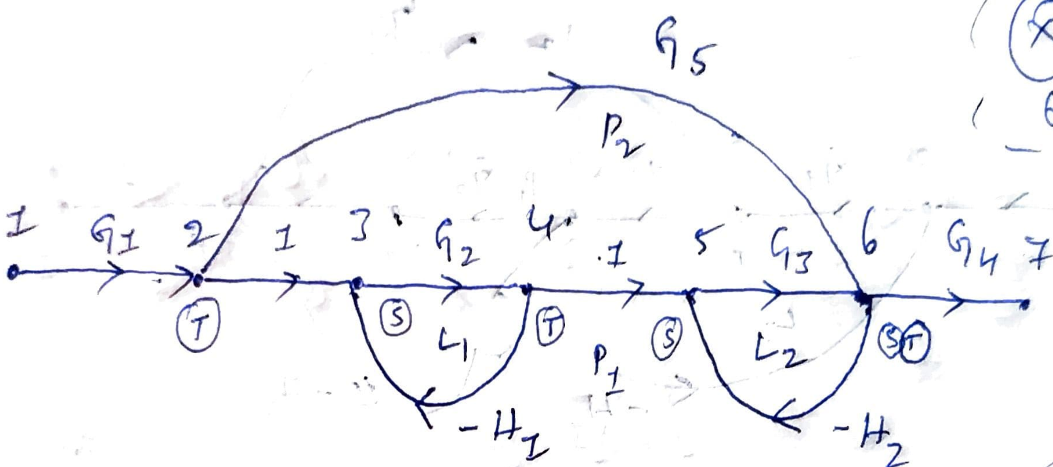
$T = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$$\therefore T = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2 - G_1G_2G_3H_1H_2}$$

Question 2:







P_1 does not have non-common loop

$P_1 = G_1 G_2 G_3 G_4 \quad | \quad \Delta_1 = 1 - \{0\}$

$P_2 = G_1 G_5 G_4 \quad | \quad \Delta_2 = 1 - \{-G_2 H_1\}$

Individual loop

$L_1 = -G_2 H_1$

$L_2 = -G_3 H_2$

Non-touching loop

$L_{NT} = L_1 \& L_2 = G_2 G_3 H_1 H_2$

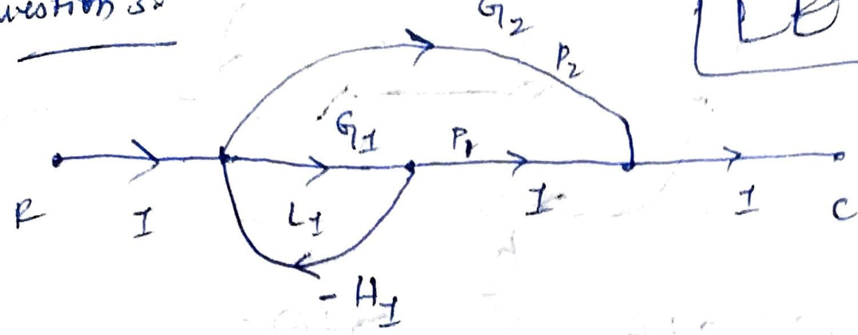
$$\Delta = 1 - (-G_2 H_1 - G_3 H_2) + G_2 G_3 H_1 H_2$$

$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\therefore T = \frac{G_1 G_2 G_3 G_4 + G_1 G_5 G_4 (1 + G_2 H_1)}{1 + G_2 H_1 + G_3 H_2 + G_2 G_3 H_1 H_2}$$

Question 3:

[LEC 26]



C/R is ?

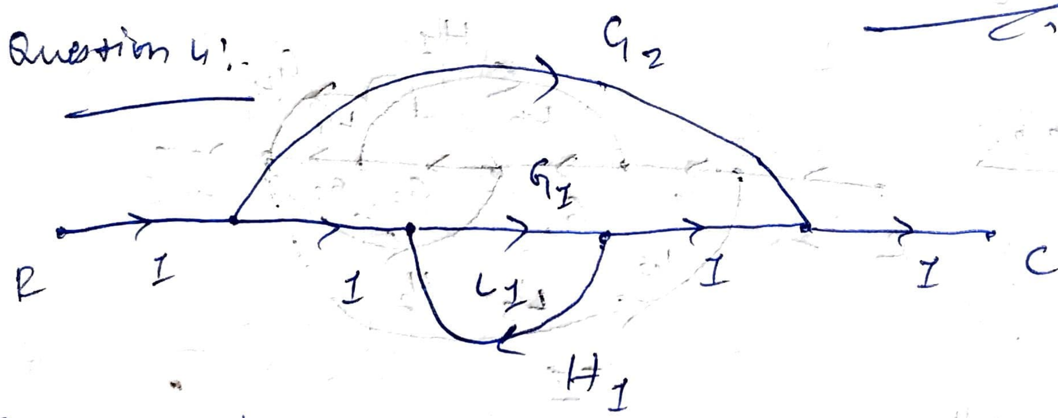
$P_1 = G_1$ | $\Delta_1 = 1 - \{0\}$

$P_2 = G_2$ | $\Delta_2 = 1 - \{0\}$

$L_1 = -G_1 H_1$

$$\therefore TF = \frac{G_1 + G_2}{1 + G_1 H_1}$$

Question 4:



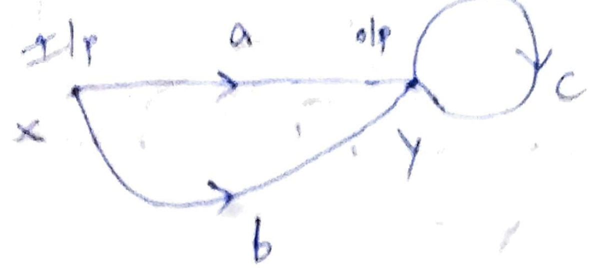
$P_1 = G_1$ | $\Delta_1 = 1 - \{0\}$

$P_2 = G_2$ | $\Delta_2 = 1 - \{G_1 H_1\}$

$L_1 = G_1 H_1$

$$\therefore T = \frac{G_1 + G_2(1 - G_1 H_1)}{1 - G_1 H_1}$$

Question 5:



$\frac{Y}{X}$ is (a) $a+b$ (b) $(a+b)c$

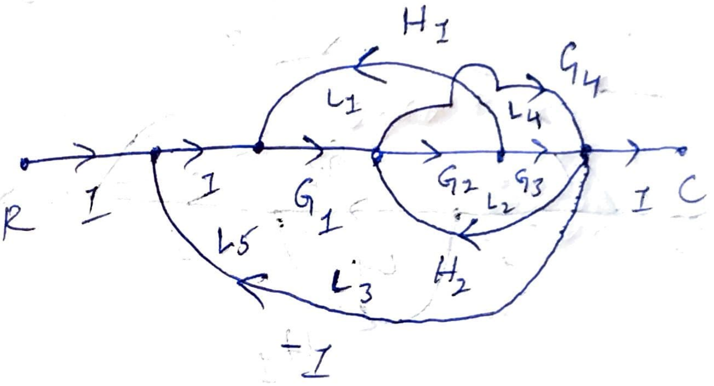
(c) $\frac{a+b}{1-c}$ (d) $\frac{a+b}{1+c}$

$P_1 = a \mid \Delta_1 = 1 - \{0\}$

$P_2 = b \mid \Delta_2 = 1 - \{0\}$ $\quad T = \frac{a+b}{1-c}$ (option c)

$L_1 = c$ $\quad \Delta = 1 - c$

Question 6:



$G_R = ?$

Path

$P_1 = G_1 G_2 G_3 \mid \Delta_1 = 1 - \{0\} = 1$

$P_2 = G_1 G_4 \mid \Delta_2 = 1 - \{0\} = 1$

~~Loop~~

Individual loop

$L_1 = G_1 G_2 H_1$

$L_2 = G_2 G_3 H_2$

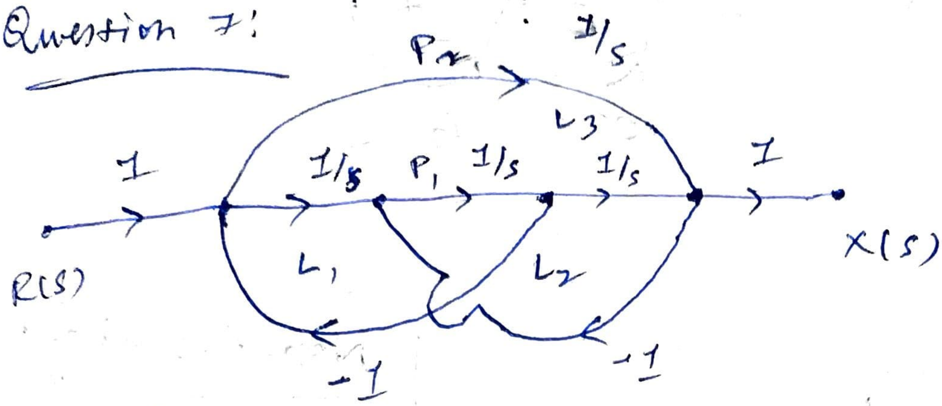
$L_3 = -G_1 G_2 G_3$, $L_5 = -G_1 G_4$

$L_4 = G_4 H_2$

Non-touching loops is not existing.

$$\frac{C}{R} = T = \frac{G_1 G_2 G_3 + G_1 G_4}{1 - G_1 G_2 H_1 - G_2 G_3 H_2 + G_1 G_2 G_3 - G_4 H_2 + G_1 G_4}$$

Question 7:



$\frac{X(s)}{R(s)}$ is ?

(a) $1/s$ ✓ (b) $\frac{s^2+1}{s(s^2+2)}$ (c) $\frac{s(s^2+1)}{s^2+2}$ (d) $1 - 1/s$

$P_1 = \frac{1}{s^3}$ | $\Delta_1 = 1 - \{0\} = 1$

$P_2 = \frac{1}{s}$ | $\Delta_2 = 1 - \{0\} = 1$

$L_1 = \frac{-1}{s^2}$

$L_2 = -\frac{1}{s^2}$

$L_3 = \frac{+1}{s^2}$

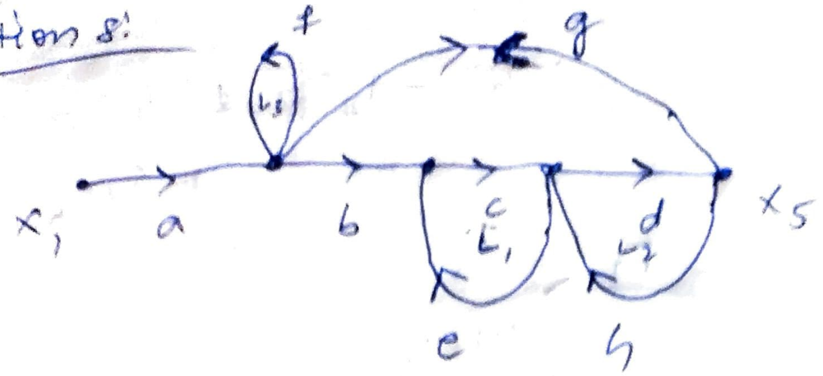
$$\frac{X(s)}{R(s)} = T = \frac{\frac{1}{s^3} + \frac{1}{s}}{1 + \frac{1}{s^2} + \cancel{\frac{1}{s^2}} - \cancel{\frac{1}{s^2}}}$$

$$T = \frac{\frac{1+s^2}{s^3}}{\frac{s^2+1}{s^2}}$$

$\therefore T = \frac{1}{s}$

Option a

Question 8:



$\frac{x_5}{x_1}$ is ?

Path:

$P_1 = abcd \mid \Delta_1 = 1 - \{0\}$

$P_2 = ag \mid \Delta_2 = 1 - ce$

Non-touching loop

$L_1 \& L_3 = cef$

$L_2 \& L_3 = dhf$

Loops:

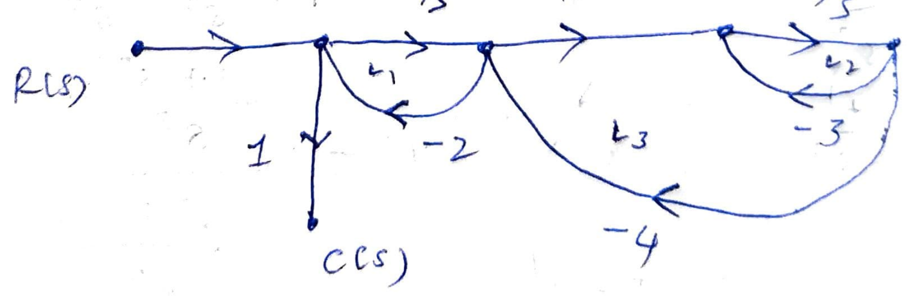
$L_1 = ce$

$L_2 = dh$

$L_3 = f$

$\therefore \frac{x_5}{x_1} = T = \frac{abcd + ag(1-ce)}{1 - ce - dh - f + cef + dhf}$

Question 9: $\boxed{\text{LEC 27}}$



(a) $\frac{6}{s^2+29s+6}$ (b) $\frac{6s}{s^2+29s+6}$ (c) $\frac{s(s+2)}{s^2+29s+6}$

✓ (d) $\frac{s(s+27)}{s^2+29s+6}$

$\frac{C(s)}{R(s)} = TF = ?$

Path:

$P_1 = 1$ | $\Delta_1 = 1 - \left[-\frac{3}{s} - \frac{24}{s} \right] = 1 + \frac{27}{s}$

Loops:

$L_1 = \frac{-2}{s}$

$L_2 = \frac{-3}{s}$

$L_3 = \frac{-24}{s}$

Non-touching loops:

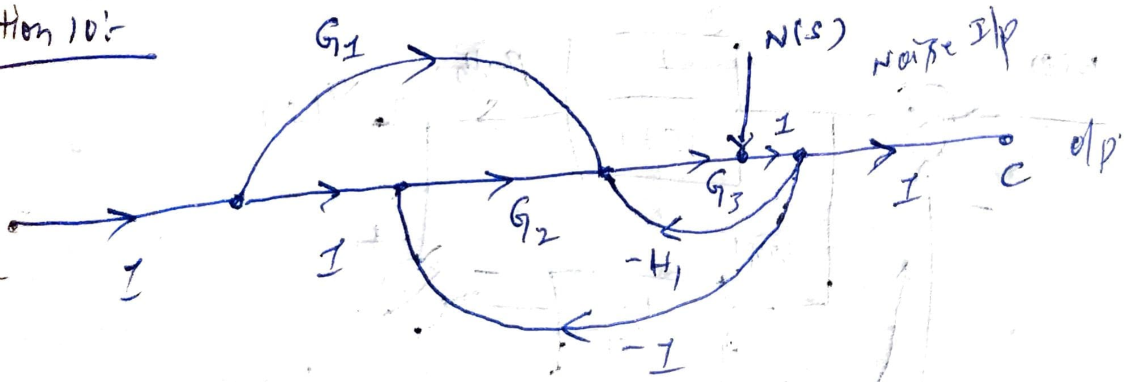
$L_1 \& L_2 = \left(-\frac{2}{s}\right)\left(-\frac{3}{s}\right) = \frac{6}{s^2}$

$$\therefore T.F = \frac{1 + \frac{27}{s}}{1 + \frac{2}{s} + \frac{3}{s} + \frac{24}{s} + \frac{6}{s^2}}$$

$$= \frac{\frac{s+27}{s}}{\frac{6 + 29s + s^2}{s^2}}$$

$$= \frac{s(s+27)}{s^2 + 29s + 6}$$ option (d)

Question 10:-



Find $\frac{C}{R}\Big|_{N(s)=0}$

Find $\frac{C}{N(s)}\Big|_{R=0}$.

Path: For C/R

$P_1 = G_2 G_3 \mid \Delta_1 = 1 - \{0\}$

$P_2 = G_1 G_3 \mid \Delta_2 = 1 - \{0\}$.

Loops:

$L_1 = -G_3 H_1$ $\therefore T = \frac{C}{R} = \frac{G_2 G_3 + G_1 G_3}{1 + G_3 H_1 + G_2 G_3}$

$L_2 = -G_2 G_3$

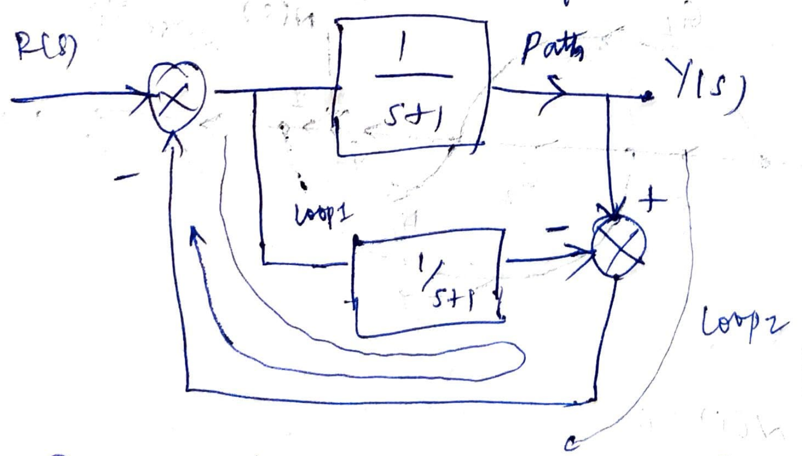
For $C/N \mid_{R=(0)}$ ← Δ' value will be fixed.

Path:

$P_1 = 1 \mid \Delta_1 = 1 - 0$.

$\therefore T = \frac{C}{N} = \frac{1}{1 + G_3 H_1 + G_2 G_3}$

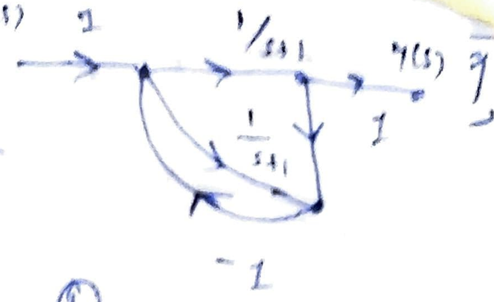
Question 11: The T.F. of the system is?



(a) 0 ✓(b) $\frac{1}{s+1}$ (c) $\frac{2}{s+1}$ (d) $\frac{2}{s+3}$

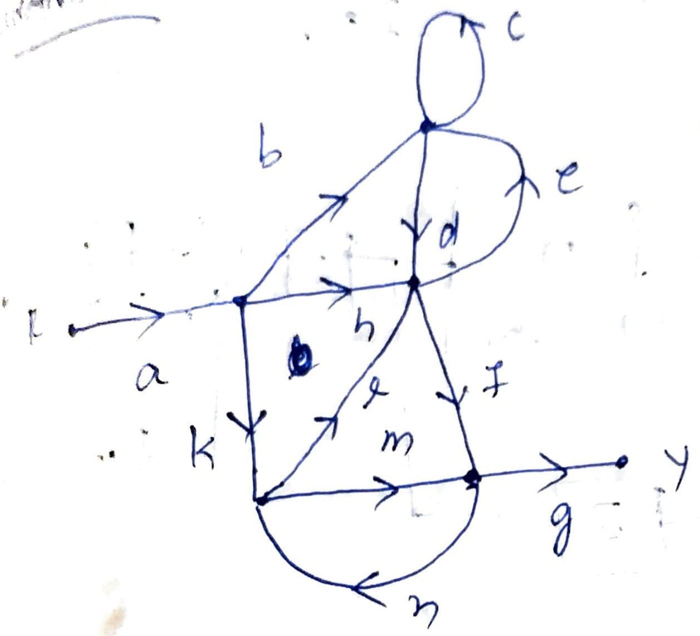
$P_1 = \frac{1}{s+1}$, $L_1 = +\frac{1}{s+1}$, $L_2 = -\frac{1}{s+1}$

$$T.F = \frac{\frac{1}{s+1}}{1 - \frac{1}{s+1} + \frac{1}{s+1}}$$



$\therefore T.F = \frac{1}{s+1}$ option (b).

Question 12:



No of paths?

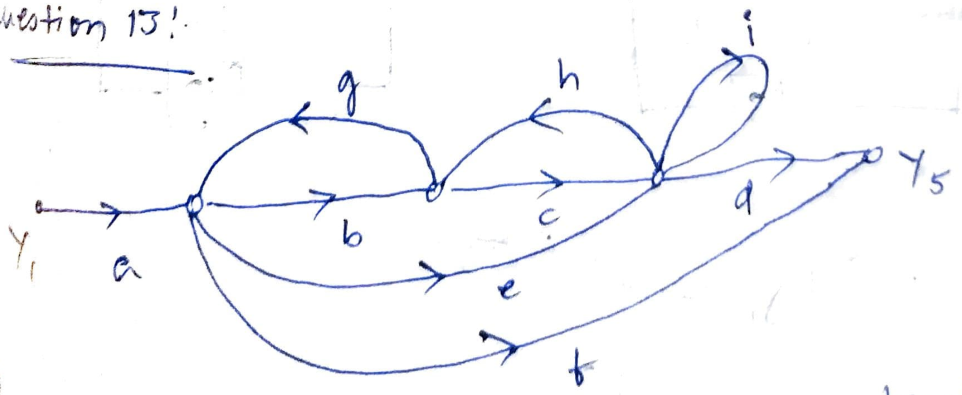
No of loops?

$P_1 = a-h-f-g$, $P_2 = a-b-d-f-g$ } 4 paths

$P_3 = a-k-l-f-g$, $P_4 = a-k-m-g$

$L_1 = c$, $L_2 = de$, $L_3 = n$, $L_4 = lfn$ } 4 loops.

Question 13:



(1) No of paths

(2) No of loops

(3) No of 2 non-touching loops