EC540 Control Systems

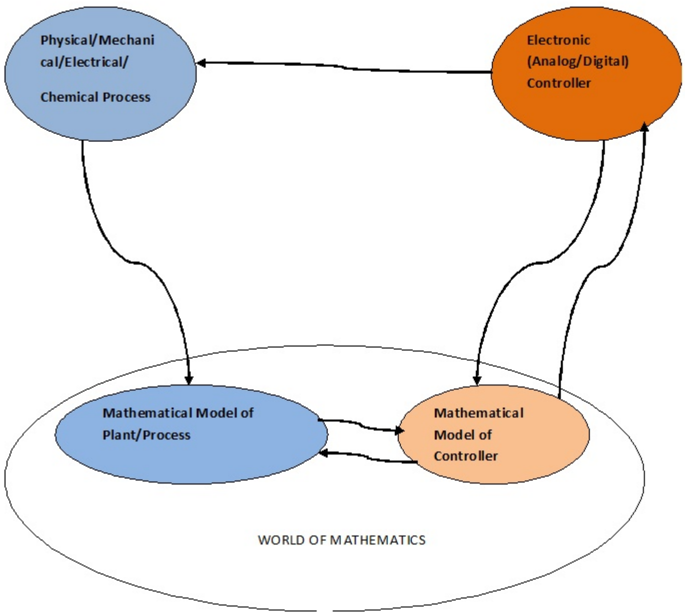
Mathematical Modeling of Mechanical Systems-II

(**Dr. S. Patilkulkarni, 15/09/2020**)

Instructions:

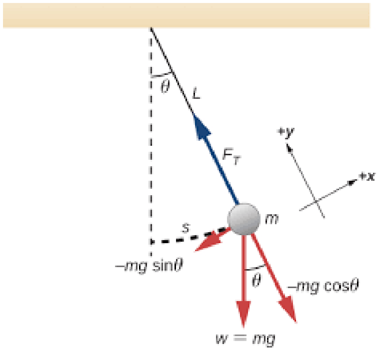
1. Lecture session will be of one hour duration.
2. Fifteen minutes session will be made available for students after the lecture for any question and answers.
3. Regularly review Signals and Systems concepts
4. Regularly visit course webpage.
5. Everyday learn new functions from Octave/Python/MATLAB software
6. Email me on any queries at sudarshan_pk@sjce.ac.in

Why Mathematical Modeling?:



Mathematical Modeling of Mechanical System with Rotational Motion

Example 1 Consider the *simple pendulum* system of length L, mass of ball m, applied torque $T(t)$ as input to the system, angular position $\theta(t)$ as output of the system. Write the differential equation and obtain the transfer function model of the system.



Solution: For mechanical systems involving rotation motion and torque
Applied Torque − Opposing Torque = Moment of Inertia × Angular Acceleration

$$T(t) - mgLsin\theta(t) = J \times \frac{d^2\theta(t)}{dt^2}$$

This is a nonlinear system, as sin() is a nonlinear function.
So we consider small angles to approximate $\sin\theta(t) \approx \theta(t)$
Since ouput is angular position: $y(t) = \theta(t)$

$$u(t) - mgLy(t) = J \times \frac{d^2y(t)}{dt^2}$$

Applying Laplace transform to above differential equation:

$$\mathcal{L}\{u(t)\} - mgL\mathcal{L}\{y(t)\} = J \times \mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\}$$

$$U(s) - mgLY(s) = J[s^2Y(s) - sy(0) - y'(0)]$$

Assuming zero intial conditions:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Js^2 + mgL}$$

For simple pendulum $J = mL^2$ with unit $Kg.m^2$

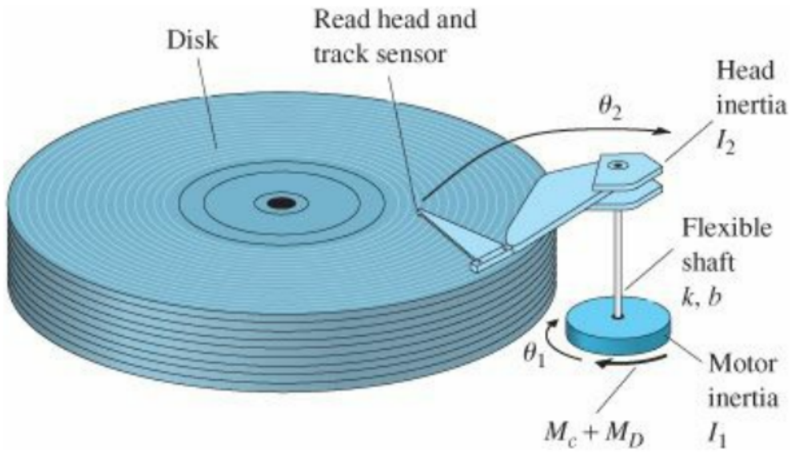
$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{mL^2}\frac{1}{s^2 + \frac{g}{L}}$$

$$\boxed{G(s) = \frac{1}{mL^2}\sqrt{\frac{L}{g}}\left(\frac{\sqrt{\frac{g}{L}}}{s^2 + \frac{g}{L}}\right)}$$

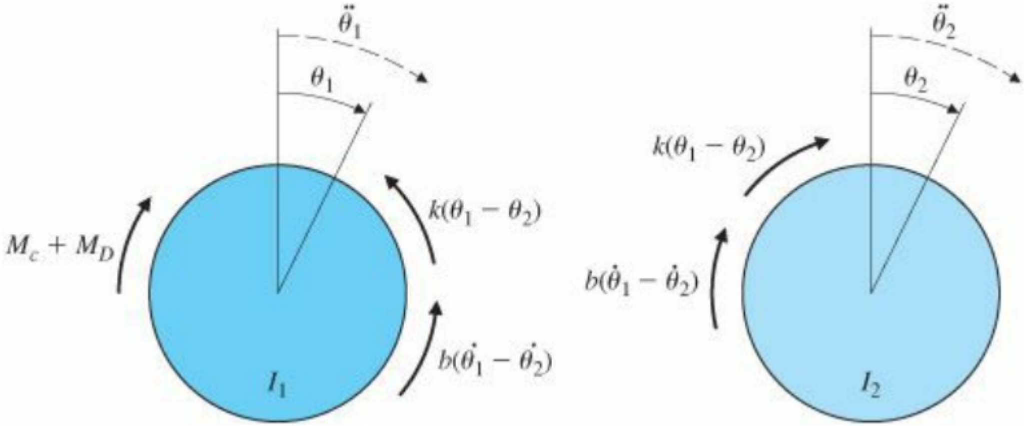
$$\boxed{g(t) = \frac{1}{mL^2}\sqrt{\frac{L}{g}}\sin\sqrt{\frac{g}{L}}t}$$

Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ cycles/sec.

Disk Read-write Head Mechanism



Disk Read-write Head Mechanism



Home Page

Title Page

Page 10 of 12

Go Back

Full Screen

Close

Quit

Torque Equations

For rotor 1:

$$T(t) - b\left(\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right) - k(\theta_1(t) - \theta_2(t)) = J_1 \frac{d^2\theta_1(t)}{dt^2}$$

where $T(t) = M_c + M_D$ in the Figure.

For rotor 2:

$$b\left(\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right) + k(\theta_1(t) - \theta_2(t)) = J_2 \frac{d^2\theta_2(t)}{dt^2}$$

Assume $b = 0$ and $y(t) = \theta_2(t)$

Applying Laplace transform to Rotor 2 Equation :

$$k\theta_1(s) = (J_2 s^2 + k)\theta_2(s)$$

Applying Laplace transform to Rotor 1 Equation:

$$T(s) + k\theta_2(s) = (J_1 s^2 + k)\theta_1(s)$$

$$T(s) + k\theta_2(s) = \frac{J_2 s^2 + k}{k}(J_1 s^2 + k)\theta_2(s)$$

$$U(s) = \left\{ \frac{J_2 s^2 + k}{k}(J_1 s^2 + k) - k \right\} Y(s)$$

Therefore Transfer Function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{J_1 J_2 s^4 + s^2(J_1 k + J_2 k)}.$$