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EC540 Control Systems
Interconnected Systems - 1

(Dr. S. Patilkulkarni, 25,28,29/09/2020)

JSS Science and Technology University Mysuru

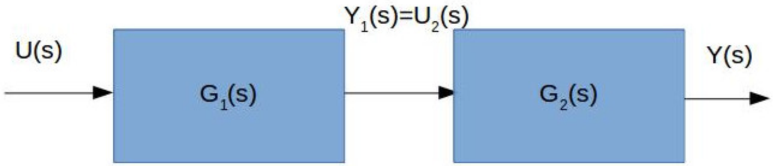
Instructions:

1. Lecture session will be of one hour duration.
2. Fifteen minutes session will be made available for students after the lecture for any question and answers.
3. Regularly review Signals and Systems concepts
4. Regularly visit course webpage.
5. Everyday learn new functions from Octave/Python/MATLAB software
6. Email me on any queries at sudarshan_pk@sjce.ac.in

Interconnected Systems

Real world systems are complex interconnection of several subsytems. If we know the transfer function of individual subsystem; it is possible to find transfer function of entire system. Therefore we first obtain transfer function of some basic primary connections.

Series Connection:



$$\begin{aligned} Y(s) &= G_2(s)U_2(s) \\ &= G_2(s)Y_1(s) \\ &= G_2(s)G_1(s)U(s). \end{aligned}$$

$$\boxed{G(s) = G_1(s).G_2(s)}$$

Time domain equations:

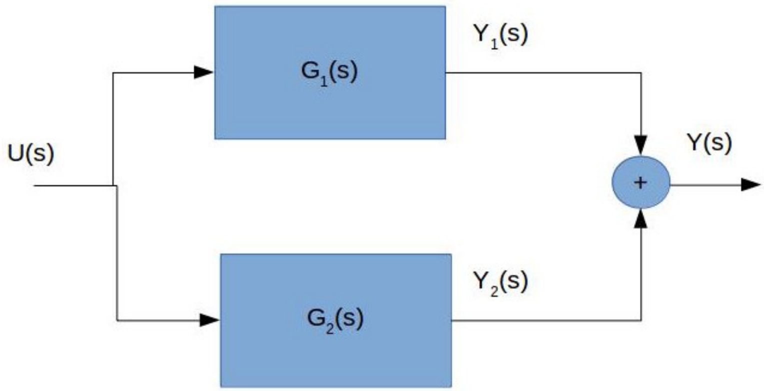
$$y(t) = \int_0^\infty g_2(\tau)u_2(t-\tau)d\tau$$

$$= \int_0^\infty g_2(\tau)y_1(t-\tau)d\tau$$

$$y_1(t) = \int_0^\infty g_1(\mu)u(t-\mu)d\mu$$

$$y(t) = \int_0^\infty g_2(\tau)\left\{\int_0^\infty g_1(\mu)u(t-\tau-\mu)d\mu\right\}d\tau$$

Parallel Connection:



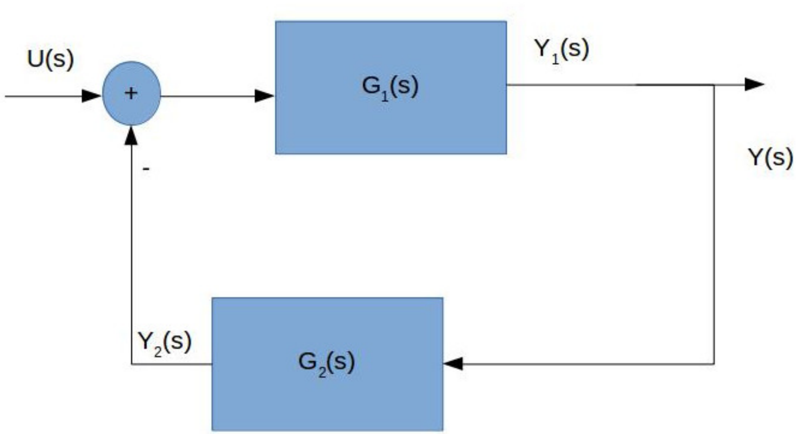
$$\begin{aligned}Y(s) &= Y_1(s) + Y_2(s)\\ &= G_1(s)U(s) + G_2(s)U(s)\\ &= \{G_1(s) + G_2(s)\}U(s).\end{aligned}$$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = G_1(s) + G_2(s)}$$

Time Domain Equations:

$$
\begin{aligned}
y(t) &= y_1(s) + y_2(t) \\
&= \int_0^\infty g_1(\tau)u(t-\tau)d\tau + \int_0^\infty g_2(\tau)u(t-\tau)d\tau \\
&= \int_0^\infty \left(g_1(\tau) + g_2(\tau)\right) u(t-\tau)d\tau.
\end{aligned}
$$

Feedback Connection:

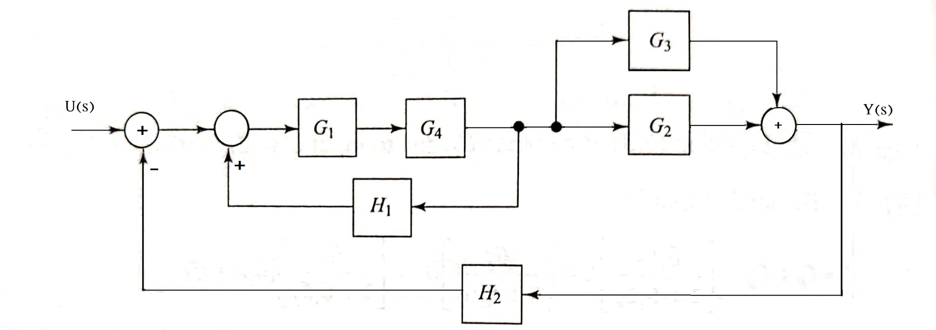


$$\begin{aligned}
U_1(s) &= U(s) - Y_2(s)\\
Y_1(s) &= Y(s)\\
Y_1(s) &= G_1(s)\{U(s) - Y_2(s)\}\\
U_2(s) &= Y(s)\\
Y_2(s) &= G_2(s)Y(s)\\
Y(s) &= G_1(s)\{U(s) - G_2(s)Y(s)\}
\end{aligned}$$

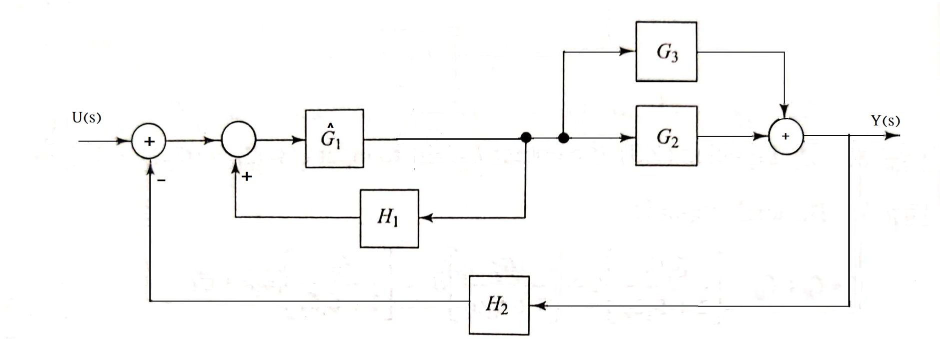
$\therefore$ Transfer function for feedback connection is:

$$\boxed{G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}}$$

Example: Find the transfer function of the following system by block reduction method.



We first observe that $G_1(s)$ and $G_4(s)$ are in series. Therefore let $\hat{G}_1(s) = G_1(s)G_4(s)$



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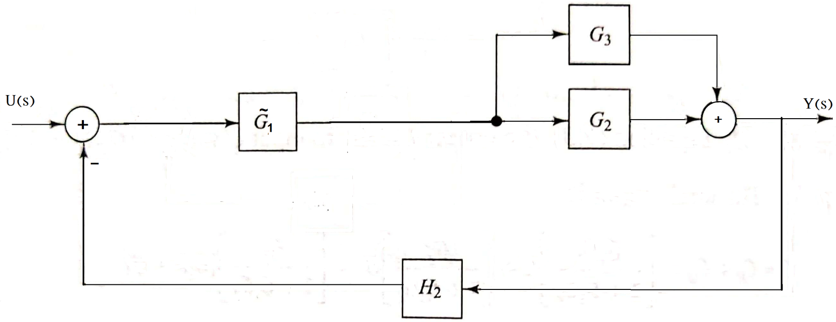
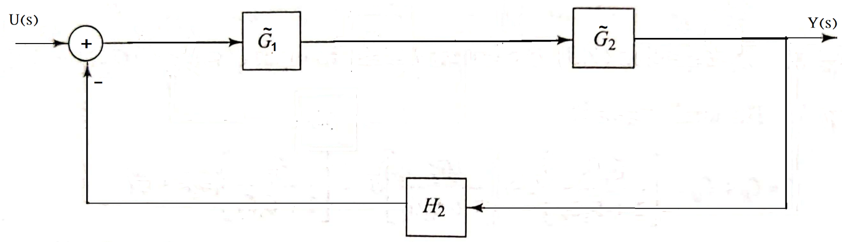
Now, observe that $\hat{G}_1(s)$ and $H_1(s)$ are in feedback connection. Therefore let

$$\tilde{G}_1(s) = \frac{\hat{G}_1(s)}{1 - \hat{G}_1(s)H_1(s)}.$$



Now, $G_2(s)$ and $G_3(s)$ are in parallel. Therefore let $\tilde{G}_2(s) = G_2(s) + G_3(s)$



Now $\tilde{G}_1(s)$ and $\tilde{G}_2(s)$ are in series.

$$\begin{aligned}
\hat{G}_2(s) &= \tilde{G}_1(s)\tilde{G}_2(s) \\
&= \frac{\hat{G}_1(s)}{1-\hat{G}_1(s)H_1(s)}.\left(G_2(s)+G_3(s)\right) \\
&= \frac{G_1(s)G_4(s)}{1-G_1(s)G_4(s)H_1(s)}.\left(G_2(s)+G_3(s)\right) \\
&= \frac{G_1(s)G_4(s)G_2(s)+G_1(s)G_4(s)G_3(s)}{1-G_1(s)G_4(s)H_1(s)}
\end{aligned}$$

Finally $\hat{G}_2(s)$ is in feedback connection with $H_2(s)$. Therefore

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\hat{G}_2(s)}{1 + \hat{G}_2(s)H_2(s)}$$

$$\begin{aligned} G(s) &= \frac{\tilde{G}_1(s)\tilde{G}_2(s)}{1 + \tilde{G}_1(s)\tilde{G}_2(s)H_2(s)} \\ &= \frac{G_1(s)G_4(s)G_2(s) + G_1(s)G_4(s)G_3(s)}{1 - G_1(s)G_4(s)H_1(s) + G_1(s)G_4(s)G_2(s)H_2(s) + G_1(s)G_4(s)G_3(s)H_2(s)} \end{aligned}$$

Mason's Gain Formula

Sketch the signal flow graph for the given interconnected system and apply this formula:

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \sum_{1}^{N} \frac{P_k \Delta_k}{\Delta}}$$

where P_k is the product of gains in k-th forward path

$$\Delta = 1 - \sum_{k=1}^{M} L_k + \sum L_j L_k - \sum L_i L_j L_k + \cdots$$

$$\Delta = 1 - \text{ sum of individual loop gains}$$

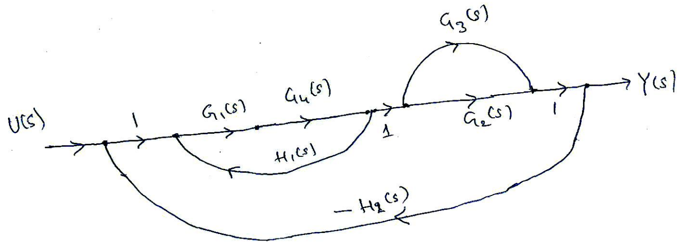
$$+ \text{ sum of products of gains of two non touching loops}$$

$$-\text{sum of products of gains of three non touching loops } + \cdots$$

Loops are nontouching if they donot share common node or common branch.

Δ_k is Δ for the signal flow graph NOT containing the k-th forward path.

Example Considering the same example:
First express the block diagram as signal flow graph:



Identify the forward paths: $P_1 = G_1(s)G_4(s)G_2(s)$,
$P_2 = G_1(s)G_4(s)G_3(s)$.

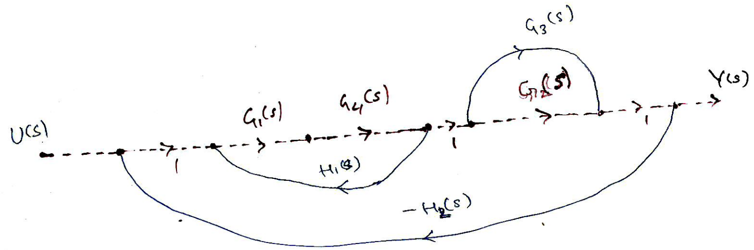
Identify the individual loop gains:

$$L_1 = G_1(s)G_4(s)H_1(s)$$
$$L_2 = -G_1(s)G_4(s)G_2(s)H_2(s)$$
$$L_3 = -G_1(s)G_4(s)G_3(s)H_2(s)$$

Identify Pair (Two) of Non Touching loops (Do not share common node or common branch)

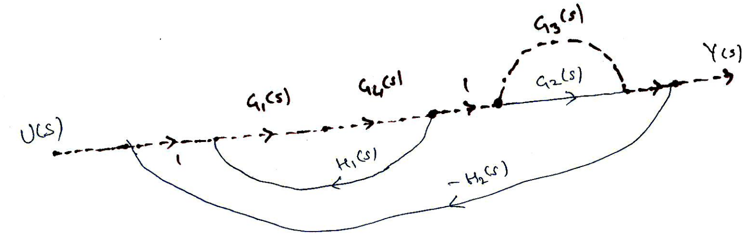
In this Example: None.

$$\therefore \Delta = 1 - (L_1 + L_2 + L_3) = 1 - G_1(s)G_4(s)H_1(s)$$
$$-(-G_1(s)G_4(s)G_2(s)H_2(s)) - (-G_1(s)G_4(s)G_3(s)H_2(s))$$



No loops exist in this graph.

$$\Delta_1 = 1$$



No loops exist in this graph.

$$\Delta_2 = 1$$

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Applying Mason's Gain Formula:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$G(s) = \frac{G_1(s)G_4(s)G_2(s) + G_1(s)G_4(s)G_3(s)}{1 - G_1(s)G_4(s)H_1(s) + G_1(s)G_4(s)G_2(s)H_2(s) + G_1(s)G_4(s)G_3(s)H_2(s)}$$