Waveform Coding Techniques

4.1. Introduction

To transport an information-bearing signal from one point to another point over a communication channel, we can use digital or analog techniques. As discussed earlier in chapter 1, the use of digital communication offers several important advantages as compared to analog communication. In particular, a digital communication system offers the following highly attractive features:

(i) ruggedness to channel noise and external interference unmatched by any analog communication system.

(ii) flexible operation of the system.

(iii) integration of diverse sources of information into a common format.

(iv) security of information in the course of its transmission from source to the destination.

In view of above reasons, digital communications have become the dominant form of communication technology in our society.

However, to handle the transmission of analog message signals (*i.e.*, voice and video signals)* by digital means, the signal has to undergo an analog-to-digital conversion. We already know about the pulse amplitude modulation (PAM) technique. We also know the disadvantages of using PAM technique. Using the waveform coding technique, we convert the analog PAM signal into a digital signal. This digital signal is in the form of a train or stream of binary digits 0 and 1. Thus, with waveform coding techniques, we enter into the world of digital communication. After sampling an analog signal, the next step in its digital transmission is the generation of the "coded version" (digital representation) of the signal. Pulse Code Modulation (PCM) provides one method to meet such a requirement.

4.2. Discretization in Time and Amplitude

As discussed earlier, pulse modulation may be classified under two heads *i.e.*, pulse analog modulation and pulse digital modulation. In case of pulse analog modulation, only time is expressed in the digital form and any one of the pulse parameters (*i.e.*, pulse amplitude, duration or position) is varied in a continuous manner in accordance with the message signal. Pulse amplitude modulation (PAM), Pulse duration (width) modulation (PWM) and Pulse position modulation (PPM) are the examples of pulse analog modulation. In these modulation schemes, information transmission is accomplished in an analog form at discrete times. On the other hand, in the pulse digital modulation, the time and the pulse parameter (usually the amplitude) occur in discrete form and digital coded form respectively. Pulse digital modulation is therefore basically a scheme which converts the analog signal to its corresponding digital form. It is for this reason that the analog-to-digital conversion is sometimes known as **pulse digital modulation.**

The simplest form of pulse digital modulation is called pulse code modulation (PCM). In this system (*i.e.*, PCM), the message signal is first sampled and then amplitude of each sample is rounded off to the nearest one of a finite set of allowable values known as quantization levels, so that both time and amplitude are in the discrete form. This means that in pulse code modulation both parameters *i.e.*, time and amplitude are expressed in discrete form. This process is called **discretization in time and amplitude.**

Concept of Quantization

In communication systems, sometimes it happens that we are available with analog signal, however, we have to transmit a digital signal for a particular application. In such cases, we have to convert an analog signal into digital signal. This means that we have to convert a continuous time signal in the form of digits. To see how a signal can be converted from analog to digital form, let us consider an analog signal as shown in figure 4.1(a). First of all, we get samples of this signal according to sampling theorem. For this purpose, we mark the time-instants t_0, t_1, t_2 and so on., at equal time-intevals along the time axis. At each of these time-instants, the magnitude of the signal is measured and thus samples of the signal are taken.

Figure 4.1(*b*) shows a representation of the signal of figure 4.1(*a*) in terms of its samples.

Now, we can say that the signal in figure 4.1(*b*) is defined only at the sampling instants. This means that it no longer is a continuous function of time, but rather, it is a discrete-time signal. However, since the magnitude of each smaple can take any value in a continuous range, the signal in figure 4.1(*b*) is still an analog signal.

This difficulty is neatly resolved by a process known as **quantization**. In quantization, the total amplitude range which the signal may occupy is divided into a number of standard levels.

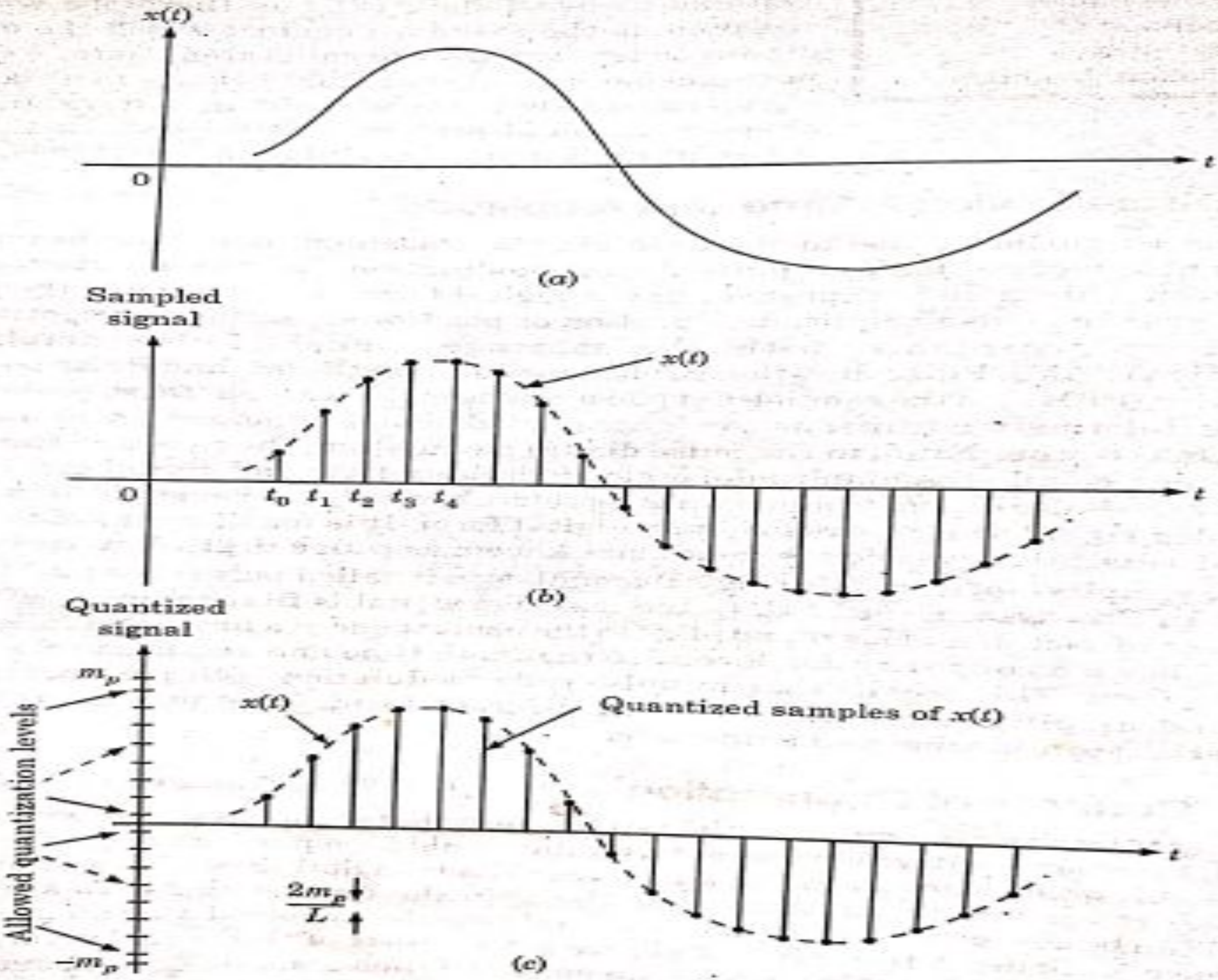


Fig. 4.1. *(a) An analog signal, (b) Samples of analog siagnal, (c) Q...*

As shown in figure 4.1(c), amplitudes of the signal $x(t)$ lie in the range $(-m_p, m_p)$ which is partitioned into L intervals, each of magnitude $\Delta v = \frac{2m_p}{L}$. Now, each smaple is approximated or rounded off to the nearest quantized level as shown in figure. Since each sample is now approximated to one of the L numbers therefore the information is digitized.

The quantized signal is an approximation of the original one. We can improve the accuracy of the quantized signal to any desired degree simply by increasing the number of levels L.

4.4. Pulse Code Modulation (PCM)

(U.P. Tech-Semester Exam. 2002-2003)

Pulse-code modulation is known as a **digital pulse modulation technique**. In fact, the pulse-code modulation (PCM) is quite complex compared to the analog pulse modulation techniques (*i.e.*, PAM, PWM and PPM) in the sense that the message signal is subjected to a great number of operations. Figure 4.2 shows the basic elements of a PCM system. It consists of three main parts *i.e.*, transmitter, transmission path and receiver. The essential operations in the transmitter of a PCM system are sampling, quantizing and encoding as shown in figure 4.2. As discussed earlier, sampling is the operation in which an analog (*i.e.*, continous-time) signal is sampled according to the sampling theorem resulting in a discrete-time signal. The quantizing and encoding operations are usually performed in the same circuit which is known as an **analog-to-digital converter (ADC)**.

Also, the essential operations in the receiver are regeneration of impaired signals, decoding and demodulation of the train of quantized samples. These operations are usually performed in the same cirucit which is known as a digital-to-analog converter (DAC).

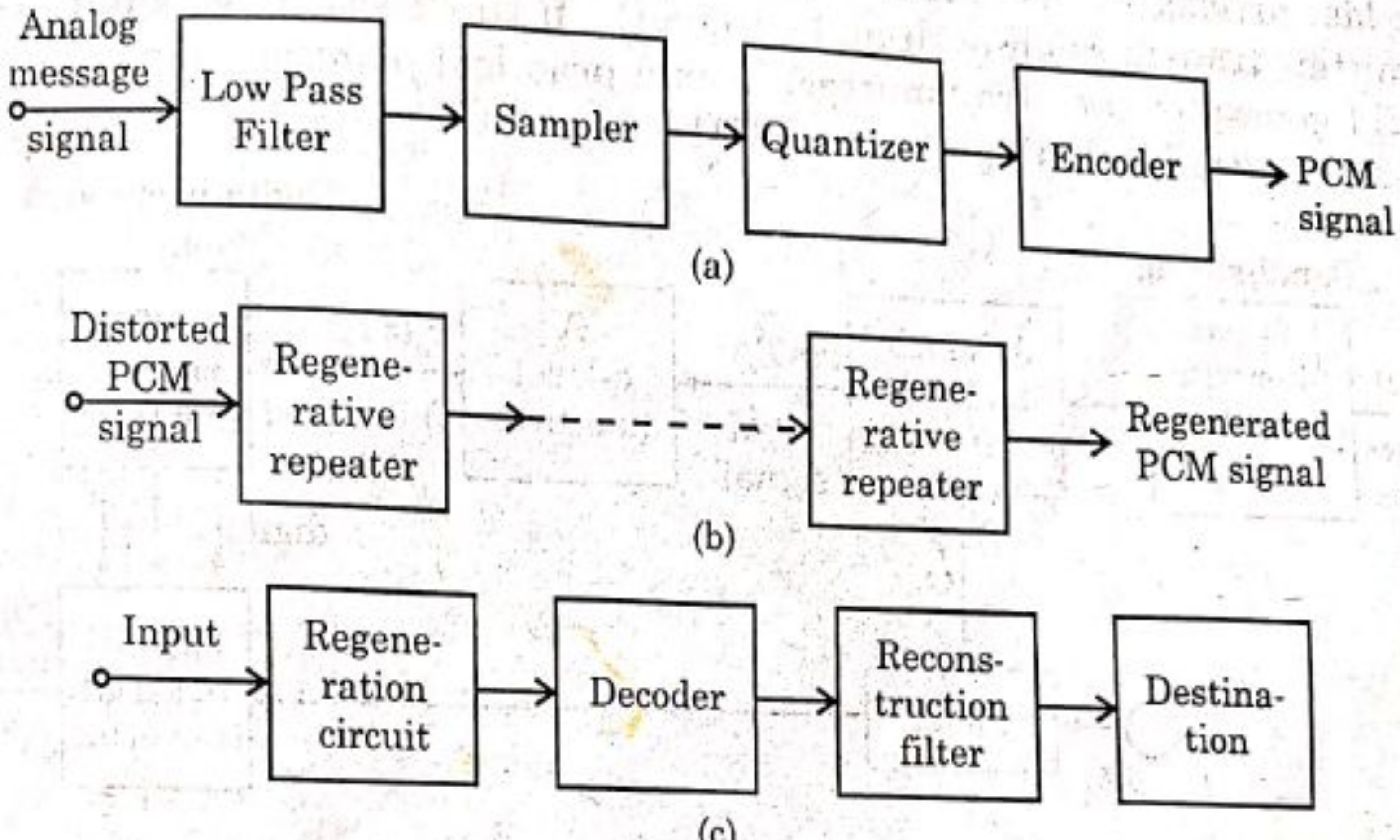


Fig. 4.2. *The basic elements of a PCM system*
(a) Transmitter (b) Transmission path (c) Receiver.

Further, at intermediate points, along the transmission route from the transmitter to the receiver, regenerative repeaters are used to reconstruct (*i.e.*, regenerate) the transmitted sequence of coded pulses in order to combat the accumulated effects of signal distortion and noise.

As discussed in article 4.3, the quantization refers to the use of a finite set of amplitude levels and the selection of a level nearest to a particular sample value of the message signal as the representation for it. In fact, this operation combined with sampling, permits the use of coded pulses for representing the message signal. *Thus, it is the combined use of quantizing and coding that distinguishes pulse code modulation form analog modulation techniques.*

Now, let us summarize PCM in the form of few points as under:

(*i*) PCM is a type of pulse modulation like PAM, PWM or PPM but there is an important difference between them PAM, PWM or PPM are "analog" pulse modulation systems whereas PCM is a "digital" pulse modulation system.

(*ii*) This means that the PCM output is in the coded digital form. It is in the form of digital pulses of constant amplitude, width and position.

(*iii*) The information is transmitted in the form of "code words". A PCM system consists of a PCM encoder (transmitter) and a PCM decoder (receiver).

(*iv*) The essential operations in the PCM transmitter are sampling, quantizing and encoding.

(*v*) All the operations are usually performed in the same circuit called as **analog-to-digital converter**.

(*vi*) It should be understood that the PCM is not modulation in the conventional sense.

(*vii*) Because in modulation, one of the characteristics of the carrier is varied in proportion with the amplitude of the modulating signal. Nothing of that sort happen in PCM.

4.5. A PCM Generator or Transmitter

In the last article, we had an overview of the elements of a PCM system (*i.e.*, transmitter, transmission-path and receiver). In this section, we shall discuss the PCM generator (*i.e.*, transmitter) from a practical point of view. Figure 4.3 shows a practical block diagram of a PCM generator.

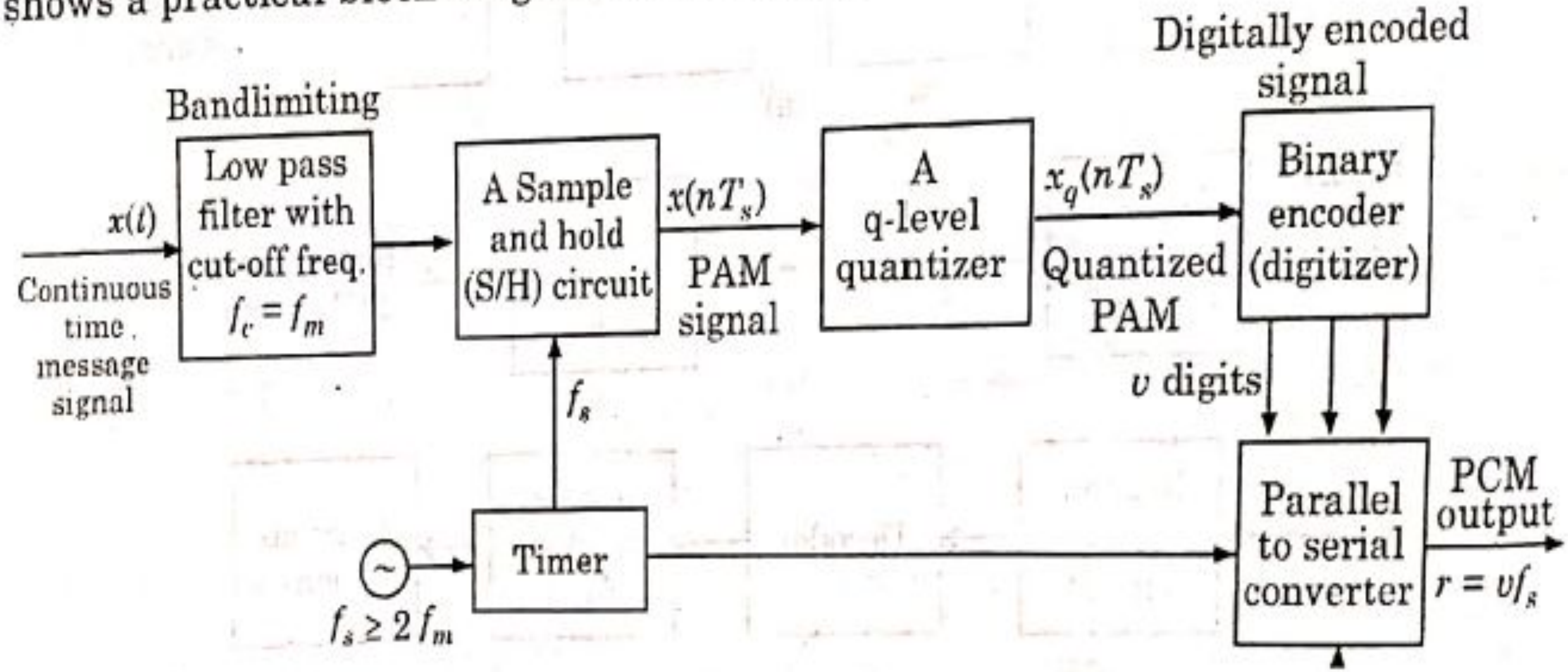


Fig. 4.3. *A practical PCM generator*

In PCM generator of figure 4.3, the signal $x(t)$ is first passed through the low-pass filter of cutoff frequency f_m Hz. This low-pass filter blocks all the frequency components which are lying above f_m Hz.*

This means that now the signal $x(t)$ is bandlimited to f_m Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above nyquist rate to aviod aliasing *i.e.*,

$$f_s \geq 2f_m$$

In figure 4.3, the output of sample and hold circuit is denoted by $x(nT_s)$. This signal $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called **quantization error**. Thus, output of quantizer is a digital level called $x_q(nT_s)$.

Now, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'v' binary bits. This encoder is also known as digitizer.

Note: *It may be noted that it is not possible to transmit each bit of the binary word separately on transmission line. Therefore 'v' binary digits are converted to serial bit stream to generate single baseband signal. In a parallel to serial converter, usually a shift register does this job. The output of PCM generator is thus a single baseband signal of binary bits.*

Also, an oscillator generates the clocks for sample and hold circuit and parallel to serial converter. In the pulse code modulation generator discussed above, sample and hold, quantizer and encoder combinely form an analog to digital converter (ADC).

4.6. PCM Transmission Path

The Path between the PCM transmitter and PCM receiver over which the PCM signal travel, is called as **PCM transmission path** and it is as shown in figure 4.4. The most important feature of PCM system lies in its ability to control the effects of distortion and noise when the PCM wave travels on the channel. PCM accomplishes this capacity by means of using a chain of regenerative repeaters as shown in figure 4.4. Such repeaters are spaced close enough to each other on the transmission path. The regenerative performs three basic operations namely equalization, timing and decision making. Hence, each repeater actually reproduces the clean noise free PCM signal from the PCM signal distorted by the channel noise. This improves the performance of PCM in presence of noise.

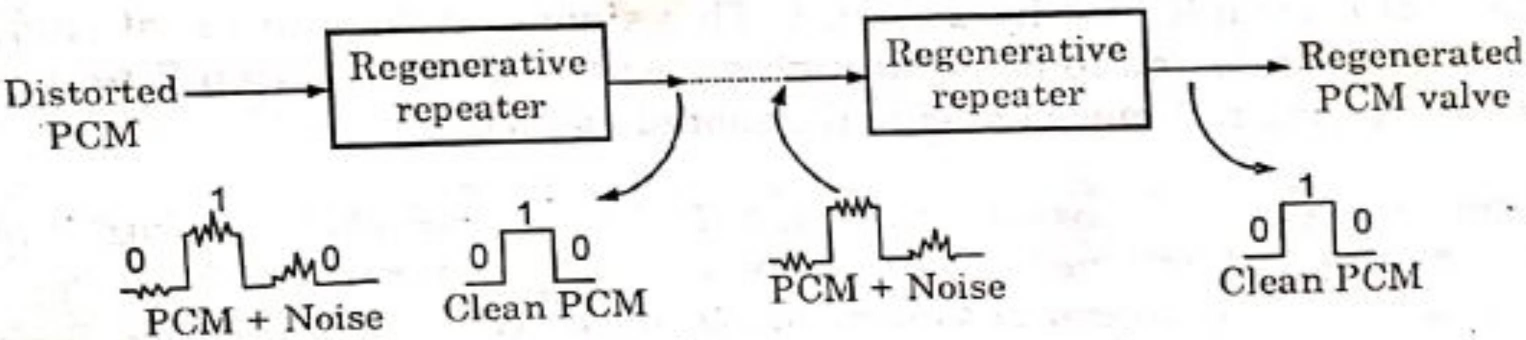


Fig. 4.4. *PCM transmission path.*

4.6.1. Block Diagram of a Repeater

Figure 4.5 shows the block diagram of a regenerative repeater.

The amplitude equalizer shapes the distorted PCM wave so as to compensate for the effects of amplitude and phase distortions. The timing circuit produces a periodic pulse train which is derived from the input PCM pulses. This pulse train is then applied to the decision making device. The decision making device uses this pulse train for sampling the equalized PCM pulses. The sampling is carried out at the instants where the signal to noise ratio is maximum.

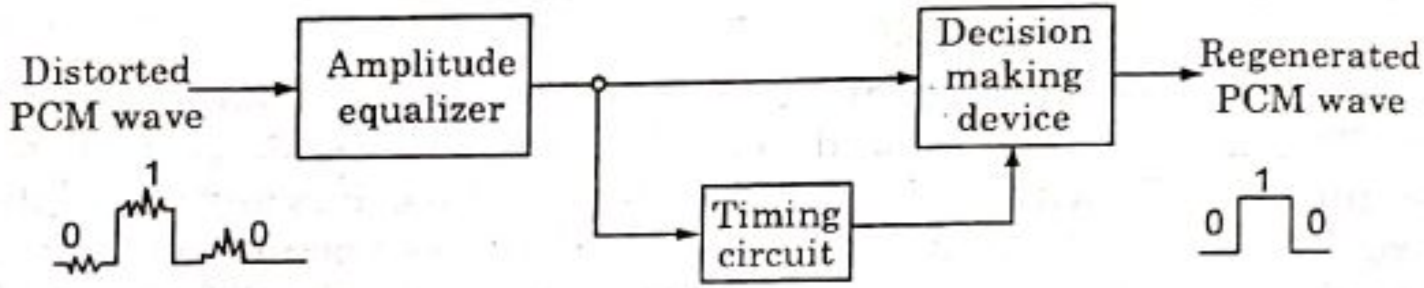


Fig. 4.5. *Block diagram of a regenerative repeater.*

The decision device makes a decision about whether the equalized PCM wave at its input has a 0 value or 1 value at the instant of sampling. Such a decision is made by comparing equalized PCM with a reference level called **decision threshold** as illustrated in figure 4.6*a*. At the output of the decision device, we get a clean PCM signal without any trace of noise.

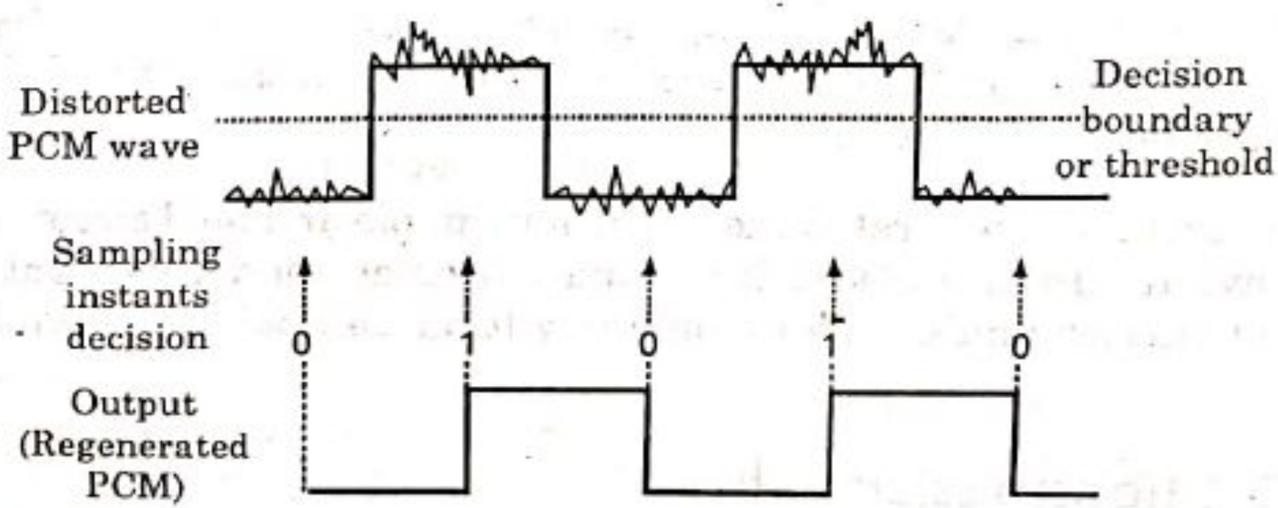


Fig. 4.6*a*. *Waveforms of a regenerative repeater.*

4.7. PCM Receiver

In this section, we shall discuss a PCM receiver from practical point of view. Figure 4.6(a) shows the block diagram of PCM receiver and figure 4.6 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulse and removes the noise. This signal is then converted to parallel digital words for each sample.

Now, the digital word is converted to its analog value denoted as $x_q(t)$ with the help of a sample and hold circuit. This signal, at the output of sample and hold circuit, is allowed to pass through a lowpass reconstruction fi'ter to get the appropriate original message signal denoted as $y(t)$.

Note: *As shown in reconstructed signal of figure 4.6 (b), it is impossible to reconstruct exact original signal x(t) because of permanent quantization error introduced during quantization at the transmitter. In fact, this quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits (bits) per sample. But increasing bits 'v' increases the singnaling rate as well as transmission bandwidth as we have observed in last article. Therefore the choice of these parameters is made, in such a manner that noise due to quantization error (i.e., also called as quantization noise) is in tolerable limits.*

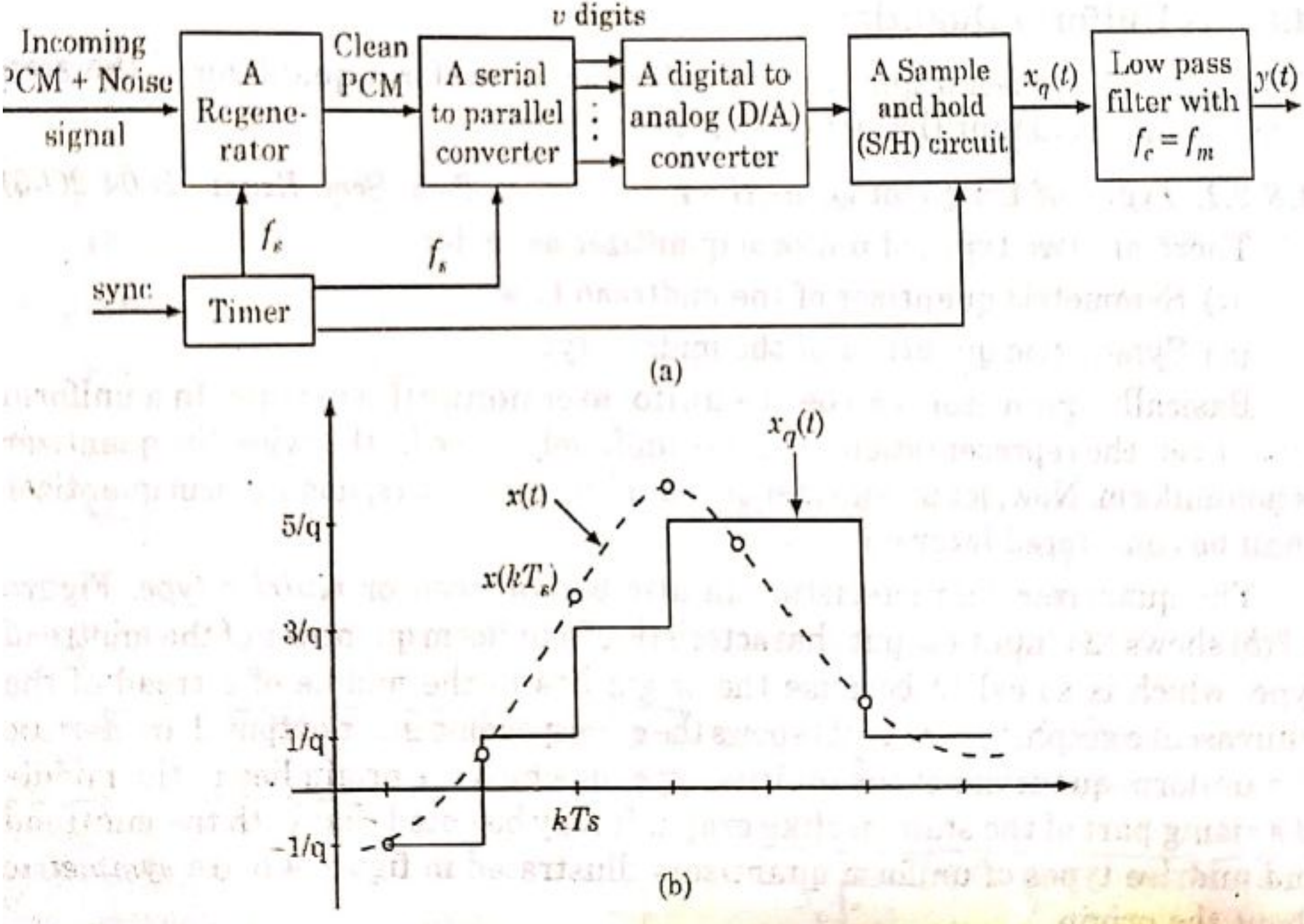


Fig. 4.6. (a) PCM receiver (b) Reconstructed waveform.

4.8. Quantizer

As discussed in article 4.5, a q-level quantizer compares the discrete-time input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called quantization error. Thus, the output of a quantizer is a digital level called $x_q(nT_s)$.

4.8.1. Classification of Quantization Process

Figure 4.7 shows the classification of quantization process.

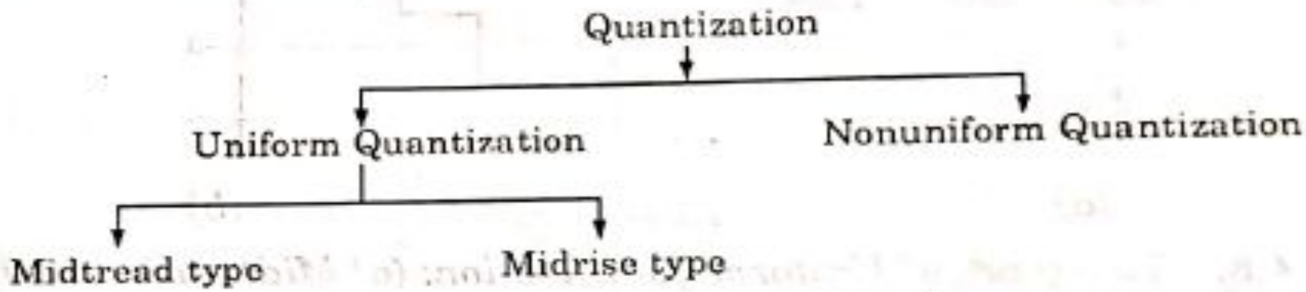


Fig. 4.7. *Classification of quantization process.*

The quantization process can be classified into two types as under:

(*i*) Uniform quantization

(*ii*) Non-uniform quantization.

This classification is based on the step size as defined earlier.

(*i*) Uniform Quantizer

A uniform quantizer is that type of quantizer in which the 'step size' remains same throughout the input range.

(*ii*) Nonuniform Quantizer

A non-uniform quantizer is that type of quantizer in which the 'step-size' varies according to the input signal values.

4.8.2. A Uniform Quantizer

As discussed earlier, a quantizer is called as an uniform quantizer if the step size remains constant throughout the input range.

4.8.2.1. Types of Uniform Quantizer *(U.P. Tech, Sem. Exam., 2004-2005)*

There are two types of uniform quantizer as under:

(*i*) Symmetric quantizer of the midtread type

(*ii*) Symmetric quantizer of the midrise type

Basically, quantizers can be of a **uniform** or **nonuniform type**. In a uniform quanitizer, the representation levels are uniformly spaced; otherwise, the quantizer is nonuniform. Now, let us consider only uniform quantizers, nonuniform quantizer shall be considered later on.

The quantizer characteristic can also be *midtread* or *midrise type*. Figure 4.8(*b*) shows the input-output characteristic of a uniform quantizer of the midtread type, which is so called because the origin lies in the middle of a tread of the staircaselike graph. Figure 4.8(*b*) shows the corresponding input-output characteristic of a uniform quantizer of the midrise type, in which the origin lies in the middle of a rising part of the staircaselike graph. It may be noted that both the midtread and midrise types of uniform quantizers illustrated in figure 4.8 are *symmetric* about the origin.

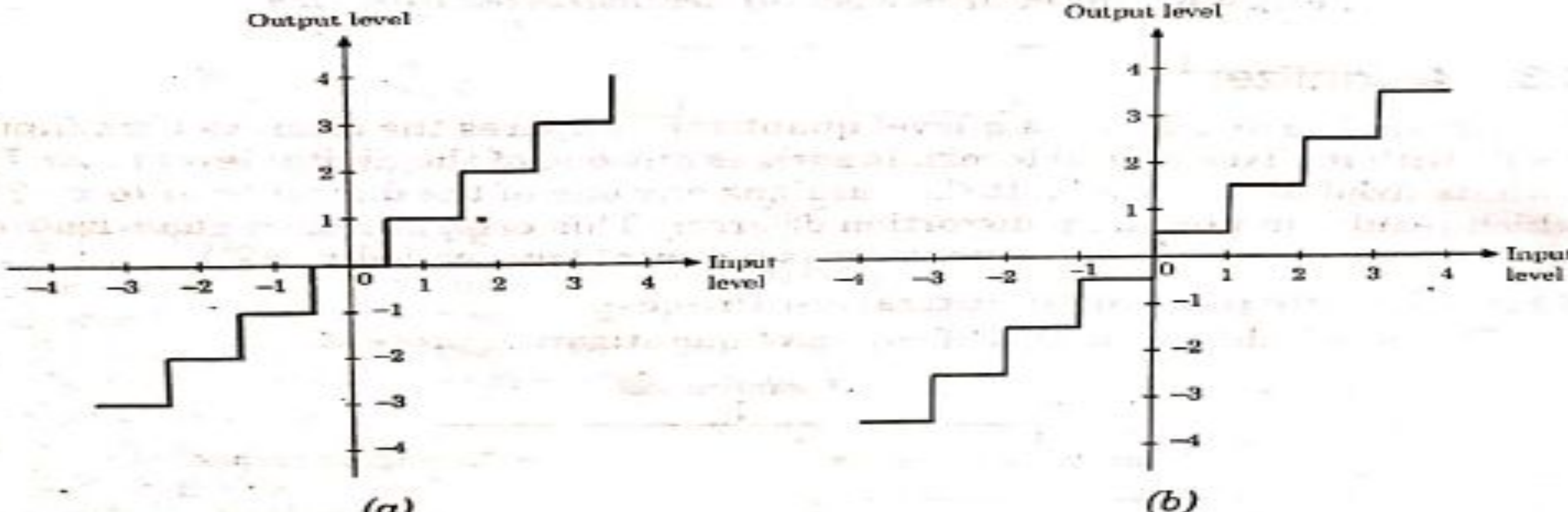


Fig. 4.8. *Two types of Uniform quantization: (a) Midtread, and (b) Midrise.*

4.9. Working Principle of Quantizer

In this section, let us see how uniform quantization takes place. For this purpose, we shall consider uniform quantizer of midrise type. Figure 4.9(a) shows the transfer characteristics of a uniform quantizer of midrise type. In figure 4.9(a), let us assume that the input to the quantizer $x(nT_s)$ varies from $-4\,\Delta$ to $+4\,\Delta$. This means that the peak to peak value of $x(nT_s)$ will be between -4Δ to $+4\Delta$. Here 'Δ' is the step size.

Thus, input $x(nT_s)$ can take any value between $-4\,\Delta$ to $+4\,\Delta$. Now, the fixed digital levels are avilable at $\pm\frac{\Delta}{2}, \pm\frac{3}{2}\Delta, \pm\frac{5}{2}\Delta$ and $\pm\frac{7}{2}\Delta$. These levels are available at quantizer because of its characteristics.

Hence, according to figure 4.9(a), we have

If $\quad x(nT_s) = 4\,\Delta$, then $x_q\,(nT_s) = \frac{7}{2}\Delta$

and if $\quad x(nT_s) = -4\,\Delta$, then $x_q\,(nT_s) = -\frac{7}{2}\Delta$

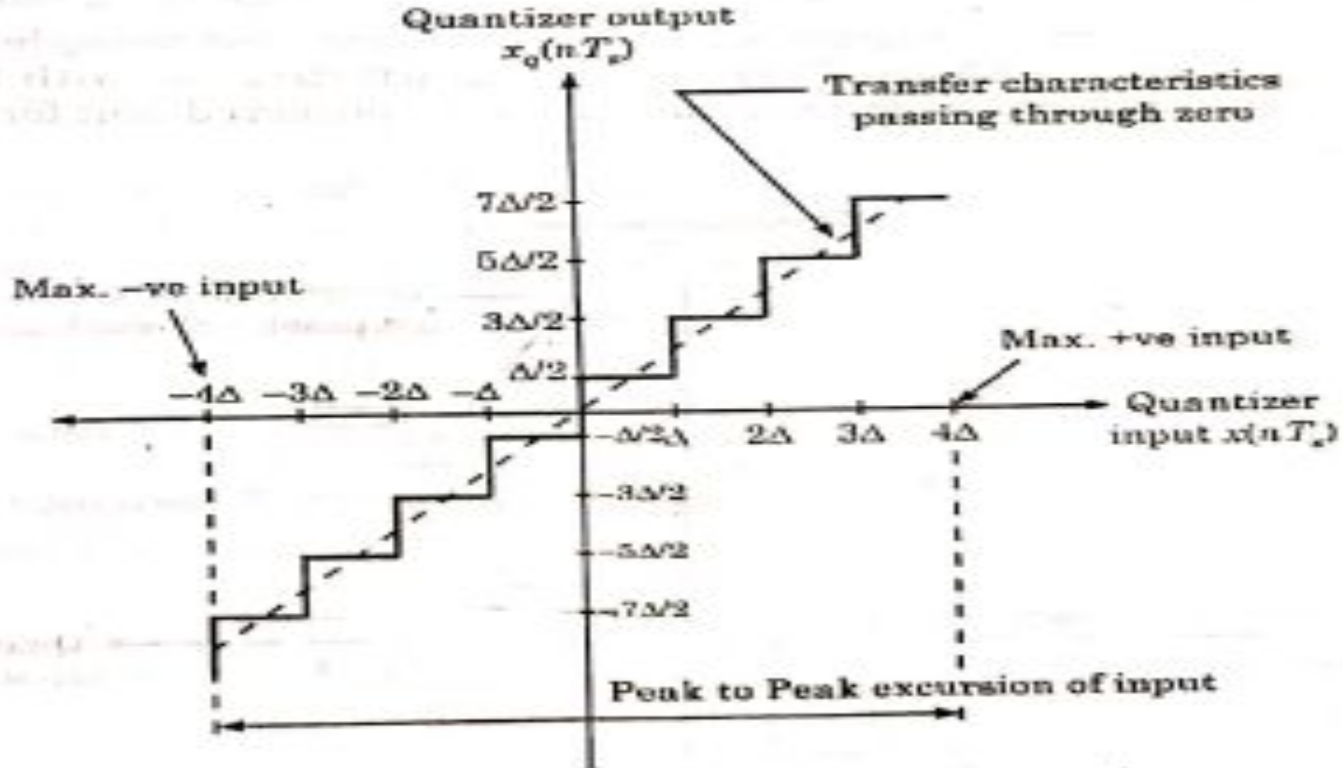
Thus, it may be observed from figure 4.9(b) that maximum quantization error would be $\pm\frac{\Delta}{2}$.

From above, we conclude that quantization error may be expressed as

$$\varepsilon = x_q(nT_s) - x(nT_s \qquad \text{...(4.1)}$$

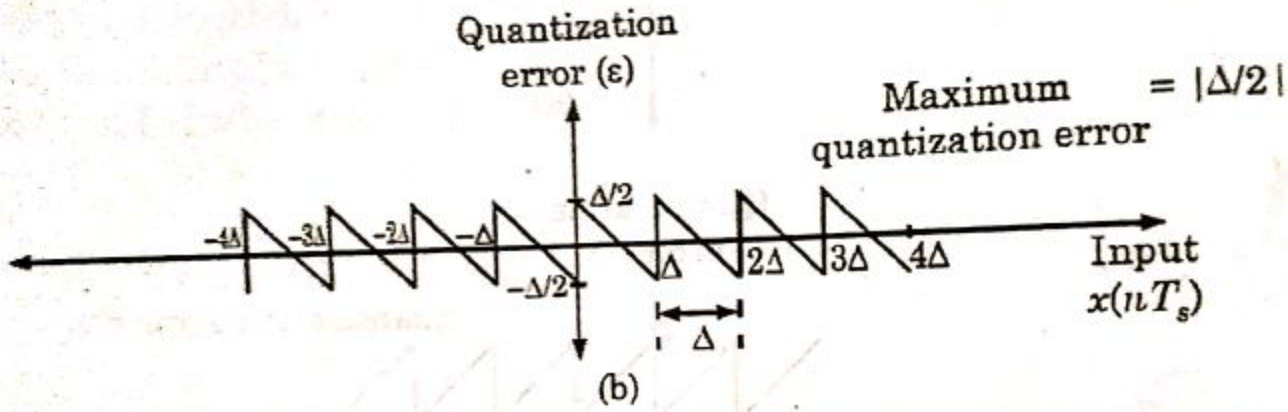
here 'ε' represents the quantization error

Now, when $x(nT_s) = 0$, quantizer will assign any one of the nearest binary levels *i.e.*, either $\Delta/2$ or $-\Delta/2$. If $\Delta/2$ is assigned, then quantization error will be,



(a)

Quantization



(b)

Fig. 4.9. *(a) Transfer characteristic of a quantizer (b) Variation of quantization error with input* $\varepsilon = x_q(nT_s) - x(nT_s) = \Delta/2 - 0 = \Delta/2$

From figure 4.9(*a*), it may also be observed that

$$\text{for} \quad \Delta < x(nT_s) < 2\Delta, \quad x_q(nT_s) = \frac{3}{2}\Delta$$

$$\text{or} \quad -\Delta < x(nT_s) < -2\Delta, \quad x_q(nT_s) = -\frac{3}{2}\Delta$$

This means that the maximum quantization error will be $\pm\,\Delta/2$.

In other words, maximum quantization error is given by

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$$

4.11. Transmission Bandwidth in a PCM System

In this section, we shall evaluate the transmission bandwidth for PCM system. Let us assume that the quantizer use 'v' number of binary digits to represent each level.

Then, the number of levels that may be represented by 'v' digits will be,

$$q = 2^{v} \quad \text{...(4.3)}$$

Here 'q' represents total number of digital levels of a q-level quantizer.

For example, if $v = 4$ bits, the total number of levels will be,

$$q = 2^4 = 16 \text{ levels}$$

Each sample is converted to 'v' binary bits. *i.e.*,

Number of bits per sample = v.

We know that,

Number of samples per second = f_s

Therefore, Number of bits per second is expressed as

(Number of bits per second) = (Number of bits per samples) × (Number of samples per second)

= v bits per sample × f_s samples per second ...(4.4)

As a matter of fact, the number of bits per second is known as **signaling rate of PCM** and is denoted by 'r' *i.e.*,

Signaling rate in PCM, $r = v f_s$ ...(4.5)

where $f_s \geq 2 f_m$

Also, since bandwidth needed for PCM transmission is given by half of the signaling rate therefore,we have

Transmission Bandwidth in PCM,

Transmission Bandwidth in PCM,

$$BW \geq \frac{1}{2} r$$

But $$r = \upsilon f_s$$

Therefore, $$BW \geq \frac{1}{2} \upsilon f_s$$

Again, since $$f_s \geq 2 f_m$$

Hence, $$BW \geq \upsilon f_m$$

This is the required expression for bandwidth of a PCM system.

4.12. Quantization Noise/Error in PCM

In this section, we shall derive an expression for quantization noise (*i.e.*, error) in a PCM system for linear quantization or uniform quantization. Because of quantization, inherent errors are introduced in the signal. This error is called **quantization error**. As defined earlier, the quantization error is given as

$$\varepsilon = x_q\,(nT_s) - x(nT_s) \qquad \text{...(4.9)}$$

Let us assume that the input $x(nT_s)$ to a linear or uniform quantizer has continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

From figure 4.9(a), it may be observed that the total excursion of input $x(nT_s)$ is mapped into 'q' levels on vertical axis. This means that when input is 4Δ, output is $\frac{7}{2}\Delta$ and when input is -4Δ, output is $-\frac{7}{2}\Delta$. Thus, $+x_{max}$ represents $\frac{7}{2}\Delta$ and $-x_{max}$ represents $-\frac{7}{2}\Delta$. Therefore, the total amplitude range becomes,

Total amplitude range $= x_{max} - (-x_{max}) = 2\,x_{max}$

Now, if this total amplitude range is divided into 'q' levels of quantizer, then the step size 'Δ' will be,

$$\text{'step size'}\ \Delta = \frac{x_{\max} - (-x_{\max})}{q} = \frac{2x_{\max}}{q} \qquad \text{...(4.10)}$$

Again, now if signal $x(t)$ is normalized to minimum and maximum values equal to 1, then we have

$$x_{max} = 1$$

$$-x_{max} = -1 \qquad \text{...(4.11)}$$

Therefore, step side would be,

$$\Delta = \frac{2}{q} \qquad \text{(for normalized signal)} \qquad \text{...(4.12)}$$

Now, if step size 'Δ' is considered as sufficiently small, then it may be assumed that the quantization error 'ε' will be an uniformly distributed random variable. We know that the maximum quantization error is given as,

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right| \qquad ...(4.13)$$

i.e.,

$$-\frac{\Delta}{2} \leq \varepsilon_{\max} \leq \frac{\Delta}{2} \qquad ...(4.14)$$

Hence, over the interval $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$ quantization error may be assumed as an uniformly distributed random variable.

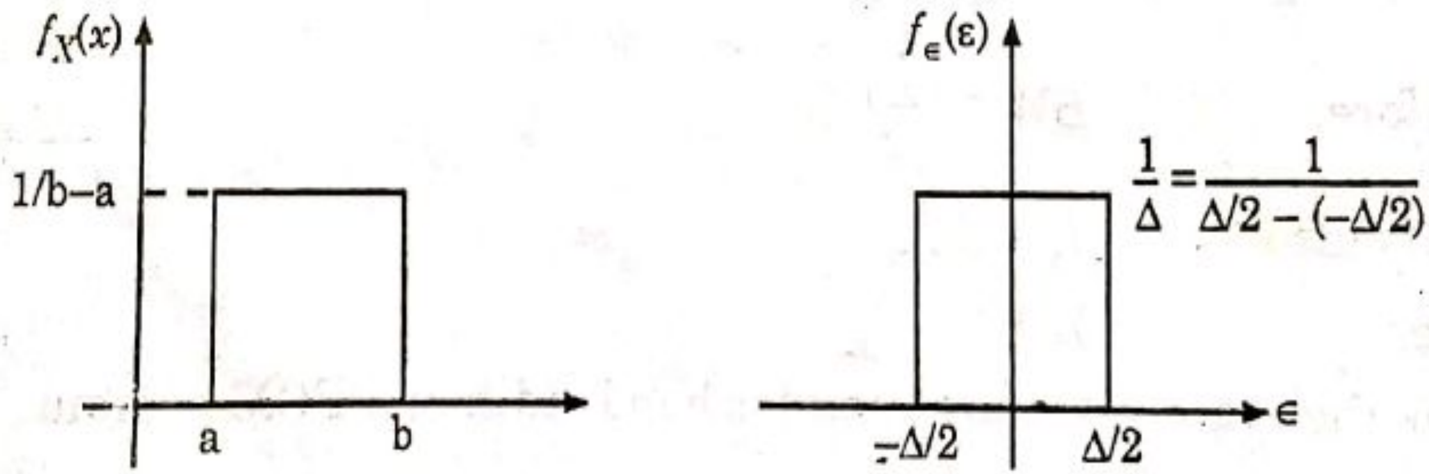


Fig. 4.11. *(a) A Uniform distribution (b) A Uniform distribution for quantization error*

Figure 4.11(a) shows an uniformly distrubuted random variable 'X' over an interval (a, b).

Recall that the PDF of uniformly distributed random variable 'X' is given as

$$f_X(x) = \begin{cases} 0 & \text{for } x \leq a \\ \dfrac{1}{b-a} & \text{for } a < x \leq b \\ 0 & \text{for } x > b \end{cases} \quad ...(4.15)$$

Thus, with the help of equation (4.15), the probability density function (PDF) for quantization error 'ε' may be defined as

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon \leq \dfrac{\Delta}{2} \\ \dfrac{1}{\Delta} & \text{for } -\dfrac{\Delta}{2} < \varepsilon \leq \dfrac{\Delta}{2} \\ 0 & \text{for } \varepsilon > \dfrac{\Delta}{2} \end{cases} \quad ...(4.16)$$

Also, from figure 4.11(b), it may be observed that quantization error 'ε' has zero average value. In other words, the mean 'm_ε' of the quantization error is zero.

Further, we know that the signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}} \quad ...(4.17)$$

If type of signal at input *i.e.*, $x(t)$ is known, then it is possible to calculate signal power.

The noise power is expressed as,

$$\text{Noise power} = \frac{{}^*V_{\text{noise}}^2}{R} \quad ...(4.18)$$

Here, V^2_{noise} is taken as the mean square value of noise voltage. Since, here noise is defined by random varible 'ε', and $PDF\ f_\varepsilon(\varepsilon)$ therefore, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \overline{\varepsilon^2} = V^2_{\text{noise}} \qquad \text{...(4.19)}$$

We know that the mean square value of a random variable 'X' is expressed as,

$$\overline{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx \qquad \text{...(4.20)}$$

Here

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

Using equation (4.16), above equation may be written as,

$$E\,[\varepsilon^2] = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \times \frac{1}{\Delta}\, d\varepsilon = \frac{1}{\Delta}\left[\frac{\varepsilon^3}{3}\right]_{-\Delta/2}^{-\Delta/2}$$

or

$$E[\varepsilon^2] = \frac{1}{\Delta}\left[\frac{(\Delta/2)^3}{3} + \frac{(\Delta/2)^3}{3}\right] = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right]$$

Simplifying, we get

or $$E[\varepsilon^2] = \frac{\Delta^2}{12} \qquad \text{...(4.22)}$$

Now, using equation (4.19), the mean square value of noise voltage would be

$$V_{noise}^2 = \text{ mean square value} = \frac{\Delta^2}{12}$$

Also, if load resistance, $R = 1$ ohm, then the noise power is normalized *i.e.*,

$$\text{Noise power (normalized)} = \frac{V_{noise}^2}{1} \qquad \text{(putting } R = 1 \text{ in equation (4.18))}$$

$$= \frac{\Delta^2/12}{1} = \frac{\Delta^2}{12}$$

Hence, finally, we write

$$\left.\begin{array}{l}\text{Normalized noise power} \\ \text{or Quantization noise power} \\ \text{or Quantization error (in terms of power)}\end{array}\right\} = \frac{\Delta^2}{12}, \text{ for linear quantization.} \qquad \text{...(4.23)}$$

4.13. Signal to Quantization Noise Ratio for Linear Quantization

We know that in a PCM system for linear quantization the signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

But, normalized noise power has been calculated as $\frac{\Delta^2}{12}$.

Therefore,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{(\Delta^2/12)} \quad \text{...(4.24)}$$

We know that the number of bits 'v' and quantization levels are related as,

$$q = 2^v \quad \text{...(4.25)}$$

Let us assume that input $x(nT_s)$ to a linear quantizer has continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

Therefore, total amplitude range

$$= x_{max} - (-x_{max}) = 2x_{max}$$

Now, the step size will be

$$\Delta = \frac{2x_{max}}{q} \quad \text{...(4.26)}$$

Here, substituting the value of q from equation (4.25) in equation (4.26), we get

$$\Delta = \frac{2x_{max}}{2^v}$$

Now substituting this value in equation (4.24) we get,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\left(\frac{2x_{max}}{2^v}\right)^2 \cdot \frac{1}{12}}$$

Let normalized signal pow_r be denoted as 'P'.

Then, $$\frac{S}{N} = \frac{P}{\frac{4x_{\max}^2}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x_{\max}^2} \cdot 2^{2v}$$

This is the required relation for signal to quantization noise ratio for linear quantization in a PCM system.

Hence, signal to quantization noise ratio:

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \cdot 2^{2v} \qquad \text{...(4.27)}$$

This expression shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

Now if we assume that inpur $x(t)$ is normalized, *i.e.*,

$$x_{max} = 1 \qquad \text{...(4.28)}$$

Then, signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P \qquad \text{...(4.29)}$$

Also, if the destination signal power 'P' is normalized, *i.e.*,

$$P \leq 1 \qquad \text{...(4.30)}$$

Then the signal to noise ratio will be given as

$$\frac{S}{N} \leq 3 \times 2^{2v} \qquad \text{...(4.31)}$$

Because $x_{max} = 1$ and $P \leq 1$, the signal to noise ratio given by equation (4.31) is said to be normalized.

Expressing the signal to noise ratio in decibels, we get

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) dB \leq 10 \log_{10} [3 \times 2^{2v}]$$

or

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6\, v) \text{ dB}$$

Thus, Signal to Quantization noise ratio for normalized values of power 'P' and amplitude of input $x(t)$, will be

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6v)\, dB \qquad \text{...(4.32)}$$

Example 4.2. A Television signal having a bandwidth of 4.2 MHz is transmitted using binary PCM system. Given that the number of quantization levels is 512. Determine:

(*i*) Code word length

(*ii*) Transmission bandwidth

(*iii*) Final bit rate

(*iv*) Output signal to quantization noise ratio.

Solution: Givn that the bandwidth is 4.2 MHz. This means that highest frequency component will have frequency of 4.2 MHz *i.e.*,

$$f_m = 4.2 \text{ MHz}$$

Also, given that Quantization levels, $q = 512$

(*i*) We know that the number of bits and quantization levels are related in binary PCM as under:

$$q = 2^v$$

i.e., $$512 = 2^v$$

or $$\log_{10} 512 = v \log_{10} 2$$

or $$v = \frac{\log_{10} 512}{\log_{10} 2}$$

Simplifying, we get, $v = 9$ bits

Hence, the code word length is 9 bits. **Ans.**

(*ii*) We know that the transmission channel bandwidth is given as,

$$BW \geq vf_m \geq 9 \times 4.2 \times 10^6 \text{ Hz} \geq 37.8 \text{ MHz} \quad \textbf{Ans.}$$

(*iii*) The final bit rate is equal to signaling rate.

Example 4.3. **The bandwidth of an input signal to the PCM is restricted to 4 kHz. The input signal varies in amplitude from – 3.8 V to + 3.8 V and has the average power of 30 mW. The required signal to noise ratio is given as 20 dB. The PCM modulator produces binary output. Assuming uniform quantization,**

(i) Find the number of bits required per sample.

(ii) Outputs of 30 such PCM coders are time multiplexed. What would be the minimum required transmission bandwidth for this multiplexed signal? *(Punjab Technical University-1999)*

Solution: The given value of signal to noise ratio is 20 dB.

This means that.

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) = 20\ dB$$

Hence,
$$\frac{S}{N} = 100$$

(i) We know that the signal to quantization noise ratio is given as,

$$\therefore \quad \frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{\max}^2}$$

Here, we are given

$$x_{max} = 3.8\ V$$

$$P = 30\ \text{mW}$$

and
$$\frac{S}{N} = 100$$

Example 4.3. The bandwidth of an input signal to the PCM is restric... kHz. The input signal varies in amplitude from – 3.8 V to + 3.8 V and ... average power of 30 mW. The required signal to noise ratio is give... dB. The PCM modulator produces binary output. Assuming u... quantization,

(i) Find the number of bits required per sample.

(ii) Outputs of 30 such PCM coders are time multiplexed. What wo... the minimum required transmission bandwidth for this multi... signal? *(Punjab Technical Universit...*

Solution: The given value of signal to noise ratio is 20 dB.

This means that.

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) = 20\ dB$$

Hence,
$$\frac{S}{N} = 100$$

(i) We know that the signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{\max}^2}$$

Here, we are given

$$x_{max} = 3.8\ V$$

$$P = 30\ \text{mW}$$

Therefore, $100 = \dfrac{3 \times 30 \times 10^{-3} \times 2^{2v}}{(3.8)^2}$

Solving, we get $v = 6.98$ bits $= 7$ bits **Ans.**

(*ii*) The maximum frequency is given as

$$f_m = 4 \text{ kHz}$$

We know that the transmission bandwidth is expressed as,

$$BW \geq v f_m$$

Since there are 30 PCM coders which are time multiplexed, the transmission andwidth must be,

$$BW \geq 30 \times v \times f_m \geq 30 \times 7 \times 4 \text{ kHz} \geq 840 \text{ kHz}$$ **Ans.**

We also know that the signaling rate is two times the transmission bandwidth, *i.e.*

Signaling rate, $r = 840 \times 2$ bits/sec $= 1680$ bits/sec. **Ans.**

Example 4.4. The information in an analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of $\pm$ 0.1% (full scale). The analog voltage waveform has a bandwidth of 100 Hz and an amplitude range of – 10 to + 10 volts.

(*i*) Find the minimum sampling rate required.

(*ii*) Find the number of bits in each PCM word.

(*iii*) Find minimum bit rate required in the PCM signal.

(*iv*) Find the minimum absolute channel bandwidth required for the transmission of the PCM signal. *(West Bengal-1999)*

Solution: Here an accuracy is given as $\pm$ 0.1%. This means that the quantization error must be $\pm$ 0.1% or the maximum quantization error must be $\pm$ 0.1%.

Thus, $$\varepsilon_{max} = \pm 0.1\% = \pm 0.001$$

We know that the maximum quantization error for an uniform quantizer is expressed as,

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$$

or $$\left|\frac{\Delta}{2}\right| = 0.001$$

Therefore,

$$\text{Step size } \Delta = 2 \times 0.001 = 0.002$$

We know that the step size, number of quantization levels and maximum value of the signal are related as

$$\Delta = \frac{2\,x_{\max}}{q} \quad ...(i)$$

Here, given $|x_{max}| = 10$ volts

Substituting, values of Δ and x_{max} in equation (*i*), we get

$$0.002 = \frac{2 \times 10}{q}$$

or $$q = \frac{20}{0.002} = 10{,}000$$

Hence, the number of levels are 10,000.

(*i*) The maximum frequency in the signal is given as 100 Hz *i.e.*,

$$f_m = 100 \text{ Hz}$$

By sampling theorem minimum sampling frequency should be,

$$f_s \geq 2 f_m \geq 2 \times 100 \geq 200 \text{ Hz} \qquad \textbf{Ans.}$$

(*ii*) We know that minimum 10,000 levels should be used to quantize the signal. If binary PCM is used, then number of bits for each samples may be calculated as under, *i.e.*

$$q = 2^{\upsilon}$$

Here, q = number of levels

υ = bits in PCM,

Thus, $$10{,}000 = 2^{\upsilon}$$

$$\log_{10} 10{,}000 = \upsilon \log_{10} 2.$$

or $$\upsilon = \frac{\log_{10} 10{,}000}{\log_{10} 2} = 13.288 = 14 \text{ bits} \qquad \textbf{Ans.}$$

(*iii*) The bit rate or signaling rate is expressed as,

$$r \geq \upsilon f_s \geq 14 \times 200 \geq 2800 \text{ bits per second.} \qquad \textbf{Ans.}$$

(*iv*) The transmission bandwidth for PCM is expressed as,

$$BW \geq \frac{1}{2} r \geq \frac{1}{2} \times 2800 \geq 1400 \text{ Hz} \qquad \textbf{Ans.}$$

Example 4.5. Twenty four voice signals are sampled uniformly and then have to be time division multiplexed. The highest frequency component for each voice signal is equal to 3.4 kHz. Now

(*i*) If the signals are pulse amplitude modulated using Nyquist rate sampling, what would be the minimum channel bandwidth required.

(*ii*) If the signals are pulse code modulated with an 8 bit encoder, what would be the sampling rate? The bit rate of system is given as 1.5×10^6 bits/sec.

Solution: (*i*) As a matter of fact, if N channels are time division multiplexed, then minimum transmission bandwidth is expressed as,

$$BW = N f_m$$

Here, f_m is the maximum frequency in the signals.

Given, $$f_m = 3.4 \text{ kHz}$$

Therefere $$BW = 24 \times 3.4 \text{ kHz} = 81.6 \text{ kHz} \qquad \textbf{Ans.}$$

(*ii*) The signaling rate of the system is given as,

$$r = 1.5 \times 10^6 \text{ bits/sec}$$

Since there are 24 channels, the bit rate of an individual channel is,

$$r\text{ (one channel)} = \frac{1.5 \times 10^6}{24} = 62500 \text{ bits/sec}$$

Further, since each sample is encoded using 8 bits, the samples per second will be,

$$\text{Sample/sec} = \frac{r\text{ (one channel) bits/sec}}{\text{bits/sample}}$$

Note that the samples per seconds is nothing but sampling frequecy.

Thus, we have $f_s = \frac{62500 \text{ bits/sec}}{8 \text{bits/sample}}$

Solving, we get, $f_s = 7812.5$ Hz or samples per second **Ans.**

Example 4.6. A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50×10^6 bits/sec.

(*i*) What is the maximum message signal bandwidth for which the system operates satisfactorily?

(*ii*) Calculate the output signal to quantization noise ratio when a full load sinusoidal modulating wave of frequency 1 MHz is applied to the input. *(U.P. Tech-Semester Exam. 2005-2006)*

Solution: (*i*) Let us assume that the message bandwidth be f_m Hz. Therefore sampling frequency should be,

$$f_s \geq 2 f_m$$

The number of bits given as $\upsilon = 7$ bits

We know that the signaling rate is given as,

$$r \geq \upsilon \cdot f_s$$

or $$r \geq 7 \times 2 f_m$$

Substituting value for r, we get

$$50 \times 10^6 \geq 14 f_m$$

or $$f_m \leq 3.57 \text{ MHz}$$ **Ans.**

Thus the maximum message bandwidth is 3.57 MHz.

(*ii*) The modulating wave is sinusoidal. For such signal, the signal to quantization noise ratio is expressed as,

$$\left(\frac{S}{N}\right) dB = 1.8 + 6\upsilon$$

Substituting the value of υ, we get

$$\left(\frac{S}{N}\right) dB = 1.8 + 6 \times 7 = 43.8 \text{ dB}$$ **Ans.**

Example 4.7. The information in an analog waveform with maximum frequency $f_m = 3$ kHz is to be transmitted over an M-level PCM system where the number of quantization levels is $M = 16$. The quantization distortion is specified not to exeed 1% of peak to peak analog signal.

(i) What would be the maximum number of bits per sample that should be used in this PCM system?

(ii) What is the minimum sampling rate and what is the resulting bit transmission rate?

Solution: (i) Since the number of quantization levels given here are $M = 16$,

$$q = M = 16$$

We know that the bits and levels in binary PCM are related as,

$$q = 2^{v}$$

Here, v = number of bits in a codeword

Thus, $$16 = 2^{v}$$

or $$v = 4 \text{ bits.} \quad \textbf{Ans.}$$

(ii) Again since $$f_m = 3 \text{ kHz}$$

By sampling theorem, we know that

$$f_s \geq 2 f_m$$

Thus, $$f_s \geq 2 \times 3 \text{ kHz} \geq 6 \text{ kHz}$$ **Ans.**

Hence, the minimum sampling rate is 6 kHz

Also, bit transmission rate or signaling rate is given as,

$$r \geq v f_s \geq 4 \times 6 \times 10^3$$

or $$r \geq 24 \times 10^3 \text{ bits per second}$$ **Ans.**

Example 4.1. Derive an expressin for signal to quantization noise ratio for a PCM system which emplys linear (*i.e.*, uniform) quantization technique. Given that input to the PCM system is a sinusoidal signal.

(U.P.S.C.I.E.S. Examination-1999)

or

A PCM system uses a uniform quantizer followed by a v bit encoder. Show that rms signal to quantization noise ratio is approximately given as $(1.8 + 6v)$ dB. *(U.P. Tech-Semester Exam. 2002-2003)*

Solution: Let us assume that the modulating signal is a sinusoidal voltage, having a peak amplitude equal to A_m. Also, let this signal cover the complete excursion of representation levels.

Then, the power of this signal will be,

$$P = \frac{V^2}{R}$$

Here, V = rms value

i.e.,
$$V = \left[A_m/\sqrt{2}\right]^2$$

Therefore, we have

$$P = \frac{A_m{}^2}{2} \cdot \frac{1}{R} \qquad \text{...(i)}$$

In case when $R = 1$, the power P is normalized *i.e*,

Normalized power $P = \frac{A_m^2}{2}$ with $R = 1$ in equation (*i*).

We know that, signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{3P}{x_{\max}^2} \times 2^{2v} \qquad \text{...(ii}$$

Here $$P = \frac{A_m^2}{2}$$

and $$x_{max} = A_m$$

Substituting, these values in the equation (*ii*), we get

$$\frac{S}{N} = \frac{3 \times \frac{A_m^2}{2} \times 2^{2v}}{A_m^2} = \frac{3}{2} \times 2^{2v} = 1.5 \times 2^{2v}$$

Expressing signal to noise power ratio in dB, we get

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) = 10 \log_{10} (1.5 \times 2^{2v})$$

or $$\left(\frac{S}{N}\right) dB = 10 \log_{10} (1.5) + 10 \log_{10} (2^{2v})$$

or $$\left(\frac{S}{N}\right) dB = 1.76 + 2v \times 10 \times 0.3$$

Therefore, we have

$$\left(\frac{S}{N}\right) dB \text{ in PCM} = \left(\frac{S}{N}\right) dB = 1.8 + 6v \text{ (for sinusoidal signal)}$$ **Ans.**

Example 4.6. A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50×10^6 bits/sec.

(*i*) What is the maximum message signal bandwidth for which the system operates satisfactorily?

(*ii*) Calculate the output signal to quantization noise ratio when a full load sinusoidal modulating wave of frequency 1 MHz is applied to the input. *(U.P. Tech-Semester Exam. 2005-2006)*

Solution: (*i*) Let us assume that the message bandwidth be f_m Hz. Therefore sampling frequency should be,

$$f_s \geq 2 f_m$$

The number of bits given as $\upsilon = 7$ bits

We know that the signaling rate is given as,

$$r \geq \upsilon \cdot f_s$$

or

$$r \geq 7 \times 2 f_m$$

Substituting value for r, we get

$$50 \times 10^6 \geq 14 f_m$$

or

$$f_m \leq 3.57 \text{ MHz} \quad \textbf{Ans.}$$

Thus the maximum message bandwidth is 3.57 MHz.

(*ii*) The modulating wave is sinusoidal. For such signal, the signal to quantization noise ratio is expressed as,

$$\left(\frac{S}{N}\right) dB = 1.8 + 6\upsilon$$

Substituting the value of υ, we get

$$\left(\frac{S}{N}\right) dB = 1.8 + 6 \times 7 = 43.8 \text{ dB} \quad \textbf{Ans.}$$

Example 4.8. A signal having bandwidth equal to 3.5 kHz is sampled, quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel of supporting a transmission rate of 50 k bits/sec. Determine the maximum signal to noise ratio that can be obtained by this system.

The input signal has peak to peak value of 4 volts and rms value of 0.2 V.

(Pune University-1998)

Solution: The maximum frequency of the signal is given as 3.5 kHz,

i.e., $$f_m = 3.5 \text{ kHz}$$

Therefore sampling frequency will be

$$f_s \geq 2 f_m \geq 2 \times 3.5 \text{ kHz} \geq 7 \text{ kHz}$$

We know that the signaling rate is given by

$$r \geq \upsilon f_s$$

Substituting values of $r = 50 \times 10^3$ bits/sec and $f_s \geq 7 \times 10^3$ Hz in above equation, we get

$$50 \times 10^3 \geq \upsilon \cdot 7 \times 10^3$$

Simplifying, we get

$$\upsilon \leq 7.142 \text{ bits} \cong 8 \text{ bits}$$

The rms value of the signal is 0.2 V. Therefore the normalized signal power will be,

$$\text{Normalized signal power } P = \frac{(0.2)^2}{1} *$$

i.e., $$P = 0.04 \; W$$

Further, the maximum signal to noise ratio is given by,

$$\frac{S}{N} = \frac{3P \cdot 2^{2\upsilon}}{x_{\max}^2}$$

Substituting the values of $P = 0.04$, $\upsilon = 8$ and $x_{max} = 2$ in above equation, we have

$$\frac{S}{N} = \frac{3 \times 0.04 \times 2^{2 \times 8}}{4} = 1966.08 \cong 33 \text{ dB} \qquad \textbf{Ans.}$$

Example 4.10. Consider an audio signal consisting of the sinusoidal term given as

$$x(t) = 3 \cos (500 \pi t)$$

(*i*) Determine the signal to quantization noise ratio when this is quantized using 10 bit PCM.

(*ii*) How many bits of quantization are needed to achieve a signal to quantization noise ratio of atleast 40 dB?

Solution: Here given that $x(t) = 3 \cos (500 \pi t)$

This is sinusoidal signal applied to the quantizer.

(*i*) Let us assume that peak value of cosine wave defined by $x(t)$ covers the complete range of quantizer.

i.e., $A_m = 3V$ covers complete range

In example 4.1, we have derived signal to noise ratio for a sinusoidal signal. It is expressed as

$$\left(\frac{S}{N}\right) dB = 1.8 + 6\upsilon$$

Since here 10 bit PCM is used *i.e.*,

$$\upsilon = 10$$

Thus,

$$\left(\frac{S}{N}\right) dB = 1.8 + 6 \times 10 = 61.8 \text{ dB} \quad \textbf{Ans.}$$

(*ii*) For sinusoidal signal, again, let us use the same relation

i.e.,

$$\left(\frac{S}{N}\right) dB = 1.8 + 6\upsilon \text{ dB}$$

To get signal to noise ratio of at least 40 dB we can write above equation as,

$$1.8 + 6\upsilon \geq 40 \text{ dB}$$

Solving this, we get $\upsilon \geq 6.36 \text{ bits} \approx 7 \text{ bits}$

Hence, at least 7 bits are required to get signal to noise ratio of 40 dB. **Ans.**

Example 4.11. A 7 bit PCM system employing uniform quantization has an overall signaling rate of 56 k bits per second. Calculate the signal to quantization noise that would result when its input is a sine wave with peak amplitude equal to 5 Volt. Find the dynamic range for the sine wave inputs in order that the signal to quantization noise ratio may be less than 30 dBs. What is the theoretical maximum frequency that this system can handle?

(Madras University-1999)

Solution: The number of bits in the PCM system are

$$v = 7 \text{ bits}$$

Assume that 5 V peak to peak voltage utilizes complete range of quantizer. Then, we can find the signal to quantization noise ratio as,

$$\left(\frac{S}{N}\right) dB = 1.8 + 6v \text{ dB} = 1.8 + 6 \times 7 = 43.8 \text{ dB}$$

We know that the signaling rate is given, as,

$$r = v\, f_s$$

Substituting $r = 56 \times 10^3$ bits/second and $v = 7$ bits in above equation, we get

$$56 \times 10^3 = 7 \cdot f_s$$

Simplifying, we get

Sampling frequency, $f_s = 8 \times 103$ Hz

Further, using sampling theorem we have,

$$f_s \geq 2\, f_m$$

Thus, maximum frequency that can be handled is given as,

$$f_m \leq \frac{f_s}{2} \leq \frac{8000}{2} \leq 4000 \text{ Hz} \leq 4 \text{ kHz} \qquad \textbf{Ans.}$$

4.17. Nonuniform Quantization

If the quantizer characteristics is nonlinear and the step size is not constant instead if it is variable, dependent on the amplitude of input signal then the quantization is known as **nonuniform quantization**. In non-uniform quantization, the step size is reduced with the reduction in signal level. For weak signals ($P << 1$), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals. The step size is thus varied according to the signal level to keep the signal to noise ratio adequately high. This is nonuniform quantization. The non-uniform quantization is practically achieved through a process called **companding**. We shall discuss companding in the next section.

4.18. Companding (i.e., Companded PCM)

As a matter of fact, companding is nonuniform quantization. It is required to be implemented to improve the signal to quantization noise ratio of weak signals.

We know that the quantization noise is given by

$$N_q = \frac{\Delta^2}{12}$$

This shows that in the uniform quantization, once the step size is fixed, the quantization noise power remains constant. However, the signal power is not constant. It is proportional to the square of signal amplitude. Hence signal power will be small for weak signals, but quantization noise power is constant. Therefore, the signal to quantization noise for the weak signals is very poor. This will affect the quality of signal. The remedy is to use companding. Companding is a term derived from two words *i.e.*, compression and expansion as under:

Companding = Compressing + Expanding

In pratice, it is difficult to implement the non-uniform quantization because it is not known in advance about the changes in the signal level. Therefore, a particular method is used. The weak signals are amplified and strong signals are attenuated before applying them to a uniform quantizer. This process is called as **compression** and the block that provides it is called as a **compressor**.

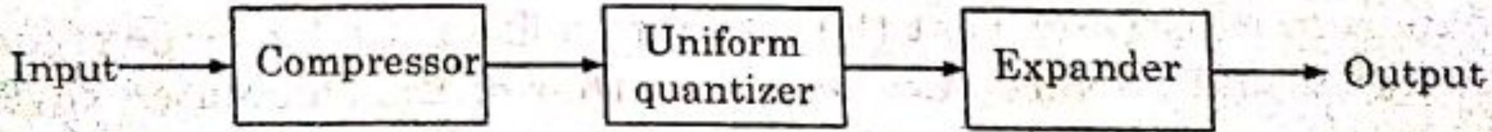


Fig. 4.14. *A companding model.*

At the receiver exactly opposite is followed which is called expansion. The circuit used for providing expansion is called as an **expander**. The compression of signal at the transmitter and expansion at the receiver is combined to be called **companding**. The process of companding has been shown in the form of a block diagram in figure 4.14.

4.18.1. Compressor Characteristic

Figure 4.15 shows the compressor characteristics. As shown in figure 4.15, the compressor provides a higher gain to the weak signals and smaller gain to the strong input signals. Thus weak signals are artificially boosted to improve the signal to quantization noise ratio. It may be noted that this compressor characteristics has been shown only for the positive input signal but we can draw it even for the negative input signals using the some principle. In fact, the compressor is included at the PCM transmitter.

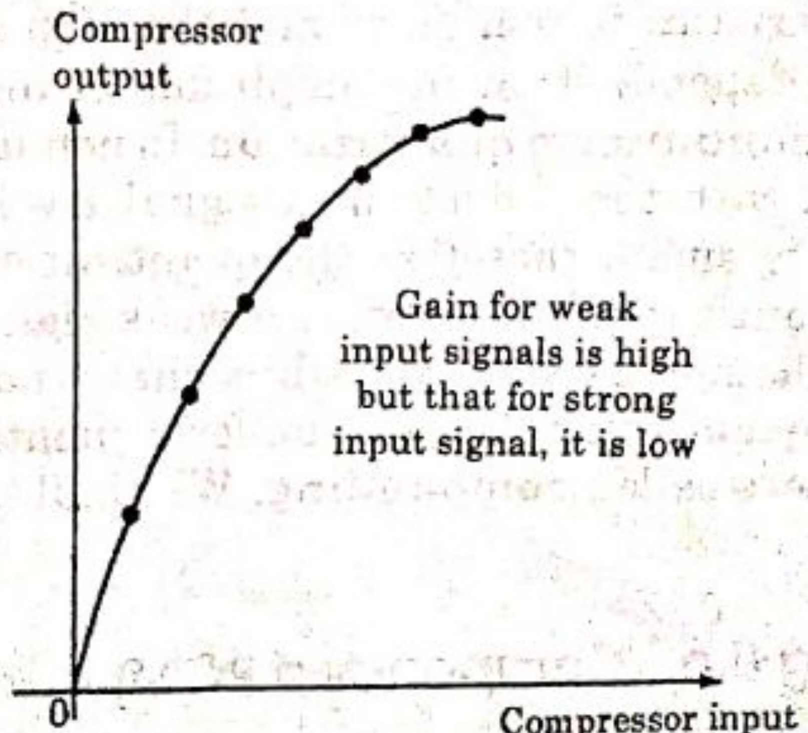
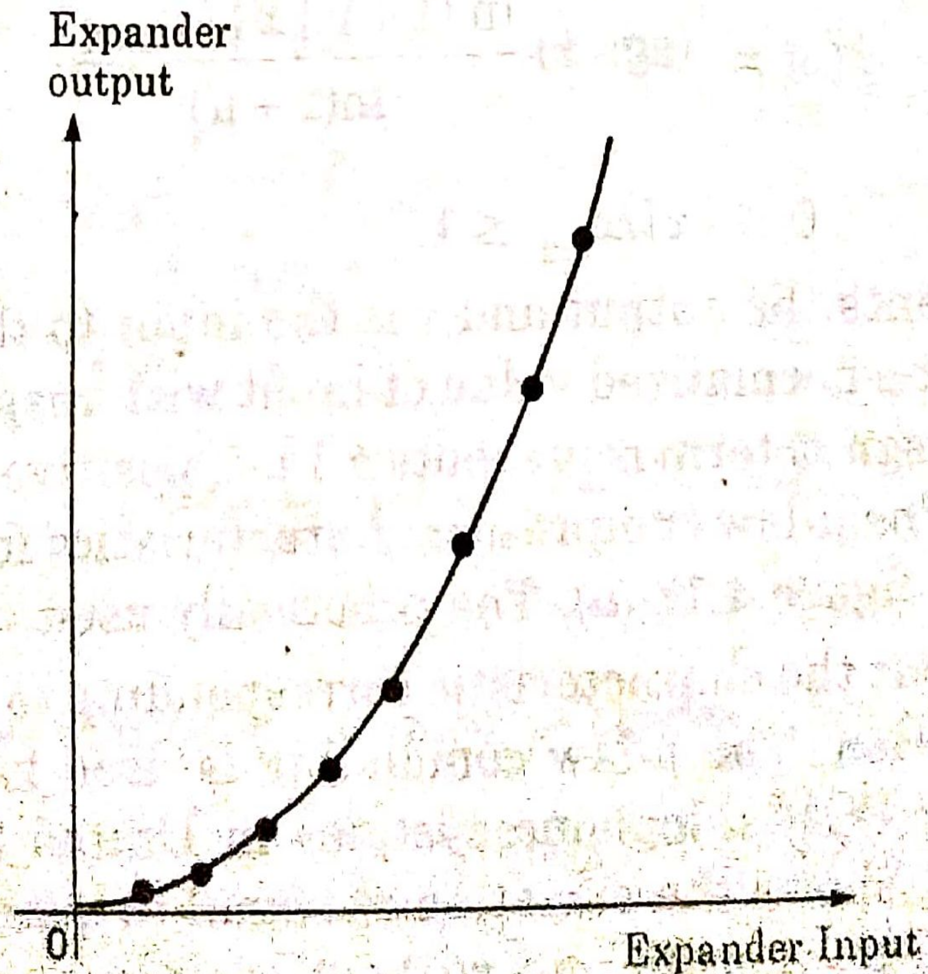


Fig. 4.15. *Compressor characteristics.*

4.18.2. Expander Characteristics

Figure 4.16 shows the expander characteristics. This characteristics is exactly the inverse of the compressor characteristics. This ensures that all the artificially boosted signals by the compressor are brought back to their original amplitudes at the receiver end.



4.19. Compander Characteristic

Figure 4.17 shows the compander characteristics which is the combination of the compressor and expander characteristics. Due to the inverse nature of compressor and expander, the overall characteristics of the compander is a straight line (dotted line in figure 4.17. This indicates that all the boosted signals are brought back to their original amplitudes.

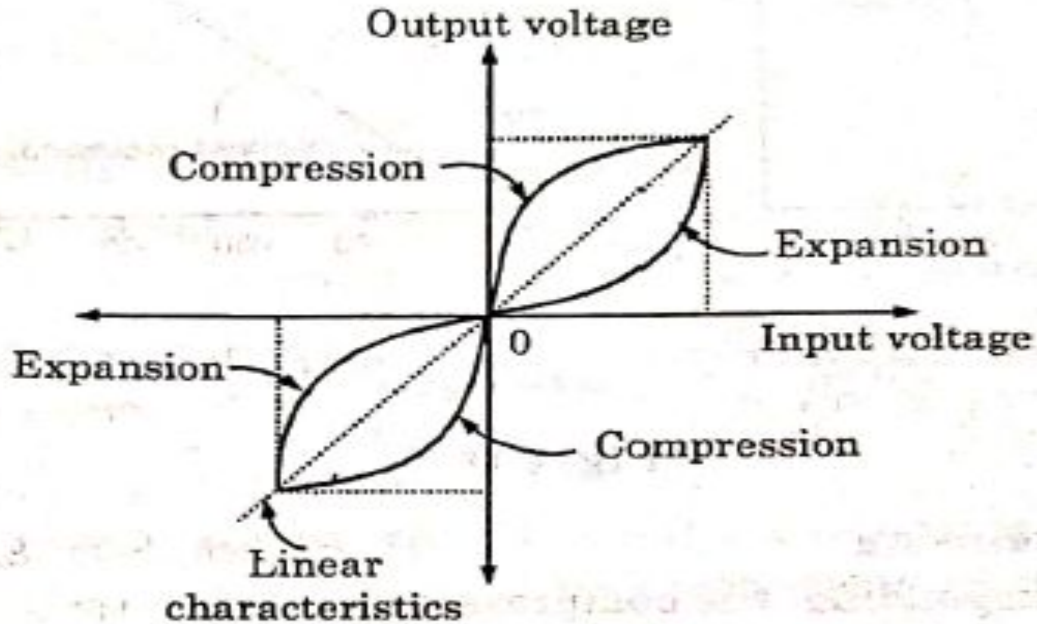


Fig. 4.17. *Companding curves for PCM system*

4.20. Different Types of Compressor Characteristics

Ideally, we need a linear compressor characteristics for small amplitudes of the input signal and a logarithmic characteristic elsewhere. In pratice, this is achieved by using following two methods:

(i) μ-law companding

(ii) A-law comapding

4.20.1. μ-law Companding

In the μ-law companding, the compressor characteristic is continuous. It is approximately linear for smaller values of input levels and logarithmic for high input levels. The μ-law compre-ssor characteristic is mathematically expressed as under:

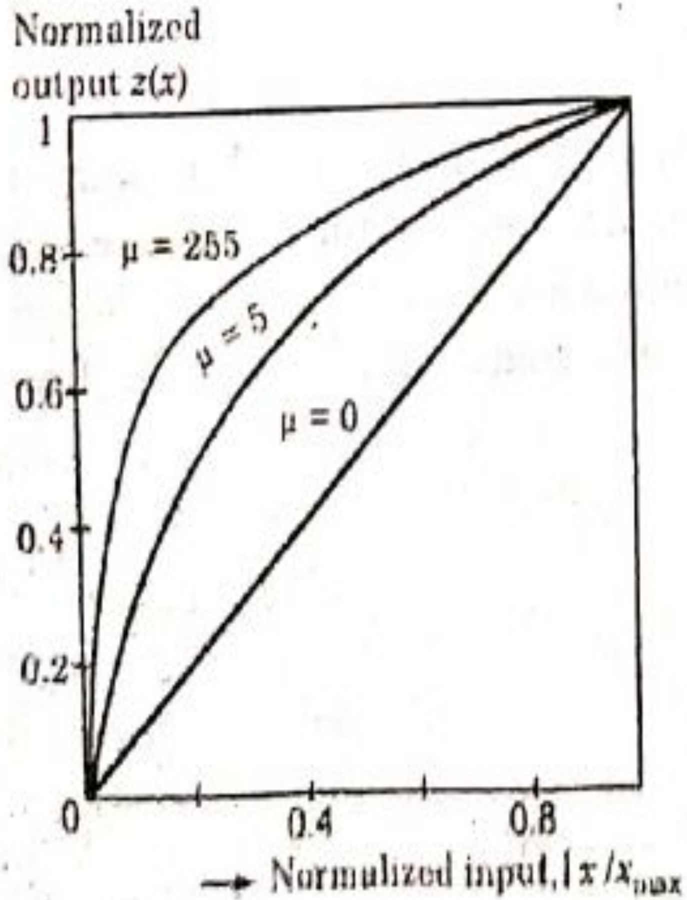
$$z(x) = (\operatorname{sgn} x)\,\frac{\ln\left(1 + \mu\,|x|/x_{\max}\right)}{\ln(1 + \mu)} \qquad \ldots(4.44)$$

$\frac{\log(1+\mu|m|)}{\log(1+\mu)}$

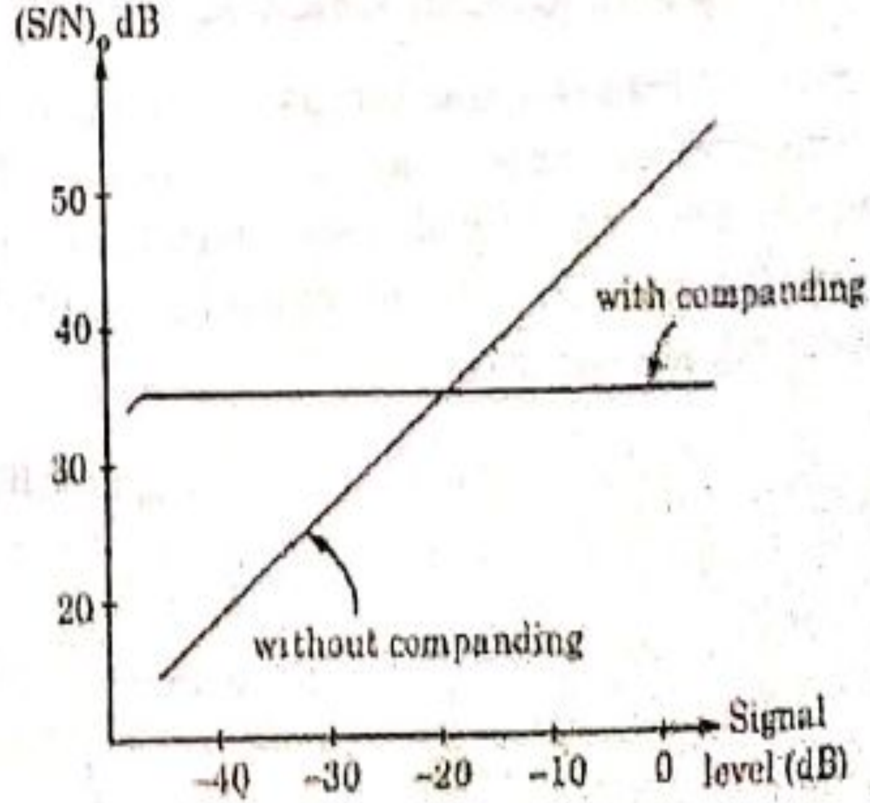
where $\qquad 0 \le |x|/x_{\max} \le 1.$

Here, $z(x)$ represents the output and x is the input to the compressor. Also, $|x|/x_{\max}$ represents the normalized value of input with respect to the maximum value $x_{\max}$. Further, $(\operatorname{sgn} x)$ term represents ± 1 *i.e.*, positive and negative values of input and output. The μ-law compressor characteristics for different values of μ have been shown in figure 4.18 (a). The practically used value of μ is 255.

It may be noted that the characteristic corresponding to $\mu = 0$ corresponds to the uniform quantization. The μ-law comanding is used for speech and music signals. It is used for PCM telephone systems in United States, Canada and Japan. Figure 4.18 (b) shows the variation of signal to quantization noise ratio with respect to signal level, with and without companding. It is obvious that SNR is almost constant at all the signal levels when companding is used.



(a) Compressor characteristic of a μ-law compressor



(b) PCM performance with μ-law companding

4.20.2. A-law Comanding *(U.P. Tech, Sem. Exam., 2004-2005)*

In the A-law companding, the compressor characteristic is piecewise, made up of a linear segment for low level inputs and a logarithmic segment for high level inputs. Figure 4.19 shows the A-law compressor characteristics for different values of A. Corresponding to $A = 1$, we observe that the characteristic is linear which corresponds to a uniform quantization. The practically used value of A is 87.56. The A-law companding is used for PCM telephone systems in Europe. The linear segment of the characteristics is for low level inputs whereas the logarithmic segments is for high level input. It is mathematically expressed as under:

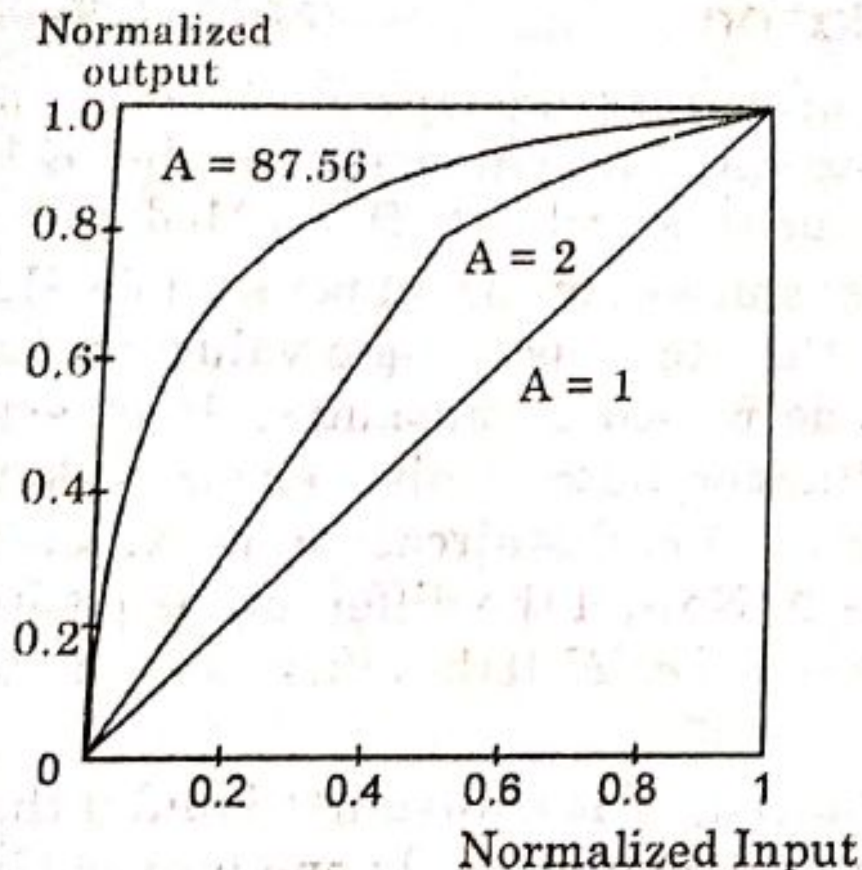


Fig. 4.19. *Compressor characteristic of A-law compressor.*

$$\frac{z(x)}{x_{\max}} = \begin{cases} \dfrac{A\,|x|/x_{\max}}{1+\log_e A} & \text{for } 0 \le \dfrac{|x|}{x_{\max}} \le 1 \\[2ex] \dfrac{1+\log_e\left[A\,|x|/x_{\max}\right]}{1+\log_e A} & \text{for } \dfrac{1}{A} \le \dfrac{|x|}{x_{\max}} \le 1 \end{cases} \qquad \text{...(4.44}a\text{)}$$

4.21. Applications of PCM

Some of the applications of PCM are as under:

(i) In telephony (with the advent of fibre optic cables).

(ii) In the space communication, space craft transmits signals to earth. Here, the transmitted power is very low (10 or 15 W) and the distances are huge (a few million km). Still due to the high noise immunity, only PCM systems can be used in such applications.

4.22. Advantages of PCM

Following are the advantages of a PCM system:

(i) Very high noise immunity.

(ii) Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers. The repeaters actually regenerate the received PCM signal. This is not possible in analog systems. Repeaters further reduce the effect of noise.

(iii) It is possible to store the PCM signal due to its digital nature.

(iv) It is possible to use various coding techniques so that only the desired person can decode the received signal.

4.23. Disadvantages of PCM

A PCM system has few drawbacks as under:

(i) The encoding, decoding and quantizing circuitry of PCM is complex.

(ii) PCM requires a large bandwidth as compared to the other systems.

4.24. Delta Modulation

(U.P. Tech-Semester Exam. 2002-200

We have observed in PCM that it transmits all the bits which are used to code sample. Hence, signaling rate and transmission channel bandwidth are qui large in PCM. To overcome this problem, Delta Modulation is used.

Delta modulation transmits only one bit per sample. Here, the present samp value is compared with the previous sample value and this result whether th amplitude is increased or decreased is transmitted. Input signal $x(t)$ is approximate to step signal by the delta modulator. This step size is kept fixed. The differen between the input signal $x(t)$ and staircase approximated signal is confined two levels, *i.e.*, $+\Delta$ and $-\Delta$. Now, if the difference is positive, then approximate signal is increased by one step, *i.e.*, 'Δ'. If the difference is negative, then approximat signl is reduced by 'Δ'.

When the step is reduced, '0' is transmitted and if the step is increased, '1' transmitted. Hence, for each sample, only one binary bit is transmitted. Figu 4.20 shows the analog signal $x(t)$ and its staircase approximated signal by th delta modulator.

Thus, The principle of delta modulation can be explained with the help few equations as under:

The error between the sampled value of $x(t)$ and last approximated sample given as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad ...(4.4$$

where $e(nT_s)$ = error at present sample

$x(nT_s)$ = sampled signal of $x(t)$

$\hat{x}(nT_s)$ = last sample approximation of the staircase wavefor

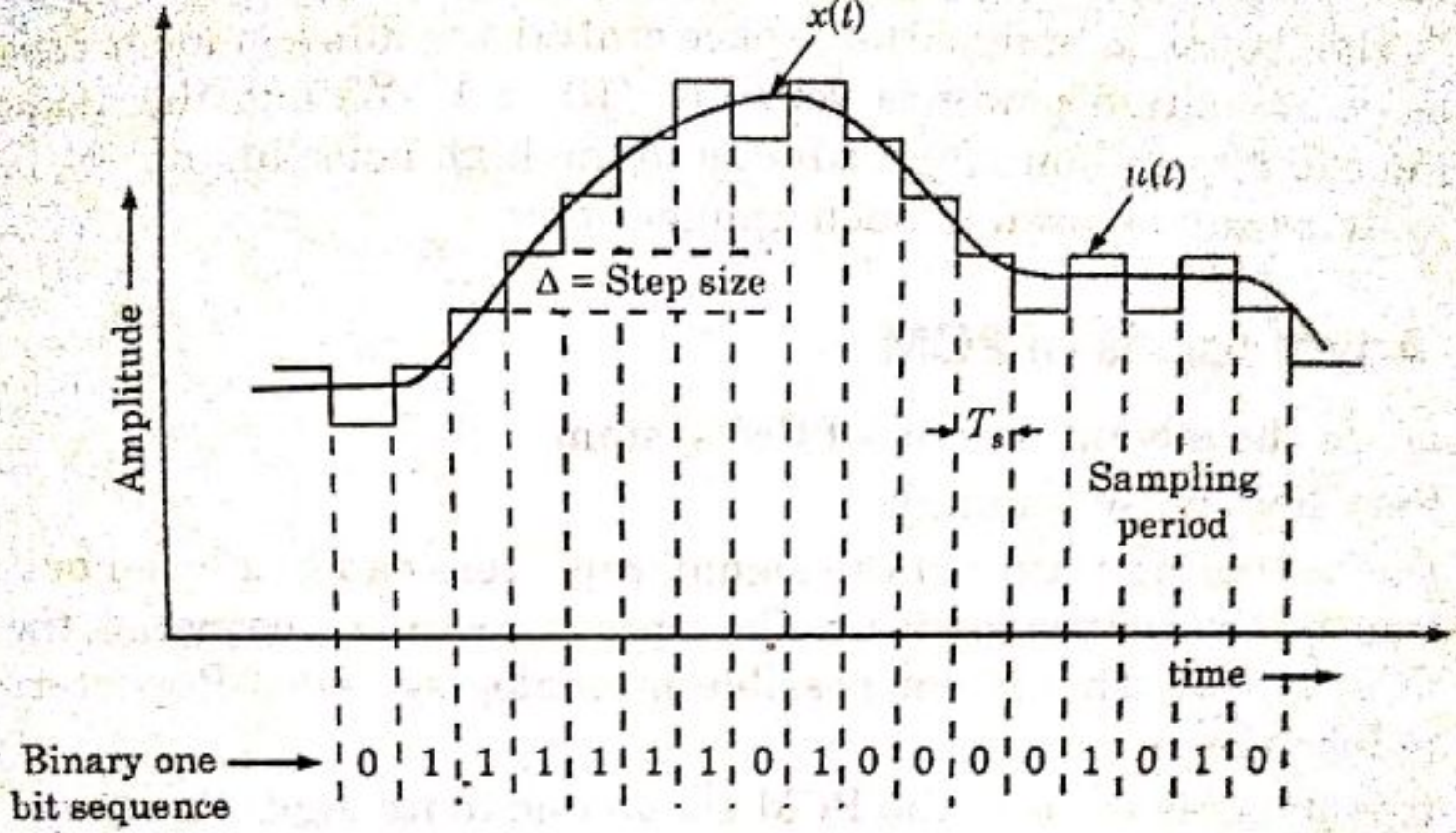


Fig 4.20. *Delta modulation waveform.*

If we assume $u(nT_s)$ as the present sample approximation of staircase output, then,

$$u[(n-1)T_s] = \hat{x}(nT_s) \quad \text{...(4.46)}$$

= last sample approximation of staircase waveform

Let us define a quantity $b(nT_s)$ in such a way that,

$$b(nT_s) = \Delta\, sgn\, [e(nT_s)] \quad \text{...(4.47)}$$

This means that depending on the sign of error $e(nT_s)$, the sign of step size Δ is decided. In other words, we can write

$$b(nT_s) = \begin{cases} +\Delta & \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ -D & \text{if } x(nT_s) < \hat{x}(nT_s) \end{cases} \quad ...(4.48)$$

Also, If $b(nT_s) = + \Delta$ then a binary '1' is transmitted

and if $b(nT_s) = - \Delta$ then a binary '0' is transmitted.

Here, T_s = Sampling interval.

Figure 4.21(a) shows the transmitter (*i.e.*, generation of Delta Modulated signal).

The summer in the accumulator adds quantizer output ($\pm \Delta$) with the previous sample approximation. This gives present sample approximation. *i.e.*,

$$u(nT_s) = u(nT_s - T_s) + [\pm \Delta]$$

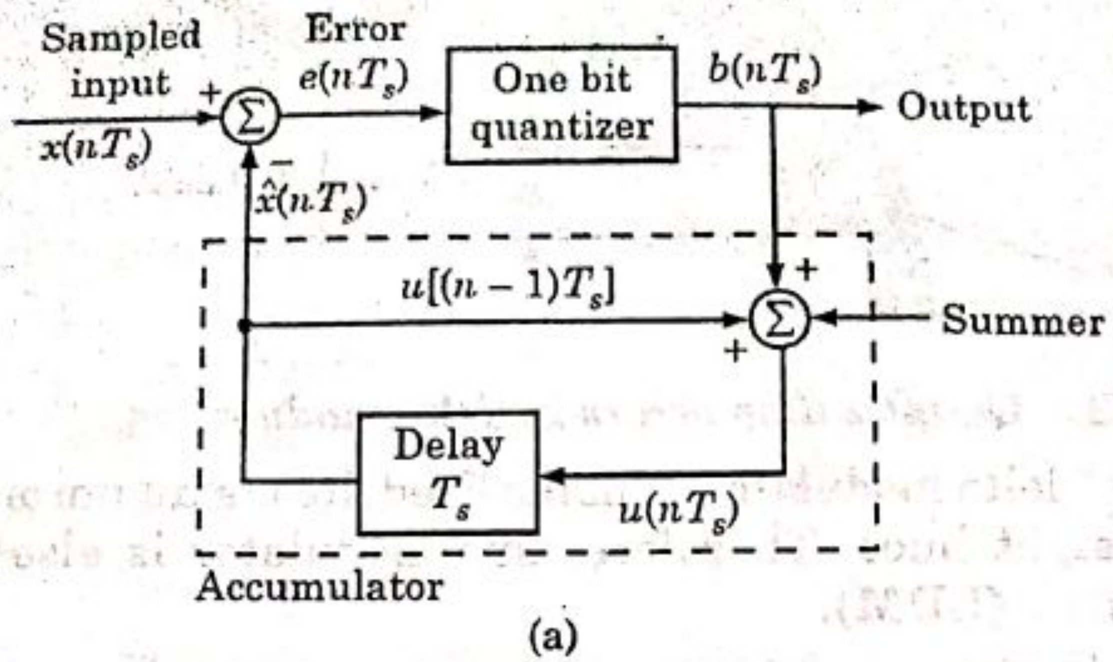
or

$$u(nT_s) = u[(n-1)\, T_s] + b(nT_s) \quad ...(4.49)$$

The previous sample approximation $u[(n-1)\, T_s]$ is restored by delaying one sample period T_s. The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.

Thus, depending on the sign of $e(nT_s)$, one bit quantizer generates an output of $+ \Delta$ or $- \Delta$. If the step size is $+ \Delta$, then binary '1' is transmitted and if it is $- \Delta$, then binary '0' is transmitted.

At the receiver end, shown in figure 4.21(b), the accumulator and low-pass filter (LPF) are used. The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s. It is then added to the input signal. If input is binary '1' then it adds $+ \Delta$ step to the previous output (which is delayed). If input is binary '0' then one step 'Δ' is subtracted from the delayed signal. Also, the low-pass filter has the cutoff frequency equal to highest frequency in $x(t)$. This low-pass filter smoothens the staircase signal to reconstruct orignal message signal $x(t)$.



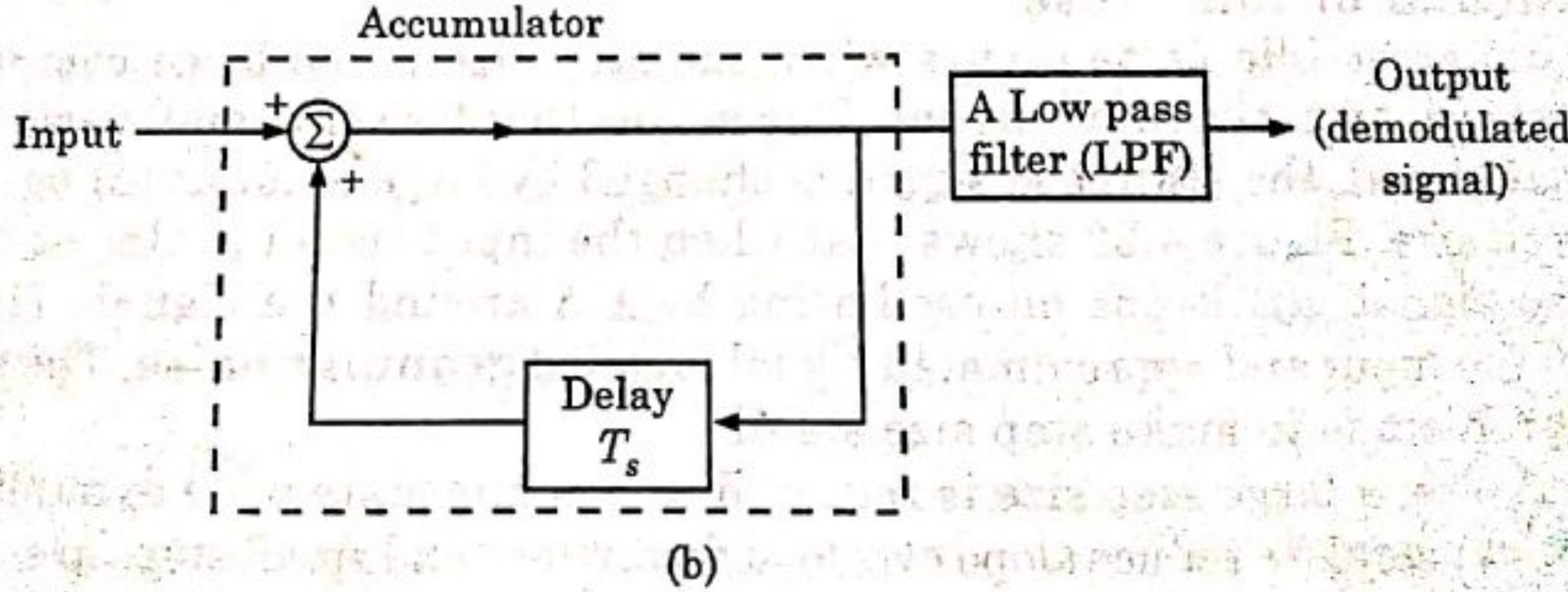


Fig. 4.21. *(a) A Delta modulation transmitter (b) A Delta modulation receiver*

4.24.1. Advantages of Delta Modulation

The delta modulation has certain advantages over PCM as under:

1. Since, the delta modulation transmits only one bit for one sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta modulation compared to PCM.
2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulation.

4.24.2. Disadvantges of Delta Modulation

(U.P. Tech., Sem. Examination, 2003-04, 2005-06)

The delta modulation has two major drawbacks as under:

(*i*) Slope overload distortion,

(*ii*) Granular or idle noise

Now, let us discuss these two drawbacks in detail.

(*i*) Slope Overload Distortion

This distortion arises because of large dynamic range of the input signal.

As can be observed from figure 4.22, the rate of rise of input signal $x(t)$ is so high that the staircase signal cannot approximate it, the step size 'Δ' becomes too small for staircase signal $u(t)$ to follow the step segment of $x(t)$. Hence, there is a large error between the staircase approximated signal and the original input signal $x(t)$. This error or noise is known as **slope overload distortion.** To reduce this error, the step size must be increased when slope of signl $x(t)$ is high.

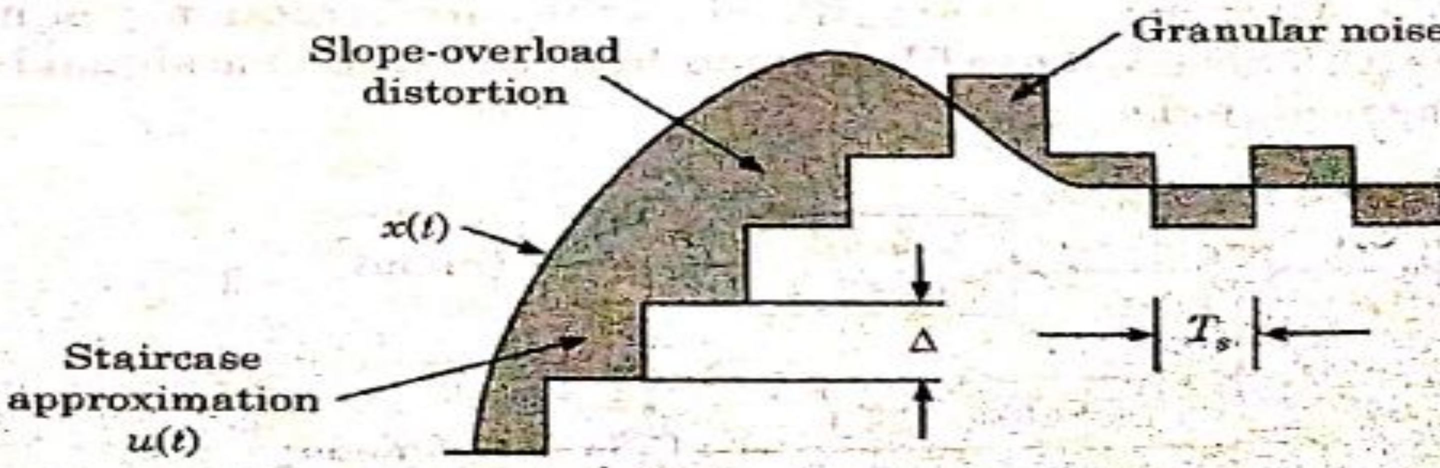


Fig. 4.22. *Quantization errors in delta modulation.*

Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines. Therefore, this modulator is also known as **Linear Delta Modulator (LDM).**

(*ii*) Granular or Idle Noise

Granular or Idle noise occurs when the step size is too large compared to small variations in the input signal. This means that for very small variations in the input signal, the staircase signal is changed by large amount (Δ) because of large step size. Figure 4.22 shows that when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\ \Delta$ around the signal. The error between the input and approximated signal is called **granular noise.** The solution to this problem is to make step size small.

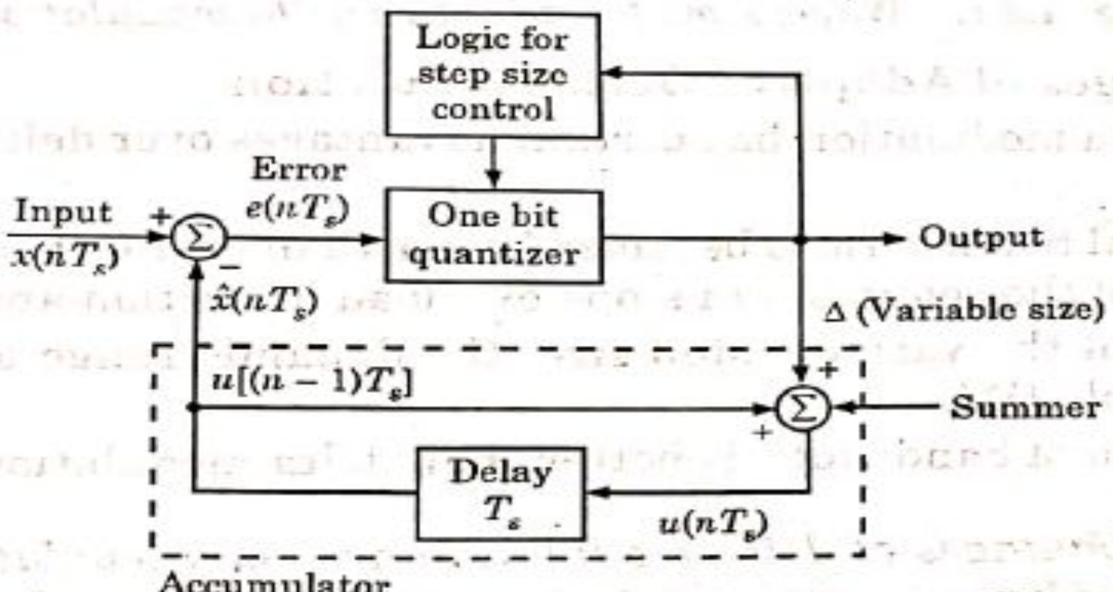
Therefore, a large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps are required to reduce granular noise. Infact, Adaptive delta modulation is the modification to overcome these errors.

4.25. Adaptive Delta Modulation

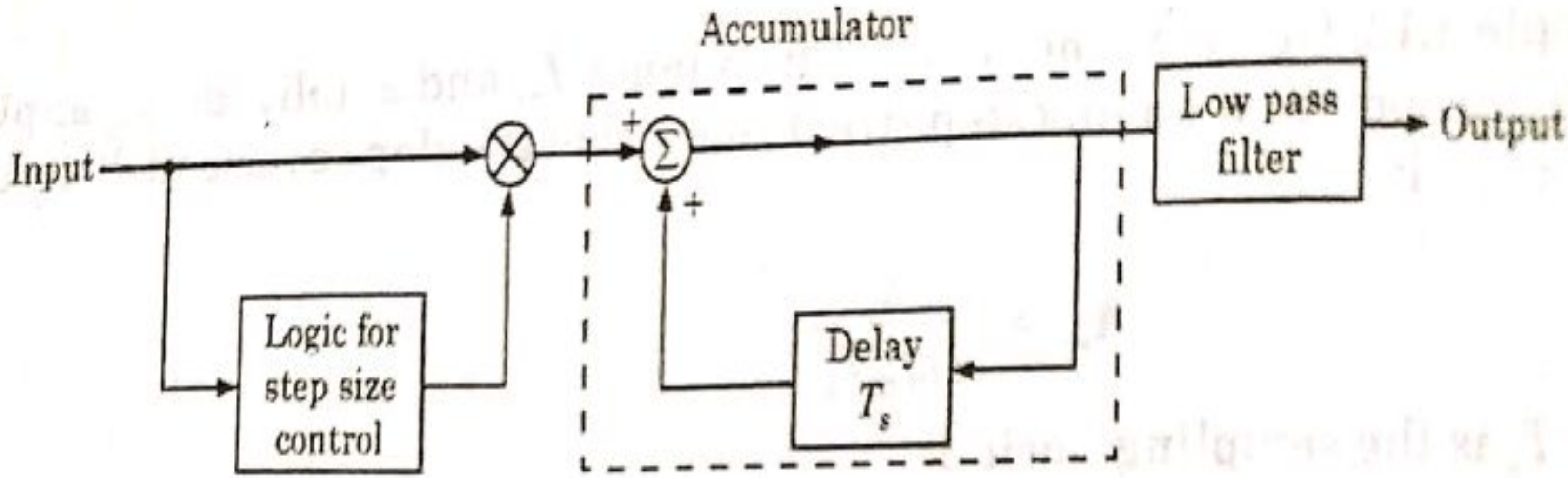
To overcome the quantization errors due to slope overload and granular noise, the step size (Δ) is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal $x(t)$, the step size is increased. Also, if the input is varying slowly, the step size is reduced. Then, this method is known as *Adaptive Delta Modulation (ADM).*

The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.

Figure 4.23(a) shows the transmitter and 4.23(b) shows receiver of adaptive delta modulator. The logic for step size control is added in the diagram. The step size increases or decreases according to a specified rule depending on one bit quantizer output. As an example, if one bit quantizer output is high ($i.e.$ 1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step. Figure 4.24 shows the staircase waveforms of adaptive delta modultor and the sequence of bits to be transmitted.



(a)



(b)

Fig. 4.23. *Adaptive Delta Modulator (a) Transmitter (b) Receiver*

In the receiver of adaptive delta modulator shown in figure 4.23(*b*), there are two portions. The first portion produces the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size. It is then applied to an accumulator which builds up staircase waveform. The low-pass filter then smoothens out the staircase waveform to reconstruct the original signal.

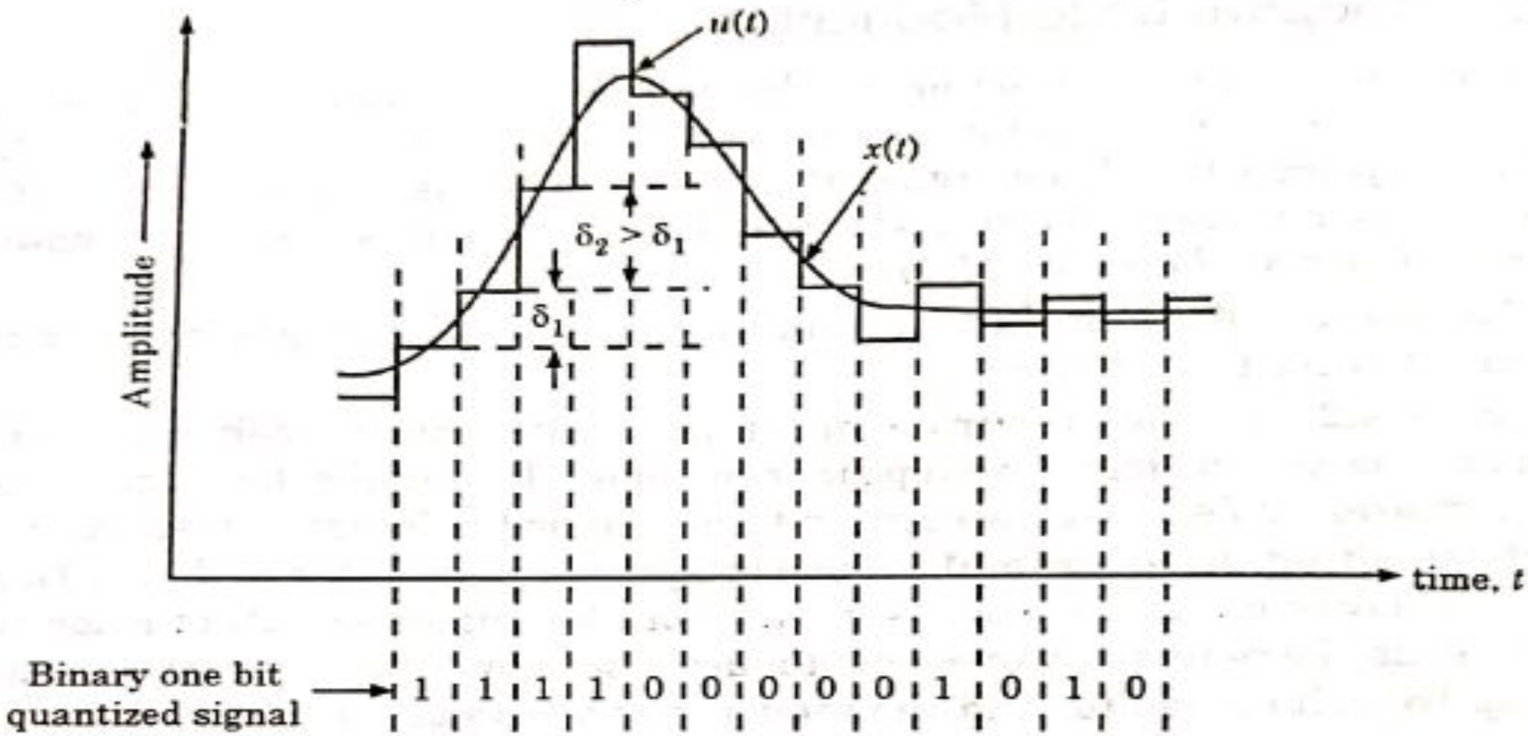


Fig. 4.24. *Waveforms for adaptive delta modulation*

4.25.1. Advantages of Adaptive Delta Modulation

Adaptive delta modulation has certain advantages over delta modulation as under:

(*i*) The signal to noise ratio becomes better than ordinary delta modulation because of the reduction in slope overload distortion and idle noise.

(*ii*) Because of the variable step size, the dynamic range of ADM is wider than simple DM.

(*iii*) Utilization of bandwidth is better than delta modulation.

Note: *Other advantages of delta modulation are, only one bit per sample is required and simplicity of implementation of transmitter and receiver.*

4.26. Differential Pulse Code Modulation (DPCM)

(U.P. Tech., Sem. Exam., 2005-2006)

It may be observed that the samples of a signal are highly correlated with each other. This is due to the fact that any signal does not change fast. This means that its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with a little difference. When these samples are encoded by a standard PCM system, the resulting encoded signal contains some redundant information. Figure 4.25 illustrates this **redundant information.**

Figure 4.25 shows a continuous time signal $x(t)$ by dotted line. This signal is sampled by flat top sampling at intervals T_s, $2T_s$, $3T_s$ ... nT_s. The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3 bit (7 levels) PCM. The sample is quantized to the nearest digital level as shown by small circles in the figure 4.25. The encoded binary value of each sample is written on the top of the samples. We can observe from figure 4.25 that the samples taken at $4T_s$, $5T_s$ and $6T_s$ are encoded to same value of (110). This information can be carried only by one sample. But three samples are carrying the same information means that it is redundant. Consider another example of samples taken at $9T_s$ and $10\ T_s$. The difference between these samples s only due to last bit and first two bits are redundant, since they do not change.

If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is known as **Differential Pulse Code Modulation (DPCM).**

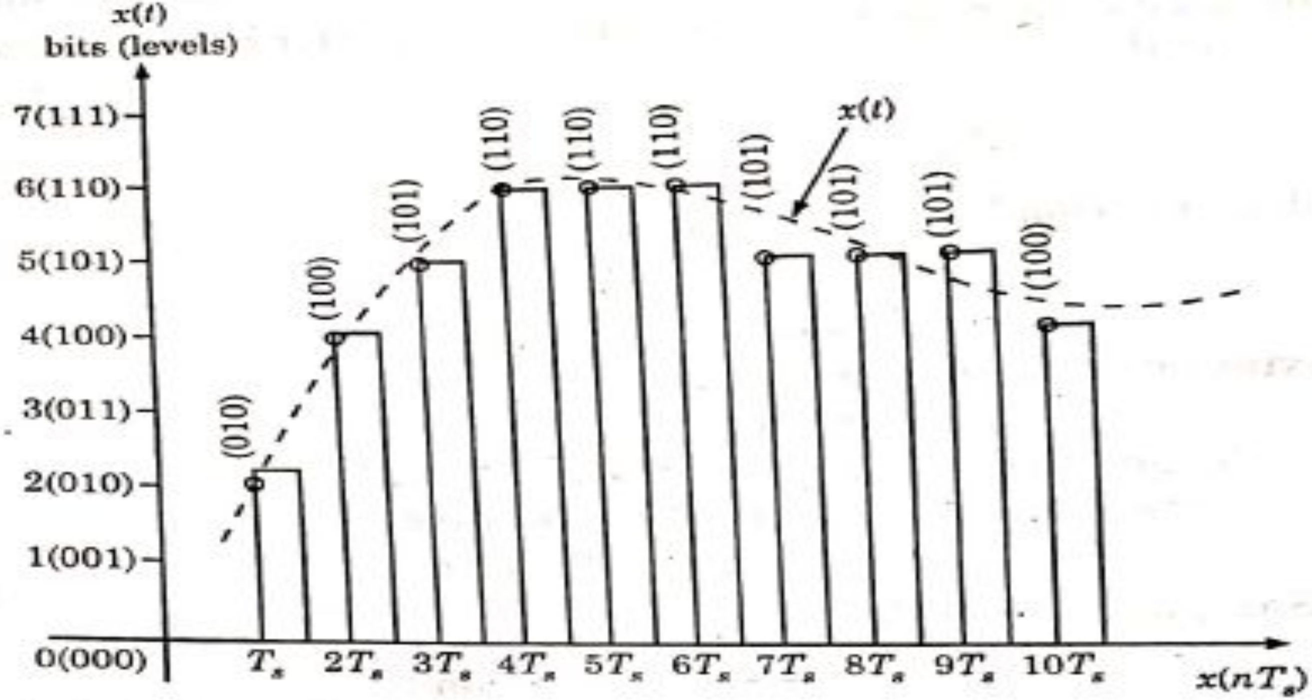


Fig 4.25. *Illustration of redundant information in PCM.*

In fact the differential pulse code modulation works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample vlaue. Figure 4.26 shows the transmitter of Differential Pulse Code Modulation (DPCM) system. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicated sample value $\hat{x}(nT_s)$. This is known as Prediction error and it is denoted by $e(nT_s)$. It can be defined as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \qquad ...(4.50)$$

Thus, error is the difference between unquantized input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer output signal gap $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$. This makes the prediction more and more close to the actual sampled signal. We can observe that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

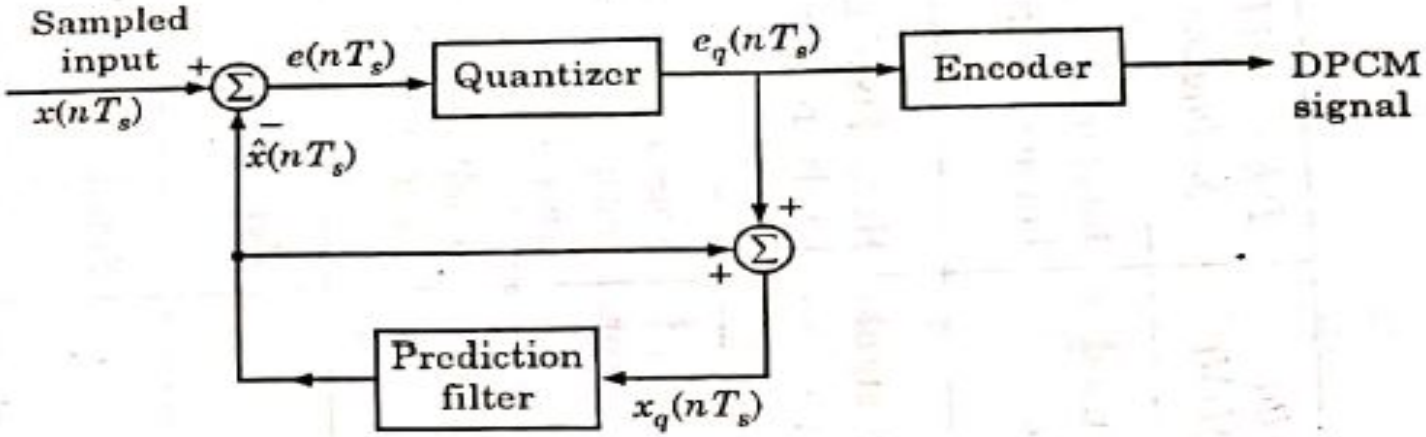


Fig. 4.26. *A Differential pulse code modulation transmitter.*

The quantizer output can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \quad \text{...(4.51)}$$

Here $q(nT_s)$ is the quantization error. As shown in figure 4.26, the prediction filter input $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer output *i.e.*,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \quad \text{...(4.52)}$$

Substituting the value of $e_q(nT_s)$ from equation (4.51) in the above equation, we get,

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad \text{...(4.53)}$$

Equation (4.50) is written as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$\therefore \quad e(nT_s) + \hat{x}(nT_s) = x(nT_s) \quad \text{...(4.54)}$$

Therefore, the value of $e(nT_s) + \hat{x}(nT_s)$ from above equation into equation (4.53), we get,

$$x_q(nT_s) = x(nT_s) + q(nT_s) \quad \text{...(4.55)}$$

Hence, the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value and quantization error $q(nT_s)$. The quantization error can be positive or negative. Thus equation (4.55) does not depend on the prediction filter characteristics.

4.26.1. Reconstruction of DPCM Signal

Figure 4.27 shows the block diagram of DPCM receiver.

The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error sigals are summed up to give the quantized version of the orginal signal. Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

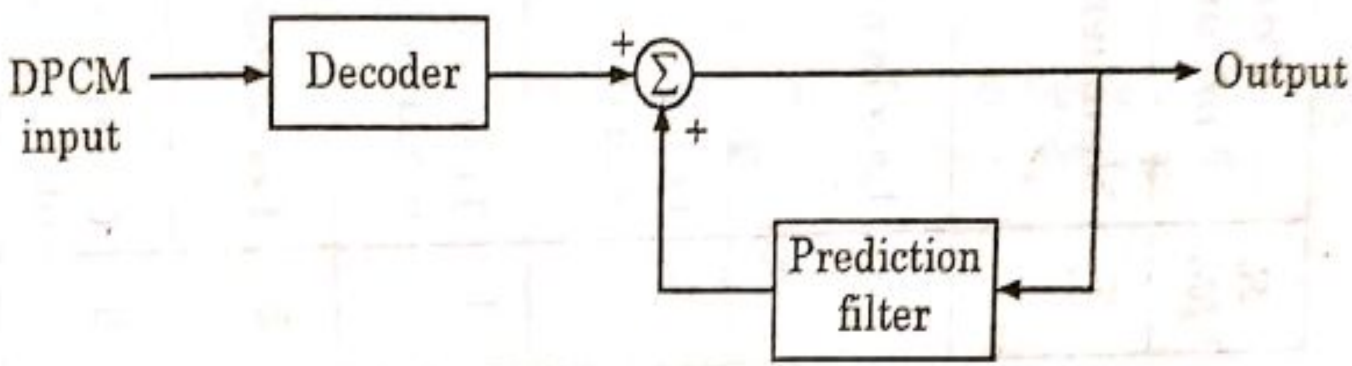


Fig. 4.27. *DPCM receiver*

Example 4.14. A binary channel with bit rate $r_b = 36000$ bits per second (b/s) is available for PCM voice transmission. Evaluate the appropriate values of the sampling rate f_s, the quantizing level q, and the number of binary digits υ. Assume $f_m = 3.2$ kHz.

Solution: Here, we require that

$$f_s \geq f_m = 6400$$

and

$$\upsilon f_s \leq r = 36000$$

Therefore, we have, $\upsilon \leq \dfrac{r}{f_s} \leq \dfrac{36000}{6400} = 5.6$

Hence, we have $\upsilon \cong 5$,

and also,

$$q = 2^\upsilon = 2^5 = 32,$$

and

$$f_s = \frac{36000}{5} = 7200 \text{ Hz} = 7.2 \text{ kHz} \quad \textbf{Ans.}$$

Example 4.15. An analog is quantized and is to be transmitted by using a PCM system. If each sample at the receiving end of the system must be known to witin ± 0.5 per cent of the peak-to-peak full-scale value, how many digits must sample contain?

Solution: Let us consider that $2m_p$ is the peak-to-peak value of the signal.

The peak error is then $0.005(2m_p) = 0.01\, m_p$, and the peak-to-peak error will be $(2(0.01\, m_p) = 0.02\, m_p$(the maximum step size Δ).

Thus, the required number of quantizing levels will be

$$q = \frac{2m_p}{\delta} = \frac{2m_p}{0.02m_p} = 100 \leq 2^\upsilon$$

$\Delta = 0.1\, m_p$.

$\Delta = \dfrac{2m_p}{q}$

Hence, the number of binary digits needed for each sample is $\upsilon = 7$. **Ans.**

Example 4.16. An analog signal is sampled at the Nyquist rate f_s and quantized into L levels. Find the time duration τ of 1 bit of the binary-encoded signal.

Solution: Let υ be number of bits per sample. Then we have

$$\upsilon = [\log_2 q]$$

where $[\log_2 q]$ indicates the next higher integer to be taken if $\log_2 q$ is not integer value *i.e,* υf_s binary pulses must be transmitted per second.

Thus, we have
$$\tau = \frac{1}{\upsilon f_s} = \frac{T_s}{\upsilon} = \frac{T}{[\log_2 q]}$$

where T_s is the Nyquist interval, **Ans**

Table 4.1. Comparison between PCM, Delta Modulation, Adaptive Delta Modulation and Differential Pulse Code Modulation

S. No.	*Parameter of comparison*	*Pulse Code Modulation (PCM)*	*Delta modulation (DM)*	*Adaptive Delta Modulation (ADM)*	*Differential Pulse Code Modulation (DPCM)*
1.	Number of bits.	It can use 4, 8 or 16 bits per sample.	It uses only one bit for one sample.	Only one bit is used to encode one sample.	Bits can be more than one but are less than PCM.
2.	Levels and step size	The number of levels depend on number of bits. Level size is kept fixed.	Step size is kept fixed and cannot be varied.	According to the signal variation, step size varies (*i.e.* Adapted).	Here, Fixed number of levels are used.
3.	Quantization error and distortion	Quantization error depends on number of levels used.	Slope overload distor-tion and granular noise are present.	Quantization noise is present but other errors are absent.	Slope overload distor-tion and quantization noise is present.
4.	Transmission bandwidth	Highest bandwidth is required since number of bits are high	Lowest bandwidth is required.	Lowest bandwidth is required.	Bandwidth required is lower than PCM.
5.	Feedback	There is no feedback in transmitter or receiver.	Feedback exists in transmitter.	Feedback exists.	Here, Feedback exists.
6.	Complexity of implementation	System complex.	Simple.	Simple.	Simple

Example 4.17. The output signal-to-quantizing-noise ratio $(SNR)_0$ in a PCM system is defined as the ratio of average signal power to average quantizing noise power. For a full-scale sinusoidal modulating signal with amplitude A, prove that *(U.P. Tech-Semester Exam. 2002-2003)*

$$(SNR)_o = \left(\frac{S}{N_q}\right)_o = \frac{3}{2}q^2 \quad \text{...(i)}$$

or

$$\left(\frac{S}{N_q}\right)_{odB} = 10log\left(\frac{S}{N_q}\right)_o = 1.76 + 20 \log q \quad \text{...(ii)}$$

where q is the number of quantizing levels.

Solution: Since, here peak-to-peak excursion of the quatizer input is $2A$. Therefore, the quantizer step size will be

$$\Delta = \frac{2A}{q}$$

$\Delta = \frac{2x_{max}}{q}$

Then, the average quantizing noise power is

$N = \frac{\Delta^2}{12} = \frac{(2x_{max})^2}{12} = \frac{4x_{max}^2}{12}$

then,

$$N_q = \langle q_e^2 \rangle = \frac{\delta^2}{12} = \frac{A^2}{3q^2}$$

$\frac{A^2}{3q^2}$

The output signal-to-quantizing-noise ratio of a PCM system for a full scale test tone is, therefore,

$$(SNR)_0 = \left(\frac{S}{N_q}\right)_o = \frac{A^2/2}{A^2/(3q^2)} = \frac{3}{2}q^2$$

$S = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$

Expressing this in decibels, we have

$$\left(\frac{S}{N_q}\right)_{0dB} = 10\log\left(\frac{S}{N_q}\right)_0 = 1.76 + 20 \log q$$ **Hence Proved.**

Example 4.18. In a binary PCM system, the output signal-to-quantizing-noise ratio is to be held to a minimum value of 40 dB. Determine the number of required levels, and find the corresponding output signal-to-quantizing-noise ratio. *(GATE Examination-1997)*

Solution: In a binary PCM system, $q = 2^{\upsilon}$, where υ is the number of binary digits. Then, we have

$$\left(\frac{S}{N_q}\right)_{0dB} = 1.76 + 20\log 2^{\upsilon} = 1.76 + 6.02\,\upsilon\ dB \qquad ...(i)$$

Now, since $$\left(\frac{S}{N_q}\right)_{0dB} = 40\ dB$$

Therefore, $$\left(\frac{S}{N_q}\right)_{0} = 10{,}000$$

Thus, we have, $$q = \sqrt{\frac{2}{3}\left(\frac{S}{N_q}\right)_0} = \sqrt{\frac{2}{3}(10000)} = [81.6] \cong 82$$

and the number of binary digits υ is

$$\upsilon = [\log_2 82] = [6.36] \cong 7$$

Then, the number of levels required is $q = 2^7 = 128$, and corresponding output signal-to-quantizing noise ratio will be

$$\left(\frac{S}{N_q}\right)_{0dB} = 1.76 + 6.02 \times 7 = 43.9\ \text{dB} \quad \textbf{Ans.}$$

SUMMARY

1. The use of digital communications offers several important advantages as compared to analog communications.
2. Digital communications have become the dominant form of communication technology in our society.
3. To handle the transmission of analog message signals (*i.e.*, voice and video signals) by digital means, the signal has to undergo an analog-to-digital conversion.
4. The simplest form of pulse digital modulation is called pulse code modulation (PCM).
5. In this system (PCM) the message signal is first sampled and then amplitude of each sample is rounded off to the nearest one of a finite set of allowable values known as quantization levels, so that both time and amplitude are in the discrete form. This means that in pulse code modulation both parameters *i.e.*, time and amplitude are expressed in discrete form. This process is called discretization in time and amplitude.
6. We can improve the accuracy of the quantized signal to any desired degree simply by increasing the number of levels L.

7. Pulse-code modulation is known as a digital pulse modulation techniques. The pulse-code modulation (PCM) is quite complex compared to the Analog pulse modulation techniques (*i.e.*, PAM, PWM and PPM) in the sense that the message signal is subjected to a great number of operations.
8. The essential operations in the receiver are regeneration of impaired signals, decoding and demodulation of the train of quantized samples.
9. Quantization refers to the use of a finite set of amplitude levels and the selection of a level nearest to a particular sample value of the message signal as the representation for it.
10. This operation combined with sampling, permits the use of coded pulses for representing the message signal.
11. It is the combined use of quantizing and coding that distinguishes pulse code modulation form analog modulation techniques.
12. Basically, the quantizers are of two types:
    (*i*) Uniform quantizer
    (*ii*) Non-uniform quantizer.
13. A uniform quantizer is that type of quantizer in which the 'step size' remains same through out the input range.
14. A non-uniform quantizer is that type of quantizer in which the 'step-wise' varies according to the input values.
15. Transmission bandwidth in PCM is given by

$$BW \geq \frac{1}{2} r$$

But $\quad r = \upsilon f_s$

Therefore, $\quad BW \geq \frac{1}{2} \upsilon f_s$

Again, since $\quad f_s \geq 2 f_m$

Hence, $\quad BW \geq \upsilon f_m$

16. Because of quantization, inherent errors are introduced in the signal. This error

16. Because of quantization, inherent errors are introduced in the signal. This error is called quantization error.
17. Normalized noise power
18. Thus, Signal to Quantization noise ratio for normalized values of power 'P' and amplitude of input $x(t)$, will be

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6v)\, dB$$

19. The compression of signal at transmitter and expansion at receiver is called combinely as *companding.*
20. The compression and expansion is obtained by passing the signal through the amplifier having non-linear transfer characteristic.
21. The combination of a compression and an expander is called a compander. Naturally in an actual PCM system, the combination of compressor and uniform quantizer is located in the transmitter whereas the expander is located in the receiver.
22. In the μ-law companding, the compressor characteristic is continuous. It is described as

$$|v| = \frac{\log(1 + \mu|m|)}{\log(1 + \mu)}$$

where m and v are the normalized input and output voltages, and μ is a positive constant.

23. Another compression law that is used in practice is the so-called A-law. In the A-law companding, the compressor characteristics is piecewise, made up of a linear segment for low-level inputs and a logarithmic segment for high level inputs. It is described as

$$|v| = \begin{cases} \dfrac{A|m|}{1+\log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{1+\log(A|m|)}{1+\log A}, & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$

24. The signal to noise ratio of PCM remains almost constant with companding.
25. Delta modulation transmits only one bit per sample. Here, the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted.
26. The delta modulation has certain advantages over PCM as under:
    (i) Since, the delta modulation transmits only one bit for one sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta modulation compared to PCM.
    (ii) The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulation.
27. The delta modulation has two major drawbacks as under:
    (i) Slope overload distortion,
    (ii) Granular or idle noise.
28. To overcome the quantization errors due to slope overload and granular noise, the step size (Δ) is made adaptive to variations in the input signal $x(t)$.
29. Adaptive delta modulation has certain advantages over delta modulation as under:
    (i) The signal to noise ratio becomes better than ordinary delta modulation because of the reduction in slope overload distortion and idle noise.
    (ii) Because of the variable step size, the dynamic range of ADM is wider than simple DM.
    (iii) Utilization of bandwidth is better than delta modulation.
30. If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is known as Differential Pulse Code Modulation.

OBJECTIVE TYPE QUESTIONS

Fill up the Blanks

1. The essential operations in the transmitter of a PCM system are ______ and ______ .
2. The quantizing and encoding are performed in the circuit which is called ______.
3. The existence of a finite number of ______ is a basic condition of PCM.
4. The conversion of an analog sample of the signal into a digital (discrete) form is called the ______ process.
5. The difference between two adjacent discrete values is called ______ (or) ______.
6. The ______ consists of difference between the input and output signals of the quantizer.
7. The use of a nonuniform quantizer is equivalent to passing the baseband signal through a ______ and then applying the compressed signal to a ______ quantizer.
8. The combination of a compressor and expander is called a ______ .
9. Any plan for representing each of the discrete set of values as particular arrangement of discrete events is called a ______ .
10. One of the discrete events in a code is called a ______ (or) ______ .
11. A particular arrangement of symbols used in a code to represent a single value of the discrete set is called a ______ (or) ______ .
12. In a ______ code, each symbol may be one of the three distinct values (or) kinds.
13. The process in which the information in a binary PCM is encoded in terms of signal transitions is referred to as ______ .

14. The capability of controlling the effects of distortion and noise produced by transmitting a PCM wave through a channel lies in reconstructing it by using ______ .

15. The ______ process involves generating a pulse the amplitude of which is the linear sum of all the pulses in the code word, with each pulse weighted by its place value (2^0, 2^1, 2^2, 2^3, ...) in the code.

16. The basic operations performed by ______ are equalization, timing and decision making.

17. If the spacing between received pulses deviates from its assigned value, a ______ is introduced into the regenerated pulse position, thereby causing distortion.

18. ______ noise may be introduced anywhere between the transmitter, output and the receiver input.

19. ______ noise may be introduced in the transmitter and is carried along to the receiver output.

20. The average probability of error in a PCM receiver depends on the ratio of ______ to ______, measured at the decoder input in the receiver.

21. The important characteristic of a PCM system is its _______ to interference.
22. The _______ is the one bit version of DPCM.
23. A Delta modulator using a fixed step size is often referred to as _______.
24. The _______ noise occurs when the step size is too large relative to the local slope characteristics of the input waveform.
25. The _______ and _______ are the two noise effects in Delta modulation.
26. The method in which the step size is adapted to the level of the input signal is called _______ .
27. In terms of SNR performance, the adaptive delta modulation is _______ to linear delta modulation by _______ dB at a bit rate of _______ .
28. Companding is used [ ]
    (*a*) to overcome quantizing noise in PCM
    (*b*) in PCM transmitters, to allow amplitude limiting in the receivers
    (*c*) to protect small signals in PCM from quantizing distortion.
    (*d*) in PCM receivers, to overcome impulse noise.
29. The biggest disadvantages of PCM is [ ]
    (*a*) its inability to handle analog signals
    (*b*) the high error rate which its quantizing noise introduces
    (*c*) its incompatibility with TDM
    (*d*) the large bandwidths that are required for it.
30. Indicate which of the following pulse modulation systems is analog [ ]
    (*a*) PCM
    (*b*) Differential PCM
    (*c*) PWM
    (*d*) Delta modulation
31. Quantizing noise occurs in [ ]
    (*a*) time-division multiplexing
    (*b*) FDM
    (*c*) PCM
    (*d*) PWM

ANSWERS

1. Sampling, quantizing and encoding
2. Analog-to-Digital converter
3. discrete amplitude levels
4. Quantizing
5. quantum, step size
6. Quantizing error
7. Compressor, uniform
8. compander
9. code
10. code element, symbol
11. codeword, character
12. ternary code
13. Differential encoding
14. regenerative repeaters
15. decoding
16. Regenerative repeater
17. jitter
18. Transmission noise
19. Quantizing noise
20. Peak pulse power to the average noise power
21. Ruggedness
22. Delta modulation
23. linear delta modulator
24. Granular noise
25. slope over-load, granular noise
26. Adaptive delta modulator
27. superior, 8 dB, 20 kilobits/sec.
28. (*a*)
29. (*a*)
30. (*c*)
31. (*c*)

□□□