Routh Stability Test

Example: Check if all the roots of

$$a(s) = s^4 + 2s^3 + 2s^2 + 4s + 8$$

are in left half of s-plane.

If all are NOT in left half of s-plane, determine how many roots are in right half of s-plane.

Soln: Necessary condition is satisfied. All coefficients are positive.

Routh Table:

s^4	1	2	8
s^3	2	4	0
s^2	$0 \to \epsilon > 0$	8	0
s	$\frac{16 - 4\epsilon}{-\epsilon}$	0	0
1	8		

2 sign changes

$\therefore$ Two roots are in right half of s-plane.

Ex: Determine the roots of following polynomial by applying the Routh Test.

$a(s) = 2.25 s^3 + 6.75 s^2 - 0.25 s - 0.75$

Soln: All coefficients are NOT of same sign. ∴ some roots are NOT in left half of s-plane.

Routh's Table:

s^3	2.25	−0.25
s^2	6.75	−0.75
s	0 ↓ 13.5	0 ↓ 0
1	−0.75	

← Replace zeros by coeffts of in $\frac{dA(s)}{ds}$

One sign change.

When all entries in a row are zero, use the auxillary polynomial constructed using previous row:

$A(s) = 6.75 s^2 - 0.75$

$\frac{dA(s)}{ds} = 13.5 s - 0$

Roots of Auxillary polynomial $A(s)=0$ are also roots of original polynomial $a(s)=0$

Roots of $A(s) = 6.75s^2 - 0.75 = 0$

$$= 27s^2 - 3 = 0$$

$$\Rightarrow s^2 = \frac{1}{9}$$

$$s_1 = +\frac{1}{3}, \quad s_2 = -\frac{1}{3}.$$

To get the third root divide the original polynomial $a(s)$ by $A(s)$.

$$\begin{array}{r|l} & \tfrac{1}{3}s + 1 \\ \hline s^2 - 0.75 & 2.25s^3 + 6.75s^2 - 0.25s - 0.75 \\ & 2.25s^3 \qquad\qquad\quad - 0.25s \\ \hline & 6.75s^2 - 0.75 \\ & 6.75s^2 - 0.75 \\ \hline & 0 \end{array}$$

$\therefore\ a(s) = A(s)\left(\frac{1}{3}s + 1\right)$

$\Rightarrow\ s_3 = -3.$