

Problem Statement

For the plant $G(s) = 1/s(s+6)$ design a **phase-lead** controller for damping ratio $\zeta = 0.4$ and natural frequency **15 rad/sec**.

What is the phase margin and gain margin of the compensated system?

Solution

Clearing Workspace

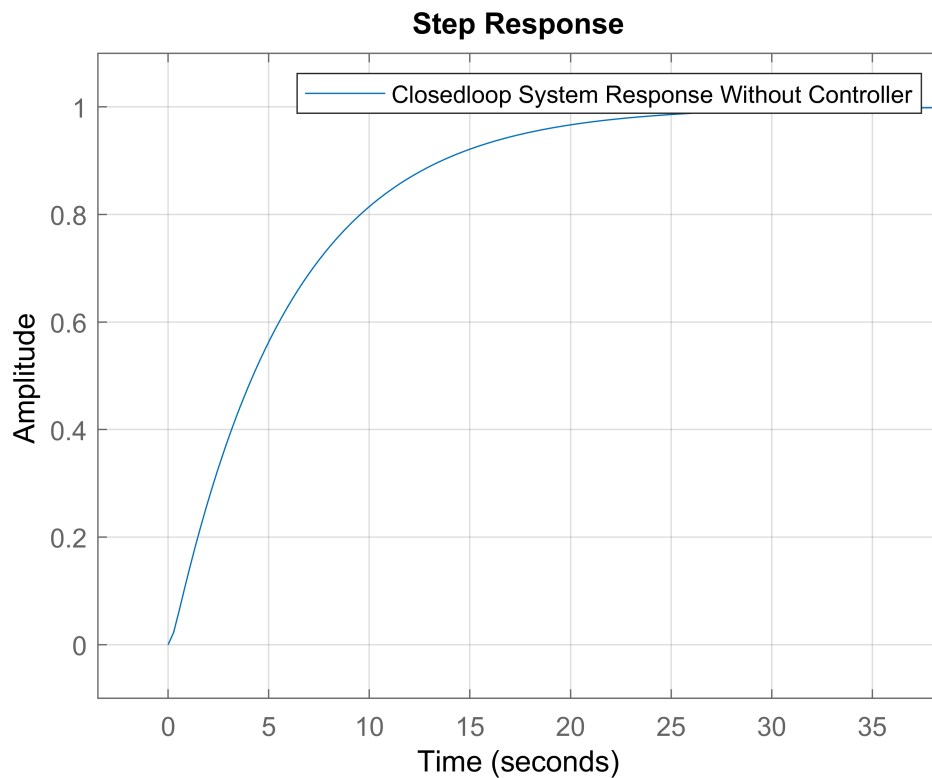
```
close all;  
clear;  
clc;
```

System without a controller

```
s=tf('s');  
G=1/(s*(s+6));
```

Gcl = Closedloop Transfer Function

```
Gcl = G/(1+G); % system without controller  
step(Gcl); % plotting step response  
grid on;  
setAxisLimits(axis);  
legend('Closedloop System Response Without Controller');
```



Poles for the system without controller

```
disp(pole(Gc1));  
  
0  
-6.0000  
-5.8284  
-0.1716
```

One of the pole is on the imaginary axis, and therefore, the system without controller is marginally stable.

Time domain parameters of system

```
stepinfo(Gc1)  
  
ans = struct with fields:  
    RiseTime: 12.8096  
    SettlingTime: 22.9766  
    SettlingMin: 0.9016  
    SettlingMax: 0.9993  
    Overshoot: 0  
    Undershoot: 0  
    Peak: 0.9993  
    PeakTime: 42.6770
```

Designing a Phase Lead Controller

$\zeta = 0.4$

$\omega_n = 15$

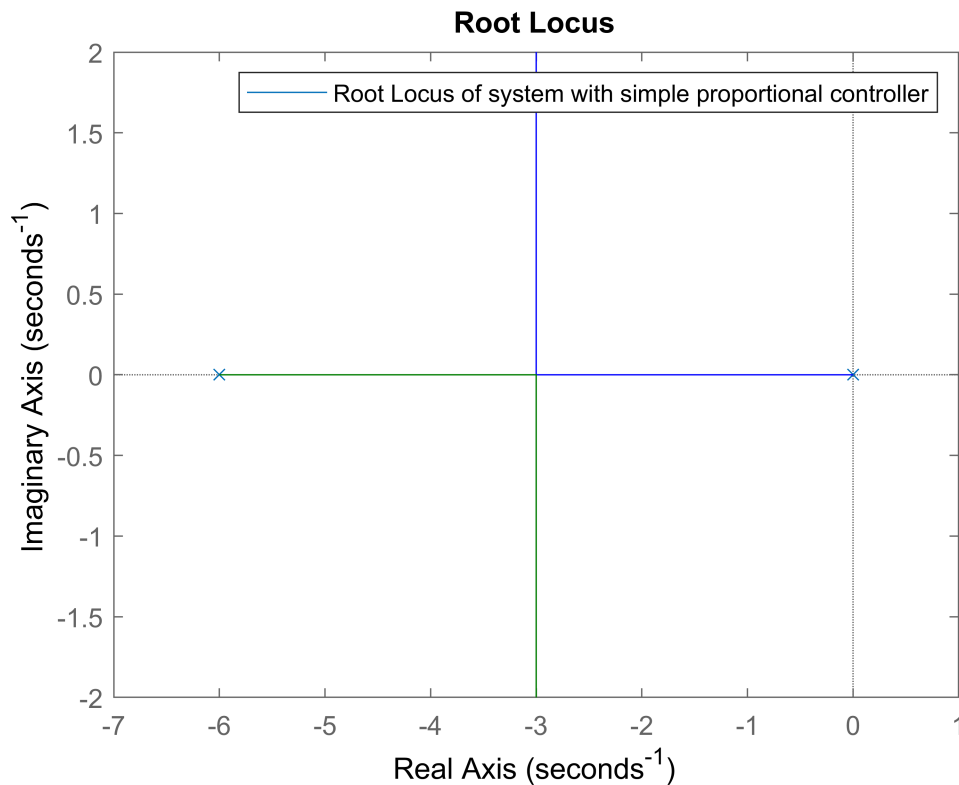
```
zita = 0.4;  
wn = 15;  
desiredPoles = roots([1 2*zita*wn wn^2]);
```

Root Locus must pass through desired poles.

```
disp(desiredPoles);  
  
-6.0000 +13.7477i  
-6.0000 -13.7477i
```

Root Locus of a system with a simple proportional controller

```
figure;  
rlocus(G);  
legend('Root Locus of system with simple proportional controller');
```



We can see that no matter what, the root locus doesn't pass through **desired poles**.

```
syms s1
G1=1/(s1*(s1+6));
phi=double(angle(subs(G1,s1,-6+13.74i)))*180/pi;
sphi=180-phi;
```

The zero of the controller is usually taken just below the desired poles, but as in this system, a pole already exists at $S = -6$.

∴ We take the zero of the controller slightly towards left of -6. i.e $S = -7$ or $Z = 7$.

```
z=-7;
p=z-13.7477/tand(90-sphi);
disp(p);
```

-13.0034

And thus we the pole of the controller as **-13** or **P = 13**.

Then we find out **k** using magnitude criteria.

```
Ds=(s1-z)/(s1-p);
k=1/(double(abs(subs(Ds*G1,-6+13.7477i))))
```

k = 230.8210

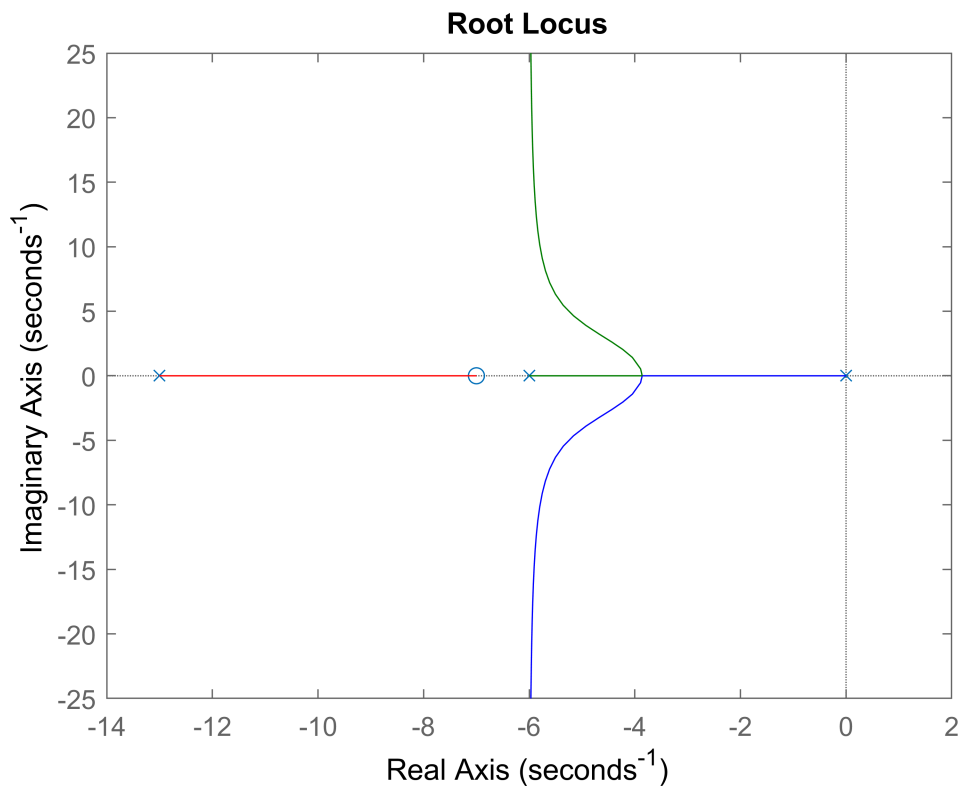
Thus at $k = 230.5863$, the RL passes through the desired pole location.

Verification of design

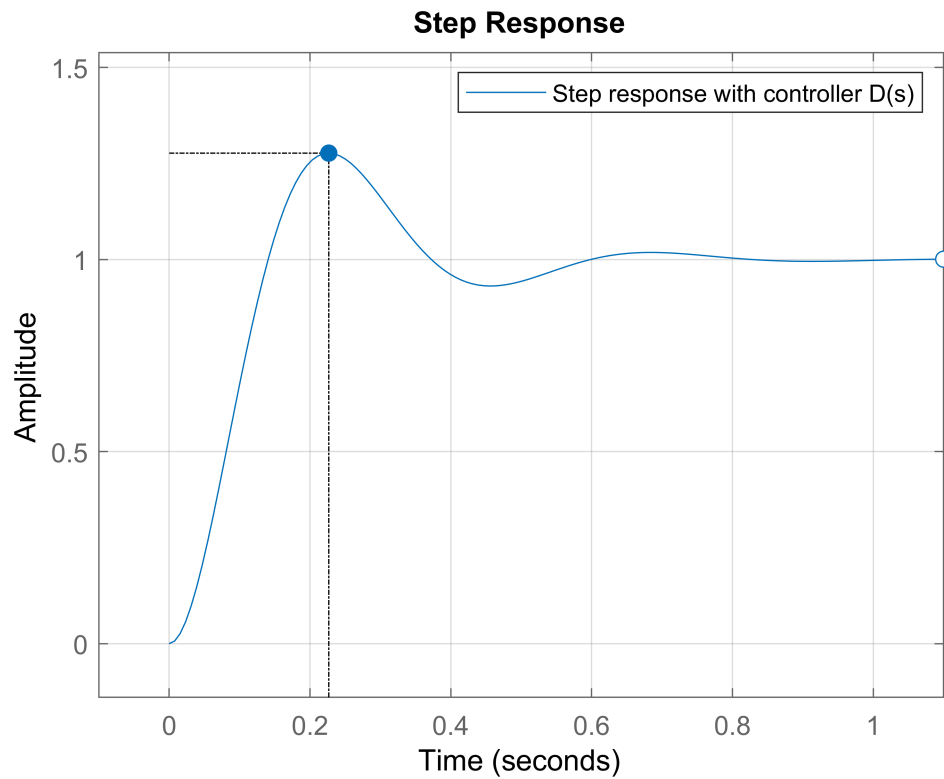
Ds = Controller Transfer Function

Ls = Closedloop Transfer Function with Controller

```
Ds = (s-z)/(s-p);  
Ls = k*Ds*G/(1+k*Ds*G);  
figure;  
rlocus(Ds*G);
```



```
figure;  
response = stepplot(Ls);  
grid on;  
response.showCharacteristic('PeakResponse');  
response.showCharacteristic('SettlingTime');  
response.showCharacteristic('RiseTime');  
response.showCharacteristic('SteadyState');  
setAxisLimits(axis);  
legend('Step response with controller D(s)');
```



Time Domain parameters of system with controller.

```
stepinfo(Ls)
```

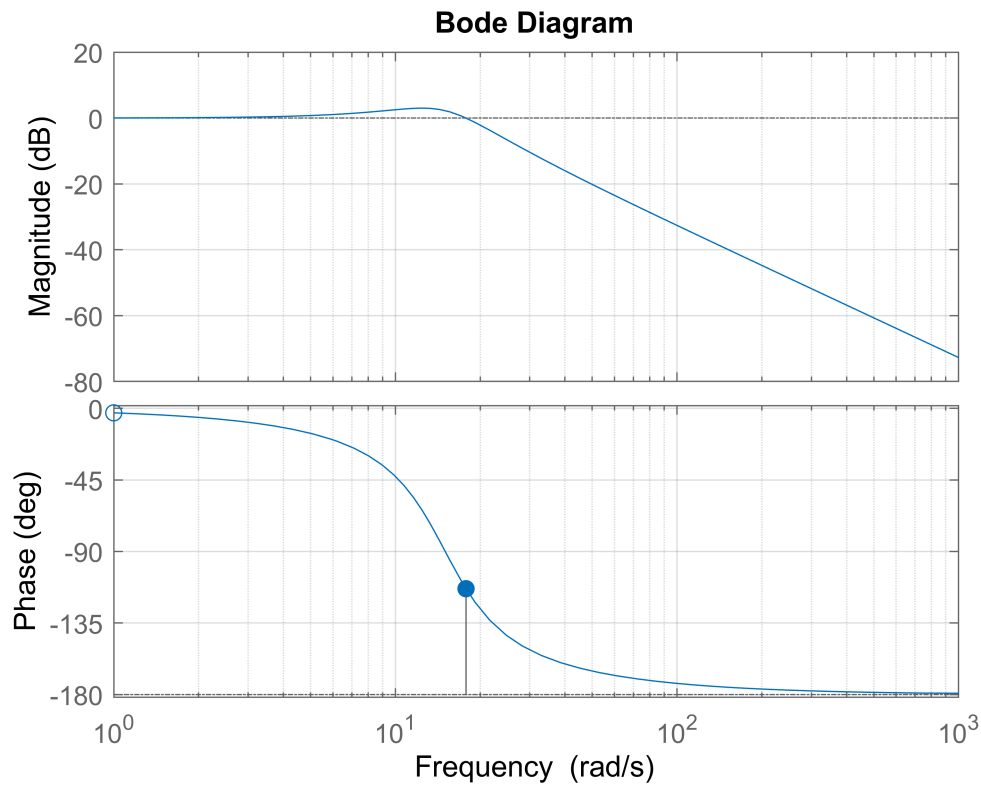
```
ans = struct with fields:
    RiseTime: 0.0952
    SettlingTime: 0.5612
    SettlingMin: 0.9312
    SettlingMax: 1.2771
    Overshoot: 27.7119
    Undershoot: 0
    Peak: 1.2771
    PeakTime: 0.2267
```

```
[gainMargin, phaseMargin, wcg, wcp] = margin(Ls)
```

```
Warning: The closed-loop system is unstable.
gainMargin = Inf
phaseMargin = 66.5761
wcp = Inf
wcp = 17.8151
```

Bode Plot of the closed loop transfer function.

```
response = bodeplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```

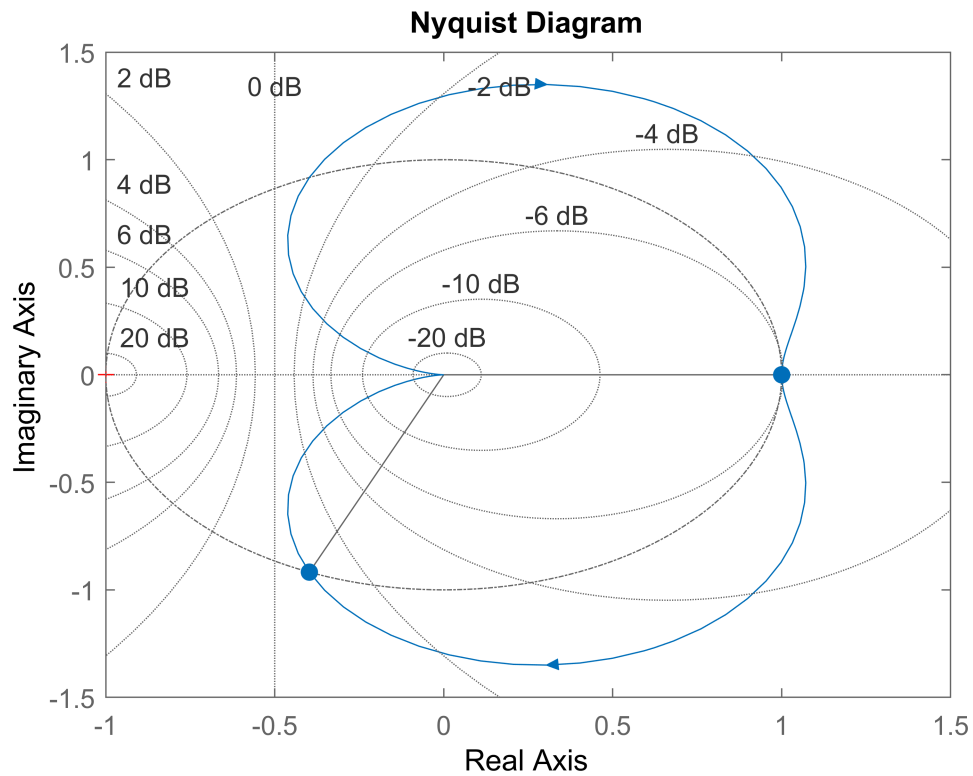


We can see from the **bode plot** that once the gain crossed the **0db** point, it never crosses it back again, hence no matter what the gain is, the system is going to remain stable.

Where as the phase when the gain crosses the **0db** is the **phase margin** and it's angle is 66.57°

Nyquist Plot of the closed loop transfer function

```
response = nyquistplot(Ls);  
response.showCharacteristic('AllStabilityMargins');  
grid on;
```



Inference

Since the **Gain Margin is infinity**, any amount of gain will not result in system getting unstable, this can be shown in the Nyquist plot.

Phase Margin is 66.5761, this means that the system is stable for any value on phase but not 66.5761. As seen on the Nyquist plot, At a gain of **0db**, the plot touches the -1 point at an angle of 66.5761 degrees.