Analog communication

unit 1

Introduction to communication system

- Modulation
- Need for modulation

1. Reduce the height of anteena

The antenna height must be multiple of $\lambda/4$,where λ is the wavelength .

$\lambda = c / f$

where c : is the velocity of light

f: is the frequency of the signal to be transmitted f = 10 kHz

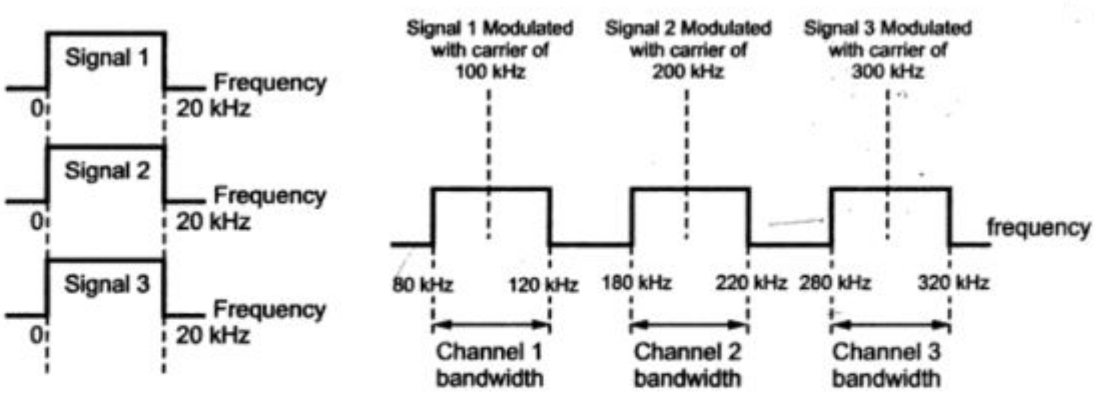
$$Minimum\ antenna\ height = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 10 \times 10^3} = 7500\ meters\ i.e. 7.5\ km$$

f = 1 MHz

$$Minimum\ antenna\ height = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 10 \times 10^6} = 75\ meters$$

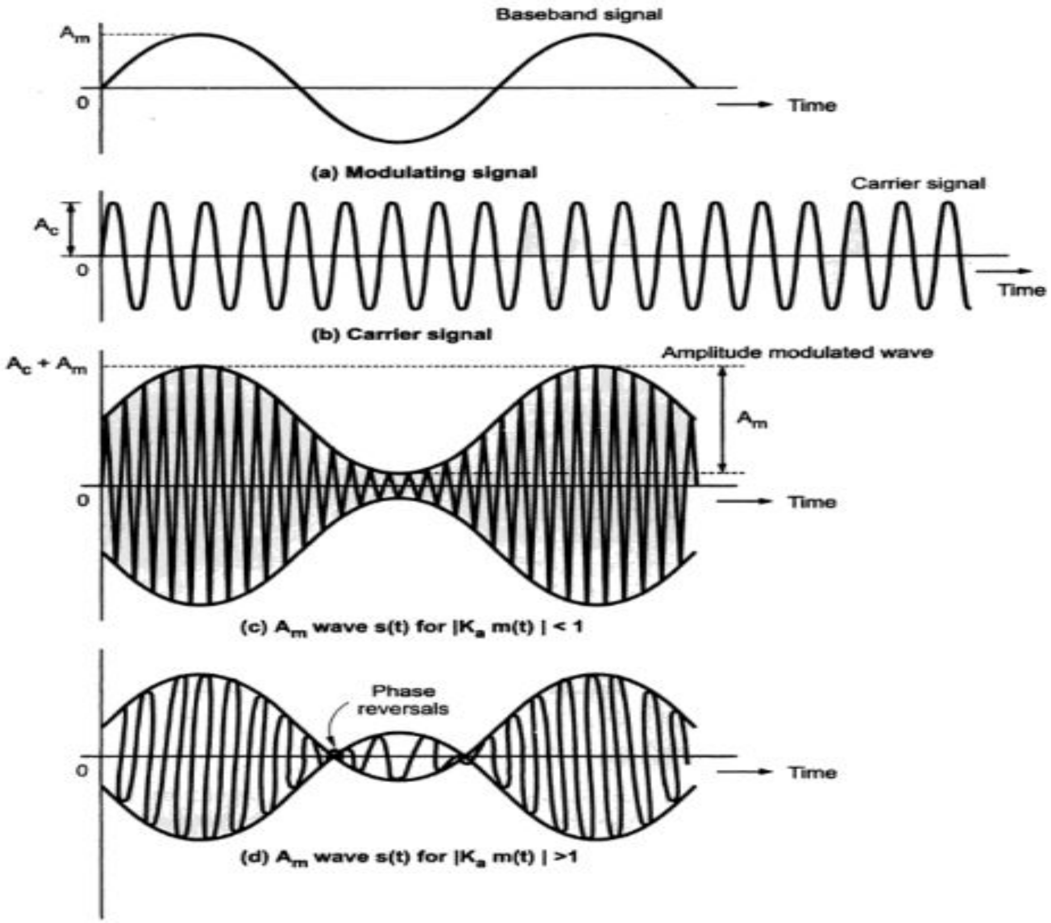
2. Avoid mixing of signal

Modulation avoids mixing of signals



- 3. Increases the range of communication
- 4. Allow multiplexing of signals
- 5. Allow adjustments in this bandwidth
- 6. Improve quality of reception

Amplitude modulation



Time domain description

The instantaneous values of modulating signal and carrier signal can be represented as given below.

Instantaneous value of modulating signal

$$m(t) = A_m \cos(2\pi f_m t) \qquad \dots (1)$$

Instantaneous value of carrier signal

$$c(t) = A_c \cos(2\pi f_c t) \qquad \dots (2)$$

where A_m and A_c are the maximum amplitudes of modulating signal and carrier signal, respectively.

The standard form of an amplitude modulated (AM) wave is defined by

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t) \qquad \dots (3)$$

where K_a is a constant called the amplitude sensitivity of the modulator. The modulated wave so defined is said to be a "standard" AM wave.

Percentage Modulation or Modulation Factor (μ)

Substituting value of m(t) from equation (1) in equation (3) we get,

$$s(t) = A_c [1 + K_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t) \qquad \ldots (4)$$

The equation 3.4 can be written as,

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \qquad \ldots (5)$$

where $\qquad \mu = K_a A_m \qquad \ldots (6)$

Frequeny domain description

The modulated carrier has new signals at different frequencies, called side frequencies or sidebands, occur in the frequency spectrum directly above and below the carrier frequency.

$$f_{USB} = f_c + f_m$$

$$f_{LSB} = f_c + f_m$$

The upper sideband is called f_{USB} and lower sideband is called f_{LSB}.

Now we will see the existence of sideband frequencies in the amplitude modulated wave with mathematical expression. Let us recall the expression for the instantaneous value of the amplitude modulated wave : (Refer equation 5)

$$\begin{aligned} s(t) &= A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \\ &= A_c \cos(2\pi f_c t) + \mu A_c \cos(2\pi f_m t) \cos(2\pi f_c t) \qquad \ldots (7) \end{aligned}$$

Equation (7) can be further expanded, by means of the trigonometrical relation :

$[\cos a \cos b = \frac{1}{2}(\cos(a - b) - \cos(a + b))]$, to give

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos(2\pi f_c - 2\pi f_m)t - \frac{\mu A_c}{2} \cos(2\pi f_c + 2\pi f_m)t \qquad \ldots (8)$$

Bandwidth of AM wave

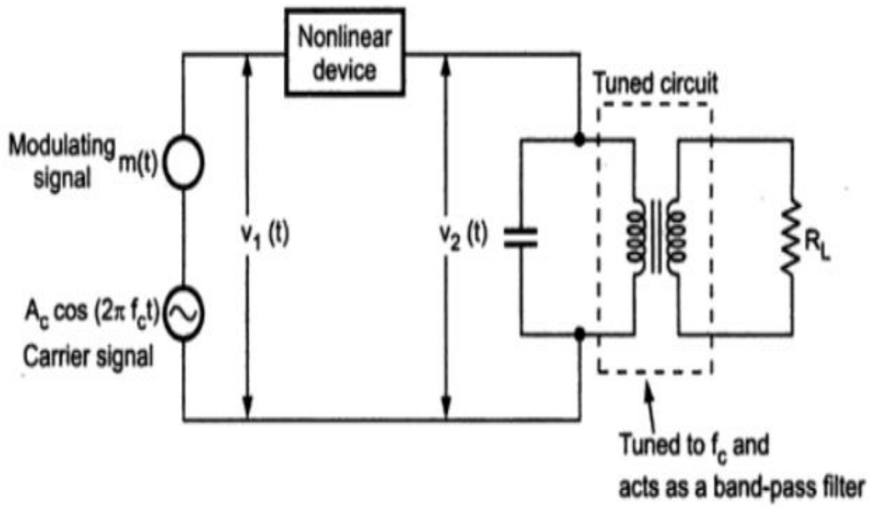
We know bandwidth can be measured by subtracting lowest frequency of the signal from the highest frequency of the signal. For amplitude modulated wave it is given by,

$$
\begin{aligned}
BW &= f_{USB} - f_{LSB} \\
&= (f_c + f_m) - (f_c - f_m) \\
&= 2\,f_m = 2W \qquad \ldots (9)
\end{aligned}
$$

where W is the message bandwidth.

Therefore the bandwidth required for the amplitude modulation is twice the frequency of the modulating signal.

Square law modulator



$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where $v_1(t)$ is the nput voltage

$v_2(t)$ is the output voltage

a_1 and a_2 are constants.

$$v_1(t) = A_c \cos(2\pi f_c t) + m(t)$$

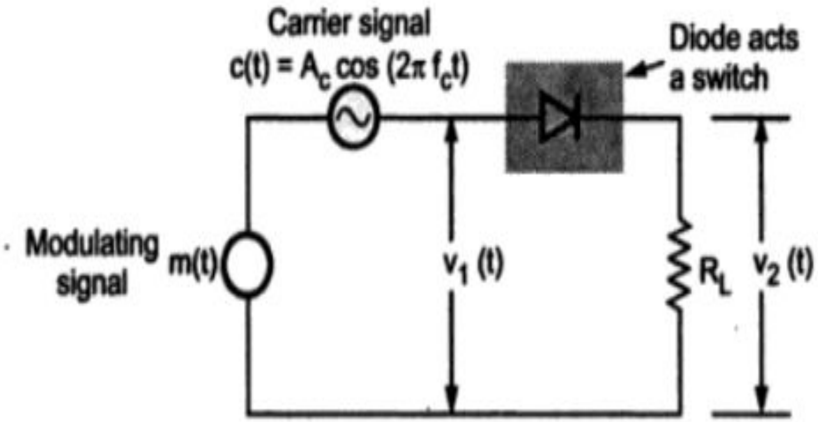
Substituting value of $v_1(t)$ in equation (1) we get,

$$v_2(t) = a_1 \left(A_c \cos(2\pi f_c t) + m(t)\right) + a_2 \left(A_c \cos(2\pi f_c t) + m(t)\right)^2$$

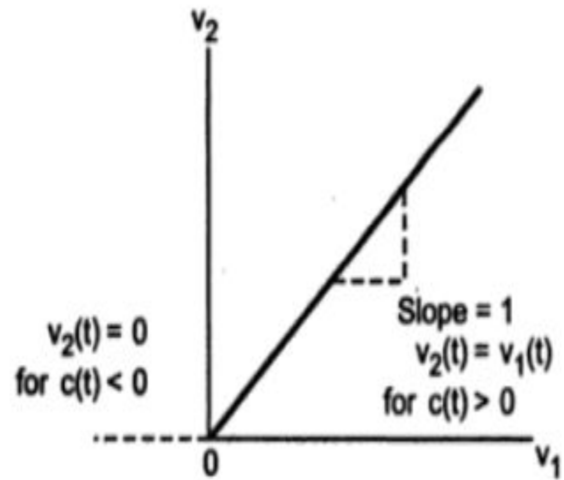
$$= a_1 A_c \cos(2\pi f_c t) + a_1 m(t) + a_2 A_c^2 \cos^2(2\pi f_c t) + 2 a_2 A_c \cos(2\pi f_c t)\, m(t) + a_2 m^2(t)$$

$$= \underbrace{a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos(2\pi f_c t)}_{\text{AM Wave}} + \underbrace{a_1 m(t) + a_2 A_c^2 \cos^2(2\pi f_c t) + a_2 m^2(t)}_{\text{Unwanted terms}}$$

Switching modulator

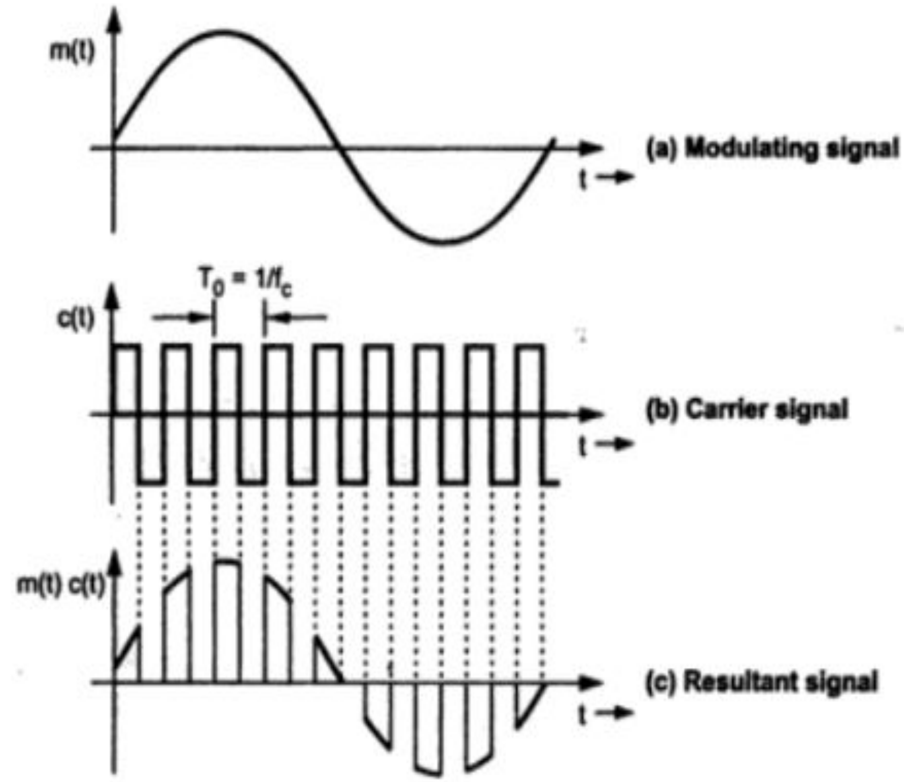


(a) Switching modulator using diode



(b) Idealized input-output relation

Waveforms for switching modulator



We know that, $$v_1(t) = A_c \cos(2\pi f_c t) + m(t)$$

where $$m(t) \ll A_c.$$

The output voltage $v_2(t)$ can be represented as

$$v_2(t) = \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

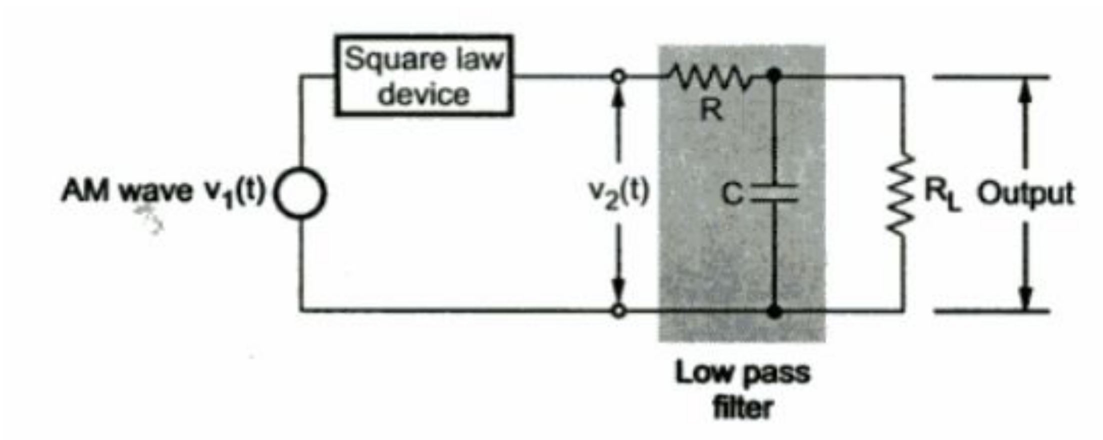
That is, the load voltage $v_2(t)$ varies periodically between the values $v_1(t)$ and zero at a rate equal to the carrier frequency f_c.

We can express equation mathematically as

$$v_2(t) \cong [A_c \cos(2\pi f_c t) + m(t)]\, g_p(t)$$

Detection of AM wave

- Square Law Detector



$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \quad \ldots (1)$$

where $v_1(t)$ and $v_2(t)$ are the input and output voltages , respectively, and a_1 and a_2 are constants. Here $v_1(t)$ represents AM wave and hence it is given as

$$v_1(t) = A_c [1 + K_a m(t)] \cos(2\pi f_c t) \quad \ldots (2)$$

Substituting $v_1(t)$ from equation (2) in equation (1) we get,

$$v_2(t) = a_1[A_c (1 + K_a m(t))] \cos(2\pi f_c t) + a_2 [A_c (1 + K_a (t)) \cos(2\pi f_c t)]^2$$

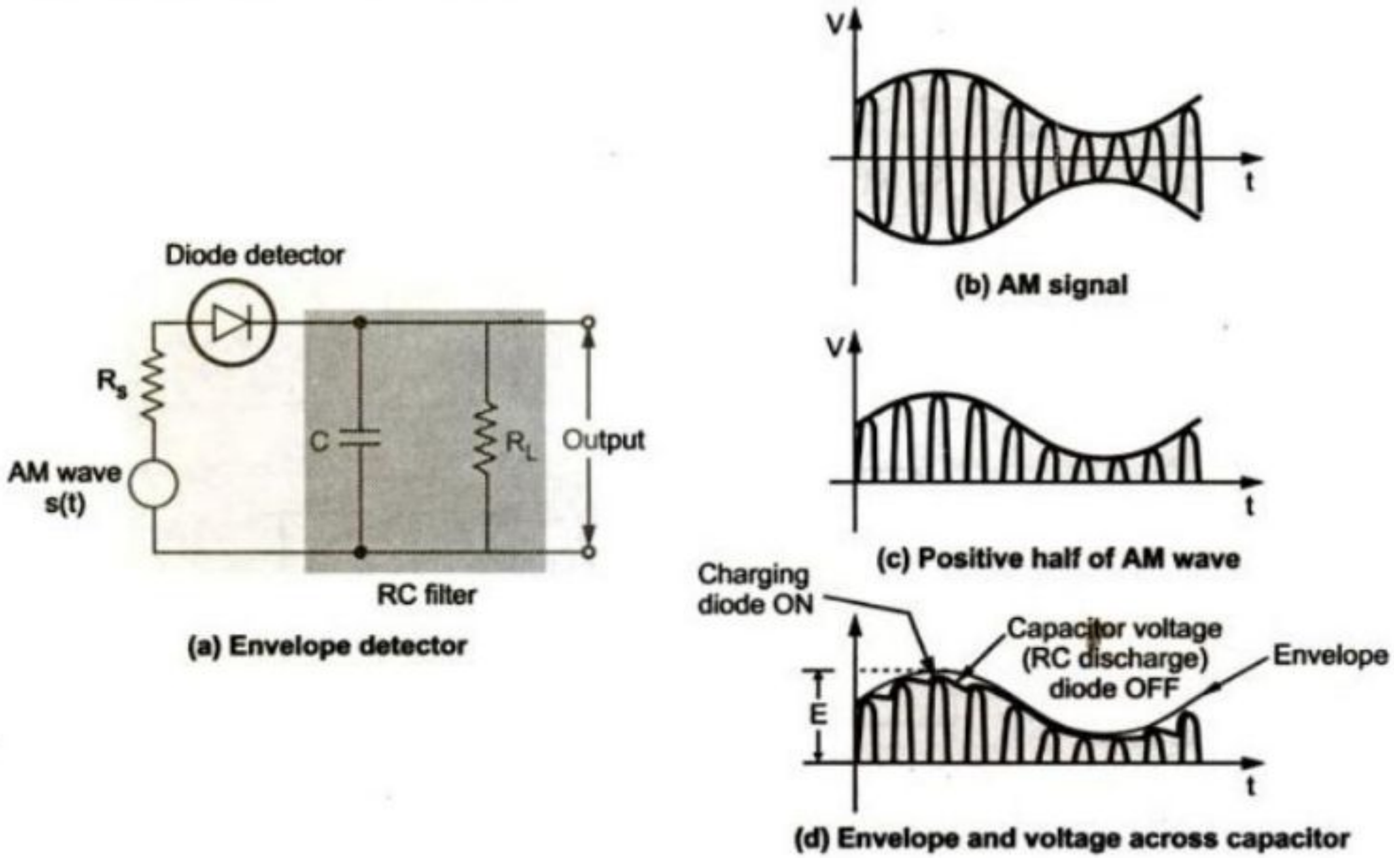
$$= a_1 A_c [1 + k_a m(t)] \cos (2\pi f_c t) + a_2 [A_c^2 (1 + 2 K_a m(t) + K_a^2 m^2 (t)) \cos^2 (2\pi f_c t)]$$

$$= a_1 A_c [1 + K_a m(t)] \cos (2\pi f_c t)$$

$$+ \frac{1}{2} a_2 A_c^2 [1 + 2 K_a m(t) + K_a^2 m^2 (t)] [1 + \cos(4 \pi f_c t)]$$

$$\because \cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)] \quad \ldots (3)$$

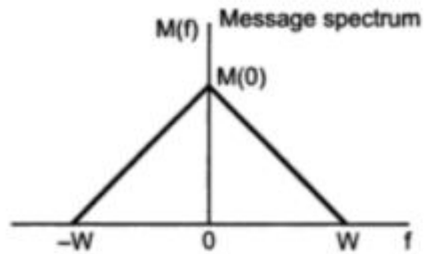
Envelope Detector



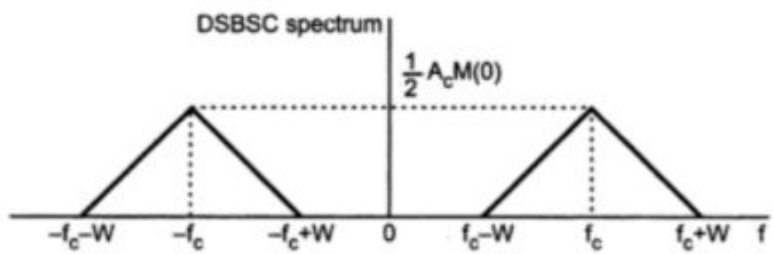
Single Sideband Modulation

- Frequency domain description

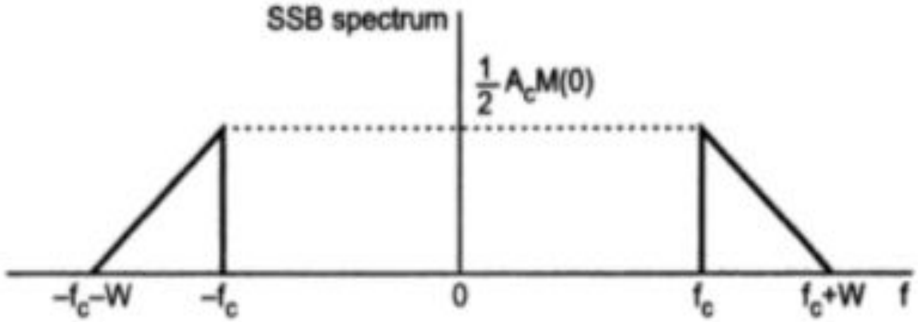
Spectrum of message signal



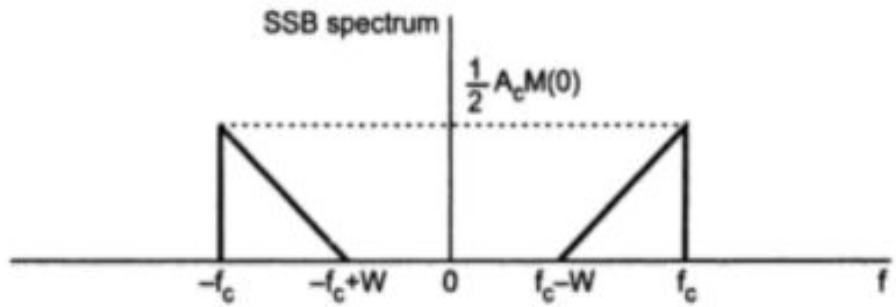
Spectrum of DSBSC modulated wave



Spectrum of SSB modulated wave with the upper sideband transmitted



Spectrum of SSB modulated wave with the lower sideband transmitted



Advantages of SSB

1. The spectrum space occupied by the SSB signal is f_m, which is only half that of AM and DSB signals. In other words, we can say that SSB required half the bandwidth required of AM and DSB signals. This reduction in frequency spectrum or bandwidth allows more signals to transmit in the same frequency range without interfering each other.
2. Due to suppression of carrier and one sideband power is saved and saved power can be used to produce a stronger signal that will carry farther and it will be reliably received at greater distances.
3. When bandwidth is less, the receiver circuits can be made with a narrower bandwidth, filtering out most of the noise. We know that, the SSB signal has less bandwidth than an AM or a DSB signal, hence, logically there will be less noise on it.
4. Fading does not occur in SSB transmission. Fading means that a signal alternately increases and decreases in strength as it is picked up by the receiver. It occurs because the carrier and sideband may reach the receiver shifted in time and phase with respect to each other. The carrier and sideband signals have different frequencies, which are affected by the ionosphere in different ways. The ionosphere bends the carrier and sideband signals at slightly different angles, resulting fading.

Disadvantages of SSB

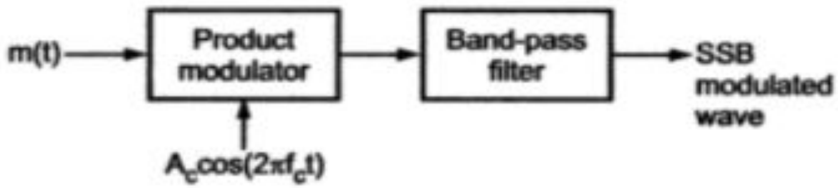
1. The generation and reception of SSB signal is a complex process. It is discussed later on.
2. Since carrier is absent, the SSB transmitter and receiver need to have an excellent frequency stability. A little change in frequency hampers the quality of transmitted and received signals. Thus SSB is not used for the transmission of good quality of signal such as music signal. It is usually used for transmission of speech signa.l

Applications of SSB

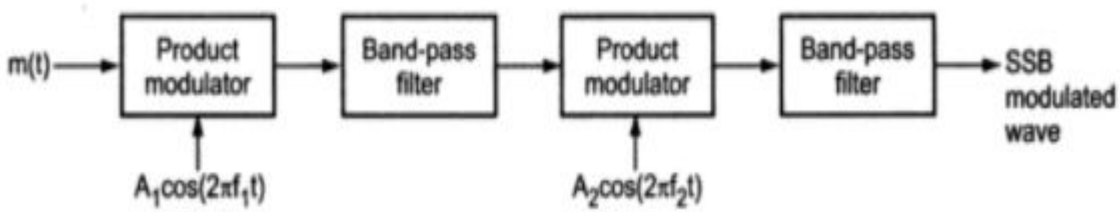
In practice, SSB is used to save power in applications where such a power saving is required, i.e. in mobile systems. Single-side band modulation is also used in applications in which bandwidth requirements are low. Point-to-point communications, land, air, and maritime mobile communications, television, telemetry, military communications, radio navigation, and amateur radio are the greatest users of SSB in one form or another.

Frequecncy discrimination method for generating SSB modulated wave

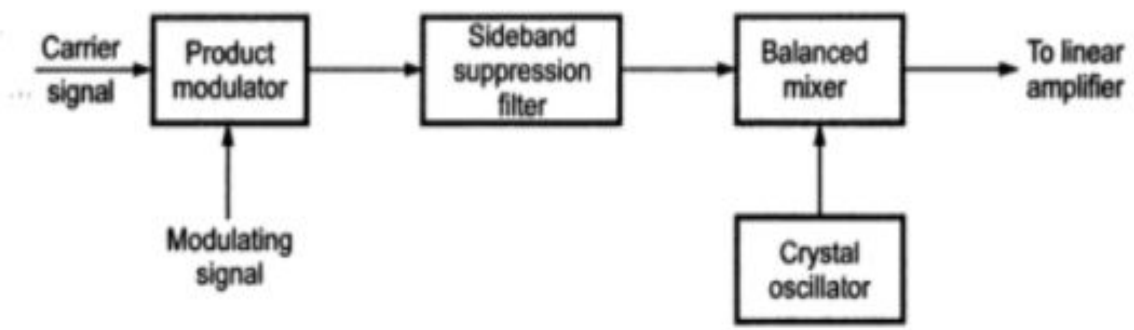
Block diagram of frequency discriminator method



Block diagram of a two-stage SSB modulator



Filter method of sideband suppression



Advantages of filter method

1. The filter method gives sideband suppression upto 50 dB which is quite adequate.
2. The sideband filters also helps to attenuate carrier if present in the output of balanced modulator.
3. Bandwidth is sufficiently flat and wide.

Disadvantages of filter method

1. They are bulky.
2. As modulation takes place at lower carrier frequency repeated mixing is required in conjunction with extremely stable oscillators to generate SSB at high radio frequencies.
3. At lower audio frequencies expensive filters are required.

Time Domain description

The SSB signal may be generated by passing a DSBSC modulated wave through a band-pass filter of transfer function $H_u(f)$. The DSBSC modulated wave is defined mathematically as,

$$s_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)$$

where $m(t)$ = message signal

and $A_c \cos(2\pi f_c t)$ = carrier wave

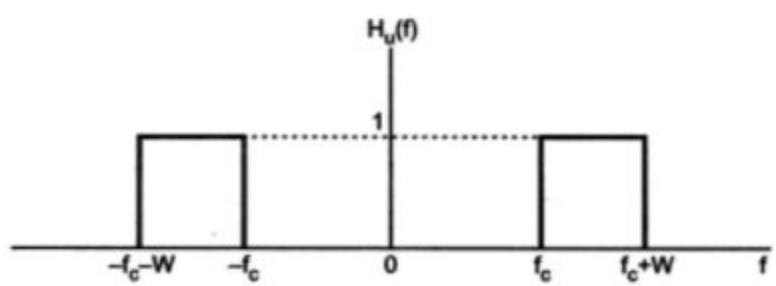
The low pass complex envelope of the DSBSC modulated wave is expressed as,

$$\tilde{s}_{DSBSC}(t) = A_c m(t)$$

Consider the SSB modulated wave $s_u(t)$, in which only the upper sideband is retained. It has quadrature as well as in-phase component. Then $\tilde{s}_u(t)$ is the complex envelope of $s_u(t)$ then from equation (2) of section 1.11 we can write,

$$s_u(t) = R_e\left[\tilde{s}_u(t) \exp(j 2\pi f_c t)\right]$$
$$= R_e\left[\tilde{s}_u(t)\, e^{(j 2\pi f_c t)}\right]$$

The frequency response of ideal band pass filter is shown in the Fig. below



$$s_u(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right]$$

where $\frac{A_c}{2}$ = scaling factor

$m(t)$ = in-phase component = message signal

$\hat{m}(t)$ = quadrature component = Hilbert transform of $m(t)$

This is time domain representation of a modulated wave containing only an upper sideband.

The time domain representation of SSB modulated wave containing only lower sideband is given by

$$s_l(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right]$$

The equations for $s_u(t)$ and $s_l(t)$ are the canonical representation of upper and lower sidebands modulated on a carrier of frequency f_c.

Single tone SSB Modulation

The equation (8) and (9) are bit complex from mathematical point of view as it involves the Hilbert transform. Hence it is difficult to sketch waveforms of SSB modulated waves. Hence to make it more simpler, single tone modulation is used in practice.

Consider the sinusoidal modulating wave,

$$m(t) = A_m \cos(2\pi f_m t)$$

Now Hilbert transform means changing phase of the input signal by 90° without changing its amplitude. Hence Hilbert transform of m(t) can be written as,

$$\hat{m}(t) = A_m \cos\left(2\pi f_m t - \frac{\pi}{2}\right)$$

$$\therefore \quad \hat{m}(t) = A_m \sin(2\pi f_m t)$$

Substituting these two equations in equation (8) we get,

$$s_u(t) = \frac{A_c}{2}\left[A_m \cos(2\pi f_m t)\cos(2\pi f_c t) - A_m \sin(2\pi f_m t)\sin(2\pi f_c t)\right]$$

$$= \frac{A_c A_m}{2}\left[\cos(2\pi f_m t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)\right]$$

Now cos A cos B – sin A sin B = cos (A + B)

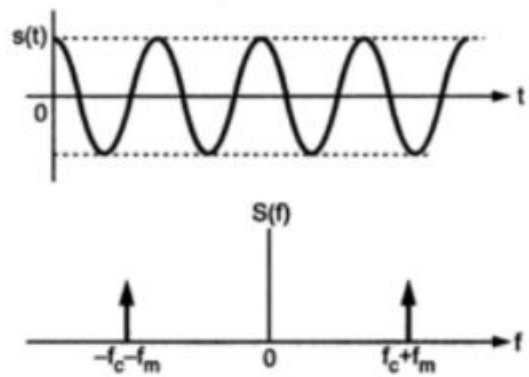
$$\therefore \qquad s_u(t) = \frac{A_c A_m}{2}\left[\cos[2\pi f_m t + 2\pi f_c t]\right]$$

$$\therefore \qquad s_u(t) = \frac{A_c A_m}{2}\cos[2\pi(f_m + f_c)t]$$

This is SSB wave obtained by transmitting only the upper side frequency.

This is exactly same as can be obtained by suppressing the lower side frequency $f_c - f_m$ of the corresponding DSBSC wave the SSB wave and its spectrum are shown Below

Spectrum of SSB with lower sideband supressed



Now substitute equations m(t) and $\hat{m}(t)$ in equation (9) to obtain SSB modulated wave obtained by transmitting only lower side frequency.

$$\therefore \quad s_f(t) = \frac{A_c}{2}[A_m \cos(2\pi f_c t)\ \cos(2\pi f_m t) + A_m \sin(2\pi f_m t)\sin(2\pi f_c t)]$$

$$= \frac{A_c A_m}{2}[\cos(2\pi f_c t)\ \cos(2\pi f_m t) + \sin(2\pi f_m t)\sin(2\pi f_c t)]$$

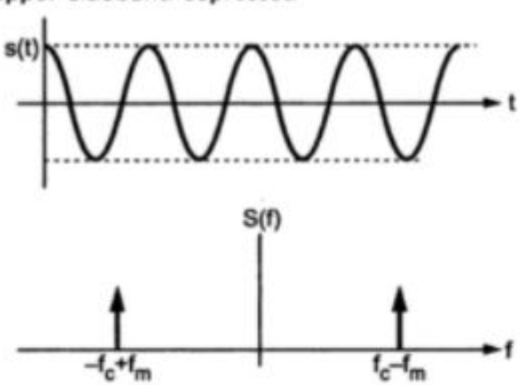
Now cos A cos B + sin A sin B = cos A − B

$$\therefore \quad s_f(t) = \frac{A_c A_m}{2}\cos[2\pi f_c t - 2\pi f_m t]$$

$$\therefore \quad s_f(t) = \frac{A_c A_m}{2}\cos[2\pi(f_c - f_m)t]$$

This is exactly same as can be obtained by suppressing the upper side frequency $(f_c + f_m)$ of the DSBSC wave. The SSB wave and its spectrum is shown in the Fig. below

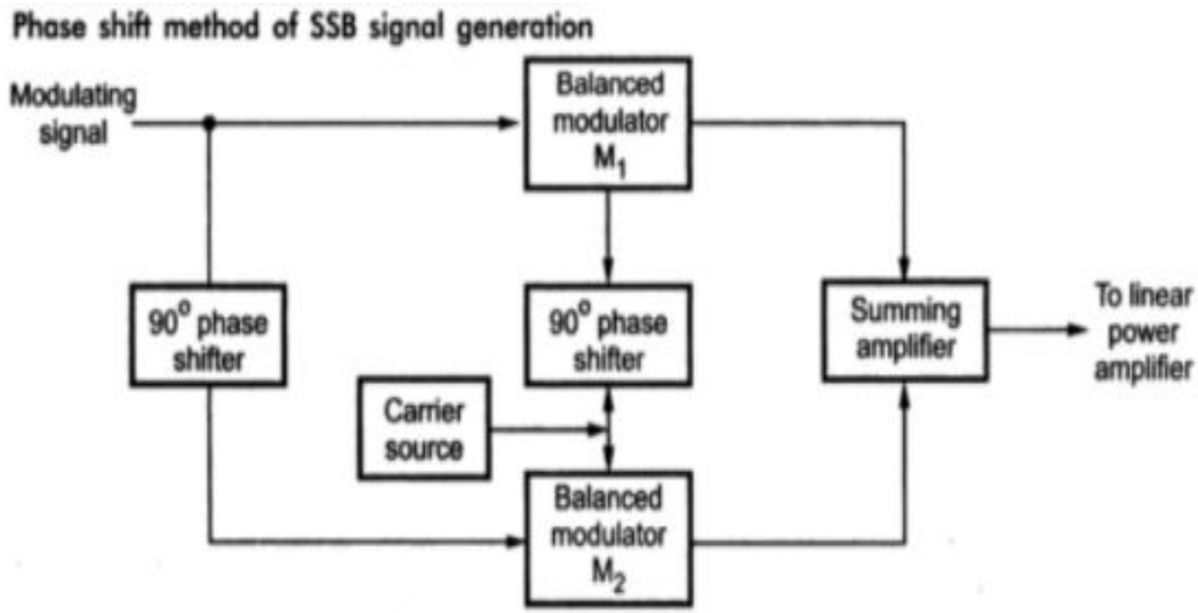
Spectrum of SSB with upper sideband supressed



$$\text{Thus,} \quad s(t) = \frac{1}{2}A_c A_m \cos[2\pi(f_c \pm f_m)t]$$

where minus sign applies to transmission of only lower side frequency while plus sign to the transmission of only upper side frequency.

Phase discrimination method for generating SSB modulated wave



This can be proved mathematically as follows :

Frequency inputs for balanced modulator (M_1) are : $\sin \omega_m t$ and $\sin (\omega_c t + 90°)$

Frequency inputs for balanced modulator (M_2) are : $\sin (\omega_m t + 90°)$ and $\sin \omega_c t$

We know that output of balanced modulator is a sum and difference frequencies. Thus

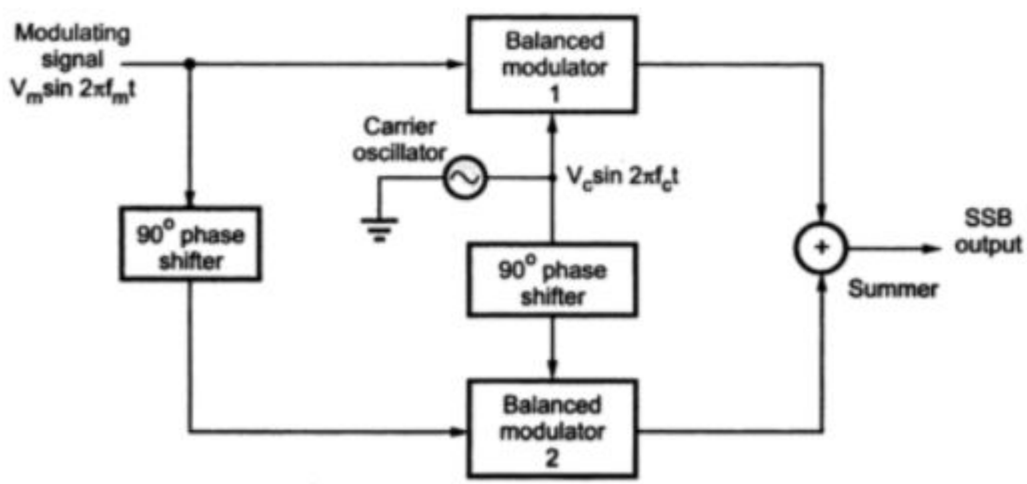
$$\text{Output of } M_1 = \cos [(\omega_c t + 90°) - \omega_m t] - \cos [(\omega_c t + 90° + \omega_m t)]$$

$$= \underbrace{\cos(\omega_c t - \omega_m t + 90°)}_{\text{(LSB)}} - \underbrace{\cos(\omega_c t - \omega_m t + 90°)}_{\text{(USB)}}$$

$$\text{Output of } M_2 = \cos [\omega_c t - (\omega_m t + 90°)] - \cos [\omega_c t + (\omega_m t + 90°)]$$

$$= \underbrace{\cos(\omega_c t - \omega_m t - 90°)}_{\text{(LSB)}} - \underbrace{\cos(\omega_c t + \omega_m t + 90°)}_{\text{(USB)}}$$

$$\text{Output of summer} = \text{Output of } M_1 + \text{Output of } M_2 = 2 \cos (\omega_c t + \omega_m t + 90°)$$



Advantages of phase shift method

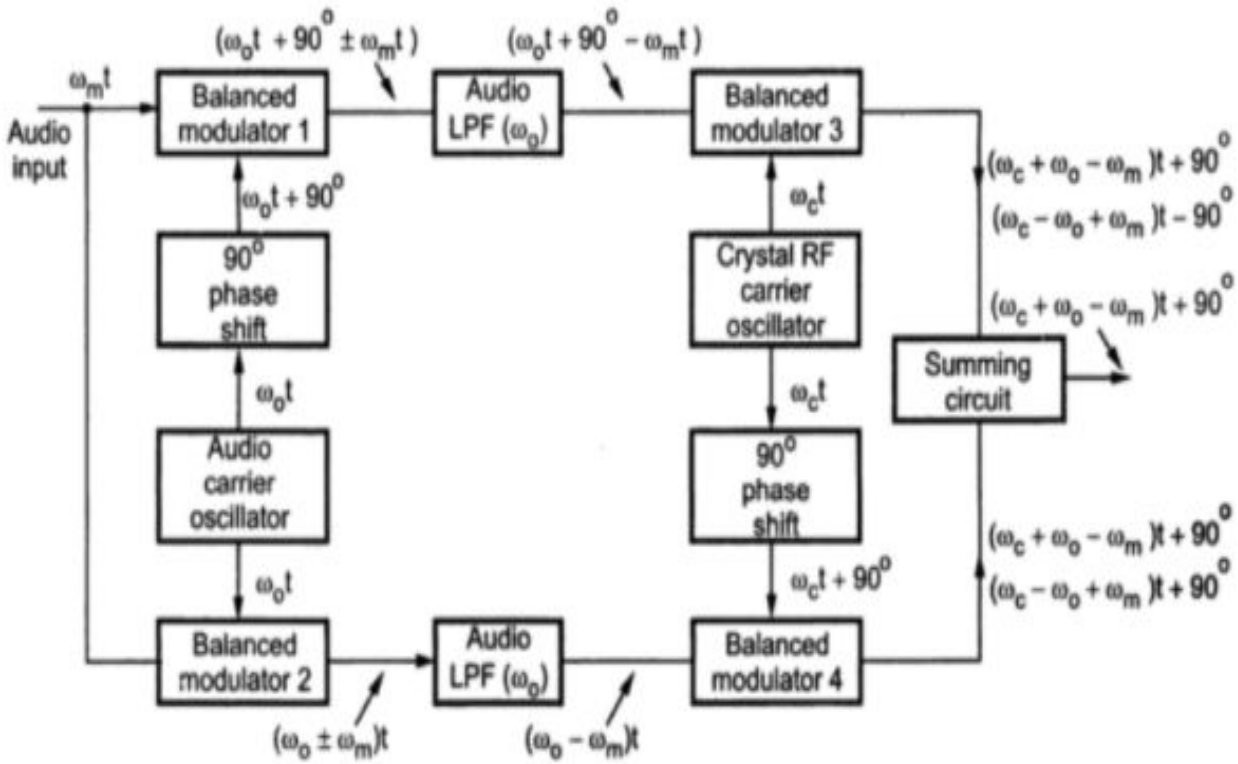
1. Bulky filters are replaced by small filters.
2. Low audio frequencies may be used for modulation.
3. It can generate SSB at any frequency.
4. Easy switching from one sideband to other sideband is possible.
5. To generate SSB at high radio frequencies up conversion and hence repeatative mixing is not necessary.

Disadvantages of phase shift method

1. It requires complex AF phase-shift network since it has to work for large frequency range.
2. If the phase shifter provides a phase change other than 90° at any audio frequency, that particular frequency will not be completely removed from the unwanted sideband. Hence great care in adjustment is necessary.
3. The output of two balanced modulators must be exactly same; otherwise cancellation will be incomplete.

Third Method for Generating SSB Modulated wave

Block diagram of third method of SSB generation



Advantages of third method

1. It does not require a sideband suppression filter and a wideband audio phase shift network.
2. As carrier frequencies are constant, the phase shift network is very simple RC circuit.
3. Correct output can be maintained simply without critical parts or adjustments.
4. Low frequencies can be transmitted.
5. Since the majority of the circuitry is at AF, layout and components tolerances are not critical.
6. Sidebands may also be switched quite easily.

Disadvantages of third method

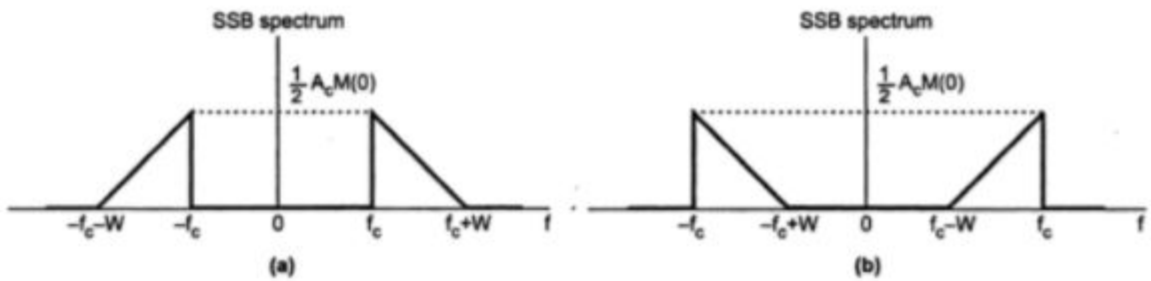
1. Most complex amongest three method.
2. Extra crystal is required for sideband switching.
3. DC coupling is required to avoid the loss of signal components close to the audio carrier frequency.

Comparison Between SSB Generation Methods

Comparison between single sideband suppression methods

S.N.	Parameter	Filter Method	Phase shift method	Third method
1.	Method used	Filter is used to remove unwanted sideband.	Phase-shifting techniques is used to remove unwanted sideband.	Similar to phase-shift method, but carrier signal is phase shifted by 90°.
2.	90° phase shift	Not required	Requires complex phase shift network	Phase shift network is simple RC circuit
3.	Possible frequency range of SSB	Not possible to generate SSB at any frequency.	Possible to generate SSB at any frequency.	Possible to generate SSB at any frequency.
4.	Need for up-conversion	Required	Not required	Not required
5.	Complexity	Less	Medium	High
6.	Design aspects	Q of tuned circuit, filter type, it size, weight and upper frequency limit.	Design of 90° phase shifter for entire modulating frequency range. Symmetry of balanced modulators.	Symmetry of balanced modulators.
7.	Bulkyness	Yes	No	No
8.	Switching ability	Not possible with existing circuit. Extra filter and switching network is necessary	Easily possible	Easily possible. But extra crystal is required

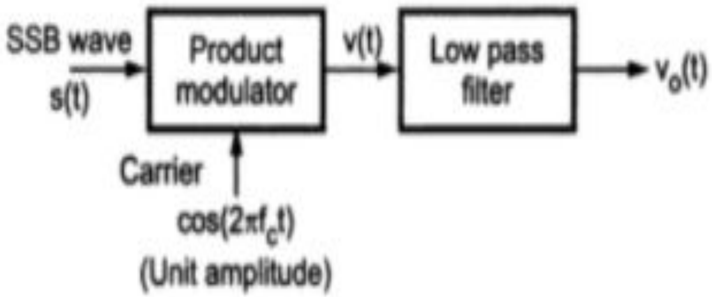
Demodulation of SSB waves



Such a recovery of m(t) from SSB wave can be achieved by the method of coherent detection. This method includes following procedure :

1. Consider a SSB wave either $s_u(t)$ or $s_l(t)$.
2. Apply it to a product modulator.
3. Apply the locally generated carrier wave $\cos(2\pi f_c t)$ of unit amplitude as a second input to the product modulator. The unit amplitude is considered for convenience only.
4. Apply the output of modulator to a low pass filter. The output of low-pass filter is the recovered signal m(t).

Demodulation of SSB wave-coherent detection



Let us find the expression for the output of product modulator shown as v(t). It is the product of SSB wave and carrier wave.

$$\therefore \quad v(t) = s(t) \cdot \cos(2\pi f_c t)$$

$$\text{Let} \quad s(t) = \frac{A_c}{2}[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t]$$

$$\therefore \quad v(t) = \frac{A_c}{2}\cos(2\pi f_c t)\,[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t]$$

$$= \frac{A_c}{2}m(t)\cos^2(2\pi f_c t) \pm \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$\text{Now} \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

while $2\sin x\cos x = \sin 2x$

Applying above trigonometric results,

$$\therefore \quad v(t) = \frac{A_c}{2}m(t)\left[\frac{1+\cos(4\pi f_c t)}{2}\right] \pm \frac{A_c}{2}\hat{m}(t)\left[\frac{\sin(4\pi f_c t)}{2}\right]$$

$$= \frac{A_c}{4}m(t) + \frac{A_c}{4}m(t)\cos(4\pi f_c t)$$

$$\pm \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)$$

$$\therefore \qquad v(t) = \underbrace{\frac{A_c}{4} m(t)}_{\text{Desired signal}} + \underbrace{\frac{A_c}{4}\left[m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)\right]}_{\text{Unwanted component}}$$

where $\frac{A_c}{4} m(t)$ = Scaled message signal with $\frac{A_c}{4}$ as a scale factor

while the remaining part is the unwanted component.

The component other than $\left[\frac{A_c}{4} m(t)\right]$ in the equation (16) is the SSB modulated wave with a carrier frequency of 2 f_c. Hence it is unwanted component and hence can be removed by filtering the signal v(t).

The method of coherent detection is based on the assumption that there is perfect synchronization between local carrier and that in the transmitter both in frequency and phase.

But in practice, there is phase error ϕ in the locally generated carrier wave which distorts the output $V_o(t)$ as,

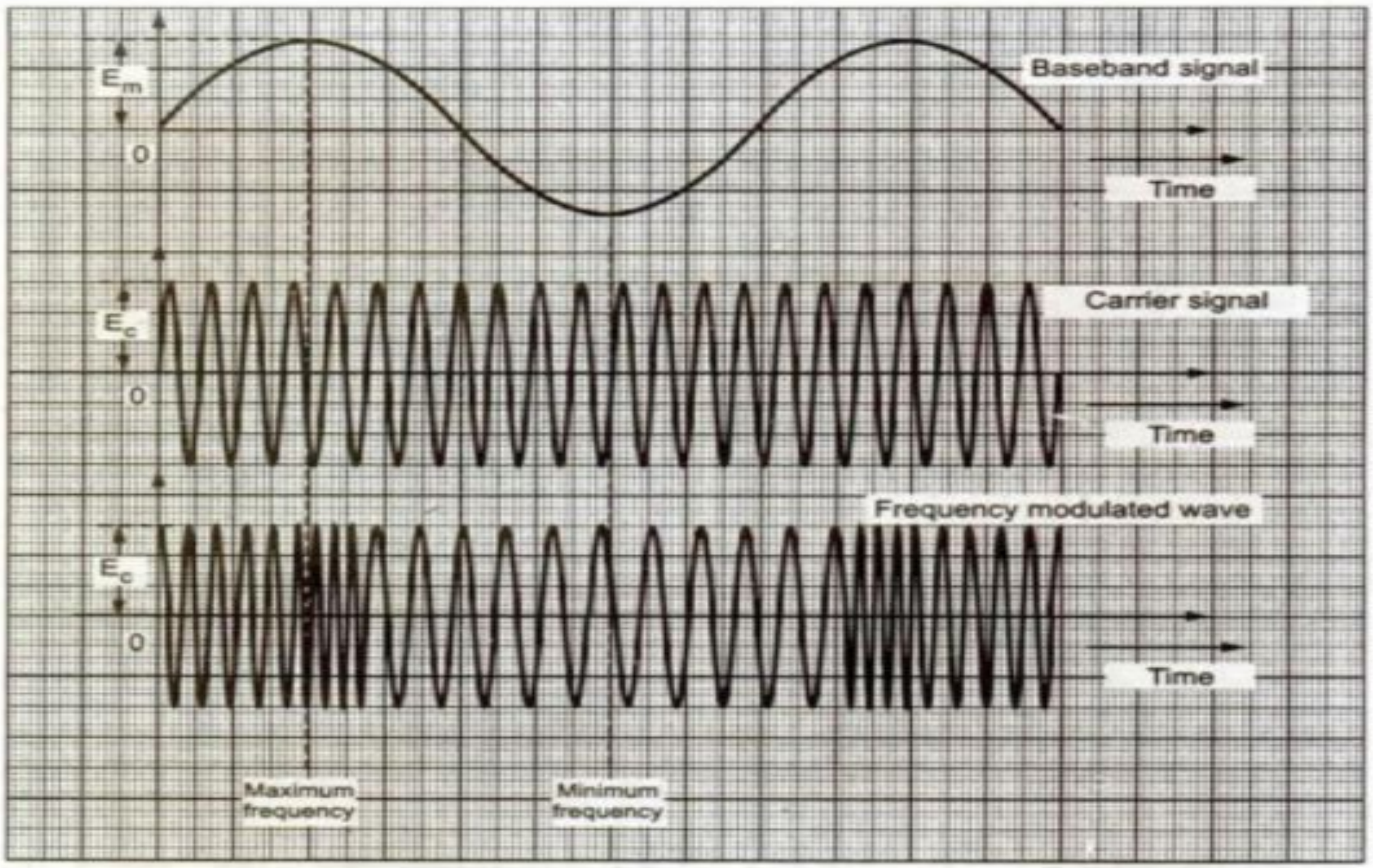
$$V_o(t) = \frac{1}{4} A_c\, m(t)\cos\phi \mp \frac{1}{4} A_c\, \hat{m}(t)\sin\phi$$

Angle Modulation

Frequency Modulation

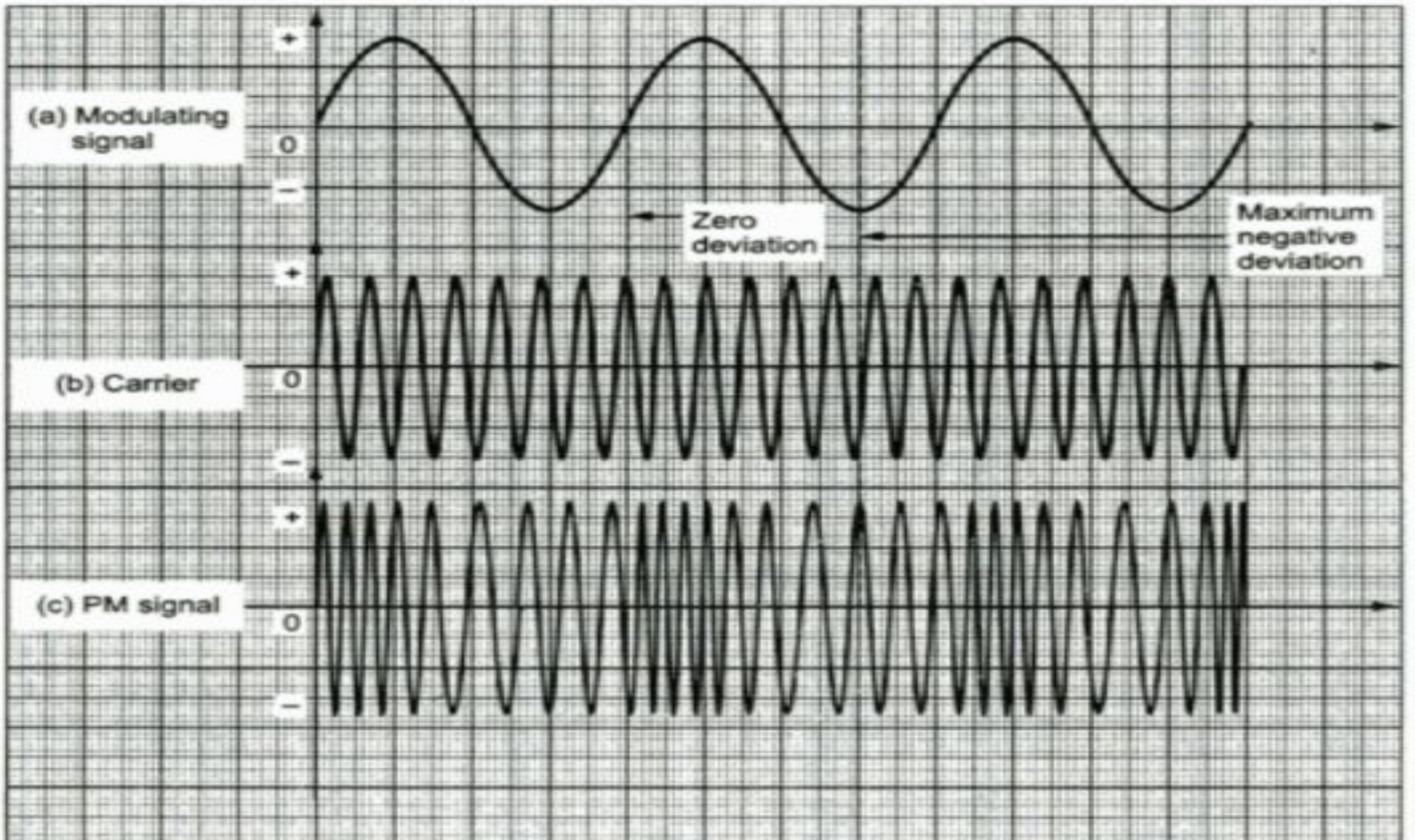
Frequency modulation is a system in which the amplitude of the modulated carrier is kept constant, while its frequency is varied by the modulating signal.

The amount of change in carrier frequency produced by the modulating signal is known as **frequency deviation**. Maximum frequency deviation occurs at the maximum amplitude of the modulating signal. Fig. 1.4 shows a single frequency sine wave modulating a higher frequency carrier signal with frequency modulation.



Phase Modulation

Another way to produce angle modulation is a phase modulation. In phase modulation, the phase of the carrier is varied in accordance with the modulating signal instead of its frequency; as in FM, the amplitude of the carrier remains constant. The greater the amplitude of the modulating signal, the greater the phase shift. Fig. 1.5 shows a single frequency sine wave modulating a higher frequency carrier signal with phase modulation.



Angle Modulation

The sinusoidal carrier wave has basically three characteristics, viz. amplitude, frequency, and phase. In amplitude modulation, the amplitude of the sinusoidal carrier is slowly varied in accordance with the baseband signal required to be transmitted.

Instead of amplitude, either frequency or phase of the sinusoidal carrier can be changed according to the message, keeping the amplitude constant. This is another method of modulation, termed "angle modulation".

Let us denote the carrier signal by

$$s(t) = A_c \cos \theta(t) \quad \text{... (1)}$$

where A_c is the carrier amplitude, which is held constant in angle modulation, while the phase angle is varied by the message signal m(t), which is the modulating signal.

The Fig. 2.1 shows the generalized angle $\theta(t)$ as a function of time, t.

The conventional sinusoidal signal is represented by

$$s(t) = A_c \cos (\omega_c t + \phi_c) \quad \text{... (2)}$$

for which the angle is given by $[\omega_c t + \phi_c]$, where ϕ_c is the value of $\theta(t)$ at $t = 0$. The angle $[\omega_c t + \phi_c]$ represents a straight line with a slope ω_c and interrupt ω_c, as shown in

Phase Modulation

and frequency modulation, employed to vary ...

1) **Phase Modulation [PM]** is defined as that method of angular modulation in which the angular argument $\theta(t)$ is varied linearly with the message signal m(t), as given by

$$\theta(t) = \omega_c t + k_p m(t) \quad \ldots (6)$$

Here the constant k_p is termed as the phase sensitivity of the modulator. It is expressed in radians per volt. Here it is assumed that message signal m(t) is a voltage waveform.

The phase modulated wave s(t) can now be written in time-domain as

$$s(t) = A_c \cos\left[\omega_c t + k_p m(t)\right] \quad \ldots (7)$$

The instantaneous angular frequency ω_i is given by

$$\omega_i(t) = \frac{d\theta}{dt} = \frac{d}{dt}\left[\omega_c t + k_p m(t)\right]$$

$$\therefore \quad \omega_i(t) = \omega_c + k_p m(t)$$

$$\therefore \quad \omega_i(t) = \omega_c + k_p \frac{d}{dt}[m(t)] \quad \ldots (8)$$

Thus in PM, the instantaneous frequency ω_i varies linearly with the derivative of the message signal, m(t).

2) **Frequency Modulation [FM]** is defined as that method of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal m(t), as expressed by

$$f_i(t) = f_c + k_f\, m(t) \qquad \ldots(9)$$

Here f_c is the frequency of unmodulated sinusoidal carrier, also termed as **center frequency** ; and the constant k_f denotes the "**Frequency sensitivity**" of the modulator, and it is expressed in Hz per volt ; assuming that m(t) is a voltage waveform.

$$\text{As per equation (4)} \quad f_i(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$$

$$\therefore \quad 2\pi f_i(t) = \frac{d\theta}{dt}$$

$$\therefore \quad \theta(t) = \int_0^t 2\pi f_i(t)\, dt \qquad \ldots(10$$

$$\therefore \quad \theta(t) = \int_0^t 2\pi\left[f_c + k_f\, m(t)\right] dt$$

In the time domain, the frequency modulated wave can be written as

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

Thus we have seen that :

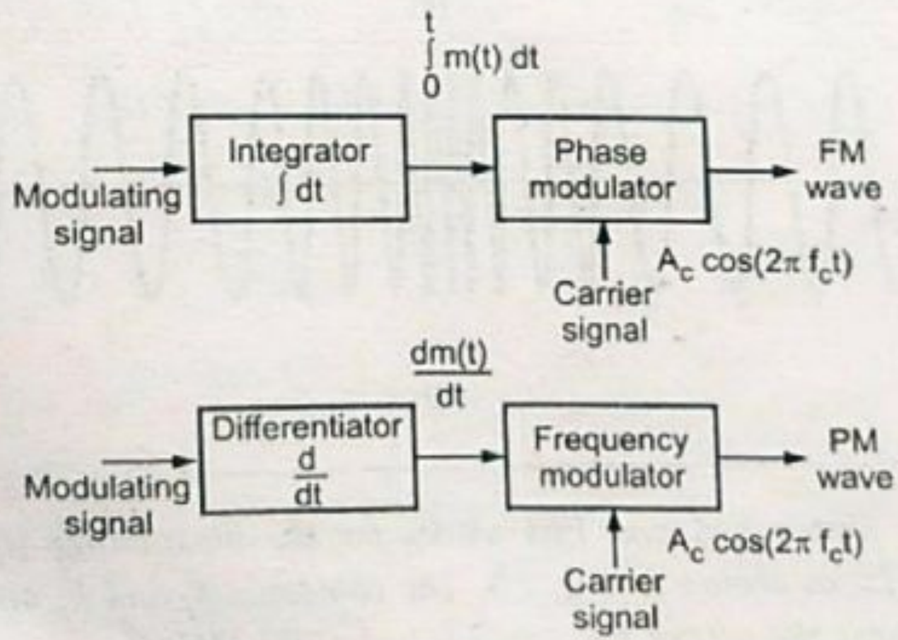
$$s(t) = A_c \cos\left[\omega_c t + k_p m(t)\right] \text{ for PM}$$

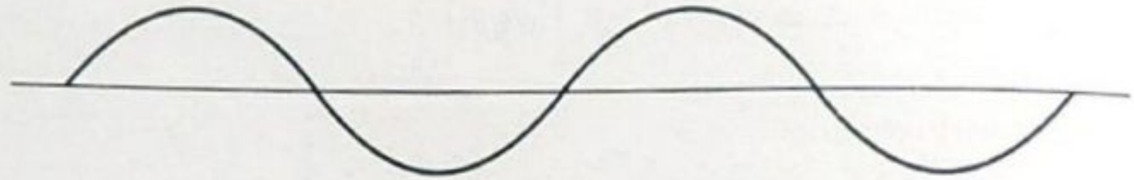
and $$s(t) = A_c \cos\left[\omega_c t + 2\pi k_f \int_0^t m(t)\,dt\right] \text{ for FM}$$

2.2.1 FM Wave Using PM and PM Wave Using FM

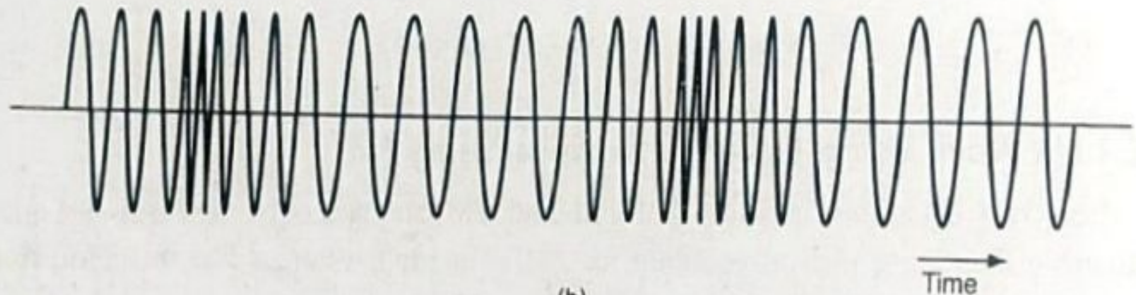
These two equations indicate that PM and FM are not only very similar but are inseparable. Replacing m(t) in equation for PM with $\int m(t)$ changes PM into FM. Thus, a signal which is an FM wave corresponding to m(t) is also the PM wave corresponding to $\int m(t)\,dt$. Similarly, a PM wave corresponding to m(t) is the FM wave corresponding to $\frac{dm(t)}{dt}$. This is illustrated in Fig. 2.3.

Figure 2.3

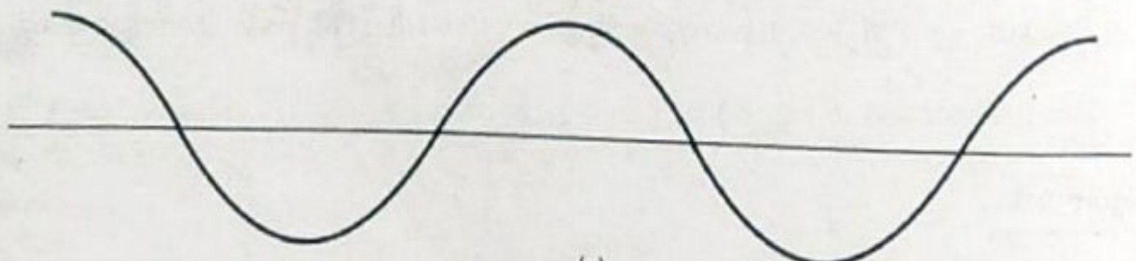




(a)



(b)



(c)

(d)

5.3 Frequency Modulation

We have seen that frequency-modulated wave in time domain is expressed as

$$s(t) = A_c \cos\left[2\pi f_c(t) + 2\pi k_f \int_0^t m(t)dt\right]$$

This equation indicates that FM wave s(t) is a nonlinear function of the modulating wave m(t). Thus, frequency modulation is basically a nonlinear modulation process. Hence, unlike AM, the spectrum of an FM wave is not related in a simple way to that of the modulating signal. To study the spectral properties of an FM wave, we start with single-tone modulation, i.e. modulation by a sinusoidal signal at single frequency.

5.3.1 Single Tone Frequency Modulation

Let the sinusoidal modulating signal be

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{... (1)}$$

The instantaneous frequency of the resulting FM wave is given by

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$$

$$= f_c + \Delta f \cos(2\pi f_m t) \quad \text{... (2)}$$

where $\Delta f = k_f A_m$. ... (3)

The quantity Δf is termed as the **"frequency deviation"**, representing the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency f_c.

We note that the frequency deviation Δf is proportional to the amplitude of the modulating signal, but is independent of the modulation frequency.

Now $\quad f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$

But $\quad f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[\theta(t)]$

$$\therefore \quad \theta(t) = 2\pi\int_0^t f_i(t)dt = 2\pi\int_0^t f_c + \Delta f \cos(2\pi f_m t)\, dt$$

$$\therefore \quad \theta(t) = 2\pi f_c t + \frac{\Delta f \sin(2\pi f_m t)}{2\pi f_m} \times 2\pi$$

$$\therefore \quad \theta(t) = 2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t) \qquad \ldots (4)$$

The ratio of frequency deviation Δf to the modulation frequency f_m is called the "modulation index" of the FM wave, which is denoted by β.

Thus $\quad \beta = \frac{\Delta f}{f_m} \qquad \ldots (5)$

and $\quad \theta(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) \qquad \ldots (6)$

Example 5.4 : *A sinusoidal modulating wave of amplitude 5 V and frequency 1 kHz is applied to a frequency modulator. The frequency sensitivity of the modulator is 50 Hz/V. The carrier frequency is 100 kHz.*

Calculate :

a) The frequency deviation,

b) Modulation index.

Sol. : a) Frequency deviation = frequency sensitivity × amplitude of modulating signal

$$= [50\ \text{Hz}\ /\text{V}]\ [5] = 250\ \text{Hz}$$

b) Modulation index

$$\beta = \frac{\Delta f}{f_m}$$

$$= \frac{250\ \text{Hz}}{1\ \text{kHz}}$$

$$= 0.25$$

Example 5.5 : *In an FM system, when the audio frequency is 500 Hz and modulating voltage 2.5V, the deviation produced is 5 kHz. If the modulating voltage is now increased to 7.5 V, calculate the new value of frequency deviation produced. If the AF voltage is raised to 10V while the modulating frequency dropped to 250 Hz, what is the frequency deviation? Calculate the modulation index in each case.*

Sol. : The modulating voltage of 5.5 V produces frequency deviation of 5 kHz.

Hence,

Frequency deviation constant $k_f = \dfrac{5\,\text{kHz}}{2.5\,\text{V}}$

$\therefore$ $k_f = 2\ \text{kHz/V}.$

The modulating voltage is now 7.5 V.

$\therefore$ Frequency deviation produced $= 2 \times 7.5$

$= 15\ \text{kHz}$

Similarly,

When modulating voltage is 10 V, then

frequency deviation $= [2\ \text{kHz/V}]\ [10\text{V}]$

$= 20\ \text{kHz}$

Δf is typically given as a peak frequency shift in Hertz (Δf). The peak to peak frequ deviation ($2\Delta f$) is sometimes called carrier swing

a) Modulation index $\beta_1 = \frac{\text{Frequency deviation}}{\text{Modulation frequency}}$

$$= \frac{5\,\text{kHz}}{500\,\text{Hz}}$$

$$= 10$$

b) $\beta_2 = \frac{15\,\text{kHz}}{500\,\text{Hz}}$

$$= 30$$

c) $\beta_3 = \frac{20\,\text{kHz}}{250\,\text{Hz}}$

$$= 80$$

Example 5.6 : *A 93.2 MHz carrier is frequency modulated by a 5 kHz sine wave. The resultant FM signal has a frequency deviation of 40 kHz.*

a) Find the carrier swing of the FM signal.

b) What are the highest and lowest frequencies attained by the frequency modulated signal?

c) Calculate the modulation index for the wave.

Sol. : a) Carrier swing = $2 \times$ frequency deviation

$$= 2 \times 40 \text{ kHz}$$

$$= 80 \text{ kHz}$$

b) The highest frequency reached

$$= \text{carrier frequency} + \text{frequency deviation}$$

$$= 93.2 \text{ MHz} + 40 \text{ kHz}$$

$$= 93.24 \text{ MHz}$$

The lowest frequency reached

$$= \text{Carrier frequency} - \text{frequency deviation}$$

$$= 93.2 \text{ MHz} - 40 \text{ kHz}$$

$$= 93.16 \text{ MHz}$$

c) $$\text{Modulation index} = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

$$\beta = \frac{40 \text{ kHz}}{5 \text{ kHz}}$$

$$= 8$$

Example 5.7 : *When a 50.4 MHz carrier is frequency modulated by a sinusoidal modulating signal, the highest frequency reached is 50.405 MHz. Calculate ;*

a) the frequency deviation produced,

b) carrier swing of the wave,

c) lowest frequency reached.

Sol: :

a) Frequency deviation = [Highest frequency reached] – [Carrier frequency]

= [50.405 MHz] – [50.4 MHz]

= 5 kHz

b) Carrier swing = 2× frequency deviation

= 2 × 5

= 10 kHz

c) Lowest frequency attained = [Carrier frequency] – [Frequency deviation]

= 50.4 MHz – 5 kHz

= 50.395 MHz

Example 5.8 : *The carrier swing of a frequency-modulated signal is 70 kHz and the modulating signal is a 7 kHz sine wave. Determine the modulation index of the FM signal.*

Sol. :

$$\text{Carrier swing} = 2 \times \text{frequency deviation}$$

$$\therefore \quad 70 \text{ kHz} = 2 \times \text{frequency deviation}$$

$$\therefore \quad \text{Frequency deviation} = \frac{70 \text{ kHz}}{2}$$

$$= 35 \text{ kHz.}$$

$$\text{Modulation index} = \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{35 \text{ kHz}}{7 \text{ kHz}}$$

$$= 5$$

$$\therefore \quad \beta = 5$$

Example 5.9 : *Calculate the carrier swing, carrier frequency, frequency deviation, and modulation index for an FM signal which reaches a maximum frequency of 99.047 MHz and a minimum frequency of 99.023 MHz. The frequency of the modulating signal is 7 kHz.*

.. : a) Carrier swing = [Highest frequency reached by FM wave]

$$- \text{[Lowest frequency reached by FM wave]}$$

$$= [99.047 \text{ MHz}] - [99.023 \text{ MHz}]$$

$$= 24 \text{ kHz}$$

b) $$\text{Frequency deviation} = \frac{1}{2}\text{[Carrier swing]} = 12 \text{ kHz.}$$

c) $$\text{Carrier frequency} = \text{[Highest frequency reached]} - \text{[Frequency deviatio}$$

$$= 99.04 \text{ MHz} - 12 \text{ kHz}$$

$$= 99.035 \text{ MHz}$$

d) $$\text{Modulation index} = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

$$= \frac{12 \text{ kHz}}{7 \text{ kHz}}$$

$$\therefore \quad \beta = 1.714.$$

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5.3.2 Spectrum Analysis of Sinusoidal FM Wave

Consider a sinusoidal modulating voltage signal, at a single frequency, frequency modulating carrier having a frequency f_c.

The equation for such an F.M. wave is

$$s(t) = A_c \cos\left[\omega_c t + 2\pi k_f \int_0^t m(t)dt\right] \quad \dots (7)$$

In order to determine the bandwidth for this FM wave, let us define

$$a(t) = \int_0^t 2\pi m(t)dt \quad \dots (8)$$

and $$\hat{s}(t) = A_c e^{j[\omega_c t + k_f a(t)]} = A_c e^{jk_f a(t)} e^{j\omega_c t} \quad \dots (9)$$

Then, $$s(t) = \text{Re}\ \hat{s}(t).$$

Expanding the exponential $e^{jk_f a(t)}$ in power series gives

$$\hat{s}(t) = A_c\left[1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \dots\dots + j^n \frac{k_f^n}{n!}a^n(t) + \dots\dots\right]e^{j\omega_c t} \quad \dots (10)$$

and $$s(t) = \text{Re}\ \hat{s}(t)$$

$$= A_c\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!}a^2(t)\cos\omega_c t + \frac{k_f^3}{3!}a^3(t)\sin\omega_c t + \dots\dots\right] \quad \dots (11)$$

The modulated wave consists of an unmodulated carrier plus various amplitude modulated terms, such as, $a(t) \sin \omega_c t$, $a^2(t)\cos \omega_c t$, $a^3(t)\sin \omega_c t$ ....the signal $a(t)$ is an integral of $m(t)$. The frequency spectrum consists of an unmodulated carrier plus spectra of $a(t)$, $a^2(t)$, ...,$a^n(t)$.., centered around ω_c. Clearly, the FM wave theoretically consists of infinite number of sideband, occupying infinite bandwidth. The bandwidth is not related to the modulating signal spectrum in any simple way, as was noted in the case of AM.

Even though the theoretical bandwidth of an FM wave is infinite, most of the modulated signal power is in a finite bandwidth.

Let us obtain the bandwidth for a specific case of single tone modulation ; that is, when $m(t)$ is a sinusoidal.

Let $$m(t) = A_m \cos \omega_m t \qquad \text{... (12)}$$

Then $$a(t) = 2\pi \int_0^t m(t)\,dt = 2\pi \int_0^t A_m \cos \omega_m t\,dt$$

$$\therefore \quad a(t) = A_m \times 2\pi \frac{\sin \omega_m t}{\omega_m} = \frac{A_m}{\omega_m} \sin \omega_m t \times 2\pi \qquad \text{... (13)}$$

Then from equation (9) we have

$$\hat{s}(t) = A_c e^{j\left[\omega_c t + k_f \times 2\pi \frac{A_m}{\omega_m} \sin \omega_m t\right]}$$

The frequency deviation $\Delta f = A_m k_f = k_f m_p$ and bandwidth of $m(t)$ is $B = f_m$.

The modulation index, in this case, is

$$\beta = \frac{\Delta f}{f_m} = \frac{A_m k_f}{f_m} = \frac{2\pi A_m k_f}{2\pi f_m} = \frac{2\pi A_m k_f}{\omega_m} \qquad \text{... (14)}$$

$$\therefore \quad \hat{s}(t) = A_c\, e^{[j\omega_c t + j\beta \sin \omega_m t]} \qquad \text{...(15)}$$

5.4 Narrow Band Frequency Modulation

We have seen that although the theoretical bandwidth of an FM wave is infinite, most of the modulated signal power resides in a finite bandwidth. In terms of bandwidth, there are two different possibilities - **narrowband FM** and **wideband FM.**

We have obtained the equation :

$$s_{FM}(t) = A_c\left[\cos\omega_c - k_f\, a(t)\sin\omega_c t - \frac{k_f^{\,2}}{2!}a^2(t)\cos\omega_c t + \frac{k_f^{\,3}}{3!}a^3(t)\sin\omega_c t + \ldots\ldots\right] \qquad \ldots (1)$$

where $\quad a(t) = \int_0^t 2\pi\, m(t)dt$

$e^{j\theta} = \cos\theta + j\sin\theta$

$R(e^{j\theta}) = \cos\theta$

If k_f is very small, i.e. if $|\, k_f\, a(t)\,| << 1$, then all the terms except the first two terms in above equation are negligible, and we have

$R(e^{j\theta}) = \sin\theta$

$$s_{FM}(t) = A_c\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right] \qquad \ldots (2)$$

This expression is similar to that of the AM wave. Thus, when β is small, similar to AM, there are only two significant sidebands, one lower and the upper.

This is known as narrow band FM

The narrow-band PM wave is similarly given by

$$s_{PM}(t) = A_c[\cos\omega_c t - k_p m(t)\sin\omega_c t] \quad \dots (3)$$

A comparison of narrow-band FM with AM indicates the similarities and differences between the two types of modulation. In both types, we have a carrier term and sidebands centered at $\pm\,\omega_c$. The modulated signal bandwidths are identical, viz, 2 × highest modulating frequency.

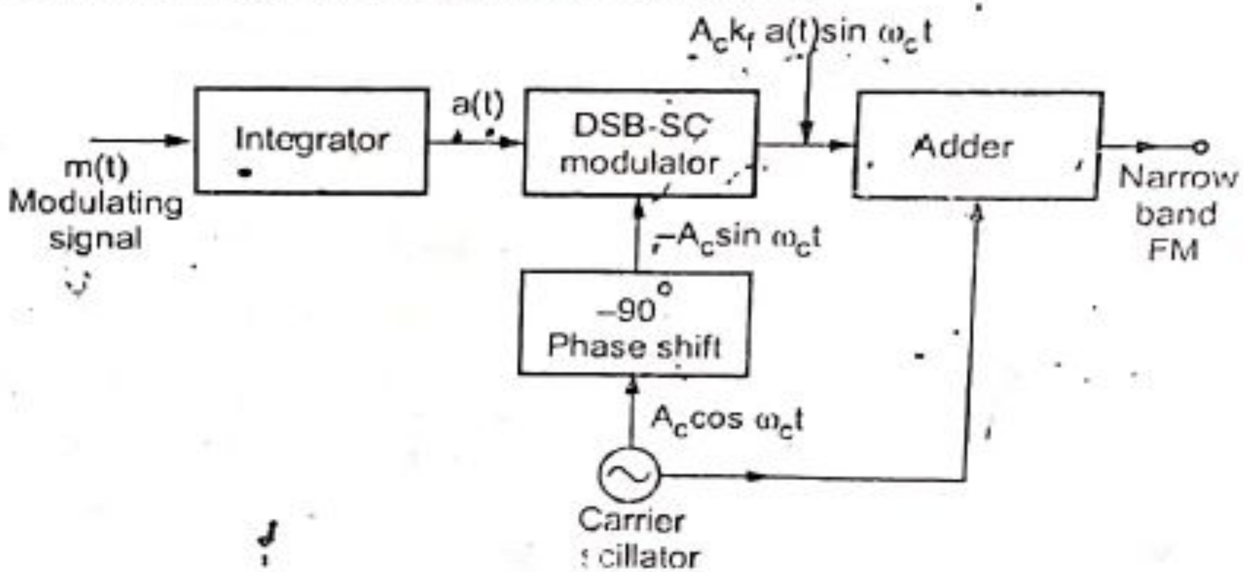
However, the AM and FM signals have very different waveforms. In an AM signal, the frequency is constant and the amplitude is varied in accordance with modulating signal, whereas in FM signal, the amplitude is constant and the frequency is varied.

The equations (2) and (3) indicate a possible way of generating narrow-band FM and PM signals by using DSB-SC modulators (balanced modulator). The block schematic of such systems is shown in the Fig. 5.10.

PM signals by using DSB-SC modulators (balanced modulator). The block schematic of such systems is shown in the Fig. 5.10.

Figure 5.10 (a)

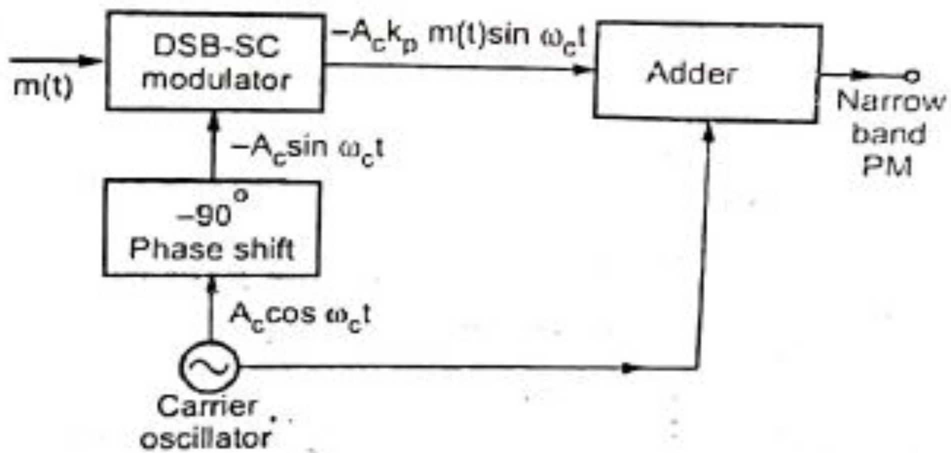
Generation of narrow band FM using DSB-SC modulator



$s(t) = A_c\left(1 + k_f \int_0^t m(t)\right) \cos \omega_c t$

Figure 5.10 (b)

Generation of narrow band PM using DSB-SC modulator



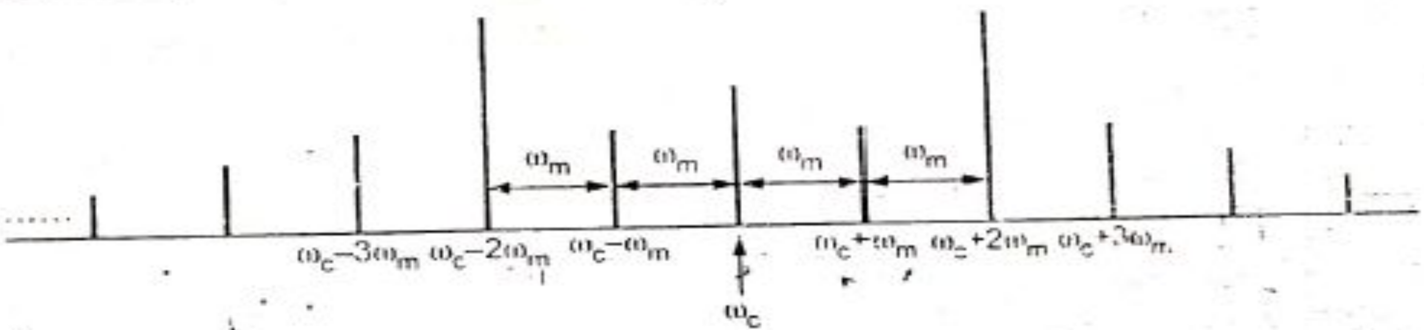
$PM = A_c\left(1 + k_p m(t)\right) \cos \omega_c t$

Using Bessel functions, it may be shown that the equation for FM is

$$\begin{aligned} s(t) = A_c\{&J_0(\beta)\sin\omega_c t + J_1(\beta)[\sin(\omega_c+\omega_m)t - \sin(\omega_c-\omega_m)t] \\ &+ J_2(\beta)[\sin(\omega_c+2\omega_m)t - \sin(\omega_c-2\omega_m)t] \\ &+ J_3(\beta)[\sin(\omega_c+3\omega_m)t - \sin(\omega_c-3\omega_m)t] \\ &+ \ldots\ldots\ldots\ldots\} \end{aligned} \quad \ldots (18)$$

Thus the modulated signal has a carrier component and an infinite number of side frequencies $\omega_c \pm \omega_m$; $\omega_c \pm \omega_m$, $\omega_c \pm 3\omega_m$.............. $\omega_c \pm n\omega_m$, ... as shown in the Fig. 5.8.

Figure 5.8



The amplitude of the n^{th} side frequency at $\omega = \omega_c + n\,\omega_m$ is $J_n(\beta)$. As $J_{-n}(\beta) = (-1)^n J_n(\beta)$, the magnitude of the lower side frequency at $\omega = \omega_c - n\,\omega_m$ is the same as that of the upper side frequency at $\omega = \omega_c + n\,\omega_m$.

From the plots of $J_n(\beta)$ it is seen that for a given β, in $J_n(\beta)$ decreases with n. For a sufficiently large n, $J_n(\beta)$ is so small that it can be neglected, and there are only a finite number of significant side frequencies. Normally $J_n(\beta)$ is negligible for $n > \beta + 1$. Hence number of significant sidebands is $(\beta + 1)$. Then the bandwidth of the FM signal is given by

$$BW_{FM} = 2nf_m = 2(\beta + 1)f_m$$

Bessel functions

x	n or Order																
(β)	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.4 2.5	–0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	–0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	–0.40	–0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	–0.18	–0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
5.45 6.0	0.15	–0.28	–0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	–0.30	–0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	–0.11	–0.29	–0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	0.01	—	—	—	—
5.165 9.0	–0.09	0.24	0.14	–0.18	–0.27	–0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03		—	—	—
10.0	–0.25	0.04	0.25	0.06	–0.22	–0.23	–0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	–0.22	–0.08	0.20	0.18	–0.07	–0.24	–0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	–0.01	0.21	0.04	–0.19	–0.12	0.13	0.21	0.03	–0.17	–0.22	–0.09	0.10	0.24	0.28	0.25	0.18	0.12

$$\begin{aligned} BW_{FM} &= 2nf_m = 2(\beta + 1)f_m \\ &= 2\beta(f_m) + 2f_m = 2\Delta f + 2BW \text{ of } m(t) \\ &= 2[\Delta f + BW \text{ of } m(t)] \qquad \ldots (19) \end{aligned}$$

where m(t) is modulating signal.

i.e. $\quad BW_{FM} = s[2\Delta f + B] \leftarrow W \qquad \ldots (20)$

where B is bandwidth of modulating signal

This relations is known as **Carson's Rule**

When $\beta << 1$, which represents narrow band FM, there is only one significant sideband and

$$BW_{FM} = 2B; \qquad \beta << 1 \qquad \ldots (21)$$

Example 5.10 : *A carrier wave of amplitude 5V and frequency 90 MHz is frequency modulated by a sinusoidal voltage of amplitude 5V and frequency 15 kHz. The frequency deviation constant is 1 kHz /V. Sketch the spectrum of the modulated FM wave.*

Sol. : Amplitude of modulating voltage is 5V and frequency deviation constant is 1kHz/V.

k_f = 1 kHz/V

Hence, Frequency deviation = [1 kHz/V] [5 V] = 5 kHz.

$$\text{Modulation index } \beta = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{5 \text{ kHz}}{15 \text{ kHz}} = 0.333.$$

For $\beta = 0.333$, from the table of Bessel functions, (Refer table 5.1).

Use approximate, values for J_0 , J_1 and J_2

For carrier : $J_0 = 0.96$

First side frequency : $J_1 = 0.18$

Second side frequency : $J_2 = 0.02$

These are the values for unmodulated carrier of 1V.

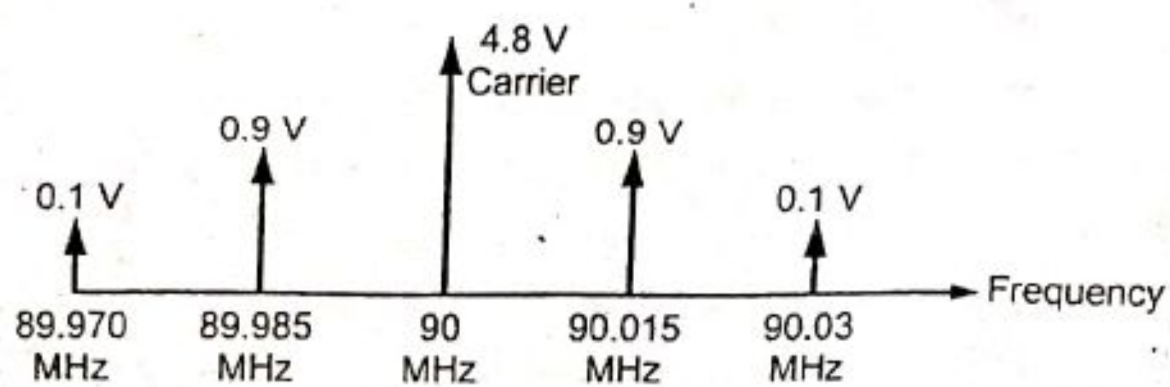
Higher order side frequencies are negligible since β is small

The amplitude of the carrier is 5V (given).

Hence :
$$J_0 = 0.96 \times 5 = 4.8 \text{ V}$$
$$J_1 = 0.18 \times 5 = 0.9 \text{ V}; \quad J_2 = 0.02 \times 5 = 0.1 \text{ V}.$$

The frequency spectrum is shown below :

Figure 5.9



Modulating frequency is 15 kHz.

Electrical noise is defined as any undesirable electrical energy that falls within the passband of the signal. For example, in audio recording, any unwanted electrical signals that fall within the audio frequency band of 0 Hz to 15 kHz will interfere with the music and therefore be considered noise. Figure 1-6 shows the effect that noise has on an electrical signal. Figure 1-6a shows a sine wave without noise, and Figure 1-6b shows the same signal except in the presence of noise. The grassy-looking squiggles superimposed on the sine wave in Figure 1-6b are electrical noise, which contains a multitude of frequencies and amplitudes that can interfere with the quality of the signal.

Noise can be divided into two general categories: *correlated* and *uncorrelated.* Correlation implies a relationship between the signal and the noise. Therefore, correlated noise exists only when a signal is present. Uncorrelated noise, on the other hand, is present all the time whether there is a signal or not.

1-7-1 Uncorrelated Noise

Uncorrelated noise is present regardless of whether there is a signal present or not. Uncorrelated noise can be further subdivided into two general categories: external and internal.

External noise. *External noise* is noise that is generated outside the device or circuit. The three primary sources of external noise are atmospheric, extraterrestrial, and man-made.

Atmospheric noise. Atmospheric noise is naturally occurring electrical disturbances that originate within Earth's atmosphere. Atmospheric noise is commonly called

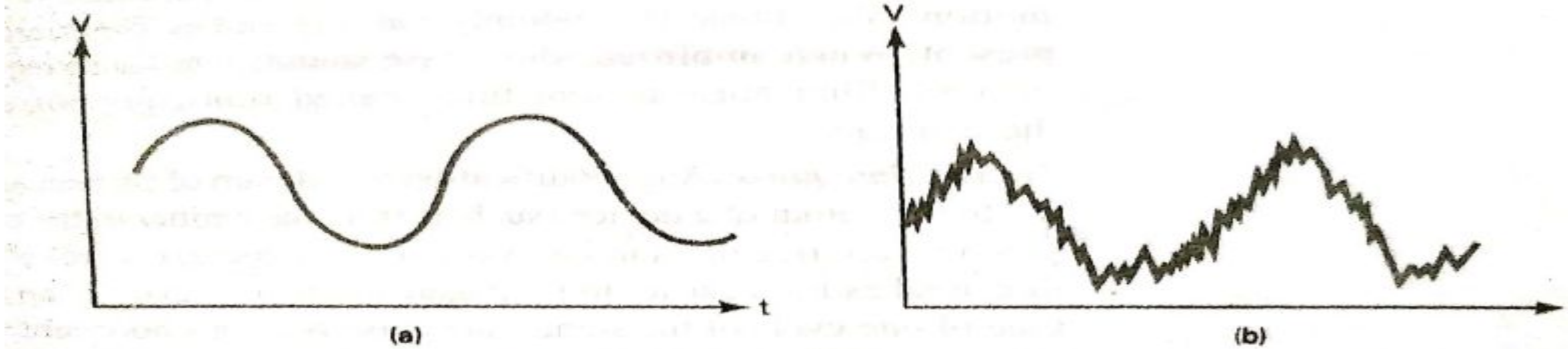


FIGURE 1-6 Effects of noise on a signal: (a) signal without noise; (b) signal with noise

static electricity and is the familiar sputtering, crackling, and so on often heard from a speaker when there is no signal present. The source of most static electricity is naturally occurring electrical conditions, such as lightning. Static electricity is often in the form of impulses that spread energy throughout a wide range of frequencies. The magnitude of this energy, however, is inversely proportional to its frequency. Consequently, at frequencies above 30 MHz or so, atmospheric noise is relatively insignificant.

Extraterrestrial noise. Extraterrestrial noise consists of electrical signals that originate from outside Earth's atmosphere and is therefore sometimes called *deep-space noise*. Extraterrestrial noise originates from the Milky Way, other galaxies, and the sun. Extraterrestrial noise is subdivided into two categories: solar and cosmic.

Solar noise is generated directly from the sun's heat. There are two parts to solar noise: a *quiet* condition, when a relatively constant radiation intensity exists, and *high intensity,* sporadic disturbances caused by *sunspot* activity and *solar flare-ups*. The magnitude of the sporadic noise caused by sunspot activity follows a cyclic pattern that repeats every 11 years.

Cosmic noise sources are continuously distributed throughout the galaxies. Because the sources of galactic noise are located much farther away than our sun, their noise intensity is relatively small. Cosmic noise is often called *black-body noise* and is distributed fairly evenly throughout the sky.

Man-made noise. Man-made noise is simply noise that is produced by mankind. The predominant sources of man-made noise are spark-producing mechanisms, such as commutators in electric motors, automobile ignition systems, ac power-generating and switching equipment, and fluorescent lights. Man-made noise is impulsive in nature and contains a wide range of frequencies that are propagated through space in the same manner as radio waves. Man-made noise is most intense in the more densely populated metropolitan and industrial areas and is therefore sometimes called *industrial noise.*

Internal noise. *Internal noise* is electrical interference generated within a device or circuit. There are three primary kinds of internally generated noise: shot, transit time, and thermal.

Shot noise. Shot noise is caused by the random arrival of carriers (holes and electrons) at the output element of an electronic device, such as a diode, field-effect transistor, or bipolar transistor. Shot noise was first observed in the anode current of a vacuum-tube amplifier and was described mathematically by W. Schottky in 1918. The current carriers (for both ac and dc) are not moving in a continuous, steady flow, as the distance they travel varies because of their random paths of motion. Shot noise is randomly varying and is superimposed onto any signal present. When amplified, shot noise sounds similar to metal pellets falling on a tin roof. Shot noise is sometimes called *transistor noise* and is additive with thermal noise.

Transit-time noise. Any modification to a stream of carriers as they pass from the input to the output of a device (such as from the emitter to the collector of a transistor) produces an irregular, random variation categorized as *transit-time noise.* When the time it takes for a carrier to propagate through a device is an appreciable part of the time of one cycle of the signal, the noise becomes noticeable. Transit-time noise in transistors is determined by carrier mobility, bias voltage, and transistor construction.

Carriers traveling from emitter to collector suffer from emitter-time delays, base transit-time delays, and collector recombination-time and propagation-time delays. If transit delays are excessive at high frequencies, the device may add more noise than amplification to the signal.

Thermal noise. Thermal noise is associated with the rapid and random movement of electrons within a conductor due to thermal agitation. The English botanist Robert Brown first noted this random movement. Brown first observed evidence for the moving-particle nature of matter in pollen grains and later noticed that the same phenomenon occurred with smoke particles. J. B. Johnson of Bell Telephone Laboratories first recognized random movement of electrons in 1927. Electrons within a conductor carry a unit negative charge, and the mean-square velocity of an electron is proportional to the absolute temperature. Consequently, each flight of an electron between collisions with molecules constitutes a short pulse of current that develops a small voltage across the resistive component of the conductor. Because this type of electron movement is totally random and in all directions, it is sometimes called random noise. With random noise, the average voltage in the substance due to electron movement is 0 V dc. However, such a random movement does produce an ac component.

Thermal noise is present in all electronic components and communications systems. Because thermal noise is uniformly distributed across the entire electromagnetic frequency spectrum, it is often referred to as *white noise* (analogous to the color white containing all colors, or frequencies, of light). Thermal noise is a form of additive noise, meaning that it cannot be eliminated, and it increases in intensity with the number of devices in a circuit and with circuit length. Therefore, thermal noise sets the upper bound on the performance of a communications system.

The ac component produced from thermal agitation has several names, including *thermal noise*, because it is temperature dependent; *Brownian noise*, after its discoverer; *Johnson noise*, after the man who related Brownian particle movement of electron movement; and *white noise* because the random movement is at all frequencies. Hence, thermal noise is the random motion of free electrons within a conductor caused by thermal agitation.

Johnson proved that thermal noise power is proportional to the product of bandwidth and temperature. Mathematically, noise power is

$$N = KTB \tag{1-13}$$

where N = noise power (watts)
B = bandwidth (hertz)
K = Boltzmann's proportionality constant (1.38×10^{-23} joules per kelvin)
T = absolute temperature (kelvin) (room temperature = 17°C, or 290 K)

To convert °C to kelvin, simply add 273°; thus, T = °C + 273°.

Example 1-10

Convert the following temperatures to kelvin: 100°C, 0°C, and −10°C

Solution The formula T = °C + 273° is used to convert °C to kelvin.

$$T = 100°\text{C} + 273° = 373\text{ K}$$
$$T = 0°\text{C} + 273° = 273\text{ K}$$
$$T = -10°\text{C} + 273° = 263\text{ K}$$

Noise power stated in dBm is a logarithmic function and equal to

$$N_{(\text{dBm})} = 10 \log \frac{KTB}{0.001} \tag{1-14}$$

Equations 1-13 and 1-14 show that at absolute zero (0 K, or −273° C), there is no random molecular movement, and the product *KTB* equals zero.

Rearranging Equation 1-14 gives

$$N_{(\mathrm{dBm})} = 10 \log \frac{KT}{0.001} + 10 \log B \tag{1-15}$$

and for a 1-Hz bandwidth at room temperature,

$$N_{(\mathrm{dBm})} = 10 \log \frac{(1.38 \times 10^{-23})(290)}{0.001} + 10 \log 1$$

$$= -174 \text{ dBm}$$

Thus, at room temperature, Equation 1-14 can be rewritten for any bandwidth as

$$N_{(\mathrm{dBm})} = -174 \text{ dBm} + 10 \log B \tag{1-16}$$

Random noise results in a constant power density versus frequency, and Equation 1-13 indicates that the available power from a thermal noise source is proportional to bandwidth over any range of frequencies. This has been found to be true for frequencies from 0 Hz to the highest microwave frequencies used today. Thus, if the bandwidth is unlimited, it appears that the available power from a thermal noise source is also unlimited. This, of course, is not true, as it can be shown that at arbitrarily high frequencies thermal noise power eventually drops to zero. Because thermal noise is equally distributed throughout the frequency spectrum, a thermal noise source is sometimes called a *white noise source*, which is analogous to white light, which contains all visible-light frequencies. Therefore, the rms noise power measured at any frequency from a white noise source is equal to the rms noise power measured at any other frequency from the same noise source. Similarly, the total rms noise power measured in any fixed bandwidth is equal to the total rms noise power measured in an equal bandwidth anywhere else in the total noise spectrum. In other words, the rms white noise power present in the band from 1000 Hz to 2000 Hz is equal to the rms white noise power present in the band from 1,001,000 Hz to 1,002,000 Hz.

Thermal noise is random and continuous and occurs at all frequencies. Also, thermal noise is predictable, additive, and present in all devices. This is why thermal noise is the most significant of all noise sources.

1-7-2 Noise Voltage

Figure 1-7 shows the equivalent circuit for a thermal noise source where the internal resistance of the source (R_I) is in series with the rms noise voltage (V_N). For the worst-case condition and maximum transfer of noise power, the load resistance (R) is made equal to R_I. Thus, the noise voltage dropped across R is equal to half the noise source ($V_R = V_N/2$),

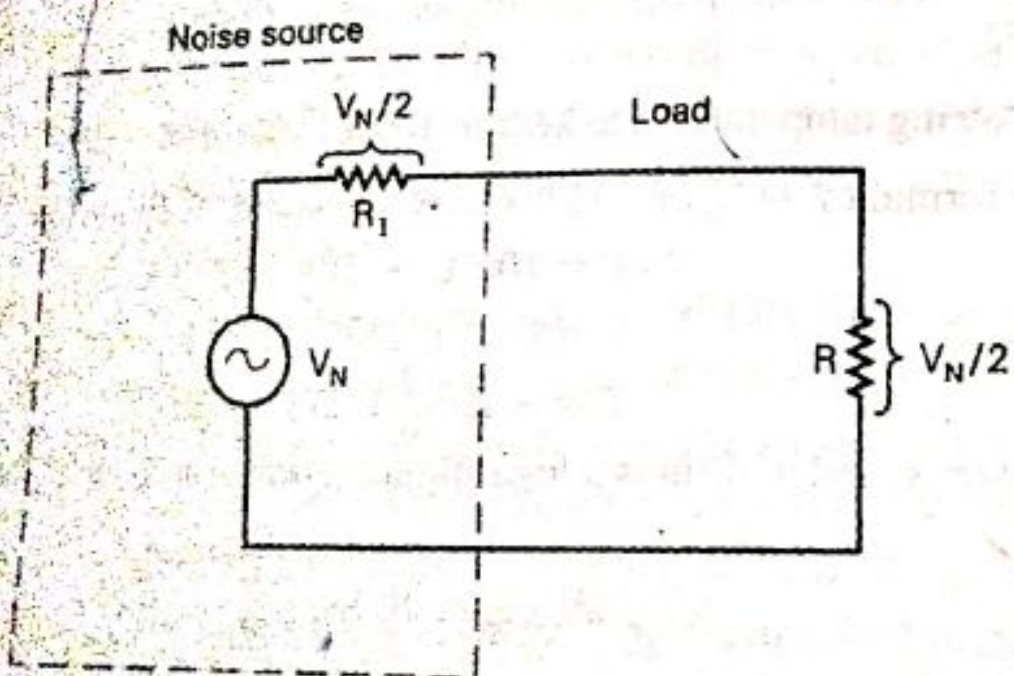


FIGURE 1-7 Noise source equivalent circuit

Random Noise result in a const power density v/s frequency

Thermal Noise is random & continuous & occurs at all frequencies. Also thermal noise is predictable, additive & present in all devices. This is why thermal Noise is the most significant of all noise sources.

Noise voltage

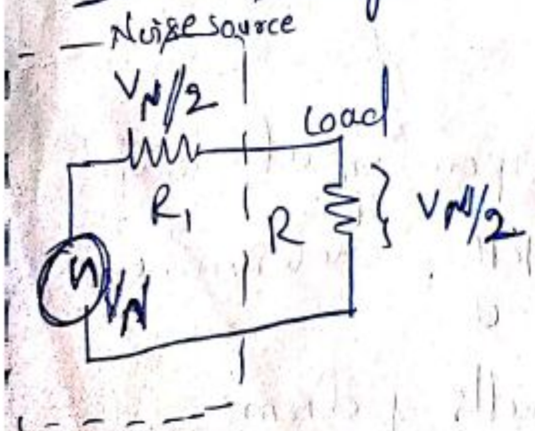


Fig shows the equivalent ckt for a thermal Noise Source where the internal resistance of the Source (R_1) is in Series with the rms noise voltage (V_N). For th

For maximum transfer of noise power

the load resistance (R) is made equal to R_I then

the noise voltage dropped across R is equal to

half the noise source ($V_R = V_N/2$)

From the Equat $N = KTB = \dfrac{(V_N/2)^2}{R}$ $\quad P = V \times \frac{V}{R} = \frac{V^2}{R}$

$$= \frac{V_N^2}{4R}$$

$$V_N^2 = 4RKTB.$$

$$V_N = \sqrt{4RKTB}$$

(Ex). For an Electronic device operating at temp^r of 17°C with a bandwidth of 10kHz. determine

(a) thermal noise power in watts & dBm

(b) rms noise voltage for a 100Ω - internal resistance & a 100Ω - load resistance.

(a) Thermal Noise power is found by substituting $N = KTB$.

$T(\text{Kelvin}) = 17^{0}c + 273^{0} = \underline{290\,K}$

$$N = \underset{K}{(1.38 \times 10^{-23})}\, \underset{T}{(290)}\, (1 \times 10^{4}) = 4 \times 10^{-17}\, W$$

substituting Eqn giving the noise power in dBm

$$N_{(dBm)} = 10 \log \left[\frac{4 \times 10^{-17}}{1 \times 10^{-3}} \right] = -13\underline{4}\ dBm$$

Substitute

$$N_{(dBm)} = -174\ dBm + 10 \log 10{,}000$$

$$= -174\ dBm + 40\ dB$$

$$= -\underline{134}\ dBm.$$

5) The rms noise voltage is found by substitut into Eq.

$$V_N = \sqrt{4RKTB} \qquad KTB = 4 \times 10^{-17}$$

$$= \sqrt{(4)(100)(4 \times 10^{-17})} = \underline{0.1265\,\mu V}$$

1-7-3 Correlated Noise

Correlated noise is a form of internal noise that is correlated (mutually related) to the signal and cannot be present in a circuit unless there is a signal—simply stated, *no signal, no noise!* Correlated noise is produced by *nonlinear amplification* and includes *harmonic* and *intermodulation distortion,* both of which are forms of *nonlinear distortion.* All circuits are nonlinear to some extent; therefore, they all produce nonlinear distortion. Nonlinear distortion creates unwanted frequencies that interfere with the signal and degrade performance.

Harmonic distortion occurs when unwanted *harmonics* of a signal are produced through nonlinear amplification (*nonlinear mixing*). Harmonics are integer multiples of the original signal. The original signal is the *first harmonic* and is called the *fundamental frequency.* A frequency two times the original signal frequency is the *second harmonic,* three times is the *third harmonic,* and so forth. *Amplitude distortion* is another name for harmonic distortion.

There are various degrees of harmonic distortion. Second-order harmonic distortion is the ratio of the rms amplitude of the second harmonic to the rms amplitude of the fundamental. Third-order harmonic distortion is the ratio of the rms amplitude of the third harmonic to the rms value of the fundamental. A more meaningful measurement is total harmonic distortion (TDH), which is the ratio of the quadratic sum of the rms values of all the higher harmonics to the rms value of the fundamental.

Figure 1-8a show the input and output frequency spectrums for a nonlinear device with a single input frequency (f_1). As the figure shows, the output spectrum contains the

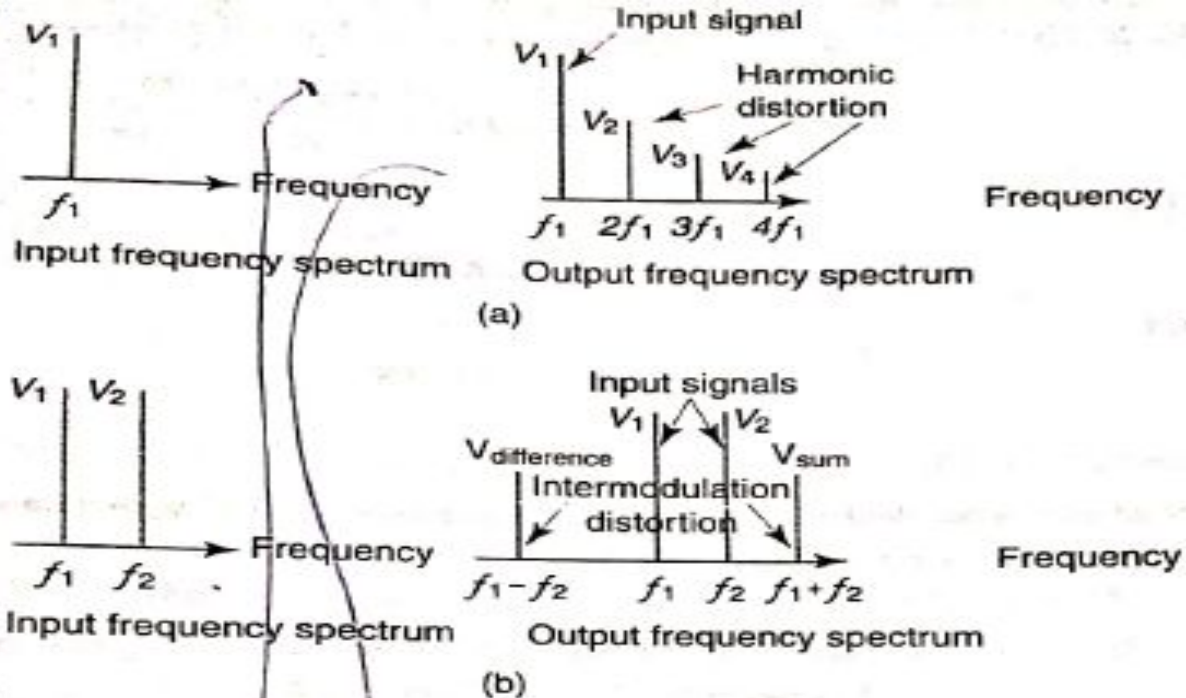


FIGURE 1-8 Correlated noise: (a) Harmonic distortion; (b) Intermodulation distortion

original input frequency plus several harmonics ($2f_1$, $3f_1$, $4f_1$) that were not part of the original signal.

Mathematically, total harmonic distortion (THD) is

$$\% \text{ THD} = \frac{v_{\text{higher}}}{v_{\text{fundamental}}} \times 100 \tag{1-18}$$

where
% THD = percent total harmonic distortion
v_{higher} = quadratic sum of the rms voltages of the harmonics above the fundamental frequency, $\sqrt{v_2^2 + v_3^2 + v_n^2}$
$v_{\text{fundamental}}$ = rms voltage of the fundamental frequency

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Intermodulation distortion is the generation of unwanted *sum* and *difference* frequencies produced when two or more signals mix in a nonlinear device. The sum and difference frequencies are called *cross products*. The emphasis here is on the word *unwanted* because in communications circuits it is often desirable to produce harmonics or to mix two or more signals to produce sum and difference frequencies. Unwanted cross-product frequencies can interfere with the information signals in a circuit or with the information signals in other circuits. Cross products are produced when harmonics as well as fundamental frequencies mix in a nonlinear device.

Mathematically, the sum and difference frequencies are

$$\text{cross products} = mf_1 \pm nf_2 \tag{1-19}$$

where f_1 and f_2 are fundamental frequencies, where $f_1 > f_2$, and m and n are positive integers between one and infinity.

Figure 1-8b shows the input and output frequency spectrums for a nonlinear device with two input frequencies (f_1 and f_2). As the figure shows, the output spectrum contains the two original frequencies plus their sum and difference frequencies ($f_1 - f_2$ and $f_1 + f_2$). In actuality, the output spectrum would also contain the harmonics of the two input frequencies ($2f_1$, $3f_1$, $2f_2$, and $3f_2$), as the same nonlinearities that caused the intermodulation distortion would also cause harmonic distortion. The harmonics have been eliminated from the diagram for simplicity.

Determine

a. 2nd, 3rd, and 12th harmonics for a 1-kHz repetitive wave.

b. Percent second-order, third-order, and total harmonic distortion for a fundamental frequency with an amplitude of 8 Vrms, a second harmonic amplitude of 0.2 Vrms, and a third harmonic amplitude of 0.1 Vrms.

Solution a. Harmonic frequencies are simply integer multiples of the fundamental frequency:

$$\text{2nd harmonic} = 2 \times \text{fundamental} = 2 \times 1\text{ kHz} = 2\text{ kHz}$$

$$\text{3rd harmonic} = 3 \times \text{fundamental} = 3 \times 1\text{ kHz} = 3\text{ kHz}$$

$$\text{12th harmonic} = 12 \times \text{fundamental} = 12 \times 1\text{ kHz} = 12\text{ kHz}$$

b.

$$\%\text{ 2nd order} = \frac{V_2}{V_1} \times 100 = \frac{0.2}{8} \times 100 = 2.5\%$$

$$\%\text{ 3rd order} = \frac{V_3}{V_1} \times 100 = \frac{0.1}{8} \times 100 = 1.25\%$$

$$\%\text{ THD} = \frac{\sqrt{(0.2)^2 + (0.1)^2}}{8} = 2.795\%$$