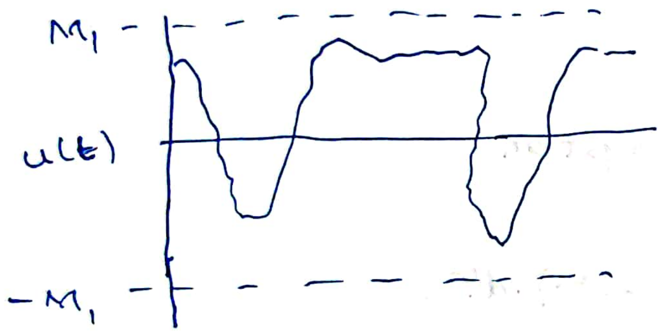
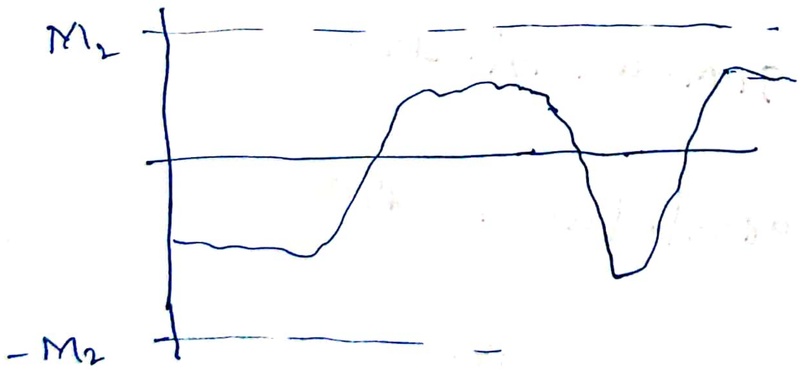
STABILITY OF LTI SYSTEM

Definition of BIBO stability

Any dynamical system is said to be BIBO stable, if for every bounded input signal $|u(t)| \leq M_1 < \infty$ for all t,



it produces a bounded output $y(t)$,



$|y(t)| \leq M_2 < \infty$, for all t.

Theorem (Time Domain Condition)

Given LTI system is BIBO stable if and only if its impulse response is absolutely integrable;

$$\int_0^\infty |g(t)|\, dt \le M < \infty .$$

Proof: For an LTI system

$$y(t) = \int g(\tau) \cdot u(t-\tau) \cdot d\tau .$$

$$|y(t)| = \left| \int g(\tau) \cdot u(t-\tau) \cdot d\tau \right|$$

$$|y(t)| \le \int |g(\tau) \cdot u(t-\tau)| \cdot d\tau .$$

$$\le \int |g(\tau)| \cdot |u(t-\tau)| \cdot d\tau$$

$$\le \int |g(\tau)| \cdot M_1 \cdot d\tau .$$

$\therefore$ $|y(t)| < \infty$ ~~only~~ if $\int |g(\tau)| \cdot d\tau \le M < \infty$.

If $\int |g(\tau)|\, d\tau$ is NOT absolutely integrable we can show that there exists a bounded input that will produce unbounded output.

Theorem (s-domain Condition)

An LTI system is BIBO stable if and only if all the poles are in left half of s-plane.

Proof:

$$G(s) = \frac{K(s^m + b_1 s^{m-1} + \dots + b_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

without loss of generality

$$G(s) = \frac{A_1}{(s-p_1)} + \frac{A_2}{(s-p_2)} + \dots \frac{A_n}{(s-p_n)}$$

when poles are all real.

$$g(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} + \dots + A_n e^{p_n t}$$

$$\int |g(t)| \cdot dt \leq M < \infty$$

if and only if all $p_k < 0$.

When complex conjugate poles

Time Domain function will be of form

$$f(t) = e^{Re(P)t} \cdot \cos(Im(P))t$$

OR

$$f(t) = e^{Re(P)t} \cdot \sin(Im(P))t .$$

This function is absolutely integrable only when $Re(P) < 0$.