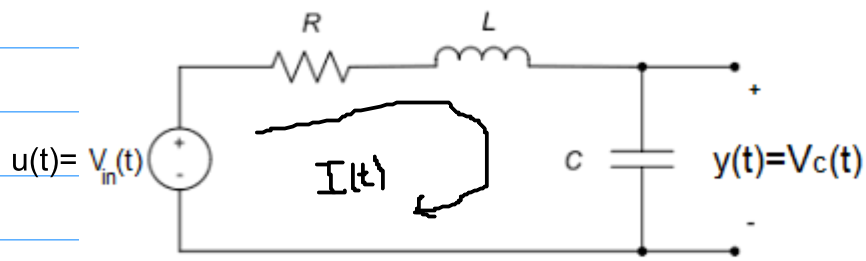
Mathematical Modeling of Electrical Circuits:

Example 2: Series RLC Circuit



By Kirchoff's Voltage Law:

$$V_{in}(t) = R\,I(t) + L\frac{dI(t)}{dt} + V_c(t)$$

$$\boxed{I(t) = C\frac{dV_c}{dt}}$$

$$= RC\frac{dV_c}{dt} + LC\frac{d^2V_c(t)}{dt^2} + V_c(t)$$

$$u(t) = RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t)$$

Apply Laplace Transform:

$$\mathcal{L}\{u(t)\} = RC\,\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + LC\left\{\frac{d^2y(t)}{dt^2}\right\}$$

$$+ \mathcal{L}\{y(t)\}$$

Assuming zero initial conditions:

$$U(s) = RCsY(s) + LCs^2Y(s) + Y(s)$$

Therefore Transfer Function of the circuit is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{RCs + LCs^2 + 1}$$

$$\boxed{G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

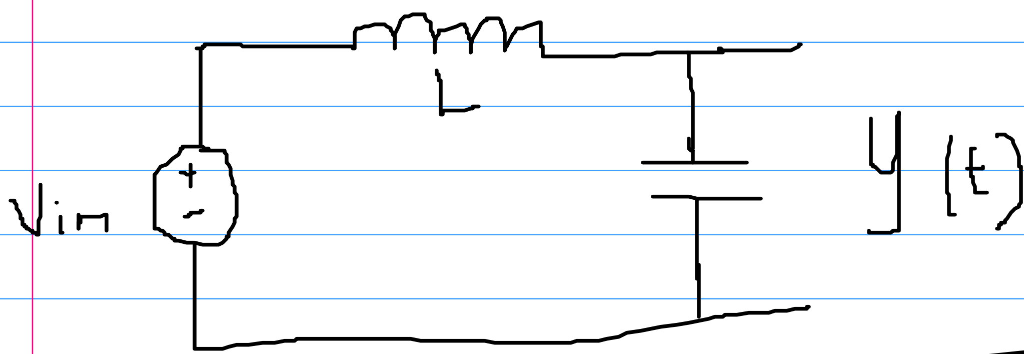
Ex: $R = 1\Omega \quad L = 0.2\,H \quad C = 0.2\,F$

$$G(s) = \frac{25}{s^2 + 5s + 25}$$

$$= \frac{(25)\sqrt{18.75}}{(s+2.5)^2 + 18.75} \times \frac{1}{\sqrt{18.75}}$$

$$g(t) = 5.773\,e^{-2.5t}\sin\sqrt{18.75}\,t$$

$$\mathcal{L}\left\{ e^{-at} \sin \omega_0 t \right\} = \frac{\omega_0}{(s+a)^2 + \omega_0^2}$$



$$G(s) = \frac{25}{s^2 + 25}$$

Impulse Response is

g(t) = 5 sin(5t)

when $U(s) = \frac{1}{s}$

$$Y(s) = \frac{25}{s\,(s^2+25)}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 25}$$

$$= \frac{1}{s} - \frac{s}{s^2 + 25}$$

Step response of LC circuit with y(t)=Voltage across capacitor

$$y(t) = 1 - \cos \frac{1}{\sqrt{LC}} t \qquad \text{for } t >= 0$$

Step response of series RLC circuit: R = 1 Ohm, L=0.2 H, C=0.2 F

$$Y(s) = \frac{25}{s(s^2 + 5s + 25)}$$

$$Y(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 5s + 25}$$

$$A = 1 \qquad B = -1 \qquad C = -5$$

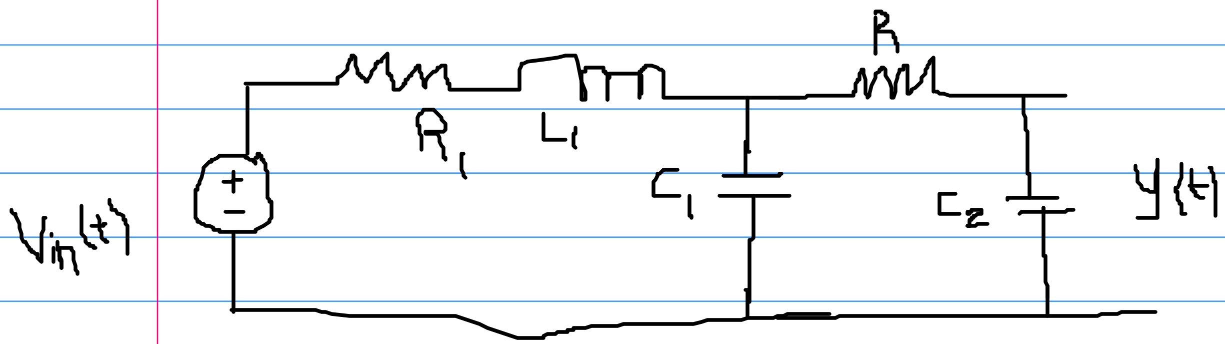
$$Y(s) = \frac{1}{s} - \frac{(s+2.5) - 2.5}{(s+2.5)^2 + 18.75}$$

$$y(t) = u_s(t) - e^{-2.5t} \cos \sqrt{18.75}\, t$$

$$- e^{-2.5t} \frac{2.5}{\sqrt{18.25}} \sin \sqrt{18.75}\, t$$

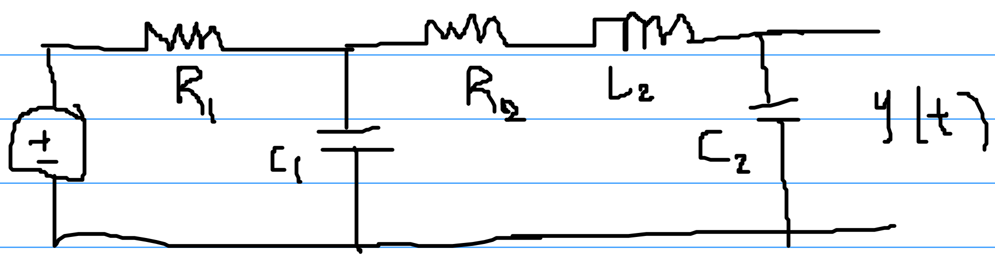
Ex1

Write the differential equation only in terms of u(t) and y(t); hence obtain transfer function G(s).



Ex2:

Write the differential equation only in terms u(t) and y(t); hence obtain transfer func
G(s)



Ex3:

Write the differential equation in terms of u(t) and y(t)

Hence obtain transfer function G(s)