Probelm Statement

For the plant **G(s) = 1/s(s+6)** design a **phase-lead** controller for damping ratio **ζ = 0.4** and natural frequency **15 rad/sec**.

What is the phase margin and gain margin of the compensated system?

Solution

Clearing Workspace

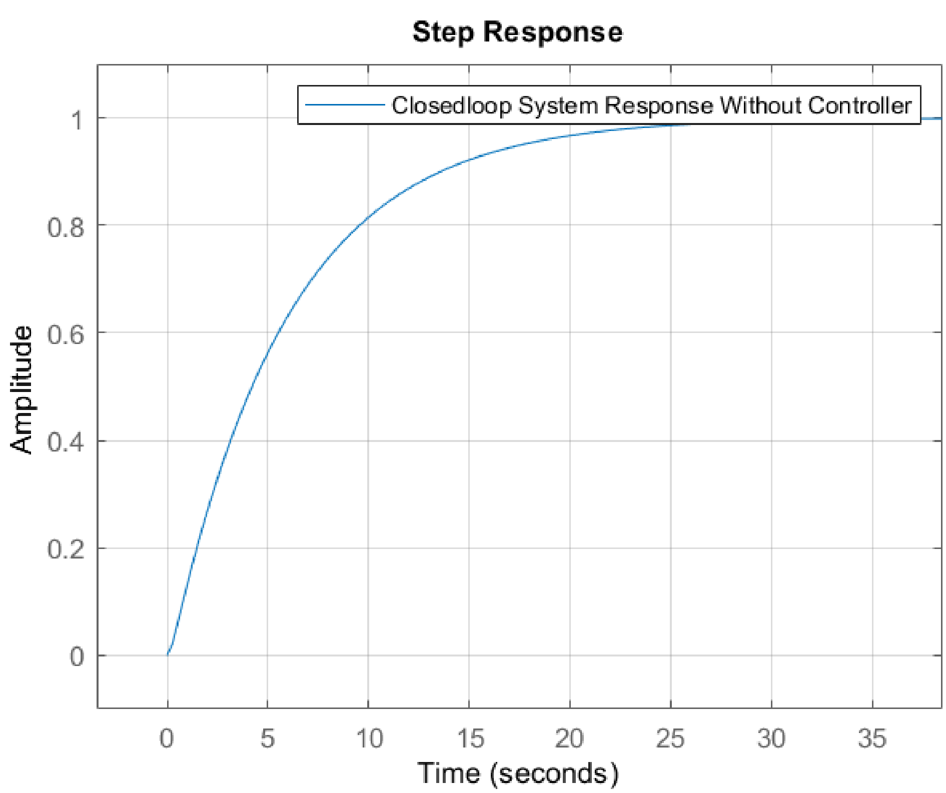
```
close all;
clear;
clc;
```

System without a controller

```
s=tf('s');
G=1/(s*(s+6));
```

Gcl = Closedloop Transfer Function

```
Gcl = G/(1+G); % system without controller
step(Gcl); % plotting step response
grid on;
setAxisLimits(axis);
legend('Closedloop System Response Without Controller');
```



Poles fo the system without controller

```
disp(pole(Gcl));
```

```
         0
   -6.0000
   -5.8284
   -0.1716
```

One of the pole is on the imaginary axis, and therefore, the system without controller is marginally stable.

Time domain parameters of system

```
stepinfo(Gcl)
```

```
ans = struct with fields:
        RiseTime: 12.8096
    SettlingTime: 22.9766
     SettlingMin: 0.9016
     SettlingMax: 0.9993
       Overshoot: 0
      Undershoot: 0
            Peak: 0.9993
        PeakTime: 42.6770
```

Designing a Phase Lead Controller

ζ = 0.4

ωn = 15

```
zita = 0.4;
wn = 15;
desiredPoles = roots([1 2*zita*wn wn^2]);
```

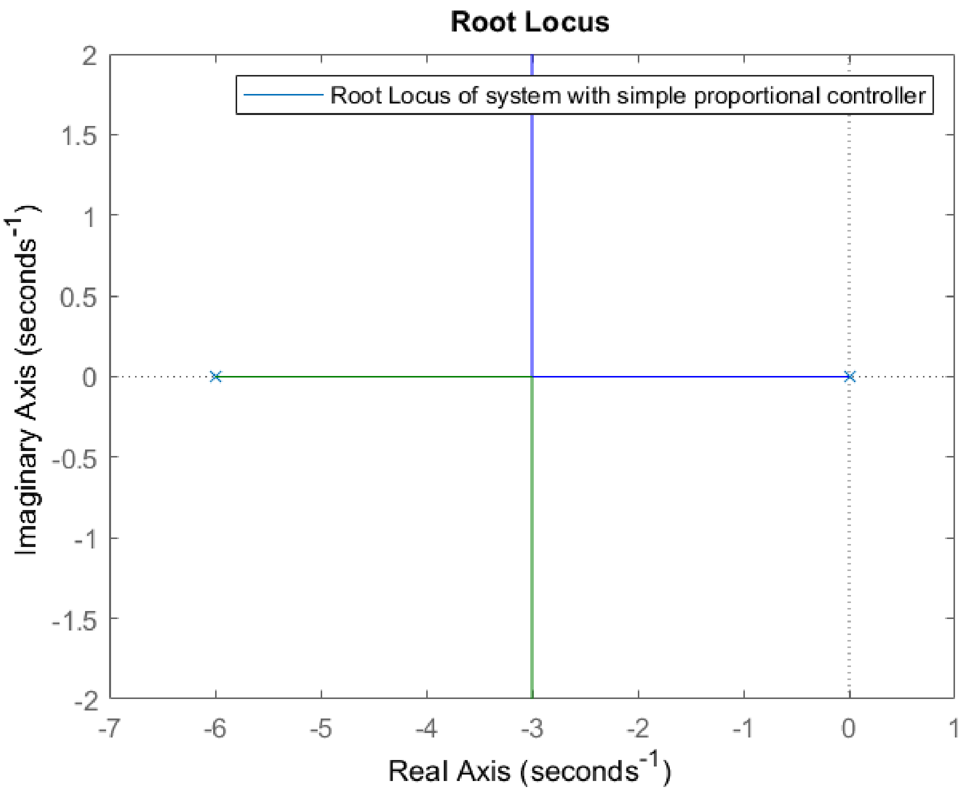
Root Locus must pass throught desired poles.

```
disp(desiredPoles);
```

```
  -6.0000 +13.7477i
  -6.0000 -13.7477i
```

Root Locus of a system with a simple proportional controller

```
figure;
rlocus(G);
legend('Root Locus of system with simple proportional controller');
```



We can see that no matter what, the root locus doesn't pass through **desired poles.**

```
syms s1
G1=1/(s1*(s1+6));
phi=double(angle(subs(G1,s1,-6+13.74i)))*180/pi;
sphi=180-phi;
```

The zero of the controller is usually taken just below the desired poles, but as in this system, a pole already exists at S = -6.

∴ We take the zero of the controller slightly towards left of -6. i.e S = -7 or **Z = 7**.

```
z=-7;
p=z-13.7477/tand(90-sphi);
disp(p);
```

```
-13.0034
```

And thus we the pole of the controller as **-13** or **P = 13**.

Then we find out **k** using magnitude criteria.

```
Ds=(s1-z)/(s1-p);
k=1/(double(abs(subs(Ds*G1,-6+13.7477i))))
```

```
k = 230.8210
```

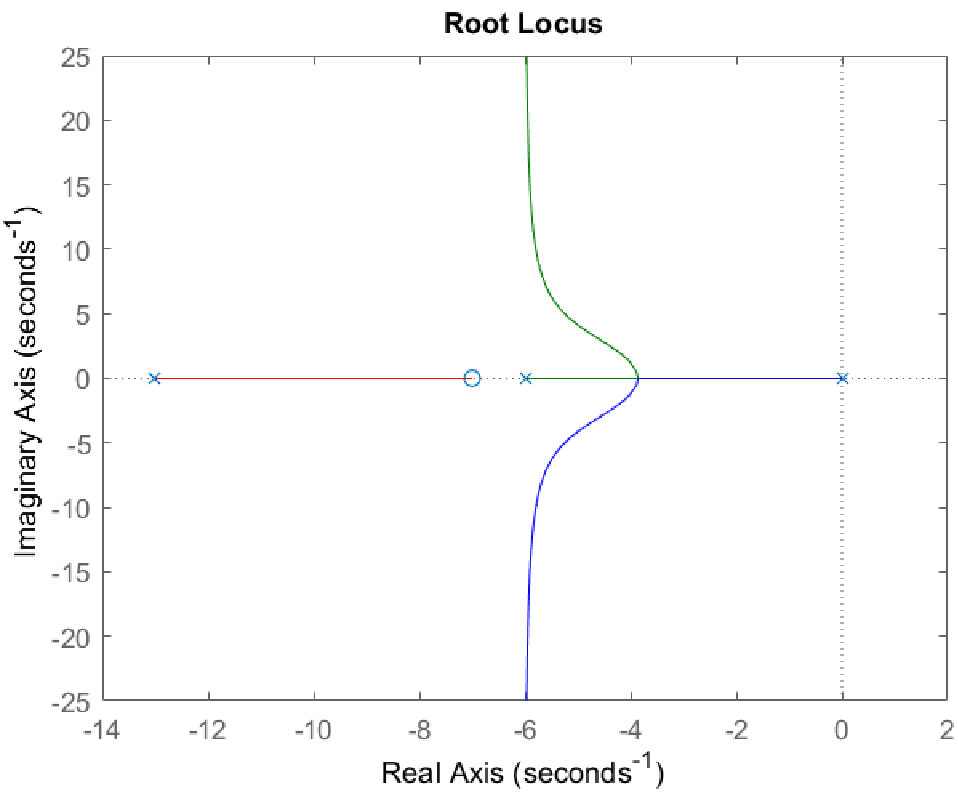
Thus at k = 230.5863, the RL passes through the desired pole location.

Verification of design

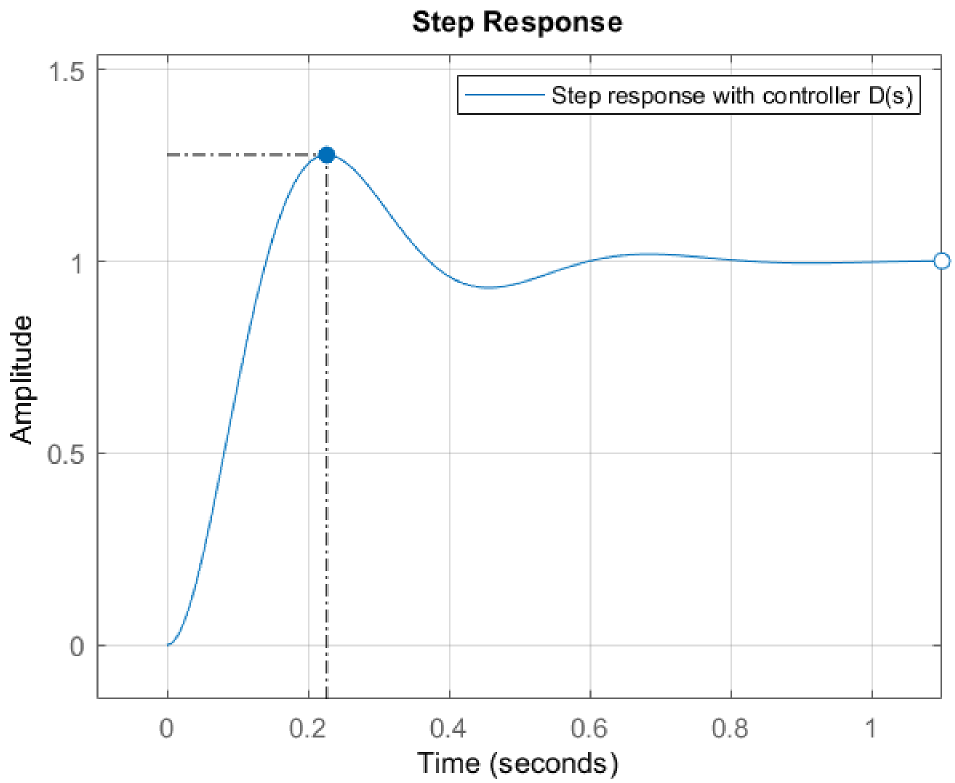
Ds = Controller Transfer Function

Ls = Closedloop Transfer Function with Controller

```
Ds = (s-z)/(s-p);
Ls = k*Ds*G/(1+k*Ds*G);
figure;
rlocus(Ds*G);
```




```
figure;
response = stepplot(Ls);
grid on;
response.showCharacteristic('PeakResponse');
response.showCharacteristic('SettlingTime');
response.showCharacteristic('RiseTime');
response.showCharacteristic('SteadyState');
setAxisLimits(axis);
legend('Step response with controller D(s)');
```



Time Domain parameters of system with controller.

```
stepinfo(Ls)
```

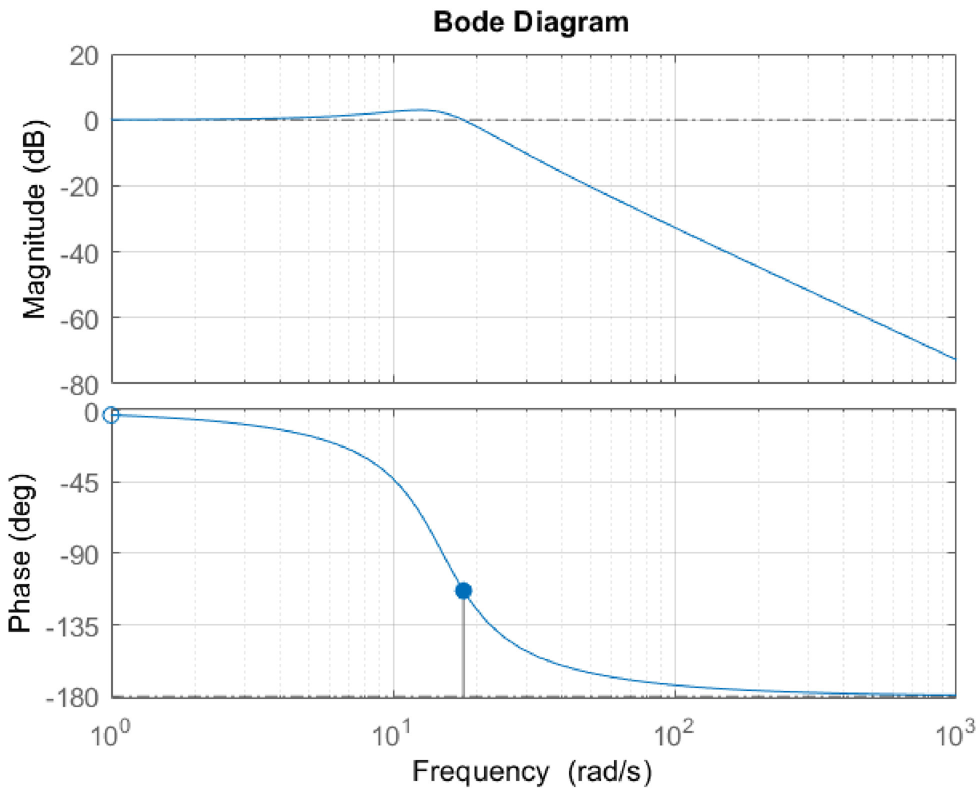
```
ans = struct with fields:
        RiseTime: 0.0952
    SettlingTime: 0.5612
     SettlingMin: 0.9312
     SettlingMax: 1.2771
       Overshoot: 27.7119
      Undershoot: 0
            Peak: 1.2771
        PeakTime: 0.2267
```

```
[gainMargin, phaseMargin, wcg, wcp] = margin(Ls)
```

```
Warning: The closed-loop system is unstable.
gainMargin = Inf
phaseMargin = 66.5761
wcg = Inf
wcp = 17.8151
```

Bode Plot of the closed loop transfer function.

```
response = bodeplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```

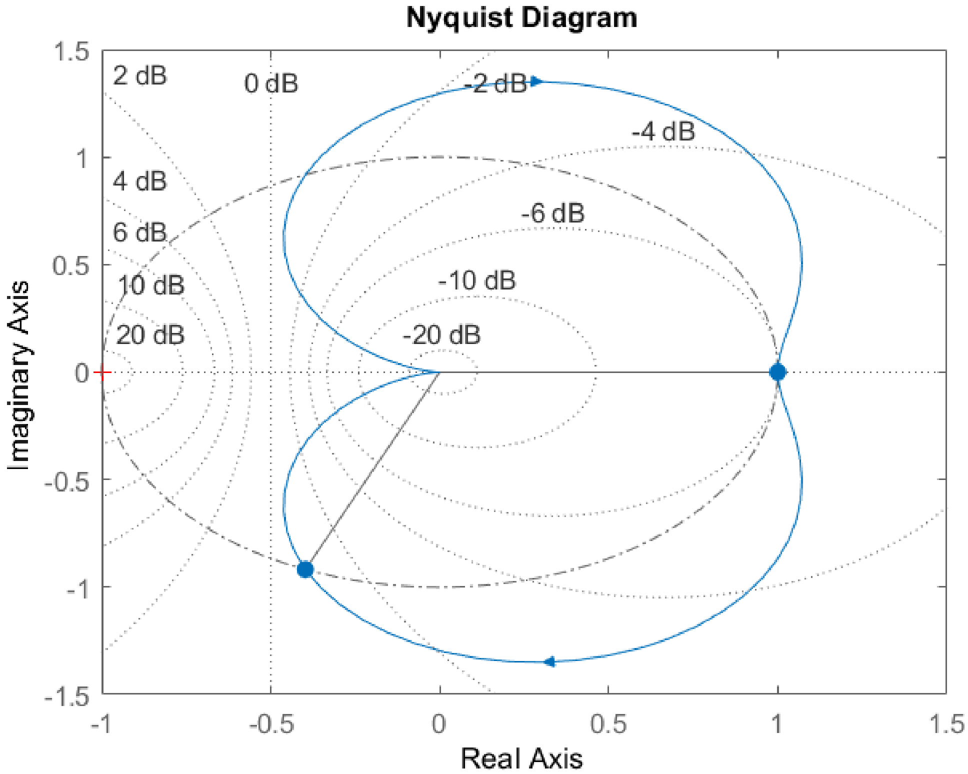


We can see from the **bode plot** that once the gain crossed the **0db** point, it never crosses it back again, hence no matter what the gain is, the system is going to remain stable.

Where as the phase when the gain crosses the **0db** is the **phase margin** and it's angle is 66.57°

Nyquist Plot of the closed loop transfer function

```
response = nyquistplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```



Since the **Gain Margin is infinity**, and the **Phase Margin is 66.5761**, the system is unstable at a phase of 66.5761 else it's stable.