MA510 USN:

JSS MAHAVIDYAPEETHA
SRI JAYACHAMARAJENDRA COLLEGE OF ENGINEERING (AUTONOMOUS), MYSORE 570 006

Affiliated to Visvesvaraya Technological University, Belgaum

Fifth Semester B.E. Degree Semester End / [redacted] Examination, Dec 2014

Branch: ECE Paper-setter: C. S. Yogananda

Linear Algebra

Duration: 3 Hours Date: 12.12.2014 Day: Friday

Time: 2.15 PM **Answer Part A & Part B** Max. Marks: 100

Part A - Answer all questions

1. Solve: $x + 4y - z = -5$; $x + y - 6z = -12$; $3x - y - z = 4$. $\boxed{6}$

2. Which of the following are subspaces of R^3?

   (a) $\{(x, y, z) \mid x + y + z = 0\}$
   (b) $\{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$
   (c) $\{(x, y, z) \mid x = 2y = 3z\}$.

   $\boxed{6}$

3. Check for linear dependence / independence of the following four matrices in M_2:

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$ $\boxed{6}$

4. In a vector space V define the *linear span* of a set of vectors $\{v_1, v_2, \ldots, v_k\}$ in V. Obtain the linear span of the vectors $2 - x + 3x^2 - x^3$, $1 + 2x - x^2 + 4x^3$ and $x + x^2$. $\boxed{6}$

5. State the *Rank-Nullity theorem* for a linear transformation, $T : V \longrightarrow W$, between two finite dimensional vector spaces V, W. Verify it for the transformation: $D : P_3(x) \longrightarrow P_3(x)$ given by

$$D(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2.$$ $\boxed{7}$

6. Define ***eigen values***, *eigen vectors* and the *characteristic polynomial* of a linear transformation from a vectorspace V to itself. $\boxed{6}$

7. Determine the characteristic polynomial of the following matrix:

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 3 & -2 \\ 5 & -3 & 4 \end{pmatrix}.$$ 7

8. Define an *inner product space* and show how to define the notions of *length* of a vector, *distance* and *angle* between two vectors in that inner product space. 6

Part B - Answer all questions

9. (a) Find the solution set of the system $A\mathrm{x} = 0$ where

$$A == \begin{pmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 5 & -2 \\ 3 & -1 & 0 & 2 \end{pmatrix}, \quad \mathrm{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$ 6

OR

(b) Show that the solution set of a homogeneous system $A\mathrm{x} = 0$, where A is a $m \times n$ matrix and X is a $n \times 1$ column vector, forms a vectorspace under suitable addition and multiplication by scalars. 6

10. (a) Define the *dimension* of a vectorspace. If W is a subspace of the vectorspace V, then show that the dimension of W is less than or equal to that of V. What can you say when their dimensions are equal? 6

OR

(b) Let W be a subspace of a vectorspace V. If $\{w_1, w_2, \ldots, w_k\}$ is a basis of W, show that it can be extended to a basis of V. 6

11. (a) Show that $\{1, 1+x, 1+x-x^2, 1-x+x^2-x^3\}$ is a linearly independent set in $P_3(x)$ and express the following polynomials as linear combinations of the elements of this set:

(i) $1 - x + x^2 - x^3$; (ii) $1 - 2x + 3x^2 - 4x^3$. 6

OR

(b) Consider the vectorspace $M_{n\times n}$ of $n \times n$ matrices with real number entries. Let U be the subset of matrices in $M_{n\times n}$ defined by:

$$U = \{A \in M_{n\times n} \mid a_{11} + a_{22} + a_{33} + \cdots + a_{nn} = 0\}.$$

Show that U is a subspace and find a basis of U. 6

12. (a) Consider ng $T : P_3(x) \longrightarrow P_3(x)$ defined by

$+ a_1x + a_2x^2 + a_3x^3) = a_1 + a_2x + a_3x^2 - a_0x^3.$

Show that iear transformation and obtain its matrix w.r.t the basis: $\{1, 1+x,$ $+ x^3\}$. $\boxed{6}$

OR

(b) If T_1 and o linear transformations from V into W (V, W are vectorspa show that the matrix of the transformation $T_1 + T_2$ is the sum of th of the transformations T_1 and T_2:

$$M_{T_1+T_2} = M_{T_1} + M_{T_2}.$$

($T_1 + T_2$: is defined by $(T_1 + T_2)(v) = T_1(v) + T_2(v)$.) $\boxed{6}$

13. (a) Let $T : V$ e a linear transformation between two finite dimensional vector spa . Let $v_1, v_2, \ldots v_k$ be vectors in V and let $T(v_1) = w_1$, $T(v_2) = w$ $k) = w_k$. If Kernel of T equals $\{0\}$ and $v_1, v_2, \ldots, v_k$ are linearly in t in V, show that $w_1, w_2, \ldots, w_k$ are linearly independent in V. $\boxed{7}$

OR

(b) Consider t space of 2×2 matrices, M_2. Let T be the function from M_2 to itself w a matrix in M_2 to its transpose.i.e.

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a & c \\ b & d \end{pmatrix}.$$

Show that iear transformation and obtain its matrix representation with respect to lard basis of M_2. $\boxed{7}$

14. (a) Define the ristic polynomial of an $n \times n$ matrix A. If S is an $n \times n$ invertible e., inverse of S exists), show that the matrices S and $S^{-1}AS$ have the s acteristic polynomial for any $n \times n$ matrix A. $\boxed{6}$

OR

(b) Verify Cay ilton theorem for the matrix

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

Use it to f verse of the matrix P. $\boxed{6}$

15. (a) Let $T : V$ a linear transformation on a finite dimensional vector space. Suppose λ are two distinct eigen values of T with corresponding eigen vectors v_1 how that v_1 and v_2 are linearly independent. $\boxed{6}$

OR

(b) Find all the eigenvalues and a corresponding eigen vector for each of them, of the following matrix.

$$A = \begin{pmatrix} -3 & 8 \\ -2 & 7 \end{pmatrix}.$$

6

16. (a) Let V be an inner product space and $\{v_1, v_2, \ldots, v_n\}$ be a basis of V. Explain Gram-Schmidt process of obtaining an *orthonormal* basis of V starting from the given basis.

7

OR

(b) In the space, $P_3(x)$, of the polynomials with real co-efficients of degree ≤ 3, show that

$$\langle f, g\rangle = f(-1)g(-1) + f(0)g(0) + f(1)g(1) + f(2)g(2)$$

defines an inner product. Find the *lengths* of the polynomials $1 - 2x + 3x^2 - 4x^3$, $2 + 3x - 4x^2 + 5x^3$ and the *angle* between them with respect to this inner product.

MA-510

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SRI JAYACHAMARAJENDRA COLLEGE OF ENGINEERING
(AUTONOMOUS), MYSORE 570 006
Affiliated to Visvesvaraya Technological University, Belgaum

Dec 2013

Fifth Semester B.E. Degree Semester End / [redacted] Examination, Jan 2014

Branch: E&C

Paper-setter: C. S. Yogananda

Linear Algebra

Duration: 3 Hours   Date: 30-12-2013   Day: Monday

Time: 2 to 5 PM   **Answer all questions**   Max. Marks: 100

1. (a) Show that $P_n(x)$, the space of polynomials with real coefficients and degree less than or equal to n is a vector space and exhibit a basis for it. [6]

   (b) Solve: [6]

$$2x + y + 3z = 1; \quad -3x - 4y - 5z = 2; \quad x + 3y - 6z = 3.$$

   (c) In the space of matrices of order $m \times n$ with real entires, $M_{m \times n}(\mathbf{R})$, consider the subset U defined as follows:

$$U = \{A = (a_{ij}) \mid a_{ij} = 0 \text{ for } i < j\}$$

   Show that it is a subspace of $M_{m \times n}(\mathbf{R})$ and determine its dimension. [8]

~~2. Answer either~~ {(a), (b), (c)} **OR** {(d), (e), (f)}.

   (a) In the following express the given vector as a linear combination of the other given vectors:

   i. In $\mathbf{R}^2$, $(1, -1)$ in terms of $(-2, 1)$ and $(1, 0)$; [3]

   ii. In $\mathbf{R}^3$, $(1, -2, 1)$ in terms of $(1, 1, 0)$, $(1, 0, 1)$ and $(0, 1, 1)$; [4]

   iii. In $\mathbf{R}^3$, $(-2, 1, 3)$ in terms of $(1, 0, 0)$, $(1, -1, 0)$ and $(-1, 1, -1)$; [4]

   iv. In $\mathrm{P}_2(x)$, $2x^2 - 4x + 7$ in terms of 1, $x + 1$, $x^2 - x + 1$. [4]

   v. In $\mathrm{P}_3(x)$, $x^3 + 2x^2 + 3x - 5$ in terms of 1, $1 - x$, $1 + x^2$ and $1 - x^3$; [5]

OR

   (b) In a vector space V, define linear independence and linear dependence of a set of vectors $\{v_1, v_2, \ldots, v_m\}$. If u, v, w are linearly independent vectors then determine whether the vectros $u + v$, $v - w$, $u + 2v - 3w$ are linearly independent / dependent. [6]

   (c) Does the set $\{(1, 1, 1), (1, 2, 3), (1, 5, 8)\}$ form a basis for $\mathbf{R}^3$? [6]

(d) Show that $\{1, 1-x, 1+x+x^2, 1-x+x^2-x^3\}$ is linearly independent in $P_3(x)$ and therefore, forms a basis of $P_3(x)$. Express the following vectors in terms of this basis:

$$\text{(i) } x^3 - x^2 + x - 1; \qquad \text{(ii) } 3x^3 + 2x^2 - 2x - 3.$$ 8

3. Answer either {(a), (b), (c)} **OR** {(d), (e), (f)}.

(a) For a linear transformation $T : V \longrightarrow W$ define the kernel and image subspacs associated with T. State the *rank-nullity theorem.* 6

(b) Let $P_2(x)$ denote the space of polynomials of degree less than or equal to 2 and define a mapping $D : P_2(x) \longrightarrow P_2(x)$ by

$$T(a_0 + a_1x + a_2x^2) = (a_0 - 2a_1) - (a_1 + 3a_2)x + (a_3 - 4a_0)x^2.$$

Show that T is a linear transformation and find its matrix by taking $\{1, 1+x, 1-x^2\}$ as the basis of $P_2(x)$. 7

(c) Verify that S as defined below is a linear transformation and find its kernel:

$$S : P_3(x) \longrightarrow P_3(x),$$

where is given by

$$S(a_0 + a_1x + a_2x^2 + a_3x^3) = (a_0 + a_1) - (a_1 - a_2)x + (a_2 - a_3)x^2 - (a_3 - a_0)x^3.$$ 7

OR

(d) Let $T : V \longrightarrow W$ be a linear transformation between two finite dimensional vector spaces V, W with the kernel of T containing only the zero vector of V. Let $v_1, v_2, \ldots v_k$ be vectors in V and let $T(v_1) = w_1$, $T(v_2) = w_2, \ldots T(v_k) = w_k$. If $v_1, v_2, \ldots, v_k$ are linearly independent in V, show that $w_1, w_2, \ldots, w_k$ are linearly independent in V. 6

(e) Consider the mapping $P : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ defined by

$$P((x, y, z)) = (x - y + z, x + y - z, x + z).$$

Show that P is a linear transformation and obtain its matrix w.r.t the basis $\{(1,0,0), (1,-1,0), (1,0,-1)\}$ of $\mathbf{R}^3$. 7

(f) For the transformation P defined above 3(e), determine the kernel and use it to find the dimension of the image space of P by using the Rank-Nullity theorem. 7

4. Answer either {(a), (b), (c)} **OR** {(d), (e), (f)}.

(a) Discuss the solution of the following systems of linear equations; find the unique solution if it exists or express the solution in terms of a parameter or show that the system does not have any solution as the case may be:

i. $x + 2y - 4z = -4; \quad 2x + 5y - 9z = -10; \quad 3x - 2y + 3z = 11.$ 5

ii. $x + 2y - 3z = -1; \quad -3x + y - 2z = -7; \quad 5x + 3y - 4z = 2.$ 5

iii. $x + 2y - 3z = 1; \quad 2x + 5y - 8z = 4; \quad 3x + 8y - 13z = 7.$ 5

iv. $x + 2y - z = 3; \quad x + 3y + z = 5; \quad 3x + 8y + 4z = 17.$ 5

OR

Define eigen values, eigen vectors and the characteristic polynomial of a linear ransformation. $\boxed{5}$

Determine all the eigen values of the following matrix by finding its characteristic polynomial:

$$B = \begin{pmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{pmatrix}.$$ $\boxed{8}$

Jsing *Cayley-Hamilton theorem*, find the matrix B,

$$B = A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$$

vhere

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}.$$ $\boxed{7}$

Let $V = \mathbf{R}^3$, $v_1 = (1,0,2)$ and $v_2 = (0,3,1)$. Find the vector in $\mathbf{R}^3$ orthogonal to both v_1 nd v_2. $\boxed{6}$

how that {(2,1,3),(1,-2,0),(6,3,-5)} forms an orthogonal basis for R^3. Express the vector $= (9,-2,4)$ as a linear combination of these vectors. $\boxed{7}$

tate and prove the *Cauchy-Schwartz inequality* for an Inner Product defined on a vector pace. $\boxed{7}$

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(AUTONOMOUS), MYSORE 570 006
Affiliated to Visvesvaraya Technological University, Belgaum

Fifth Semester B.E. Degree Semester End / Make-Up Examination, Dec. 2015

Branch: ECE Paper-setter: C. S. Yogananda

Linear Algebra

Duration: 3 Hours Date: 15-12-2015 Day:

Time: 2:15 pm **Answer Part A & Part B** Max. Marks: 100

Part A - Answer all questions

1. Determine whether or not the following subsets of the space of 3×3 matrices with real number entries, $M_{3\times3}$, are subspaces:

   (a) the subset, U, of matrices with determinant 1: $U = \{A \mid \det(A) = 1\}$;

   (b) the subset, V, of matrices with trace 0: $V = \{A = (a_{ij}) \mid a_{11} + a_{22} + a_{33} = 0\}$; **[6]**

2. Determine the values of a and b for which the system:

$$x + 2y + 3z = 6; \quad x + 3y + 5z = 9; \quad 2x + 5y + az = b$$

~~has (i) no solutions and~~ (ii) unique solution, and find that solution. **[6]**

3. In the following express the given vector as a linear combination of the other given vectors:

   (a) In $\mathbf{R}^3$, $(13, 23, -10)$ in terms of $(1, 1, 0)$, $(1, 0, 1)$ and $(0, 1, 1)$. **[3]**

   (b) In $\mathbf{P}_3(x)$, $2x^3 - x^2 - 4x + 5$ in terms of 1, $x + 1$, $x^2 + x + 1$ and $x^3 + x^2 + 1$. **[4]**

4. In a vector space V, define linear independence and linear dependence of a set of vectors $\{v_1, v_2, \ldots, v_m\}$. If u, v, w are linearly independent vectors in V then check if the vectors $4u + 3v - 2w$, $u - 3v + 5w$, $3u - 4v - 6w$ are linearly dependent / independent. **[6]**

5. For a linear transformation $T : V \longrightarrow W$ define the kernel and the image of T. State the *rank-nullity theorem*. **[6]**

6. Define *eigenvalue* and *eigenvector* for a linear transformation from a vectorspace to itself. Find the eigenvalues and, for each eigenvalue, a corresponding eigenvector, of the linear transformation $R : P_3(x) \longrightarrow P_3(x)$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

$\boxed{6}$

7. Verify Cayley-Hamilton theorem for the matrix

$$R = \begin{pmatrix} -3 & 5 & 6 \\ 1 & 3 & -4 \\ 2 & 4 & -5 \end{pmatrix}.$$

Use it to find the inverse of the matrix R. $\boxed{7}$

8. In the space, $M_{m\times n}$, of $m \times n$ matrices with real number entries, show that

$$\langle A, B\rangle = \text{Trace}(AB')$$

defines an inner product. Find the *length* of the matrix $R = \begin{pmatrix} 2 & 5 & 3 \\ -1 & 3 & -4 \end{pmatrix}$ with respect to this inner product.

$\boxed{6}$

Part B - Answer all questions

9. (a) Show that the set of solutions of the differential equation $y'' + Ay' + B = 0$, where A, B are some real numbers, is a vector space over the field of real numbers. $\boxed{6}$

OR

(b) Obtain solutions of the following system of equations in parametric form:

$$3x_1 - 2x_2 + 5x_3 + 4x_4 = 2; \quad 6x_1 - 4x_2 + 4x_3 + 3x_4 = 3; \quad 9x_1 - 6x_2 + 3x_3 + 2x_4 = 4.$$ $\boxed{6}$

10. (a) Define the *dimension* of a vector space. Give an example, with justification, of an infinite dimensional vector space.

$\boxed{6}$

OR

(b) Define the *basis* of a vector space. If $\{v_1, v_2, ..., v_n\}$ is a basis for a vector space V, show that any vector v in V can be expressed uniquely as a linear combination of the basis vectors, $v_1, v_2, ..., v_n$.

$\boxed{6}$

11. (a) If U and W are two subspaces of a vector space V, then show that $U \cap W$ is also a subsapce of V.

$\boxed{6}$

OR

(b) In $M_{3\text{x}3}(\mathbf{R})$ consider the subset, U, of all lower triangular matrices:

$$U = \{A = (a_{ij}) \in M_{3\text{x}3}(\mathbf{R}) \mid a_{ij} = 0 \quad \text{if} \quad i \leq j\}$$

Show that it is a subspace and determine its dimension.

$\boxed{6}$

12. (a) In a vector space V define the linear span of a set of vectors $\{v_1, v_2, \ldots, v_k\}$ in V. Check if the polynomial $1 - 2x + 3x^2 - 4x^3$ is in the linear span of $\{1, x^2, 2x + x^2, x + x^3\}$ in $P_3(x)$. $\boxed{6}$

OR

(b) Let V be a vector space and W be a subspace of V. Show that the dimension of W is less than or equal to that of V. Further, show that $W = V$ if dimension of W is same as that of V. $\boxed{6}$

13. (a) Define a linear transformation $T : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$, such that the kernel of T is spanned by the points $(1, -2, 3)$ and $(-1, -1, 2)$ $\boxed{7}$

OR

(b) Define the kernel of a linear transformation. Let $P_3(x)$ denote the space of polynomials with real coefficients and of digree less than or equal to 3. Find the image of $P_3(x)$, under the following linear transformation T:

$$T : P_3(x) \longrightarrow P_3(x),$$

where

$$S(a_0 + a_1x + a_2x^2 + a_3x^3) = (a_0 - a_1) + (a_1 + a_2)x + (a_2 - a_3)x^2 + (a_3 - a_0)x^3.$$ $\boxed{7}$

14. (a) Consider the mapping $L : R^3 \longrightarrow R^3$ defined by

$$L((x, y, z)) = (x - y - z, x - y + z, x + y - z).$$

Show that L is a linear transformation and obtain its matrix w.r.t the basis: $\{(1, 0, -1), (-1, 1, 0), (1, -1, 0)\}$. $\boxed{6}$

OR

(b) Let $T : V \longrightarrow W$ be a linear transformation between two finite dimensional vector spaces V, W. Let $v_1, v_2, \ldots v_k$ be vectors in V and let $T(v_1) = w_1, T(v_2) = w_2, \ldots T(v_k) = w_k$. If $w_1, w_2, \ldots, w_k$ are linearly independent in W, show that $v_1, v_2, \ldots, v_k$ are linearly independent in V. $\boxed{6}$

15. (a) Determine the characteristic polynomial of the matrix A; find an eigenvalue and a corresponding eigenvector of A:

$$A = \begin{pmatrix} 2 & -1 & -3 \\ -3 & 0 & -1 \\ -2 & -3 & 1 \end{pmatrix}.$$ $\boxed{6}$

OR

(b) Define the *characteristic polynomial* of an $n \times n$ matrix A. State the *Cayley-Hamilton theorem* and explain how to find the inverse of A using it. $\boxed{6}$

16. (a) Define an *inner product space* (IPS). Using the inner product define the notions of length of vectors, distance and angle between two vetors in an IPS. $\boxed{7}$

OR

(b) Consider the space of 2×3 matrices as an inner product space with the inner product given by: $\langle A, B \rangle = \text{Trace}(AB^t)$. Find (i) the distance between the matrices P and Q and (ii) the angle between P and Q, where

$$P = \begin{pmatrix} 6 & 2 & 2 \\ 2 & -3 & -1 \end{pmatrix}, \quad Q = \begin{pmatrix} -3 & 2 & 1 \\ 2 & -3 & 7 \end{pmatrix}.$$ $\boxed{7}$

PART- A

Q No	CO	Cognitive Domain	Question	Marks
1(a)	CO3	L2	Suppose the mapping $F : R^2 \to R^2$ is defined by $F(x, y) = (x+y, x)$. Show that F is a linear transformation.	05
(b)	CO3	L3	Let $F : R^4 \to R^3$ be the linear mapping defined by $F(x, y, x, t) = (x-y+x+t,\ x+2z-t,\ x+y+3z-3t)$. Find a basis and dimension of i) the image of F (ii) the kernel of F.	05
2(a)	CO2	L1	Determine whether or not the vectors $u = (1, 1, 2)$, $v = (2, 3, 1)$, $w = (4, 5, 5)$ in $1R^3$ are linearly dependent.	05
(b)	CO2	L2	Find the dimension and a basis of the solution space W of following homogenous system $x + 2y + z - 2t = 0$ $2x + 4y + 4z - 3t = 0$ $3x + 6y + 7z - 4t = 0$	05
3(a)	CO3	L1	Find the parametric solution of the following system of equations $x + 2y - 3z = 1$ $2x + 5y - 8z = 4$ $3x + 8y - 13z = 7$	05
(b)	CO1	L1	Solve . $x + 2y - x = 3$ $x + 3y + z = 5$ $3x + 8y + 4z = 17$	05
4(a)	CO4	L5	Find the characteristic and minimal polynomial of $\begin{pmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{pmatrix}$	05
(b)	CO4	L4	Find Jordan canonical forms for the matrices whose characteristic polynomial $\Delta(t)$ and minimal polynomial m(t). $\Delta(t) = (t-2)^4 (t-3)^2$ $M(t) = (t-2)^2 (t-3)^2$	05

5	CO5	L2	Consider vectors $u = (1, 2, 4)$ $v = (2, -3, 5)$ $w = (4, 2, -3)$ in R^3. Find (a) u. V (b) u. W (c) V. w (d) w. w (e) (u +v). w (f) $\|\|u\|\|$ (g) $\|\|V\|\|$ (h) $\|\|w\|\|$.	10

PART- B

6(a)	CO3	L3	Show that the following mapping are not linear i) $F : R^2 \to R^2$ defined by $F(x,y) = (xy,x)$ ii) $F : R^2 \to R^3$ defined by $F(x,y) = (x+3, 2y, x+y)$ iii) $F : R^3 \to R^2$ defined by $F(x,y,z) = (\|x\|, y + z)$	05
(b)	CO3	L3	Let T be the linear transformation on R^3 defined by $T(1, 0, 0) = (1, 1, 1)$ $T(0, 1, 0) = (1, 3, 5)$ $T(0, 0, 1) = (2, 2, 2)$ Find the matrix A representing T relative to the basis $\{(1, 1, 0), (1, 2, 3), (1, 3, 5)\}$	05
OR				
7	CO3	L3	Consider the following two basis of R^3. $E = \{e, e_2, e_3\} = \{(1,0,0), (0, 1, 0), (0, 0, 1)\}$ and $S = \{u_1, u_2, u_3\} = \{(1, 0, 1), (2, 1, 2), (1, 2, 2)\}$ a) Find the change of basis matrix P from the basis E to the basis S. b) Find the change of basis matrix from the basis S to the basis E. c) Verify $Q = P^{-1}$.	~~05~~ 05 05
8(a)	CO2	L3	Show that the function $f(t) = \sin(t)$, $g(t) = \cos t$ f and $l(t) = t$ form R into R are linearly independent.	05
(b)	CO2	L3	Relative to the basis $S = \{u_1, u_2\} = \{(1, 1), (2,3)\}$ of R^2, Find the co ordinate vector of V, when (i) $v = (4, -3)$ (ii) $v = (5,-6)$	05

			OR	
9(a)	CO2	L2	Let W be the subspace of R^4 spanned by the vectors $u_1 = (1, -2, 5, -3)$ $u_2 = (2, 3, 1, -4)$ $u_3 = (3, 8, -3, -5)$ i) Find a basis and dimension of W ii) Extend the basis of W to a basis of R^4	06
(b)	CO2	L2	Extend $\{ u_1 = (1, 1, 1, 1)\ V_2 = (2, 3, 1; 4) \}$ to basis of R^4.	04
10(a)	CO1	L2	Find the inverse of $A = \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$ by using row canonical form.	05
(b)	CO1	L2	Obtain the LU decomposition of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 4 & -3 \\ 2 & 8 & 1 \\ -5 & -9 & 7 \end{pmatrix}$	05
			OR	
11(a)	CO1	L1	Reduce the following matrix into row canonical form $\begin{pmatrix} 2 & 2 & -1 & 6 & 4 \\ 4 & 4 & 1 & 10 & 13 \\ 8 & 8 & -1 & 26 & 23 \end{pmatrix}$	05
(b)	CO1	L1	Obtain the LDU factorization of A $A = \begin{pmatrix} 1 & -3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{pmatrix}$	05
12(a)	CO4	L1	Let $V_1, V_2, \ldots V_n$ be non zero Eigen vectors of a linear mapping T belonging to distinct Eigen values $\lambda_1, \lambda_2, \ldots \lambda_n$. Show that $v_1, v_2 \ldots v_n$ are linearly independent.	05

(b)	CO4	L3	Let $A = \begin{pmatrix} 8 & -6 & 2 \\ 6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ i) Find the characteristic polynomid of A and hence all the Eigen values of A ii) For each Eigen values of A, find a corresponding Eigen vectors.	05
OR				
13	CO4	L5	Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. i) Find all Eigen values and Eigen vectors. ii) Find a non singular matrix P such that $D = P^{-1} AP$ is digonal and P^{-1} iii) Find A^6 and F(A) where $t^4 - 3t^3 - 6t^2 + 7t + 3$ iv) Find a " real cube root" of A, ie a matrix B such that $B^3 = A$ and B has real Eigen values.	10

14	CO5	L4	Find a singular value decompsite of matrix $A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$	10
OR				
15(a)	CO5	L2	Explain the Gram Schmidt process to obtain the orthonormal basis for a finite dimensional	05
(b)	CO5	L1	What is Support vector machine? Explain with an example.	05