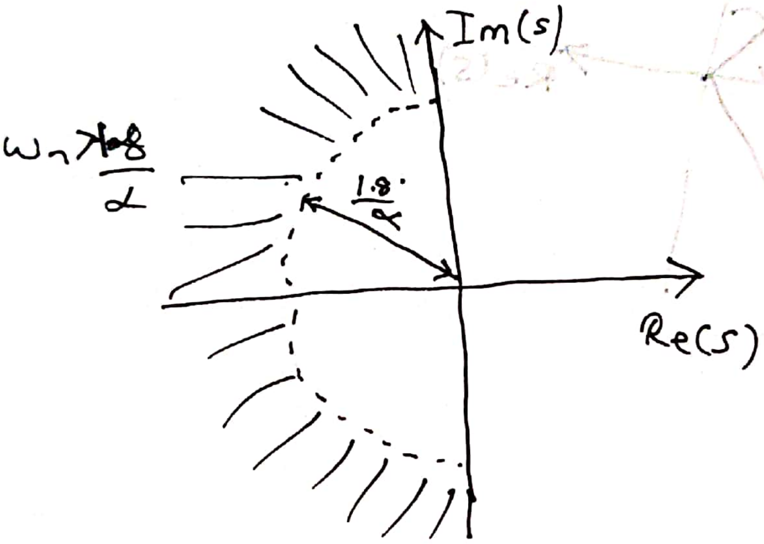
Time Domain Parameters and Region of Pole Location

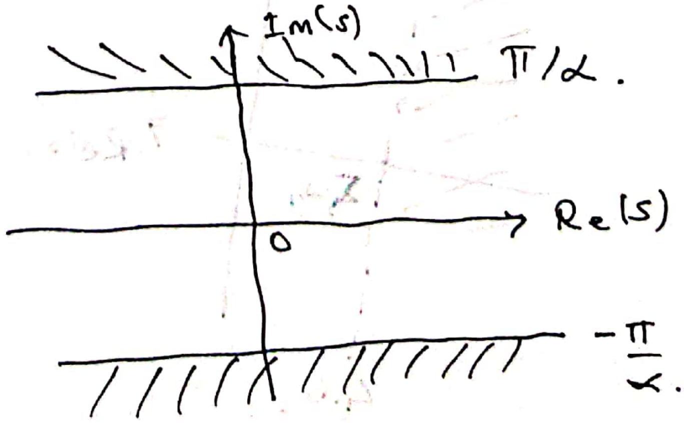
Rise Time } $t_r \simeq \dfrac{1.8}{\omega_n}$ $\qquad t_r \le \alpha$ sec.

$$\omega_n > \frac{1.8}{\alpha}$$



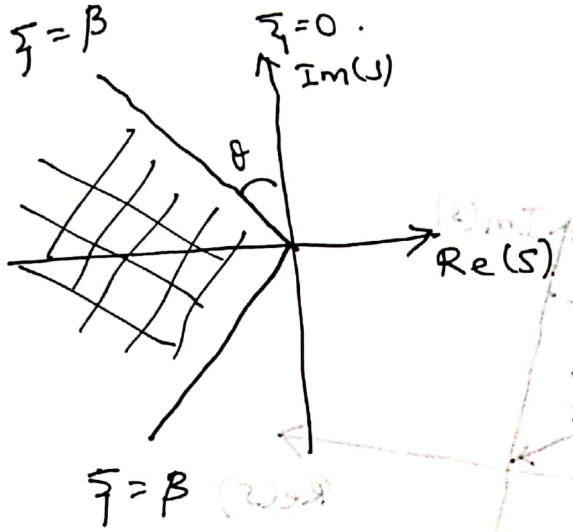
Peak Time $\quad t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\qquad t_p < \alpha$ sec.

$$\omega_n\sqrt{1-\zeta^2} > \frac{\pi}{\alpha}.$$



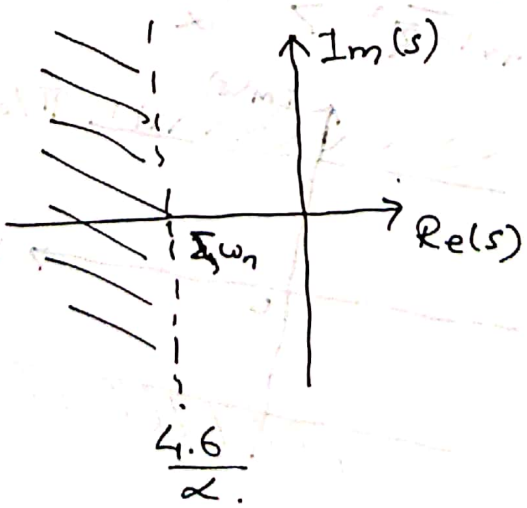
Overshoot $M_p < \alpha$, $\Rightarrow \xi > \beta$

$$\beta = \frac{|\ln \alpha|}{\sqrt{\pi^2 + (\ln \alpha)^2}}$$



Settling Time $t_s = \frac{4.6}{\xi \omega_n}$ $t_s < \alpha$

$$\frac{4.6}{\xi \omega_n} < \alpha$$



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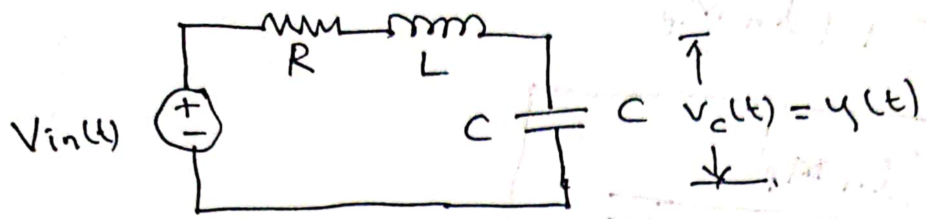
Given series RLC circuit, with $C = 0.1F$



$$\text{Transfer function} = G(s) = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Design R, L such that step response of circuit has 40% overshoot and settling time less than 2 sec.

Soln:

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$|\ln M_p| = \frac{\pi\xi}{\sqrt{1-\xi^2}}$$

$$|\ln M_p|^2 = \frac{\pi^2\xi^2}{(1-\xi^2)}$$

$$|\ln M_p|^2 - \xi^2 |\ln M_p|^2 = \pi^2\xi^2$$

$$|\ln M_p|^2 = \pi^2\xi^2 + \xi^2 |\ln M_p|^2$$

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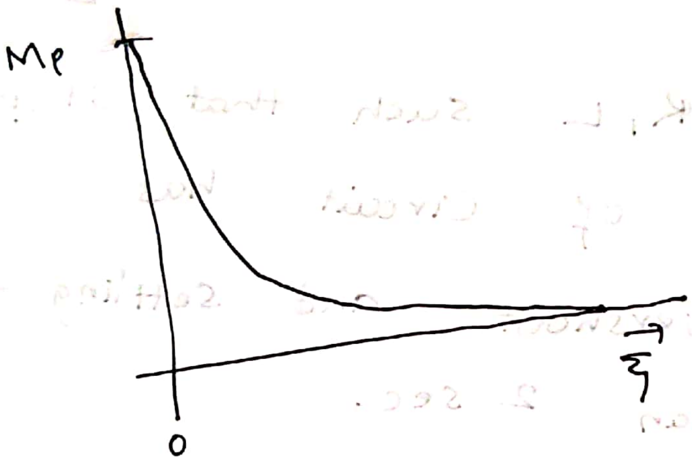
$$\zeta^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$$

$$\boxed{\zeta = \frac{|\ln M_p|}{\sqrt{\pi^2 + (\ln M_p)^2}}}$$



For overshoot $< 40\%$, $\zeta > 0.28$

Settling time $2 = t_s < \dfrac{4.6}{\zeta \omega_n}$

$$\zeta \omega_n > 2.3$$

Let $\omega_n = 10$, $\zeta = 0.3$

Since $C = 0.1\,F \Rightarrow L = 0.1\,H.$

$$\omega_n^2 = \frac{1}{LC} = \frac{1}{(0.1)(0.1)} = 100$$

Comparing Transfer function with standard second order system

$$G(s) = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{1}{LC} \qquad \omega_n = \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = \frac{R}{L} \qquad \xi = \frac{1}{2\omega_n} \cdot \frac{R}{L}$$

$$\xi = \frac{R}{2} \cdot \sqrt{\frac{C}{L}}$$

we want $\xi = 0.3$

$$0.3 = \frac{R}{2}\sqrt{\frac{0.1}{0.1}}$$

$$\boxed{R = 0.6\,\Omega}$$