Radix - 2 8 - point DIT FFT.

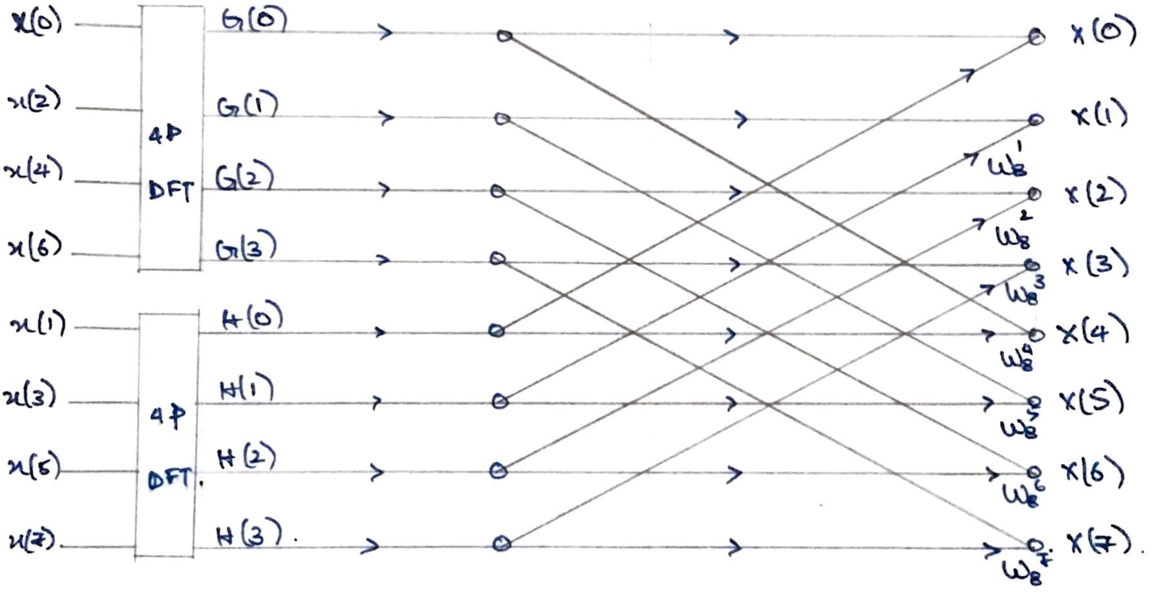
$N = 8 . = 2 \times 2 \times 2 .$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{7} x(n) W_8^{nk} = \sum_{n \to even} x(n) W_8^{nk} + \sum_{n \to odd} x(n) W_8^{nk}.$$

$$x(k) = \sum_{n=0}^{3} x(2n) W_8^{2nk} + \sum_{n=0}^{3} x(2n+1) W_8^{(2n+1)k}$$

$$= \sum_{n=0}^{3} x(2n) \cdot W_4^{nk} + \sum_{n=0}^{3} x(2n+1) W_4^{nk} . W_8^{k}$$

$$= G(k) + H(k) . W_8^{k}$$



$X(0) = G(0) + H(0).$

$X(1) = G(1) + W_8^1 H(1).$

$X(2) = G(2) + W_8^2 H(2).$

$X(3) = G(3) + W_8^3 H(3).$

$X(4) = G(0) + W_8^4 H(4)$

$X(5) = G(1) + W_8^5 H(1)$

$X(6) = G(2) + W_8^6 H(2).$

$X(7) = G(3) + W_8^7 H(3).$

$$G(k) = \sum_{n=0}^{3} x(2n)\, W_4^{nk} \qquad H(k) = \sum_{n=0}^{3} x(2n+1)\, W_4^{kn}$$

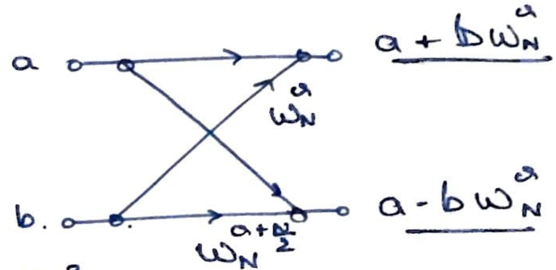
Σ odd + Σ even.

$$\sum_{n=0}^{1} x(2(2n))\, W_4^{2nk} + \sum_{n=0}^{1} x(2(2n+1))\, W_4^{nk(2n+1)}$$

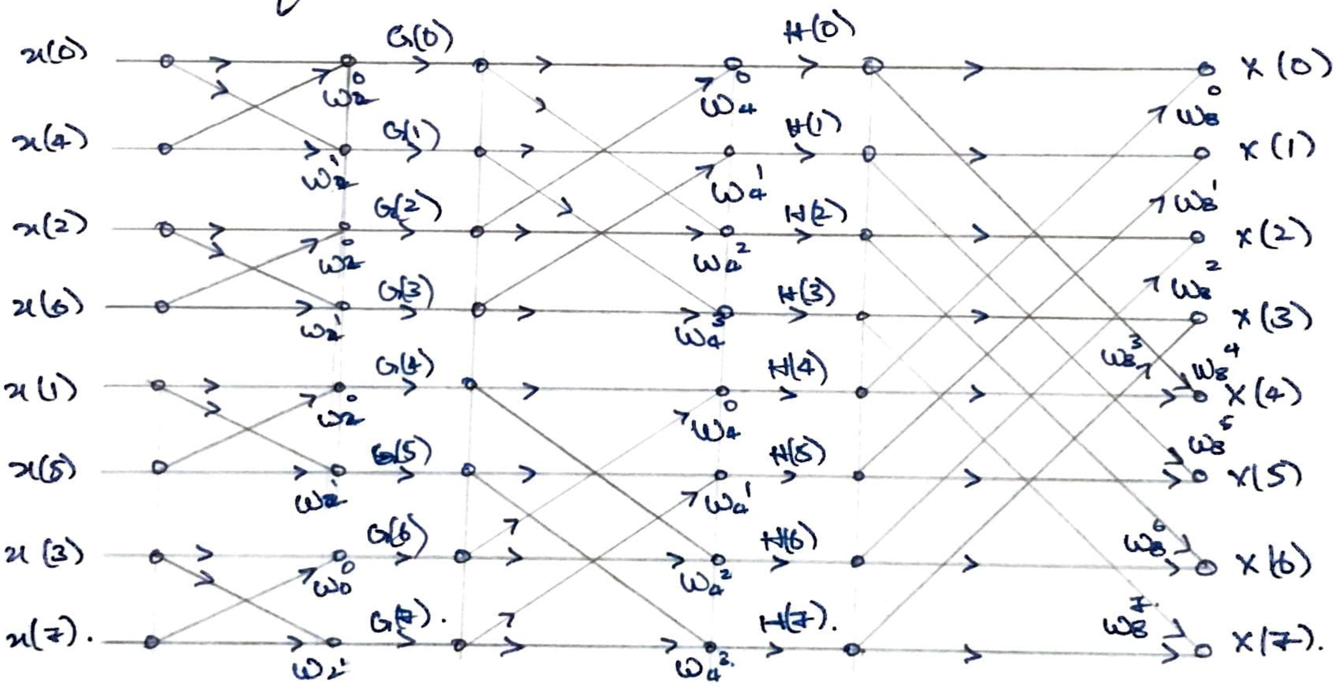
$$\sum_{n=0}^{1} x(4n)\, W_2^{nk} + \sum_{n=0}^{1} x(4n+2)\, W_2^{nk} \cdot W_4^{k}$$

$$P(k) + Q(k)\, W_4^{k}$$

Slly $R(k) + S(k)\, W_4^{k}$



DFT of. $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
0 1 2 3 4 5 6 7



$G(0) = x(0) + x(4) = 6.$

$G(1) = x(0) - x(4) = 1 - 5 = \underline{\underline{-4.}}$

$G(2) = x(2) + x(6) = 3 + 7 = 10$

$G(3) = x(2) - x(6) = 3 - 7 = -4.$

$G(4) = x(1) + x(5). = 2 + 6 = 8.$

$G(5) = x(1) - x(5) = 2 - 6 = -4.$

$G(6) = x(3) + x(7). = 4 + 8 = 12.$

$G(7) = x(3) - x(7) = 4 - 8 = \underline{\underline{-4.}}$

$H(0) = G(0) + G(2) = 6 + 10 = 16.$ $\boxed{W_4^1 = -j.}$

$H(1) = G(1) + W_4^1 G(3) = -4 - j(-4) = -4 + 4j.$

$H(2) = G(0) - G(2) = 6 - 10 = -4.$

$H(3) = G(1) - W_4^1 G(3). = -4 + j(-4) = \underline{\underline{-4 - 4j}}.$

$H(4) = G(4) + G(6). = 8 + 12 = 20$

$H(5) = G(5) + W_4^1 G(7) = -4 - j(-4) = -4 + 4j.$

$H(6) = G(4) - G(6) = 8 - 12 = -4.$

$H(7) = G(5) - W_4^1 G(7) = -4 + j(-4) = \underline{\underline{-4 - 4j}}.$

$X(0) = H(0) + H(4) = 16 + 20 = \underline{\underline{36.}}$

$X(1) = H(1) + W_8^1 H(5) = -4 + 4j + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)(-4 + 4j) = \underline{\underline{-4 + 9.65j}}.$

$X(2) = H(2) + W_8^2 H(6) = -4 + (-j)(-4) = \underline{\underline{-4 + 4j}}.$

$X(3) = H(3) + W_8^3 H(7) = -4 - 4j + \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)(-4 - 4j) = -4 + 1.65j$

$X(4) = H(0) - H(4). = -4$

$X(5) = H(1) - W_8^1 H(5) = -4 - 1.657j$

$X(6) = H(2) - W_8^2 H(6). = -4 - 4j$

$X(7) = H(3) - W_8^3 H(7) = -4 - 9.657j.$

Radix 2 - DIF FFT

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn}. \qquad \underline{N=8.}$$

$$X(K) = \sum_{n=0}^{7} x(n) W_N^{Kn} = \sum_{n=0}^{3} x(n) W_8^{Kn} + \sum_{4}^{7} x(n) W_8^{Kn}.$$

$$= \sum_{n=0}^{3} x(n) W_8^{Kn} + \sum_{n=0}^{3} x\left(n+\frac{N}{2}\right) W_N^{K\left(n+\frac{N}{2}\right)}.$$

$$\sum_{n=0}^{3} x(n) W_8^{Kn} + \sum_{n=0}^{3} x\left(n+\frac{N}{2}\right) W_8^{Kn} \cdot W_8^{K\frac{N}{2}}.$$

$$X(K) = X_1(K) + X_2(K) W_8^{K\frac{N}{2}} = X_1(K) + X_2(K)(-1)^K.$$

For even sequence. $= \sum_{n=0}^{3} \left[x(n) + x\left(n+\frac{N}{2}\right)(-1)^K\right] W_8^{Kn}.$

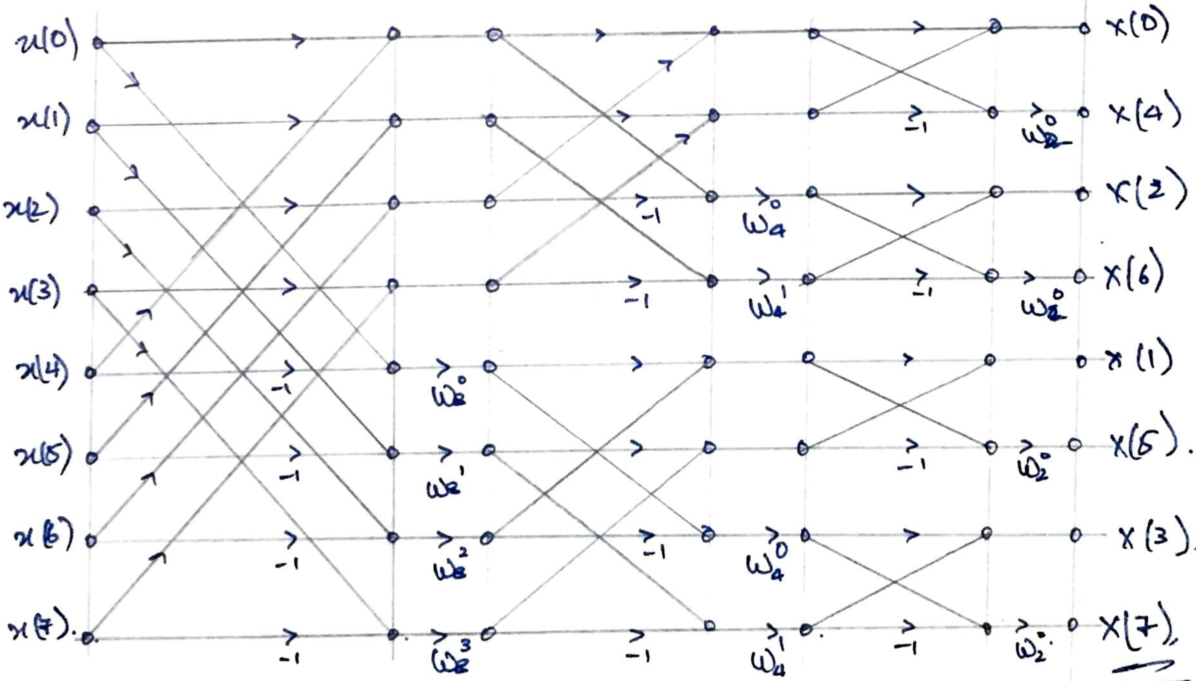
$K = 2K.$

$$X(2K) = X_1(2K) + X_2(2K)(-1)^{2K} = X_1(2K) + X_2(2K).$$

For odd.

$K = 2K+1.$

$$X(2K+1) = X_1(2K+1) + X_2(2K+1)(-1)^{2K+1} = X_1(2K+1) - X_2(2K+1).$$



$X_1(0) = x(0) + x(4) = 1 + 5 = 6$ $\boxed{x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}}$

$X_1(1) = x(1) + x(5) = 8$

$X_1(2) = x(2) + x(6) = 10$

$X_1(3) = x(3) + x(7) = 12$

$X_1(4) = x(0) - W_8^0\, x(4) = -4$

$X_1(5) = x(1) - W_8^1\, x(5) = -2.82 + 2.82j$

$X_1(6) = x(2) - W_8^2\, x(6) = 4j$

$X_1(7) = x(3) - W_8^3\, x(7) = 2.82 + 2.82j.$

$\longrightarrow$

$X_2(0) = X_1(0) + X_1(2). = 16$

$X_2(1) = X_1(1) + X_1(3). = 20$

$X_2(2) = X_1(0) - W_4^0\, X_1(2) = -4$

$X_2(3) = X_1(1) - W_4^1\, X_1(3) = 4j$

$X_2(4) = X_1(4) + X_1(6). = -4 + 4j$

$X_2(5) = X_1(5) + X_1(7). = 5.65j$

$X_2(6) = X_1(4) - W_4^0\, X_1(6). = -4 - 4j$

$X_2(7) = X_1(5) - W_4^1\, X_1(7) = 5.65j.$

$\longrightarrow$

$X(0) = X_2(0) + X_2(1). = 36$

$X(4) = X_2(0) - X_2(1)\, W_2^0 = -4$

$X(2) = X_2(2) + X_2(3) = -4 + 4j$

$X(6) = X_2(2) - X_2(3) = -4 - 4j$

$X(1) = X_2(4) + X_2(5) = -4 + 9.65j$

$X(5) = X_2(4) - X_2(5) = -4 - 1.65j$

$X(3) = X_2(6) + X_2(7). = -4 + 1.65j$

$X(7) = X_2(6) - X_2(7) = -4 - 9.65j.$

$X(K) = \{ 36$ $\leftarrow$ 0th place.

$-4 + 9.65j$

$-4 + 4j$

$-4 + 1.65j$.

-4

$-4 - 1.65j$

$-4 - 4j$

$-4 - 9.65j \}$.

Complex Radix $\quad N=6 = 3\times 2$. DIT FFT.

$x(n) = \{7, 8, 9, 8, 7, 6\}$

2 - Stages.

Radix 2 in 1st stage.

— " — 3 — " — 2nd " .

Sol: $X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK}$.

$$x(K) = \sum_{n=0}^{1} x(3n) W_N^{3nK} + \sum_{n=0}^{1} x(3n+1) W_N^{(3n+1)K} + \sum_{n=0}^{1} x(3n+2) W_N^{(3n+2)K}.$$

N=6.

$$= \sum_{n=0}^{1} x(3n) W_2^{nK} + \sum_{n=0}^{1} x(3n+1) W_2^{nK} \cdot W_6^{K} + \sum_{n=0}^{1} x(3n+2) W_2^{nK} \cdot W_6^{2K}$$

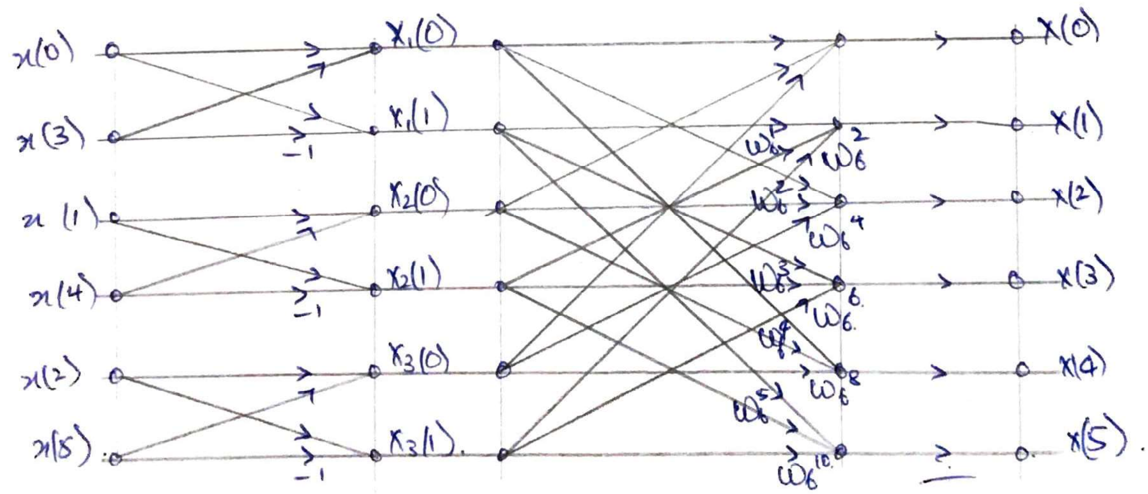
$$= X_1(K) + X_2(K) W_6^{K} + X_3(K) W_6^{2K}.$$

$$X_1(K) = \sum_{n=0}^{1} x(3n) W_2^{nK} = x(0) + x(3) W_2^{K}$$

$$X_2(K) = \sum_{n=0}^{1} x(3n+1) W_2^{nK} = x(1) + x(4) W_2^{K}$$

$$X_3(K) = \sum_{n=0}^{1} x(3n+2) W_2^{nK} = x(2) + x(5) W_2^{K}.$$

$$X(K) = X_1(K) + X_2(K) W_6^{K} + X_3(K) W_6^{2K}.$$



$X_1(0) = x(0) + x(3) = 15$

$X_1(1) = x(0) - x(3) = -1$

$X_2(0) = x(4) + x(1). = 8+7 = 15$

$X_2(1) = x(1) - x(4). = 1$

$X_3(0) = x(2) + x(5). = 9+6 = 15$

$X_3(1) = x(2) - x(5). = 9-6 = \underline{3.}$

$X(0) = X_1(0) + X_2(0) + X_3(0) = 15+15+15 = \underline{45}$

$X(1) = X_1(1) + X_2(1)W_6^1 + X_3(1)W_6^2 = -2 - 3.46j.$

$X(2) = X_1(0) + X_2(0)W_6^2 + X_3(0)W_6^4 = 0$

$X(3) = X_1(1) + X_2(1)W_6^3 + X_3(1)W_6^6 = 1$

$X(4) = X_1(0) + X_2(0)W_6^4 + X_3(0)W_6^8 = 0$

$X(5) = X_1(1) + X_2(1)W_6^5 + X_3(1)W_6^{10} = -2+3.46j.$

<u>Direct cal:</u>

$W_6^0 = 1 \qquad W_6^1 = 0.5 - 0.866j. \qquad W_6^2 = -0.5 - 0.866j.$

$W_6^3 = -1 \qquad W_6^4 = -0.5 + 0.866j \qquad W_6^5 = 0.5 + 0.866j.$

$$X(K) = \sum_{n=0}^{5} x(n) W_6^{nK}.$$

$X(0) = 7+8+9+8+7+6 = 45.$

$X(1) = 7 + 8W_6^1 + 9W_6^2 + 8W_6^3 + 7W_6^4 + 6W_6^5 = -2 - 3.46j.$

$X(2) = 7 + 8W_6^2 + 9W_6^4 + 8W_6^6 + 7W_6^8 + 6W_6^{10} = 0$

$X(3) = 7 + 8(-1) + 9 - 8 + 7 - 6 = 1$

$X(4) = 7 + 8W_6^4 + 9W_6^8 + 8W_6^{12} + 7W_6^{16} + 6W_6^{20} = 0.$

$X(5) = 7 + 8W_6^5 + 9W_6^{10} + 8W_6^{15} + 7W_6^{20} + 6W_6^{25} = \underline{\underline{-2+3.46j}}.$