① ~~Periodic~~ Similarity Property: $DFT\{x_1(n) + x_2(n)\} = X_1(K) + X_2(K)$

Sequence 1 $x_1(n) = \{\underset{\uparrow}{2}, 3, 4, 5\}$.

Sequence 2 $x_2(n) = \{\underset{\uparrow}{1}, 3, 5, 7\}$.

$$X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 14 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}.$$

$$X_2(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 16 \\ -4+4j \\ -4 \\ -4-4j \end{bmatrix}.$$

$$X_1(K) + X_2(K) = \begin{bmatrix} 30 \\ -6+6j \\ -6 \\ -6-6j \end{bmatrix}.$$

$x_3(n) = x_1(n) + x_2(n)$.

$$\begin{array}{ccccc} = & 2 & 3 & 4 & 5 \\ + & 1 & 3 & 5 & 7 \end{array}$$

$x_3(n) = \{3, 6, 9, 12\}$.

$$DFT\{x_3(n)\} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 9 \\ 12 \end{bmatrix} = \begin{bmatrix} 36 \\ -6+6j \\ -6 \\ -6-6j \end{bmatrix}$$

$\therefore$ $DFT\{x_3(n)\} = X_1(K) + X_2(K)$.

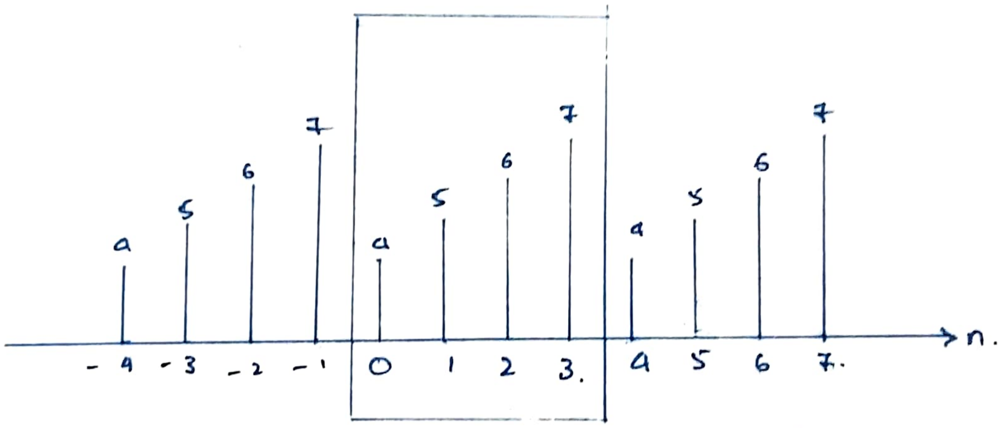
② **Periodicity Property:** If $X(k) = DFT\{x(n)\}$ & $x(n+N) = x(n)$

$$X(k) = X(k+n). \quad \text{for all } k.$$

$x(n) = \{\underset{\uparrow}{4} \quad 5 \quad 6 \quad 7\}$ $\qquad N \to$ Period $= 4.$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 22 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$x(N+n) = x(4+n).$$



$x(4+n) = x(n)$ $\qquad N = 4.$

$$\therefore X(4+k) = X(k) = [\underset{\uparrow}{22}, \; -2+2j, \; -2, \; -2-2j]$$

(3) Circular Shifting Property :-

$x(n) = \{\underset{\uparrow}{1}, 3, 5, 7\}$ $\qquad m = 2.$

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 16 \\ -4+4j \\ -4 \\ -4-4j \end{bmatrix}$$

$$x(n-m)_N = x(n-2)_4 = \{5, 7, 1, 3\}$$

$$X(K-2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 16 \\ 4-4j \\ -4 \\ 4+4j \end{bmatrix}$$

But $DFT\{x((n-m))_N\} = W_N^{mK} X(K) = W_4^{2K} X(K).$

$$X_1(K) = W_4^{2K} X(K).$$

$$X_1(0) = X(0) = 16.$$

$$X_1(1) = W_4^2 X(1) = -1(-4+4j) = 4-4j.$$

$$X_1(2) = W_4^4 X(2) = 1(-4) = -4.$$

$$X_1(3) = W_4^6 X(3) = -1(-4-4j) = 4+4j$$

$$X_1(K) = \{\underset{\uparrow}{16}, 4-4j, -4, 4+4j\}$$

$$X_1(K) = DFT\{x((n-2))_4\} = X(K-2)$$