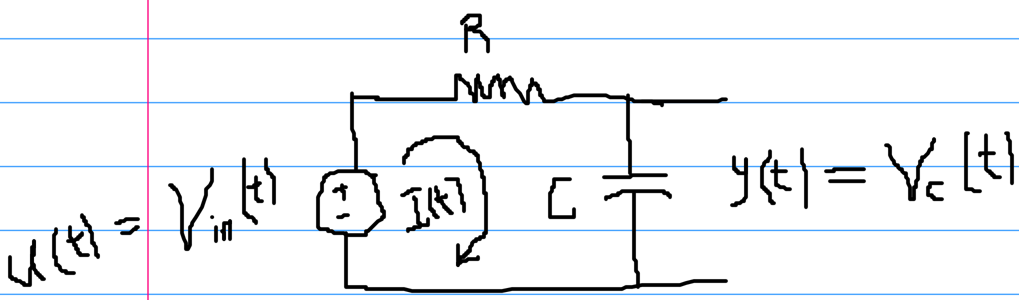
Mathematical Modeling of Electrical Systems/Circuits



By Kirchoff's Voltage Law:

$$V_{in}(t) = V_R(t) + V_C(t)$$

$$= I(t)R + V_c(t)$$

$$I(t) = C\frac{dV_c(t)}{dt} \qquad V_c(t) = \int_0^t \frac{I(\tau)}{C}\, d\tau$$

$$V_{in}(t) = RC\frac{dV_c(t)}{dt} + V_c(t)$$

$$u(t) = RC\frac{dy(t)}{dt} + y(t)$$

$$\mathcal{L}\{u(t)\} = RC\,\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + \mathcal{L}\{y(t)\}$$

Assuming zero initial conditions

$$U(s) = RCsY(s) + Y(s)$$

$$U(s) = Y(s)\{RCs + 1\}$$

$$\therefore \quad G(s) = \frac{Y(s)}{U(s)} = \frac{1}{RCs + 1}$$

$$= \frac{1/RC}{s + \frac{1}{RC}}$$

In mass spring system

$x(t)$ position $\dot{x}(t) = $ Velocity

$$F(t) - Kx(t) = M\ddot{x}(t)$$

$$y(t) = x(t)$$

Spring

$K = 0 \qquad K_f \neq 0$

$$y(t) = \text{Velocity} = \frac{dx}{dt}$$

$$F(t) - K_f \frac{dx(t)}{dt} = M \frac{d^2x}{dt^2}$$

$$u(t) - K_f\, y(t) = M \frac{dy(t)}{dt}$$

$$u(s) = Y(s)\left\{ Ms + K_f \right\}$$

Back to RC circuit

Impulse Response

$$g(t) = \frac{1}{RC} e^{-t/RC} \quad \text{for } t \geq 0$$

$$Y(s) = G(s) \cdot U(s)$$

When $u(t) =$ unit step

$$U(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s} \cdot \frac{1/RC}{s + \frac{1}{RC}}$$

$$= \frac{A}{s} + \frac{B}{s + \frac{1}{RC}}$$

$$A = Y(s) \cdot s \quad \text{when } s = 0$$

$$= 1$$

$$B = Y(s) \cdot \left(s + \frac{1}{RC}\right) \quad \text{when } s = -\frac{1}{RC}$$

$$= -1$$

$$Y(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{RC}}$$

$$y(t) = 1 - e^{-t/RC} \quad \text{for } t \geq 0$$