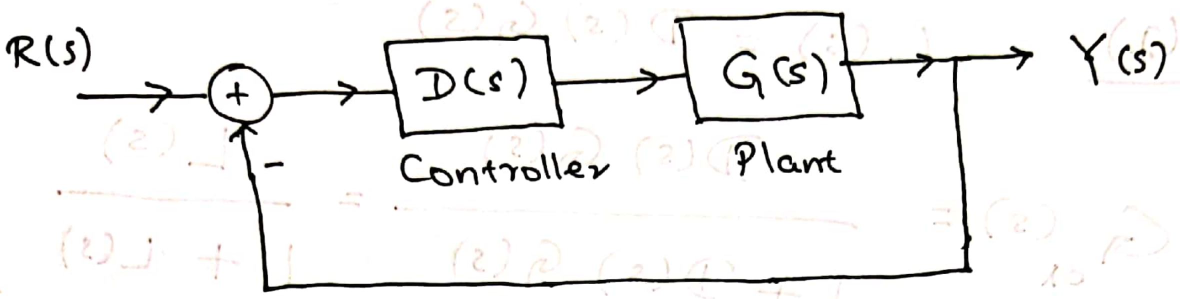
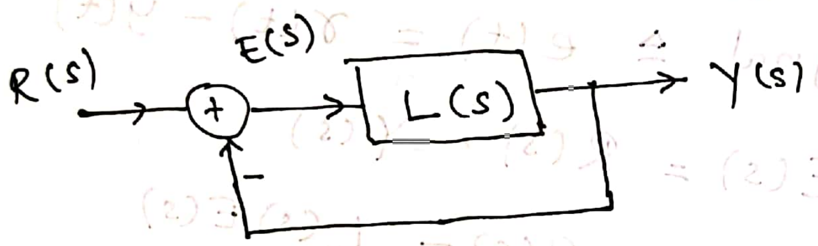
PERFORMANCE OF CLOSED-LOOP SYSTEM

STEADY STATE ERROR ANALYSIS



Unity Feedback System



$$L(s) = \frac{K(s-z_1)(s-z_2)\dots(s-z_m)}{s^{\lambda}(s-p_1)(s-p_2)\dots(s-p_n)}$$

is called type-λ system.

λ indicates number of poles of $L(s)$ at $s=0$.

Note that $p_1, p_2, \dots p_n$ may be positive or negative.

Our interest is performance of closed-loop system.

Transfer function of closed-loop system.

<u>case (1).</u> $L(s) = D(s).G(s)$

$$G_{cl}(s) = \frac{D(s)\,G(s)}{1 + D(s)\,G(s)} = \frac{L(s)}{1 + L(s)}$$

(For stability of CLS: Roots of $(1+L(s)) = 0$ must be in LHS)

Error signal $\triangleq e(t) = r(t) - y(t)$

$$E(s) = R(s) - Y(s)$$
$$= R(s) - L(s).E(s)$$

$$E(s) = \frac{R(s)}{1 + L(s)} = \frac{R(s)}{1+D(s)G(s)}$$

From final value theorem:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(s)$$

We consider three standard reference signals

$r(t) = u_s(t)$ Unit step

$r(t) = t.u_s(t)$ Unit Ramp

$r(t) = \frac{t^2}{2}.u_s(t)$ Parabola

and analyze the steady state error in the closed-loop system output.

Type-0 system

$$L(s) = \frac{K (s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots\ldots(s-p_n)}$$

When reference input is step signal,

$r(t) = u_s(t)$ $\quad R(s) = \frac{1}{s}$.

$$E(s) = \frac{R(s)}{1+L(s)} = \frac{1/s}{1 + K\frac{(s-z_1)(s-z_2)\ldots}{(s-p_1)(s-p_2)\ldots}}$$

Steady state error

$$\lim_{s\to 0} s.E(s) = \lim_{s\to 0} \frac{\cancel{s}.\,1/\cancel{s}}{1 + K\frac{(s-z_1)(s-z_2)\ldots}{(s-p_1)(s-p_2)\ldots}}$$

$$= \frac{1}{1+K_p}$$

$K_p \rightarrow$ Position Error constant

Type-0 system and Ramp signal input

$$R(s) = \frac{1}{s^2} \qquad r(t) = t \cdot u_s(t)$$

$$E(s) = \frac{R(s)}{1 + L(s)} = \frac{1/s^2}{1 + \frac{K(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)\cdots}}$$

Steady. state Error

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} \frac{s \cdot 1/s^2}{1 + \frac{K(s-z_1)\cdots}{(s-p_1)(s-p_2)\cdots}}$$

$$= \lim_{s \to 0} \frac{1}{s + \frac{s \cdot K(s-z_1)\cdots}{(s-p_1)(s-p_2)\cdots}}$$

$$= \frac{1}{0} = \infty .$$

Similar situation when $r(t) = \frac{t^2}{2} \cdot u_s(t)$

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \infty .$$

Type-1 system

$$L(s) = \frac{K(s-z_1)(s-z_2)\ldots(s-z_m)}{s(s-p_1)(s-p_2)\ldots(s-p_n)}$$

when reference input $r(t) = u_s(t)$
$R(s) = \frac{1}{s}$

$$E(s) = \frac{1/s}{1 + \frac{K(s-z_1)(s-z_2)\ldots}{s(s-p_1)(s-p_2)\ldots}}$$

steady state Error

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s}.1/\cancel{s}}{1 + \frac{K(s-z_1)(s-z_2)\ldots}{s(s-p_1)(s-p_2)\ldots}}$$

$$= \frac{1}{1+\infty} = \underline{\underline{0}}.$$

Type-1 or higher type system sholud be preferred to get zero steady state error for ~~ref~~ step input.

When Reference Input is ramp signal.

$$r(t) = t \cdot u_s(t) \qquad R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{1/s^2}{1 + \frac{K(s-z_1)\ldots}{s(s-p_1)(s-p_2)\ldots}}$$

Steady State Error

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} \frac{\cancel{s} \cdot 1/s^{\cancel{2}}}{1 + \frac{K(s-z_1)\ldots}{s(s-p_1)\ldots}}$$

$$= \lim_{s \to 0} \frac{1}{s + \cancel{s} \cdot \frac{K(s-z_1)\ldots}{\cancel{s}(s-p_1)(s-p_2)\ldots}}$$

$$= \frac{1}{K_v} \rightarrow \text{Velocity Error Constant.}$$