Given: $G(s) = \frac{1}{s(s+6)}$

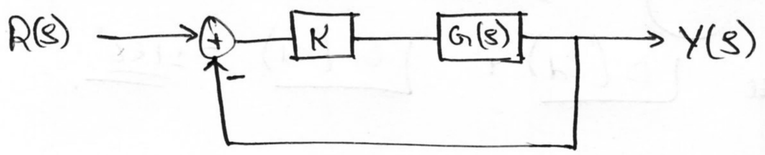
To Do: Design a phase lead controller.

$\zeta = 0.4$

$\omega_n = 15$ rad/s.

$$S_d = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2} = -6 \pm 3\sqrt{21}j.$$

Step 1: We check if a simple proportional controller can solve.



$$\alpha(s) = 1 + K\,G(s) = 1 + \frac{K}{s(s+6)} = \frac{s^2+6s+K}{s^2+6s}.$$

Finding roots of s^2+6s+K.

$a = 1$
$b = 6$.
$c = K$.

Poles are $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$$= \frac{-6 \pm \sqrt{36-4K}}{2} = -3 \pm \sqrt{9-K}.$$

Since the real part is always -3, it never passes through S_d i.e. $-6 \pm 3\sqrt{21}j$. So we use a phase lead controller.

Step 2: Phase lead Controller.

$$D(s) = \frac{K(s+z)}{s+p}.$$

'Zero' of the controller is chosen below Sd, but in this system, a pole already exists at $s = -6$.

$\therefore$ We chose the pole location to be -7.

ie $z = 7$.

$$\therefore \boxed{z = 7.}$$

→ Finding p using angle criteria.

ie $\angle D(s_d) + \angle G(s_d) = \pm 180^\circ$.

ie. $\angle s_d + z - \angle s_d + p - \angle s_d - \angle s_d + 6 = \pm 180^\circ$

$\angle -6 + 3\sqrt{21}j + 7 - \angle s_d + p - \angle -6 + 3\sqrt{21}j - \angle -6 + 3\sqrt{21}j + 6 = \pm 180^\circ$

$\pm 180^\circ = 85.839^\circ - \angle s_d + p - 113.57^\circ - 90 = \pm 180$

$$\angle s_d + p = 62.269^\circ = \tan^{-1}\left(\frac{3\sqrt{21}}{p-6}\right).$$

$$p = \frac{3\sqrt{21}}{\tan(62.269)} + 6 = 13.22. \quad \therefore \boxed{p = 13.22.}$$

Finding K using magnitude criteria.

$$L(s) = \frac{K(s+7)}{s(s+6)(s+13.22)}$$

$$K = \frac{1}{|L(s)|} = \frac{|s_d|\,|s_d+6|\,|s_d + 13.22|}{|s_d + 7|}$$

$$K = \frac{|-6+3\sqrt{21}j|\,|3\sqrt{21}j|\,|-6+3\sqrt{21}j + 13.22|}{|-6+3\sqrt{21}j + 7|} = \boxed{232.31.}$$

$$\therefore \; L(s) = \frac{232.31\,(s+7)}{s(s+6)(s+13.22)}$$

Sketching the RL of $L(s)$ above, we can see that the point $-6 \pm 3\sqrt{21}j$ passes through.