DSP- Lab E-4.

① Circular Frequency shift:

Let $x(n)$ be $\{ \underset{\uparrow}{4} \quad 5 \quad 6 \quad 7 \}$

$$x(n) = \{ \underset{\uparrow}{4}, 5, 6, 7 \}$$

$$X(K) = \sum_{n=0}^{3} x(n)\, W_4^{nK} \qquad \underline{\underline{N=4.}}$$

Using matrix method.

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 22 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Now if we circularly shift $X(K)$ by l ie $X(l-K)_N$ should be same as $DFT[x(n) \times e^{(j \times 2\pi n \times l/N)}]$

$$X(l-K)_N = \{ \underset{\uparrow}{-2-2j}, 22, -2+2j, -2 \} \text{ Suppose } \underline{\underline{l=1.}}$$

Now. $x_1(n) = x(n) \times e^{(j 2\pi n \times l/N)}$.

$x_1(0) = 4 \quad x_1(1) = 5 \times j = 0 + 5j$.

$x_1(2) = 6(-1) = -6. \quad x_1(3) = 7(-j) = 0 - 7j$.

$$x_1(n) = \{ \underset{\uparrow}{4}, 5j, -6, -7j \}.$$

$$X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 5j \\ -6 \\ -7j \end{bmatrix} = \underline{\underline{\begin{bmatrix} -2-2j \\ 22 \\ -2+2j \\ -2 \end{bmatrix}}}$$

Hence Verified

2

Circular convolution of $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$.

$h(n) = \{\underset{\uparrow}{5}, 6, 7\}$

1 → Make lengths equal.

$x(n) = \{1, 2, 3, 4\}$ $\qquad$ $h(n) = \{5, 6, 7, 0\}$

Time domain method:

$\cancel{h(n-m)}\; h(m-n)$ when $m=0$

$h(-n) = \{\underset{\uparrow}{5}, 0, 7, 6\}$

$y(n) = h(n) \circledast x(n)$.

$= x(n) . h(m-n)$.

$$\begin{bmatrix} 5 & 0 & 7 & 6 \\ 6 & 5 & 0 & 7 \\ 7 & 6 & 5 & 0 \\ 0 & 7 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 50 \\ 44 \\ 34 \\ 52 \end{bmatrix}$$

Freq Domain.

$X(K) = \{1 + 2W_4^{1K} + 3W_4^{2K} + 4W_4^{3K}\}$.

$H(K) = \{5 + 6W_4^{K} + 7W_4^{2K}\}$.

$5 + 6W_4^{K} + 7W_4^{2K} + 10W_4^{K} + 12W_4^{2K} + 14W_4^{3K} + 15W_4^{2K} + 12W_4^{3K}$

$+ 21W_4^{0} + 20W_4^{3K} + 24W_4^{0} + 28W_4^{K}$

$50W_4^{0} + 44W_4^{K} + 34W_4^{2K} + 52W_4^{3K}$

$y(n) = \{\underset{\uparrow}{50}, 44, 34, 52\}$.

(3) Conjugate Symmetry:

(a) Real sequence. $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$.

$$DFT[x(n)] = X(K) = \sum_{n=0}^{3} x(n)\, W_4^{nK}$$

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

According to conjugate symmetric property $X(K) = X^*((-K))_N$ for a real sequence.

$$X^*((-K))_N = \{DFT[x((-n))_N]\}^*$$ $\quad x((-n))_4 = \{\underset{\uparrow}{1}, 4, 3, 2\} = x_1(n)$.

$$DFT[x((-n))] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ -2-2j \\ -2 \\ -2+2j \end{bmatrix} = X_1(K).$$

$$X_1^*(K) = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = X(K).$$

Hence verified.

(b) Complex sequence.

$$x(n) = \{\underset{\uparrow}{1j},\ 2,\ 3,\ 4\}$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1j \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 + 1j \\ -3 + 3j \\ -3 + j \\ -3 - j \end{bmatrix}$$

$$x_1(n) = x(-n)_N = \{\underset{\uparrow}{1j}\ \ 4\ \ 3\ \ 2\}.$$

$$X_1(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1j \\ 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 + 1j \\ -3 - 1j \\ -3 + 1j \\ -3 + 3j \end{bmatrix}$$

$$X_1^*(k) = \begin{bmatrix} 9 - 1j \\ -3 + 1j \\ -3 - 1j \\ -3 - 3j \end{bmatrix} \neq X(k).$$

Hence sequence is not conjugate symmetric Since x(n) is not real.