① Time Reversal Property:

Let $x(n) = \{2, 3, 5, 4, 1\}$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nK} \qquad N = 5$$

$$\therefore X(K) = \sum_{n=0}^{4} x(n)\, W_5^{Kn}$$

$W_5^0 = 1 \qquad W_5^1 = 0.309 - 0.951j \qquad W_5^2 = -0.809 - 0.587j$

$W_5^3 = -0.8090 + 0.587j \qquad W_5^4 = 0.3090 + 0.9511j$

$X(0) = 2 + 3 + 5 + 4 + 1 = \underline{15}$

$X(1) = 2 + 3W_5^1 + 5W_5^2 + 4W_5^3 + W_5^4 = \underline{-4.04 - 2.49j}$

$X(2) = 2 + 3W_5^2 + 5W_5^4 + 4W_5^1 + W_5^3 = \underline{1.54 - 0.22j}$

$X(3) = 2 + 3W_5^3 + 5W_5^1 + 4W_5^4 + W_5^2 = \underline{1.54 + 0.22j}$

$X(4) = 2 + 3W_5^4 + 5W_5^3 + 4W_5^2 + W_5^1 = \underline{-4.04 + 2.49j}$

$$X^*(K) = \{\underset{\uparrow}{15}, -4.04 + 2.49j, 1.54 + 0.22j, 1.54 - 0.22j, -4.04 - 2.49j\}$$

$$x(-n) = \{\underset{\uparrow}{2}\ 1\ 4\ 5\ 3\} = x_1(n)$$

$$X_1(K) = \sum_{n=0}^{4} x_1(n)\, W_5^{nK}$$

$X_1(0) = 2 + 1 + 4 + 5 + 3 = \underline{15}$

$X_1(1) = 2 + W_5^1 + 4W_5^2 + 5W_5^3 + 3W_5^4 = \underline{-4.04 + 2.49j}$

$X_1(2) = 2 + W_5^2 + 4W_5^4 + 5W_5^6 + 3W_5^8 = \underline{1.54 + 0.22j}$

$$X_1(3) = 2 + W_5^{3} + 4W_5^{6} + 5W_5^{9} + 3W_5^{12} = 1.54 - 0.22j$$

$$X_1(4) = 2 + W_5^{4} + 4W_5^{8} + 5W_5^{12} + 3W_5^{16} = -4.04 - 2.49j$$

Since $X_1(k) = X^*(k) = \text{DFT}\{x(-n)\}$.

Time Reversal Property is verified.

Parsevals Theorem: $\sum_{n=0}^{N-1} (x(n))^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$

$$x(n) = \{\underset{\uparrow}{2}\ \ 3\ \ 5\ \ 4\ \ 1\}$$

$$P_T = 2^2 + 3^2 + 5^2 + 4^2 + 1^2 = 4 + 9 + 25 + 16 + 1 = \boxed{55}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$X(k) = \{\underset{\uparrow}{15},\ -4.04 - 2.49j,\ 1.54 - 0.22j,\ 1.54 + 0.22j,\ -4.04 + 2.49j\}$$

$$\sum_{k=0}^{4} X(k) X^*(k) = 15^2 + (-4.04 - 2.49j)(-4.04 + 2.49j)$$
$$+ (1.54 - 0.22j)(1.54 + 0.22j)$$
$$+ (1.54 + 0.22j)(1.54 - 0.22j)$$
$$+ (-4.04 + 2.49j)(-4.04 - 2.49j)$$

$$= 225 + 22.56 + 2.43 + 2.43 + 22.86 = 275$$

$$P_F = \frac{1}{N} \sum_{k=0}^{4} X(k) \times X^*(k) = \frac{1}{5} \times 275 = \boxed{55}$$

Circular conv using linear :

Linear conv = $x(n) * h(n) = \sum x(k)\, h(n-k)$

Circular conv = $x(n) \circledast h(n) = \sum x((k))_N\, h((n-k))_N$

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$

$h(n) = \{\underset{\uparrow}{6}, 7, 8\}$

Length of linear conv = 5+3-1
= 7.

$7 \times 3 \quad 3\times 1 = 7\times 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \\ 4 & 3 & 2 \\ 5 & 4 & 3 \\ 0 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ 40 \\ 61 \\ 82 \\ 67 \\ 40 \end{bmatrix}$$

Circular conv =

	6	19	40	61	82
+	67	40			
	73	59	40	61	82.

$x(n) \circledast h(n) = \{\underset{\uparrow}{73}\ \ 59\ \ 40\ \ 61\ \ 82.\}$

Linear Convolution using Circular.

Circular conv $y(n) = h(n) \circledast x(n)$.

$= x((k))_N \, h((n-k))_N$

Linear conv $y(n) = h(n) * x(n)$

$x(k) \cdot h(n-k)$.

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ $h(n) = \{\underset{\uparrow}{6}, 7, 8\}$

For linear using circular, first we make the lengths equal to

$x(n) = \{\underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \quad 5 \quad 0 \quad 0\}$ length of linear conv.

$h(n) = \{\underset{\uparrow}{6} \quad 7 \quad 8 \quad 0 \quad 0 \quad 0 \quad 0\}$ $5+3-1 = \underline{\underline{7}}$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 5 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ 40 \\ 61 \\ 82 \\ 67 \\ 40 \end{bmatrix}$$