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JSS Mahavidyapeetha

JSS Science And Technology University

(Established Under JSS Science and Technology University Act No. 43 of 2013)

(Formerly Known as SJCE)

JSS SCIENCE AND TECHNOLOGY UNIVERSITY MYSURU

“Event 4 Report”

Submitted for the fulfillment of the CIE (Event-4) for the course

CONTROL SYSTEMS

(EC540)

Submitted by

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Given: $G(s) = \frac{1}{s(s+6)}$

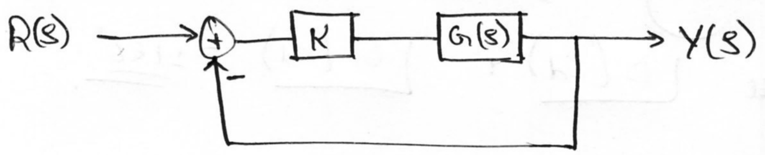
To Do: Design a phase lead controller.

$\zeta = 0.4$

$\omega_n = 15$ rad/s.

$$S_d = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2} = -6 \pm 3\sqrt{21}j.$$

Step 1: We check if a simple proportional controller can solve.



$$\alpha(s) = 1 + K\,G(s) = 1 + \frac{K}{s(s+6)} = \frac{s^2+6s+K}{s^2+6s}.$$

Finding roots of s^2+6s+K.

$a = 1$
$b = 6$.
$c = K$.

Poles are $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$$= \frac{-6 \pm \sqrt{36-4K}}{2} = -3 \pm \sqrt{9-K}.$$

Since the real part is always -3, it never passes through S_d i.e. $-6 \pm 3\sqrt{21}j$. So we use a phase lead controller.

Step 2: Phase lead Controller.

$$D(s) = \frac{K(s+z)}{s+P}.$$

'Zero' of the controller is chosen below Sd, but in this system, a pole already exists at $s = -6$.

∴ We chose the pole location to be -7.

ie $z = 7$.

$$\therefore \boxed{z = 7.}$$

→ Finding P using angle criteria.

ie $\angle D(sd) + \angle G(sd) = \pm 180°$.

ie $\angle sd + z - \angle sd + P - \angle sd - \angle sd + 6 = \pm 180°$

$\angle -6 + 3\sqrt{21}j + 7 - \angle sd + P - \angle -6 + 3\sqrt{21}j - \angle -6 + 3\sqrt{21}j + 6 = \pm 180°$

$\pm 180° = 85.839° - \angle sd + P - 113.57° - 90 = \pm 180$

$$\angle sd + P = 62.269° = \tan^{-1}\left(\frac{3\sqrt{21}}{P-6}\right).$$

$$P = \frac{3\sqrt{21}}{\tan(62.269)} + 6 = 13.22. \qquad \therefore \boxed{P = 13.22.}$$

Finding K using magnitude criteria.

$$L(s) = \frac{K(s+7)}{s(s+6)(s+13.22)}$$

$$K = \frac{1}{|L(s)|} = \frac{|sd|\,|sd+6|\,|sd+13.22|}{|sd+7|}$$

$$K = \frac{|-6+3\sqrt{21}j|\,|3\sqrt{21}j|\,|-6+3\sqrt{21}j+13.22|}{|-6+3\sqrt{21}j+7|} = \boxed{232.31.}$$

$$\therefore \; L(s) = \frac{232.31\,(s+7)}{s(s+6)(s+13.22)}$$

Sketching the RL of L(s) above, we can see that the point $-6 \pm 3\sqrt{21}j$ passes through.

Probelm Statement

For the plant **G(s) = 1/s(s+6)** design a **phase-lead** controller for damping ratio **ζ = 0.4** and natural frequency **15 rad/sec**.

What is the phase margin and gain margin of the compensated system?

Solution

Clearing Workspace

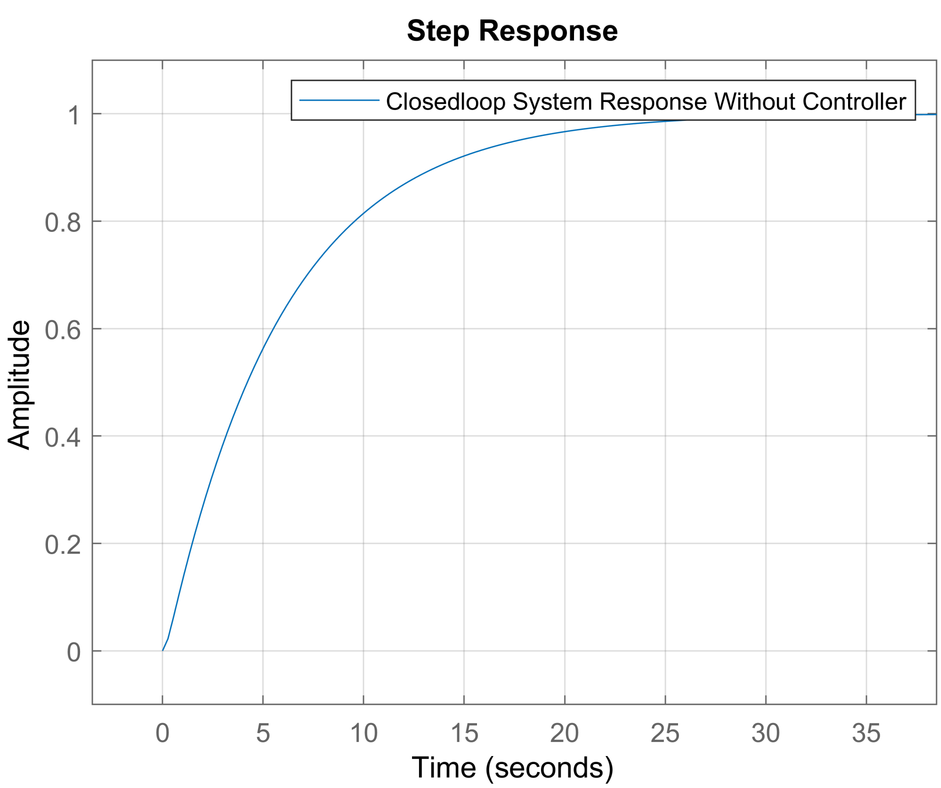
```
close all;
clear;
clc;
```

System without a controller

```
s=tf('s');
G=1/(s*(s+6));
```

Gcl = Closedloop Transfer Function

```
Gcl = G/(1+G); % system without controller
step(Gcl); % plotting step response
grid on;
setAxisLimits(axis);
legend('Closedloop System Response Without Controller');
```



Poles fo the system without controller

```
disp(pole(Gcl));
```

```
         0
   -6.0000
   -5.8284
   -0.1716
```

One of the pole is on the imaginary axis, and therefore, the system without controller is marginally stable.

Time domain parameters of system

```
stepinfo(Gcl)
```

```
ans = struct with fields:
        RiseTime: 12.8096
    SettlingTime: 22.9766
     SettlingMin: 0.9016
     SettlingMax: 0.9993
       Overshoot: 0
      Undershoot: 0
            Peak: 0.9993
        PeakTime: 42.6770
```

Designing a Phase Lead Controller

ζ = 0.4

ωn = 15

```
zita = 0.4;
wn = 15;
desiredPoles = roots([1 2*zita*wn wn^2]);
```

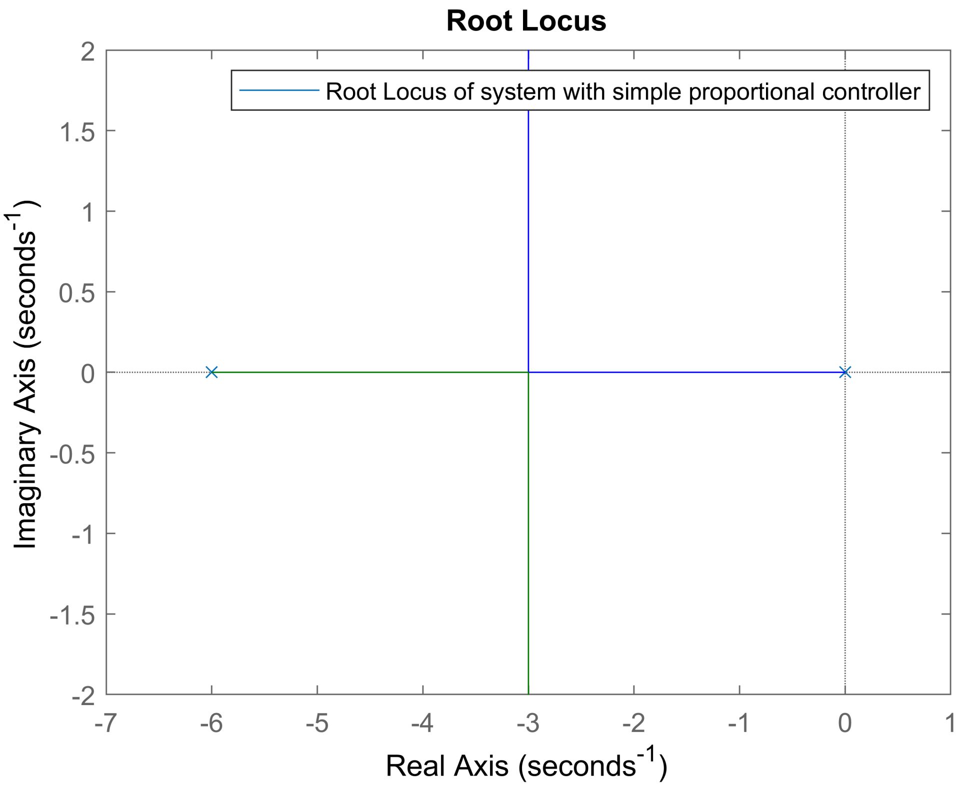
Root Locus must pass throught desired poles.

```
disp(desiredPoles);
```

```
  -6.0000 +13.7477i
  -6.0000 -13.7477i
```

Root Locus of a system with a simple proportional controller

```
figure;
rlocus(G);
legend('Root Locus of system with simple proportional controller');
```



We can see that no matter what, the root locus doesn't pass through **desired poles.**

```
syms s1
G1=1/(s1*(s1+6));
phi=double(angle(subs(G1,s1,-6+13.74i)))*180/pi;
sphi=180-phi;
```

The zero of the controller is usually taken just below the desired poles, but as in this system, a pole already exists at S = -6.

∴ We take the zero of the controller slightly towards left of -6. i.e S = -7 or **Z = 7**.

```
z=-7;
p=z-13.7477/tand(90-sphi);
disp(p);
```

```
-13.0034
```

And thus we the pole of the controller as **-13** or **P = 13**.

Then we find out **k** using magnitude criteria.

```
Ds=(s1-z)/(s1-p);
k=1/(double(abs(subs(Ds*G1,-6+13.7477i))))
```

```
k = 230.8210
```

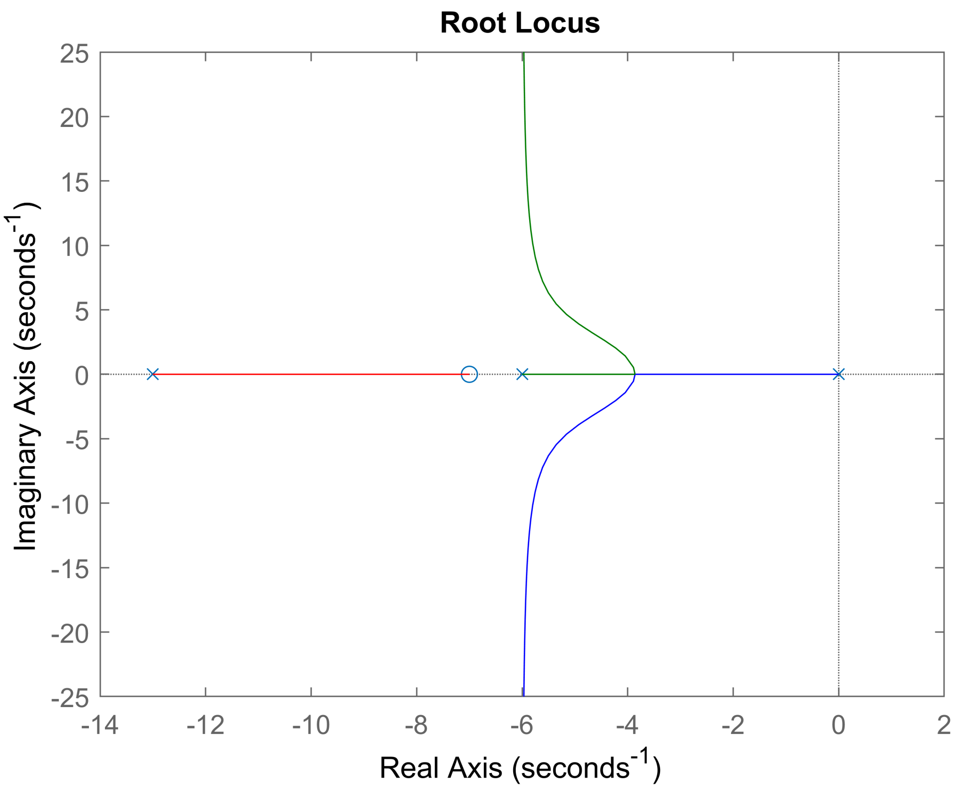
Thus at k = 230.5863, the RL passes through the desired pole location.

Verification of design

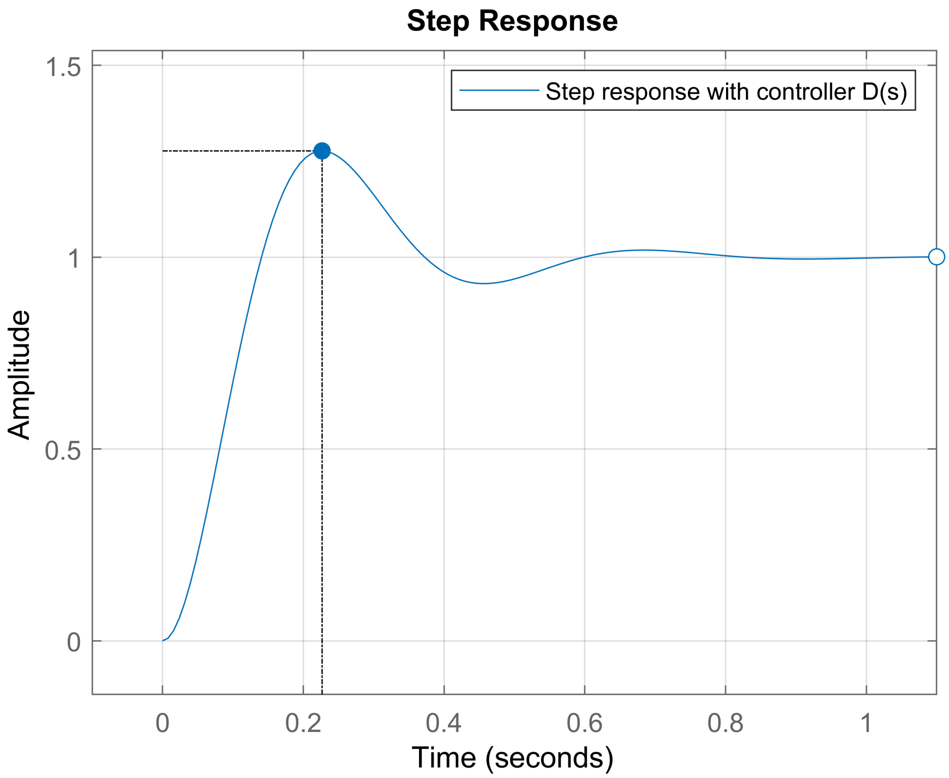
Ds = Controller Transfer Function

Ls = Closedloop Transfer Function with Controller

```
Ds = (s-z)/(s-p);
Ls = k*Ds*G/(1+k*Ds*G);
figure;
rlocus(Ds*G);
```




```
figure;
response = stepplot(Ls);
grid on;
response.showCharacteristic('PeakResponse');
response.showCharacteristic('SettlingTime');
response.showCharacteristic('RiseTime');
response.showCharacteristic('SteadyState');
setAxisLimits(axis);
legend('Step response with controller D(s)');
```



Time Domain parameters of system with controller.

```
stepinfo(Ls)
```

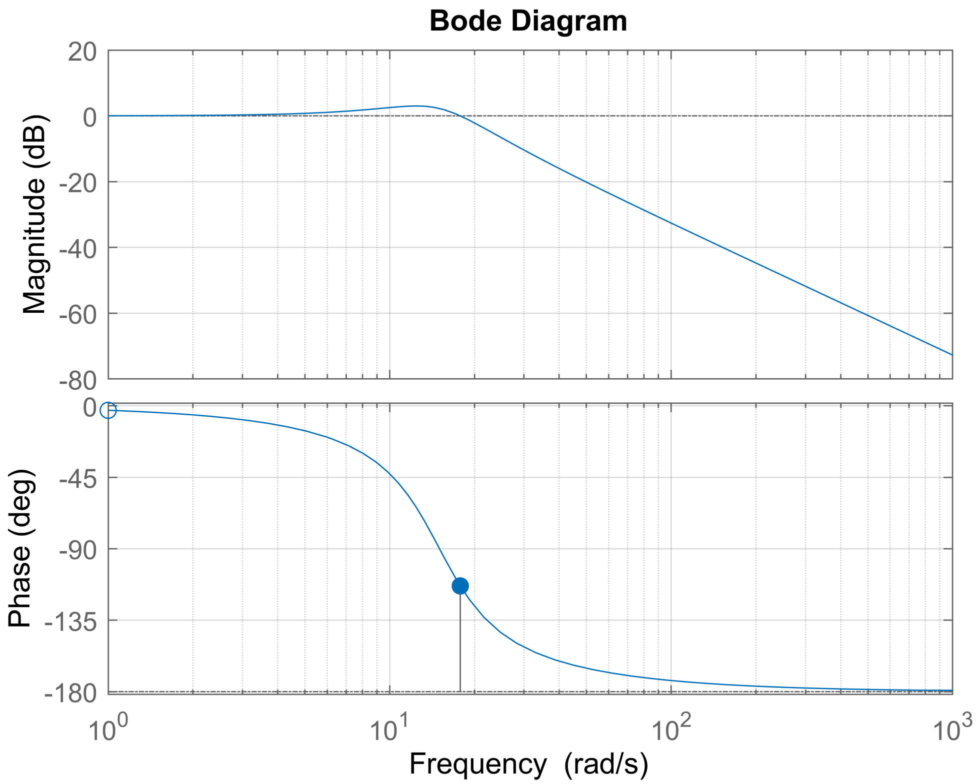
```
ans = struct with fields:
        RiseTime: 0.0952
    SettlingTime: 0.5612
     SettlingMin: 0.9312
     SettlingMax: 1.2771
       Overshoot: 27.7119
      Undershoot: 0
            Peak: 1.2771
        PeakTime: 0.2267
```

```
[gainMargin, phaseMargin, wcg, wcp] = margin(Ls)
```

```
Warning: The closed-loop system is unstable.
gainMargin = Inf
phaseMargin = 66.5761
wcg = Inf
wcp = 17.8151
```

Bode Plot of the closed loop transfer function.

```
response = bodeplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```

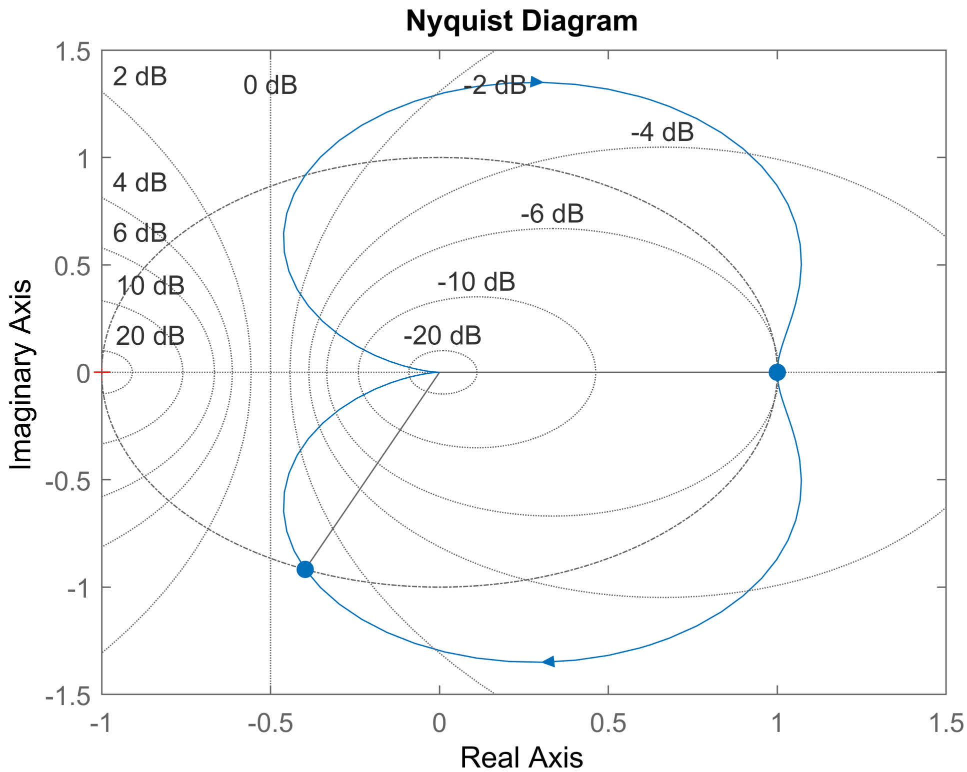


We can see from the **bode plot** that once the gain crossed the **0db** point, it never crosses it back again, hence no matter what the gain is, the system is going to remain stable.

Where as the phase when the gain crosses the **0db** is the **phase margin** and it's angle is 66.57°

Nyquist Plot of the closed loop transfer function

```
response = nyquistplot(Ls);
response.showCharacteristic('AllStabilityMargins');
grid on;
```



Inference

Since the **Gain Margin is infinity**, any amount of gain will not result in system getting unstable, this can be shown in the Nyquist plot.

Phase Margin is 66.5761, this means that the system is stable for any value on phase but not 66.5761. As seen on the Nyquist plot, At a gain of **0db**, the plot touches the -1 point at an angle of 66.5761 degrees.