ROUTH-HURWITZ TEST

Routh test to chech whether roots of polynomial: $a(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n$ are in left half of s-plane.

Necessary Condition: All coefficients a_k must be of same sign.
(Easy to check)

Sufficient Condition: All entries in the first column of Routh Table must be of same sign.

Routh Table:

$$
\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \; \dots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \; \dots \\
s^{n-2} & b_1 & b_2 & b_3 & \\
\vdots & c_1 & c_2 & c_3 & \\
 & d_1 & d_2 & & \\
 & \vdots & & & \\
s^0 & x & & &
\end{array}
$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} \div (-a_1)$$

$$b_2 = \begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix} \div (-a_1)$$

$$b_3 = \begin{vmatrix} a_0 & a_6 \\ a_1 & a_7 \end{vmatrix} \div (-a_1)$$

$$c_1 = \begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix} \div (-b_1)$$

$$c_2 = \begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix} \div (-b_1)$$

$\vdots$

$$d_1 = \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \div (-c_1)$$

$$d_2 = \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} \div (-c_1)$$

$\vdots$

Remark:

For second order polynomial:

Necessary Condition $\Leftrightarrow$ Sufficient Condition.

$$a(s) = a_0 s^2 + a_1 s + a_2$$

$$\begin{array}{c|cc} s^2 & a_0 & a_2 \\ s & a_1 & 0 \\ 1 & a_2 & \end{array}$$

First column entry are: a_0, a_1, a_2

Example:

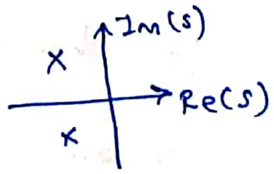
$$a(s) = s^2 + 4s + 8$$

All coefficients 1, 4, 8 are positive.

$\therefore$ Both roots are in LH of s-plane.

Actual roots are:

$$s_{1,2} = \frac{-4}{2} \pm \frac{\sqrt{4^2 - 4(8)}}{2} = -2 \pm j2.$$

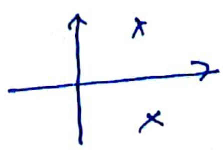


$$a(s) = s^2 - 6s + 25$$

All coefficients are NOT of same sign.

$\therefore$ Some roots NOT in LH of s-plane.

$$s_{1,2} = \frac{6}{2} \pm \frac{\sqrt{36 - 100}}{2} = 3 \pm j4$$



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Ex:

$$a(s) = s^3 + 18s^2 + 97s + 300$$

Necessary Condition: All coefficients 1, 18, 97, 300 are positive.

Sufficient Condition:

Routh Table:

s^3	1	97
s^2	18	300
s	$\frac{1446}{18}$	0
1	300	

All entries in the first coloumn are of same sign.

$\therefore$ All roots of $a(s)$ are in left half of s-plane.

Actual roots are: $-3 \pm j4, -12$.

Ex: $a(s) = s^4 + 4s^3 + 8s^2 + 8s + 12$

Necessary Condition: satisfied.

All coefficients are positive

$\therefore$ May be all roots in LH of s-plane.

Sufficient Condition:

Routh Table:

s^4	1	8	12
s^3	4	8	0
s^2	$b_1 = 6$	$b_2 = 12$	
s	$c_1 = 0$	$c_2 = 0$	
1	d		

All first coloumn entries NOT of same sign.

$$b_1 = \begin{vmatrix} 1 & 8 \\ 4 & 8 \end{vmatrix} \div (-4) = 6 \qquad b_2 = \begin{vmatrix} 1 & 12 \\ 4 & 0 \end{vmatrix} \div (-4) = 12$$

$$c_1 = \begin{vmatrix} 4 & 8 \\ 6 & 12 \end{vmatrix} \div (-6) = 0 \qquad c_2 = 0$$

At least some roots NOT in left half of s-plane.