Trees

A data structure is said to be linear if its elements form a sequence or a linear list. Previous linear data structures that we have studied like an array, stacks, queues and linked lists organize data in linear order. A data structure is said to be non linear if its elements form a hierarchical classification where, data items appear at various levels.

Trees and Graphs are widely used non-linear data structures. Tree and graph structures represents hierarchial relationship between individual data elements. Graphs are nothing but trees with certain restrictions removed.

In this chapter in particular, we will explain special type of trees known as binary trees, which are easy to maintain in the computer.

TREES:

A tree is hierarchical collection of nodes. One of the nodes, known as the root, is at the top of the hierarchy. Each node can have at most one link coming into it. The node where the link originates is called the parent node. The root node has no parent. The links leaving a node (any number of links are allowed) point to child nodes. Trees are recursive structures. Each child node is itself the root of a subtree. At the bottom of the tree are leaf nodes, which have no children.

Trees represent a special case of more general structures known as graphs. In a graph, there is no restrictions on the number of links that can enter or leave a node, and cycles may be present in the graph. The figure 1.1 shows a tree and a non-tree.

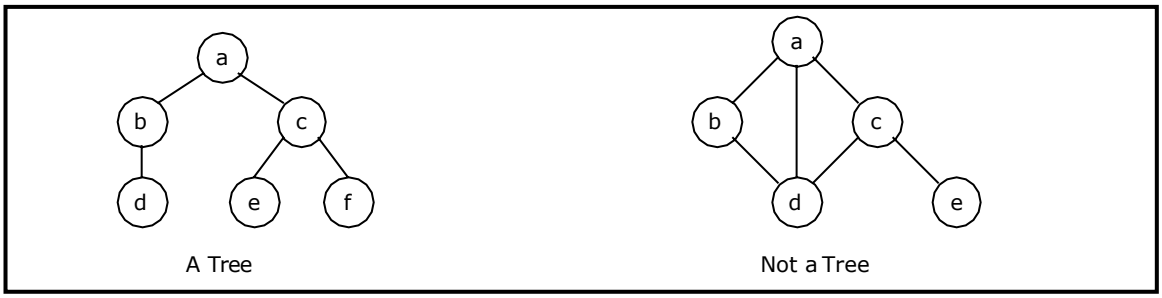


Figure 1.1 A Tree and a not a tree

In a tree data structure, there is no distinction between the various children of a node i.e., none is the "first child" or "last child". A tree in which such distinctions are made is called an **ordered tree**, and data structures built on them are called **ordered tree data structures**. Ordered trees are by far the commonest form of tree data structure.

BINARY TREE:

In general, tree nodes can have any number of children. In a binary tree, each node can have at most two children. A binary tree is either **empty** or consists of a node called the **root** together with two binary trees called the **left subtree** and the **right subtree**.

A tree with no nodes is called as a **null** tree. A binary tree is shown in figure 5.2.1.

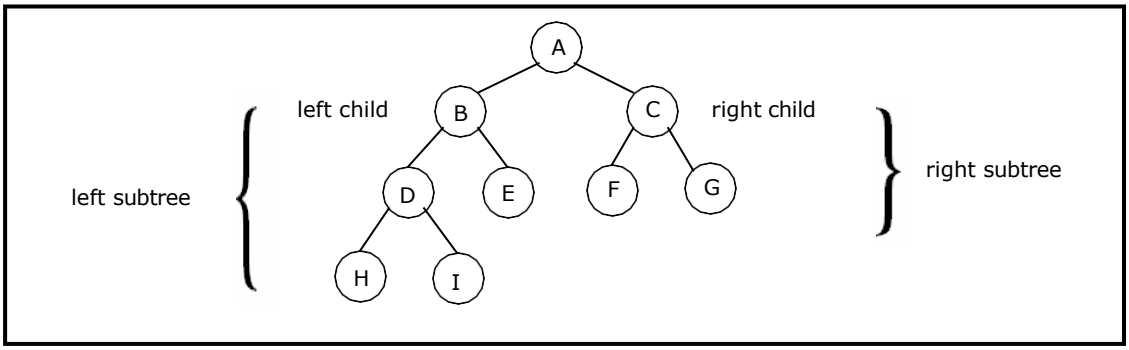


Figure 1.2. Binary Tree

Binary trees are easy to implement because they have a small, fixed number of child links. Because of this characteristic, binary trees are the most common types of trees and form the basis of many important data structures.

Tree Terminology:

Leaf node

A node with no children is called a *leaf* (or *external node*). A node which is not a leaf is called an *internal node*.

Path

A sequence of nodes n_1, n_2, . . ., n_k, such that n_i is the parent of n_{i+1} for i = 1, 2,. . ., k - 1. The length of a path is 1 less than the number of nodes on the path. Thus there is a path of length zero from a node to itself.

For the tree shown in figure 1.2, the path between A and I is A, B, D, I.

Siblings

The children of the same parent are called siblings.

For the tree shown in figure 1.2, F and G are the siblings of the parent node C and H and I are the siblings of the parent node D.

Ancestor and Descendent

If there is a path from node A to node B, then A is called an ancestor of B and *B* is called a descendent of A.

Subtree

Any node of a tree, with all of its descendants is a subtree.

Level The level of the node refers to its distance from the root. The root of the tree has level O, and the level of any other node in the tree is one more than the level of its parent. For example, in the binary tree of Figure 1.2 node F is at level 2 and node H is at level 3. *The maximum number of nodes at any level is* 2^n.

Height

The maximum level in a tree determines its height. The height of a node in a tree is the length of a longest path from the node to a leaf. The term depth is also used to denote height of the tree. The height of the tree of Figure 1.2 is 3.

Depth The depth of a node is the number of nodes along the path from the root to that node. For instance, node 'C' in figure 5.2.1 has a depth of 1.

Assigning level numbers and Numbering of nodes for a binary tree:

The nodes of a binary tree can be numbered in a natural way, level by level, left to right. The nodes of a complete binary tree can be numbered so that the root is assigned the number 1, a left child is assigned twice the number assigned its parent, and a right child is assigned one more than twice the number assigned its parent. For example, see Figure 1.3

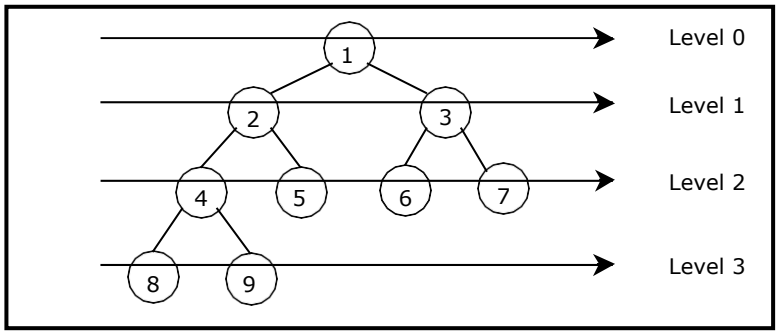


Figure 1.3 Level by level numbering of binary tree

Properties of binary trees:

Some of the important properties of a binary tree are as follows:

1. If *h* = height of a binary tree, then
    a. Maximum number of leaves = 2^h
    b. Maximum number of nodes = $2^{h+1} - 1$
2. If a binary tree contains m nodes at level l, it contains at most 2m nodes at level l + 1.
3. Since a binary tree can contain at most one node at level 0 (the root), it can contain at most 2^l node at level l.
4. The total number of edges in a full binary tree with n node is n - 1.

Strictly Binary tree:

If every non-leaf node in a binary tree has nonempty left and right subtrees, the tree is termed as strictly binary tree. Thus the tree of figure 1.4(a) is strictly binary. A strictly binary tree with n leaves always contains 2n - 1 nodes.

Full Binary tree:

A full binary tree of height h has all its leaves at level h. Alternatively; All non leaf nodes of a full binary tree have two children, and the leaf nodes have no children.

A full binary tree with height *h* has $2^{h+1} - 1$ nodes. A full binary tree of height h is a *strictly binary tree* all of whose leaves are at level h. Figure 1.4(d) illustrates the full binary tree containing 15 nodes and of height 3.
A full binary tree of height h contains 2^h leaves and, $2^h - 1$ non-leaf nodes.
Thus by induction, total number of nodes (tn) $= \sum_{l=0}^{h} 2^l = 2^{h+1} - 1$.

For example, a full binary tree of height 3 contains $2^{3+1} - 1 = 15$ nodes.

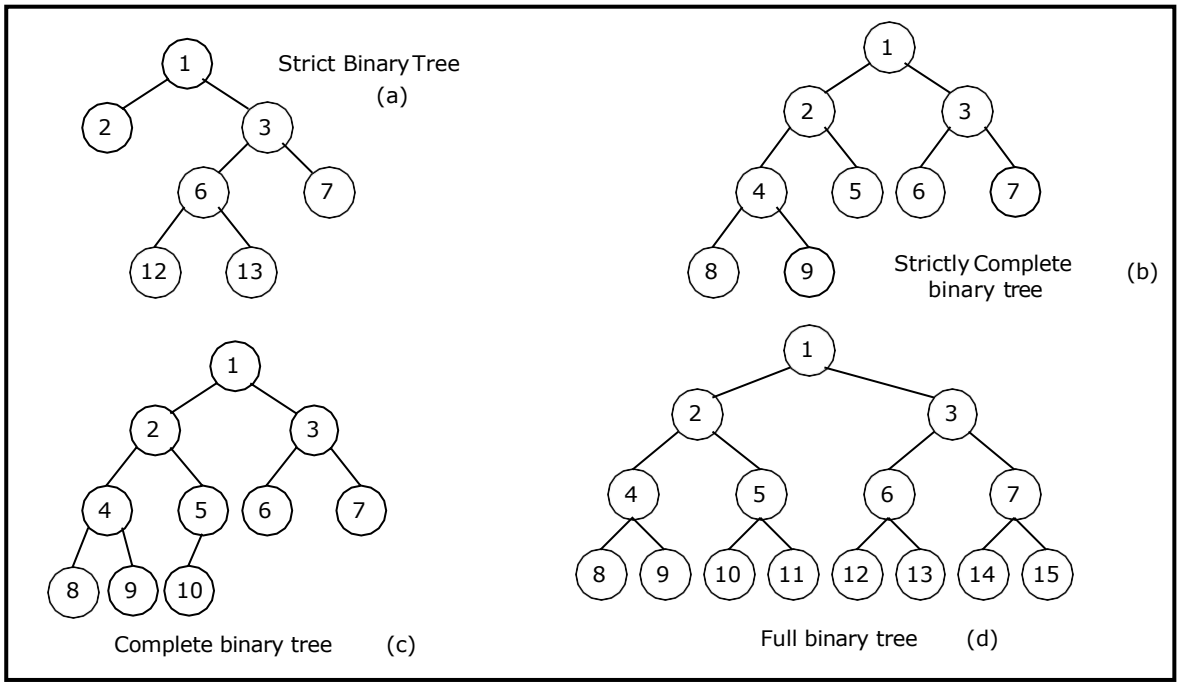


Figure 1.4 Examples of binary trees

Complete Binary tree:

A binary tree with *n* nodes is said to be **complete** if it contains all the first *n* nodes of the above numbering scheme. Figure 1.5 shows examples of complete and incomplete binary trees.

A complete binary tree of height h looks like a full binary tree down to level h-1, and the level h is filled from left to right.

A complete binary tree with n leaves that is *not strictly* binary has 2n nodes. For example, the tree of Figure 1.5(c) is a complete binary tree having 5 leaves and 10 nodes.

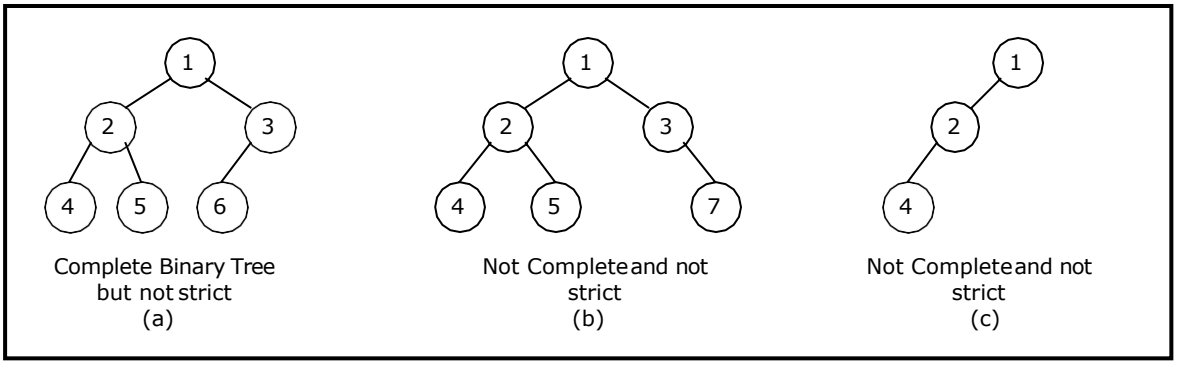


Figure 1.5 Examples of complete and incomplete binary trees

Internal and external nodes:

We define two terms: Internal nodes and external nodes. An internal node is a tree node having at least one–key and possibly some children. It is some times convenient to have another types of nodes, called an external node, and pretend that all null child links point to such a node. An external node doesn't exist, but serves as a conceptual place holder for nodes to be inserted.

We draw internal nodes using circles, with letters as labels. External nodes are denoted by squares. The square node version is sometimes called an extended binary tree. A binary tree with n internal nodes has n+1 external nodes. Figure 1.6 shows a sample tree illustrating both internal and external nodes.

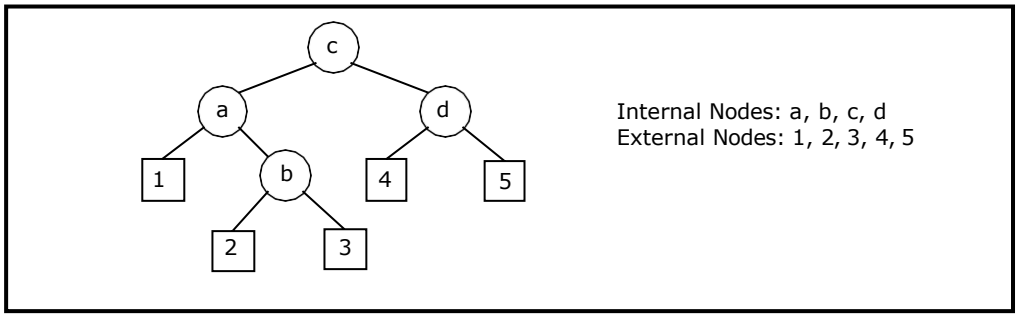


Figure 1.6. Internal and external nodes

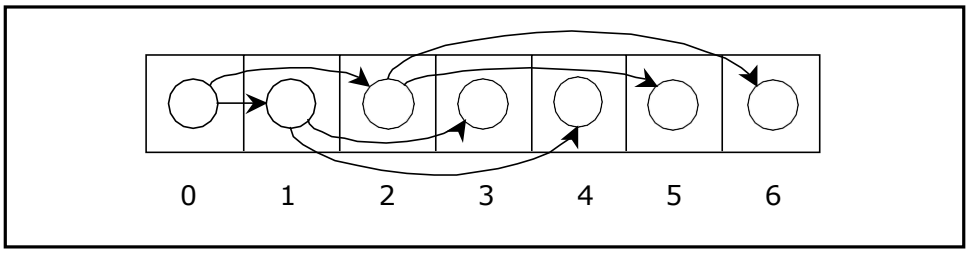
Data Structures for Binary Trees:

1. Arrays; especially suited for complete and full binary trees.
2. Pointer-based.

Array-based Implementation:

Binary trees can also be stored in arrays, and if the tree is a complete binary tree, this method wastes no space. In this compact arrangement, if a node has an index *i*, its children are found at indices 2*i*+1 and 2*i*+2, while its parent (if any) is found at index *floor((i-1)/2)* (assuming the root of the tree stored in the array at an index zero).

This method benefits from more compact storage and better locality of reference, particularly during a preorder traversal. However, it requires contiguous memory, expensive to grow and wastes space proportional to 2^h - n for a tree of height h with n nodes.



Linked Representation (Pointer based):

Array representation is good for complete binary tree, but it is wasteful for many other binary trees. The representation suffers from insertion and deletion of node from the middle of the tree, as it requires the moment of potentially many nodes to reflect the change in level number of this node. To overcome this difficulty we represent the binary tree in linked representation.
In linked representation each node in a binary has three fields, the left child field denoted as *LeftChild*, data field denoted as *data* and the right child field denoted as *RightChild*. If any sub-tree is empty then the corresponding pointer's LeftChild and RightChild will store a NULL value. If the tree itself is empty the root pointer will store a NULL value.

The advantage of using linked representation of binary tree is that:

- Insertion and deletion involve no data movement and no movement of nodes except the rearrangement of pointers.

The disadvantages of linked representation of binary tree includes:

- Given a node structure, it is difficult to determine its parent node.
- Memory spaces are wasted for storing NULL pointers for the nodes, which have no subtrees.

The structure definition, node representation empty binary tree is shown in figure 1.6 and the linked representation of binary tree using this node structure is given in figure 1.7.

```
struct binarytree
{
        struct binarytree *LeftChild;
        int data;
        struct binarytree *RightChild;
};

typedef struct binarytree node;

node *root = NULL;
```

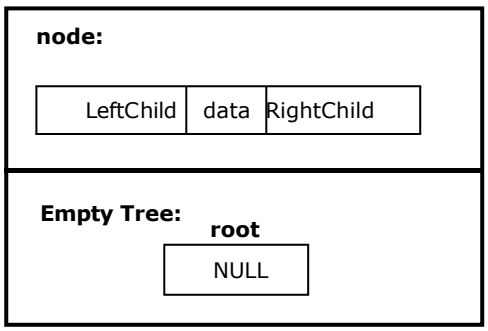


Figure 1.6. Structure definition, node representation and empty tree

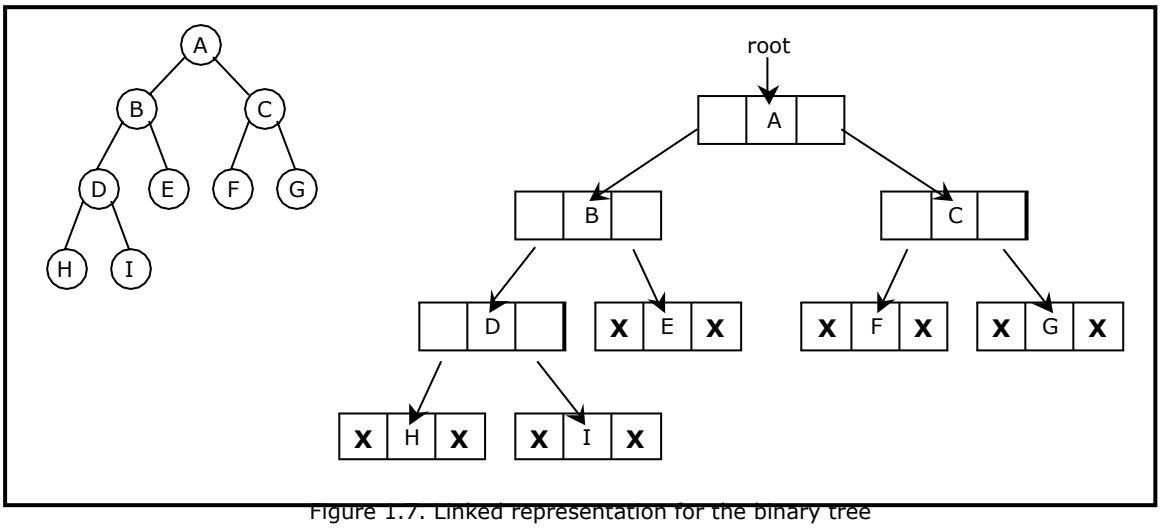


Figure 1.7: Linked representation for the binary tree

Binary Tree Traversal Techniques:

A tree traversal is a method of visiting every node in the tree. By visit, we mean that some type of operation is performed. For example, you may wish to print the contents of the nodes.

There are four common ways to traverse a binary tree:

5. *Preorder*
6. *Inorder*
7. *Postorder*
8. *Level order*

In the first three traversal methods, the left subtree of a node is traversed before the right subtree. The difference among them comes from the difference in the time at which a root node is visited.

Recursive Traversal Algorithms:

Inorder Traversal:

In the case of inorder traversal, the root of each subtree is visited after its left subtree has been traversed but before the traversal of its right subtree begins. The steps for traversing a binary tree in inorder traversal are:

1. Visit the left subtree, using inorder.
2. Visit the root.
3. Visit the right subtree, using inorder.

The algorithm for inorder traversal is as follows:

```
void inorder(node *root)
{
        if(root != NULL)
        {
                inorder(root->lchild);
```

```
                print root -> data;
                inorder(root->rchild);
        }
}
```

Preorder Traversal:

In a preorder traversal, each root node is visited before its left and right subtrees are traversed. Preorder search is also called **backtracking**. The steps for traversing a binary tree in preorder traversal are:

1. Visit the root.
2. Visit the left subtree, using preorder.
3. Visit the right subtree, using preorder.

The algorithm for preorder traversal is as follows:

```
void preorder(node *root)
{
        if( root != NULL )
        {
                print root -> data;
                preorder (root -> lchild);
                preorder (root -> rchild);
        }
}
```

Postorder Traversal:

In a postorder traversal, each root is visited after its left and right subtrees have been traversed. The steps for traversing a binary tree in postorder traversal are:

1. Visit the left subtree, using postorder.
2. Visit the right subtree, using postorder
3. Visit the root.

The algorithm for postorder traversal is as follows:

```
void postorder(node *root)
{
        if( root != NULL )
        {
                postorder (root -> lchild);
                postorder (root -> rchild);
                print (root -> data);
        }
}
```

Level order Traversal:

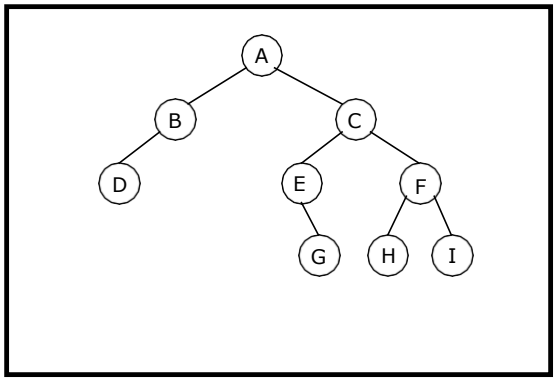
In a level order traversal, the nodes are visited level by level starting from the root, and going from left to right. The level order traversal requires a queue data structure. So, it is not possible to develop a recursive procedure to traverse the binary tree in level order. This is nothing but a breadth first search technique.

The algorithm for level order traversal is as follows:

```
void levelorder()
{
        int j;
        for(j = 0; j < ctr; j++)
        {
                if(tree[j] != NULL)
                        print tree[j] -> data;
        }
}
```

Example 1:

Traverse the following binary tree in pre, post, inorder and level order.



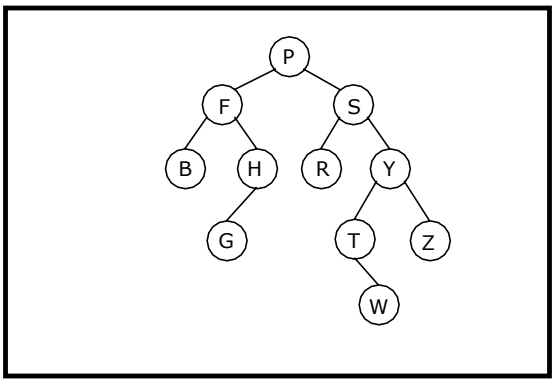
Binary Tree

- Preorder traversal yields:
  A, B, D, C , E, G , F , H, I
- Postorder traversal yields:
  D, B, G , E, H, I, F , C , A
- Inorder traversal yields:
  D, B, A, E, G , C , H, F , I
- Level order traversal yields:
  A, B, C , D, E, F , G , H, I

Pre, Post, Inorder and level order Traversing

Example 2:

Traverse the following binary tree in pre, post, inorder and level order.



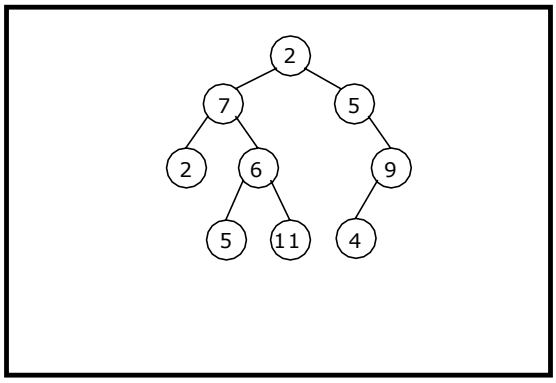
Bin ary Tree

- Preorder traversal yields:
  P , F , B, H, G , S, R, Y, T, W , Z
- Postorder traversal yields:
  B, G , H, F , R, W , T, Z, Y, S, P
- Inorder traversal yields:
  B, F , G , H, P , R, S, T, W , Y, Z
- Level order traversal yields:
  P , F , S, B, H, R, Y, G , T, Z, W

Pre, Post, Inorder and level order Traversing

Example 3:

Traverse the following binary tree in pre, post, inorder and level order.



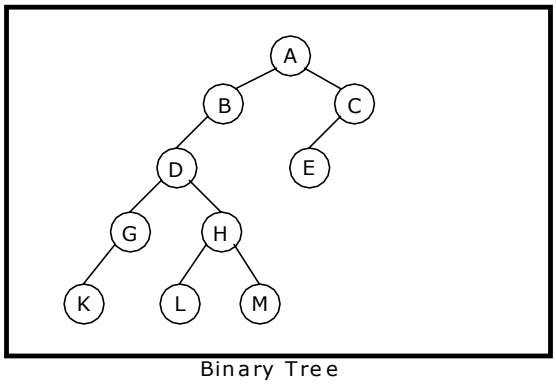
Bin ary Tree

- Preo rder traversal yields:
  2 , 7, 2, 6, 5, 11 , 5, 9, 4
- Posto rder trav arsal yields:
  2 , 5, 11 , 6, 7, 4, 9, 5, 2
- Ino rder trav arsal yields:
  2 , 7, 5, 6, 11 , 2, 5, 4, 9
- Level o rder traversal yields:
  2 , 7, 5, 2, 6, 9, 5, 11, 4

Pre, Post, Inorder and level order Traversing

Example 4:

Traverse the following binary tree in pre, post, inorder and level order.



Binary Tree

- Preo rder traversal yields:
  A, B, D, G , K, H, L, M , C , E
- Posto rder trav arsal yields:
  K, G , L, M , H, D, B, E, C , A
- Ino rder trav arsal yields:
  K, G , D, L, H, M , B, A, E, C
- Level o rder traversal yields:
  A, B, C , D, E, G , H, K, L, M

Pre, Post, Inorder and level order Traversing

Building Binary Tree from Traversal Pairs:

Sometimes it is required to construct a binary tree if its traversals are known. From a single traversal it is not possible to construct unique binary tree. However any of the two traversals are given then the corresponding tree can be drawn uniquely:

- Inorder and preorder
- Inorder and postorder
- Inorder and level order

The basic principle for formulation is as follows:

If the preorder traversal is given, then the first node is the root node. If the postorder traversal is given then the last node is the root node. Once the root node is identified, all the nodes in the left sub-trees and right sub-trees of the root node can be identified using inorder.

Same technique can be applied repeatedly to form sub-trees.

It can be noted that, for the purpose mentioned, two traversal are essential out of which one should be inorder traversal and another preorder or postorder; alternatively, given preorder and postorder traversals, binary tree cannot be obtained uniquely.

Example 1:

Construct a binary tree from a given preorder and inorder sequence:

Preorder: A B D G C E H I F
Inorder: D G B A H E I C F

Solution:

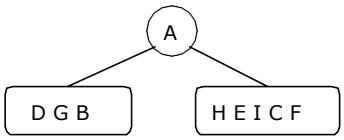
*From Preorder sequence **A** B D G C E H I F, the root is: A*

From Inorder sequence *D G B* ***A*** *H E I C F*, we get the left and right sub trees:

Left sub tree is: D G B

Right sub tree is: H E I C F

The Binary tree upto this point looks like:

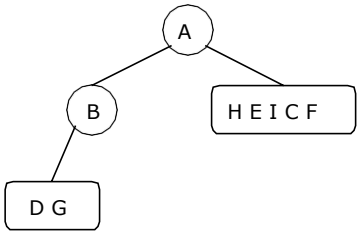


To find the root, left and right sub trees for D G B:

*From the preorder sequence **B** D G, the root of tree is: B*

From the inorder sequence D G **B,** we can find that D and G are to the left of B.

The Binary tree upto this point looks like:

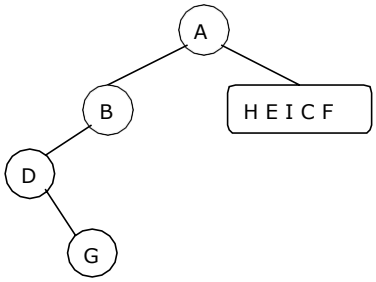


To find the root, left and right sub trees for D G:

From the preorder sequence **D** *G, the root of the tree is: D*

From the inorder sequence ***D*** *G*, we can find that there is no left node to D and G is at the right of D.

The Binary tree upto this point looks like:

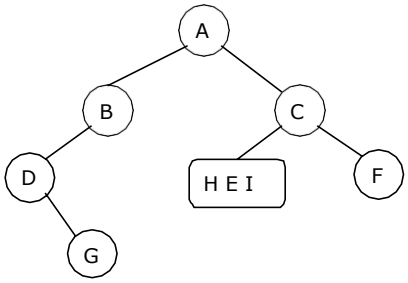


To find the root, left and right sub trees for H E I C F:

*From the preorder sequence **C** E H I F, the root of the left sub tree is: C*

From the inorder sequence *H E I* ***C*** *F*, we can find that H E I are at the left of C and F is at the right of C.

The Binary tree upto this point looks like:

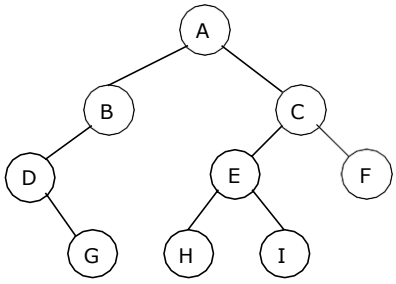


To find the root, left and right sub trees for H E I:

*From the preorder sequence **E** H I, the root of the tree is: E*

From the inorder sequence *H* ***E*** *I*, we can find that H is at the left of E and I is at the right of E.

The Binary tree upto this point looks like:



Example 2:

Construct a binary tree from a given postorder and inorder sequence:

Inorder: D G B A H E I C F
Postorder: G D B H I E F C A

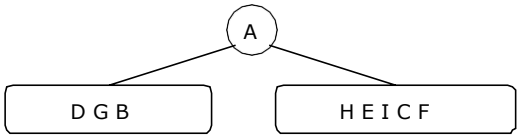
Solution:

From Postorder sequence G D B H I E F C **A**, *the root is: A*

From Inorder sequence *D G B* **A** *H E I C F*, we get the left and right sub trees:

Left sub tree is: D G B
Right sub tree is: H E I C F

The Binary tree upto this point looks like:

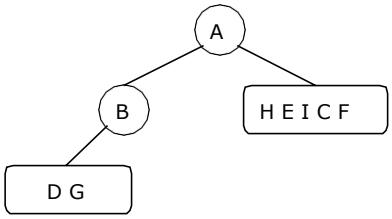


To find the root, left and right sub trees for D G B:

From the postorder sequence G D B, the root of tree is: B

From the inorder sequence *D G* **B,** we can find that D G are to the left of B and there is no right subtree for B.

The Binary tree upto this point looks like:

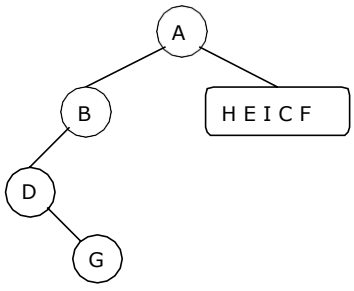


To find the root, left and right sub trees for D G:

*From the postorder sequence G **D**, the root of the tree is: D*

From the inorder sequence **D** *G*, we can find that is no left subtree for D and G is to the right of D.

The Binary tree upto this point looks like:

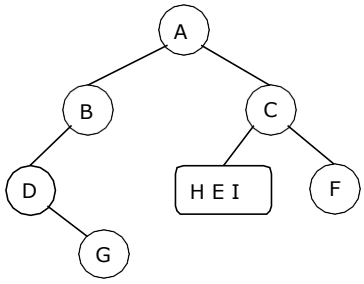


To find the root, left and right sub trees for H E I C F:

*From the postorder sequence H I E F **C**, the root of the left sub tree is: C*

From the inorder sequence H E I ***C*** F, we can find that H E I are to the left of C and F is the right subtree for C.

The Binary tree upto this point looks like:

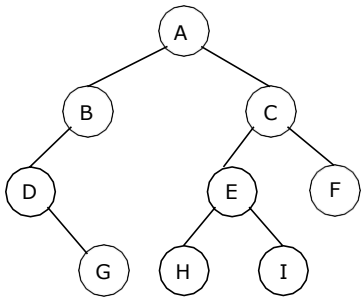


To find the root, left and right sub trees for H E I:

*From the postorder sequence H I **E**, the root of the tree is: E*

From the inorder sequence *H* **E** *I*, we can find that H is left subtree for E and I is to the right of E.

The Binary tree upto this point looks like:



Example 3:

Construct a binary tree from a given preorder and inorder sequence:

Inorder: n1 n2 n3 n4 n5 n6 n7 n8 n9
Preorder: n6 n2 n1 n4 n3 n5 n9 n7 n8

Solution:

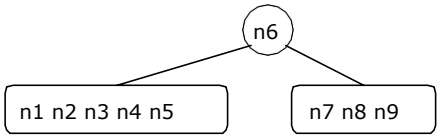
From Preorder sequence **n6** n2 n1 n4 n3 n5 n9 n7 n8, *the root is: n6*

From Inorder sequence *n1 n2 n3 n4 n5* **n6** *n7 n8 n9*, we get the left and right sub trees:

Left sub tree is: n1 n2 n3 n4 n5

Right sub tree is: n7 n8 n9

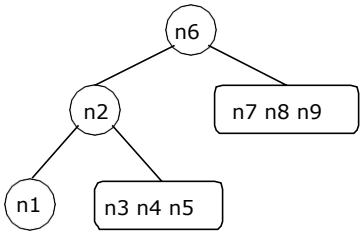
The Binary tree upto this point looks like:



To find the root, left and right sub trees for n1 n2 n3 n4 n5:

*From the preorder sequence **n2** n1 n4 n3 n5, the root of tree is: n2*

From the inorder sequence *n1* **n2** *n3 n4 n5*, we can find that n1 is to the left of n2 and n3 n4 n5 are to the right of n2. The Binary tree upto this point looks like:

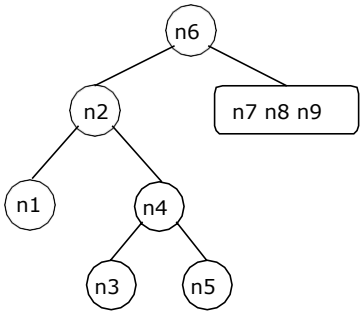


To find the root, left and right sub trees for n3 n4 n5:

*From the preorder sequence **n4** n3 n5, the root of the tree is: n4*

From the inorder sequence *n3* **n4** *n5*, we can find that n3 is to the left of n4 and n5 is at the right of n4.

The Binary tree upto this point looks like:

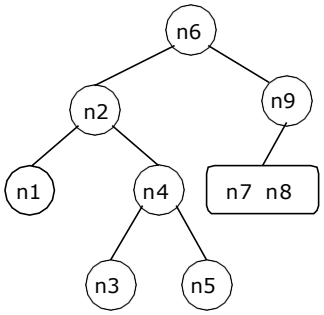


To find the root, left and right sub trees for n7 n8 n9:

*From the preorder sequence **n9** n7 n8, the root of the left sub tree is: n9*

From the inorder sequence *n7 n8* **n9**, we can find that n7 and n8 are at the left of n9 and no right subtree of n9.

The Binary tree upto this point looks like:

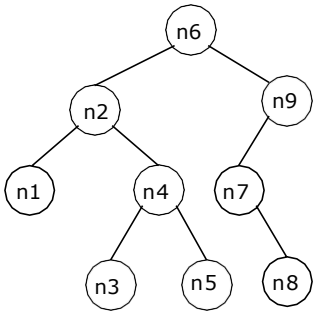


To find the root, left and right sub trees for n7 n8:

*From the preorder sequence **n7** n8, the root of the tree is: n7*

From the inorder sequence **n7** *n8*, we can find that is no left subtree for n7 and n8 is at the right of n7.

The Binary tree upto this point looks like:



Example 4:

Construct a binary tree from a given postorder and inorder sequence:

Inorder: n1 n2 n3 n4 n5 n6 n7 n8 n9
Postorder: n1 n3 n5 n4 n2 n8 n7 n9 n6

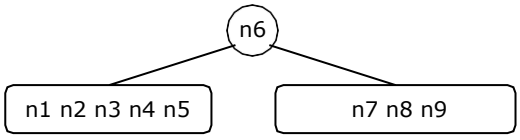
Solution:

From Postorder sequence n1 n3 n5 n4 n2 n8 n7 n9 **n6**, *the root is: n6*

From Inorder sequence *n1 n2 n3 n4 n5* **n6** *n7 n8 n9*, we get the left and right sub trees:

Left sub tree is: n1 n2 n3 n4 n5
Right sub tree is: n7 n8 n9

The Binary tree upto this point looks like:

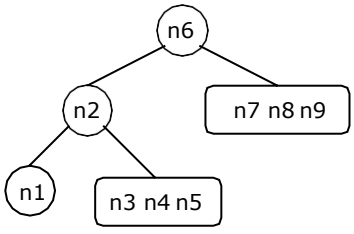


To find the root, left and right sub trees for *n1 n2 n3 n4 n5*:

From the postorder sequence n1 n3 n5 n4 **n2**, *the root of tree is: n2*

From the inorder sequence *n1* **n2** *n3 n4 n5***,** we can find that n1 is to the left of n2 and n3 n4 n5 are to the right of n2.

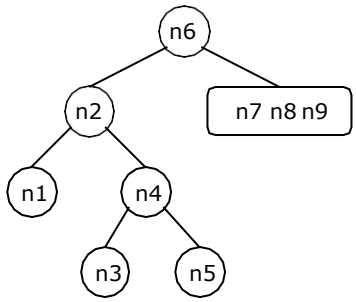
The Binary tree upto this point looks like:



To find the root, left and right sub trees for n3 n4 n5:

From the postorder sequence n3 n5 **n4***, the root of the tree is: n4*

From the inorder sequence *n3* **n4** *n5*, we can find that n3 is to the left of n4 and n5 is to the right of n4. The Binary tree upto this point looks like:

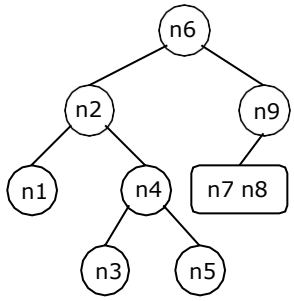


To find the root, left and right sub trees for n7 n8 and n9:

From the postorder sequence n8 n7 **n9***, the root of the left sub tree is: n9*

From the inorder sequence *n7 n8* **n9**, we can find that n7 and n8 are to the left of n9 and no right subtree for n9.

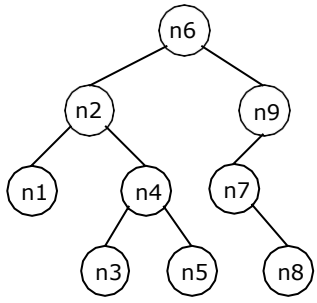
The Binary tree upto this point looks like:



To find the root, left and right sub trees for n7 and n8:

From the postorder sequence n8 **n7***, the root of the tree is: n7*

From the inorder sequence **n7** *n8*, we can find that there is no left subtree for n7 and n8 is to the right of n7. The Binary tree upto this point looks like:



Expression Trees:

Expression tree is a binary tree, because all of the operations are binary. It is also possible for a node to have only one child, as is the case with the unary minus operator. The leaves of an expression tree are operands, such as constants or variable names, and the other (non leaf) nodes contain operators.

Once an expression tree is constructed we can traverse it in three ways:

- Inorder Traversal
- Preorder Traversal
- Postorder Traversal

Figure 5.4.1 shows some more expression trees that represent arithmetic expressions given in infix form.

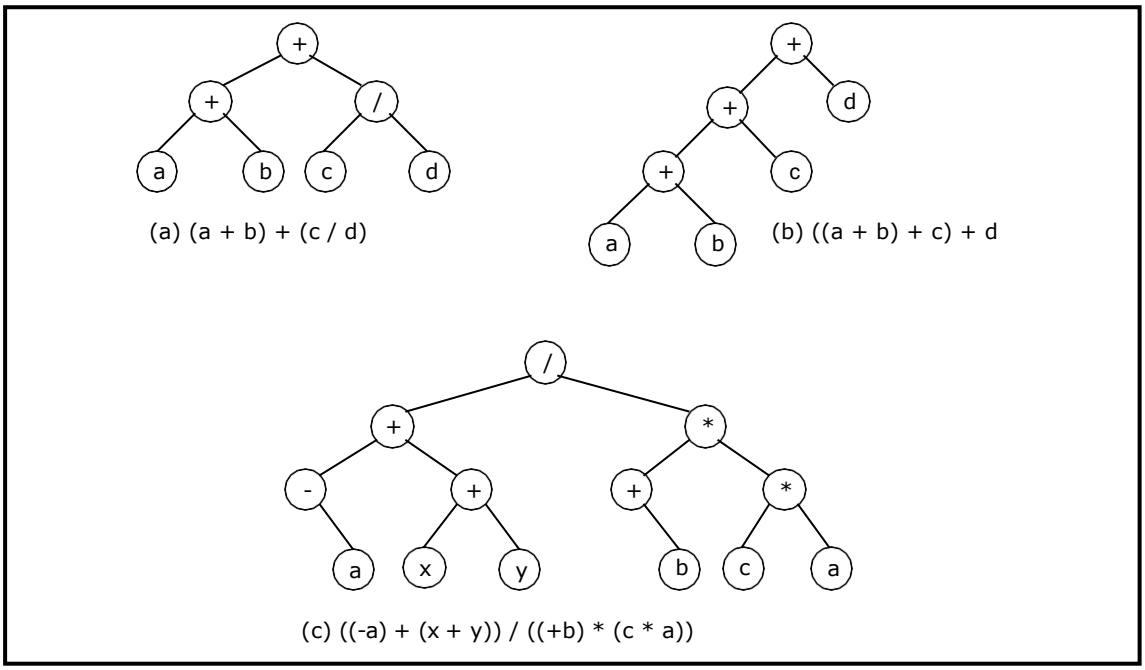


Figure 5.4.1 Expression Trees

An expression tree can be generated for the infix and postfix expressions.

An algorithm to convert a postfix expression into an expression tree is as follows:

9. Read the expression one symbol at a time.

10. If the symbol is an operand, we create a one-node tree and push a pointer to it onto a stack.

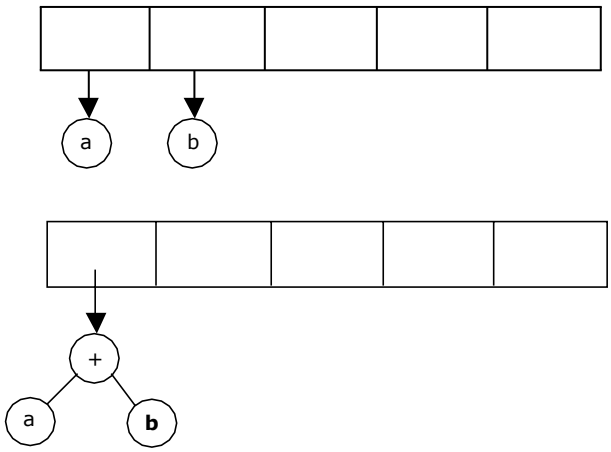
11. If the symbol is an operator, we pop pointers to two trees T1 and T2 from the stack (T1 is popped first) and form a new tree whose root is the operator and whose left and right children point to T2 and T1 respectively. A pointer to this new tree is then pushed onto the stack.

Example 1:

Construct an expression tree for the postfix expression: a b + c d e + * *

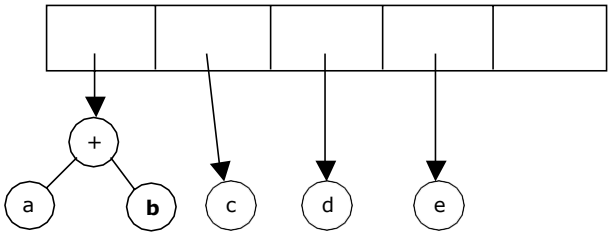
Solution:

The first two symbols are operands, so we create one-node trees and push pointers to them onto a stack.

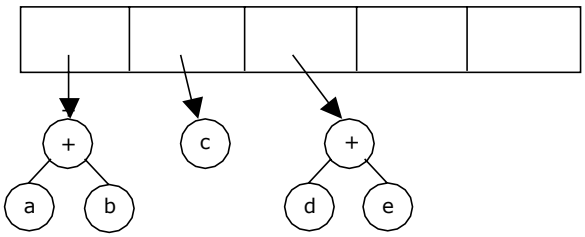


Next, a '+' is read, so two pointers to trees are popped, a new tree is formed, and a pointer to it is pushed onto the stack.

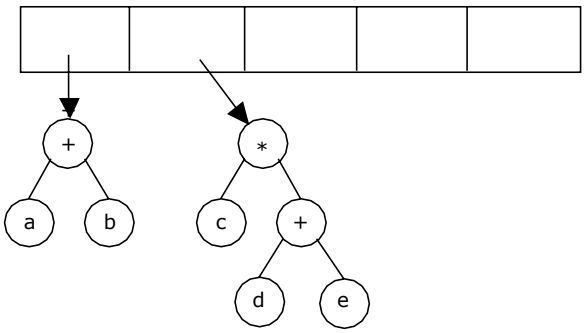
Next, c, d, and e are read, and for each one–node tree is created and a pointer to the corresponding tree is pushed onto the stack.



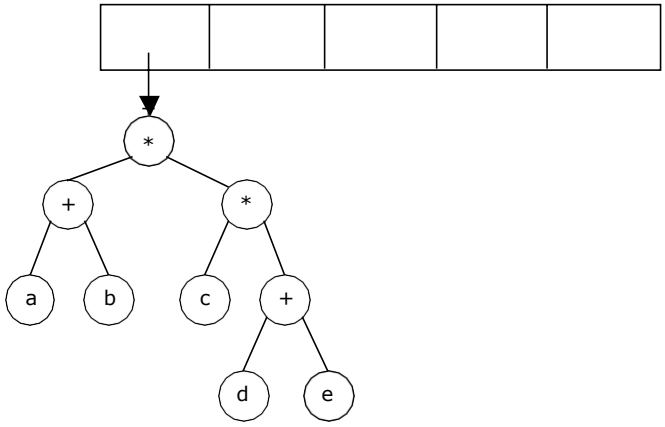
Now a '+' is read, so two trees are merged.



Continuing, a '*' is read, so we pop two tree pointers and form a new tree with a '*' as root.



Finally, the last symbol is read, two trees are merged, and a pointer to the final tree is left on the stack.



For the above tree:

Inorder form of the expression: a + b * c * d + e

Preorder form of the expression: * + a b * c + d e

Postorder form of the expression: a b + c d e + * *

Example 2:

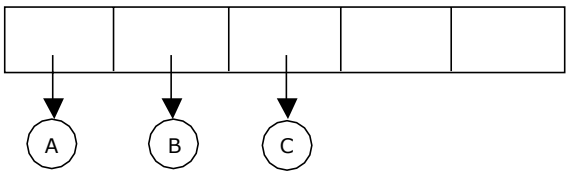
Construct an expression tree for the arithmetic expression:

(A + B * C) – ((D * E + F) / G)

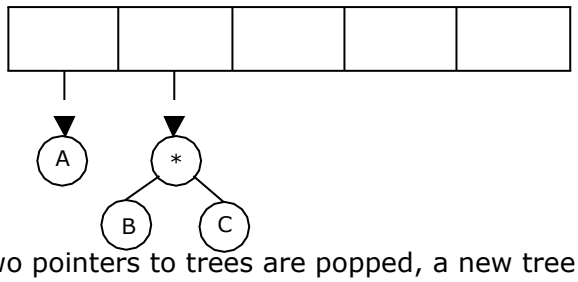
Solution:

First convert the infix expression into postfix notation. Postfix notation of the arithmetic expression is: A B C * + D E * F + G / -

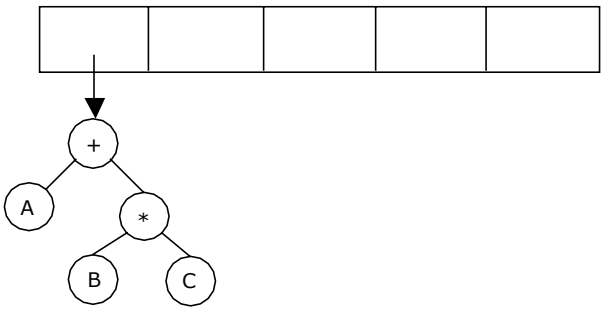
The first three symbols are operands, so we create one-node trees and pointers to three nodes pushed onto the stack.



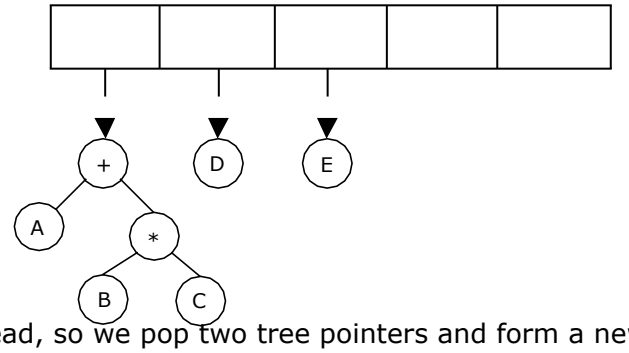
Next, a '*' is read, so two pointers to trees are popped, a new tree is formed, and a pointer to it is pushed onto the stack.



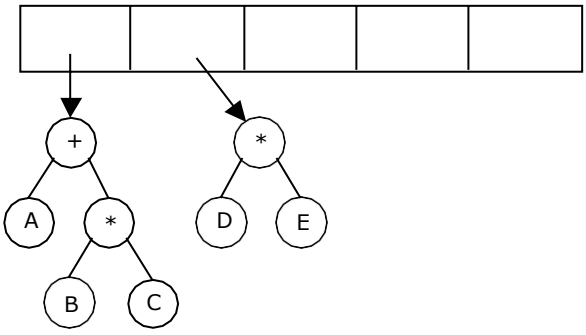
Next, a '+' is read, so two pointers to trees are popped, a new tree is formed, and a pointer to it is pushed onto the stack.



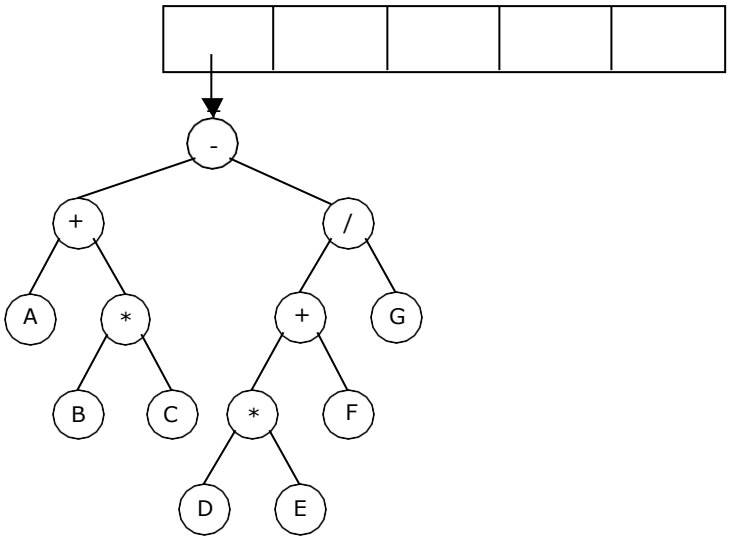
Next, D and E are read, and for each one–node tree is created and a pointer to the corresponding tree is pushed onto the stack.



Continuing, a '*' is read, so we pop two tree pointers and form a new tree with a '*' as root.



Proceeding similar to the previous steps, finally, when the last symbol is read, the expression tree is as follows:



Converting expressions with expression trees:

Let us convert the following expressions from one type to another. These can be as follows:

1. Postfix to infix
2. Postfix to prefix
3. Prefix to infix
4. Prefix to postfix

1. Postfix to Infix:

The following algorithm works for the expressions whose infix form does not require parenthesis to override conventional precedence of operators.

A. Create the expression tree from the postfix expression
B. Run inorder traversal on the tree.

2. Postfix to Prefix:

The following algorithm works for the expressions to convert postfix to prefix:

A. Create the expression tree from the postfix expression
B. Run preorder traversal on the tree.

3. Prefix to Infix:

The following algorithm works for the expressions whose infix form does not require parenthesis to override conventional precedence of operators.

A. Create the expression tree from the prefix expression
B. Run inorder traversal on the tree.

4. Prefix to postfix:

The following algorithm works for the expressions to convert postfix to prefix:

A. Create the expression tree from the prefix expression
B. Run postorder traversal on the tree.

Binary Search Tree:

A binary search tree is a binary tree. It may be empty. If it is not empty then it satisfies the following properties:

1. Every element has a key and no two elements have the same key.
2. The keys in the left subtree are smaller than the key in the root.
3. The keys in the right subtree are larger than the key in the root.
4. The left and right subtrees are also binary search trees.

Figure 5.2.5(a) is a binary search tree, whereas figure 5.2.5(b) is not a binary search tree.

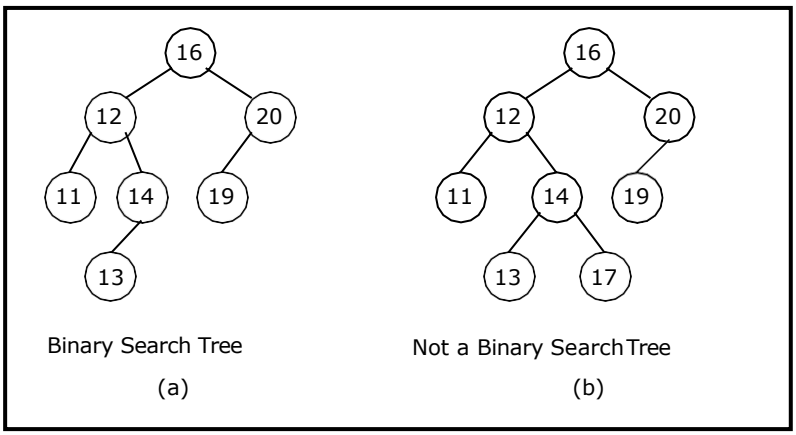


Figure 5.2.5. Examples of binary search trees

5.2. General Trees (m-ary tree):

If in a tree, the outdegree of every node is less than or equal to ***m***, the tree is called general tree. The general tree is also called as an *m-ary* tree. If the outdegree of every node is exactly equal to m or zero then the tree is called *a full or complete m-ary* tree. For m = 2, the trees are called *binary* and *full binary trees.*

Differences between trees and binary trees:

TREE	BINARY TREE
Each element in a tree can have any number of subtrees.	Each element in a binary tree has at most two subtrees.
The subtrees in a tree are unordered.	The subtrees of each element in a binary tree are ordered (i.e. we distinguish between left and right subtrees).