Circular convolution.

(1) $x(n) = \{1, 2, 3, 4\}$ $h(n) = \{1, -1, 1, -1\}$

Solution:

$x(n) = \{1, 2, 3, 4\}$ $h(-n) = \{1, -1, 1, -1\}$

Time Domain Approach:

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1-2+3-4 \\ -1+2-3+4 \\ 1-2+3-4 \\ -1+2-3+4 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -2 \\ 2 \end{bmatrix}$$

Freq Domain Approach.

$$X(K) = 1 + 2W_4^K + 3W_4^{2K} + 4W_4^{3K}.$$

$$H(K) = 1 - W_4^K + W_4^{2K} - W_4^{3K}.$$

$X(K) \times H(K)$

W_4^0	W_4^1	W_4^2	W_4^3
1	-1	1	-1
-2	+2	-2	+2
3	-3	3	-3
-4	4	-4	4
-2	2	-2	2

$\therefore Y(K) = -2W_4^0 + 2W_4^1 - 2W_4^2 + 2W_4^3.$

$\therefore y(n) = \{-2, 2, -2, 2\}$

Co-relation
- Auto-correlation (Same sequence)
- Cross-correlation (Different sequence)

Cross correlation

$$R_{xy} = \sum_{n=-\infty}^{\infty} x(n)\, y(n-k)$$

$$R_{yx} = \sum_{n=-\infty}^{\infty} y(n)\, x(n-k)$$

$R_{xy} \neq R_{yx}$

Auto Correlation

$$R_{xx} = \sum_{n=-\infty}^{\infty} x(n)\, x(n-k)$$

(1) Auto-correlation

$x(n) = \{5, 6, 7, 8, 1, 2, 3, 4\}$

$x(-n) = \{5, 4, 3, 2, 1, 8, 7, 6, \}$

$$\begin{bmatrix}
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 6 & 5 & 0 & 0 & 0 & 0 & 0 \\
8 & 7 & 6 & 5 & 0 & 0 & 0 & 0 \\
1 & 8 & 7 & 6 & 5 & 0 & 0 & 0 \\
2 & 1 & 8 & 7 & 6 & 5 & 0 & 0 \\
3 & 2 & 1 & 8 & 7 & 6 & 5 & 0 \\
4 & 3 & 2 & 1 & 8 & 7 & 6 & 5 \\
0 & 4 & 3 & 2 & 1 & 8 & 7 & 6 \\
0 & 0 & 4 & 3 & 2 & 1 & 8 & 7 \\
0 & 0 & 0 & 4 & 3 & 2 & 1 & 8 \\
0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 4 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4
\end{bmatrix}
\begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 8 \\ 7 \\ 6 \end{bmatrix}
=
\begin{bmatrix} 20 \\ 39 \\ 56 \\ 70 \\ 88 \\ 117 \\ 156 \\ 204 \\ 156 \\ 117 \\ 88 \\ 70 \\ 56 \\ 39 \\ 20 \end{bmatrix}$$

Cross correlation

$x(n) = \{1, 2, 3, 4\}$

$h(n) = \{5, 6, 7, 8\}$

Solution

$$R_{xy} = \sum_{n=-\infty}^{\infty} x(n)\, h(n-k)$$

$\therefore h(-n) = \{5, 8, 7, 6\}$

$x(n) = \{1, 2, 3, 4\}$

R_{xy} =

$x(n)$ \ $h(-n)$	8	7	6	5
1	8	7	6	5
2	16	14	12	10
3	24	21	18	15
4	32	28	24	20

$R_{xy} = \{8, 23, 44, 70, 56, 39, 20\}$

$x(-n)$ \ $h(n)$	5	6	7	8
4	20	24	28	32
3	15	18	21	24
2	10	12	14	16
1	5	6	7	8

$R_{yx} = \{20, 39, 56, 70, 44, 23, 8\}$