An end-to-end data-driven optimisation framework for constrained trajectories

Florent Dewez, Benjamin Guedj, Arthur Talpaert, Vincent Vandewalle

December 2020

Motivation

Polluting air traffic

Figure 1: Air traffic responsible of 3% of the total CO2 emissions.

Polluting air traffic

The current solutions are based on

- ✈ New technologies
- ✈ Alternative fuels
- ✈ Taxes
- ✈ Optimised operations

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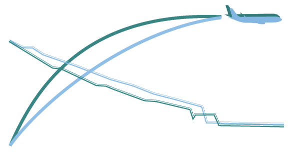
Trajectory optimisation



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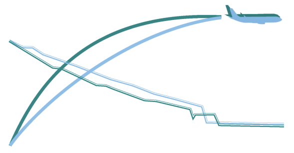

A classical approach : Optimal control

Let $y = (x, u)$ be a trajectory with states x and controls u.

- Cost function : Total fuel consumption, traveled distance,...

$$F(y) = \int_0^T f\big(y(t)\big)\, dt$$

- Constraints : Flight domain, initial and final conditions,...

$$y \in \mathcal{G} \cap \mathcal{D}(y_0, y_T)$$

- Dynamics :

$$\dot{y}(t) = g\big(t, y(t)\big)$$

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$$\dot{y}(t) = g\big(t, y(t)\big) + \varepsilon(t)$$

$\longrightarrow$ System identification ! [‡]

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$$\Longrightarrow \quad \dot{y}(t) = \hat{g}\big(t, y(t)\big)$$

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Limitations of optimal control methods

An end-to-end framework based on optimal control methods involves two steps which may require significant computational resources :

- Identification : not always straightforward ;
- Optimisation : potentially affected by the statistical errors.

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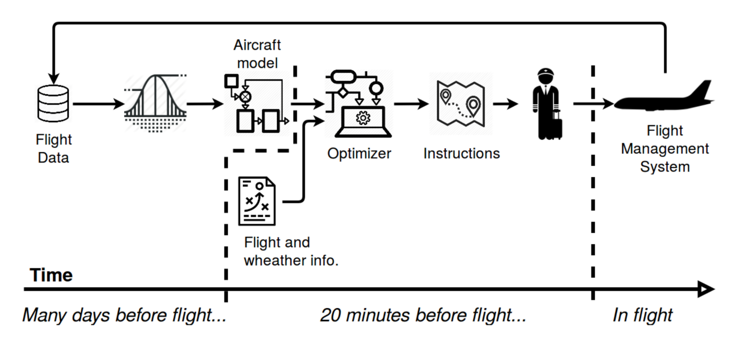
- <u>Identification :</u> not always straightforward ;
- <u>Optimisation :</u> potentially affected by the statistical errors.

→ Sometimes unacceptable for applications !



PERF-AI project

PERF-AI project which is

- funded by Clean Sky 2, a European research program developing innovative technology for CO_2 emissions reduction ;
- led by Safety Line, a Paris based start-up specialised in Big Data solutions for aviation safety and efficiency ;
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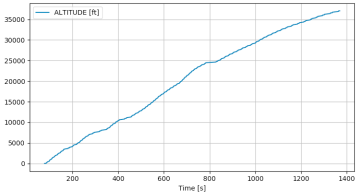
Research project aiming at **leveraging huge amounts of aeronautic data** to improve

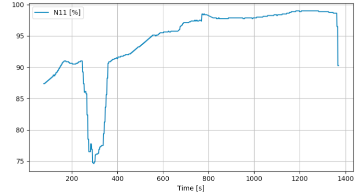
- aerodynamic performance models ;
- trajectory optimisation methods.

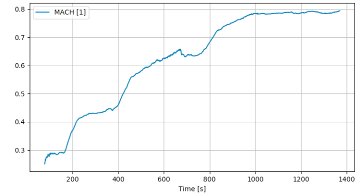
Aeronautic data

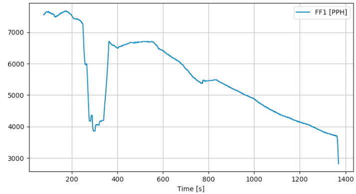
Aeronautic data

	ALTITUDE	AOA	FF1	FF2	GS	GW	IAS	LEVEL_OFF	MACH	N11	N12	PITCH	SAT
1200	34243.0	1.06640625	4144.0	4144.0	450.0	144080.0	271.75	1	0.7859999999999999	98.875	99.0	3.69140625	-53.4654464917
1201	34252.0	1.06640625	4144.0	4144.0	450.0	144080.0	271.75	1	0.7859999999999999	98.875	99.0	3.69140625	-53.4654464917
1202	34262.0	1.06640625	4144.0	4128.0	450.0	144080.0	271.75	1	0.787	98.875	99.0	3.69140625	-53.5255699099
1203	34271.0	1.06640625	4144.0	4128.0	450.0	144080.0	272.0	1	0.787	98.875	98.875	3.69140625	-53.5255699099
1204	34280.0	1.06640625	4144.0	4128.0	450.0	144080.0	271.75	1	0.787	98.875	98.875	3.69140625	-53.5255699099
1205	34289.0	1.06640625	4144.0	4128.0	450.0	144080.0	271.75	1	0.787	98.875	98.875	3.69140625	-53.748629457
1206	34299.0	1.06640625	4128.0	4112.0	450.0	144080.0	271.75	1	0.7879999999999999	98.875	98.875	3.69140625	-53.80873524850001
1207	34308.0	1.06640625	4128.0	4128.0	450.0	144080.0	271.75	1	0.7879999999999999	99.0	99.0	3.69140625	-53.80873524850001
1208	34317.0	1.06640625	4128.0	4112.0	450.0	144080.0	271.5	1	0.7879999999999999	99.0	99.0	3.69140625	-53.80873524850001
1209	34327.0	1.06640625	4128.0	4112.0	450.0	144080.0	271.5	1	0.7879999999999999	99.0	99.0	3.69140625	-54.0317336876









Overall objectives

The aim is to develop a methodology to derive trajectory optimisation problems which :

- leverage information from data ;
- may improve the efficiency of optimisation algorithms ;
- is enough generic and flexible for other applications ;
- can be intuitively understood by domain experts.

Generic methodology

Definitions – Notations (general setting)

- Trajectory : $y = \big(y^{(1)}, \dots, y^{(D)}\big) \in C\big([0,T], \mathbb{R}^D\big)$
- Endpoints conditions :

$$y \in \mathcal{D}(y_0, y_T) \quad \Longleftrightarrow \quad \begin{cases} y(0) = y_0 \\ y(T) = y_T \end{cases}$$

- Additionnal constraints :

$$y \in \mathcal{G} \quad \Longleftrightarrow \quad \forall\, \ell = 1, \dots, L \quad g_\ell\big(y(t)\big) \leqslant 0$$

- Cost function : $F : C\big([0,T], \mathbb{R}^D\big) \longrightarrow \mathbb{R}$
- Reference trajectories : $\mathrm{Y}_R := \{y_{R_1}, \dots, y_{R_I}\} \subset \mathcal{D}(y_0, y_T) \cap \mathcal{G}$

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Reference trajectories

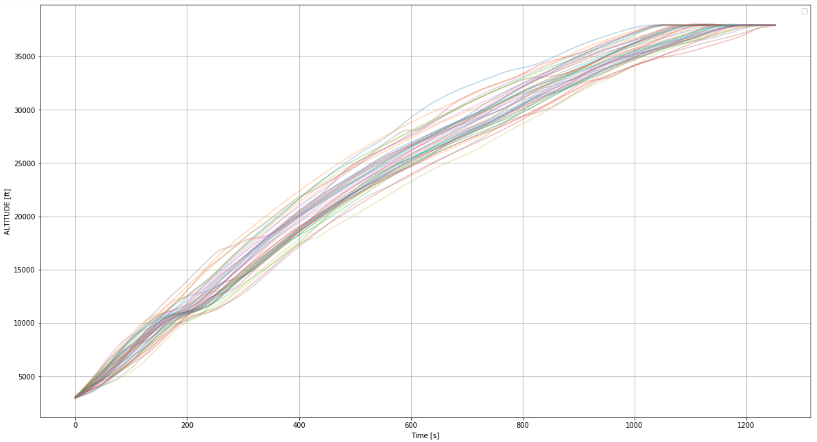

Figure 2: 48 reference climbs with the same initial and final states – Here the altitude is displayed.

Guidelines

- Aim : Find an optimised and constrained trajectory with a realistic pattern without involving the dynamics of the system !

$$\min_y F(y) \quad \text{s.t.} \begin{cases} y \in \mathcal{G} \cap \mathcal{D}(y_0, y_T) \\ \cancel{\dot{y}(t) = \hat{g}\big(t, y(t)\big)} \end{cases}$$

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- Key point : Use differently the data to derive a new problem of type

$$\min_y \Big\{ F(y) + \kappa\, \mathrm{pen}_{\mathrm{Y}_R}(y) \Big\} \quad \text{s.t. } y \in \mathcal{G} \cap \mathcal{D}(y_0, y_T)$$

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→ Consider a statistical framework and interpret the optimised trajectory as a maximum a posteriori.

- Technical workflow :
  1. Finite representation of trajectories → $\mathbb{R}^K$
  2. Model thoroughly the reference trajectories Y_R → Likelihood
  3. Assume a priori low cost trajectories → Prior
  4. Apply Bayes's rule → Posterior
  5. Find the mode of the posterior → Optimised trajectory

Finite representation of trajectories

- Orthonormal basis of L^2 : $\{\varphi_k\}_k$, each element being continuous

- Finite dimensional space : $\mathcal{Y} := \prod_{d=1}^{D} \operatorname{span} \{\varphi_k\}_{k=1}^{K_d}$

- Finite representation : $y^{(d)}(t) = \sum_{k=1}^{K_d} c_k^{(d)} \, \varphi_k(t) \, , \quad d = 1, \ldots, D$

- Equivalence : $\Phi y = c \quad \Longleftrightarrow \quad c = \Phi^{-1} y$, with

$$c = \left(c_1^{(1)}, \ldots, c_{K_1}^{(1)}, \; c_1^{(2)}, \ldots, c_{K_2}^{(2)}, \ldots, \; c_1^{(D)}, \ldots, c_{K_D}^{(D)} \right)^T \in \mathbb{R}^K$$

- Endpoints conditions : There exists a matrix $A \in \mathbb{R}^{2D \times K}$ such that

$$y \in \mathcal{Y} \cap \mathcal{D}(y_0, y_T) \qquad \Longleftrightarrow \qquad Ac = \begin{pmatrix} y_0 \\ y_T \end{pmatrix}$$

Reference trajectories modelling

- Hypothesis : The reference trajectories are noisy observations of an efficient trajectory $y = \Phi^{-1}c \in \mathcal{G} \cap \mathcal{D}(y_0, y_T)$.
- Technical point : Derive a model in $\mathbb{R}^K$ based on this hypothesis.
- Choice for the noise : Centered Gaussian with intensity depending on the reference trajectory.
- Additional property : The noise should not modify the endpoints.

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- Modelling : There exists $c \in \mathbb{R}^K$ such that

$$\begin{cases} c_{R_i} = c + \varepsilon_i \\ \varepsilon_i \sim \mathcal{N}(0_{\mathbb{R}^K}, \Sigma_i)\ , \quad \text{with } \Sigma_i = \frac{1}{2\omega_i} \Sigma \\ \varepsilon_i \in \ker A \end{cases}$$

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⚠ The covariance matrix Σ is here singular !

Reference trajectories modelling

- <u>(Sub)-goal :</u> Separate deterministic and stochastic parts and make explicit the correlations.
- <u>Method :</u> Use the fact that the matrices Σ and $A^T A$ are simultaneously diagonalisable to change the basis.
- <u>Equivalent modelling :</u> We can find an explicit basis V such that

$$\begin{cases} \widetilde{c}_{R_i,1} = \widetilde{c}_1 + \widetilde{\varepsilon}_{i,1} & \longrightarrow \text{Constraints-free} \\ \widetilde{\varepsilon}_{i,1} \sim \mathcal{N}\left(0_{\mathbb{R}^\sigma}, \dfrac{1}{2\omega_i}\Lambda_\Sigma\right) & \\ {\color{red} V_2^T\, c_{R_i} = \widetilde{c}_2} & \longrightarrow \text{Implicit constraints} \\ {\color{blue} V_3^T A^\dagger (y_0 \quad y_T)^T = \widetilde{c}_3} & \longrightarrow \text{Endpoints constraints} \end{cases}$$

with $\widetilde{c} = V^T c = (V_1\ V_2\ V_3)^T c$ and Λ_Σ diagonal and non-singular.

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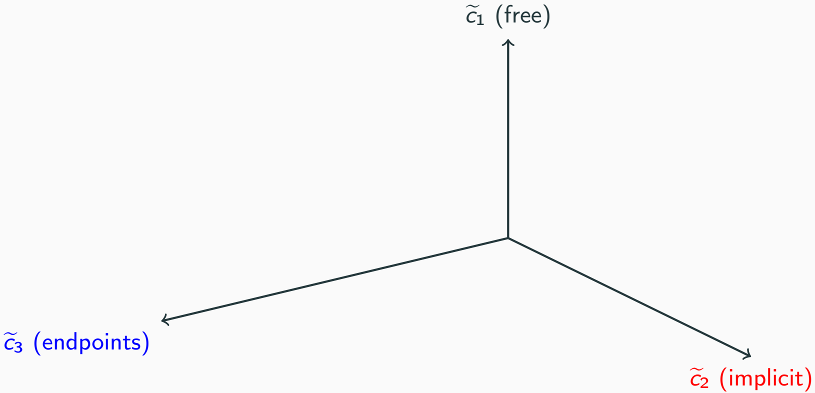
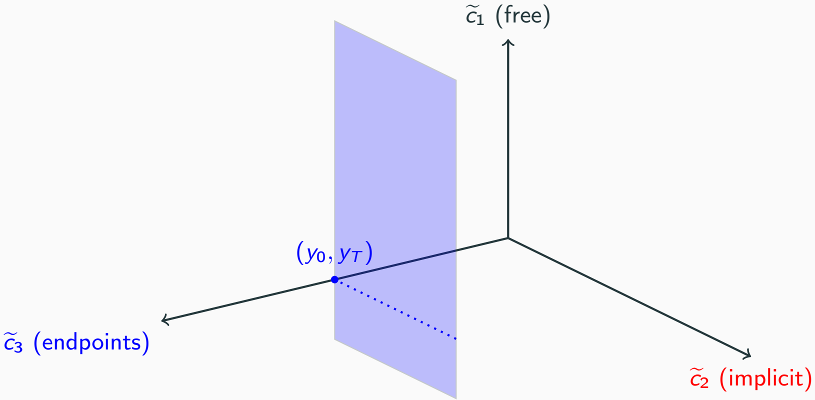
with $\widetilde{c} = V^T c = (V_1 \ V_2 \ V_3)^T c$ and Λ_Σ diagonal and non-singular.

$$\longrightarrow \quad \mathcal{V} := \left\{ c \in \mathbb{R}^K \;\middle|\; \widetilde{c}_2 = V_2 \, c_{R_i} \, , \; \widetilde{c}_3 = V_3^T A^\dagger (y_0 \quad y_T)^T \right\}$$

Visualisation of the modelling




$\widetilde{c}_1$ (free)
(y_0, y_T)
$\widetilde{c}_3$ (endpoints)
$\widetilde{c}_2$ (implicit)

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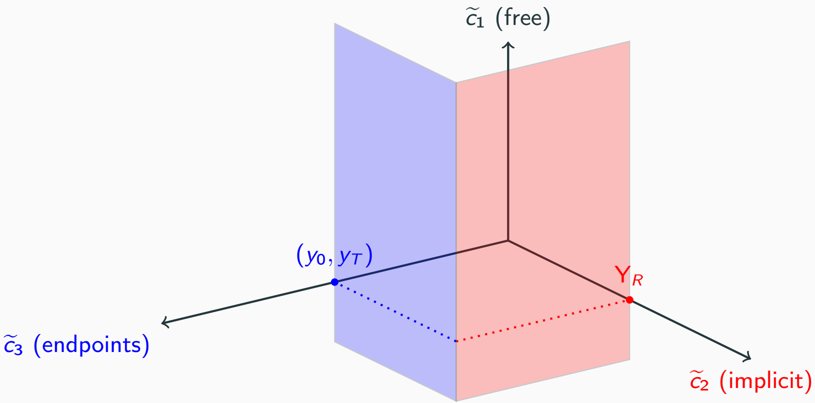

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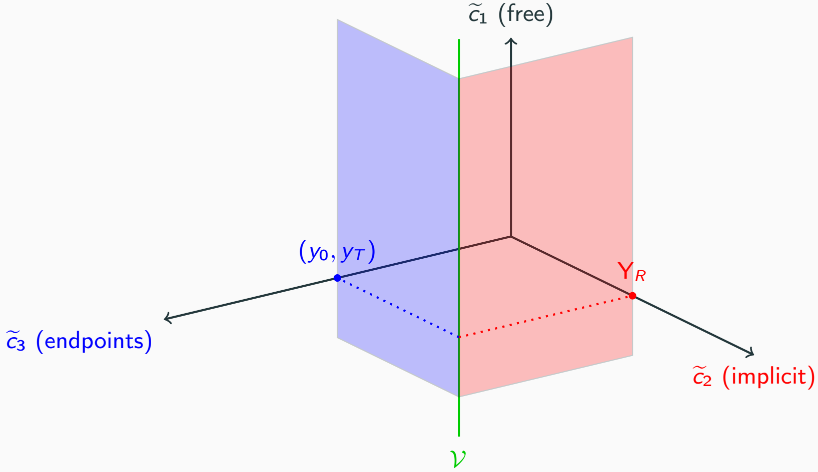

A priori model for the trajectory distribution

- A priori knowledge : Efficient trajectories with respect to the cost F are the most likely ones.

- Reminder : A trajectory depends only on the component $\widetilde{c}_1$!

- Restricted cost function : Let $\widetilde{F}$ be the restriction of the cost function to $\mathcal{V}$.

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- Prior : For $\kappa > 0$,

$$u(\widetilde{c}_1) \propto \exp\Big(-\kappa^{-1}\widetilde{F}(\widetilde{c}_1)\Big)$$

Bayes's rule and MAP

- Likelihood : Under independence assumption, we have

$$u(\widetilde{c}_{R_{\mathbf{1}},1},\dots,\widetilde{c}_{R_I,1}\,|\,\widetilde{c}_1) = \prod_{i=1}^{I} u(\widetilde{c}_{R_i,1}\,|\,\widetilde{c}_1)$$
$$\propto \prod_{i=1}^{I} \exp\Big(-\omega_i\left(\widetilde{c}_1-\widetilde{c}_{R_i,1}\right)^T \Lambda_{\Sigma,1}^{-1}\big(\widetilde{c}_1-\widetilde{c}_{R_i,1}\big)\Big)$$

- Posterior : Apply Bayes's rule to obtain

$$u(\widetilde{c}_1\,|\,\widetilde{c}_{R_{\mathbf{1}},1},\dots,\widetilde{c}_{R_I,1}) \propto u(\widetilde{c}_{R_{\mathbf{1}},1},\dots,\widetilde{c}_{R_I,1}\,|\,\widetilde{c}_1)\,u(\widetilde{c}_1)$$

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- Maximum A Posteriori : Take the negative of the logarithm

$$\begin{cases}\widetilde{c}_1^\star\in\underset{\widetilde{c}_{\mathbf{1}}\in\mathbb{R}^\sigma}{\arg\min}\,\widetilde{F}(\widetilde{c}_1)+\kappa\displaystyle\sum_{i=1}^{I}\omega_i\left(\widetilde{c}_1-\widetilde{c}_{R_i,1}\right)^T\Lambda_{\Sigma,1}^{-1}\left(\widetilde{c}_1-\widetilde{c}_{R_i,1}\right)\\ \widetilde{c}_2^\star=V_2^T\,c_{R_i}\\ \widetilde{c}_3^\star=V_3^TA^\dagger(y_0\quad y_T)^T\end{cases}$$

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\begin{aligned}
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\end{aligned}
$$

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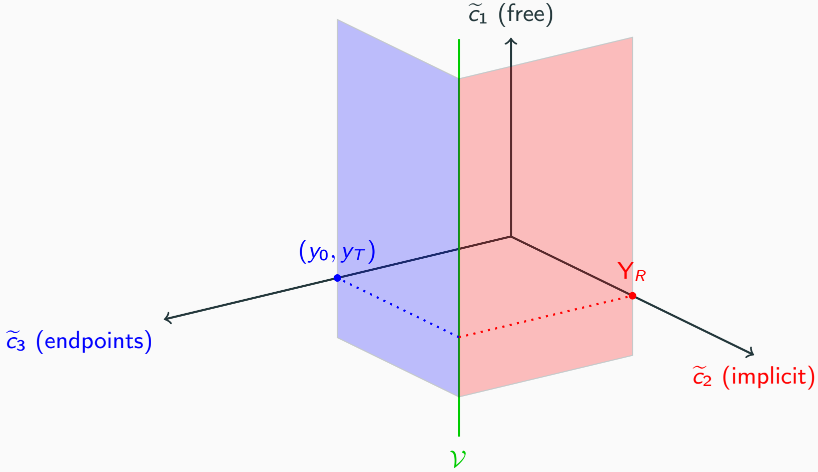
$$

- Maximum A Posteriori : Take the negative of the logarithm and change the basis to obtain

$$
c^\star \in \operatorname*{arg\,min}_{c \in \mathcal{V}} \check{F}(c) + \kappa \sum_{i=1}^{I} \omega_i \left(c - c_{R_i}\right)^T \Sigma^\dagger \big(c - c_{R_i}\big)
$$

Visualisation of the optimisation problem



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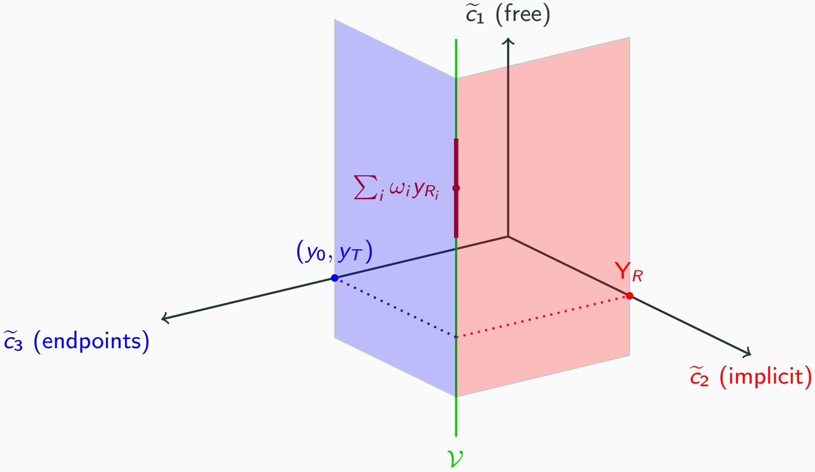

Iterative approach to satisfy additional constraints

- Additional constraints : The solution $y_\kappa^\star = \Phi^{-1} c_\kappa^\star$ should belong to the set $\mathcal{G}$! Not explicitly taken into account in

$$c_\kappa^\star \in \underset{c \in \mathcal{V}}{\arg\min}\, \check{F}(c) + \kappa \sum_{i=1}^{I} \omega_i \left(c - c_{R_i}\right)^T \Sigma^\dagger \left(c - c_{R_i}\right)$$

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- <u>Observation :</u> In the limit cases, we have

	$\kappa = 0$	$\kappa = +\infty$
$y^\star_\kappa \in \mathcal{G}$	✗	✓
$\min \check{F}$	✓	✗

Iterative approach to satisfy additional constraints

- Additional constraints : The solution $y_\kappa^\star = \Phi^{-1} c_\kappa^\star$ should belong to the set $\mathcal{G}$! Not explicitly taken into account in

$$c_\kappa^\star \in \underset{c \in \mathcal{V}}{\arg\min}\, \check{F}(c) + \kappa \sum_{i=1}^{I} \omega_i \left(c - c_{R_i}\right)^T \Sigma^\dagger \left(c - c_{R_i}\right)$$

- Observation : In the limit cases, we have

	$\kappa = 0$	$\kappa = +\infty$
$y_\kappa^\star \in \mathcal{G}$	✗	✓
$\min \check{F}$	✓	✗

- Idea : Tune κ to find a trade-off between optimisation and constraints.
  $\longrightarrow$ Iterative approach (linear search, binary search,...)

Quadratic case for a convex problem

- Hypothesis : Quadratic instantaneous cost [‡]

$$F(y) = \int_0^T y(t)^T Q y(t) + w^T y(t) + r \, dt$$

- Quadratic optimisation problem :

$$c^\star \in \arg\min_{c \in \mathcal{V}} c^T \Big(\check{Q} + \kappa\, \Sigma^\dagger\Big) c + \left(\check{w} - 2\,\kappa \sum_{i=1}^{I} \omega_i\, \Sigma^\dagger c_{R_i}\right)^T c$$

- Convex optimisation problem : If

$$\kappa \geqslant -\lambda_{min}\big(V_1^T \check{Q} V_1\big)\, \lambda_{max}(\Sigma)$$

then the above problem is convex.

‡. F. Dewez, B. Guedj, V. Vandewalle. *From industry-wide parameters to aircraft-centric on-flight inference : improving aeronautics performance prediction with machine learning*. Data-Centric Engineering, 2020.

PyRotor library

- **Py**thon **Ro**ute **t**rajectory **o**ptimise**r** ‡
- Preceding generic method developed in Python to be used in a wide range of fields
- Based on well-known librairies (SciPy, NumPy, sklearn,...)
- Developed on GitHub : https://github.com/bguedj/pyrotor

‡. Thank you Arthur !

Application 1 : Minimisation of aircraft fuel consumption during climb phase

Aeronautic setting

- Setting : Climbing phase of an aircraft in a vertical plane.
- Cost function : Total fuel consumption

$$\mathrm{TFC}(y) := \int_0^T \widehat{\mathrm{FF}}\big(y(t)\big)\,dt$$

where FF is the fuel flow.

- Trajectory : Altitude h, Mach M, engines power $\mathrm{N1}$

$$\forall\, t \in [0, T] \qquad y(t) := \big(h(t), \mathrm{M}(t), \mathrm{N1}(t)\big)$$

- Endpoints conditions :
  1. Altitude : from 3,000 ft to 38,000 ft ;
  2. Mach : from 0.3 to 0.78.
- Additional constraints :
  1. Rate of climb smaller than 3,600 ft/min ;
  2. Mach smaller than 0.82.

Aeronautic setting

- Data : 2,162 recorded flights ; 48 satisfy endpoints conditions (used to estimate Σ).
- Fuel flow : Fitted to the climb data by a quadratic model ($\sim 500,000$ observations) ; error : 1.73 %.
- Reference trajectories : The five most fuel-efficient trajectories among the 48 above ones.
- Climb duration : Given by the first time where endpoints are reached by the optimised flight.
- Basis : Legendre polynomials for each state ; overall dimension $= 20$.

Visualisation

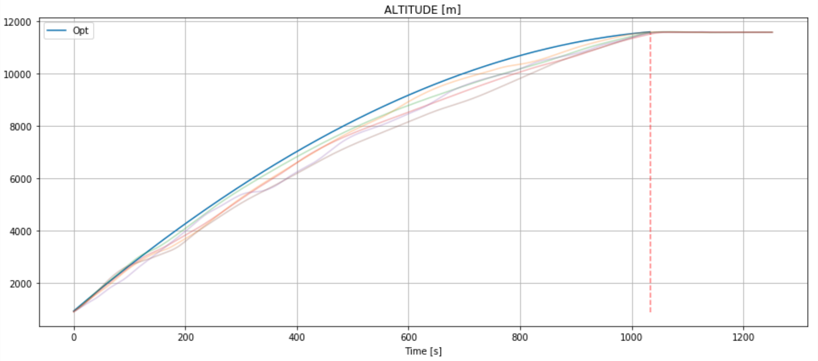

Figure 3: Altitude.

Visualisation

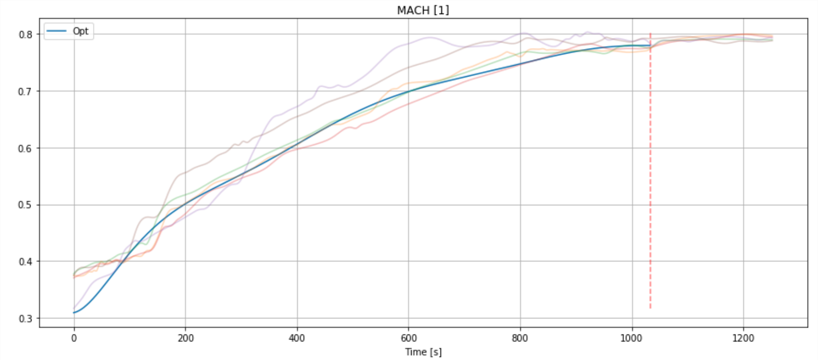

Figure 4: Mach.

Visualisation

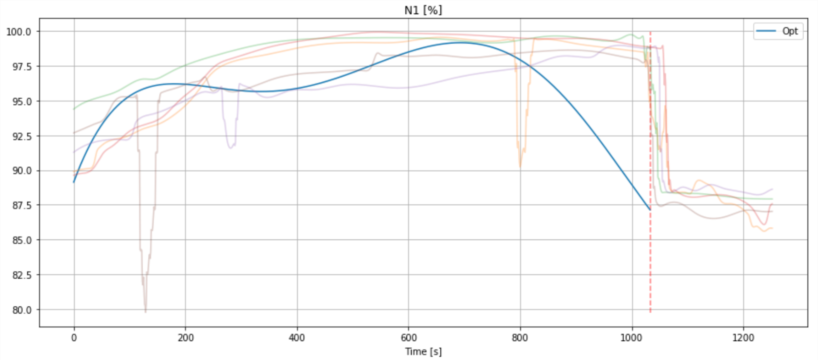

Figure 5: Engines power.

Quantitative results

	Mean	Std	Min	Med	Max
Fuel savings [kg]	260.38	86.21	71.79	261.87	393.73
Percentage [%]	16.54	4.73	5.27	16.88	23.39

Table 1: Statistical description of the fuel savings from the optimised trajectory with respect to the 48 reference ones.

<u>Execution time :</u> 3.76 ± 0.11 seconds (Intel Core i7 6 cores running at 2.2 GHz)

Application 2 : Optimisation of the work of a force field

Physical setting

- Setting : Point moving in a force fiel $V : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by

$$V\left(x^{(1)}, x^{(2)}\right) = \left(0, x^{(1)}\right)^T$$

- Cost function :

$$F_\alpha(y, \dot{y}) = \int_0^1 \alpha \left\|\dot{y}(t)\right\|_2^2 - V\big(y(t)\big)^T \dot{y}(t)\, dt$$

where $\alpha > 0$.

- Trajectory : Cartesian coordinates

$$\forall\, t \in [0, 1] \qquad y(t) := \left(y^{(1)}(t), y^{(2)}(t)\right)$$

- Endpoints conditions :
    1. $y^{(1)}$: from 0.111 to 0.912 ;
    2. $y^{(2)}$: from 0.926 to 0.211.

- Additional constraints : Stay in the square $[0, 1]^2$.

Physical setting

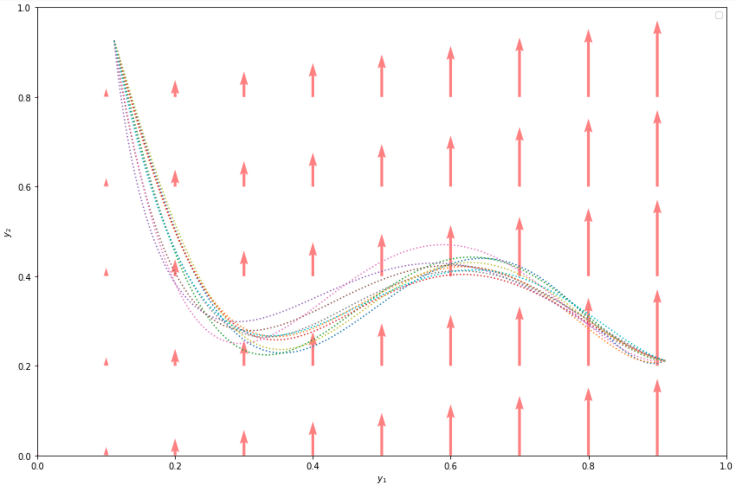

Figure 6: Illustration of the setting with reference trajectories.

Physical setting

- Data : 122 generated trajectories, each of them satisfying endpoints conditions and additional constraints.

- States derivatives :

$$y^{(d)} = \sum_{k=1}^{K_d} c_k^{(d)} \varphi_k \quad \longrightarrow \quad \dot{y}^{(d)} = \sum_{k=1}^{K_d} c_k^{(d)} \dot{\varphi}_k$$

  Do not increase the problem dimension !

- Reference trajectories : The 10 best trajectories with respect to the cost function F_α.

- Basis : Legendre polynomials for each state ; overall dimension = 10.

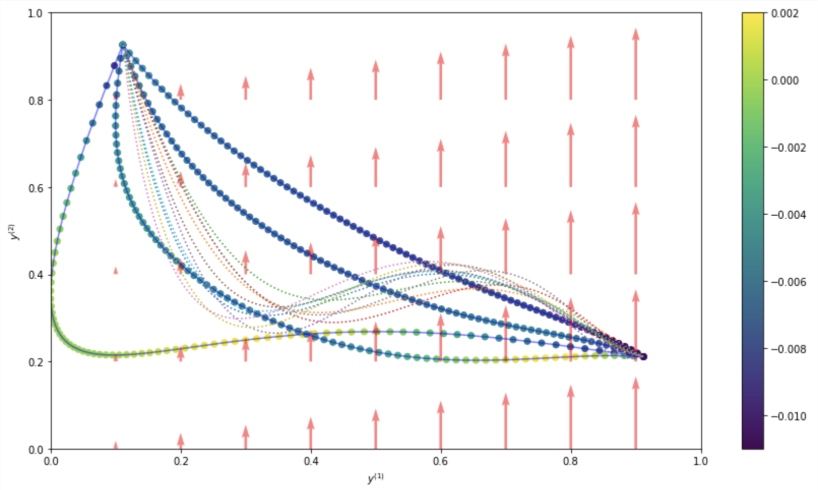

Figure 7: Optimised trajectories in the space $\left(y^{(1)}, y^{(2)}\right)$.

Quantitative results

	Mean	Std	Min	Med	Max
$\alpha = 0$	73.43	2.36	68.63	73.25	80.69
$\alpha = 0.35$	45.88	4.81	36.09	45.49	60.66
$\alpha = 1$	-6.12	9.43	-25.31	-6.88	22.87
$\alpha = 10$	-34.54	11.96	-58.87	-35.50	2.22

Table 2: Statistical description of the work gains [%] along the optimised trajectory with respect to the 122 reference ones for different values of α.

Execution time : 0.44 ± 0.03 seconds (Intel Core i7 6 cores running at 2.2 GHz)

On the horizon

Outlook

- Improvements :
  - 'Best' choice for the basis (e.g. FPCA)
  - Different modellings for the reference trajectories (e.g. no longer Gaussian noise)
  - Other numerical methods to obtain the solution (e.g. sampling methods)
  - Clustering step applied to the reference trajectories

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- Theoretical studies :
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- Applications :
  - Regatta (e.g. Vendée Globe)
  - Decision making process (e.g. patient path)

$\rightarrow$ Improvements for PyRotor

Bibliography

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The End

Quadratic fuel flow model (aeronautics)

- Estimated model : $\widehat{\mathrm{FF}}\big(y(t)\big) = y(t)^T \widehat{Q} y(t) + \widehat{w}^T y(t) + \widehat{r}$