

1. How do the basis functions of a first order (= degree 1) B-Spline look like with knot vector $(-1, 0, 1, 2, 3, 4, 5)$?
 Given the control points $(0, 0), (1, 3), (2, 1), (0, 3)$ and $(4, 1)$, draw the first order B-Spline. Explain the **connection** between degree, continuity, and support of B-Splines at this example. (1 Points)

Def grad 0: $N_i^0(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases}$

$$N_0^0(t) = \begin{cases} 1 & \text{if } t \in [-1, 0] \\ 0 & \text{otherwise} \end{cases}$$

$$N_1^0(t) = \begin{cases} 1 & \text{if } t \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

$$N_2^0(t) = \begin{cases} 1 & \text{if } t \in [1, 2] \\ 0 & \text{otherwise} \end{cases}$$

$$N_3^0(t) = \begin{cases} 1 & \text{if } t \in [2, 3] \\ 0 & \text{otherwise} \end{cases}$$

$$N_4^0(t) = \begin{cases} 1 & \text{if } t \in [3, 4] \\ 0 & \text{otherwise} \end{cases}$$

$$N_5^0(t) = \begin{cases} 1 & \text{if } t \in [4, 5] \\ 0 & \text{otherwise} \end{cases}$$

Def grad 1: $N_i^1(t) = \frac{t-t_i}{t_{i+1}-t_i} N_{i-1}^0(t) + \frac{t_{i+2}-t}{t_{i+2}-t_{i+1}} N_{i+1}^0(t)$

$$\begin{aligned} N_0^1(t) &= \frac{t-(-1)}{0-(-1)} N_0^0(t) + \frac{1-t}{1-0} N_1^0(t) \\ &= (t+1) N_0^0(t) + (1-t) N_1^0(t) \end{aligned}$$

$$\begin{aligned} N_1^1(t) &= \frac{t-0}{1-0} N_1^0(t) + \frac{2-t}{2-1} N_2^0(t) \\ &= t N_1^0(t) + (2-t) N_2^0(t) \end{aligned}$$

$$\begin{aligned} N_2^1(t) &= \frac{t-1}{2-1} N_2^0(t) + \frac{3-t}{3-2} N_3^0(t) \\ &= (t-1) N_2^0(t) + (3-t) N_3^0(t) \end{aligned}$$

$$\begin{aligned} N_3^1(t) &= \frac{t-2}{3-2} N_3^0(t) + \frac{4-t}{4-3} N_4^0(t) \\ &= (t-2) N_3^0(t) + (4-t) N_4^0(t) \end{aligned}$$

$$\begin{aligned} N_4^1(t) &= \frac{t-3}{4-3} N_4^0(t) + \frac{5-t}{5-4} N_5^0(t) \\ &= (t-3) N_4^0(t) + (5-t) N_5^0(t) \end{aligned}$$

□

Viz:

