1. How do the basis functions of a first order (= degree 1) B-Spline look like with knot vector $(-1, 0, 1, 2, 3, 4, 5)$? Given the control points $(0,0), (1,3), (2,1), (0,3)$ and $(4,1)$, draw the first order B-Spline. Explain the **connection** between degree, continuity, and support of B-Splines at this example. (1 Points)

Degree 0: $N_i^0(t) = \begin{cases} 1 & \text{if } t_i \le t < t_{i+1} \\ 0 & \text{otherwise} \end{cases}$

$N_0^0(t) = \begin{cases} 1 & \text{if } t \in [-1, 0[\\ 0 & \text{otherwise} \end{cases}$

$N_1^0(t) = \begin{cases} 1 & \text{if } t \in [0, 1[\\ 0 & \text{otherwise} \end{cases}$

$N_2^0(t) = \begin{cases} 1 & \text{if } t \in [1, 2[\\ 0 & \text{otherwise} \end{cases}$

$N_3^0(t) = \begin{cases} 1 & \text{if } t \in [2, 3[\\ 0 & \text{otherwise} \end{cases}$

$N_4^0(t) = \begin{cases} 1 & \text{if } t \in [3, 4[\\ 0 & \text{otherwise} \end{cases}$

$N_5^0 = \begin{cases} 1 & \text{if } t \in [4, 5[\\ 0 & \text{otherwise} \end{cases}$

Degree 1: $N_i^r(t) = \frac{t - t_i}{t_{i+r} - t_i} N_i^{r-1}(t) + \frac{t_{i+1+r} - t}{t_{i+1+r} - t_{i+1}} N_{i+1}^{r-1}(t)$

$$N_0^1(t) = \frac{t-(-1)}{0-(-1)} N_0^0(t) + \frac{1-t}{1-0} N_1^0(t)$$

$$= (t+1) N_0^0(t) + (1-t) N_1^0(t)$$

$$N_1^1(t) = \frac{t-0}{1-0} N_1^0(t) + \frac{2-t}{2-1} N_2^0$$

$$= t N_1^0(t) + (2-t) N_2^0$$

$$N_2^1(t) = \frac{t-1}{2-1} N_2^0(t) + \frac{3-t}{3-2} N_3^0$$

$$(t-1) N_2^0(t) + (3-t) N_3^0$$

$$N_3^1(t) = \frac{t-2}{3-2} N_3^0(t) + \frac{4-t}{4-3} N_4^0$$

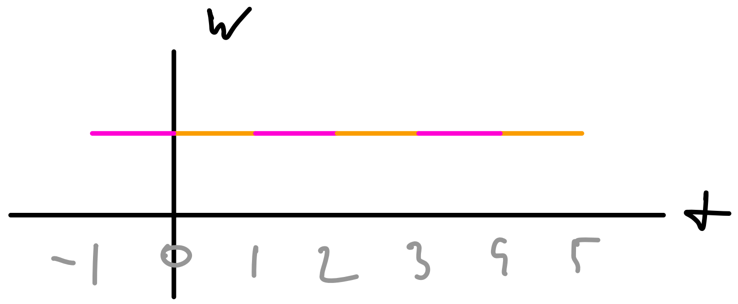
$$= (t-2) N_3^0(t) + (4-t) N_4^0$$

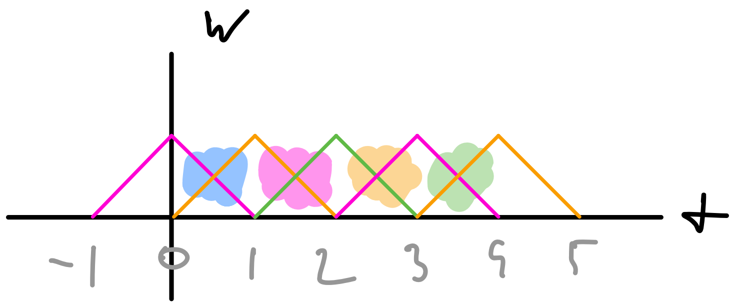
$$N_4^1 = \frac{t-3}{4-3} N_4^0(t) + \frac{5-t}{5-4} N_5^0$$

$$= (t-3) N_4^0(t) + (5-t) N_5^0$$

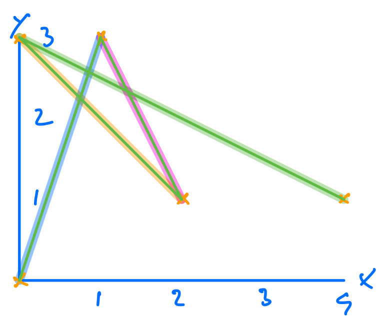
$\square$

Viz:





control points: [(0,0), (1,3), (2,1), (0,3), (4,1)]



The degree of a B-Spline impacts it's support. Specifically, the support of a spline, which is the measurement of how big an area a single control point affects, is degree+1.
In this example, since the degree is just 1, a control point just affects the section from the previous to the next control point.

A requirement for a B-Spline to be C1 continuous, the first derivative the polynomial sections it is made of need to match. For a B-Spline of degree 1, this means that the Spline can only be continuous if it is a line. The Spline from this exercise is obviously not a straight line, so it is not C1 continuous.