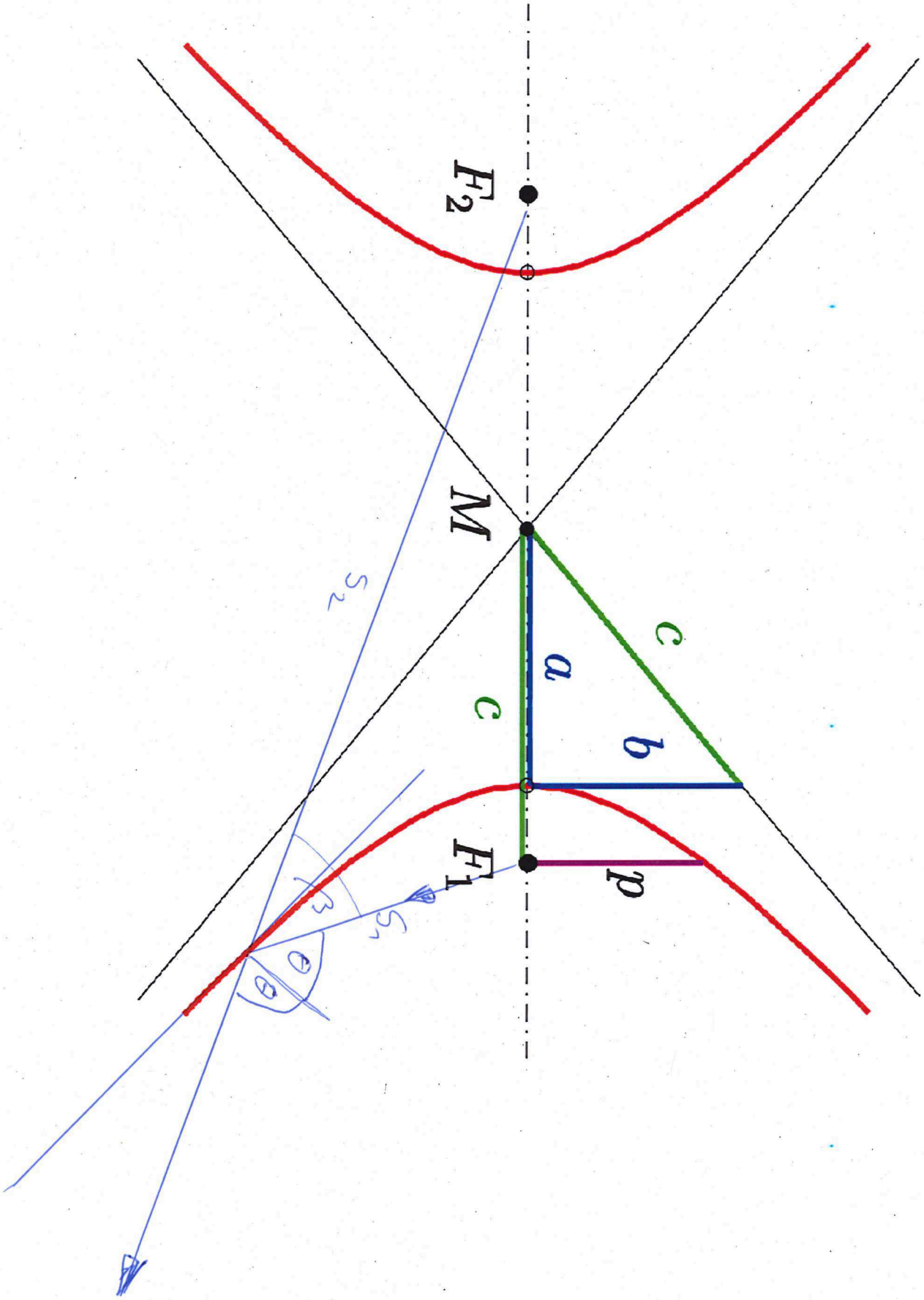
F_2
M
S_2
c
a
c
b
F_1
p
S_1
β
θ
θ

$$a = \frac{1}{2}(s_2 - s_1)$$

$$\beta = \pi - 2\theta$$

$$(2c)^2 = s_1^2 + s_2^2 - 2 s_1 s_2 \cos\beta$$

$$c^2 = a^2 + b^2$$

$$4a^2 + 4b^2 = s_1^2 + s_2^2 - 2 s_1 s_2 \cdot \cos(\pi - 2\theta)$$

$$a^2 = \frac{1}{4}(s_1^2 - 2 s_1 s_2 + s_2^2)$$

$$\cos(\pi - 2\theta) = \underbrace{\cos\pi}_{-1} \cdot \cos 2\theta + \underbrace{\sin\pi}_{0} \sin 2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$s_1^2 - 2 s_1 s_2 + s_2^2 + 4b^2 = s_1^2 + s_2^2 + 2 s_1 s_2 \cdot \cos 2\theta$$

$$4b^2 = 2 s_1 s_2 \underbrace{(\cos 2\theta + 1)}_{2\cos^2\theta}$$

$$4b^2 = 4 s_1 s_2 \cos^2\theta$$

$$b^2 = s_1 s_2 \cos^2\theta$$

$$b = \cos\theta \cdot \sqrt{s_1 s_2}$$

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hyperbola:

$$\frac{(s_2 - s_1)^2}{(2x)^2} - \frac{s_1 s_2 \cos^2\theta}{y^2} = 1$$