Introduction to Choreographies

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Abstract

This document contains lecture notes on choreographies for the course on DM861 Concurrency Theory (2018) at the University of Southern Denmark. It is organised as a working book. Different parts get updated during the course.

The chapters in advance of the lectures are given as a preview on what is to come, but **remember to download the updated version of this book every week to stay up to date!**

Contents

Preface:
Alice, Bob, and Choreographies

We live in the era of concurrency and distribution. Concurrency gives us the ability to perform multiple tasks at a time. Distribution allows us to do so at a distance. It is the era of the web, mobile computers, cloud computing, and microservices. The number of computer programs that communicate with other programs over a network is exploding. By 2025, the Internet alone might is expected to connect from 25 to 100 billion devices [OECD, 2016].

Modern computer networks and their applications are key drivers of our technological advancement. They give us better citizen services, a more efficient industry (Industry 4.0), new ways to connect socially, and even better health with smart medical devices.

Modern computer networks and their applications are complex. Services are becoming increasingly dependent on other services. For example, a web store might depend on an external service provided by a bank to carry out customer payments. The web store, the customer's web brower, and the bank service are thus integrated: they communicate with each other to reach the goal of transferring the right amount of money to the right recipient, such that the customer can get the product she wants from the store. In distributed systems, the heart of integration is the notion of *protocol*: a document that prescribes the communications that the involved parties should perform in order to reach a goal.

It is important that protocols are clear and precise, because each party needs to know what it is supposed to do such that integration is successful and the whole system works. At the same time, it is important that protocols are as concise as possible: the bigger a protocol, the higher the chance that we made some mistake in writing it. Computer scientists and mathematicians might get a familiar feeling when presented with the necessity of achieving clarity, precision, and conciseness in writing. A computer scientist could point out that we need a good *language* to write protocols. A mathematician might say that we need a good *notation*.

Needham and Schroeder [1978] introduced an interesting notation for writing protocols. A communication of a message M from the participant A to the participant B is written

$$A \rightarrow B : M.$$

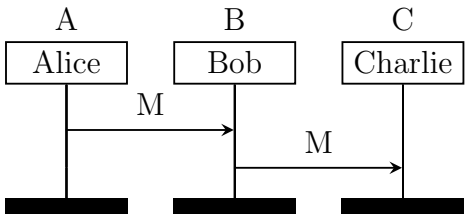
To define a protocol where A sends a message M to B, and then B passes the same message to C, we can just compose communications in a sequence:

$$\begin{aligned} A &\rightarrow B : M \\ B &\rightarrow C : M. \end{aligned}$$

This notation is called "Alice and Bob notation", due to a presentational style found in security research whereby the participants A and B represent the fictional characters Alice and Bob, respectively, who use the protocol to perform some task. There might be more participants, like C in our example—typically a shorthand for Carol, or Charlie. The first mention of Alice and Bob appeared in the seminal paper by Rivest et al. [1978] on their public-key cryptosystem:

> "For our scenarios we suppose that A and B (also known as Alice and Bob) are two users of a public-key cryptosystem".

Over the years, researchers and developers created many more protocol notations. Some of these notations are graphical rather than textual, like Message Sequence Charts [International Telecommunication Union, 1996]. The message sequence chart of our protocol for Alice, Bob, and Charlie looks as follows.



For our particular example, the graphical representation of our protocol (as a message sequence chart) and our previous textual representation (in Alice and Bob notation) are equivalent, in the sense that they contain the

same information. This is not the case in general: not all notations are equally expressive, as we will see in the rest of this book.[1]

In the beginning of the 2000s, researchers and practitioners took the idea of protocol notations even further, and developed the idea of *choreography*. A choreography is essentially still a protocol, but the emphasis is on detail. Choreographies might include details like the following.

- The kind of data being transmitted, or even the actual functions used to compute the data to be transmitted.
- Explicit cause-effect relations (also known as causal dependencies) between the data being transmitted and the choices made by participants in a protocol.
- The state of participants, e.g., their memory states.

Choreographies are typically written in languages designed to be readable *mechanically*, which also make them amenable to be used by a computer. In this book, we will start with a very simple choreography language and then progressively extend it with more sophisticated features, like parallelism and recursion. We will see that it is possible to define mathematically a *semantics* for choreographies, which gives us an interpretation of what running a protocol means. We will also see that it is possible to translate choreographies to a theoretical model of executable programs, which gives us an interpretation of how choreographies can be correctly implemented in the real world.

Although choreographies still represent a young and active area of research, they have already started emerging in many places. In 2005, the World Wide Web Consortium (W3C)—the main international standards organisation for the web—drafted the Web Services Choreography Description Language (WS-CDL), for defining interactions among web services. In 2011, the global technology standards consortium Object Management Group (OMG) introduced choreographies in their notation for business processes [BPMN]. The recent paradigm of microservices [Dragoni et al., 2017] advocates for the use of choreographies to achieve better scalability. All this momentum is motivating a lot of research on both the theory of choreographies and its application to programming [Ancona et al., 2016, Hüttel et al., 2016].

It is the time of Alice and Bob. It is the time of choreographies.

[1] Message sequence charts in particular support many features, including timeouts and alternative behaviours.

This book

This book is an introduction to the theory of choreographies. It explains what choreographies are and how we can model them mathematically. Its primary audiences are computer science students and researchers. However, the book should be approachable by anybody interested in studying choreographies, e.g., for theoretical purposes or the development of choreography-based tools. The aim of this book is to be pedagogical. It is not an aim to be comprehensive. References to alternative notations and techniques to model choreographies are given where appropriate. It is assumed that the reader is familiar with the notion of concurrency and how distributed systems are programmed.

Prerequisites To read this book, you should be familiar with:

- discrete mathematics and the induction proof method;
- context-free grammars (only basic knowledge is required);
- basic data structures, like trees and graphs;
- concurrent and distributed systems.

These prerequisites are attainable in most computer science B.Sc. degrees, or with three years of relevant experience.

To study choreographies, we are going to define choreography languages and then write choreographies as terms of these languages. The syntax of languages is going to be defined in terms of context-free grammars. To give meaning to choreographies, we are going to use extensively Plotkin's structural approach to operational semantics.

The rules defining the semantics of choreographies are going to be rules of inference, borrowing from deductive systems. Knowing formal systems based on rules of inference is not a requirement to read this book: chapter 1 provides an introduction to the essential knowledge on these systems that we need for the rest of the book. The reader familiar with inference systems or

structural operational semantics can safely skip the first chapter and jump straight to **??**.

An important aspect of choreographies is determining how they can be executed correctly in concurrent and distributed systems, in terms of independent programs for *processes*. To model process programs, we will borrow techniques from the area of process calculi. We will introduce the necessary notions on process calculi as we go along, so knowing this area is not a requirement for reading this book. The reader familiar with process calculi will recognise that we borrow ideas from Milner's seminal calculus of communicating systems [Milner, 1980].

Chapter 1

Inference systems

Before we venture into the study of choreographies, we need to become familiar with the formalism that we are going to use throughout this book: inference systems. Inference systems are widely used in formal logic and the specification of programming languages (our case).[1]

An inference system is a set of *inference rules* (also called rules of inference). An inference rule has the form

$$\frac{\text{Premise 1} \quad \text{Premise 2} \quad \cdots \quad \text{Premise } n}{\text{Conclusion}}$$

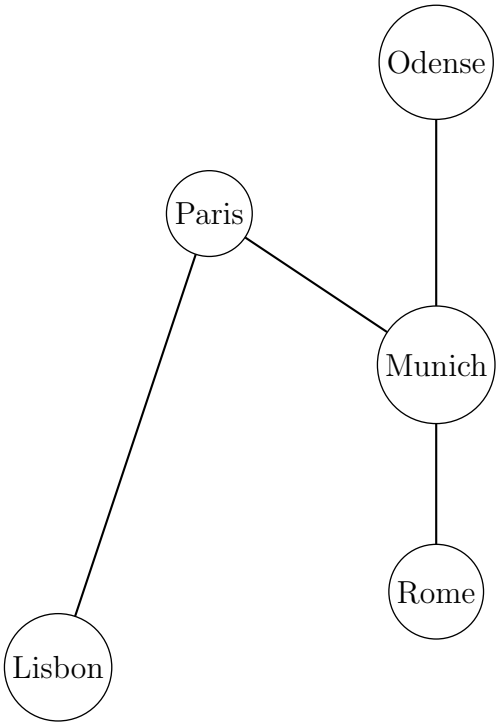
and reads "If the premises Premise 1, Premise 2, ..., and Premise n hold, then Conclusion holds". It is perfectly valid for an inference rule to have no premises: we call such rules *axioms*, because their conclusions always hold. An inference rule has always exactly one conclusion.

1.1 Example: Flight connections

This example is inspired by the usage of rules of inference to model graphs by Pfenning [2012].

Consider the following undirected graph of direct flights between cities.

[1] For further reading on these systems, see the lecture notes by Martin-Löf [1996].



Let A, B, and C range over cities. We denote that two cities A and B are connected by a direct flight with the *proposition* $\mathsf{conn}(A, B)$. Then, we can represent our graph as the set of axioms below.

$$\frac{}{\mathsf{conn}(\text{Odense}, \text{Munich})} \quad \frac{}{\mathsf{conn}(\text{Munich}, \text{Rome})} \quad \frac{}{\mathsf{conn}(\text{Paris}, \text{Munich})}$$

$$\frac{}{\mathsf{conn}(\text{Paris}, \text{Lisbon})}$$

Notice that our graph is undirected: in an ideal world, all direct flights are available also in the opposite direction. This is not faithfully represented by our axioms: if $\mathsf{conn}(A, B)$, it should also be the case that $\mathsf{conn}(B, A)$. One option to solve this discrepancy is to double the number of our axioms, to include their symmetric versions—e.g., for $\mathsf{conn}(\text{Munich}, \text{Odense})$). A more elegant option is to include a rule of inference for symmetry, as follows.

$$\frac{\mathsf{conn}(A, B)}{\mathsf{conn}(B, A)}\ \textsc{Sym}$$

The label Sym is the name of our new inference rule; it is just a decoration to remember what the rule does (Sym is a shorthand for symmetry). Rule Sym tells us that if we have a connection from any A to any B (premise),

$$\frac{}{\mathsf{conn}(\text{Odense}, \text{Munich})} \quad \frac{}{\mathsf{conn}(\text{Munich}, \text{Rome})} \quad \frac{}{\mathsf{conn}(\text{Paris}, \text{Munich})}$$

$$\frac{}{\mathsf{conn}(\text{Paris}, \text{Lisbon})}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{conn}(B, A)}\ \textsc{Sym} \qquad \frac{\mathsf{conn}(A, B)}{\mathsf{path}(A, B)}\ \textsc{Dir} \qquad \frac{\mathsf{path}(A, B) \quad \mathsf{path}(B, C)}{\mathsf{path}(A, C)}\ \textsc{Trans}$$

Figure 1.1: An inference system for flights.

then we have a connection from B to A (conclusion). In this rule, A and B are *schematic variables*: they can stand for any of our cities.

Now that we are satisfied with how our rule system captures the graph, we can use it to find flight paths from a city to another. For any two cities A and B, the proposition $\mathsf{path}(A, B)$ denotes that there is a path from A to B. Finding paths is relatively easy. First, we observe that direct connections give us a path. (Dir stands for direct.)

$$\frac{\mathsf{conn}(A, B)}{\mathsf{path}(A, B)}\ \textsc{Dir}$$

Second, we formulate a rule for multi-step paths. If there is a path from A to B, and a path from B to C, then we have a path from A to C. In other words, paths are transitive. (Trans in the following rule stands for transitivity.)

$$\frac{\mathsf{path}(A, B) \quad \mathsf{path}(B, C)}{\mathsf{path}(A, C)}\ \textsc{Trans}$$

The whole system is displayed in fig. 1.1.

1.2 Derivations

The key feature of an inference system is the capability of performing *derivations*. Suppose that we wanted to answer the following question.

> Is there a flight path from Odense to Rome?

Answering this question in our inference system for flight connections corresponds to showing that $\mathsf{path}(\text{Odense}, \text{Rome})$ holds. To do this, we build a *derivation tree* (also simply called *derivation*, or *proof tree*). The problem tackled in the following is also known as proof search.

We start our derivation from what we want to conclude with (our conclusion is what we want to prove).

$$\mathsf{path}(\text{Odense}, \text{Rome})$$

Observe that we have only two inference rules that can conclude something of this form: Dir and Trans. Dir cannot be applied: we would need to prove $\mathsf{conn}(\text{Odense}, \text{Rome})$, which is impossible. So our only choice is rule Trans, by instantiating A as Odense and C as Rome. How should we instantiate B in rule Trans, though?

$$\frac{\mathsf{path}(\text{Odense}, ??B??) \quad \mathsf{path}(??B??, \text{Rome})}{\mathsf{path}(\text{Odense}, \text{Rome})}\ \textsc{Trans}$$

We need to guess a correct instantiation for B and hope that it will allow us to continue our derivation. Looking at the definition of our graph, it is easy to see that the right choice is to pick Munich.

$$\frac{\mathsf{path}(\text{Odense}, \text{Munich}) \quad \mathsf{path}(\text{Munich}, \text{Rome})}{\mathsf{path}(\text{Odense}, \text{Rome})}\ \textsc{Trans}$$

Our derivation is not complete yet, because we are left with the tasks of proving (deriving) $\mathsf{path}(\text{Odense}, \text{Munich})$ and $\mathsf{path}(\text{Munich}, \text{Rome})$. Thanks to the notation of inference rules, we can just "dig deeper" with further rule applications. Since we know that Odense and Munich are connected, we can try using rule Dir.

$$\frac{\dfrac{\mathsf{conn}(\text{Odense}, \text{Munich})}{\mathsf{path}(\text{Odense}, \text{Munich})}\ \textsc{Dir} \quad \mathsf{path}(\text{Munich}, \text{Rome})}{\mathsf{path}(\text{Odense}, \text{Rome})}\ \textsc{Trans}$$

We now have to prove $\mathsf{conn}(\text{Odense}, \text{Munich})$. We have that as the conclusion of one of our axioms, so we can conclude that branch of our derivation by applying the related axiom.

$$\frac{\dfrac{\dfrac{}{\mathsf{conn}(\text{Odense}, \text{Munich})}}{\mathsf{path}(\text{Odense}, \text{Munich})}\ \textsc{Dir} \quad \mathsf{path}(\text{Munich}, \text{Rome})}{\mathsf{path}(\text{Odense}, \text{Rome})}\ \textsc{Trans}$$

We can finish our derivation for the right premise $\mathsf{path}(\text{Munich}, \text{Rome})$ likewise.

$$\dfrac{\dfrac{\dfrac{}{\mathsf{conn}(\text{Odense}, \text{Munich})}}{\mathsf{path}(\text{Odense}, \text{Munich})}\ \textsc{Dir} \quad \dfrac{\dfrac{}{\mathsf{conn}(\text{Munich}, \text{Rome})}}{\mathsf{path}(\text{Munich}, \text{Rome})}\ \textsc{Dir}}{\mathsf{path}(\text{Odense}, \text{Rome})}\ \textsc{Trans}$$

Our derivation is done! We can tell by the fact that there are no more premises left to prove.

If you look carefully, you can see that our derivation is a tree. The conclusion $\mathsf{path}(\text{Odense}, \text{Rome})$ is the root of the tree, and the leaves are empty nodes (the premises of our axioms). Rule applications connect the nodes of the tree. In fact, even our previous partial derivations are all trees. This is a very useful property that we will use throughout the book.

We impose that derivations are finite: we are interested in proofs that can be checked mechanically in finite time.

1.3 Proof non-existence

Proof, or proof of no proof. There is no try.
- Yoda, to an exhausted Luke

Showing that a proposition holds requires showing a proof, i.e., a derivation, in the inference systems of interest. Once the proof is shown, then we just need to check that all rules have been applied correctly. If we are convinced that this is the case, then we are convinced that the proof is correct and that the proposition indeed holds.

What about showing that a proposition *cannot* be derived? This can be far trickier, because it requires us to reason about all the possible proofs that could, potentially, conclude with our proposition and showing that none of those proofs can actually be built.

Consider a simple example: showing that $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ is not derivable. Intuitively, there is no direct connection between Lisbon and Rome in our graph. But how can we show this formally?

First, we observe that the only rules that can have a conclusion of the form $\mathsf{conn}(A, B)$ for any A and B are our axioms and rule Sym. None of the axioms can be applied for $A = \text{Lisbon}$ and $B = \text{Rome}$, as in our case. We are left with rule Sym: any search of a derivation of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ must begin as follows.

$$\dfrac{\mathsf{conn}(\text{Rome}, \text{Lisbon})}{\mathsf{conn}(\text{Lisbon}, \text{Rome})}\ \textsc{Sym}$$

By similar reasoning (there is no rule to apply for $\mathsf{conn}(\text{Rome}, \text{Lisbon})$ but Sym), we obtain that the proof *must* continue with another application of Sym.

$$\dfrac{\dfrac{\dfrac{}{\mathsf{conn}(\text{Lisbon}, \text{Rome})}}{\mathsf{conn}(\text{Rome}, \text{Lisbon})}\ \textsc{Sym}}{\mathsf{conn}(\text{Lisbon}, \text{Rome})}\ \textsc{Sym}$$

We got back to where we started: we have to prove $\mathsf{conn}(\text{Lisbon}, \text{Rome})$. Since we have only one way to prove it, and we have just shown that it leads to the exact same premise, this points out that our proof search will go on indefinitely and that we will never reach a finite derivation. So, $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ cannot be proven.

Let us be more formal, to convince ourselves that $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ cannot be derived more decisively. Since every derivation is a finite tree, we can measure the height of a derivation as a natural number (it is the height of the tree). Thus, among all the proofs of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$, there is at least one of minimal height, in the sense that there is no other proof of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ with lower height. We attempt at finding this minimal proof. The reasoning goes as before, and we soon end up seeing that a minimal proof necessarily starts as follows.

$$\dfrac{\dfrac{\dfrac{}{\mathsf{conn}(\text{Lisbon}, \text{Rome})}}{\mathsf{conn}(\text{Rome}, \text{Lisbon})}\ \textsc{Sym}}{\mathsf{conn}(\text{Lisbon}, \text{Rome})}\ \textsc{Sym}$$

So we have to find some proof of our premise $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ on top. But if such a proof exists, it would be a proof of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ that is *smaller* than the proof that we are building (because it would not have the first two applications of Sym of our proof). Thus our minimal proof must be bigger than another proof, and we reach a contradiction.

Another way of proving that $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ is by looking at how proofs are constructed from the top, rather than the bottom. For our system, we can prove the following result. We use $\mathcal{P}$ to range over proofs (derivations).

Proposition 1. *For any natural number n, there exists no proof of $\mathsf{conn}(\textit{Lisbon}, \textit{Rome})$ of height n.*

Proof. We prove the stronger result that there exists no proof of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ and there exists no proof of $\mathsf{conn}(\text{Rome}, \text{Lisbon})$, either.

We proceed by induction on n.

$$\frac{}{\mathsf{conn}(\text{Odense}, \text{Munich})} \quad \frac{}{\mathsf{conn}(\text{Munich}, \text{Rome})} \quad \frac{}{\mathsf{conn}(\text{Paris}, \text{Munich})}$$

$$\frac{}{\mathsf{conn}(\text{Paris}, \text{Lisbon})}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{conn}(B, A)}\ \textsc{Sym}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{path}(A, B, 1)}\ \textsc{DirW} \quad \frac{\mathsf{path}(A, B, n) \quad \mathsf{path}(B, C, m)}{\mathsf{path}(A, C, n + m)}\ \textsc{TransW}$$

Figure 1.2: Weighted rules for flight paths.

Base case: $n = 1$. All proofs with height 1 must necessarily be an application of an axiom, and there is no axiom that can prove either $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ or $\mathsf{conn}(\text{Rome}, \text{Lisbon})$.

Inductive case: $n = m + 1$ for some natural number m. By induction hypothesis, we know that there is no proof $\mathcal{P}$ such that the height of $\mathcal{P}$ is m and that the conclusion of $\mathcal{P}$ is $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ or $\mathsf{conn}(\text{Rome}, \text{Lisbon})$. Thus, all proofs $\mathcal{Q}$ of height m conclude with either: i) $\mathsf{conn}(A, B)$ for some A and B such that the set $\{A, B\}$ is different from $\{\text{Lisbon}, \text{Rome}\}$; or ii) $\mathsf{path}(A, B)$ for some A and B. For i), we observe that there is no rule that allows us to derive $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ from a premise that is not $\mathsf{conn}(\text{Rome}, \text{Lisbon})$, and likewise for the symmetric case where the premise is $\mathsf{conn}(\text{Rome}, \text{Lisbon})$. For ii), we observe that there is no rule that, given a conn proposition as one of its premises, allows us to conclude $\mathsf{conn}(\text{Lisbon}, \text{Rome})$ or $\mathsf{conn}(\text{Rome}, \text{Lisbon})$.

□

It follows from proposition 1 that there is no proof of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$. Assume that there were such a proof. Since it would have to be finite, it would have a height n. Thus we reach a contradiction, because proposition 1 states that for all n there is no proof of $\mathsf{conn}(\text{Lisbon}, \text{Rome})$.

$$\frac{}{\mathsf{conn}(\text{Odense}, \text{Munich})} \quad \frac{}{\mathsf{conn}(\text{Munich}, \text{Rome})} \quad \frac{}{\mathsf{conn}(\text{Paris}, \text{Lisbon})}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{conn}(B, A)}\ \textsc{Sym}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{path}(A, B, 1)}\ \textsc{DirW} \quad \frac{\mathsf{path}(A, B, n) \qquad \mathsf{path}(B, C, m)}{\mathsf{path}(A, C, n + m)}\ \textsc{TransW}$$

Figure 1.3: A limited and weighted flight system.

$$\frac{}{\mathsf{conn}(\text{Odense}, \text{Munich})} \quad \frac{}{\mathsf{conn}(\text{Munich}, \text{Rome})} \quad \frac{}{\mathsf{conn}(\text{Paris}, \text{Munich})}$$

$$\frac{}{\mathsf{conn}(\text{Paris}, \text{Lisbon})}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{conn}(B, A)}\ \textsc{Sym}$$

$$\frac{\mathsf{conn}(A, B)}{\mathsf{path}(A, B)}\ \textsc{Dir} \quad \frac{\mathsf{conn}(A, B) \qquad \mathsf{path}(B, C)}{\mathsf{path}(A, C)}\ \textsc{Step}$$

Figure 1.4: An alternative way of constructing paths.

1.4 Exercises on graphs

Exercise 1. *Consider the system in fig. 1.2, which replaces rules* Dir *and* Trans *with alternative rules that measure the length of a path (paths are "weighted", with each connection having weight* 1*).*

Prove that, for any A *and* B*, if* $\mathsf{path}(A, B)$ *is derivable in the system in fig. 1.1, then there exists* n *such that* $\mathsf{path}(A, B, n)$ *is derivable in the system in fig. 1.2.*

Suggestion: proceed by structural induction on the proof of $\mathsf{path}(A, B)$*.*

Exercise 2 (!)**.** *Consider the system in fig. 1.3, which removes the direct flight from Paris to Munich.*

Prove that it is not possible to derive $\mathsf{path}(\mathit{Lisbon}, \mathit{Munich}, n)$ *for any* n*.*

Exercise 3 (!)**.** *Consider the system in fig. 1.4, which replaces rule* Trans *from fig. 1.1 with rule* Step*.*

Prove that, for any A *and* B*,* $\mathsf{path}(A, B)$ *is derivable in the system in fig. 1.1 if and only if it is derivable in the system in fig. 1.4.*

Appendix A

Solution to selected exercises

We give the solutions to some selected exercises. Some solutions are given in full detail, to serve as examples of exposition. All solutions should still provide enough information for the reader to figure out the remaining parts.

Solution of exercise 1. We prove only the direction from the system in fig. 1.1 to the system in fig. 1.2.

To prove this direction, we actually prove the stronger statement:

- $\mathsf{conn}(A, B)$ provable in fig. 1.1 implies $\mathsf{conn}(A, B)$ provable in fig. 1.2;
- $\mathsf{path}(A, B)$ provable in fig. 1.1 implies $\mathsf{path}(A, B, n)$ for some n provable in fig. 1.2.

The proof is by induction on the structure of the derivation of $\mathsf{conn}(A, B)$ or $\mathsf{path}(A, B)$. We proceed by cases on the last applied rule of the derivation.

Base cases (axioms) If the last applied rule is an axiom, then the proof is valid also in the other system.

Case Sym The derivation has this shape:

$$\dfrac{\dfrac{\mathcal{P}}{\mathsf{conn}(B, A)}}{\mathsf{conn}(A, B)}\ \textsc{Sym}.$$

By induction hypothesis, we know that there exists $\mathcal{P}'$ in the other system such that:

$$\dfrac{\mathcal{P}'}{\mathsf{conn}(B, A).}$$

The thesis follows by applying rule Sym.

$$\dfrac{\dfrac{\mathcal{P}'}{\mathsf{conn}(B,A)}}{\mathsf{conn}(A,B)}\ \textsc{Sym}.$$

Case Dir The derivation has this shape:

$$\dfrac{\dfrac{\mathcal{P}}{\mathsf{conn}(A,B)}}{\mathsf{path}(A,B)}\ \textsc{Dir}.$$

By induction hypothesis, we know that there exists $\mathcal{P}'$ in the other system such that:

$$\dfrac{\mathcal{P}'}{\mathsf{conn}(A,B)}.$$

The thesis follows by applying rule DirW.

$$\dfrac{\dfrac{\mathcal{P}'}{\mathsf{conn}(A,B)}}{\mathsf{path}(A,B,1)}\ \textsc{DirW}.$$

Case Trans The derivation has this shape:

$$\dfrac{\dfrac{\mathcal{P}}{\mathsf{path}(A,B)}\quad\dfrac{\mathcal{Q}}{\mathsf{path}(B,C)}}{\mathsf{path}(A,C)}\ \textsc{Trans}.$$

By induction hypothesis on $\mathcal{P}$ and by induction hypothesis on $\mathcal{Q}$, we know that there exist n and m, $\mathcal{P}'$ and $\mathcal{Q}'$ in the other system such that:

$$\dfrac{\mathcal{P}'}{\mathsf{path}(A,B,n)}\qquad\dfrac{\mathcal{Q}'}{\mathsf{path}(A,B,m)}.$$

The thesis follows by applying rule TransW.

$$\dfrac{\dfrac{\mathcal{P}'}{\mathsf{path}(A,B,n)}\quad\dfrac{\mathcal{Q}'}{\mathsf{path}(A,B,m)}}{\mathsf{path}(A,B,n+m)}\ \textsc{TransW}.$$

■

Solution of exercise 3. Proceed in two steps.

First, prove by induction that if $\mathsf{path}(A, B)$ is provable in the system in fig. 1.1, then there exists a nonempty sequence of provable propositions $\mathsf{conn}(C_1, C_1'), \ldots, \mathsf{conn}(C_n, C_n')$ for some $C_1, \ldots, C_n$ such that $n > 0$, $C_1 = A$ (the sequence starts from A), and $C_n' = B$ (the sequence ends at B).

Second, prove that given a nonempty sequence of provable propositions $\mathsf{conn}(C_1, C_1'), \ldots, \mathsf{conn}(C_n, C_n')$, then $\mathsf{path}(C, C_n')$ is provable in the system in fig. 1.4. To build the proof for $\mathsf{path}(C, C_n')$, apply rule Step as needed by using the proofs $\mathcal{P}_1, \ldots, \mathcal{P}_n$ of each conn proposition in the sequence as premises. ■

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