

# Online Support Vector Regression for Non-Linear Control

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## ABSTRACT

Model Predictive Control (MPC) can provide robust control for non-linear processes. In this paper, we propose an MPC using online Support Vector Machine (SVM) which incorporates learning guided by error minimization as new data samples arrive. The tracking errors of future plant behavior are minimized by Differential Evolution, a global optimizer. With these features, an online SVR based MPC controller can be implemented to improve the model and controller performance in the presence of disturbances. The simulation results for the cases of two chemical processes are shown to demonstrate the performance of the proposed scheme.

**KEYWORDS:** differential evolution, model predictive control, non-linear system, on-line support vector regression

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## INTRODUCTION

Model Predictive Control (MPC) is an advanced method of model control used in process industries for the past three decades. MPC is a robust control algorithm that controls the future behavior of a plant through a model (Morari [1]). At each control interval, MPC algorithm predicts system output at N future sampling times into the future. Then, open-loop sequence of manipulated variable is computed in order to optimize future plant behavior. The first value in the optimal sequence of manipulated variable is implemented. The next round of calculations starts from the new current state by sampling the plant state once again.

Several authors have worked on MPC with different optimizers and system identifiers. During the last decade, MPC using Genetic Algorithm (GA) as an optimizer and artificial neural network as a process model have been studied extensively. Al-Duwaish [2] had proposed an MPC along with GA for systems identified by Hammerstein and Weiner models. A neural network based Weiner identification method with linear and nonlinear MPC was used by Arefi [3] for controlling a pH neutralization process. Chen [4] had presented a suboptimal nonlinear MPC based on GA in order to reduce the computational complexity. A radial basis function (RBF) network and an auto regressive exogenous (ARX) model based MPC algorithm was used by Abdulrahman [5] to improve the productivity of a yeast fermentation process. A good system identifier and a global optimization technique are essential for an MPC.

One of the most recognized machine learning algorithms is Support Vector Machines for regression (SVR) which uses a training set for successfully predicting output. But a major limitation of SVR is that it works in batch mode only. Online SVR which updates periodically is a good candidate for MPC. In the case of an optimizer, as compared to GA, Differential Evolution (DE) has a better selection criteria and mutation scheme that makes it self-adaptive, thus outperforming GA in terms of convergence speed (Rahman [6]).

This paper presents an MPC with online SVR as a process model and DE as an optimizer for set point tracking and disturbances rejection. A cost function of the tracking errors at five future instances in time is minimized using DE. The efficacy of MPC is shown for two nonlinear systems, namely a continuous fermenter and a multivariable pH neutralization system.

## SUPPORT VECTOR MACHINE

Support vector machines (SVMs) (Vapnik [7]) can be used for system identification of both linear and non-linear dynamic systems. Support vector machines form a part of supervised machine learning, which is used to create a function from the training data. It derives from the statistical learning theory, with the training data consisting of multiple inputs being transformed by the kernels, giving an output which is a linear function of the weights and the kernels. The output of SVMs can be used for either regression or classification. In the case of regression, the solution of the SVM is a real number. The prediction algorithm works with the aim of constrained minimization of the modelling errors for a given precision level,  $\epsilon$ . The constrained minimization removes data points within the error bounds, while the retained ones form the support vectors. It is the structural risk minimization principle implemented by SVMs that helps it achieve an optimum networks structure by minimizing an upper bound to the generalization error rather than minimizing the training error. SVMs are an equivalent of solving a linear constrained quadratic programming problem that always gives a unique and globally optimal solution. SVR is basically the mapping of input data  $x$  into a higher dimensional feature space  $F$  via a nonlinear mapping  $\phi(x)$  resulting in a linear regression problem in that feature space (Jain [8]).

## ONLINE SUPPORT VECTOR REGRESSION

As described by Parrella [9], addition of new training data to an already trained SVR model requires the following inputs to the program:

1. The trained SVR model
2. Weights and bias of SVR
3. SVR parameters:  $\epsilon$ ,  $C$  (error multiplier), Kernel type and Kernel parameters
4. New sample

The SVR then performs an update of the weights and bias while adhering to the constraints set initially, and returns the following output:

1. New trained SVR model
2. Updated weights and bias

## DIFFERENTIAL EVOLUTION

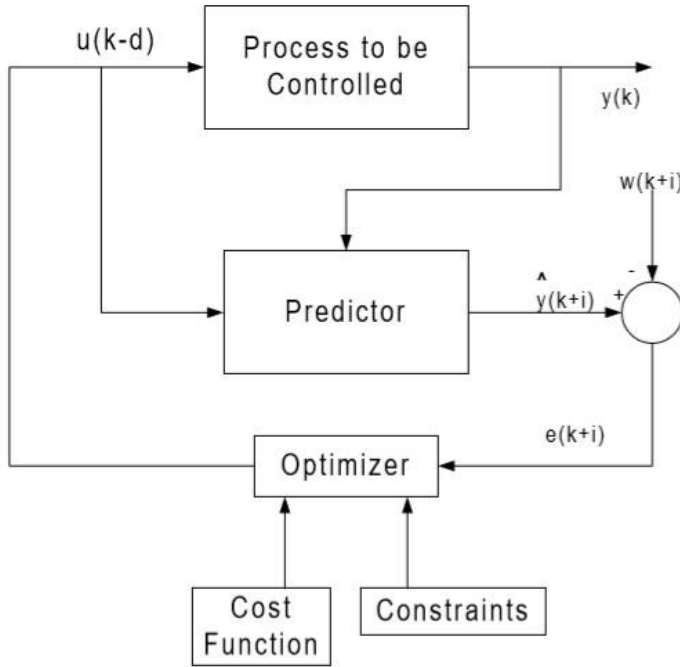
Differential Evolution (DE) (Storn [10]) is a reliable and versatile function optimizer that gives solutions quite close to the global optimum reasonably fast. DE is a population-based optimizer that attacks the starting point problem by sampling the objective function at multiple, randomly chosen initial points using a random number generator. This is the initialization process and the random initial points are chosen from the bounds specified to the DE. Once initialized, DE generates trial vectors, also called population members, by mutating and recombining the existing population. Mutation and recombination essentially involve the perturbations of existing points by scaling the difference of two randomly selected population vectors which is then added to a third randomly selected population vector to generate a trial vector.

Complementary to mutation, DE also performs uniform crossovers. The crossover strategy is to build trial vectors out of parameter values copied from two different vectors. In this way, each vector is crossed with a mutant vector. The trial vector then competes against the population vectors of the same index in the selection process. The vector with the lower objective function value is marked as a member of the next generation. Once all the trial vectors have been tested, the survivors of the current generation become parents for the next generation in the evolutionary cycle. Thus, the optimum value evolves over the generations.

## MODEL PREDICTIVE CONTROL

Model predictive algorithm (Fig. 1) can be described as:

1. Prediction of process output by use of a model at a number of future times.
2. Developing a control sequence along a future time horizon in order to optimize a cost function.
3. A receding horizon strategy, which moves the horizon toward the future at every step while implementing only the first control signal at each step.



(Figure 1: MPC strategy)

The cost function  $J$ , supplied to the optimizer considered in this paper has the form:

$$J = \sum_{k=1}^n \text{abs}(y(t+k) - SP) \quad (1)$$

Where  $SP$  is the set-point of the system,  $n$  is the number of time instances in future for which the control sequence is calculated and  $y$  is the model output.

Historic data were generated by varying the manipulated variable in an open loop system. During simulation, the new data is fed to the SVR model at each iteration to update the parameters online. The optimizer in the MPC controller is switched off in the event of the difference of two consecutive outputs of the controller being less than 0.0001.

## CONTROL OF A CONTINUOUS FERMENTOR

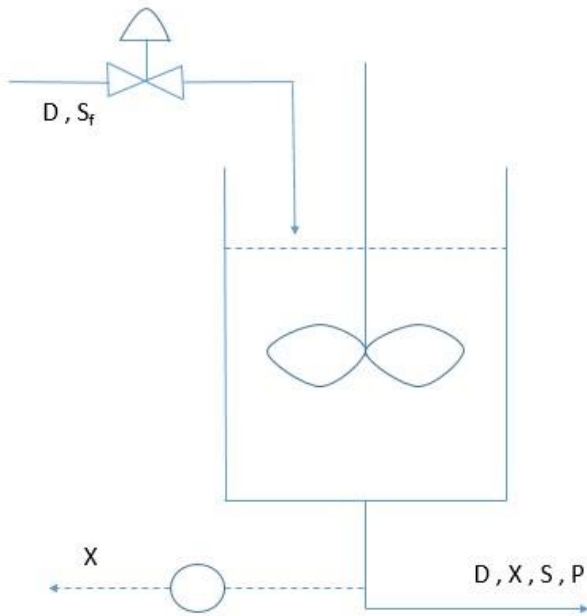
A continuous fermentor system consisting of a feed, a stirred tank and an outlet stream is considered here as shown in Fig. 2. The volume of the tank is taken to be constant. Following non-linear ordinary differential equations were used to model the system (Henson [24]):

$$\frac{dX}{dt} = -DX + \mu X \quad (2)$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{\mu X}{Y_{X/S}} \quad (3)$$

$$\frac{dP}{dt} = -DP + (\alpha\mu + \beta)X \quad (4)$$

$$\text{Where } \mu = \frac{\mu_m \left[1 - \left(\frac{P}{P_m}\right)\right] S}{K_m + S + \left(\frac{S^2}{K_i}\right)} \quad (5)$$



(Figure 2: Continuous fermenter)

Variable	Nominal Value
$D$	$0.202 \text{ h}^{-1}$
$K_i$	$22 \text{ gl}^{-1}$
$K_m$	$1.2 \text{ gl}^{-1}$
$P$	$19.14 \text{ gl}^{-1}$
$P_m$	$50 \text{ gl}^{-1}$
$S$	$5 \text{ gl}^{-1}$
$S_f$	$20 \text{ gl}^{-1}$
$X$	$6 \text{ gl}^{-1}$
$Y_{x/s}$	$0.4 \text{ gg}^{-1}$
$\alpha$	$2.2 \text{ gg}^{-1}$
$\beta$	$0.2 \text{ h}^{-1}$
$\mu_m$	$0.48 \text{ h}^{-1}$

**(Table 1:** Nominal parametric values for continuous fermenter)

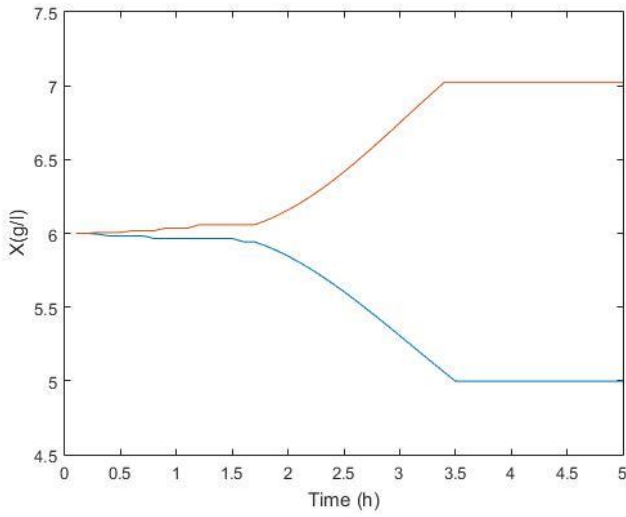
The nominal values of the terms in the equation are provided in Tab. 1 and the parameters used are defined in the nomenclature. Values of dilution rate,  $D$  and corresponding cell concentration,  $X$  used to train the SVR model were in the range (0.1495-0.22) and [7.044-4.988] respectively. The SVR predicted future values through an input of the form:

$$y(t+k) = f[ u(t+k-3), u(t+k-2), y(t+k-3), y(t+k-2) ] \quad (6)$$

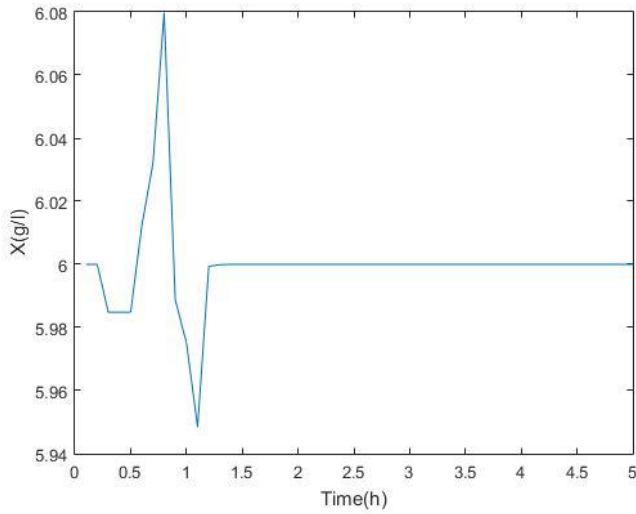
The above data was generated using ODEs simulated in MATLAB with a time span of 0.1h. In order to improve the performance of MPC, the model error was added to the cost function of DE. The SVR parameters were tuned to get the best model ( $\epsilon = 0.01$ ,  $C = 3$ , Kernel type = Gaussian RBF and Kernel parameters = 30).

The performance of MPC for the set point changes of  $X$  from its nominal value of 6 to 5 and 7 is shown in Fig. 3. A -50% disturbance in both maximum growth rate  $\mu_m$  and cell mass yield  $Y_{x/s}$  is introduced in the system simultaneously, unlike the independent disturbances of -12.5% in  $\mu_m$  and -20% in  $Y_{x/s}$  handled by the controller proposed by Rangaiah [26]. Fig. 4 shows the controller performance for handling the two disturbances simultaneously.

MPC was able to control the system very efficiently. When compared with the results reported by Rangaiah [26], set point tracking was achieved in just 3.5h. Moreover, online SVR based MPC tracked disturbance changes of larger magnitude in less than 1.5h as compared to the nearly 10h required by their controller.



(Figure 3: SP changes of 5 and 7)



(Figure 4: Disturbance changes in fermenter)

## pH & LEVEL CONTROL FOR A NEUTRALIZATION PROCESS

The aim of this multivariable process is to control:

- a) the pH of the outlet stream and
- b) the liquid level in the tank,

by manipulating acid ( $q_1$ ) and base ( $q_3$ ) flow rates. The nominal values of the parameters used in the equations are tabulated in Tab. 2. The reaction invariants  $W_a$  and  $W_b$  (Gustafsson [27], Hall [28] and Henson [29]) are used to model the system where,

$$W_a = [H^+] - [OH^-] - [HCO_3^-] - 2[CO_3^{2-}] \quad (7)$$

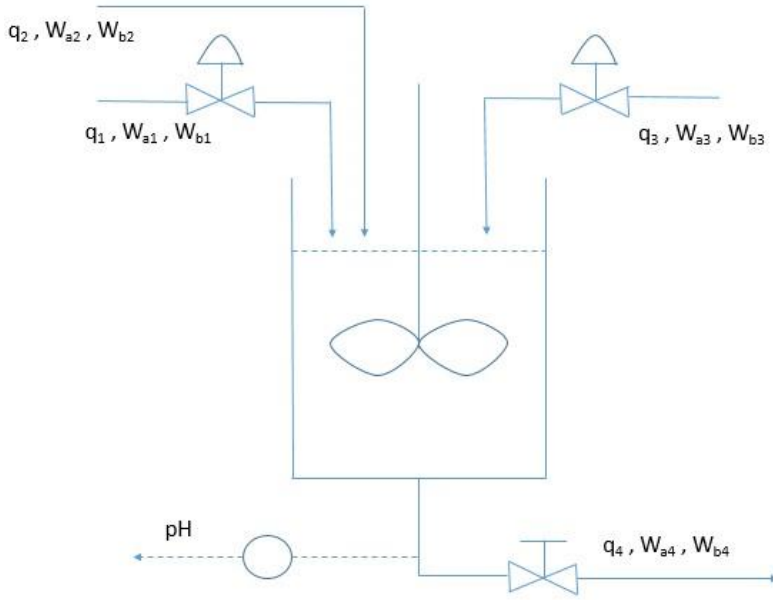
$$W_b = [H_2CO_3] + [HCO_3^-] + [CO_3^{2-}] \quad (8)$$

The three non-linear model equations (Hall [28]) are,

$$\frac{dh}{dt} = (q_1 + q_2 + q_3 - C_v h^{0.5}) \quad (9)$$

$$\frac{dW_{a4}}{dt} = [(W_{a1} - W_{a4})q_1 + (W_{a2} - W_{a4})q_2 + (W_{a3} - W_{a4})q_3] \quad (10)$$

$$\frac{dW_{b4}}{dt} = [(W_{b1} - W_{b4})q_1 + (W_{b2} - W_{b4})q_2 + (W_{b3} - W_{b4})q_3] \quad (11)$$



**(Figure 5: Neutralization system)**

Variable	Nominal value
$q_1$ composition	0.003M HNO <sub>3</sub>
$q_2$ composition	0.03M NaHCO <sub>3</sub>
$q_3$ composition	0.003M NaOH + 0.00005M NaHCO <sub>3</sub>
$A$	207 cm <sup>2</sup>
$C_v$	8.75 ml cm <sup>-1</sup> s <sup>-1</sup>
$pK_1$	6.35
$pK_2$	10.25
$W_{a1}$	0.003M
$W_{a2}$	-0.03M
$W_{a3}$	-0.00305M
$W_{a4}$	-0.000432M
$W_{b1}$	0
$W_{b2}$	0.03M
$W_{b3}$	0.00005M
$W_{b4}$	0.000528M
$q_1$	16.6 ml s <sup>-1</sup>
$q_2$	0.55 ml s <sup>-1</sup>
$q_3$	15.6 ml s <sup>-1</sup>
$h$	14.0 cm
$pH_4$	7.0

**(Table 2:** Nominal parametric values for neutralization system)

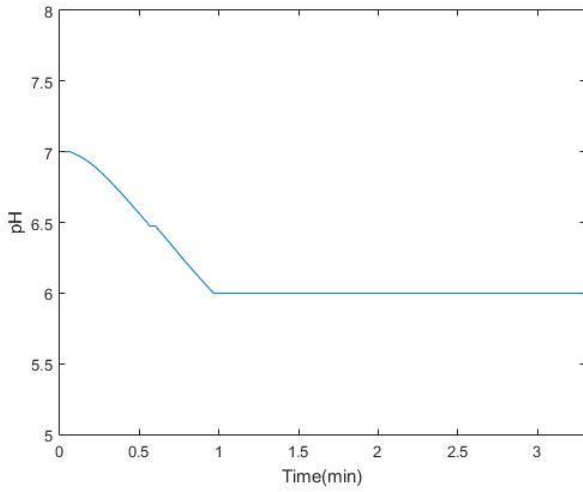
Training data for SVR was obtained from the values of base flow rate and corresponding pH in the range (12.6 – 16.63) and (5.98 – 8.32) respectively. The ODEs were simulated with a time span of 2s. The level of liquid in the tank was maintained by manipulating the acid flow rate in such a way that the sum of the flow rates of the acid, buffer and base streams was always constant. The future values of control variable were predicted by SVR as:

$$y(t + k) = f[ u(t + k - 3), u(t + k - 2), u(t + k - 1), y(t + k - 3), y(t + k - 2) ] \quad (12)$$

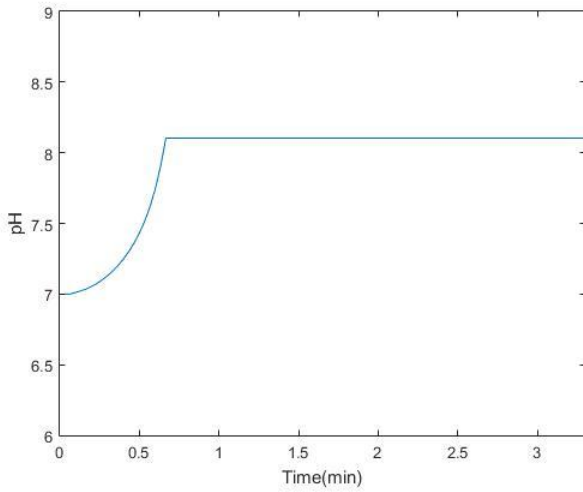
The best SVR model was given by the parameters:  $\epsilon = 0.12$ ,  $C = 500$ , Kernel type = Linear and Kernel parameters = 30.

Set-point changes of 6 and 8 were given to the system. In addition to this, disturbance changes were introduced in the system by changing value of buffer flow rate,  $q_2$  to 0.001 from its nominal value.





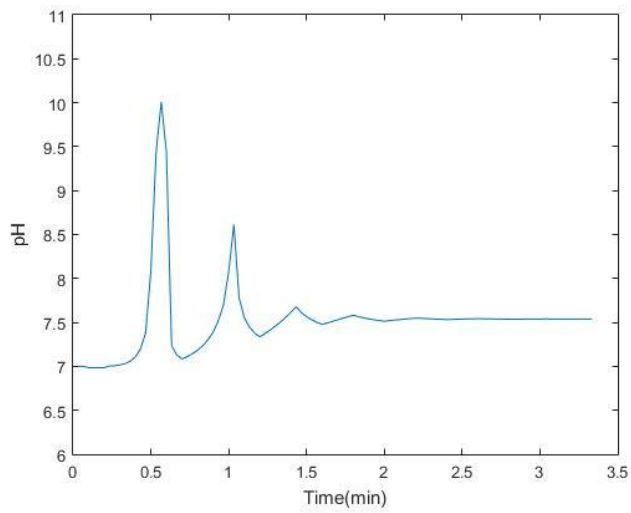
(a)



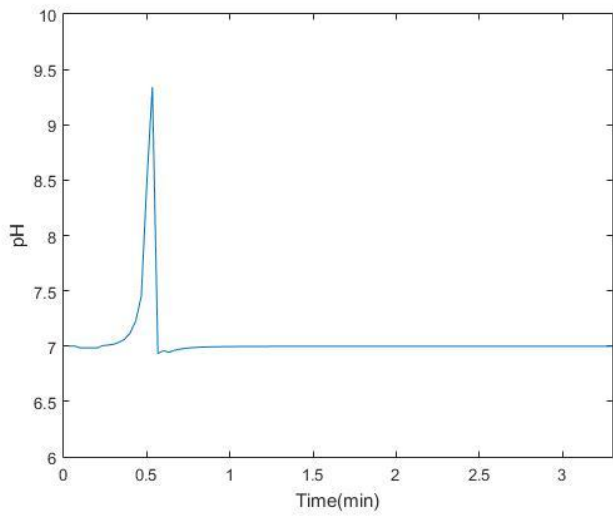
(b)

**(Figure 6: Response of MPC to step changes in set point of (a) 6 and (b) 8.)**

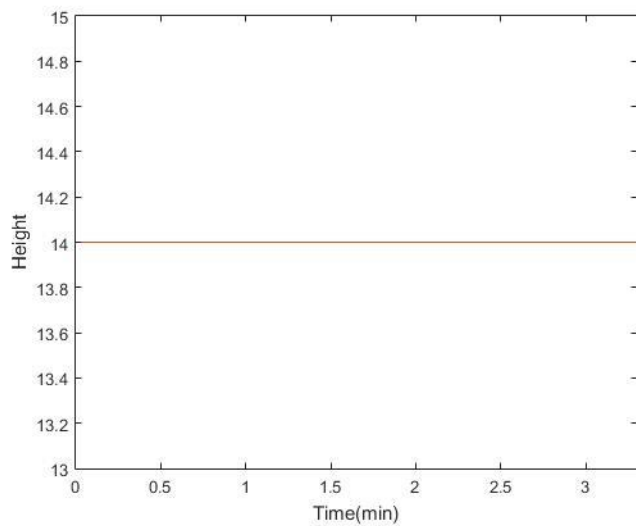
The MPC is able to track set point changes extremely fast (Fig. 6). When subjected to disturbances, the performance of the controller was calculated for two cases, one with SVR (Fig. 7) as the process model and the other with online SVR (Fig. 8). It can be seen from Fig. 7 & 8 that online SVR allows MPC to achieve a far better disturbance rejection than SVR. MPC with online SVR stabilizes at a pH of 7 with almost no fluctuations, while MPC with SVR stabilizes at a pH of 7.5 with quite many fluctuations in the pH value and thus takes a larger time to stabilize as well. Fig. 9 shows the height of the liquid in the tank, which was observed to be the same for both set point tracking and disturbances rejection.



**(Figure 7: Response of SVR to disturbances)**



**(Figure 8: Response of online SVR to disturbances)**



**(Figure 9: Height of the liquid in the tank with time for changes in set point as well as disturbances.)**

## CONCLUSION

This paper presents a method for robust control of a non-linear system using MPC in the presence of disturbances. The process models were developed using online SVR and DE was used to optimize a sequence of the manipulated variable in future space. Simulation results for two nonlinear systems show excellent disturbances rejection.

## SYMBOLS USED

### Symbols

$A$	$[h^{-1}]$	area
$C_v$	$[ml\ cm^{-1}\ s^{-1}]$	valve coefficient
$D$	$[h^{-1}]$	dilution rate
$h$	$[cm]$	liquid level in the neutralization process
$K_i$	$[gl^{-1}]$	substrate inhibition constant
$K_m$	$[gl^{-1}]$	substrate saturation constant
$P$	$[gl^{-1}]$	product concentration
$pH_4$		effluent pH from the neutralization tank
$pK$		log of equilibrium constant
$P_m$	$[gl^{-1}]$	product saturation constant
$S$	$[gl^{-1}]$	substrate concentration in the outlet stream
$S_f$	$[gl^{-1}]$	substrate concentration in the feed
$u$		manipulated variables
$W_a$	$[M]$	reaction invariant a
$W_b$	$[M]$	reaction invariant b
$X$	$[gl^{-1}]$	cell concentration
$y$		controlled outputs
$Y_{x/s}$	$[gg^{-1}]$	cell-mass yield

### Greek-symbols

$\alpha$	$[gg^{-1}]$	kinetic parameter in the fermenter
$\beta$	$[h^{-1}]$	kinetic parameter in the fermenter
$\mu$	$[h^{-1}]$	growth rate
$\mu_m$	$[h^{-1}]$	maximum growth rate

## Abbreviations

SVR	Support vector machine for regression
DE	Differential Evolution
MPC	Model predictive controller

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## TABLES

**Tab. 1:** Nominal parametric values for continuous fermenter

**Tab. 2:** Nominal parametric values for neutralization system

## FIGURES

**Fig. 1:** MPC strategy

**Fig. 2:** Continuous fermenter

**Fig. 3:** SP changes of 5 and 7

**Fig. 4:** Disturbance changes in fermenter

**Fig. 5:** Neutralization system

**Fig. 6:** Response of MPC to step changes in set point of (a) 6 and (b) 8.

**Fig. 7:** Response of SVR to disturbances

**Fig. 8:** Response of online SVR to disturbances

**Fig. 9:** Height of the liquid in the tank with time for changes in set point as well as disturbances.