Introduction to Machine Learning for Social Scientists

Class 5: Intro to GLM

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Summer 2018

Homework 3 Due Friday July 20th at midnight

Available tomorrow Start early!

In-class midterm Monday July 23rd

Final project

We'll talk more about this after midterm but you should start forming teams: 6 teams of 5 in total

All teams should be organized by Wed July 25th

Issues with the Room!

Next Monday old room, this one during the rest of the quarter

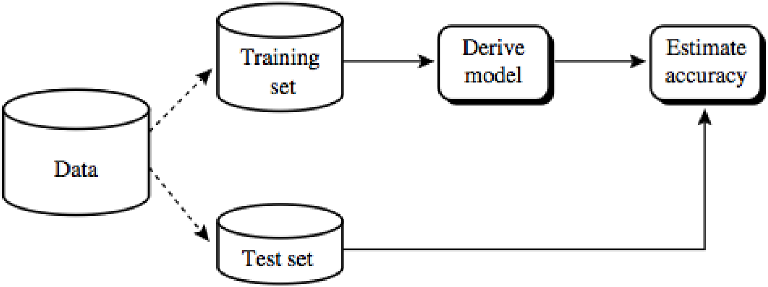
Today's Goals

1. Key concepts
    - Training and test sets
    - Linear Probability Model
    - Logit function and logit inverse function, logistic regression
2. Key techniques and R functions
    - glm
    - Natural logarithm, log
    - table

Divide your data

- **Training set:** A set of examples used to fit the parameters and learn. These are already classified by human coders, or produced in a semi-automated way, to create a 'gold standard'.
  Conventionally 80% of available data.
- **Test set:** A set that follows the same probability distribution and is used to test de model.
  Conventionally 20% of available data.

Divide your data



R!

Regression vs Classification

1. *Regression*
   - Quantitative responses
   - Example: Age, height, salary, price, vote share, etc.

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2. *Classification*
   - Qualitative responses
   - Example: Election result (win, lose), fake news (yes, no, maybe).

NOTE: The distinction is not always that clear-cut: Logistic regression (a type of non-linear regression) is often used for classification.

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For simplicity, in this class we are going to classify binary outcomes.

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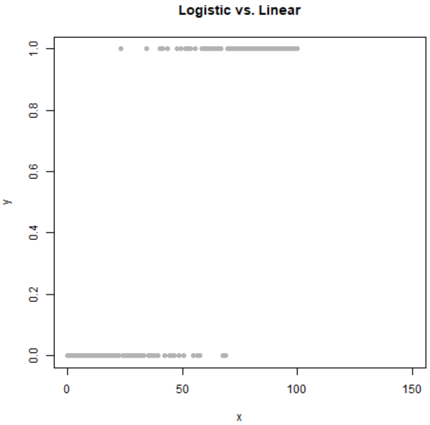
$$y \in \{0, 1\} \qquad \begin{bmatrix} 0: & \text{"Negative class"} \\ 1: & \text{"Positive class"} \end{bmatrix}$$

Predicting qualitative responses



- $y \in \{0, 1\}$, X continuous

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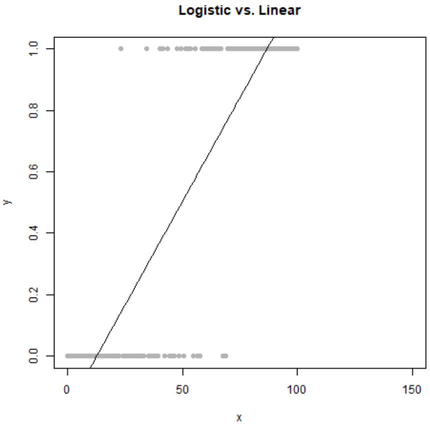

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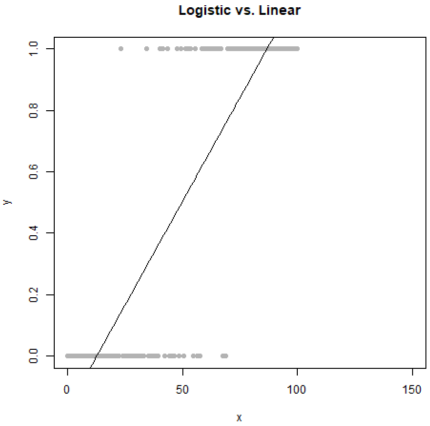

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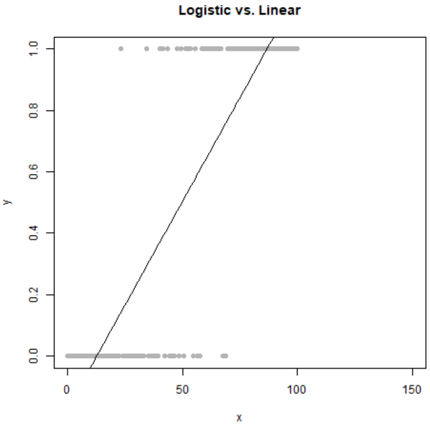

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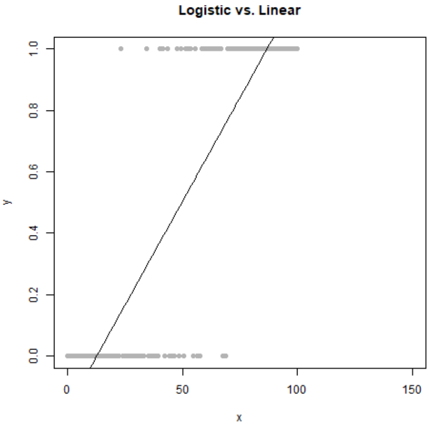

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- PROBLEMS?
- Predictions smaller than zero and larger than 1!

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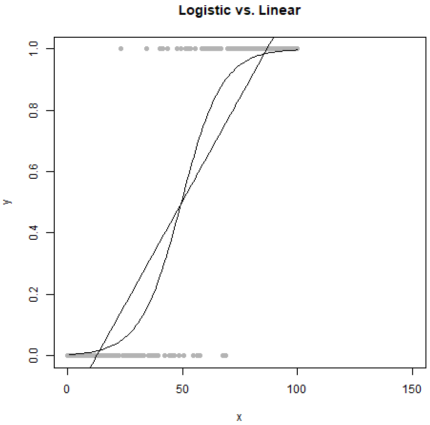
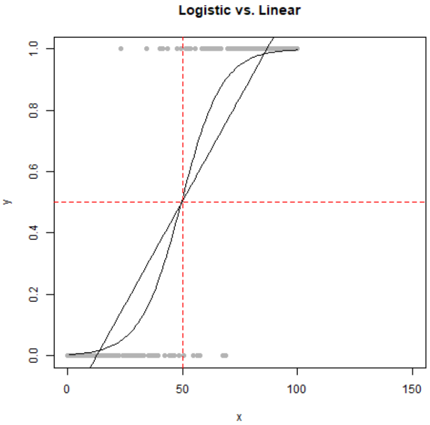


- Logistic model
- Estimates the probability of yes: $Pr(\hat{Vote_i} = 1|x_i)$ using a logarithmic transformation

Predicting qualitative responses



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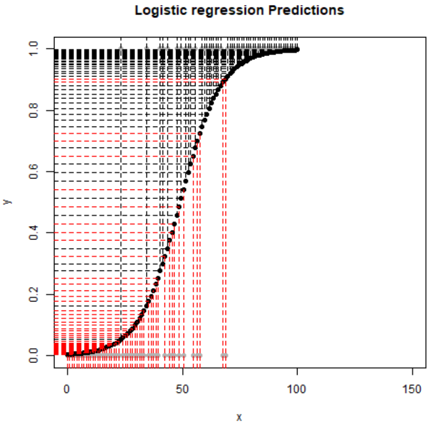

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- We can associate each point to a probability

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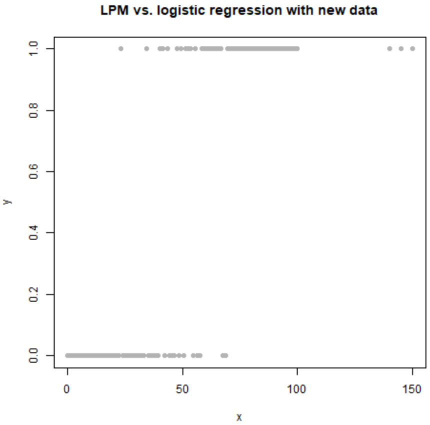

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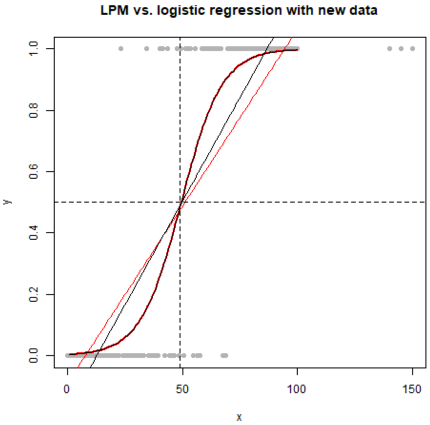

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\text{odds}(p) &= \frac{p}{1-p} \\
\text{log odds or logit}(p) &= \log\left(\frac{p}{1-p}\right) \\
\text{logistic function or logit}^{-1}(a) &= \frac{1}{1+\exp(-a)}
\end{aligned}$$

More on tutorial

R!

Some Context: Iraq Vote

- In 2002 President George Bush announced the Joint Resolution to Authorize the Use of United States Armed Forces Against Iraq.
- Congressional opposition.

NEXT

- Assessing model predictions for classification
  - Precision
  - Accuracy
  - Recall