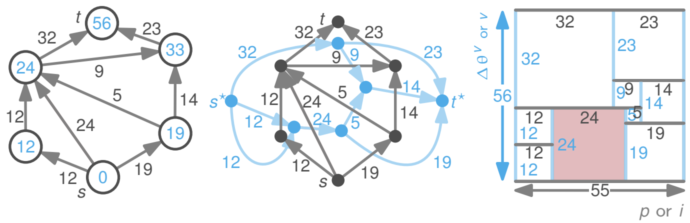
Problem Statements

Feasible electrical flows



Dynamic models

Static models

Alternating Current (AC) model

Direct Current (DC) model

Placement of control units

Discrete control decision

[Grastien, Rutter, Wagner, W., and Wolf, 2018]

Continuous control decision

[Leibfried, Mchedlidze, Meyer-Hübner, Nöllenburg, Rutter, Sanders, Wagner, and W., 2015]

[Mchedlidze, Nöllenburg, Rutter, Wagner, and W., 2015]

Cable layout

Wind farm cabling with multiple cable types

[Lehmann, Rutter, Wagner, and W., 2017]

Formulations

Vertex-based Formulation		Vertex-/Cycle-based (Kirchhoff's) Formulation
$f_{\text{net}}(u) = \sum_{\{u,v\} \in E} f(u,v) = 0$ $-\infty \leq f_{\text{net}}(u) \leq -d$ $0 \leq f_{\text{net}}(u) \leq \infty$	**KCL**	$\mathbf{I}\vec{f} = \vec{0}$
$\theta^v(v) - \theta^v(u) = f(u,v)$	**KVL**	$\mathbf{B}\overrightarrow{\Delta\theta^v} = \vec{0}$
$\lvert f(u,v) \rvert \leq \text{cap}(u,v)$		$\vec{f} \leq \overrightarrow{\text{cap}}$

I – incidence matrix
B – circuite matrix

Structure of the Incidence and Circuit Matrix

7

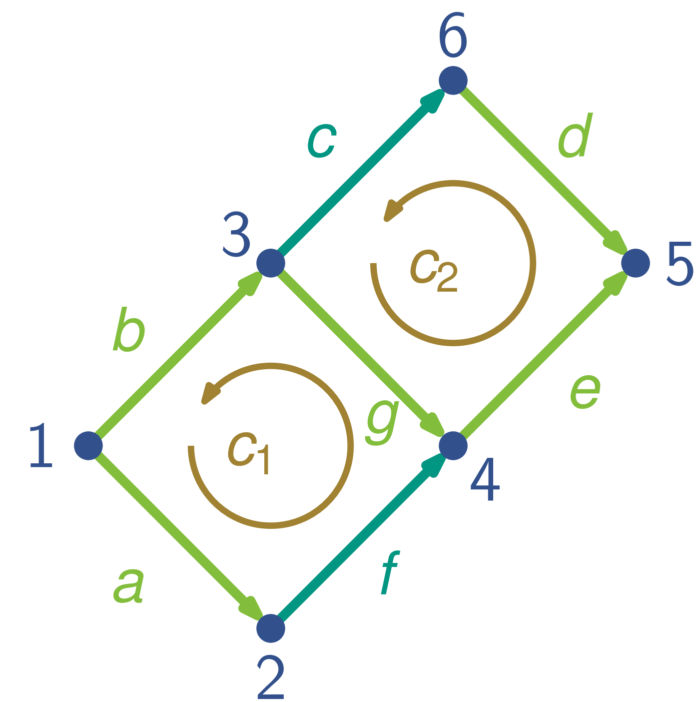
	5					2	
	Spanning Tree					Chords	
	g	e	d	b	a	f	c
3	−1	0	0	1	0	0	−1
4	1	−1	0	0	0	1	0
5	0	1	1	0	0	0	0
6	0	0	−1	0	0	0	1
1	0	0	0	−1	−1	0	0
2	0	0	0	0	1	−1	0
c_1	−1	0	0	−1	1	1	0
c_2	1	1	−1	0	0	0	1

5

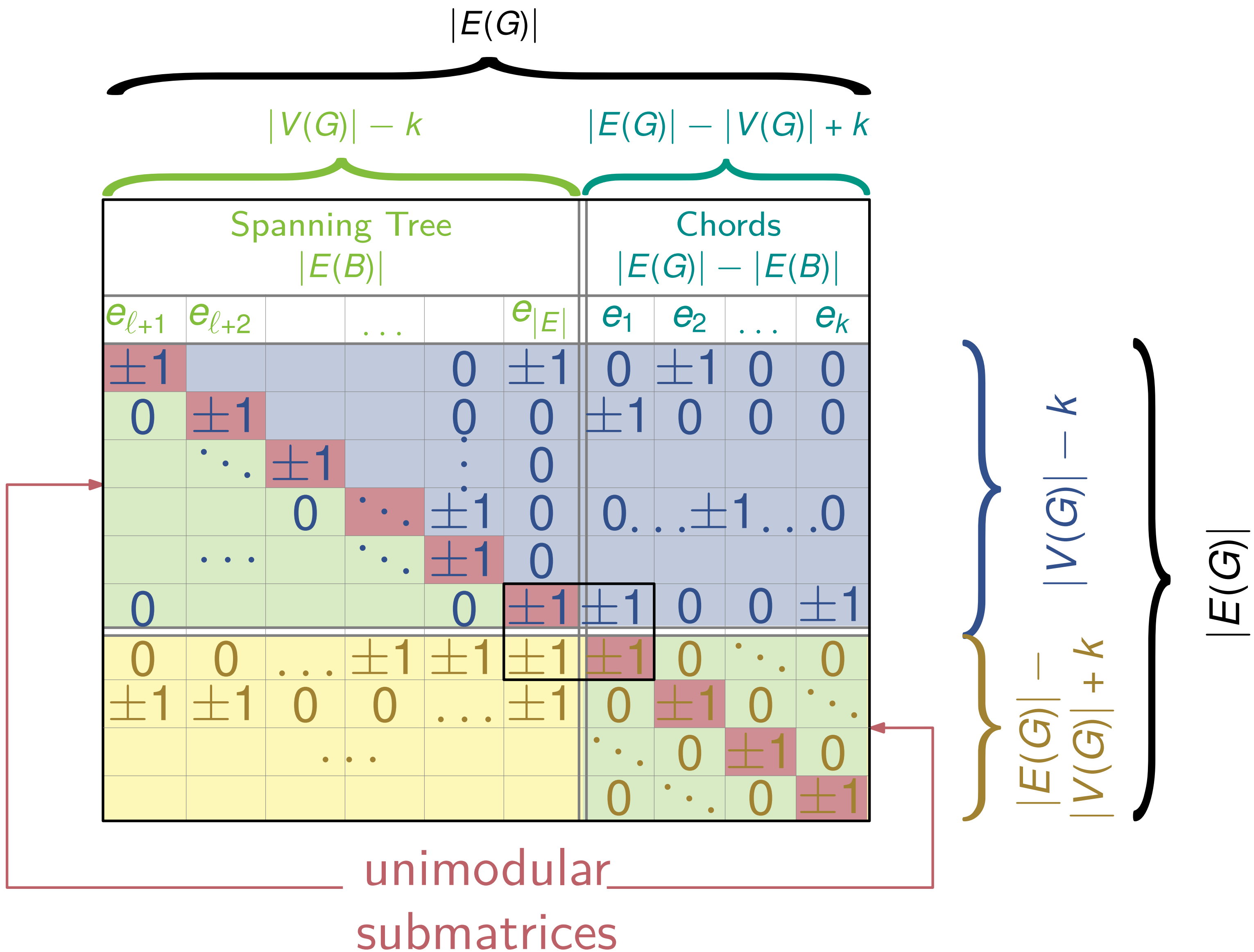
2

7

unimodular submatrices



General Structure



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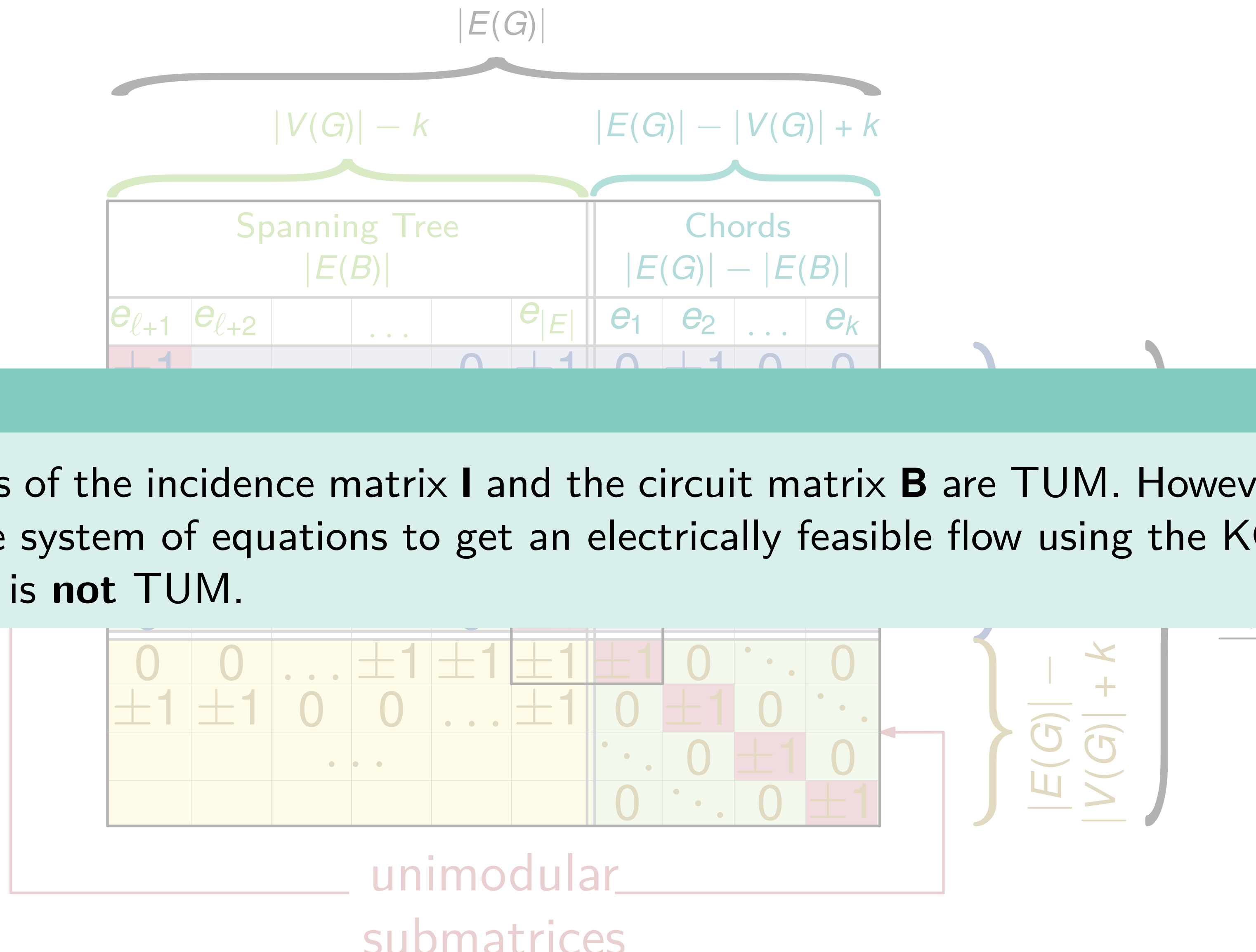
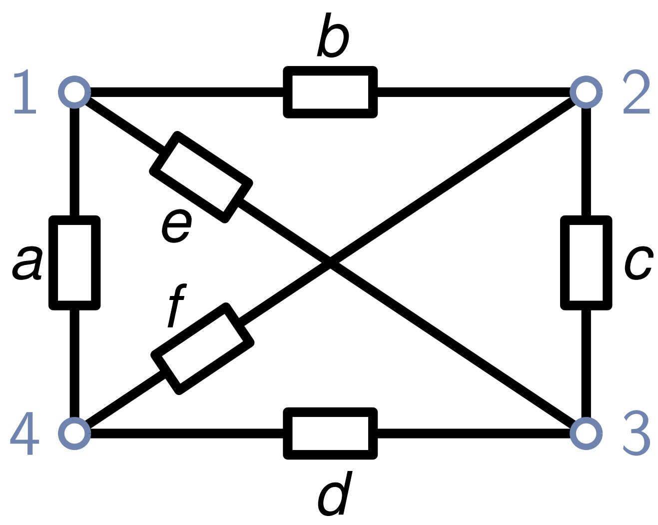


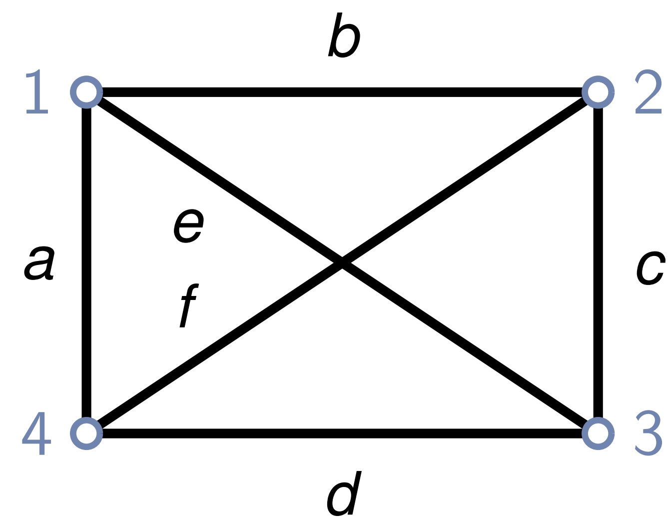
Lemma 1

The bases of the incidence matrix **I** and the circuit matrix **B** are TUM. However, the whole system of equations to get an electrically feasible flow using the KCL and KVL is **not** TUM.

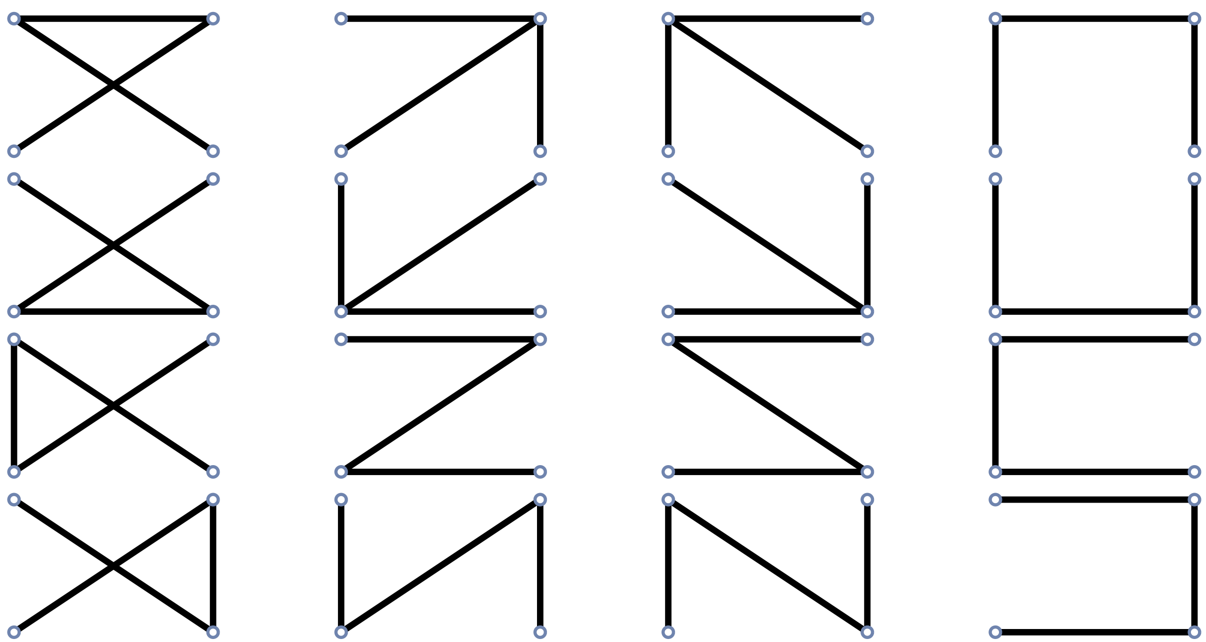
Circuits and Spanning Trees



$\Rightarrow$



All spanning trees T



An Ancient Algorithm for the Power Flow

- A first algorithm that represents a structural result

Lemma 2 [p.36, Lemma 1; Shapiro, 1987]

Let every edge of G have a resistor of 1 ohm. Let N denote the number of spanning trees and let $N(s, a \rightarrow b, t)$ be the spanning trees that contain the edge (a, b) in that particular direction. Put a 1-ampere current between s and t and let $i(a, b) = {}^{(N(s,a \rightarrow b,t) - N(s,b \rightarrow a,t))}/_{N}$. Then $i(a, b)$ is the current in the edge ab oriented from a to b.

- Apply Binet-Cauchy theorem on matrix $\mathbf{Y_n} = \mathbf{I}\ \mathbf{Y_e}\ \mathbf{I}^{\mathsf{T}}$
- $\triangle_n = \det(\mathbf{Y_n}) = \det(\mathbf{I}\ \mathbf{Y_n}\ \mathbf{I}^{\mathsf{T}}) = \sum_{T \in \mathcal{T}}$ (Tree-Admittance Product of T)
- Tree-Admittance Product $\sum_{(u,w) \in E(T)} \mathbf{Y_{u,w}}$

Primal and Dual Graphs

Theorem 3 [p.522, Theorem 23; Whitney, 1935]

Let $\mathcal{E}$ be a planar embedding of a primal graph G with $G(\mathcal{E})$ being isomorph to G. The graphs G and $G^\star$ are duals if and only if there is a bijection $\mu_{\text{dual}} \colon E(G) \to E(G^\star)$ between their edges such that bases in one correspond to base complements in the other.

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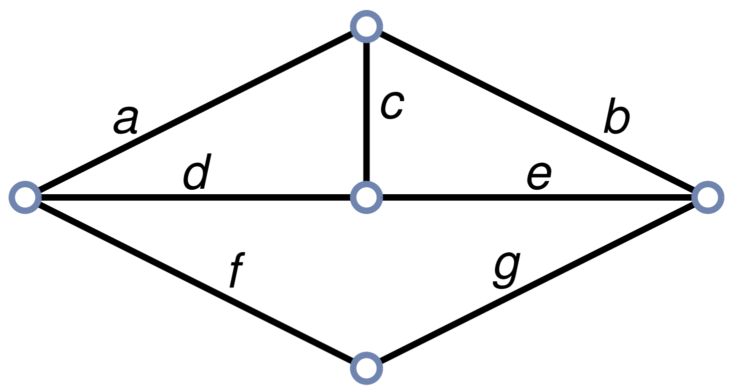

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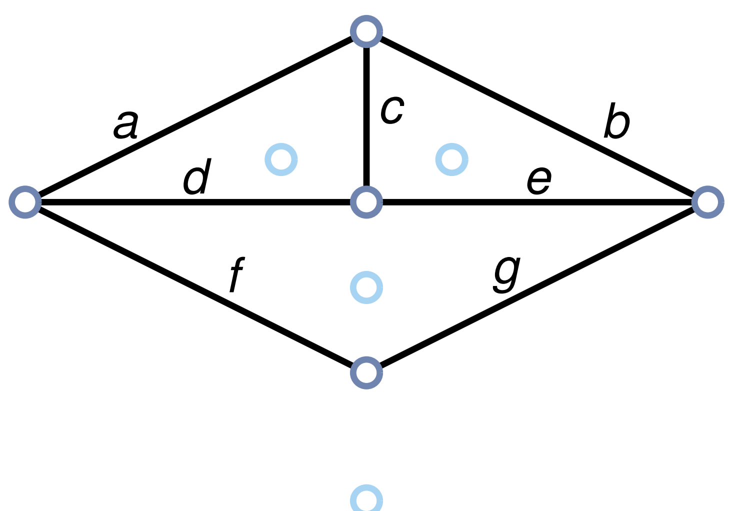

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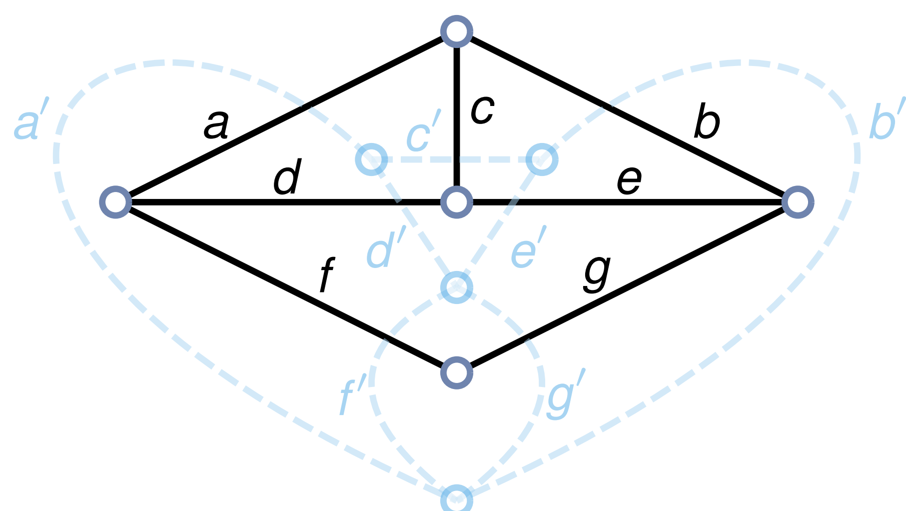

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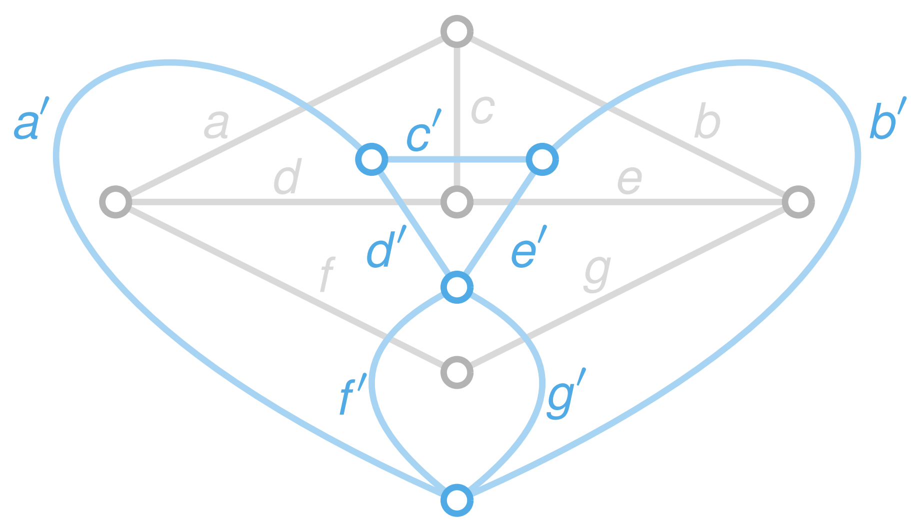

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Corollary 4 [p.85, Corollary 4-24; Seshu and Reed, 1961]

If G and $G^\star$ are dual graphs, the incidence matrix of either graphs is a circuit matrix of the other (with the proper rank, and each row representing a cycle); that is

$$\mathbf{I}_1 = \mathbf{B}_2 \text{ and } \mathbf{I}_2 = \mathbf{B}_1 .$$

Power Flow Problem Reformulation

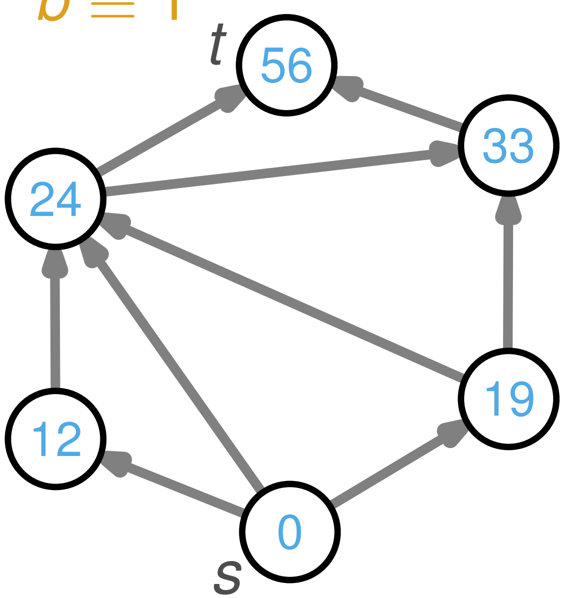
Planar s-t PF and MPF

Instance: A plane s-t-graph G, its dual graph $G^\star$ and a bijection $\mu_{\text{dual}}\colon E(G) \to E(G^\star)$.

Objective: Find feasible flows $f_G, f_{G^\star} : E \to \mathbb{R}_{\geq 0}$ in G and $G^\star$ such that for every edge $e \in E(G)$ we have

$$f_G(e) = f_{G^\star}\left(\mu_{\text{dual}}(e)\right) \cdot b(e).$$

$b \equiv 1$



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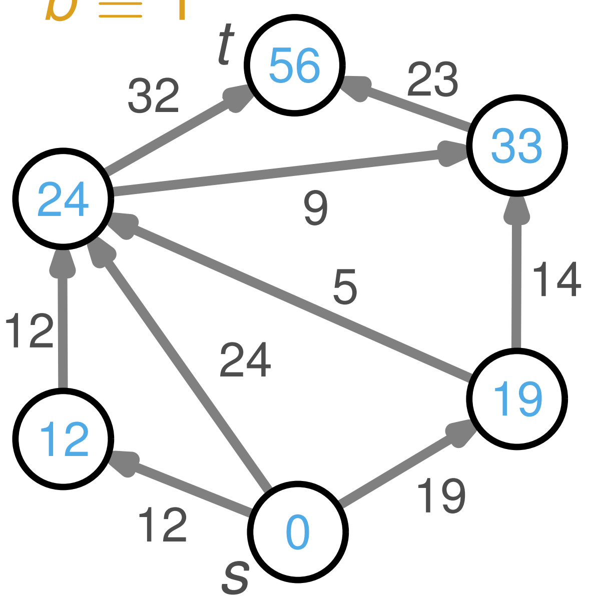

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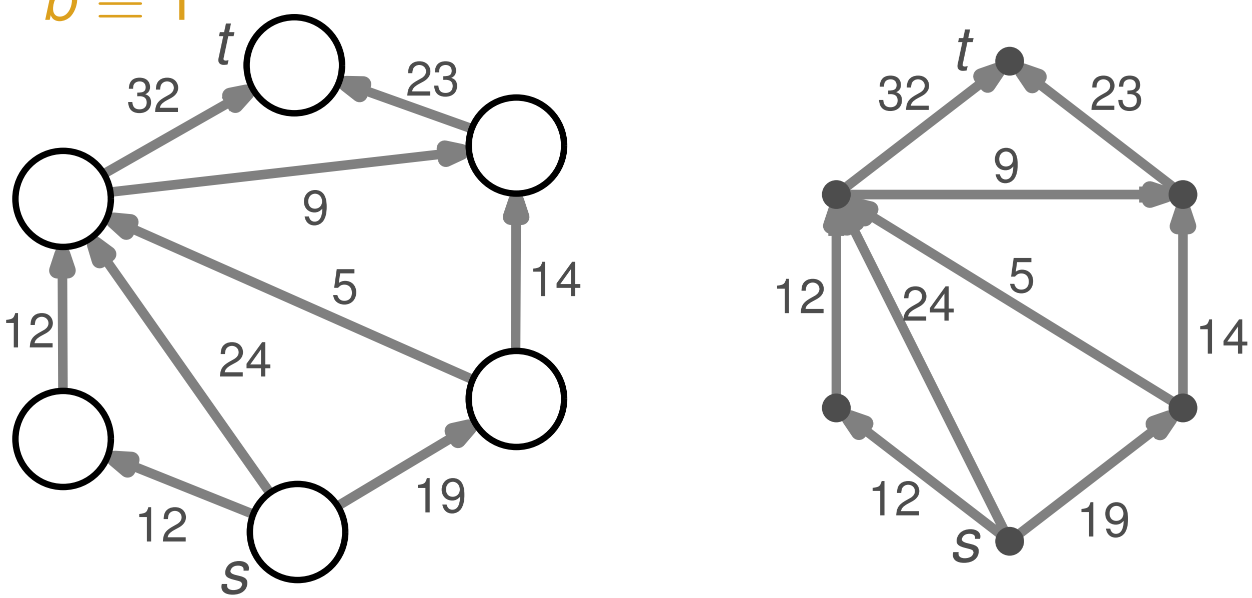

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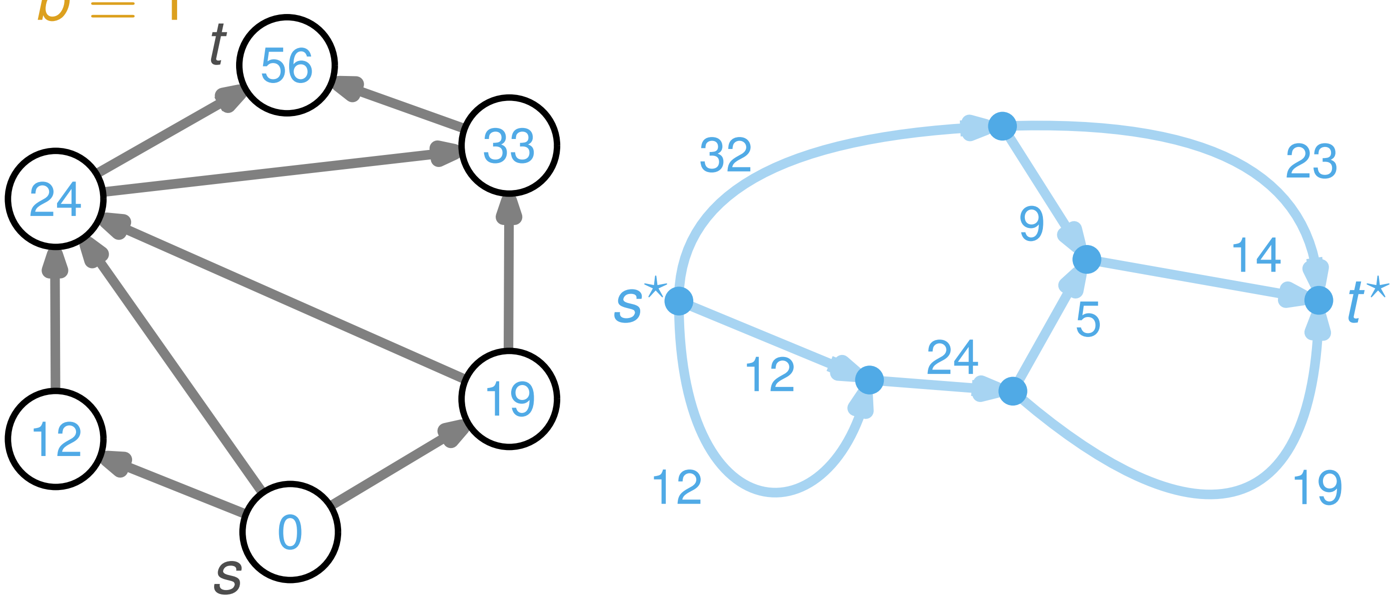

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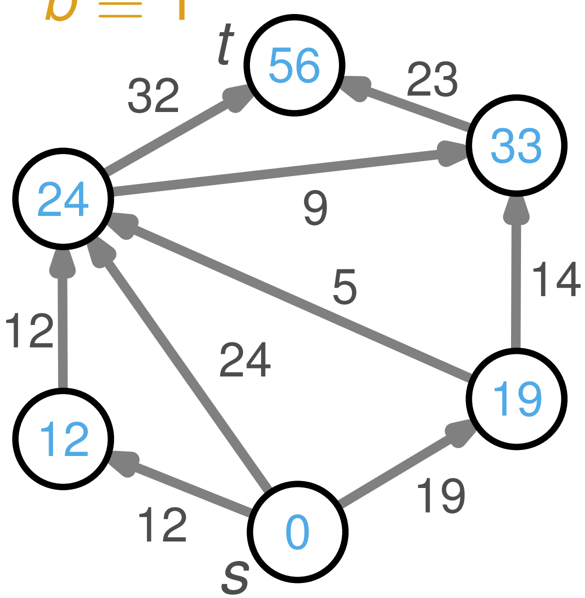

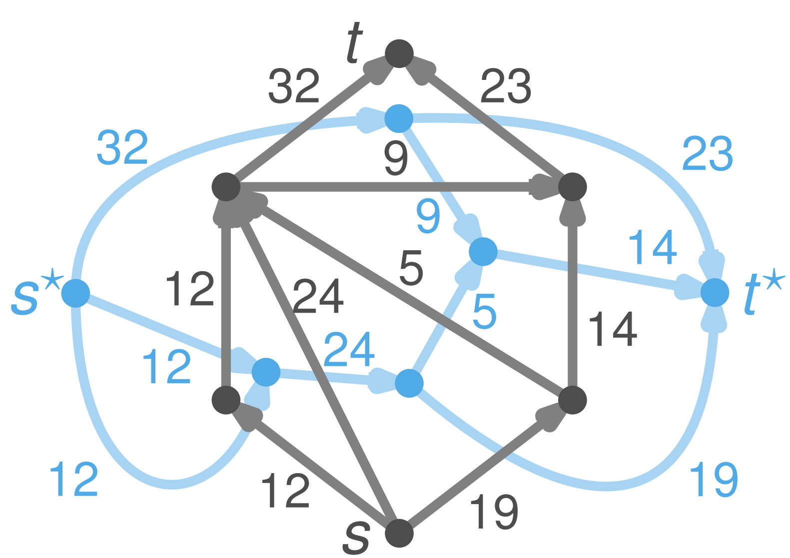

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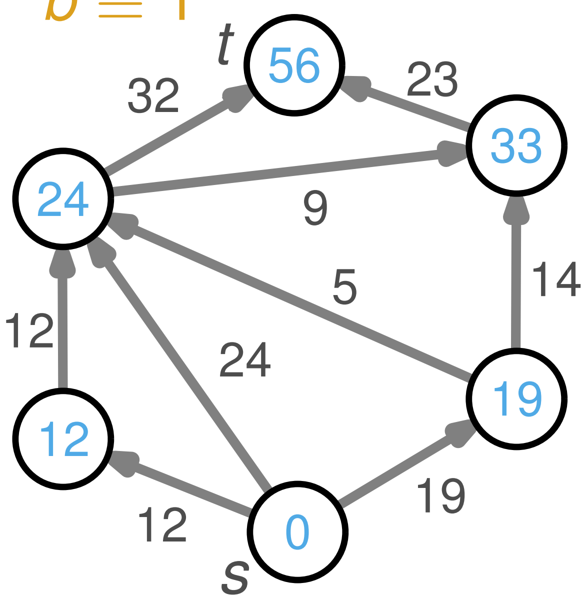

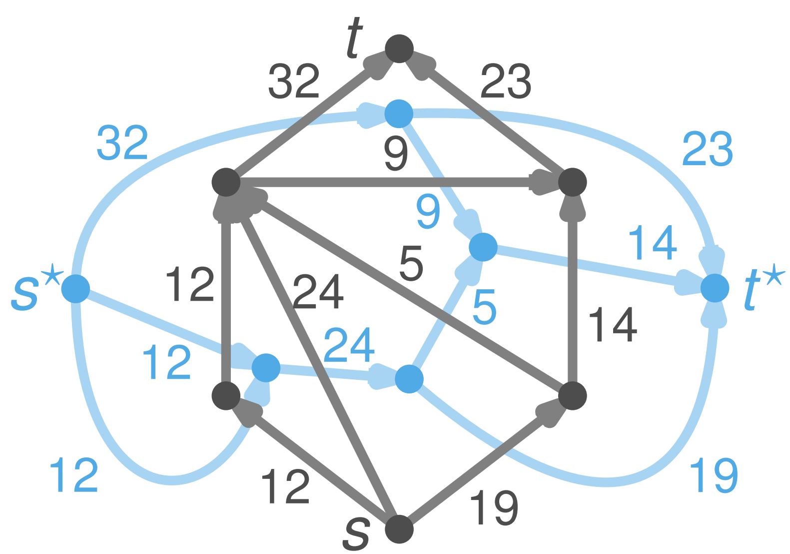

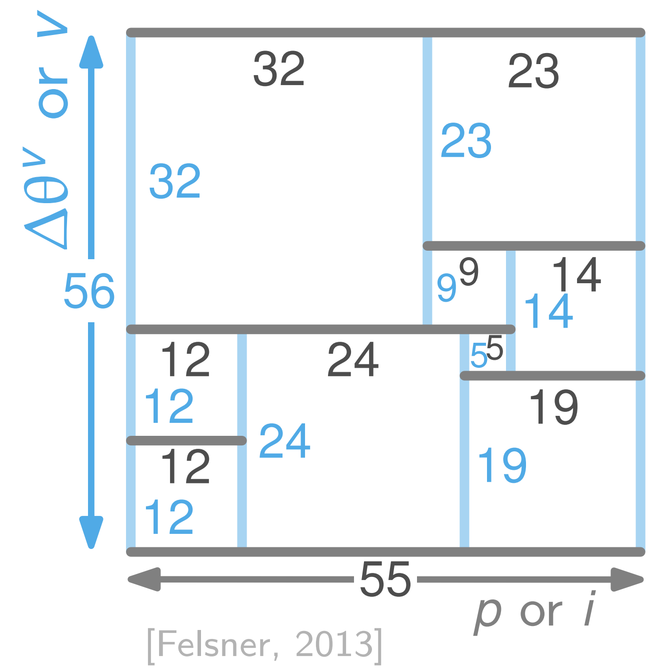

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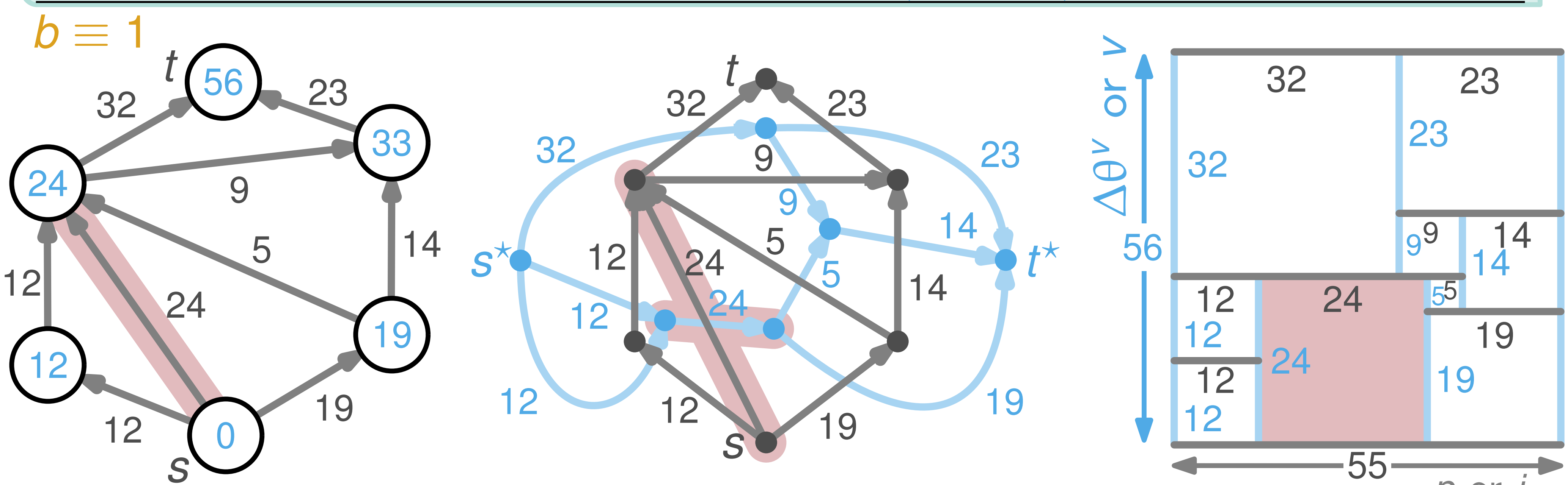

[Felsner, 2013]

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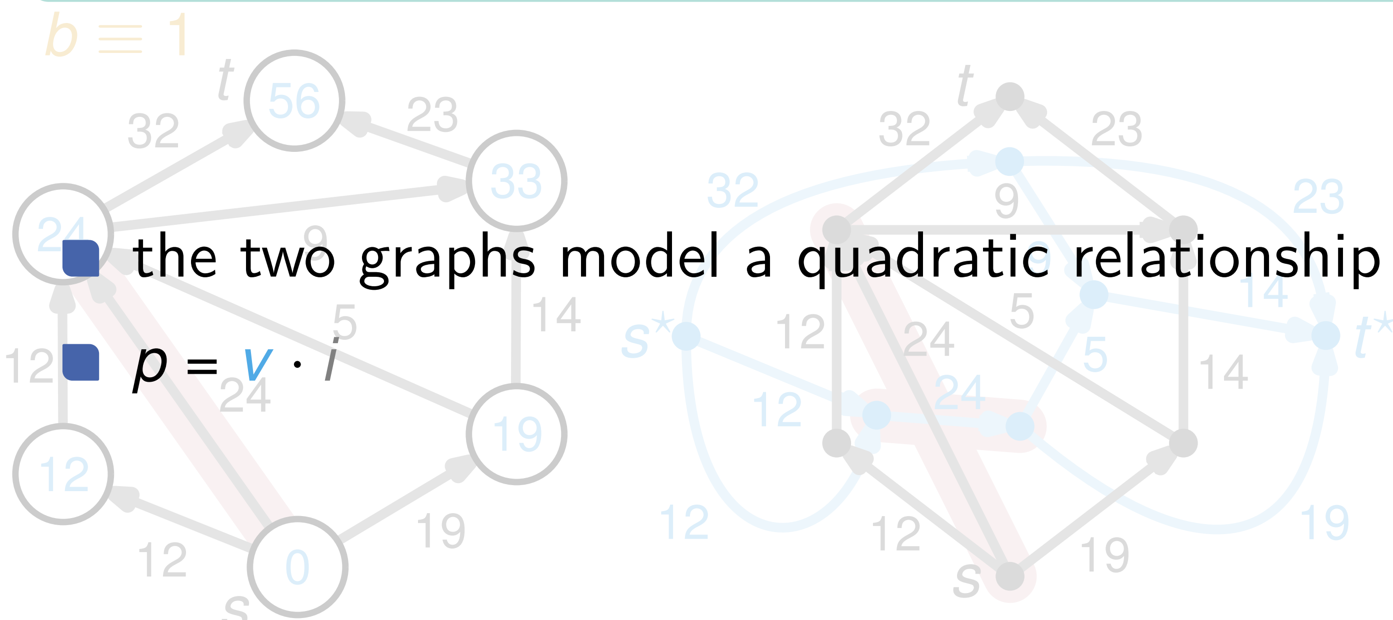
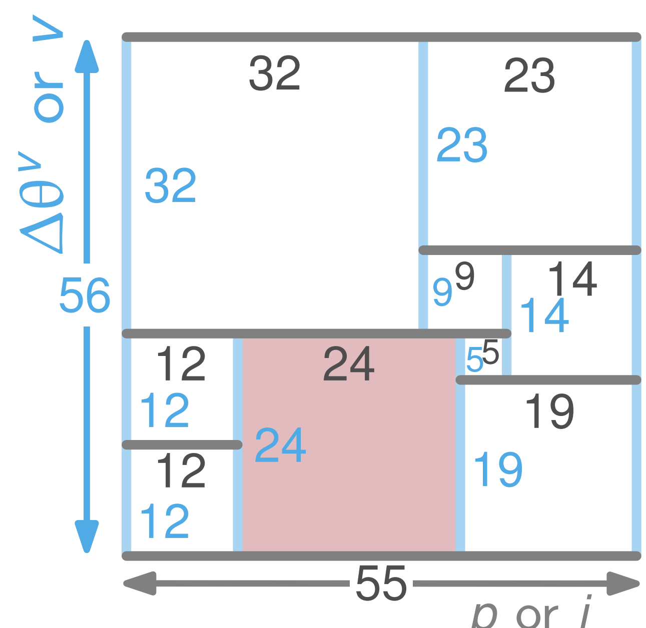
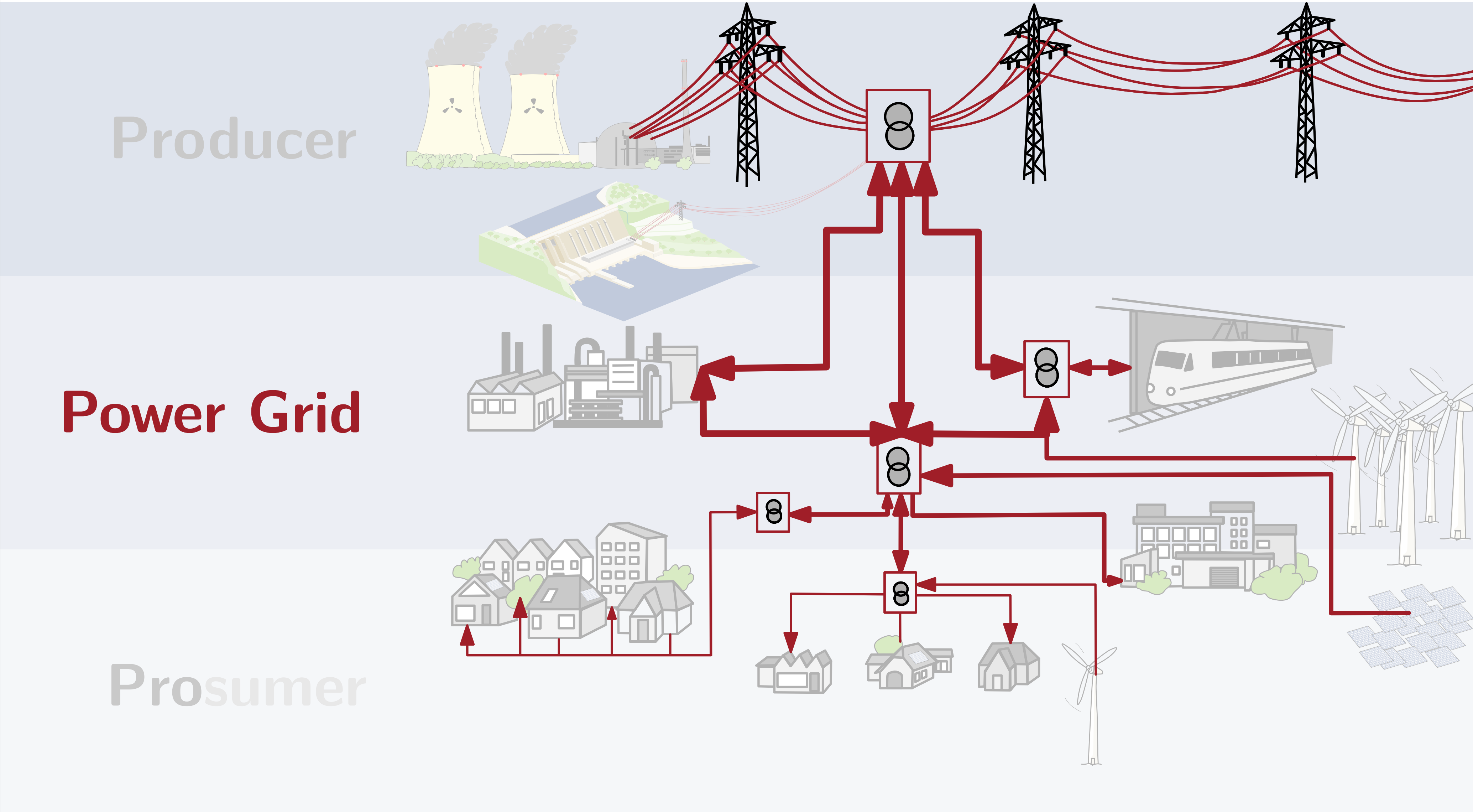


- the two graphs model a quadratic relationship
- $p = v \cdot i$

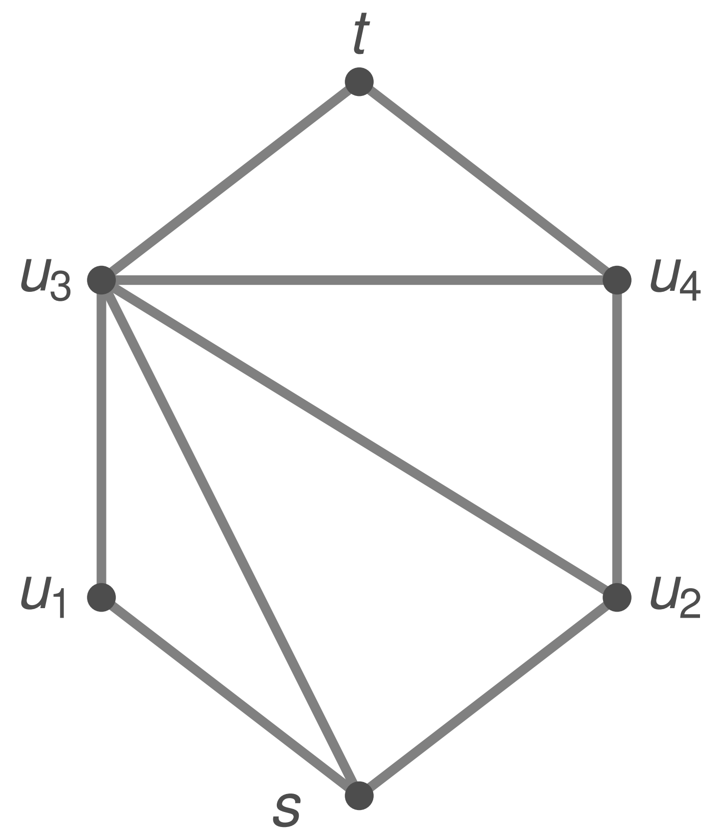


[Felsner, 2013]

Towards an Algorithm for the Power Flow

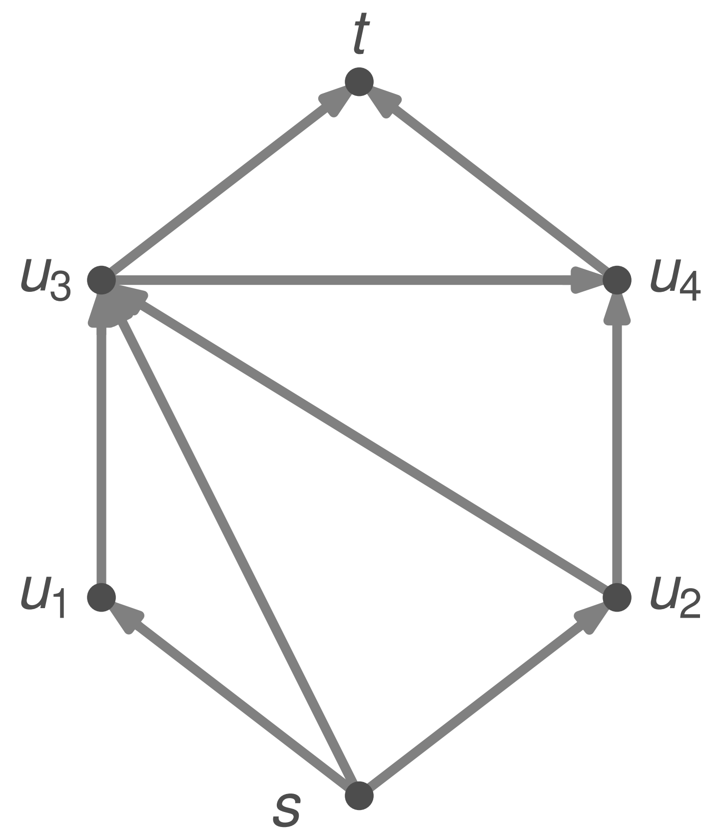


Algorithmic Sketch



Algorithmic Sketch

$G = \texttt{bipolarSubgraphOf}(G, s, t)$



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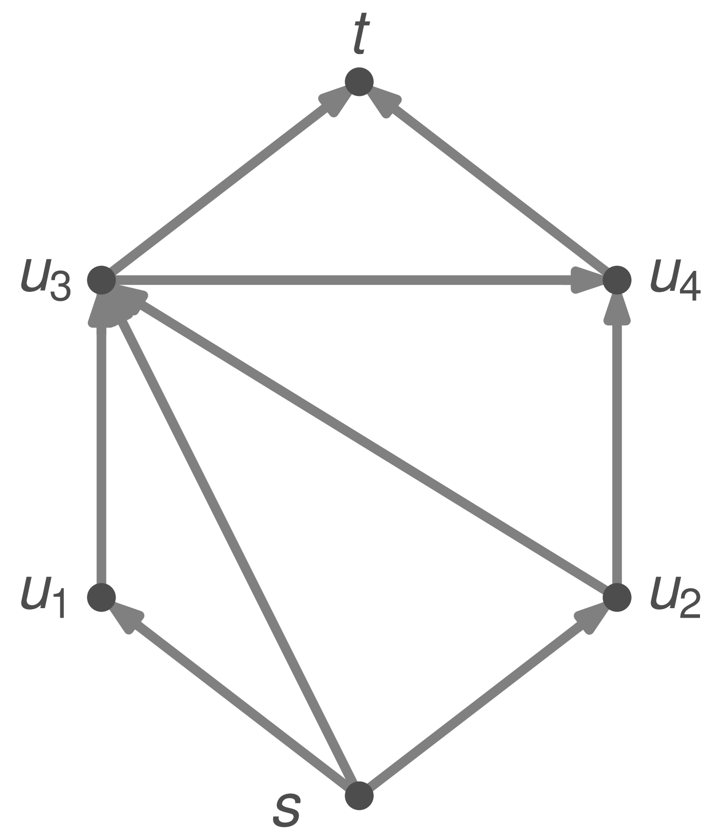

$\triangleright$ PQ-Tree; Invariant $G(\mathcal{E}) \cong G$

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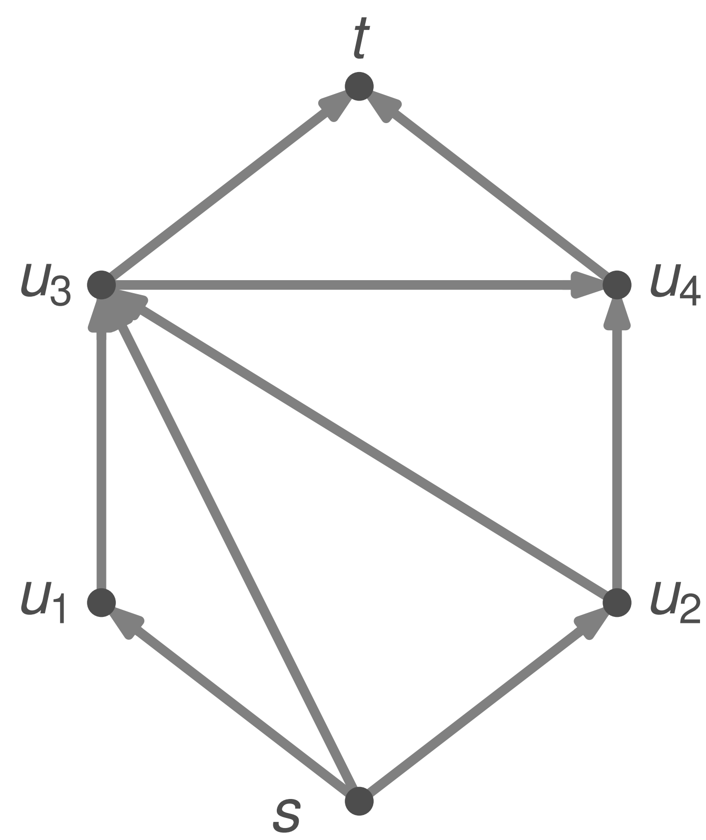

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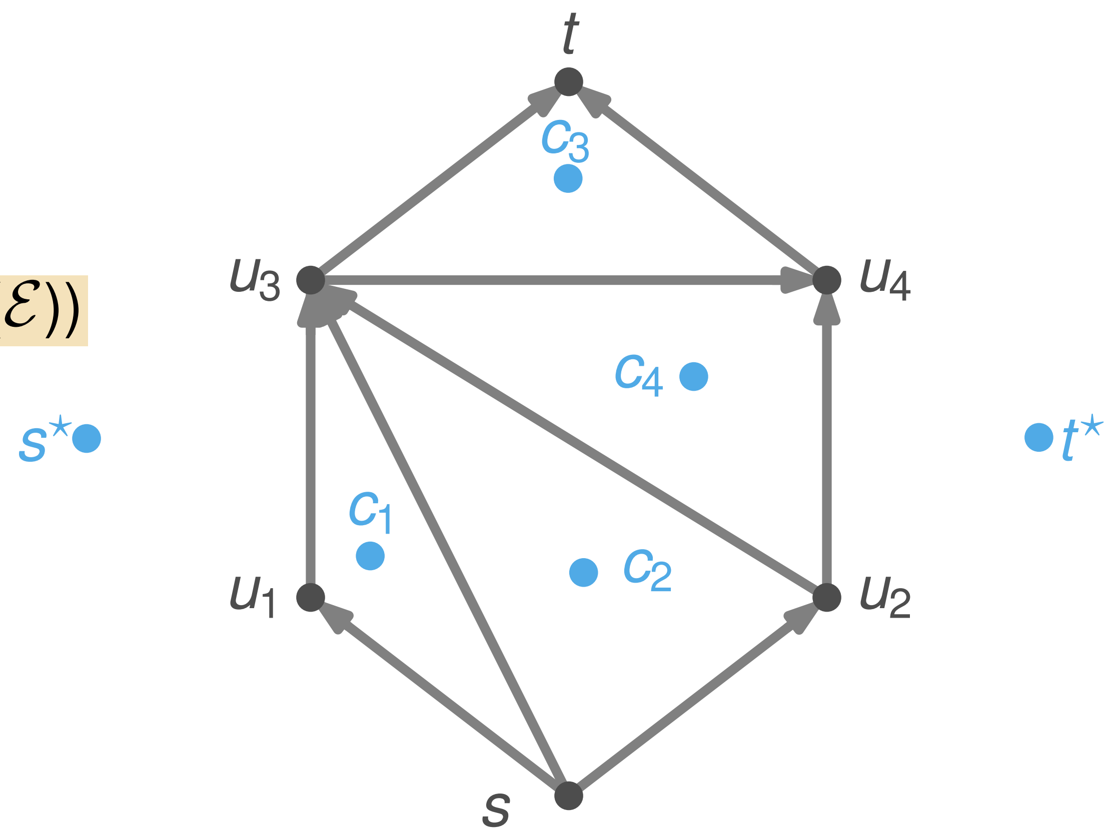

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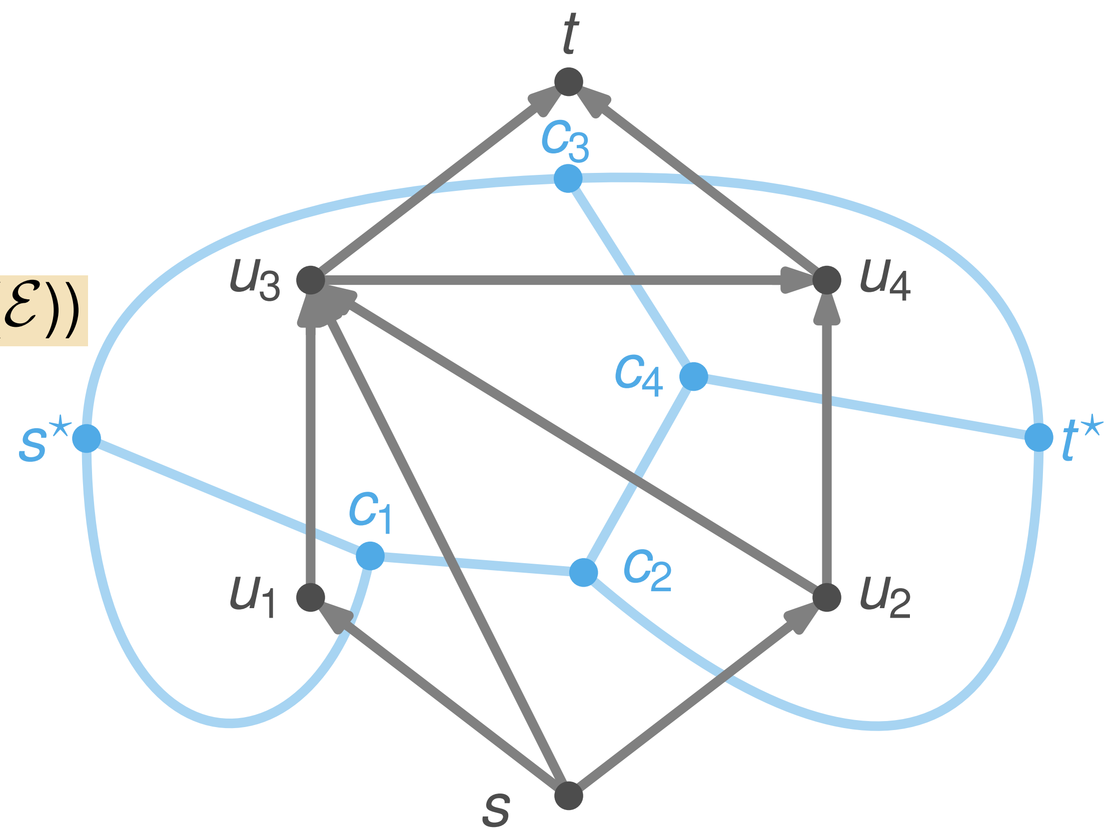

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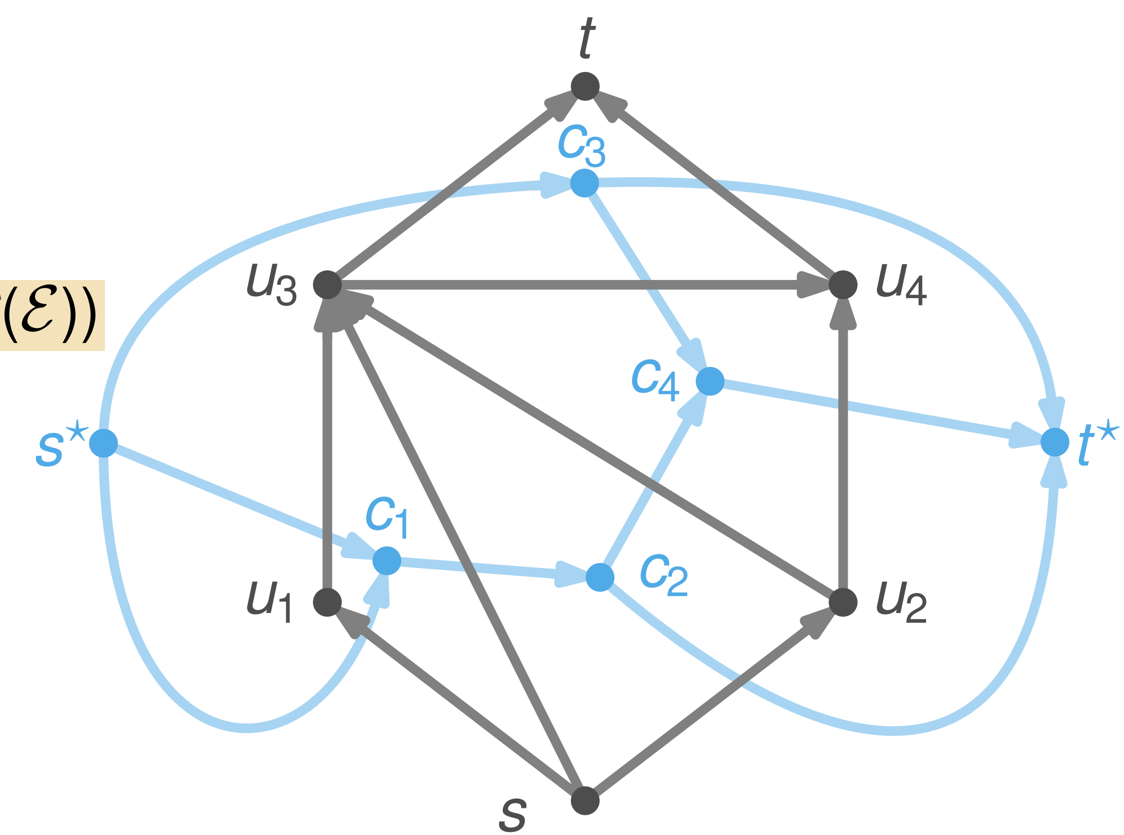

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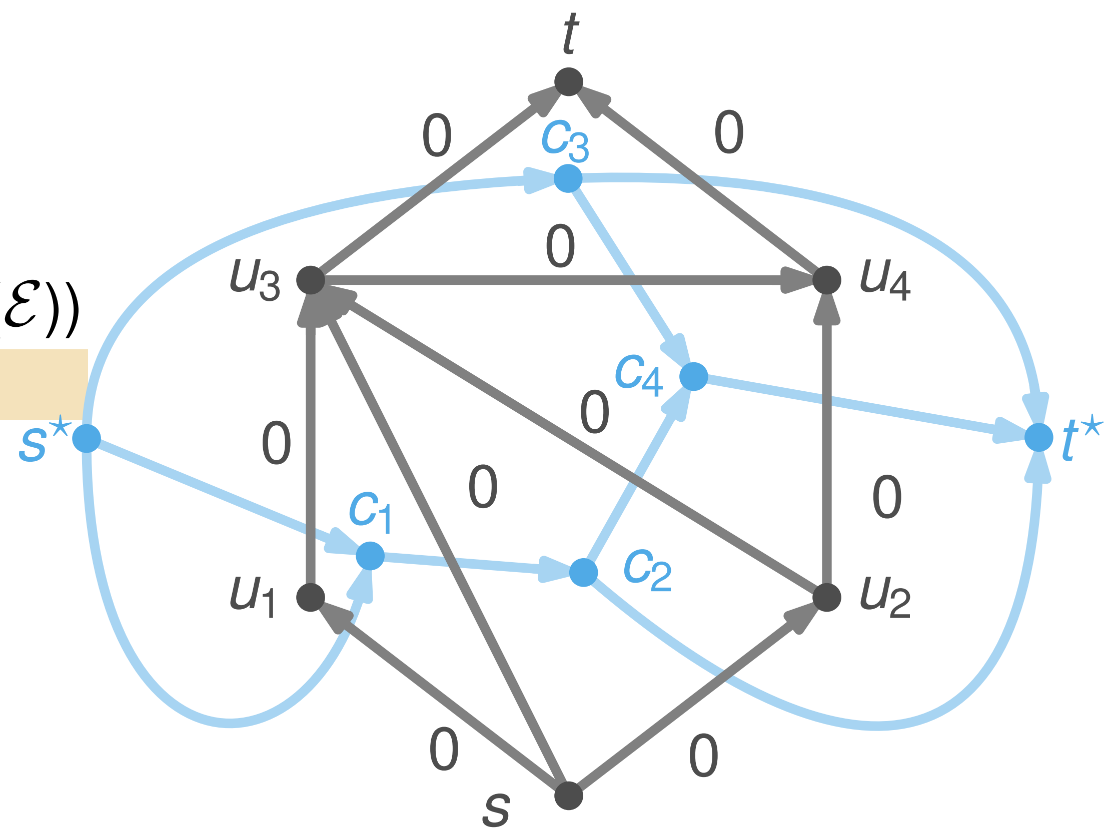

▷ Augment flow along an incident edge at source s

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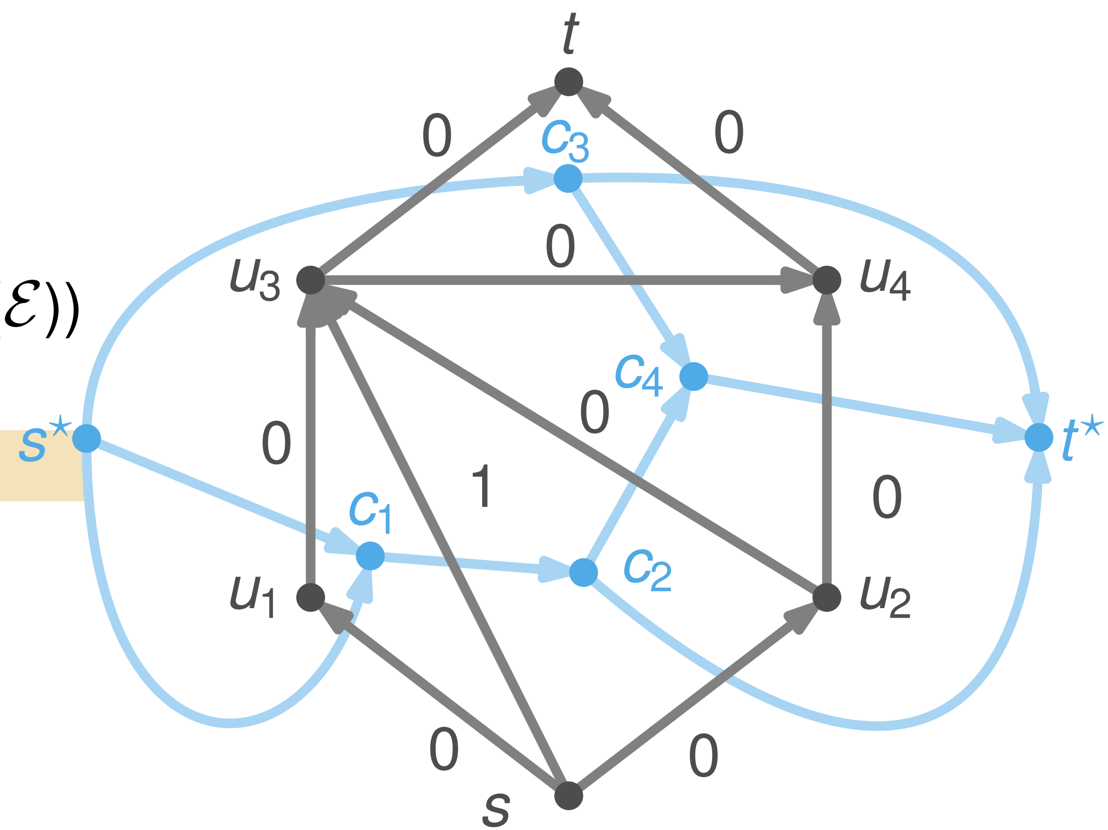

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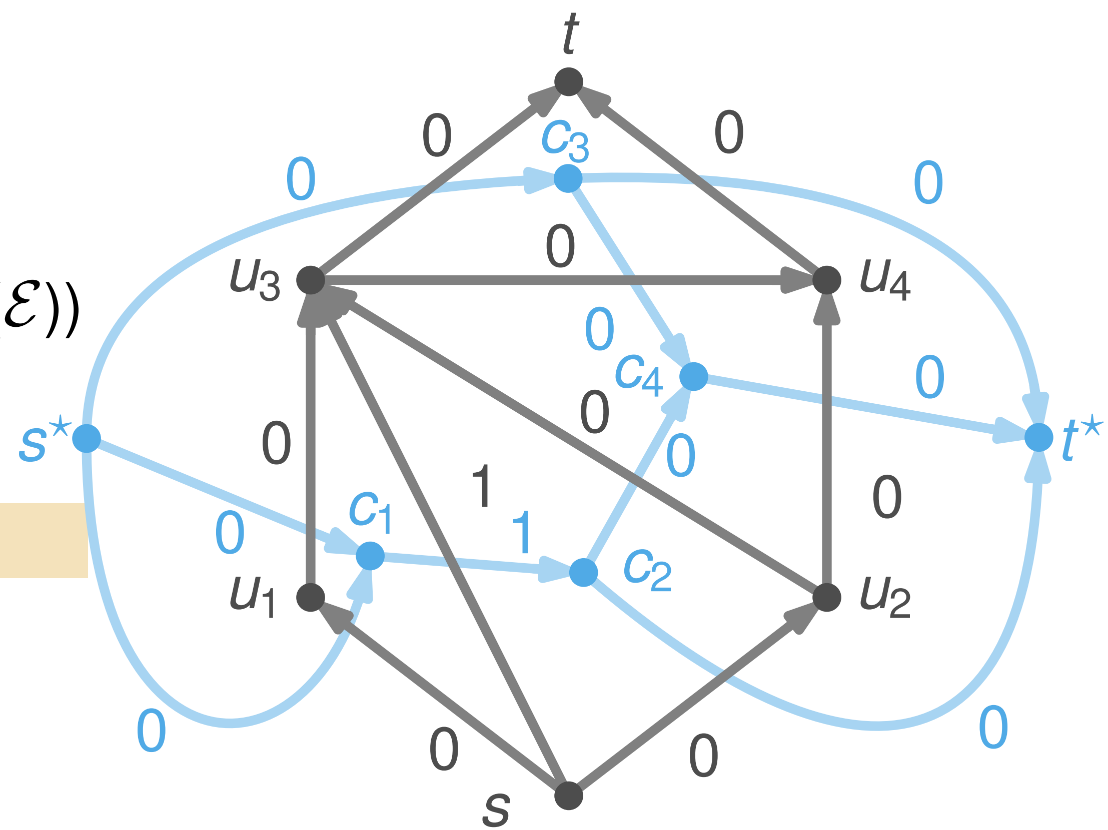

▷ Bijective function

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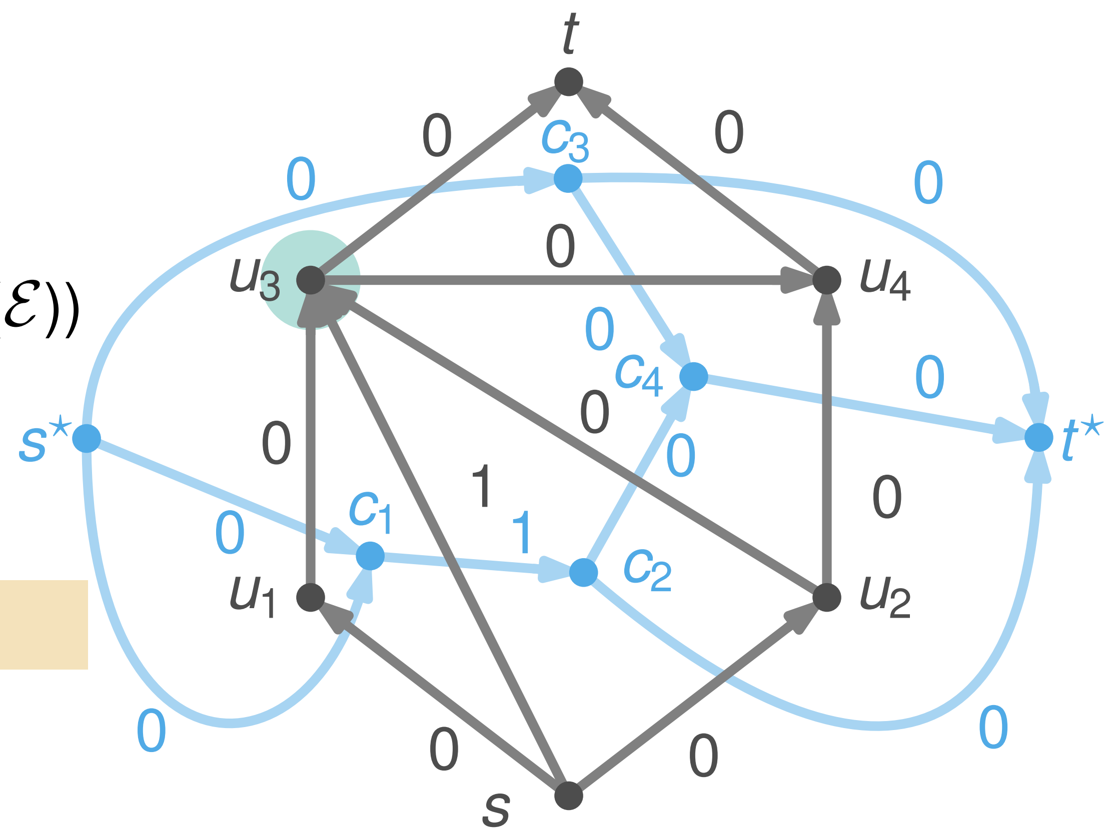

▷ Check KCL property in graph G

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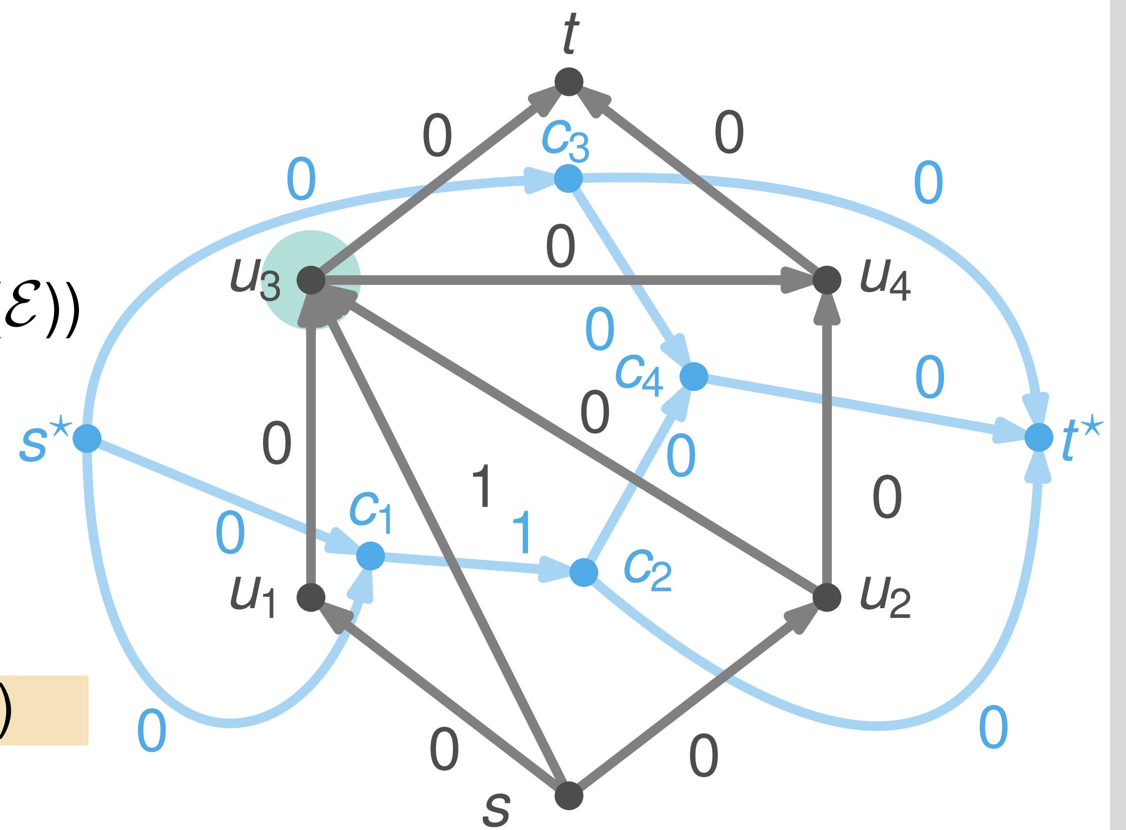
$f = \texttt{resolveConflict}(G, s, t, f, X)$



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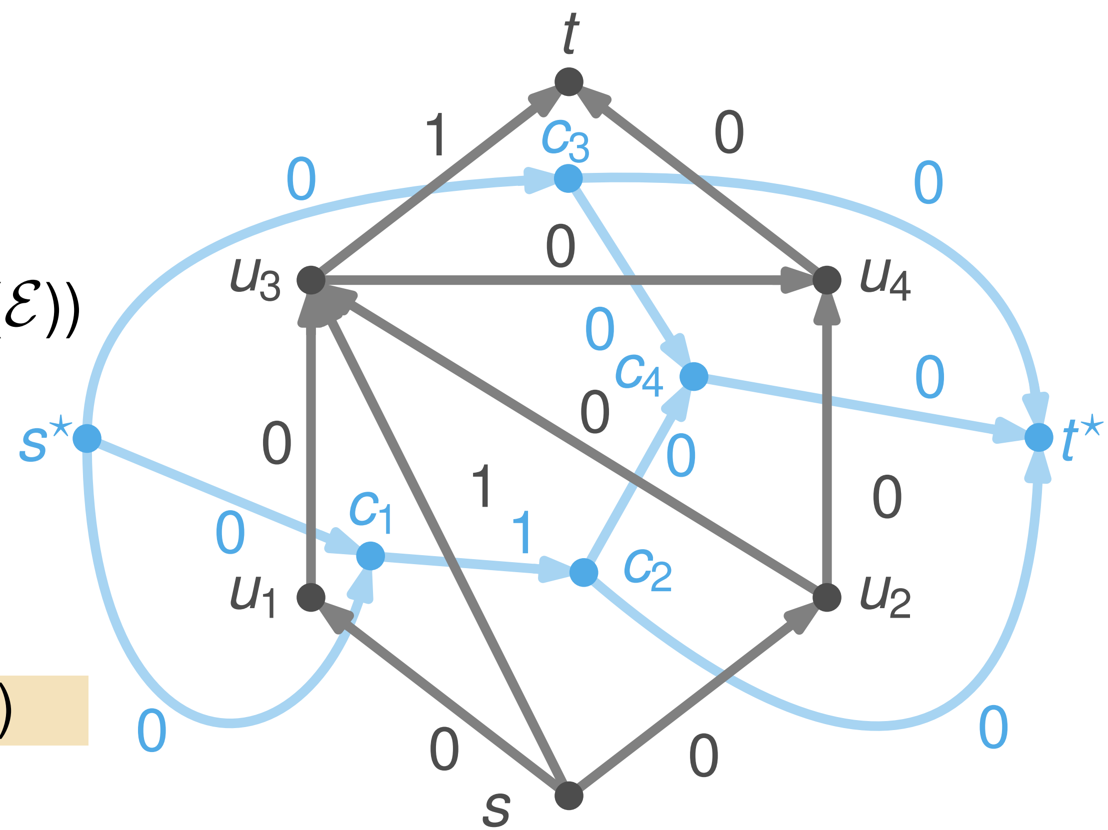

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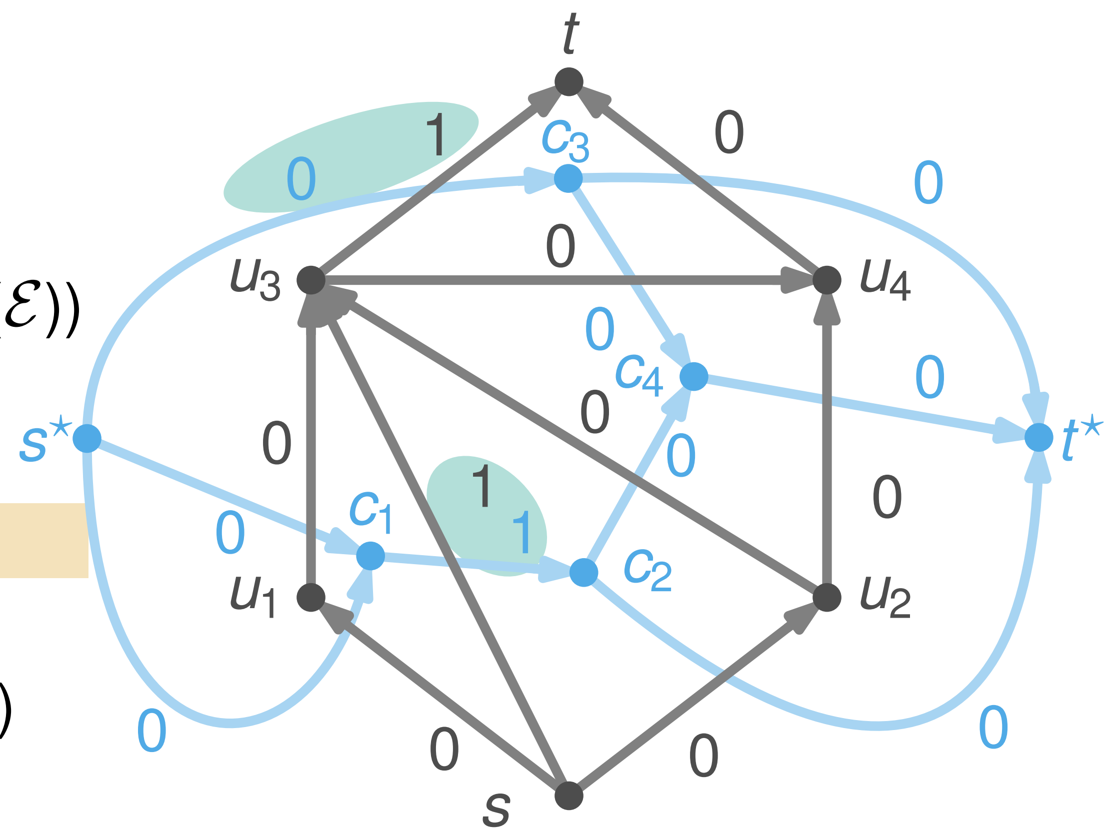

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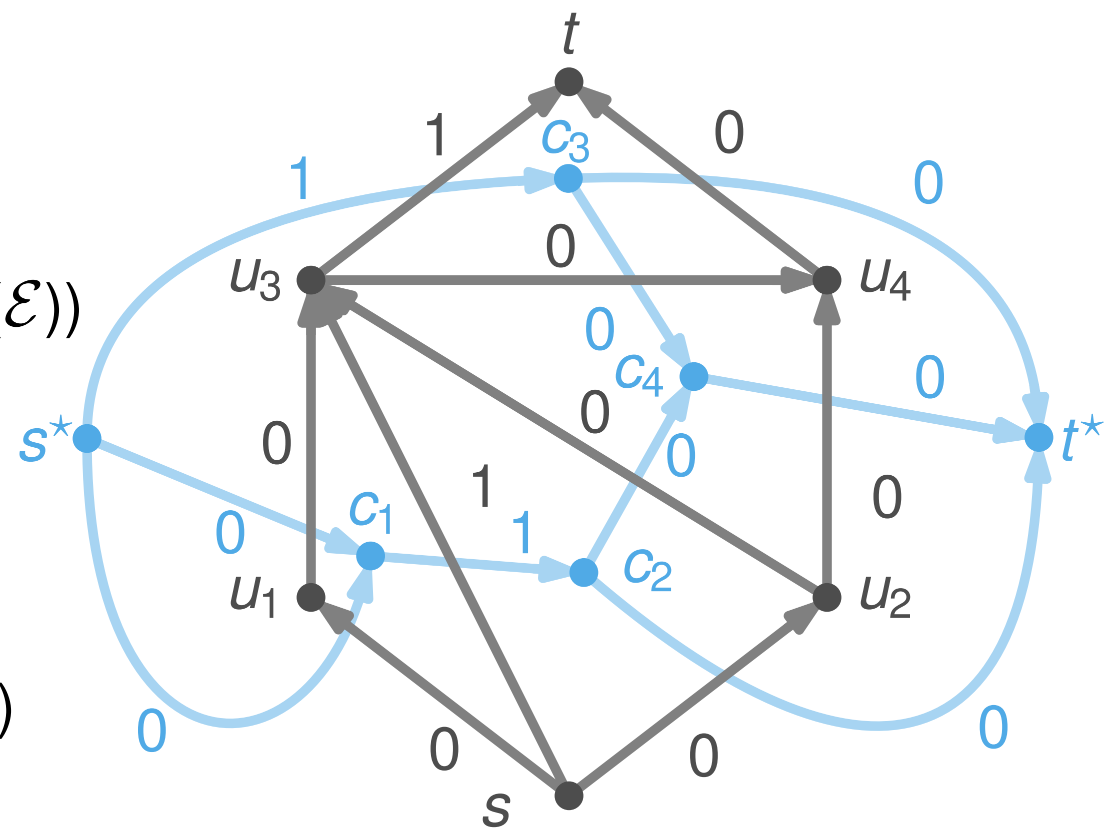

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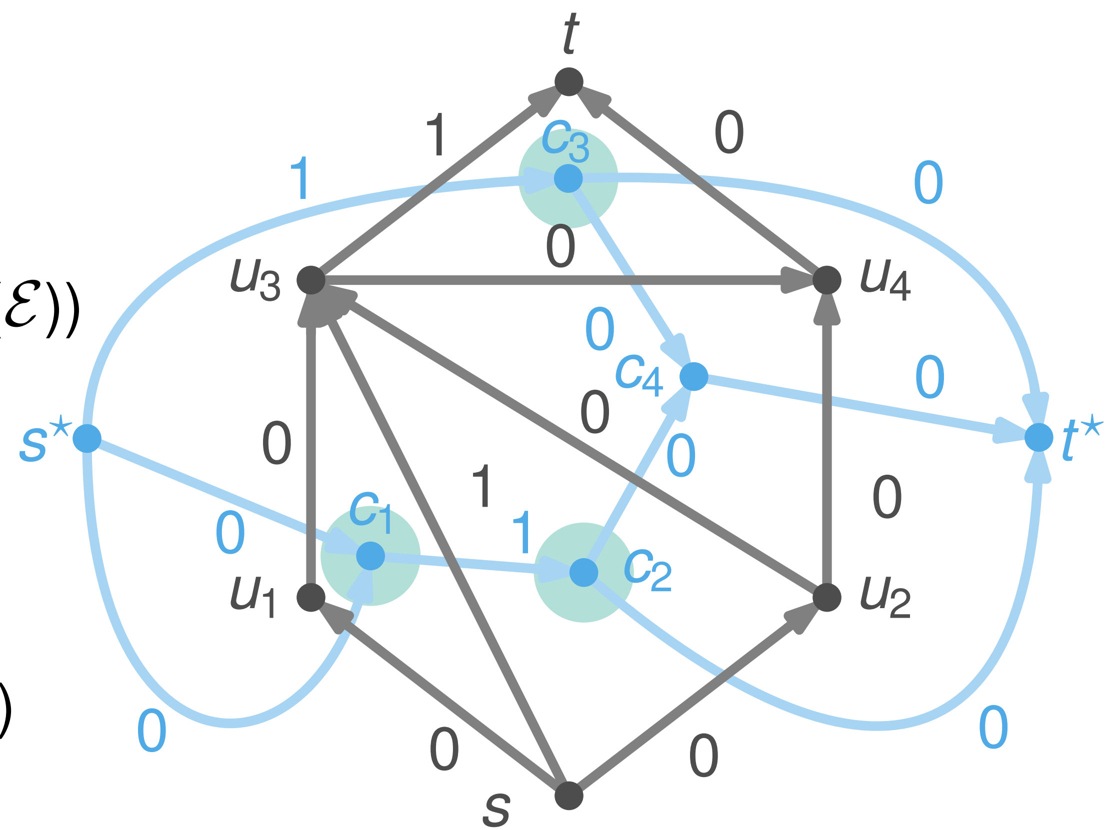

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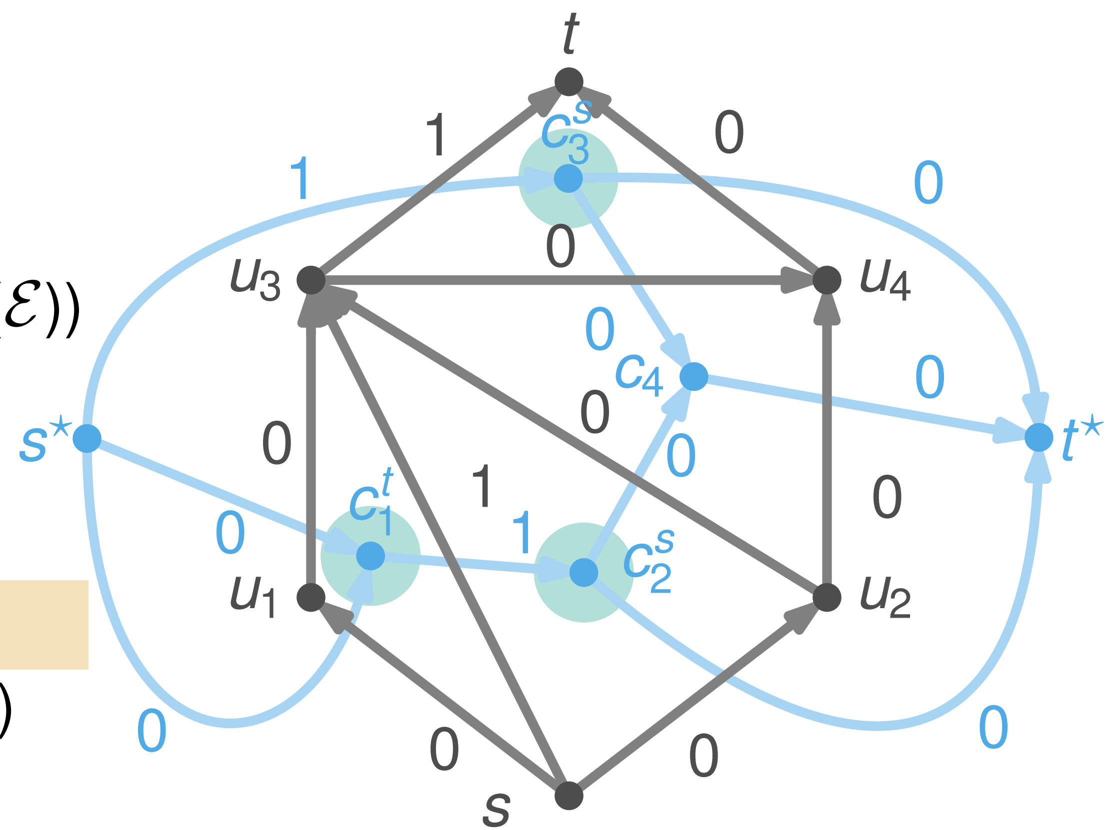

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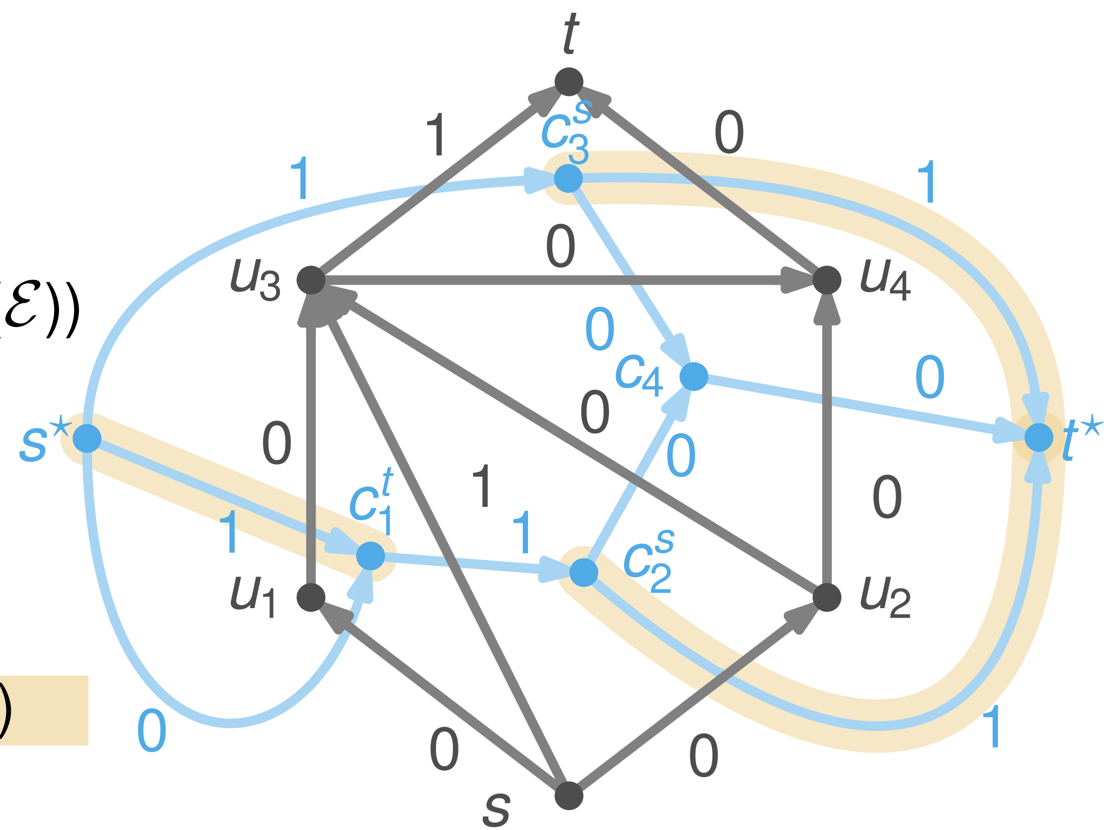

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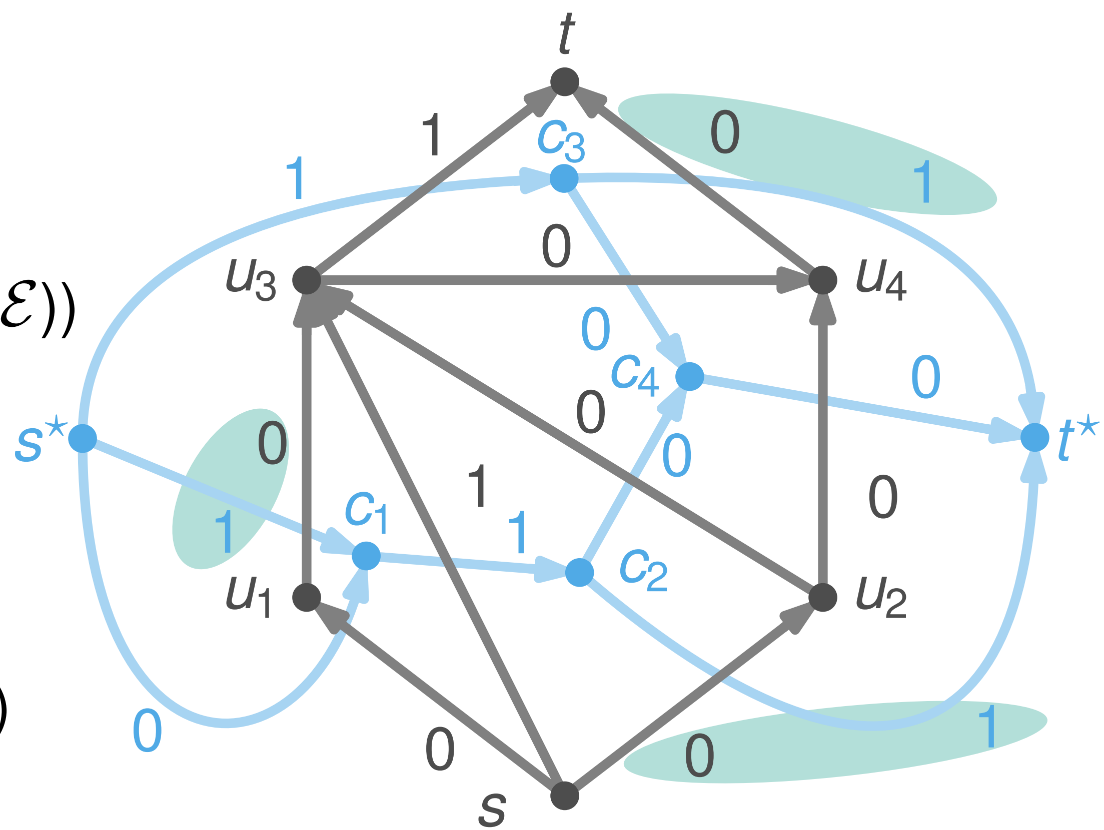

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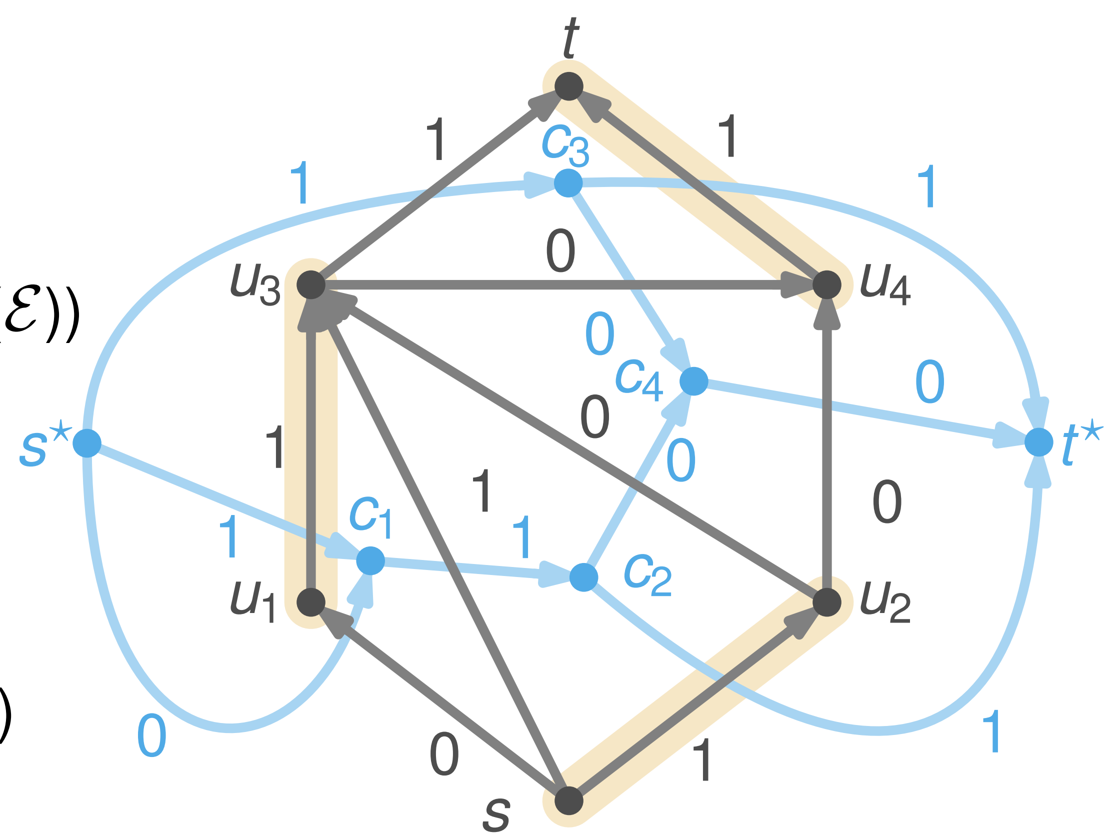

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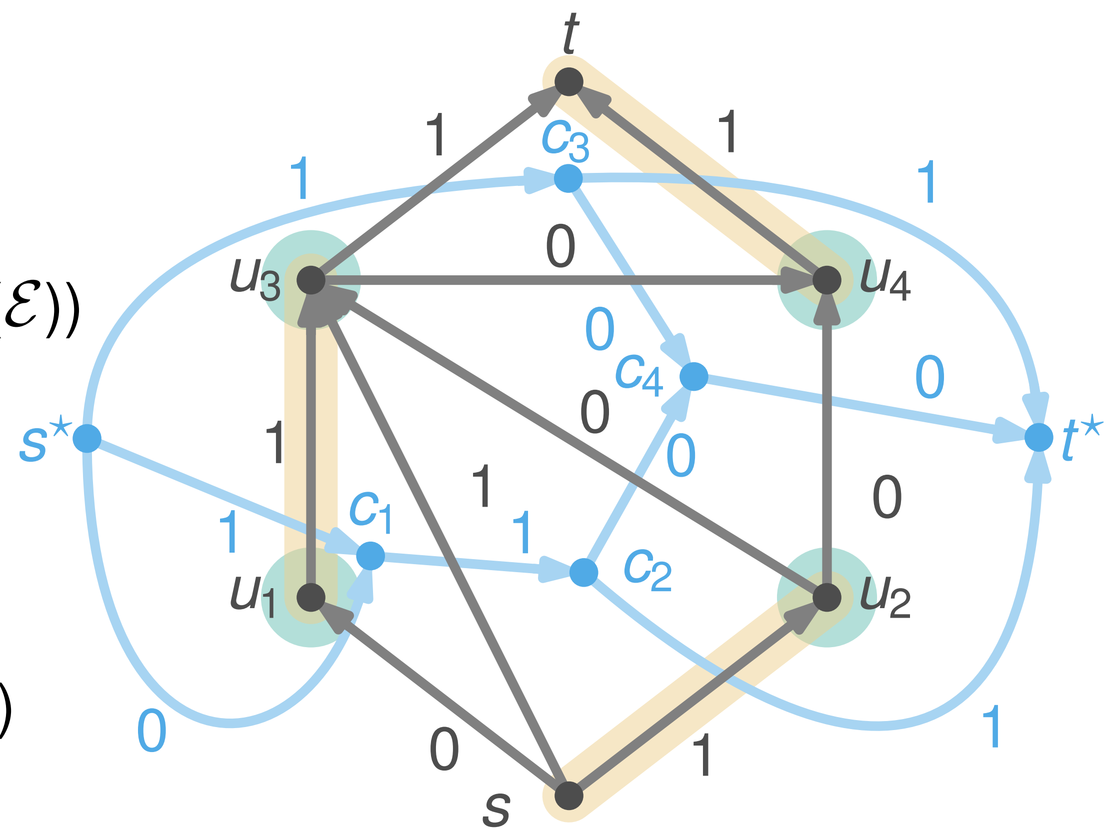

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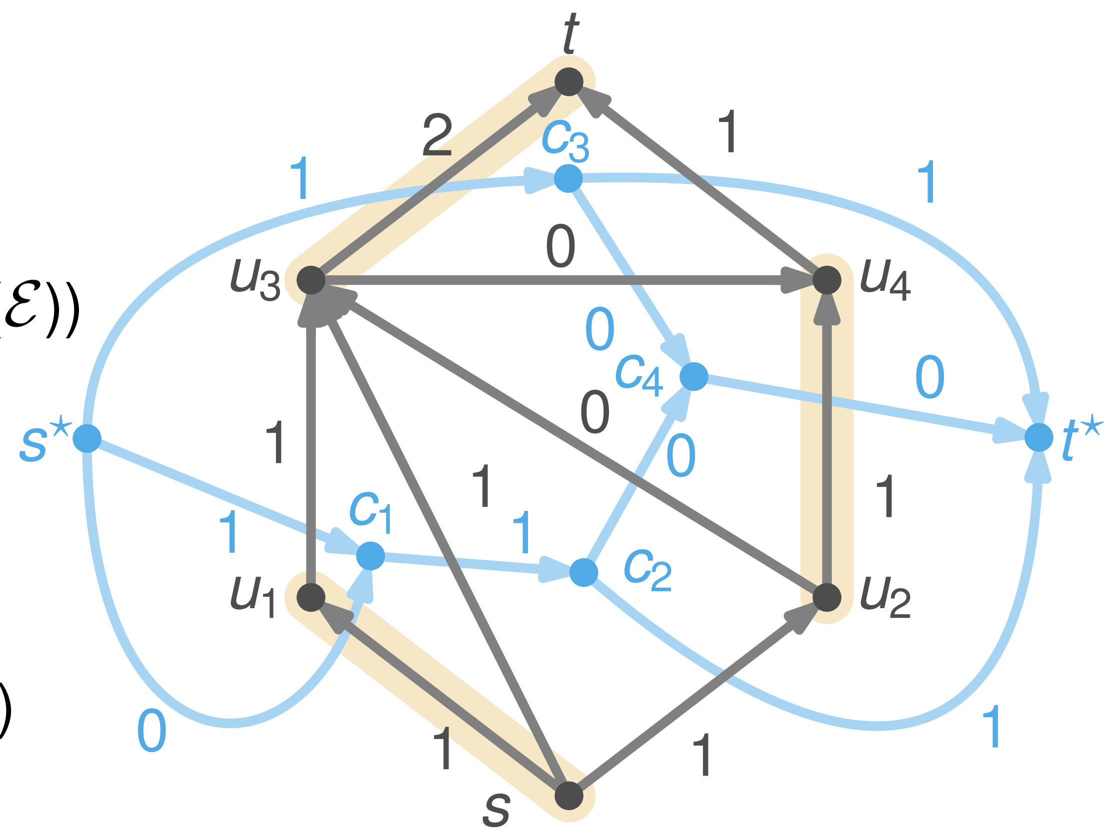

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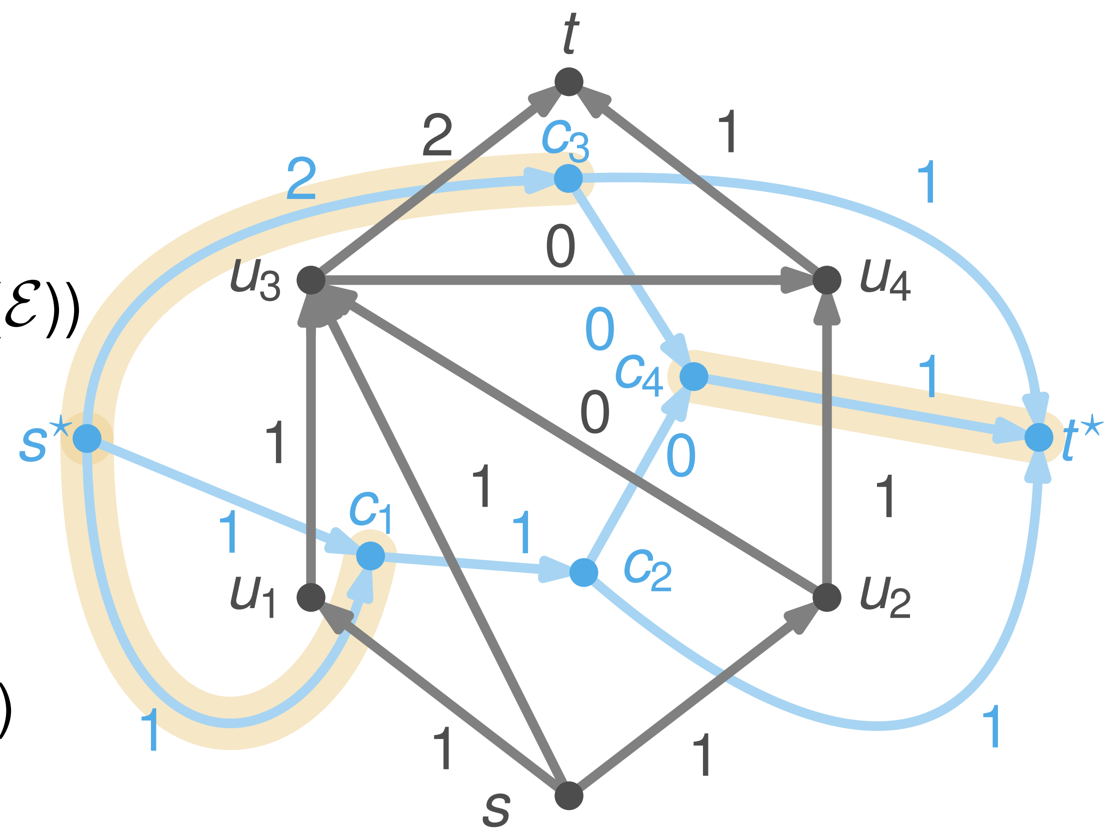

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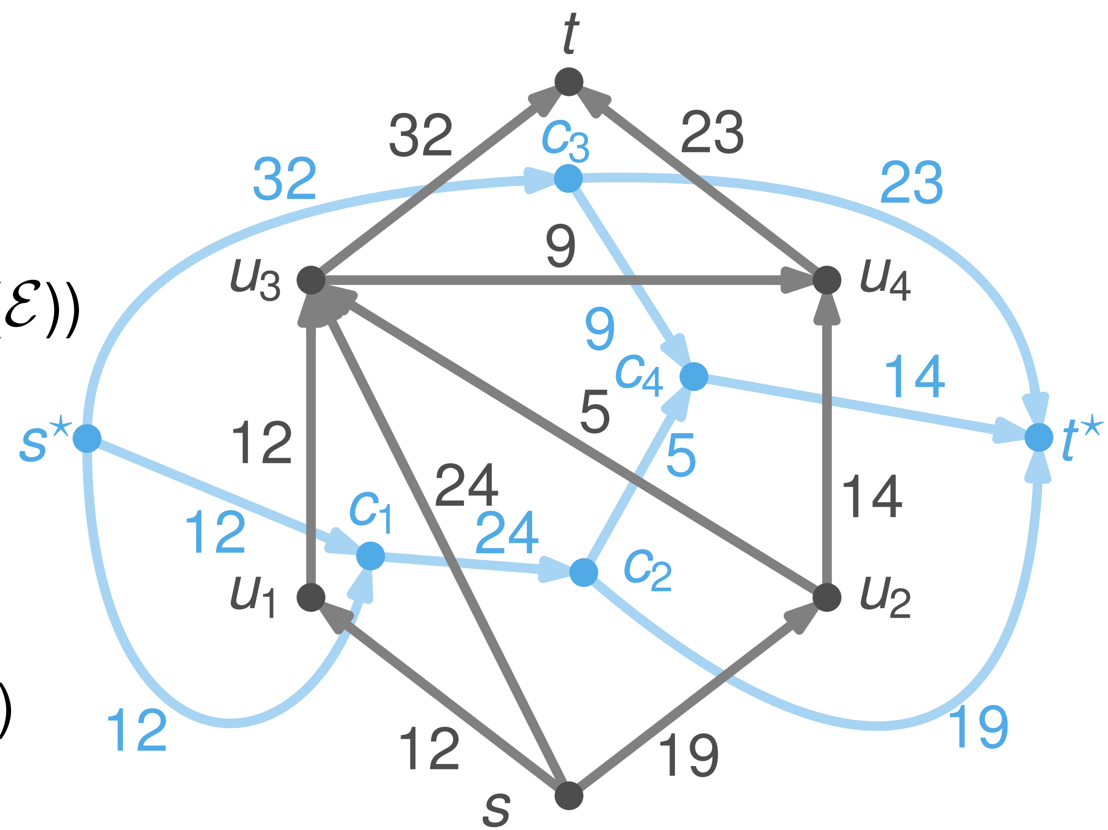
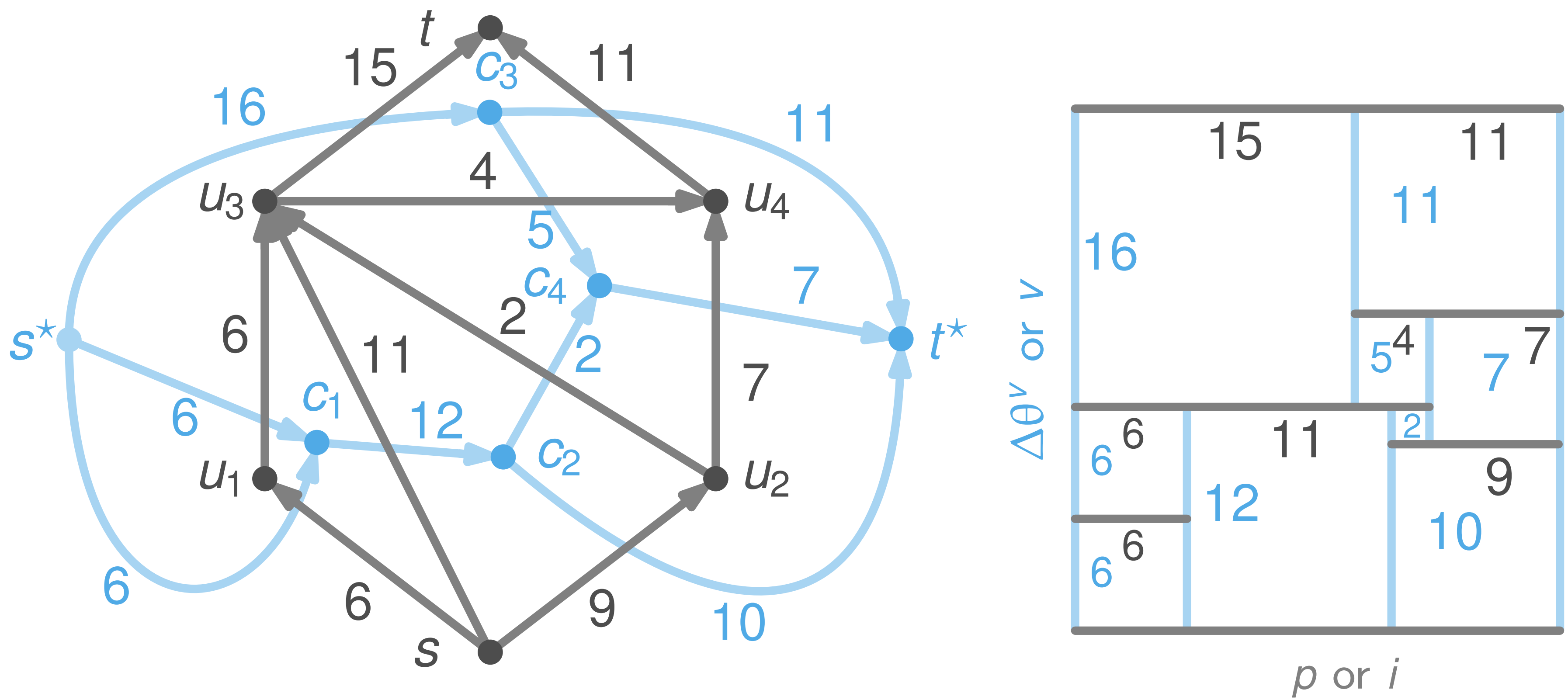


Geometric Interpretation of a KCL Conflict



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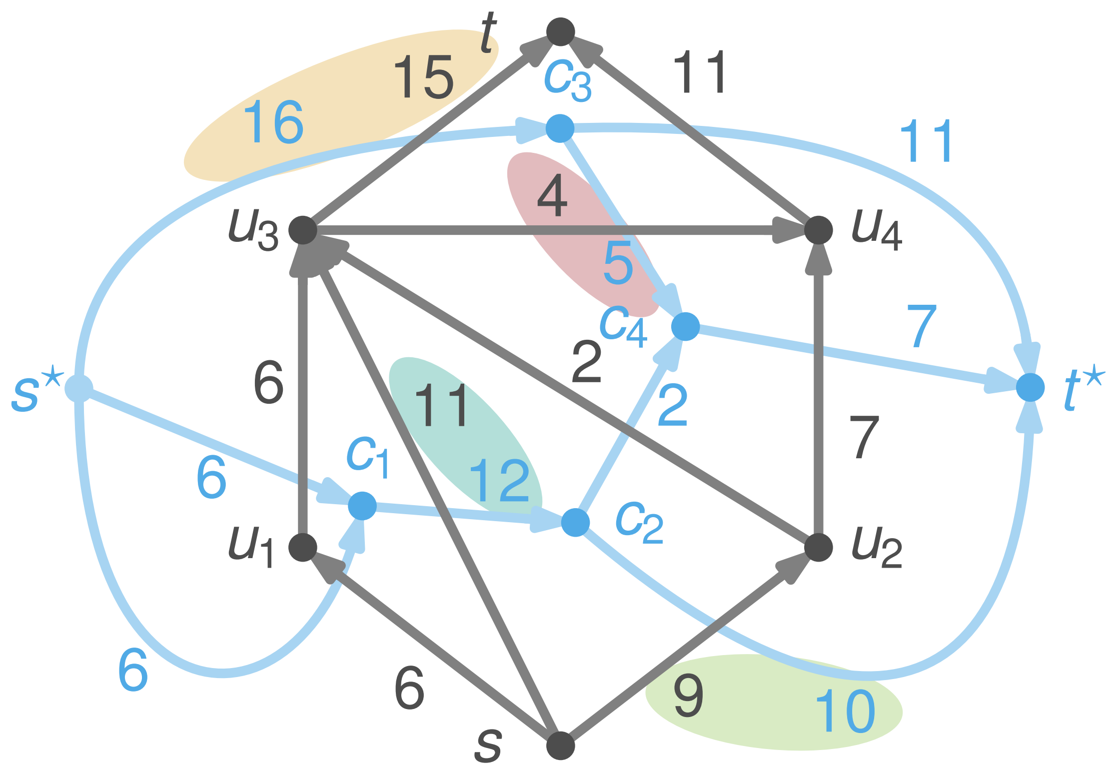

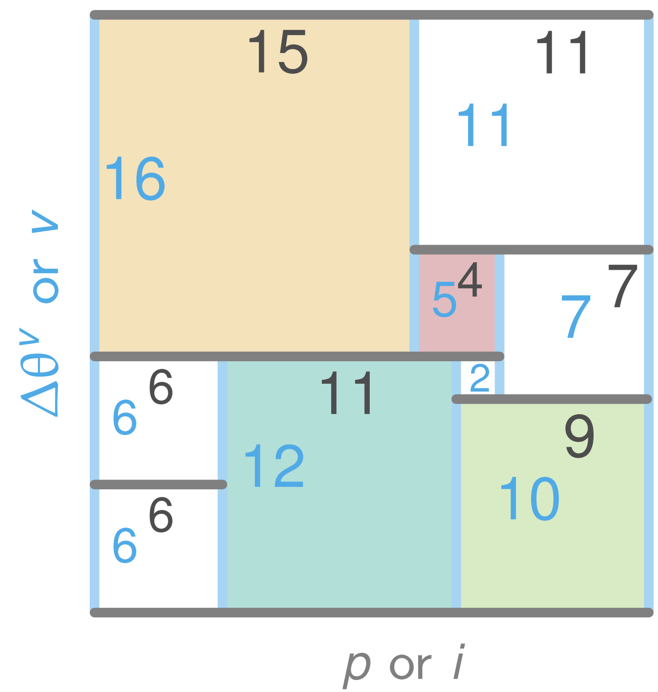

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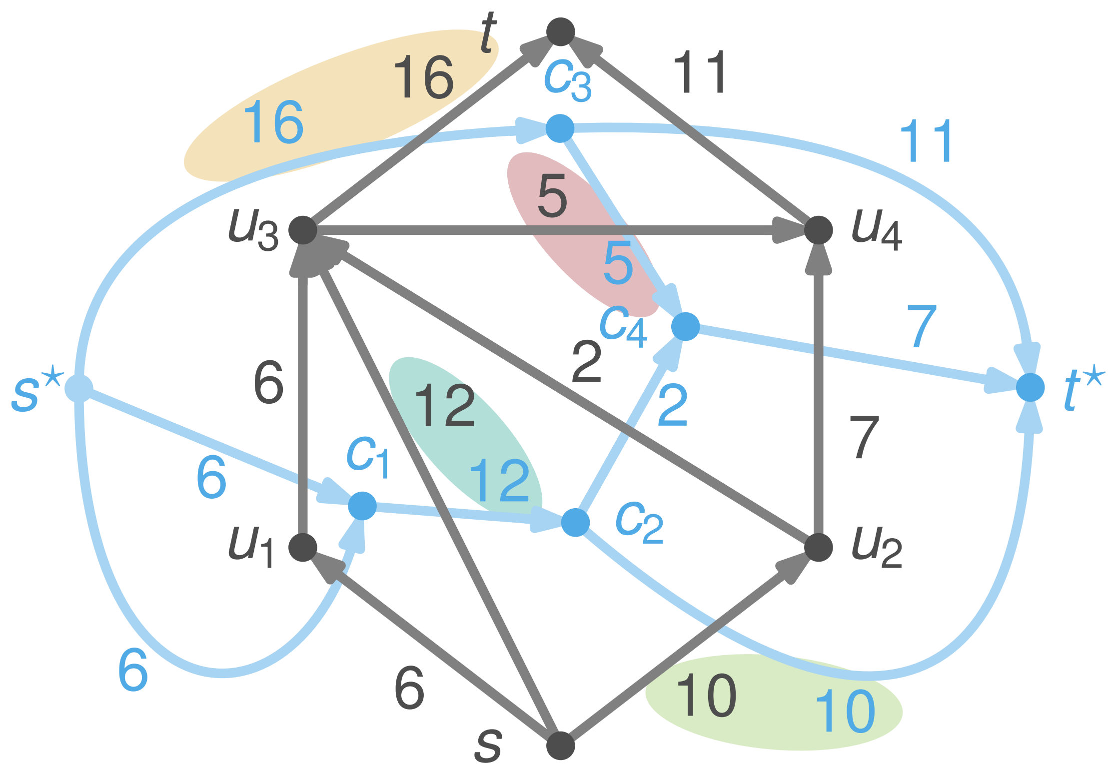

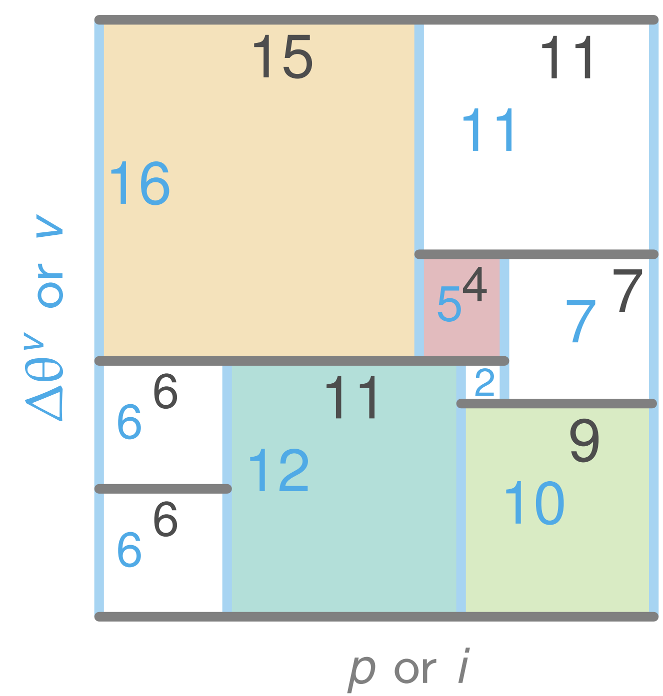

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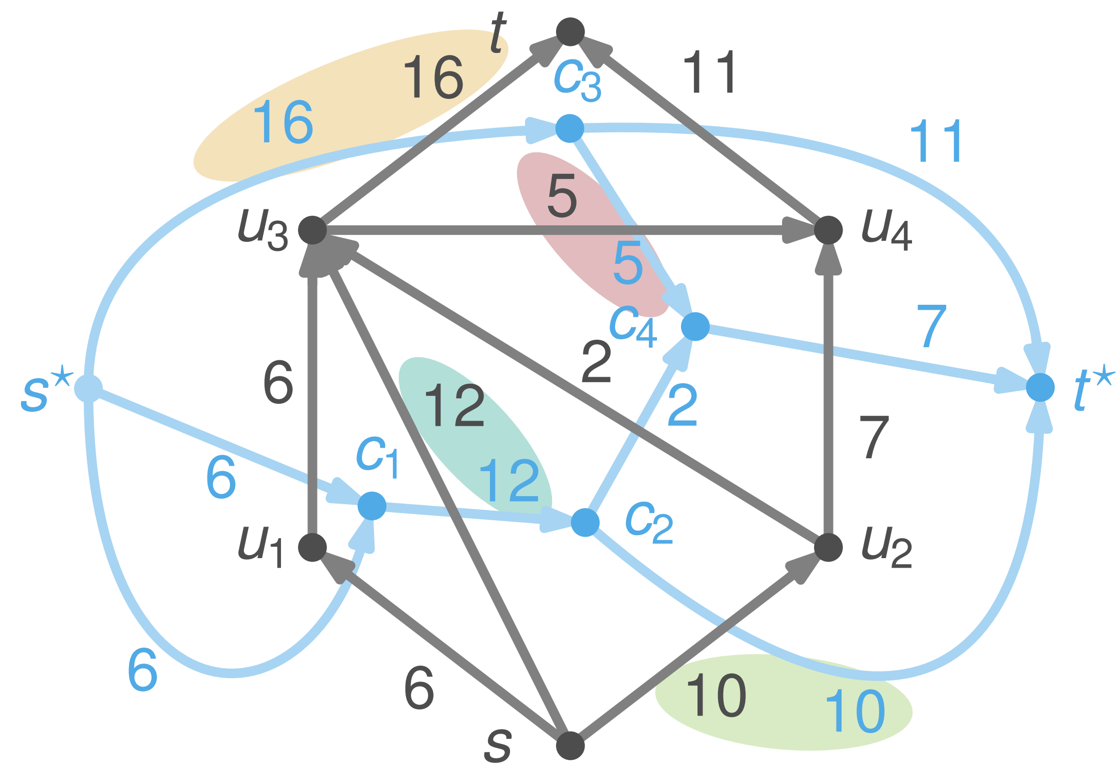

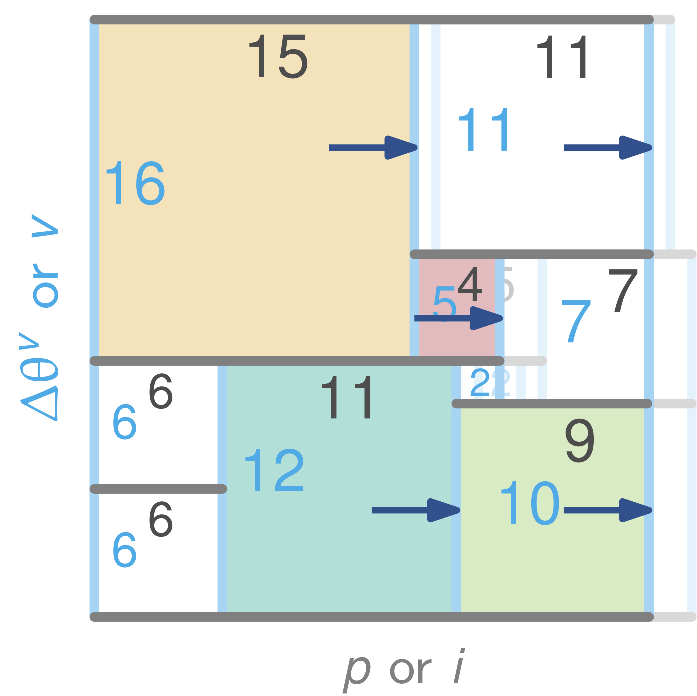

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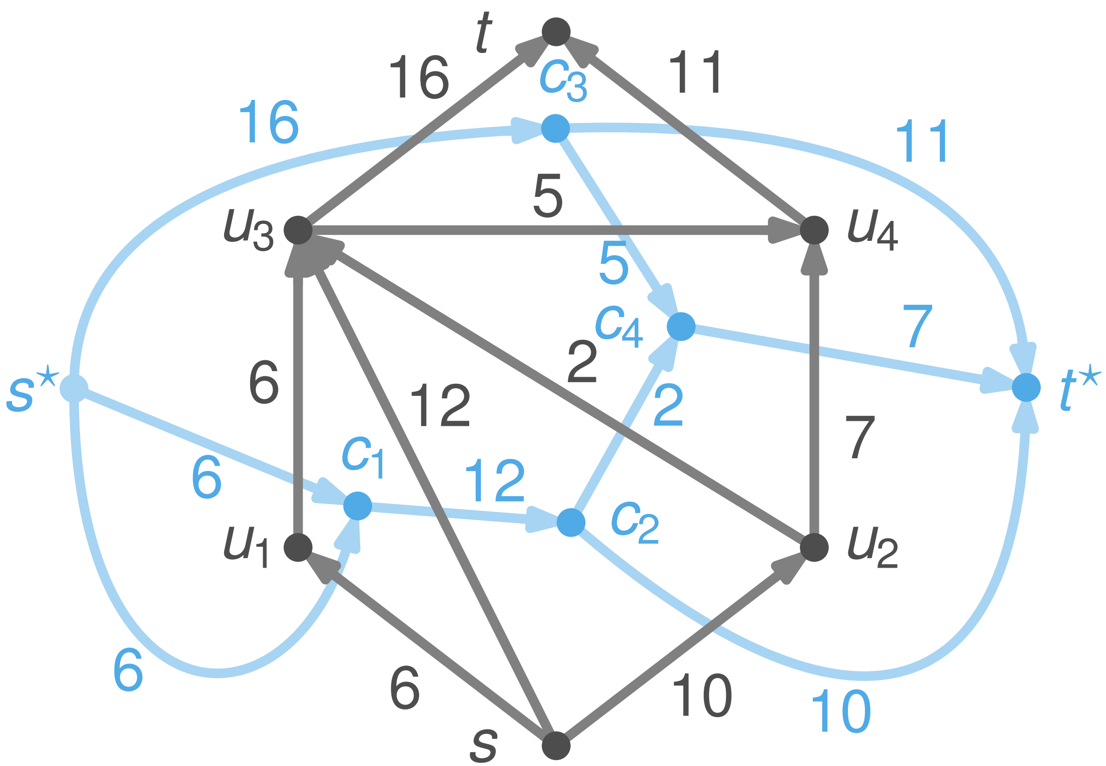

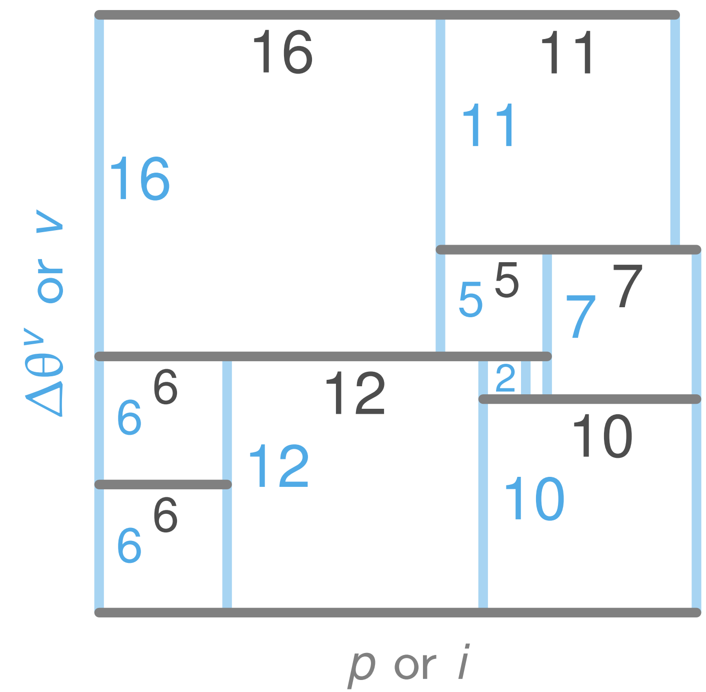

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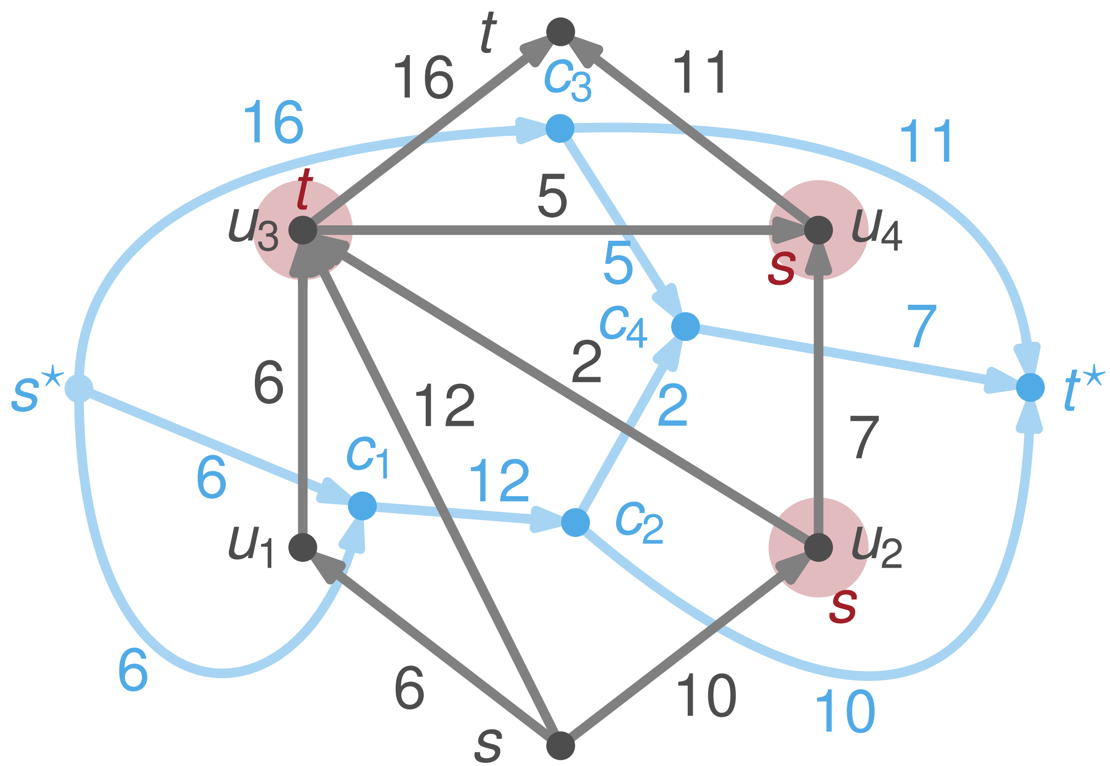

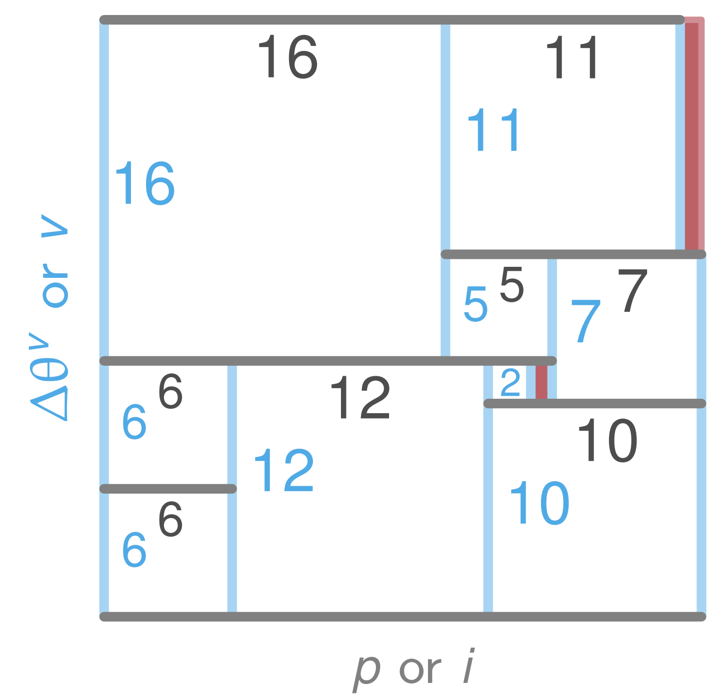

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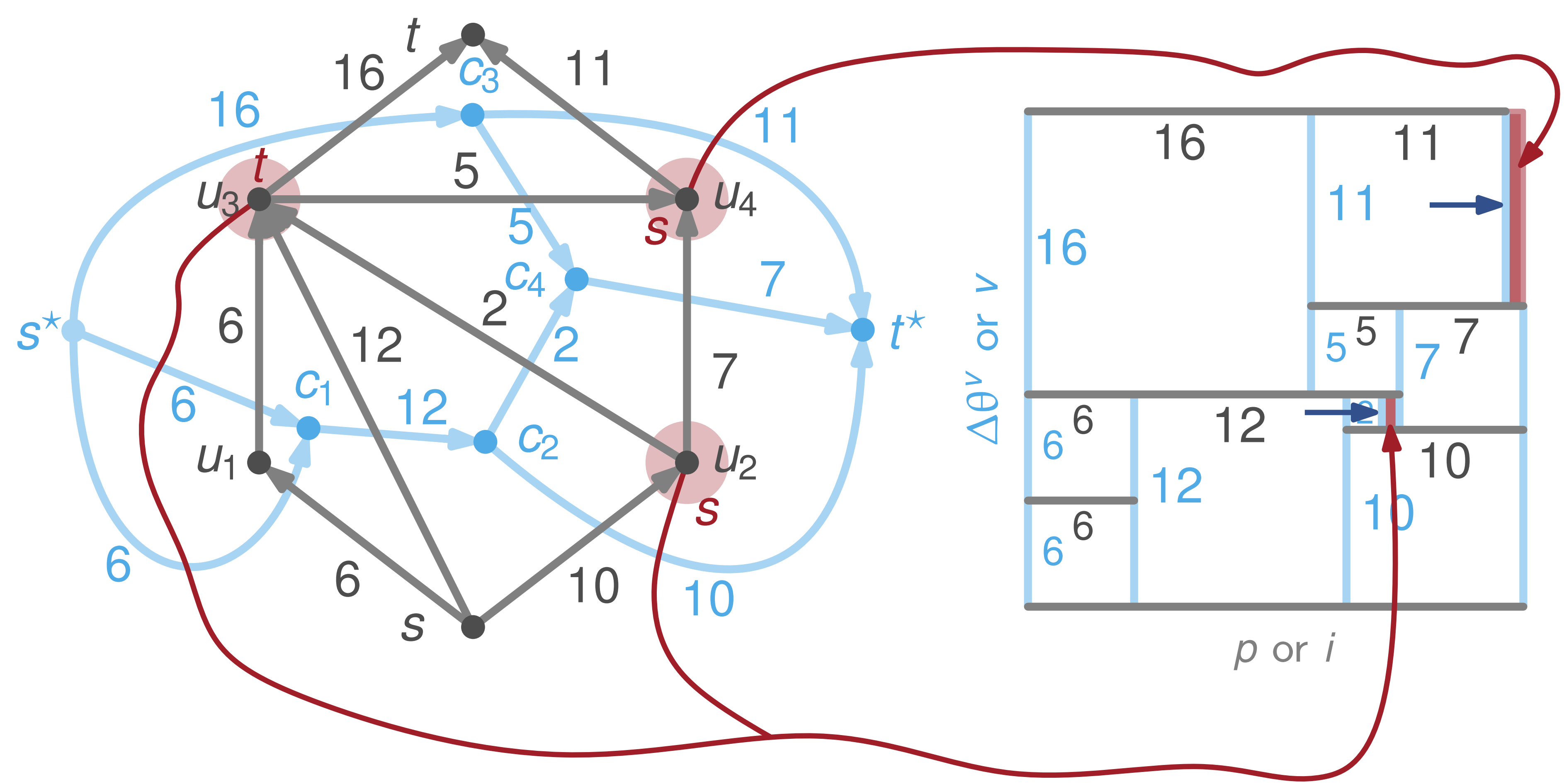

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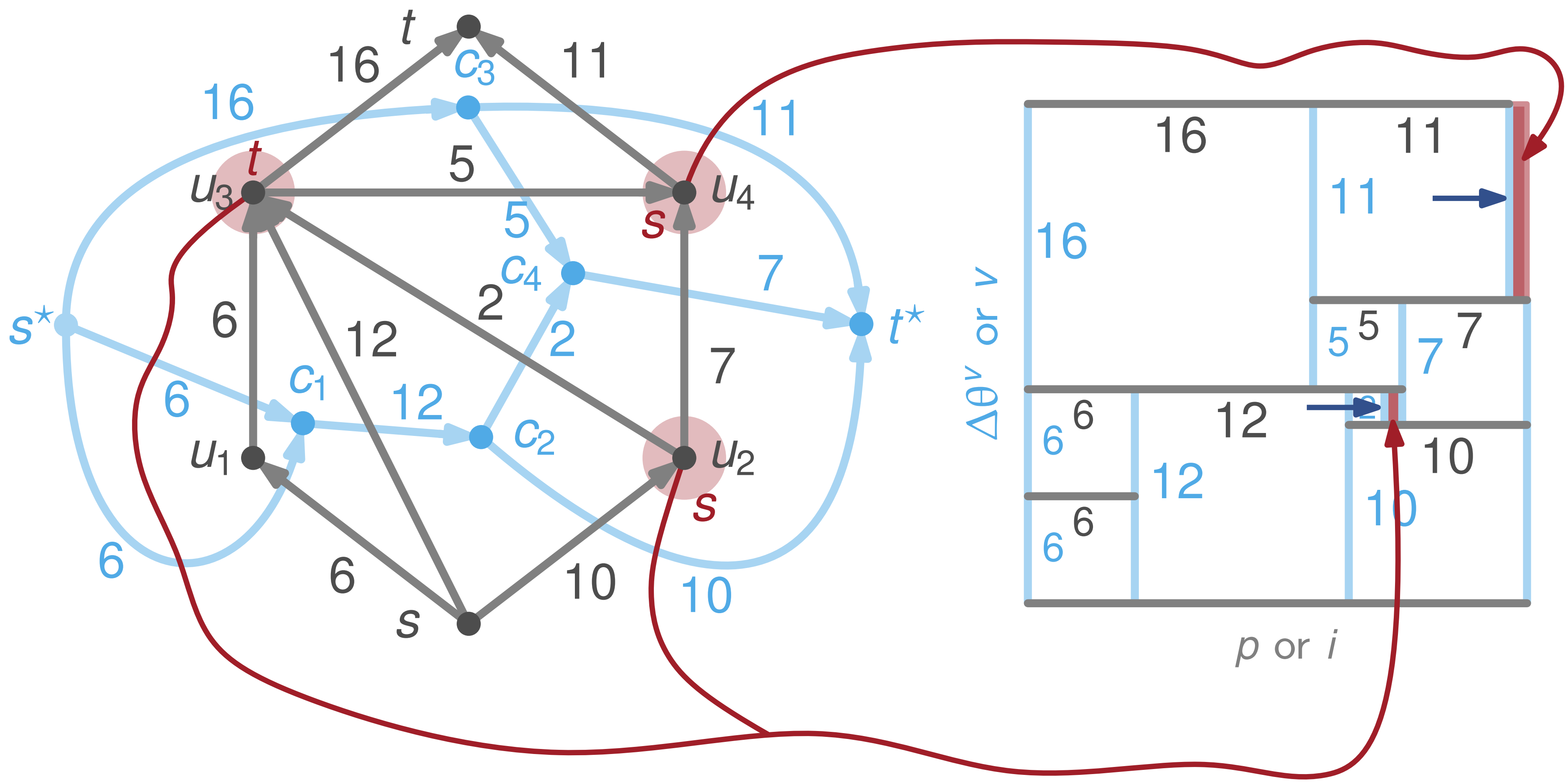

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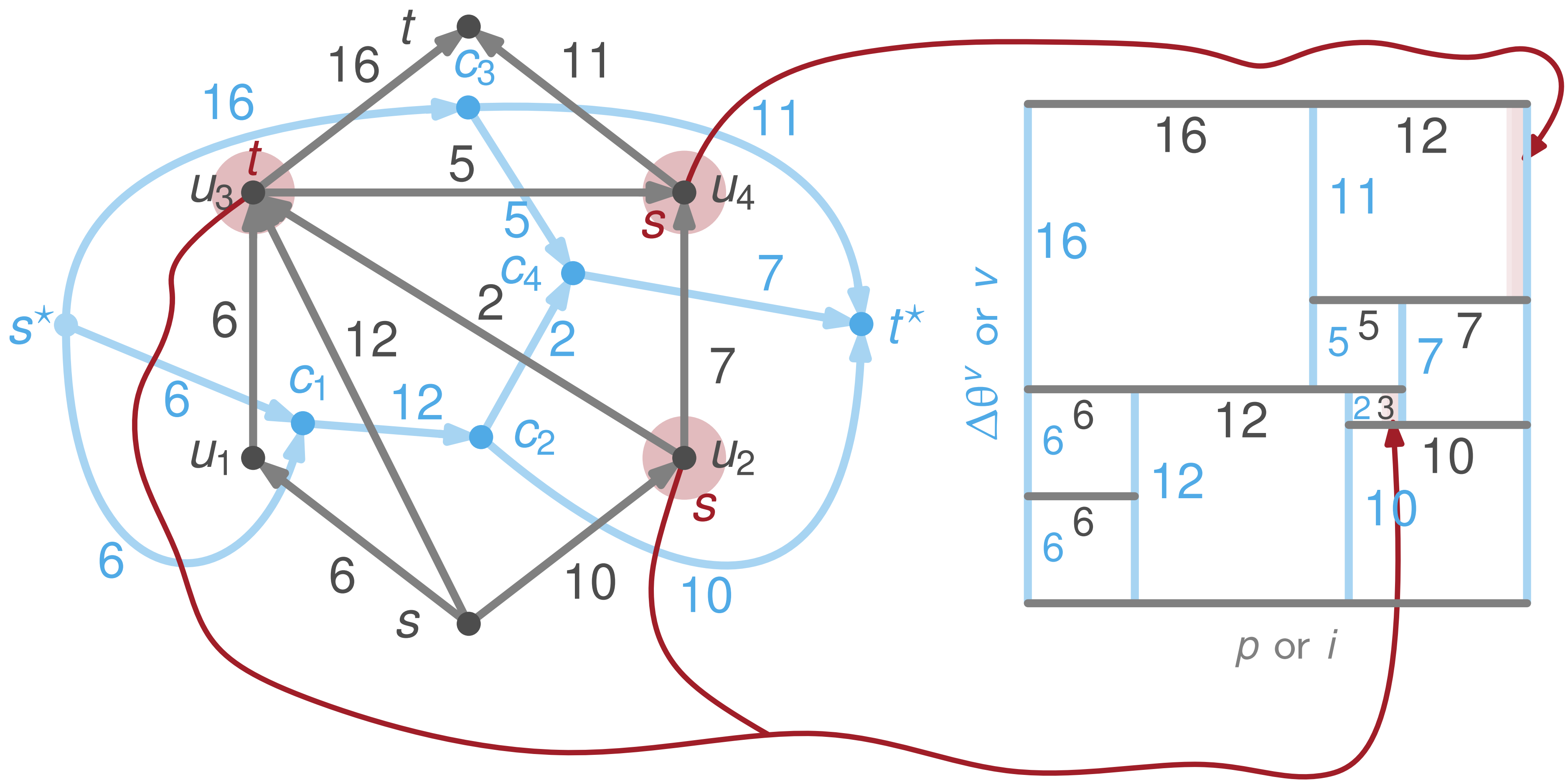

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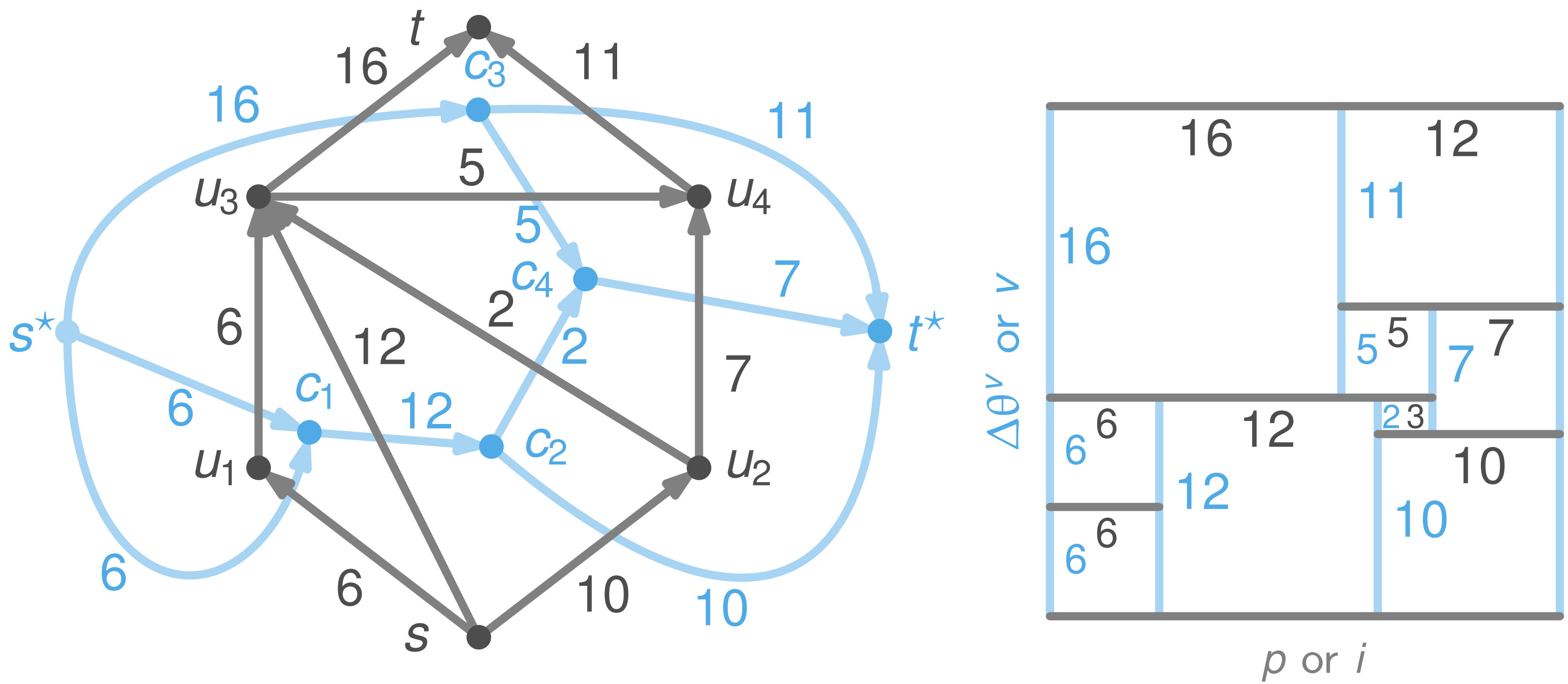

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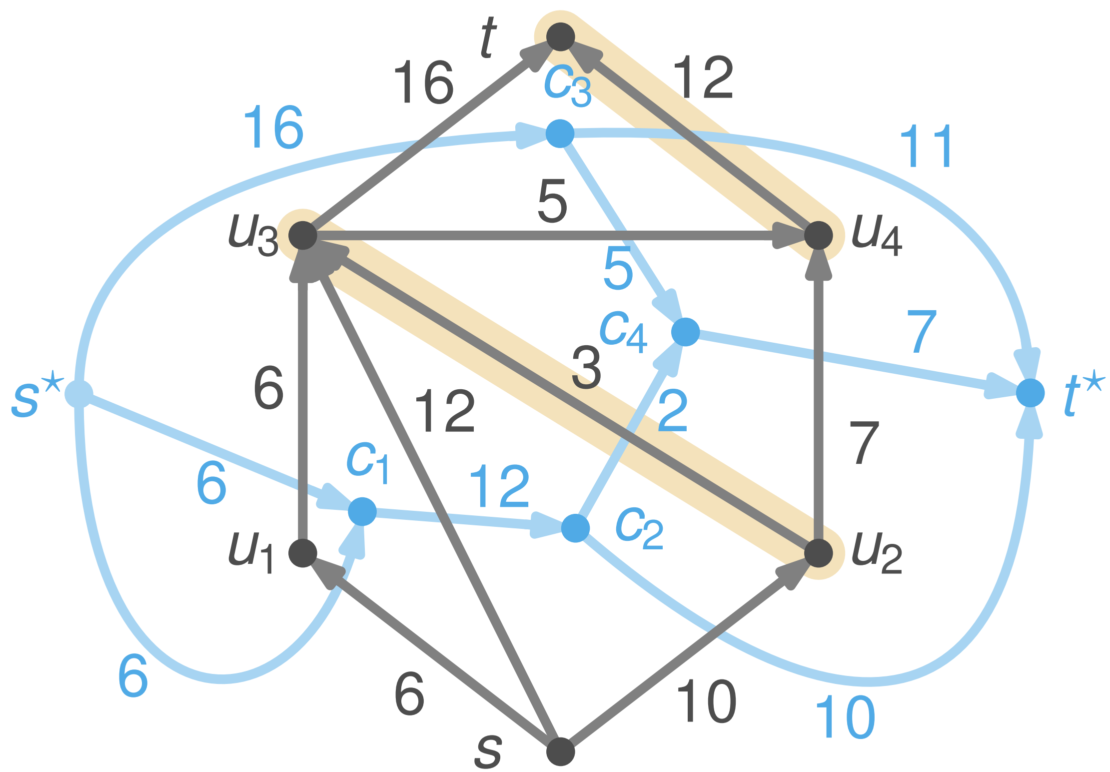

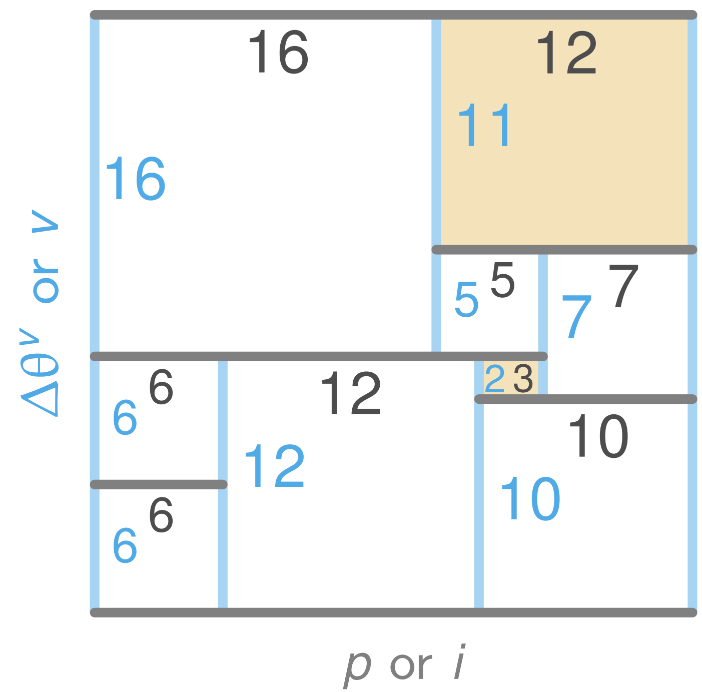

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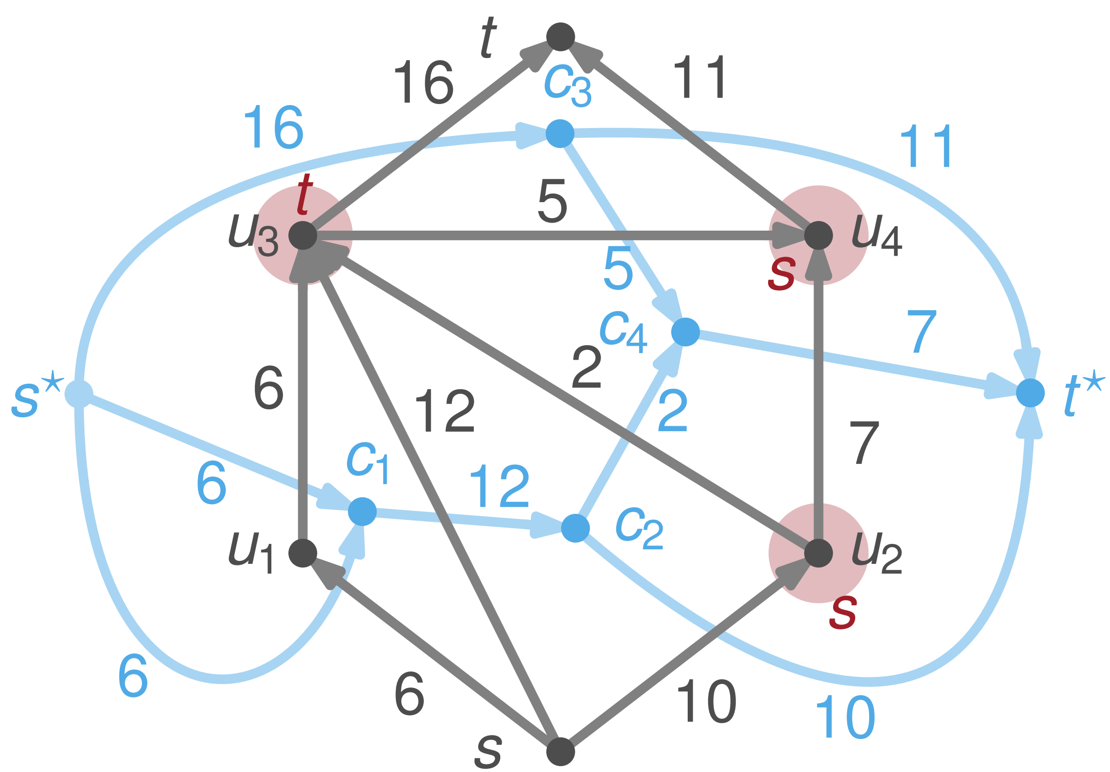

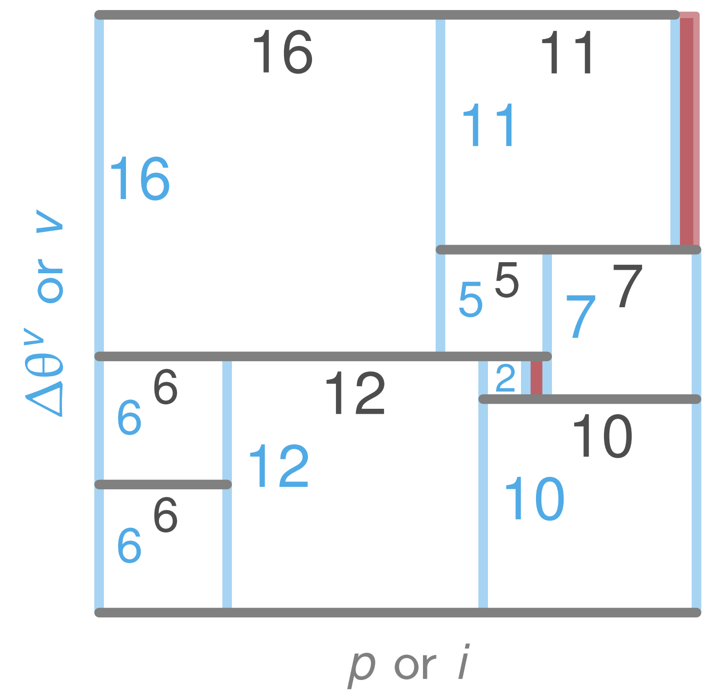

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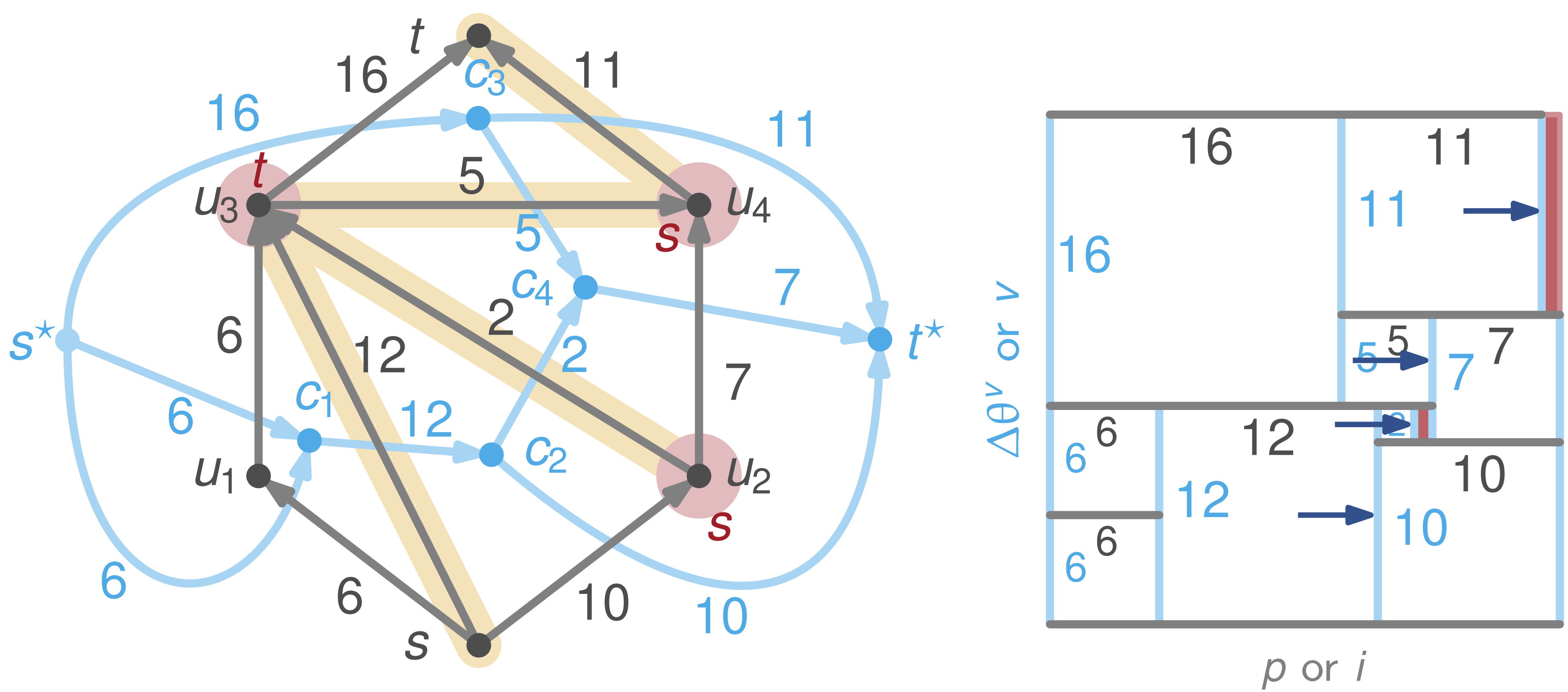

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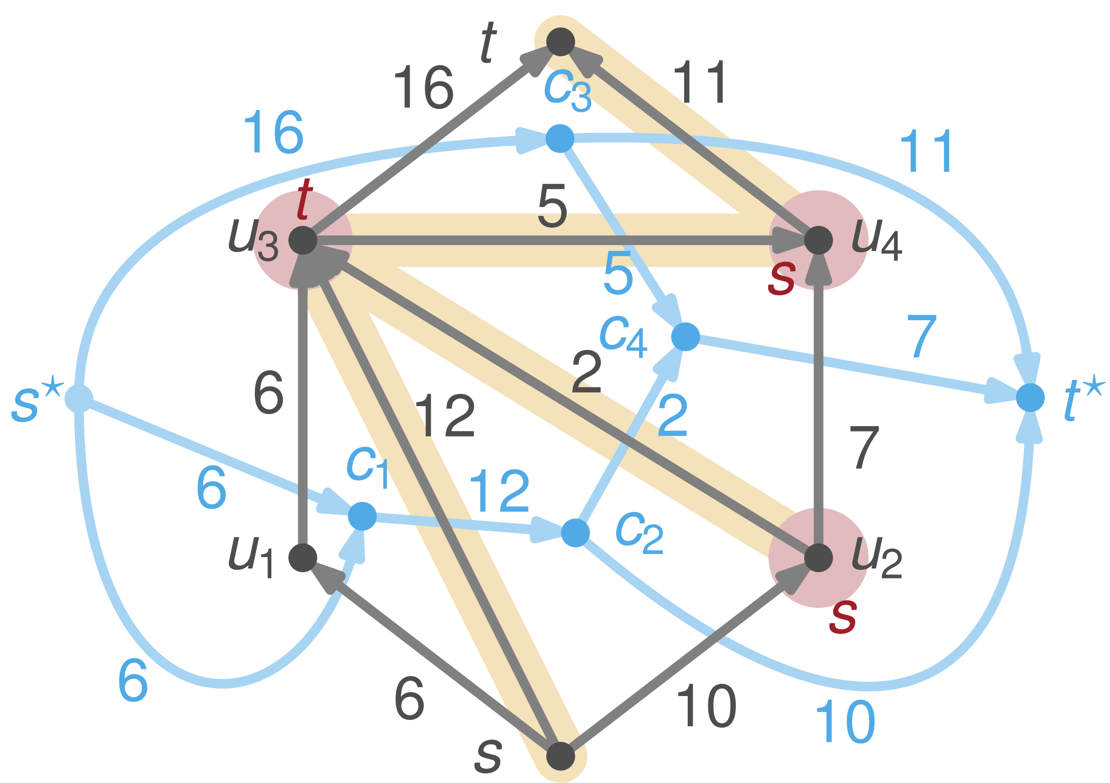
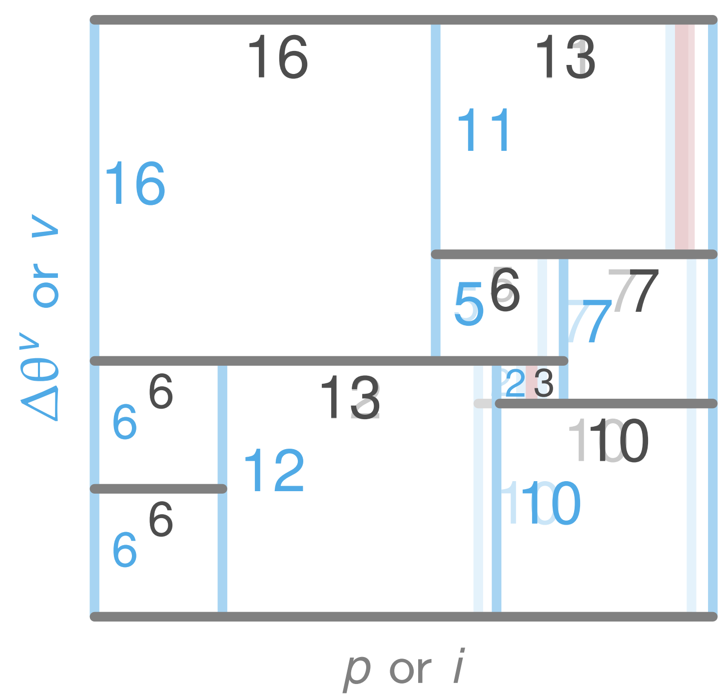
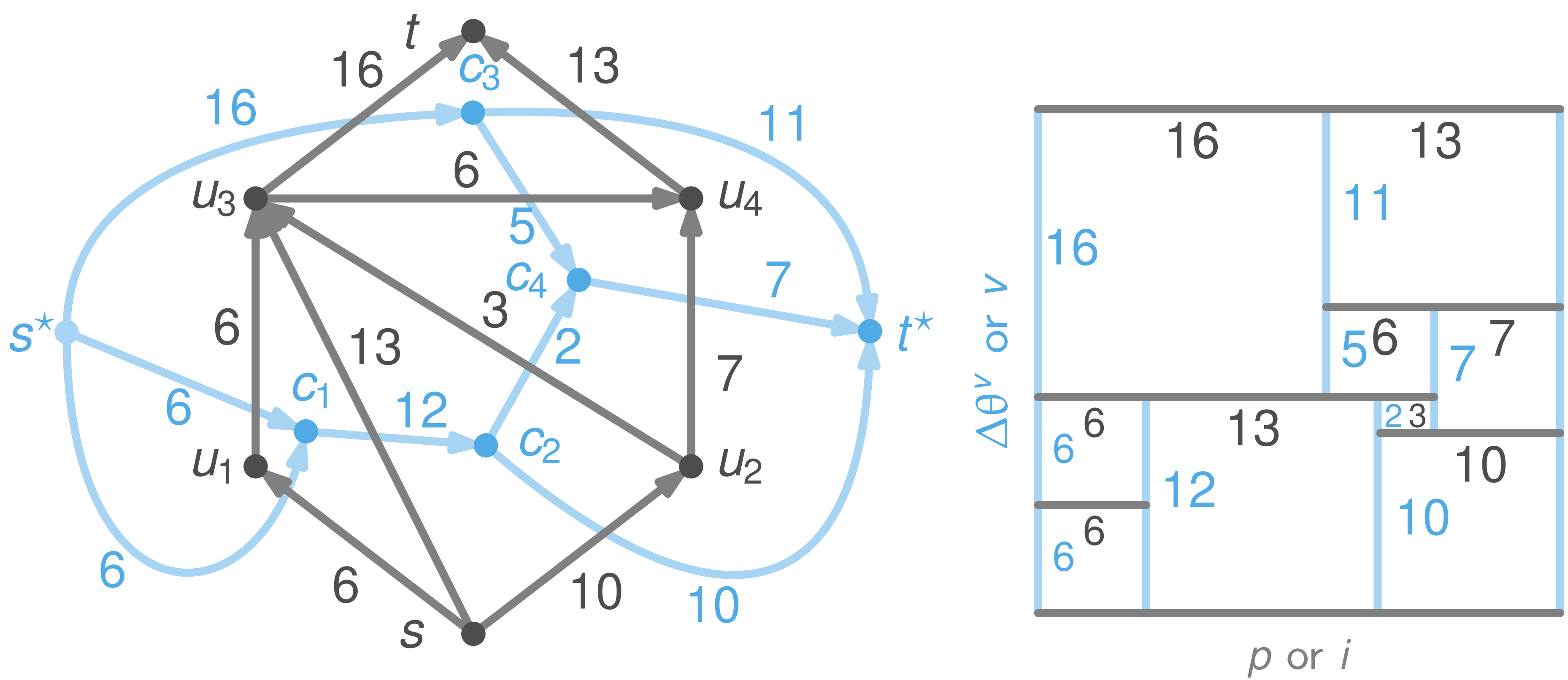




Observation 5 [Resolve KCL Conflicts]

Minimize in each `resolveConflict` step the total resizing of the outer rectangle, since a too large increase might skip a valid solution.

Geometric Interpretation of a KCL Conflict



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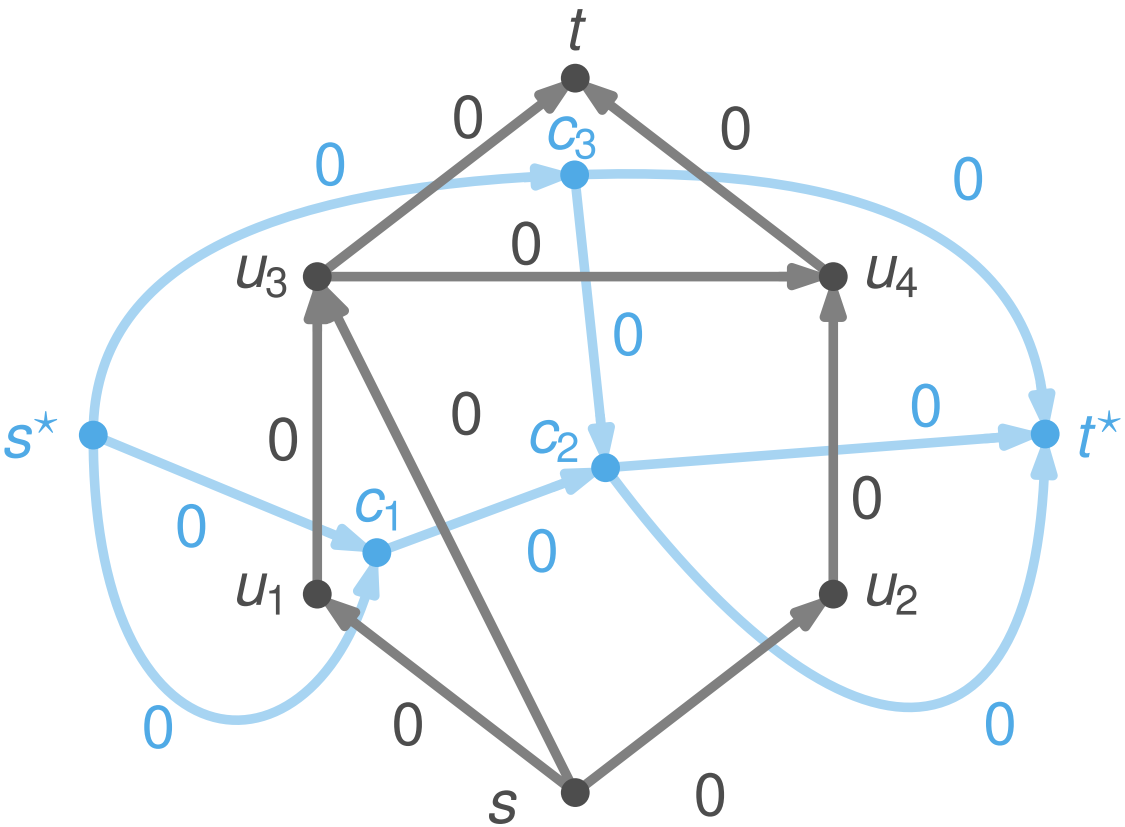
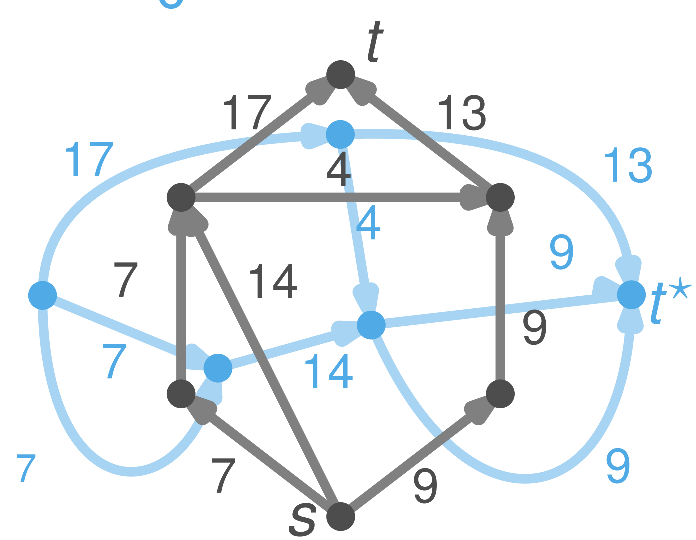
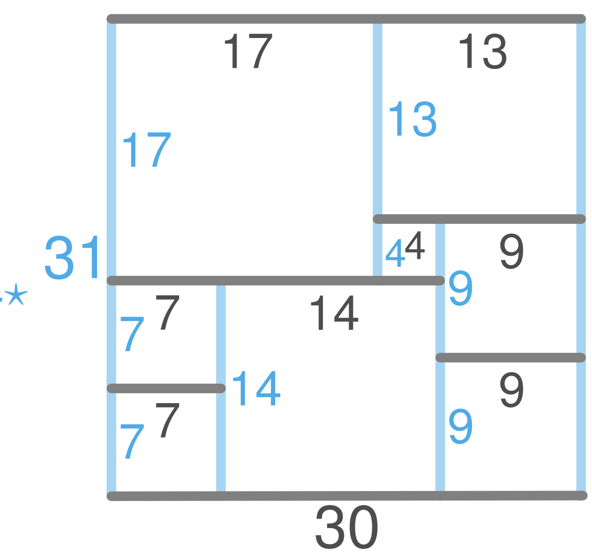
Wrong Conflict Resolution







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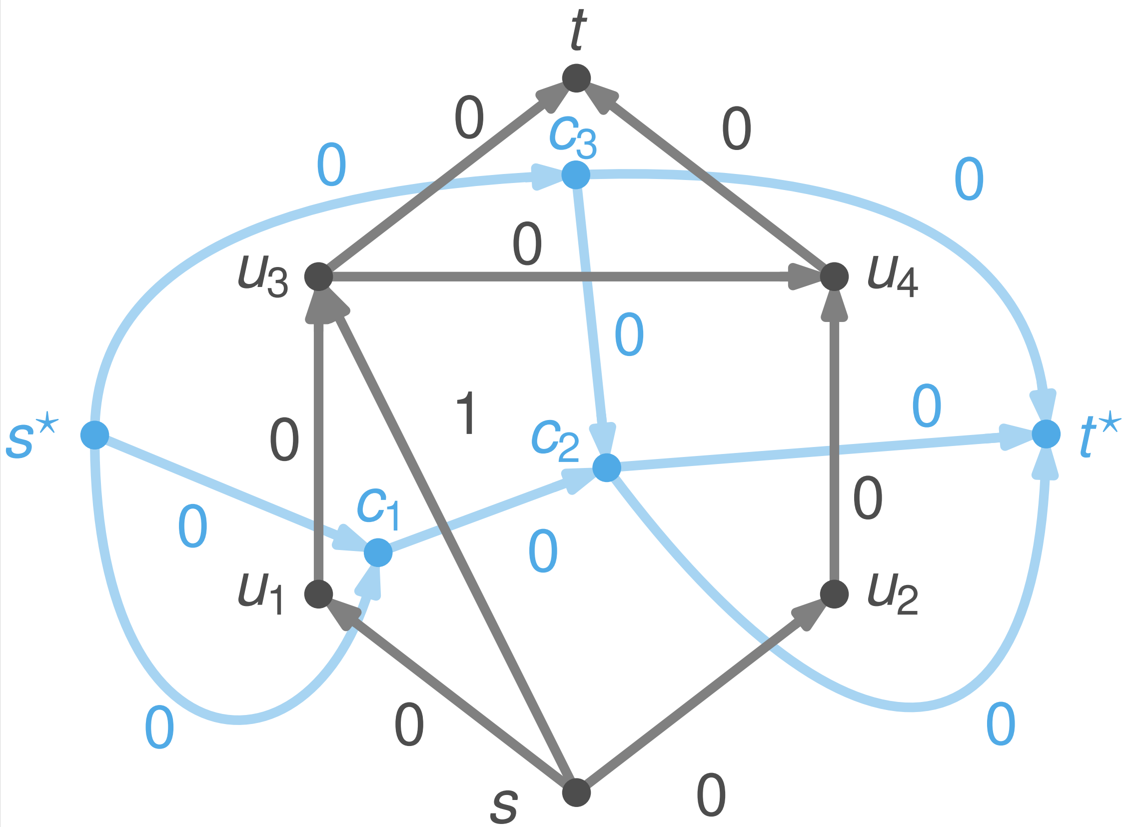

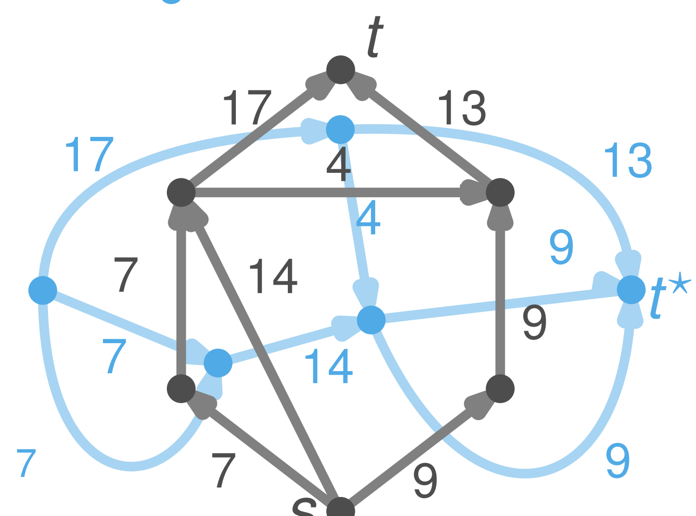

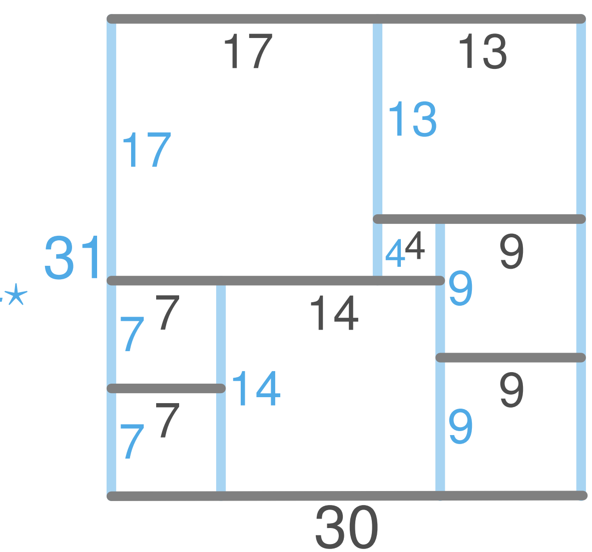

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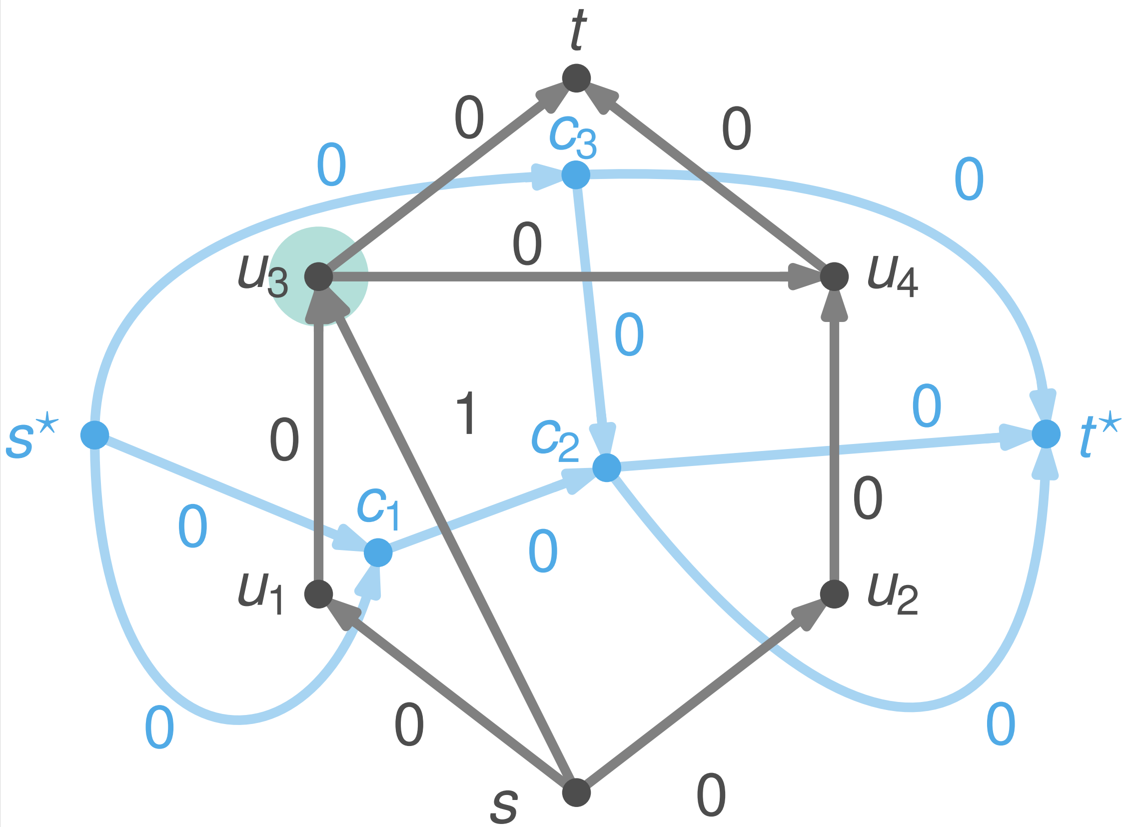

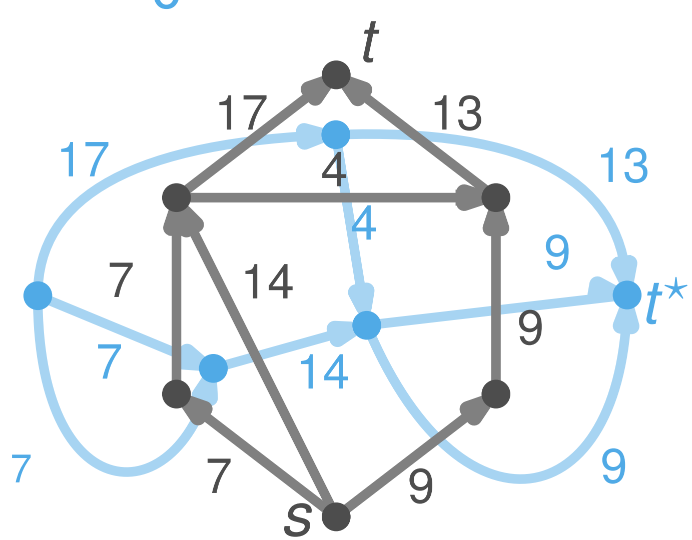

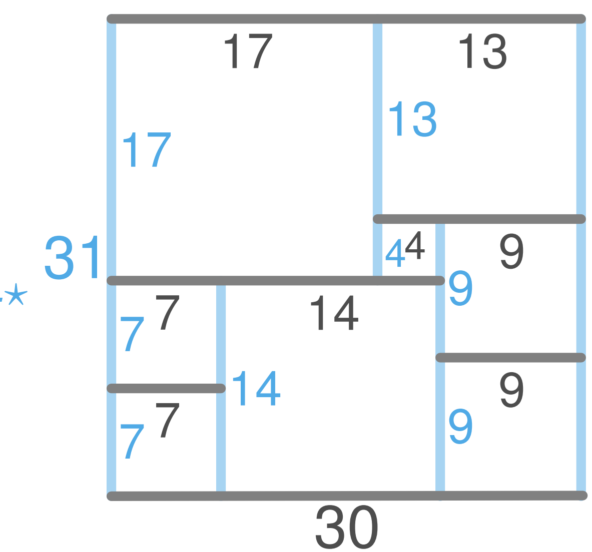

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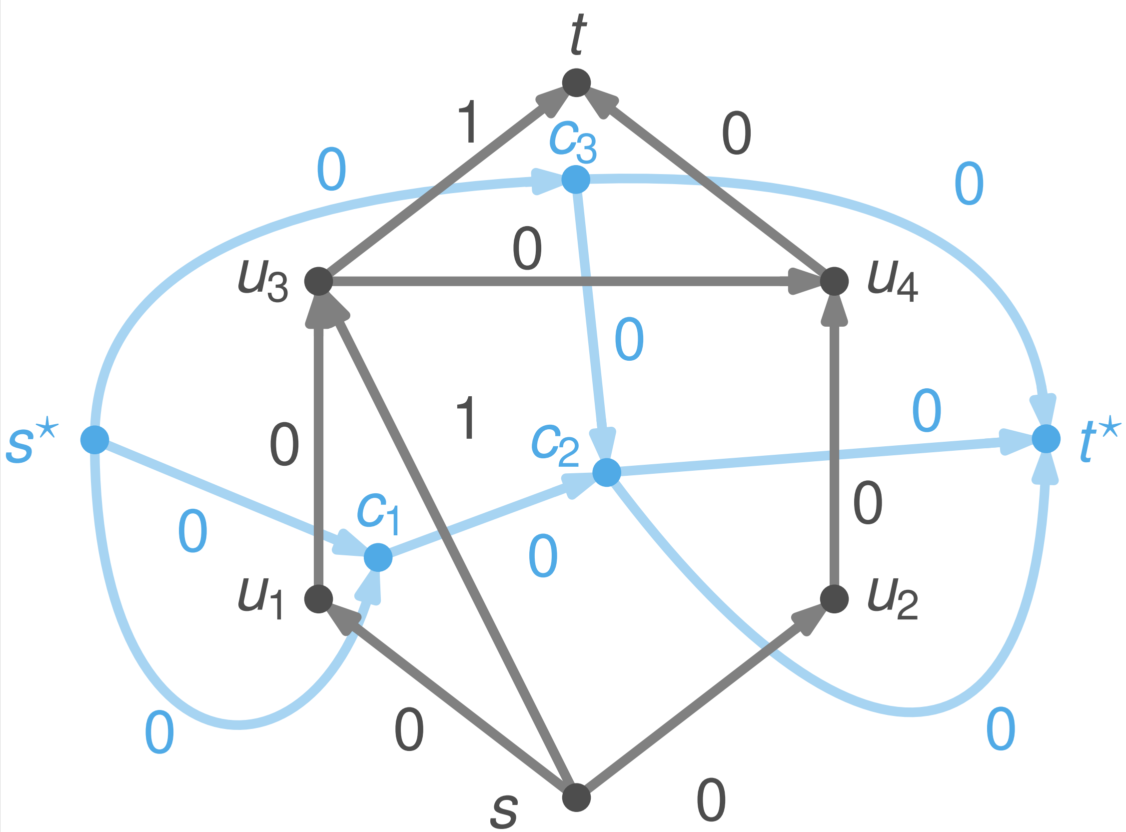

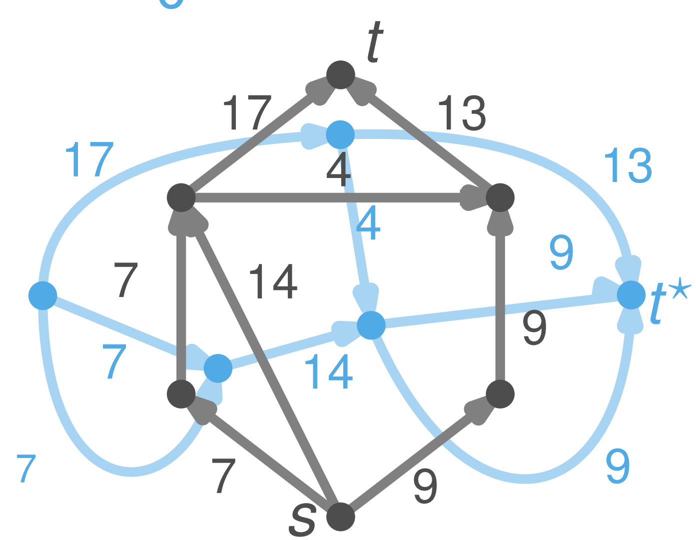

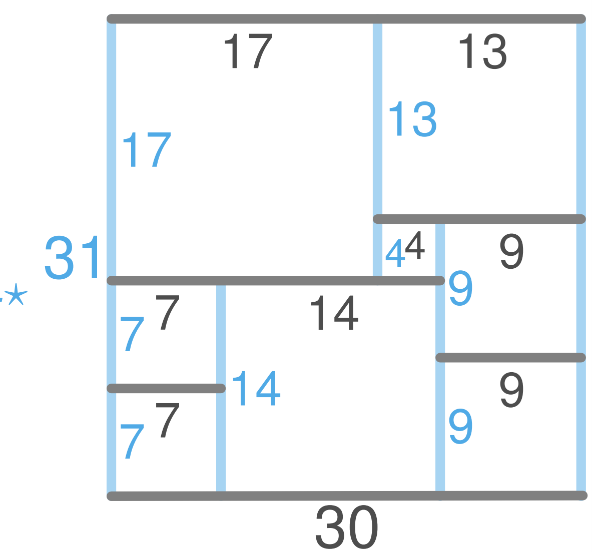

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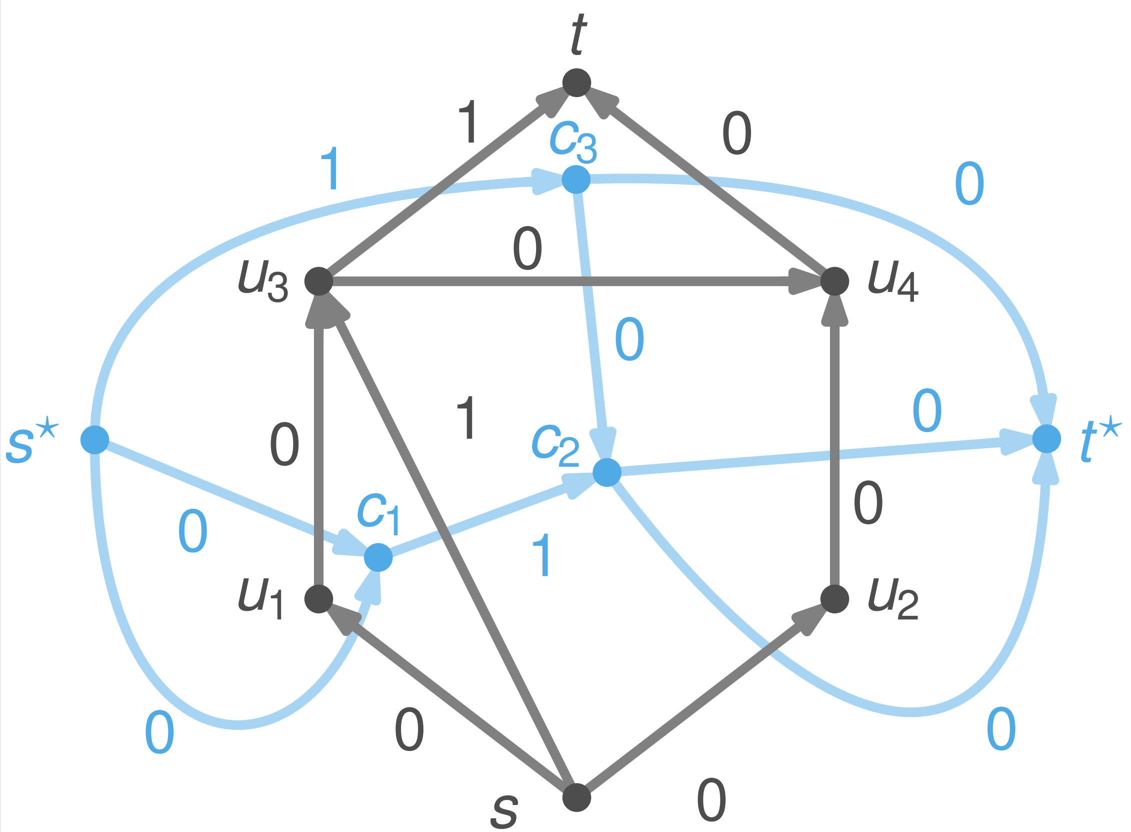

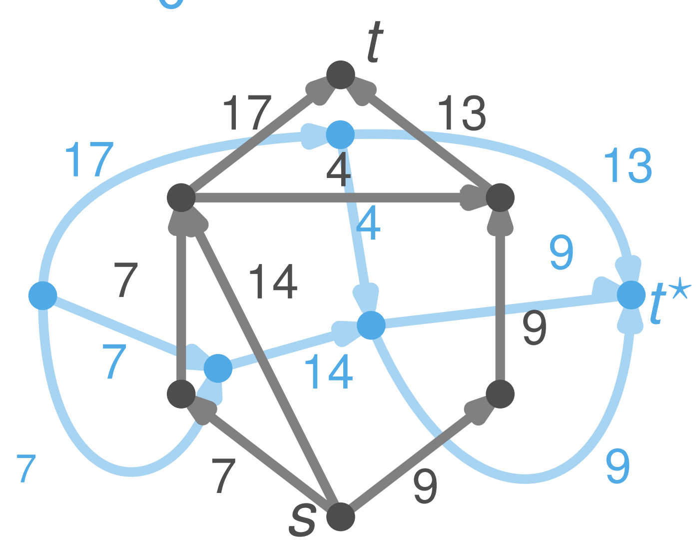

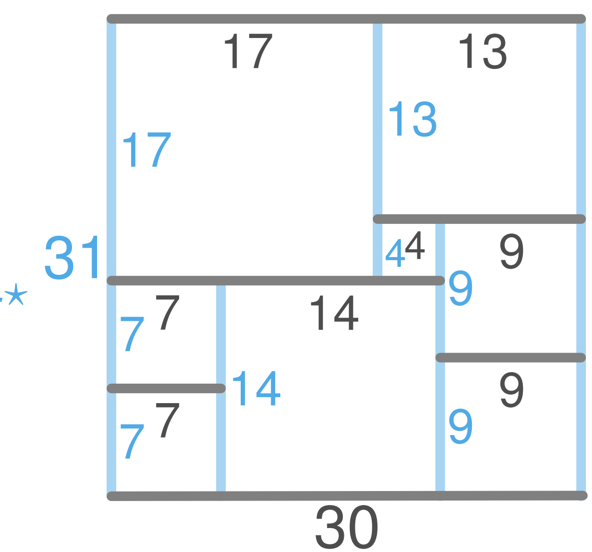

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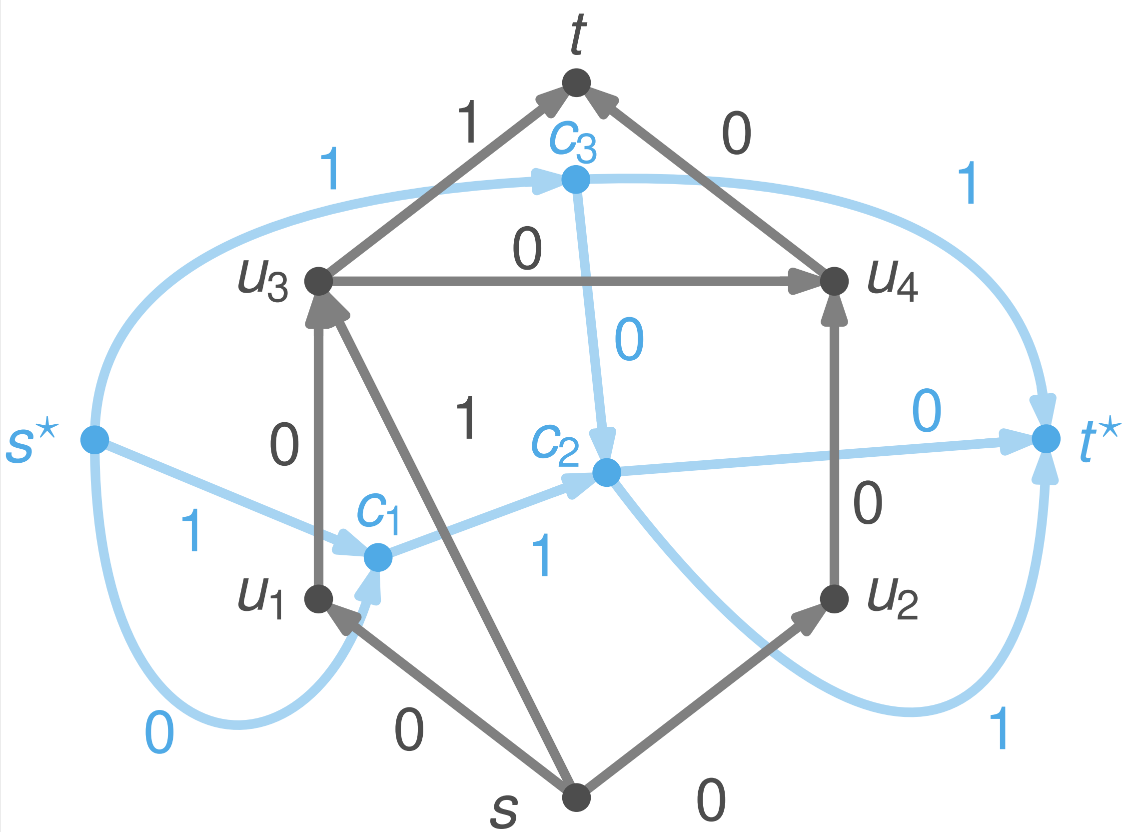

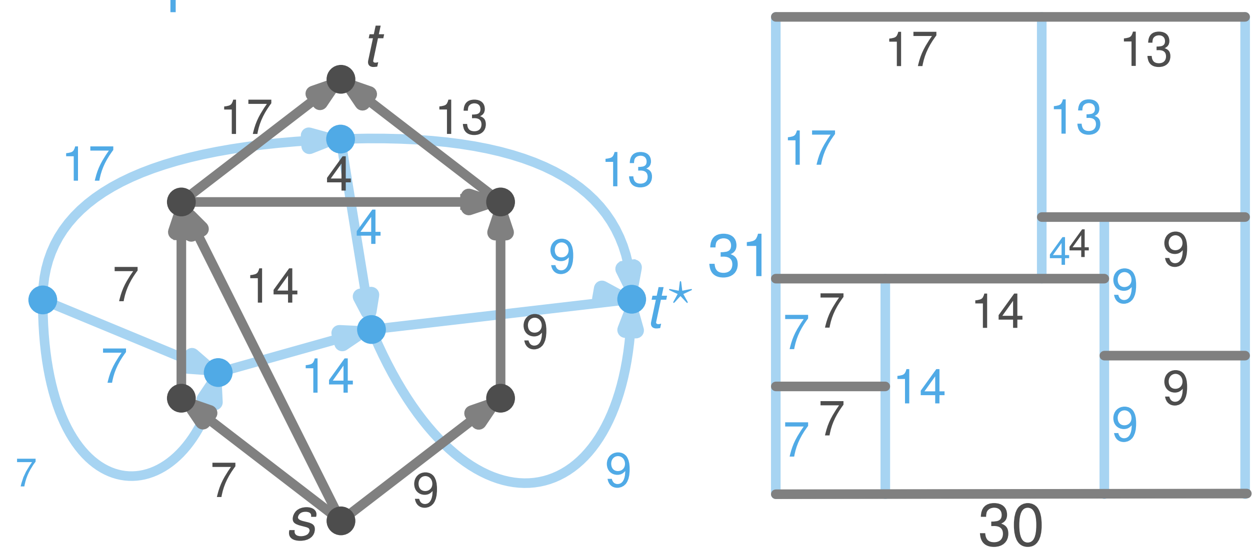

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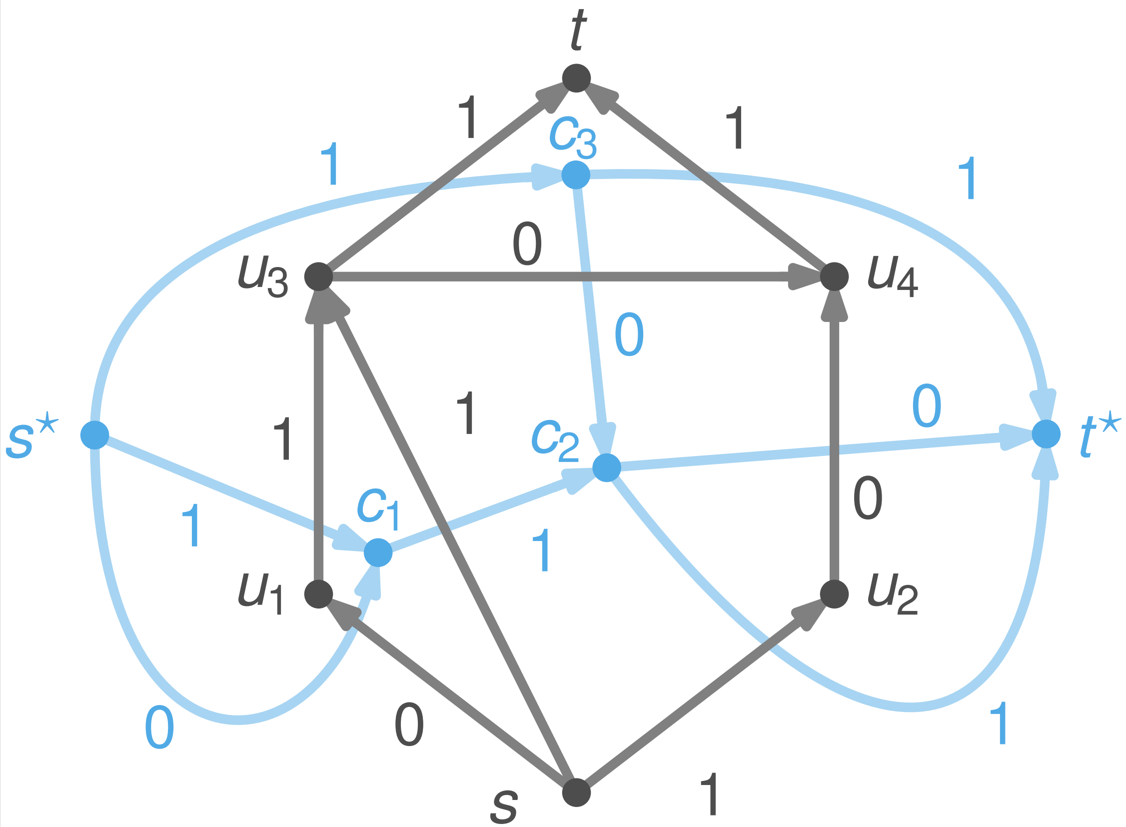

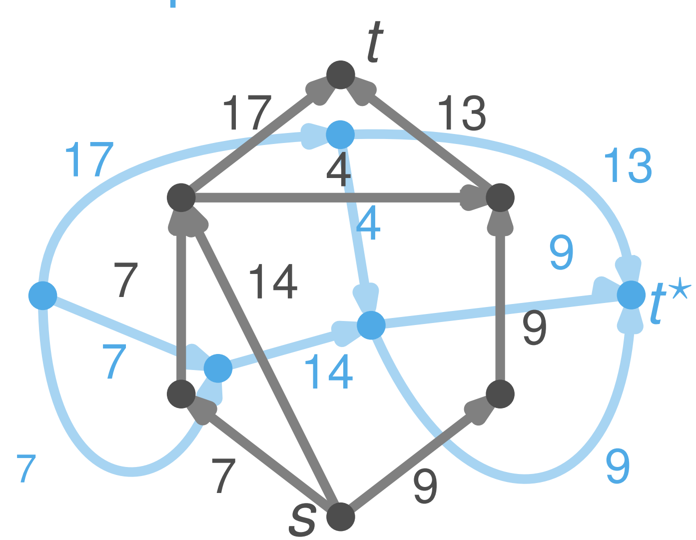

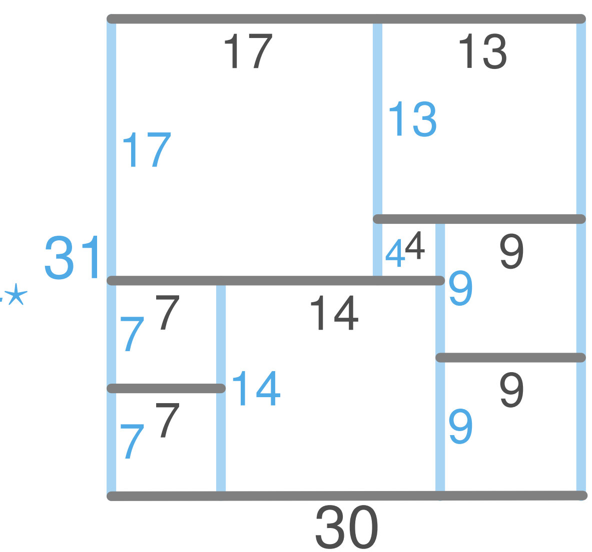

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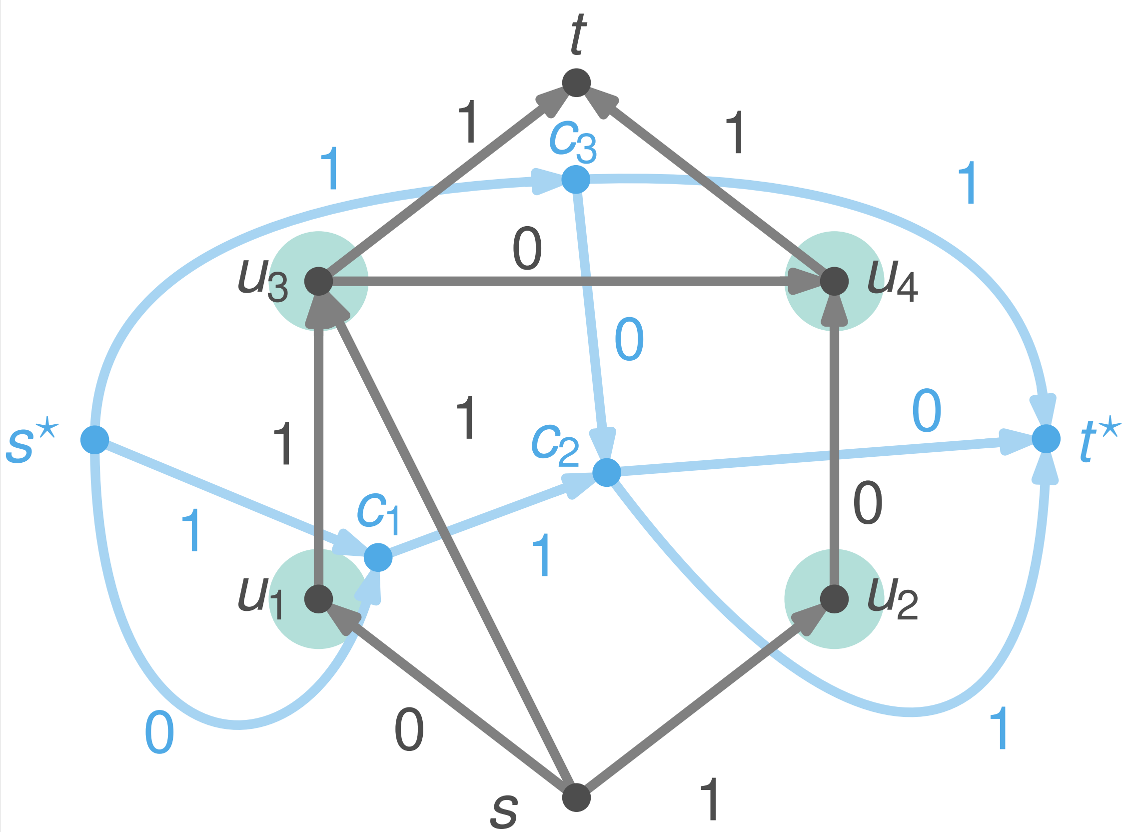
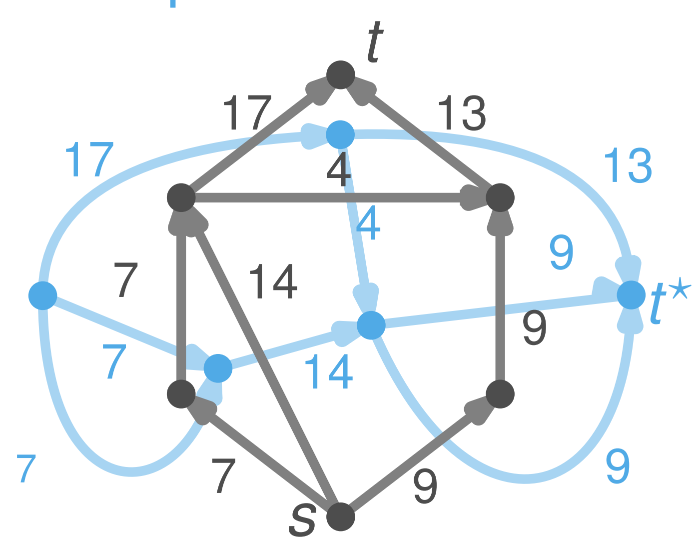
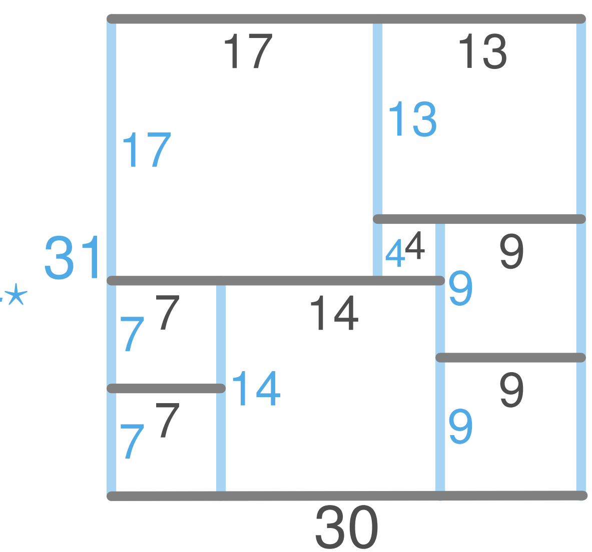
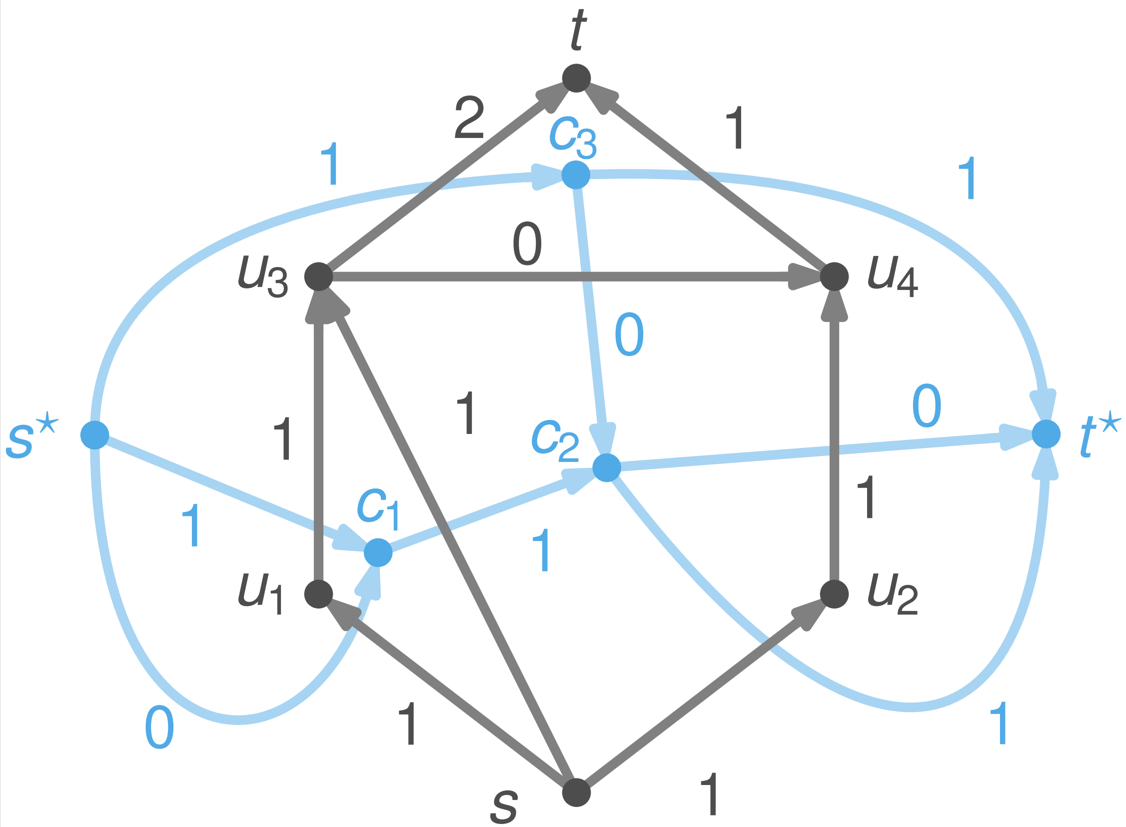
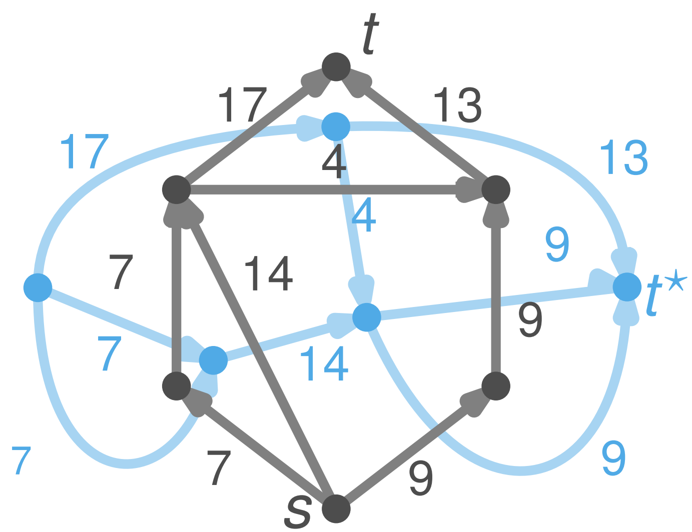


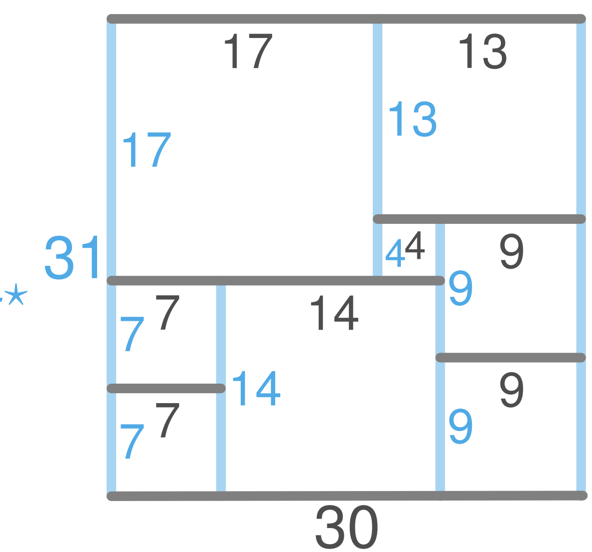




Wrong Conflict Resolution







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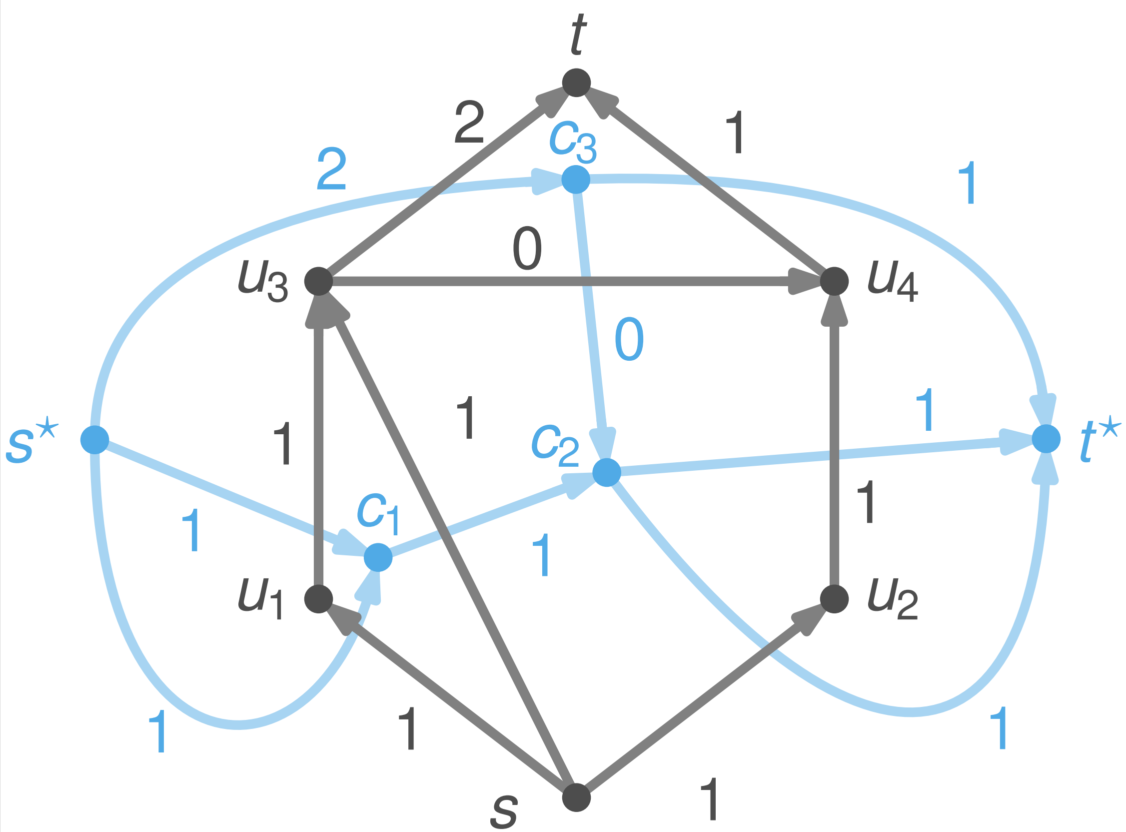

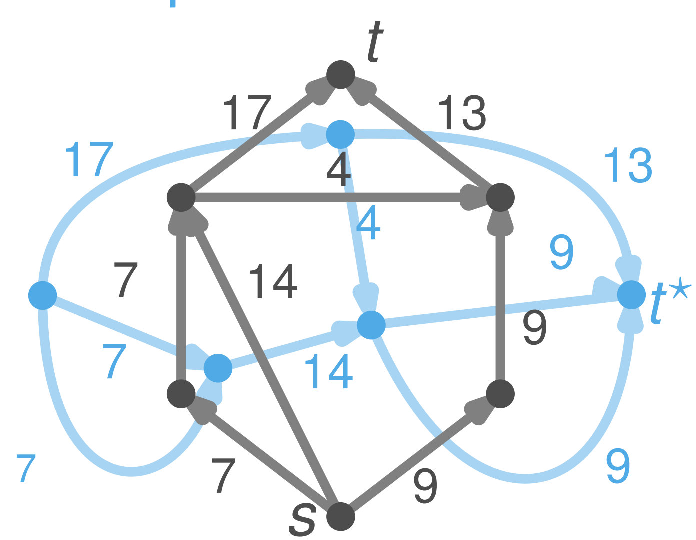

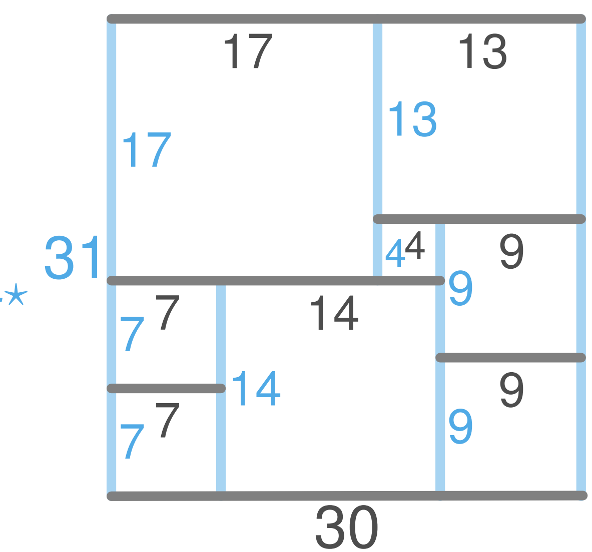

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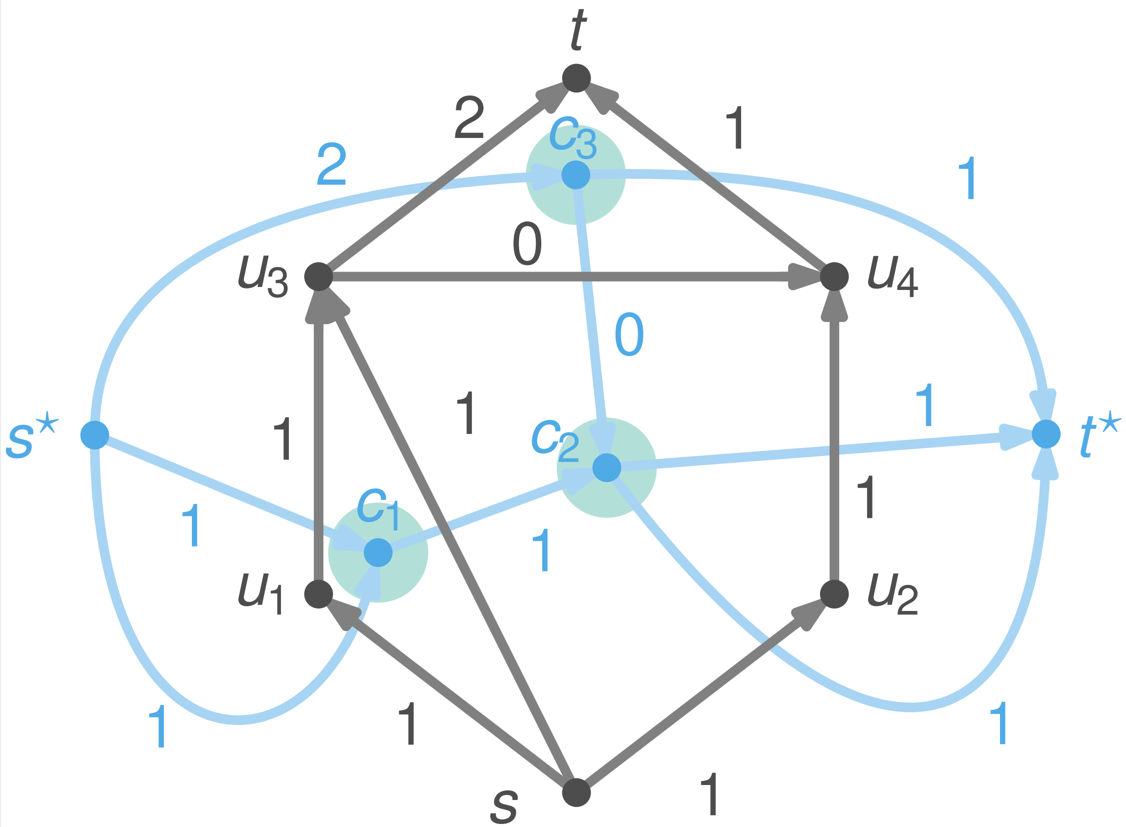

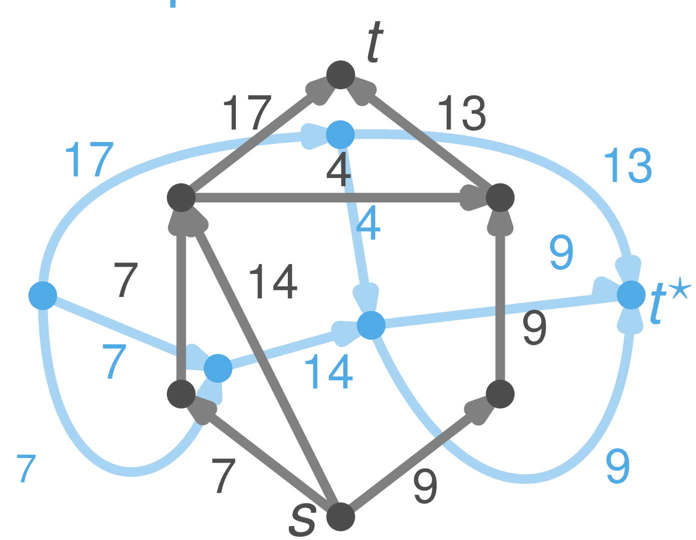

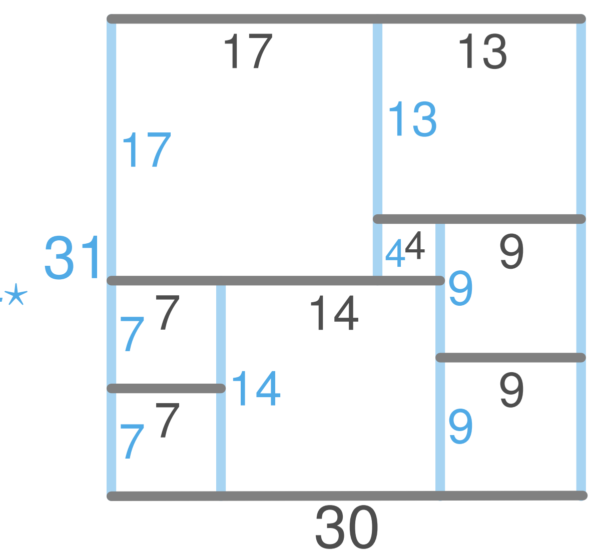

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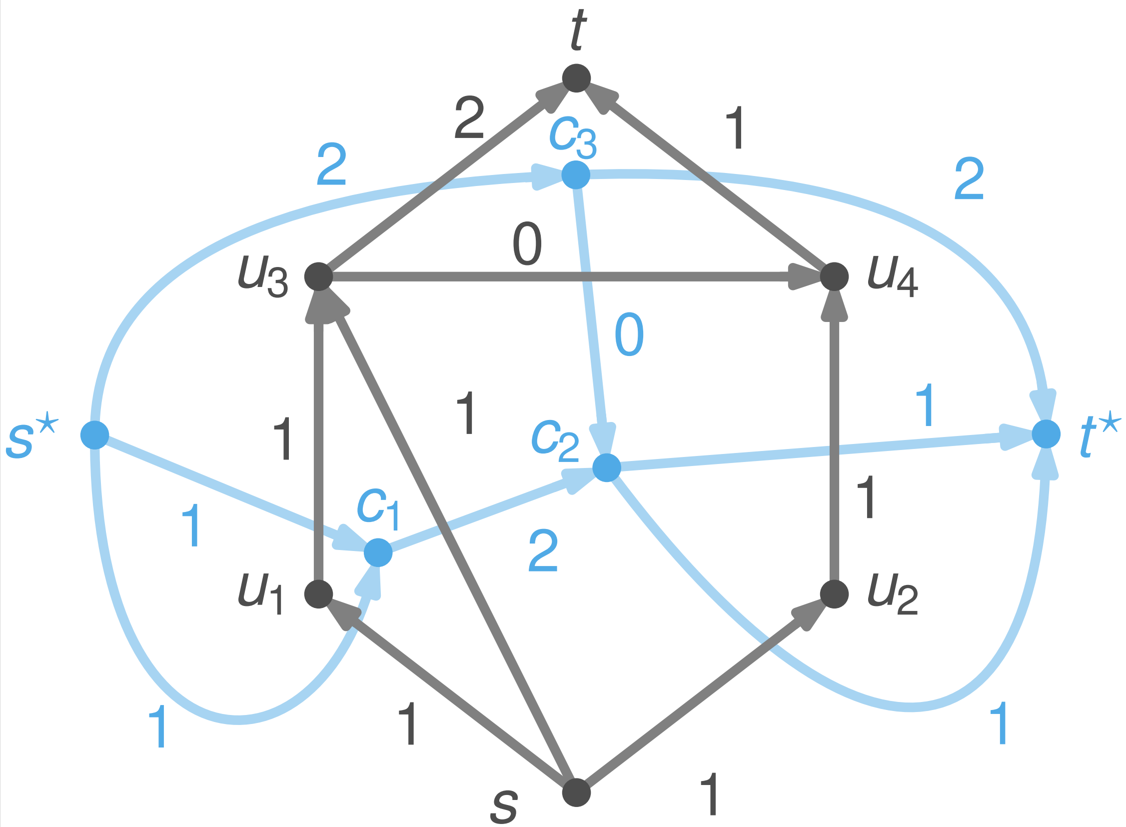

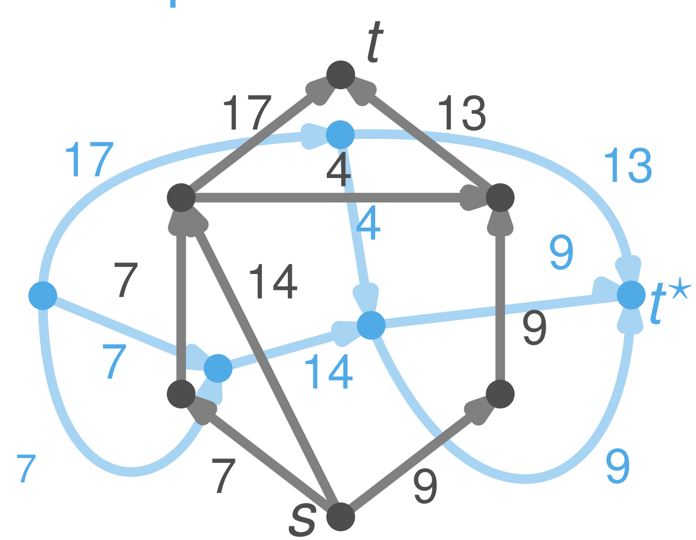

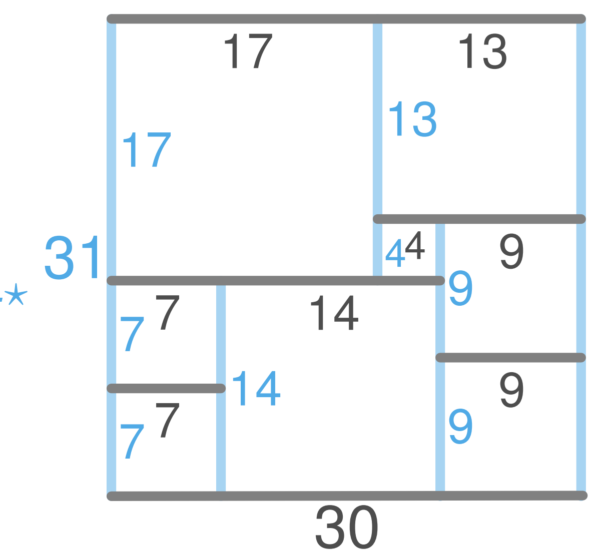

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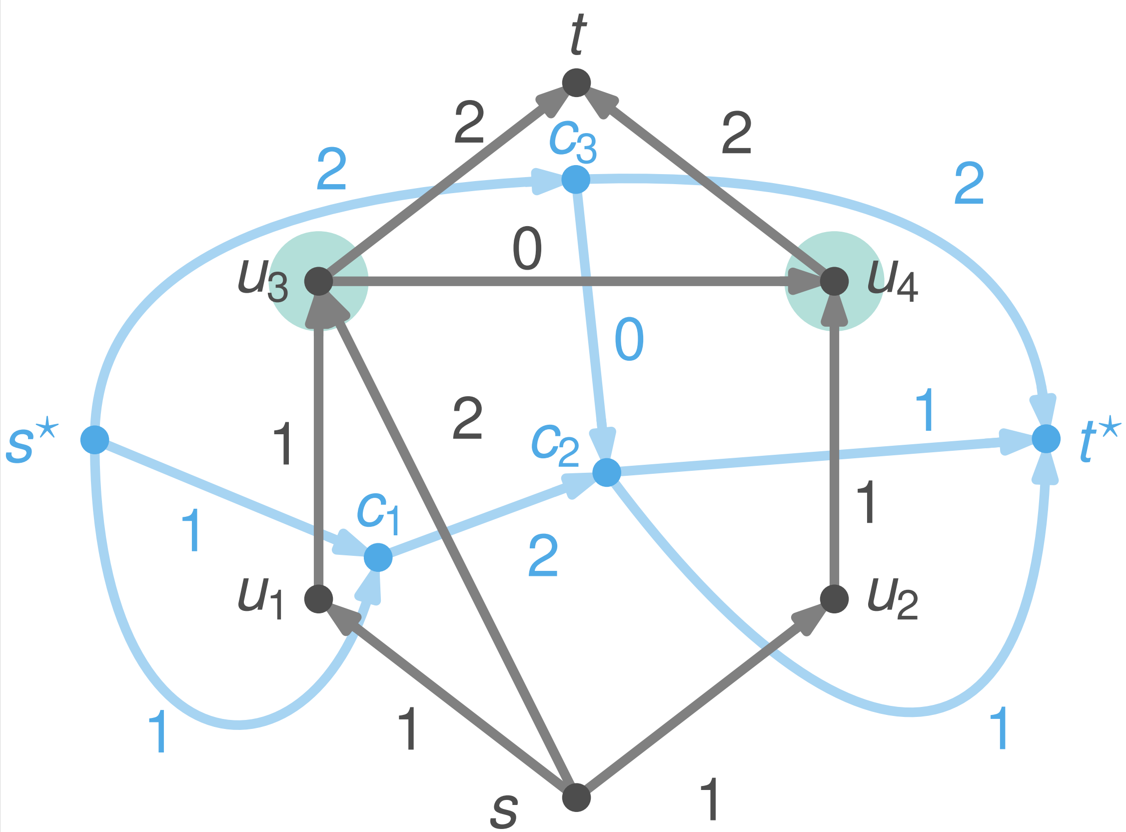

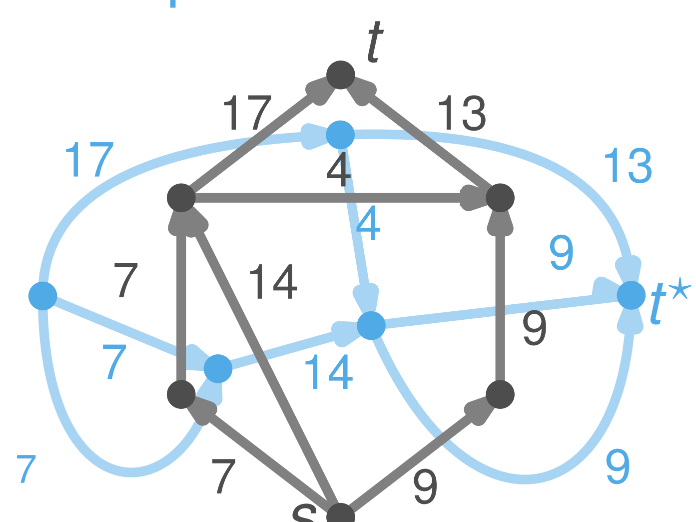

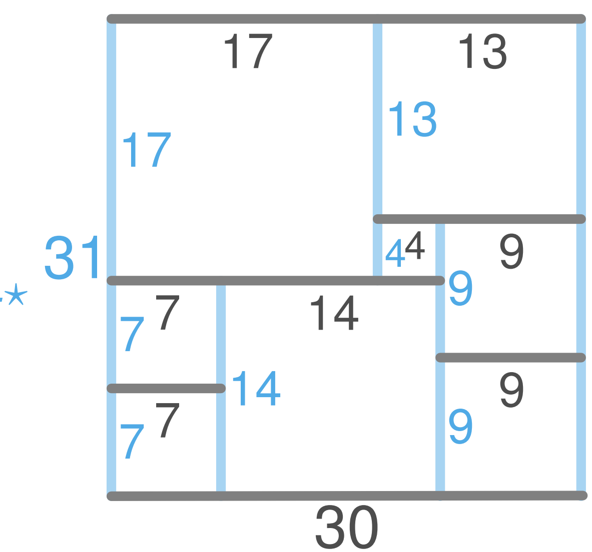

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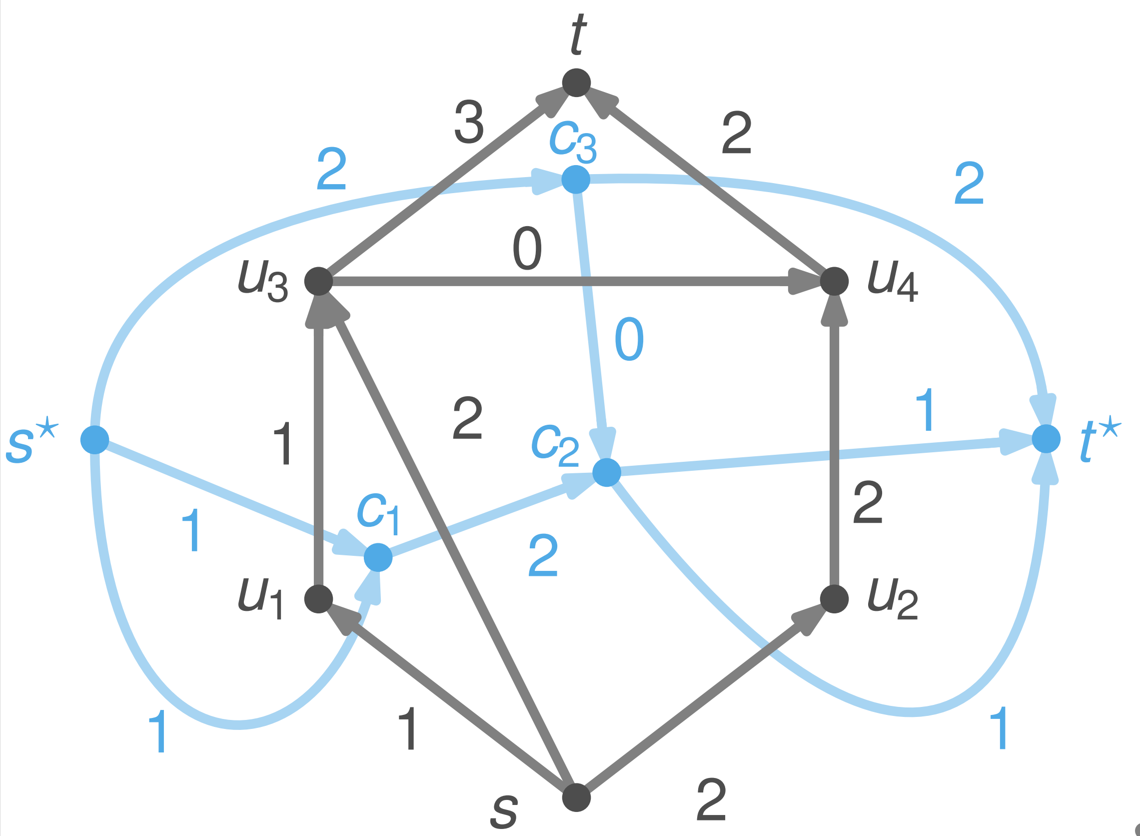

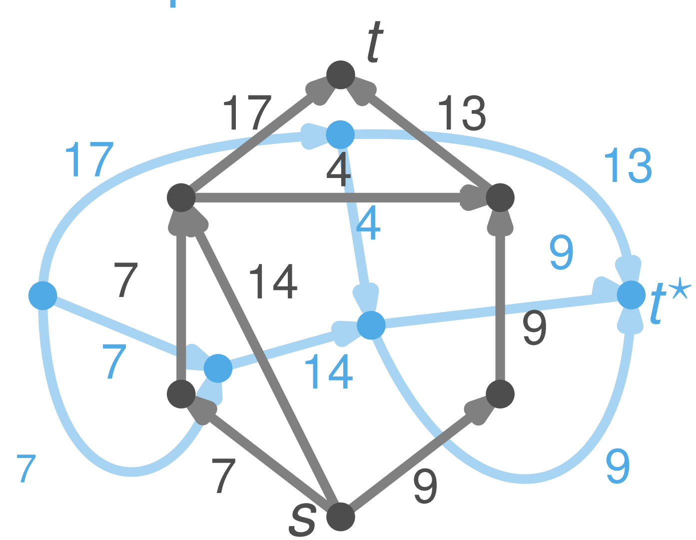

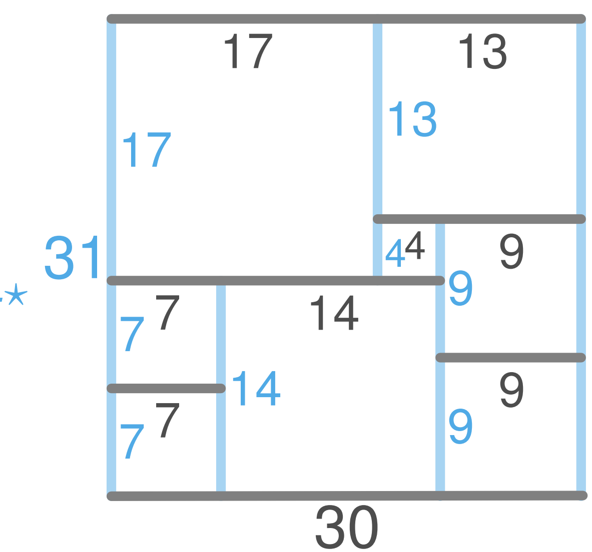

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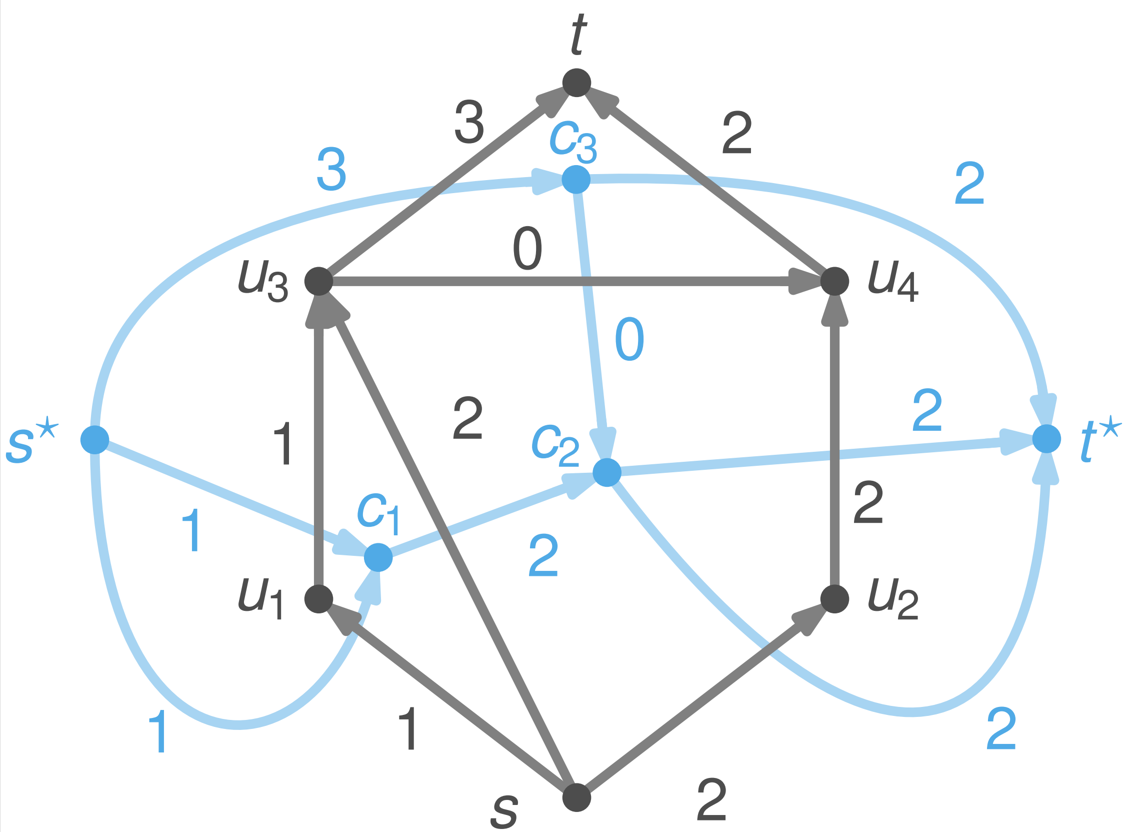

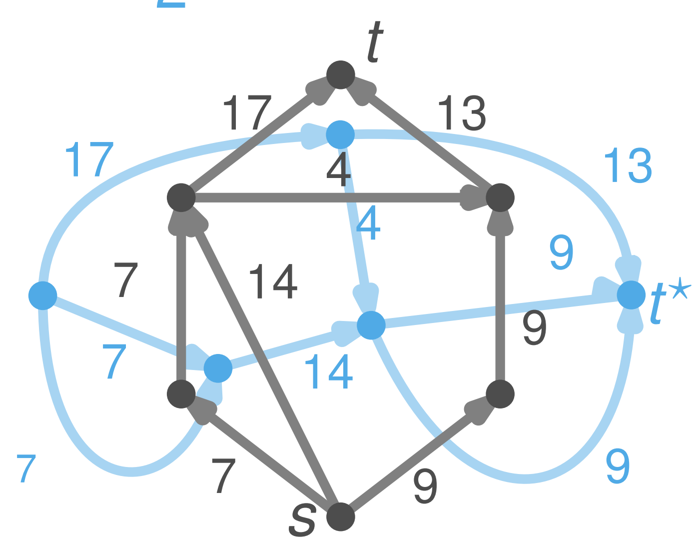

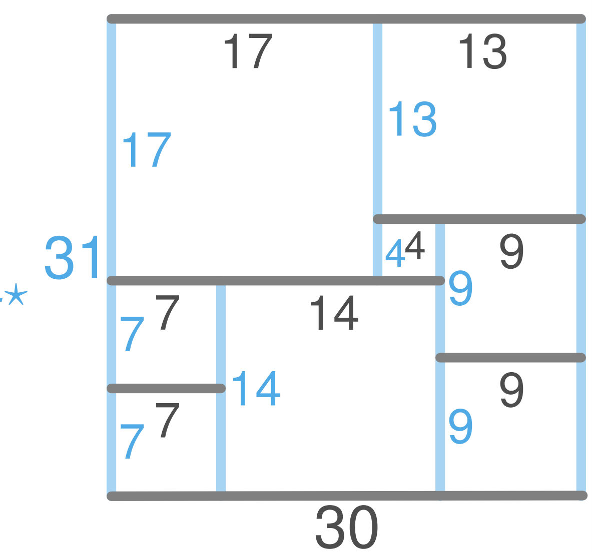

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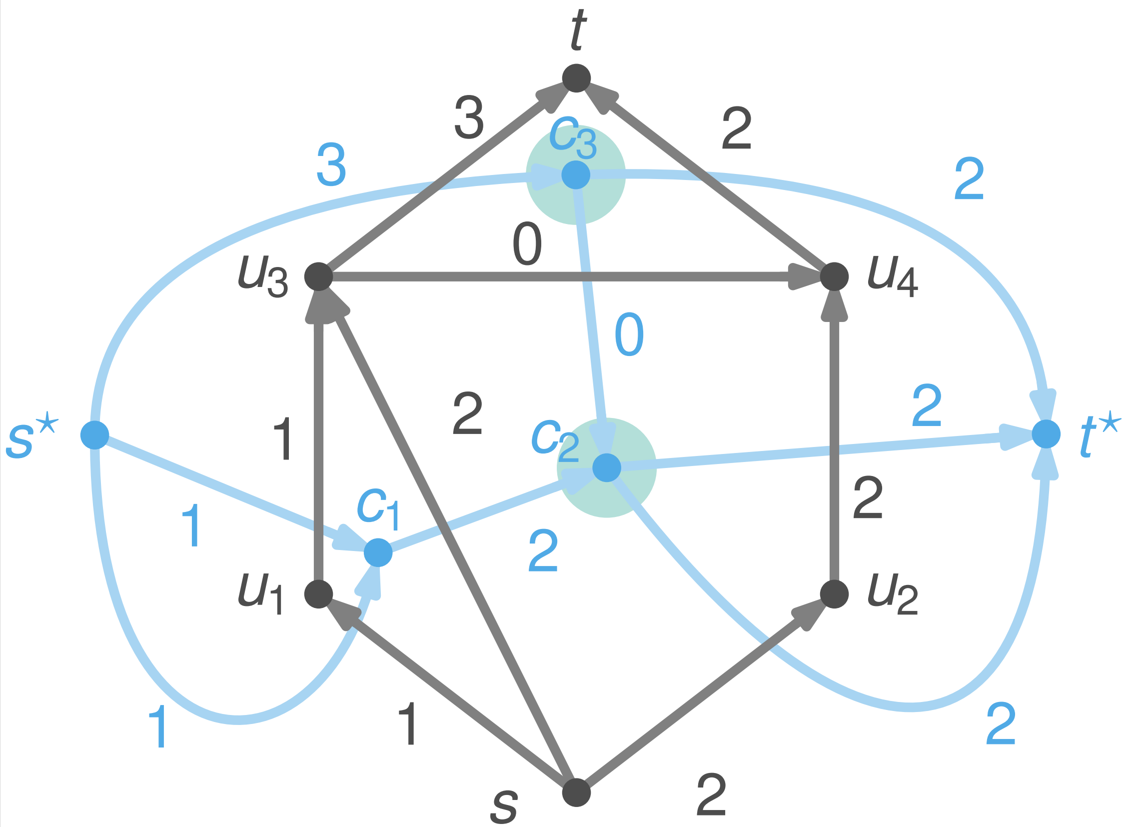

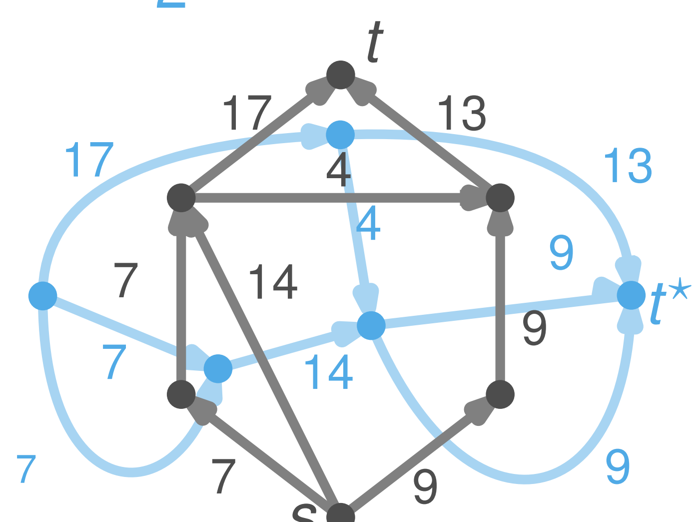

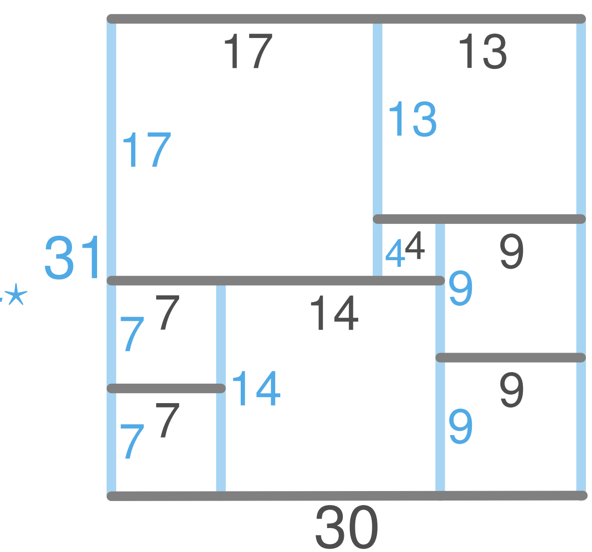

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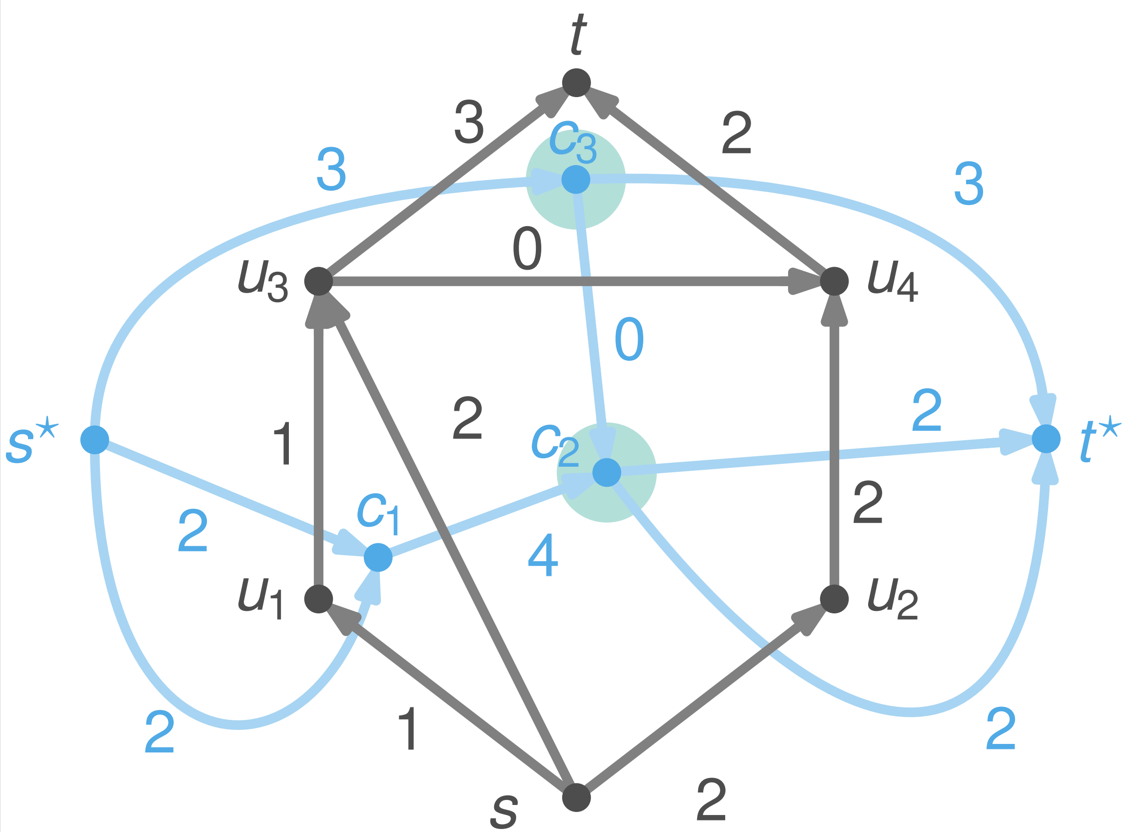

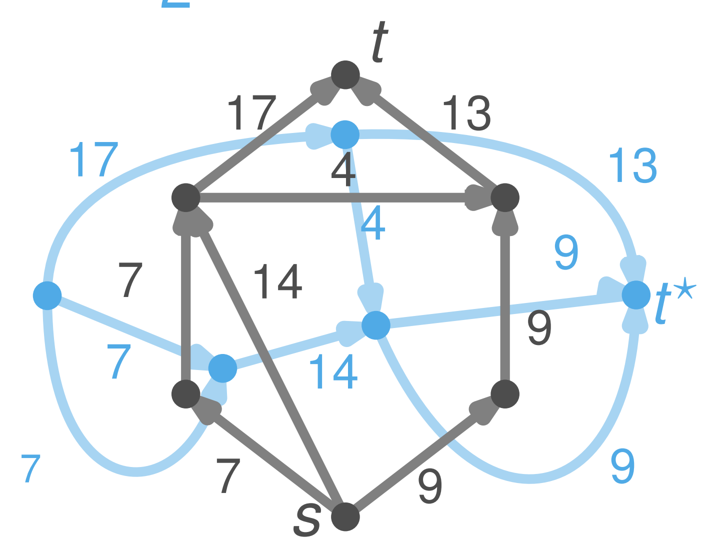

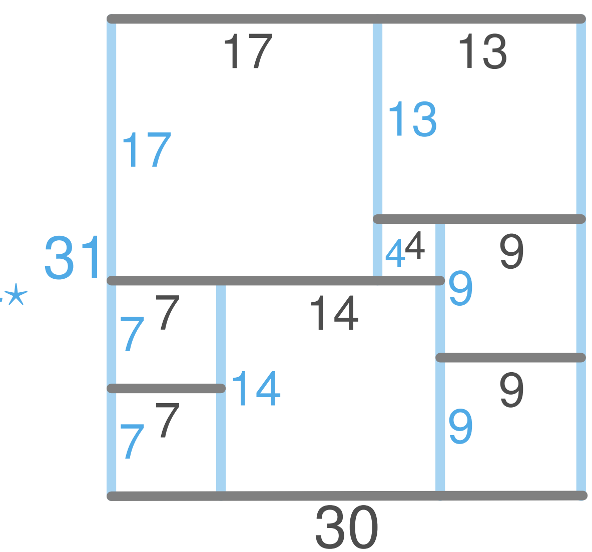

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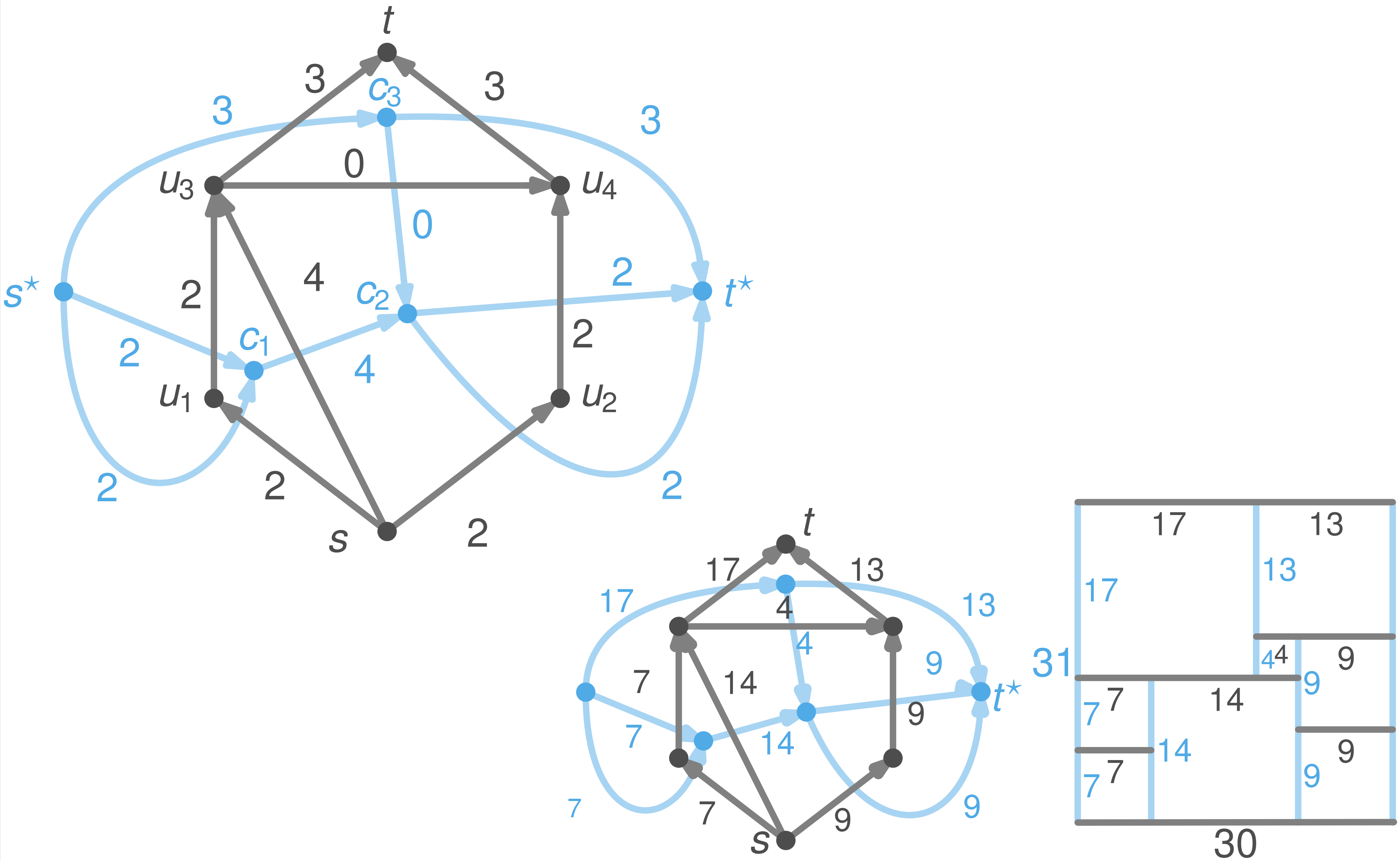

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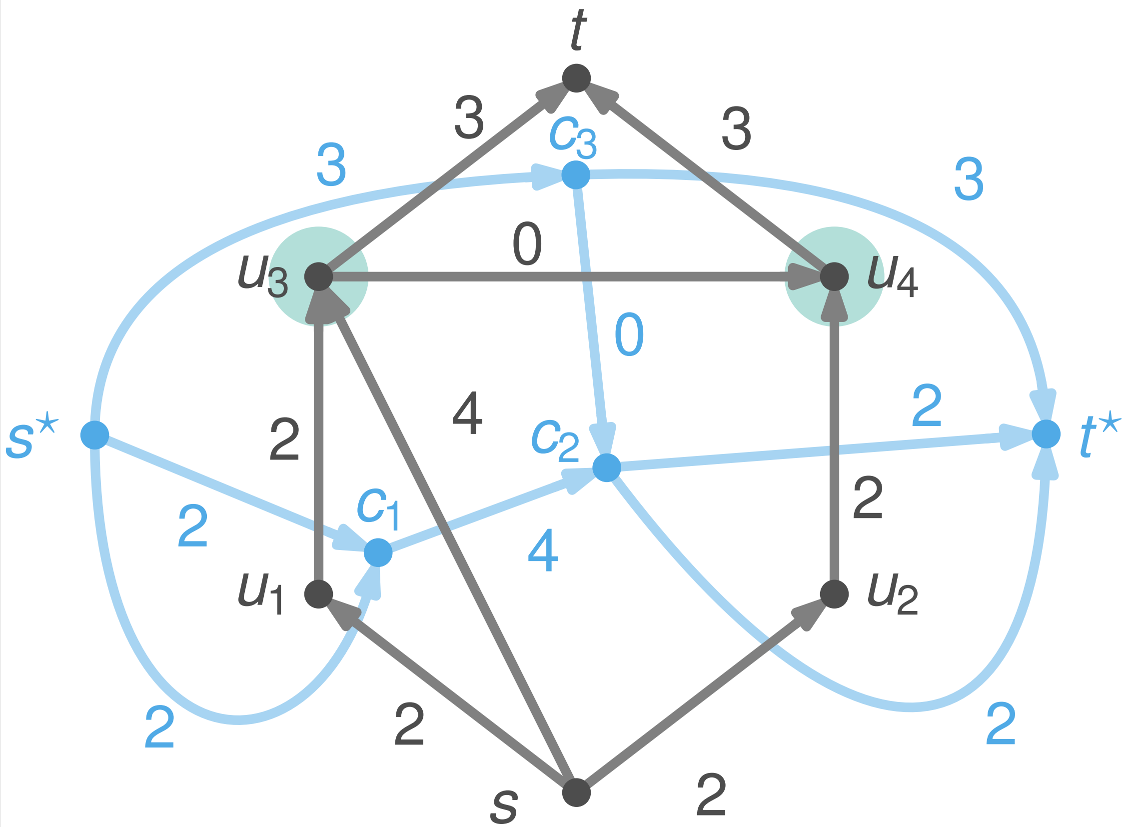

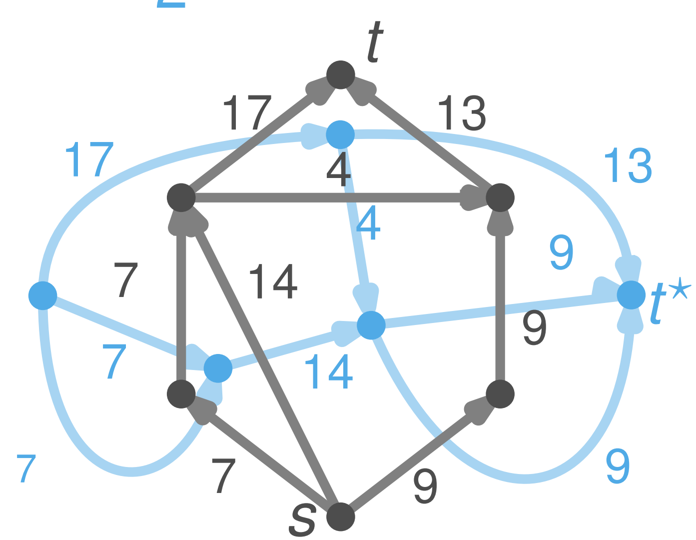

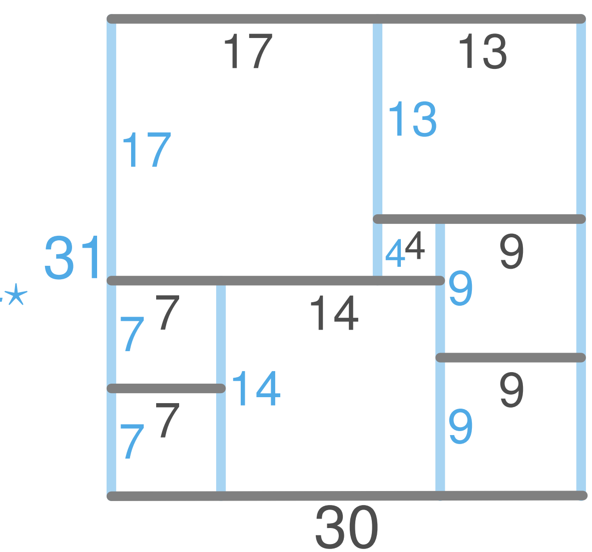

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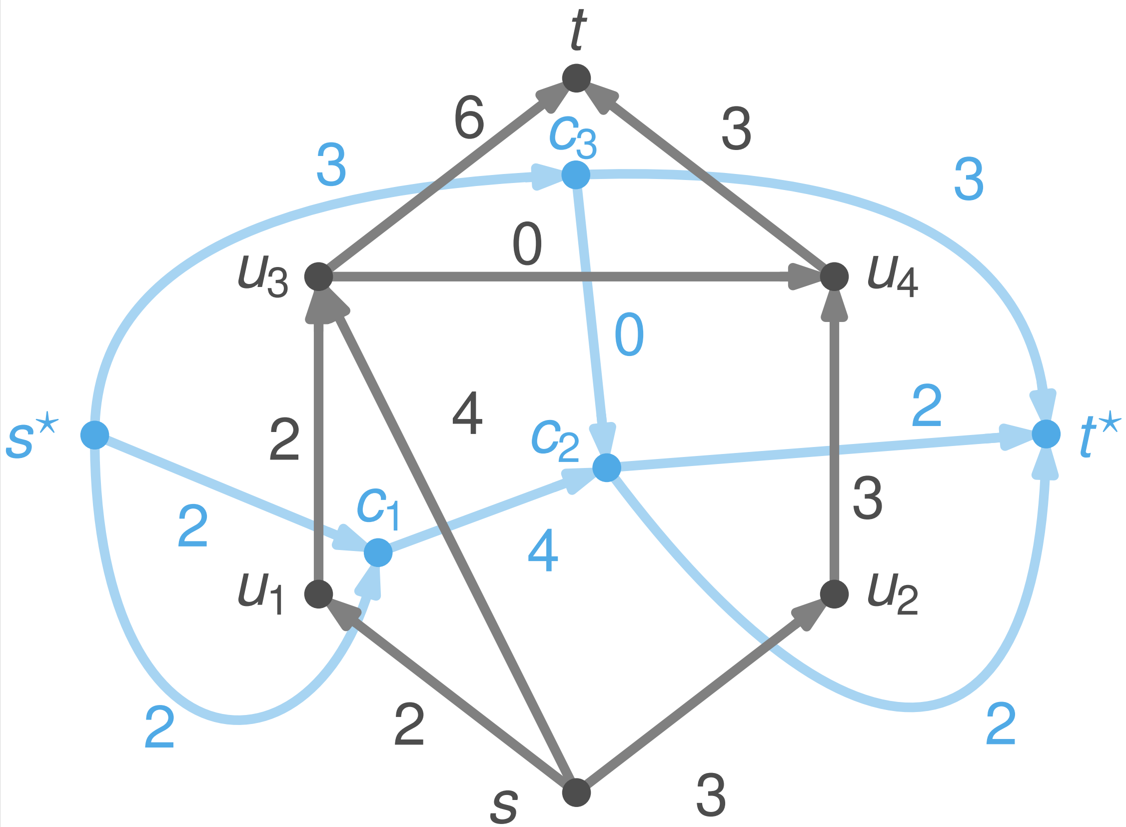

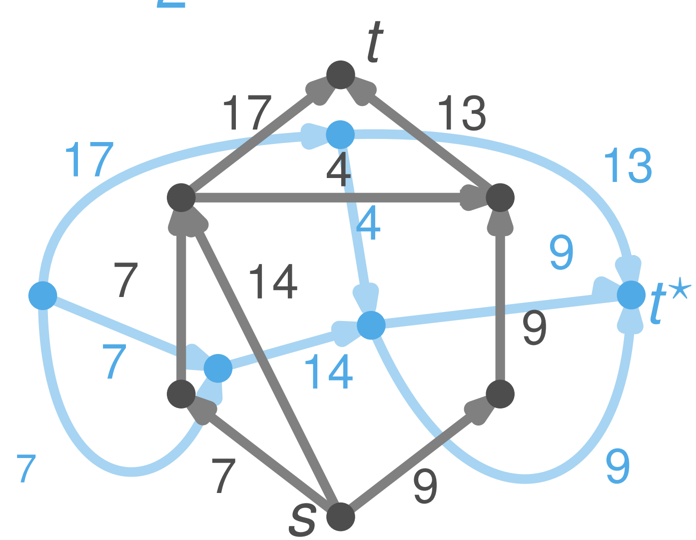

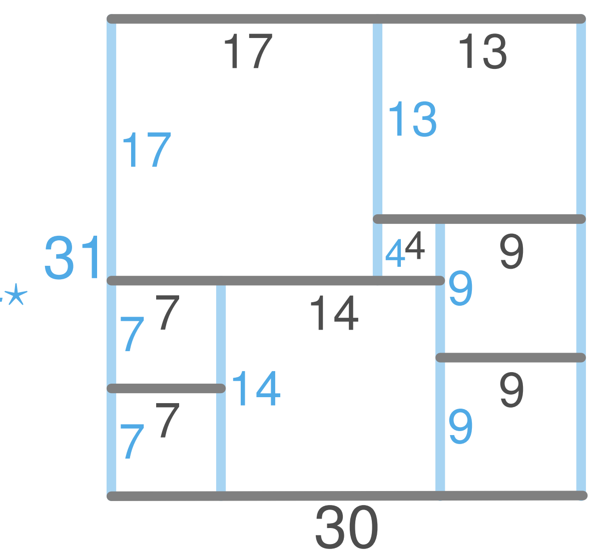

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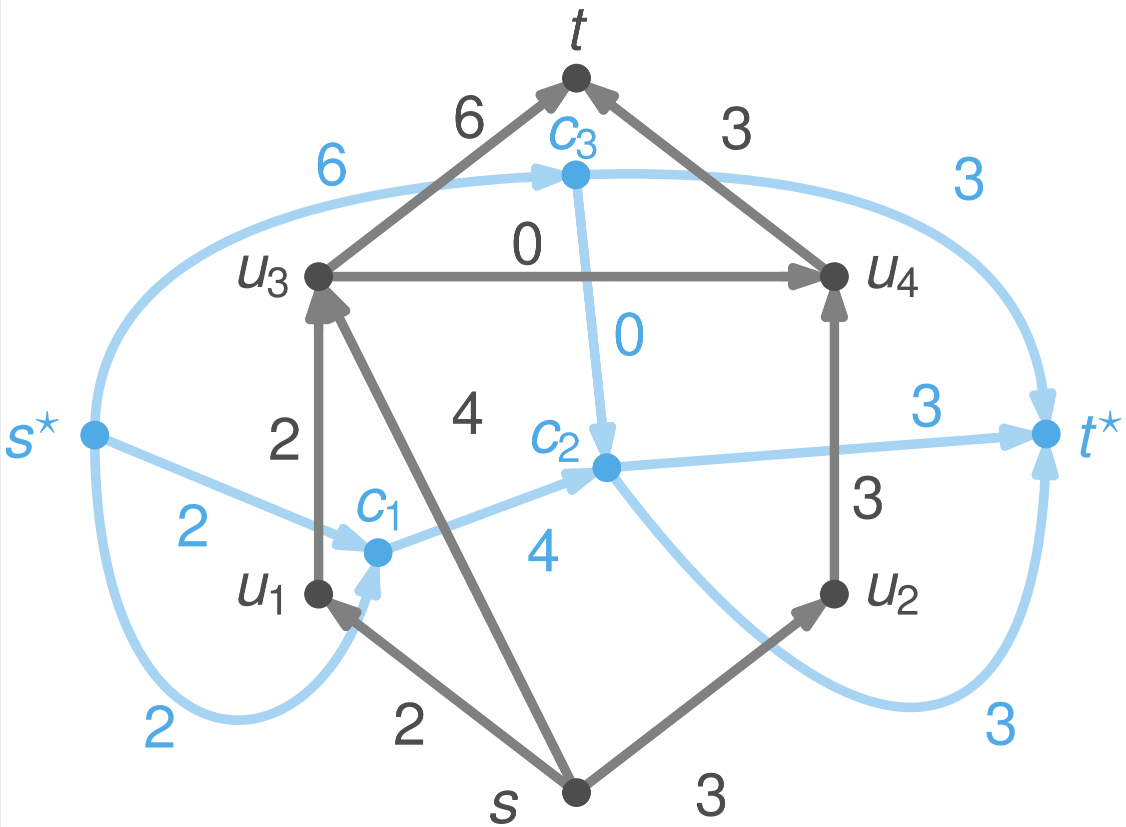

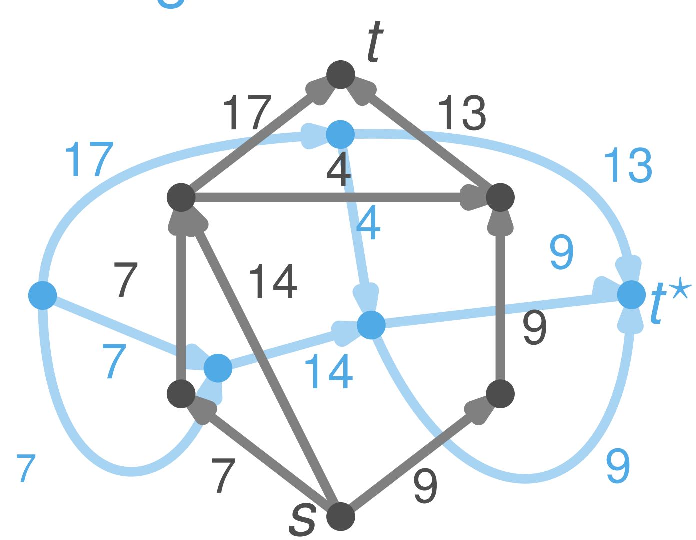

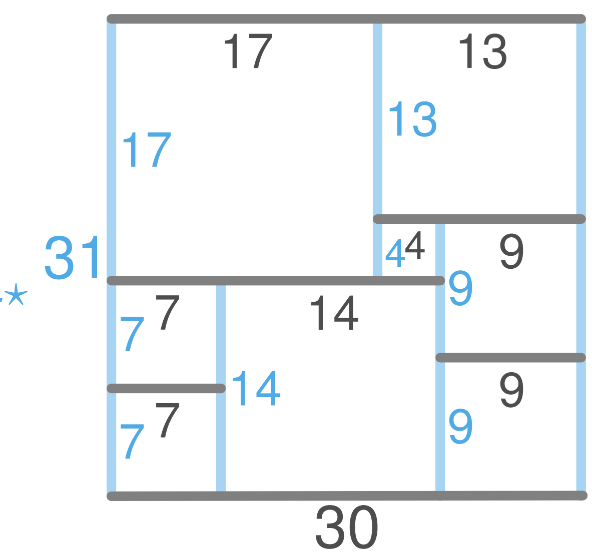

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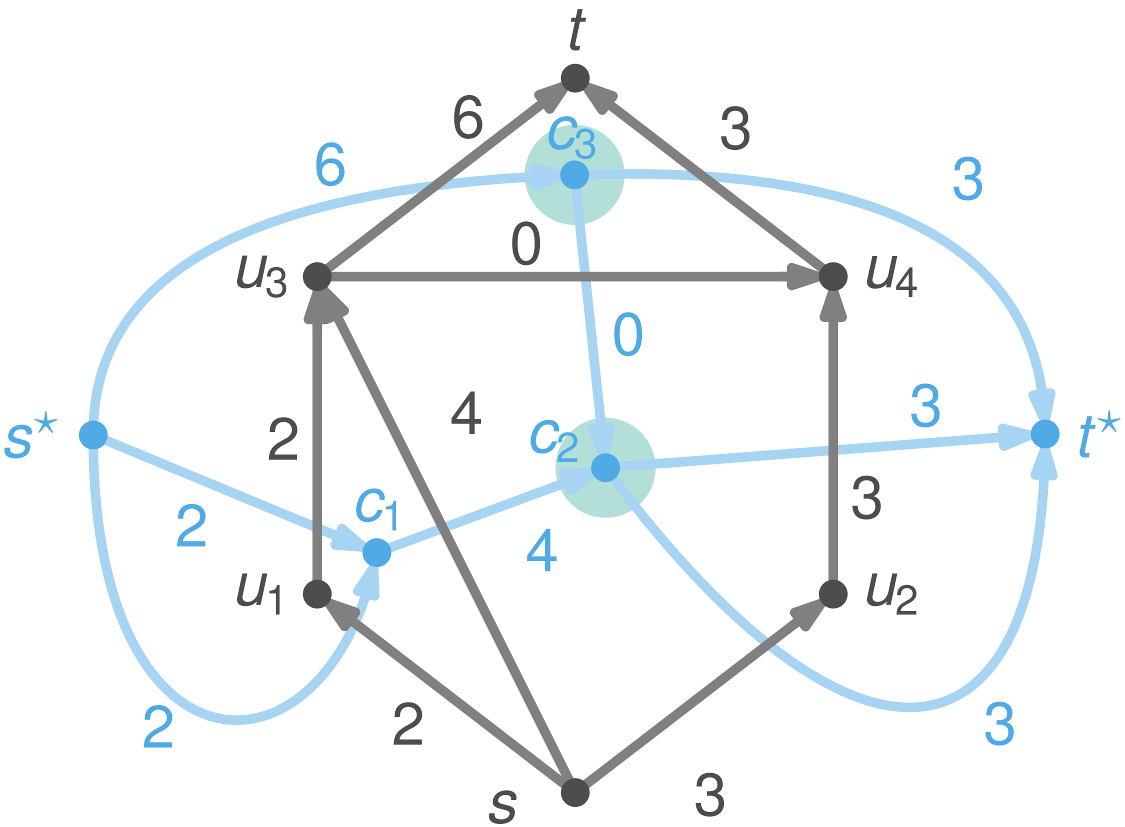

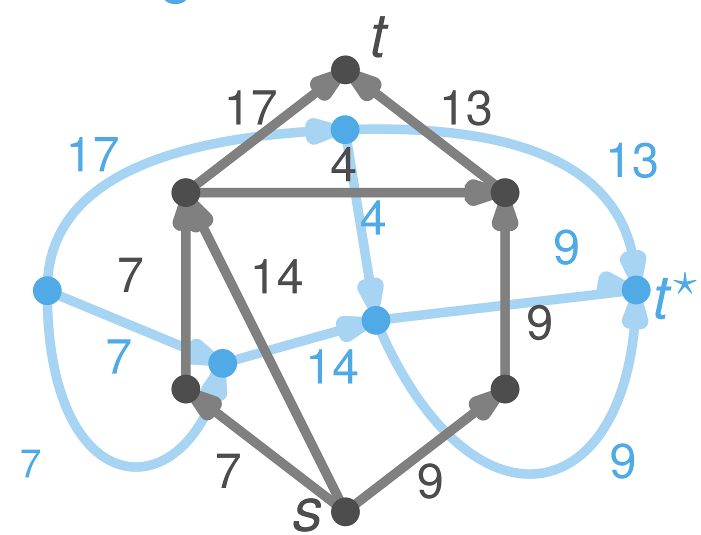

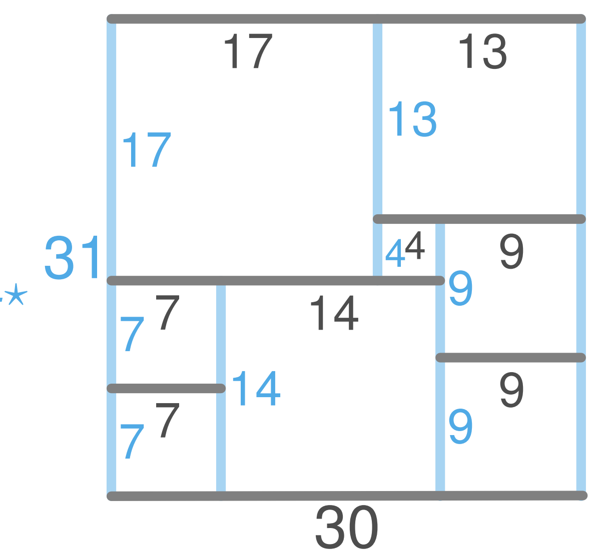

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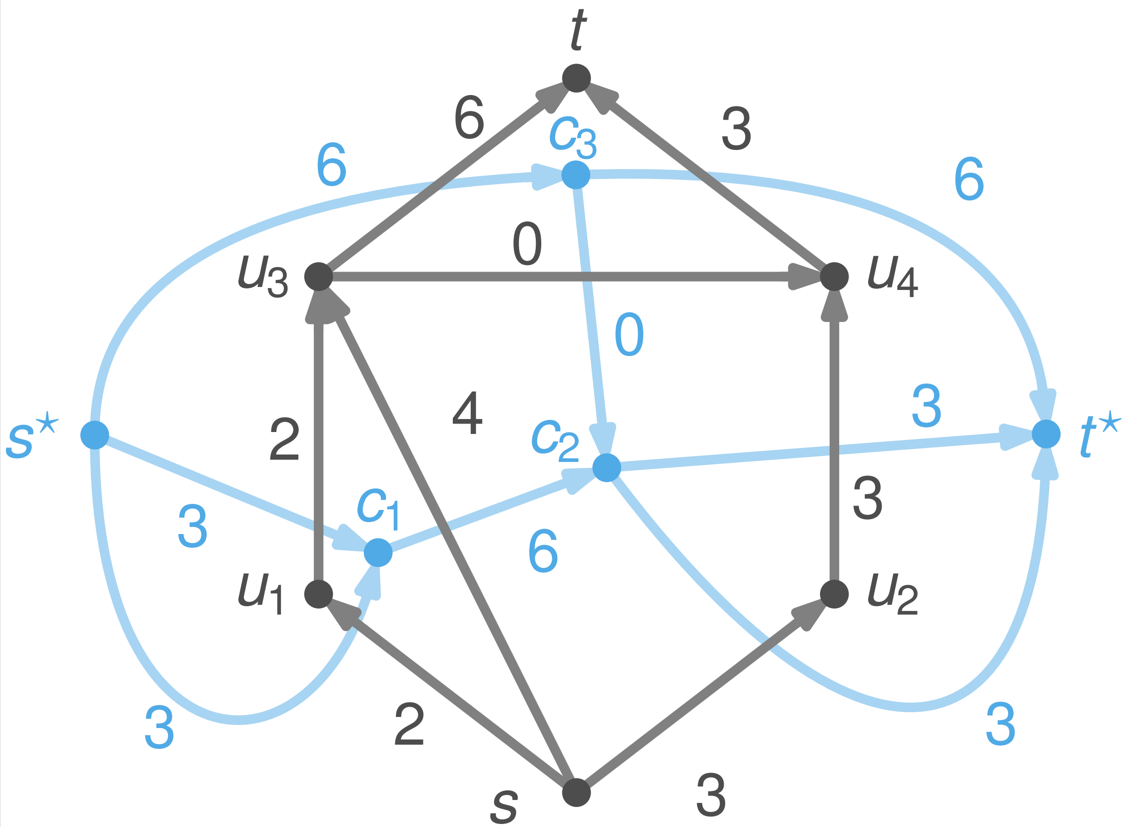

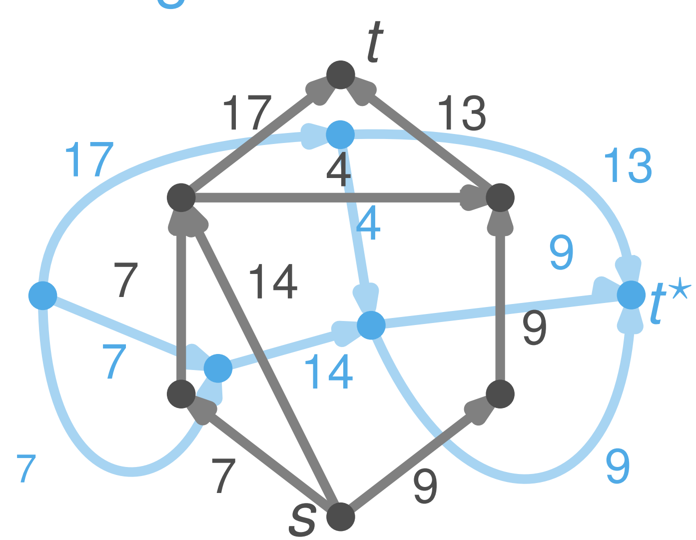

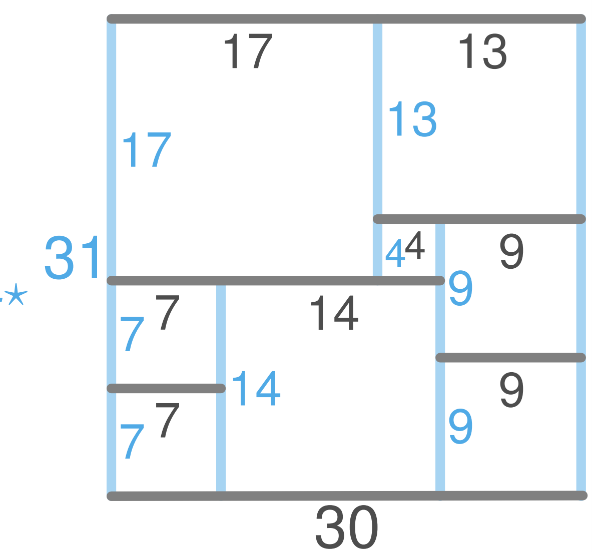

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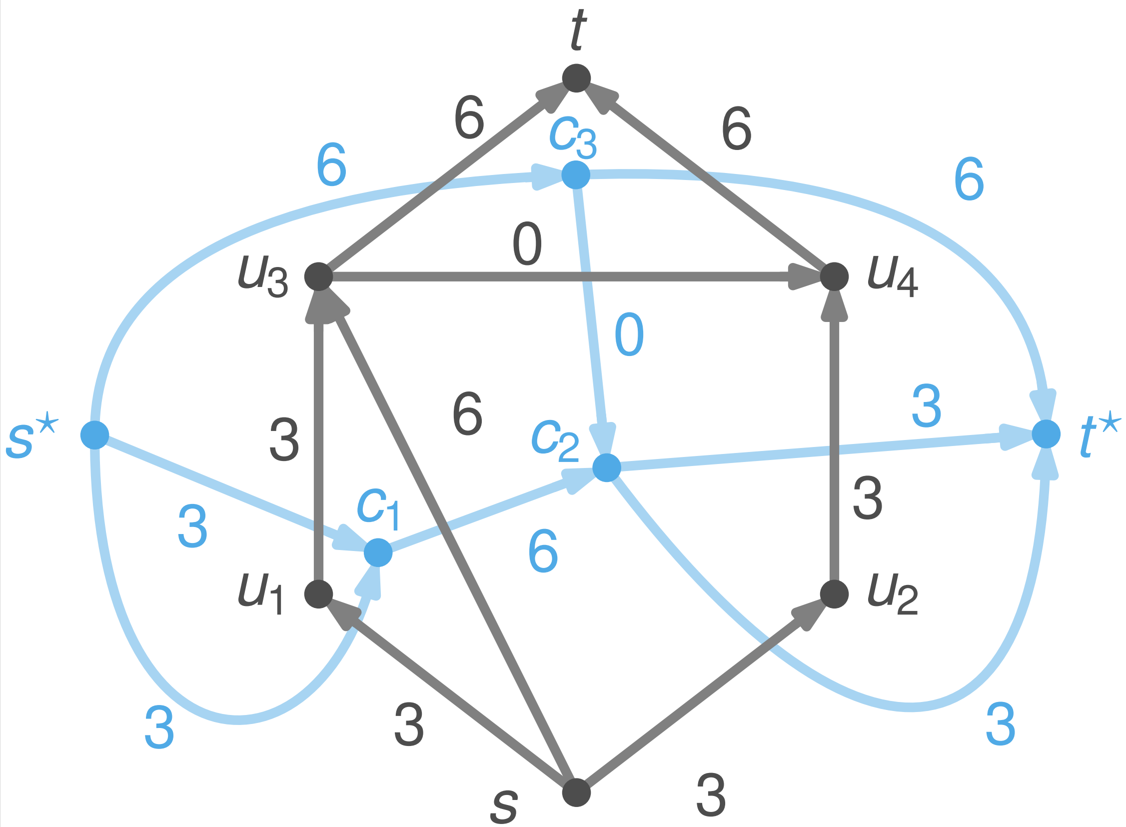

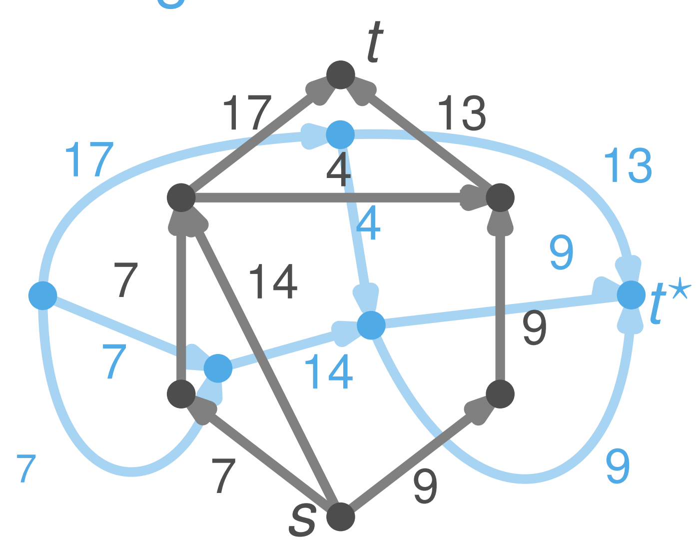

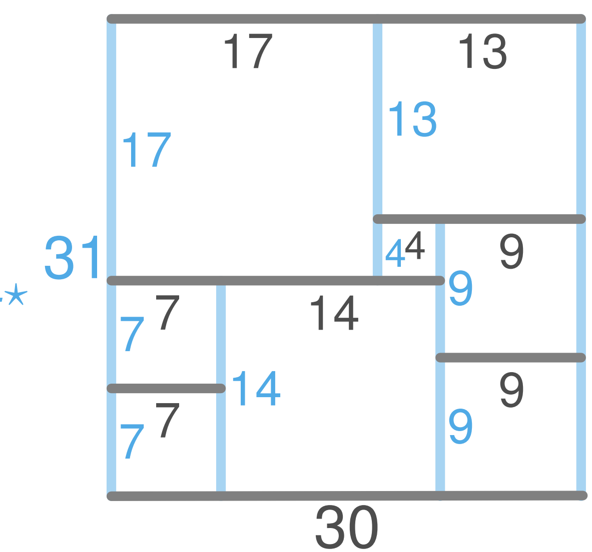

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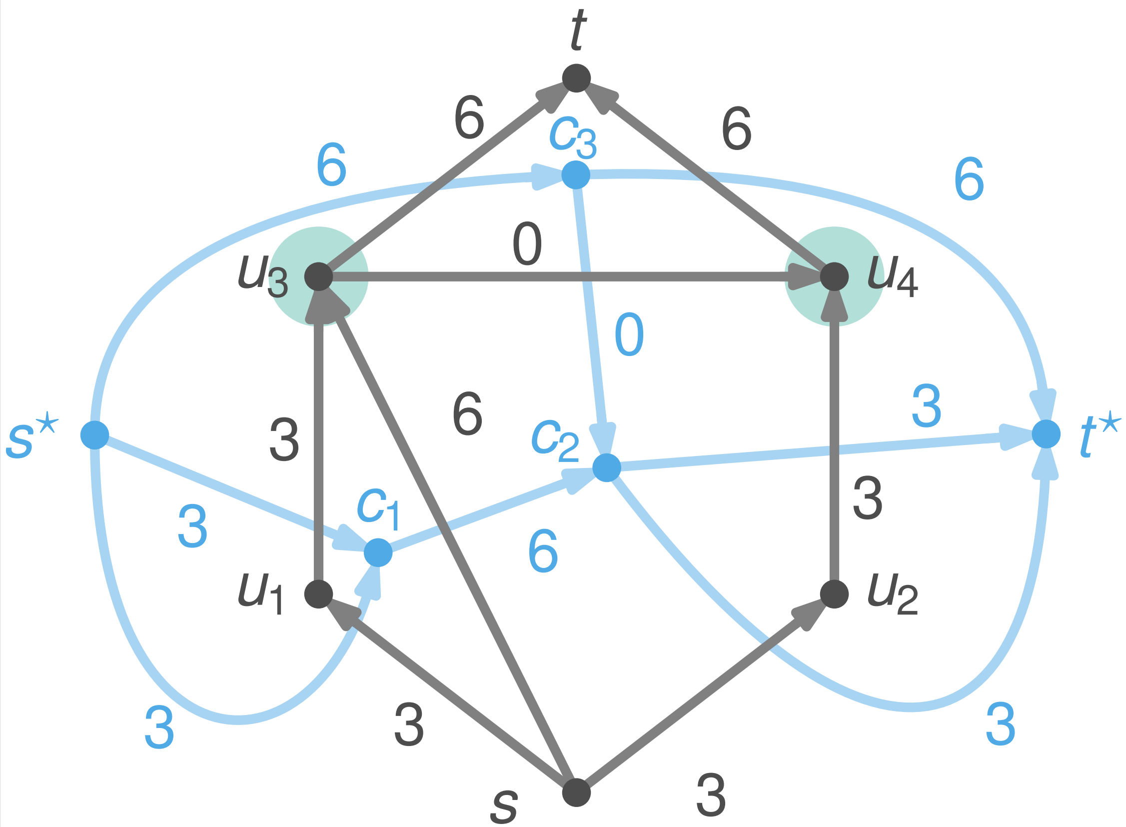

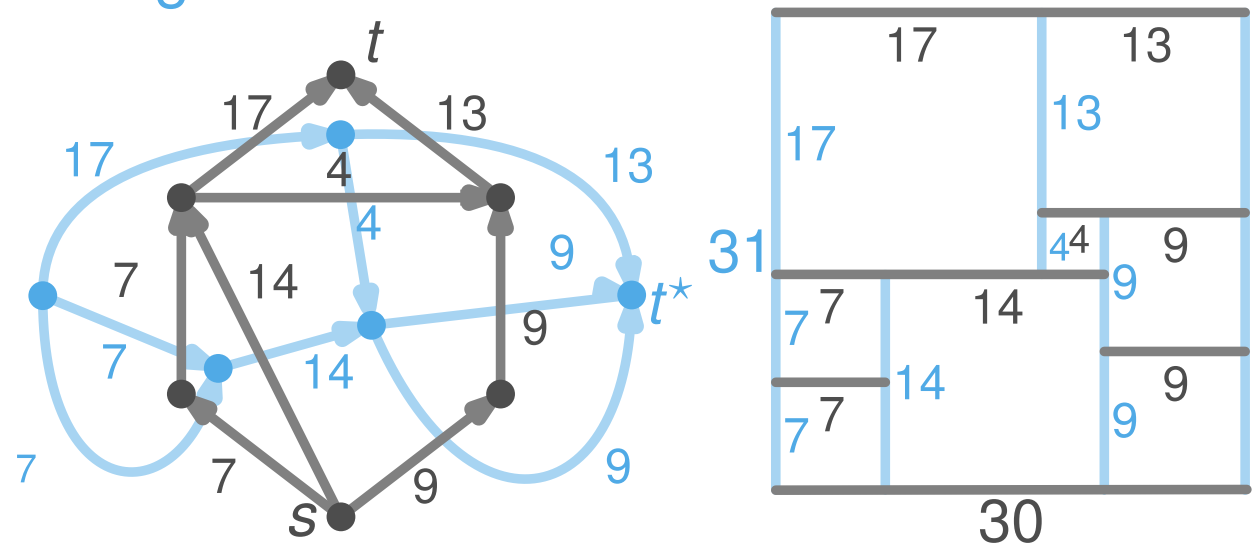

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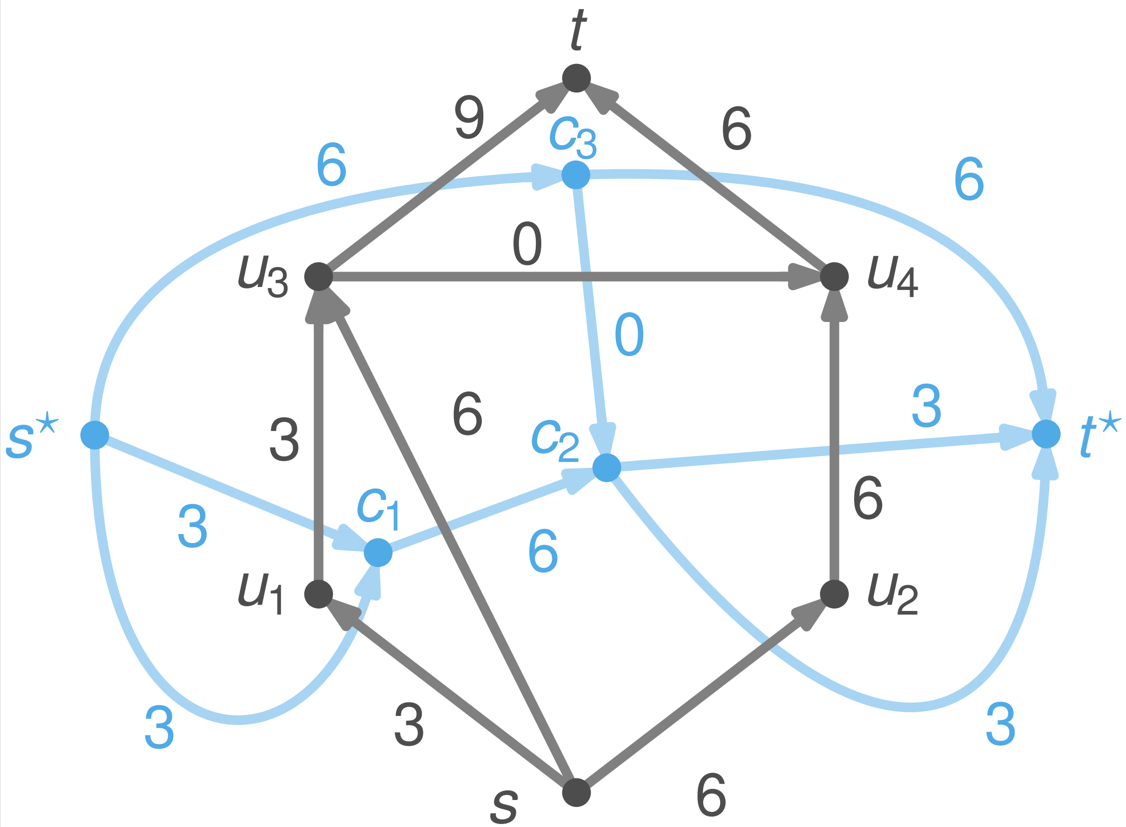

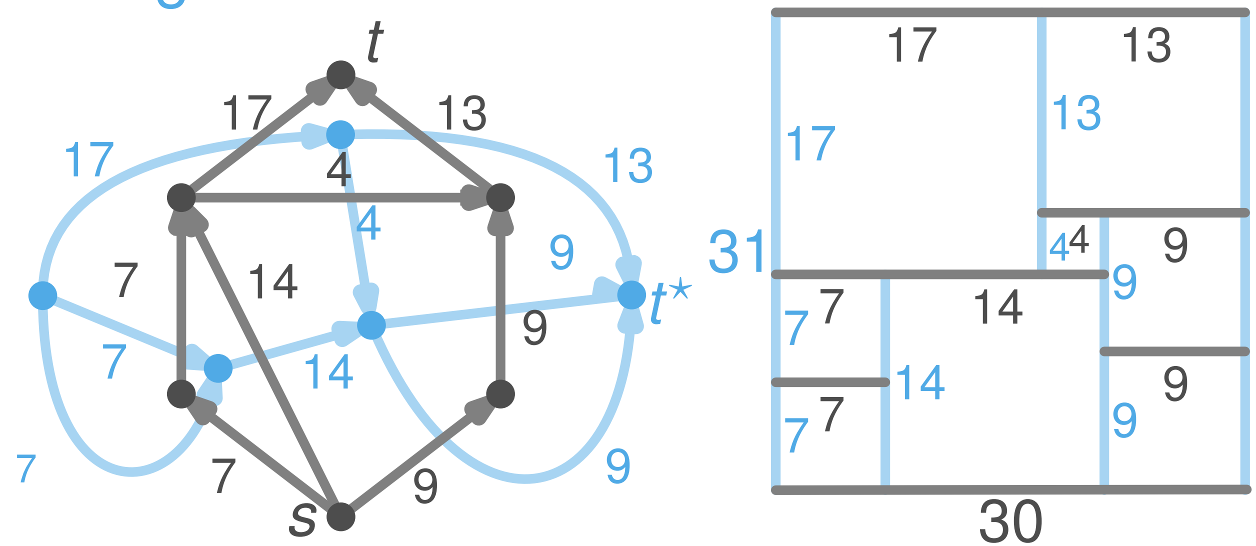

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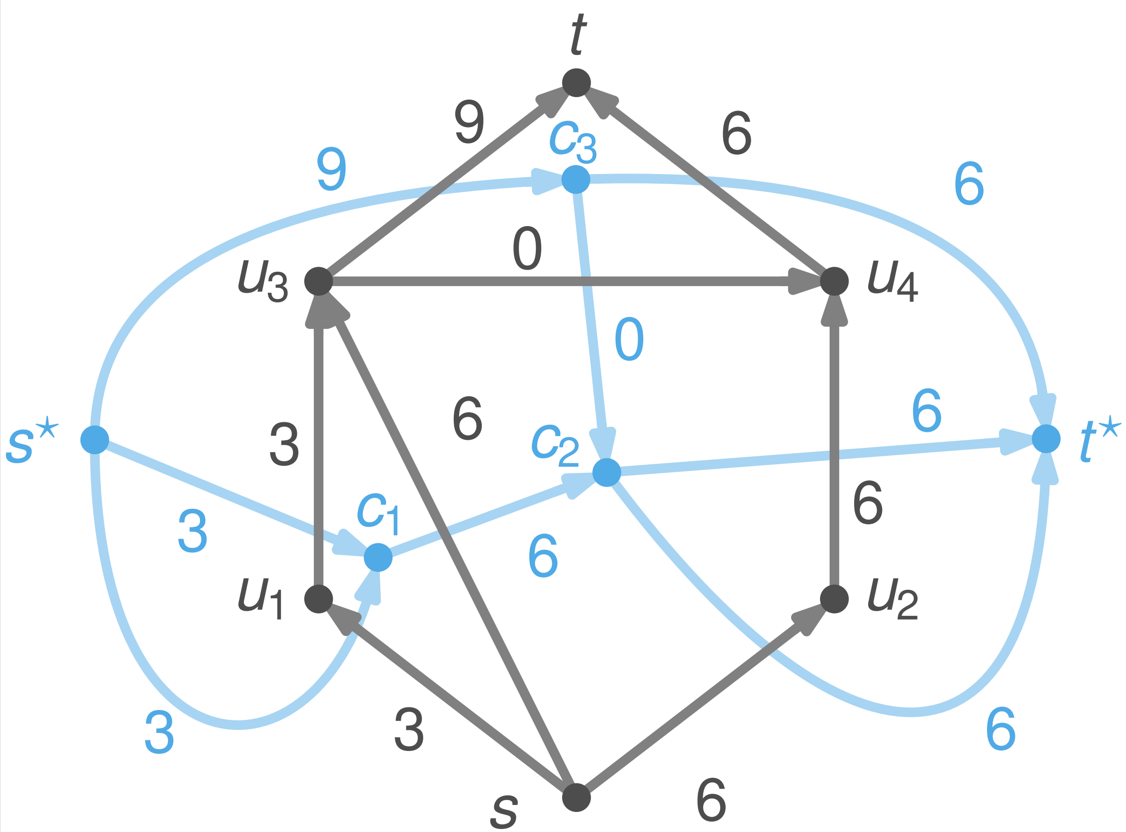

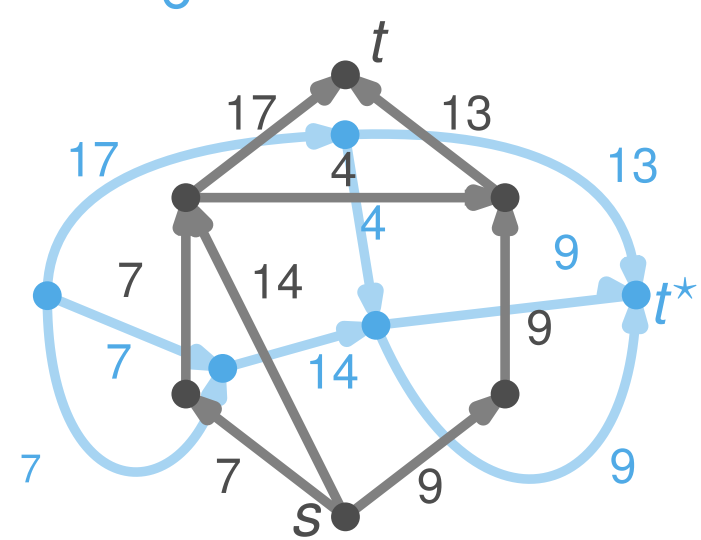

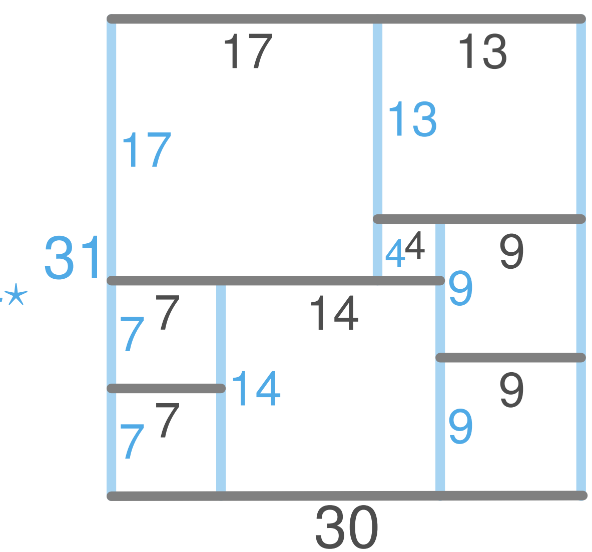

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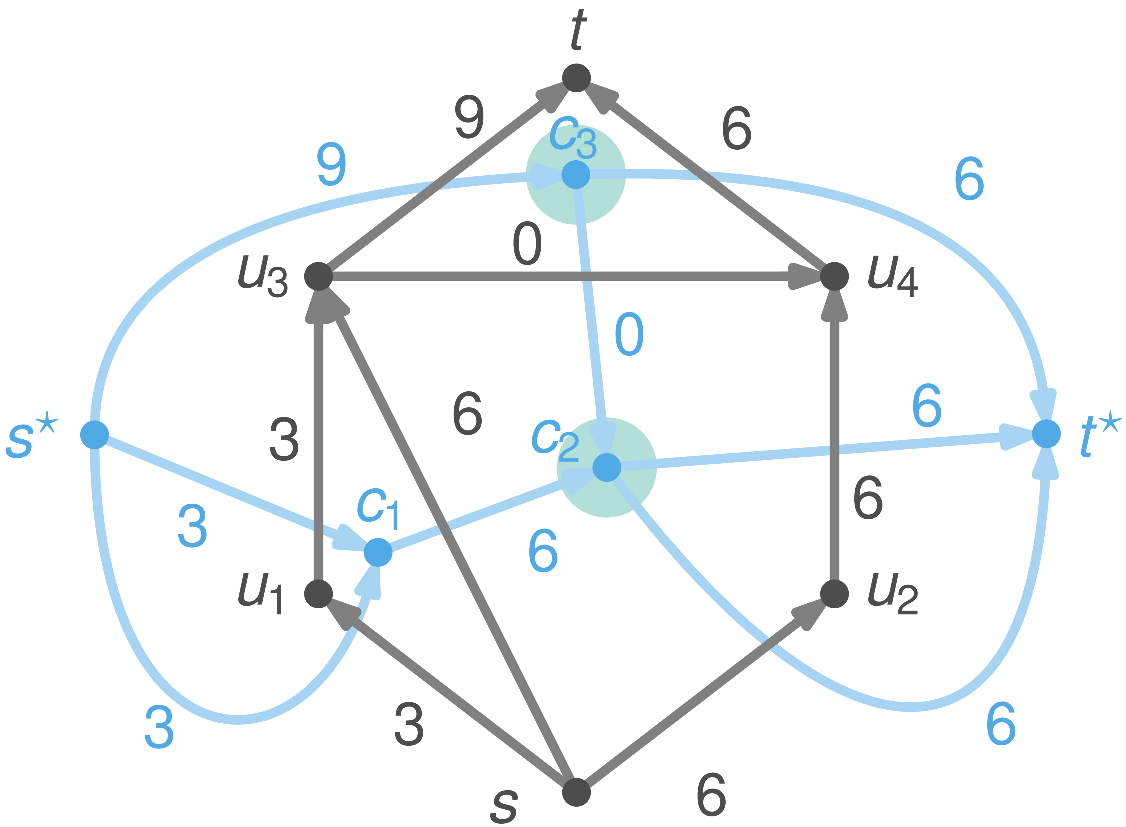

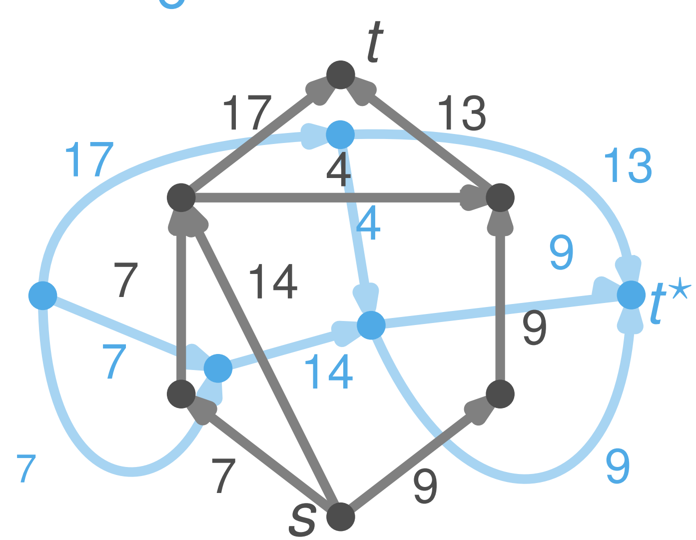

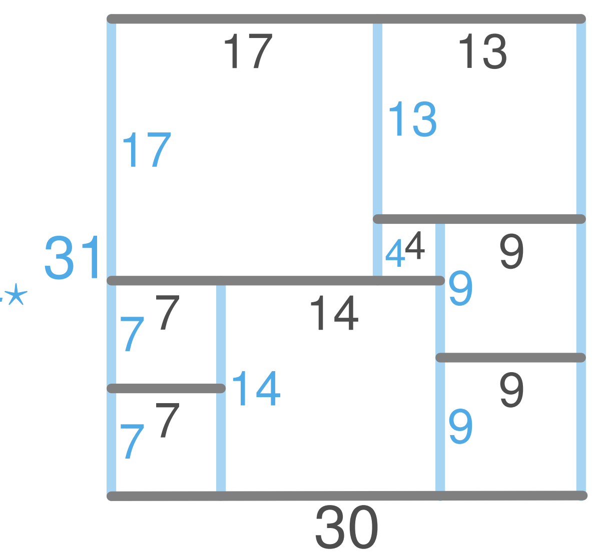

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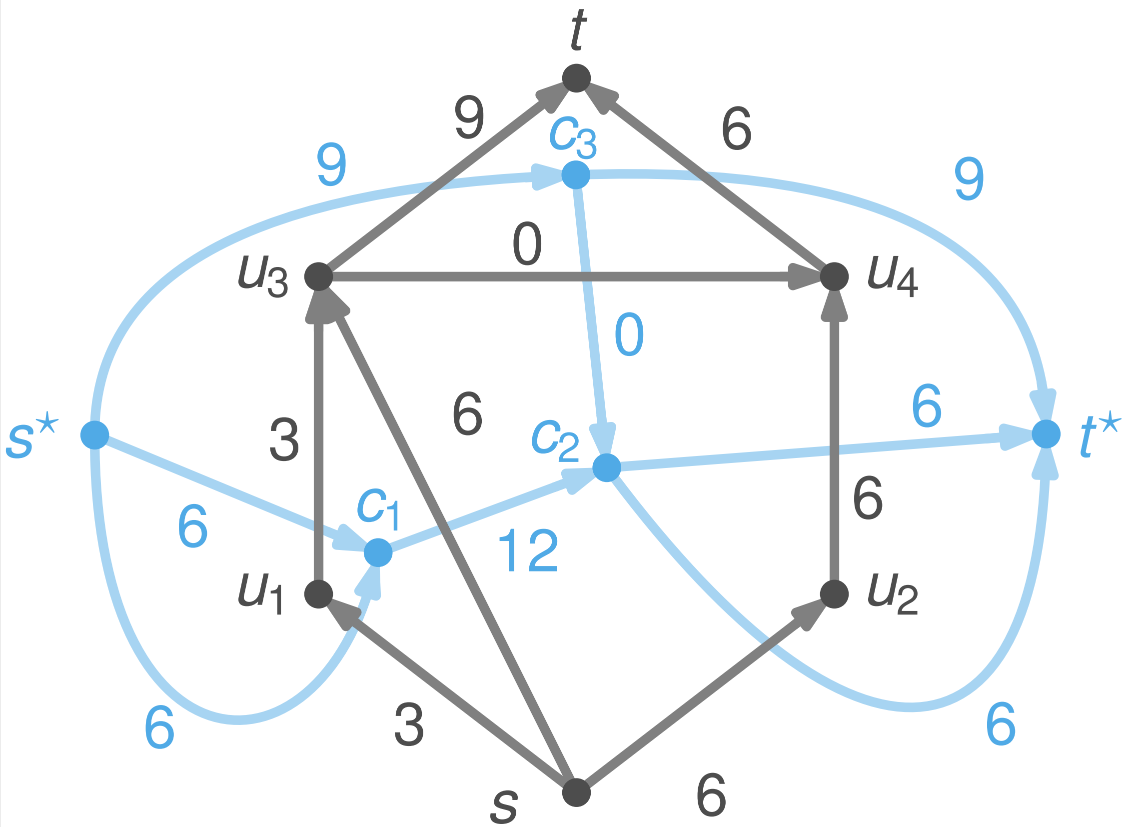

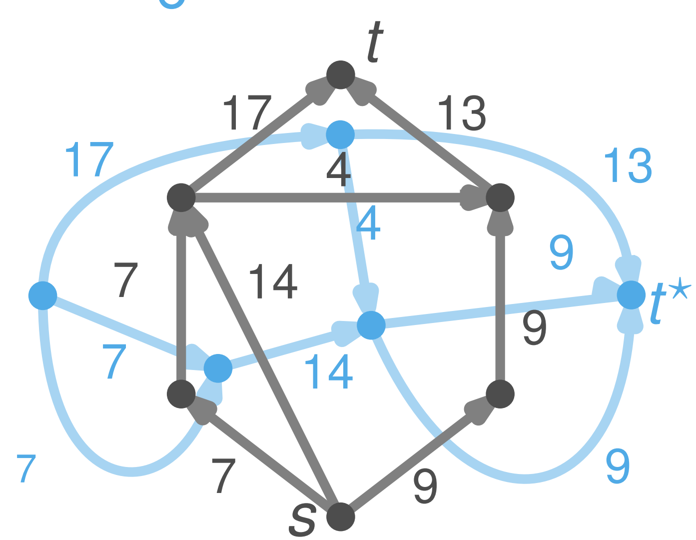

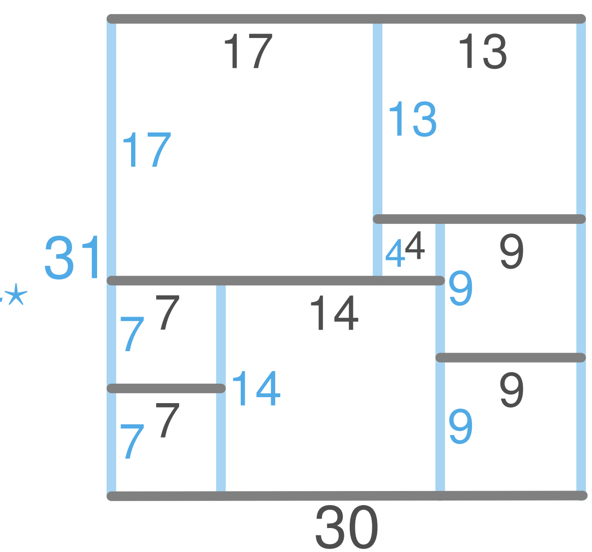

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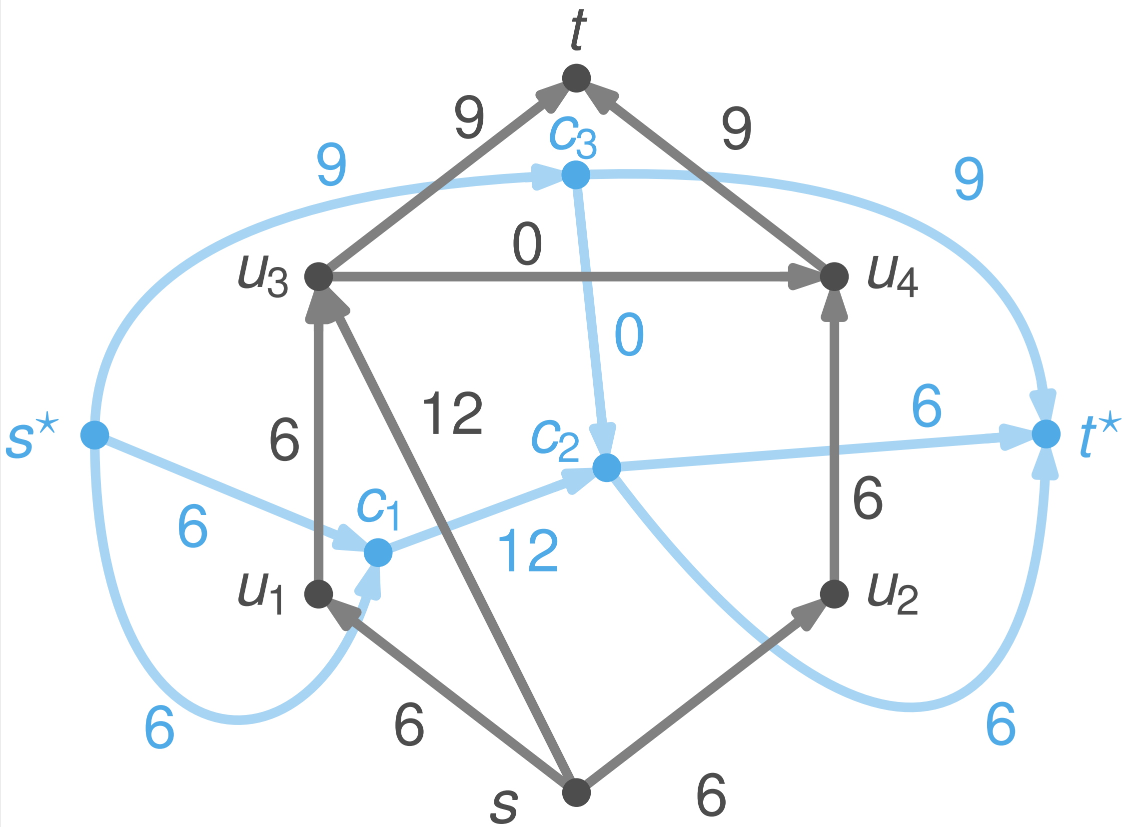

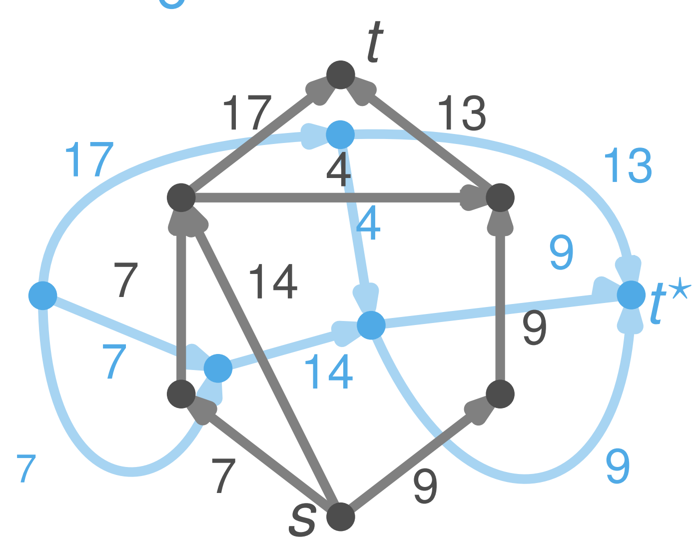

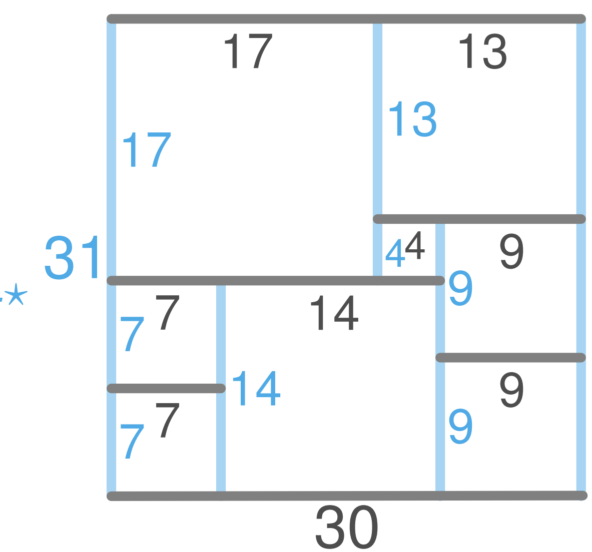

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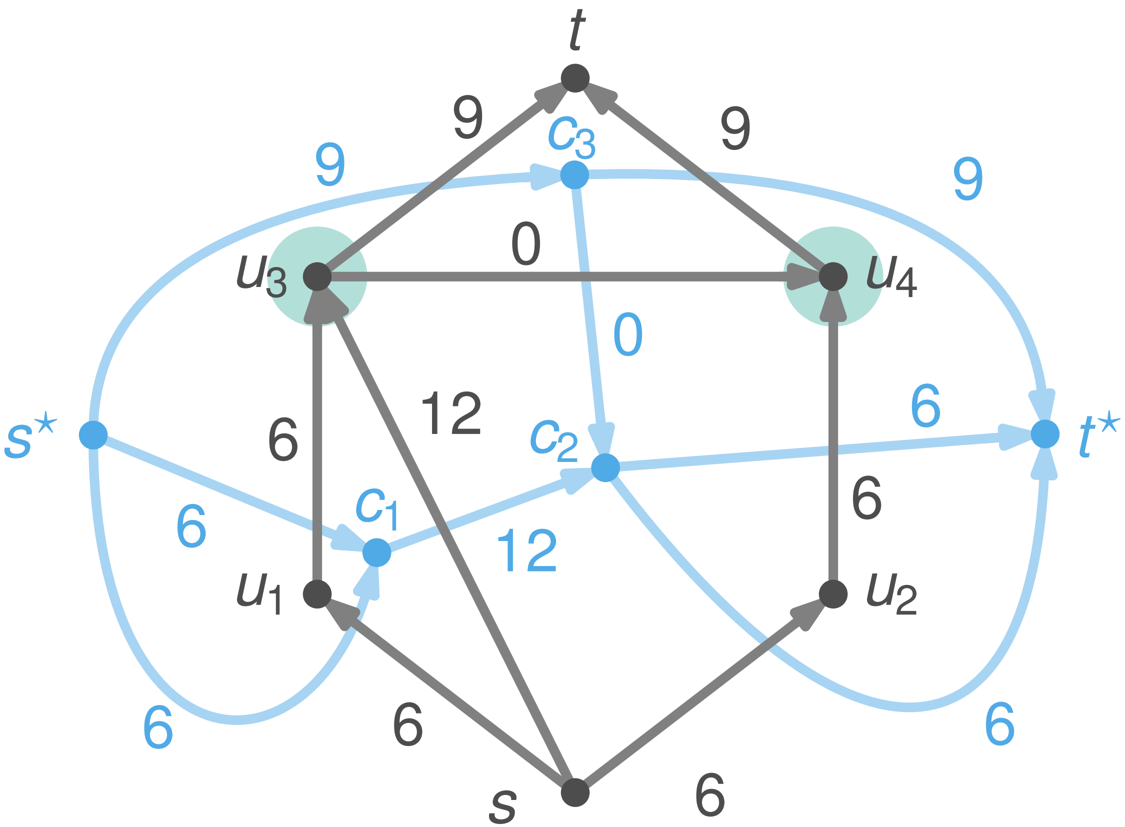

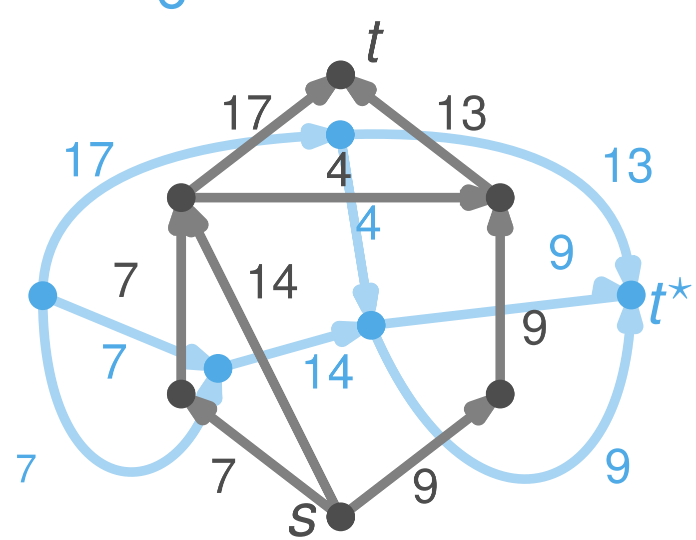

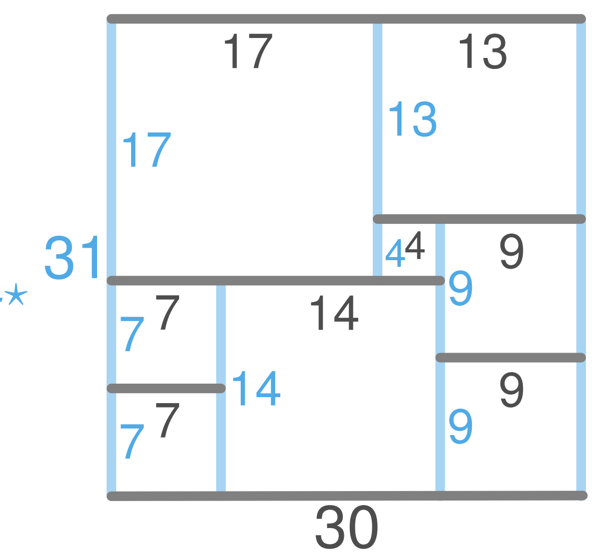

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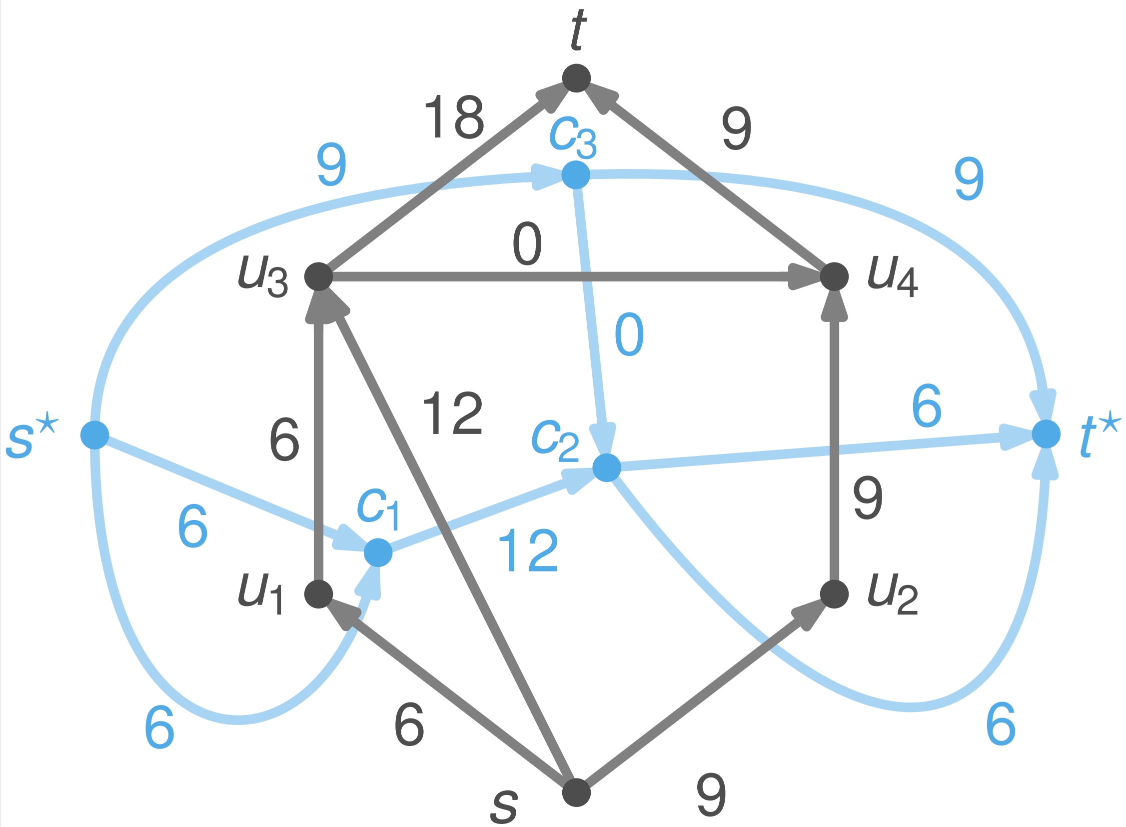

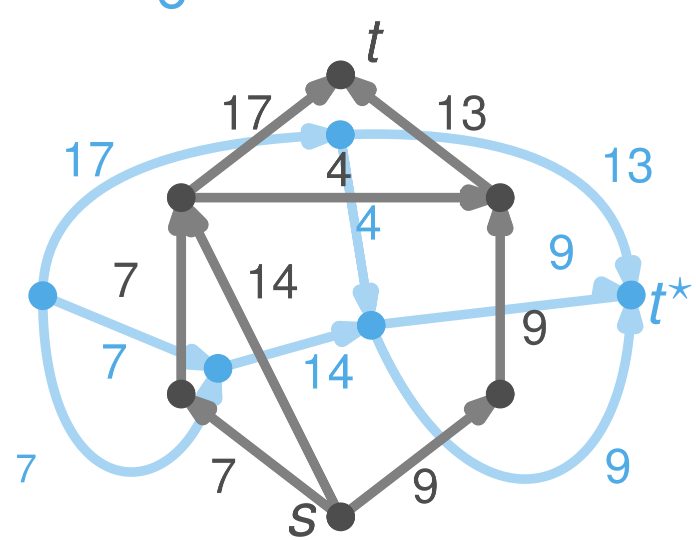

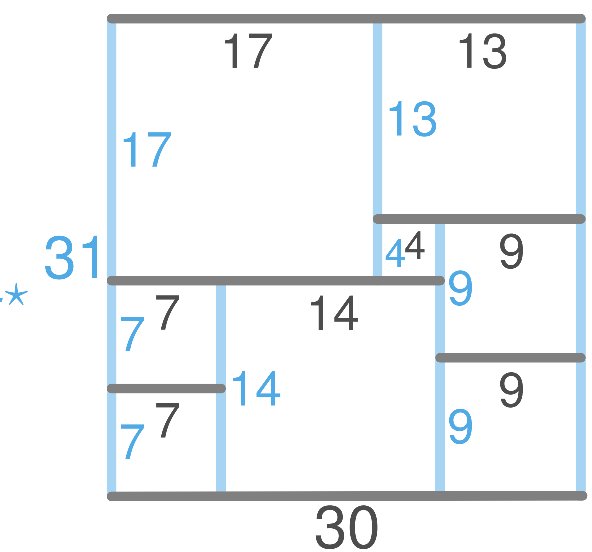

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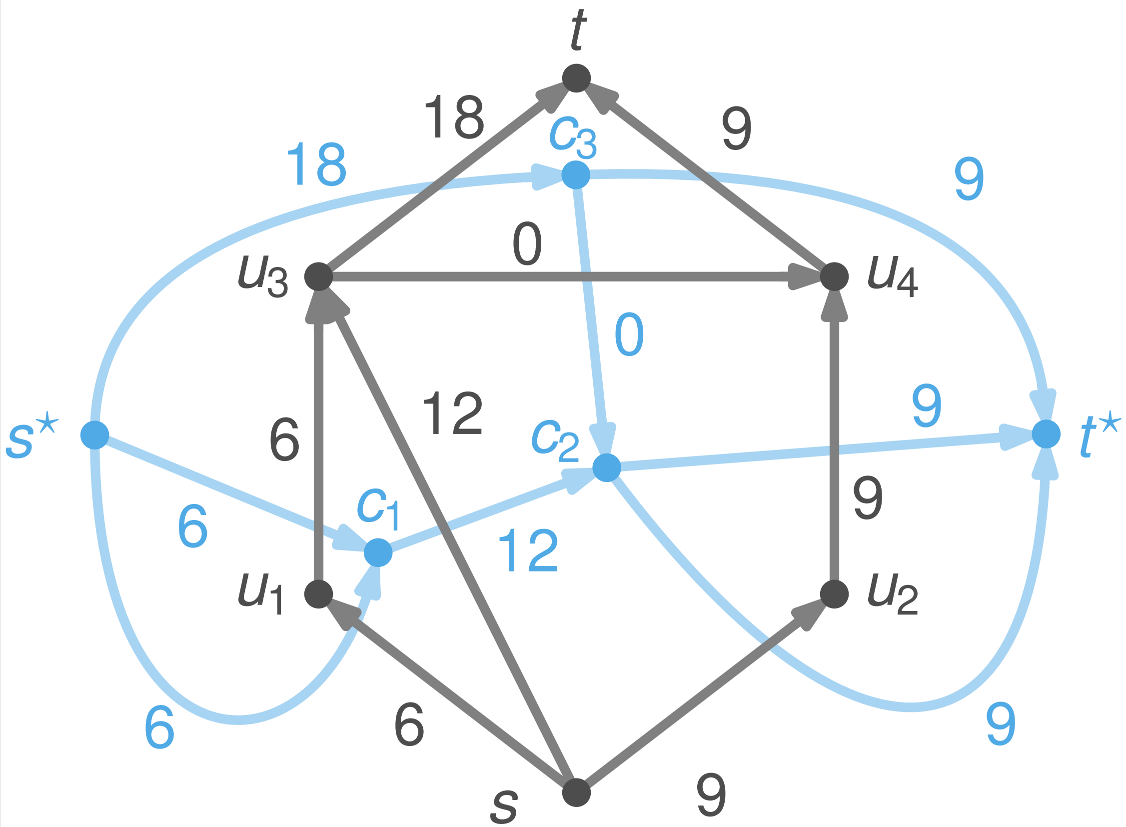

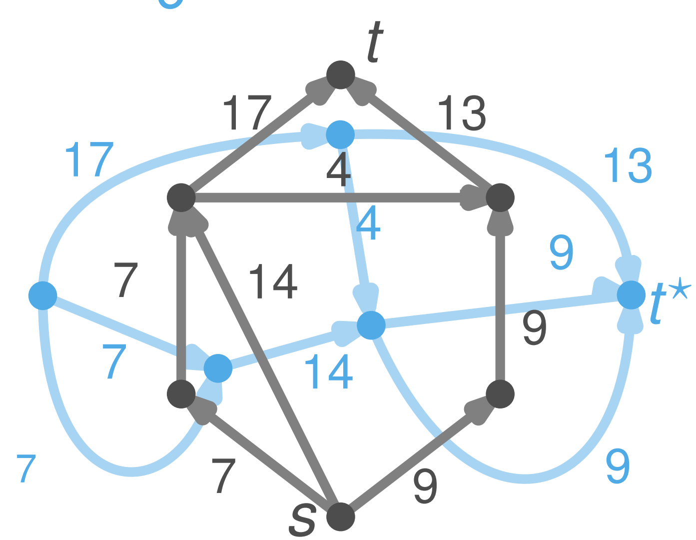

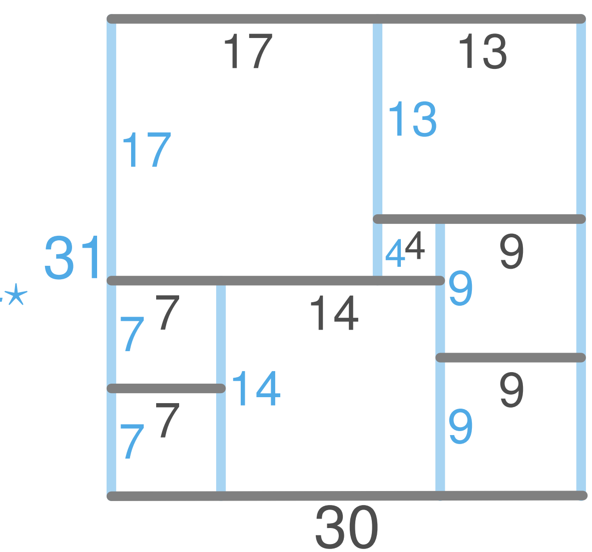

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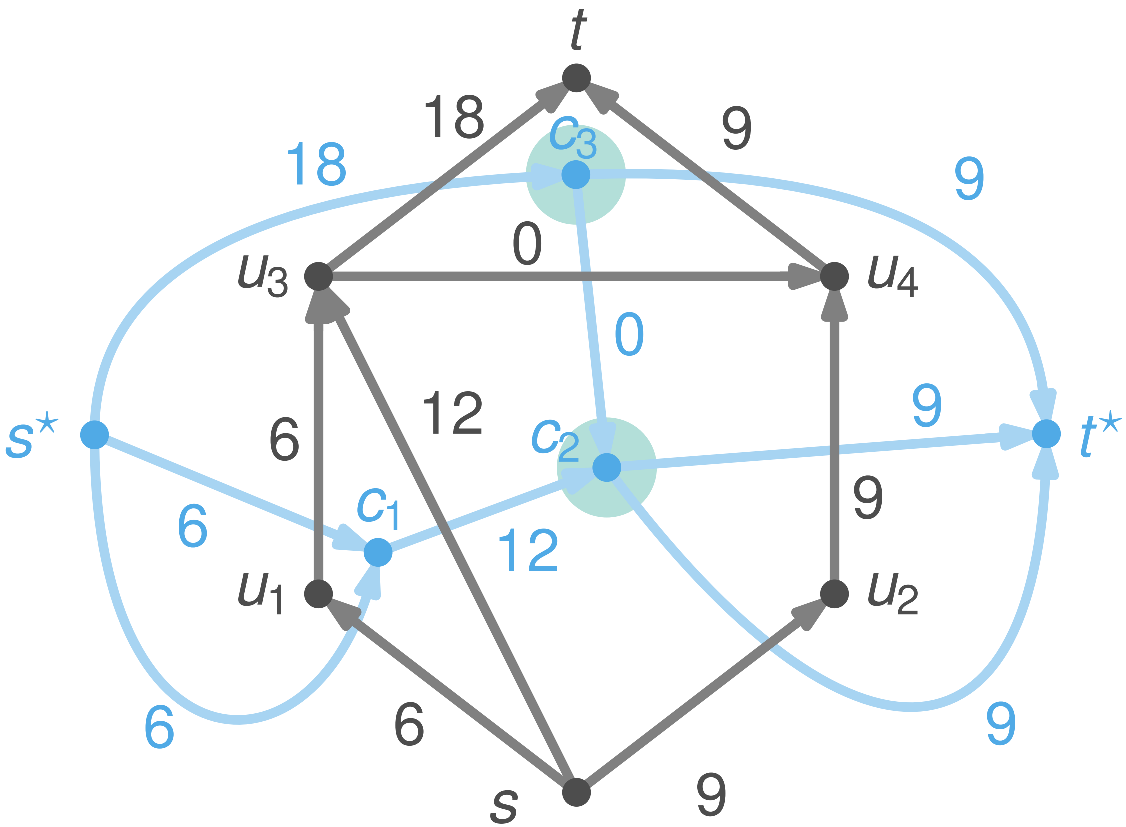

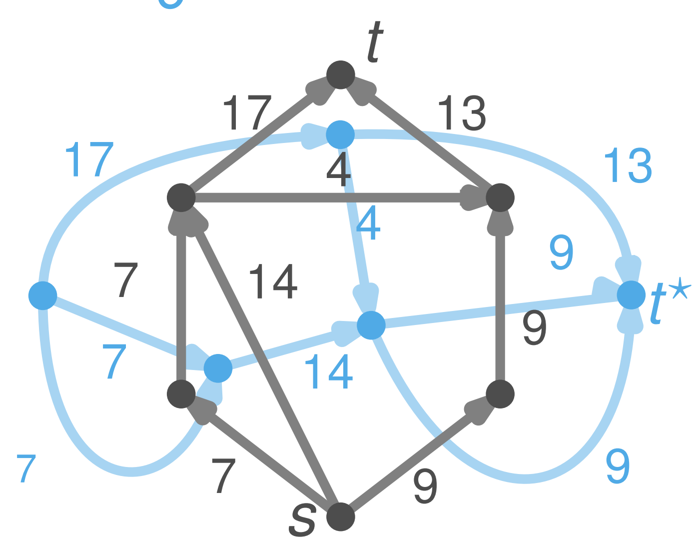

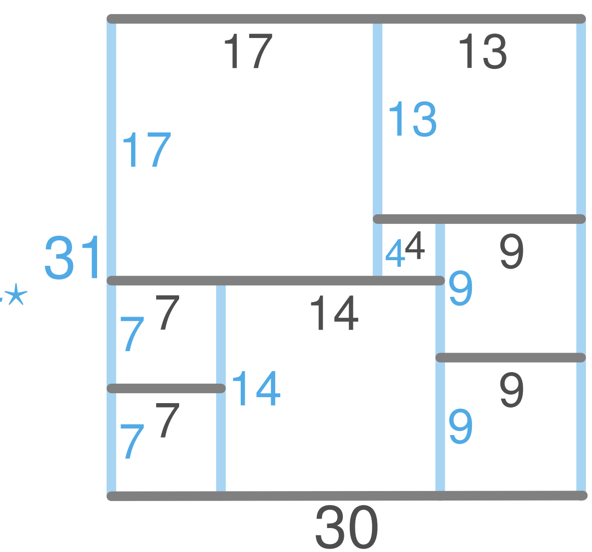

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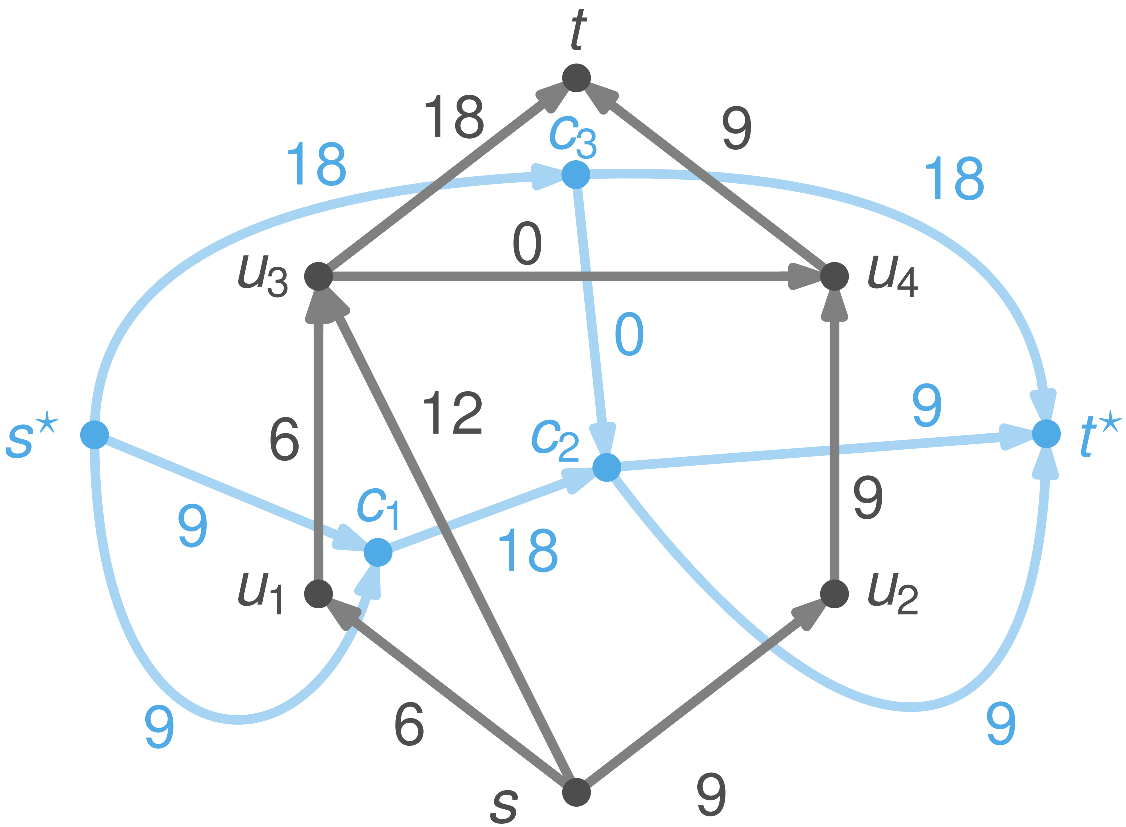
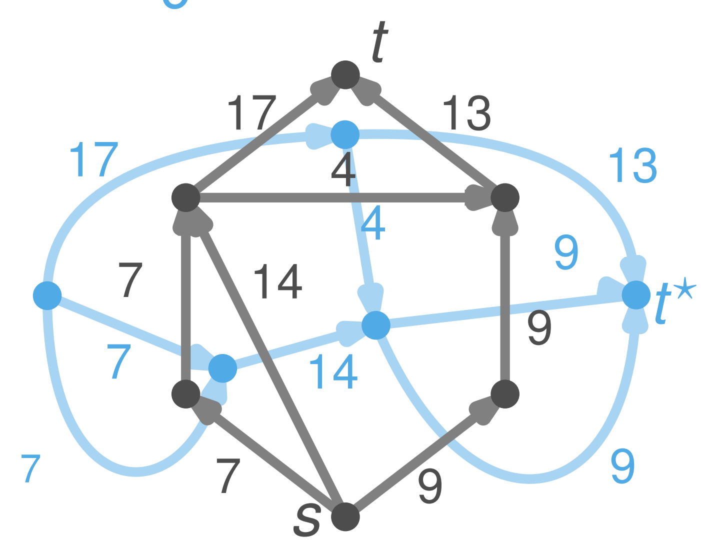
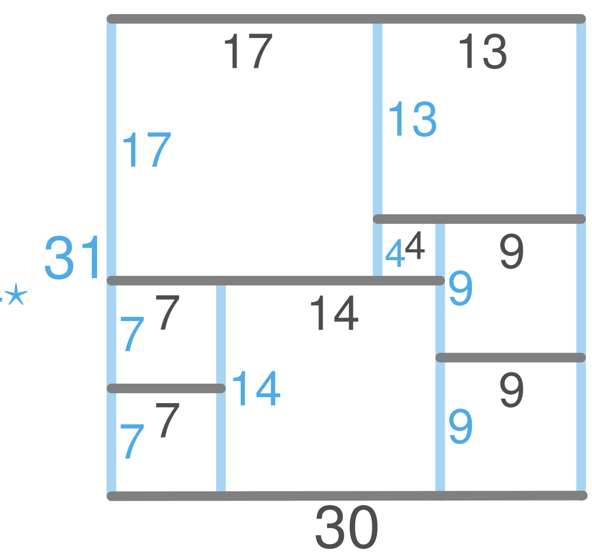
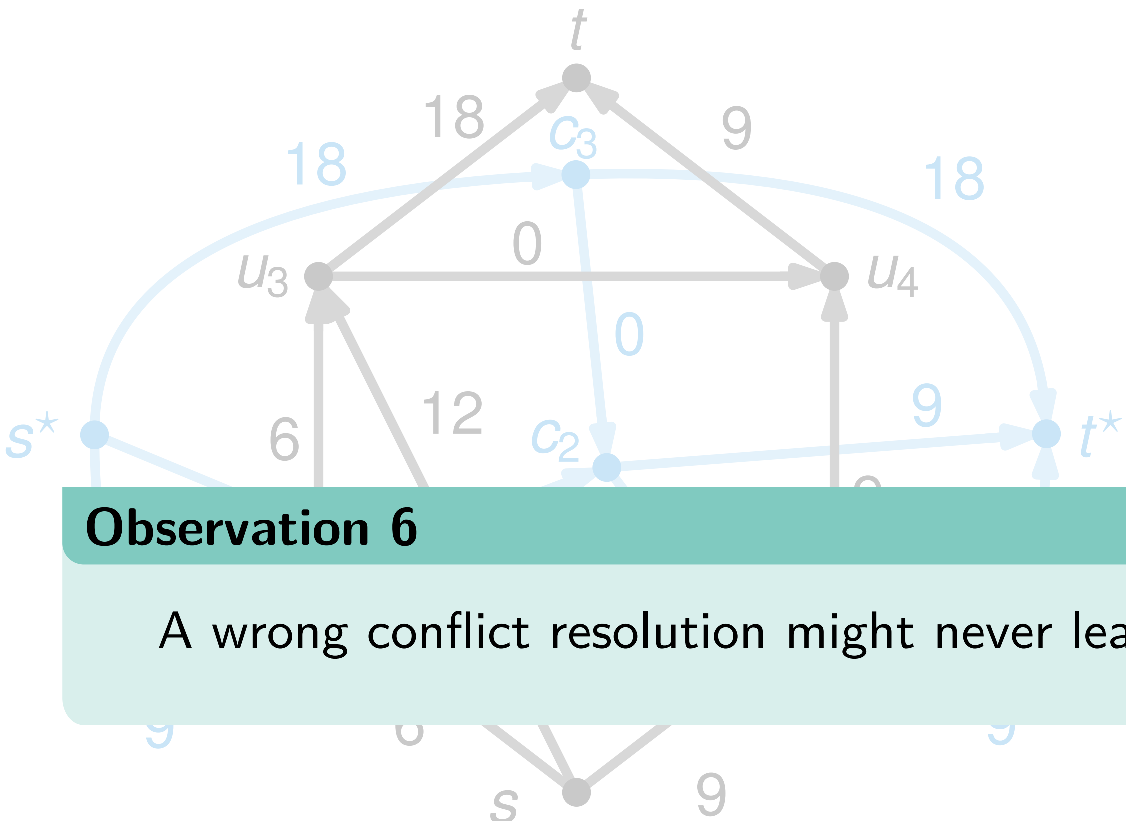






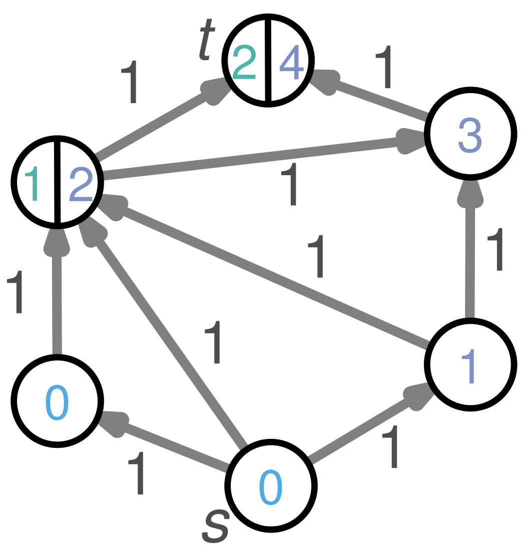
Wrong Conflict Resolution

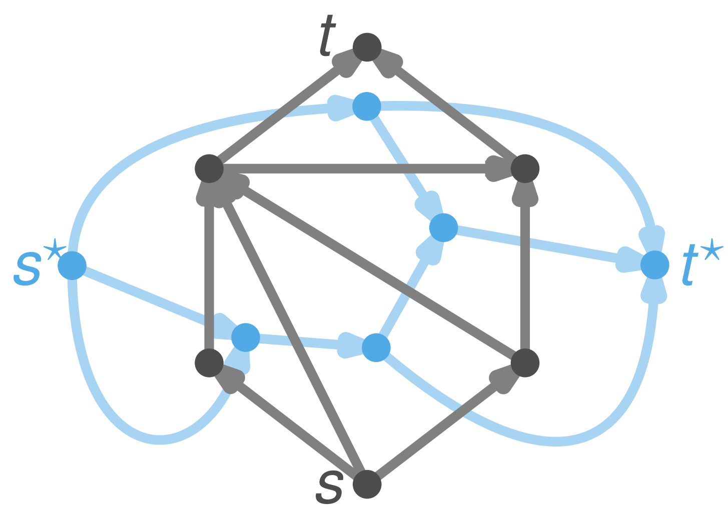


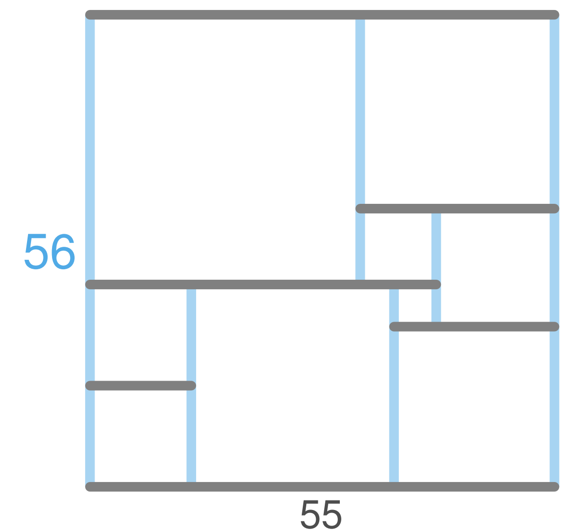
Observation 6

A wrong conflict resolution might never lead to a feasible power flow.

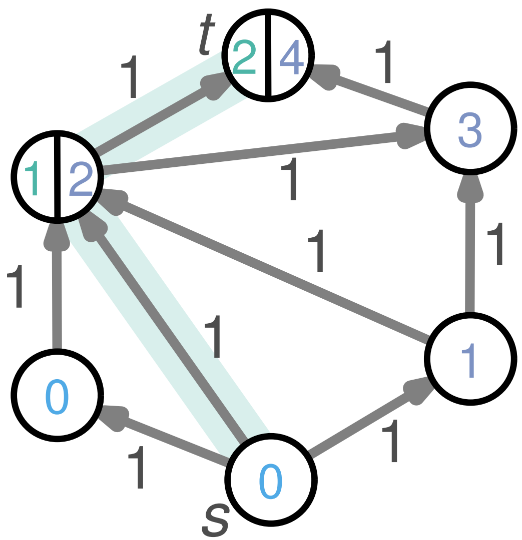
The Property of Balancing

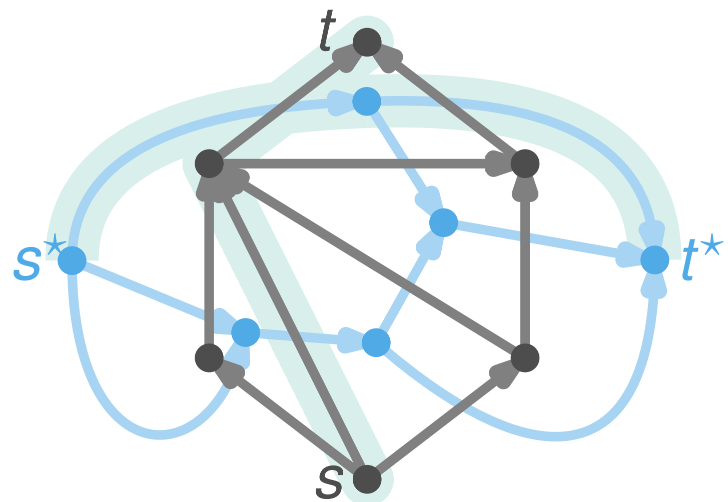


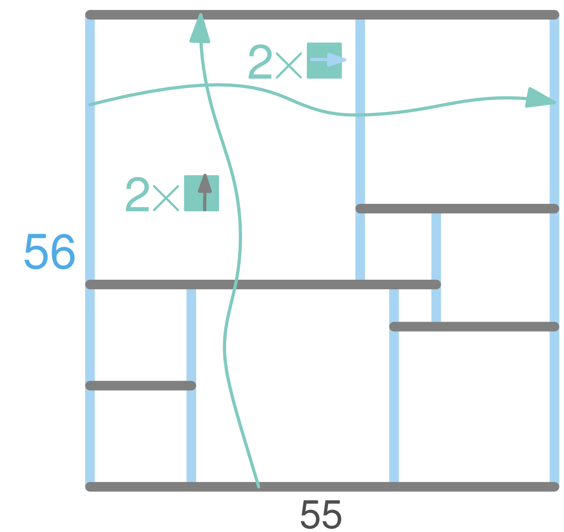




The Property of Balancing

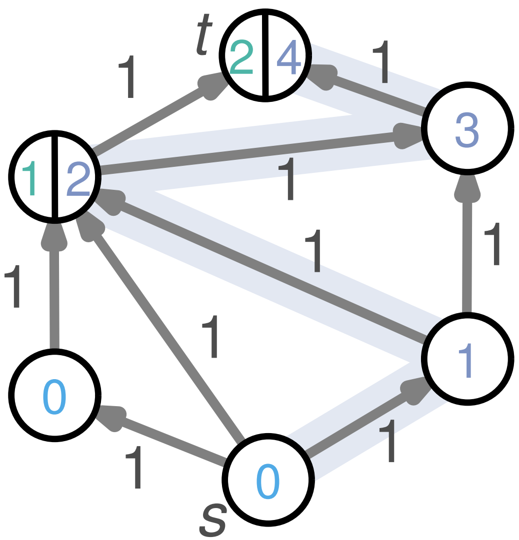


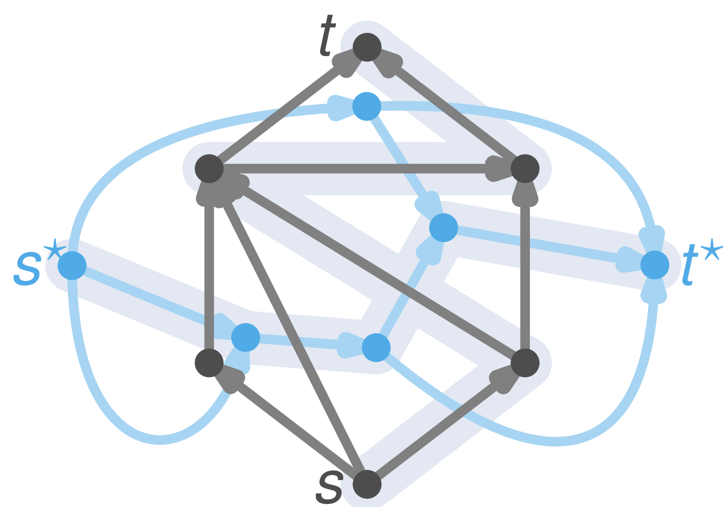


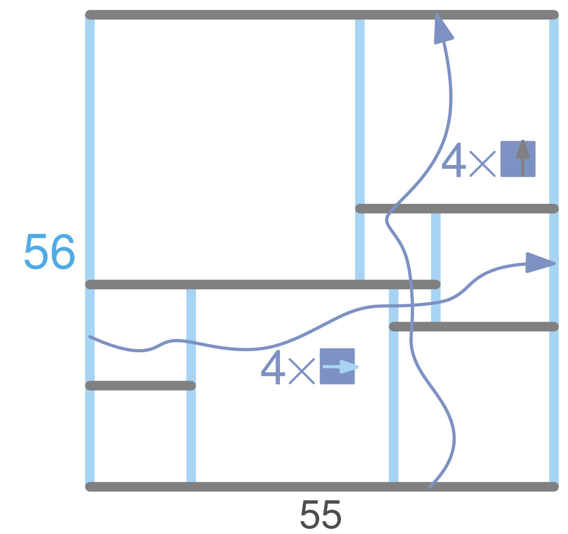


shortest path

The Property of Balancing

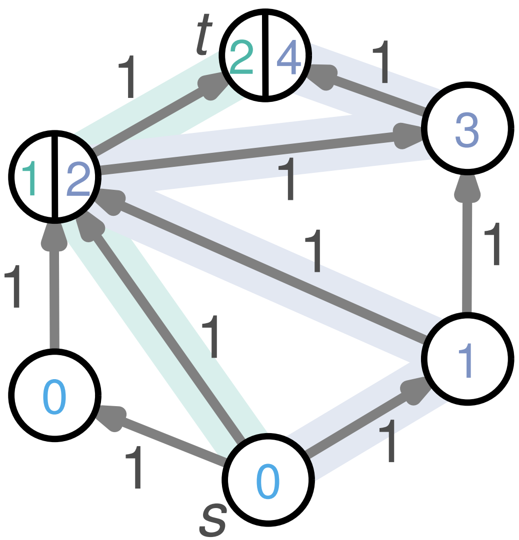


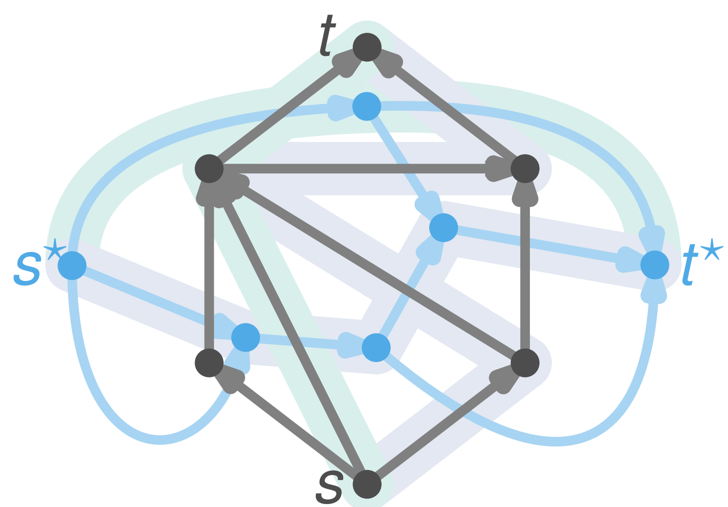


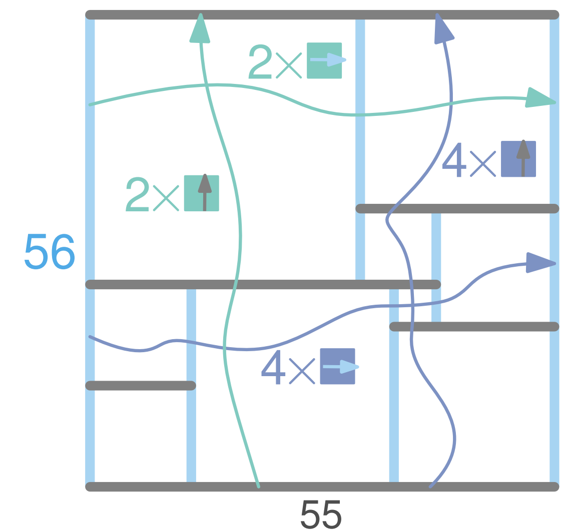


longest path

The Property of Balancing



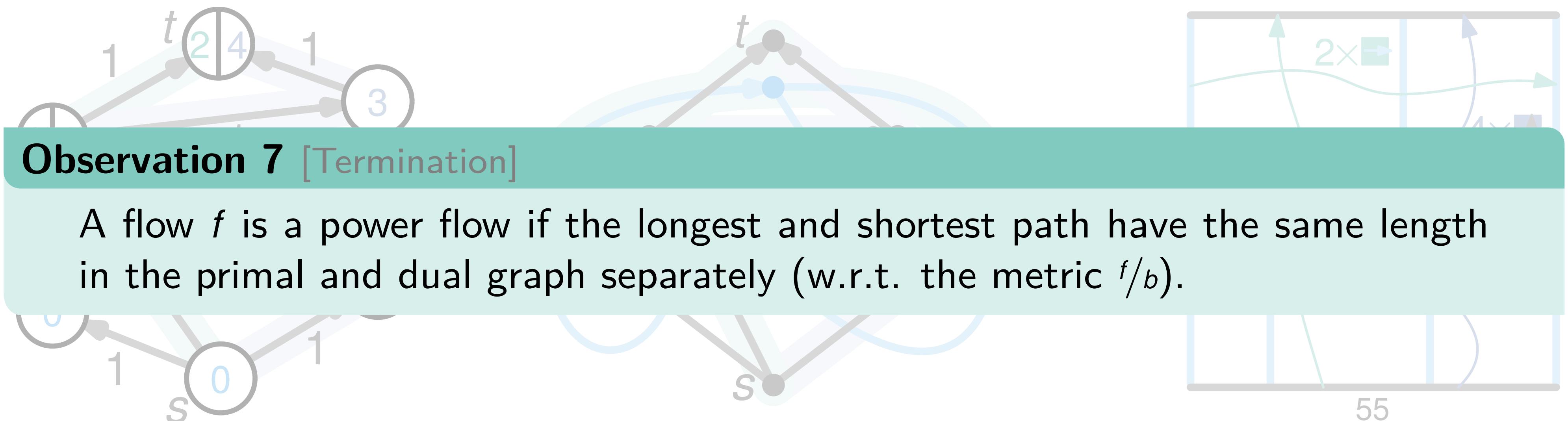




shortest path

longest path

The Property of Balancing



Observation 7 [Termination]

A flow f is a power flow if the longest and shortest path have the same length in the primal and dual graph separately (w.r.t. the metric f/b).

shortest path

longest path

Scalability of Power Flows

- Graph-theoretical flow algorithms use scaling techniques
  1. Capacity scaling [Edmonds and Karp, 1972]
  2. Excess scaling [Ahuja and Orlin, 1989]
- Power flows excluding a trivial power flow $(f \equiv 0)$ can be scaled up and down by a factor χ

Lemma 8 [Scaling]

Every non-zero electrical flow $f' \colon E \to \mathbb{R}_{>0}$ can be rescaled to a new feasible electrical flow f by applying a scaling factor

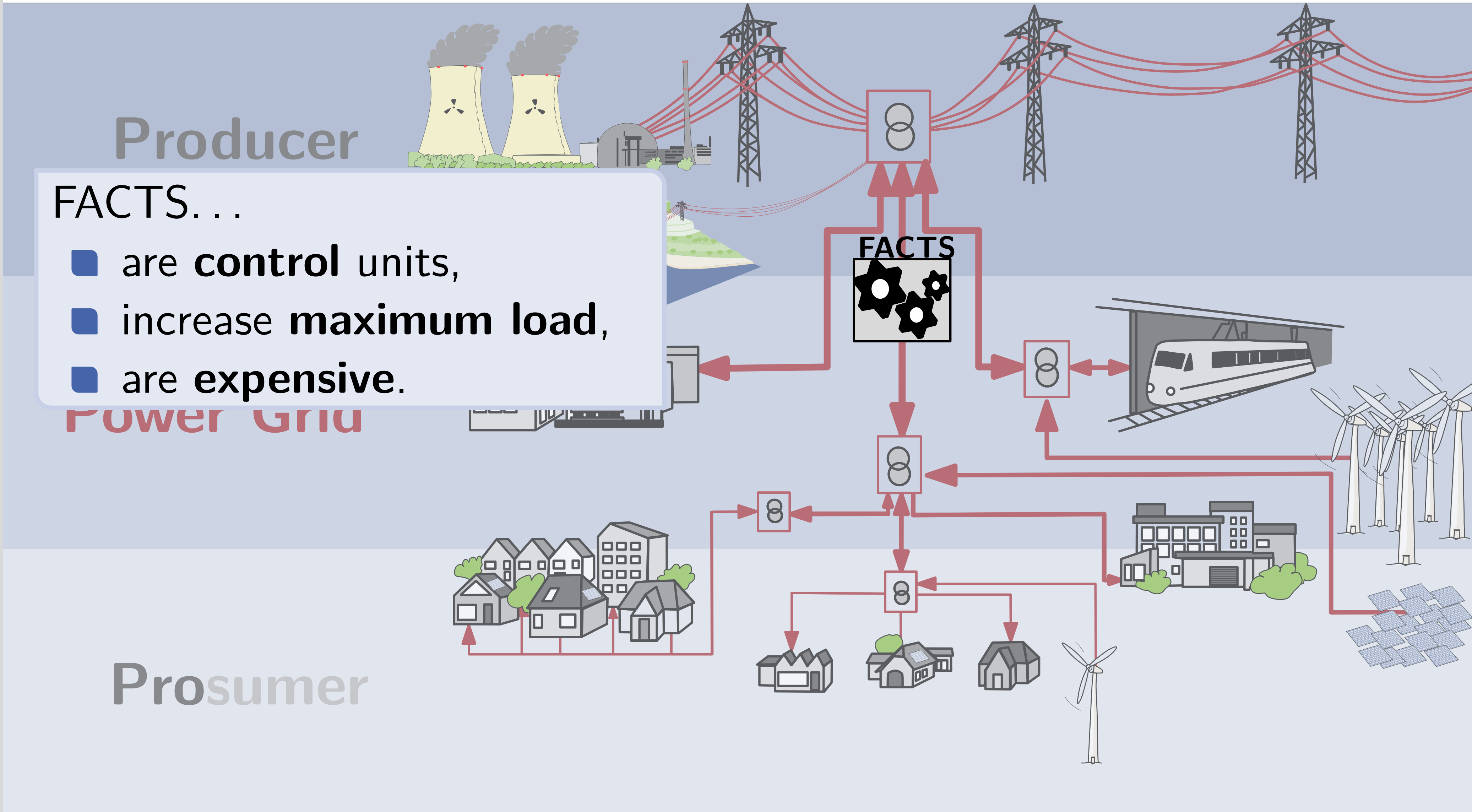
$$0 \leq \chi \leq \min_{e' \in E} \frac{\mathrm{cap}(e')}{f'(e')} =: \overline{\chi} \qquad (1)$$

to $f(e) = f'(e) \cdot \chi$ for all $e \in E$.

Continous Changes to the Power Grid

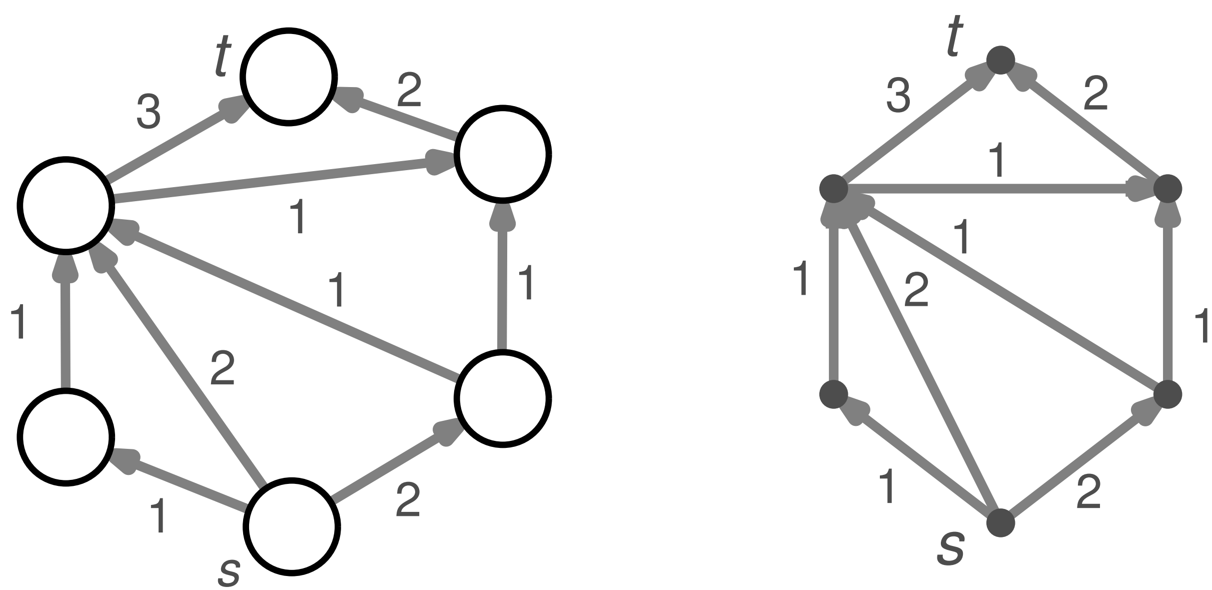
FACTS. . .

- are **control** units,
- increase **maximum load**,
- are **expensive**.



Susceptance Scaling

- Apply a feasible flow

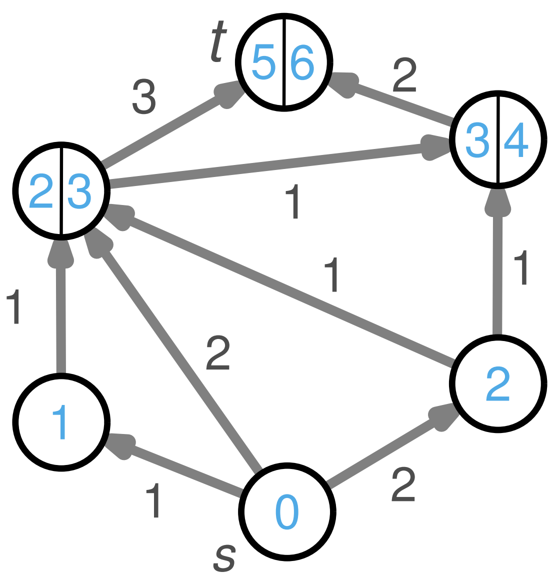


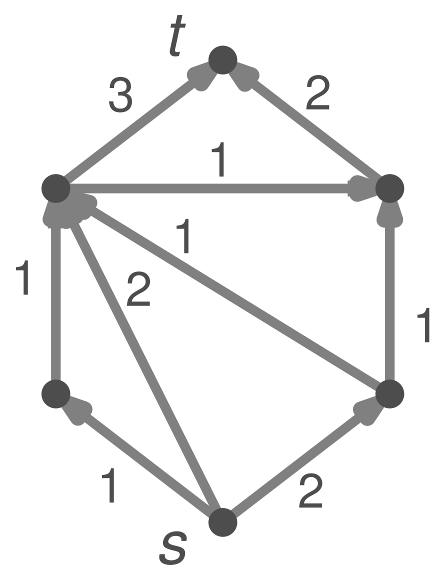
Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^\star$

Susceptance Scaling

- Apply a feasible flow



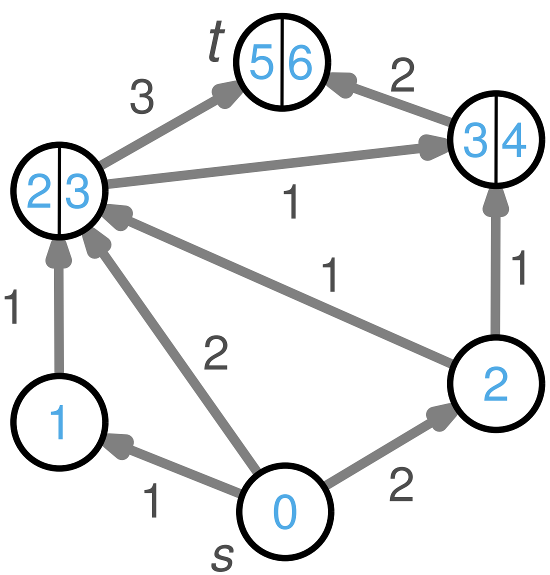


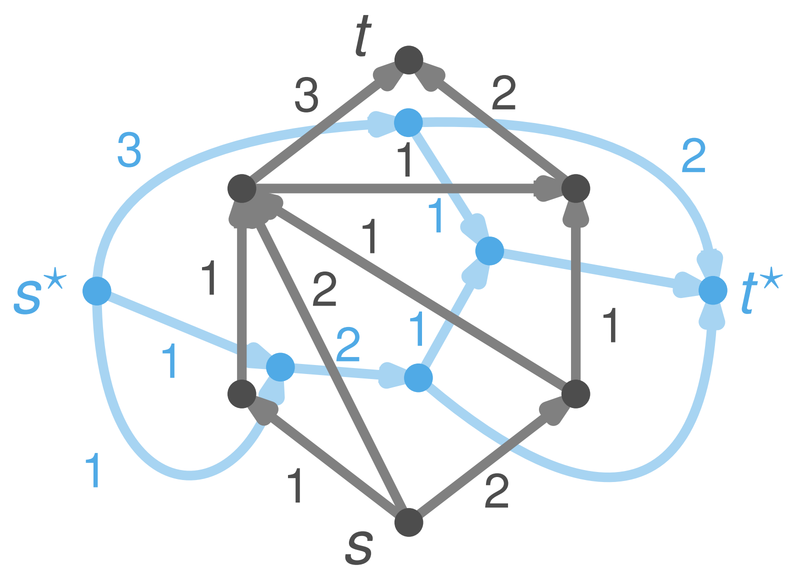
Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^{\star}$

Susceptance Scaling

- Apply a feasible flow
- Can we make it electrical feasible?





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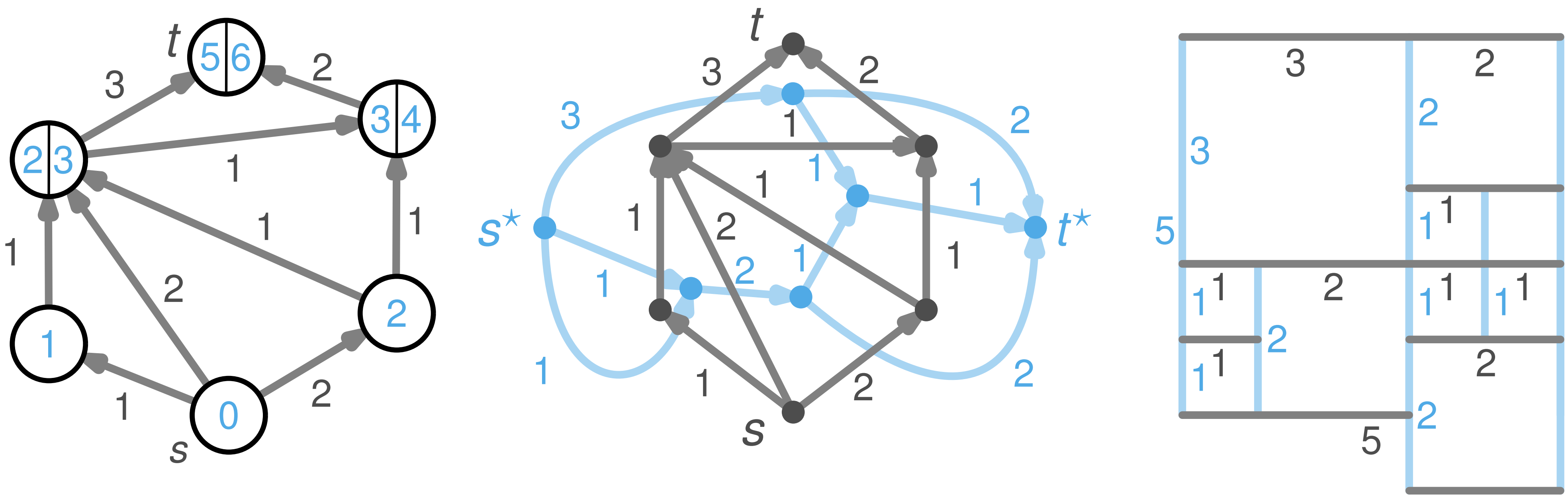

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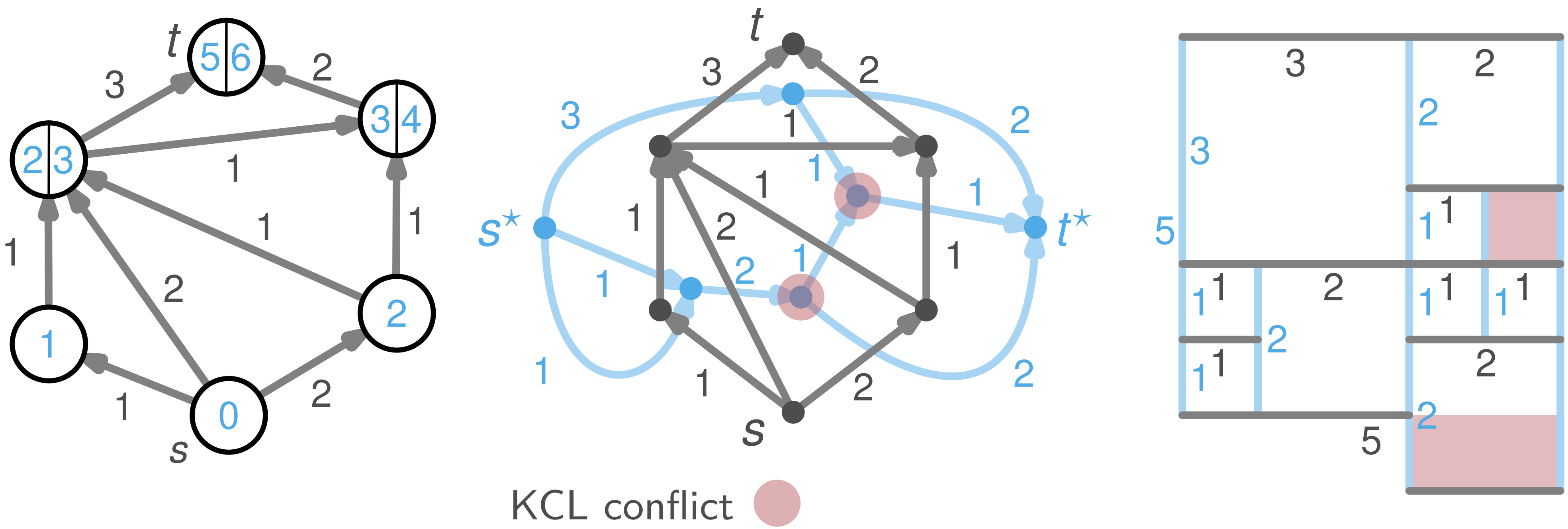
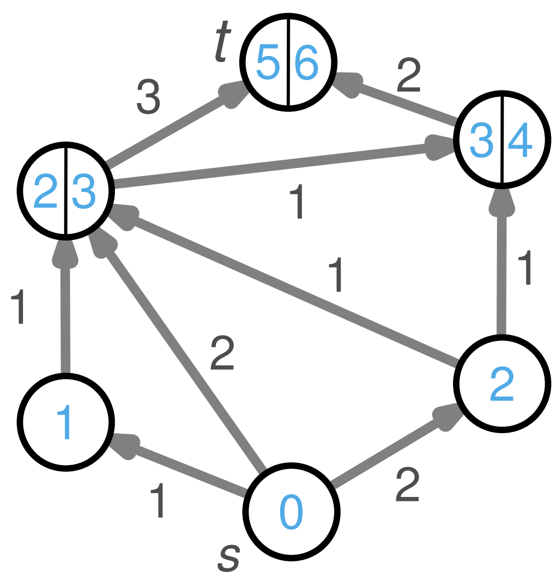


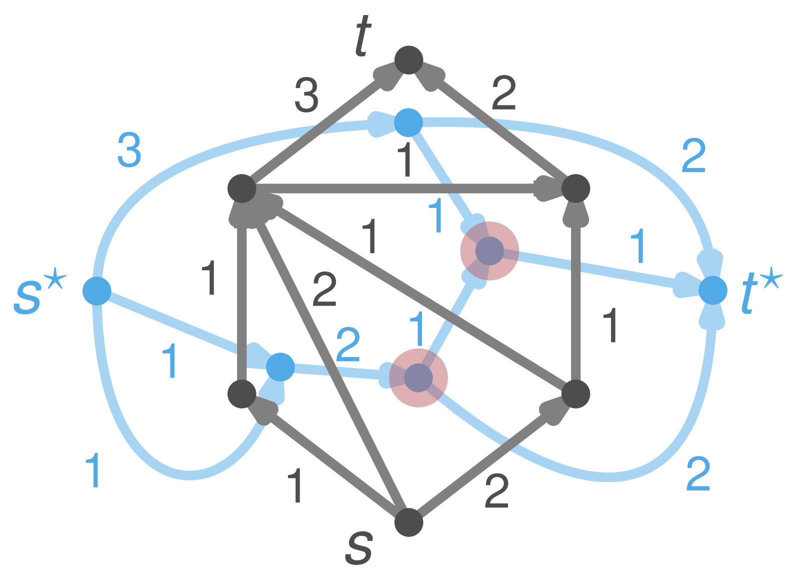
Observation 9 [KCL and KVL Duality]

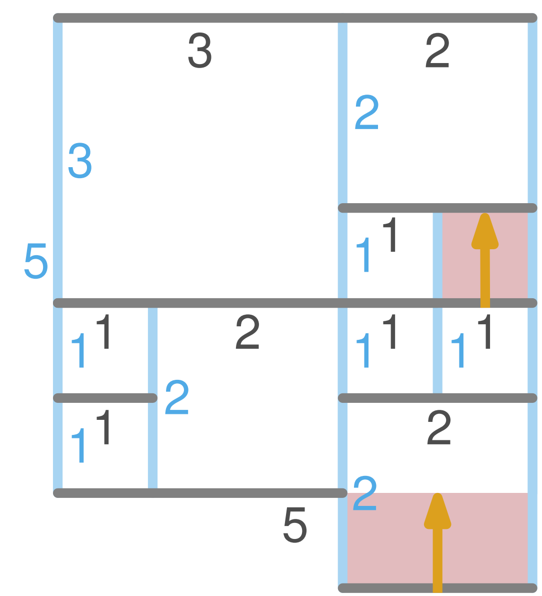
KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^\star$

Susceptance Scaling

- Apply a feasible flow
- Can we make it electrical feasible? ✓







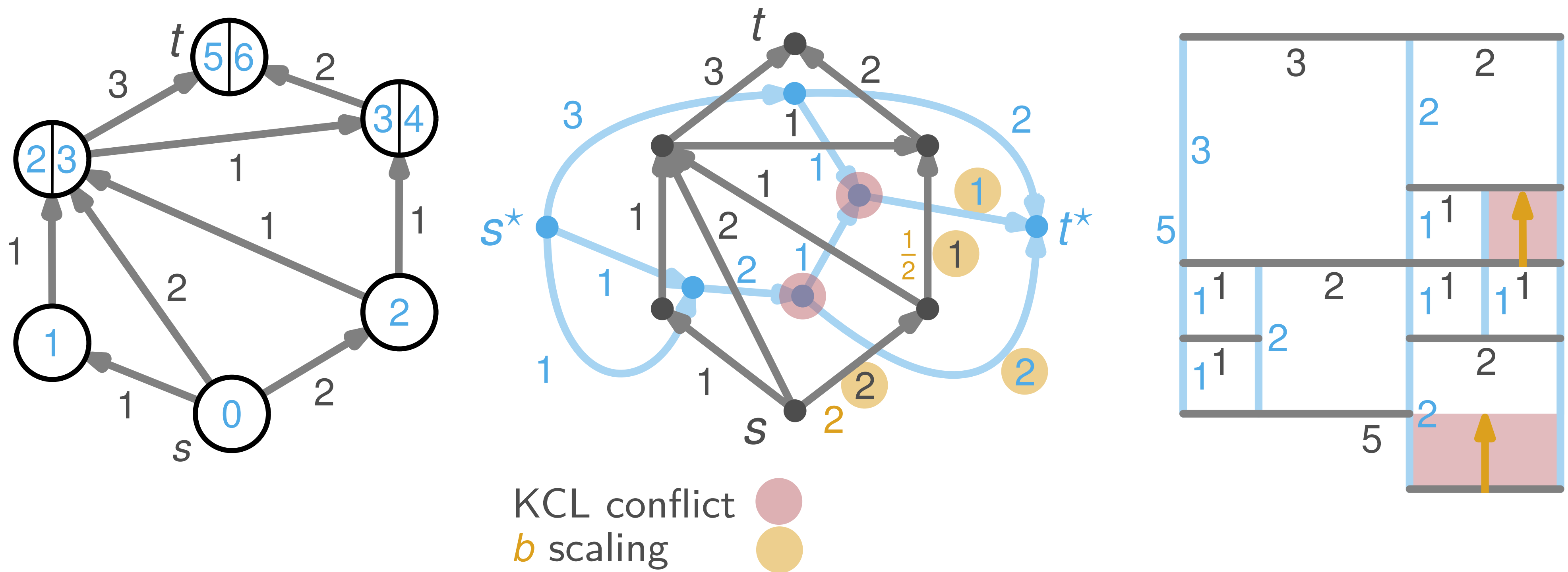
KCL conflict

Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^{\star}$

Susceptance Scaling

- Apply a feasible flow
- Can we make it electrical feasible? ✓

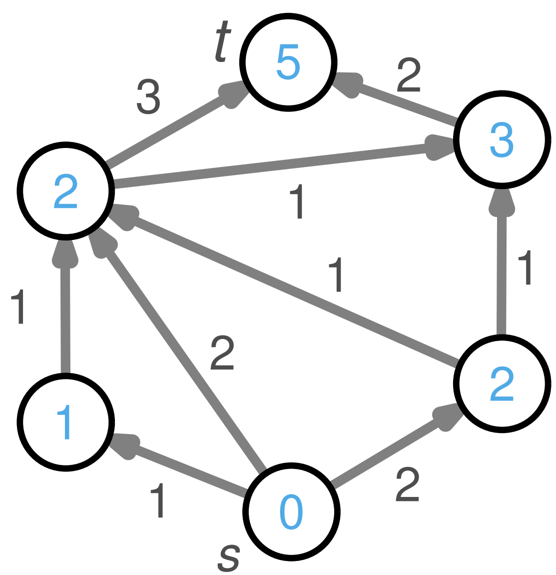


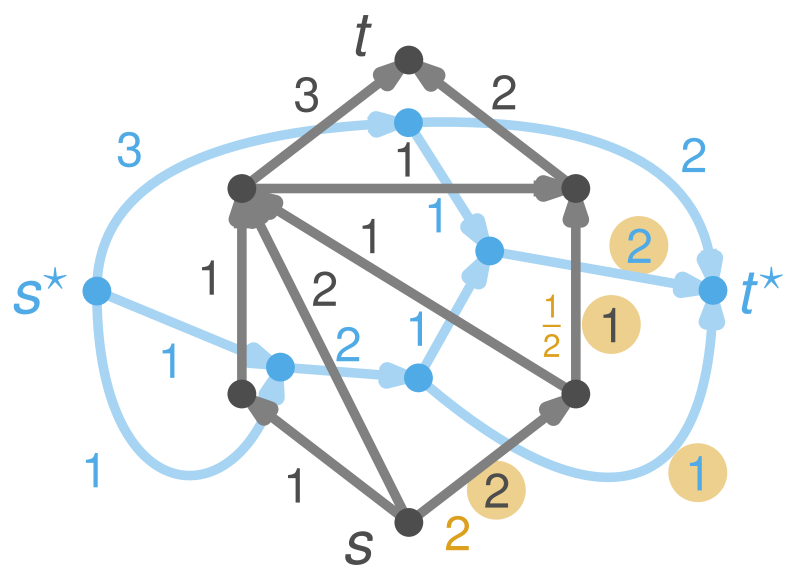
Observation 9 [KCL and KVL Duality]

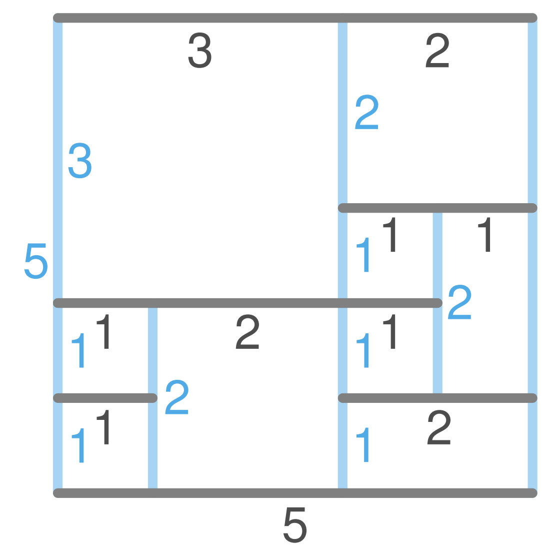
KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^\star$

Susceptance Scaling

- Apply a feasible flow
- Can we make it electrical feasible?





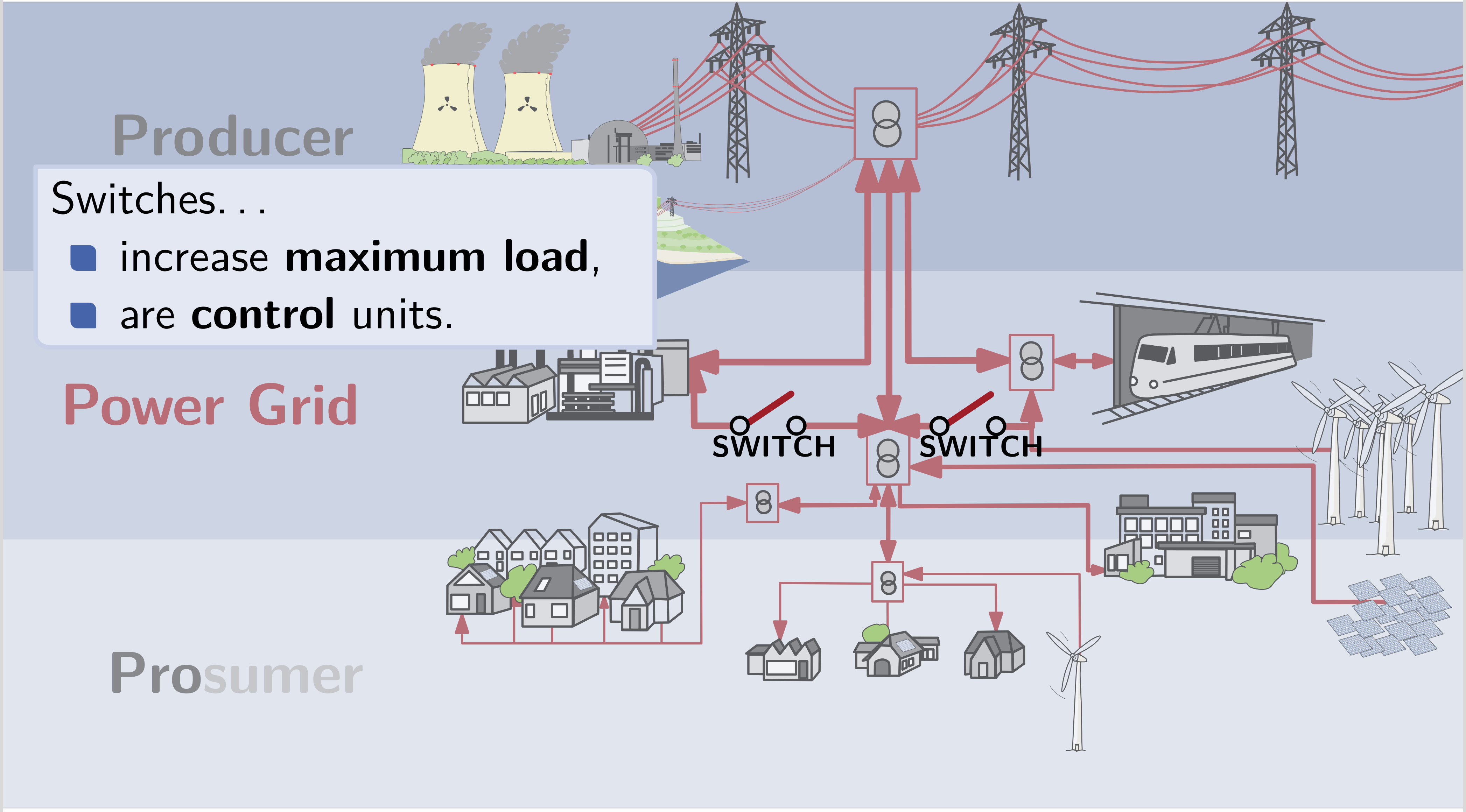


b scaling

Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^\star$

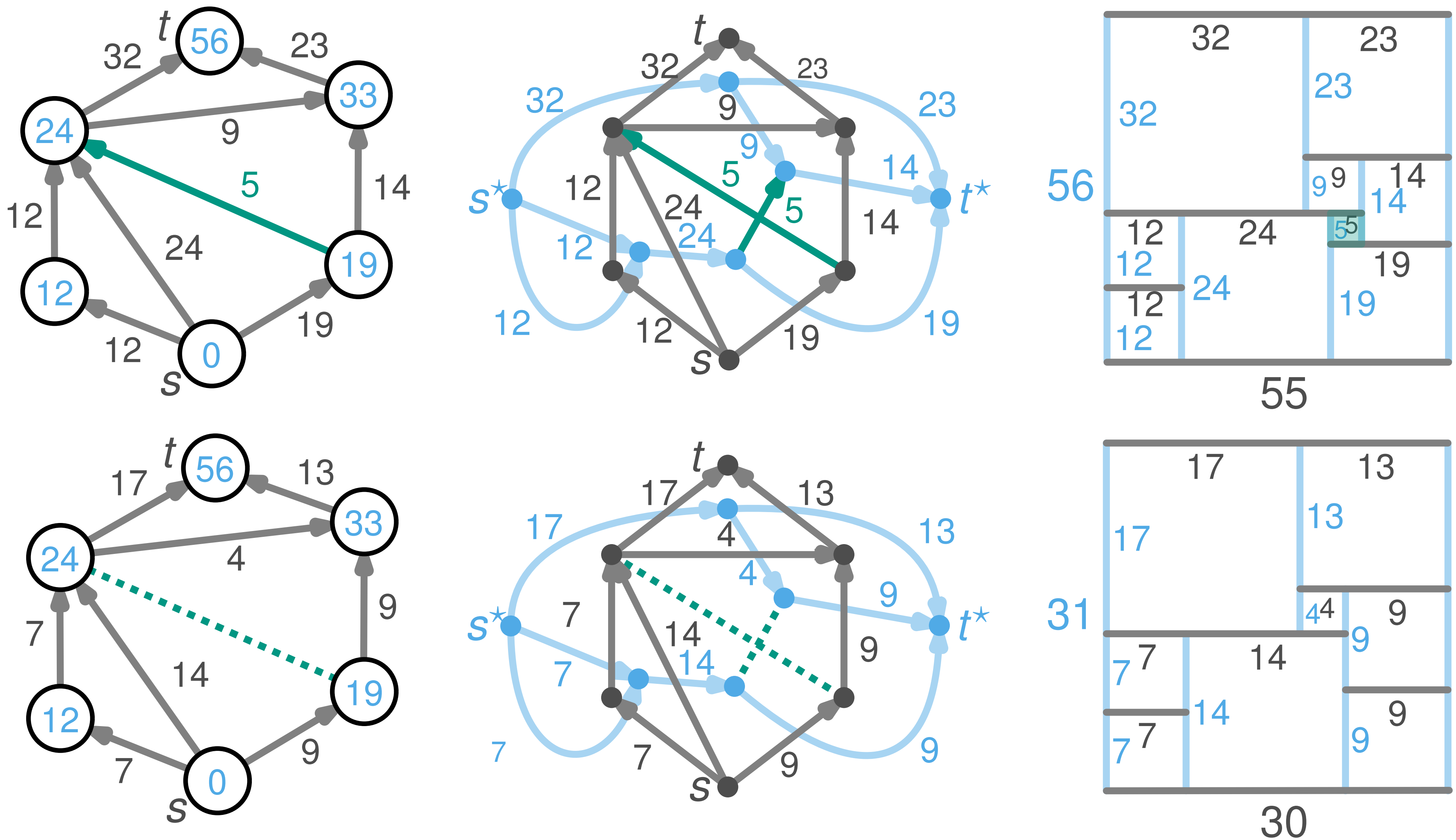
Discrete Changes to the Power Grid



Switches. . .

- increase **maximum load**,
- are **control** units.

Switching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t 32 23 32 9 23 9 5 14 s* 12 24 5 t* 24 14 12 12 19 19 12 s
32 23 23 32 56 9 9 14 14 12 24 5 5 12 19 24 12 19 12 55
t 56 17 13 33 24 4 7 9 14 19 12 0 7 9 s
t 17 13 17 4 13 4 9 s* 7 14 t* 14 9 7 7 9 7 s 9
17 13 13 17 31 4 4 9 7 7 14 9 14 7 7 9 9 30

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