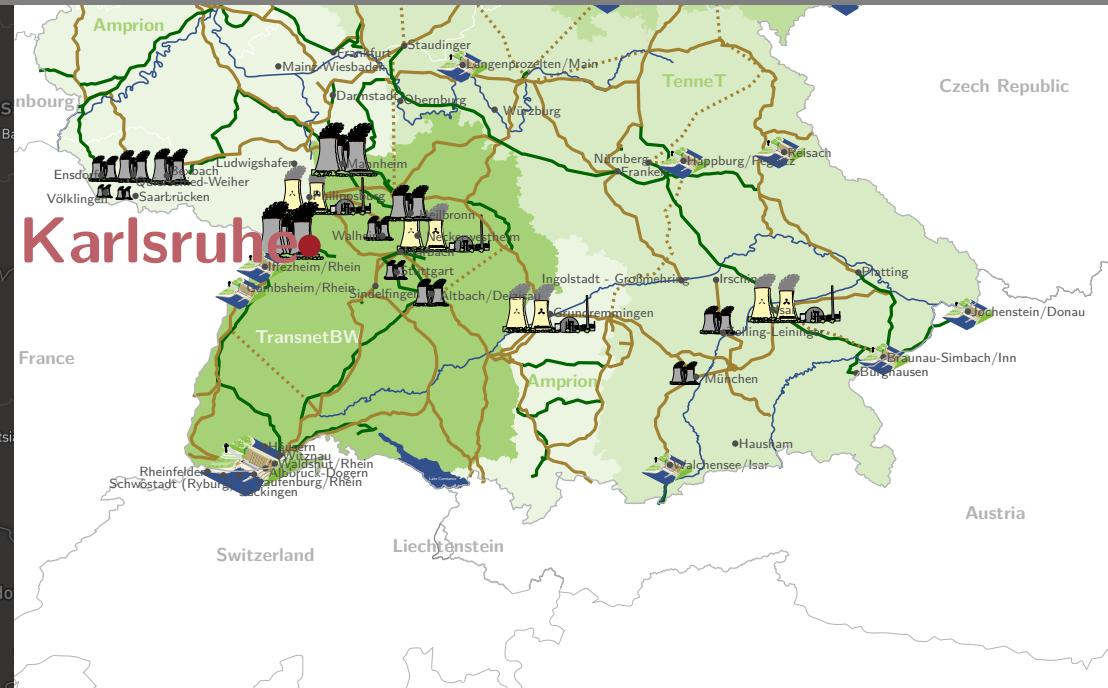
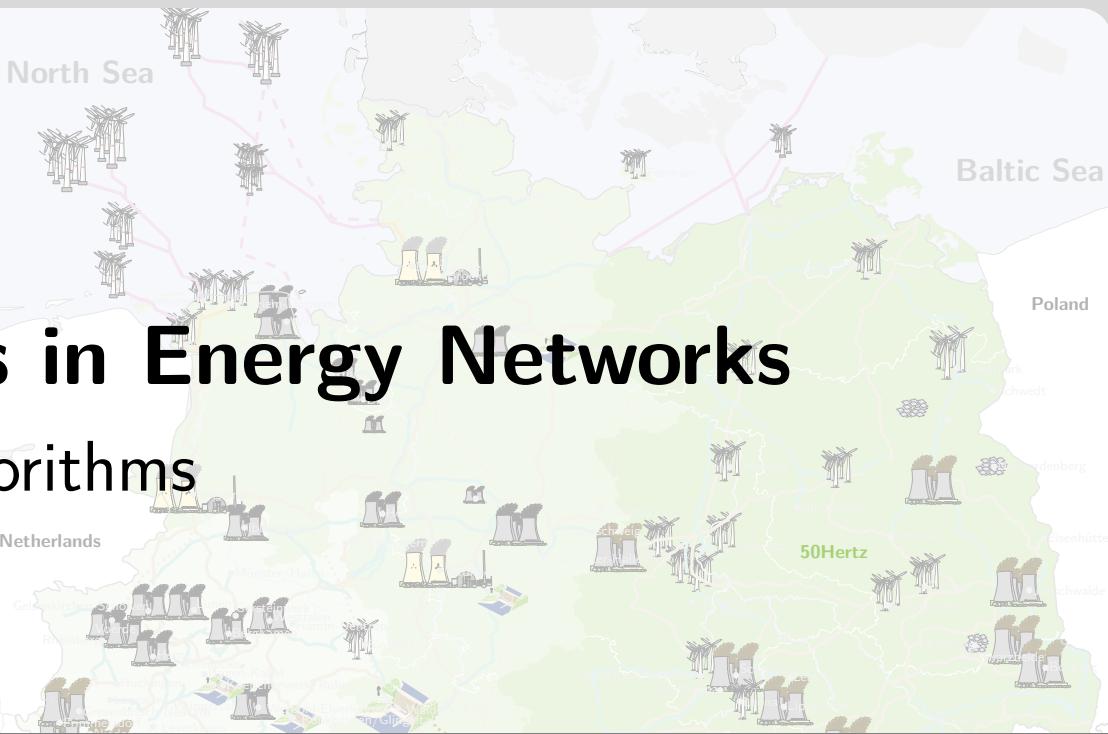
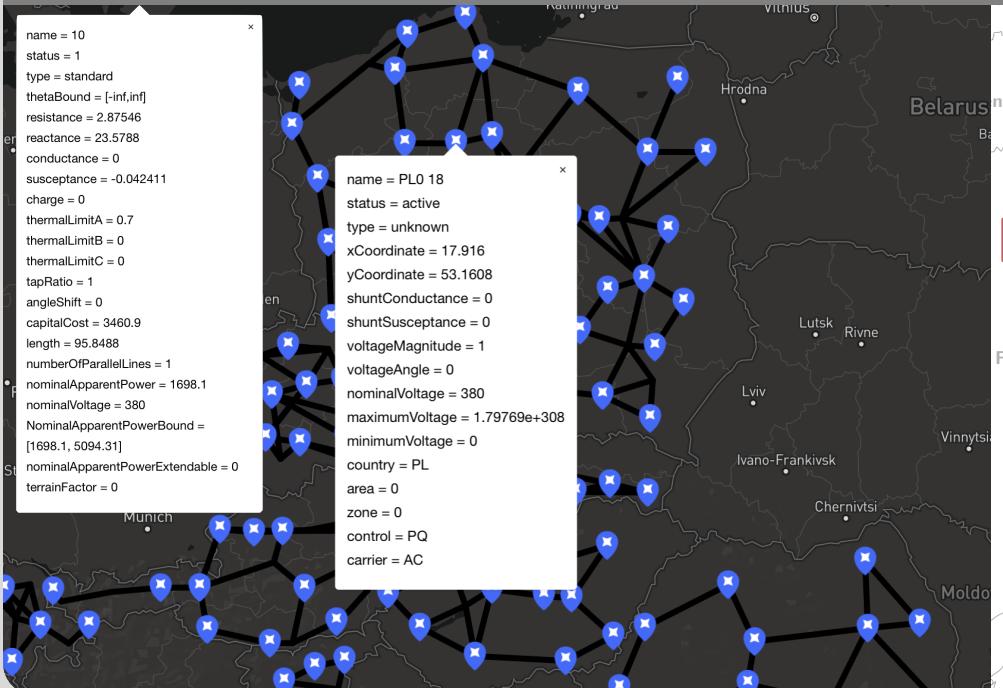


Combinatorial Problems in Energy Networks

Graph-theoretic Models and Algorithms

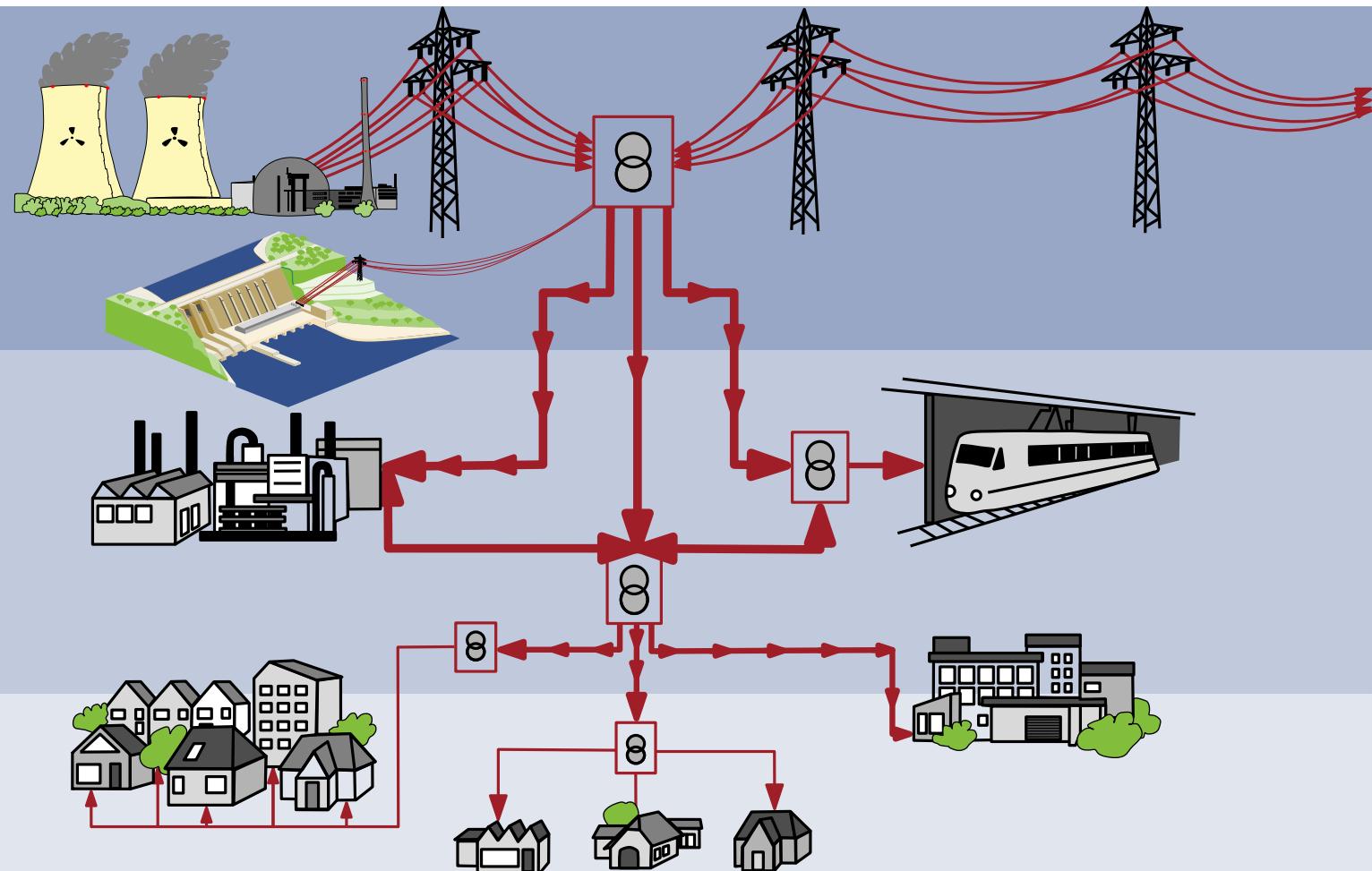
Franziska Wegner

Doctoral Examination



Recent Development in Power Grids

Producer

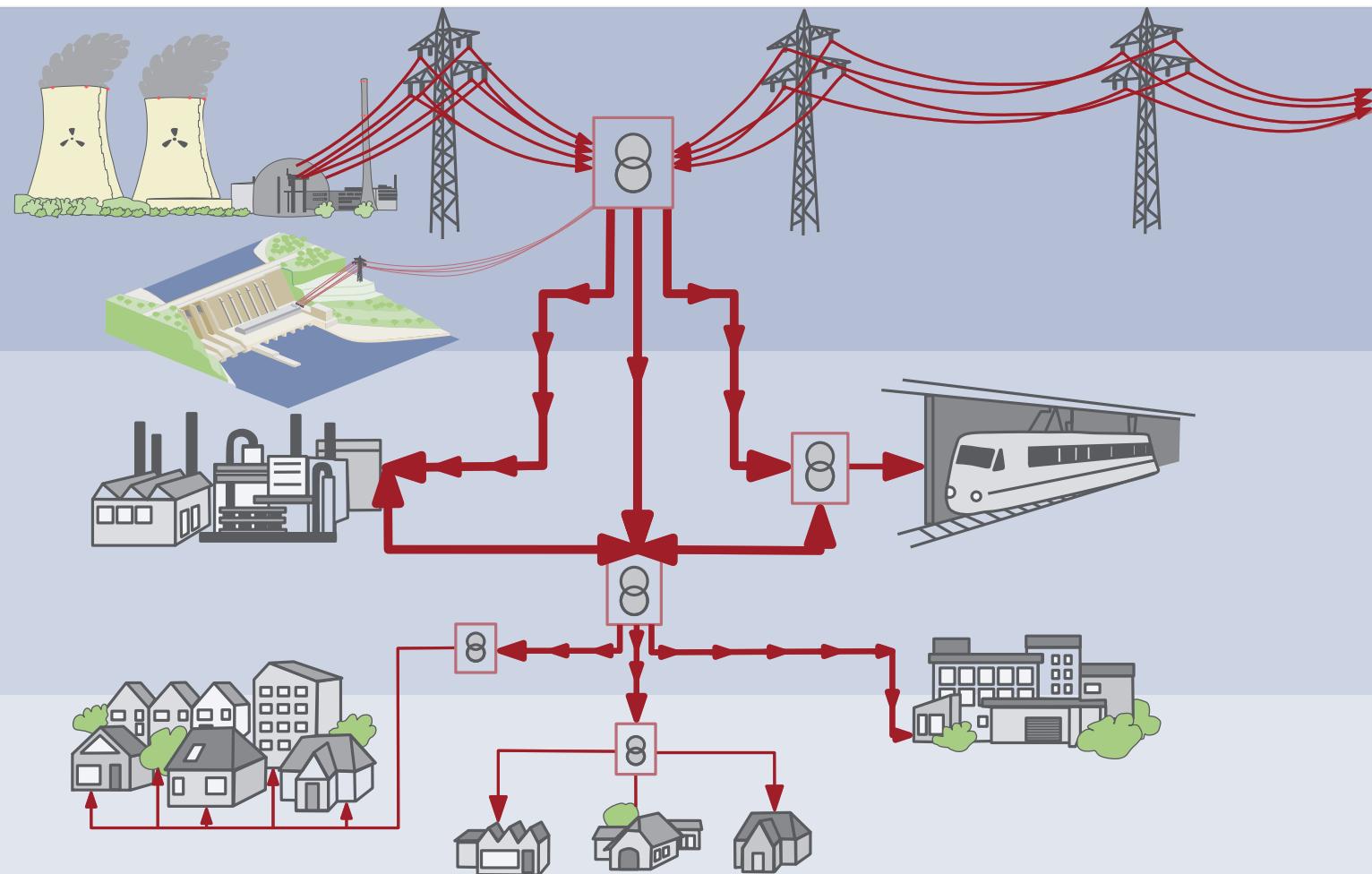


Power Grid

Consumer

Recent Development in Power Grids

Producer



Power Grid

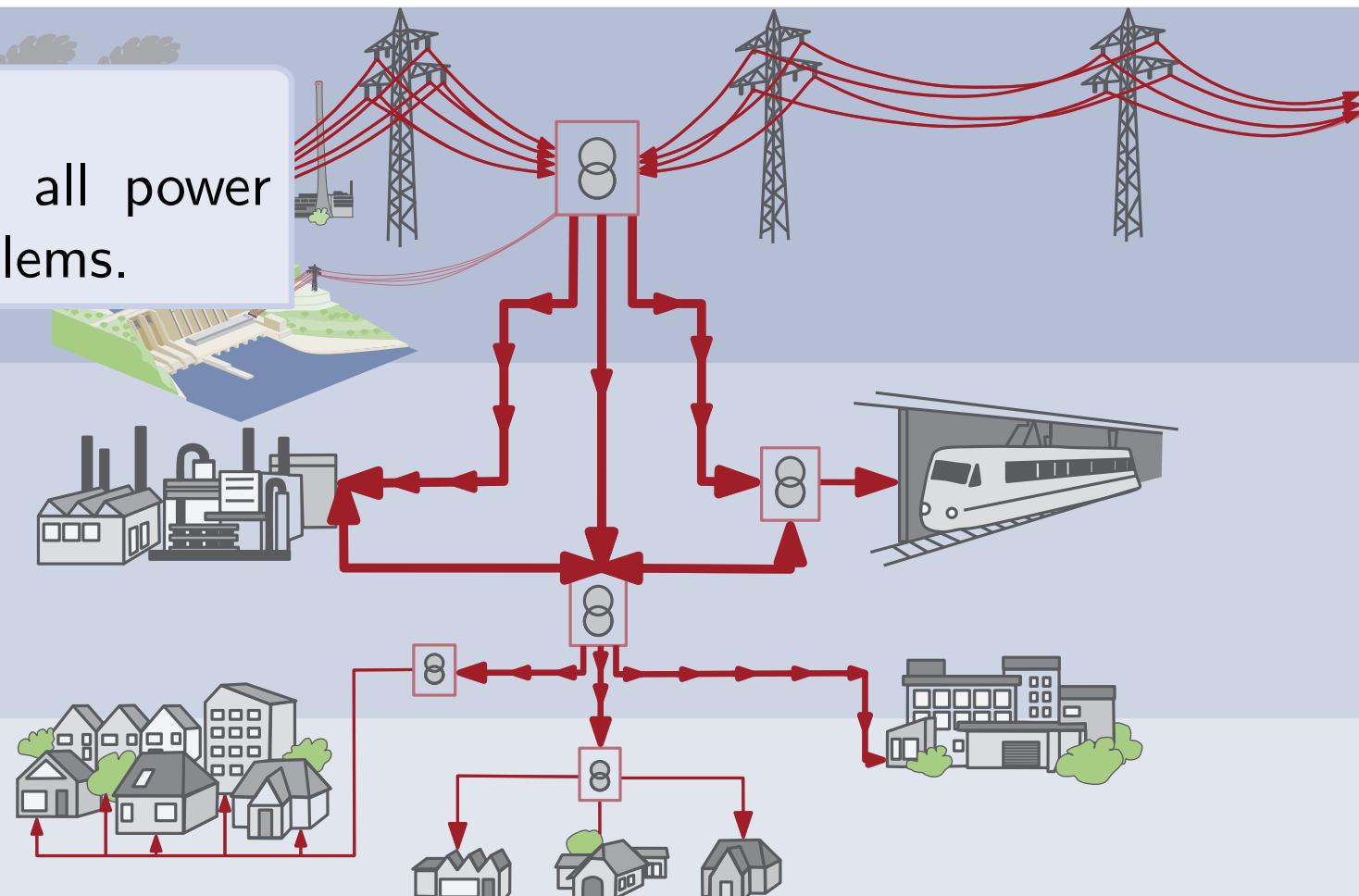
Consumer

Recent Development in Power Grids

Power Flows...

- are essential for all power grid related problems.

Power Grid



Consumer

Recent Development in Power Grids

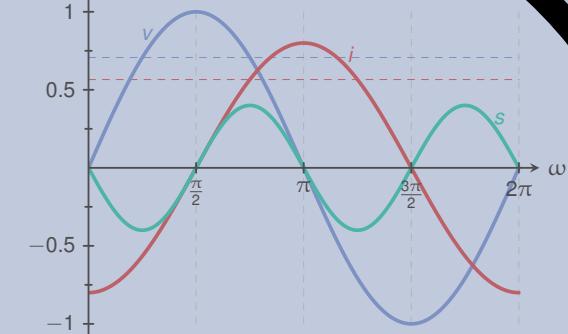
Power Flows...

- are essential for all power grid related problems.

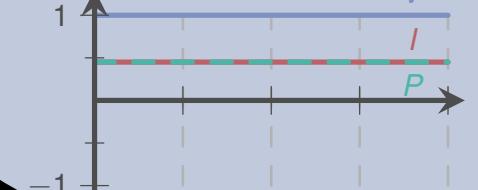
Power Grid

Consumer

$v(u, t), i(u, t), s(u, t)$

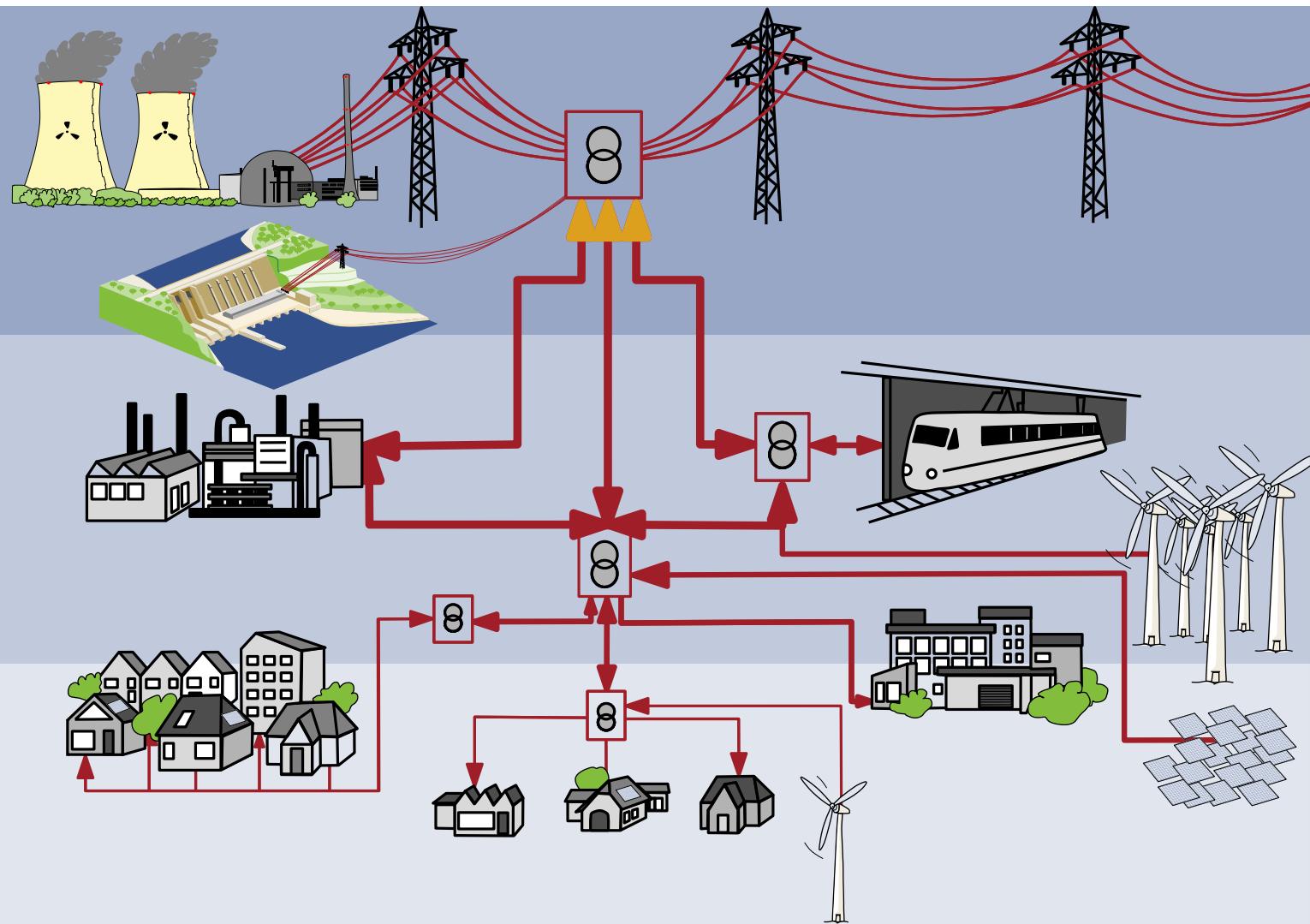


V, I, P



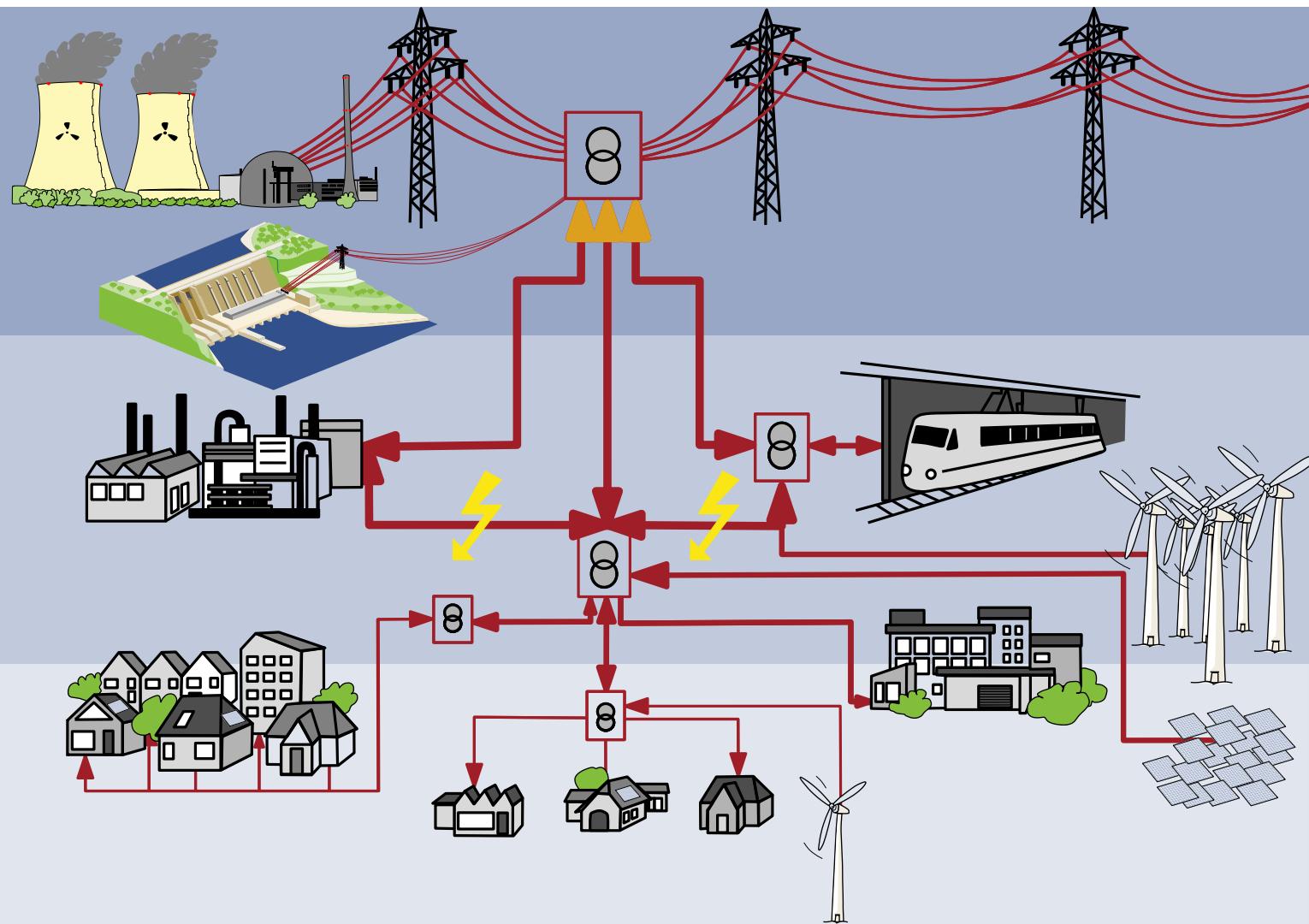
Recent Development in Power Grids

Producer



Recent Development in Power Grids

Producer

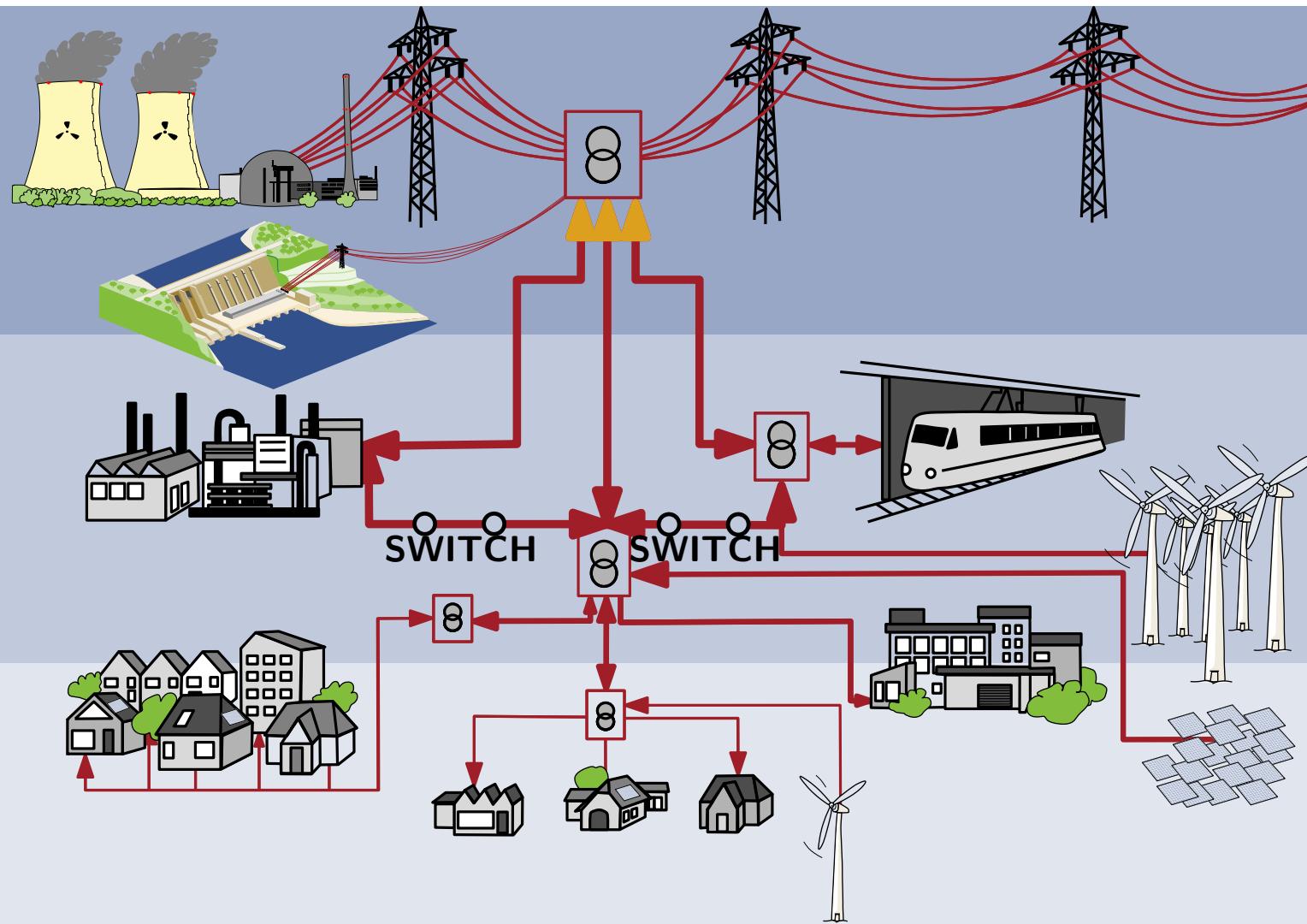


Power Grid

Prosumer

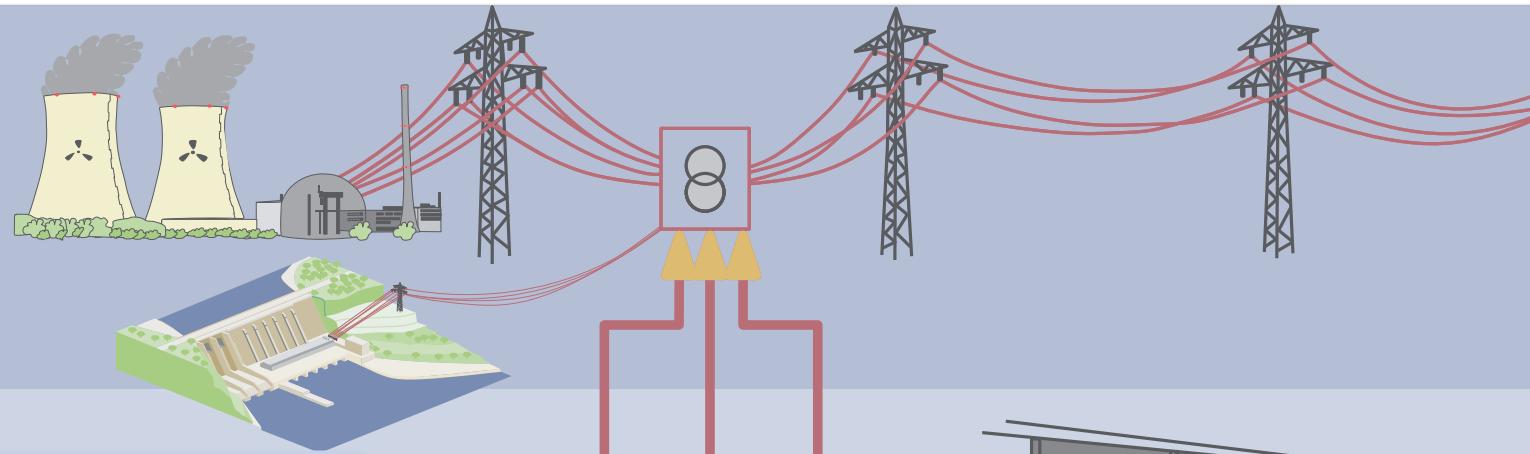
Recent Development in Power Grids

Producer



Recent Development in Power Grids

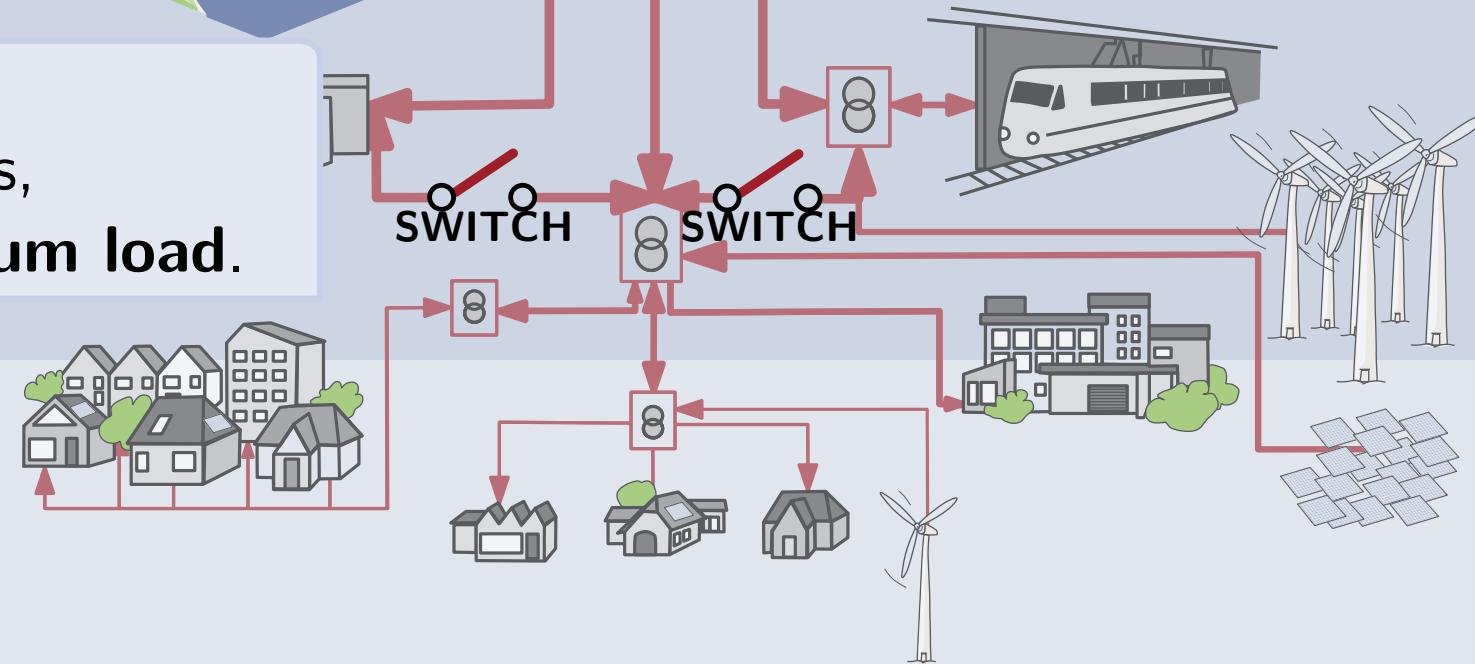
Producer



Switches...

- are **control units**,
- increase **maximum load**.

Prosumer



Recent Development in Power Grids

Producer



Switches...

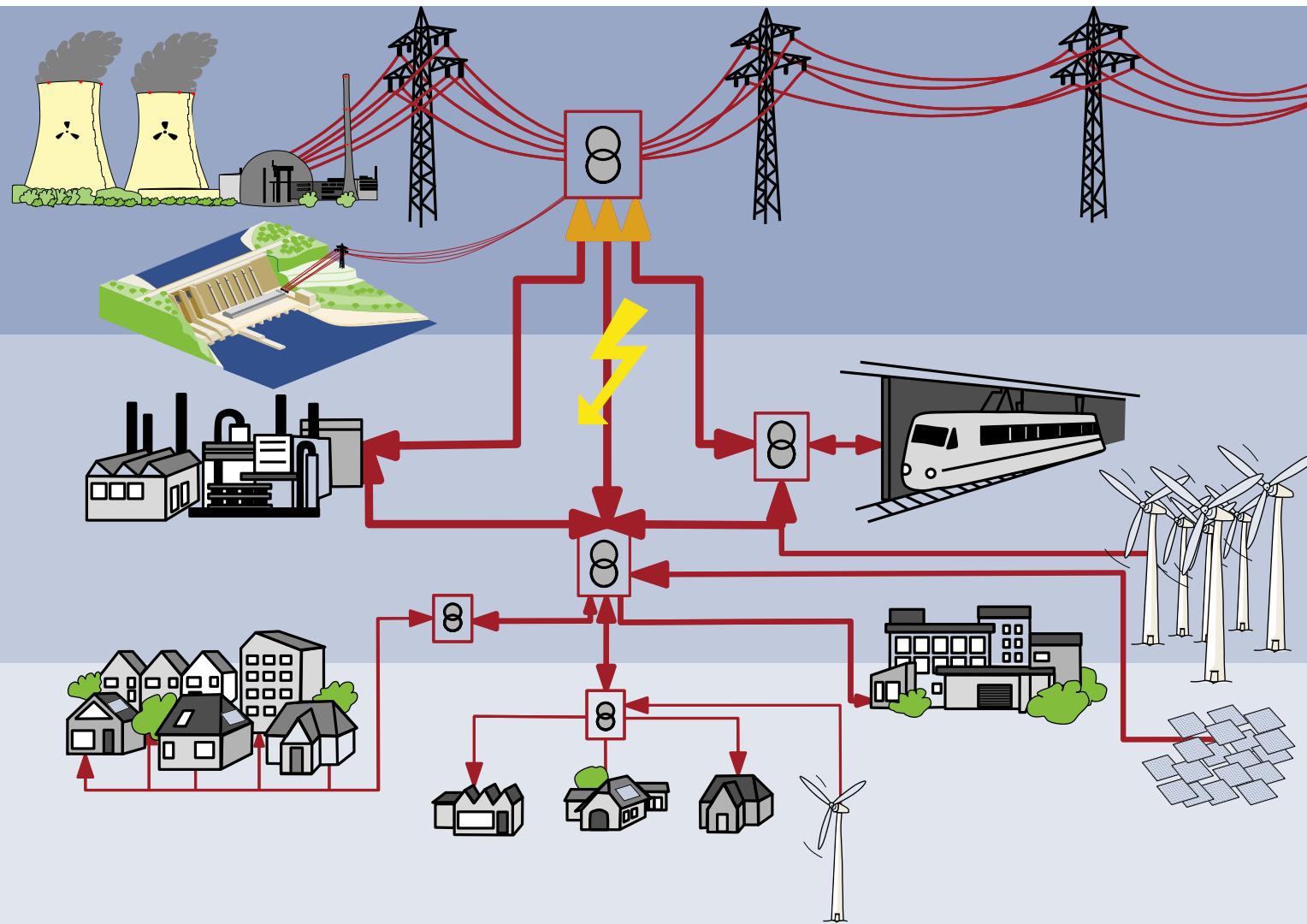
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Prosumer



Recent Development in Power Grids

Producer

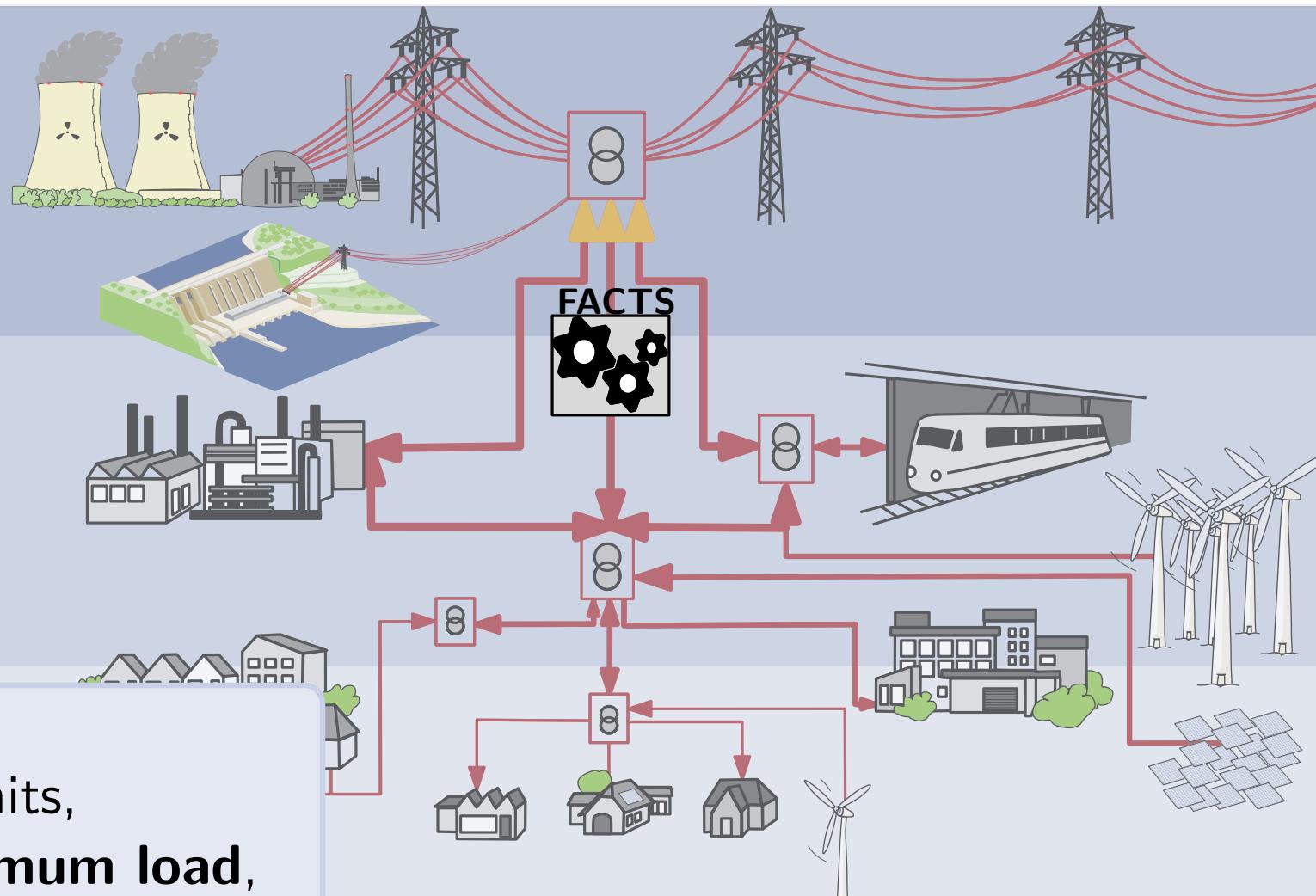


Power Grid

Prosumer

Recent Development in Power Grids

Producer



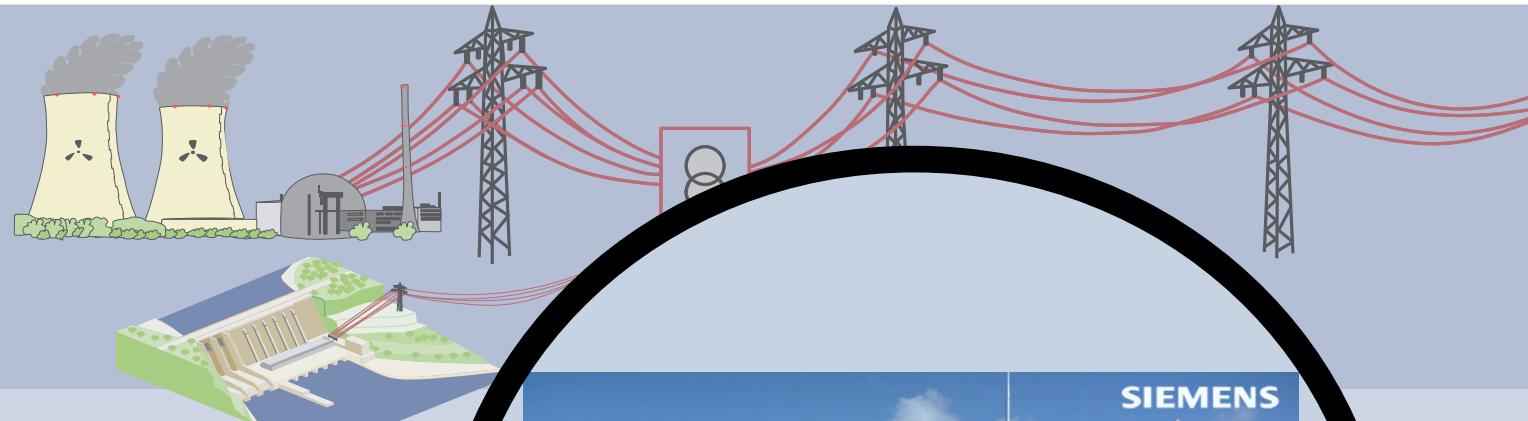
Power Grid

FACTS...

- are **control** units,
- increase **maximum load**,
- are **expensive**.

Recent Development in Power Grids

Producer



Power Grid



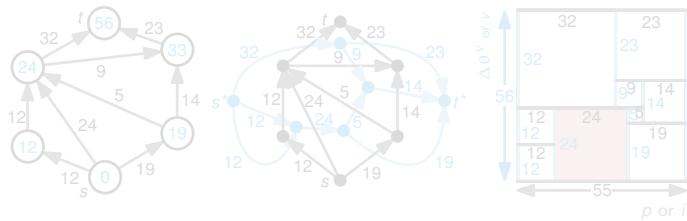
FACTS...

- are **control units**,
- increase **maximum load**,
- are **expensive**.



Problem Statements

Feasible electrical flows



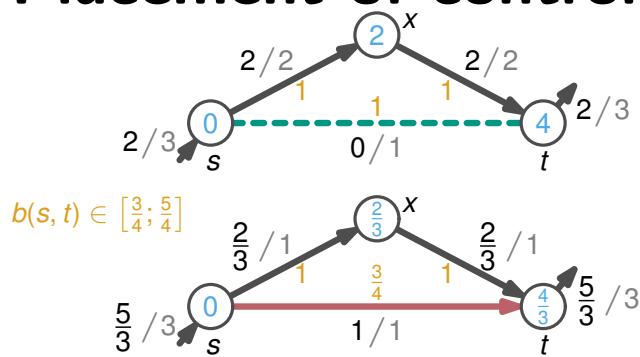
Dynamic models

Static models

Alternating Current (AC) model

Direct Current (DC) model

Placement of control units



Discrete control decision

[Grastien, Rutter, Wagner, W., and Wolf, 2018]

Continuous control decision

[Leibfried, Mchedlidze, Meyer-Hübner, Nöllenburg, Rutter, Sanders, Wagner, and W., 2015]

[Mchedlidze, Nöllenburg, Rutter, Wagner, and W., 2015]

Cable layout

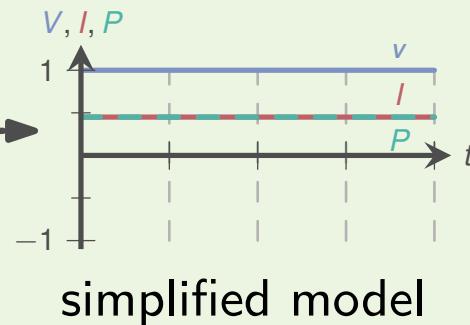
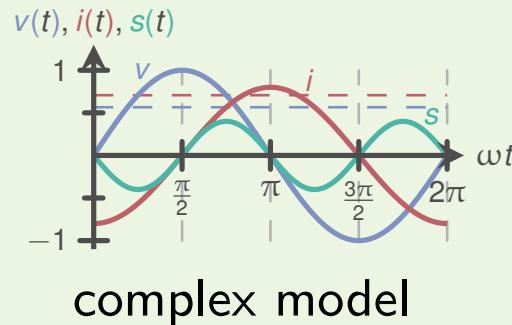


Wind farm cabling with multiple cable types

[Lehmann, Rutter, Wagner, and W., 2017]

Methodology

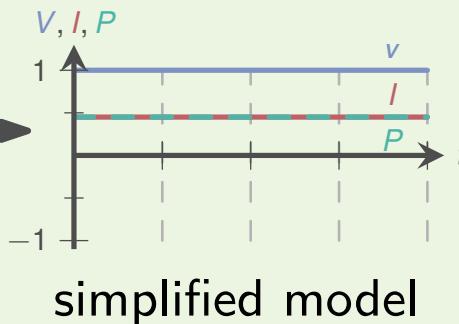
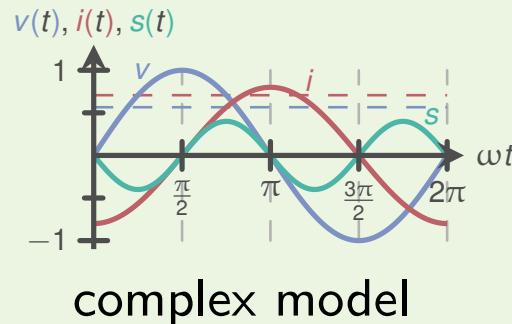
Model



combinatorial
optimization problem

Methodology

Model



combinatorial
optimization problem

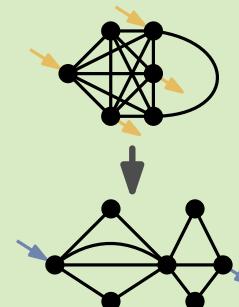
Theory



problem and
complexity analysis



efficient algorithms
with guarantees



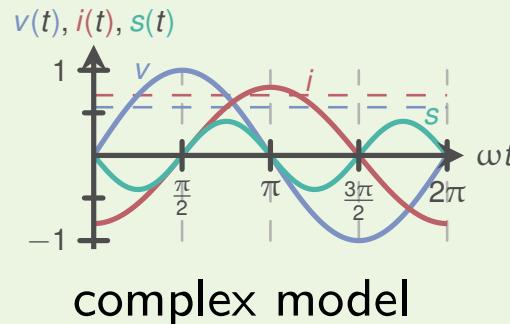
simplified graphs

non-linear
program
↓
mixed-integer
linear program

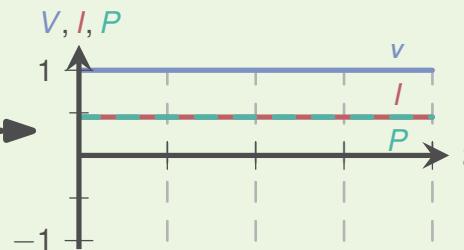
mathematical model

Methodology

Model



complex model



simplified model



combinatorial
optimization problem

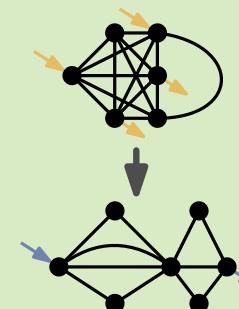
Theory



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non-linear
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mixed-integer
linear program

Implementation

exact algorithms

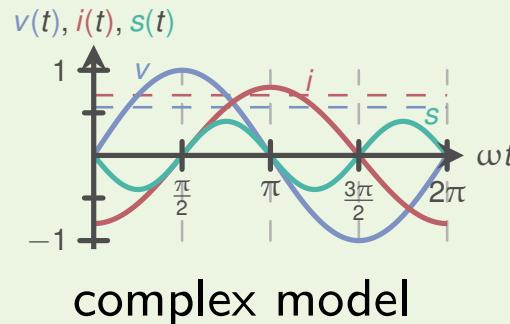
approximation algorithms

mathematical model

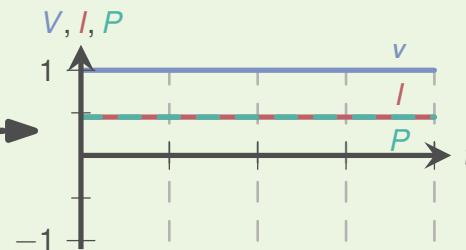
optimal solution

Methodology

Model



complex model



simplified model



combinatorial
optimization problem

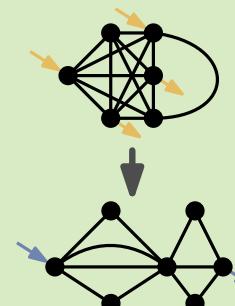
Theory



problem and
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with guarantees



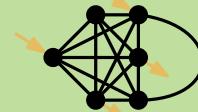
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Implementation

exact algorithms



approximation algorithms

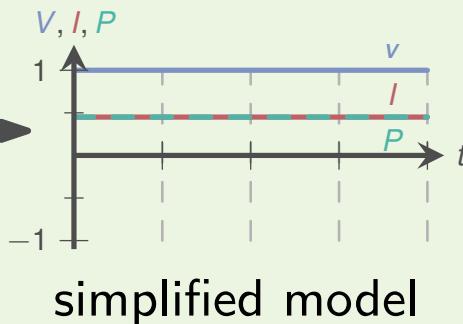
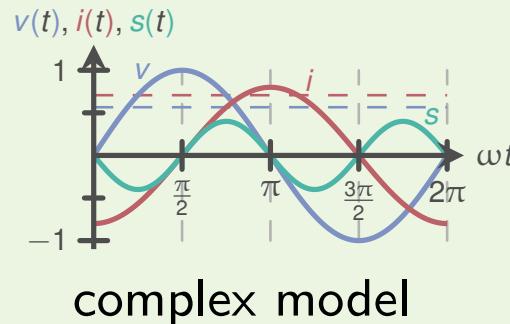
heuristics

mathematical model

optimal solution

Methodology

Model



combinatorial
optimization problem

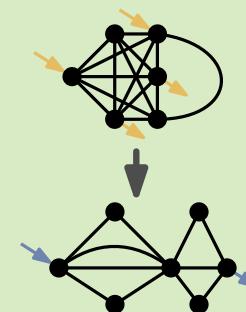
Theory



problem and
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efficient algorithms
with guarantees



simplified graphs

non-linear
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Implementation

exact algorithms



approximation algorithms

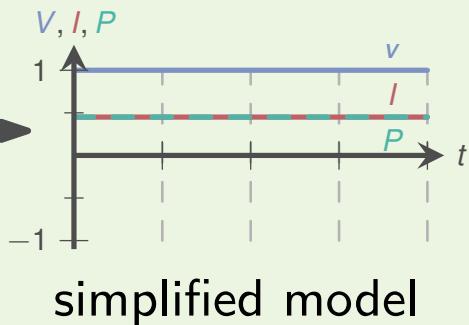
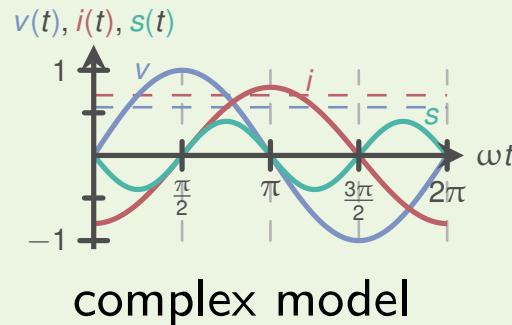
heuristics

real/realistic
data

optimal solution

Methodology

Model



combinatorial
optimization problem

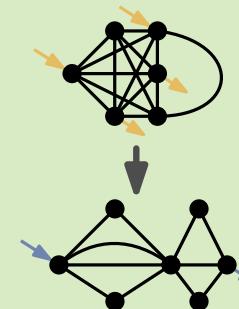
Theory



problem and
complexity analysis



efficient algorithms
with guarantees



simplified graphs

non-linear
program
mixed-integer
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Implementation

exact algorithms



approximation algorithms

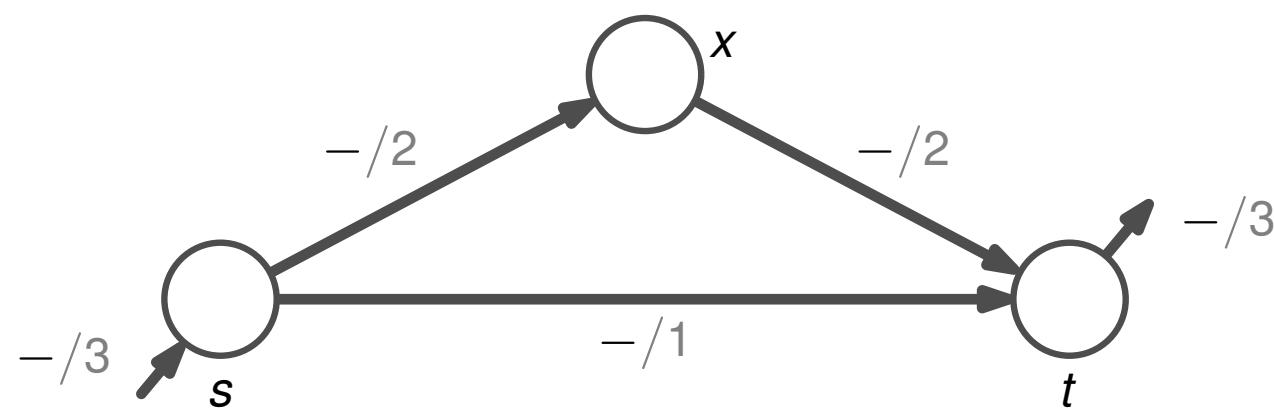
heuristics

real/realistic
data

experimental
evaluation

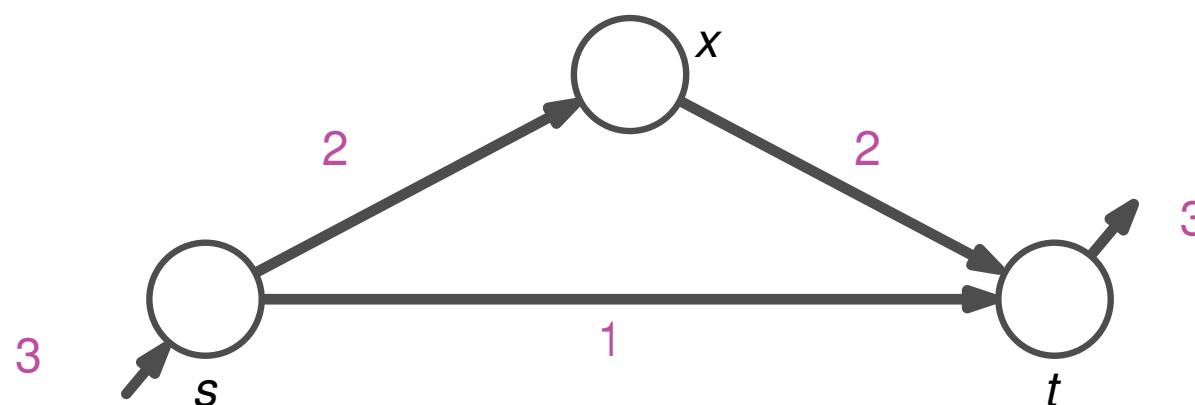
optimal solution

The Maximum Flow (MF) Problem



The Maximum Flow (MF) Problem

- Flow $f: E \rightarrow \mathbb{R}$ with $f_{\text{net}}: V \rightarrow \mathbb{R}$ defined as $f_{\text{net}}(u) := \sum_{\{u,v\} \in E} f(u, v)$ and flow value $F(\mathcal{N}, f) := \sum_{u \in V_G} f_{\text{net}}(u)$
- The value of the maximum flow is defined as
$$\text{MF}(\mathcal{N}) = \max F(\mathcal{N}, f)$$
with f being a feasible flow meaning



The Maximum Flow (MF) Problem

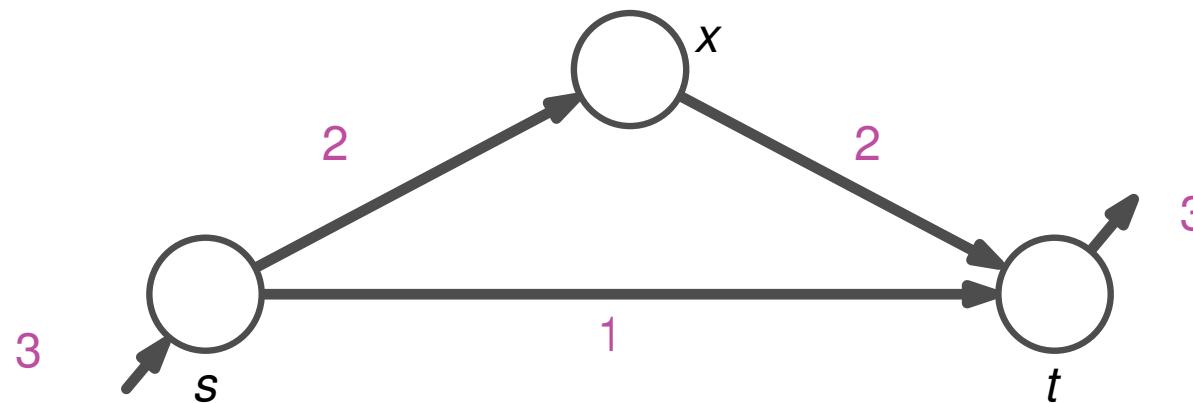
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$$f_{\text{net}}(u) = \sum_{\{u,v\} \in E} f(u, v) = 0 \quad \forall u \in V \setminus (V_G \cup V_D)$$
$$-\infty \leq f_{\text{net}}(u) \leq -d \quad \forall u \in V_D$$
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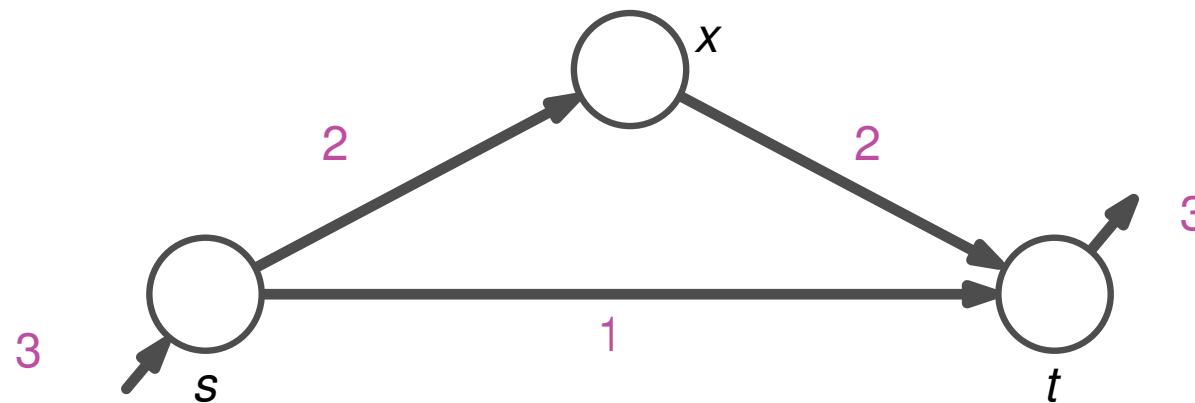


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(node)
conservation of
flow

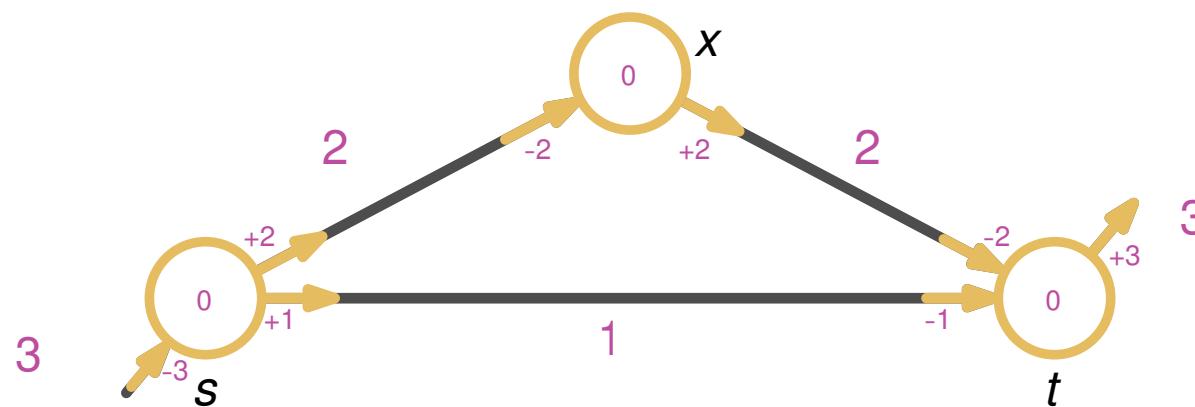
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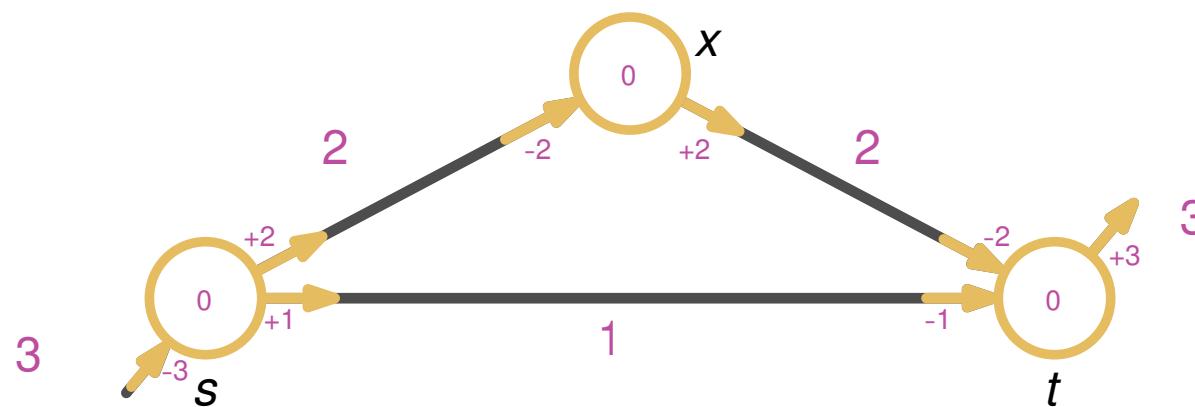
(node) conservation of flow	$f_{\text{net}}(u) = \sum_{\{u,v\} \in E} f(u, v) = 0 \quad \forall u \in V \setminus (V_G \cup V_D)$
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The Maximum Flow (MF) Problem

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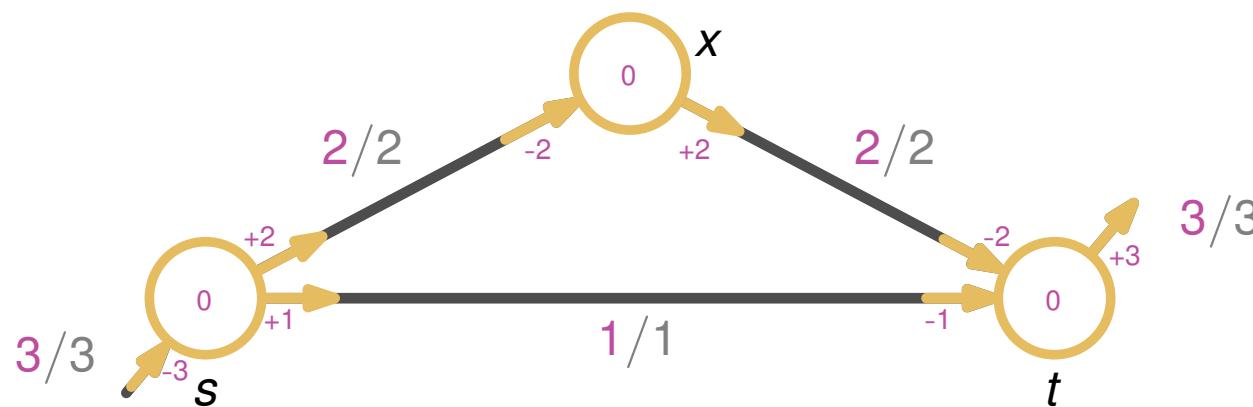
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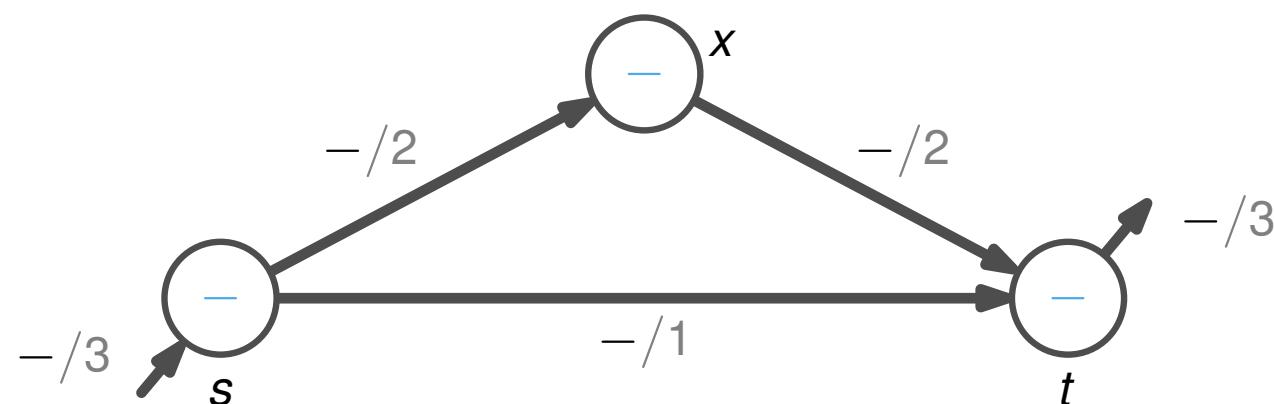
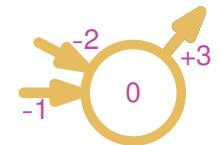


The Maximum Power Flow (MPF) Problem

[Zimmerman et al., 2011]

A feasible power flow has to satisfy (additional) physical constraints:

- The Kirchhoff's Current Law (KCL) which relates to flow conservation, i.e., $f_{\text{net}}(u) = 0$ for all $u \in V \setminus (V_G \cup V_D)$

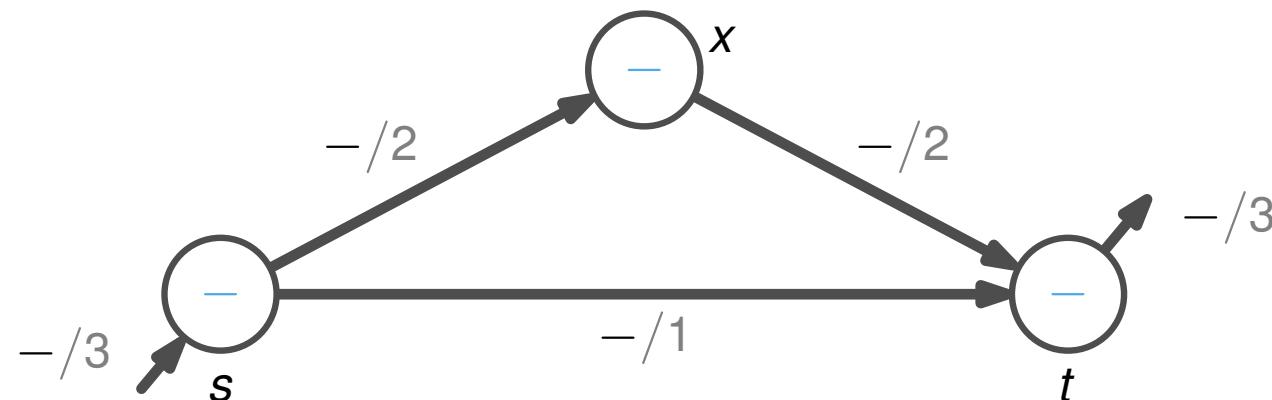
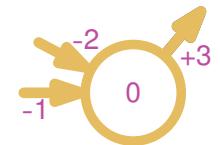


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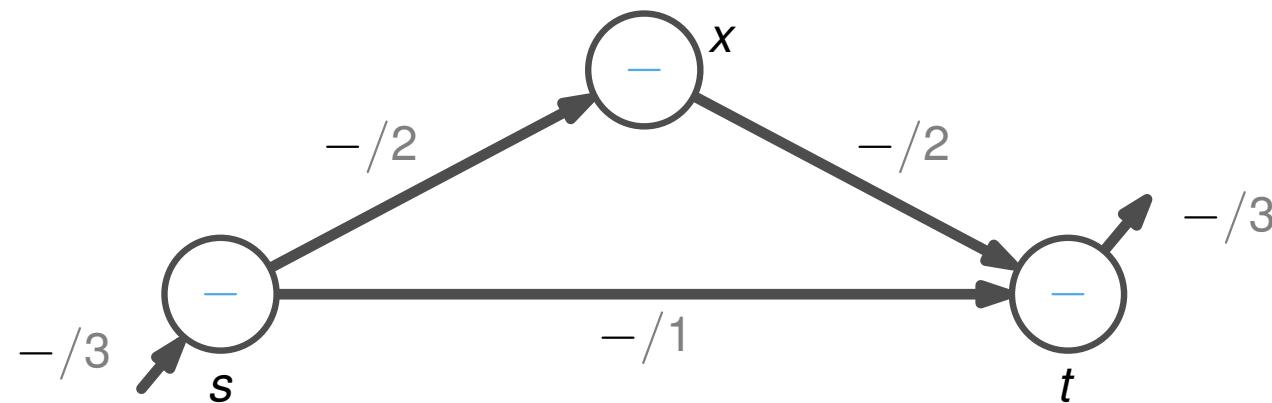
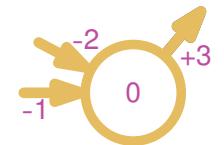
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$$\theta(v) - \theta(u) = f(u, v) \quad \forall (u, v) \in E$$

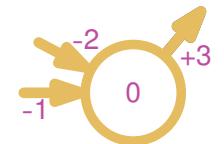


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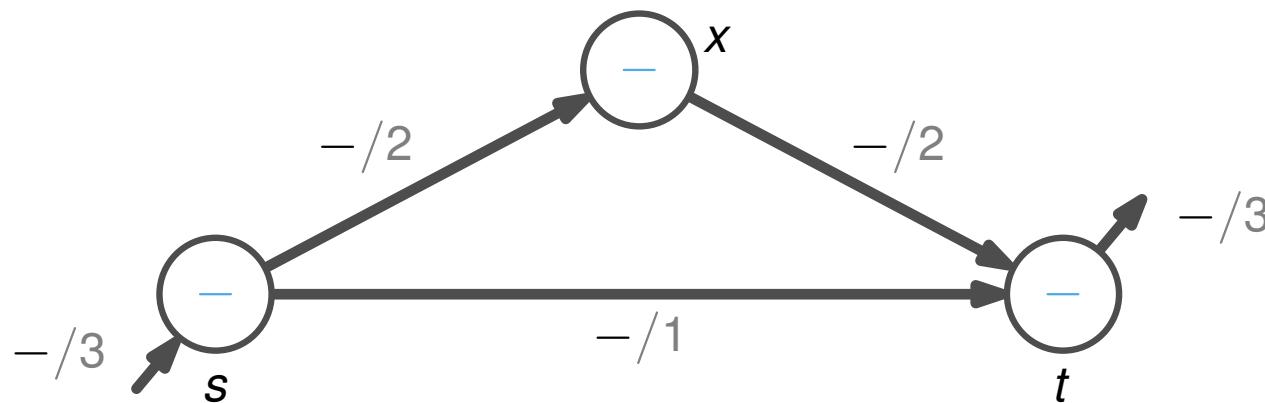
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The Maximum Power Flow (MPF) Problem

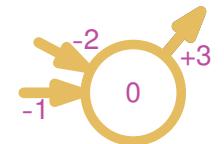
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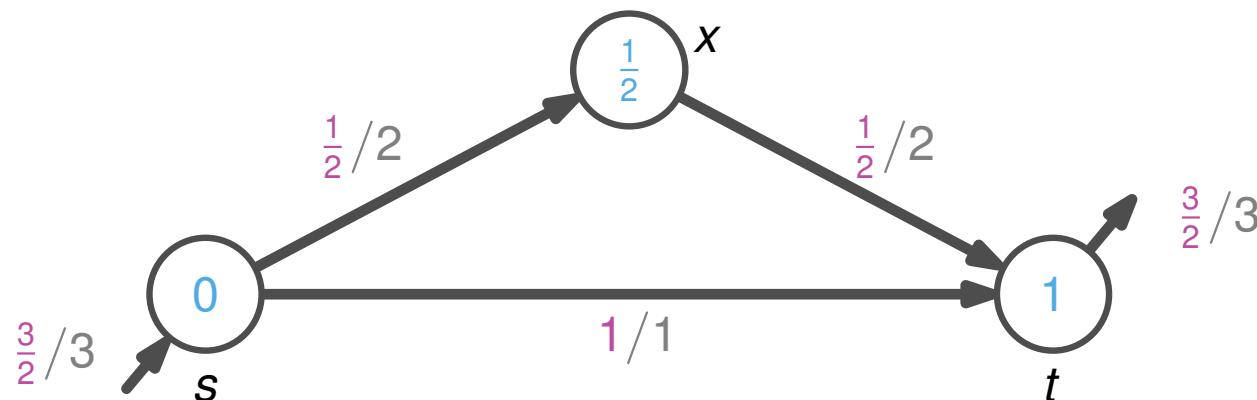
with f being a feasible power flow meaning

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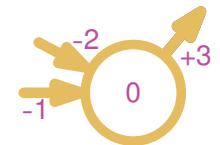
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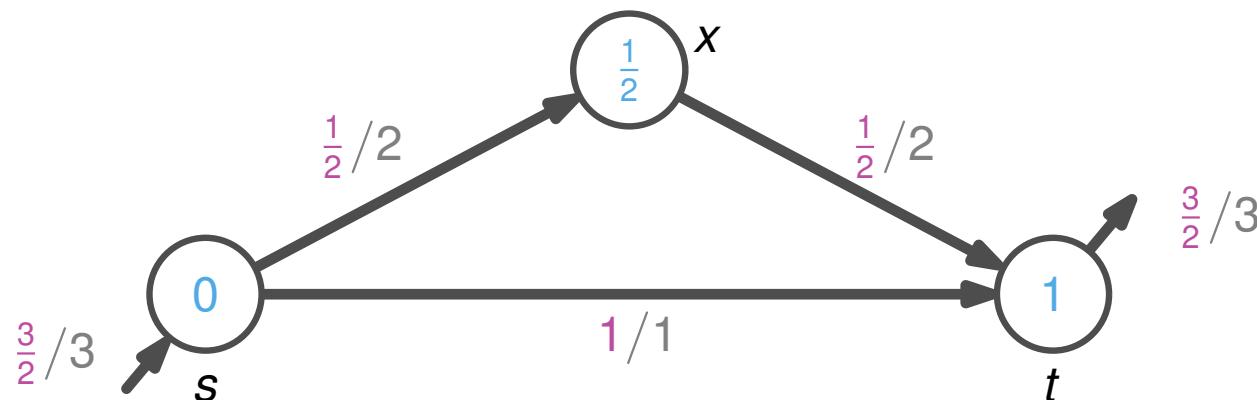
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cycle
conservation of
flow

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The Maximum Power Flow (MPF) Problem

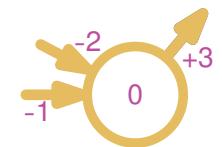
[Zimmerman et al., 2011]

- The value of the maximum power flow is defined as

$$\text{MPF}(\mathcal{N}) := \max F(\mathcal{N}, f)$$

with f being a feasible power flow meaning

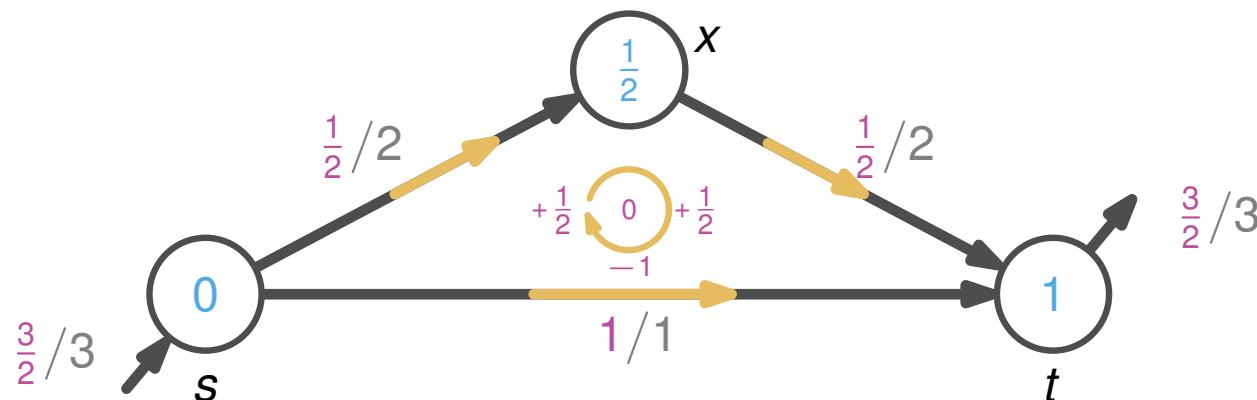
- The Kirchhoff's Current Law (KCL) which relates to flow conservation, i.e., $f_{\text{net}}(u) = 0$ for all $u \in V \setminus (V_G \cup V_D)$



- In addition, the Kirchhoff's Voltage Law (KVL) with assignment of potentials (voltage angles) $\theta: V \rightarrow \mathbb{R}$

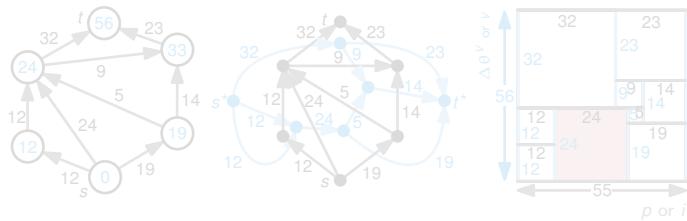
cycle
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Classification

Feasible electrical flows



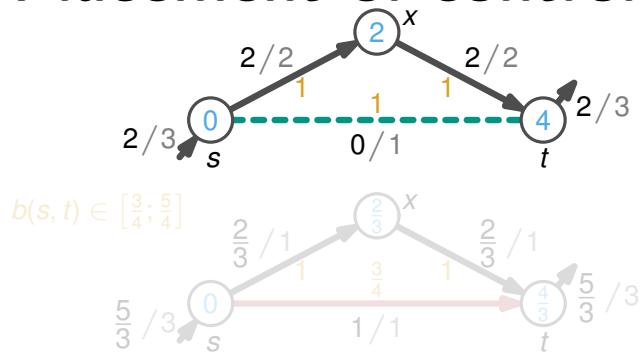
Dynamic models

Static models

Alternating Current (AC) model

Direct Current (DC) model

Placement of control units



Discrete control decision

[Grastien, Rutter, Wagner, W., and Wolf, 2018]

Continuous control decision

[Leibfried, Mchedlidze, Meyer-Hübner, Nöllenburg, Rutter, Sanders, Wagner, and W., 2015]

[Mchedlidze, Nöllenburg, Rutter, Wagner, and W., 2015]

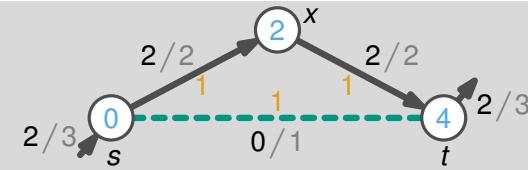
Cable layout



Wind farm cabling with multiple cable types

[Lehmann, Rutter, Wagner, and W., 2017]

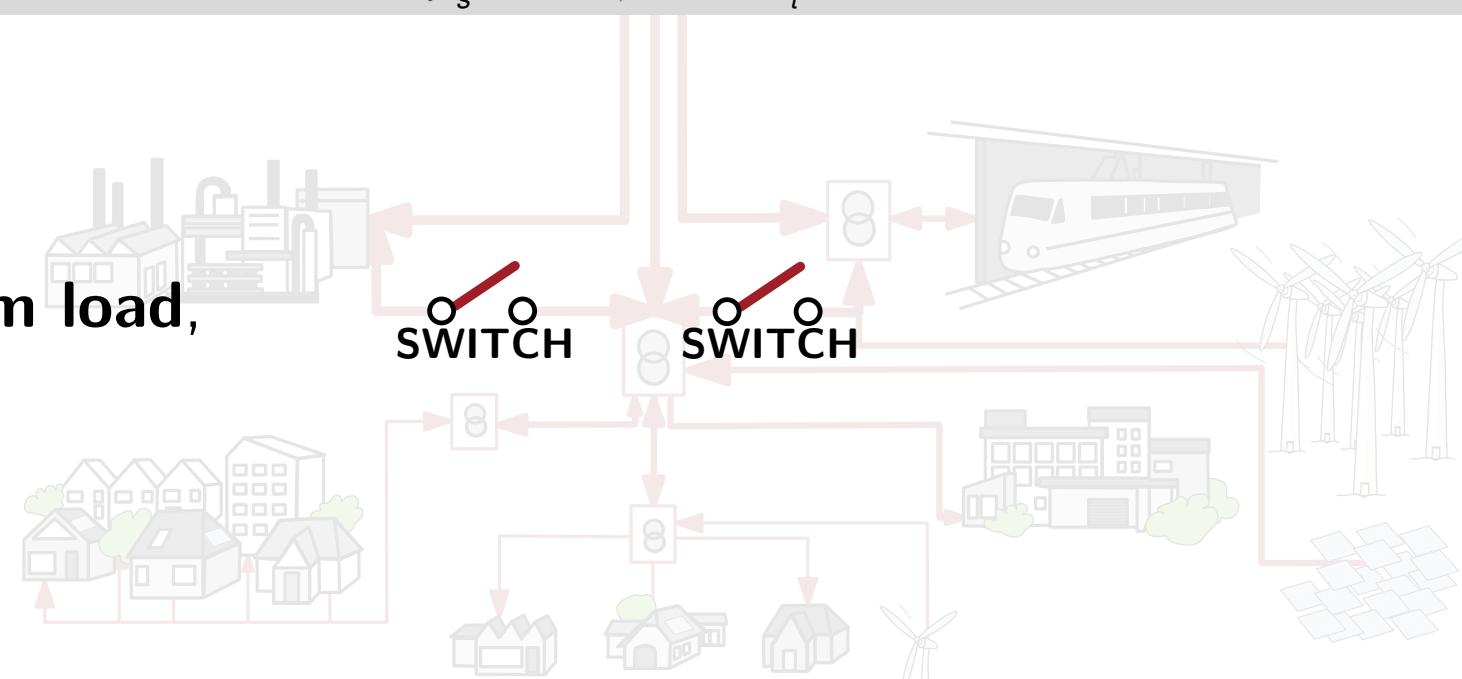
Placement of control units Discrete control decision



[Grastien, Rutter, Wagner, W., and Wolf, 2018]

Switches...

- increase **maximum load**,
- are **control units**.



The Maximum Transmission Switching Flow (MTSF) Problem

[Fisher et al., 2008]

- The value of the MAXIMUM TRANSMISSION SWITCHING FLOW is defined as

$$\text{MTSF}(\mathcal{N}) := \max_{S \subseteq E} \text{MPF}(\mathcal{N} - S)$$

with f being a feasible power flow meaning

$$f_{\text{net}}(u) = 0$$

$$\forall u \in V \setminus (V_G \cup V_D)$$

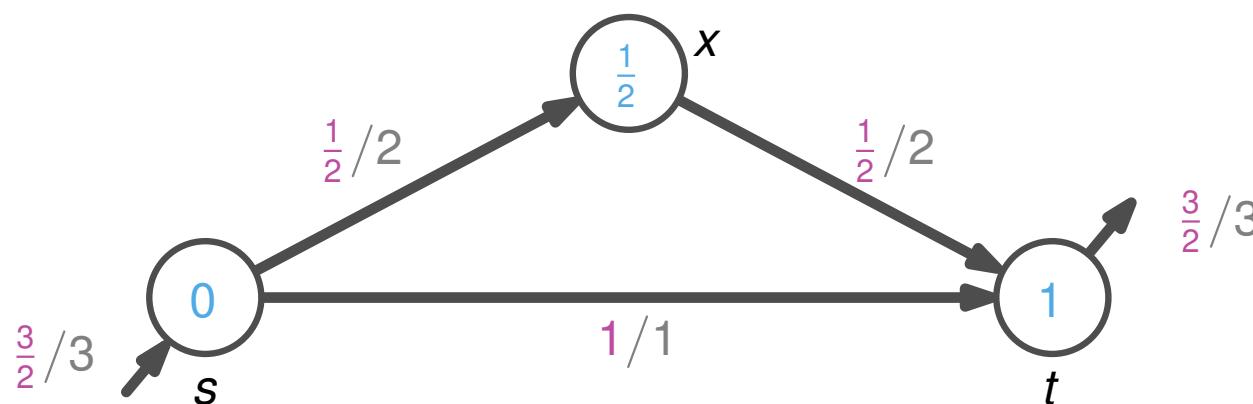
$$|f(u, v)| \leq z(u, v) \cdot \text{cap}(u, v)$$

$$\forall (u, v) \in E$$

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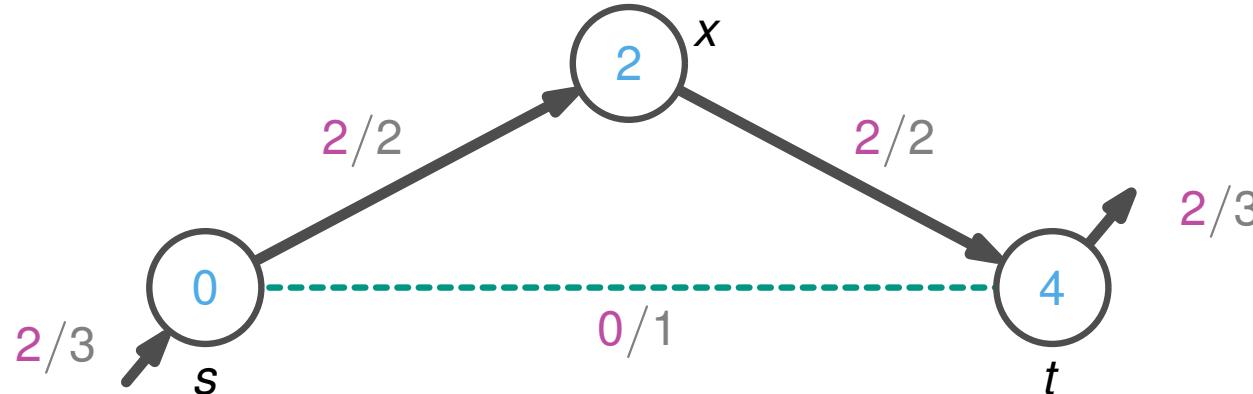
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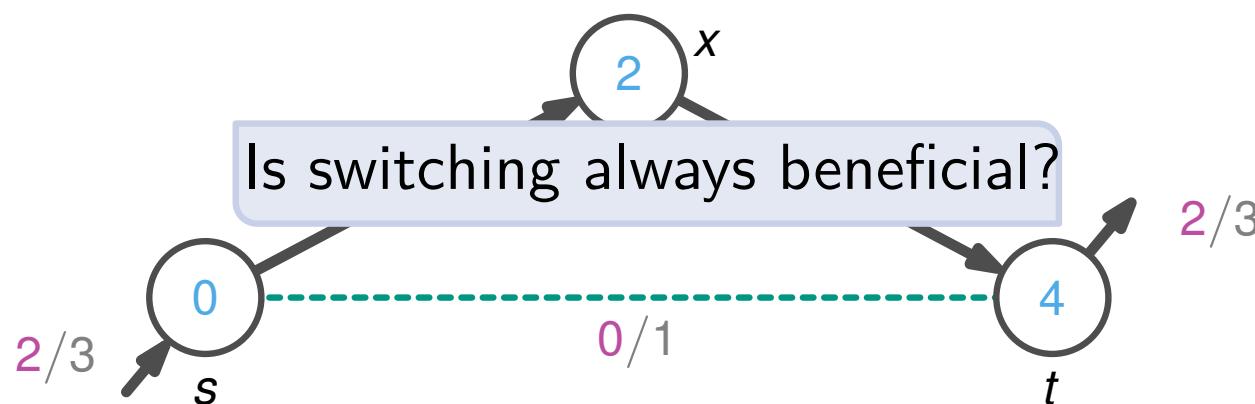
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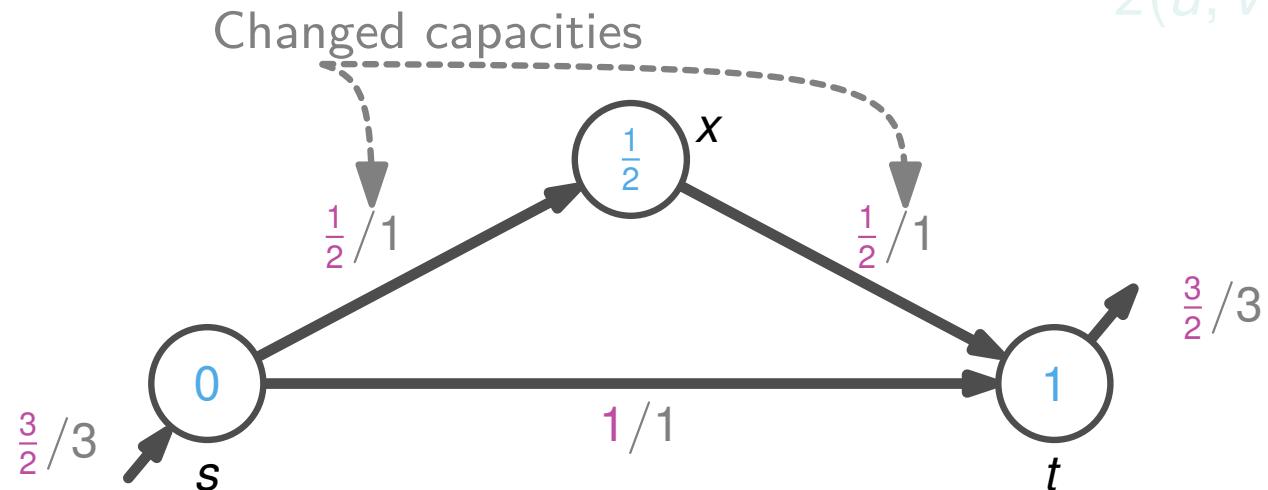
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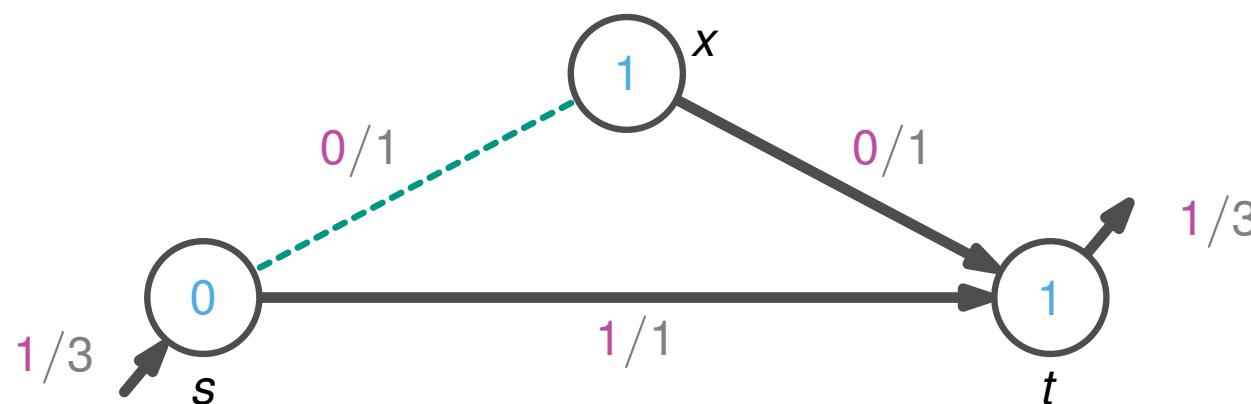
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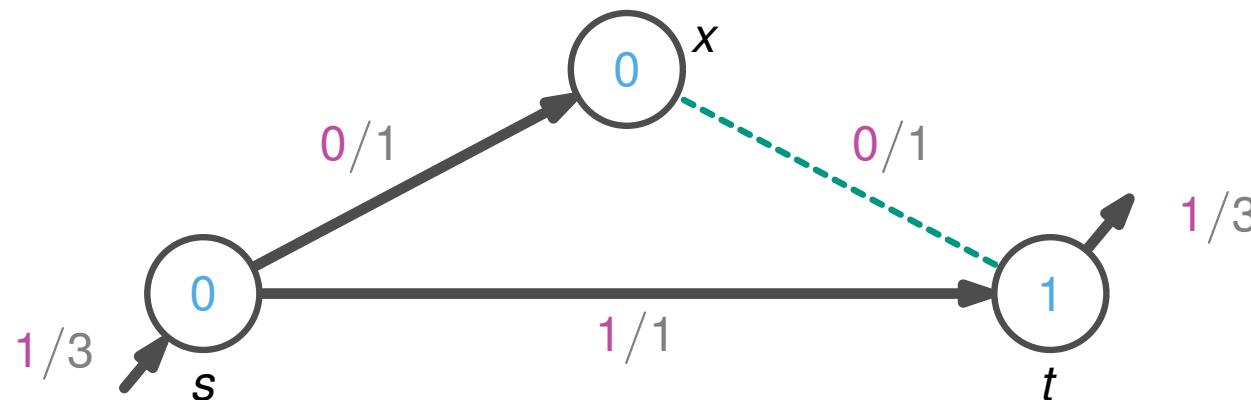
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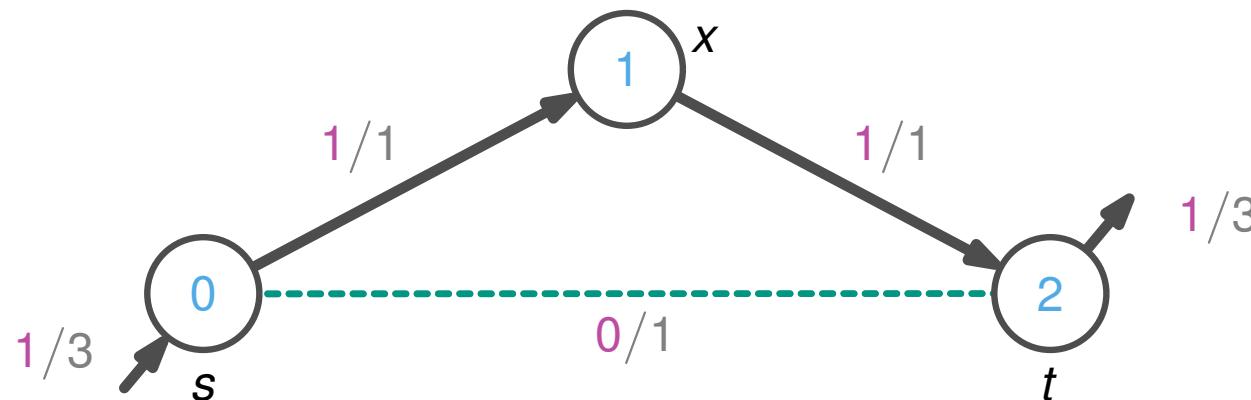
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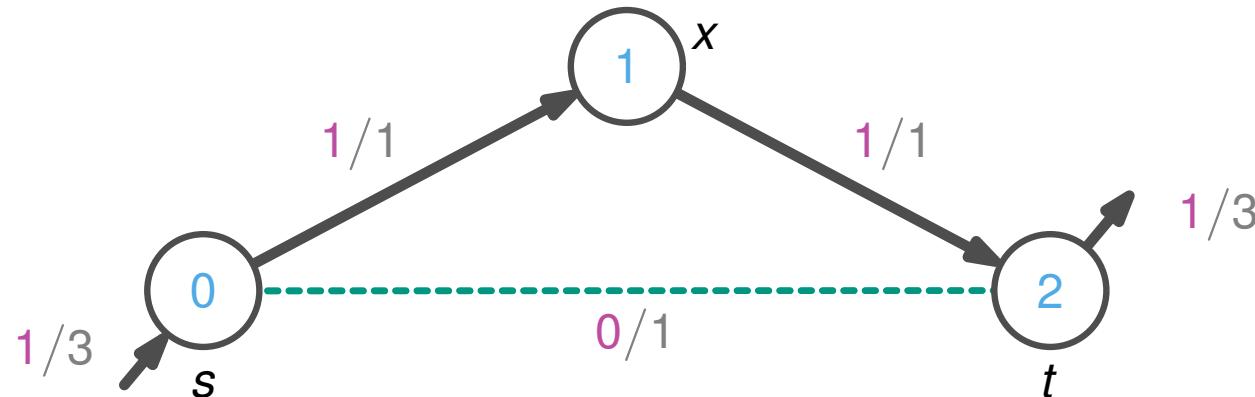
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Optimization Problem MTSF

Instance: A power grid \mathcal{N} .

Objective: Find a set $S \subseteq E$ of switched edges such that $\text{MPF}(\mathcal{N} - S)$ is maximum among all choices of switched edges S .

$$z(u, v) \in \{0, 1\}$$



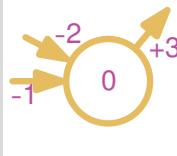
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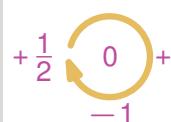


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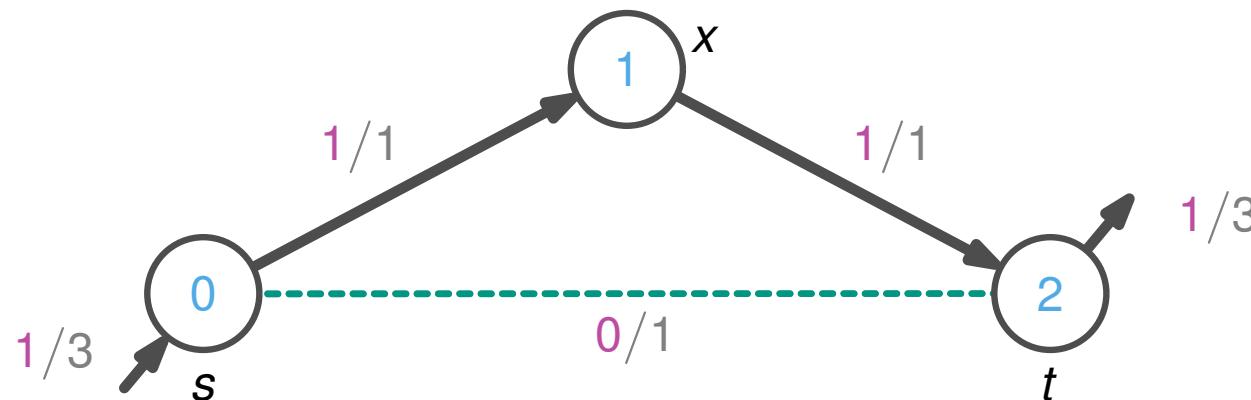
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Overview of the MTSF Results

Graph Structure	Complexity	Algorithm
penrose-minor-free graphs series-parallel graphs	polynomial-time solvable	DTP
cacti with max degree of 3	NP-hard	
2-level trees	NP-hard <small>[Lehmann et al., 2014]</small>	2-approx.
planar graphs with max degree of 3	NP-hard <small>[Lehmann et al., 2014]</small>	
arbitrary graphs	strongly NP-hard <small>[Lehmann et al., 2014]</small>	
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Dominating Theta Path (DTP)

[Section 5; Grastien, Rutter, Wagner, W., and Wolf, 2018]

Fix $u, v \in V$ and a u - v -path π .

Susceptance Norm:

$$\|\pi\|_b := \sum_{e \in E(\pi)} \frac{1}{b(e)}$$

Minimum Capacity:

$$\underline{\text{cap}}(\pi) := \min\{\text{cap}(e) \mid e \in \pi\}$$

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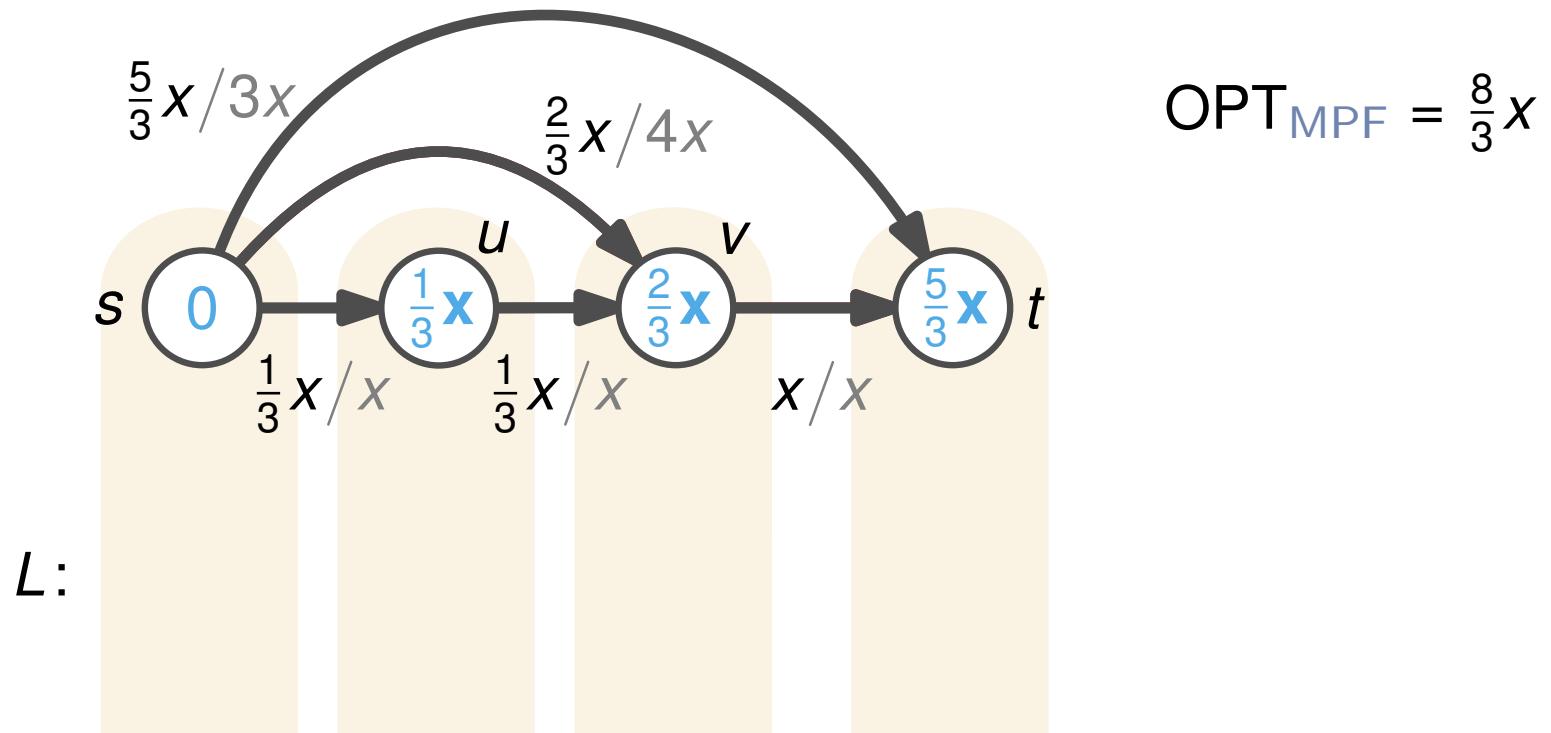
Computing DTP

[Section 5; Grastien, Rutter, Wagner, W., and Wolf, 2018]

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \text{cap}(\pi))$
- At most $|E|$ labels per vertex

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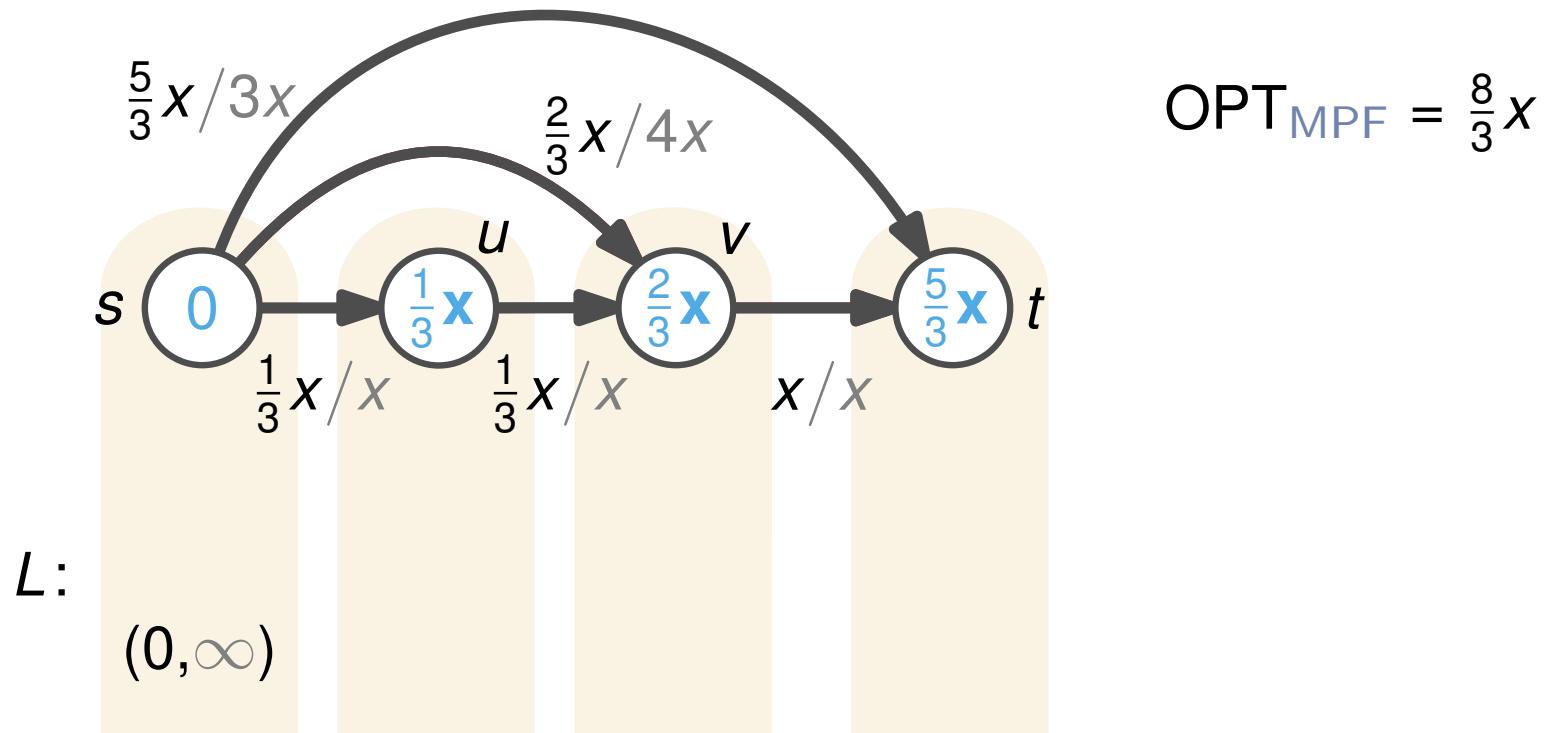
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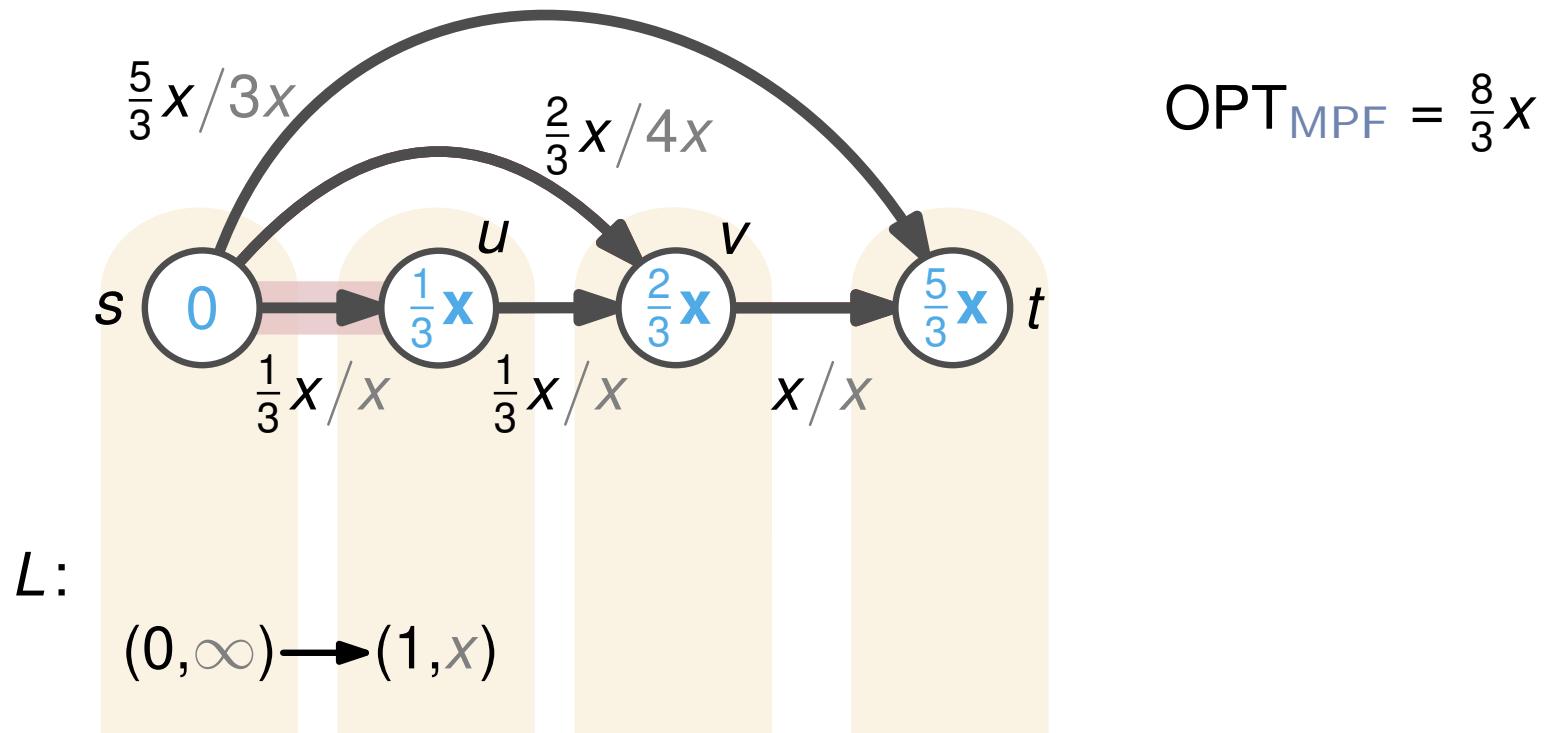
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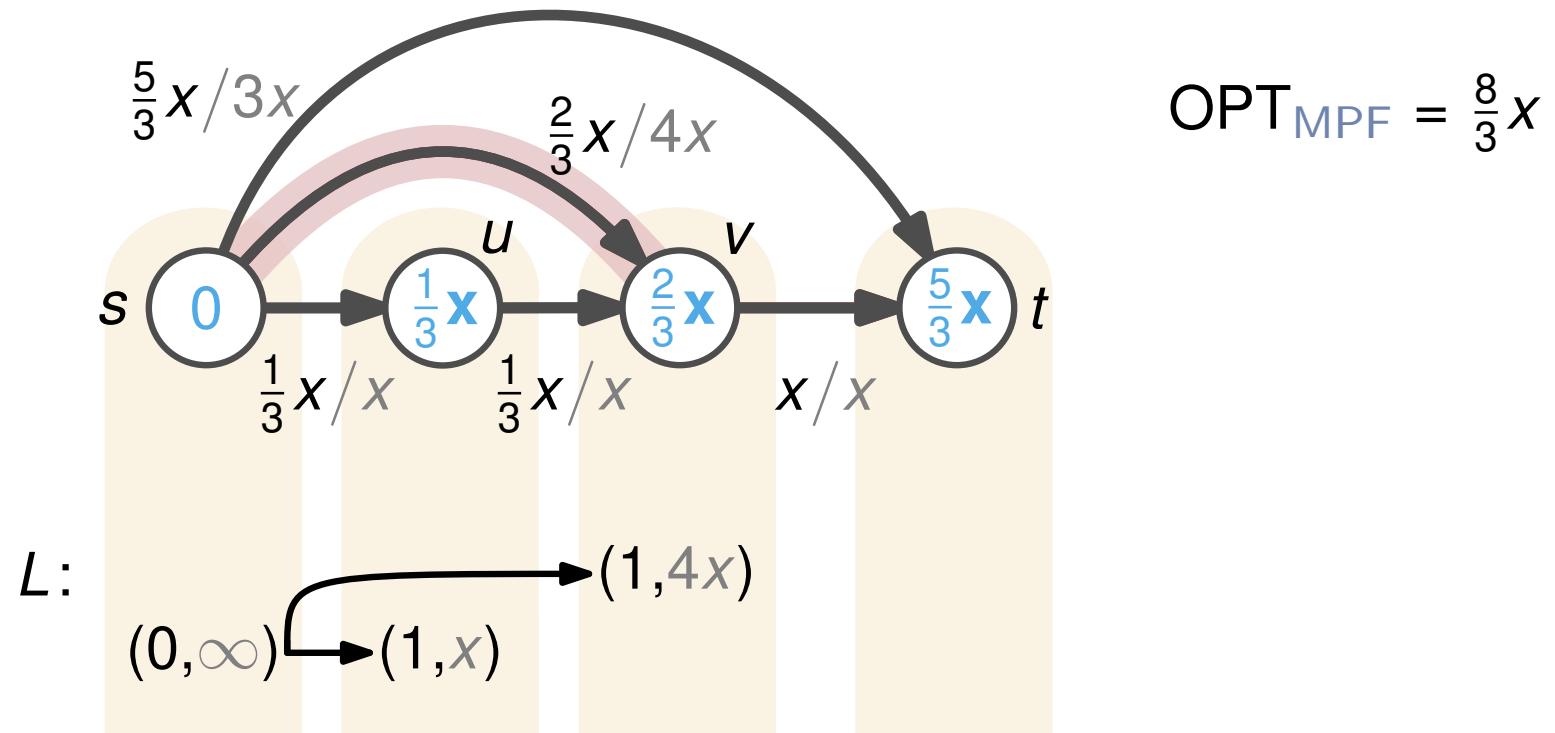
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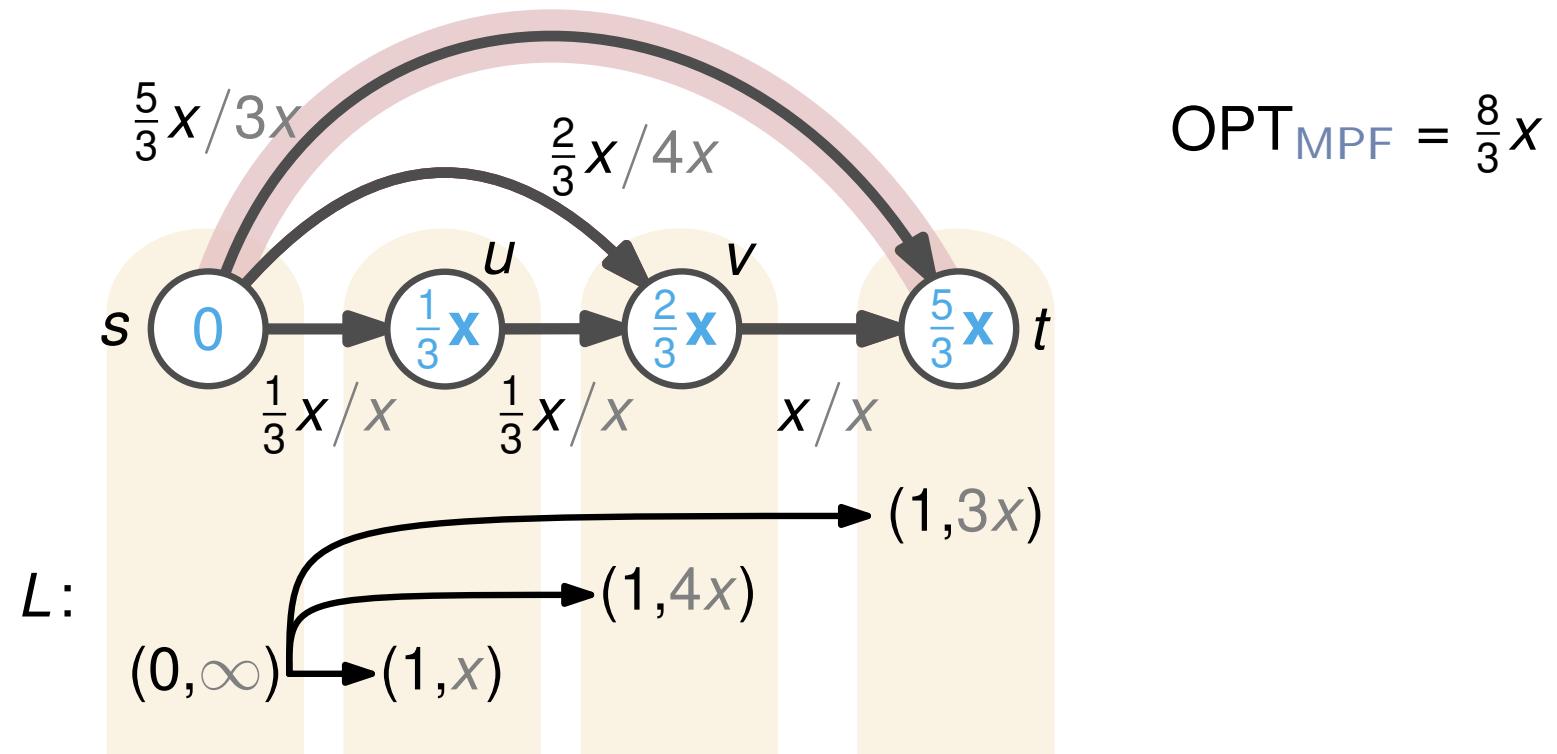
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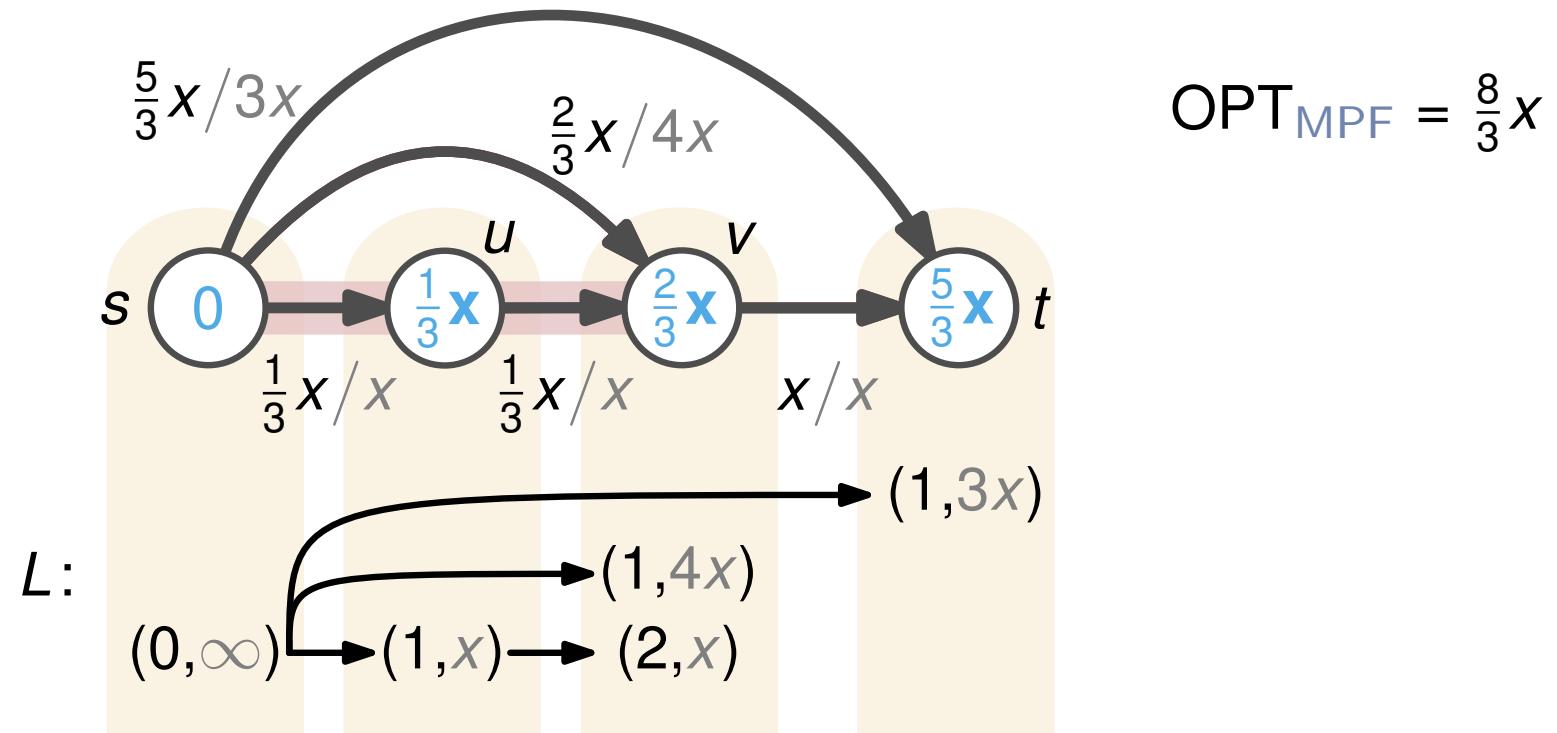
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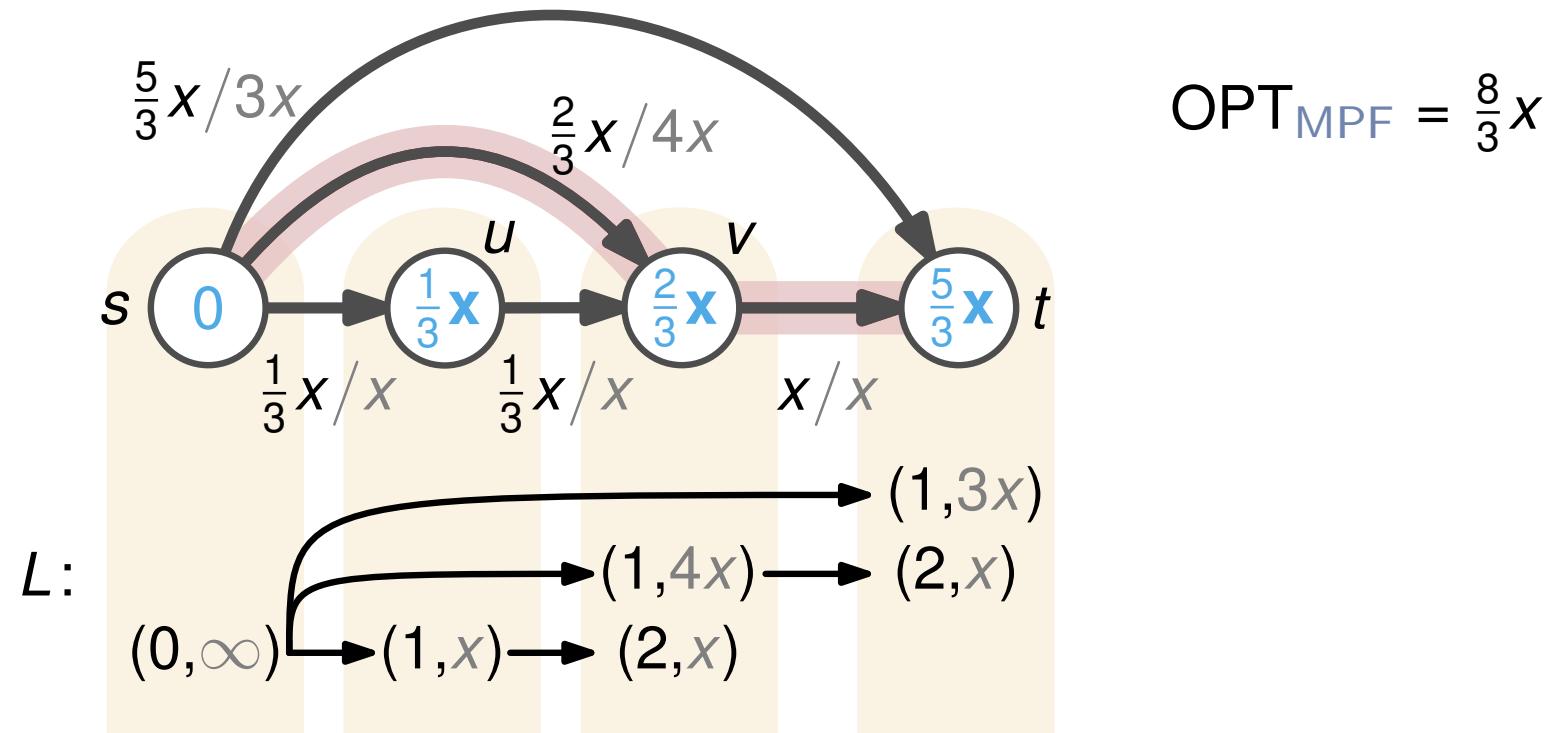
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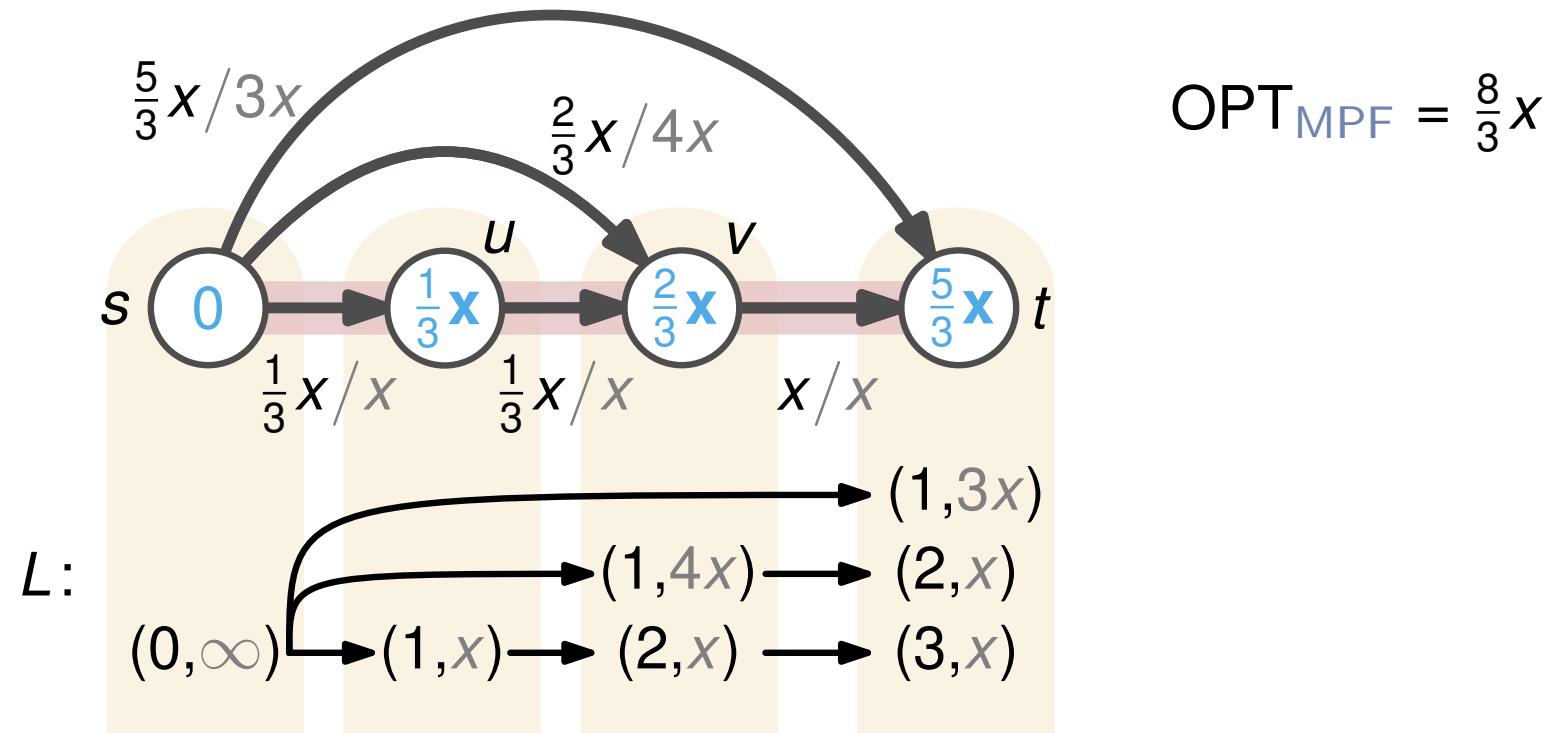
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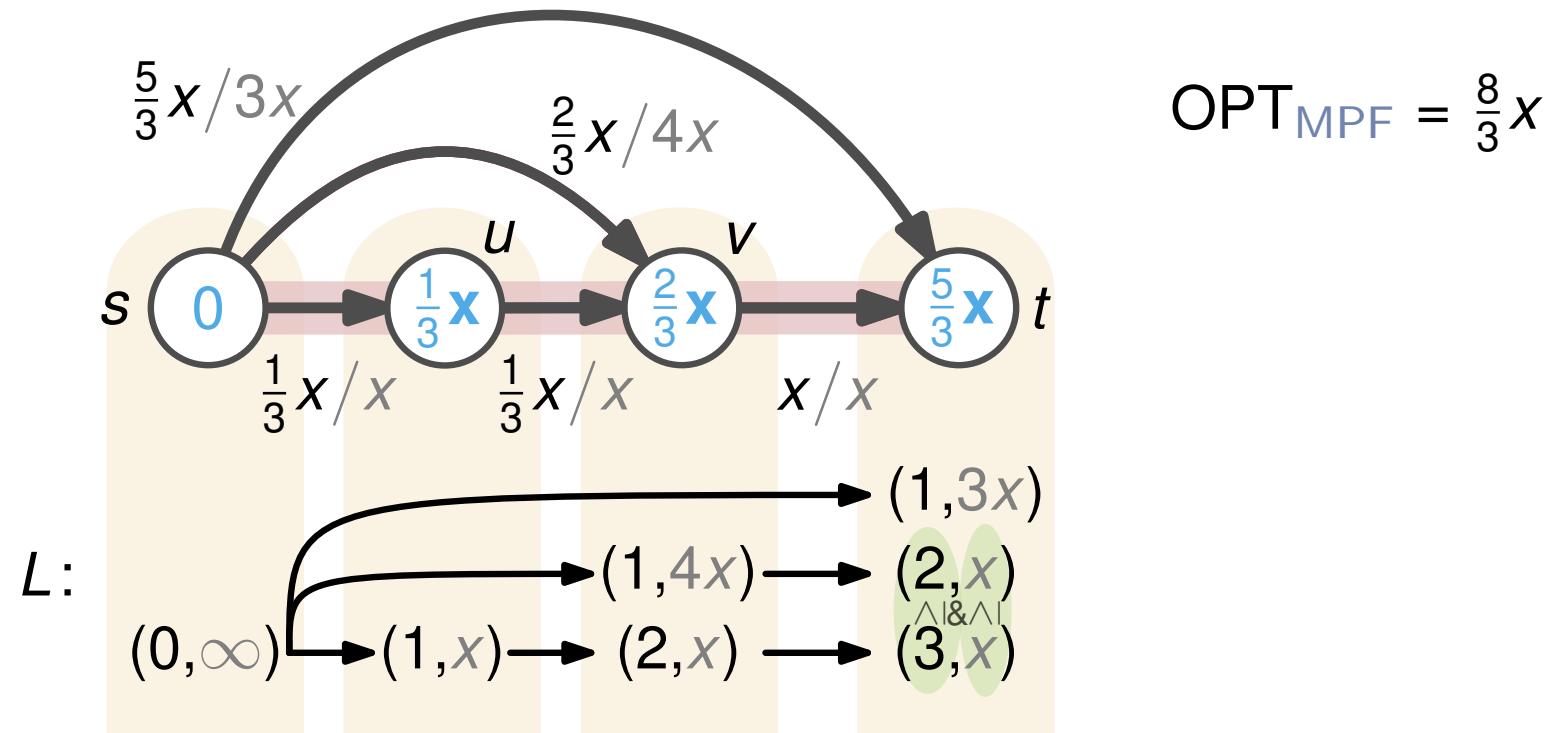
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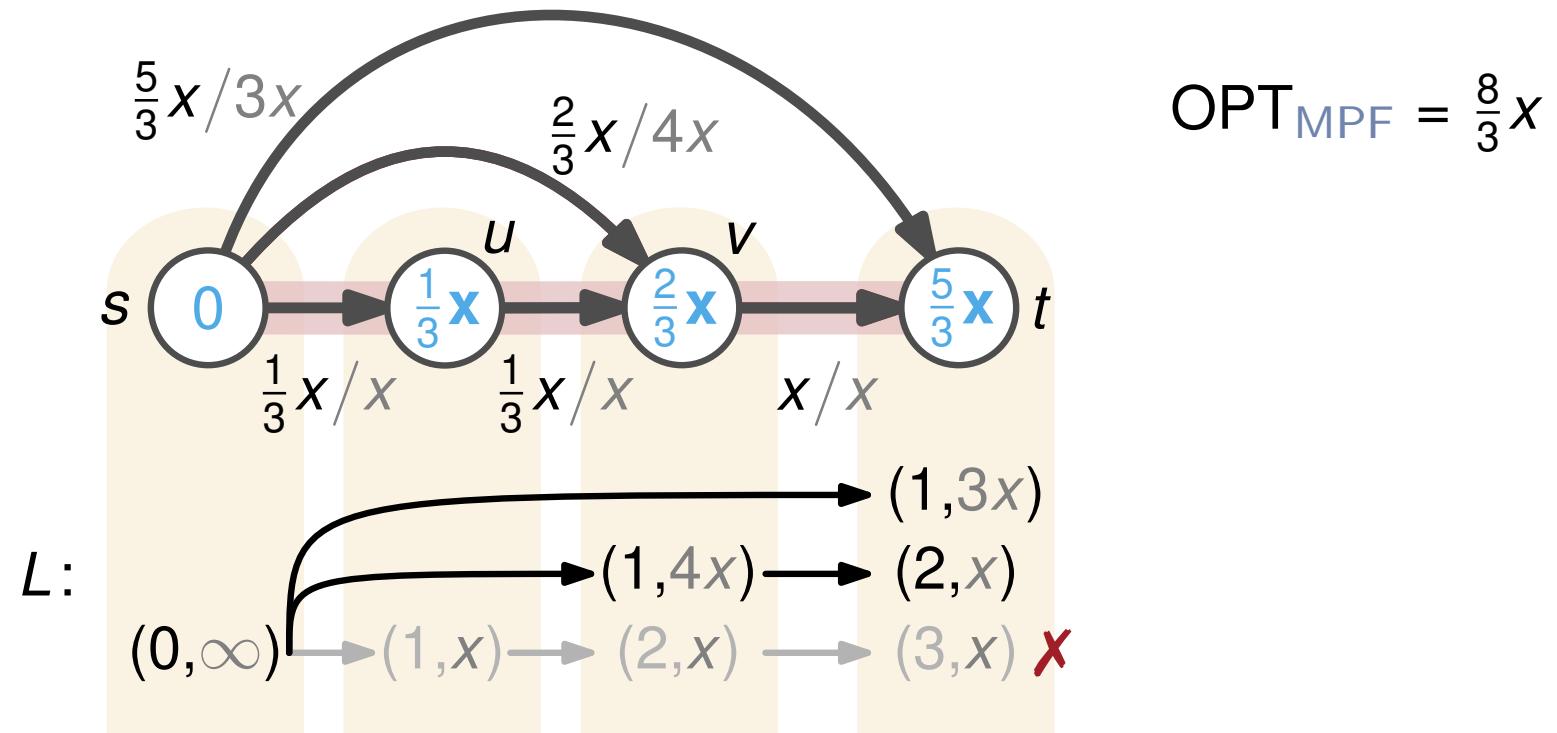
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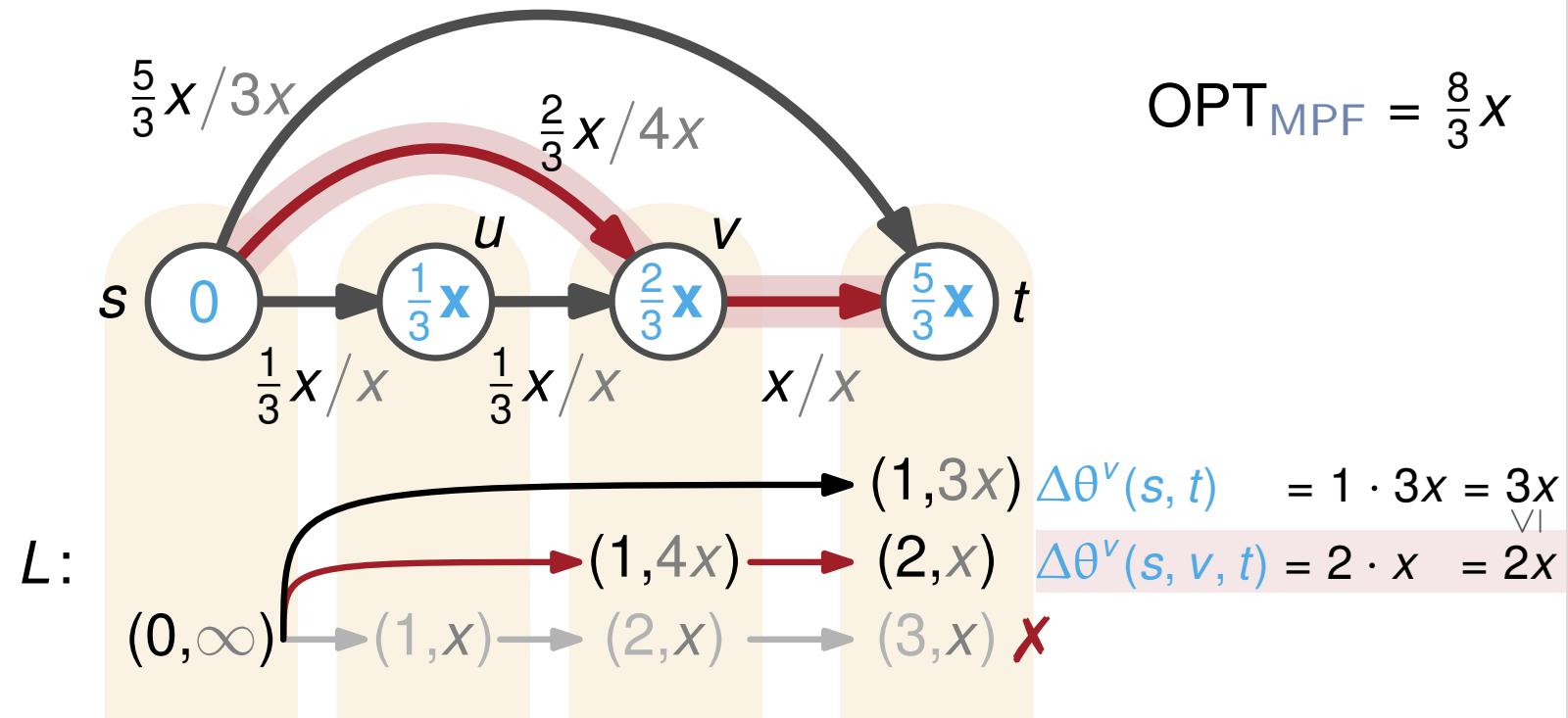
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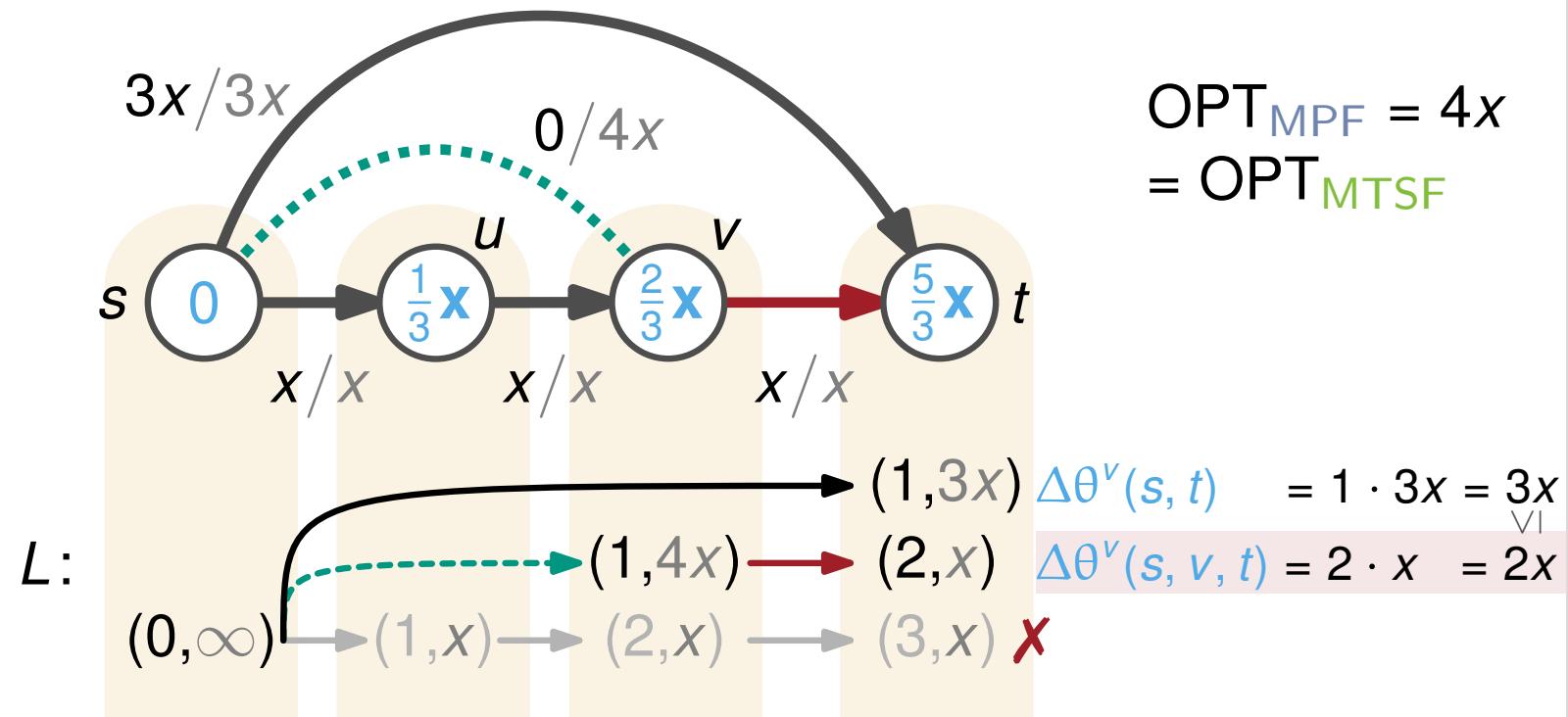
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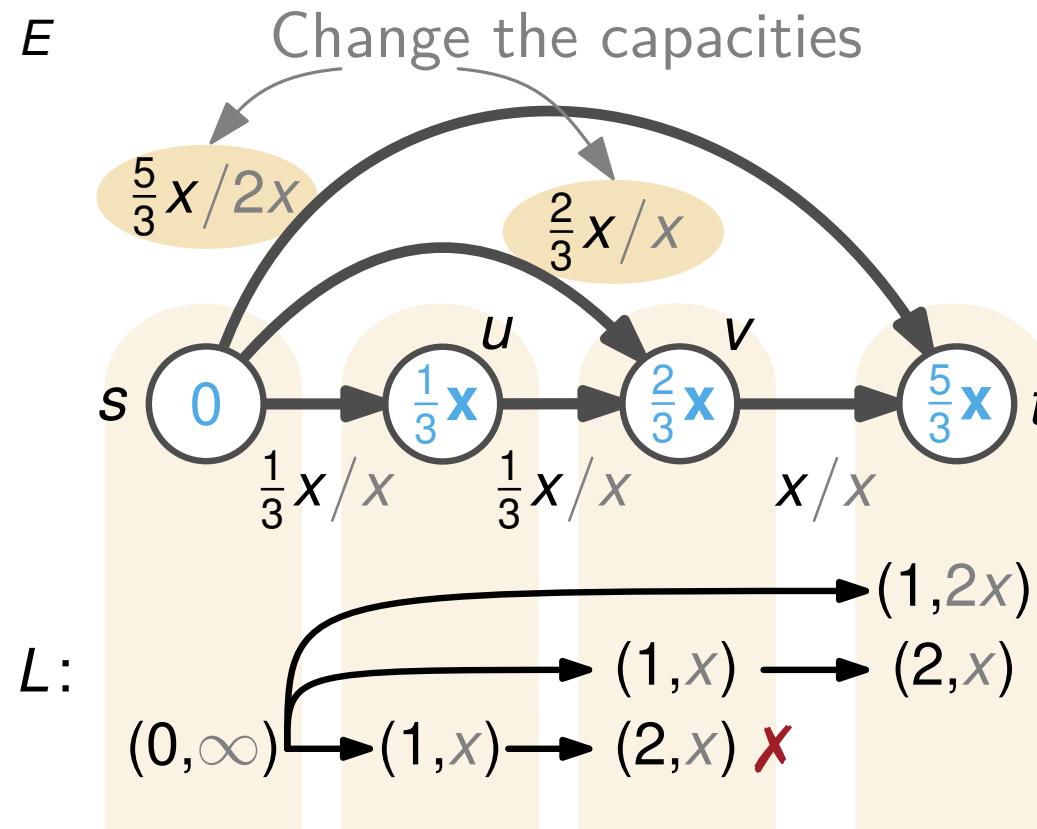
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$$\text{OPT}_{\text{MPF}} = \frac{8}{3}x$$

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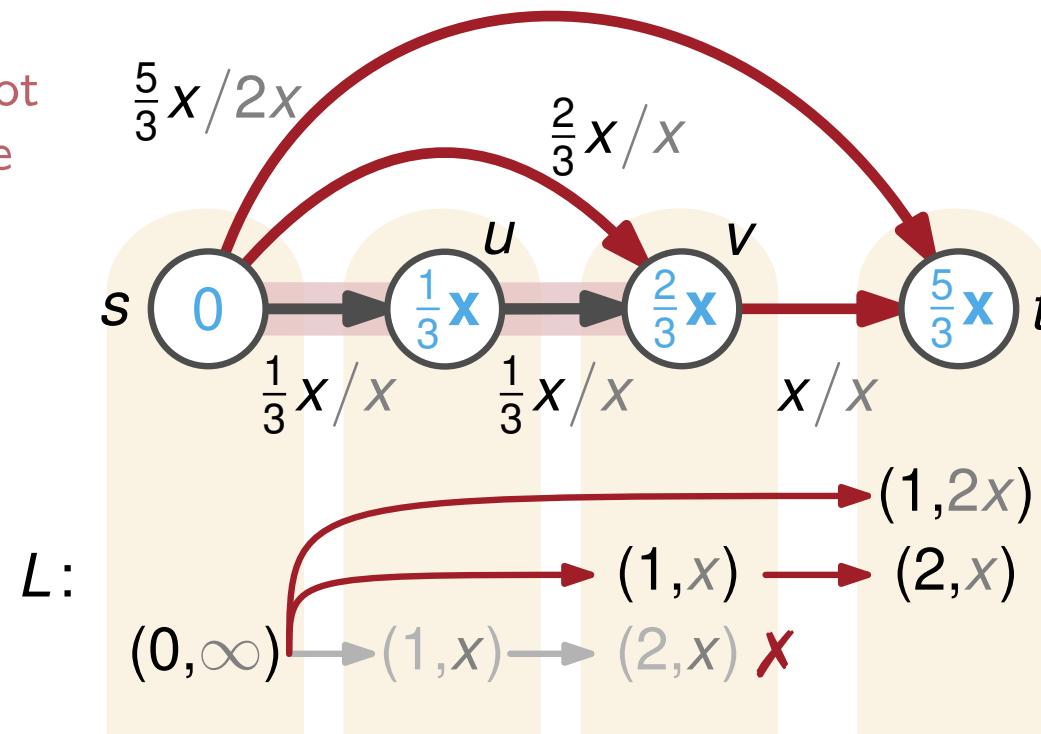
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Computing DTP

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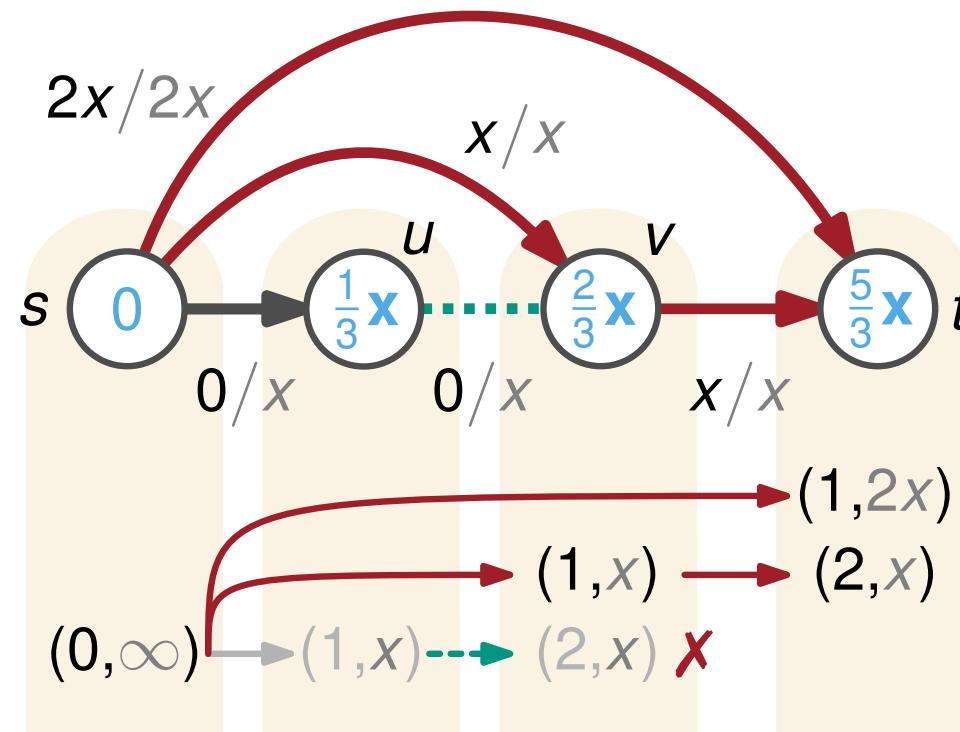
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- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free

$L:$



Computing DTP

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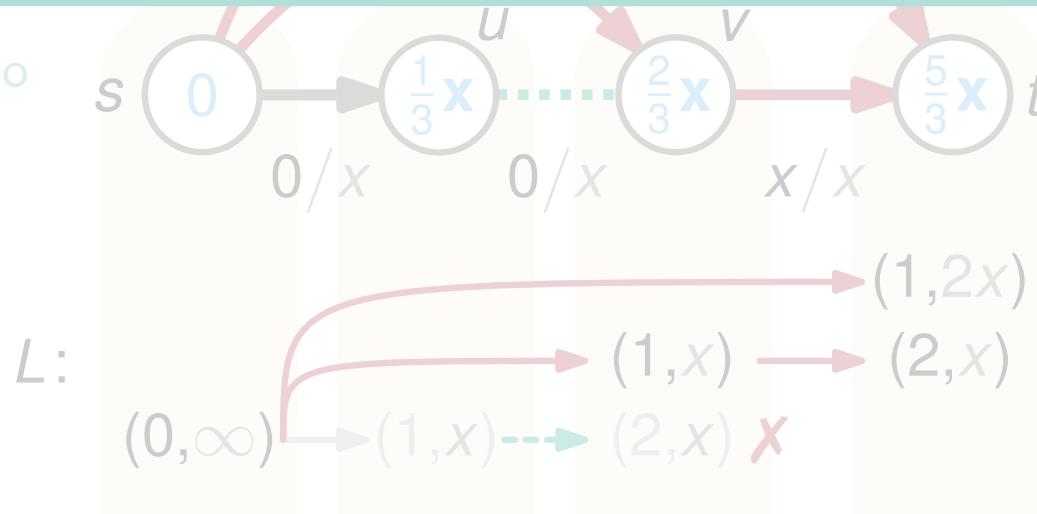
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On general graphs the DTP algorithm runs in time $\mathcal{O}(2^{|V|}|V| \cdot |E|^3)$.

- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free



Computing DTP

[Section 5; Grastien, Rutter, Wagner, W., and Wolf, 2018]

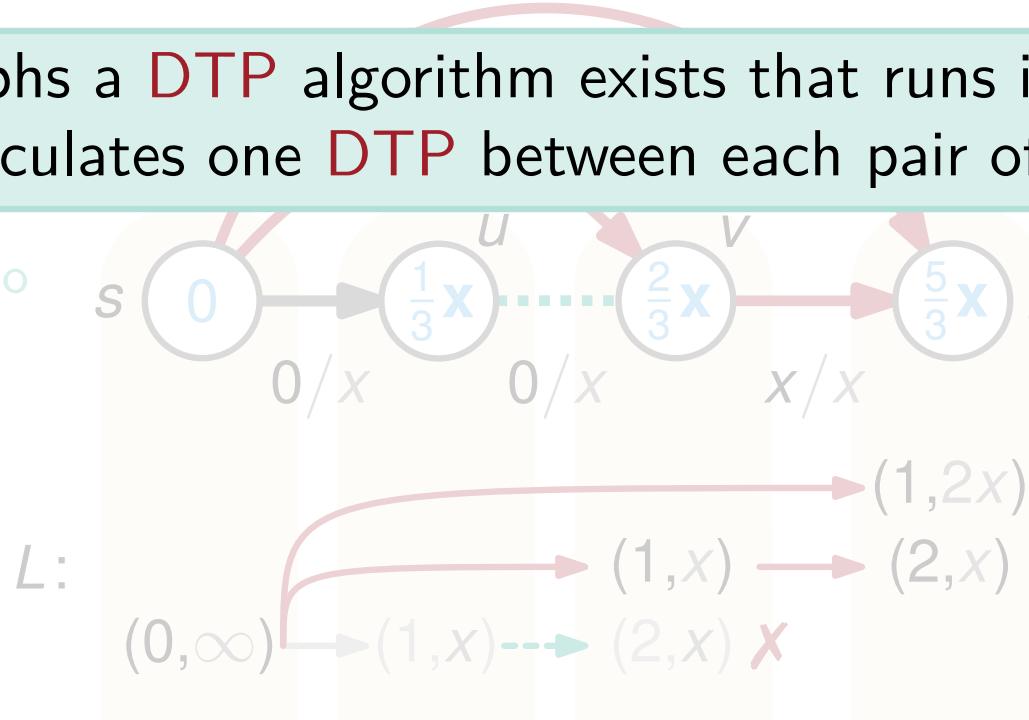
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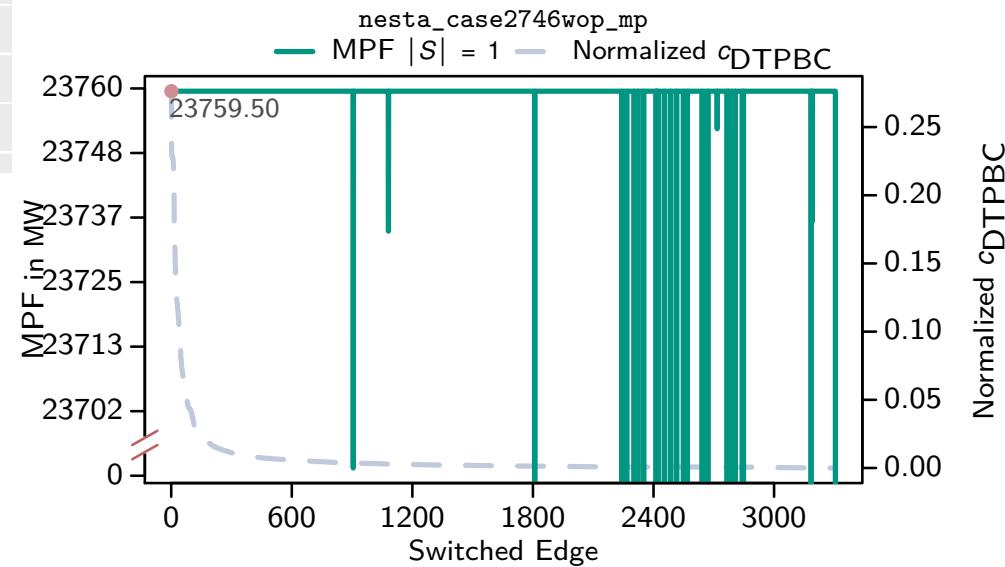
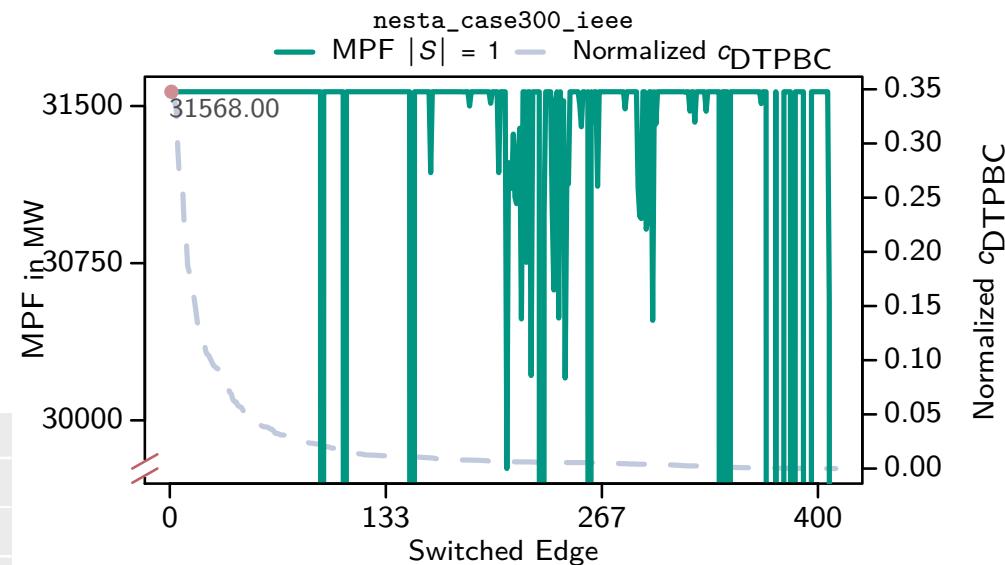
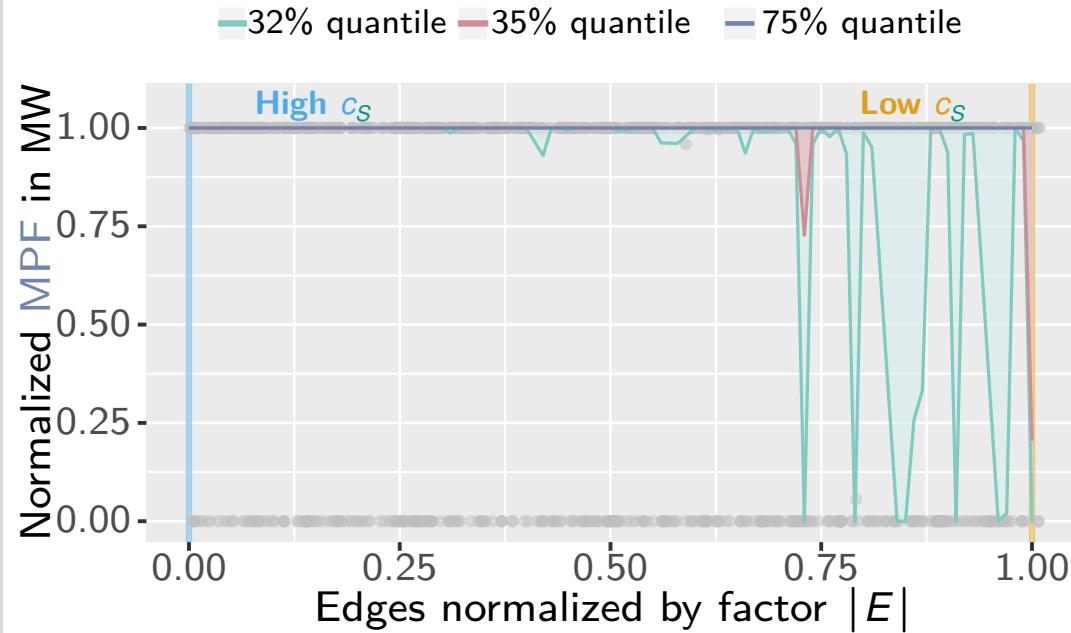
On general graphs a **DTP** algorithm exists that runs in polynomial time and calculates one **DTP** between each pair of u and v .

- Optimal switches do not have to lie on the **DTP** if the structure is not penrose-minor free

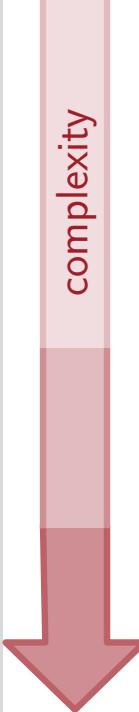


Computing DTP

[Section 5; Grastien, Rutter, Wagner, W., and Wolf, 2018]



Overview of the MTSF Results

	Graph Structure	Complexity	Algorithm
complexity 	penrose-minor-free graphs	polynomial-time solvable	DTP 
	series-parallel graphs	NP-hard	
	cacti with max degree of 3	NP-hard [Lehmann et al., 2014]	2-approx. 
	2-level trees	NP-hard [Lehmann et al., 2014]	
	planar graphs with max degree of 3	strongly NP-hard [Lehmann et al., 2014]	
	arbitrary graphs	non-APX [Lehmann et al., 2014]	

Overview of the MTSF Results

	Graph Structure	Complexity	Algorithm
complexity ↓	penrose-minor-free graphs series-parallel graphs	polynomial-time solvable NP-hard	DTP ✗
	cacti with max degree of 3	NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees planar graphs with max degree of 3 arbitrary graphs	NP-hard [Lehmann et al., 2014] strongly NP-hard [Lehmann et al., 2014] non-APX [Lehmann et al., 2014]	✗ ✗ ✗
$ V_G = 2, V_D = 2$			

2-approximation on Cacti

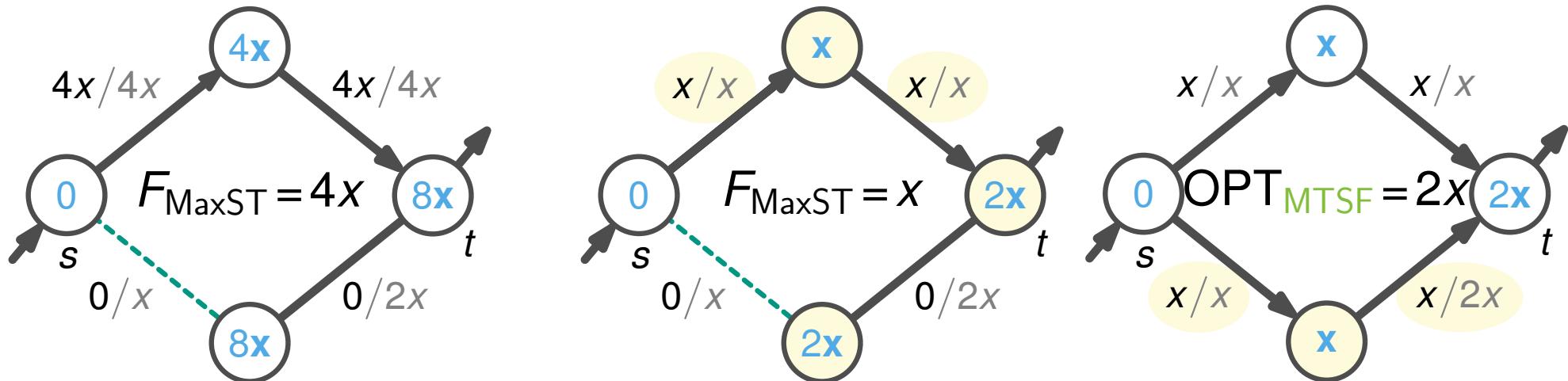
[page 349; Grastien, Rutter, Wagner, W., and Wolf, 2018]

Description

- Remove from each cycle the edge with the smallest capacity
 \Leftrightarrow the MAXIMUM SPANNING TREE (MaxST)

MaxST on Cacti

- MTSF is NP-hard on cacti [Lehmann et al., 2014]



Theorem 1

[page 348; Grastien et al., 2018]

MaxST is a factor 2-approximation algorithm for the MF and MTSF problem on cacti.

2-approximation on Cacti

[page 349; Grastien, Rutter, Wagner, W., and Wolf, 2018]

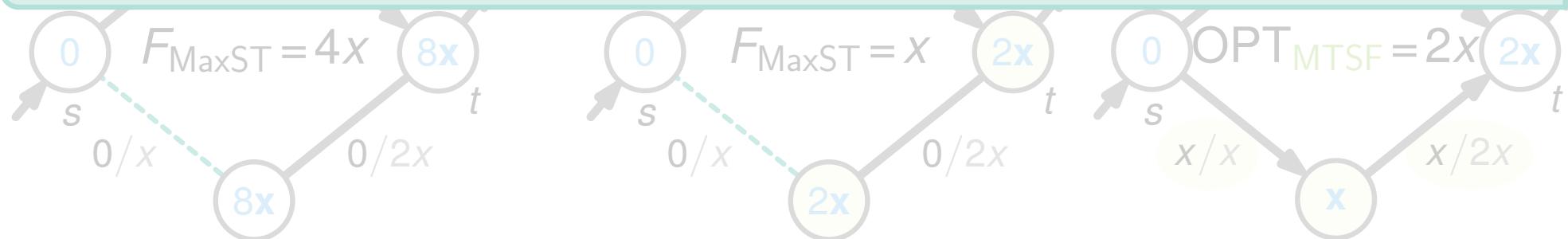
Description

- Remove from each cycle the edge with the smallest capacity
↔ the MAXIMUM SPANNING TREE (MaxST)

MaxST on Cacti

- MTSF is NP-hard on cacti [Lehmann et al., 2014]

On cacti the MaxST algorithm runs in time $\mathcal{O}(|V|)$.

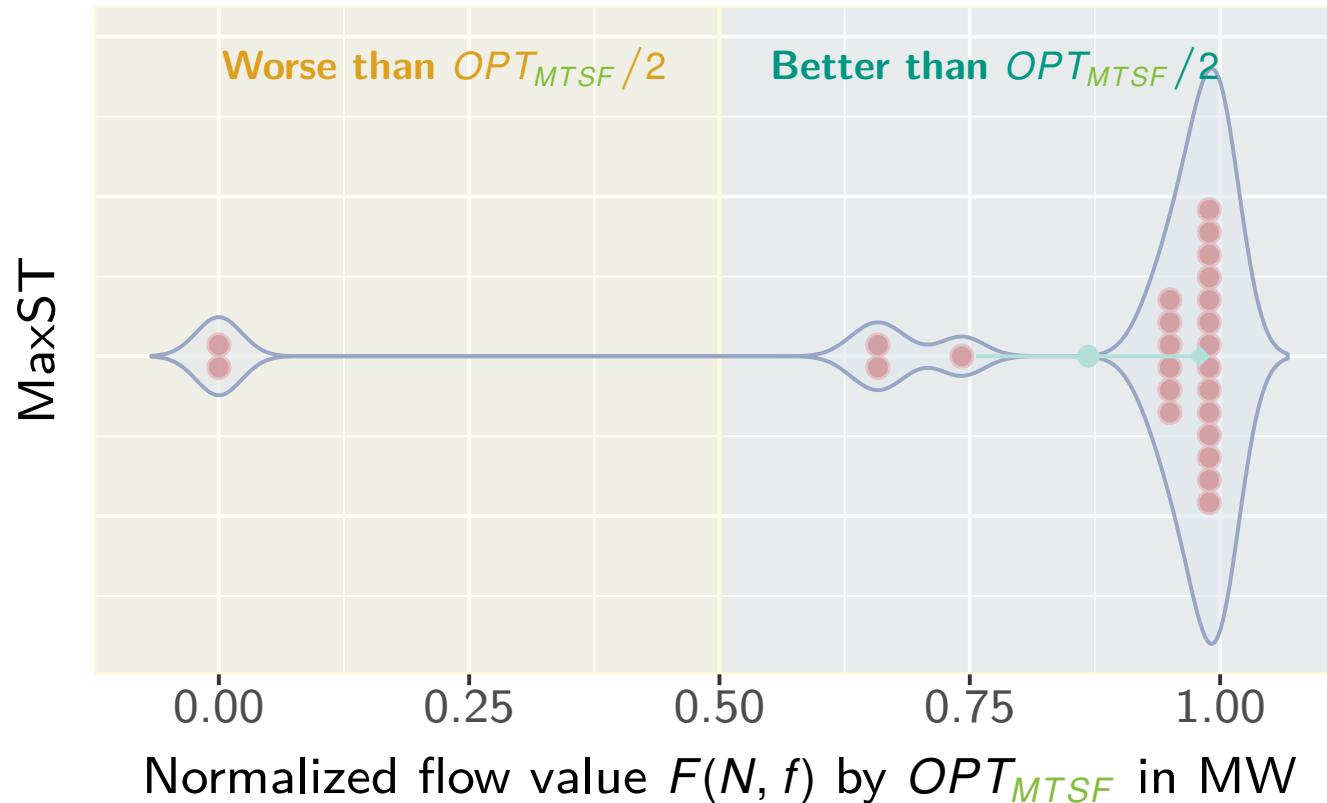


Theorem 1 [page 348; Grastien et al., 2018]

MaxST is a factor 2-approximation algorithm for the MF and MTSF problem on cacti.

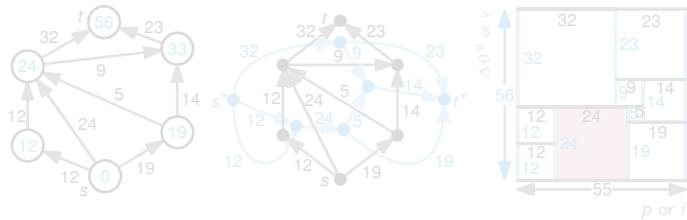
2-approximation on Cacti

[page 349; Grastien, Rutter, Wagner, W., and Wolf, 2018]



Classification

Feasible electrical flows



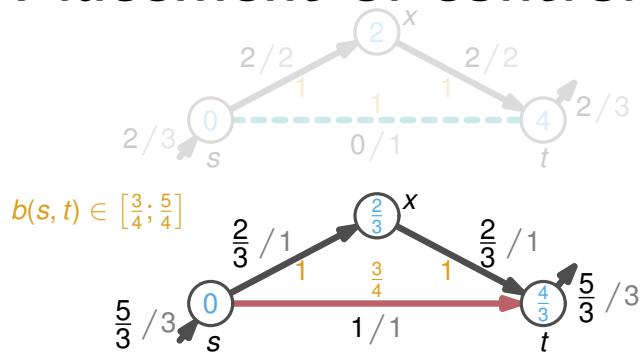
Dynamic models

Static models

Alternating Current (AC) model

Direct Current (DC) model

Placement of control units



Discrete control decision

[Grastien, Rutter, Wagner, W., and Wolf, 2018]

Continuous control decision

[Leibfried, Mchedlidze, Meyer-Hübner, Nöllenburg, Rutter, Sanders, Wagner, and W., 2015]

[Mchedlidze, Nöllenburg, Rutter, Wagner, and W., 2015]

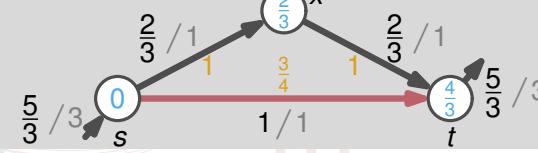
Cable layout



Wind farm cabling with multiple cable types

[Lehmann, Rutter, Wagner, and W., 2017]

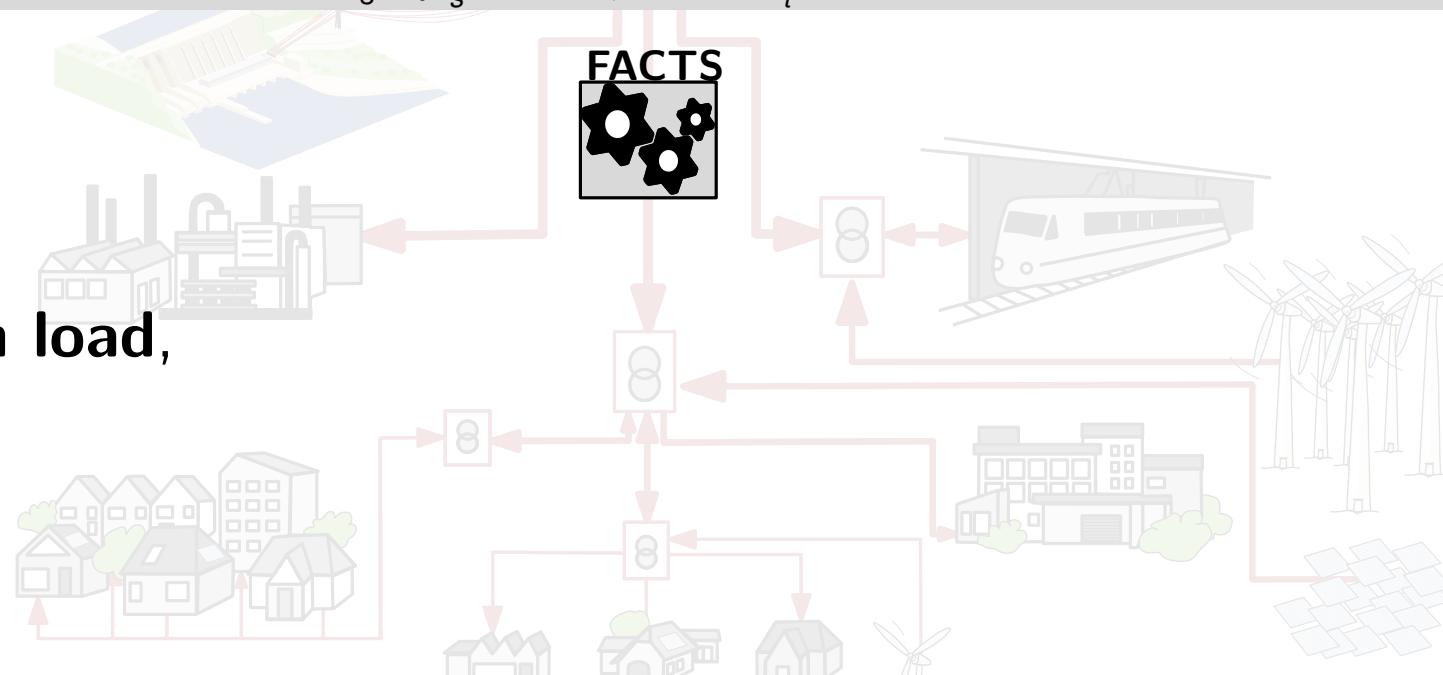
Placement of control units Continuous control decision



[Leibfried, Mchedlidze, Meyer-Hübler, Nöllenburg, Rutter, Sanders, Wagner, and W., 2015]
[Mchedlidze, Nöllenburg, Rutter, Wagner, and W., 2015]

FACTS...

- are **control** units,
- increase **maximum load**,
- are **expensive**.



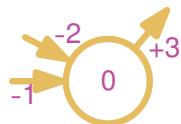
The Maximum FACTS Flow (MFF) Problem

[Lehmann et al., 2015]

- The value of the Maximum Flexible AC Transmission Switching Flow (MFF) is defined as

$$\text{MFF}(\mathcal{N}, k) := \max_{E' \subseteq E, b} \text{MPF}(\mathcal{N}) \quad |E'| \leq k$$

with f being a feasible power flow meaning



$$f_{\text{net}}(u) = 0 \quad \forall u \in V \setminus (V_G \cup V_D)$$

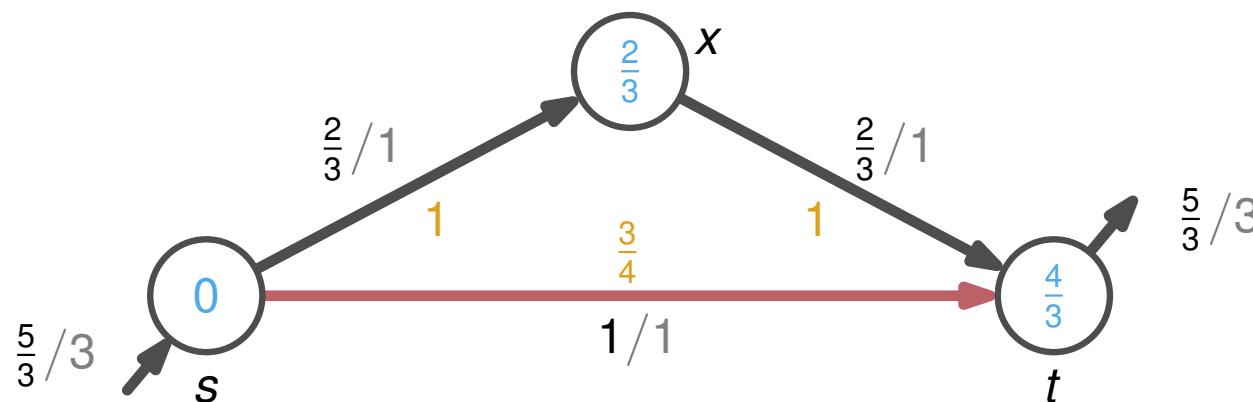
$$|f(u, v)| \leq \text{cap}(u, v) \cdot z(u, v) \quad \forall (u, v) \in E$$



$$z(u, v) \cdot (\theta(v) - \theta(u)) = f(u, v) \quad \forall (u, v) \in E$$

$$z(u, v) \in \{0, 1\} \quad \forall (u, v) \in E'$$

$$k = 1$$



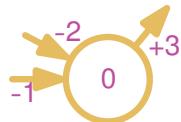
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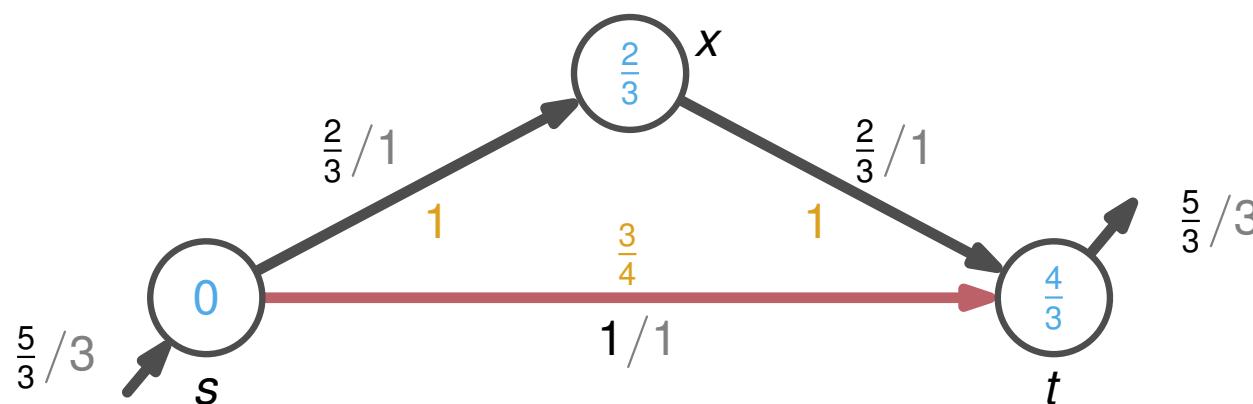
$$|f(u, v)| \leq \text{cap}(u, v) \quad \forall (u, v) \in E$$



$$b(u, v) \cdot (\theta(v) - \theta(u)) = f(u, v) \quad \forall (u, v) \in E$$

$$b(u, v) \in \left[\frac{3}{4}, \frac{5}{4}\right] \quad \forall (u, v) \in E'$$

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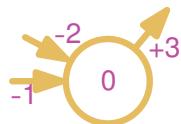
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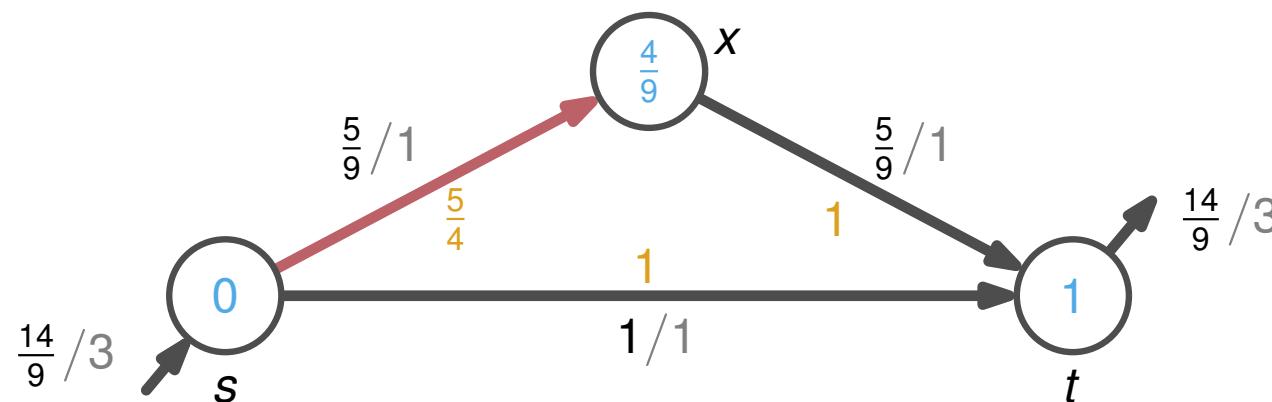
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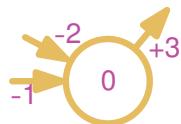
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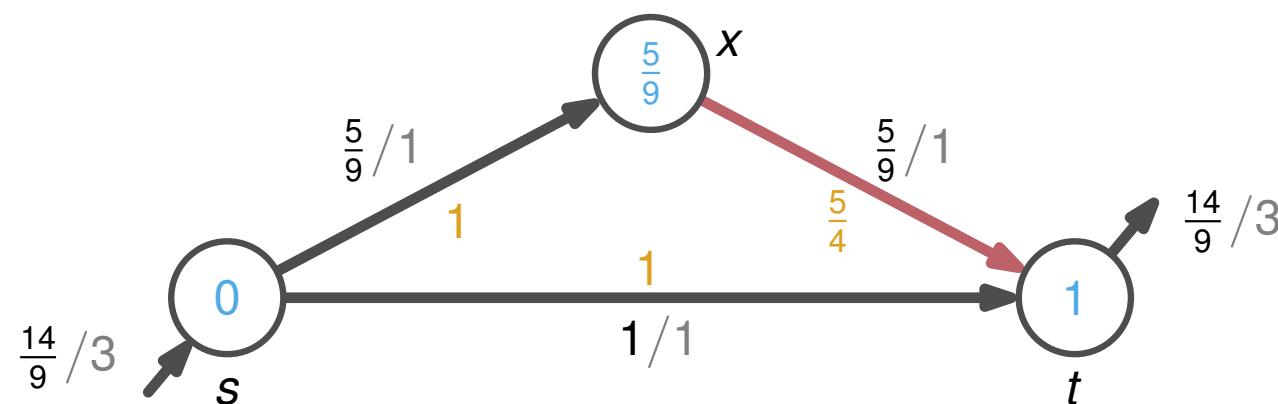
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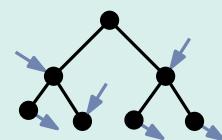
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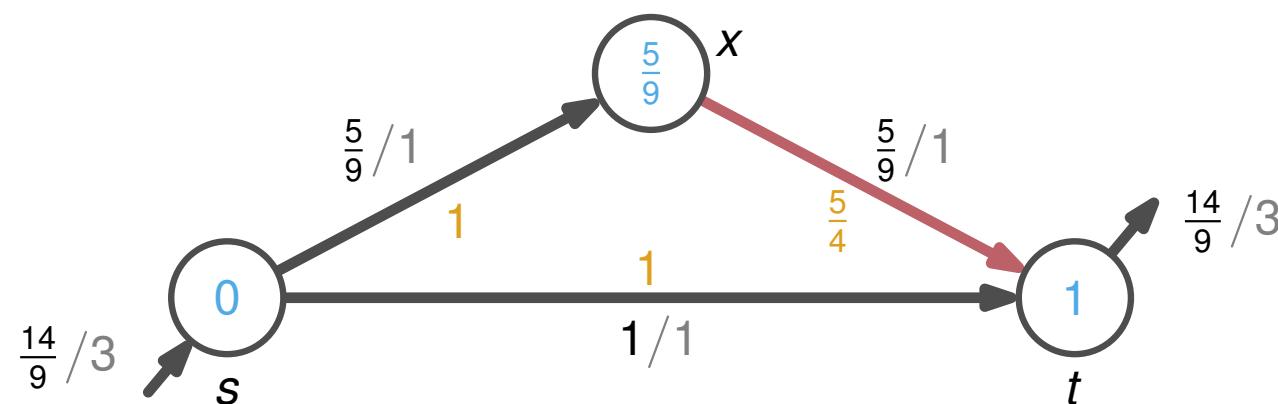


Physical Model = Maximum FACTS Flow = Flow Model
(MPF) (MFF) (MF)

$$b(u, v) \cdot (\theta(v) - \theta(u)) = f(u, v) \quad \forall (u, v) \in E$$

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$$k = 1$$



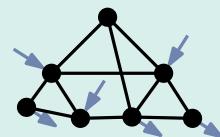
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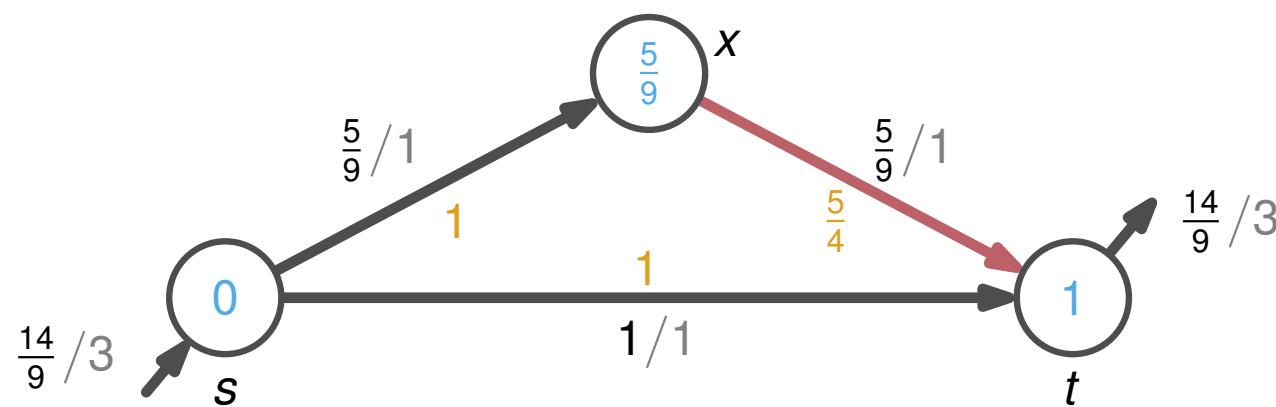


Physical Model \leq Maximum FACTS Flow \leq Flow Model
(MPF) \leq (MFF) \leq (MF)

$$b(u, v) \cdot (\theta(v) - \theta(u)) = f(u, v) \quad \forall (u, v) \in E$$

$$b(u, v) \in \left[\frac{3}{4}, \frac{5}{4}\right] \quad \forall (u, v) \in E'$$

$$k = 1$$



The Optimal FACTS Flow (OFF) Problem

- The value of the OPTIMAL POWER FLOW (OPF) is defined as

$$\text{OPF}(\mathcal{N}) = \min \gamma(\mathcal{N}, \mathbf{f})$$

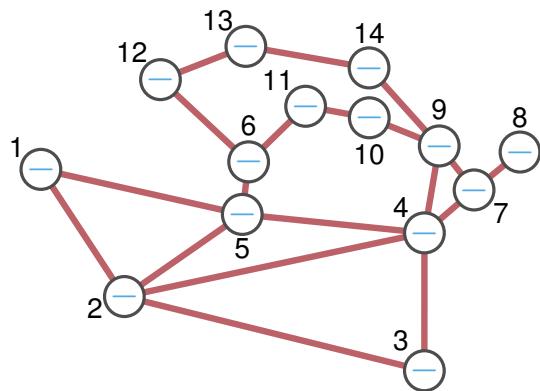
with \mathbf{f} being a feasible power flow and the generator cost function γ .

Optimization Problem OFF

Instance: A power grid \mathcal{N} .

Objective: Find a set $E' \subseteq E$ of edges with FACTS and a susceptance configuration $b(e)$ with $e \in E'$ such that $\text{OPF}(\mathcal{N})$ is minimum among all choices of FACTS placements and susceptance configurations while complying with $|E'| \leq k$.

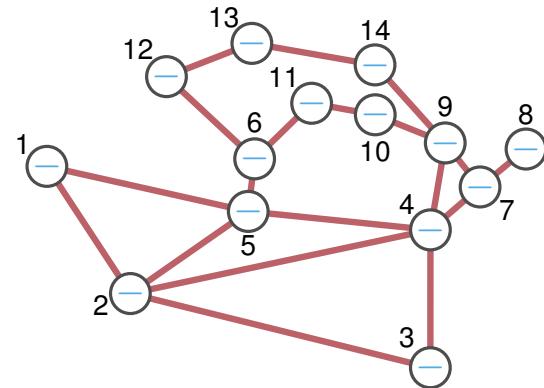
Optimal FACTS Flow (OFF)



— Full Control

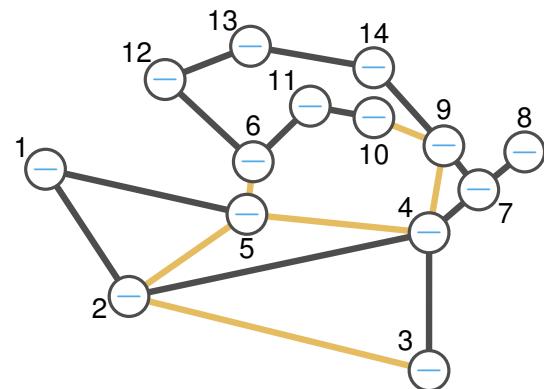
Can we become as good as the Flow Model
with fewer FACTS?

Optimal FACTS Flow (OFF)



— Full Control

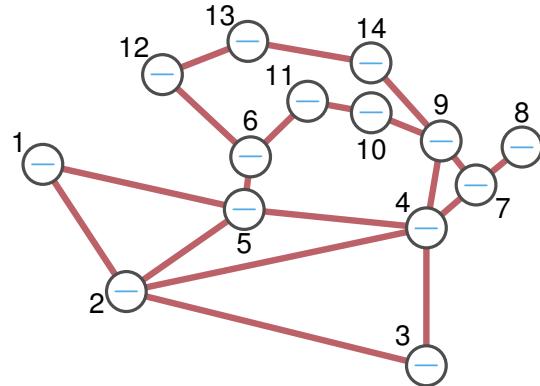
Can we become as good as the **Flow Model** with fewer FACTS?



— Feedback Edge Set

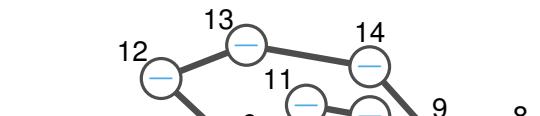
If the graph without FACTS represents a forest all flows represent **feasible power flows**.

Optimal FACTS Flow (OFF)



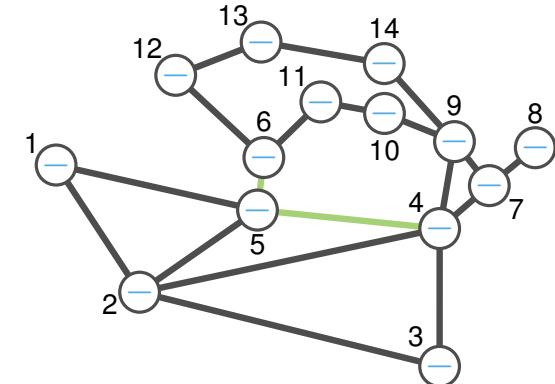
— Full Control

Can we become as good as the **Flow Model** with fewer FACTS?



— Feedback Edge Set

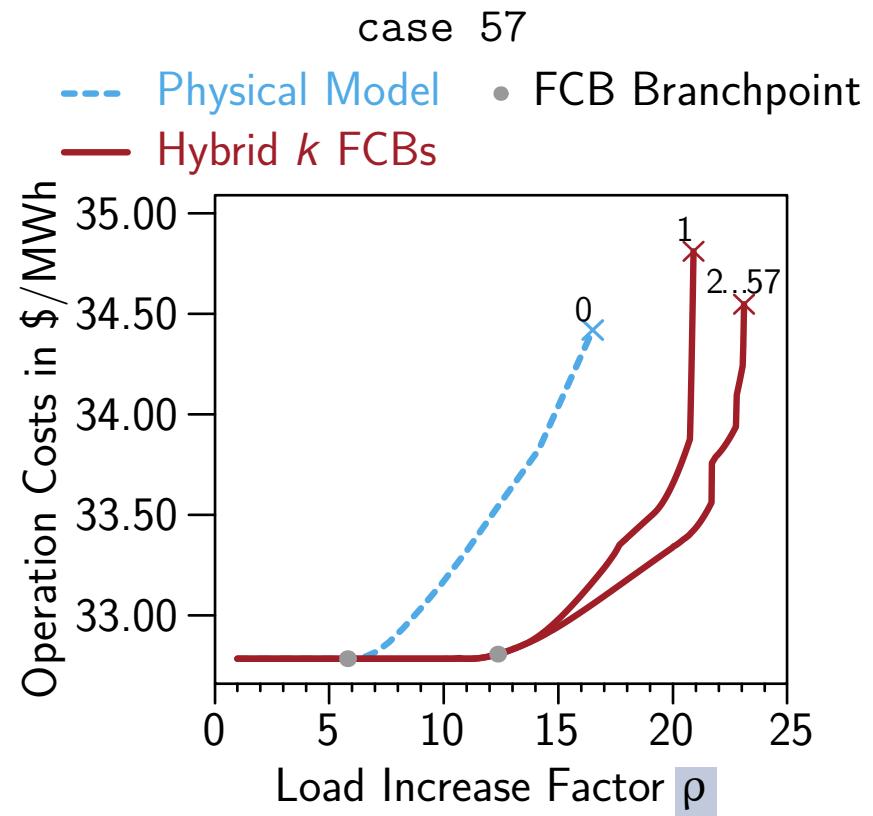
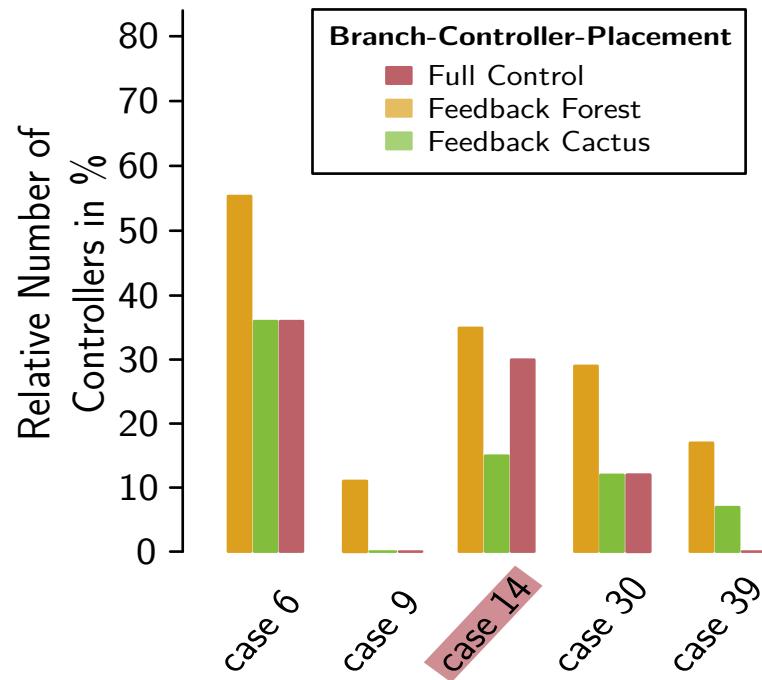
If the graph without FACTS represents a forest all flows represent **feasible power flows**.



— Diamond Hitting Set

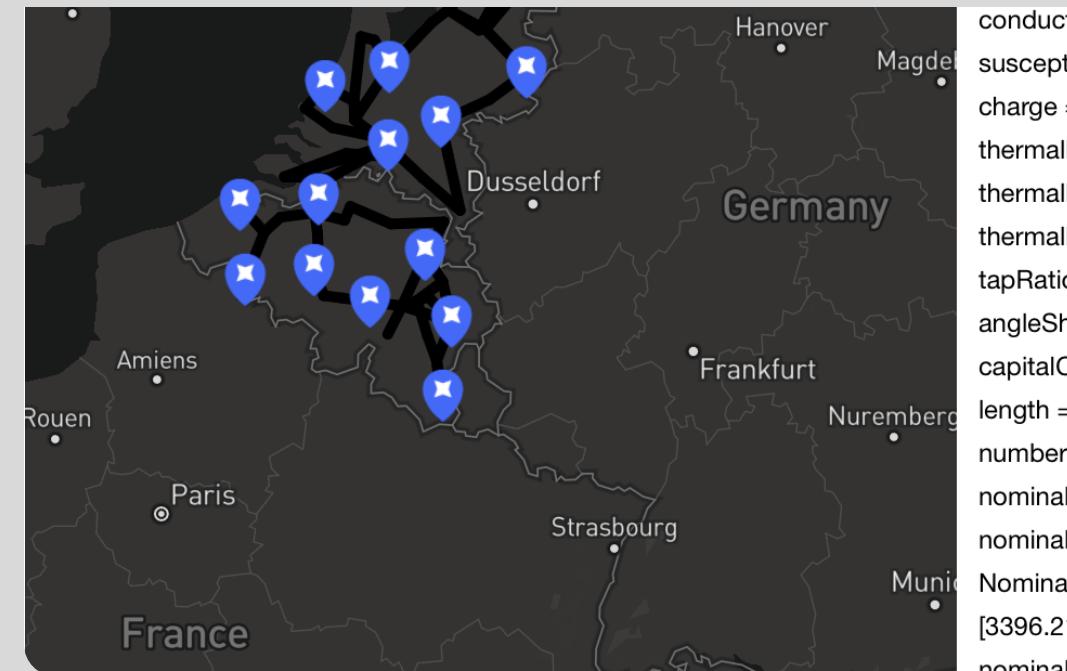
If the remaining graph is a cactus and the capacities on the cycles are suitably bounded then there is for every flow a cost-equivalent **feasible power flow**.

Optimal FACTS Flow (OFF)



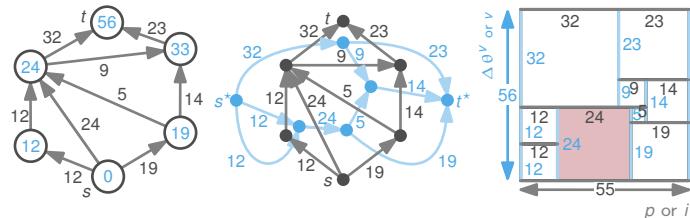


Summary



Summary

Feasible electrical flows



Dynamic models

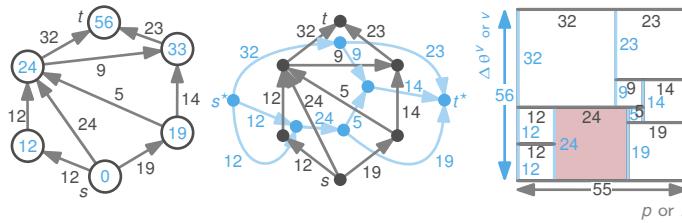
Static models

Alternating Current (AC) model

Direct Current (DC) model

Summary

Feasible electrical flows



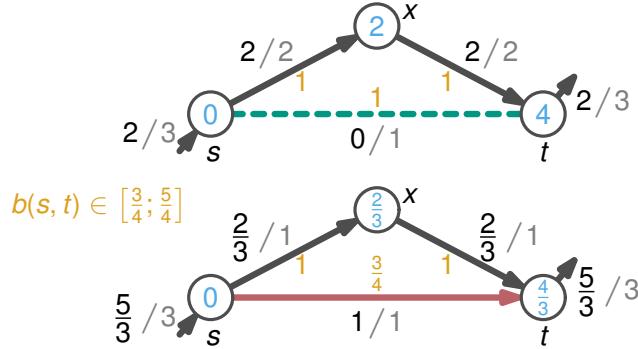
Dynamic models

Static models

Alternating Current (AC) model

Direct Current (DC) model

Placement of control units



Discrete control decision

[Grastien, Rutter, Wagner, W., and Wolf, 2018]

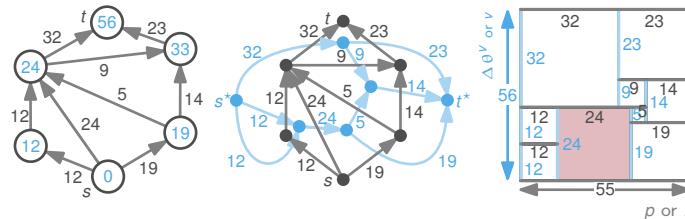
Continuous control decision

[Leibfried, Mchedlidze, Meyer-Hübnér, Nöllenburg, Rutter, Sanders, Wagner, and W., 2015]

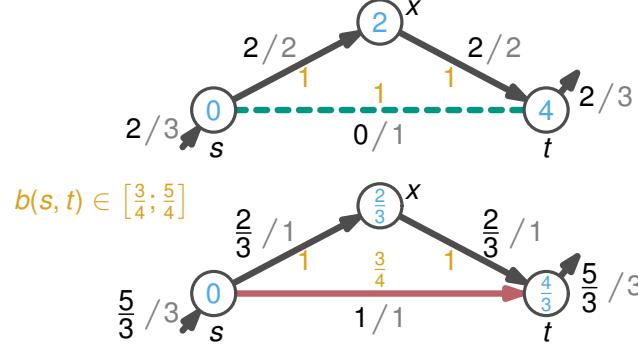
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Summary

Feasible electrical flows



Placement of control units



Cable layout



Dynamic models

Static models

Alternating Current (AC) model

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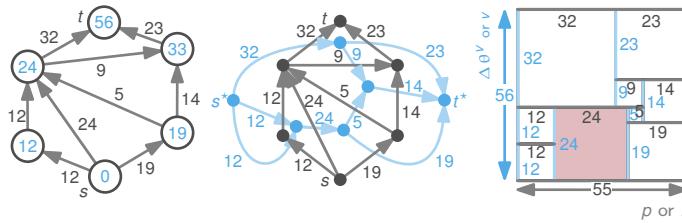
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Wind farm cabling with multiple cable types

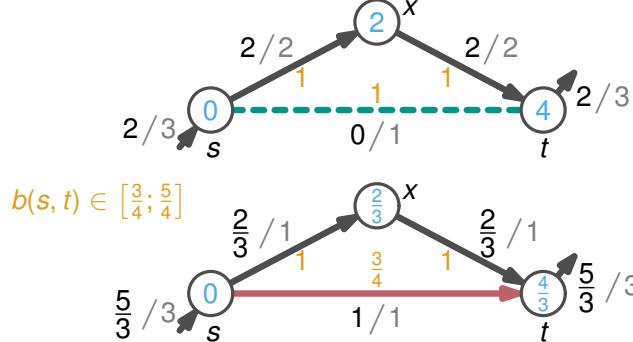
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Summary

Feasible electrical flows



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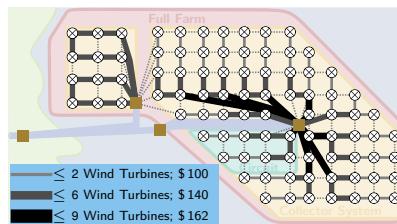
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Cable layout



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Algorithm Engineering

- Quality and efficiency **guarantee**,
- **Practical** methods.

References

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