Probabilities & Natural Catastrophes

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Chaire PARI,
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The Challenge of Predictive Models (for Rare Events)

"*A 30% chance of rain tomorrow: How does the public understand probabilistic weather forecasts?* " Gigerenzer *et al.* (2005)
See Nate Silver's "*there's an awful lot of room to debate what 'probably' means*" 538.com, or Ronald Fisher on predicting probabilities on 'one-shot' events,
(or more provocative "*probability does not exist*" by Bruno de Finetti, Nau (2011))
Classical issue in risk management : predict 1% chance of occurrence, and occurs.
Was 1% wrong ?

Disasters are not *per se* 'one-shot' events, but we have to deal with changing environment...

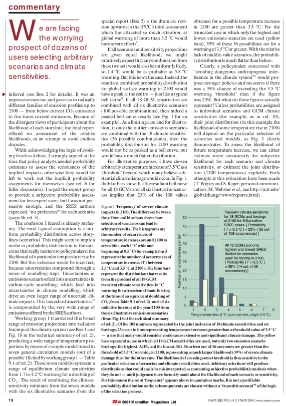
Here we focus on rare events :
(1) hard to find a good model (2) hard to assess if the model is good

How Likely Catastrophe Is To Occur ?

"*to combat global warming, we must first assess just* ***how likely*** *it is to occur*"

Schneider (2001)

"*Does climate adaptation policy need probabilities?* " Dessai & Hulme (2004)

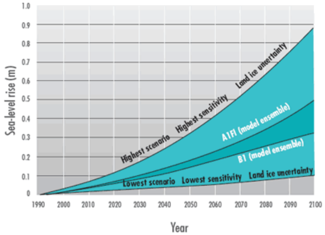


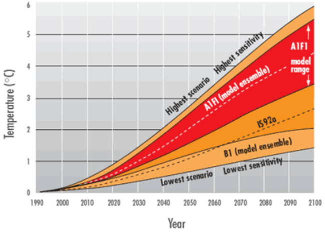
So what...?

In this talk, "*natural catastrophe*" is seen from an insurance perspective

• quantitative analysis of natural events
difficult part : climate is changing (warming)
classical curves from IPCC on temperature or sea level
(and references therein)

• impact on the insurance industry ?
In 2015, French Federation of Insurers claimed that "*Catastrophes naturelles : la facture des assureurs pourrait doubler d'ici 2040*" (in 25 years, +3% per year,
see https://www.argusdelassurance.com/)





Insurability of Climate Risks ?

see Berliner (1985) and Charpentier (2008)

Table 3 Insurability criteria and related requirements according to Berliner

Insurability Criteria			**Requirements**
Actuarial	(1)	Randomness of loss occurrence	Independence and predictability of loss exposures
	(2)	Maximum possible loss	Manageable
	(3)	Average loss per event	Moderate
	(4)	Loss exposure	Loss exposure must be large
	(5)	Information asymmetry	Moral hazard and adverse selection not excessive
Market	(6)	Insurance premium	Cost recovery and affordable
	(7)	Cover limits	Acceptable
Societal	(8)	Public policy	Consistent with societal value
	(9)	Legal restrictions	Allow the coverage

(Table from Bienier *et al.* (2015) - for cyber risk)
climate disasters will be insured if there is an insurance market for that...

Insurance and Uncertainty : the Framework

Classical framework : agent, with utility $u(\cdot)$ facing random loss L, purchases insurance at price π if

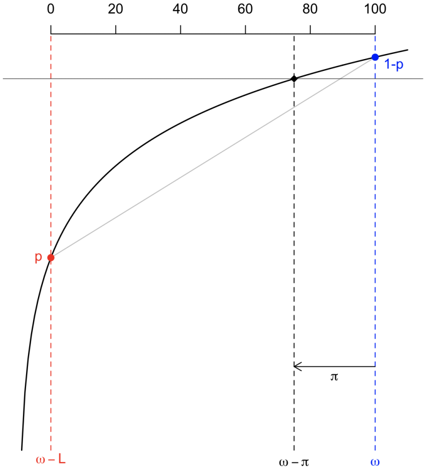
$$\mathbb{E}_{\mathbb{P}}\big[u(\omega-\pi-L+I(L))\big] > \mathbb{E}_{\mathbb{P}}\big[u(\omega-L)\big]$$

(von-Neumann expected utility).
Let $\pi^\star$ denote the highest premium

$$\mathbb{E}_{\mathbb{P}}\big[u(\omega-\pi^\star-L+I(L))\big] = \mathbb{E}_{\mathbb{P}}\big[u(\omega-L)\big]$$

where I denotes the indemnity function, $I(\ell)\in[0,\ell]$.

see e.g. Goussebaïle & Louaas (2015)



Insurance and Uncertainty : the Framework

Suppose that L is a binary loss, $L \in \{0, \ell\}$ with probabilities $1-p$ and p, or $L = X \cdot \ell$, $X \sim \mathcal{B}(p)$

$$(1-p) \cdot u(\omega - \pi^\star) + p \cdot u(\omega - \pi^\star - \ell + I(\ell)) = (1-p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

If $I(\ell) = \ell$, $u(\omega - \pi^\star) = (1-p) \cdot u(\omega) + p \cdot u(\omega - \ell)$
If u is linear (no risk aversion), $\pi^\star = \mathbb{E}[I(L)] = p \cdot I(\ell)$
If u is concave (risk aversion), $\pi^\star > p \cdot I(\ell)$
$p \cdot I(\ell)$ is the Actuarial (pure) Premium, also called Learned Hand formula in law business (see Grossman *et al.* (2006)) or simply "*probability times consequence*" in climate literature (see Schneider (2002))

Consider the simplistic case of homogeneous agents.
An agent, with utility u, will purchase insurance if

$$(1-p)\cdot u(\omega-\pi)+p\cdot u(\omega-\pi-\ell+I(\ell))>(1-p)\cdot u(\omega)+p\cdot u(\omega-\ell)$$

Assume that this inequality is satisfied.
An insurance company, with utility v, will sell insurance if

$$\mathbb{E}_p\big[v\big(\kappa+n\pi-S\big)\big]>u(\kappa),\ S=\sum_{1=1}^n L_i=\sum_{1=1}^n X_i\cdot\ell,\ \text{where } X_i\sim\mathcal{B}(p),$$

where κ is the capital of the company, and S is the total indemnity.
The actuarial fair premium is obtained when v is linear :

$$\pi^\star=\mathbb{E}[I(L)]=p\cdot I(\ell)$$

If risks are exchangeable $X = L_1 + \cdots + L_n = N_n \cdot I(\ell)$,

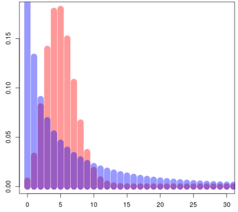
$$\mathbb{P}[N_n = k] = \int_0^1 \binom{n}{k} \theta^k (1-\theta)^{n-k} dG(\theta),\ \mathbb{P}[L_i > 0] = p$$

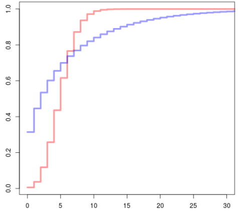
from de Finetti (1921). See Charpentier & le Maux (2014) for correlated risks and disaster (with endogeneous default probability of the insurer).

Classical framework : binomial-beta model (G is a Beta distribution $\mathcal{B}(\alpha,\beta)$), then $r = \text{corr}(X_i, X_j) = (1+\alpha+\beta)^{-1}$. Then

$$\mathbb{E}[N_n] = np \text{ while } \text{Var}[N_n] = (n + n(n-1)r)p(1-p)$$

(the insurance company has correlation aversion, Richard (1975), not clear for insured...)





Insurance and Uncertainty : the disaster puzzle

Earthquakes and floods cause potentially large losses, rational people should find actuarially fair insurance policies attractive, see Kunreuther (1996)
Individuals *underestimate* the true probability of a disaster event occurring and/or have fairly high discount rates for the benefits of uncertain future reimbursements due to losses.
Kunreuther & Pauly (2004) proved that even when insurance for low-probability, high-loss events is offered at favorable premiums, the search costs associated with obtaining the information on premiums and disaster probabilities may prevent from purchasing insurance.
Individuals lack information about the expected harm, before the disaster, see Botzen (2013). Insurance prices can be a signal used to correct for possible bias (unless market distortion), see Thieken *et al.* (2006).

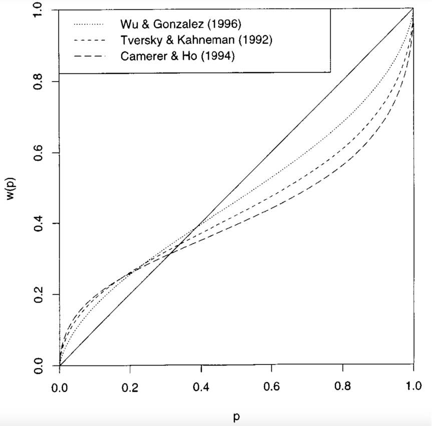
Loss Probability p and Perceived Loss Probability $\omega(p)$

RDEU framework, or Prospect theory, Kahneman & Tversky (1979) or Wu & Gonzalez (1996)

$$\omega(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

(experimental) evidence of convexity of ω

extreme risk aversion in situations that involve low stakes see Huysentruyt & Read (2010), but inadequate insurance coverage against disaster, Viscusi (2010), even if it is highly subsidized : underweighting of adverse tail events.



Loss Probability p and Perceived Loss Probability $\omega(p)$

people are aware of the risk, see Barnes (2011) (no one knows the odds)

Vesuvius last awoke with a small blast in 1944. A large eruption could unleash incendiary avalanches and ash that would threaten millions of people.

BETTMANN/CORBIS

EUROPE'S TICKING TIME BOMB

Vesuvius is one of the most dangerous volcanoes in the world — but scientists and the civil authorities can't agree on how to prepare for a future eruption.

BY KATHERINE BARNES

It starts with a blast so strong that a column of ash and stone rockets 40 kilometres up into the stratosphere. The debris then drops to Earth, pelting the surface with boiling hot fragments of pumice and covering the ground with a thick layer of ash. Roofs crumble and vehicles grind to a halt. Yet the worst is still to come. Soon, avalanches of molten ash, pumice and gas roar down the slopes of the volcano, pulverizing buildings and burying everything in their path. Almost overnight, a packed metropolis becomes a volcanic wasteland.

This is Naples, Italy, in the throes of a cataclysmic eruption of Vesuvius — the volcano that destroyed the city of Pompeii in AD 79. The scenario may sound far-fetched, but in the wake of Japan's recent earthquake and tsunami, many areas are reassessing the risks from their own 'black swans', a term used to describe unlikely but potentially devastating disasters. And Naples stands out as particularly vulnerable, with a population of 3 million living in the shadow of Vesuvius.

The volcano has been eerily dormant since a small eruption in 1944, but recent studies suggest that Vesuvius could be more dangerous than previously assumed, which has prompted a vigorous debate about the risk and scale of future disasters. Local authorities face the difficult task of deciding how to protect a large population in the event of earthquakes and other signs heralding the volcano's reawakening. "There would be no modern precedent for an evacuation of this magnitude," says Giuseppe Mastrolorenzo at the Vesuvius Volcano Observatory in Naples. "This is why Vesuvius is the most dangerous volcano in the world."

RUMBLINGS OF DISSENT

The slumbering giant won't stay quiet forever. Seismic imaging studies have detected an unusual layer about 8–10 kilometres deep under the mountain's surface. Mastrolorenzo and his colleague Lucia Pappalardo interpret this layer as an active magma reservoir[1], which could produce large-scale 'plinian'-style explosions — named after Pliny the Younger, who described the AD 79 eruption.

NATURE.COM
For more on volcanic eruptions see:
go.nature.com/Glkjak

The first rumblings of activity at Vesuvius could come weeks to years before an eruption, but there might be little, if any, warning of the eruption itself. Pappalardo and Mastrolorenzo analysed the geochemistry of rock fragments from past eruptions, and found evidence that magma ascended rapidly — in just a few hours — from its deep chamber to the surface.

For many years the largest known eruption of Vesuvius was that of AD 79. But in 2006, Mastrolorenzo and Michael Sheridan at the University at Buffalo in New York described geological evidence for a much larger blast, about 3,800 years ago in the Bronze Age[2]. Fiery avalanches of ash and debris called pyroclastic flows travelled 20 kilometres and covered the whole of the area of present-day Naples. "The deposits right in the centre of Naples are 4 metres thick," says Sheridan. "Even a few inches would be enough to kill everyone."

Given these concerns, the Vesuvius observatory team has urged the Neapolitan authorities to base their emergency plan on a worst-case 'maximum possible' eruption similar to the Bronze Age blast. "A crisis could start today," says Mastrolorenzo. "The trouble is that nobody would be able to tell how long it would last, what type of eruption it would be, or how the event would evolve." The researchers recommend the complete evacuation of an area 20 kilometres around Vesuvius if earthquakes and other signs of unrest hint that it is coming back to life.

Not all scientists share this doom-laden outlook. Some groups have even proposed that Vesuvius is becoming less explosive. Bruno Scaillet and his colleagues at the University of Orleans in France argue that the eruptive style of Vesuvius has changed as the magma chambers feeding the eruptions have migrated upwards, with the small 1944 eruption coming from a relatively shallow level 3 kilometres below the surface[3]. Evidence suggests the magma stored there is less viscous, so it is less prone to causing large explosions. If the past trend holds, says Scaillet, the next eruption could be similar to the most recent ones.

Scaillet adds that the seismically unusual layer 10 kilometres below the surface could be magma, but it could also be some other fluid such as water or brine. "These various issues are far from being settled," he says.

EMERGENCY PLANNING

With the size of any future eruption in doubt, and a public more concerned about day-to-day problems such as traffic and crime, mitigating the hazard of Vesuvius is an enormous task shared by researchers and the civil authorities.

Scientists keep constant tabs on Vesuvius through a network of sensors that monitor for earthquakes, ground deformation and changes in the chemistry of escaping gases.

And Italy's Department of Civil Protection (DPC) maintains a National Emergency Plan for Vesuvius. The plan, first developed in 1995, is based on a scenario for an intermediate-sized eruption, similar to one that occurred in 1631. That sub-plinian blast killed 6,000 people but affected an area much smaller than the earlier plinian eruptions.

The plan divides the area around the volcano into three regions according to the type of hazard expected. The red zone, closest to Vesuvius, is deemed most at risk from pyroclastic flows, so the plan calls for the evacuation of all 600,000 residents in this area before an eruption starts (see 'In the line of fire'). The main danger in the yellow zone comes from falling ash and small rocks. Officials would wait until the eruption starts, and the wind direction is known, before ordering an evacuation of regions in yellow zones downwind of the volcano. The blue zone is at risk from floods and mud flows triggered by the eruption, and would be evacuated according to the same plan. The city of Naples was excluded from any of the hazard zones because the prevailing wind typically blows ash to the east, away from the city.

In 2003, the DPC announced that it would constantly update the emergency plan to take account of new scientific information. The red zone is being expanded to include the eastern districts of Naples and officials reduced the evacuation time from two weeks to 72 hours, recognizing that there may be less of a warning before the eruption.

Nevertheless, some researchers argue that the plan has ignored important scientific evidence. Last year, Mastrolorenzo and Pappalardo[4] and Giuseppe Rolandi[5] of the University of Naples found that even with an intermediate-sized eruption, pyroclastic flows would threaten several municipalities not currently included in the red zone. Mastrolorenzo says that officials should also not wait to evacuate the yellow zone, because fine ash would rapidly fill the air and plunge the area into total darkness. "You have to get people out before it starts," he says. And the wind does sometimes blow towards Naples, so the authorities cannot rule out heavy ashfall in the city, say both Mastrolorenzo and Rolandi.

Putting all the evidence together, they and other researchers insist that the emergency plan should correspond to the 'worst-case scenario', which means including metropolitan Naples and its 3 million inhabitants.

That makes sense for planning, says Jonathan Fink, a volcanologist at Portland State University in Oregon. Once the volcano shows signs of unrest, authorities and scientists can re-evaluate. "If there is an error on the high side, there is less lost than would be the case in the opposite situation," he says.

In a written response to *Nature*, the DPC advocates evaluating the eruption risk "on the basis of the present state of the volcano and not simply assuming the largest eruption event that ever occurred in the volcanic history". Some scientists agree. "You can't spend [everything] on the absolute worst case. You need to reduce the risk in a rational way," says Warner Marzocchi at the National Institute for Geophysics and Volcanology (INGV) in Rome. A complete evacuation of Naples' 3 million residents, he says, "would be impossible to manage".

"You need to reduce the risk in a rational way."

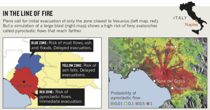
Marzocchi and other researchers are developing modelling tools — based on the probabilities of different scenarios — that could help civil authorities evaluate the evidence during a crisis and choose a course of action. Peter Baxter, an expert in emergency planning at the University of Cambridge, UK, specializes in the impacts of volcanic eruptions and used this type of method successfully during the 1997 eruption in Montserrat in the Caribbean to predict which regions would be affected. A complete evacuation of the island was avoided.

For Vesuvius, Baxter and his colleagues have used geological data and models of eruptive processes to develop an 'event-tree' to display the full range of possible eruptions[6]. If sensors on the volcano pick up signs of magmatic unrest, the analysis suggests a 70% probability of an explosive eruption but only a 4% chance of a catastrophic plinian one. The most likely event is a violent but smaller blast, like the one in 1944, with lava flows and moderate ash emissions.

For now, this kind of probabilistic approach seems the only way forward for volcanologists and disaster planners, as there is no recipe for accurate eruption forecasting on the horizon. "It's an extremely complex problem to solve," says Augusto Neri of the INGV's laboratories in Pisa. "We simply do not know how the volcano works." ■

IN THE LINE OF FIRE

Plans call for initial evacuation of only the zone closest to Vesuvius (left map, red). But a simulation of a large blast (right map) shows a high risk of fiery avalanches called pyroclastic flows that reach farther.



SOURCE: LEFT, DEPT CIVIL PROTECTION; RIGHT, REF. 4

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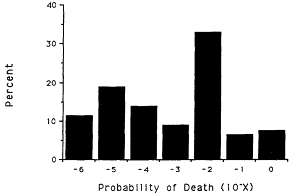
1. Pappalardo, L. & Mastrolorenzo, G. *Earth Planet. Sci. Lett.* **296,** 133–143 (2010).
2. Mastrolorenzo, G. *et al. Proc. Natl Acad. Sci. USA* **103,** 4366–4370 (2006).
3. Scaillet, B., Pichavant, M. & Cioni, R. *Nature* **455,** 216–219 (2008).
4. Mastrolorenzo, G. & Pappalardo, L. *J. Geophys. Res.* **115,** B12212 (2010).
5. Rolandi, G. *J. Volcanol. Geotherm. Res.* **189,** 347–362 (2010).
6. Baxter, P. J. *et al. J. Volcanol. Geotherm. Res.* **178,** 454–473 (2008).

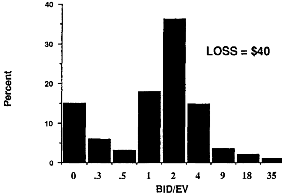
Loss Probability p and Perceived Loss Probability $\omega_i(p)$

Experimental study of McClelland, Schulze & Coursey (1993) : bimodal distribution of willingness to pay for insurance, with two goups

- neglect low-probability risks, do not purchase insurance
- willingness to pay higher than expected loss



link between the bimodal risk judgments and protective behavior McClelland, Schulze & Hurd (1990), over and underweighting rare extreme events, Epper & Fehr-Duda (2017). See also Charpentier & le Maux (2014) on optimal (Pareto) vs. equilibrium (Nash)



Loss Probability p and Perceived Loss Probability $\omega(p)$

"*The probability of tail events is overestimated, which is consistent with probability weighting in prospect theory*" Botzen, Kunreuther & Michel-Kerjan (2015) (but "*potential damage is underestimated*")
Myopic loss aversion or myopic probability weighting, see Barberis, Huang & Thaler (2006), and Barberis (2013) on psychology of "tail events".

See also Gilboa & Schmeidler (1989) on the use of capacities : instead of computing $\mathbb{E}_{\mathbb{P}}[u(\omega - \pi - L + I(L))]$, consider

$$\min_{\mathbb{Q}\in\mathcal{P}} \{\mathbb{E}_{\mathbb{Q}}[u(\omega - \pi - L + I(L))]\} \text{ for some set of beliefs } \mathcal{P}$$

Can capture uncertainty associated with climate change, that makes it difficult to respond optimally, see Dessai & Hulme (2006) or Tompkins & Adger (2005).

Loss Probability p : Meteorological Perspective

For hurricanes, see Gray *et al.* (1992), the seasonal number of intense hurricanes is

$$\widehat{N} = 3.571 + 0.042\big(U_{50} + 0.103U_{30} - 1.415|U_{50} - U_{30}|\big) + 0.717\big(R_S + 2.455R_G\big)$$

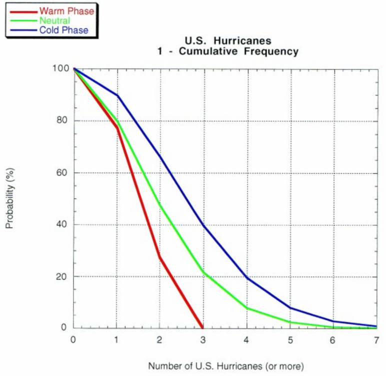
where U's are upper-air zonal winds at 50 and 30 mb and R's are composite functions of August-September western Sahel (R_S) and August-November Gulf of Guinea (R_G) rainfall.

See also Klotzbach & Bell (2017) on the Landfalling Hurricane Probability Project (with all landfall probability calculations)

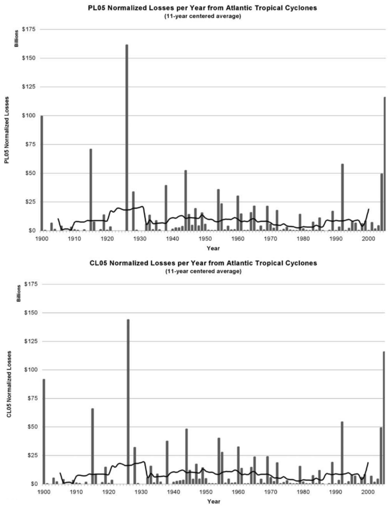
	A	N	O	P	Q	R
3		Climatological Probability			Current-Year Probability	
4		(Using Poisson)			(Using Poisson)	
5	State	H	MH		H	MH
6	Texas	33%	12%		21%	7%
7	Louisiana	30%	12%		19%	7%
8	Mississippi	11%	4%		6%	3%
9	Alabama	16%	3%		10%	2%
10	Florida	51%	21%		35%	13%
11	Georgia	11%	1%		7%	1%
12	South Carolina	17%	4%		11%	2%
13	North Carolina	28%	8%		18%	5%

Statistical Perspective

Need (long) historical data,
(if possible with normalized losses)
see Pielke Jr. *et al.* (2008)



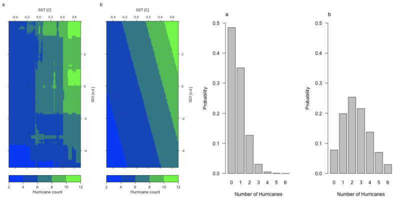
From Bove *et al.* (1998) for hurricanes.

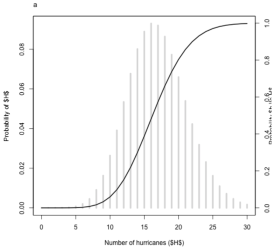


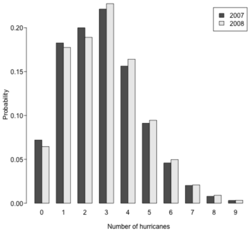
Loss Probability : Statistical Perspective

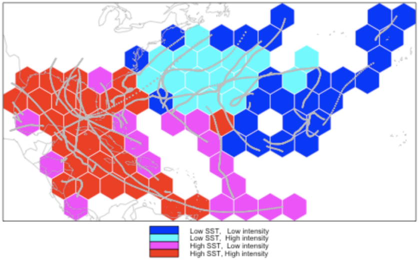
See Elsner & Jagger (2013) for a exhaustive statistical analysis of hurricanes,

- Poisson regression for counts
  - Southern Oscillation Index (SOI)
  - sea-surface temperature (SST)
- regression for intensity
- spatial models for trajectories
- Bayesian models for one year prediction









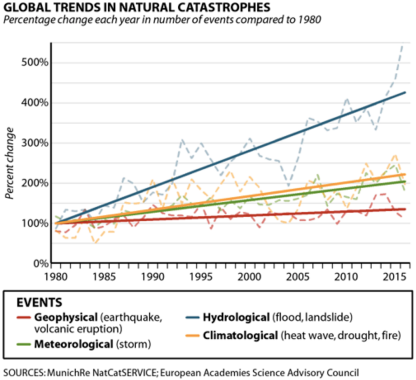
Loss Probability : Statistical Perspective

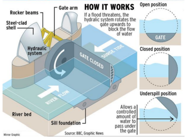
hard to justify in a changing environment

“In order to apply any theory we have to suppose that the data are homogeneous, i.e. that no systematical change of climate and no important change in the basin have occurred within the observation period and that no such changes will take place in the period for which extrapolations are made”, Gumbel (1941)

see flood risk (dams, reservoirs, etc)

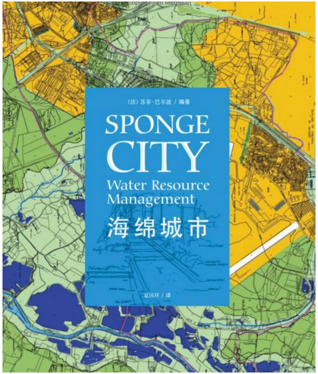
see storms or heatwaves (climate change) Charpentier (2011)





The Architect / Urban Planner Perspective ?

See Dong & Han (2011), Bardaux (2016), Li *et al.* (2017) Jiang *et al.* (2018) on sponge cities (urban underground water system operates like a sponge to absorb, store, leak and purify rainwater, and release it for reuse when necessary)



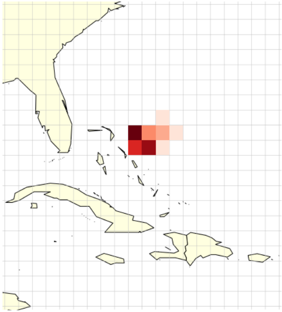
Loss Probability p : Actuarial Perspective

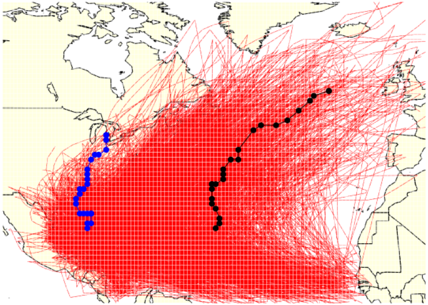
Actuaries use catastrophe softwares

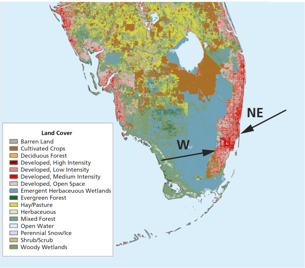
- Risk Management Solutions (RMS)
- AIR Worldwide (AIR)
- Risk Quantification & Engineering (RQE) EQECAT

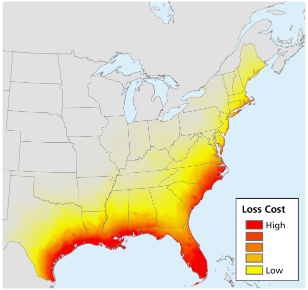
see Cole, Macpherson & McCullough (2010).

Generation of climatic scenarios (+ losses)









see for Markovian generation Charpentier (2014).

Predictive Markets

Peer prediction systems

Wolfers & Zitzewitz (2004) for a description

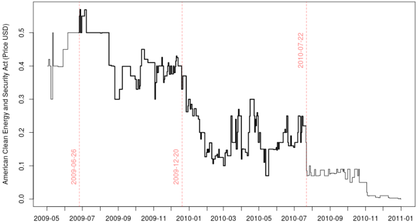
• elections (e.g. popes, XVIth Century) Rhode & Strumpf (2008)

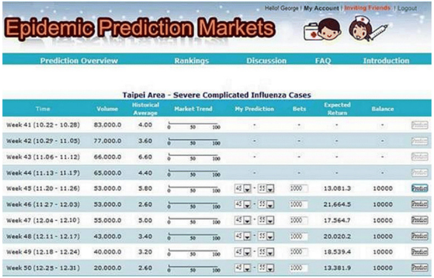
• politics, Meng (2016) Waxman-Markey bill (2009-2010, finally rejected)

• sports (racetrack bets) Ali (1977)

• infectious diseases, Polgreen, Nelson & Neumann (2006) or Tung, Chou & Lin (2015)

• climate risk Hallstrom & Smith (2005) for hurricanes





Taipei Area - Severe Complicated Influenza Cases

Time	Volume	Historical Average	Market Trend	My Prediction	Bets	Expected Return	Balance
Week 41 (10.22 - 10.28)	83,000.0	4.00	0 50 100	-	-	-	-
Week 42 (10.29 - 11.05)	77,000.0	3.60	0 50 100	-	-	-	-
Week 43 (11.06 - 11.12)	66,000.0	6.60	0 50 100	-	-	-	-
Week 44 (11.13 - 11.19)	65,000.0	4.40	0 50 100	-	-	-	-
Week 45 (11.20 - 11.26)	53,000.0	5.80	0 50 100	45 - 55	1000	13,081.3	10000
Week 46 (11.27 - 12.03)	53,000.0	2.60	0 50 100	45 - 55	1000	21,664.5	10000
Week 47 (12.04 - 12.10)	55,000.0	5.00	0 50 100	45 - 55	1000	17,564.7	10000
Week 48 (12.11 - 12.17)	43,000.0	3.40	0 50 100	45 - 55	1000	20,020.2	10000
Week 49 (12.18 - 12.24)	40,000.0	3.20	0 50 100	45 - 55	1000	18,539.4	10000
Week 50 (12.25 - 12.31)	20,000.0	2.60	0 50 100	45 - 55	1000	13,381.9	10000

(Perceived) Loss Probability p : Betting Markets

See Eisenberg & Gale (1959) model, on consensus and subjective probabilities (used in Manski (2004) on all-or-nothing contracts)
Individual i (wealth b_i) can bet on horse j an amount $\beta_{i,j}$ (so that $\beta_{i,1}+\cdots+\beta_{i,J}=b_i$). Assume that $b_1+\cdots+b_I=1$.
Let π_j denote the sum bet on horse j, $\pi_j=\beta_{1,j}+\cdots+\beta_{I,j}$. Observe that from the budget constraint $\pi_1+\cdots+\pi_J=1$, hence π_j's are probabilities.
Individual beliefs can be related expressed through a probability vector $\boldsymbol{p}_i=(p_{i,1},\cdots,p_{i,J})$, then there is an equilibrium if

$$p_{i,j}=\pi_j\cdot\max_s\left\{\frac{p_{i,s}}{\pi_s}\right\}\text{ as soon as }\beta_{i,j}>0$$

(so called Eisenberg-Gale equilibrium).

(Perceived) Loss Probability p : Betting Markets

Consider a contract that pay \$1 if an event A occurs (e.g. major flood), sold at price π_A. If there is a contract on $\bar{A}$, then $\pi_{\bar{A}} = 1 - \pi_A$ (no arbitrage).
Assume that individual i believe that A will occur with probability p_i. If $p_i > \pi_A$, he should purchase b_i units of A (otherwise b_i units of $\bar{A}$).
There is equilibrium if

$$\sum_{i=1}^{I} b_i = \frac{1}{\pi_A}\sum_{i=1}^{I} b_i \mathbb{P}[p_i > \pi_A] = \frac{1}{\pi_{\bar{A}}}\sum_{i=1}^{I} b_i \mathbb{P}[p_i < \pi_A]$$

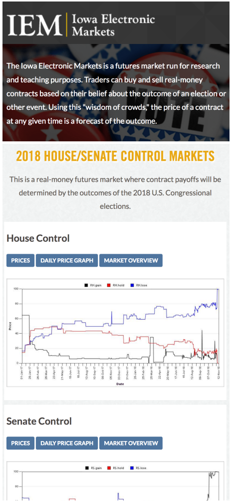
hence, if wealth b_i is independent of belief p_i, then prices are probabilities

$$\pi_A = \mathbb{P}[p_i > \pi_A] = 1 - \pi_{\bar{A}}.$$

see Wolfers & Zitzewitz (2004) for further interpretation.

Parimutuel and Betting Markets

- 2004 and 2005, ICAP, Goldman and Nymex (see IFR) with parimutuel derivative on energy (changes in crude oil and natural gas inventories)
- 2004, CME launched futures trading on the US Consumer Price Index (CPI), see Filippov (2005)
- 2008, LLC (see Ou-Yang (2010)) with parimutuel derivative on hurricanes
- The longitude plateforme ("*real time calculation of the odds*").
- The Iowa Electronic Markets for elections, "*using this* **wisdom of crowds** *the price of a contract at any given time is a forecast of the outcome*"



HuRLOs (Hurricane Risk Landfall Option) options

The exist the Hurricane Futures Market (HFM), see Kelly, Letson, Nelson, Nolan & Solis (2010) for a description

Hurricane Risk Landfall Options (HuRLOs) were launched in October 2008 by Weather Risk Solutions, LLC (WRS) and promised a mutualized marketplace for hurricane options, using parimutuel options

From French pari mutuel, literally, mutual stake :

"*A system of betting on races whereby the winners divide the total amount bet, after deducting management expenses, in proportion to the sums they have wagered individually*", see Baron & Lang (2007), also called universal Dutch auction in Syz (2008)

"*A Shapley-Shubik market game for contingent claims within a probability state space with secured selling is a parimutuel market*" (Lange & Economides (2005))

A matching / pricing mecanism is necessary to get financial efficiency (self-hedging). Participant i submit orders, state bids β_i, b_i a limit share quantity and $\bar{\pi}_i$ a limit price per share. The market organizer determines the order fill x_i (and state prices π_j), using call auction mechanism, see Peters, So and Ye (2005) and Agrawal, Wang & Ye (2008).
Using a Linear Programming Market Mechanism, market organizer should solve

$$\text{maximize}\left\{\sum_{i=1}^{I}\bar{\pi}_i x_i - z\right\} \text{ where } z \text{ is some worst case cost}$$

$$\text{subject to} \quad \text{cost if state } j \text{ occurs} = \sum_{i=1}^{I}\beta_{i,j}x_i \leq z \quad \forall j$$

$$0 \leq \bar{\pi}_i x_i \leq b_i \quad \forall i$$

More realistically, one can consider some dynamic parimutuel market maker as in Agrawal *et al.* (2014) for recent advances, and Chen & Pennock (2010) for an overview.

See also techniques to get some peer prediction Bayesian Nash equilibrium solvers (see Jurca & Faltings (2008) on elections and bets) and the related collective revelation mechanism (see Goel, Reeves & Pennock (2009))

"*Peer prediction systems are designed for eliciting information on events where ground truth does not exist or is unobtainable*", Chen & Pennock (2010)

Collective Dimension of Insurance (Climate) Risk

"*Insurance is the contribution of the many to the misfortune of the few* "

"[L'assurance distingue] *entre le dommage que subit tel ou tel individu — c'est affaire de chance ou de malchance — et la perte liée au dommage dont l'attribution est, quant à elle, toujours collective et sociale*", Ewald (1986).

- pricing perspective (zero-sum game, ex-post subsidizing)
- modeling perspective (strongly correlated risks - spatially)
- prevention for climate risks is essentially collective

see Charpentier, Barry & Gallic (2019) for additional thoughts

Take-Away Conclusion

- knowing real probabilities of occurrence disasters is either complicated (hurricanes) or very complicated (flood)
- those probabilities are necessary to assess solvency of insurance companies (central limit theorem is based on true probabilities)
- insurance pricing is based on beliefs of insured, and insurance companies
- predictive markets can be an interesting revelation mechanism of crowd beliefs (can be used to assess if an insurance market can actually exist, or not)
- it is difficult to think of climate risk without its collective dimension ...