Machine Learning in Actuarial Science & Insurance

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Machine learning for economists and applied social scientists, 2020

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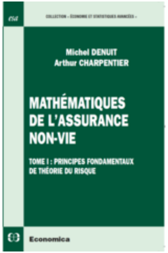
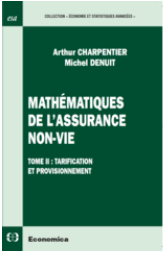
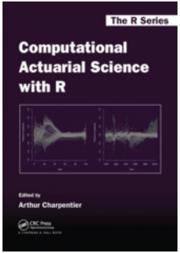
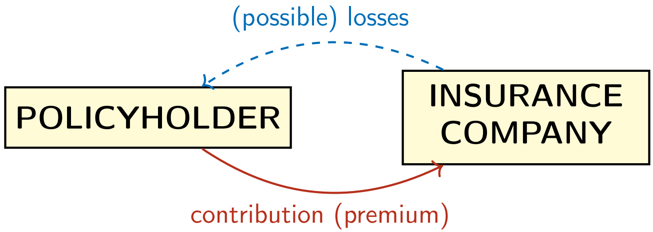






"*Insurance is the contribution of the many to the misfortune of the few*"



What would be a "*fair contribution*" ? see O'Neill (1997)

- pure actuarial fairness contributions for individual policyholders should perfectly reflect their predicted risk levels $\rightarrow$ predictive modeling
- choice-sensitive fairness contributions should take into account only risks that result from choices - luck-egalitarianism (Cohen (1989) or Arneson (2011))

Agenda

General Insurance & Predictive Modeling (1)

- fraud detection & network data

see e.g. Óskarsdóttir et al. (2019)

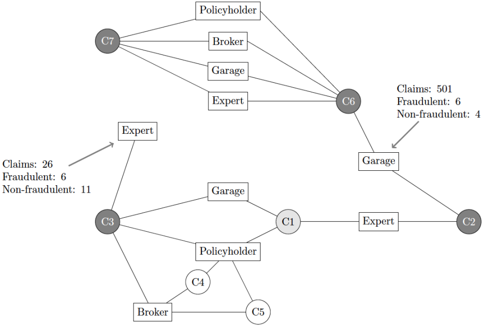
- parametric insurance & satellite pictures

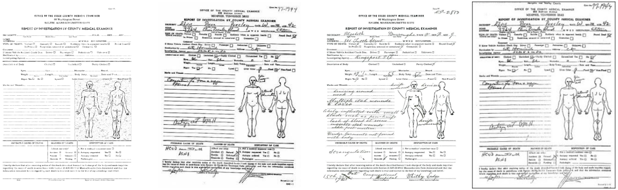
see e.g. de Leeuw et al. (2014)

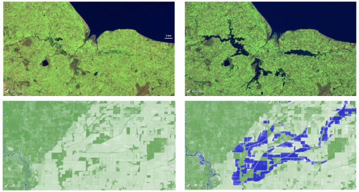
- claims reserving

see e.g. Wüthrich (2018)

- automatic reading (medical reports)



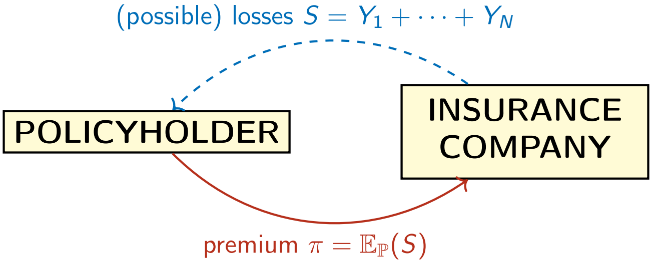




See also Denuit et al. (2019a, 2020, 2019b)

General Insurance & Predictive Modeling (2)

- premium computation, $\pi = \mathbb{E}_{\mathbb{P}}(S)$



or, given some features $\boldsymbol{X} = \{X_1, \cdots, X_p\}$, premium $\pi(\boldsymbol{x}) = \mathbb{E}_{\mathbb{P}_{\boldsymbol{X}}}(S) = \mathbb{E}[S|\boldsymbol{X}]$

Under standard assumptions,

$$\pi(\boldsymbol{x}) = \mathbb{E}[S|\boldsymbol{X}] = \underbrace{\mathbb{E}[N|\boldsymbol{X}]}_{\text{frequency}} \cdot \underbrace{\mathbb{E}[Y|\boldsymbol{X}]}_{\text{average cost}} = \underbrace{\mathbb{E}[\mathbf{1}_{S>0}|\boldsymbol{X}]}_{\text{occurence}} \cdot \underbrace{\mathbb{E}[S|\boldsymbol{X}, S>0]}_{\text{individual cost}}$$

From Econometric Techniques to Machine Learning (1)

Merging claims & underwriters databases (per policy), e.g. $(n_i, e_i, \boldsymbol{x}_i)$, $i = 1, \cdots, n_p$

	id	exposure	zone	pwr	agecar	agedrv	model	gas	dsty	region	nbr	occ
1	3227	0.87	C	7	0	56	12	D	93	13	0	0
2	4115	0.72	D	5	0	45	22	R	54	13	0	0
3	5121	0.05	C	6	3	37	17	D	11	13	0	0
4	5142	0.90	C	10	10	42	7	D	93	13	0	0
5	6255	0.12	C	7	0	59	11	R	73	13	0	0
6	8486	0.83	C	5	0	75	7	R	42	13	2	1

Standard model, Poisson regression (possibly zero-inflated, possibly over-dispesersed, etc), related to the Poisson process

$$N_i \sim \mathcal{P}(e_i \cdot \exp[\theta_i]), \quad \text{where } \theta_i = \alpha_0 + \alpha_1 x_{1,i} + \alpha_2 x_{2,i} + \cdots + \alpha_p x_{p,i} = \boldsymbol{x}_i^\top \boldsymbol{\alpha}$$

or possibly GAMs (for nonlinearities, or spatial component)

$$\theta_i = \alpha_0 + s_1(x_{1,i}) + \alpha_2 x_{2,i} + \cdots + \alpha_p x_{p,i}, \ s_1(x) = \sum_j \gamma_j \varphi_j(x)$$

From Econometric Techniques to Machine Learning (2)

or $(y_i \boldsymbol{x}_i)$, $i = 1, \cdots, n_c$

```
      id    no cover    cost  zone pwr agecar agedrv model gas dsty
1   1870 17219   1TP 1692.29     C   5      0     52    12   R   93
2   1963 16336   1TP  422.05     E   9      0     78    12   R   27
3   4263 17089   2MT  549.21     C  10      7     27    17   D   19
4   5181 17801   1TP  191.15     D   5      2     26     3   D   91
5   6375 17485   1TP 2031.77     B   7      4     46     6   R   48
```

Standard model, Gamma regression (with a log link function)

$$Y_i \sim \mathcal{G}(\exp[\vartheta_i], b), \quad \text{where } \vartheta_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \cdots + \beta_p x_{p,i} = \boldsymbol{x}_i^\top \boldsymbol{\beta}$$

possible excluding large claims (and pooling on the entiere population)

$$\mathbb{E}[Y|\boldsymbol{X}] = \underbrace{\mathbb{P}[Y \leq s|\boldsymbol{X}]}_{p_s} \cdot \underbrace{\mathbb{E}[Y|\boldsymbol{X}, Y \leq s]}_{\mu_s(\boldsymbol{x})} + \underbrace{\mathbb{P}[Y > s|\boldsymbol{X}]}_{1-p_s} \cdot \underbrace{\mathbb{E}[Y|\boldsymbol{X}, Y > s]}_{\overline{y}}$$

or some Tweedie regression on S (but less flexible), Tweedie (1984).

From Econometric Techniques to Machine Learning (3)

Probabilistic interpretations : estimation using maximum likelihood - Generalized (Mixed) Linear Models (see McCullagh and Nelder (1989) or Jørgensen (1997))

- Individual model: $S = \mathbf{1}_{S>0} \cdot \widetilde{S}$
- Collective model: $S = \sum_{i=1}^{N} Y_i$, $N \sim \mathcal{P}(\lambda)$, $Y_i \sim \mathcal{G}(\alpha, \beta)$ in the exponential family,

with variance function $V(\mu) = \mu^k$, where $k \in [1, 2]$

$$\log \mathcal{L} = \sum_{i=1}^{n} \underbrace{y_i \frac{\mu_i^{1-k}}{1-k} - \frac{\mu_i^{2-k}}{2-k}}_{-\ell(y_i,\mu_i)}, \text{ where } \mu_i = \exp[\boldsymbol{x}_i^\top \boldsymbol{\beta}]$$

Machine learning : interpret $-\log \mathcal{L}$ as a loss

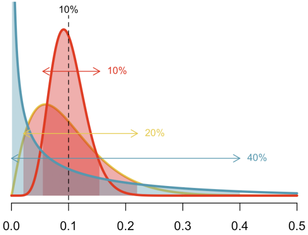
One can also include some penalty if the number of possible covariates is too large...

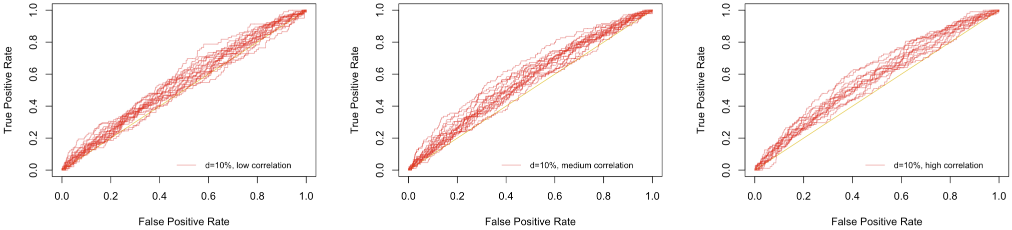
Goodness of Fit & Uncertainty (1)

Consider a simple model: number of claims $N \in \{0, 1\}$ and fixed cost, say \$1,000.
Simple classification problem.
Assume that $N \sim \mathcal{B}(\theta)$ and we have some covariate x.
Two important features:

- the dispersion of the heterogeneity, i.e. the variance of θ

  e.g. assume that θ has a Beta distribution on $[0, 1]$
- the dependence between heterogeneity θ and covariate x





← low correlation high correlation →

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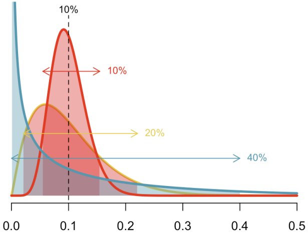
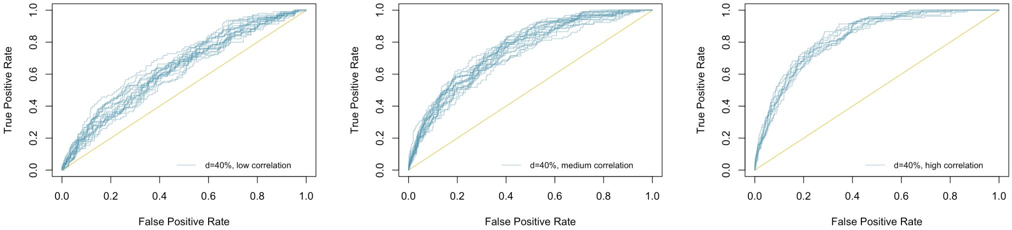


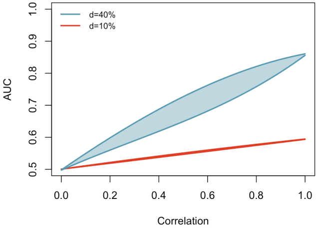


$\leftarrow$ low correlation high correlation $\rightarrow$

Goodness of Fit & Uncertainty (2)

- the dispersion of the heterogeneity, $d = 10\%$ or 40%
- the dependence between heterogeneity θ and covariate x (correlation of the underlying copula function)

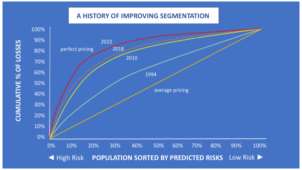
very difficult to reach high AUC (area under the ROC curve)

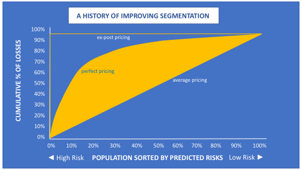


More complicated for insurance premiums...

Frees et al. (2014) defined a ROC-type curve, inspired by Lorenz curve: given observed losses s_i and premiums $\widehat{\pi}(\boldsymbol{x}_i)$, policyholders ordered by premiums, $\widehat{\pi}(\boldsymbol{x}_1) \geq \widehat{\pi}(\boldsymbol{x}_2) \geq \cdots \geq \widehat{\pi}(\boldsymbol{x}_n)$, plot

$$\{F_i, L_i\} \text{ with } \underbrace{F_i = \frac{i}{n}}_{\text{proportion of insured}} \text{ and } \underbrace{L_i = \frac{\sum_{j=1}^{i} s_j}{\sum_{j=1}^{n} s_j}}_{\text{proportion of losses}}$$



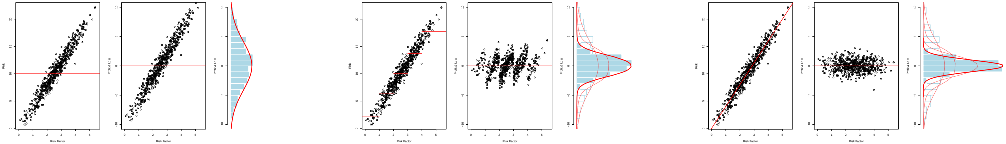


Goodness of Fit & Uncertainty (3)

Parallel with classical OLS models, $S \sim \mathcal{N}(\theta, \sigma^2)$, with covariates $\boldsymbol{X} = \{X_1, \cdots, X_p\}$

$$\text{Var}\Big[S\Big] = \mathbb{E}\Big[\text{Var}[S|\boldsymbol{X}]\Big] + \text{Var}\Big[\underbrace{\mathbb{E}[S|\boldsymbol{X}]}_{\text{premium}}\Big]$$

$$\mathbb{E}\Big[\text{Var}[S|\boldsymbol{X}]\Big] = \underbrace{\mathbb{E}\Big[\text{Var}[S|\Theta]\Big]}_{\text{perfect pricing } = \sigma^2} + \underbrace{\mathbb{E}\Big\{\text{Var}\Big[\mathbb{E}[S|\Theta]\Big|\boldsymbol{X}\Big]\Big\}}_{\text{misfit}}$$



(1) (θ_i, s_i) and $\mathbb{E}(S_i|\boldsymbol{X_i})$ (2) $(\theta_i, s_i - \mathbb{E}(S_i|\boldsymbol{X_i}))$ (3) distribution of $S - \mathbb{E}(S|\boldsymbol{X})$)

Model Interpretation & Explainability (1)

Most machine linearing algorithms are black boxes, Pasquale (2015)
"*providing transparency and explanations of algorithmic outputs might serve a number of goals, including scrutability, trust, effectiveness, persuasiveness, efficiency, and satisfaction*" Diakopoulos (2016)

- Ceteris paribus can be translated into "*all other things being equal*" or "*holding other factors constant*"
- Mutatis mutandis approximately translates as "*allowing other things to change accordingly*" or "*the necessary changes having been made*"

Guidotti et al. (2018) for a survey on methods for explaining back boxes

- explaining the model or explaining the outcome
- providing a *transparent* design (locally or globally)

Model Interpretation & Explainability (2)

Various tools :

- Local Interpretable Model-Agnostic Explanations (LIME) for general models,
- and for regression types, partial dependence plots (PDP), Individual Conditional Expectation (ICE), or Accumulated Local Effects (ALE)

$$pdp(\boldsymbol{x}_1)=\mathbb{E}\big[\pi(\boldsymbol{x}_1,\boldsymbol{X}_2)\big] \text{ and } \widehat{pdp}(\boldsymbol{x}_1)=\frac{1}{n}\sum_{i=1}^{n}\widehat{\pi}(\boldsymbol{x}_1,\boldsymbol{x}_{i,2})$$

Or define $ale(\boldsymbol{x}_1)=\displaystyle\int_{-\infty}^{x_1}\mathbb{E}\left[\frac{\partial\pi(\boldsymbol{z}_1,\boldsymbol{X}_2)}{\partial\boldsymbol{x}_1}\right]d\boldsymbol{z}_1$, as in Apley and Zhu (2016).

For more details, see https://christophm.github.io/interpretable-ml-book/,
or Lakkaraju et al. (2019), Samek et al. (2017), Ribeiro et al. (2016), Lipton (2016), Lundberg and Lee (2017), Gilpin et al. (2018), etc.

Insurance Premium Fairness

Various concepts and definitions,

- pure actuarial fairness: insurance costs for individuals should directly reflect their level of risk
- choice-sensitive equity: insurance costs for individuals should reflect only the risks that result from individual choices (see luck-egalitarian)
- equity as social justice: insurance of goods that are basic requirements of social justice should be provided irrespective of the risks and choices of individuals

Economic and philosophical issues.... but also statistical ones.
Formally, $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2)$ where only variables $\boldsymbol{x}_2$ can be used.

Algorithmic Fairness (for classifiers)

Feldman et al. (2015), Bonchi et al. (2017) or Corbett-Davies and Goel (2018)
Let s denote the (underlying) score of a classifier $s(\boldsymbol{x}) = \mathbb{P}[Y = 1|\boldsymbol{X} = \boldsymbol{x}]$, and p the classifier $p(\boldsymbol{x}) = \mathbf{1}_{[t,\infty)}(s(\boldsymbol{x}))$,

- anti-classification: some attributes $\boldsymbol{x}_1$ cannot be used

$$p(\boldsymbol{x}_1, \boldsymbol{x}_2) = p(\boldsymbol{x}_1', \boldsymbol{x}_2) \text{ whenever } \boldsymbol{x}_1 \neq \boldsymbol{x}_1'$$

- classification parity: standard metric do not depend on $\boldsymbol{x}_1$,

  e.g. for a classifier, the false positive rate should not depend on $\boldsymbol{x}_1$

$$\mathbb{P}[p(\boldsymbol{X}_1, \boldsymbol{X}_2) = 1|Y = 0, \boldsymbol{X}_1 = \boldsymbol{x}_1] = \mathbb{P}[p(\boldsymbol{X}_1, \boldsymbol{X}_2) = 1|Y = 0], \ \forall \boldsymbol{x}_1.$$

- calibration equity:

$$\mathbb{P}[Y = 1|S(\boldsymbol{X}_1, \boldsymbol{X}_2) = s, \boldsymbol{X}_1 = \boldsymbol{x}_1] = \mathbb{P}[Y = 1|S(\boldsymbol{X}_1, \boldsymbol{X}_2) = s]$$

.

Wrap-Up & Open Challenges

See Charpentier and Denuit (2020) for a recent state-of-the-art

- predictive models (econometrics or ML) are everywhere in insurance
- impossible to use black boxes for ratemaking
- more and more regulation towards transparency, fairness, equity
- so far, only actuarial modeling, nothing about economics of insurance (moral hazard, competition, price elasticity, etc)
- how to include *ex-post* observations, e.g. telematics (endogeneity issues)?
- how to include competition? (learning games)

- to go further freakonometrics.hypotheses.org or charpentier.arthur@uqam.ca

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