

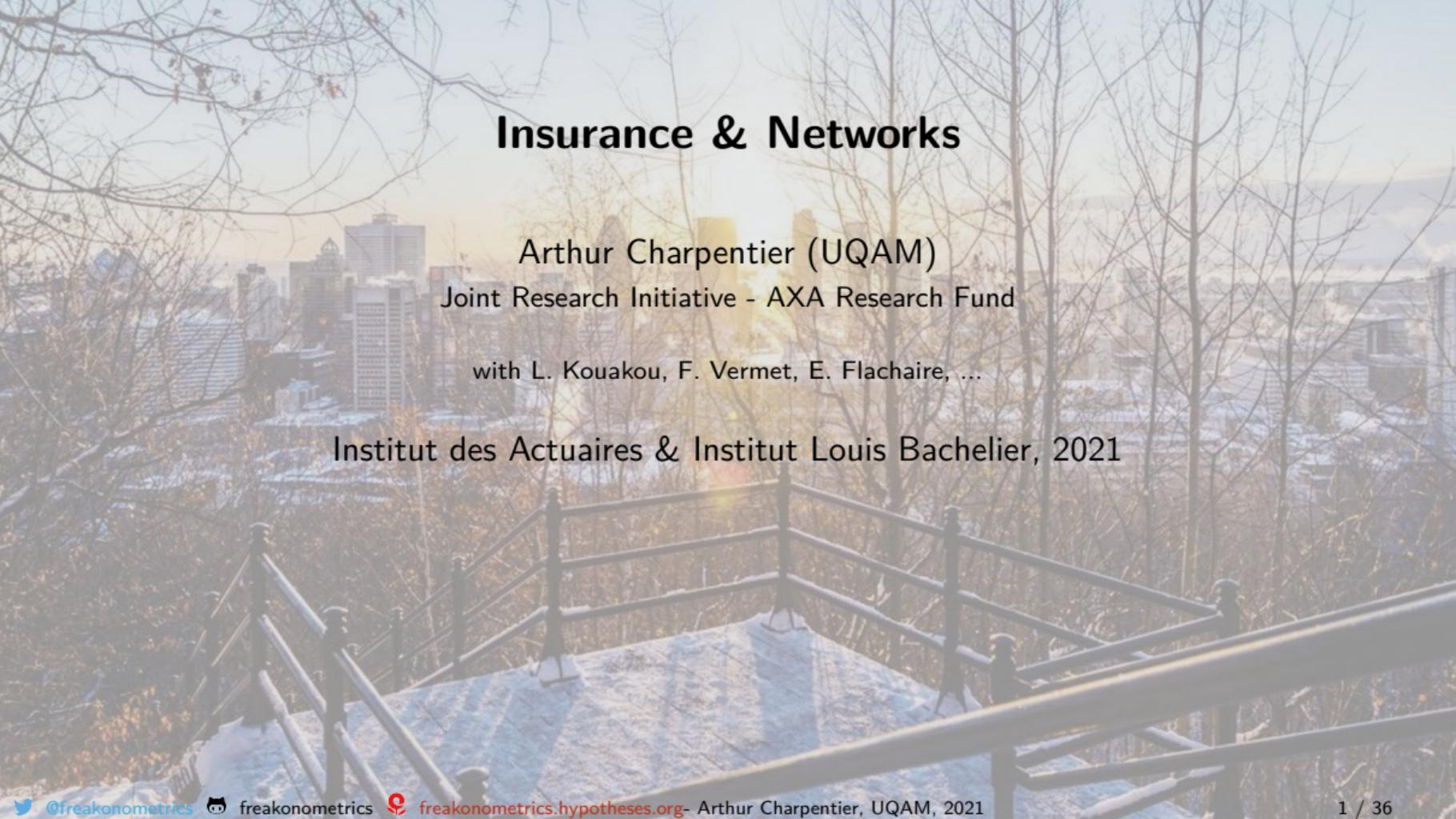
Insurance & Networks

Arthur Charpentier (UQAM)

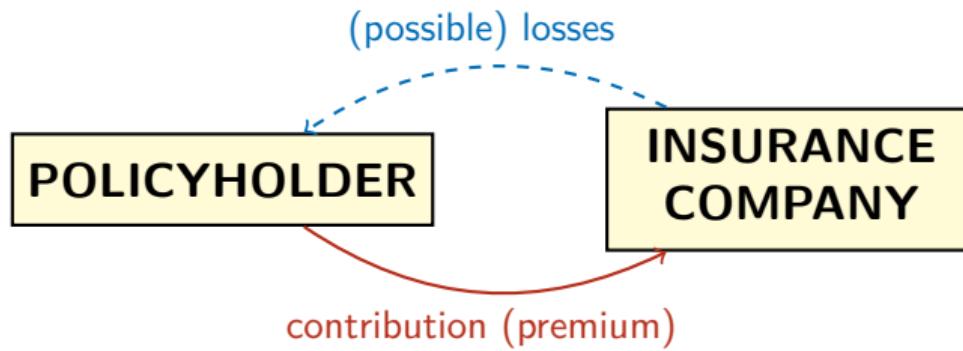
Joint Research Initiative - AXA Research Fund

with L. Kouakou, F. Vermet, E. Flachaire, ...

Institut des Actuaires & Institut Louis Bachelier, 2021

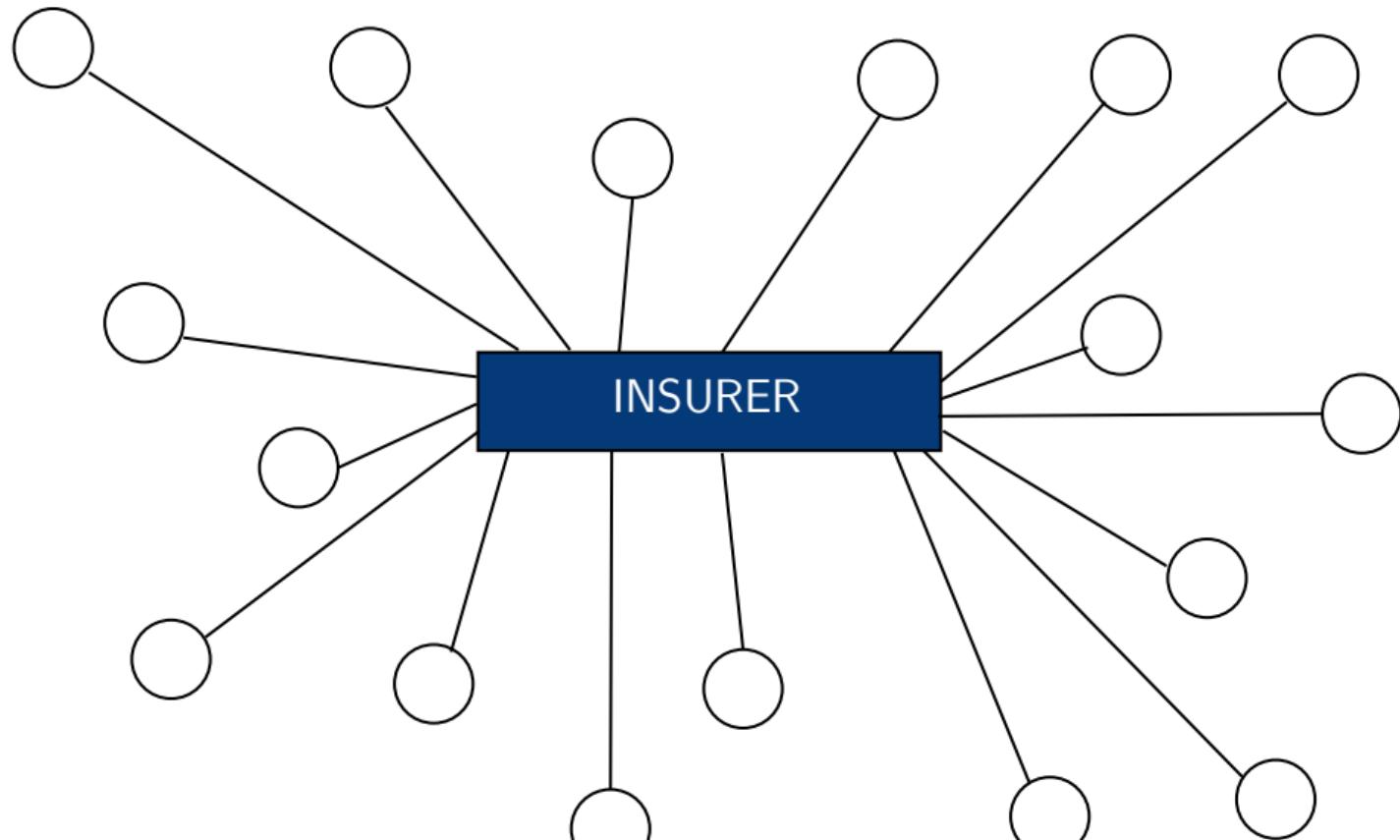


Insurance, in a nutshell

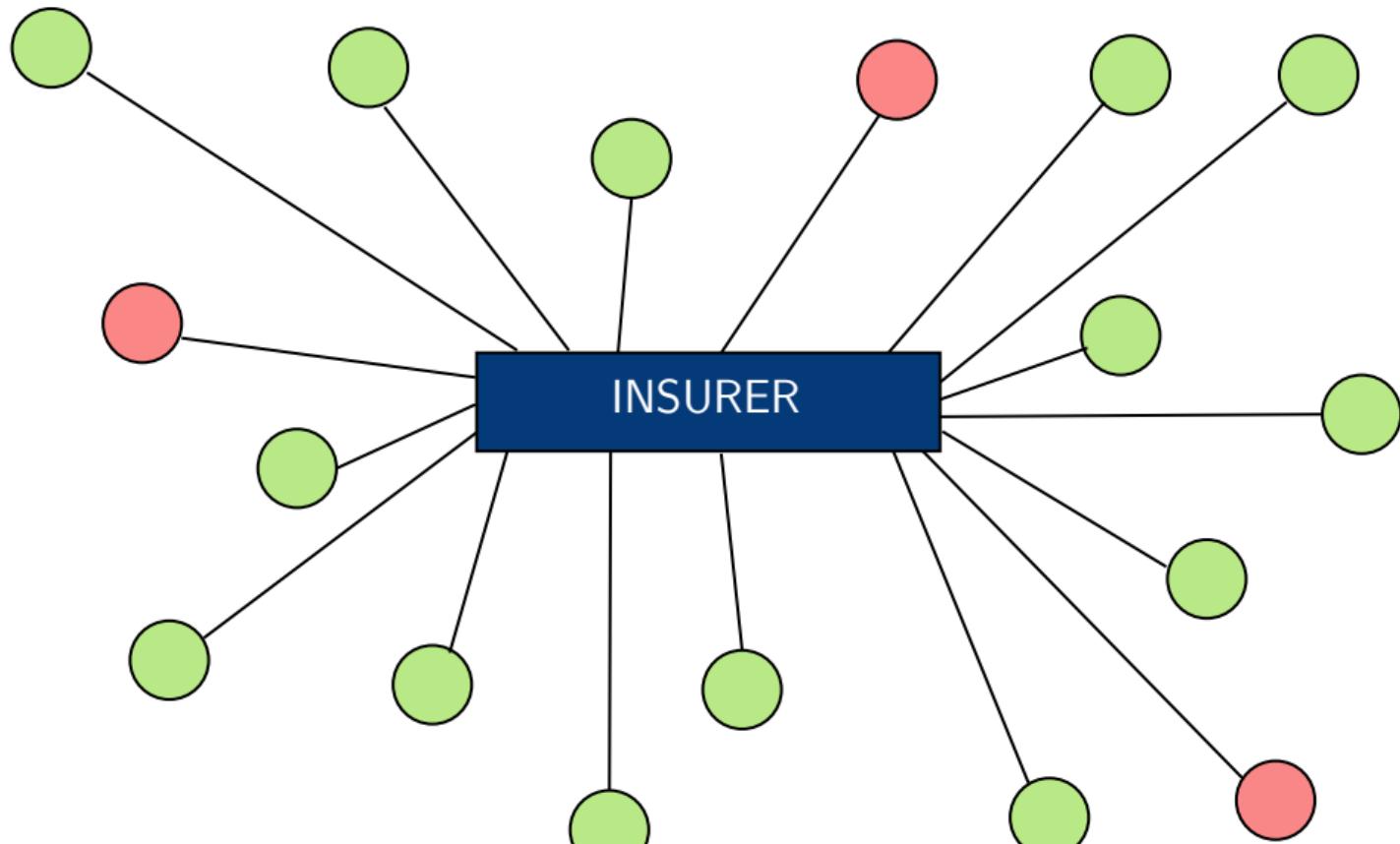


“Insurance is the contribution of the many to the misfortune of the few”
Insurance works only with a large number of policyholders

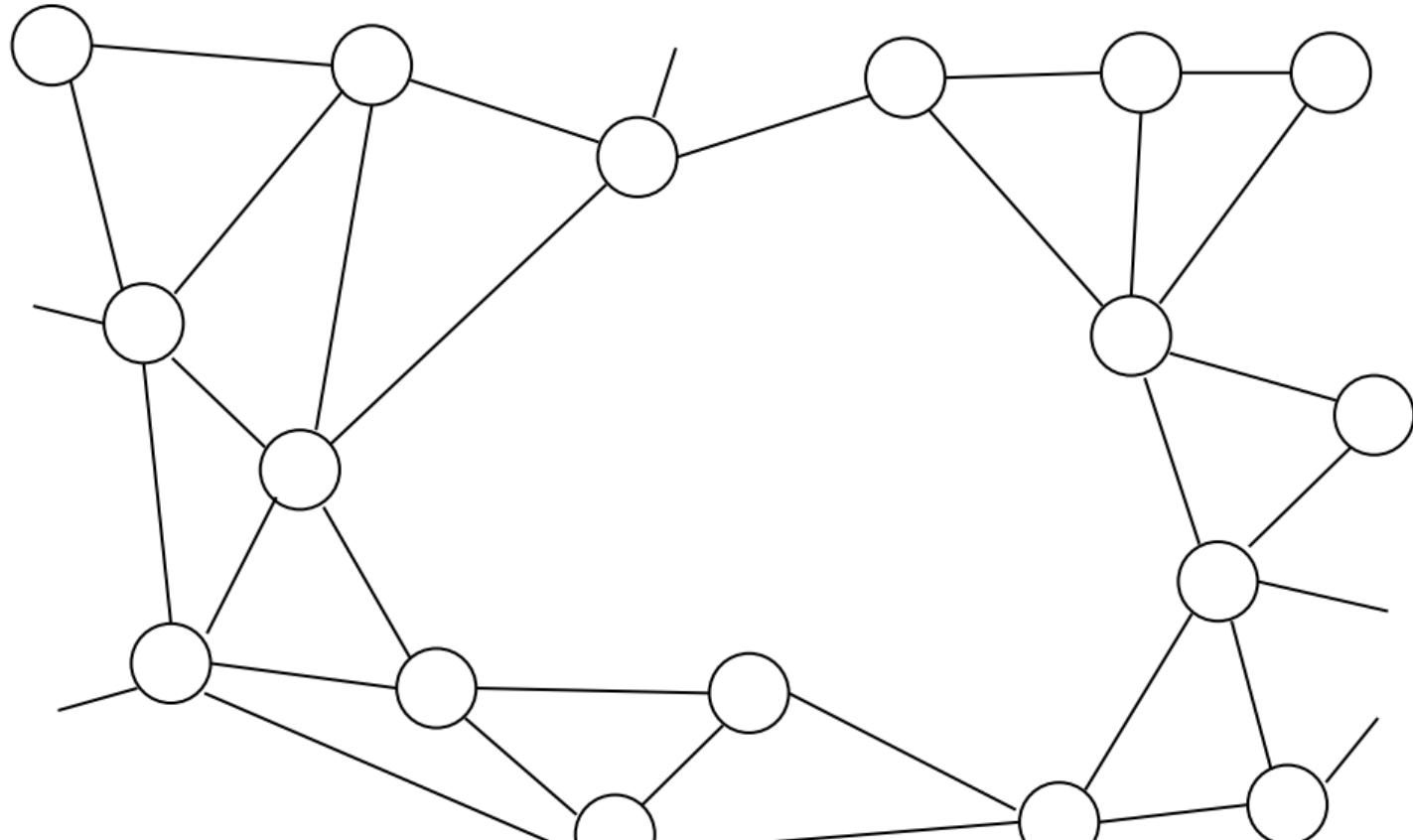
Insurance



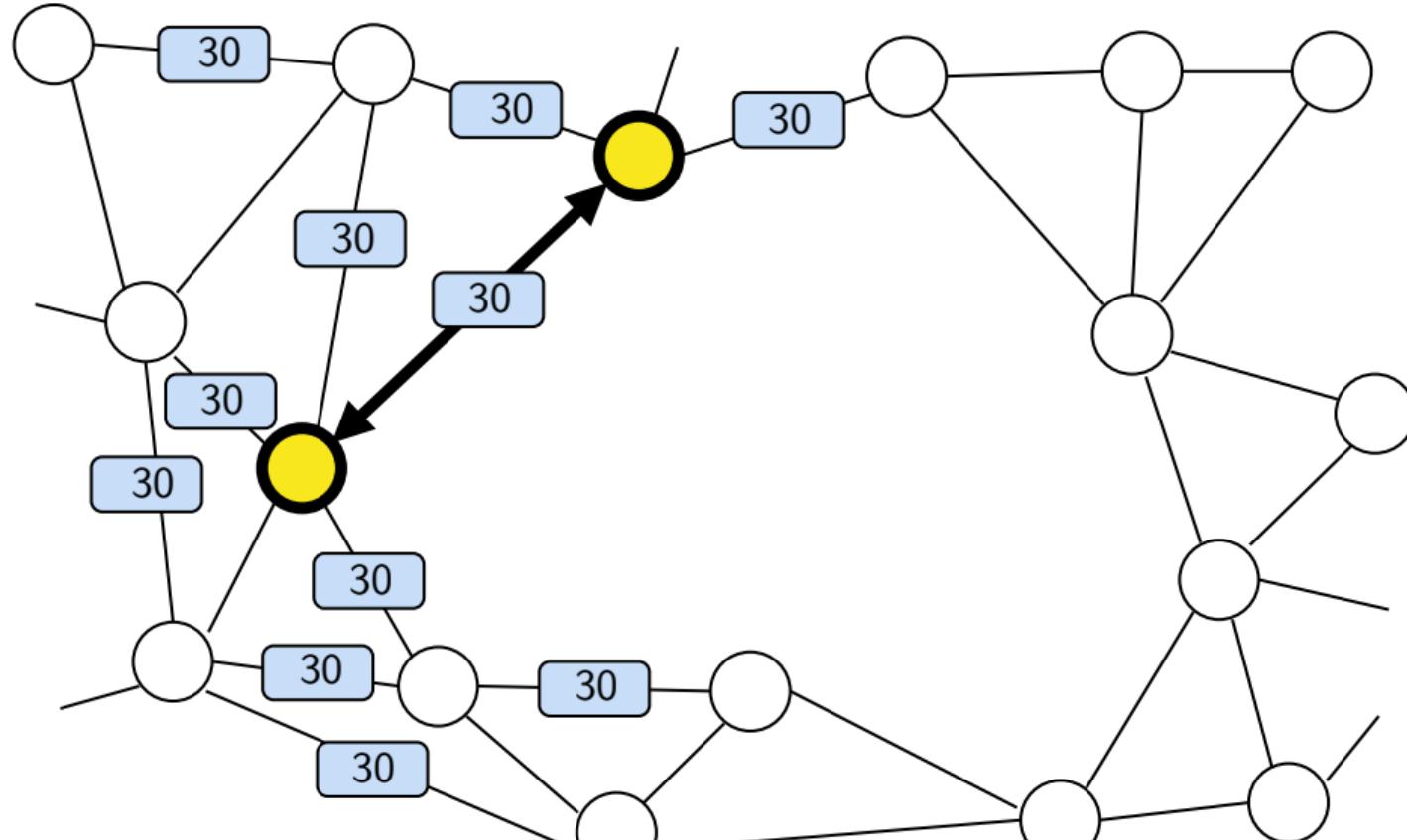
Insurance



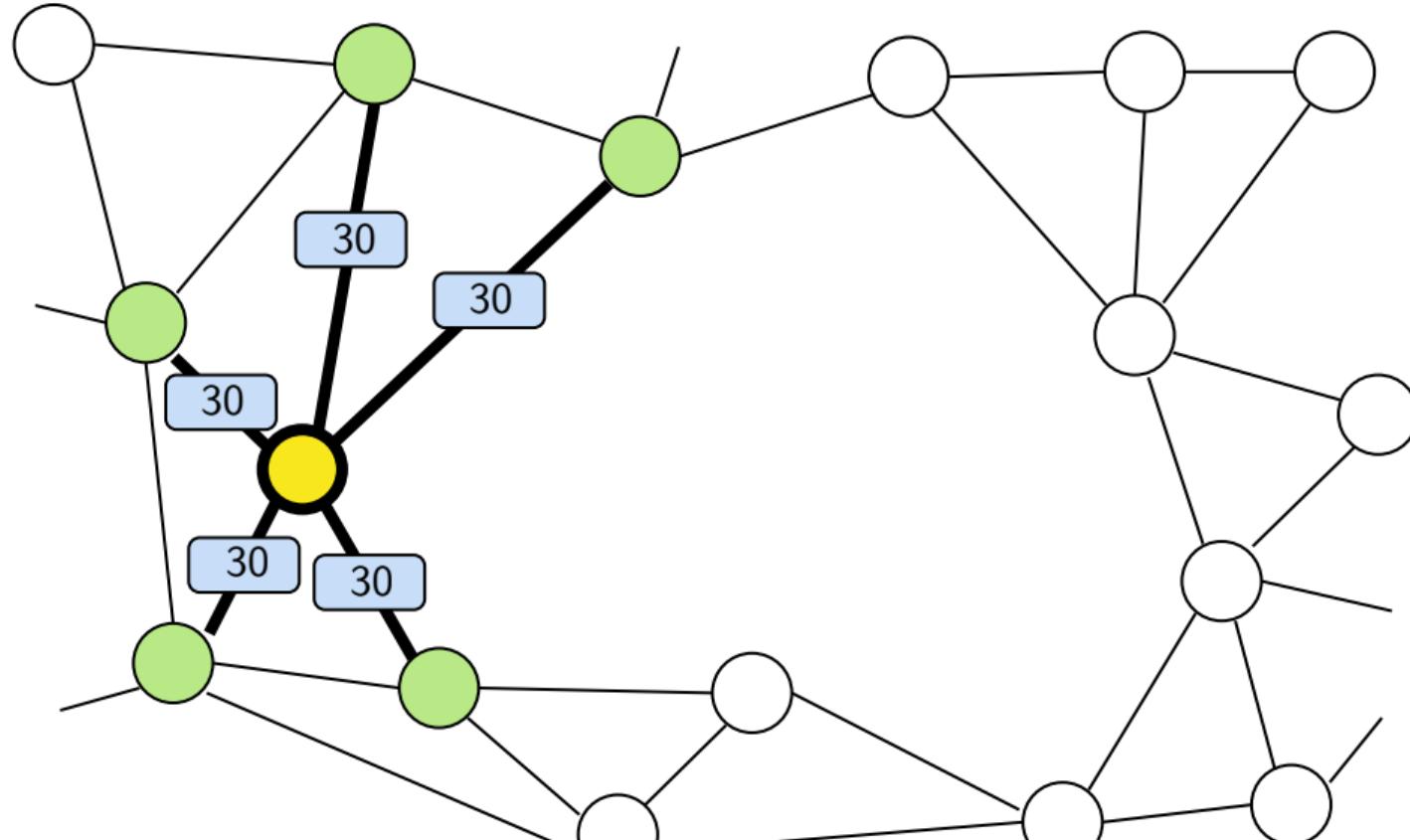
Insurance on a Network



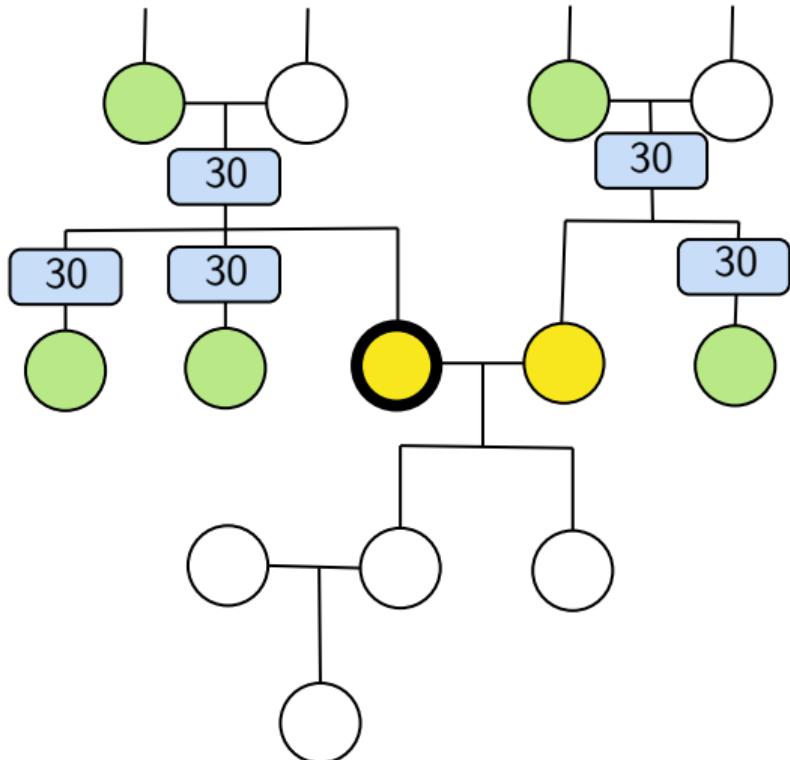
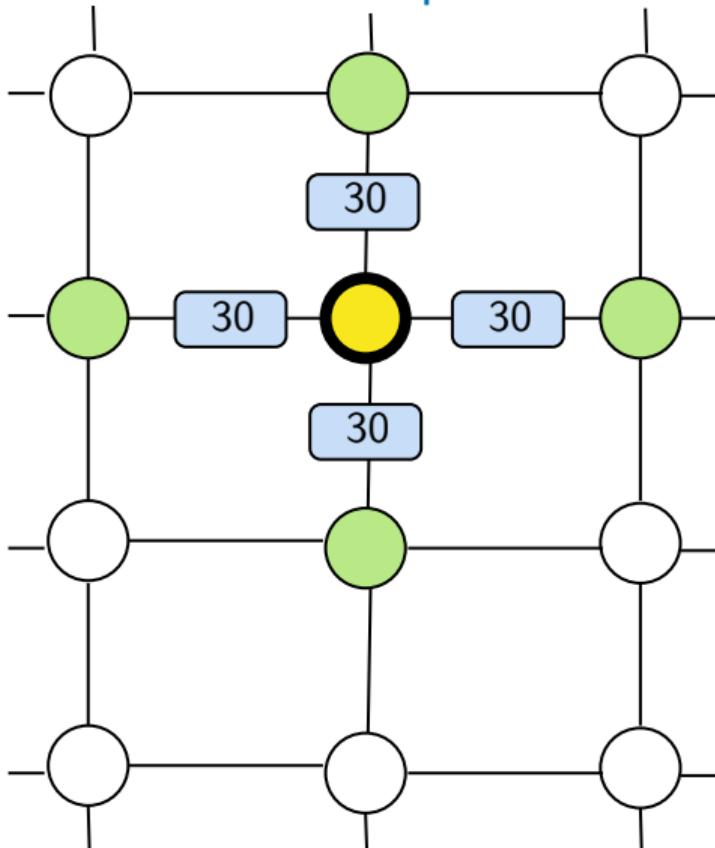
Collaborate Insurance on a Network



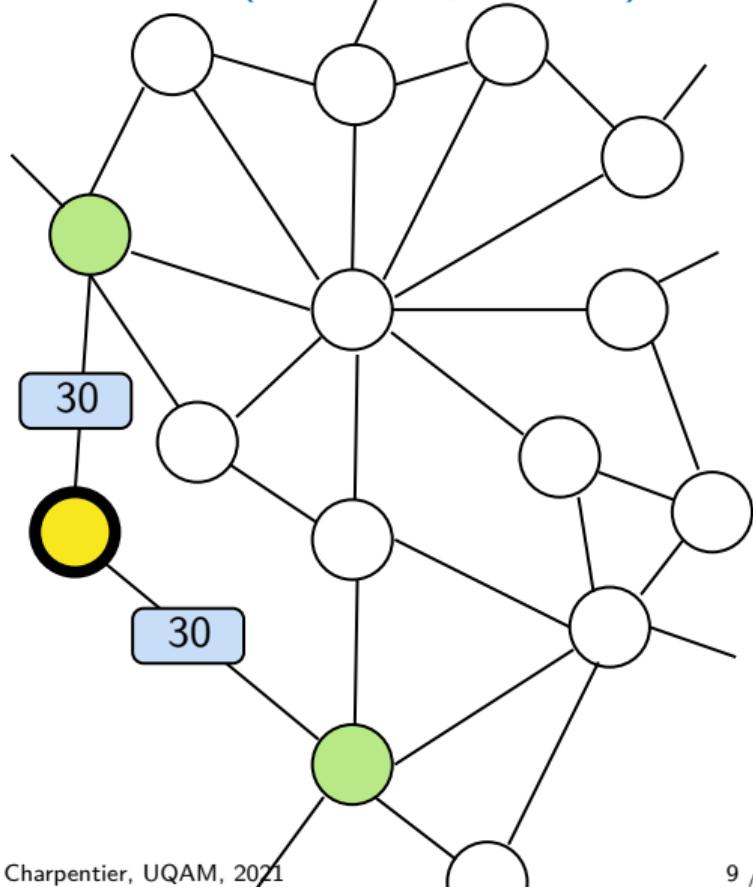
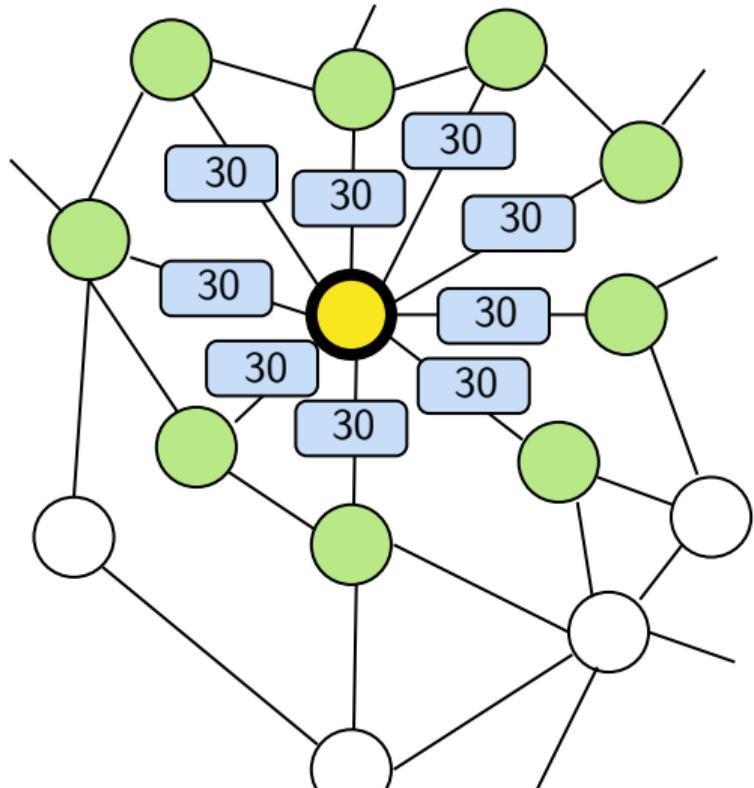
Collaborate Insurance on a Network



Shapes of Networks (regular mesh & family tree)

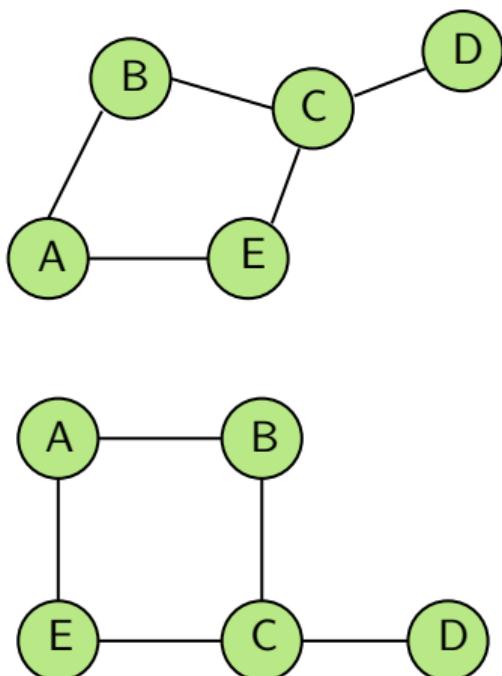


Insurance on a Network (centrality issues)



Network, some mathematical formalism

Vertice $i \in \mathcal{V} = \{A, B, C, D, E\}$ and edges $(i, j) \in \mathcal{E}$
Adjacency matrix \mathbf{A} , $A_{i,j} = 1$ indicates a link (i, j)

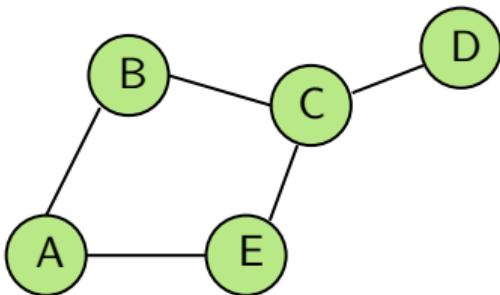


$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 0 & 0 \\ C & 0 & 1 & 0 & 1 & 1 \\ D & 0 & 0 & 1 & 0 & 0 \\ E & 1 & 0 & 1 & 0 & 0 \end{array}$$

$\mathbf{d} = (\mathbf{d}_i) = \mathbf{A}\mathbf{1}$ is called degree

“There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement”,
Freeman (1979))

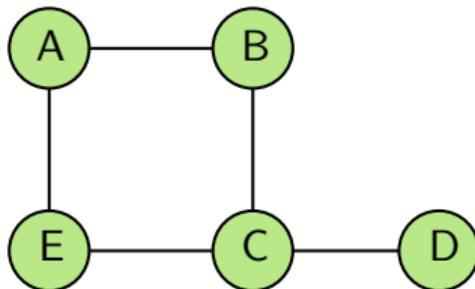
Network, Friendship Paradox



The average degree of a friend is strictly greater than the average degree of a random node, see [Feld \(1991\)](#) and [Zuckerman & Jost \(2001\)](#).

$(A, 2), (B, 2), (C, 3), (D, 1), (E, 2)$

$$\frac{2 + 2 + 3 + 1 + 2}{5} = 2$$



$A : (B, 2), (E, 2), B : (A, 2), (C, 3),$
 $C : (B, 2), (D, 1), (E, 2), D : (C, 3), E : (A, 2), (C, 3)$

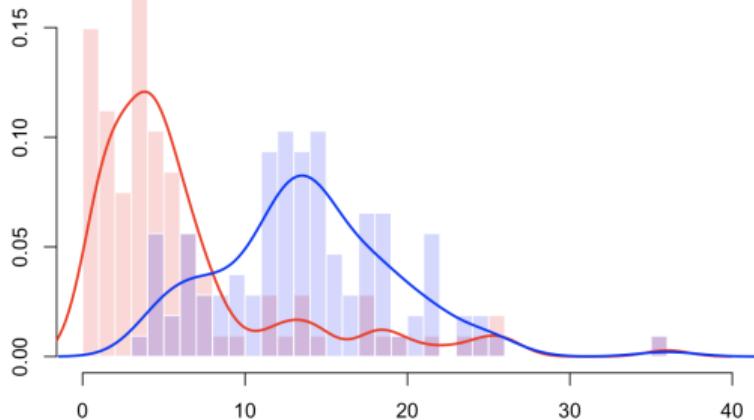
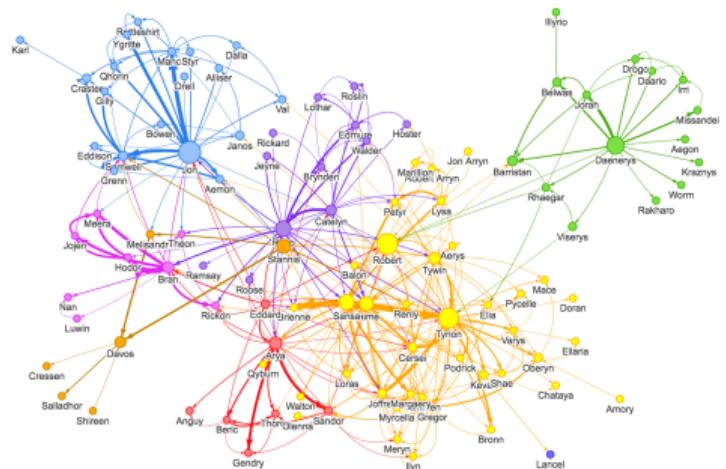
$$\frac{1}{5} \left(\frac{2+2}{2} + \frac{2+3}{2} + \frac{2+2+1}{3} + \frac{3}{1} + \frac{2+3}{2} \right) = \frac{7}{3} > 2$$

Network, Friendship Paradox

The average number of friends of a random person in the graph is

$$\mu = \frac{1}{n_{\mathcal{V}}} \sum_{v \in \mathcal{V}} d(v) = \frac{2n_{\mathcal{E}}}{n_{\mathcal{V}}} = \frac{1}{n_{\mathcal{V}}} \|\mathbf{d}\|_1 \quad (= \mathbb{E}(D))$$

where \mathbf{d} is the vector of $d(v)$'s (or $\mathbf{d} = \mathbf{A}\mathbf{1}$).



Friendship Paradox

The average number of friends that a typical friend has is

$$\frac{1}{n_{\mathcal{V}}} \sum_{v \in \mathcal{V}} \left(\frac{1}{d(v)} \sum_{v' : (v, v') \in \mathcal{E}} d(v') \right) = \frac{1}{\|\mathbf{d}\|_1} \mathbf{d}^\top \mathbf{d} \quad \left(= \frac{\mathbb{E}(D^2)}{\mathbb{E}(D)} \right)$$

We can prove that

$$\frac{1}{\|\mathbf{d}\|_1} \mathbf{d}^\top \mathbf{d} \geq \frac{1}{n_{\mathcal{V}}} \|\mathbf{d}\|_1$$

See also

$$\frac{\mathbb{E}[D^2]}{\mathbb{E}[D]} = \mathbb{E}[D] + \frac{\text{Var}[D]}{\mathbb{E}[D]} \geq \mathbb{E}[D]$$

because $\text{Var}[D] \geq 0$.

CNN BUSINESS Markets Tech Media Success Video

Facebook patent: Your friends could help you get a loan - or not

by Ananya Bhattacharya @CNNTech

🕒 August 4, 2015: 6:58 PM ET



“You apply for a loan and your would-be lender somehow examines the credit ratings of your Facebook friends. If the average credit rating of these members is at least a minimum credit score, the lender continues to process the loan application. Otherwise, the loan application is rejected,” the patent states.”

See [Facebook patent: Your friends could help you get a loan - or not](#)

Consider some positive variable y observed at each node v . It can be the credit score of friends, or activity of people on twitter, as in [Hodas et al. \(2013\)](#)

Friendship Paradox

Using the extension of König-Huyghens, $\text{Cov}[D, Y] = \mathbb{E}[DY] - \mathbb{E}[D]\mathbb{E}[Y]$, write $\mathbf{d}^\top \mathbf{y}$ as

$$\mathbf{d}^\top \mathbf{y} = \frac{\|\mathbf{d}\|_1 \|\mathbf{y}\|_1}{n_V} + n_V \text{Cov}[D, Y]$$

or

$$\frac{1}{\|\mathbf{d}\|_1} \mathbf{d}^\top \mathbf{y} = \frac{1}{n_V} \|\mathbf{y}\|_1 + \frac{n_V}{\|\mathbf{d}\|_1} \text{Cov}[D, Y]$$

This can be written also

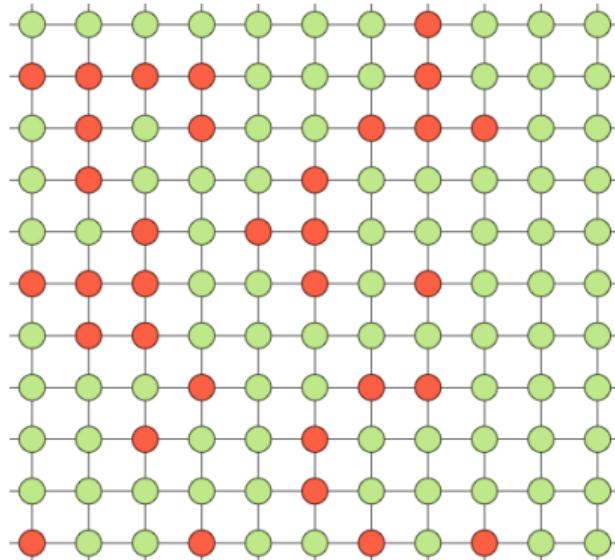
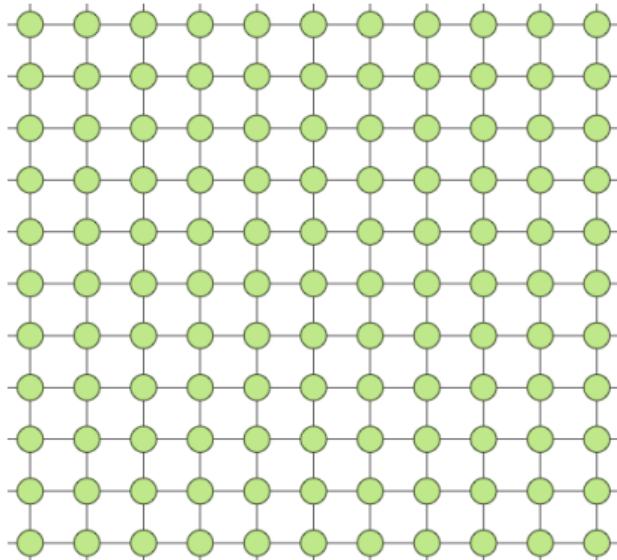
$$\frac{\mathbb{E}[DY]}{\mathbb{E}[D]} = \mathbb{E}[Y] + \frac{\text{Cov}[D, Y]}{\mathbb{E}[D]}$$

Hence,

$$\frac{\mathbb{E}[DY]}{\mathbb{E}[D]} = \mathbb{E}[Y] \text{ if } Y \perp D \text{ and } \frac{\mathbb{E}[DY]}{\mathbb{E}[D]} > \mathbb{E}[Y] \text{ if } \text{Cov}[D, Y] > 0.$$

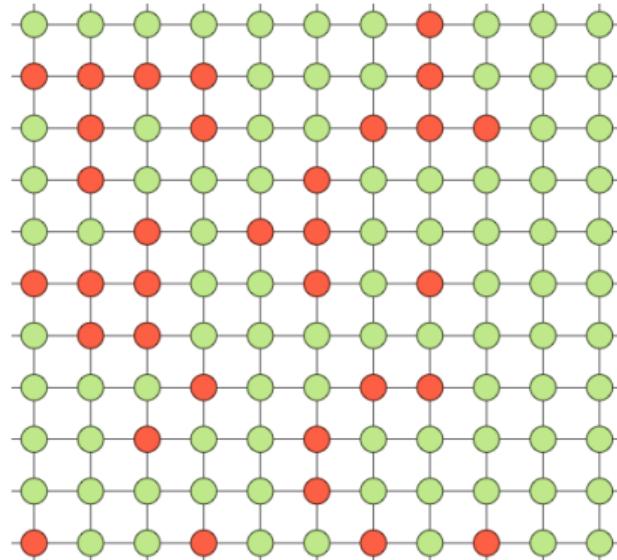
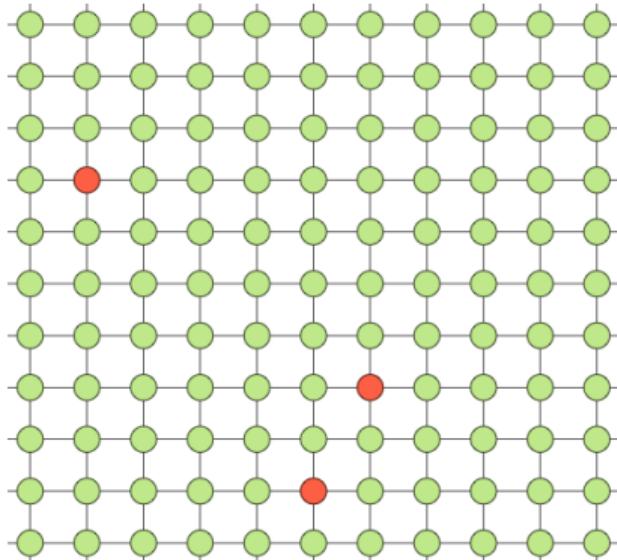
Networks, from Probability to Statistics

Sampling issues...



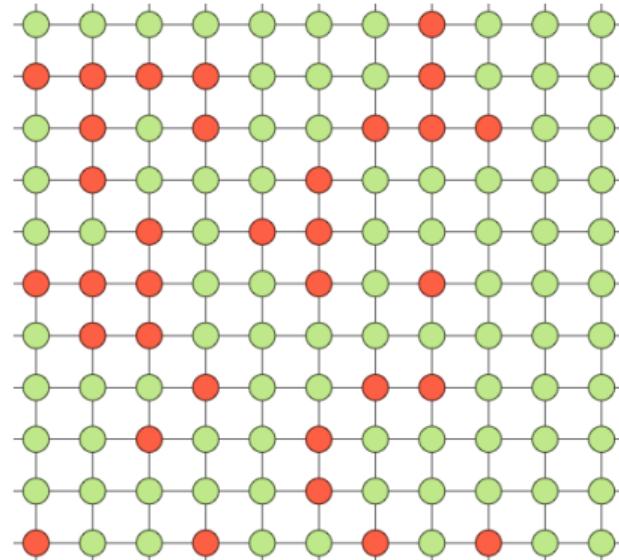
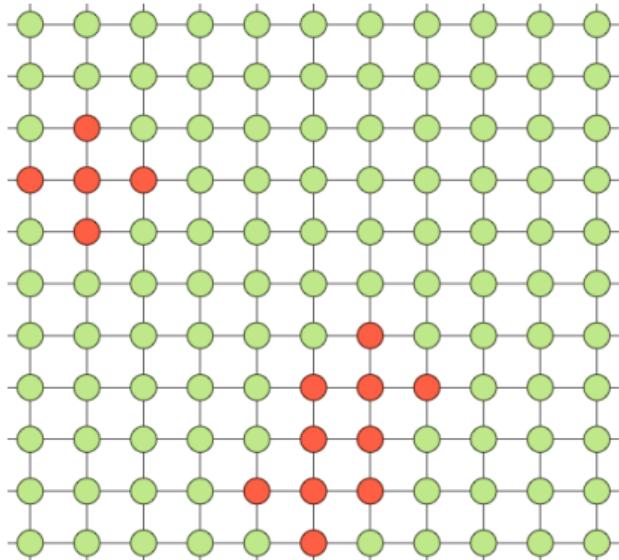
Networks, from Probability to Statistics

Snowball: draw randomly $k \ll s$ nodes



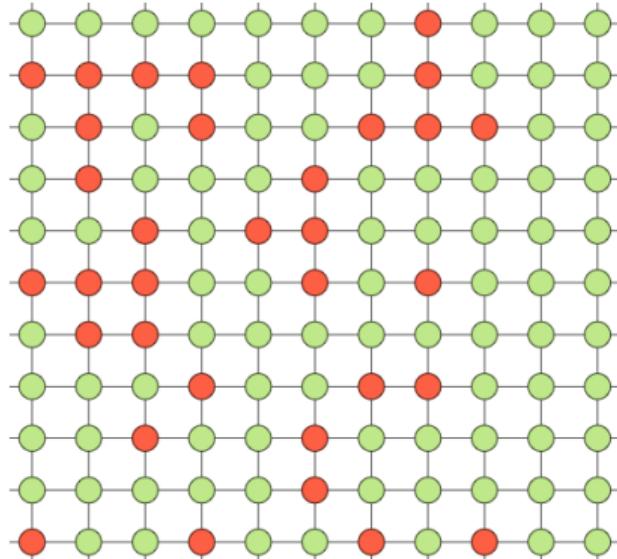
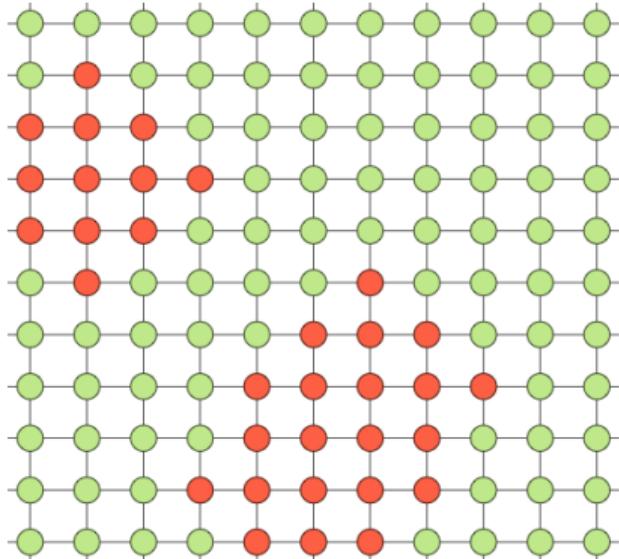
Networks, from Probability to Statistics

keep neighbors of previously selected nodes



Networks, from Probability to Statistics

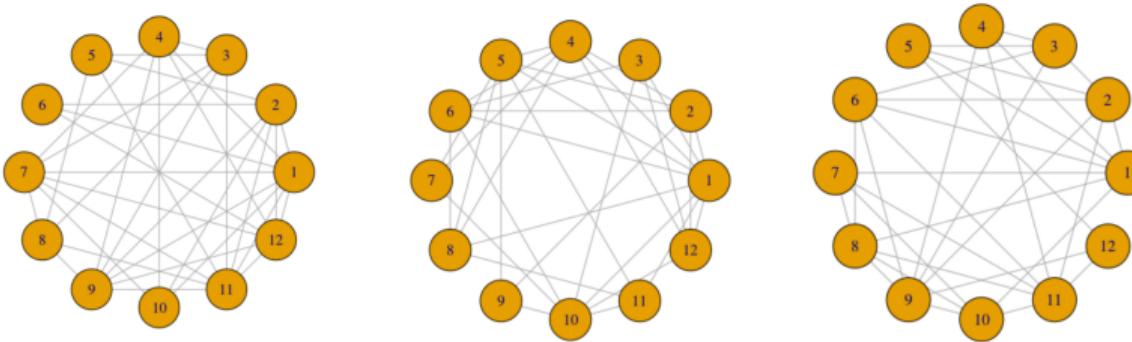
keep neighbors of previously selected nodes



see **Respondent-driven sampling (RDS)**

Classical Random Graphs

The Erdős-Renyi random graph model $G_{n,p}$ is an undirected graph with n vertices, such that edge (v, v') is present with probability p , independent of other edges.



$$f_\delta = \mathbb{P}[d(v) = \delta] = \binom{n-1}{\delta} p^\delta (1-p)^{nV-1-\delta}$$

For a large network ($n_V \rightarrow \infty$), $d(v) \sim \mathcal{N}(n_V p, n_V p(1 - p))$ from the law of large numbers.

For a large network with $p \sim \lambda/n_V$, $d(v) \sim \mathcal{P}(\lambda)$ from the law of small numbers.

Classical Random Graphs

Scale-free network: degree distribution with power-law tail,
see also Barabasi-Albert graphs

The normalized power-law degree distribution is

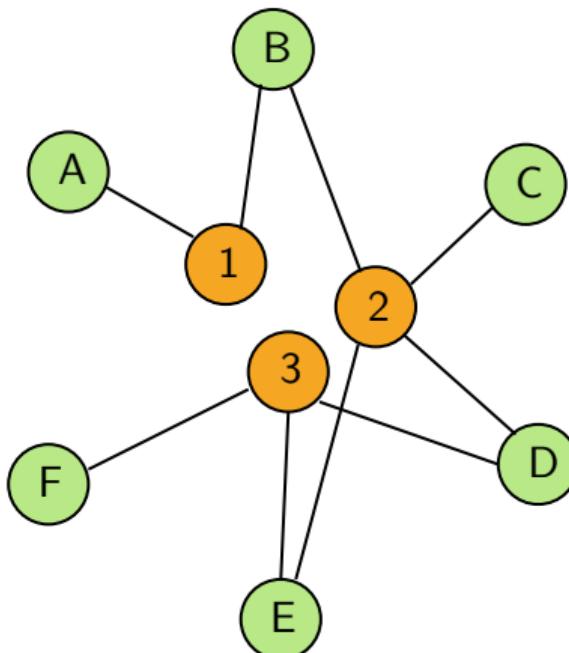
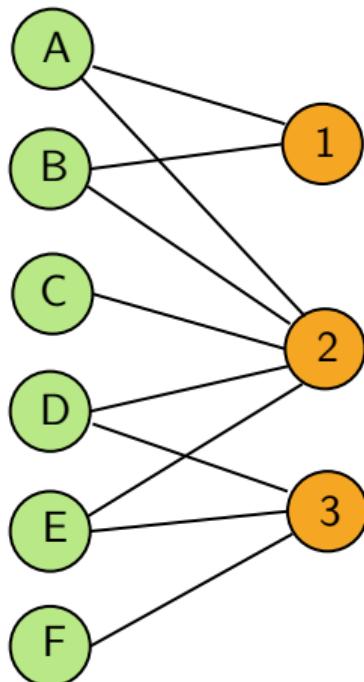
$$f_\delta = \mathbb{P}[d(v) = \delta] = \frac{\alpha - 1}{\delta_0} \left(\frac{\delta}{\delta_0} \right)^{-\alpha}, \text{ for } \delta \geq \delta_0.$$

See Pareto distribution, $\bar{F}(x) = \mathbb{P}[X > x] = x^{-\alpha+1}$

“Scale-free networks are rare“, Broido & Clauset (2019).

Sampling issue, and second order property, see Stumpf, Wiuf & May (2005) or Charpentier & Flachaire (2019).

Networks & Fraud

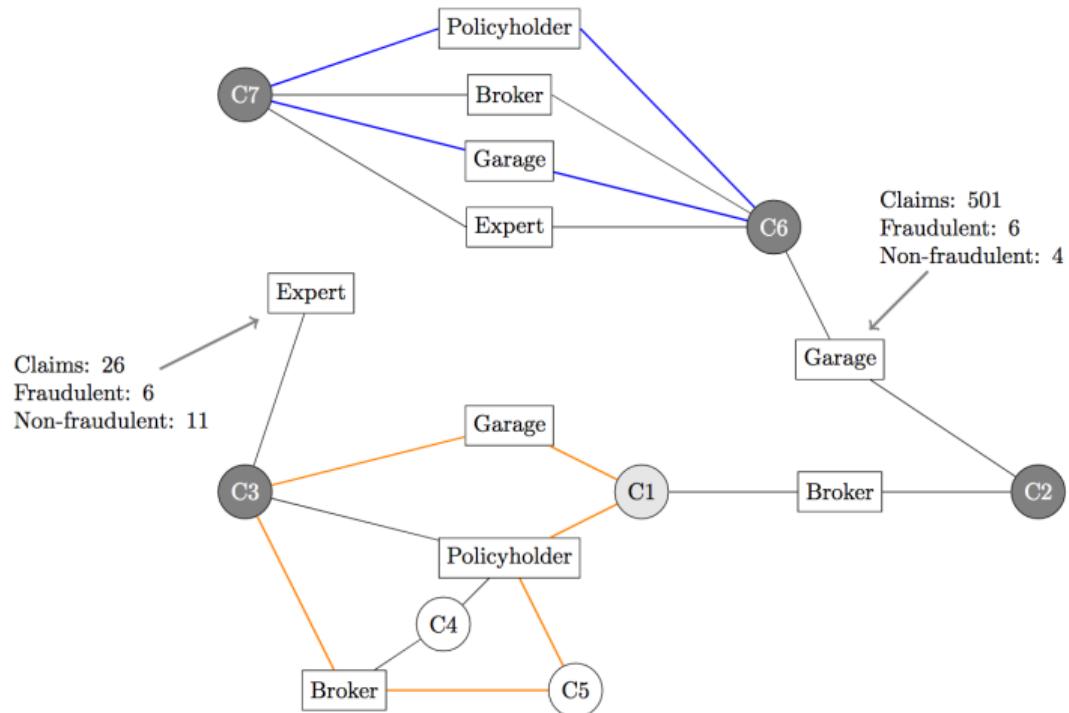


Bipartite graphs

vertices can be divided into two disjoint and independent sets
workers and employers
matching problem

Networks & Fraud

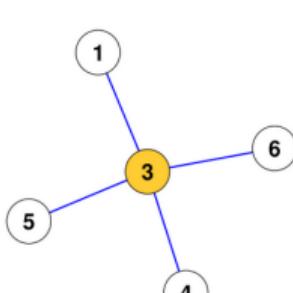
See Óskarsóttir *et al.* (2020) on fraud detection in insurance



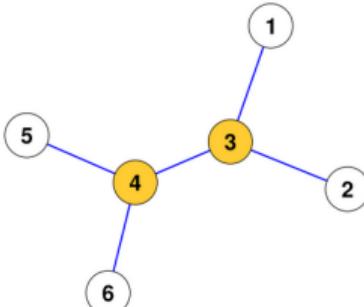
Networks & Security

“There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement”, Freeman (1979)

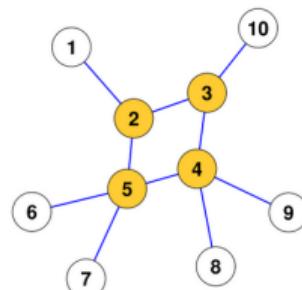
See Wang et al. (2017) on robustness of metro networks



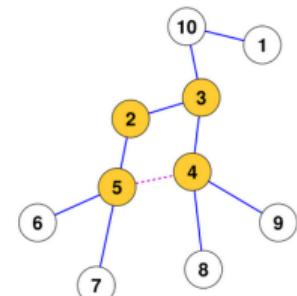
(a) Rome.



(b) Cairo and Marseille.



(c) Montreal.



(d) Toronto.

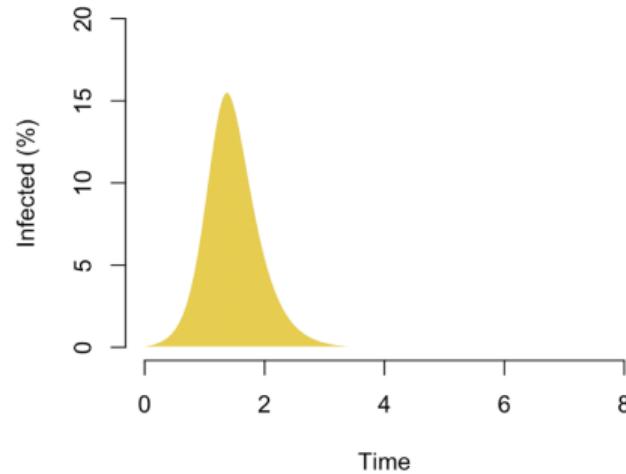
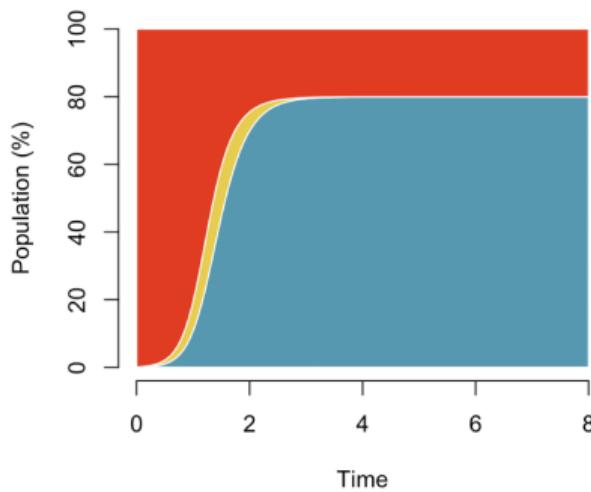
or on edges, as in cyber-risk, see Cohignac & Kazi-Tani (2020)

Networks & Pandemic

Kermack & McKendrick (1927), three groups, S_t (susceptible) I_t (infected) R_t (recovered)

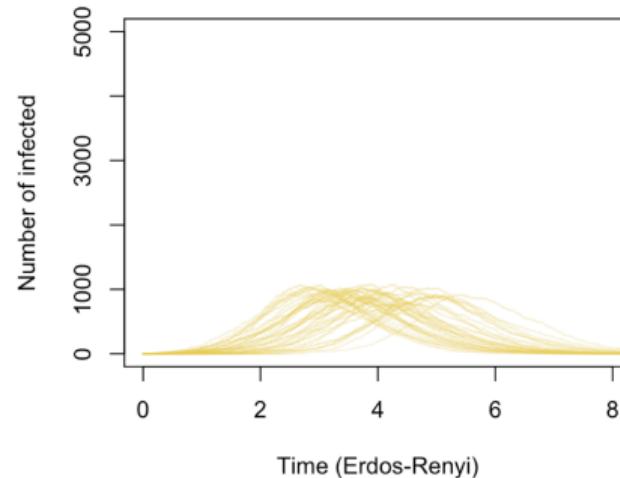
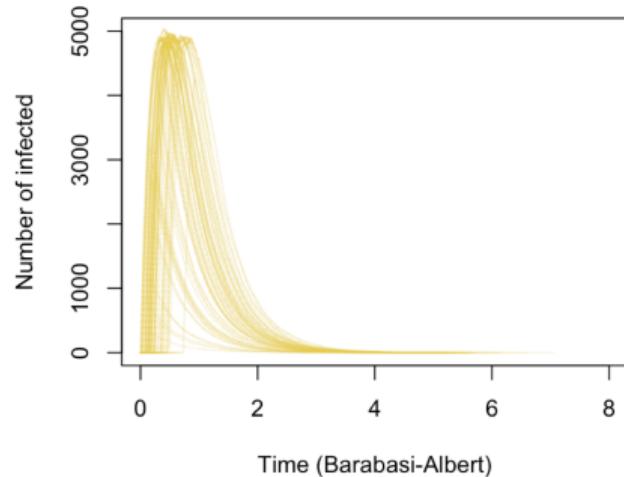
$$\frac{dS_t}{dt} = -\beta I_t S_t, \frac{dI_t}{dt} = \beta I_t S_t - \gamma I_t, \frac{dR_t}{dt} = \gamma I_t$$

■ S (susceptible) ■ I (infected) ■ R (recovered)

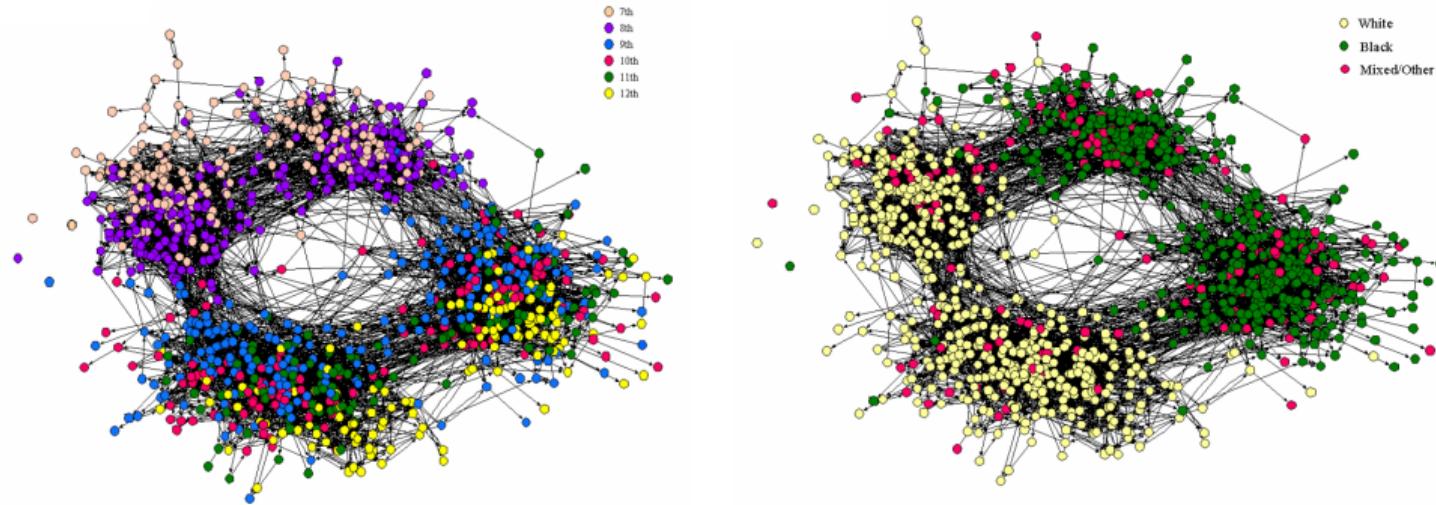


Networks & Pandemic

With Erdős-Renyi random graphs, or Barabasi-Albert graphs (scale free / power law)



Networks, Race & Friendship in Schools



Homophily: tendency of individuals to associate and bond with similar others, “*birds of a feather flock together*”, from **Moody (2001)**

Networks & Collaborative Insurance

FEATURES

PEER-TO-PEER INSURANCE

GOING BACK TO BASICS

Innovators have set their sights on simplifying insurance by adopting new peer-to-peer business models. Who are the key players and does the Australian industry need to pay attention?

DISCUSSION OF the sharing economy has taken up considerable column space in recent times.

PriovateinsuranceCoopers estimates the five main sharing sectors (peer-to-peer finance, online staffing, peer-to-peer accommodation, car sharing and music video streaming) will potentially generate global revenues of US\$833bns (A\$849bns) by 2025. Today, PwC says revenue generated is around US\$1bn (A\$1.197bns).

One insurance sector still very much in its infancy is peer-to-peer insurance. But Amy Gibbs, digital communications and client strategy manager at ANZTIF, says popularity of the concept is increasing.

"When it takes off, it will likely happen much quicker than we expect," she says.

"P2P insurance is not about a new technology threatening an industry but about people demanding an industry that gives them what they believe they deserve. Consumers expect different things in 2016 and technology now allows them the power to get what they want or go elsewhere."

The current crop in P2P

On 23 March, Germany's high-profile P2P player, Friendsurance, announced it had collected US\$15.3m (A\$20.26m) from investors in its latest round of financing.

Tim Kunkel, Friendsurance co-founder and managing director, says the organisation intends to use that fresh capital to grow further in

Germany and expand internationally. He says the first expansion target for 2016 is Australia, and expansion opportunities for other markets are currently being considered.

Friendsurance is one of the players Gibbs has been keeping a close eye on.

"Friendsurance and Guevara are ones that I watch closely," she says.

"They have interesting models and appear to have put a lot of thought into them, plus they get the marketing/growing angle, which is crucial."

Friendsurance, based in Germany in 2009, operates as an independent insurance broker. It operates in mission to "make insurance easier and more affordable for customers, and to reduce the number of fraudulent claims".

"Our idea is inspired by insurance in its original form, when people got together in small groups... and supported each other in case of damage," Tim Kunkel, Friendsurance co-founder and managing director, tells Insurance Business.

"This was easy and efficient but also limited in the extent of coverage. Today, big insurance



John Major-Perth, Tim Kunkel and Tim Gibbs of Friendsurance

"P2P insurance is not about a new technology threatening an industry but about people demanding an industry that gives them what they believe they deserve"

Amy Gibbs, ANZTIF

companies can carry claims of any amount, but marketing, administration and fraud cause retributive costs.

"Against this background, we developed an insurance concept that again [creates] smaller groups within bigger insurance societies and rewards remaining claimless within [these groups] with annual cashbacks."

Friendsurance customers with the same insurance type form small groups online. Part of

their insurance premium is paid into a cashback pool, and part of it is provided to the group (trustee or retributor).

When no claims are made, customers are reimbursed from the general fund. Larger claims still go through the insurer. Groups that have no claims during a year receive a cashback bonus the following January. Friendsurance says its claims-free bonus is available for private liability, home contents and legal expenses insurance.

"So far, more than 80% of users received some of their insurance 'for back,'" says Kunkel.

"In the property insurance line, the average cashback is 33% of the insurance fees."

Kunkel reports that, in 2015, Friendsurance engaged 75,000 new customers.

"Today, we have a six-digit number of customers, 70 insurance partners, 15 corporates and 80 employees."

Over in the UK, start-up Guevara has received

via Insurance Business (2016), see Charpentier (2019)

Networks & Collaborative Insurance



insPeer

insPeer: car insurance in France, 2014

Lemonade

Lemonade: household insurance, in the U.S., 2015

G groupago
by Solidas

groupago: car & household insurance, in Belgium, 2015

besure

besure: general insurance, in Canada, 2017

Networks & Collaborative Insurance

Lariosse (2020) considered risk sharing (transfer) of the **deductible**, under strong assumptions (identical risks, identical deductible)

Question : How to set the amount of reciprocal commitments in order to optimize the collaborative franchise product?

Question : What social graph model(s) can be used to optimize the collaborative franchise product?

very sensitive to the structure of the network...

Networks & Collaborative Insurance

Frequency $\mathcal{B}(1/10)$ (i.i.d.)

Loss $\mathcal{G}(100, 250)$

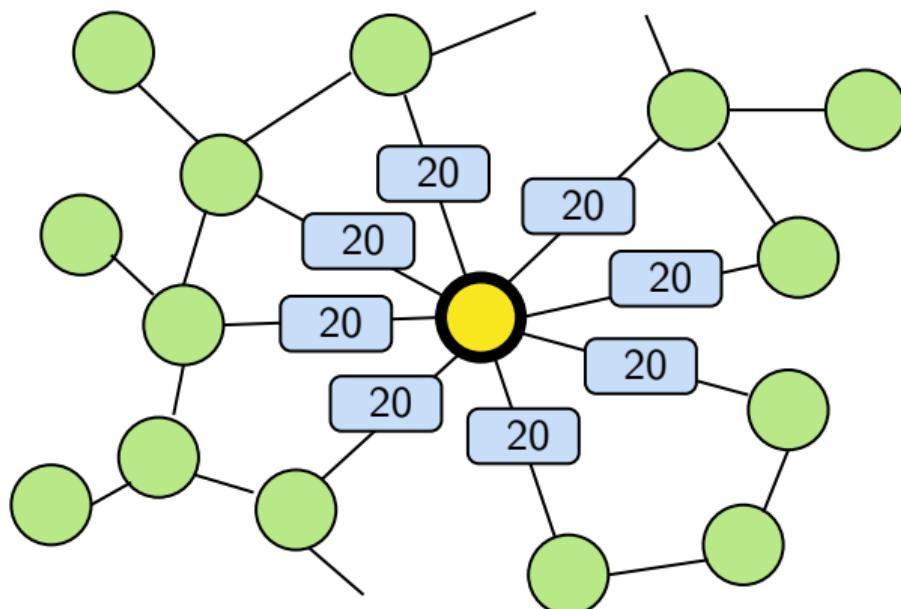
Deductible 100

Network: $\mathbb{E}[D] = 5$

$$\text{Contribution } \frac{100}{5} = 20$$

(uniform, constant and reciprocal)

We generate some random networks
given a distribution of degrees
average $\mathbb{E}[D]$
and standard deviation σ



Networks & Collaborative Insurance

Frequency $\mathcal{B}(1/10)$ (i.i.d.)

Loss $\mathcal{G}(100, 250)$

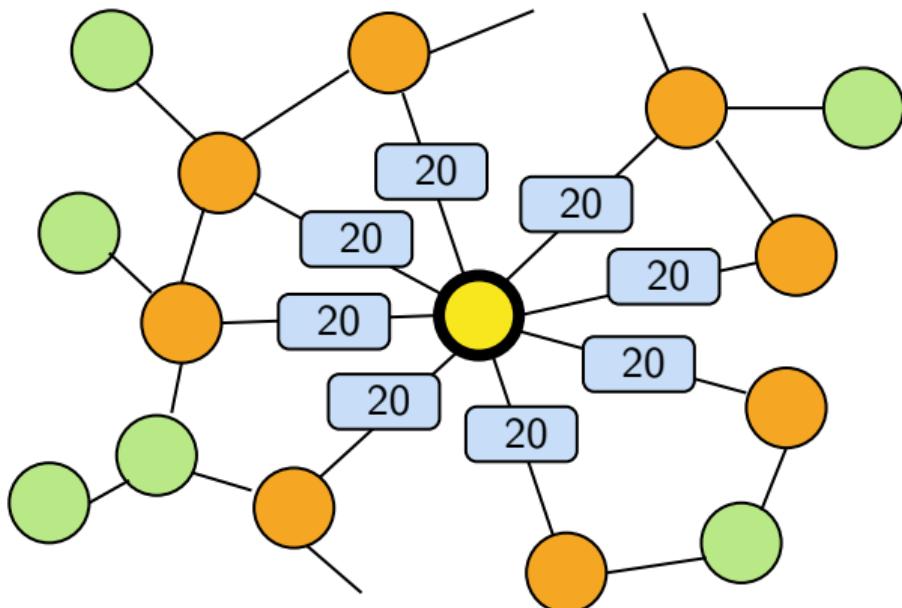
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Networks & Collaborative Insurance

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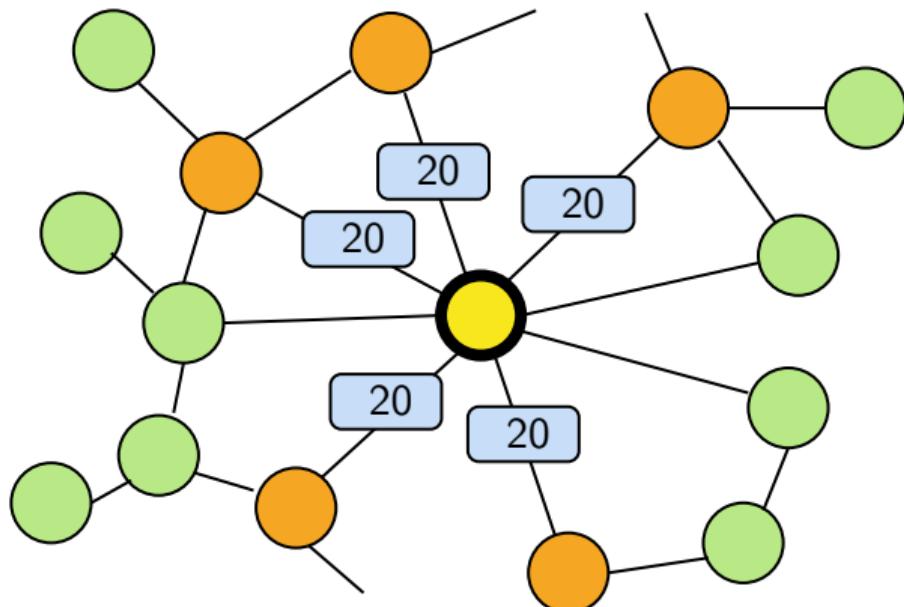
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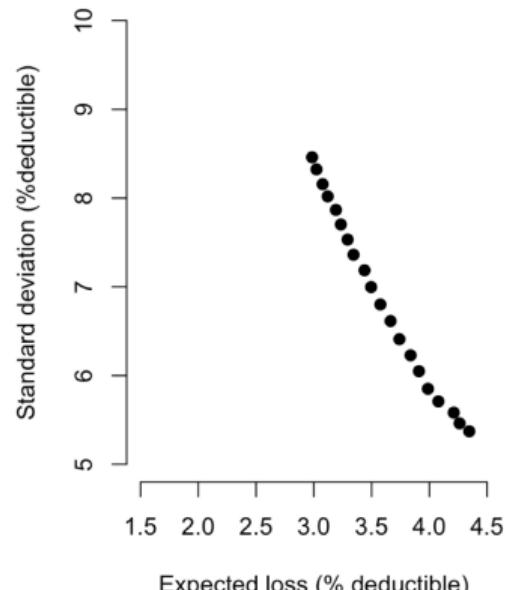
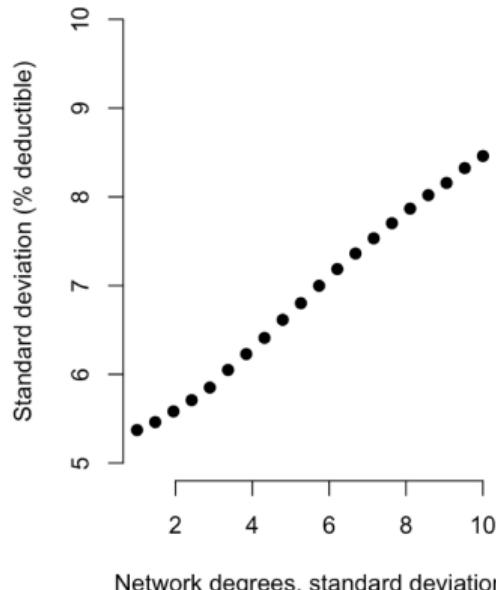
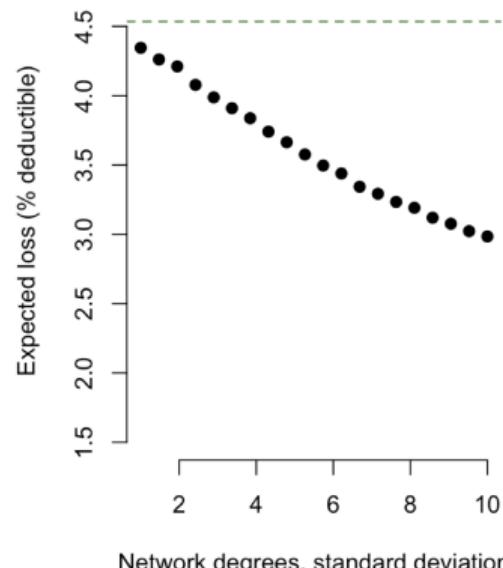
We generate some random networks
given a distribution of degrees
average $\mathbb{E}[D]$
and standard deviation σ



Networks & Collaborative Insurance

Contribution of all connections.

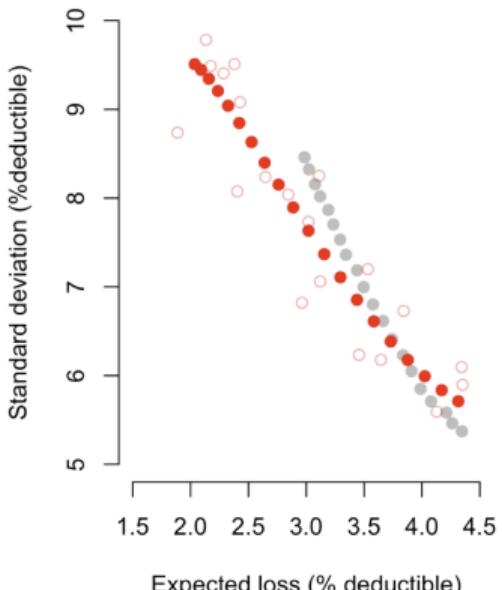
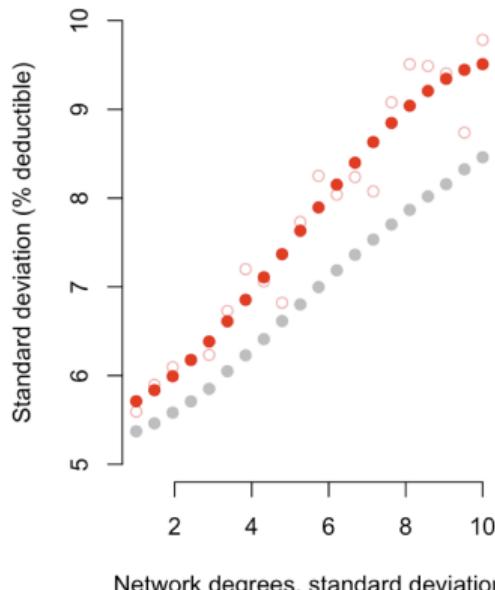
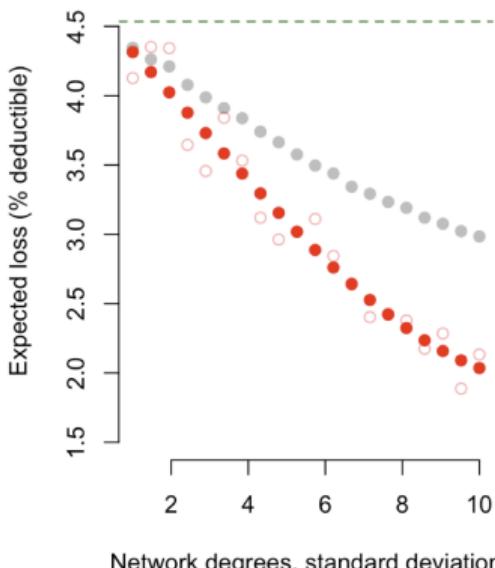
Here $\mathbb{E}[X] = 4.5\%$ and $\text{sd}[X] = 17\%$



Networks & Collaborative Insurance

Contribution of **some** connections, $d = \max\{d, \bar{d}\}$.

Here $\mathbb{E}[X] = 4.5\%$ and $\text{sd}[X] = 17\%$



Networks & Collaborative Insurance

Contribution of some (optimized) connections, $d = \max\{d, \bar{d}\}$.
Here $\mathbb{E}[X] = 4.5\%$ and $\text{sd}[X] = 17\%$

