

How to Estimate the Occurrence Probability of Natural Catastrophes

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ETH Zürich, January 2019



How Likely Catastrophe Is To Occur?

“to combat global warming, we must first assess just how likely it is to occur”

Schneider (2001) - see also Dessai & Hulme (2004) against it

Box1: Glossary of terminology

Working groups. Writing team of 100–200 lead authors, dozens of review editors and hundreds of peer reviewers from around the world who, as part of the Intergovernmental Panel on Climate Change (IPCC), produce assessments of the science for about 100 governments. Working group 1 deals primarily with the science of climate change, while working groups 2 and 3 deal with adaptation and mitigation, respectively, and the future climate changes and events. Working group 2 assesses the potential impacts of such projections, prospects for adaptability and policy vulnerability, and the costs of adaptation and mitigation and other policy options for dealing with climate change. There have been three cycles of assessment reports since 1990.

Special reports. In-depth assessments of topics that determine the need for additional treatment. The special report on emission scenarios (SRES) is discussed in this article, but others have been prepared, for example on ecosystem impacts and carbon sinks.

Storylines. A framework from the SRES explicitly to recognize different demographic, social, economic and environmental developments. Each scenario represents a specific quantitative interpretation of four storyline elements based on their respective constitutive family.

Scenarios. According to the SRES, “Scenarios are alternative images of what the future might unfold and are an appropriate tool with which to analyse how driving forces may influence future emission outcomes and to assess the associated uncertainties.”

General circulation models (GCMs). The most comprehensive climate models used for projecting climate change available. They often include submodels of the atmosphere to account for ecological systems and ice systems. Criterion of explicit treatment of processes on a smaller scale than the grain size of the model creates uncertainty in their outputs, especially for rapidly changing processes.

Probabilistic approach to climate-change projections. An approach to climate-change projections that attempts to estimate the probability of various outcomes in the Earth's surface and space as the atmosphere can be modified. The scenarios are fed into biogeochemical cycle models to translate the emissions into projected concentrations of greenhouse gases, which in turn, “force” the climate system by perturbing the heat balance of the Earth's climate system. Working group 1 has translated the scenario information into probabilistic projections used by working group 2 to assess potential impacts.

The likelihood controversy

In the third assessment report of the SRES, led by Nobuyuki Nakicenovic at the International Institute for Applied Systems Analysis near Vienna, I was impressed by the broad representation of the group: academic scientists, environmental organizations, industry and government, and even systems analysts. Their task was to imagine plausible alternative future societies and technologies that would translate the scenario projections into climate-change projections by working group 2 (ref. 4). Because there is a lag of a few years between emission predictions, climate models' responses, and analysis of possible impacts, the IPCC decided to

commentary

What is ‘dangerous’ climate change?

To combat global warming, we must first assess just how likely it is to occur.

Stephen H. Schneider

On preparing a special report on emissions scenarios (SRES) to produce a family of updated projections in time for the working group 1 review cycle for the third assessment report (see Box 1, overleaf). For this reason, preliminary scenarios were released, and several ultimate-modelling groups produced general circulation model (GCM) runs of the six “illustrative storylines” for 40 emission projections in the special report. Unfortunately, the time lag persisted, and the impacts-assessment literature still did not have enough time to incorporate the updated climate runs for the latest round of the IPCC.

The likelihood controversy

I am sure that many of the SRES’s supporters, led by Nobuyuki Nakicenovic, at the International Institute for Applied Systems Analysis near Vienna, I was impressed by the broad representation of the group: academic scientists, environmental organizations, industry and government, and even systems analysts. Their task was to imagine plausible alternative future societies and technologies that would translate the scenario projections into climate-change projections by working group 2 (ref. 4). Because there is a lag of a few years between emission predictions, climate models' responses, and analysis of possible impacts, the IPCC decided to

commentary

We are facing the worrying prospect of dozens of users selecting arbitrary scenarios and climate sensitivities.

► inferred (see Box 2 for details). It was an interesting exercise, and gave rise to radically different findings. In one scenario run to 2100 — from below current CO₂ emissions to five times current emissions. Because of the divergent views of participants about the likelihood of each storyline, the final report offered no assessment of the relative likelihoods, in an attempt to avoid disputes.

While acknowledging the logic of applying fruitless efforts to argue about the timing of the policy analysis or projection estimates to assess the seriousness of the implied impacts; otherwise they would be left to work out the implicit probability assessments for themselves (see ref. 6 for further discussion). I urged the working group to provide a possible probability assessment for less expert users, but I was not persuasive enough, and the SRES makes no specific quantitative interpretation for each scenario.

It is

special report (Box 2) is the dramatic revision upward in the IPCC third assessment, which has attracted so much attention, as global warming of more than 3.5 °C would have severe effects.⁷

If all scenarios and climate sensitivities for all 18 GCMs were used, the joint distribution for global surface warming in 2100 would have a peak at the centre — just like a typical bell curve⁸. If all 18 GCM sensitivities are assigned to the six illustrative scenarios (108 possible combinations), then a peaked bell curve results (see Fig. 1 for an example). As a limiting case and for illustration, if only the outlier emission scenario were used with the 18 climate sensitivities (36 possible outcomes), then the probability distribution for 2100 warming would not be as peaked as a bell curve, but would have much flatter distributions.

The illustration shows that I have chosen arbitrarily a temperature increase of 3.5 °C beyond which many believe substantial climate damage would occur. In Fig. 1, the blue bars show that the resultant bell curve for 18 GCMs and six illustrative scenarios implies that 23% of the 108 values

obtained for a possible temperature increase in 2100 are greater than 3.5 °C. For the scenario in which only the highest and lowest emissions scenarios are used (yellow bars), 30% of these 36 possibilities are for a warming of 3.5 °C greater. With the relative lack of middle-value scenarios, the probability distribution is much flatter than before.

Clearly, a policymaker concerned with “avoiding dangerous anthropogenic interference in the climate system” would propose quite different climate policies if there were a 30% chance of exceeding the 3.5 °C warming⁹ than if the figure was 23%. But what do these figures actually represent? If the 18 GCMs had been assigned to individual scenarios and GCM climate sensitivities (for example, as in ref. 10), then their joint distribution (in this example the likelihood of some temperature rise in 2100) would depend on the particular selection of scenarios and models. As Fig. 1 clearly demonstrates. To assess the likelihood of future temperature increase, we can either consider more generally the subjective likelihood of each scenario and climate sensitivity, or estimate the joint distribution (2100 temperatures) explicitly. Early attempts at this estimation have been made (T. Wigley et al., personal communication; M. Webster et al., <http://mit.edu/globchange/www/reports.html>).

Emissions, impacts and the implications of policy responses all depend on how society is structured over time.

Social conditions and emissions

One of the most important points of the SRES storyline approach is that the socio-economic conditions driving emissions would also help to define the adaptive and mitigative capacity of different countries or regions. Emissions, impacts on the calculations of various policy responses are all mutually dependent on how society is structured over time. This is one of the major strengths of the SRES approach, and will occupy integrated assessments of climate change for the foreseeable future. It is essential that the storyline approach should continue, and that the scenario work should attempt to move forthrightly with the difficult issue of the relative likelihood of each scenario, and to provide more guidance as to the independence of each storyline.

■

What next? The special report leadership was not wrong, of course, about how difficult it would be to assign subjective probabilities to reduce the uncertainty about the future. But in the probability vacuum that followed its assertion that all scenarios are “equally sound”, we are facing the even more perplexing task of how to select scenarios. Selecting arbitrary scenarios and climate sensitivities to construct frequency charts that, like the histogram of Fig. 1, are built on implicit assumptions. In the risk-management literature, this is called “the art of policymaking”. I would definitely put more trust in the probability estimates of the SRES team — however subjective — than those of the myriad special interests that have been encouraged to make their own selection.

Meanwhile, as we wait for the IPCC to decide whether to resemble the team



commentary

Box2: Summary of special report's findings

The most compelling conclusions of the special report on emission scenarios are:

- (1) Alternative combinations of main economic driving forces can lead to similar levels of greenhouse-gas emissions by the end of this century. Scenarios with different underlying assumptions can result in very similar climate changes.
- (2) Technology is as least as important a driving force as population and emissions as population and economic development across the end of 40 scenarios.

The 40 scenarios used for the most “representative” and “useful” storylines, each have distinct features when combining different assumptions on population, economic and technological growth; orientation towards a global economy; and attitudes toward economic and social equity.

The scenarios cover the full range of greenhouse-gas and SO₂ emissions that SRES authors could imagine as plausible.

One prominent difference is the relatively low emphasis on coal (A1C), oil and gas (A1B), non-fossil energy sources (A1T) or reliance across all sources (A1B). In the working group 1 summary for policymakers, A1C and A1B groups are merged into one fossil-intense family, A1F. All scenario families are said to be equally sound.

The scenarios are also grouped into two sets of “illustrative” scenarios, which indicate that scenarios with different driving forces can lead to similar cumulative emissions, and those with similar driving forces can branch out into different categories of cumulative emissions.

Four scenarios are designated as “markets”.

Together with two scenarios from the A1 family, the six “illustrative” scenarios form the backbone of the range of projections working group 1.

The scenarios are also grouped into three sets of “regions” in which any evaluation should include at least the six illustrative scenarios.

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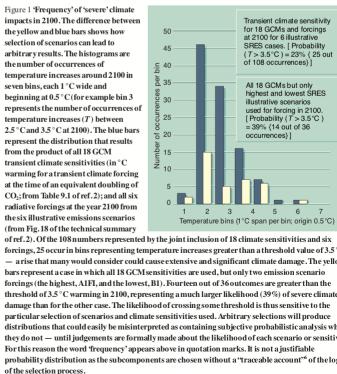
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Insurance and Uncertainty : the Framework

Classical framework : agent, with utility $u(\cdot)$ facing random loss L , purchases insurance at price π if

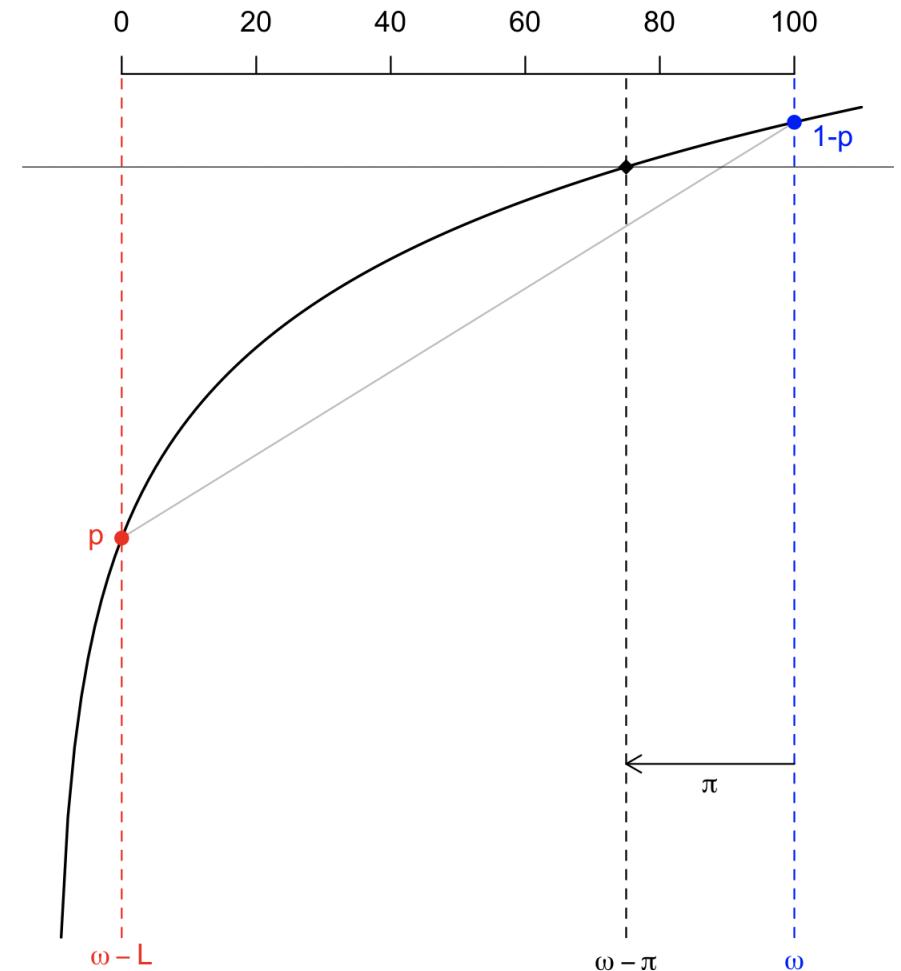
$$\mathbb{E}[u(\omega - \pi - L + I(L))] > \mathbb{E}[u(\omega - L)]$$

(von-Neumann expected utility).

Let π^* denote the highest premium

$$\mathbb{E}[u(\omega - \pi^* - L + I(L))] = \mathbb{E}[u(\omega - L)]$$

where I denotes the indemnity function,
 $I(\ell) \in [0, \ell]$.



Insurance and Uncertainty : the Framework

Suppose that L is a binary loss, $L \in \{0, \ell\}$ with probabilities $1 - p$ and p , or
 $L = X \cdot \ell$, $X \sim \mathcal{B}(p)$

$$(1 - p) \cdot u(\omega - \pi^*) + p \cdot u(\omega - \pi^* - \ell + I(\ell)) = (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

If $I(\ell) = \ell$, $u(\omega - \pi^*) = (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$

If u is linear (no risk aversion), $\pi^* = \mathbb{E}[I(L)] = p \cdot I(\ell)$

If u is concave (risk aversion), $\pi^* > p \cdot I(\ell)$

$p \cdot I(\ell)$ is the actuarial (pure) premium, also called [Learned Hand formula](#) in law business (see [Grossman et al. \(2006\)](#)) or simply “*probability times consequence*” in climate literature (see [Schneider \(2002\)](#))

Insurance and Uncertainty : the Framework

Consider the simplistic case of homogeneous agents.

An **agent**, with utility u , will purchase insurance if

$$(1 - p) \cdot u(\omega - \pi) + p \cdot u(\omega - \pi - \ell + I(\ell)) > (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

Assume that this inequality is satisfied.

An **insurance company**, with utility v , will sell insurance if

$$\mathbb{E}[v(\kappa + n\pi - S)] > u(\kappa), \quad S = \sum_{i=1}^n L_i = \sum_{i=1}^n X_i \cdot \ell, \quad \text{where } X_i \sim \mathcal{B}(p),$$

where κ is the capital of the company, and S is the total indemnity.

The actuarial fair premium is obtained when v is linear :

$$\pi^* = \mathbb{E}[I(L)] = p \cdot I(\ell)$$

Insurance and Uncertainty : the Framework

If risks are exchangeable $X = L_1 + \cdots + L_n = N_n \cdot I(\ell)$,

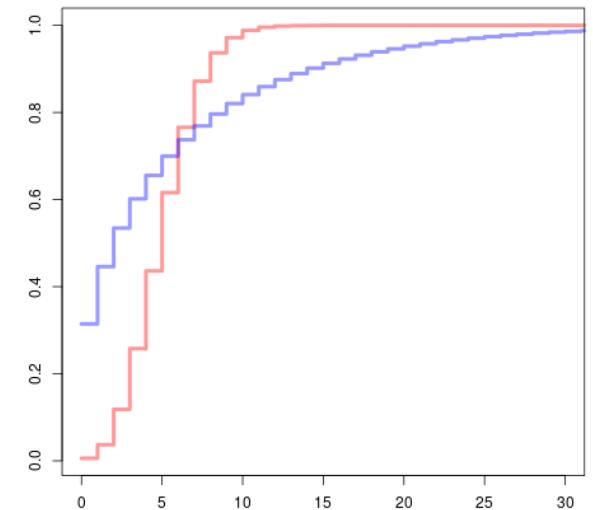
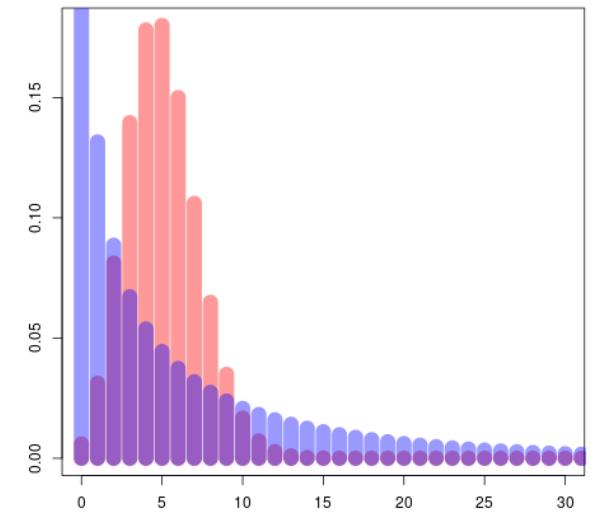
$$\mathbb{P}[N_n = k] = \int_0^1 \binom{n}{k} \theta^k (1-\theta)^{n-k} dG(\theta), \quad \mathbb{P}[L_i > 0] = p$$

from [de Finetti \(1921\)](#). See [Charpentier & le Maux \(2014\)](#) for correlated risks and disaster (with endogeneous default probability of the insurer).

Classical framework : [binomial-beta](#) model (G is a Beta distribution $\mathcal{B}(\alpha, \beta)$), then $r = \text{corr}(X_i, X_j) = (1 + \alpha + \beta)^{-1}$. Then

$$\mathbb{E}[N_n] = np \text{ while } \text{Var}[N_n] = (n + n(n-1)r)p(1-p)$$

(the insurance company has correlation aversion, [Richard \(1975\)](#))



Insurance and Uncertainty : the disaster puzzle

Earthquakes and floods cause potentially large losses, rational people should find actuarially fair insurance policies attractive, see [Kunreuther \(1996\)](#)

Individuals *underestimate* the true probability of a disaster event occurring and/or have fairly high discount rates for the benefits of uncertain future reimbursements due to losses.

[Kunreuther & Pauly \(2004\)](#) prove that even when insurance for low-probability, high-loss events is offered at favorable premiums, the search costs associated with obtaining the information on premiums and disaster probabilities may prevent from purchasing insurance.

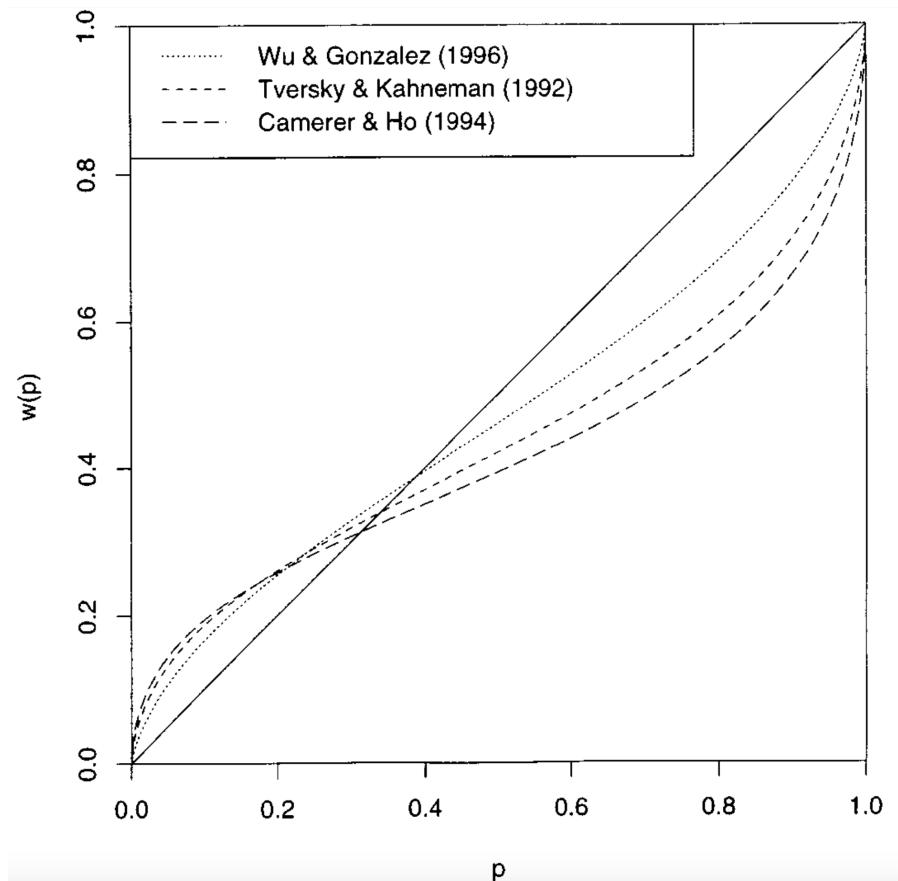
Individuals lack information about the expected harm, before the disaster, see [Botzen \(2013\)](#). Insurance prices can be a signal used to correct for possible bias (unless market distortion), see [Thieken et al. \(2006\)](#).

Loss Probability p and Perceived Loss Probability $\omega(p)$

RDEU framework, or Prospect theory,
 Kahneman & Tversky (1979) or Wu & Gonzalez (1996)

$$\omega(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

(experimental) evidence of convexity of ω
 extreme risk aversion in situations that involve low stakes see Huysentruyt & Read (2010), but inadequate insurance coverage against disaster, Viscusi (2010), even if it is highly subsidized : underweighting of adverse tail events.



Loss Probability p and Perceived Loss Probability $\omega(p)$

“rare extreme events are underweighted because people are not aware of their existence” : no, see Barnes (2011)



Vesuvius last awoke with a small blast in 1944. A large eruption could unleash incendiary avalanches and ash that would threaten millions of people.

EUROPE'S TICKING TIME BOMB

Vesuvius is one of the most dangerous volcanoes in the world — but scientists and the civil authorities can't agree on how to prepare for a future eruption.

BY KATHERINE BARNES

It starts with a blast so strong that a column of ash and stone rockets 40 kilometres up into the stratosphere. The debris then drops to Earth, peltig the surface with boiling hot fragments of pumice and covering the ground with a thick layer of ash. Roots crumble and vehicles grind to a halt. Yet Vesuvius is still too tame. It has had tsunamis, ash, mud, ice and gas roar down the slopes of the volcano, pulverizing buildings and burying everything in their path. Almost overnight, a packed metropolis becomes a volcanic wasteland.

This is Naples, Italy, in the throes of a cataclysmic eruption of Vesuvius — the volcano that gave us Pompeii. The scenario may sound far-fetched, but in the wake of Japan's recent earthquake and tsunami, many areas are reassessing the risks from their own "black swans," a term used to describe unlikely but potentially devastating disasters. And Naples stands out particularly vulnerable, with a population of 3 million living in the shadow of Vesuvius.

The volcano has been eerily dormant since a

small eruption in 1944, but recent studies suggest that Vesuvius could be more dangerous than previously assumed, which has prompted a vigorous debate about the risk and scale of future eruptions. The scientific community is split on whether to focus on the potential for a catastrophe in the event of earthquakes and other signs heralding the volcano's reawakening. "There would be no modern precedent for an evacuation of this magnitude," says Giuseppe Mastrolorenzo at the Vesuvius Volcano Observatory in Naples. "This is why Vesuvius is the most dangerous volcano in the world."

BRUMBLINGS OF DISSENT
The slumbering giant won't stay quiet forever. Seismic imaging studies have detected an unusual layer about 8–10 kilometres deep under the mountain's surface. Mastrolorenzo and his colleague Lucia Pappalardo

For more on volcanic eruptions see: go.nature.com/GINjyk

interpret this layer as an active magma reservoir, which could produce large-scale "plinian"-style explosions — named after Pliny the Younger, who described the AD 79 eruption. The first rumblings of activity at Vesuvius could come weeks to years before an eruption, but until now, the scientific community of the eruption itself. Pappalardo and Mastrolorenzo analysed the geochemistry of rock fragments from past eruptions, and found evidence that magma ascended rapidly — in just a few hours — from its deep chamber to the surface.

For many years the largest known eruption of Vesuvius was that of AD 79. But in 2006, Mastrolorenzo and colleagues at the State University at Buffalo in New York described geological evidence for a much larger blast, about 3,800 years ago in the Bronze Age¹. Fiery avalanches of ash and debris called pyroclastic flows travelled 20 kilometres and covered the whole of the area of present-day Naples. "The deposits left in the centre of Naples are 4 metres thick," says Sheridan. "Even a few inches would be enough to kill everyone."

Given these concerns, the Vesuvius observatory has developed an emergency plan for the worst-case "maximum possible" eruption similar to the Bronze Age blast. "A crisis could start today," says Mastrolorenzo. "The trouble is that nobody would be able to tell how long it would last, what type of eruption it would be, or how the event would evolve." The researchers recommend the construction of a new national observatory to monitor Vesuvius if earthquakes and other signs of unrest hint that it is coming back to life.

Not all scientists share this doom-laden outlook. Some groups have even proposed that Vesuvius is becoming less explosive. Bruno Scaillet and his colleagues at the University of Orleans in France argue that the eruptive style of Vesuvius has changed as the magma chamber has shifted upwards, causing the magma to rise upwards from a relatively shallow level 3 kilometres below the surface². Evidence suggests the magma stored there is less viscous, so it is less prone to causing large explosions. If the past trend holds, says Scaillet, the next eruption could be a relatively minor one.

Scaillet adds that the seismically unusual layer 10 kilometres below the surface could be magma, but it could also be some other fluid such as water or brine. "These various issues are not yet settled," he says.

EMERGENCY PLANNING

With the size of any future eruption in doubt, and a public more concerned about day-to-day problems such as traffic and crime, mitigating the hazard of Vesuvius is an enormous task shared by researchers and the civil authorities.

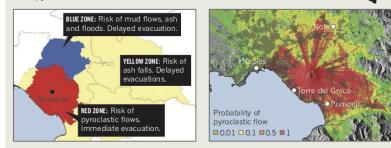
Scientists keep constant tabs on Vesuvius through a network of sensors that monitor for earthquakes and volcanic deformation and changes in the chemistry of erupting gases.

And Italy's Department of Civil Protection (DPC) maintains a National Emergency Plan for Vesuvius. The plan, first developed in 1995, is based on a scenario for an intermediate-sized eruption, similar to one that occurred

SOURCE LEFT: DEPT CIVIL PROTECTION, ROME/REF. 4

IN THE LINE OF FIRE

Plans call for initial evacuation of only the zone closest to Vesuvius (left map, red). But a simulation of a large blast (right map) shows a high risk of fiery avalanches called pyroclastic flows that reach farther.



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scenario³, which means including metropolitan Naples and 3 million inhabitants in the evacuation zone for planning, says Jonathan Fink, a volcanologist at Portland State University in Oregon. Once the volcano shows signs of unrest, authorities and scientists can re-evaluate. "If there is an error on the high side, there is less lost than would be the case in the opposite situation," he says.

In a report published in 2006, the DPC

advocates evaluating the eruption risk "on the basis of the present state of the volcano and not simply assuming the largest eruption event that ever occurred in the volcano's history." Some scientists agree. "You can't spend [everything]

on the absolute worst

case. You need to reduce

the risk in a rational

way," says Giuseppe Marzocchi at the National Institute for Geophysics and Volcanology (INGV) in Rome. A complete evacuation of Naples' 3 million residents, he says, "would be impossible to manage".

Marzocchi and other researchers are developing a probabilistic approach to the probabilities of different scenarios — that could help civil authorities evaluate the evidence during a crisis and choose a course of action.

Peter Baxter, an expert in emergency planning at the University of Cambridge, UK, specializing in the impacts of volcanic eruptions and used this type of model successfully during the 1997 eruption in Mount Pinatubo, Philippines, to predict which regions would be affected. A complete evacuation of the island was avoided.

For Vesuvius, Baxter and his colleagues have used geological data and models of eruptive processes to develop an "event-tree" to display the full range of possible eruptions⁴. If sensors detect an increase in seismicity, they and other researchers insist that the emergency plan should correspond to the "worst-case

scenario" of a large blast, which has a 1% chance of an explosive eruption but only a 0.1% chance of a catastrophic plinian one. The most likely event is a violent but smaller blast, like the one in 1944, with lava flows and moderate ash emissions.

For now, this kind of probabilistic approach seems the only way forward for volcanologists and disaster managers, as there is no hope for accurate eruption forecasting in the horizon.

"It's an extremely complex problem to solve," says Augusto Neri of the INGV's laboratories in Pisa. "We simply do not know how the volcano works." ■

Katherine Barnes is a freelance writer in London.

1. Pappalardo, L. & Mastrolorenzo, G. *Earth Planet. Sci. Lett.* **296**, 133–143 (2010).
2. Mastrolorenzo, G. et al. *Proc. Natl Acad. Sci. USA* **103**, 4390–4395 (2006).
3. Scaillet, B., Pichavant, M. & Cioni, R. *Nature* **455**, 216–219 (2008).
4. Houghton, J. & Pappalardo, L. *J. Geophys. Res.* **115**, B12212 (2010).
5. Rolandi, G. *J. Volcanol. Geotherm. Res.* **178**, 432–438 (2009).
6. Baxter, P. J. et al. *J. Volcanol. Geotherm. Res.* **178**, 454–473 (2008).

Loss Probability p and Perceived Loss Probability $\omega(p)$

Myopic loss aversion or myopic probability weighting, see [Barberis, Huang & Thaler \(2006\)](#)

See also [Gilboa & Schmeidler \(1989\)](#) on the use of capacities : instead of computing $\mathbb{E}_{\mathbb{P}}[u(\omega - \pi - L + I(L))]$, consider

$$\min_{\mathbb{Q} \in \mathcal{P}} \left\{ \mathbb{E}_{\mathbb{Q}}[u(\omega - \pi - L + I(L))] \right\} \text{ for some set of beliefs } \mathcal{P}$$

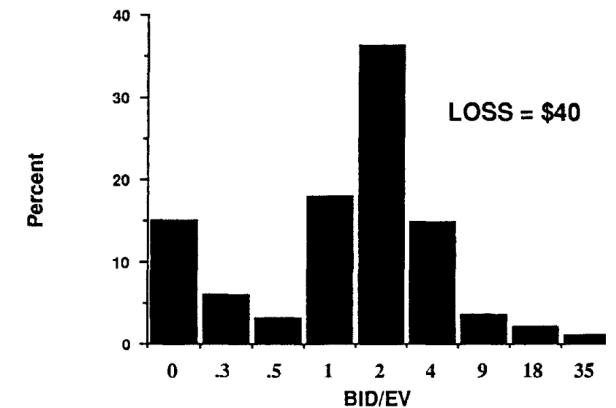
Can capture uncertainty associated with climate change, that makes it difficult to respond optimally, see [Dessai & Hulme \(2006\)](#) or [Tompkins & Adger \(2005\)](#).

Perceived Loss Probabilities $\omega_i(p)$

Experimental study of McClelland, Schulze & Coursey (1993) : bimodal distribution of willingness to pay for insurance, with two groups

- neglect low-probability risks, do not purchase insurance
- willingness to pay higher than expected loss

link between the bimodal risk judgments and protective behavior McClelland, Schulze & Hurd (1990), see also Charpentier & le Maux (2014) on optimal (Pareto) vs. equilibrium (Nash)



Loss Probability p : Meteorological Perspective

For hurricanes, see [Gray et al. \(1992\)](#), the seasonal number of intense hurricanes is

$$\hat{N} = 3.571 + 0.042(U_{50} + 0.103U_{30} - 1.415|U_{50} - U_{30}|) + 0.717(R_S + 2.455R_G)$$

where U 's are upper-air [zonal winds](#) at 50 and 30 mb and R 's are composite functions of August-September western Sahel (R_S) and August-November Gulf of Guinea (R_G) [rainfall](#).

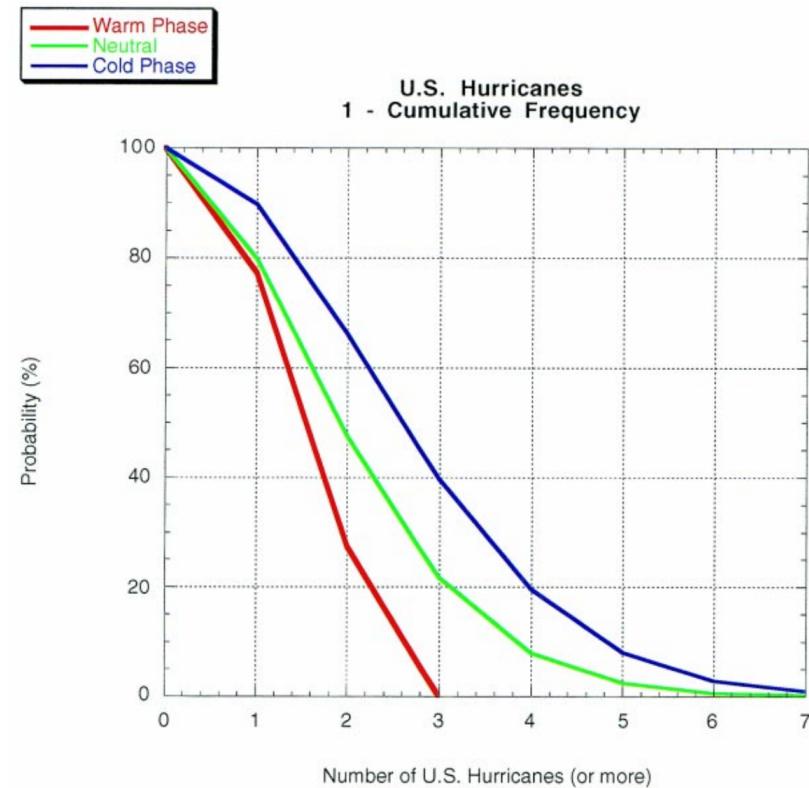
See also [Klotzbach & Bell \(2017\)](#) on the
Landfalling Hurricane Probability Project
(with all landfall probability calculations)

	A	N	O	P	Q	R
3		Climatological Probability		Current-Year Probability		
4		(Using Poisson)		(Using Poisson)		
5	State	H	MH	H	MH	
6	Texas	33%	12%	21%	7%	
7	Louisiana	30%	12%	19%	7%	
8	Mississippi	11%	4%	6%	3%	
9	Alabama	16%	3%	10%	2%	
10	Florida	51%	21%	35%	13%	
11	Georgia	11%	1%	7%	1%	
12	South Carolina	17%	4%	11%	2%	
13	North Carolina	28%	8%	18%	5%	

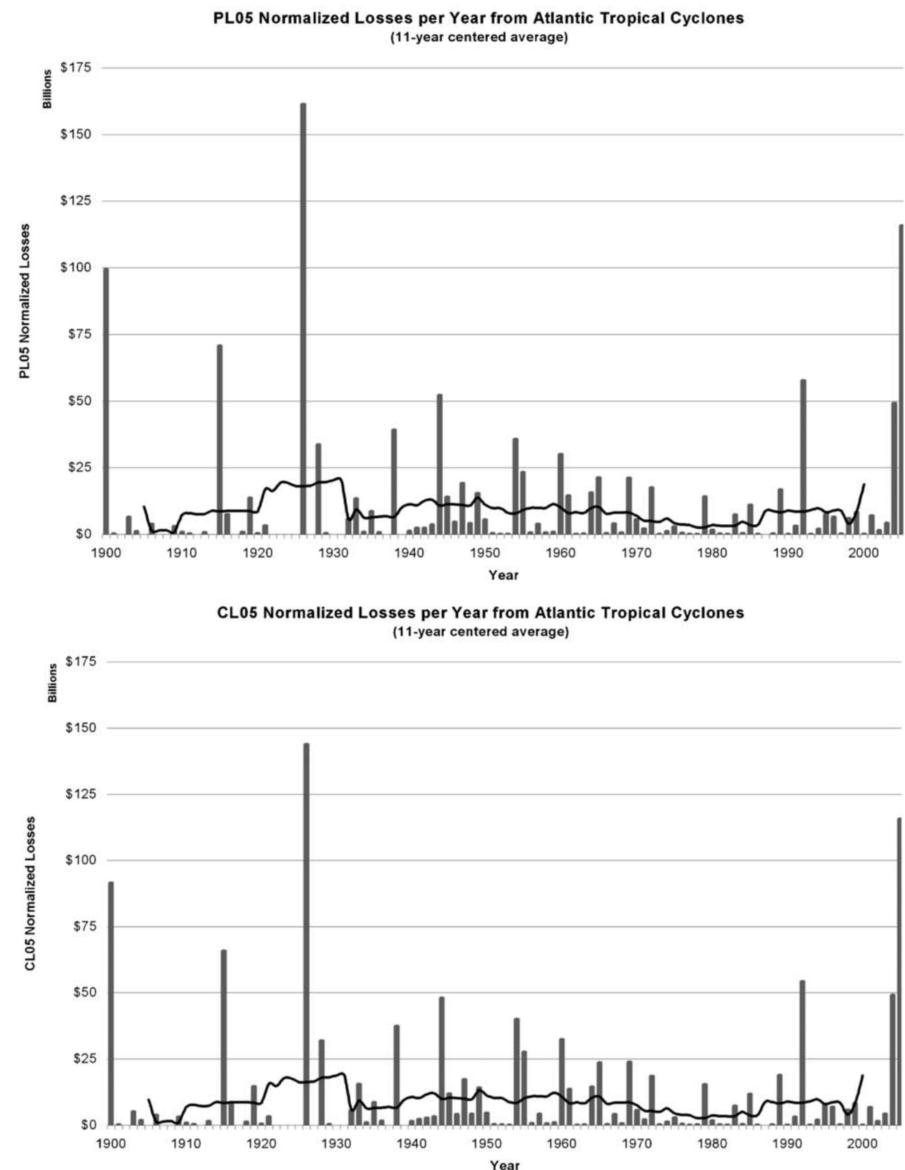
Statistical Perspective

Need (long) historical data,
(if possible with normalized losses)

see Pielke Jr. *et al.* (2008)



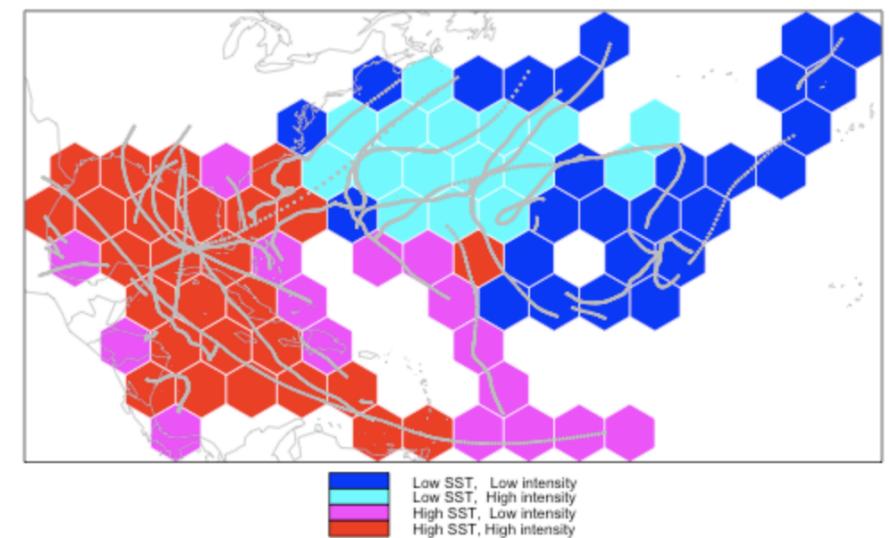
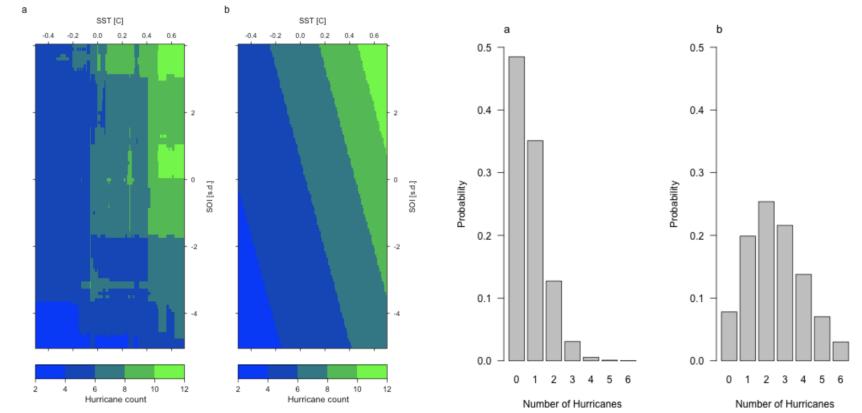
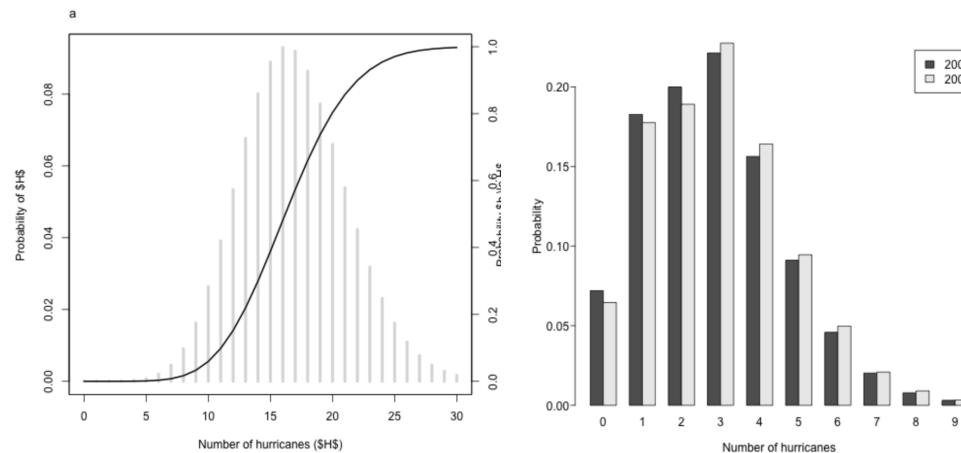
From Bove *et al.* (1998) for hurricanes.



Loss Probability : Statistical Perspective

See Elsner & Jagger (2013) for a exhaustive statistical analysis of hurricanes,

- Poisson regression for **counts**
 - Southern Oscillation Index (SOI)
 - sea-surface temperature (SST)
- regression for **intensity**
- spatial models for **trajectories**
- Bayesian models for **one year prediction**



Statistical Perspective

hard to justify in a changing environment
In order to apply any theory we have to suppose that the data are homogeneous, i.e. that no systematical change of climate and no important change in the basin have occurred within the observation period and that no such changes will take place in the period for which extrapolations are made

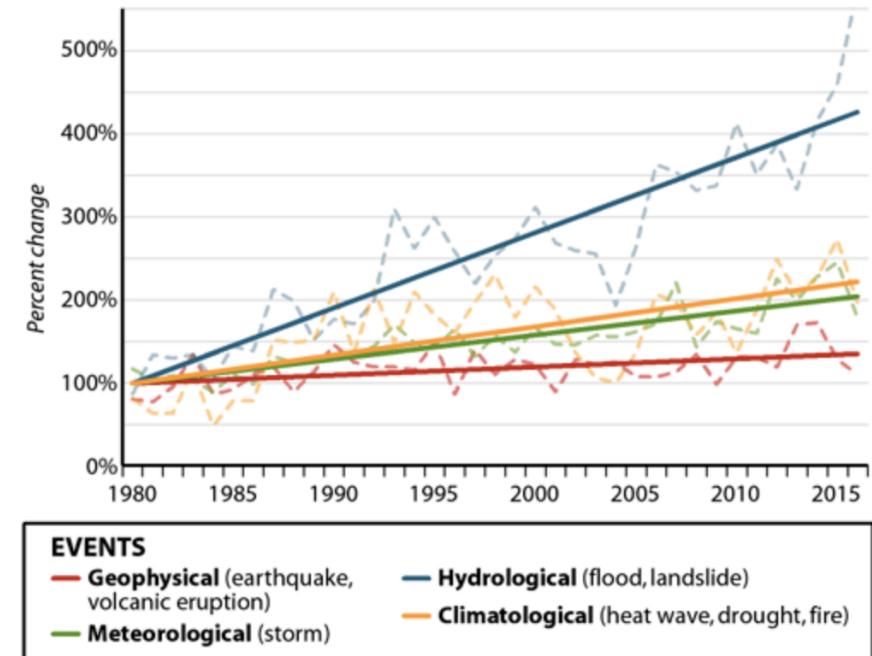
Gumbel (1941)

see flood risk (dams, reservoirs, etc)

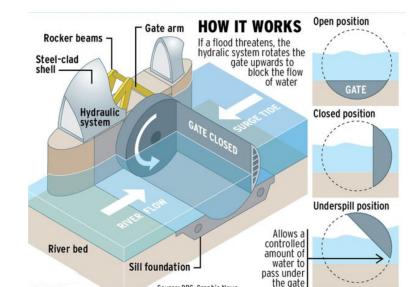
see storms or heatwaves (climate change)

Charpentier (2011)

GLOBAL TRENDS IN NATURAL CATASTROPHES
Percentage change each year in number of events compared to 1980



SOURCES: MunichRe NatCatSERVICE; European Academies Science Advisory Council



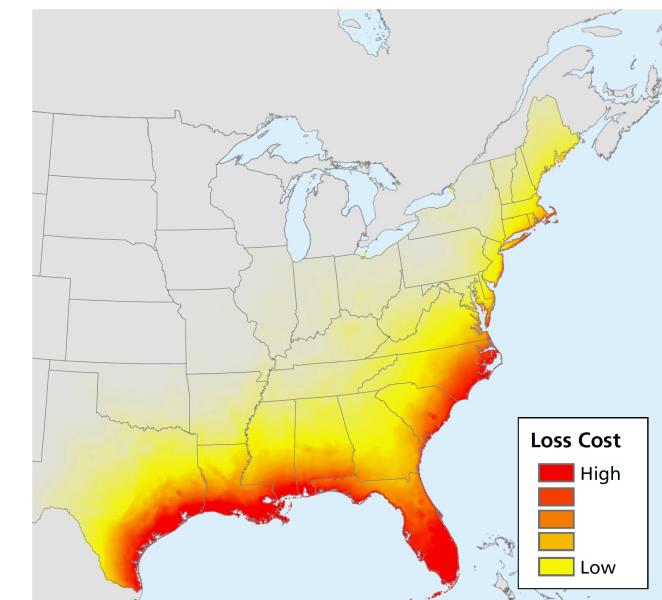
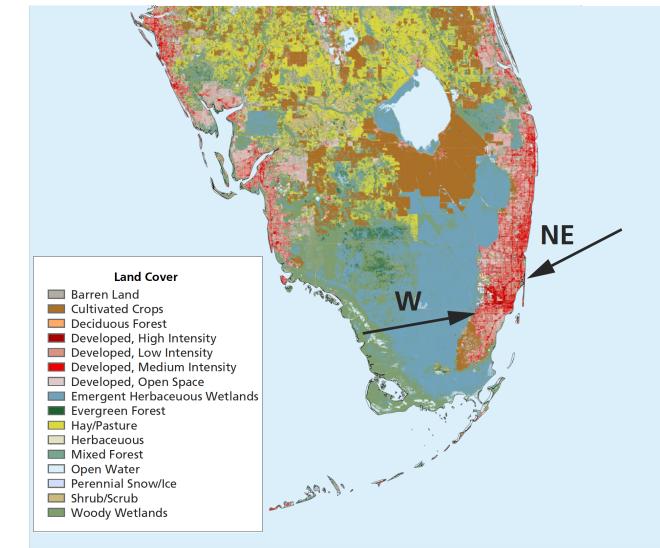
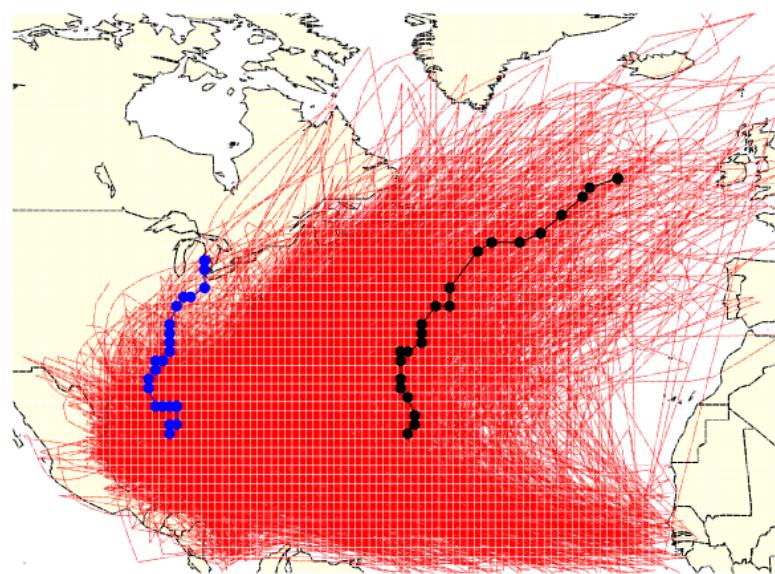
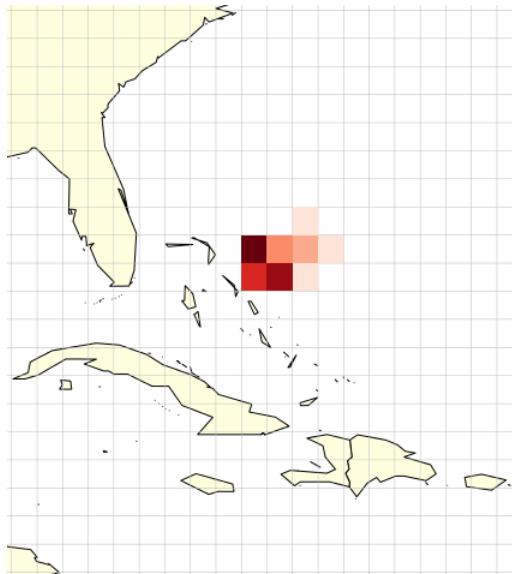
Loss Probability p : Actuarial Perspective

Actuaries use catastrophe softwares

- Risk Management Solutions (**RMS**)
- AIR Worldwide (**AIR**)
- Risk Quantification Engineering (**RQE**) EQECAT

see Cole, Macpherson & McCullough (2010).

Generation of climatic scenarios (+ losses)



see for Markovian generation Charpentier (2014).

Predictive Markets

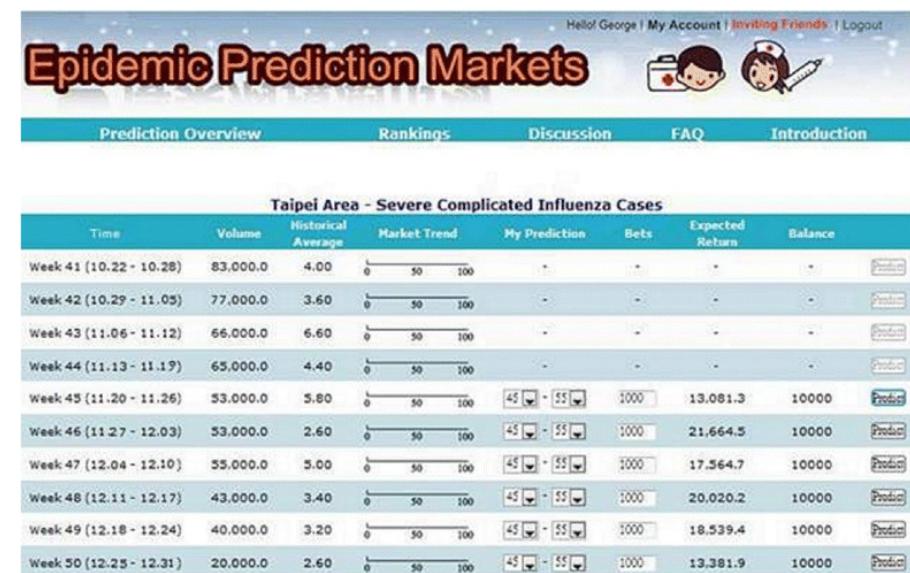
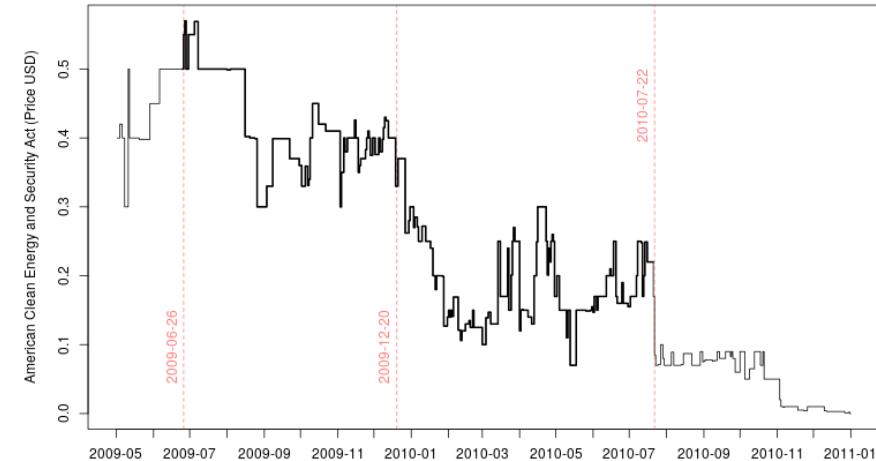
Peer prediction systems

[Wolfers & Zitzewitz \(2004\)](#) for a description

- elections (e.g. popes, XVIth Century)

[Rhode & Strumpf \(2008\)](#)

- politics, [Meng \(2016\)](#) Waxman-Markey bill (2009-2010, finally rejected)
- sports (racetrack bets) [Ali \(1977\)](#)
- infectious diseases, [Polgreen, Nelson & Neumann \(2006\)](#) or [Tung, Chou & Lin \(2015\)](#)
- climate risk [Hallstrom & Smith \(2005\)](#) for hurricanes



Loss Probability p : Betting Markets

See [Eisenberg & Gale \(1959\)](#) model, on consensus and subjective probabilities (used in [Manski \(2004\)](#) on all-or-nothing contracts)

Individual i (wealth b_i) can bet on horse j an amount $\beta_{i,j}$ (so that $\beta_{i,1} + \dots + \beta_{i,J} = b_i$). Assume that $b_1 + \dots + b_I = 1$.

Let π_j denote the sum bet on horse j , $\pi_j = \beta_{1,j} + \dots + \beta_{I,j}$. Observe that from the budget constraint $\pi_1 + \dots + \pi_J = 1$, hence π_j 's are probabilities.

Individual beliefs can be related expressed through a probability vector $\mathbf{p}_i = (p_{i,1}, \dots, p_{i,J})$, then there is an equilibrium if

$$p_{i,j} = \pi_j \cdot \max_s \left\{ \frac{p_{i,s}}{\pi_s} \right\} \text{ as soon as } \beta_{i,j} > 0$$

(so called Eisenberg-Gale equilibrium).

Loss Probability p : Betting Markets

Consider a contract that pay \$1 if an event A occurs (e.g. major flood), sold at price π_A . If there is a contract on \bar{A} , then $\pi_{\bar{A}} = 1 - \pi_A$ (no arbitrage).

Assume that individual i believe that A will occur with probability p_i . If $p_i > \pi_A$, he should purchase b_i units of A (otherwise b_i units of \bar{A}).

There is equilibrium if

$$\sum_{i=1}^I b_i = \frac{1}{\pi_A} \sum_{i=1}^I b_i \mathbb{P}[p_i > \pi_A] = \frac{1}{\pi_{\bar{A}}} \sum_{i=1}^I b_i \mathbb{P}[p_i < \pi_A]$$

hence, if wealth b_i is independent of belief p_i , then prices are probabilities

$$\pi_A = \mathbb{P}[p_i > \pi_A] = 1 - \pi_{\bar{A}}.$$

see [Wolfers & Zitzewitz \(2004\)](#) for further interpretation.

Parimutuel and Betting Markets

- 2004 and 2005, ICAP, Goldman and Nymex (see [IFR](#)) with parimutuel derivative on energy (changes in crude oil and natural gas inventories)
- 2004, CME launched futures trading on the US Consumer Price Index (CPI), see [Filippov \(2005\)](#)
- 2008, LLC (see [Ou-Yang \(2010\)](#)) with parimutuel derivative on [hurricanes](#)
- The [longitude](#) plateforme (“*real time calculation of the odds*”).
- The [Iowa Electronic Markets](#) for elections, “*using this wisdom of crowdsthe price of a contract at any given time is a forecast of the outcome*”



HuRLOs (Hurricane Risk Landfall Option) options

The exist the Hurricane Futures Market (HFM), see [Kelly, Letson, Nelson, Nolan & Solis \(2010\)](#) for a description

Hurricane Risk Landfall Options (HuRLOs) were launched in October 2008 by Weather Risk Solutions, LLC (WRS) and promised a mutualized marketplace for hurricane options, using [parimutuel options](#)

From French [pari mutuel](#), literally, mutual stake :

“A system of betting on races whereby the winners divide the total amount bet, after deducting management expenses, in proportion to the sums they have wagered individually”, see [Baron & Lang \(2007\)](#).

also calle [universal Dutch auction](#) in [Syz \(2008\)](#)

Parimutuel and Betting Markets

“A Shapley-Shubik market game for contingent claims within a probability state space with secured selling is a parimutuel market” ([Lange & Economides \(2005\)](#))

A matching / pricing mechanism is necessary to get [financial efficiency](#) (self-hedging). Participant i submit orders, state bids β_i , b_i a limit share quantity and $\bar{\pi}_i$ a limit price per share

The market organizer determines the order fill x_i (and state prices π_j), using call auction mechanism, see [Peters, So and Ye \(2005\)](#) and [Agrawal, Wang & Ye \(2008\)](#).

Using a [Linear Programming Market Mechanism](#), market organizer should solve

$$\text{maximize} \left\{ \sum_{i=1}^I \bar{\pi}_i x_i - z \right\} \text{ where } z \text{ is some worst case cost}$$

$$\text{subject to} \quad \text{cost if state } j \text{ occurs} = \sum_{i=1}^I \beta_{i,j} x_i \leq z \quad \forall j$$

$$0 \leq \bar{\pi}_i x_i \leq b_i \quad \forall i$$

Parimutuel and Betting Markets

More realistically, one can consider some dynamic parimutuel market maker as in [Agrawal et al. \(2014\)](#) for recent advances, and [Chen & Pennock \(2010\)](#) for an overview.

See also techniques to get some peer prediction Bayesian Nash equilibrium solvers (see [Jurca & Faltings \(2008\)](#) on elections and bets) and the related collective revelation mechanism (see [Goel, Reeves & Pennock \(2009\)](#))

“Peer prediction systems are designed for eliciting information on events where ground truth does not exist or is unobtainable”, [Chen & Pennock \(2010\)](#)

Take-Away Conclusion

- knowing **real probabilities** of occurrence disasters is either complicated (hurricanes) or very complicated (flood)
- those probabilities are necessary to assess solvency of insurance companies (central limit theorem is based on true probabilities)
- insurance pricing is based on **beliefs** of insured, and insurance companies
- **predictive markets** can be an interesting revelation mechanism of crowd beliefs