How to Estimate the Occurrence Probability of Natural Catastrophes

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The Challenge of Predictive Models (for Rare Events)

"*A 30% chance of rain tomorrow : How does the public understand probabilistic weather forecasts ?*" Gigerenzer *et al.* (2005)

Here we focus on rare events :

(1) hard to find a good model (2) hard to assess if the model is good

See Nate Silver's "*there's an awful lot of room to debate what 'probably' means*" 538.com, or Ronald Fisher on predicting probabilities on 'one-shot' events,

(or more provocative "*probability does not exist*" by Bruno de Finetti, Nau (2011))

Classical issue in risk management : predict 1% chance of occurrence, and occurs. Was 1% wrong ?

Disasters are not *per se* 'one-shot' events, but we have to deal with changing environement...

How Likely Catastrophe Is To Occur ?

"*to combat global warming, we must first assess just* **how likely** *it is to occur*" Schneider (2001)

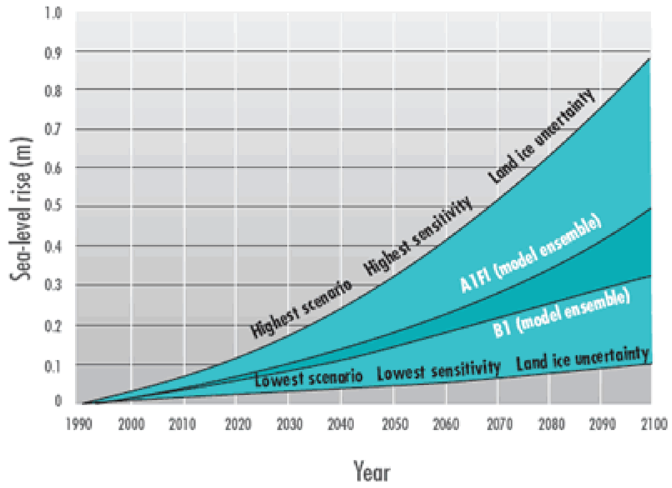
"*Does climate adaptation policy need probabilities ?* " Dessai & Hulme (2004)

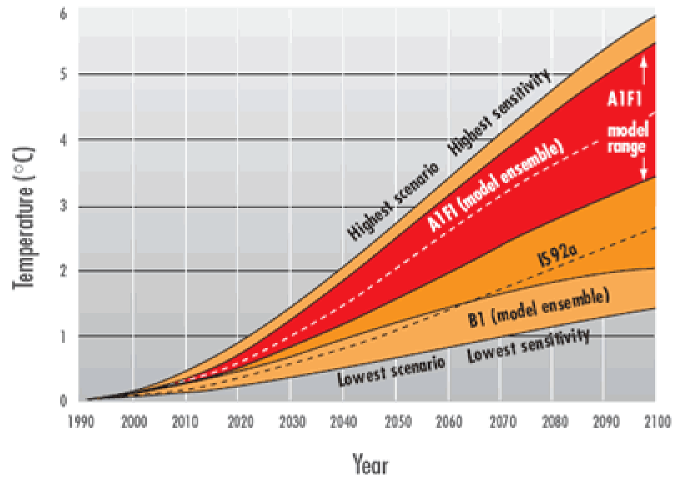
So what ?

In this talk, "*natural catastrophe*" is seen from an insurance perspective

• quantitative analysis of natural events
difficult part : climate is changing (warming)
classical curves from IPCC on temperature or sea level (and references therein)

• impact on the insurance industry ?
In 2015, French Federation of Insurers claimed "*natural disasters : insurers' bills could double by 2040*" (in 25 years, +3% per year
see https://www.argusdelassurance.com/)





Insurability of Climate Risks ?

see Berliner (1985) and Charpentier (2008)

Table 3 Insurability criteria and related requirements according to Berliner

Insurability Criteria			**Requirements**
Actuarial	(1)	Randomness of loss occurrence	Independence and predictability of loss exposures
	(2)	Maximum possible loss	Manageable
	(3)	Average loss per event	Moderate
	(4)	Loss exposure	Loss exposure must be large
	(5)	Information asymmetry	Moral hazard and adverse selection not excessive
Market	(6)	Insurance premium	Cost recovery and affordable
	(7)	Cover limits	Acceptable
Societal	(8)	Public policy	Consistent with societal value
	(9)	Legal restrictions	Allow the coverage

(Table from Bienier *et al.* (2015) - for cyber risk)

climate disasters will be insured if there is an insurance market for that...

Insurance and Uncertainty : the Framework

Classical framework : agent, with utility $u(\cdot)$ facing random loss L, purchases insurance at price π if

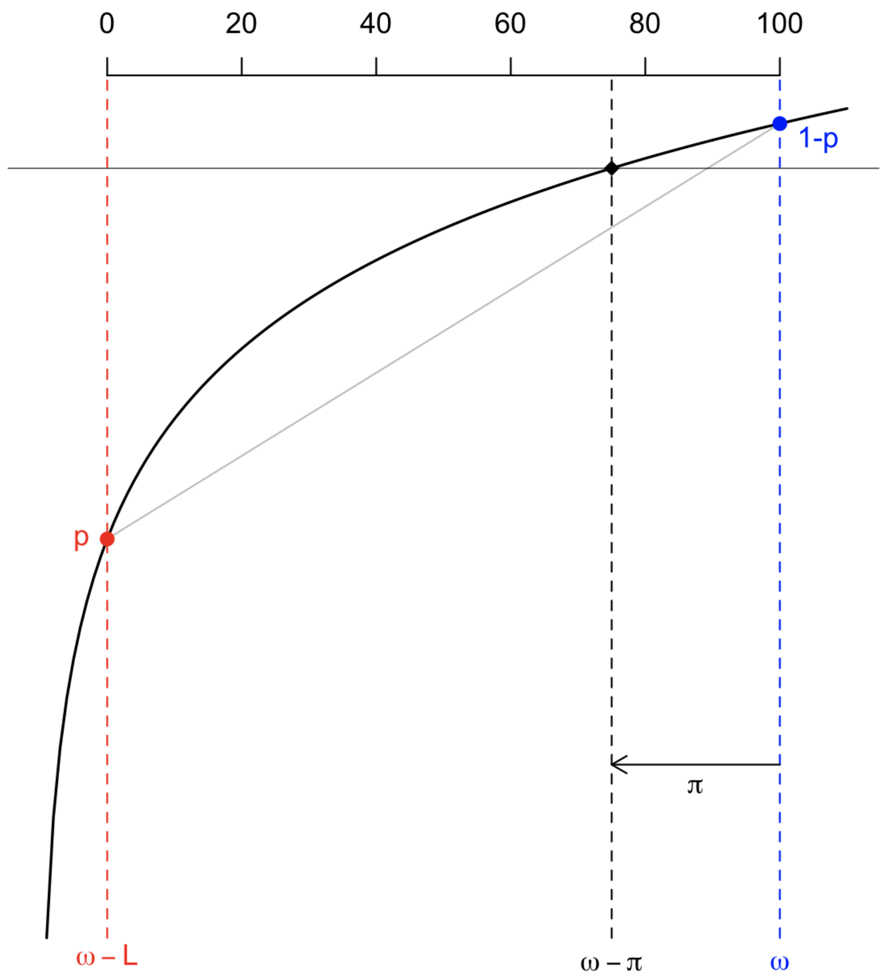
$$\mathbb{E}_{\mathbb{P}}\big[u\big(\omega - \pi - L + I(L)\big)\big] > \mathbb{E}_{\mathbb{P}}\big[u(\omega - L)\big]$$

(von-Neumann expected utility).
Let $\pi^\star$ denote the highest premium

$$\mathbb{E}_{\mathbb{P}}\big[u\big(\omega - \pi^\star - L + I(L)\big)\big] = \mathbb{E}_{\mathbb{P}}\big[u(\omega - L)\big]$$

where I denotes the indemnity function, $I(\ell) \in [0, \ell]$.

see e.g. Goussebaïle & Louaas (2015)



Insurance and Uncertainty : the Framework

Suppose that L is a binary loss, $L \in \{0, \ell\}$ with probabilities $1-p$ and p, or $L = X \cdot \ell$, $X \sim \mathcal{B}(p)$

$$(1-p) \cdot u(\omega - \pi^\star) + p \cdot u\big(\omega - \pi^\star - \ell + I(\ell)\big) = (1-p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

If $I(\ell) = \ell$, $u(\omega - \pi^\star) = (1-p) \cdot u(\omega) + p \cdot u(\omega - \ell)$

If u is linear (no risk aversion), $\pi^\star = \mathbb{E}[I(L)] = p \cdot I(\ell)$

If u is concave (risk aversion), $\pi^\star > p \cdot I(\ell)$

$p \cdot I(\ell)$ is the Actuarial (pure) Premium, also called Learned Hand formula in law business (see Grossman *et al.* (2006)) or simply "*probability times consequence*" in climate literature (see Schneider (2002))

Insurance and Uncertainty : the Framework

Consider the simplistic case of homogeneous agents.

An agent, with utility u, will purchase insurance if

$$(1-p)\cdot u(\omega-\pi)+p\cdot u\big(\omega-\pi-\ell+I(\ell)\big)>(1-p)\cdot u(\omega)+p\cdot u(\omega-\ell)$$

Assume that this inequality is satisfied.

An insurance company, with utility v, will sell insurance if

$$\mathbb{E}_p\big[v\big(\kappa+n\pi-S\big)\big]>u(\kappa),\ S=\sum_{1=1}^{n}L_i=\sum_{1=1}^{n}X_i\cdot\ell,\ \text{where } X_i\sim\mathcal{B}(p),$$

where κ is the capital of the company, and S is the total indemnity.

The actuarial fair premium is obtained when v is linear :

$$\pi^\star=\mathbb{E}[I(L)]=p\cdot I(\ell)$$

Insurance and Uncertainty : the Framework

If risks are exchangeable $X = L_1 + \cdots + L_n = N_n \cdot I(\ell)$,

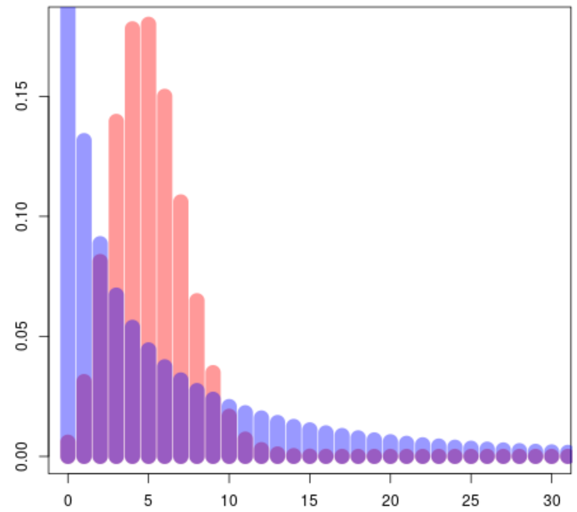
$$\mathbb{P}[N_n = k] = \int_0^1 \binom{n}{k} \theta^k (1-\theta)^{n-k} dG(\theta),\ \mathbb{P}[L_i > 0] = p$$

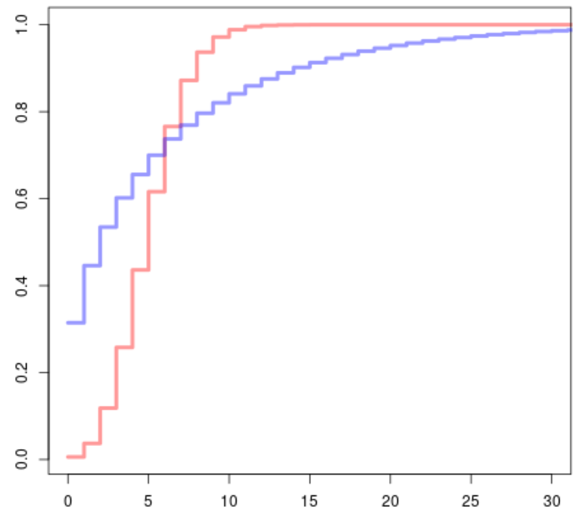
from de Finetti (1921). See Charpentier & le Maux (2014) for correlated risks and disaster (with endogeneous default probability of the insurer).

Classical framework : binomial-beta model (G is a Beta distribution $\mathcal{B}(\alpha,\beta)$), then $r = \text{corr}(X_i, X_j) = (1+\alpha+\beta)^{-1}$. Then

$$\mathbb{E}[N_n] = np \text{ while } \text{Var}[N_n] = (n + n(n-1)r)p(1-p)$$

(the insurance company has correlation aversion, Richard (1975), not clear for insured...)





Insurance and Uncertainty : the disaster puzzle

Earthquakes and floods cause potentially large losses, rational people should find actuarially fair insurance policies attractive, see Kunreuther (1996)

Individuals *underestimate* the true probability of a disaster event occurring and/or have fairly high discount rates for the benefits of uncertain future reimbursements due to losses.

Kunreuther & Pauly (2004) proved that even when insurance for low-probability, high-loss events is offered at favorable premiums, the search costs associated with obtaining the information on premiums and disaster probabilities may prevent from purchasing insurance.

Individuals lack information about the expected harm, before the disaster, see Botzen (2013). Insurance prices can be a signal used to correct for possible bias (unless market distortion), see Thieken *et al.* (2006).

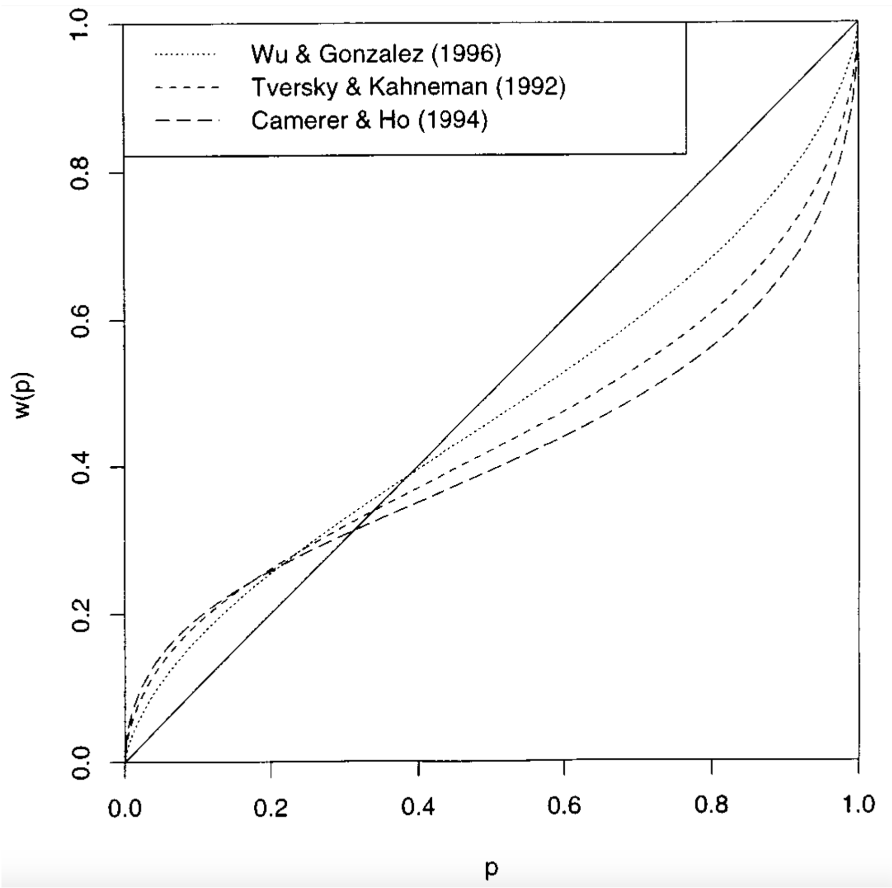
Loss Probability p and Perceived Loss Probability $\omega(p)$

RDEU framework, or Prospect theory, Kahneman & Tversky (1979) or Wu & Gonzalez (1996)

$$\omega(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

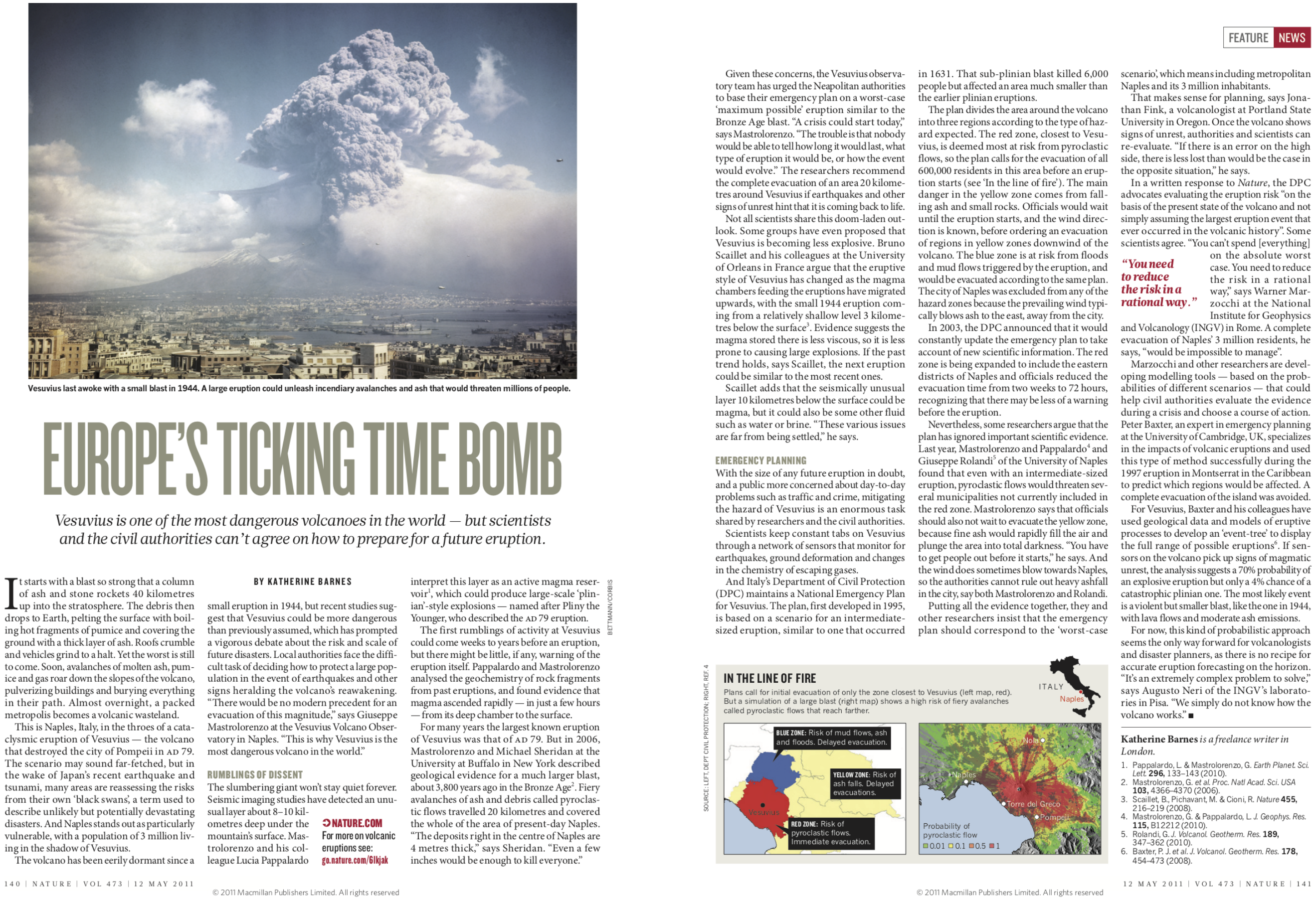
(experimental) evidence of convexity of ω

extreme risk aversion in situations that involve low stakes see Huysentruyt & Read (2010), but inadequate insurance coverage against disaster, Viscusi (2010), even if it is highly subsidized : underweighting of adverse tail events.



Loss Probability p and Perceived Loss Probability $\omega(p)$

people are aware of the risk, see Barnes (2011) (no one knows the odds)

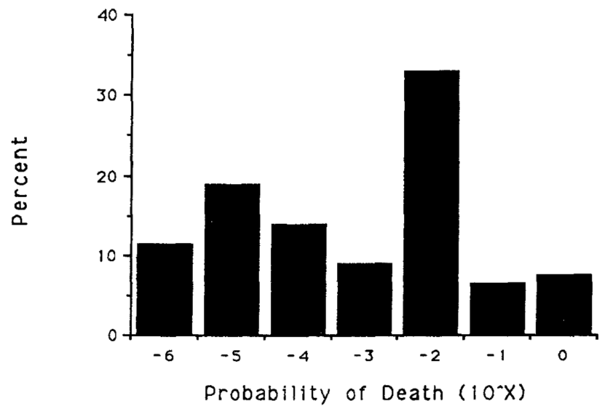


Perceived Loss Probabilities $\omega_i(p)$

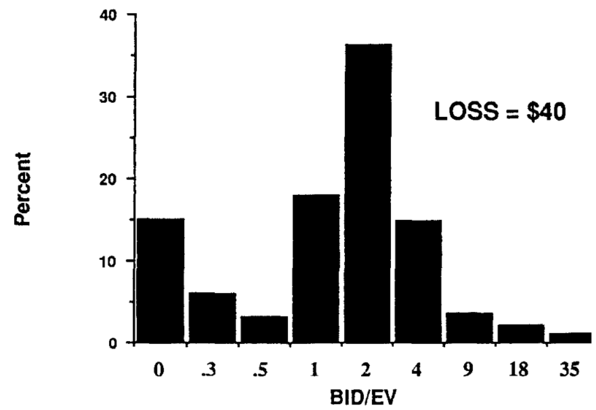
Experimental study of McClelland, Schulze & Coursey (1993) : bimodal distribution of willingness to pay for insurance, with two goups

• neglect low-probability risks, do not purchase insurance

• willingness to pay higher than expected loss



link between the bimodal risk judgments and protective behavior McClelland, Schulze & Hurd (1990), over and underweighting rare extreme events, Epper & Fehr-Duda (2017). See also Charpentier & le Maux (2014) on optimal (Pareto) vs. equilibrium (Nash)



Loss Probability p and Perceived Loss Probability $\omega(p)$

"*The probability of tail events is overestimated, which is consistent with probability weighting in prospect theory*" Botzen, Kunreuther & Michel-Kerjan (2015) (but "*potential damage is underestimated*")

Myopic loss aversion or myopic probability weighting, see Barberis, Huang & Thaler (2006), and Barberis (2013) on psychology of "tail events".

See also Gilboa & Schmeidler (1989) on the use of capacities : instead of computing $\mathbb{E}_{\mathbb{P}}[u(\omega-\pi-L+I(L))]$, consider

$$\min_{\mathbb{Q}\in\mathcal{P}}\left\{\mathbb{E}_{\mathbb{Q}}[u(\omega-\pi-L+I(L))]\right\} \text{ for some set of beliefs } \mathcal{P}$$

Can capture uncertainty associated with climate change, that makes it difficult to respond optimally, see Dessai & Hulme (2006) or Tompkins & Adger (2005).

Loss Probability p : Meteorological Perspective

For hurricanes, see Gray *et al.* (1992), the seasonal number of intense hurricanes is

$$\widehat{N} = 3.571 + 0.042\big(U_{50} + 0.103U_{30} - 1.415|U_{50} - U_{30}|\big) + 0.717\big(R_S + 2.455R_G\big)$$

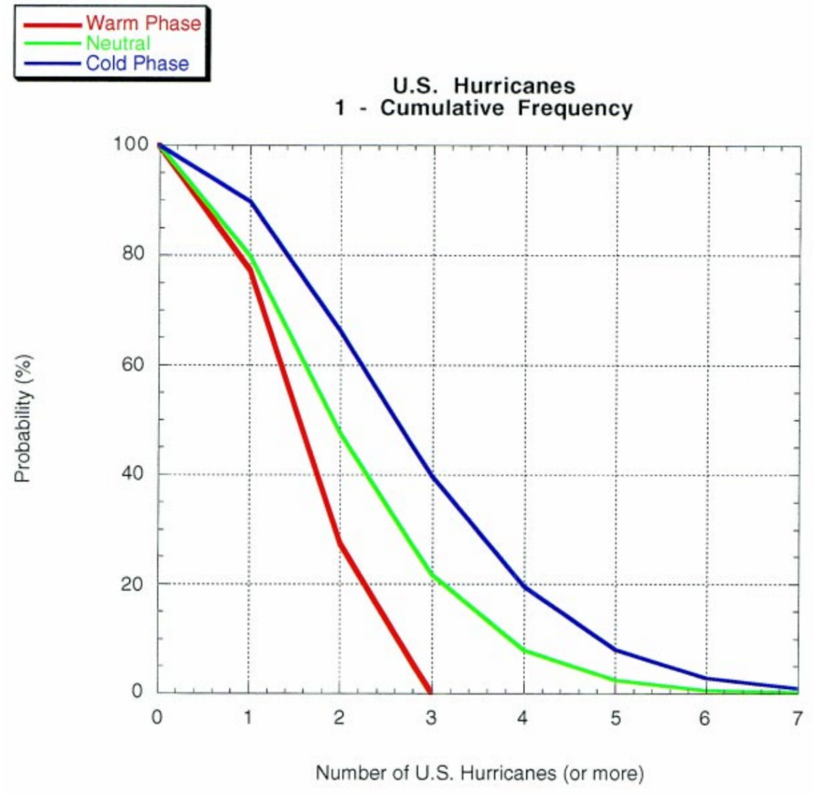
where U's are upper-air zonal winds at 50 and 30 mb and R's are composite functions of August-September western Sahel (R_S) and August-November Gulf of Guinea (R_G) rainfall.

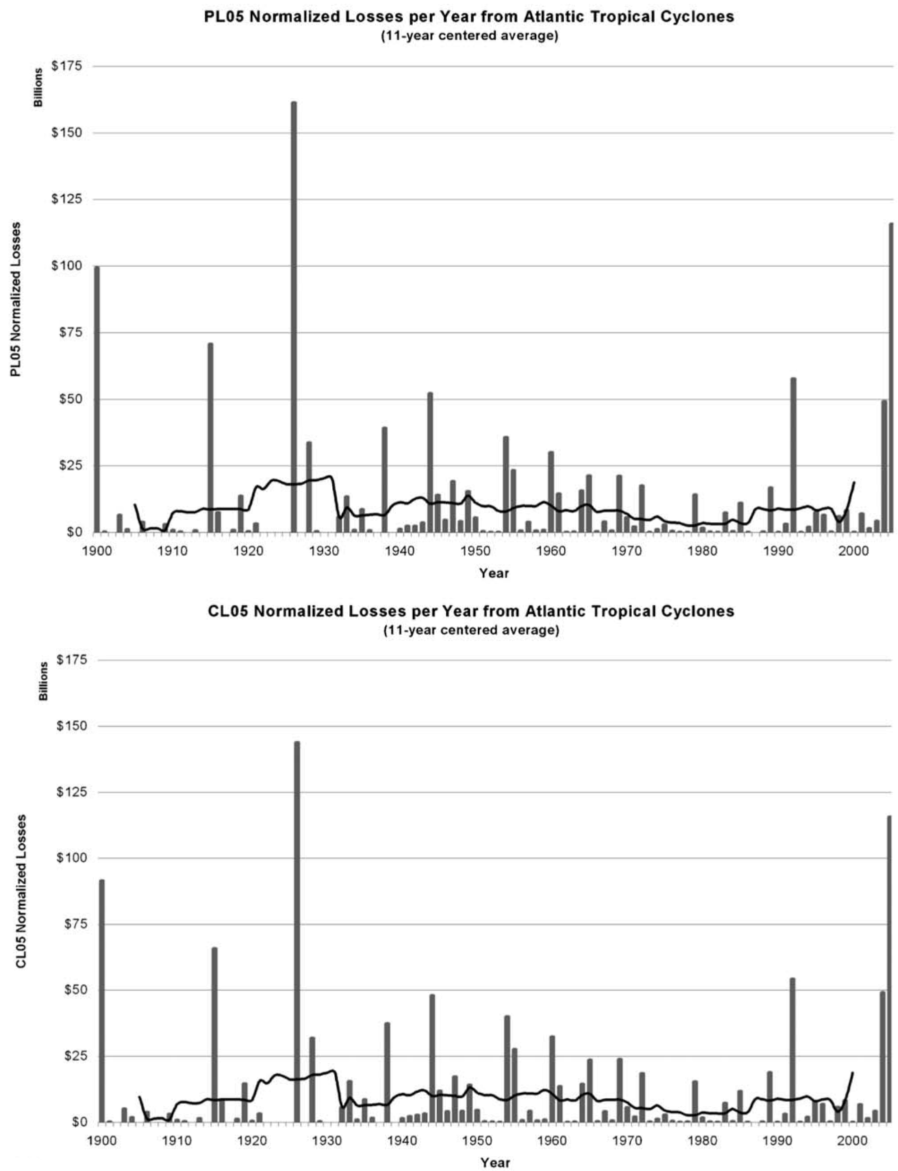
See also Klotzbach & Bell (2017) on the Landfalling Hurricane Probability Project (with all landfall probability calculations)

	A	N	O	P	Q	R
3		Climatological Probability			Current-Year Probability	
4		(Using Poisson)			(Using Poisson)	
5	State	H	MH		H	MH
6	Texas	33%	12%		21%	7%
7	Louisiana	30%	12%		19%	7%
8	Mississippi	11%	4%		6%	3%
9	Alabama	16%	3%		10%	2%
10	Florida	51%	21%		35%	13%
11	Georgia	11%	1%		7%	1%
12	South Carolina	17%	4%		11%	2%
13	North Carolina	28%	8%		18%	5%

Statistical Perspective

Need (long) historical data,
(if possible with normalized losses)
see Pielke Jr. *et al.* (2008)



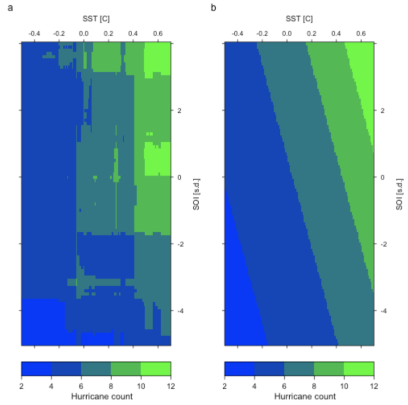


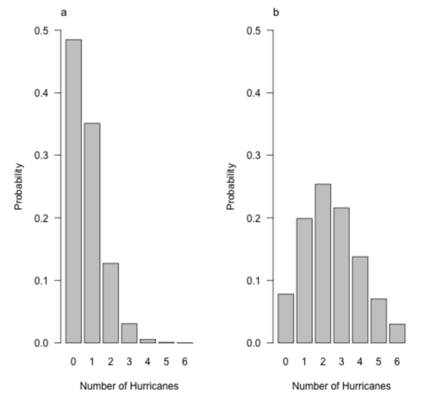
From Bove *et al.* (1998) for hurricanes.

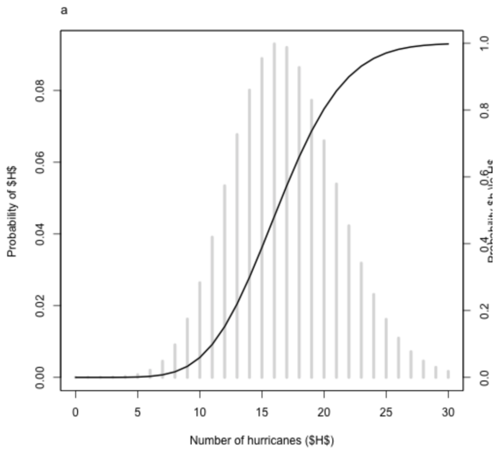
Loss Probability : Statistical Perspective

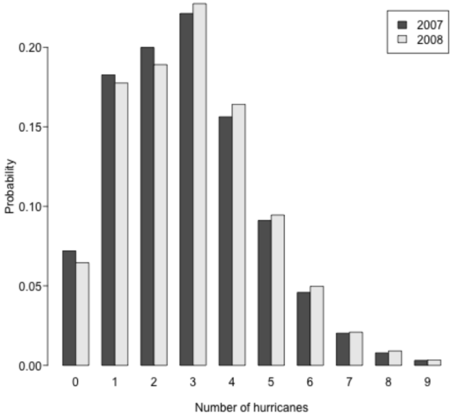
See Elsner & Jagger (2013) for a exhaustive statistical analysis of hurricanes,

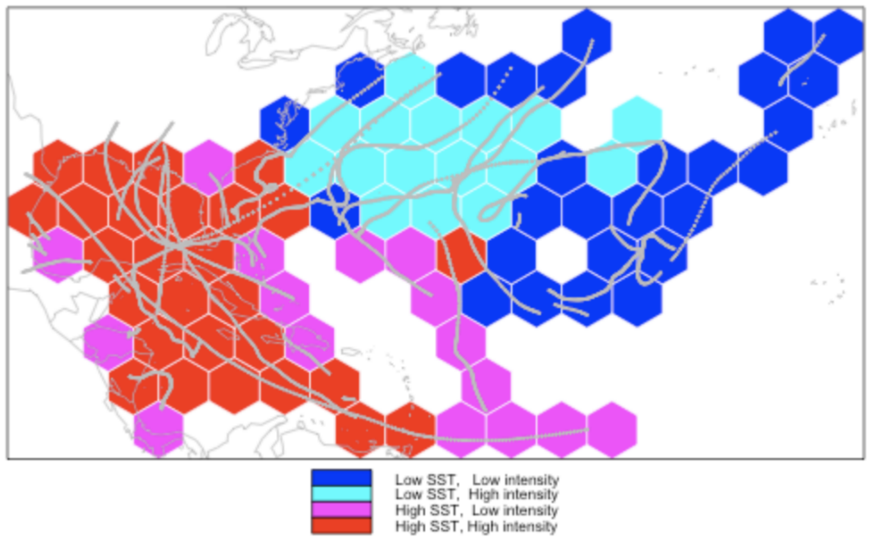
- Poisson regression for counts
– Southern Oscillation Index (SOI)
– sea-surface temperature (SST)
- regression for intensity
- spatial models for trajectories
- Bayesian models for one year prediction











Statistical Perspective

hard to justify in a changing environment

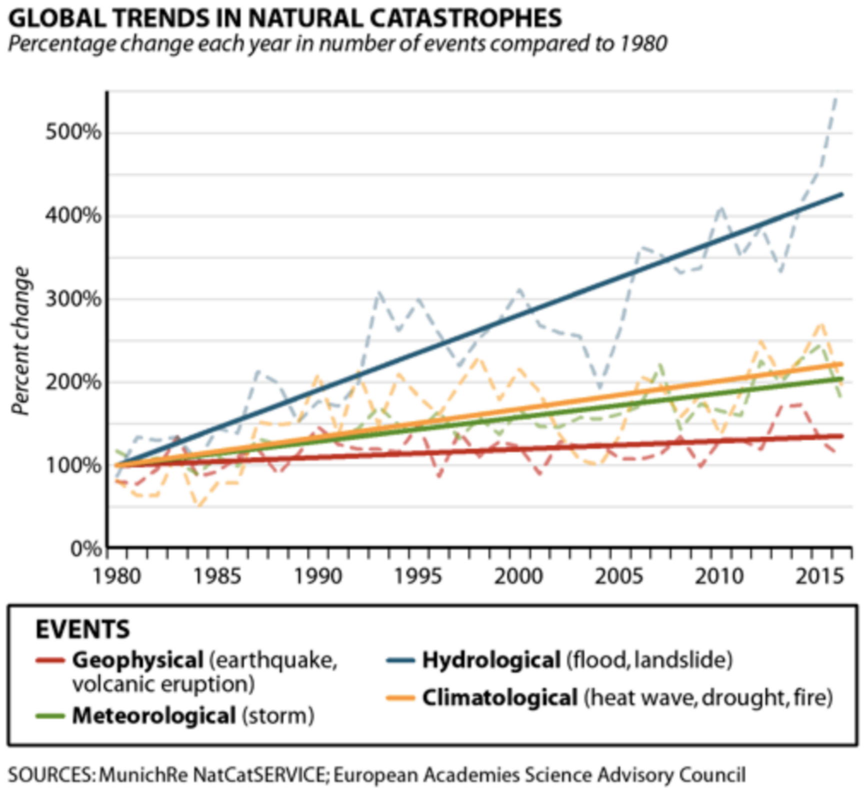
In order to apply any theory we have to suppose that the data are homogeneous, i.e. that no systematical change of climate and no important change in the basin have occurred within the observation period and that no such changes will take place in the period for which extrapolations are made

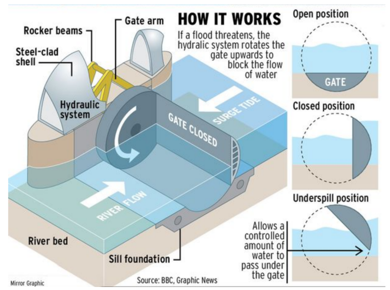
Gumbel (1941)

see flood risk (dams, reservoirs, etc)

see storms or heatwaves (climate change)

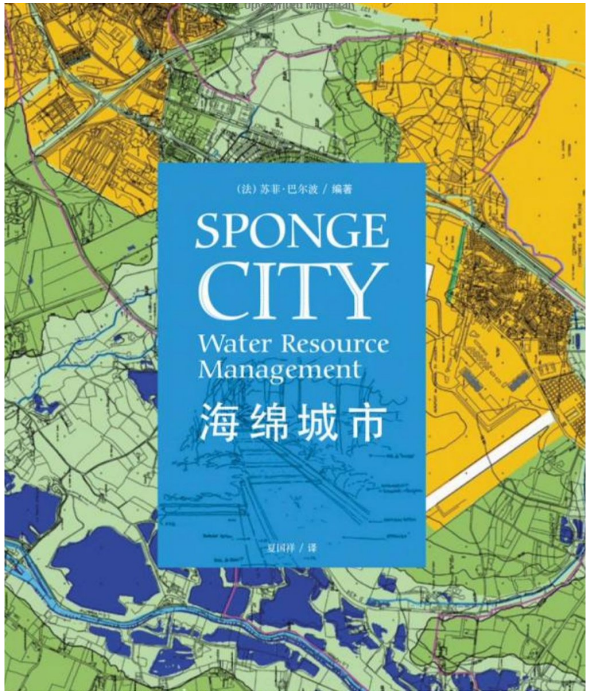
Charpentier (2011)





The Architect / Urban Planner Perspective ?

See Dong & Han (2011), Bardaux (2016), Li *et al.* (2017) Jiang *et al.* (2018) on sponge cities (urban underground water system operates like a sponge to absorb, store, leak and purify rainwater, and release it for reuse when necessary)



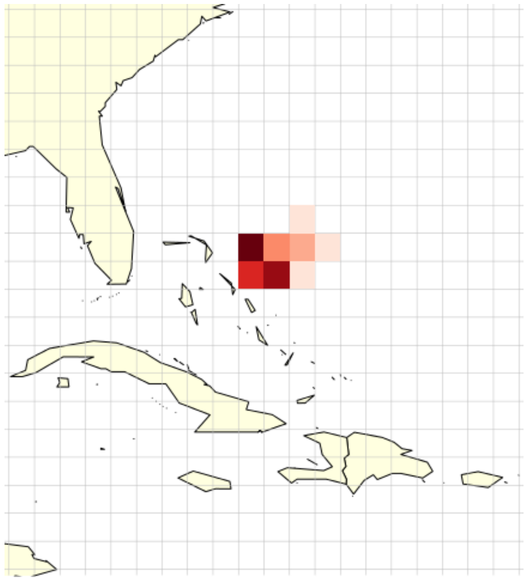
Loss Probability p : Actuarial Perspective

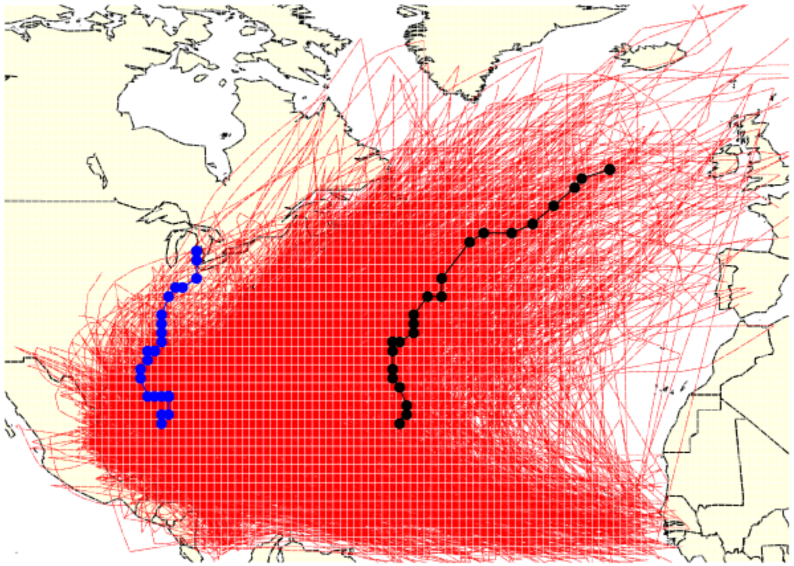
Actuaries use catastrophe softwares

- Risk Management Solutions (RMS)
- AIR Worldwide (AIR)
- Risk Quantification & Engineering (RQE) EQECAT

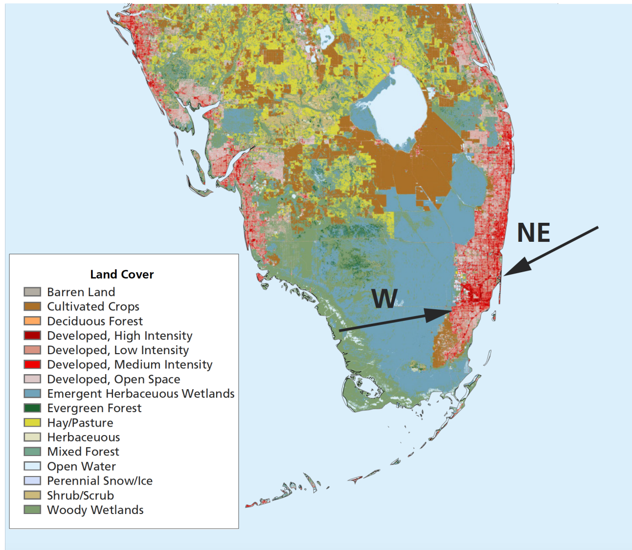
see Cole, Macpherson & McCullough (2010).

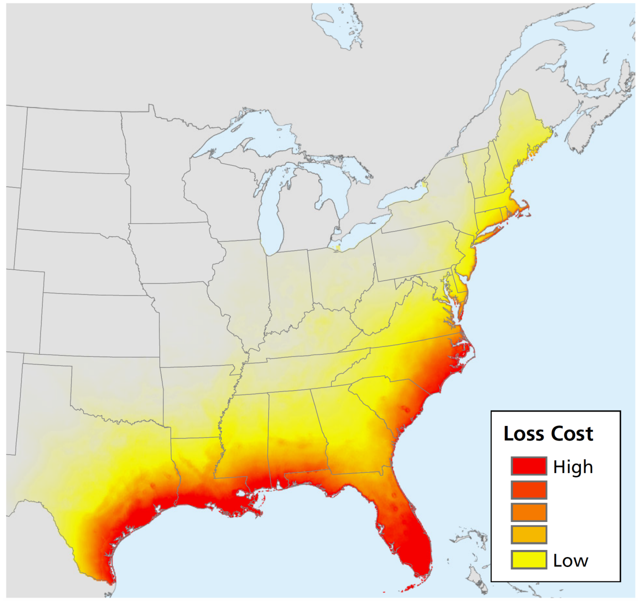
Generation of climatic scenarios (+ losses)





see for Markovian generation Charpentier (2014).



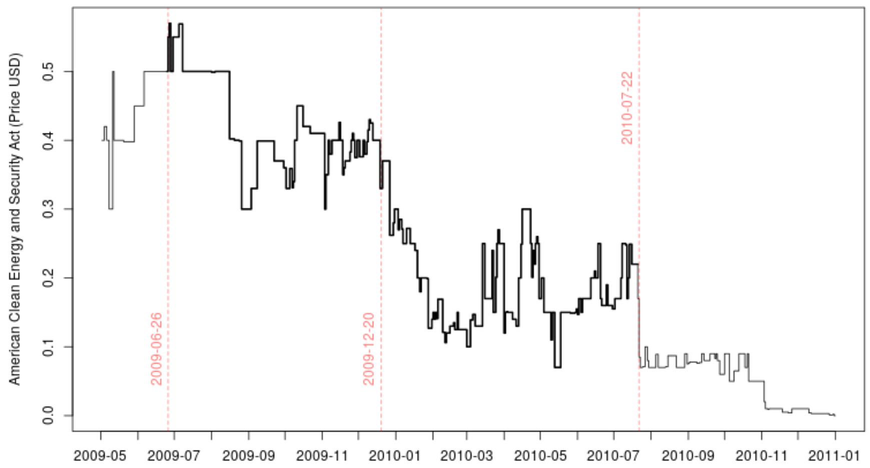


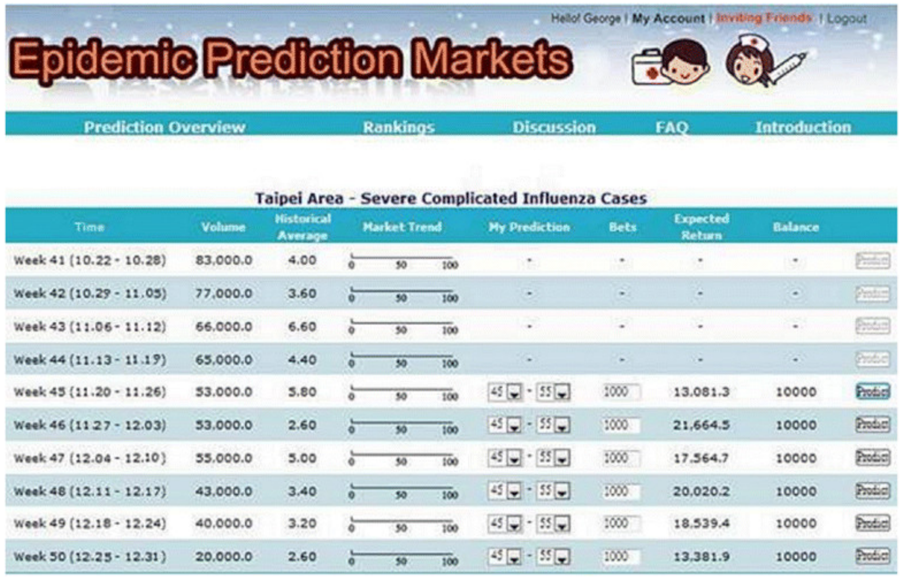
Predictive Markets

Peer prediction systems

Wolfers & Zitzewitz (2004) for a description

- elections (e.g. popes, XVIth Century) Rhode & Strumpf (2008)
- politics, Meng (2016) Waxman-Markey bill (2009-2010, finally rejected)
- sports (racetrack bets) Ali (1977)
- infectious diseases, Polgreen, Nelson & Neumann (2006) or Tung, Chou & Lin (2015)
- climate risk Hallstrom & Smith (2005) for hurricanes





(Perceived) Loss Probability p : Betting Markets

See Eisenberg & Gale (1959) model, on consensus and subjective probabilities (used in Manski (2004) on all-or-nothing contracts)

Individual i (wealth b_i) can bet on horse j an amount $\beta_{i,j}$ (so that $\beta_{i,1}+\cdots+\beta_{i,J}=b_i$). Assume that $b_1+\cdots+b_I=1$.

Let π_j denote the sum bet on horse j, $\pi_j=\beta_{1,j}+\cdots+\beta_{I,j}$. Observe that from the budget constraint $\pi_1+\cdots+\pi_J=1$, hence π_j's are probabilities.

Individual beliefs can be related expressed through a probability vector $\boldsymbol{p}_i=(p_{i,1},\cdots,p_{i,J})$, then there is an equilibrium if

$$p_{i,j}=\pi_j\cdot\max_s\left\{\frac{p_{i,s}}{\pi_s}\right\}\text{ as soon as }\beta_{i,j}>0$$

(so called Eisenberg-Gale equilibrium).

(Perceived) Loss Probability p : Betting Markets

Consider a contract that pay \$1 if an event A occurs (e.g. major flood), sold at price π_A. If there is a contract on $\bar{A}$, then $\pi_{\bar{A}} = 1 - \pi_A$ (no arbitrage).

Assume that individual i believe that A will occur with probability p_i. If $p_i > \pi_A$, he should purchase b_i units of A (otherwise b_i units of $\bar{A}$).

There is equilibrium if

$$\sum_{i=1}^{I} b_i = \frac{1}{\pi_A} \sum_{i=1}^{I} b_i \mathbb{P}[p_i > \pi_A] = \frac{1}{\pi_{\bar{A}}} \sum_{i=1}^{I} b_i \mathbb{P}[p_i < \pi_A]$$

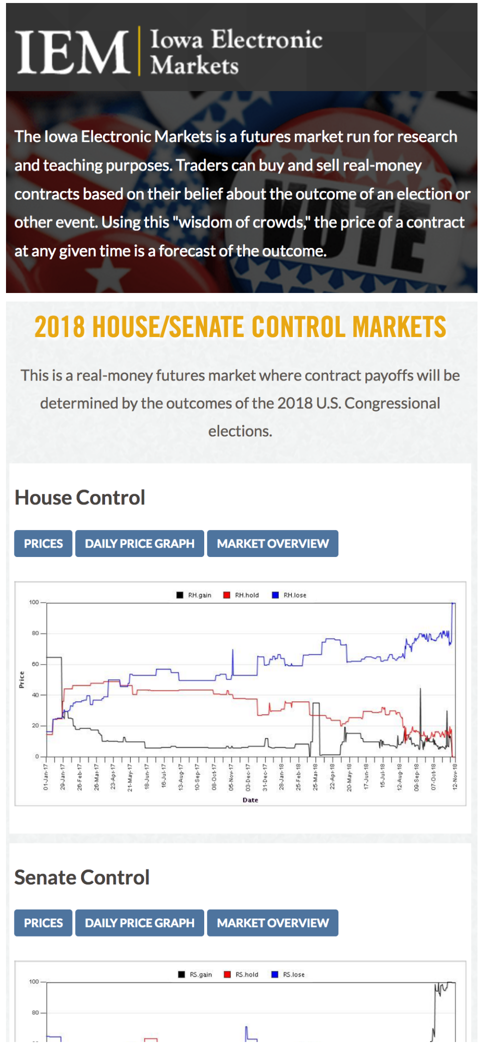
hence, if wealth b_i is independent of belief p_i, then prices are probabilities

$$\pi_A = \mathbb{P}[p_i > \pi_A] = 1 - \pi_{\bar{A}}.$$

see Wolfers & Zitzewitz (2004) for further interpretation.

Parimutuel and Betting Markets

- 2004 and 2005, ICAP, Goldman and Nymex (see IFR) with parimutuel derivative on energy (changes in crude oil and natural gas inventories)
- 2004, CME launched futures trading on the US Consumer Price Index (CPI), see Filippov (2005)
- 2008, LLC (see Ou-Yang (2010)) with parimutuel derivative on hurricanes
- The longitude plateforme (“*real time calculation of the odds*”).
- The Iowa Electronic Markets for elections, “*using this* **wisdom of crowds** *the price of a contract at any given time is a forecast of the outcome*”



HuRLOs (Hurricane Risk Landfall Option) options

The exist the Hurricane Futures Market (HFM), see Kelly, Letson, Nelson, Nolan & Solis (2010) for a description

Hurricane Risk Landfall Options (HuRLOs) were launched in October 2008 by Weather Risk Solutions, LLC (WRS) and promised a mutualized marketplace for hurricane options, using parimutuel options

From French pari mutuel, literally, mutual stake :

"*A system of betting on races whereby the winners divide the total amount bet, after deducting management expenses, in proportion to the sums they have wagered individually*", see Baron & Lang (2007).

also calle universal Dutch auction in Syz (2008)

Parimutuel and Betting Markets

"*A Shapley-Shubik market game for contingent claims within a probability state space with secured selling is a parimutuel market*" (Lange & Economides (2005))

A matching / pricing mecanism is necessary to get financial efficiency (self-hedging). Participant i submit orders, state bids $\boldsymbol{\beta}_i$, b_i a limit share quantity and $\bar{\pi}_i$ a limit price per share

The market organizer determines the order fill x_i (and state prices π_j), using call auction mechanism, see Peters, So and Ye (2005) and Agrawal, Wang & Ye (2008).

Using a Linear Programming Market Mechanism, market organizer should solve

$$\text{maximize} \left\{ \sum_{i=1}^{I} \bar{\pi}_i x_i - z \right\} \text{ where } z \text{ is some worst case cost}$$

$$\text{subject to} \quad \text{cost if state } j \text{ occurs} = \sum_{i=1}^{I} \beta_{i,j} x_i \leq z \quad \forall j$$

$$0 \leq \bar{\pi}_i x_i \leq b_i \quad \forall i$$

Parimutuel and Betting Markets

More realistically, one can consider some dynamic parimutuel market maker as in Agrawal *et al.* (2014) for recent advances, and Chen & Pennock (2010) for an overview.

See also techniques to get some peer prediction Bayesian Nash equilibrium solvers (see Jurca & Faltings (2008) on elections and bets) and the related collective revelation mechanism (see Goel, Reeves & Pennock (2009))

"*Peer prediction systems are designed for eliciting information on events where ground truth does not exist or is unobtainable*", Chen & Pennock (2010)

Take-Away Conclusion

- knowing real probabilities of occurrence disasters is either complicated (hurricanes) or very complicated (flood)
- those probabilities are necessary to assess solvency of insurance companies (central limit theorem is based on true probabilities)
- insurance pricing is based on beliefs of insured, and insurance companies
- predictive markets can be an interesting revelation mechanism of crowd beliefs (can be used to assess if an insurance market can actually exist, or not)