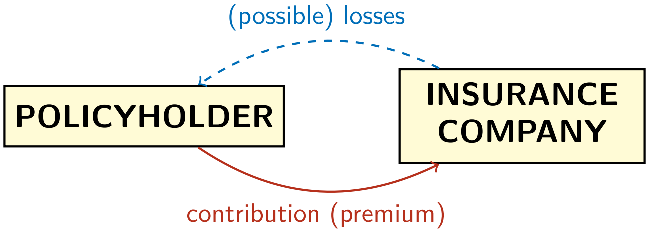
Insurance & Networks

Arthur Charpentier (UQAM)
Joint Research Initiative - AXA Research Fund

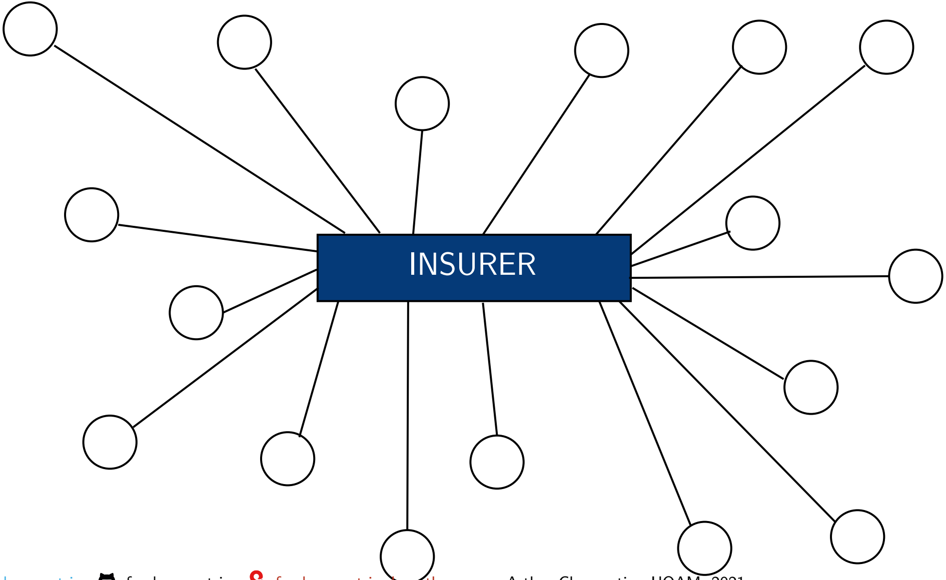
with L. Kouakou, F. Vermet, E. Flachaire, ...

Institut des Actuaires & Institut Louis Bachelier, 2021

Insurance, in a nutshell



"*Insurance is the contribution of the many to the misfortune of the few* "
Insurance works only with a large number of policyholders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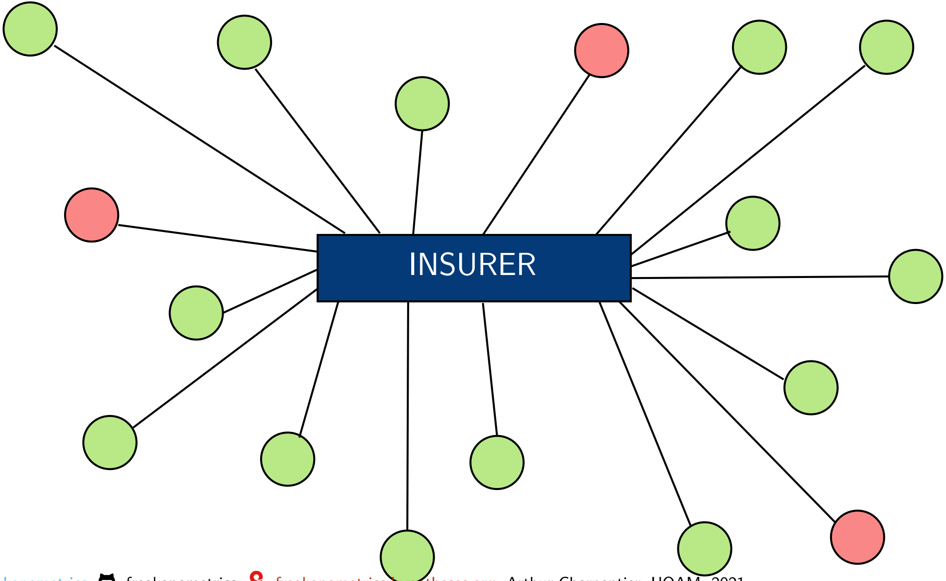
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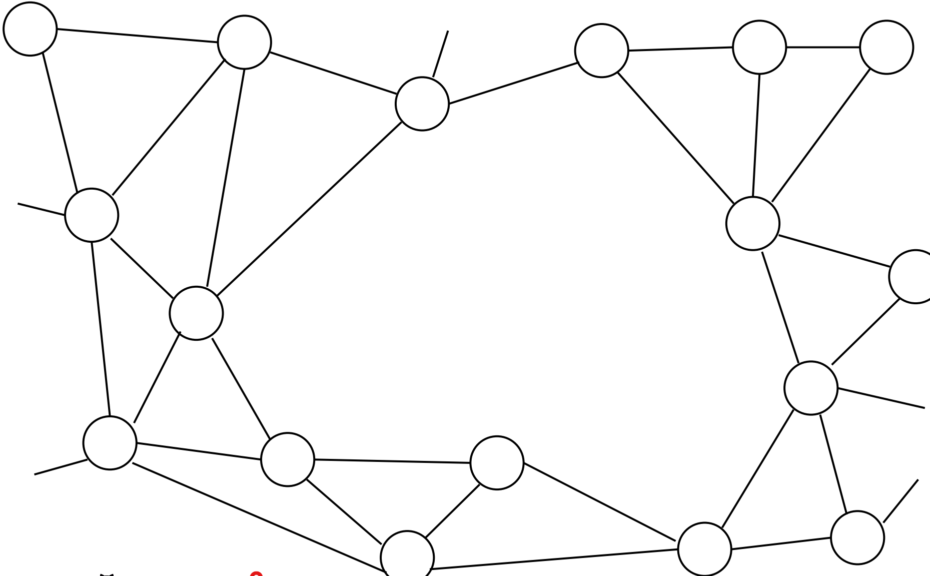
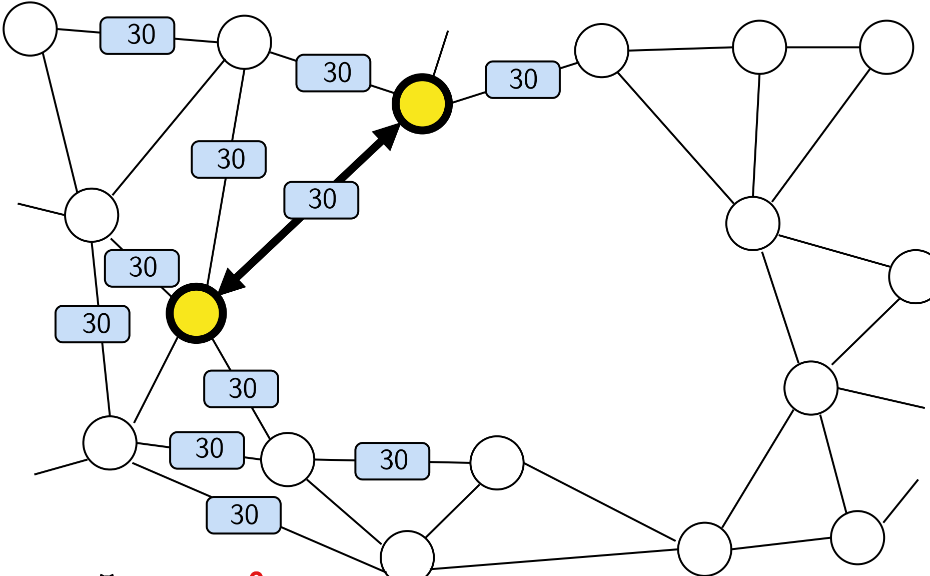
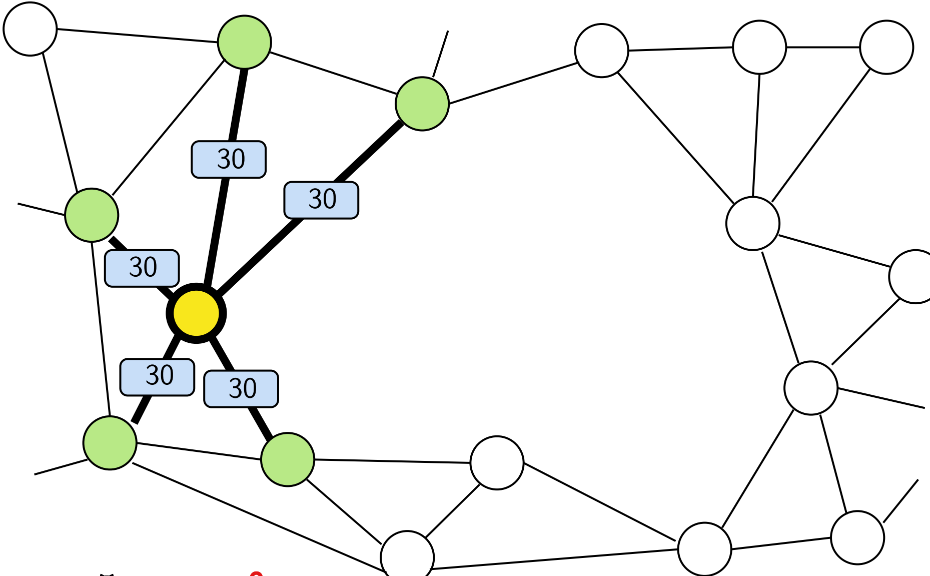
Insurance on a Network



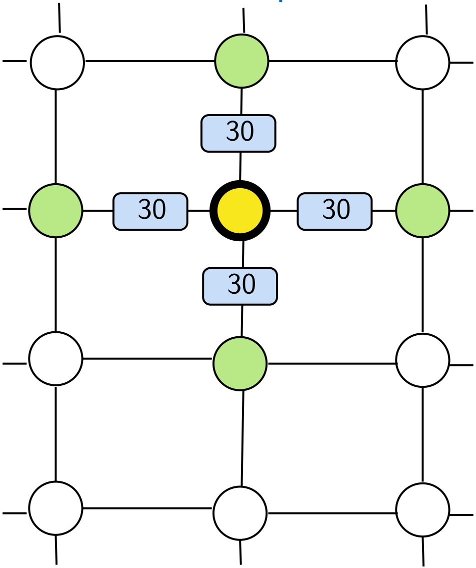
Collaborate Insurance on a Network

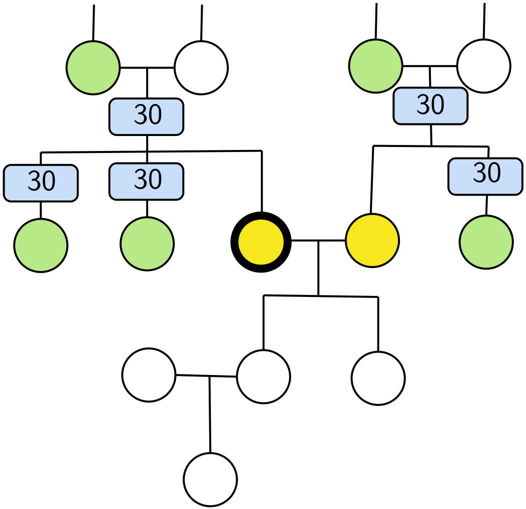


Collaborate Insurance on a Network

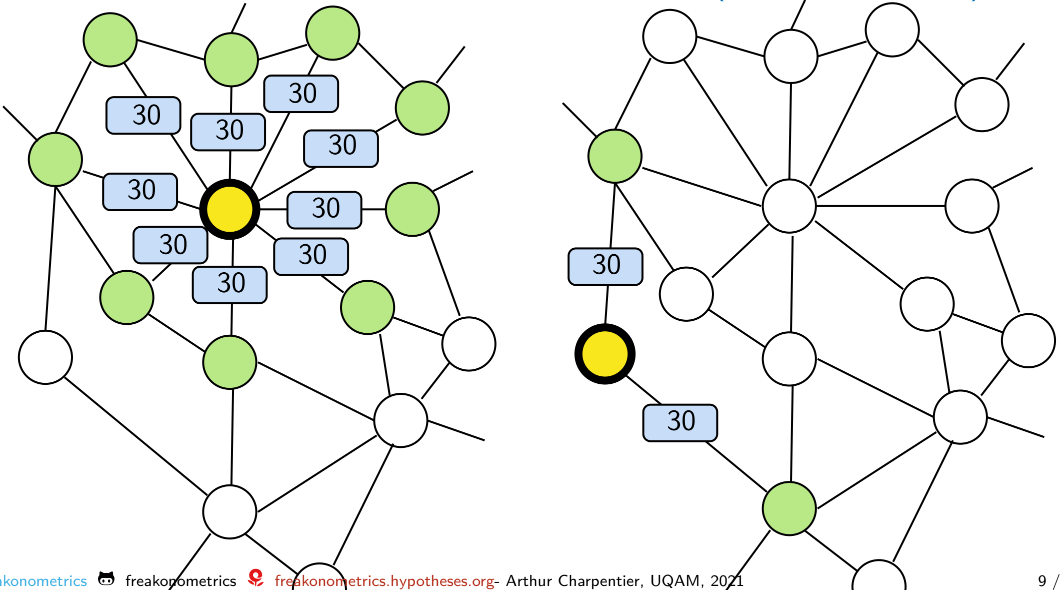


Shapes of Networks (regular mesh & family tree)





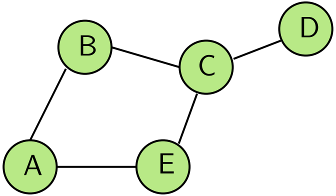
Insurance on a Network (centrality issues)



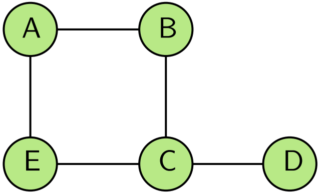
Network, some mathematical formalism

Vertice $i \in \mathcal{V} = \{A, B, C, D, E\}$ and edges $(i, j) \in \mathcal{E}$

Adjacency matrix $\boldsymbol{A}$, $A_{i,j} = 1$ indicates a link (i, j)



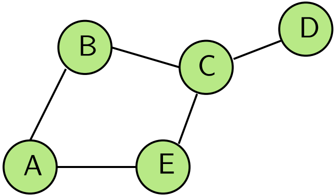
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

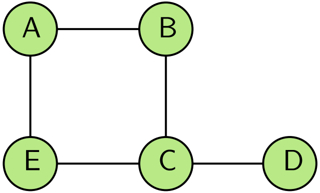


$\boldsymbol{d} = (\boldsymbol{d}_i) = \boldsymbol{A1}$ is called degree

"*There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement*", Freeman (1979))

Network, Friendship Paradox





The average degree of a friend is strictly greater than the average degree of a random node, see Feld (1991) and Zuckerman & Jost (2001).

$(A, 2)$, $(B, 2)$, $(C, 3)$, $(D, 1)$, $(E, 2)$

$$\frac{2+2+3+1+2}{5} = 2$$

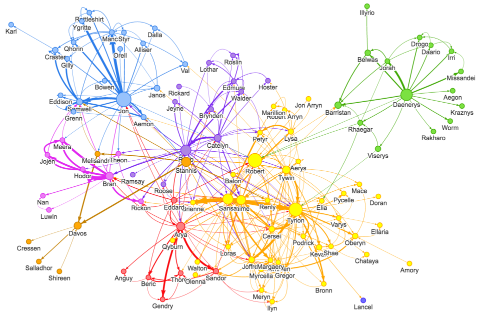
$A : (B, 2), (E, 2)$, $B : (A, 2), (C, 3)$,
$C : (B, 2), (D, 1), (E, 2)$, $D : (C, 3)$, $E : (A, 2), (C, 3)$

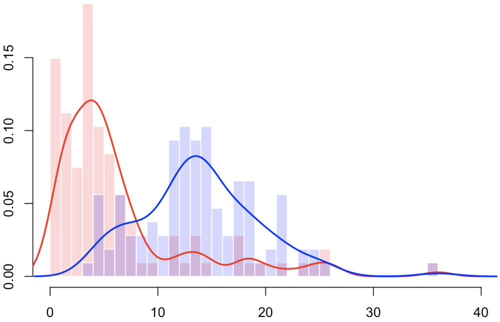
$$\frac{1}{5}\left(\frac{2+2}{2} + \frac{2+3}{2} + \frac{2+2+1}{3} + \frac{3}{1} + \frac{2+3}{2}\right) = \frac{7}{3} > 2$$

The average number of friends of a random person in the graph is

$$\mu = \frac{1}{n_{\mathcal{V}}}\sum_{v\in\mathcal{V}} d(v) = \frac{2n_{\mathcal{E}}}{n_{V}} = \frac{1}{n_{\mathcal{V}}}\|\boldsymbol{d}\|_1 \quad (= \mathbb{E}(D))$$

where $\boldsymbol{d}$ is the vector of $d(v)$'s (or $\boldsymbol{d} = \boldsymbol{A}\mathbf{1}$).





The average number of friends that a typical friend has is

$$\frac{1}{n_{\mathcal{V}}}\sum_{v\in\mathcal{V}}\left(\frac{1}{d(v)}\sum_{v':(v,v')\in\mathcal{E}}d(v')\right)=\frac{1}{\|\boldsymbol{d}\|_1}\boldsymbol{d}^\top\boldsymbol{d}\quad\Big(=\frac{\mathbb{E}(D^2)}{\mathbb{E}(D)}\Big)$$

We can prove that

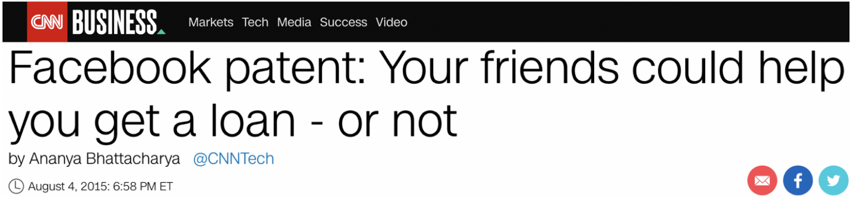
$$\frac{1}{\|\boldsymbol{d}\|_1}\boldsymbol{d}^\top\boldsymbol{d}\geq\frac{1}{n_{\mathcal{V}}}\|\boldsymbol{d}\|_1$$

See also

$$\frac{\mathbb{E}[D^2]}{\mathbb{E}[D]}=\mathbb{E}[D]+\frac{\text{Var}[D]}{\mathbb{E}[D]}\geq\mathbb{E}[D]$$

because $\text{Var}[D]\geq 0$.

Friendship Paradox


CNN BUSINESS Markets Tech Media Success Video

Facebook patent: Your friends could help you get a loan - or not

by Ananya Bhattacharya @CNNTech

August 4, 2015: 6:58 PM ET

"*You apply for a loan and your would-be lender somehow examines the credit ratings of your Facebook friends.* **If the average credit rating of these members is at least a minimum credit score, the lender continues to process the loan application**. *Otherwise, the loan application is rejected," the patent states.*"

See Facebook patent: Your friends could help you get a loan - or not

Consider some positive variable y observed at each node v. It can be the credit score of friends, or activity of people on twitter, as in Hodas *et al.* (2013)

Using the extension of König-Huyghens, $\text{Cov}[D,Y]=\mathbb{E}[DY]-\mathbb{E}[D]\mathbb{E}[Y]$, write $\boldsymbol{d}^\top\boldsymbol{y}$ as

$$\boldsymbol{d}^\top\boldsymbol{y}=\frac{\|\boldsymbol{d}\|_1\|\boldsymbol{y}\|_1}{n_V}+n_V\text{Cov}[D,Y]$$

or

$$\frac{1}{\|\boldsymbol{d}\|_1}\boldsymbol{d}^\top\boldsymbol{y}=\frac{1}{n_V}\|\boldsymbol{y}\|_1+\frac{n_v}{\|\boldsymbol{d}\|_1}\text{Cov}[D,Y]$$

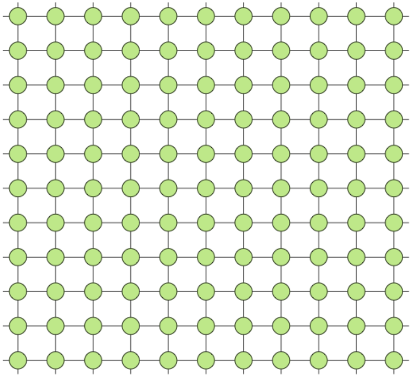
This can be written also

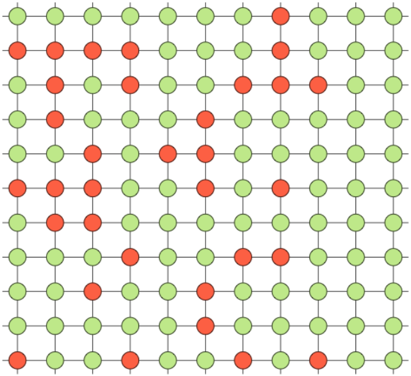
$$\frac{\mathbb{E}[DY]}{\mathbb{E}[D]}=\mathbb{E}[Y]+\frac{\text{Cov}[D,Y]}{\mathbb{E}[D]}$$

Hence,

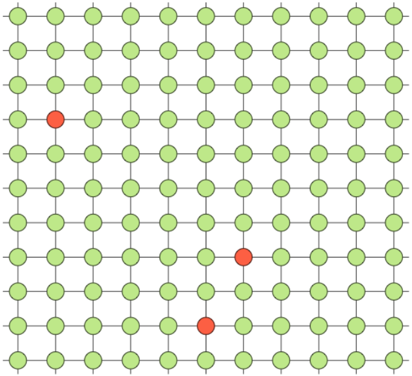
$$\frac{\mathbb{E}[DY]}{\mathbb{E}[D]}=\mathbb{E}[Y]\text{ if }Y\perp D,\text{ and }\frac{\mathbb{E}[DY]}{\mathbb{E}[D]}>\mathbb{E}[Y]\text{ if }\text{Cov}[D,Y]>0.$$

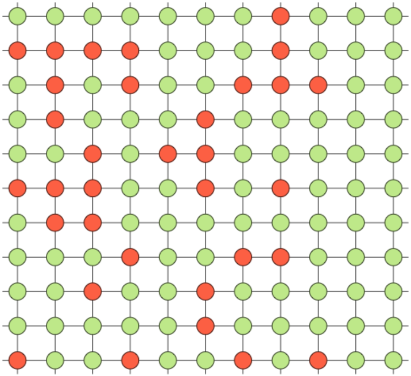
Sampling issues...



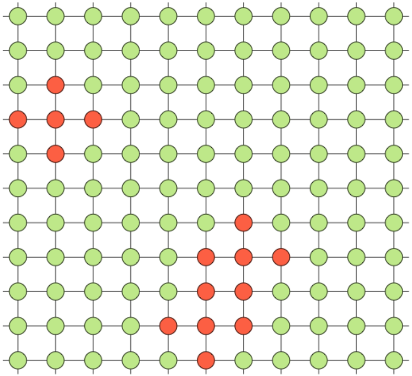


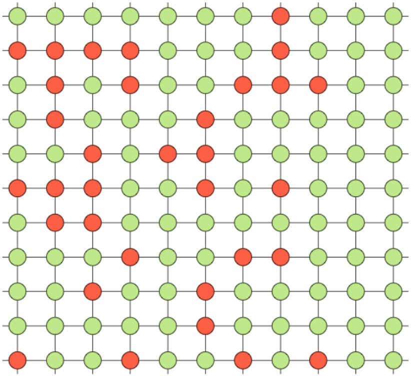
Snowball: draw randomly $k \ll s$ nodes





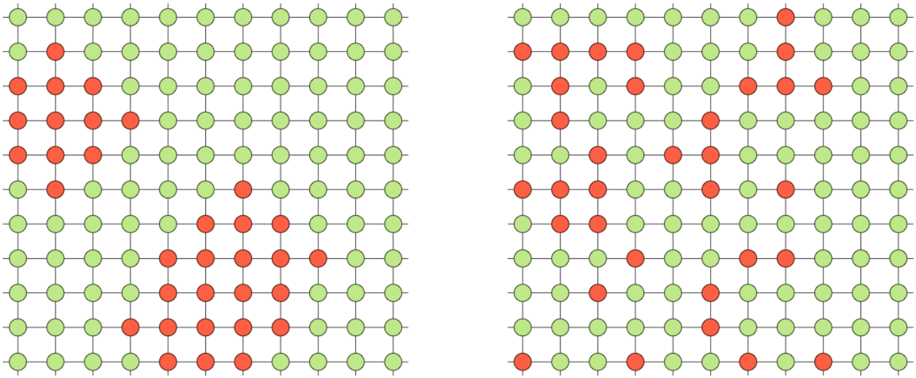
keep neighbors of previously selected nodes





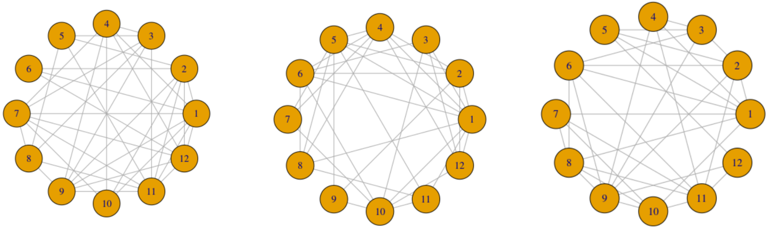
Networks, from Probability to Statistics

keep neighbors of previously selected nodes



see Respondent-driven sampling (RDS)

The Erdös-Renyi random graph model $G_{n,p}$ is an undirected graph with n vertices, such that edge (v, v') is present with probability p, independent of other edges.



$$f_\delta = \mathbb{P}[d(v) = \delta] = \binom{n-1}{\delta} p^\delta (1-p)^{n_V - 1 - \delta}$$

For a large network ($n_V \to \infty$), $d(v) \sim \mathcal{N}(n_V p, n_V p(1-p))$ from the law of large numbers.

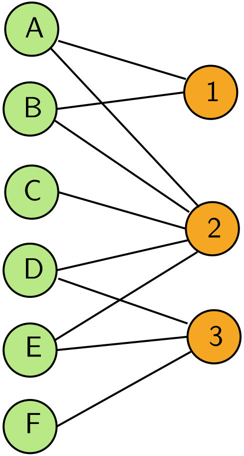
For a large netword with $p \sim \lambda / n_V$, $d(v) \sim \mathcal{P}(\lambda)$ from the law of small numbers.

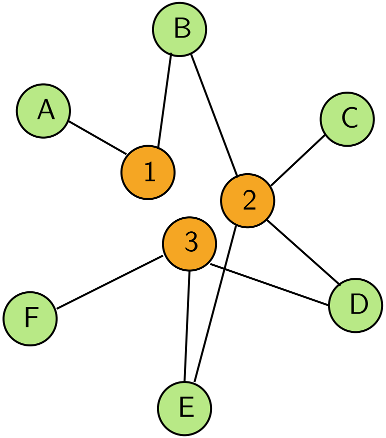
Scale-free network: degree distribution with power-law tail,
see also Barabasi-Albert graphs
The normalized power-law degree distribution is

$$f_\delta = \mathbb{P}[d(v) = \delta] = \frac{\alpha - 1}{\delta_0}\left(\frac{\delta}{\delta_0}\right)^{-\alpha}, \text{ for } \delta \geq \delta_0.$$

See Pareto distribution, $\overline{F}(x) = \mathbb{P}[X > x] = x^{-\alpha+1}$
"Scale-free networks are rare", Broido & Clauset (2019).
Sampling issue, and second order property, see Stumpf, Wiuf & May (2005) or Charpentier & Flachaire (2019).

Networks & Fraud

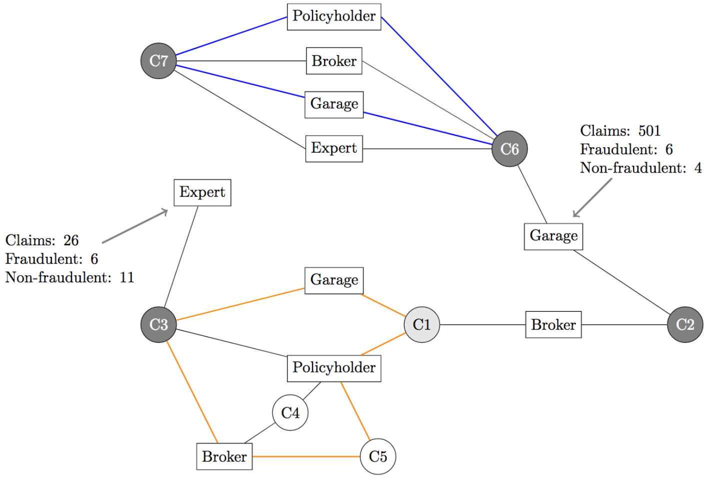




Bipartite graphs
vertices can be divided into two disjoint and independent sets
workers and employers
matching problem

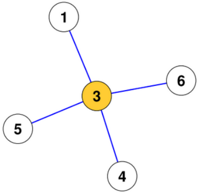
see Charpentier, Galichon & Vernet (2019)

See Óskarsóttir *et al.* (2020) on fraud detection in insurance

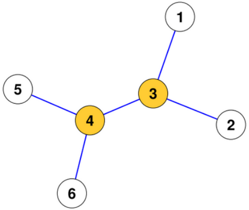


"*There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement*", Freeman (1979)

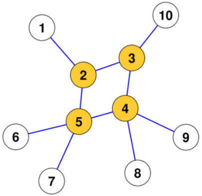
See Wang *et al.* (2017) on robustness of metro networks



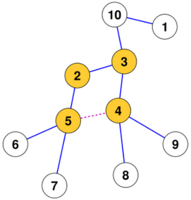
(a) Rome.



(b) Cairo and Marseille.



(c) Montreal.

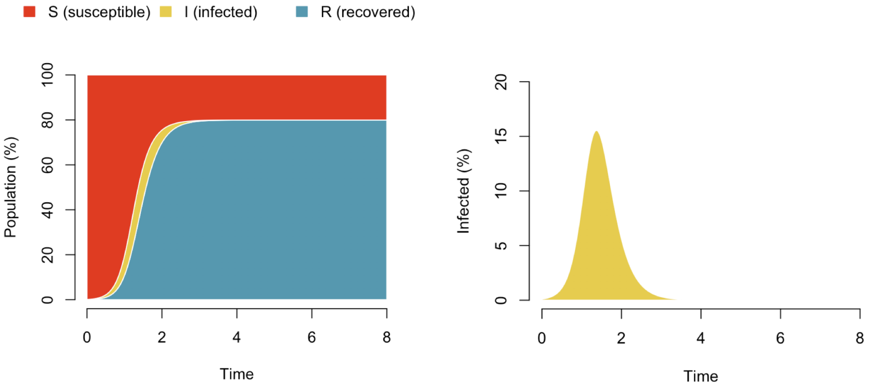


(d) Toronto.

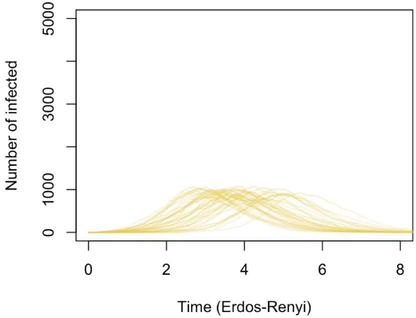
or on edges, as in cyber-risk, see Cohignac & Kazi-Tani (2020)

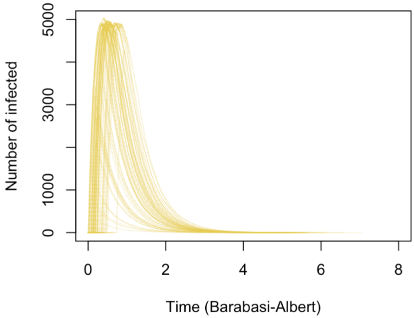
Kermack & McKendrick (1927), three groups, S_t (susceptible) I_t (infected) R_t (recovered)

$$\frac{dS_t}{dt}=-\beta I_t S_t,\ \frac{dI_t}{dt}=\beta I_t S_t-\gamma I_t,\ \frac{dR_t}{dt}=\gamma I_t$$

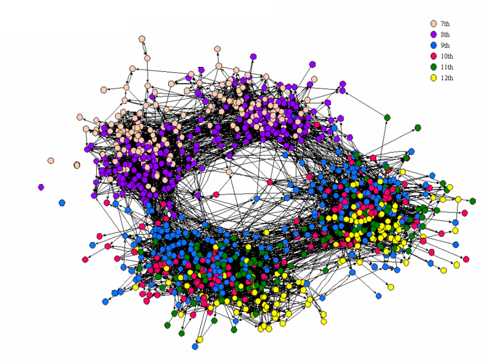


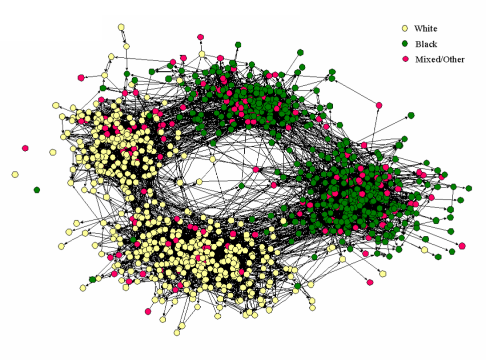
With Erdös-Renyi random graphs, or Barabasi-Albert graphs (scale free / power law)





Networks, Race & Friendship in Schools





Homophily: tendency of individuals to associate and bond with similar others, "*birds of a feather flock together*", from Moody (2001)

Networks & Collaborative Insurance

FEATURES

PEER-TO-PEER INSURANCE

GOING BACK TO BASICS

Innovators have set their sights on simplifying insurance by adopting new peer-to-peer business models. Who are the key players and does the Australian industry need to pay attention?

Janis Meyer-Plath, Sebastian Herfurth and Tim Kunde of Friendsurance

DISCUSSION OF the sharing economy has taken up considerable column space in recent times.

PricewaterhouseCoopers estimates the five main sharing sectors (peer-to-peer finance, online staffing, peer-to-peer accommodation, car sharing and music video streaming) will potentially generate global revenues of US$335bn (A$439bn) by 2025. Today, PwC says revenue generated is around US$15bn (A$19.67bn).

One sharing economy sector still very much in its infancy is peer-to-peer insurance. But Amy Gibbs, digital communications and content strategy manager at ANZIIF, says popularity of the concept is increasing.

"When it takes off, it will likely happen much quicker than we expect," she says.

"P2P insurance is not about a new technology threatening an industry but about people demanding an industry that gives them what they believe they deserve. Consumers expect different things in 2016, and technology now allows them the power to get what they want or go elsewhere."

The current crop in P2P

On 24 March, Germany's high-profile P2P player, Friendsurance, announced it had collected US$15.3m (A$20.26m) from investors in its latest round of financing.

Tim Kunde, Friendsurance's co-founder and managing director, says the organisation intends to use that fresh capital to grow further in Germany and expand internationally. He says the first expansion target for 2016 is Australia, and expansion opportunities for other markets are currently being considered.

Friendsurance is one of the players Gibbs has been keeping a close eye on.

"Friendsurance and Guevara are ones that I watch keenly," she says.

"They have interesting models and appear to have put a lot of thought into them, plus they get the marketing/consumer angle, which is crucial."

Friendsurance, founded in Germany in 2010, operates as an independent insurance broker. It describes its mission as to make insurance easier and more affordable for customers, and to reduce the number of fraudulent claims.

"Our idea is inspired by insurance in its original form, when people got together in small groups … and supported each other in case of damage," Tim Kunde, Friendsurance's co-founder and managing director, tells *Insurance Business*.

"This was easy and efficient but also limited in the extent of coverage. Today, big insurance companies can carry claims of any amount, but marketing, administration and fraud cause remarkable costs.

"P2P insurance is not about a new technology threatening an industry but about people demanding an industry that gives them what they believe they deserve"

Amy Gibbs, ANZIIF

"Against this background, we developed an insurance concept that again [creates] smaller groups within bigger insurance societies and rewards remaining claimless within [these groups] with annual cashbacks."

Friendsurance customers with the same insurance type form small groups online. Part of their insurance premium is paid into a cashback pool, and part of it is provided to the group's insurer (or reinsurer).

When small claims are made, customers are reimbursed from the general fund. Larger claims still go through the insurer. Groups that have no claims during a year receive a cashback bonus the following January. Friendsurance says its claims-free bonus is available for private liability, home contents and legal expenses insurance.

"So far, more than 80% of users received some of their insurance fees back," says Kunde.

"In the property insurance line, the average cashback was 33% of the paid insurance fees."

Kunde reports that, in 2015, Friendsurance engaged 75,000 new customers.

"Today, we have a six-digit number of customers, 70 insurance partners, 15 copycats and 80 employees."

Over in the UK, start-up Guevara has received

34 www.insurancebusinessonline.com.au

www.insurancebusinessonline.com.au 35

via Insurance Business (2016), see Charpentier (2019)

Networks & Collaborative Insurance

insPeer: car insurance in France, 2014

Lemonade: household insurance, in the U.S., 2015

groupago: car & household insurance, in Belgium, 2015

besure: general insurance, in Canada, 2017

Networks & Collaborative Insurance

Kouakou (2020) considered risk sharing (transfer) of the deductible, under strong assumptions (identical risks, identical deductible)

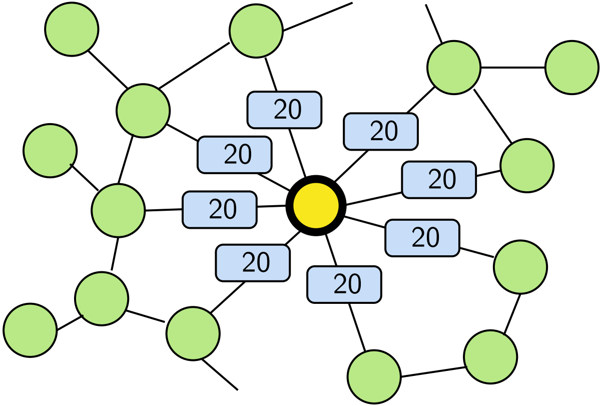
Question : How to set the amount of reciprocal commitments in order to optimize the collaborative franchise product?
Question : What social graph model(s) can be used to optimize the collaborative franchise product?

very sensitive to the structure of the network...

Frequency $\mathcal{B}(1/10)$ (i.i.d.)
Loss $\mathcal{G}(100, 250)$
Deductible 100
Network: $\mathbb{E}[D] = 5$

Contribution $\dfrac{100}{5} = 20$
(uniform, constant and reciprocal)

We generate some random networks given a distribution of degrees average $\mathbb{E}[D]$ and standard deviation σ



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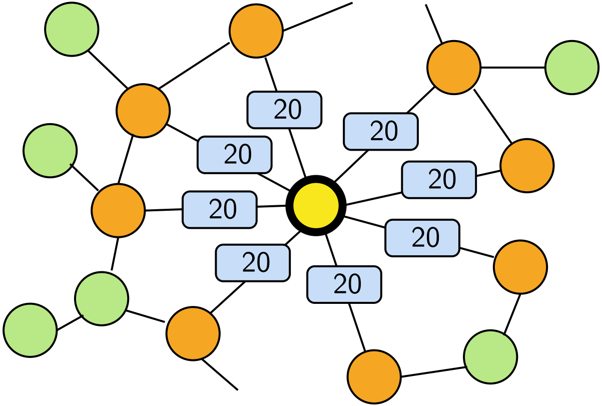

Networks & Collaborative Insurance

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