Insurance pricing in a competitive market

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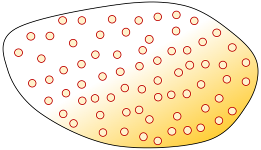
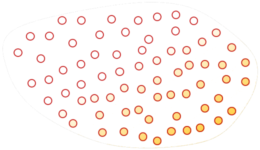
"*Insurance is the contribution of the many to the misfortune of the few* "

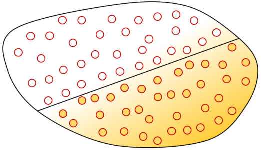
What should be that "*contribution*" ?



mutualization (of risk) : *spread of risk among several parties* see pooling, sharing $\pi = \mathbb{E}_{\mathbb{P}}[S_1]$



(market) segmentation : *division of a market into identifiable groups* see differentiation, customization $\pi(\omega) = \mathbb{E}_{\mathbb{P}}[S_1 | \Omega = \omega]$ for some (unobservable) risk factor Ω



Use of features (covariates) $\boldsymbol{x}$ as a proxy

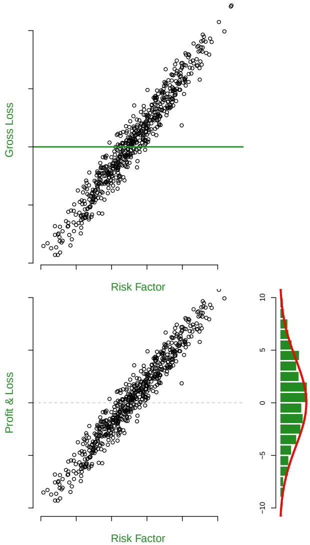
$\pi(\boldsymbol{x}) = \mathbb{E}_{\mathbb{P}}[S_1 | \boldsymbol{X} = \boldsymbol{x}] = \mathbb{E}_{\mathbb{P}_{\boldsymbol{x}}}[S_1]$

Risk Transfert without Segmentation

	Insured	Insurer
Loss	$\mathbb{E}[S]$	$S - \mathbb{E}[S]$
Average Loss	$\mathbb{E}[S]$	0
Variance	0	$\text{Var}[S]$

All the risk - $\text{Var}[S]$ - is kept by the insurance company.

Remark: interpretations are discussed in Denuit & Charpentier (2004).



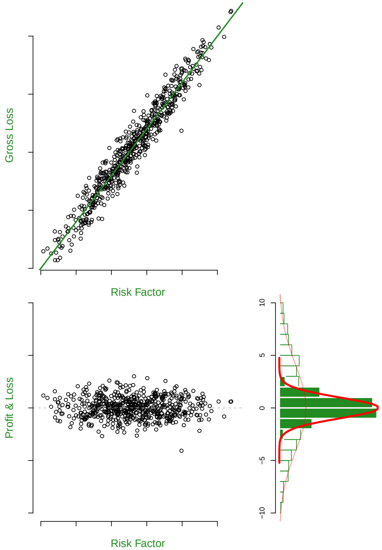
Risk Transfert with Segmentation and Perfect Information

Assume that information $\boldsymbol{\Omega}$ is observable,

	Insured	Insurer
Loss	$\mathbb{E}[S\vert\boldsymbol{\Omega}]$	$S-\mathbb{E}[S\vert\boldsymbol{\Omega}]$
Average Loss	$\mathbb{E}[S]$	0
Variance	$\text{Var}\Big[\mathbb{E}[S\vert\boldsymbol{\Omega}]\Big]$	$\text{Var}\Big[S-\mathbb{E}[S\vert\boldsymbol{\Omega}]\Big]$

Observe that $\text{Var}\Big[S-\mathbb{E}[S|\boldsymbol{\Omega}]\Big]=\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]$, so that

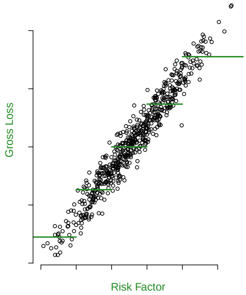
$$\text{Var}[S]=\underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\to\text{ insurer}}+\underbrace{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big]}_{\to\text{ insured}}.$$



Segmentation and Imperfect Information

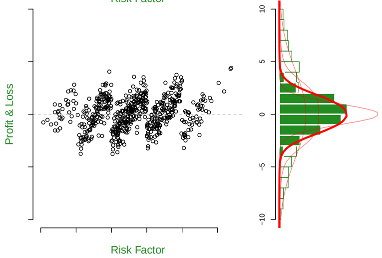
Assume that $\boldsymbol{X} \subset \boldsymbol{\Omega}$ is observable

	Insured	Insurer
Loss	$\mathbb{E}[S\|\boldsymbol{X}]$	$S - \mathbb{E}[S\|\boldsymbol{X}]$
Average Loss	$\mathbb{E}[S]$	0
Variance	$\text{Var}\big[\mathbb{E}[S\|\boldsymbol{X}]\big]$	$\mathbb{E}\big[\text{Var}[S\|\boldsymbol{X}]\big]$



Now

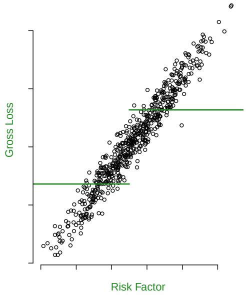
$$
\begin{aligned}
\mathbb{E}\Big[\text{Var}[S|\boldsymbol{X}]\Big] &= \mathbb{E}\Big[\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big] + \mathbb{E}\Big[\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big] \\
&= \underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\text{perfect pricing}} + \underbrace{\mathbb{E}\Big\{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big\}}_{\text{misfit}}.
\end{aligned}
$$



spiral of segmentation...

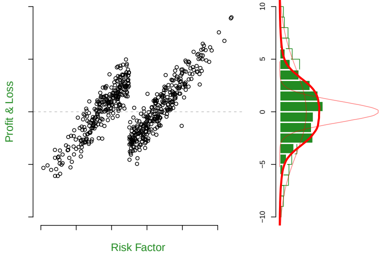
Assume that $\boldsymbol{X} \subset \boldsymbol{\Omega}$ is observable

	Insured	Insurer
Loss	$\mathbb{E}[S\|\boldsymbol{X}]$	$S - \mathbb{E}[S\|\boldsymbol{X}]$
Average Loss	$\mathbb{E}[S]$	0
Variance	$\text{Var}\big[\mathbb{E}[S\|\boldsymbol{X}]\big]$	$\mathbb{E}\big[\text{Var}[S\|\boldsymbol{X}]\big]$



Now

$$\begin{aligned}\mathbb{E}\Big[\text{Var}[S|\boldsymbol{X}]\Big] &= \mathbb{E}\Big[\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big] + \mathbb{E}\Big[\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big] \\ &= \underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\text{perfect pricing}} + \underbrace{\mathbb{E}\Big\{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big\}}_{\text{misfit}}.\end{aligned}$$



spiral of segmentation...

Actuarial Pricing Model

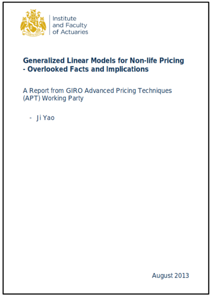
Premium is $\mathbb{E}[S|\boldsymbol{X}=\boldsymbol{x}] = \mathbb{E}\left[\sum_{i=1}^{N} Y_i \middle| \boldsymbol{X}=\boldsymbol{x}\right] = \mathbb{E}[N|\boldsymbol{X}=\boldsymbol{x}] \cdot \mathbb{E}[Y|\boldsymbol{X}=\boldsymbol{x}]$

Statistical and modeling issues to approximate based on some training datasets, with claims frequency $\{n_i, \boldsymbol{x}_i\}$ and individual losses $\{y_i, \boldsymbol{x}_i\}$.

Use GLM to approximate $\mathbb{E}[N|\boldsymbol{X}=\boldsymbol{x}]$ and $\mathbb{E}[Y|\boldsymbol{X}=\boldsymbol{x}]$

"*Most firms ... rely on traditional generalised linear models (GLMs)* [...]
A small number of firms use non-linear methods (e.g. decision trees) as input to GLMs" FCA (2016)

(see also Yiao (2013))



From Econometric to 'Machine Learning'

In a competitive market, insurers can use different sets of variables and different models, e.g. GLMs, $N_t|\boldsymbol{X} \sim \mathcal{P}(\lambda_{\boldsymbol{X}} \cdot t)$ and $Y|\boldsymbol{X} \sim \mathcal{G}(\mu_{\boldsymbol{X}}, \varphi)$

$$\widehat{\pi}_j(\boldsymbol{x}) = \widehat{\mathbb{E}}[N_1|\boldsymbol{X} = \boldsymbol{x}] \cdot \widehat{\mathbb{E}}[Y|\boldsymbol{X} = \boldsymbol{x}] \;=\; \underbrace{\exp(\widehat{\boldsymbol{\alpha}}^{\mathsf{T}}\boldsymbol{x})}_{\text{Poisson } \mathcal{P}(\lambda_{\boldsymbol{x}})} \cdot \underbrace{\exp(\widehat{\boldsymbol{\beta}}^{\mathsf{T}}\boldsymbol{x})}_{\text{Gamma } \mathcal{G}(\mu_{\boldsymbol{X}},\varphi)}$$

that can be extended to GAMs,

$$\widehat{\pi}_j(\boldsymbol{x}) = \underbrace{\exp\left(\sum_{k=1}^{d} \widehat{s}_k(x_k)\right)}_{\text{Poisson } \mathcal{P}(\lambda_{\boldsymbol{x}})} \cdot \underbrace{\exp\left(\sum_{k=1}^{d} \widehat{t}_k(x_k)\right)}_{\text{Gamma } \mathcal{G}(\mu_{\boldsymbol{X}},\varphi)}$$

or Tweedie models on S_t (compound Poisson, see Tweedie (1984)) conditional on $\boldsymbol{X}$

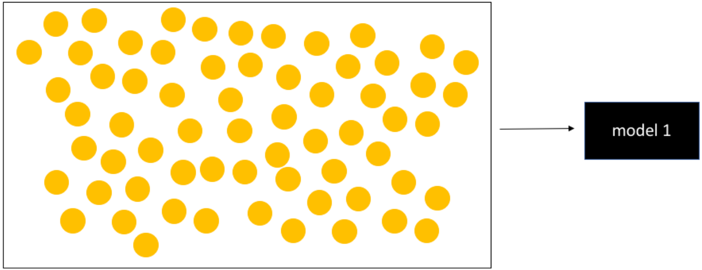
From Econometric to 'Machine Learning'

(see Charpentier & Denuit (2005) or Kaas *et al.* (2008)) or any other statistical model

$$\widehat{\pi}_j(\boldsymbol{x}) \text{ where } \widehat{\pi}_j \in \underset{m\in\mathcal{F}_j:\mathcal{X}_j\to\mathbb{R}}{\text{argmin}} \left\{\sum_{i=1}^{n}\ell(s_i, m(\boldsymbol{x}_i)) + \lambda\cdot\text{penalty}(m)\right\}$$

For some loss function $\ell:\mathbb{R}^2\to\mathbb{R}_+$ (usually an L_2 based loss, $\ell(s,y)=(s-y)^2$ since $\text{argmin}\{\mathbb{E}[\ell(S,m)],\ m\in\mathbb{R}\}$ is $\mathbb{E}(S)$, interpreted as the pure premium).
For instance, consider regression trees, forests, neural networks, or boosting based techniques to approximate $\pi(\boldsymbol{x})$, and various techniques for variable selection, such as LASSO (see Hastie *et al.* (2009) or Charpentier *et al.* (2017) for a description and a discussion).

Competitive Insurance Markets



$\widehat{\pi}_1(\cdot)$

Before discussing the use of those models in insurance, note that the same issues exist in credit, see Hardt, Price & Srebro (2017).

"*the shift from traditional to machine learning lending models may have important distributional effects for consumers* [...] *machine learning would offer lower rates to racial groups who already were at an advantage under the traditional model, but it would also benefit disadvantaged groups by enabling them to obtain a mortgage in the first place* " Fuster, Goldsmith-Pinkham, Ramadorai & Walther (2017)

Field experiment: actuarial pricing games

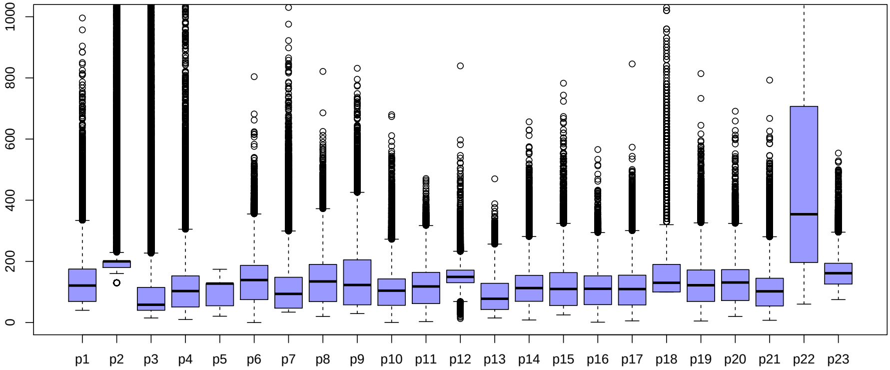
Actuarial pricing is data based,
and model based

To understand how model influence pricing
we ran some actuarial pricing games

With d competitors, each insured i has to choose among d premiums,

$$\boldsymbol{\pi}_i = (\widehat{\pi}_1(\boldsymbol{x}_i), \cdots, \widehat{\pi}_d(\boldsymbol{x}_i)) \in \mathbb{R}_+^d$$

Insurance Ratemaking Before Competition



(impact of various categorical variables)

Premiums Tail Correlations (strong)

Strong tail dependence between (say) π_1 and π_2

$$\lambda(u)=\begin{cases}\mathbb{P}[X_1\leq F_1^{-1}(u)|X_2\leq F_2^{-1}(u)] \text{ if } u\in(0,1/2)\\ \mathbb{P}[X_1> F_1^{-1}(u)|X_2> F_2^{-1}(u)] \text{ if } u\in(1/2,1),\end{cases}$$

e.g.

$$\lambda(u)=\frac{\mathbb{P}[X_1\leq F_1^{-1}(u),X_2\leq F_2^{-1}(u)]}{\mathbb{P}[X_2\leq F_2^{-1}(u)]} \text{ if } u\in(0,1/2)$$

estimated by (with $U_{1,i}=\widehat{F}_1(\pi_1(\boldsymbol{x}_i))$ and $U_{2,i}=\widehat{F}_1(\pi_2(\boldsymbol{x}_i))$)

$$\widehat{\lambda}(u)=\begin{cases}\displaystyle\frac{1}{nu}\sum_{i=1}^n \mathbf{1}[U_{1,i}\leq u,U_{2,i}\leq u] \text{ if } u\in(0,1/2)\\ \displaystyle\frac{1}{n(1-u)}\sum_{i=1}^n \mathbf{1}[U_{1,i}> u,U_{2,i}> u] \text{ if } u\in(1/2,1),\end{cases}$$

from Joe (1990), see also Charpentier (2012).

Premiums Tail Correlations (weak)

Weak tail dependence between (say) π_1 and π_2

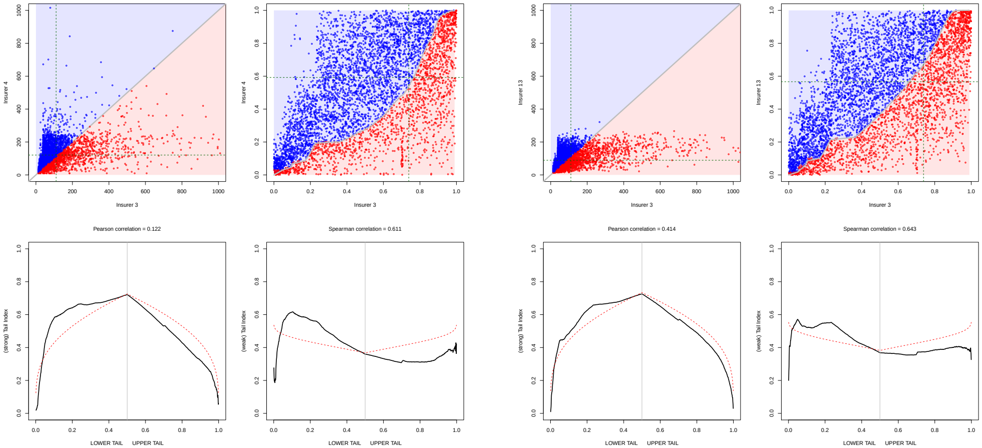
$$\chi(u) = \begin{cases} \frac{\log \mathbb{P}[X_2 \leq F_2^{-1}(u)]}{\log \mathbb{P}[X_1 \leq F_1^{-1}(u), X_2 \leq F_2^{-1}(u)]} & \text{if } u \in (0, 1/2) \\ \frac{\log \mathbb{P}[X_2 > qF_2^{-1}(u)]}{\log \mathbb{P}[X_1 > F_1^{-1}(u), X_2 > F_2^{-1}(u)]} & \text{if } u \in (1/2, 1), \end{cases}$$

estimated by

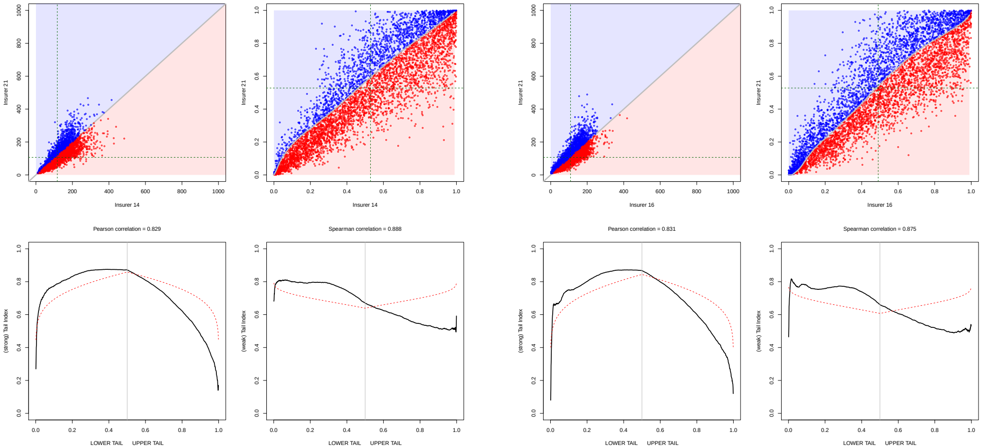
$$\widehat{\chi}(u)^{-1} = \begin{cases} \frac{1}{u} \log\left(\frac{1}{n}\sum_{i=1}^{n} \mathbf{1}[U_{1,i} \leq u, U_{2,i} \leq u]\right) & \text{if } u \in (0, 1/2) \\ \frac{1}{1-u} \log\left(\frac{1}{n}\sum_{i=1}^{n} \mathbf{1}[U_{1,i} > u, U_{2,i} > u]\right) & \text{if } u \in (1/2, 1), \end{cases}$$

from Ledford & Tawn (1996), see also Charpentier (2012).

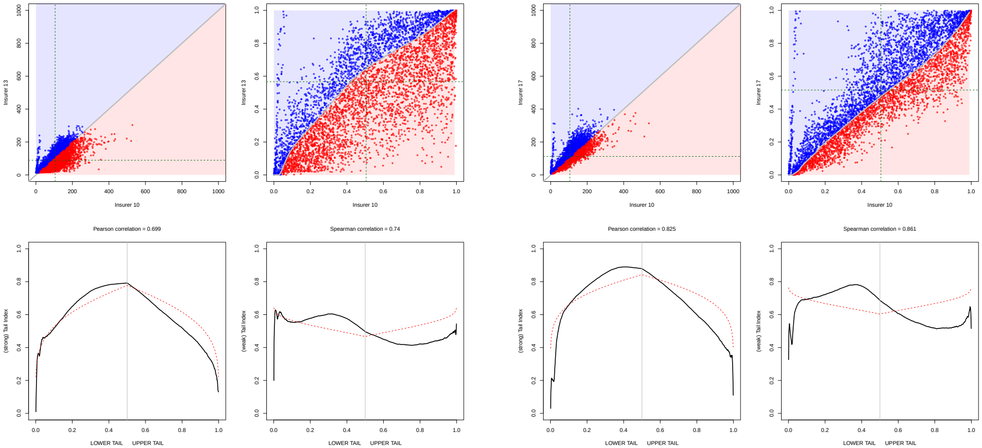
Premiums Correlations



Premiums Correlations



Premiums Correlations



We need a **Decision Rule** to select premium chosen by insured i

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Insurance Ratemaking Competition

Basic '**rational rule**' $\pi_i = \min\{\widehat{\pi}_1(\boldsymbol{x}_i), \cdots, \widehat{\pi}_d(\boldsymbol{x}_i)\} = \widehat{\pi}_{1:d}(\boldsymbol{x}_i)$

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Insurance Ratemaking Competition

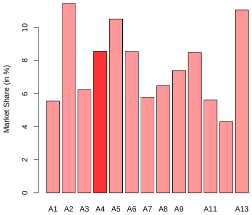
A more **realistic rule** $\pi_i \in \{\widehat{\pi}_{1:d}(\boldsymbol{x}_i), \widehat{\pi}_{2:d}(\boldsymbol{x}_i), \widehat{\pi}_{3:d}(\boldsymbol{x}_i)\}$

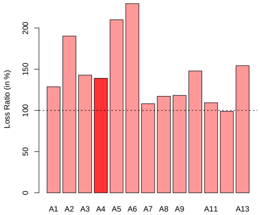
	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Actuarial Pricing Game, 2015

Insurer 4

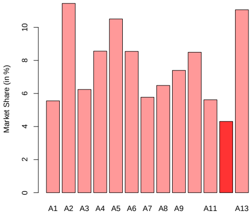
GLM for frequency and standard cost (large claimes were removed, above 15k), Interaction Age and Gender
Actuary working for a *mutuelle* company

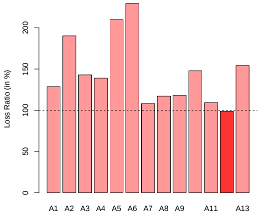




Insurer 11

Use of two XGBoost models (bodily injury and material), with correction for negative premiums
Actuary working for a private insurance company

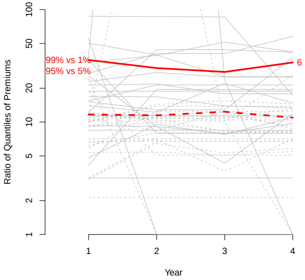


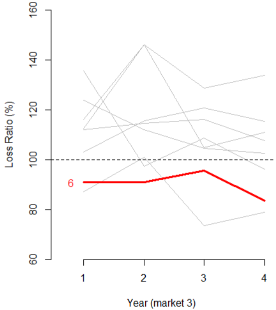


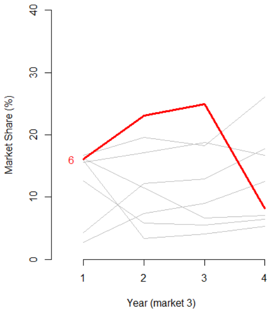
Actuarial Pricing Game, 2017

Insurer 6 (market 3)

Team of two actuaries (degrees in Engineering and Physics), in Vancouver, Canada. Used GLMs (Tweedie), no territorial classification, no use of information about other competitors, "*Segments with high market share and low loss ratios were also given some premium increase*"

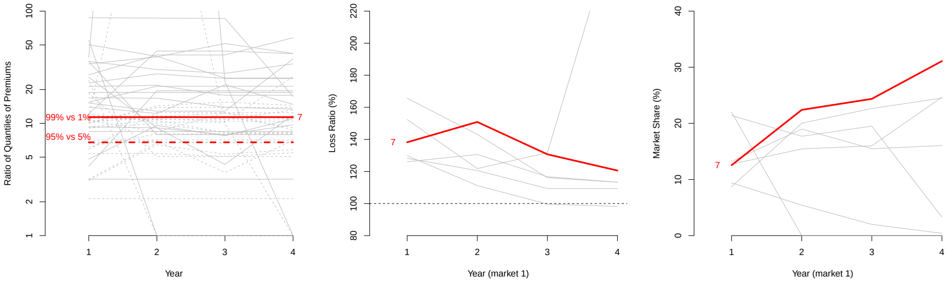






Insurer 7 (market 1)

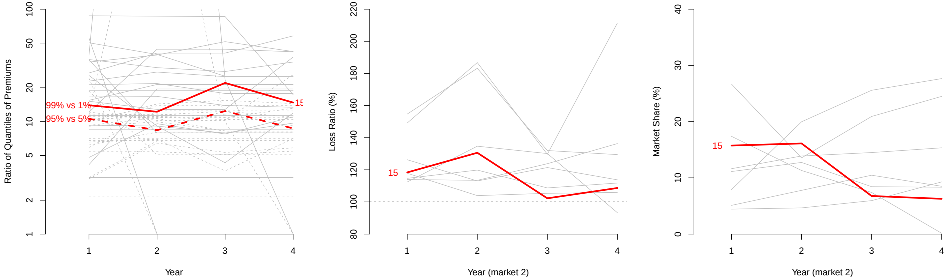
Actuary in France, used random forest for variable selection, and GLMs



Actuarial Pricing Game, 2017

Insurer 15 (market 2)

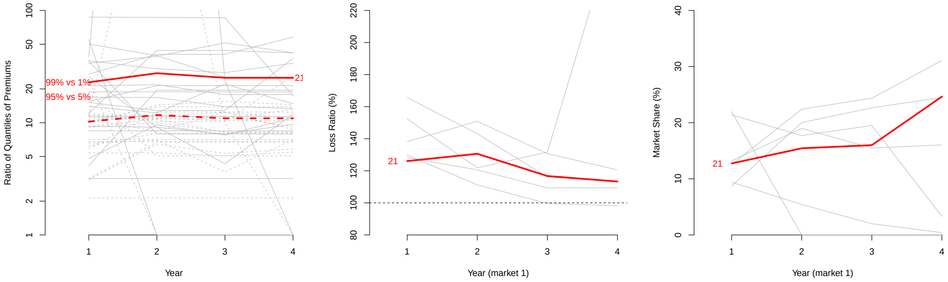
Actuary,working as a consultant, Margin Method with iterations, MS Access & MS Excel



Insurer 21 (market 1)

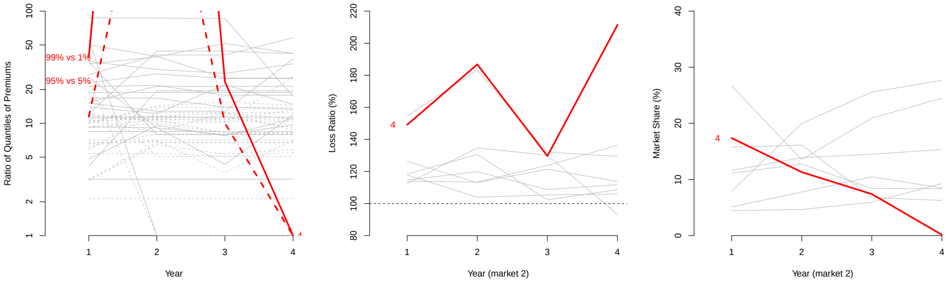
Actuary, working as a consultant, used GLMs, with variable selection using LARS and LASSO

Iterative learning algorithm (codes available on github)

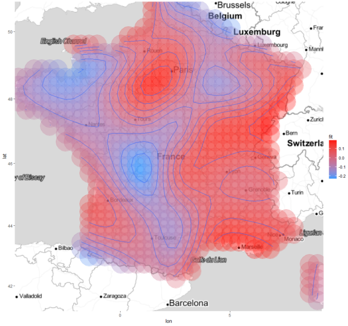


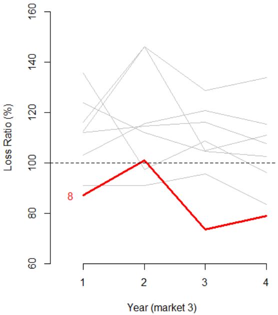
Insurer 4 (market 2)

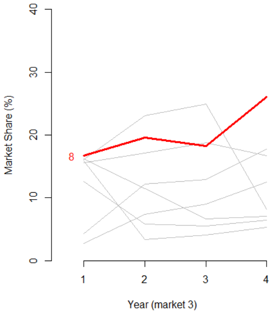
Actuary, working as a consultant,used XGBOOST, used GLMs for year 3.



Insurer 8 (market 3)
Mathematician, working on Solvency II software in Austria
Generalized Additive Models with spatial variable



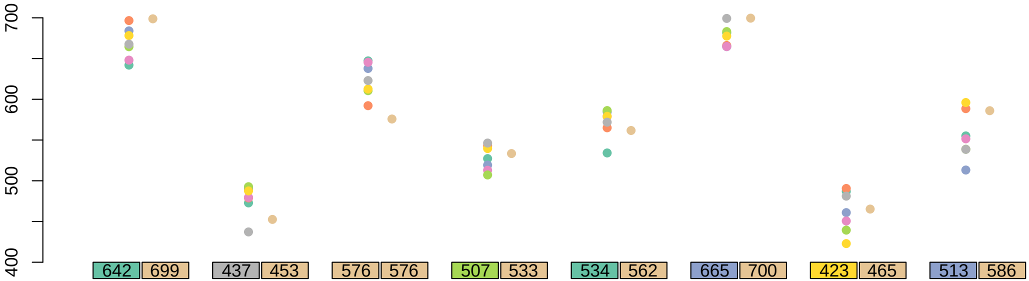




Segmentation + Overall Level

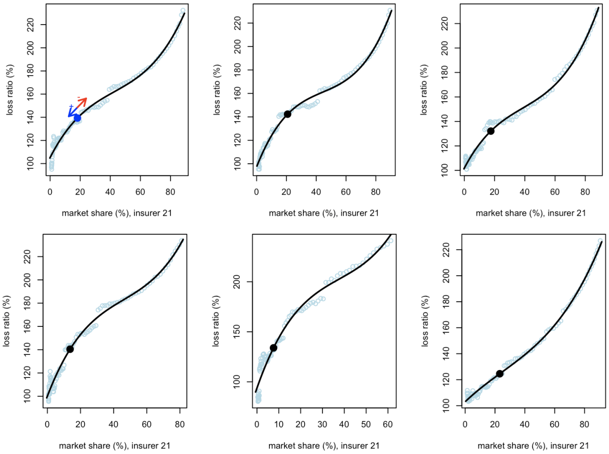
$$\widehat{\pi}(\boldsymbol{x}) = \underbrace{\exp(\widehat{\boldsymbol{\alpha}}^{\mathsf{T}}\boldsymbol{x})}_{\text{Poisson } \mathcal{P}(\lambda_{\boldsymbol{x}})} \cdot \underbrace{\exp(\widehat{\boldsymbol{\beta}}^{\mathsf{T}}\boldsymbol{x})}_{\text{Gamma } \mathcal{G}(\mu_{\boldsymbol{x}},\varphi)} = \underbrace{e^{\gamma_0}}_{\text{level}} \cdot \underbrace{e^{\gamma_1 x_1} e^{\gamma_2 x_2} \cdots e^{\gamma_k x_k}}_{\text{marginal effects}}$$

Why not try to change e^{γ_0} ?



Actuarial Pricing Game, 2017 (static)

What could we do when we observe competitors' prices ?



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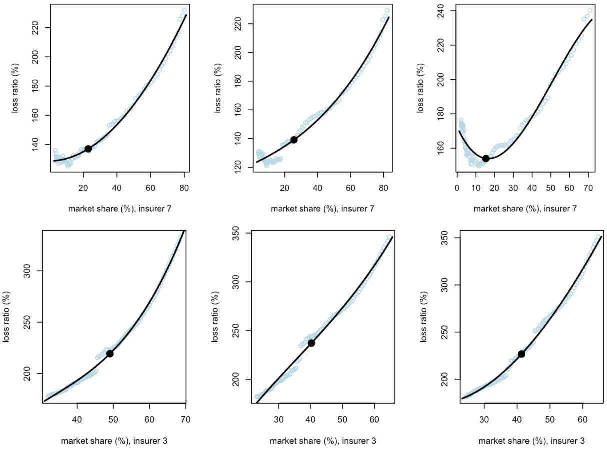
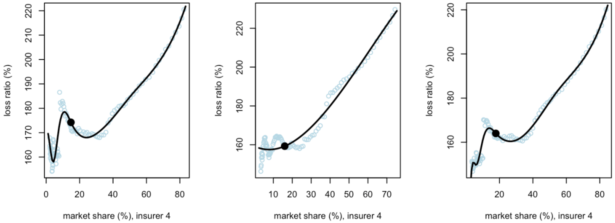
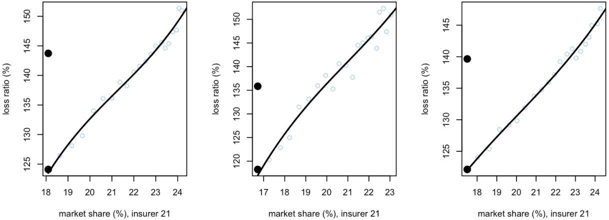


Actuarial Pricing Game, 2017 (static)

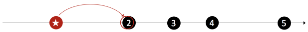


In some cases, it seems possible to increase the market share *and* decrease the loss ratio...

Ajust to Competitors' Prices



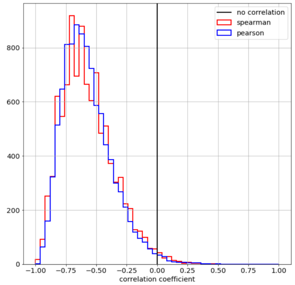
- if $\pi_\star(\boldsymbol{x}_i)=\pi_{1:d}(\boldsymbol{x}_i)$ then $\pi_\star(\boldsymbol{x}_i)=\pi_{2:d}(\boldsymbol{x}_i)-\epsilon$
- if $\pi_\star(\boldsymbol{x}_i)=\pi_{2:d}(\boldsymbol{x}_i)$
  and if $(\pi_\star(\boldsymbol{x}_i)-\pi_{1:d}(\boldsymbol{x}_i))\leq\alpha$ then $\pi_\star(\boldsymbol{x}_i)=\pi_{1:d}(\boldsymbol{x}_i)-\epsilon$



Linear vs Nonlinear Markets

Using simulations, we created a few models, some *linear* and some *non-linear*.
We created markets of 10 companies.

The correlation between the total earned premium (of the market) and the proportion of *non-linear* models is strongly negative.



The more nonlinear models are used, the lower the total earned premium (the worst the market loss ratio...)

ongoing research with Ali Farzanehfar, Florimond Houssiau and Yves-Alexandre de-Montjoye (Imperial College)

Key takeaways

On-going research,

- hard to derive theoretical properties of competition market
- use field studies (but hard to get players...)
- use simulated models and markets

- 2017 game, too complicated design with dynamics
- 2018 game, too complicated for CS students
- new design in 2019... more to come !