

Pricing Fairness

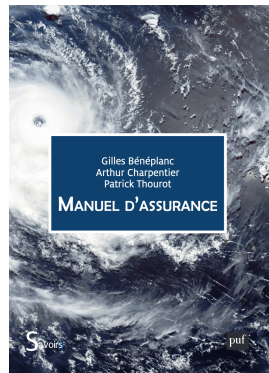
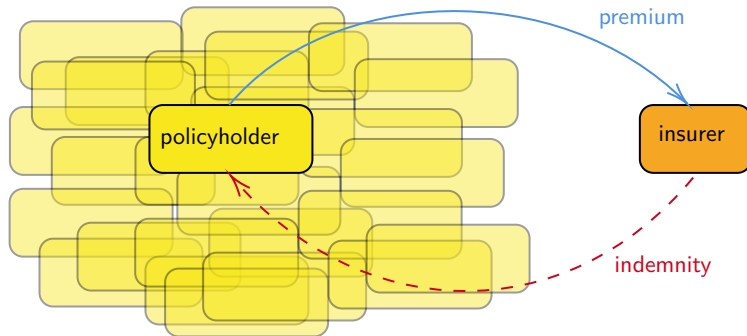
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Chief Actuary Meeting, AXA, December 2022

Preliminaries and Motivation

- Insurance is the contribution of the many to the misfortune of the few



There is no "*unique price*" for an individual... actuaries will use $\pi(\mathbf{x}) = \mathbb{E}_{\hat{\mathbb{P}}_n}[Y|\mathbf{X} = \mathbf{x}]$ as a proxy for $\pi(\mathbf{x}) = \mathbb{E}_{\mathbb{P}}[Y|\Omega]$ to compute a "*fair actuarial premium*".

Preliminaries and Motivation

- ▶ *"Technology is neither good nor bad; nor is it neutral"* , Kranzberg (1986)
- ▶ *"Machine learning won't give you anything like gender neutrality 'for free' that you didn't explicitly ask for"* , Kearns and Roth (2019)
- ▶ *"at the core of insurance business lies discrimination between risky and non-risky insureds"* , Avraham (2017)
- ▶ **Accuracy** : $\pi(\mathbf{x}) = \mathbb{E}_{\hat{\mathbb{P}}_n}[Y|\mathbf{X} = \mathbf{x}]$ ($\hat{\mathbb{P}}_n$ historical probability) (**is**)
- ▶ **Fairness** : $\pi^*(\mathbf{x}) = \mathbb{E}_{\mathbb{P}^*}[Y|\mathbf{X} = \mathbf{x}]$ (\mathbb{P}^* targeted probability) (**ought**, Hume (1739))



- ▶ Charpentier (2022a) Assurance: biais, discrimination et équité
- ▶ Charpentier (2022b) Insurance: biases, discrimination and fairness

Gender directive 2011-2012



EUROPEAN COMMISSION
PRESS RELEASE

Brussels, 20 December 2012

EU rules on gender-neutral pricing in insurance industry enter into force

Brussels, 20 December 2012 – Under new rules which enter force tomorrow, insurers in Europe will have to charge the same price to women and men for the same insurance products without distinction on the grounds of sex. This means that insurance prices could rise or fall in the short term for certain categories of customers while they are likely to balance out over time. The change comes after the Court of Justice of the European Union ruled that different premiums for men and women purely on the grounds of sex were incompatible with the principle of 'unisex' pricing included in EU gender equality legislation, and with the EU Charter of Fundamental Rights.

"Gender equality is a fundamental right in the European Union and the Court of Justice made clear that this also applies to insurance pricing," said Vice-President Reding, the EU's Justice Commissioner. "The insurance sector has had over a year to prepare the switch over to unisex pricing and the European Commission has helped the industry to adapt during this period. The Commission will monitor how the industry will implement these new rules in practice."

In its ruling on 1 March 2011 in the *Test-Achats* case (C-236/09), the Court of Justice of the EU gave insurers until 21 December 2012 to change their pricing policies in order to treat individual male and female customers equally in terms of insurance premiums and benefits (MEMO/11/1223).

Following the Court's judgement, Vice-President Viviane Reding, the EU's Justice Commissioner, met with leading EU insurers in September 2011 to discuss how the Commission can help the industry to adapt to the Court's ruling (MEMO/11/624). As a result on 22 December 2011, the Commission gave the industry concrete guidance on implementing the ruling (IP/11/1581).



IP/12/1430

EUROPEAN COMMISSION - PRESS RELEASE

European Commission gives guidance to Europe's insurance industry to ensure non-discrimination between women and men in insurance premiums

Brussels, 22 December 2011 – The European Commission has today adopted guidelines to help the insurance industry implement unisex pricing, after the Court of Justice of the European Union ruled that different premiums for men and women constitute sex discrimination. In its ruling on the *Test-Achats* case on 1 March 2011, the Court of Justice gave insurers until 21 December 2012 to treat individual male and female customers equally in terms of insurance premiums and benefits (MEMO/11/1223).

Vice-President Viviane Reding, the EU's Justice Commissioner, met with leading EU insurers in September 2011 to discuss how the industry should adapt to the Court's ruling (MEMO/11/624). Following consultations with national governments, insurers and consumers, the new Commission guidelines respond to the need for practical guidance on the implications of the ruling. They aim to benefit both consumers and insurance companies.

"When the Court of Justice issued its decision in the *Test-Achats* case on 1 March this year, I promised that the Commission would help insurers and consumers adapt to the ruling," said EU Justice Commissioner Viviane Reding, the Commission's Vice-President. "By adopting these guidelines a full year ahead of the deadline to comply with the Court's ruling, we have lived up to our commitment. It is now up to the insurance industry to ensure that there is a smooth transition to fully equal treatment of men and women in insurance. The Commission will remain vigilant in how the industry implements the Court's ruling. I expect that insurers that move to a unisex tariff first will have a competitive advantage on the European market."

EU Commissioner for the Internal Market and Services, Michel Barnier said: "There have been some concerns among insurers as to the impact and consequences of this important judgment, in particular at this time when insurers as all other financial market participants face important challenges. I believe that these guidelines will be helpful for the industry and assist them in adapting their contracts and premiums to be able to ensure timely and full compliance with the judgment. This will be beneficial for both the industry and policyholders."

The guidelines adopted today cover a series of issues which emerged from in-depth consultations with Member States and stakeholders. For example, they clarify that the ruling applies only to new contracts, in particular to contracts concluded as from 21 December 2012. They also give specific examples of what is considered a 'new contract' to ensure a comprehensive application of the unisex rule at EU level from the same date.

IP/11/1581



EUROPEAN COMMISSION
MEMO

Brussels, 20 December 2012

Factsheet: EU rules on gender-neutral pricing in insurance

What will change on 21 December?

From 21 December 2012, insurance companies in the European Union will have to charge the same price to men and women for the same insurance products, without distinction on the grounds of sex. The change will apply to all new contracts for insurance products, including car insurance, life insurance and annuities.

This 'unisex' or gender-neutral pricing means men and women with the same characteristics (e.g. age, state of health depending on the product) should pay the same price for the same product. Pricing will have to be based on other risk factors than sex, such as driving behaviour in the case of car insurance. This means people will no longer have to pay more, or less, simply because of their gender.

The change will apply to contracts concluded from 21 December, but some insurers offer customers the possibility of concluding contracts before this date for policies which start later – up to 90 days in some cases.

Why is this happening now?

Gender equality is a fundamental right in the EU. The European Court of Justice ruled on 1 March 2011 that differences in insurance pricing based purely on a person's sex are discriminatory (MEMO/11/1223). The EU's Court of Justice ruled that different insurance premiums for women and men constitute discrimination on the grounds of sex and are thus not compatible with the EU's Charter of Fundamental Rights. Member States are not allowed to derogate from this important principle in their national legislation.

The Court gave the insurance sector a transitional period of 21 months to adapt their pricing structures to the new rules, with a final deadline of 21 December 2012. One year ago, on 21 December 2011, the European Commission issued guidelines to help the sector adapt to the new rules (IP/11/1581).

How will this affect prices for customers?

The change will have an impact on individual premiums for certain insurance products. In the past, men and women sometimes paid a different price for different types of insurance because of their sex. Those customers who previously paid less (such as young women for car insurance) will likely see rises in their insurance premiums. But by the same measure, those who paid more (such as men for term-life insurance), are likely to pay less in future.

MEMO/11/123

Brussels, 1 March 2011

Sex Discrimination in Insurance Contracts: Statement by European Commission Vice-President Viviane Reding, the EU's Justice Commissioner, on the European Court of Justice's ruling in the *Test-Achats* case

The Court of Justice of the European Union today delivered its ruling in the *Test-Achats* case (C-236/09) concerning sex discrimination in insurance premiums. Commenting on the judgement, EU Justice Commissioner Viviane Reding (who is in charge of gender equality at the European Commission) said:

"Today is an important moment for gender equality in the European Union.

30 years ago, the Supreme Court of the United States ruled that the Civil Rights Act of 1964 prohibits different treatment of insured persons on the basis of their sex in connection with pension funds.

Today, the EU's Court of Justice ruled that different insurance premiums for women and men constitute sex discrimination and are not compatible with the EU's Charter of Fundamental Rights. Member States are not allowed to derogate from this important principle in their national legislation. The relevant "opt out" clause in the *Council's 2004 Directive on gender equality* is thus illegal.

This is an important step towards clarifying the fundamental right of gender equality under EU law. Today's ruling also underlines the power and importance of our Charter of Fundamental Rights. It has the same legal value as our EU Treaties. No EU legislation can be adopted that conflicts with the rights and principles guaranteed by the Charter.

The European Commission issued a 'fundamental rights checklist' last October to make sure that all laws proposed comply with the EU Charter (see IP/10/1348). This checklist ensures that our rules are beyond any reproach. We have also called on the European Parliament and the Council to take a similar fundamental-rights-friendly approach when they add amendments in the EU law-making process. Today's ruling confirms how essential this is. It is important to note that the derogation for insurers was not part of the Commission's initial proposal for the 2004 Directive; it was only added later by the Council.

So what happens next? The European Commission will now carefully examine the implications of the Court's decision for the EU's law on equal access to goods and services for women and men, as well as for the insurance sector and consumers.

The insurance industry will certainly be affected by the ruling. For products such as life assurance and annuities, all 27 EU countries currently allow insurers to use sex as a risk-rating factor.

However, I also would like to underline that parts of the insurance industry have already started to move in the direction of gender equality. Insurers have already shown flexibility as Belgium, Bulgaria, Cyprus, Estonia, Latvia, Lithuania, the Netherlands and Slovenia apply unisex premiums for car insurance.

MEMO/10/1012



Differential pricing reports 2021-2022



RAPPORT Får lojala försäkringstagare betala mer?

1 juli 2022



Discrimination and Protected Attributes

California

Allowed (with applicable limitations): driving experience, marital status, address/zip code

Prohibited (or effectively prohibited): gender, age, credit history, education, occupation, employment status, residential status, insurance history

Notes & Clarifications: California's insurance commissioner banned gender as of January 2019. Occupation and education are permitted for use in group plans (i.e. for alumni associations and other membership programs).

Georgia

Allowed (with applicable limitations): gender, age, years of driving experience, credit history, marital status, residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): occupation, education, and employment status

Notes & Clarifications: none

Hawaii

Allowed (with applicable limitations): address/zip code, insurance history

Prohibited (or effectively prohibited): gender, age, years of driving experience, credit history, education, occupation, employment status, marital status, residential status

Notes & Clarifications: none

Illinois

Allowed (with applicable limitations): gender, age, years of driving experience, credit history, education, occupation, employment status, marital status, residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): none

Notes & Clarifications: none

Massachusetts

Allowed (with applicable limitations): years of driving experience, address/zip code, insurance history

Prohibited (or effectively prohibited): gender, age, credit history, education, occupation, employment status, marital status, residential status

Notes & Clarifications: none

Michigan

Allowed (with applicable limitations): gender (group-rated policies), age, years of driving experience, credit history, education, occupation, employment status, marital status (group-rated policies), residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): gender (non-group policies), marital status (non-group policies)

Notes & Clarifications: Gender and marital status are permitted only in rate-making for group plans (i.e. for alumni associations and other membership programs). **UPDATE:** [Michigan lawmakers approved a major insurance reform bill](#) in May 2019 that will ban insurers in the state from using gender, marital status, address/zipcode, residential status, education and occupation in rate setting. The ban will be enforced starting in July 2020. Insurers will be permitted to use "territory" as approved by the state regulators instead of zip code.

New York

Allowed (with applicable limitations): gender, age, years of driving experience, credit history, marital status, residential status, address/zip code, insurance history

Prohibited (or effectively prohibited): occupation, education, employment status

Notes & Clarifications: none

via [The Zebra \(2022\)](#)

Discrimination and Protected Attributes

	CA	HI	GA	NC	NY	MA	PA	FL	TX	AL	ON	NB	NL	QC
Gender	X	X	●	X	●	X	X	●	●	●	●	X	X	●
Age	X	X	●	X*	●	X	●	●	●	●*	●	X	X	●
Driving experience	●	X	●	●	●	●	●	●	●	●	●	●	●	●
Credit history	X	X	●	●	●	X	●*	●	●	X*	X	●*	X	●
Education	X	X	X	X	X	X	●	●	●	●	●	●	●	●
Occupation	X	X	X	●	X	X	●	●	●	●	●	●	●	●
Employment status	X	X	X	●	X	X	●	●	●	●	●	●	●	●
Marital status	●	X	●	●	●	X	●	●	●	●	●	●	●	●
Housing situation	X	X	●	●	●	X	●	●	●	X	X	●	●	●
Address/ZIP code	●	●	●	●	●	●	●	●	●	X	X	●	●	●
Insurance history	●	●	●	●	●	●	●	●	●	●	●	●	●	●

CA: Californie, HI: Hawaii, GA: Georgia, NC: Caroline du nord, NY: New York, MA: Massachusetts, PA: Pennsylvanie, FL: Floride, TX: Texas, AL: Alberta, ON: Ontario, NB: Nouveau-Brunswick, NL: Terre-Neuve-et-Labrador, QC: Québec

Price Walking

- “Price walking, or the loyalty penalty, is a form of price discrimination whereby longstanding, loyal customers of a service provider are charged higher prices for the same services compared to customers that have just switched to that provider” Wikipedia (2022)

Figure 3: Variation of the average Actual Premium, average Technical Premium, and the average APTP with tenure on private car insurance. (Includes policies from 2017-2019).

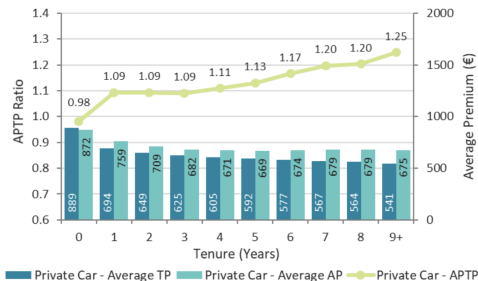
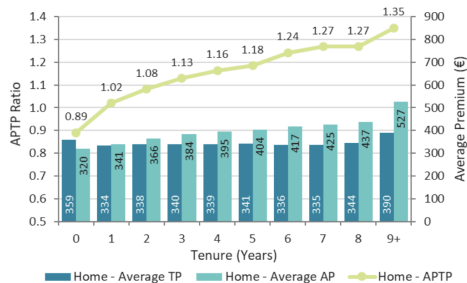


Figure 4: Variation of the average Actual Premium, average Technical Premium, and the average APTP with tenure on home insurance. (Includes policies from 2017-2019).



Actuarial Formalism I

$$\left\{ \begin{array}{l} \mathbf{x} \in \mathcal{X} \subset \mathbb{R}^d : \text{'explanatory' variables} \\ s \in \{0, 1\} : \text{'sensitive' variable} \\ y \in \{0, 1\} : \text{classification, or } \mathcal{Y} \subset \mathbb{R} : \text{regression} \\ \hat{y} = \mathbf{1}_{m(\mathbf{x}) > t} : \text{classification or } m(\mathbf{x}) : \text{regression (prediction)} \end{array} \right.$$

Demographic Parity, (Corbett-Davies et al. (2017), Agarwal (2021))

Decision function \hat{y} satisfies demographic parity if $\hat{Y} \perp\!\!\!\perp S$, i.e.

$$\mathbb{P}[\hat{Y} = y | S = 0] = \mathbb{P}[\hat{Y} = y | S = 1], \forall y \text{ or } \mathbb{E}[\hat{Y} | S = 0] = \mathbb{E}[\hat{Y} | S = 1]$$

Equal Opportunity, Hardt et al. (2016)

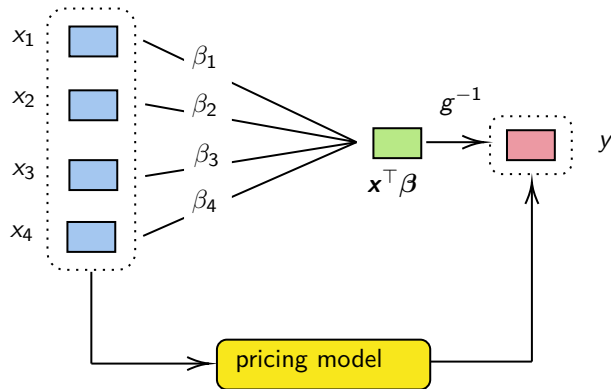
Function \hat{y} satisfies equal opportunity if $\hat{Y} \perp\!\!\!\perp S$ given Y , e.g. true positive parity

$$\mathbb{P}[\hat{Y} = 1 | S = 0, Y = 1] = \mathbb{P}[\hat{Y} = 1 | S = 1, Y = 1]$$

Actuarial Formalism II

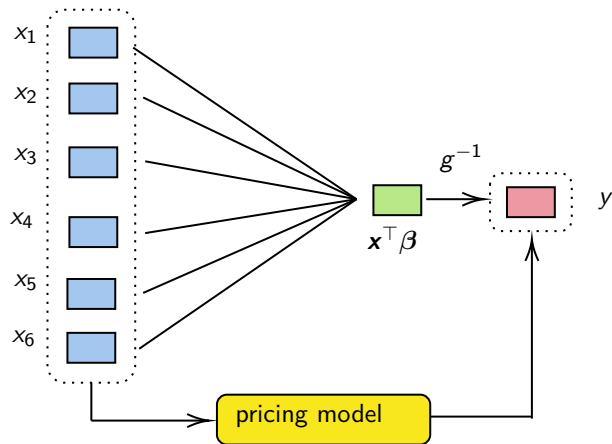
statistical parity, Dwork et al. (2012)	$\mathbb{P}[\hat{Y} = 1 S = s] = \text{cst}, \forall s$	independence
conditional stat. parity, Corbett-Davies et al. (2017)	$\mathbb{P}[\hat{Y} = 1 S = s, X = x] = \text{cst}_x, \forall s, y$	$\hat{Y} \perp\!\!\!\perp P$
equalized odds, Hardt et al. (2016)	$\mathbb{P}[\hat{Y} = 1 S = s, Y = y] = \text{cst}_y, \forall s, y$	separation
equalized opportunity, Hardt et al. (2016)	$\mathbb{P}[\hat{Y} = 1 S = s, Y = 1] = \text{cst}, \forall s$	$\hat{Y} \perp\!\!\!\perp S Y$ $S \perp\!\!\!\perp P Y$
predictive equality, Corbett-Davies et al. (2017)	$\mathbb{P}[\hat{Y} = 1 S = s, Y = 0] = \text{cst}, \forall s$	
balance (positive), Kleinberg et al. (2017)	$\mathbb{E}[m(\mathbf{X}) S = s, Y = 1] = \text{cst}, \forall s$	
balance (negative), Kleinberg et al. (2017)	$\mathbb{E}[m(\mathbf{X}) S = s, Y = 0] = \text{cst}, \forall s$	
conditional accuracy equality, Berk et al. (2017)	$\mathbb{P}[Y = y S = s, \hat{Y} = y] = \text{cst}_y, \forall s, y$	sufficiency
predictive parity, Chouldechova (2017)	$\mathbb{P}[Y = 1 S = s, \hat{Y} = 1] = \text{cst}, \forall s$	$Y \perp\!\!\!\perp S \hat{Y}$
calibration, Chouldechova (2017)	$\mathbb{P}[Y = 1 S = s, m(\mathbf{X}) = m] = \text{cst}_s, \forall s, m$	
well-calibration, Chouldechova (2017)	$\mathbb{P}[Y = 1 S = s, m(\mathbf{X}) = m] = s, \forall s, m$	
accuracy equality, Berk et al. (2017)	$\mathbb{P}[\hat{Y} = Y m(\mathbf{X}) = m] = \text{cst}, \forall m$	
treatment equality, Berk et al. (2017)	$\frac{\text{FN}_s}{\text{FP}_s} = \text{cst}_s, \forall s$	

Actuarial Modeling I



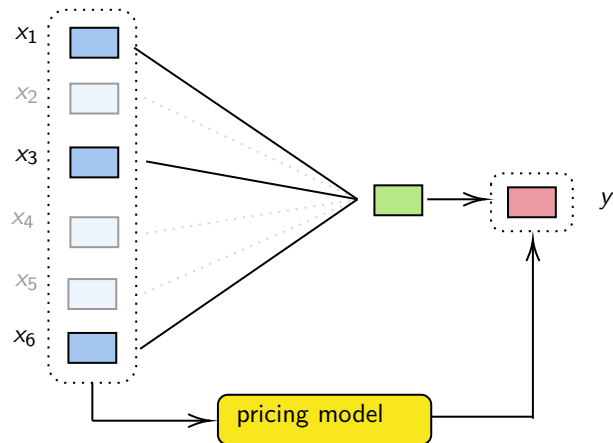
$$\text{GLM: } \min \sum_{i=1}^n \ell(y_i, g^{-1}(\mathbf{x}_i^\top \boldsymbol{\beta})) \text{ where } \mathbf{x}_i^\top \boldsymbol{\beta} = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i},$$

Actuarial Modeling II



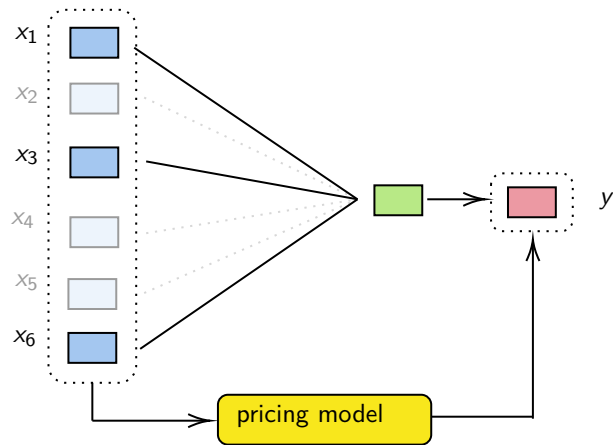
Data enrichment: $\min \sum_{i=1}^n \ell(y_i, g^{-1}(\mathbf{x}_i^\top \boldsymbol{\beta}))$ where $\mathbf{x}_i^\top \boldsymbol{\beta} = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki}$,

Actuarial Modeling III



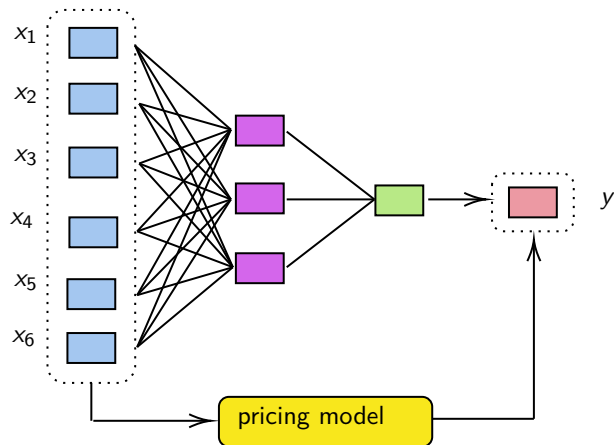
$$\text{LASSO and penalty: } \min \left\{ \sum_{i=1}^n \ell(y_i, g^{-1}(\mathbf{x}_i^\top \boldsymbol{\beta})) \right\} \text{ subject to } \dim(\boldsymbol{\beta}) \leq s$$

Actuarial Modeling IV



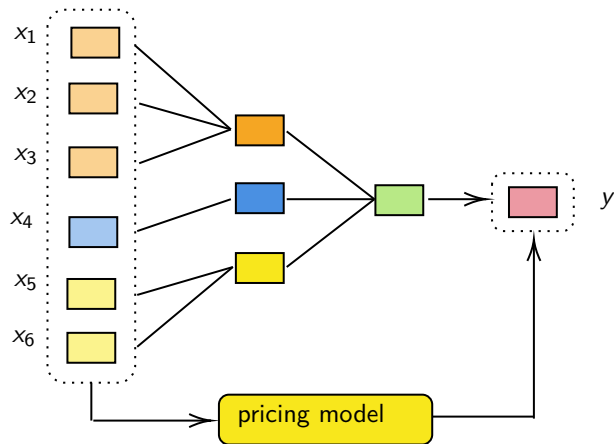
$$\text{LASSO and penalty: } \min \left\{ \sum_{i=1}^n \ell(y_i, g^{-1}(\mathbf{x}_i^{\top} \boldsymbol{\beta})) + \lambda \cdot \dim(\boldsymbol{\beta}) \right\}$$

Actuarial Modeling V



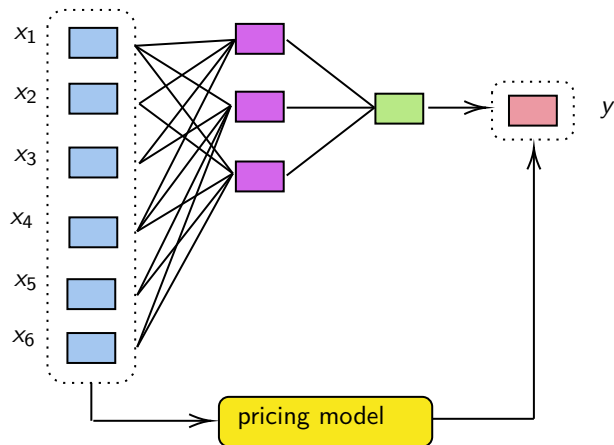
Neural Nets: $\min \sum_{i=1}^n \ell(y_i, g^{-1}(\omega_1 z_{1i} + \omega_2 z_{2i} + \omega_3 z_{3i}))$ where $z_{ji} = \mathbf{x}_i^\top \boldsymbol{\beta}_j$.

Actuarial Modeling VI



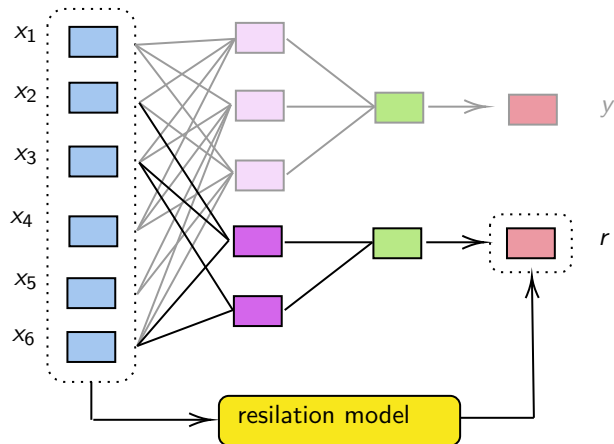
Embedding & Feature engineering: $\min \sum_{i=1}^n \ell(y_i, g^{-1}(\mathbf{z}_i^\top \boldsymbol{\omega}))$ where $z_{ji} = \mathbf{x}_i^\top \boldsymbol{\beta}_j$.

Actuarial Modeling VII



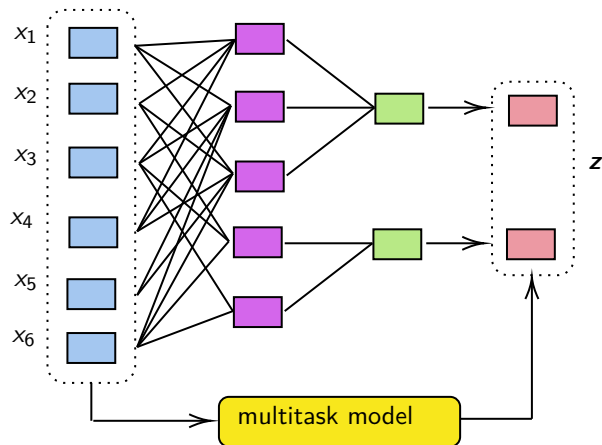
$$\text{Neural Nets } (y): \min \sum_{i=1}^n \ell_1(y_i, g_1^{-1}(\omega_1 z_{1i} + \omega_2 z_{2i} + \omega_3 z_{3i})) \text{ where } z_{ji} = \mathbf{x}_i^\top \boldsymbol{\beta}_j.$$

Actuarial Modeling VIII



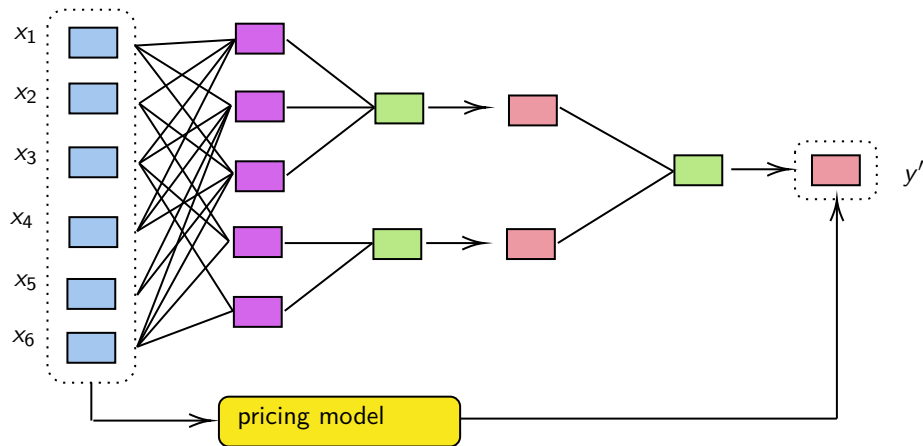
Neural Nets (r): $\min \sum_{i=1}^n \ell_2(r_i, g_2^{-1}(\omega_4 z_{4i} + \omega_5 z_{5i}))$ where $z_{ji} = \mathbf{x}_i^\top \boldsymbol{\beta}_j$.

Actuarial Modeling IX



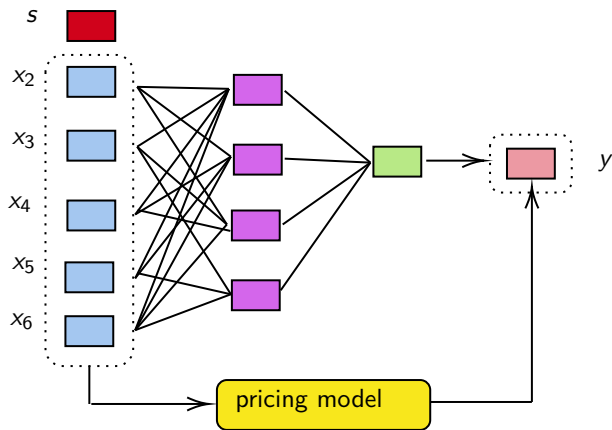
Multi-task: $\min \sum_{i=1}^n \ell(\mathbf{z}_i, \mathbf{g}^{-1}(\mathbf{x}_i^\top \boldsymbol{\omega}))$, $\mathbf{z}_i = (y_i, r_i)$, for some multivariate loss function ℓ

Actuarial Modeling X



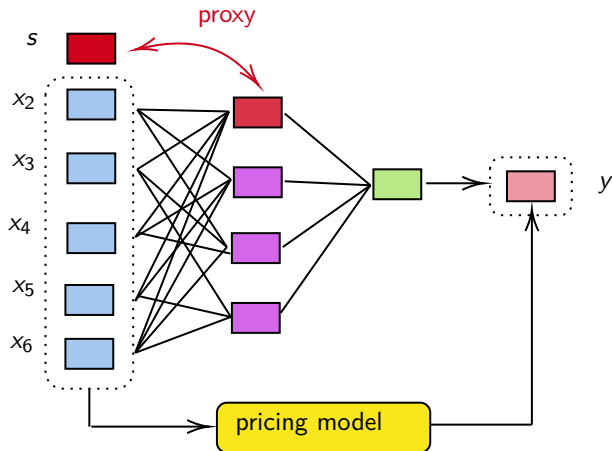
$y' = h(\mathbf{z})$ for some function $h : \mathbb{R}^2 \rightarrow \mathbb{R}$.

Actuarial Modeling XI



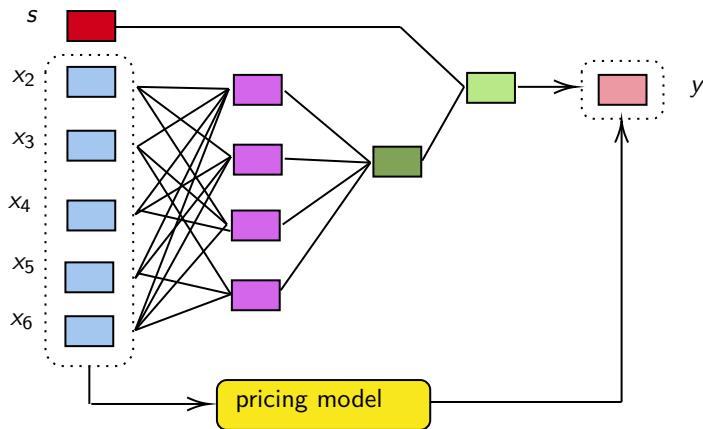
Fairness by unawareness (remove the sensitive variable s)

Actuarial Modeling XII



Possible statistical discrimination if z_1 and s are highly correlated (demographic parity)

Actuarial Modeling XIII

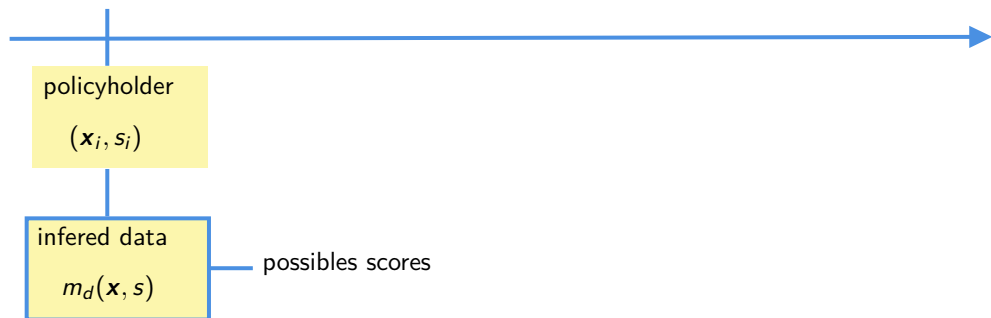


$$\text{Penalizing discrimination } \min \left\{ \sum_{i=1}^n \ell(y_i, \hat{y}_i) + \lambda \cdot \text{cor}(\hat{y}, s) \right\} \text{ (adversarial learning)}$$

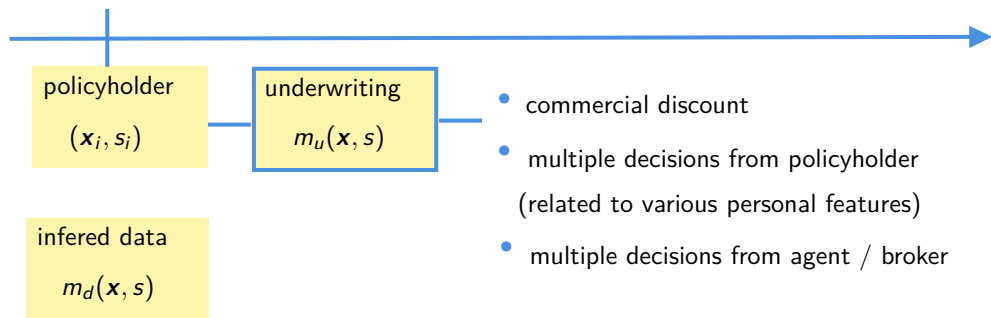
Insurance process, where a discrimination might come from... I



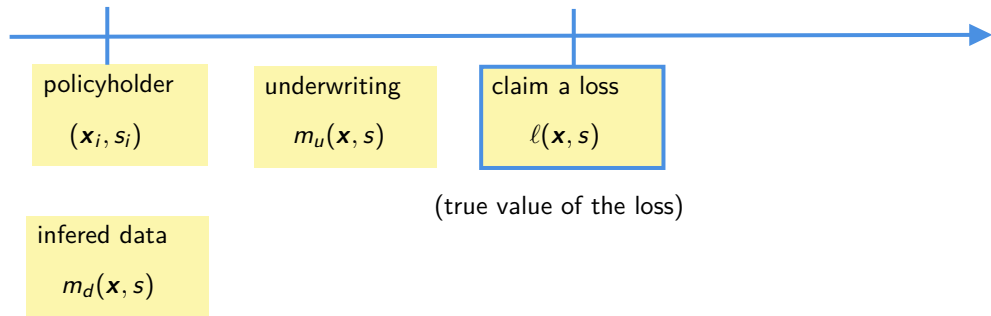
Insurance process, where a discrimination might come from... II



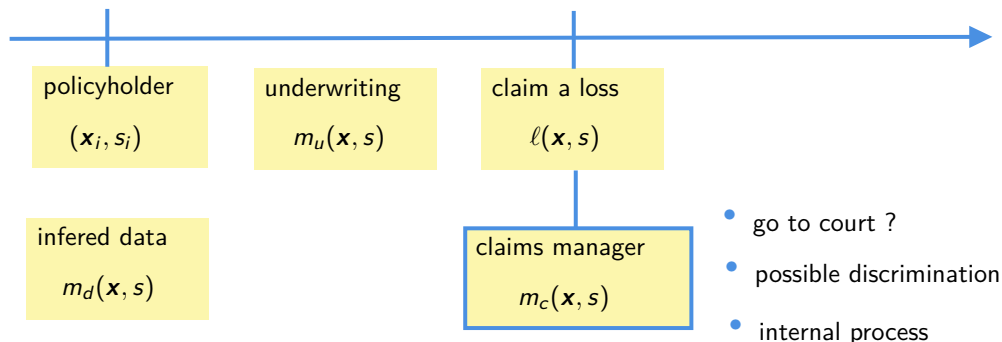
Insurance process, where a discrimination might come from... III



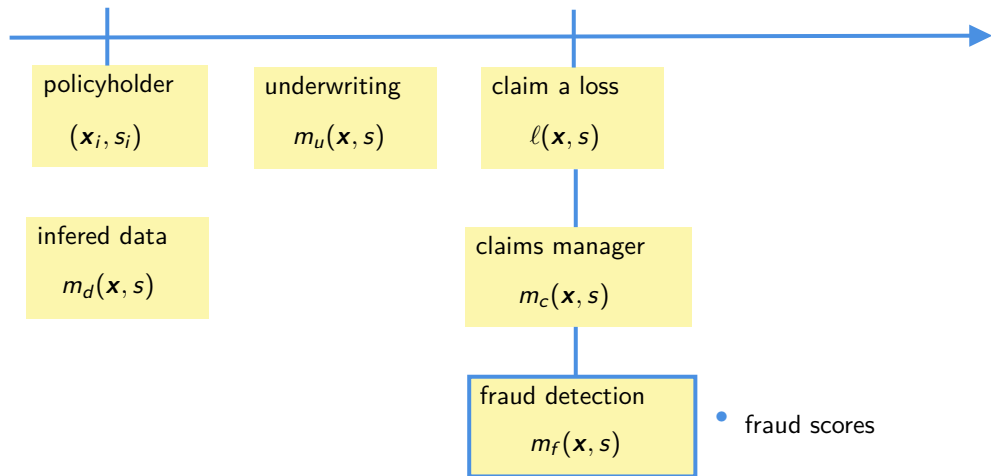
Insurance process, where a discrimination might come from... IV



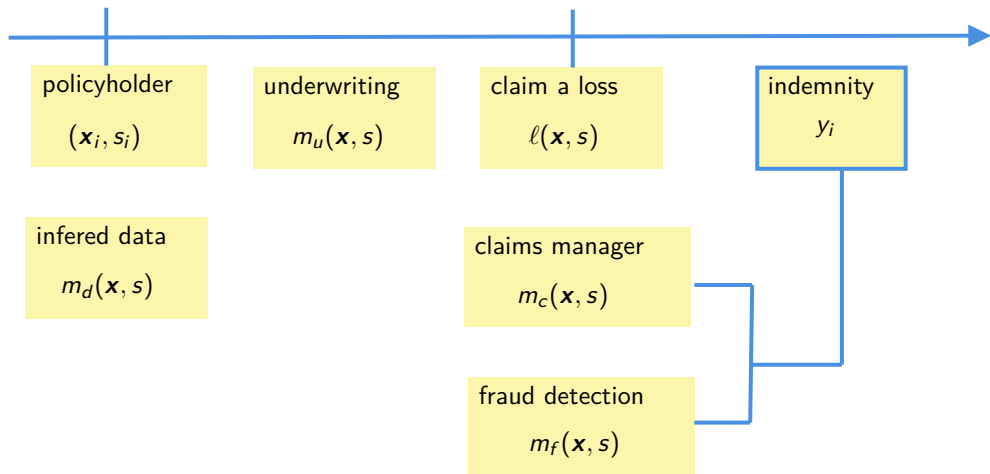
Insurance process, where a discrimination might come from... V



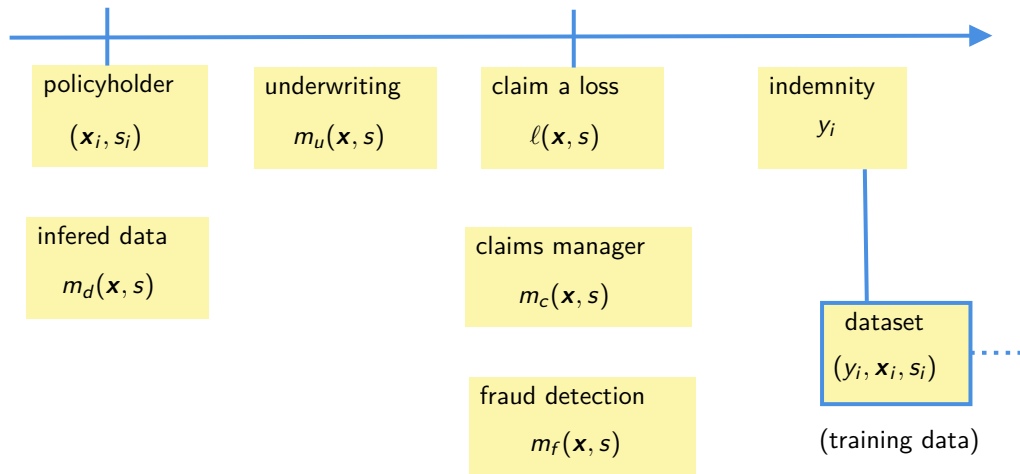
Insurance process, where a discrimination might come from... VI



Insurance process, where a discrimination might come from... VII



Insurance process, where a discrimination might come from... VIII



Causality & Interpretation

- ▶ *“Member States may decide (...) to allow proportionate differences in premiums and benefits for individuals where the use of sex is a determining factor in the assessment of risk, on the basis of relevant and accurate actuarial and statistical data”*

causal model ?

- ▶ *“the myth of the actuary, a powerful rhetorical situation in which decisions appear to be based on objectively determined criteria when they are also largely based on subjective ones”, Glenn (2000)*

“virtually every aspect of the insurance industry is predicated on stories first and then numbers”, Glenn (2003)

importance of the narrative part of modeling

“all models are wrong but some models are useful”, Box et al. (2011)

References I

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