

# How to Estimate the Occurrence Probability of Natural Catastrophes

Arthur Charpentier (UQAM),

ETH Zürich, January 2019



## The Challenge of Predictive Models (for Rare Events)

“A 30% chance of rain tomorrow : How does the public understand probabilistic weather forecasts ?” [Gigerenzer et al. \(2005\)](#)

Here we focus on [rare events](#) :

- (1) hard to find a good model
- (2) hard to assess if the model is good

See Nate Silver’s “*there’s an awful lot of room to debate what ‘probably’ means*”

[538.com](#), or Ronald Fisher on predicting probabilities on ‘one-shot’ events,

(or more provocative “*probability does not exist*” by Bruno de Finetti, [Nau \(2011\)](#))

Classical issue in risk management : predict 1% chance of occurrence, and occurs.

Was 1% wrong ?

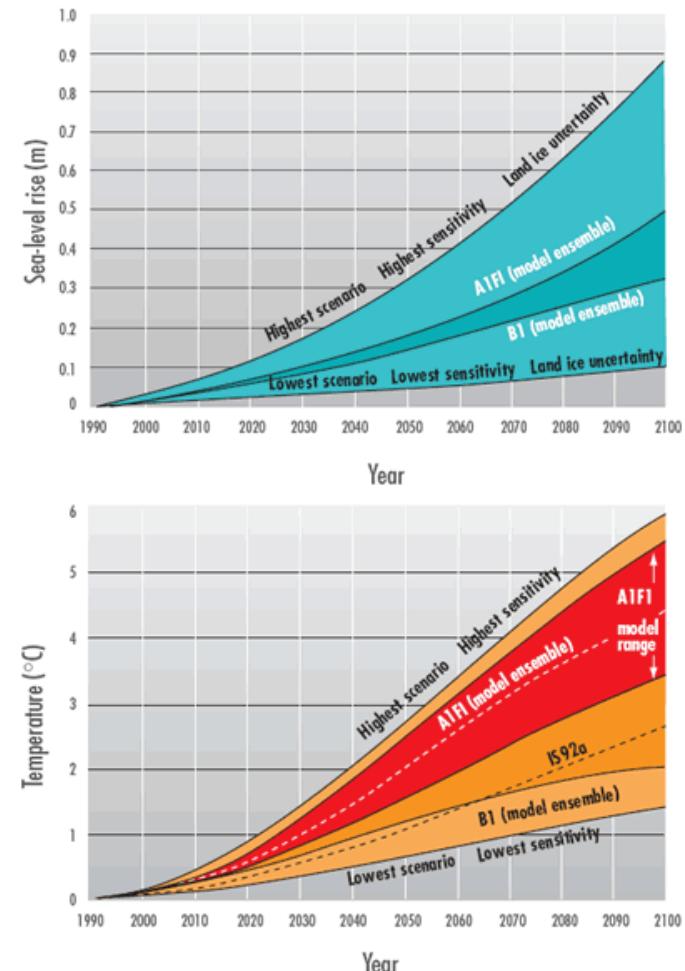
Disasters are not *per se* ‘one-shot’ events, but we have to deal with [changing environment](#)...



## So what ?

In this talk, “*natural catastrophe*” is seen from an insurance perspective

- quantitative analysis of natural events  
difficult part : climate is changing (warming)  
classical curves from **IPCC** on temperature or sea level  
(and references therein)
- impact on the insurance industry ?  
In 2015, French Federation of Insurers claimed  
“*natural disasters : insurers’ bills could double by 2040*”  
(in 25 years, +3% per year  
see <https://www.argusdelassurance.com/>)



## Insurability of Climate Risks ?

see Berliner (1985) and Charpentier (2008)

**Table 3** Insurability criteria and related requirements according to Berliner

<b>Insurability Criteria</b>		<b>Requirements</b>
<i>Actuarial</i>	(1) Randomness of loss occurrence	Independence and predictability of loss exposures
	(2) Maximum possible loss	Manageable
	(3) Average loss per event	Moderate
	(4) Loss exposure	Loss exposure must be large
	(5) Information asymmetry	Moral hazard and adverse selection not excessive
<i>Market</i>	(6) Insurance premium	Cost recovery and affordable
	(7) Cover limits	Acceptable
<i>Societal</i>	(8) Public policy	Consistent with societal value
	(9) Legal restrictions	Allow the coverage

(Table from Bienier *et al.* (2015) - for cyber risk)

climate disasters will be insured if there is an insurance market for that...

## Insurance and Uncertainty : the Framework

Classical framework : agent, with utility  $u(\cdot)$  facing random loss  $L$ , purchases insurance at price  $\pi$  if

$$\mathbb{E}_{\mathbb{P}}[u(\omega - \pi - L + I(L))] > \mathbb{E}_{\mathbb{P}}[u(\omega - L)]$$

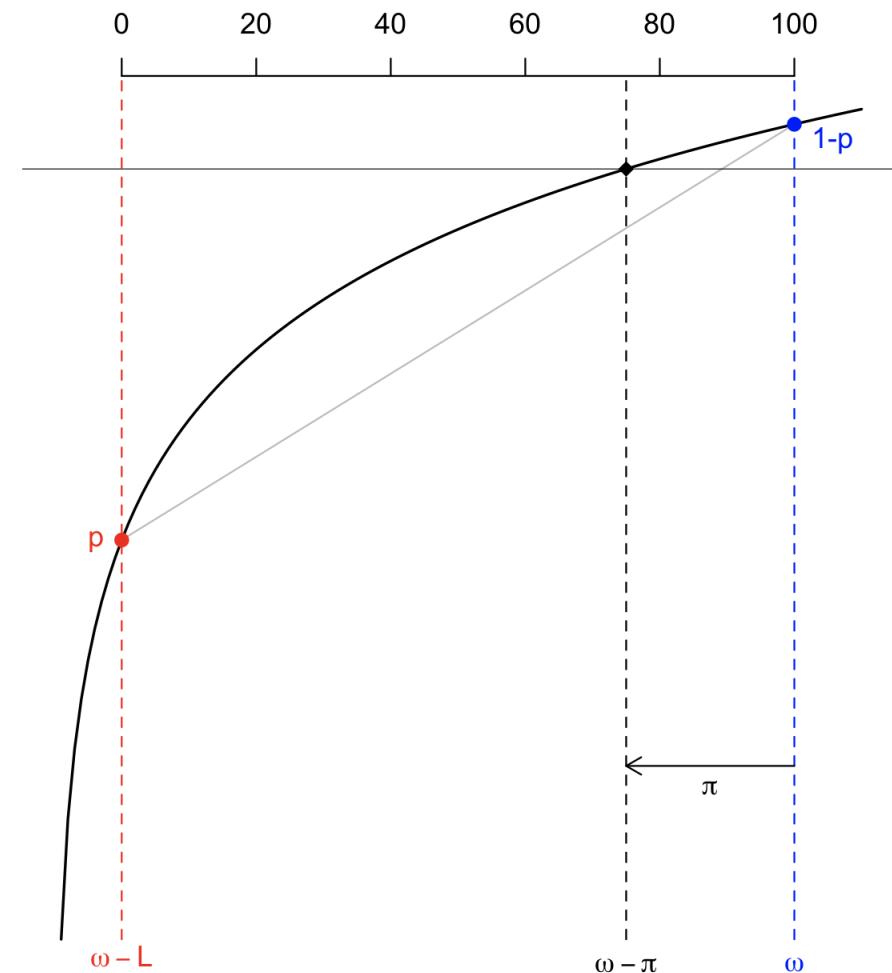
(von-Neumann expected utility).

Let  $\pi^*$  denote the highest premium

$$\mathbb{E}_{\mathbb{P}}[u(\omega - \pi^* - L + I(L))] = \mathbb{E}_{\mathbb{P}}[u(\omega - L)]$$

where  $I$  denotes the indemnity function,  
 $I(\ell) \in [0, \ell]$ .

see e.g. Goussebaïle & Louaas (2015)



## Insurance and Uncertainty : the Framework

Suppose that  $L$  is a binary loss,  $L \in \{0, \ell\}$  with probabilities  $1 - p$  and  $p$ , or  $L = X \cdot \ell$ ,  $X \sim \mathcal{B}(p)$

$$(1 - p) \cdot u(\omega - \pi^*) + p \cdot u(\omega - \pi^* - \ell + I(\ell)) = (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

If  $I(\ell) = \ell$ ,  $u(\omega - \pi^*) = (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$

If  $u$  is linear (no risk aversion),  $\pi^* = \mathbb{E}[I(L)] = p \cdot I(\ell)$

If  $u$  is concave (risk aversion),  $\pi^* > p \cdot I(\ell)$

$p \cdot I(\ell)$  is the Actuarial (pure) Premium, also called Learned Hand formula in law business (see Grossman et al. (2006)) or simply “*probability times consequence*” in climate literature (see Schneider (2002))

## Insurance and Uncertainty : the Framework

Consider the simplistic case of homogeneous agents.

An **agent**, with utility  $u$ , will purchase insurance if

$$(1 - \textcolor{red}{p}) \cdot u(\omega - \pi) + p \cdot u(\omega - \pi - \ell + I(\ell)) > (1 - \textcolor{red}{p}) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

Assume that this inequality is satisfied.

An **insurance company**, with utility  $v$ , will sell insurance if

$$\mathbb{E}_{\textcolor{red}{p}}[v(\kappa + n\pi - S)] > u(\kappa), \quad S = \sum_{i=1}^n L_i = \sum_{i=1}^n X_i \cdot \ell, \quad \text{where } X_i \sim \mathcal{B}(\textcolor{red}{p}),$$

where  $\kappa$  is the capital of the company, and  $S$  is the total indemnity.

The actuarial fair premium is obtained when  $v$  is linear :

$$\pi^* = \mathbb{E}[I(L)] = \textcolor{red}{p} \cdot I(\ell)$$

## Insurance and Uncertainty : the Framework

If risks are exchangeable  $X = L_1 + \cdots + L_n = N_n \cdot I(\ell)$ ,

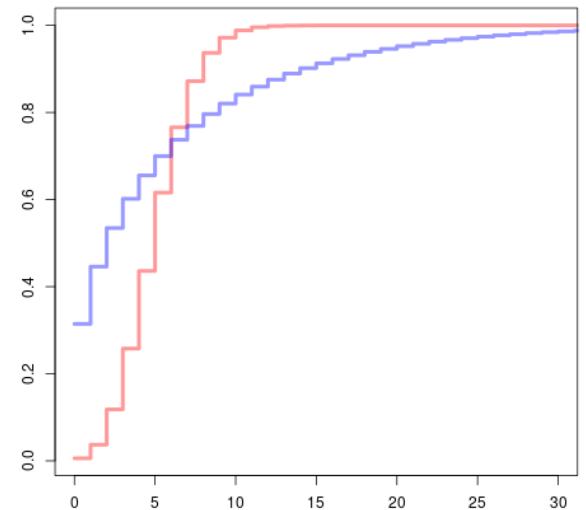
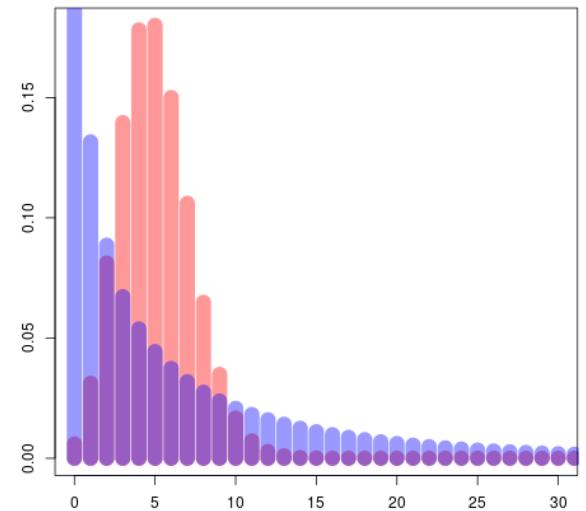
$$\mathbb{P}[N_n = k] = \int_0^1 \binom{n}{k} \theta^k (1-\theta)^{n-k} dG(\theta), \quad \mathbb{P}[L_i > 0] = p$$

from [de Finetti \(1921\)](#). See [Charpentier & le Maux \(2014\)](#) for correlated risks and disaster (with endogeneous default probability of the insurer).

Classical framework : [binomial-beta](#) model ( $G$  is a Beta distribution  $\mathcal{B}(\alpha, \beta)$ ), then  $r = \text{corr}(X_i, X_j) = (1 + \alpha + \beta)^{-1}$ . Then

$$\mathbb{E}[N_n] = np \text{ while } \text{Var}[N_n] = (n + n(n-1)r)p(1-p)$$

(the insurance company has correlation aversion, [Richard \(1975\)](#), not clear for insured...)



## Insurance and Uncertainty : the disaster puzzle

Earthquakes and floods cause potentially large losses, rational people should find actuarially fair insurance policies attractive, see [Kunreuther \(1996\)](#)

Individuals *underestimate* the true probability of a disaster event occurring and/or have fairly high discount rates for the benefits of uncertain future reimbursements due to losses.

[Kunreuther & Pauly \(2004\)](#) proved that even when insurance for low-probability, high-loss events is offered at favorable premiums, the search costs associated with obtaining the information on premiums and disaster probabilities may prevent from purchasing insurance.

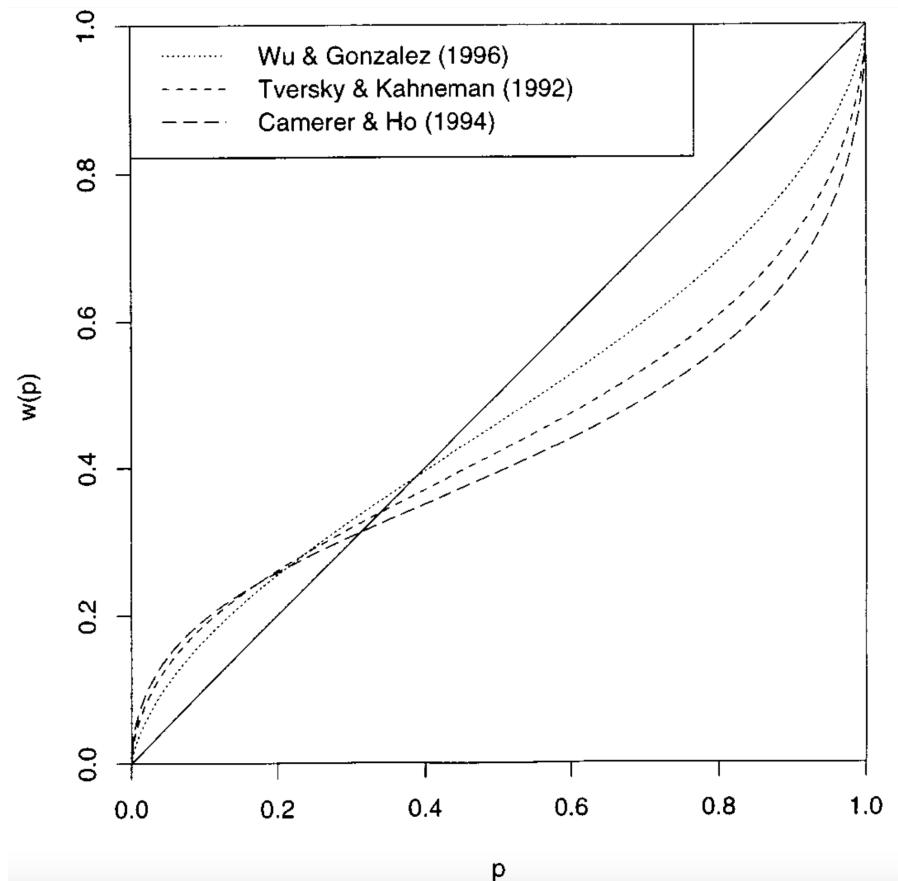
Individuals lack information about the expected harm, before the disaster, see [Botzen \(2013\)](#). Insurance prices can be a signal used to correct for possible bias (unless market distortion), see [Thieken et al. \(2006\)](#).

## Loss Probability $p$ and Perceived Loss Probability $\omega(p)$

RDEU framework, or Prospect theory,  
 Kahneman & Tversky (1979) or Wu & Gonzalez (1996)

$$\omega(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

(experimental) evidence of convexity of  $\omega$   
 extreme risk aversion in situations that involve low stakes see Huysentruyt & Read (2010), but inadequate insurance coverage against disaster, Viscusi (2010), even if it is highly subsidized : underweighting of adverse tail events.



# Loss Probability $p$ and Perceived Loss Probability $\omega(p)$

people are aware of the risk, see Barnes (2011) (no one knows the odds)



Vesuvius last awoke with a small blast in 1944. A large eruption could unleash incendiary avalanches and ash that would threaten millions of people.

## EUROPE'S TICKING TIME BOMB

Vesuvius is one of the most dangerous volcanoes in the world — but scientists and the civil authorities can't agree on how to prepare for a future eruption.

BY KATHERINE BARNES

**I**t starts with a blast so strong that a column of ash and stone rockets 40 kilometres up into the stratosphere. The debris then drops to Earth, peltig the surface with boiling hot fragments of pumice and covering the ground with a thick layer of ash. Roots crumble and vehicles grind to halt. Yet the worst is still to come. Soon, avalanches of molten ash, pumice and gas will descend the volcano's flanks, pulverising buildings and burying everything in their path. Almost overnight, a packed metropolis becomes a volcanic wasteland.

This is Naples, Italy, in the throes of a cataclysmic eruption of Vesuvius — the volcano that destroyed the city of Pompeii in AD 79. The scenario may sound far-fetched, but in the wake of Japan's tsunami and tsunamis, major areas are reassessing the risks from their own "black swan," a term used to describe unlikely but potentially devastating disasters. And Naples stands out as particularly vulnerable, with a population of 3 million living in the shadow of Vesuvius.

The volcano has been eerily dormant since a

BETTMANN/CORBIS

small eruption in 1944, but recent studies suggest that Vesuvius could be more dangerous than previously assumed, which has prompted a vigorous debate about the risk and scale of future disasters. Local authorities face the difficult task of trying to protect a large population in the event of a future eruption, without sending false alarms or causing panic.

There would be no modern precedent for an evacuation of this magnitude," says Giuseppe Mastrolorenzo at the Vesuvius Volcano Observatory in Naples. "This is why Vesuvius is the most dangerous volcano in the world."

### SHRUBS OF DUST

The slumbering giant won't stay quiet forever. Seismic imaging studies have detected an unusual layer about 8–10 km deep under the mountain's surface. Mastrolorenzo and his colleague Lucia Pappalardo

interpret this layer as an active magma reservoir, which could produce large-scale "plinian"-style explosions — named after Pliny the Younger, who described the AD 79 eruption. The first rumblings of activity at Vesuvius could come weeks to years before an eruption, but there might be little, if any, warning of the eruption itself. Pappalardo and Mastrolorenzo collected seismic measurements from past eruptions, and found evidence that magma ascended rapidly — in just a few hours — from its deep chamber to the surface.

For many years the largest known eruption of Vesuvius was that of AD 79. But in 2006, Mastrolorenzo and Michael Sheridan at the University at Buffalo in New York described details of another, much larger eruption, about 3,800 years ago in the Bronze Age. Fiery avalanches of ash and debris detected pyroclastic flows travelled 20 kilometres and covered the whole of the area of present-day Naples. "The deposits right in the centre of Naples are 4 metres thick," says Sheridan. "Even a few inches would be enough to kill everyone."

Given these concerns, the Vesuvius observatory team has urged the Neapolitan authorities to base their emergency plan on a worst-case scenario. "It's a very dangerous volcano," says Bruno Agus Mast. "A crisis could start today," says Mastrolorenzo. "The trouble is that nobody would be able to tell how long it would last, what type of eruption it would be, or how the event would evolve." The researchers recommend the complete evacuation of an area 20 kilometres around Vesuvius if earthquakes and other signs of unrest merit that the volcano has to life. Not all scientists share this doomsday scenario. Some have even proposed that Vesuvius is becoming less explosive. Bruno Scaillet and his colleagues at the University of Orleans in France argue that the eruptive style of Vesuvius has changed as the magma chambers feeding the eruptions have migrated upwards. In the latest scenario, an eruption starting from a relatively shallow level 3 kilometre below the surface, "evidence suggests the magma stored there is less viscous, so it is less prone to causing large explosions. If the past trend holds, says Scaillet, the next eruption could be similar to the most recent ones."

Scaillet adds that the seismically unusual layer of ash and debris below the surface could be magma, but it could also be some other fluid such as water or brine. "These various issues are far from being settled," he says.

### EMERGENCY PLANNING

With the size of a future eruption in doubt, and a public that is concerned about day-to-day problems such as traffic and crime, mitigating the hazard of Vesuvius is an enormous task shared by researchers and the civil authorities.

Scientists keep constant tabs on Vesuvius through a network of sensors that monitor for earthquakes, ground deformation and changes in the chemistry of escaping gases.

And Italy's Civil Protection Department (DPC) maintains a National Emergency Plan for Vesuvius. The plan, first developed in 1995, is based on a scenario for an intermediate-sized eruption, similar to one that occurred

in 1631. That sub-plinian blast killed 6,000 people but affected an area much smaller than the earlier plinian eruptions.

Now the DPC has divided the volcano

into three regions according to the type of hazard expected. The red zone, closest to Vesuvius, is deemed most at risk from pyroclastic flows, so the plan calls for the evacuation of all 600,000 residents in this area before an eruption starts ('In the line of fire'). The main danger in the yellow zone comes from falling ash and small rocks. Officials would fall back to the green zone if the eruption started and the direction is known, before ordering an evacuation of regions in yellow zones downwind of the volcano. The blue zone is at risk from floods and mud flows triggered by the eruption, and would be evacuated according to the same plan. The red zones were selected from the hazard maps because they are the places where pyroclastic flows blow to the east, away from the city.

In 2003, the DPC announced that it would constantly update the emergency plan to take account of new scientific information. The red zone is being expanded to include the eastern districts of Naples and officials reduced the evacuation period from two weeks to 72 hours, recognizing that there may be less of a warning before the eruption.

Nevertheless, some researchers argue that the plan has ignored important scientific evidence. Last year, Mastrolorenzo and Pappalardo<sup>1</sup> and Giuseppe Rolandi<sup>2</sup> at the University of Naples found that even with an intermediate-sized eruption, pyroclastic flows could travel several miles and inundate the northern part of the city, as both Mastrolorenzo and Rolandi had predicted. A complete evacuation of the island was avoided.

For Vesuvius, Baxter and his colleagues have used geological data and models of eruptive processes to develop an "event-tree" to display the full range of possible eruptions.<sup>3</sup> If sensors on the volcano pick up signs of magmatic unrest, the analysis suggests a 70% probability of a small, intermediate-sized eruption and a 30% chance of a catastrophic plinian one. The most likely event is a violent but smaller blast, like the one in 1944, with lava flows and moderate ash emissions.

For now, this kind of probabilistic approach seems the only way forward for volcanologists and disaster planners, as there is no recipe for accurate eruption forecasting on the horizon. "It's an extremely complex problem to solve," says Antonio Neri of the INGV's laboratories in Pisa. "We simply do not know how the volcano works."<sup>4</sup>

Katherine Barnes is a freelance writer in London.

1. Pappalardo, L. & Mastrolorenzo, G. *Earth Planet. Sci. Lett.* **269**, 143–153 (2008).
2. Mastrolorenzo, G. et al. *Proc. Natl Acad. Sci. USA* **103**, 4366–4370 (2006).
3. Scaillet, B. et al. *Nature* **455**, 216–219 (2008).
4. Mastrolorenzo, G. & Pappalardo, L. *J. Geophys. Res.* **115**, B12212 (2010).
5. Rolandi, G. *J. Volcanol. Geotherm. Res.* **189**, 140–147 (2010).
6. Baxter, P. J. et al. *J. Volcanol. Geotherm. Res.* **178**, 454–473 (2008).

FEATURE NEWS



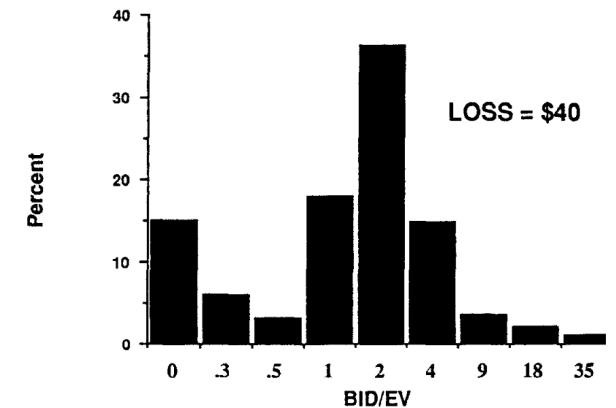
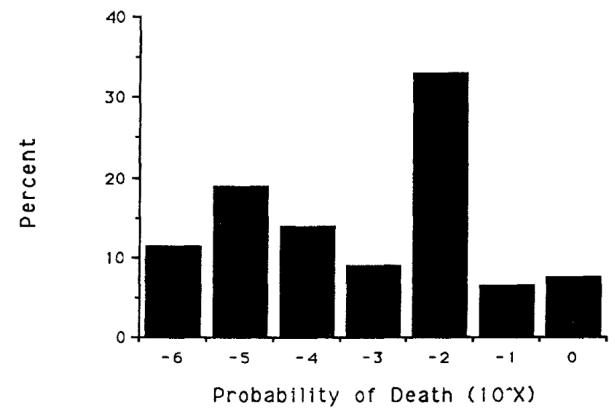
SOURCE: LEFT: DPC/CIVIL PROTECTION; RIGHT: 4. MASTROLORENZO, G. ET AL. *Proc. Natl Acad. Sci. USA* **103**, 4366–4370 (2006); 2. MASTROLORENZO, G. ET AL. *Proc. Natl Acad. Sci. USA* **103**, 4366–4370 (2006); 3. SCAILLET, B. ET AL. *NATURE* **455**, 216–219 (2008); 4. MASTROLORENZO, G. & PAPPALARDO, L. *J. GEOPHYS. RES.* **115**, B12212 (2010); 5. ROLANDI, G. *J. VOLCANOL. GEOTHERM. RES.* **189**, 140–147 (2010); 6. BAXTER, P. J. ET AL. *J. VOLCANOL. GEOTHERM. RES.* **178**, 454–473 (2008).

## Perceived Loss Probabilities $\omega_i(p)$

Experimental study of McClelland, Schulze & Coursey (1993) : bimodal distribution of willingness to pay for insurance, with two groups

- neglect low-probability risks, do not purchase insurance
- willingness to pay higher than expected loss

link between the bimodal risk judgments and protective behavior McClelland, Schulze & Hurd (1990), over and underweighting rare extreme events, Epper & Fehr-Duda (2017). See also Charpentier & le Maux (2014) on optimal (Pareto) vs. equilibrium (Nash)



## Loss Probability $p$ and Perceived Loss Probability $\omega(p)$

*“The probability of tail events is overestimated, which is consistent with probability weighting in prospect theory”* Botzen, Kunreuther & Michel-Kerjan (2015) (but “potential damage is underestimated”)

Myopic loss aversion or myopic probability weighting, see Barberis, Huang & Thaler (2006), and Barberis (2013) on psychology of “tail events”.

See also Gilboa & Schmeidler (1989) on the use of capacities : instead of computing  $\mathbb{E}_{\mathbb{P}}[u(\omega - \pi - L + I(L))]$ , consider

$$\min_{\mathbb{Q} \in \mathcal{P}} \{\mathbb{E}_{\mathbb{Q}}[u(\omega - \pi - L + I(L))]\} \text{ for some set of beliefs } \mathcal{P}$$

Can capture uncertainty associated with climate change, that makes it difficult to respond optimally, see Dessai & Hulme (2006) or Tompkins & Adger (2005).

## Loss Probability $p$ : Meteorological Perspective

For hurricanes, see [Gray et al. \(1992\)](#), the seasonal number of intense hurricanes is

$$\hat{N} = 3.571 + 0.042(U_{50} + 0.103U_{30} - 1.415|U_{50} - U_{30}|) + 0.717(R_S + 2.455R_G)$$

where  $U$ 's are upper-air [zonal winds](#) at 50 and 30 mb and  $R$ 's are composite functions of August-September western Sahel ( $R_S$ ) and August-November Gulf of Guinea ( $R_G$ ) [rainfall](#).

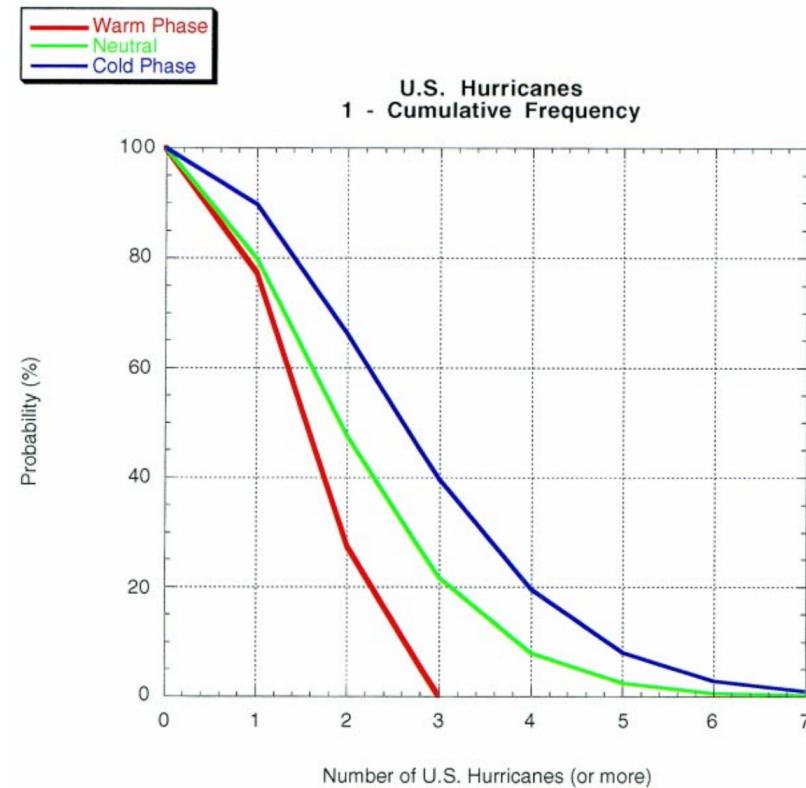
See also [Klotzbach & Bell \(2017\)](#) on the  
Landfalling Hurricane Probability Project  
(with all landfall probability calculations)

	A	N	O	P	Q	R
3		Climatological Probability		Current-Year Probability		
4		(Using Poisson)		(Using Poisson)		
5	State	H	MH	H	MH	
6	Texas	33%	12%	21%	7%	
7	Louisiana	30%	12%	19%	7%	
8	Mississippi	11%	4%	6%	3%	
9	Alabama	16%	3%	10%	2%	
10	Florida	51%	21%	35%	13%	
11	Georgia	11%	1%	7%	1%	
12	South Carolina	17%	4%	11%	2%	
13	North Carolina	28%	8%	18%	5%	

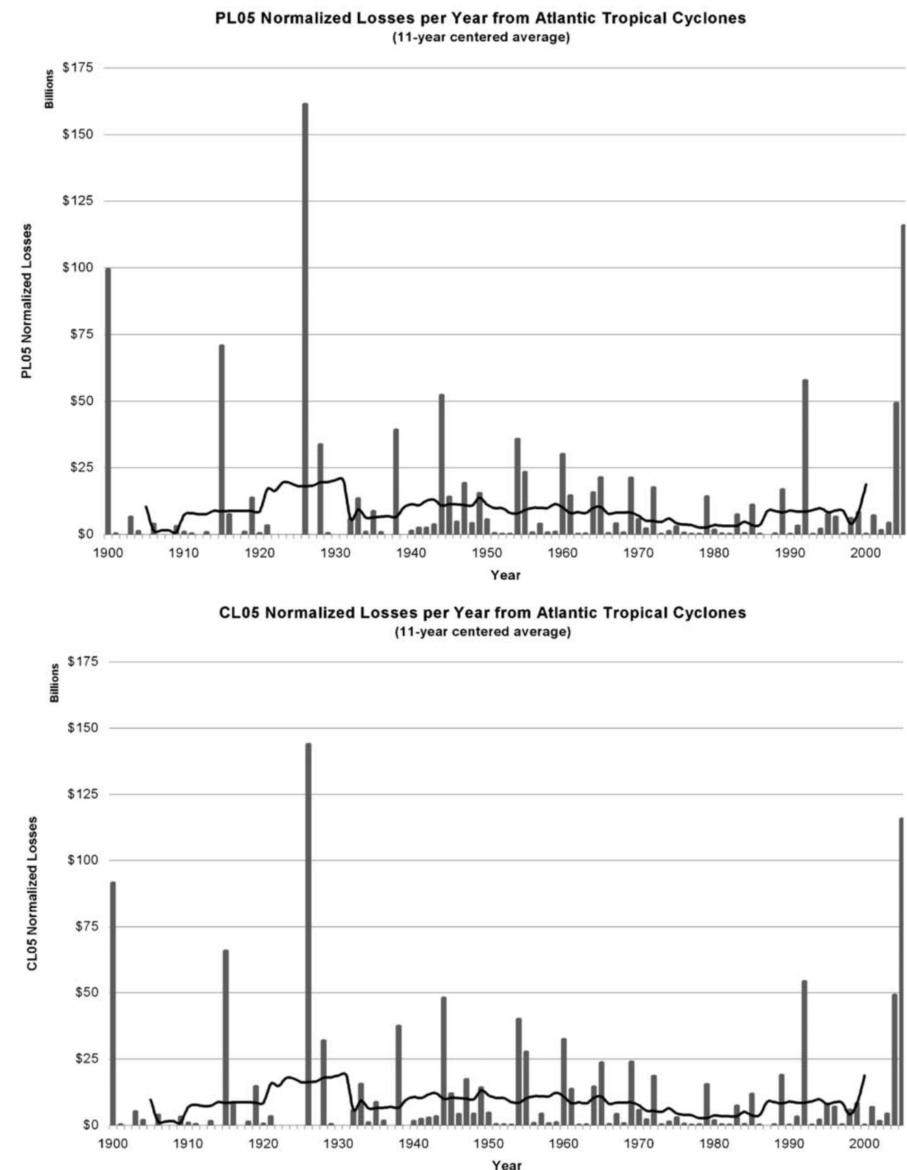
## Statistical Perspective

Need (long) historical data,  
(if possible with normalized losses)

see Pielke Jr. *et al.* (2008)



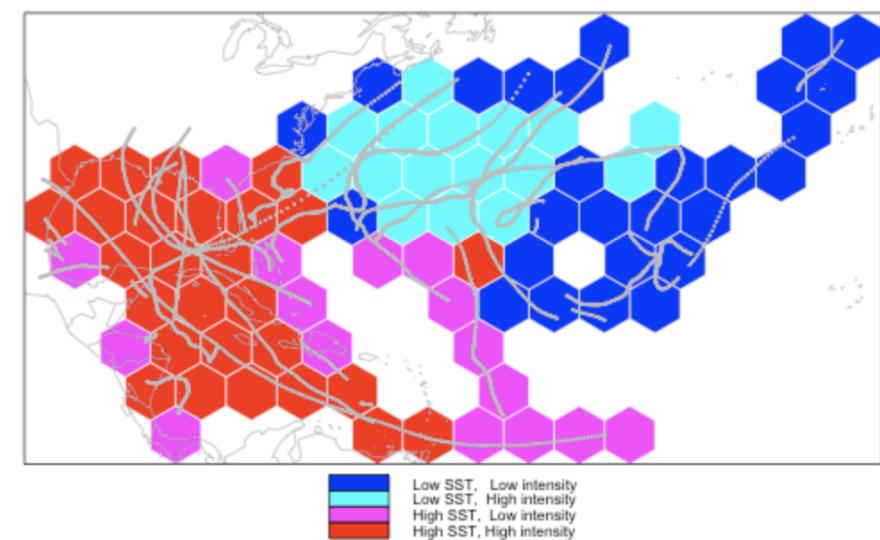
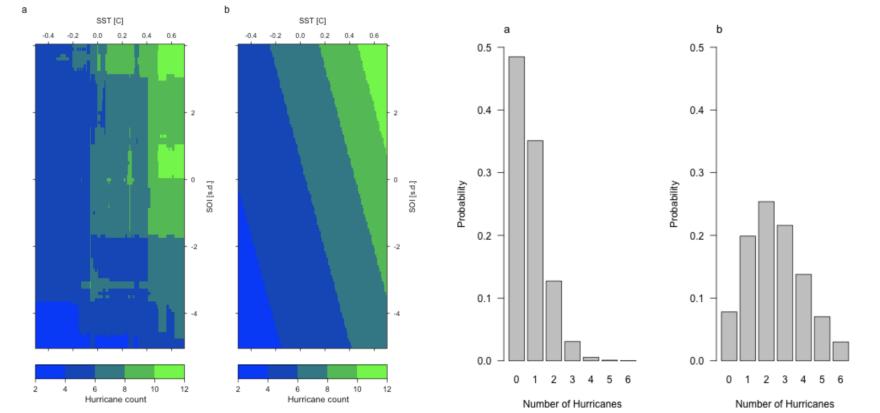
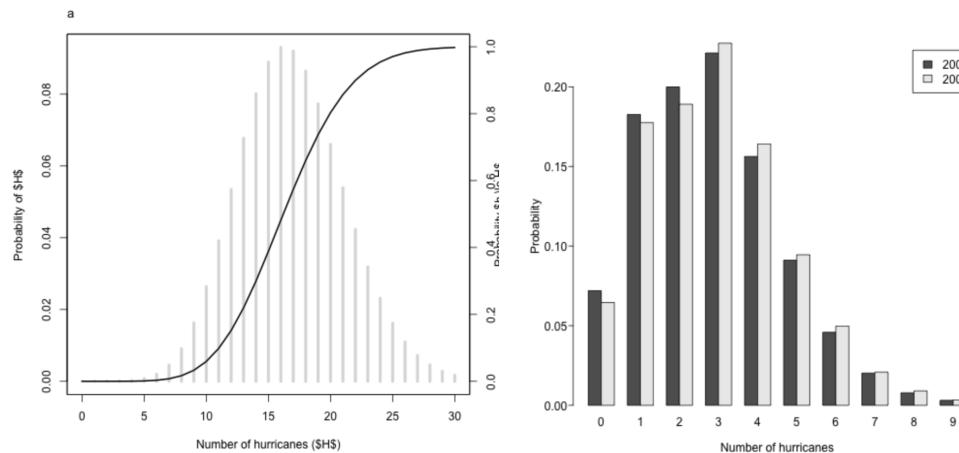
From Bove *et al.* (1998) for hurricanes.



## Loss Probability : Statistical Perspective

See Elsner & Jagger (2013) for a exhaustive statistical analysis of hurricanes,

- Poisson regression for **counts**
  - Southern Oscillation Index (SOI)
  - sea-surface temperature (SST)
- regression for **intensity**
- spatial models for **trajectories**
- Bayesian models for **one year prediction**



# Statistical Perspective

hard to justify in a changing environment  
*In order to apply any theory we have to suppose that the data are homogeneous, i.e. that no systematical change of climate and no important change in the basin have occurred within the observation period and that no such changes will take place in the period for which extrapolations are made*

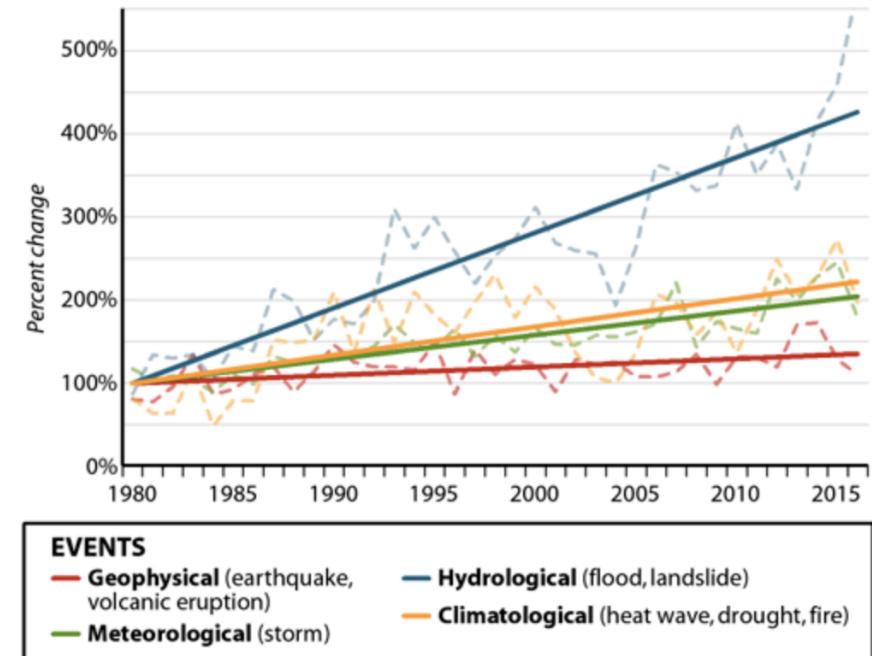
Gumbel (1941)

see flood risk (dams, reservoirs, etc)

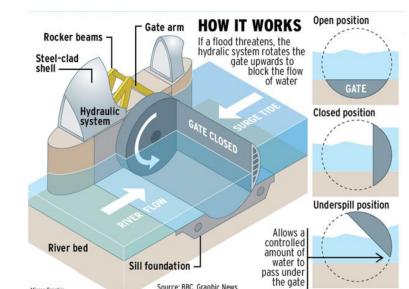
see storms or heatwaves (climate change)

Charpentier (2011)

**GLOBAL TRENDS IN NATURAL CATASTROPHES**  
*Percentage change each year in number of events compared to 1980*

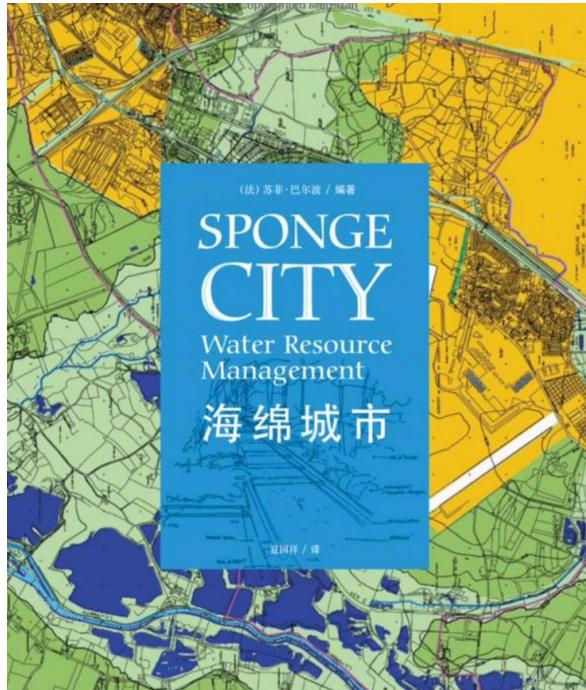


SOURCES: MunichRe NatCatSERVICE; European Academies Science Advisory Council



## The Architect / Urban Planner Perspective ?

See Dong & Han (2011), Bardaux (2016), Li *et al.* (2017) Jiang *et al.* (2018) on sponge cities (urban underground water system operates like a sponge to absorb, store, leak and purify rainwater, and release it for reuse when necessary)



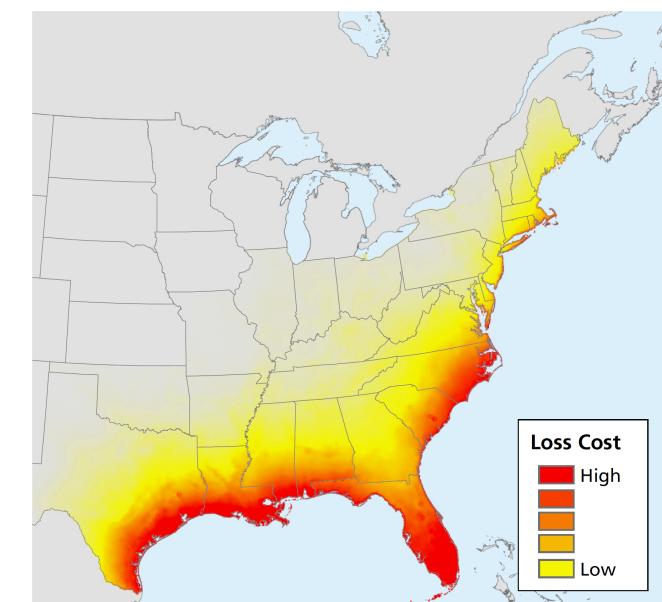
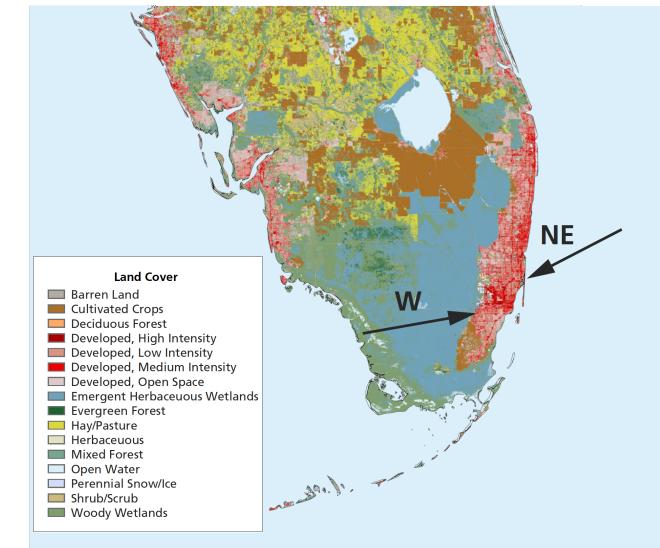
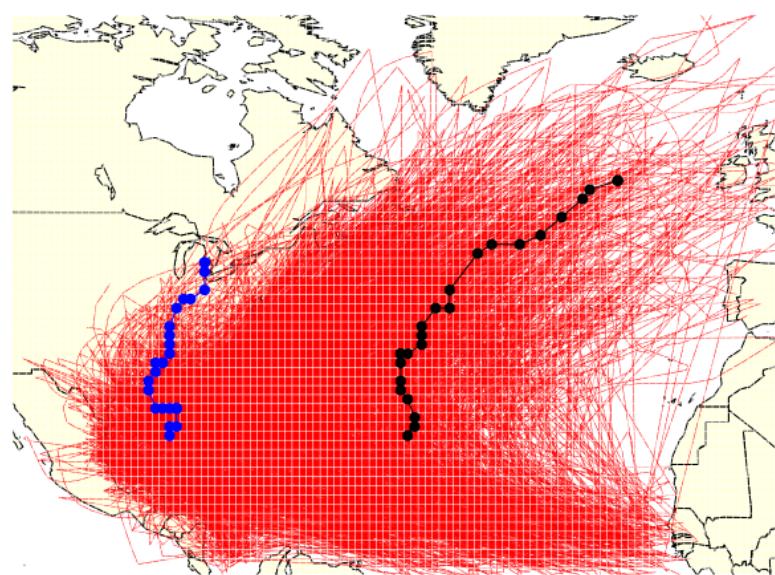
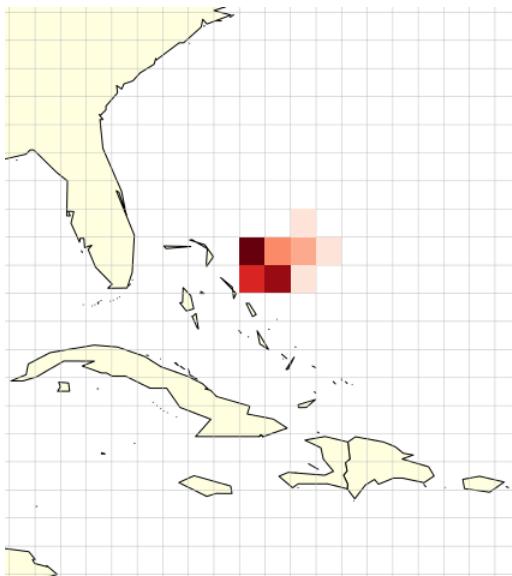
## Loss Probability $p$ : Actuarial Perspective

Actuaries use catastrophe softwares

- Risk Management Solutions (**RMS**)
- AIR Worldwide (**AIR**)
- Risk Quantification & Engineering (**RQE**) EQECAT

see Cole, Macpherson & McCullough (2010).

Generation of climatic scenarios (+ losses)



see for Markovian generation Charpentier (2014).

## Predictive Markets

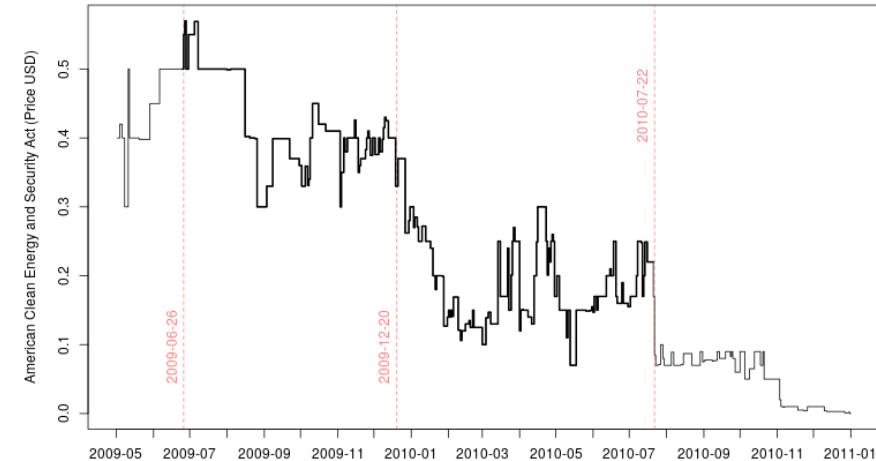
Peer prediction systems

[Wolfers & Zitzewitz \(2004\)](#) for a description

- elections (e.g. popes, XVIth Century)

[Rhode & Strumpf \(2008\)](#)

- politics, [Meng \(2016\)](#) Waxman-Markey bill (2009-2010, finally rejected)
- sports (racetrack bets) [Ali \(1977\)](#)
- infectious diseases, [Polgreen, Nelson & Neumann \(2006\)](#) or [Tung, Chou & Lin \(2015\)](#)
- climate risk [Hallstrom & Smith \(2005\)](#) for hurricanes



## (Perceived) Loss Probability $p$ : Betting Markets

See Eisenberg & Gale (1959) model, on consensus and subjective probabilities (used in Manski (2004) on all-or-nothing contracts)

Individual  $i$  (wealth  $b_i$ ) can bet on horse  $j$  an amount  $\beta_{i,j}$  (so that  $\beta_{i,1} + \dots + \beta_{i,J} = b_i$ ). Assume that  $b_1 + \dots + b_I = 1$ .

Let  $\pi_j$  denote the sum bet on horse  $j$ ,  $\pi_j = \beta_{1,j} + \dots + \beta_{I,j}$ . Observe that from the budget constraint  $\pi_1 + \dots + \pi_J = 1$ , hence  $\pi_j$ 's are probabilities.

Individual beliefs can be related expressed through a probability vector  $\mathbf{p}_i = (p_{i,1}, \dots, p_{i,J})$ , then there is an equilibrium if

$$p_{i,j} = \pi_j \cdot \max_s \left\{ \frac{p_{i,s}}{\pi_s} \right\} \text{ as soon as } \beta_{i,j} > 0$$

(so called Eisenberg-Gale equilibrium).

## (Perceived) Loss Probability $p$ : Betting Markets

Consider a contract that pay \$1 if an event  $A$  occurs (e.g. major flood), sold at price  $\pi_A$ . If there is a contract on  $\bar{A}$ , then  $\pi_{\bar{A}} = 1 - \pi_A$  (no arbitrage).

Assume that individual  $i$  believe that  $A$  will occur with probability  $p_i$ . If  $p_i > \pi_A$ , he should purchase  $b_i$  units of  $A$  (otherwise  $b_i$  units of  $\bar{A}$ ).

There is equilibrium if

$$\sum_{i=1}^I b_i = \frac{1}{\pi_A} \sum_{i=1}^I b_i \mathbb{P}[p_i > \pi_A] = \frac{1}{\pi_{\bar{A}}} \sum_{i=1}^I b_i \mathbb{P}[p_i < \pi_A]$$

hence, if wealth  $b_i$  is independent of belief  $p_i$ , then prices are probabilities

$$\pi_A = \mathbb{P}[p_i > \pi_A] = 1 - \pi_{\bar{A}}.$$

see [Wolfers & Zitzewitz \(2004\)](#) for further interpretation.

## Parimutuel and Betting Markets

- 2004 and 2005, ICAP, Goldman and Nymex (see [IFR](#)) with parimutuel derivative on energy (changes in crude oil and natural gas inventories)
- 2004, CME launched futures trading on the US Consumer Price Index (CPI), see [Filippov \(2005\)](#)
- 2008, LLC (see [Ou-Yang \(2010\)](#)) with parimutuel derivative on [hurricanes](#)
- The [longitude](#) plateforme (“*real time calculation of the odds*”).
- The [Iowa Electronic Markets](#) for elections, “*using this wisdom of crowds the price of a contract at any given time is a forecast of the outcome*”



## HuRLOs (Hurricane Risk Landfall Option) options

The exist the Hurricane Futures Market (HFM), see [Kelly, Letson, Nelson, Nolan & Solis \(2010\)](#) for a description

Hurricane Risk Landfall Options (HuRLOs) were launched in October 2008 by Weather Risk Solutions, LLC (WRS) and promised a mutualized marketplace for hurricane options, using [parimutuel options](#)

From French [pari mutuel](#), literally, mutual stake :

*“A system of betting on races whereby the winners divide the total amount bet, after deducting management expenses, in proportion to the sums they have wagered individually”*, see [Baron & Lang \(2007\)](#).

also calle [universal Dutch auction](#) in [Syz \(2008\)](#)

## Parimutuel and Betting Markets

“A Shapley-Shubik market game for contingent claims within a probability state space with secured selling is a parimutuel market” ([Lange & Economides \(2005\)](#))

A matching / pricing mechanism is necessary to get [financial efficiency](#) (self-hedging). Participant  $i$  submit orders, state bids  $\beta_i$ ,  $b_i$  a limit share quantity and  $\bar{\pi}_i$  a limit price per share

The market organizer determines the order fill  $x_i$  (and state prices  $\pi_j$ ), using call auction mechanism, see [Peters, So and Ye \(2005\)](#) and [Agrawal, Wang & Ye \(2008\)](#).

Using a [Linear Programming Market Mechanism](#), market organizer should solve

$$\text{maximize} \left\{ \sum_{i=1}^I \bar{\pi}_i x_i - z \right\} \text{ where } z \text{ is some worst case cost}$$

$$\text{subject to} \quad \text{cost if state } j \text{ occurs} = \sum_{i=1}^I \beta_{i,j} x_i \leq z \quad \forall j$$

$$0 \leq \bar{\pi}_i x_i \leq b_i \quad \forall i$$

## Parimutuel and Betting Markets

More realistically, one can consider some dynamic parimutuel market maker as in [Agrawal et al. \(2014\)](#) for recent advances, and [Chen & Pennock \(2010\)](#) for an overview.

See also techniques to get some peer prediction Bayesian Nash equilibrium solvers (see [Jurca & Faltings \(2008\)](#) on elections and bets) and the related collective revelation mechanism (see [Goel, Reeves & Pennock \(2009\)](#))

*“Peer prediction systems are designed for eliciting information on events where ground truth does not exist or is unobtainable”, [Chen & Pennock \(2010\)](#)*

## Take-Away Conclusion

- knowing **real probabilities** of occurrence disasters is either complicated (hurricanes) or very complicated (flood)
- those probabilities are necessary to assess solvency of insurance companies (central limit theorem is based on true probabilities)
- insurance pricing is based on **beliefs** of insured, and insurance companies
- **predictive markets** can be an interesting revelation mechanism of crowd beliefs (can be used to assess if an insurance market can actually exist, or not)