

How to Estimate the Occurrence Probability of Natural Catastrophes

Arthur Charpentier (UQAM),

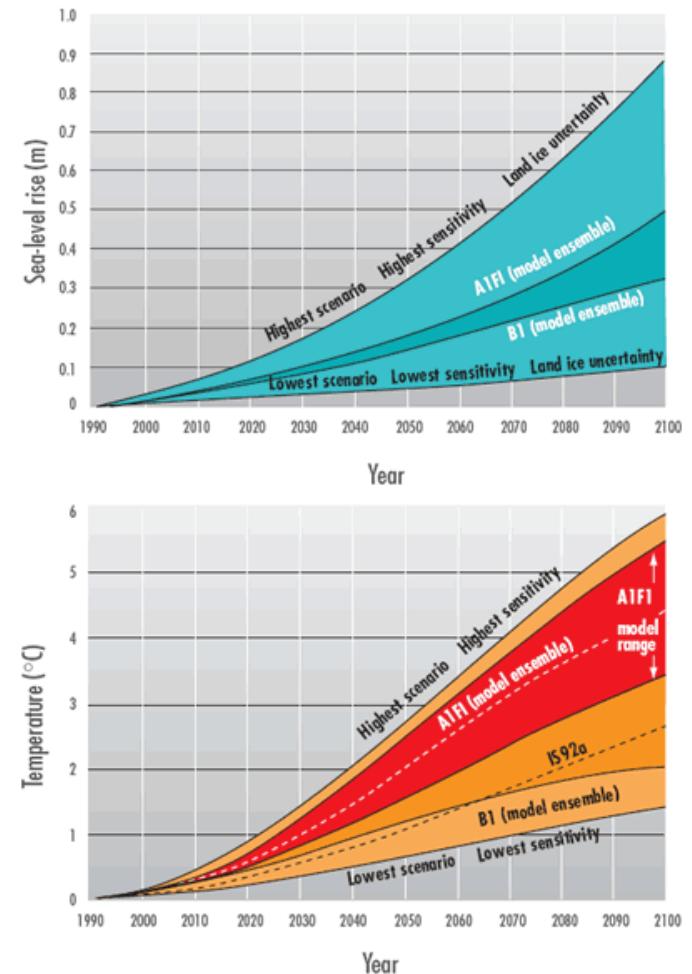
ETH Zürich, January 2019



So what ?

In this talk, “*natural catastrophe*” is seen from an insurance perspective

- quantitative analysis of natural events
difficult part : climate is changing (warming)
classical curves from **IPCC** on temperature or sea level
(and references therein)
- impact on the insurance industry ?
In 2015, French Federation of Insurers claimed
“*natural disasters : insurers’ bills could double by 2040*”
(in 25 years, see <https://www.argusdelassurance.com/>)



Insurance and Uncertainty : the Framework

Classical framework : agent, with utility $u(\cdot)$ facing random loss L , purchases insurance at price π if

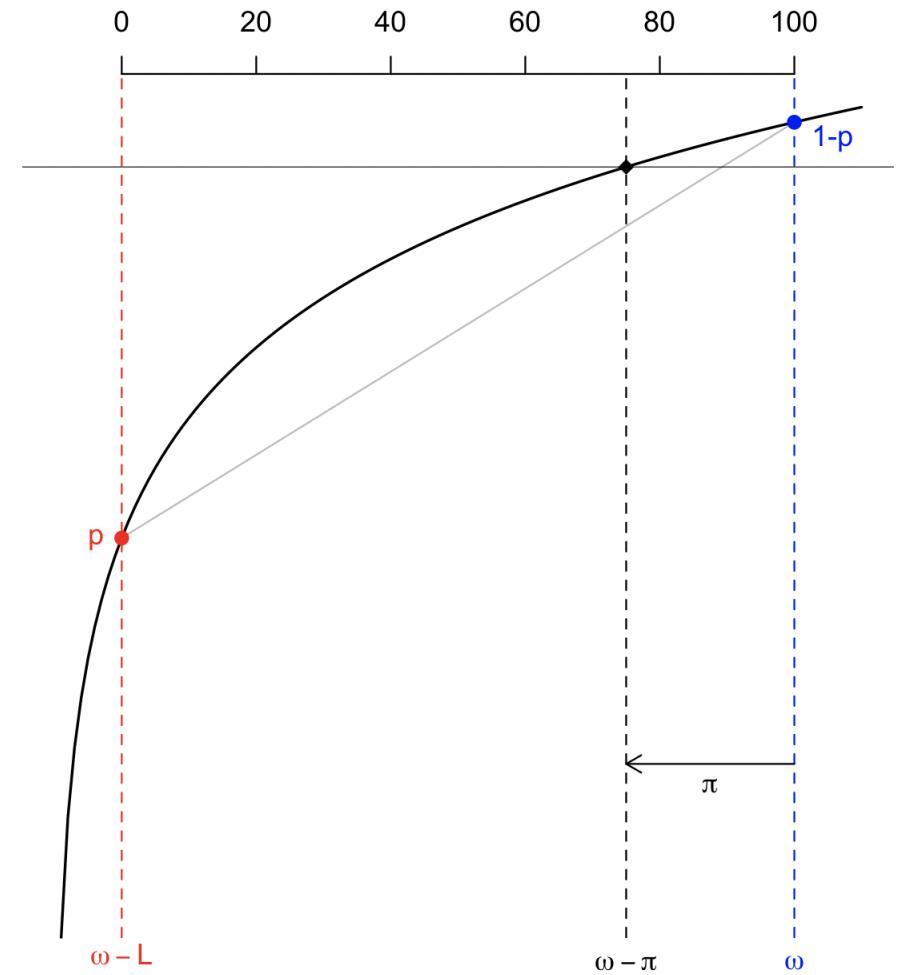
$$\mathbb{E}[u(\omega - \pi - L + I(L))] > \mathbb{E}[u(\omega - L)]$$

(von-Neumann expected utility).

Let π^* denote the highest premium

$$\mathbb{E}[u(\omega - \pi^* - L + I(L))] = \mathbb{E}[u(\omega - L)]$$

where I denotes the indemnity function,
 $I(\ell) \in [0, \ell]$.



Insurance and Uncertainty : the Framework

Suppose that L is a binary loss, $L \in \{0, \ell\}$ with probabilities $1 - p$ and p , or
 $L = X \cdot \ell$, $X \sim \mathcal{B}(p)$

$$(1 - p) \cdot u(\omega - \pi^*) + p \cdot u(\omega - \pi^* - \ell + I(\ell)) = (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

If $I(\ell) = \ell$, $u(\omega - \pi^*) = (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$

If u is linear (no risk aversion), $\pi^* = \mathbb{E}[I(L)] = p \cdot I(\ell)$

If u is concave (risk aversion), $\pi^* > p \cdot I(\ell)$

$p \cdot I(\ell)$ is the actuarial (pure) premium, also called [Learned Hand formula](#) in law business (see [Grossman et al. \(2006\)](#)) or simply “*probability times consequence*” in climate literature (see [Schneider \(2002\)](#))

Insurance and Uncertainty : the Framework

Consider the simplistic case of homogeneous agents.

An **agent**, with utility u , will purchase insurance if

$$(1 - p) \cdot u(\omega - \pi) + p \cdot u(\omega - \pi - \ell + I(\ell)) > (1 - p) \cdot u(\omega) + p \cdot u(\omega - \ell)$$

Assume that this inequality is satisfied.

An **insurance company**, with utility v , will sell insurance if

$$\mathbb{E}[v(\kappa + n\pi - S)] > u(\kappa), \quad S = \sum_{i=1}^n L_i = \sum_{i=1}^n X_i \cdot \ell, \quad \text{where } X_i \sim \mathcal{B}(p),$$

where κ is the capital of the company, and S is the total indemnity.

The actuarial fair premium is obtained when v is linear :

$$\pi^* = \mathbb{E}[I(L)] = p \cdot I(\ell)$$

Insurance and Uncertainty : the Framework

If risks are exchangeable $X = L_1 + \cdots + L_n = N_n \cdot I(\ell)$,

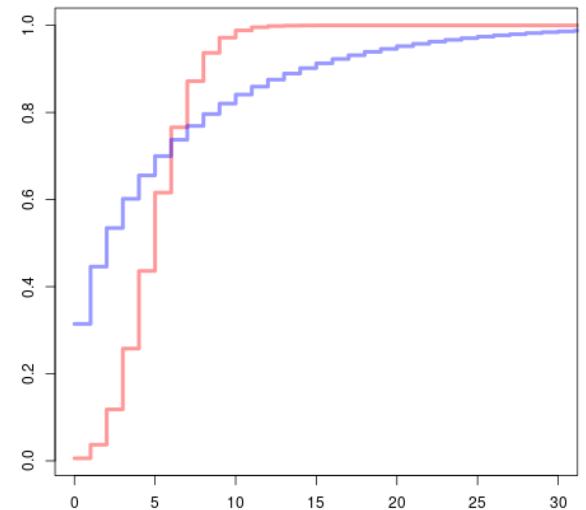
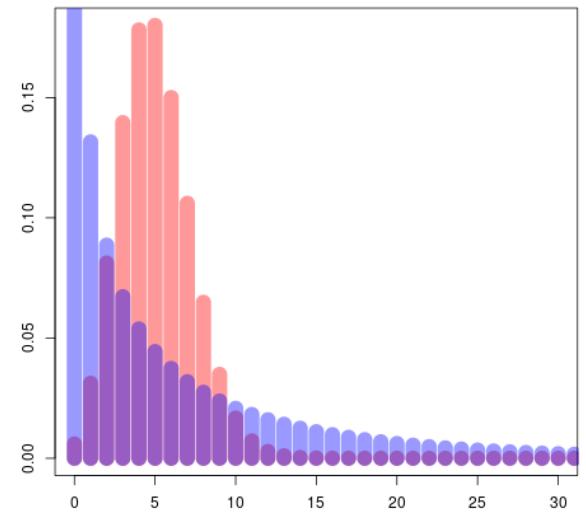
$$\mathbb{P}[N_n = k] = \int_0^1 \binom{n}{k} \theta^k (1-\theta)^{n-k} dG(\theta), \quad \mathbb{P}[L_i > 0] = p$$

from [de Finetti \(1921\)](#). See [Charpentier & le Maux \(2014\)](#) for correlated risks and disaster (with endogeneous default probability of the insurer).

Classical framework : [binomial-beta](#) model (G is a Beta distribution $\mathcal{B}(\alpha, \beta)$), then $r = \text{corr}(X_i, X_j) = (1 + \alpha + \beta)^{-1}$. Then

$$\mathbb{E}[N_n] = np \text{ while } \text{Var}[N_n] = (n + n(n-1)r)p(1-p)$$

(the insurance company has correlation aversion, [Richard \(1975\)](#))



Insurance and Uncertainty : the disaster puzzle

Earthquakes and floods cause potentially large losses, rational people should find actuarially fair insurance policies attractive, see [Kunreuther \(1996\)](#)

Individuals *underestimate* the true probability of a disaster event occurring and/or have fairly high discount rates for the benefits of uncertain future reimbursements due to losses.

[Kunreuther & Pauly \(2004\)](#) prove that even when insurance for low-probability, high-loss events is offered at favorable premiums, the search costs associated with obtaining the information on premiums and disaster probabilities may prevent from purchasing insurance.

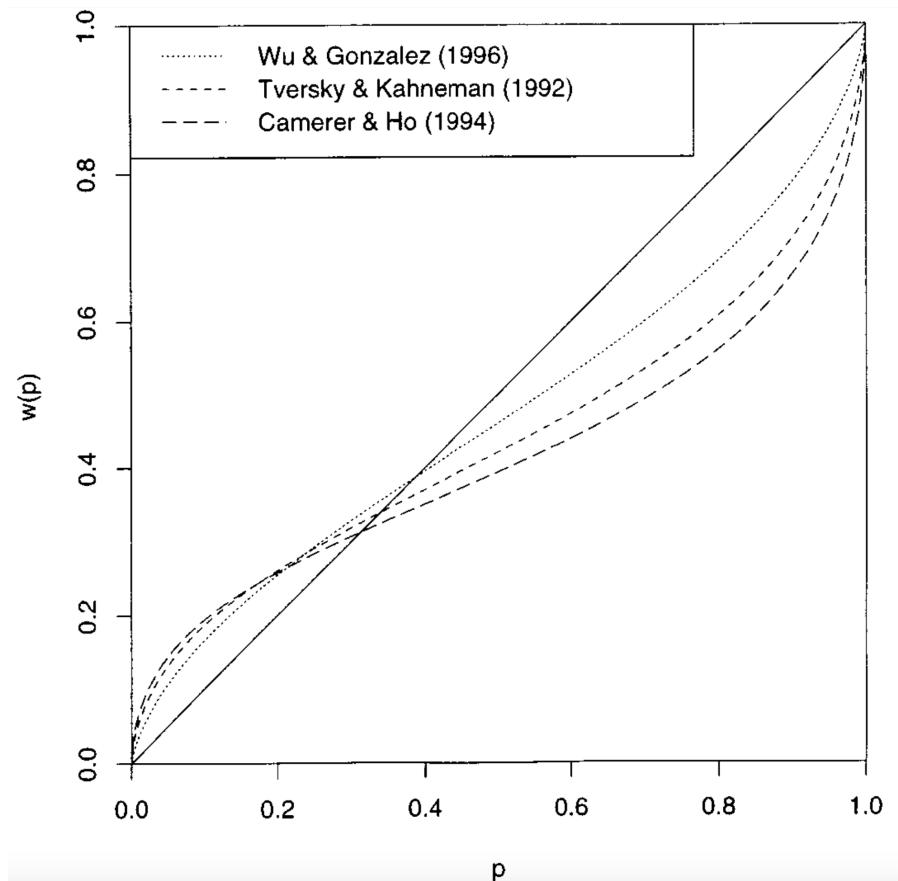
Individuals lack information about the expected harm, before the disaster, see [Botzen \(2013\)](#). Insurance prices can be a signal used to correct for possible bias (unless market distortion), see [Thieken et al. \(2006\)](#).

Loss Probability p and Perceived Loss Probability $\omega(p)$

RDEU framework, or Prospect theory,
 Kahneman & Tversky (1979) or Wu & Gonzalez (1996)

$$\omega(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

(experimental) evidence of convexity of ω
 extreme risk aversion in situations that involve low stakes see Huysentruyt & Read (2010), but inadequate insurance coverage against disaster, Viscusi (2010), even if it is highly subsidized : underweighting of adverse tail events.



Loss Probability p and Perceived Loss Probability $\omega(p)$

“rare extreme events are underweighted because people are not aware of their existence” : no, see Barnes (2011)



Vesuvius last awoke with a small blast in 1944. A large eruption could unleash incendiary avalanches and ash that would threaten millions of people.

EUROPE'S TICKING TIME BOMB

Vesuvius is one of the most dangerous volcanoes in the world — but scientists and the civil authorities can't agree on how to prepare for a future eruption.

BY KATHERINE BARNES

It starts with a blast so strong that a column of ash and stone rockets 40 kilometres up into the stratosphere. The debris then drops to Earth, pelting the surface with boiling hot fragments of pumice and covering the ground with a thick layer of ash. Roots crumble and vehicles grind to a halt. Yet Vesuvius is still too tame. It has had tsunamis, ash, mud, incandescent lava, mudslides, pyroclastic flows and gas run down the slopes of the volcano, pulverizing buildings and burying everything in their path. Almost overnight, a packed metropolis becomes a volcanic wasteland.

This is Naples, Italy, in the throes of a cataclysmic eruption of Vesuvius — the volcano that buried Pompeii. The scenario may sound far-fetched, but in the wake of Japan's recent earthquake and tsunami, many areas are reassessing the risks from their own "black swans," a term used to describe unlikely but potentially devastating disasters. And Naples stands out particularly vulnerable, with a population of 3 million living in the shadow of Vesuvius.

The volcano has been eerily dormant since a

small eruption in 1944, but recent studies suggest that Vesuvius could be more dangerous than previously assumed, which has prompted a vigorous debate about the risk and scale of future eruptions. The scientific community is split on whether to focus on the potential for a catastrophe in the event of earthquakes and other signs heralding the volcano's reawakening. "There would be no modern precedent for an evacuation of this magnitude," says Giuseppe Mastrolorenzo at the Vesuvius Volcano Observatory in Naples. "This is why Vesuvius is the most dangerous volcano in the world."

BRUMBLINGS OF DISSENT
The slumbering giant won't stay quiet forever. Seismic imaging studies have detected an unusual layer about 8–10 kilometres deep under the mountain's surface. Mastrolorenzo and his colleague Lucia Pappalardo

For more on volcanic eruptions see:
go.nature.com/GINjyk

interpret this layer as an active magma reservoir, which could produce large-scale "plinian"-style explosions — named after Pliny the Younger, who described the AD 79 eruption. The first rumblings of activity at Vesuvius could come weeks to years before an eruption, but until now, the main concern of the eruption itself. Pappalardo and Mastrolorenzo analysed the geochemistry of rock fragments from past eruptions, and found evidence that magma ascended rapidly — in just a few hours — from its deep chamber to the surface.

For many years the largest known eruption of Vesuvius was that of AD 79. But in 2006, Mastrolorenzo and his colleagues at the University at Buffalo in New York described geological evidence for a much larger blast, about 3,800 years ago in the Bronze Age¹. Fiery avalanches of ash and debris called pyroclastic flows travelled 20 kilometres and covered the whole of the area of present-day Naples. "The deposits right in the centre of Naples are 4 metres thick," says Sheridan. "Even a few inches would be enough to kill everyone."

Given these concerns, the Vesuvius observatory has developed an emergency plan for the worst-case "maximum possible" eruption similar to the Bronze Age blast. "A crisis could start today," says Mastrolorenzo. "The trouble is that nobody would be able to tell how long it would last, what type of eruption it would be, or how the event would evolve." The researchers recommend the construction of a new seismic network around Vesuvius if earthquakes and other signs of unrest hint that it is coming back to life.

Not all scientists share this doom-laden outlook. Some groups have even proposed that Vesuvius is becoming less explosive. Bruno Scaillet and his colleagues at the University of Orleans in France argue that the eruptive style of Vesuvius has changed as the magma chamber has shifted upwards, causing the magma to rise upwards from a relatively shallow level 3 kilometres below the surface². Evidence suggests the magma stored there is less viscous, so it is less prone to causing large explosions. If the past trend holds, says Scaillet, the next eruption could be a relatively minor one.

Scaillet adds that the seismically unusual layer 10 kilometres below the surface could be magma, but it could also be some other fluid such as water or brine. "These various issues are far from being settled," he says.

EMERGENCY PLANNING

With the size of any future eruption in doubt, and a public more concerned about day-to-day problems such as traffic and crime, mitigating the hazard of Vesuvius is an enormous task shared by researchers and the civil authorities.

Scientists keep constant tabs on Vesuvius through a network of sensors that monitor for earthquakes and volcanic deformation and changes in the chemistry of erupting gases.

And Italy's Department of Civil Protection (DPC) maintains a National Emergency Plan for Vesuvius. The plan, first developed in 1995, is based on a scenario for an intermediate-sized eruption, similar to one that occurred

in 1631. That sub-plinian blast killed 6,000 people and affected an area much smaller than the earlier plinian eruption in 79.

The plan divides the area around the volcano into three regions according to the type of hazard expected. The red zone, closest to Vesuvius, is deemed most at risk from pyroclastic flows, so the plan calls for the evacuation of all 600,000 residents in this area before an eruption begins. The yellow zone is the next to be put on edge in the event of an eruption. Danger in the yellow zone comes from falling ash and small rocks. Officials would wait until the eruption starts, and the wind direction is known, before ordering an evacuation of regions in yellow zones downwind of the volcano. The blue zone at risk from floods and mud flows triggered by the eruption, and would be evacuated according to the same principles. The blue zone excluded the red zone, as hazard ash because the prevailing wind typically blows ash to the east, away from the city.

In 2003, the DPC announced that it would constantly update the emergency plan to take account of new scientific information. The red zone was being expanded to include the eastern districts of Pompeii and Herculaneum, and the evacuation time from two weeks to 72 hours, recognizing that there may be less of a warning before the eruption.

Nevertheless, some researchers argue that the plan has ignored important scientific evidence. Last year, Mastrolorenzo and Pappalardo³ and Giuseppe Rolandi at the University of Naples Federico II in Italy simulated a supereruption, pyroclastic flows would threaten several municipalities not currently included in the red zone. Mastrolorenzo says that officials should also not wait to evacuate the yellow zone, because ash would rapidly fill the air and plunge the area into total darkness. "You have to get people out before they are buried," says Mastrolorenzo.

Putting all the evidence together, they and other researchers insist that the emergency plan should correspond to the "worst-case

scenario", which means including metropolitan Naples and 3 million inhabitants in the red zone. In other words, for planning, says Jonathan Fink, a volcanologist at Portland State University in Oregon, once the volcano shows signs of unrest, authorities and scientists can re-evaluate. "If there is an error on the high side, there is less lost than would be the case in the opposite situation," he says.

In a paper published last year, the DPC advocates evaluating the eruption risk "on the basis of the present state of the volcano and not simply assuming the largest eruption event that ever occurred in the volcano's history." Some scientists agree. "You can't spend [everything]

on the absolute worst case. You need to reduce the risk in a rational way," says Giuseppe Marzocchi at the National Institute for Geophysics and Volcanology (INGV) in Rome. A complete evacuation of Naples' 3 million residents, he says, "would be impossible to manage".

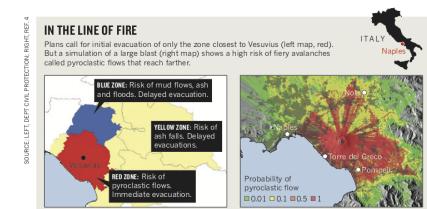
Marzocchi and others researchers are developing a probabilistic approach to the probabilities of different scenarios — that could help civil authorities evaluate the evidence during a crisis and choose a course of action. Peter Baxter, an expert in emergency planning at the University of Cambridge, UK, specializing in the impacts of volcanic eruptions and used this type of model successfully during the 1997 eruption in Mount Pinatubo, Philippines, to predict which regions would be affected. A complete evacuation of the island was avoided.

For Vesuvius, Baxter and his colleagues have used geological data and models of eruptive processes to develop an "event-tree" to display the full range of possible eruptions⁴. If sensors detect certain patterns, they can pick up signs of magmatic intrusions and assess the probability of an explosive eruption but only the chance of a catastrophic plinian one. The most likely event is a violent but smaller blast, like the one in 1944, with lava flows and moderate ash emissions.

For now, this kind of probabilistic approach seems the only way forward for volcanologists and disaster managers, as there is no recipe for accurate eruption forecasting in the horizon. "It's an extremely complex problem to solve," says Augusto Neri of the INGV's laboratories in Pisa. "We simply do not know how the volcano works."⁵

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Loss Probability p and Perceived Loss Probability $\omega(p)$

Myopic loss aversion or myopic probability weighting, see [Barberis, Huang & Thaler \(2006\)](#)

See also [Gilboa & Schmeidler \(1989\)](#) on the use of capacities : instead of computing $\mathbb{E}_{\mathbb{P}}[u(\omega - \pi - L + I(L))]$, consider

$$\min_{\mathbb{Q} \in \mathcal{P}} \left\{ \mathbb{E}_{\mathbb{Q}}[u(\omega - \pi - L + I(L))] \right\} \text{ for some set of beliefs } \mathcal{P}$$

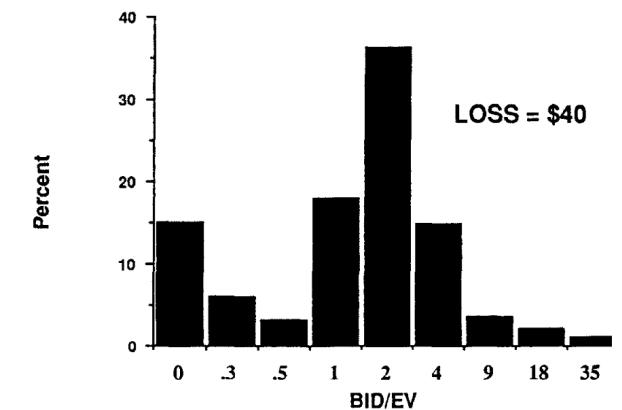
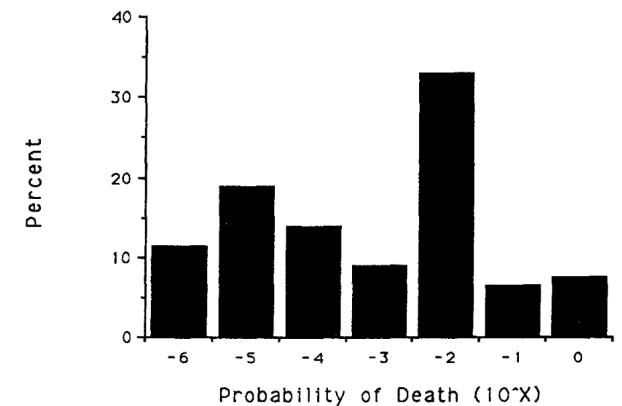
Can capture uncertainty associated with climate change, that makes it difficult to respond optimally, see [Dessai & Hulme \(2006\)](#) or [Tompkins & Adger \(2005\)](#).

Perceived Loss Probabilities $\omega_i(p)$

Experimental study of McClelland, Schulze & Coursey (1993) : bimodal distribution of willingness to pay for insurance, with two groups

- neglect low-probability risks, do not purchase insurance
- willingness to pay higher than expected loss

link between the bimodal risk judgments and protective behavior McClelland, Schulze & Hurd (1990), see also Charpentier & le Maux (2014) on optimal (Pareto) vs. equilibrium (Nash)



Loss Probability p : Meteorological Perspective

For hurricanes, see [Gray et al. \(1992\)](#), the seasonal number of intense hurricanes is

$$\hat{N} = 3.571 + 0.042(U_{50} + 0.103U_{30} - 1.415|U_{50} - U_{30}|) + 0.717(R_S + 2.455R_G)$$

where U 's are upper-air [zonal winds](#) at 50 and 30 mb and R 's are composite functions of August-September western Sahel (R_S) and August-November Gulf of Guinea (R_G) [rainfall](#).

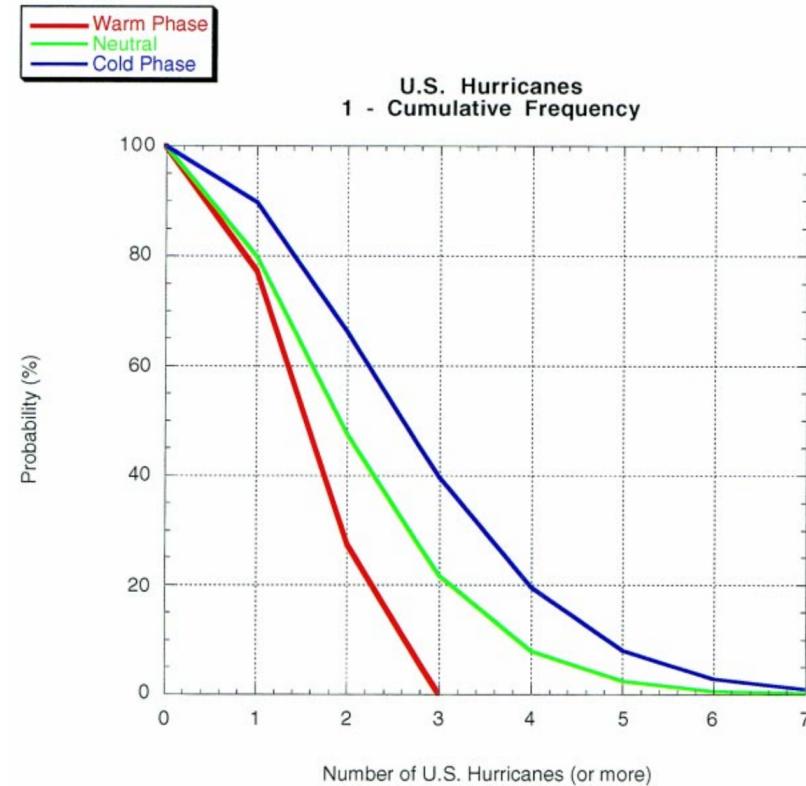
See also [Klotzbach & Bell \(2017\)](#) on the
Landfalling Hurricane Probability Project
(with all landfall probability calculations)

	A	N	O	P	Q	R
3		Climatological Probability		Current-Year Probability		
4		(Using Poisson)		(Using Poisson)		
5	State	H	MH	H	MH	
6	Texas	33%	12%	21%	7%	
7	Louisiana	30%	12%	19%	7%	
8	Mississippi	11%	4%	6%	3%	
9	Alabama	16%	3%	10%	2%	
10	Florida	51%	21%	35%	13%	
11	Georgia	11%	1%	7%	1%	
12	South Carolina	17%	4%	11%	2%	
13	North Carolina	28%	8%	18%	5%	

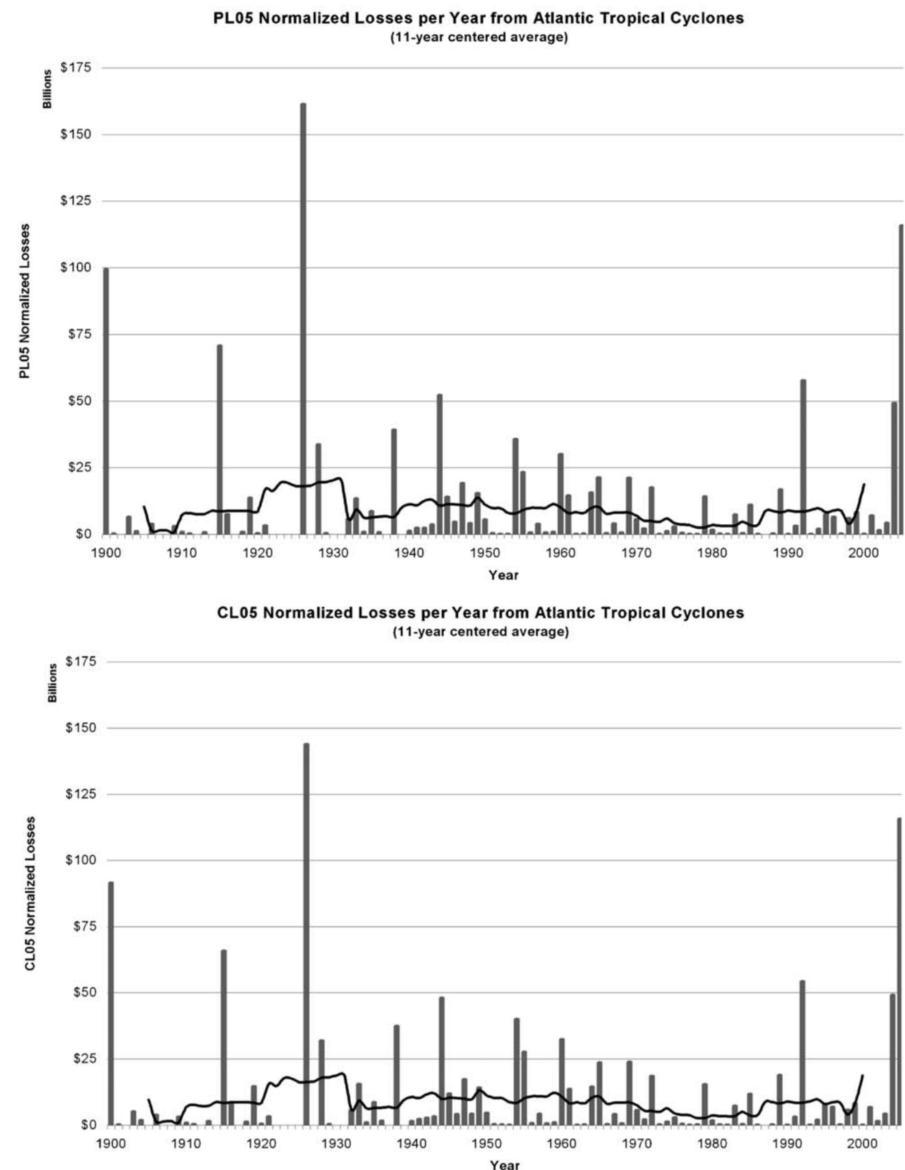
Statistical Perspective

Need (long) historical data,
(if possible with normalized losses)

see Pielke Jr. *et al.* (2008)



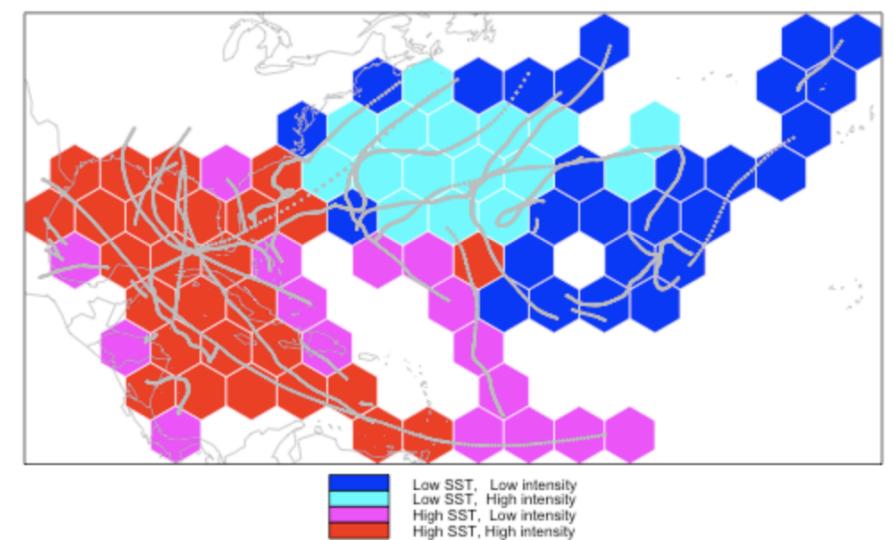
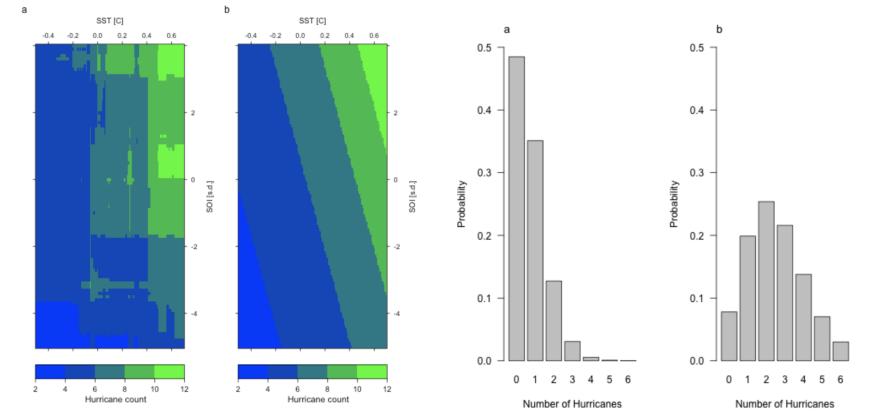
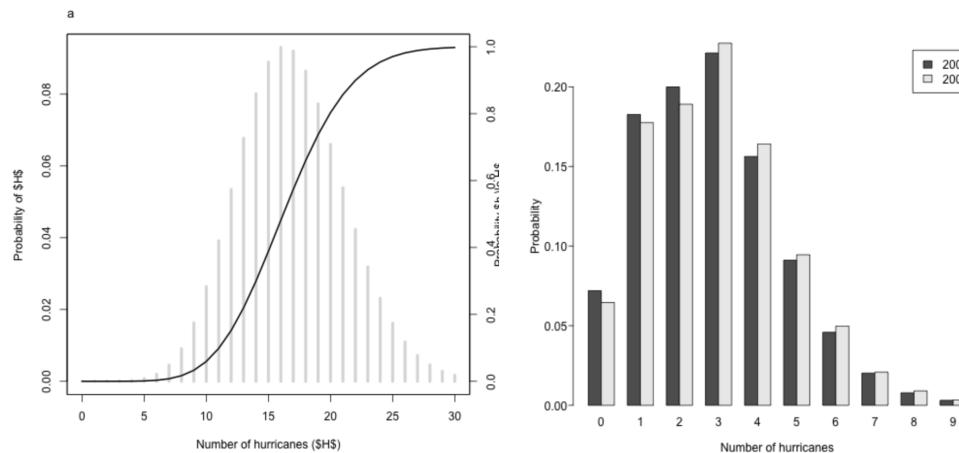
From Bove *et al.* (1998) for hurricanes.



Loss Probability : Statistical Perspective

See Elsner & Jagger (2013) for a exhaustive statistical analysis of hurricanes,

- Poisson regression for **counts**
 - Southern Oscillation Index (SOI)
 - sea-surface temperature (SST)
- regression for **intensity**
- spatial models for **trajectories**
- Bayesian models for **one year prediction**



Statistical Perspective

hard to justify in a changing environment
In order to apply any theory we have to suppose that the data are homogeneous, i.e. that no systematical change of climate and no important change in the basin have occurred within the observation period and that no such changes will take place in the period for which extrapolations are made

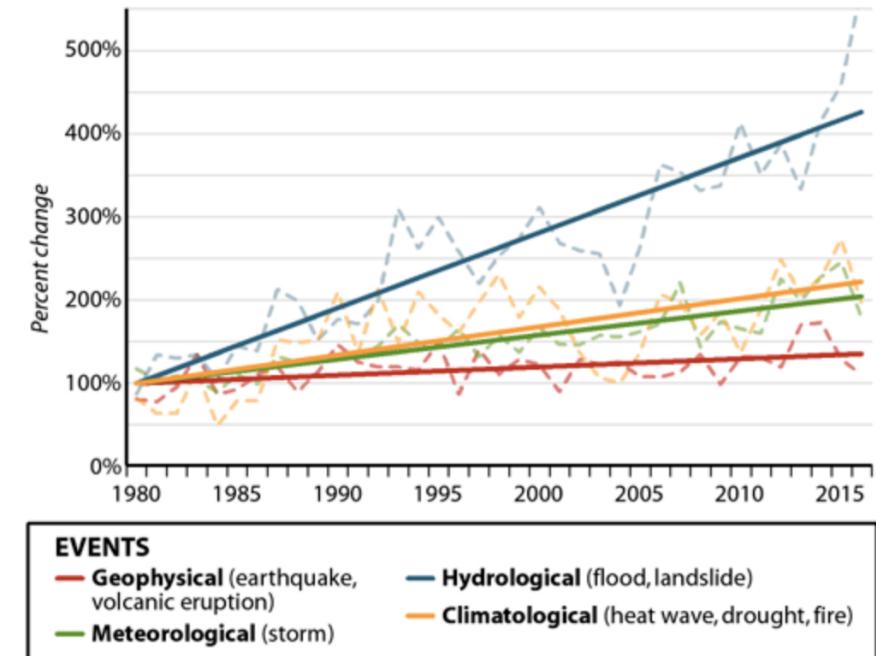
Gumbel (1941)

see flood risk (dams, reservoirs, etc)

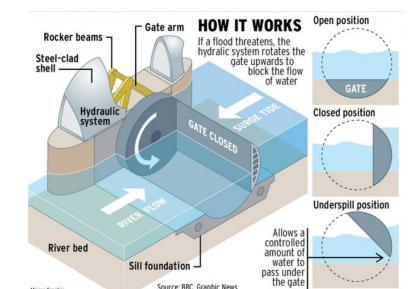
see storms or heatwaves (climate change)

Charpentier (2011)

GLOBAL TRENDS IN NATURAL CATASTROPHES
Percentage change each year in number of events compared to 1980

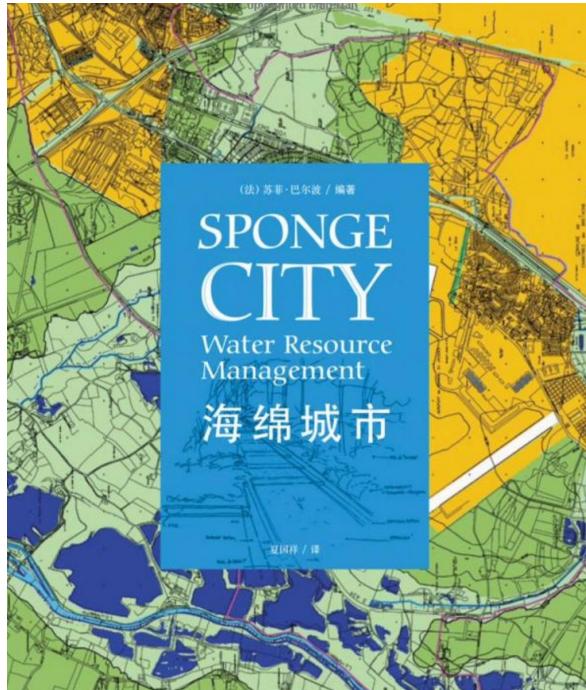


SOURCES: MunichRe NatCatSERVICE; European Academies Science Advisory Council



The Architect / Urbanist Perspective ?

See Dong & Han (2011), Bardaux (2016), Li *et al.* (2017) Jiang *et al.* (2018) on sponge cities (urban underground water system operates like a sponge to absorb, store, leak and purify rainwater, and release it for reuse when necessary)



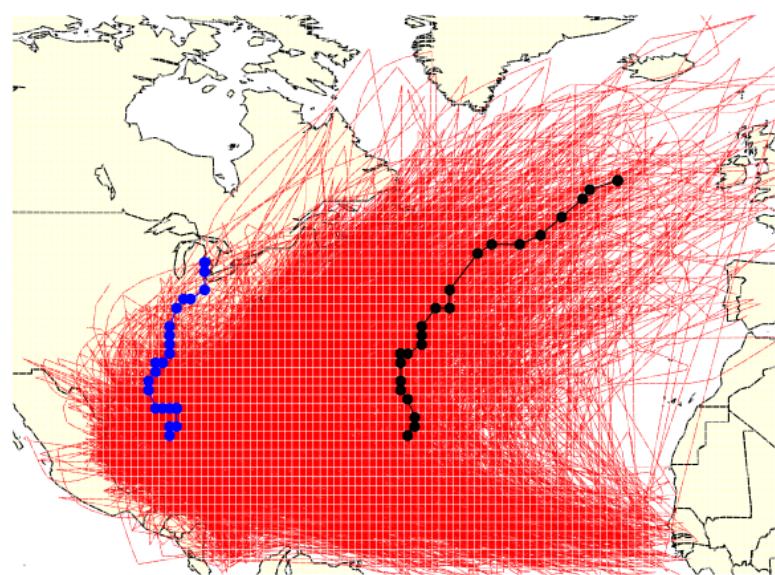
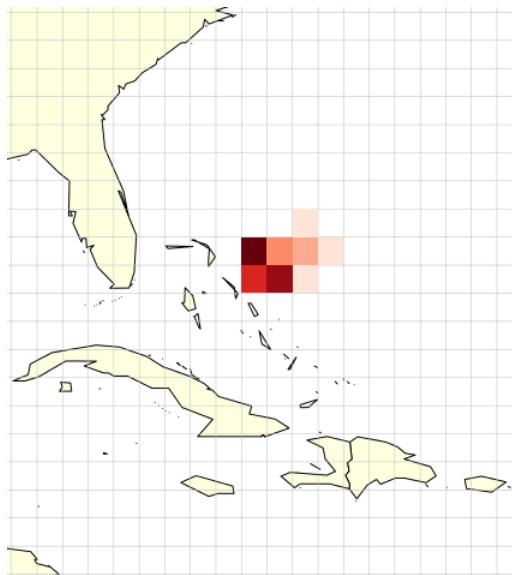
Loss Probability p : Actuarial Perspective

Actuaries use catastrophe softwares

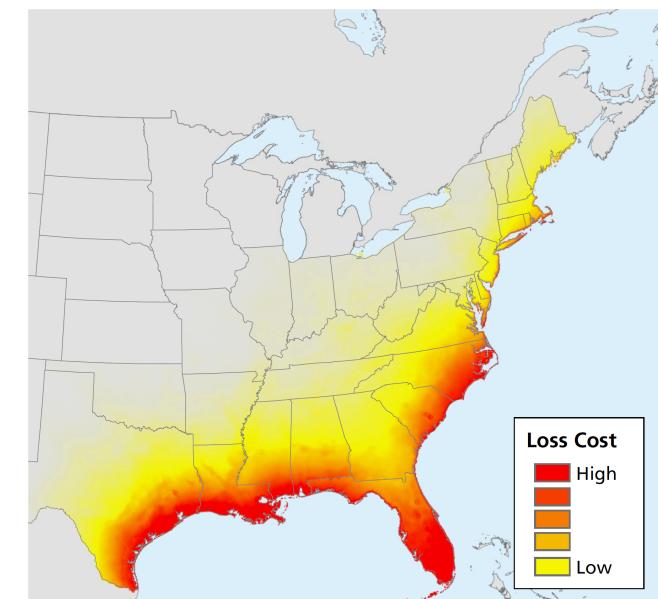
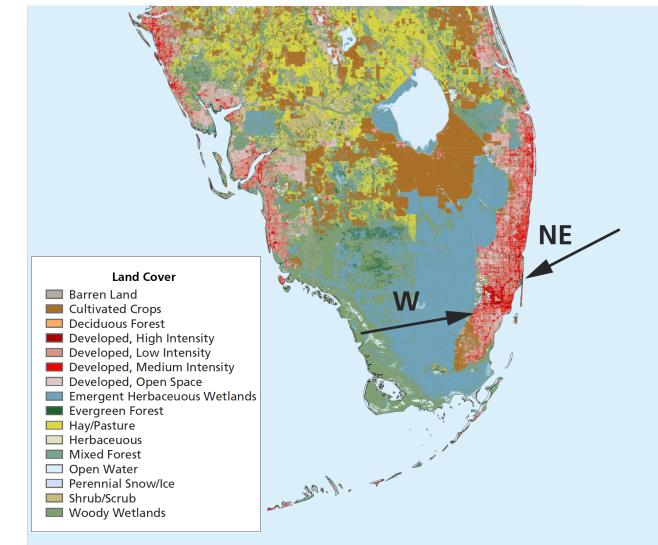
- Risk Management Solutions (**RMS**)
- AIR Worldwide (**AIR**)
- Risk Quantification Engineering (**RQE**) EQECAT

see Cole, Macpherson & McCullough (2010).

Generation of climatic scenarios (+ losses)



see for Markovian generation Charpentier (2014).



Predictive Markets

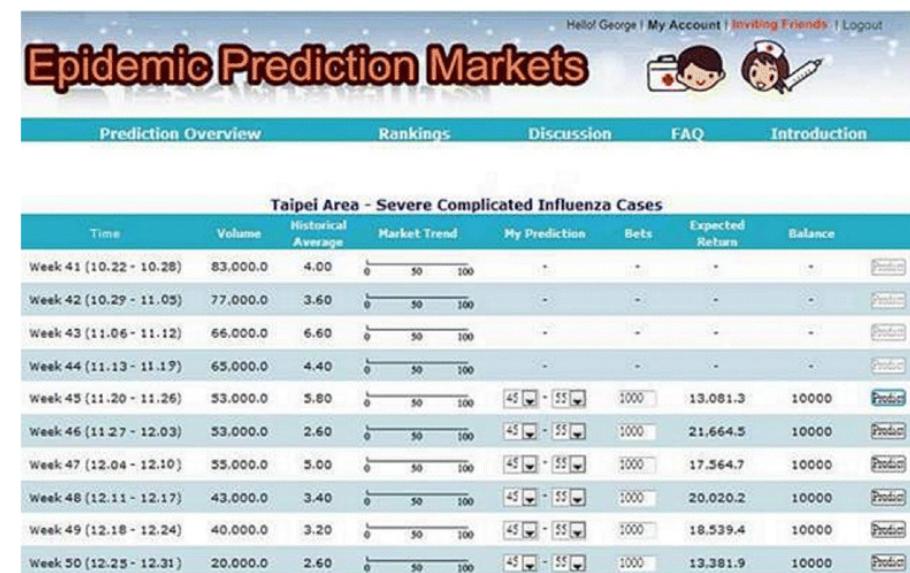
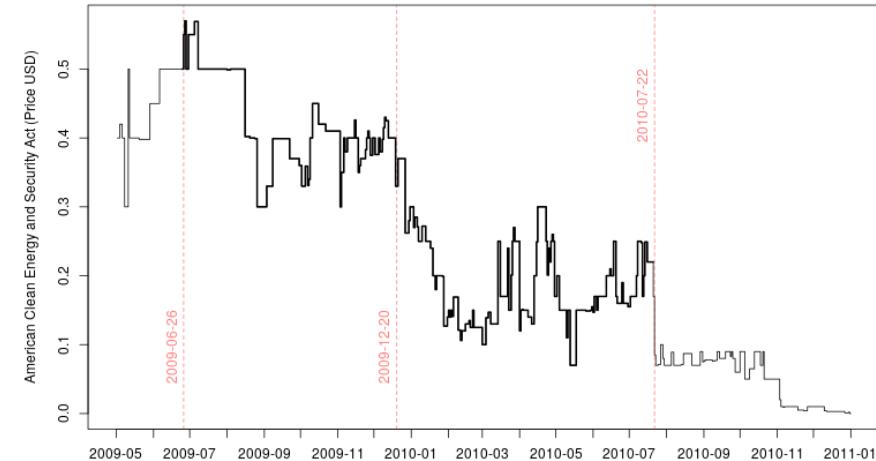
Peer prediction systems

[Wolfers & Zitzewitz \(2004\)](#) for a description

- elections (e.g. popes, XVIth Century)

[Rhode & Strumpf \(2008\)](#)

- politics, [Meng \(2016\)](#) Waxman-Markey bill (2009-2010, finally rejected)
- sports (racetrack bets) [Ali \(1977\)](#)
- infectious diseases, [Polgreen, Nelson & Neumann \(2006\)](#) or [Tung, Chou & Lin \(2015\)](#)
- climate risk [Hallstrom & Smith \(2005\)](#) for hurricanes



Loss Probability p : Betting Markets

See [Eisenberg & Gale \(1959\)](#) model, on consensus and subjective probabilities (used in [Manski \(2004\)](#) on all-or-nothing contracts)

Individual i (wealth b_i) can bet on horse j an amount $\beta_{i,j}$ (so that $\beta_{i,1} + \dots + \beta_{i,J} = b_i$). Assume that $b_1 + \dots + b_I = 1$.

Let π_j denote the sum bet on horse j , $\pi_j = \beta_{1,j} + \dots + \beta_{I,j}$. Observe that from the budget constraint $\pi_1 + \dots + \pi_J = 1$, hence π_j 's are probabilities.

Individual beliefs can be related expressed through a probability vector $\mathbf{p}_i = (p_{i,1}, \dots, p_{i,J})$, then there is an equilibrium if

$$p_{i,j} = \pi_j \cdot \max_s \left\{ \frac{p_{i,s}}{\pi_s} \right\} \text{ as soon as } \beta_{i,j} > 0$$

(so called Eisenberg-Gale equilibrium).

Loss Probability p : Betting Markets

Consider a contract that pay \$1 if an event A occurs (e.g. major flood), sold at price π_A . If there is a contract on \bar{A} , then $\pi_{\bar{A}} = 1 - \pi_A$ (no arbitrage).

Assume that individual i believe that A will occur with probability p_i . If $p_i > \pi_A$, he should purchase b_i units of A (otherwise b_i units of \bar{A}).

There is equilibrium if

$$\sum_{i=1}^I b_i = \frac{1}{\pi_A} \sum_{i=1}^I b_i \mathbb{P}[p_i > \pi_A] = \frac{1}{\pi_{\bar{A}}} \sum_{i=1}^I b_i \mathbb{P}[p_i < \pi_A]$$

hence, if wealth b_i is independent of belief p_i , then prices are probabilities

$$\pi_A = \mathbb{P}[p_i > \pi_A] = 1 - \pi_{\bar{A}}.$$

see [Wolfers & Zitzewitz \(2004\)](#) for further interpretation.

Parimutuel and Betting Markets

- 2004 and 2005, ICAP, Goldman and Nymex (see [IFR](#)) with parimutuel derivative on energy (changes in crude oil and natural gas inventories)
- 2004, CME launched futures trading on the US Consumer Price Index (CPI), see [Filippov \(2005\)](#)
- 2008, LLC (see [Ou-Yang \(2010\)](#)) with parimutuel derivative on [hurricanes](#)
- The [longitude](#) plateforme (“*real time calculation of the odds*”).
- The [Iowa Electronic Markets](#) for elections, “*using this wisdom of crowdsthe price of a contract at any given time is a forecast of the outcome*”



HuRLOs (Hurricane Risk Landfall Option) options

The exist the Hurricane Futures Market (HFM), see [Kelly, Letson, Nelson, Nolan & Solis \(2010\)](#) for a description

Hurricane Risk Landfall Options (HuRLOs) were launched in October 2008 by Weather Risk Solutions, LLC (WRS) and promised a mutualized marketplace for hurricane options, using [parimutuel options](#)

From French [pari mutuel](#), literally, mutual stake :

“A system of betting on races whereby the winners divide the total amount bet, after deducting management expenses, in proportion to the sums they have wagered individually”, see [Baron & Lang \(2007\)](#).

also calle [universal Dutch auction](#) in [Syz \(2008\)](#)

Parimutuel and Betting Markets

“A Shapley-Shubik market game for contingent claims within a probability state space with secured selling is a parimutuel market” ([Lange & Economides \(2005\)](#))

A matching / pricing mechanism is necessary to get [financial efficiency](#) (self-hedging). Participant i submit orders, state bids β_i , b_i a limit share quantity and $\bar{\pi}_i$ a limit price per share

The market organizer determines the order fill x_i (and state prices π_j), using call auction mechanism, see [Peters, So and Ye \(2005\)](#) and [Agrawal, Wang & Ye \(2008\)](#).

Using a [Linear Programming Market Mechanism](#), market organizer should solve

$$\text{maximize} \left\{ \sum_{i=1}^I \bar{\pi}_i x_i - z \right\} \text{ where } z \text{ is some worst case cost}$$

$$\text{subject to} \quad \text{cost if state } j \text{ occurs} = \sum_{i=1}^I \beta_{i,j} x_i \leq z \quad \forall j$$

$$0 \leq \bar{\pi}_i x_i \leq b_i \quad \forall i$$

Parimutuel and Betting Markets

More realistically, one can consider some dynamic parimutuel market maker as in [Agrawal et al. \(2014\)](#) for recent advances, and [Chen & Pennock \(2010\)](#) for an overview.

See also techniques to get some peer prediction Bayesian Nash equilibrium solvers (see [Jurca & Faltings \(2008\)](#) on elections and bets) and the related collective revelation mechanism (see [Goel, Reeves & Pennock \(2009\)](#))

“Peer prediction systems are designed for eliciting information on events where ground truth does not exist or is unobtainable”, [Chen & Pennock \(2010\)](#)

Take-Away Conclusion

- knowing **real probabilities** of occurrence disasters is either complicated (hurricanes) or very complicated (flood)
- those probabilities are necessary to assess solvency of insurance companies (central limit theorem is based on true probabilities)
- insurance pricing is based on **beliefs** of insured, and insurance companies
- **predictive markets** can be an interesting revelation mechanism of crowd beliefs