Assurance collaborative, théorie des graphes et actuariat

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SCOR FONDATION POUR LA SCIENCE en partenariat avec Institut des ACTUAIRES

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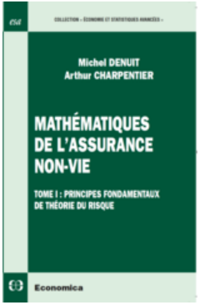
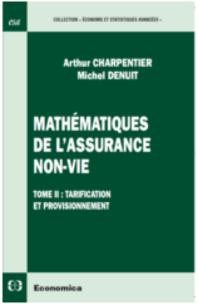
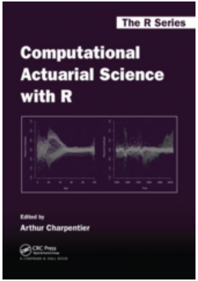
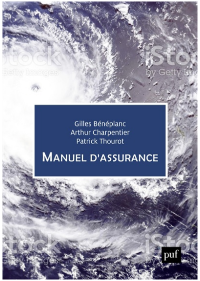
freakonometrics & freakonometrics.hypotheses.org

Modélisation prédictive, Science actuarielle,
Économie mathématique, Risque, Inégalités,
Économétrie, statistiques, apprentissage automatique
Modélisation du climat, Extrêmes, Équité

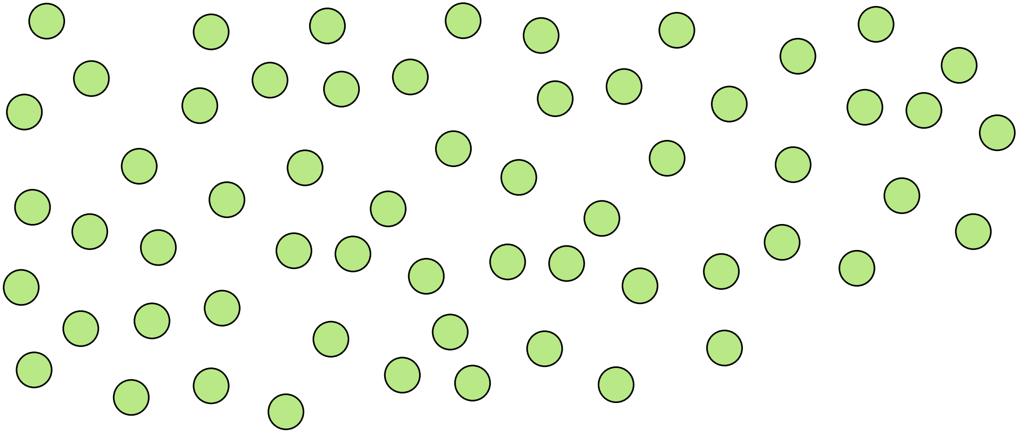






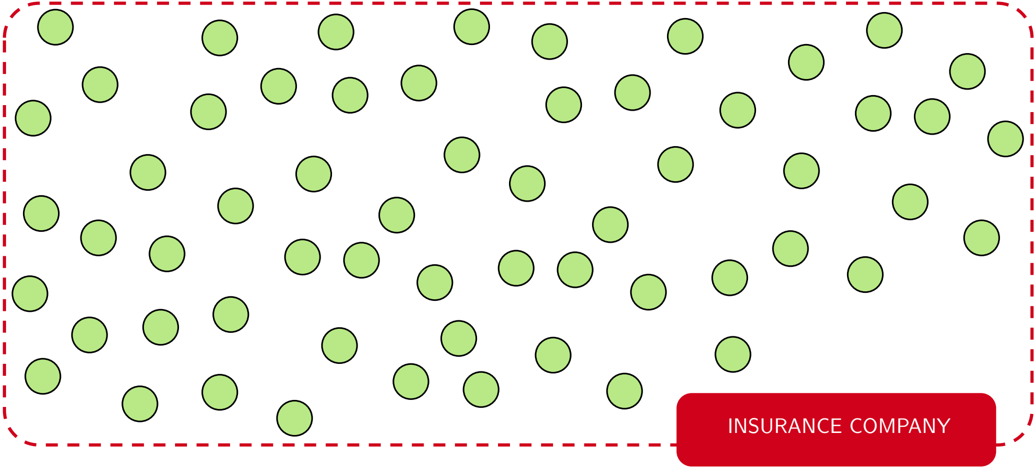


Risk Transfert



"Insurance is the contribution of the many to the misfortune of the few"

Risk Transfert



Risk Sharing & Networks

Def Consider two random variables X and Y, $X \preceq_{CX} Y$ if $\mathbb{E}[h(X)] \leq \mathbb{E}[h(Y)]$ for any convex function h

$\Longleftrightarrow$ Y is a mean-preserving spread of X, i.e. $Y \overset{\mathcal{L}}{=} X + Z$, where $\mathbb{E}[Z|X] = 0$.

$\Longleftrightarrow$ $\mathbb{E}[(X - s)_+] \leq \mathbb{E}[(Y - s)_+]$ for all $s \in \mathbb{R}$.

$\Longrightarrow$ $\mathbb{E}[X] = \mathbb{E}[Y]$ and $\text{Var}[X] \preceq \text{Var}[Y]$.

$\Longleftrightarrow$ Pigou-Dalton transfert, majorization order, etc

Following Denuit and Dhaene (2012) and Carlier et al. (2012),

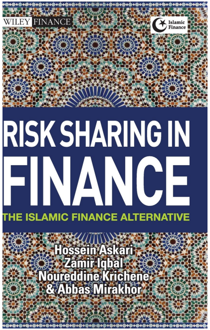
Def Consider two random vectors $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_n)$ and $\boldsymbol{X} = (X_1, \cdots, X_n)$ on $\mathbb{R}_+^n$. $\boldsymbol{\xi}$ is a risk-sharing scheme of $\boldsymbol{X}$ if $X_1 + \cdots + X_n = \xi_1 + \cdots + \xi_n$ almost surely.

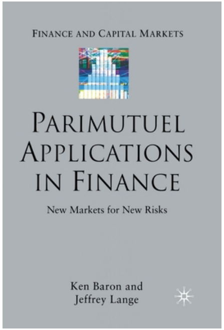
Def Consider two random vectors $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_n)$ and $\boldsymbol{X} = (X_1, \cdots, X_n)$ on $\mathbb{R}_+^n$. $\boldsymbol{\xi} \preceq_{CCX} \boldsymbol{X}$ if $\xi_i \preceq_{CX} X_i$.

Risk Sharing & Networks

Peer-to-peer insurance is a risk sharing network where a group of individuals pool their premiums together to insure against a risk. Peer-to-Peer Insurance mitigates the conflict that inherently arises between a traditional insurer and a policyholder when an insurer keeps the premiums that it doesn't pay out in claims

- Takaful التكافل
- Wakalah وَكَالَة
- Musharakah مشاركة
- Xiang Hu Bao 相互保
- Parimutuel





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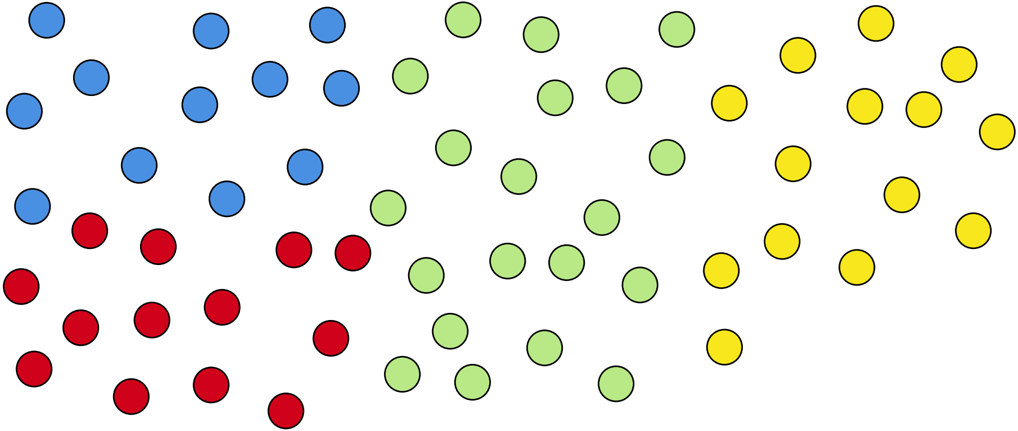

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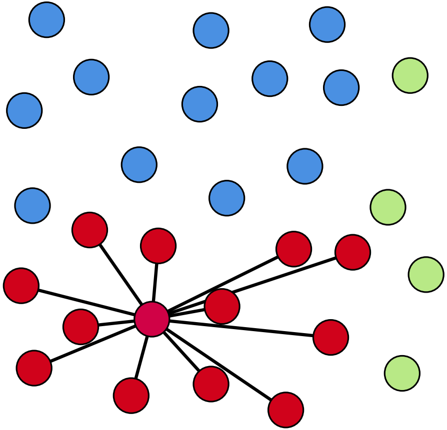


Let $\xi_j = \dfrac{1}{n}\sum_{i=1}^{n} X_i, \;\forall j$

- Risk sharing

$\xi_1 + \cdots + \xi_n = X_1 + \cdots + X_n$

- Componentwise convex-order

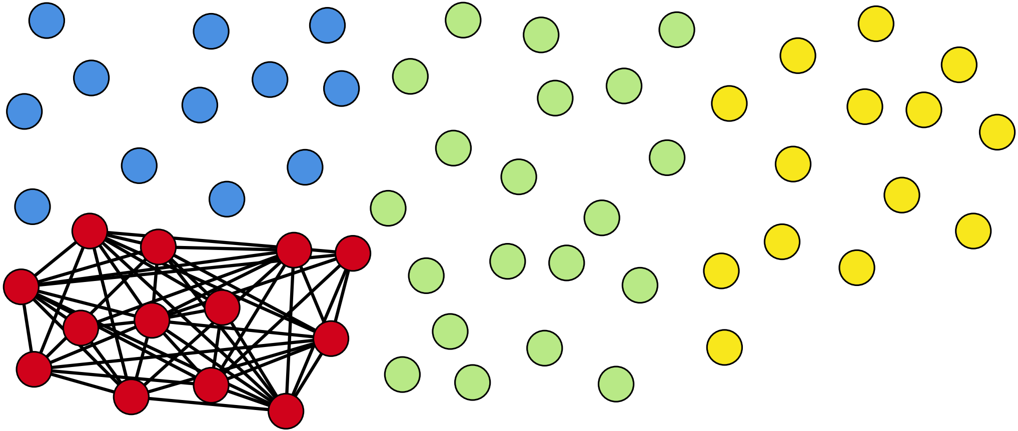
$\xi_j \preceq_{CX} X_j, \;\forall j$

More generally, consider some linear risk sharing $\boldsymbol{\xi} = M\boldsymbol{X}$, for some $n \times n$ matrix

$$M = \begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{M}_k \end{bmatrix}, \;\mathbf{M}_k = \frac{1}{n_k}\mathbf{1}_k$$

where $\mathbf{1}_k$ is the $n_k \times n_k$ matrix full of 1's.

Risk Sharing



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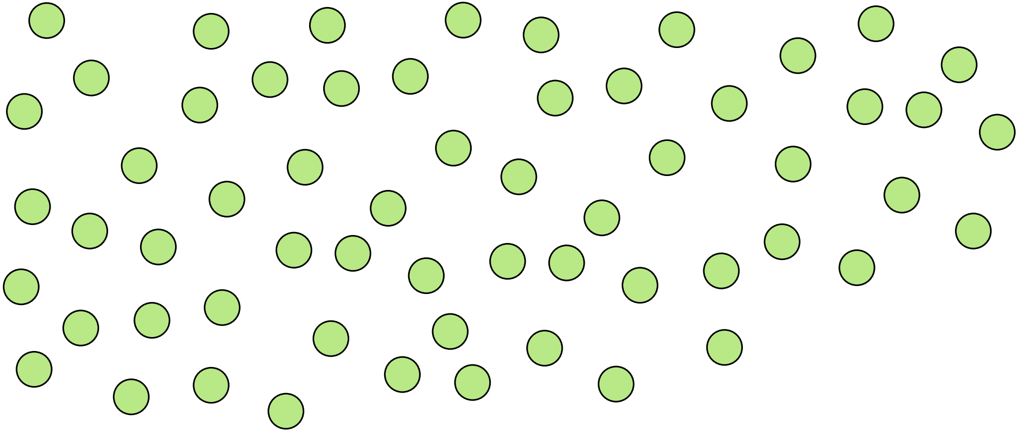
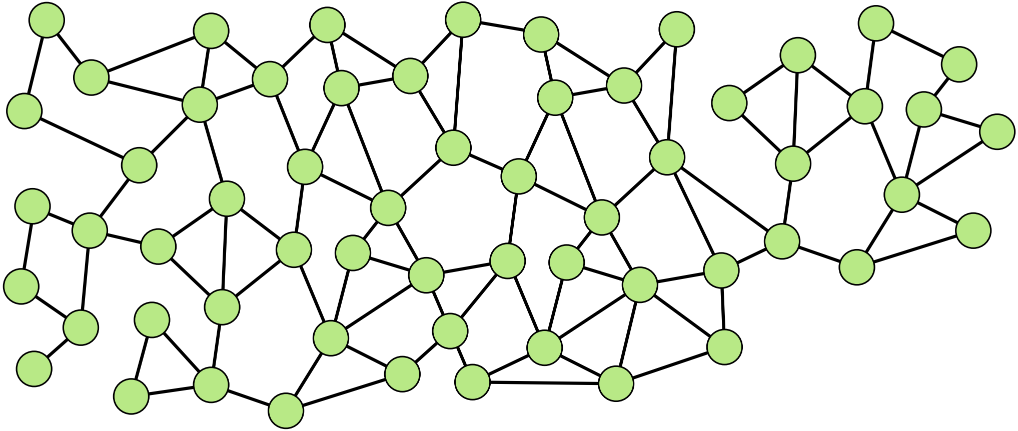


Risk Sharing on a Network



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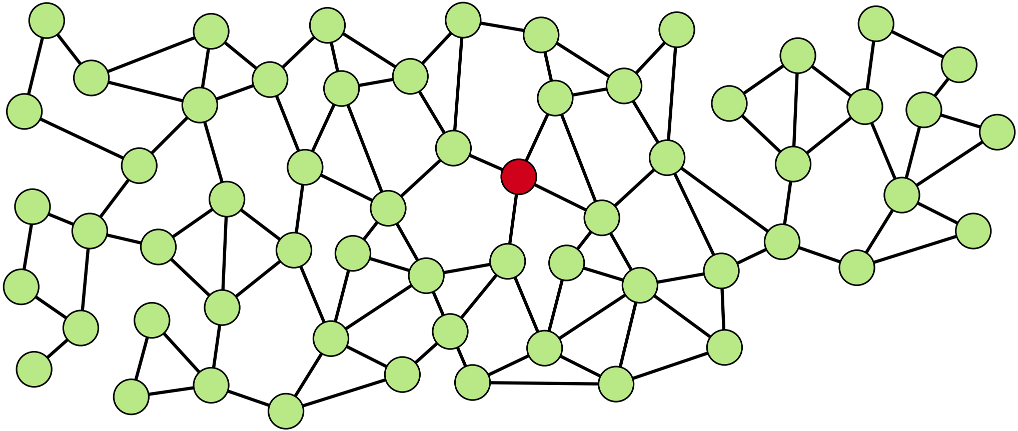

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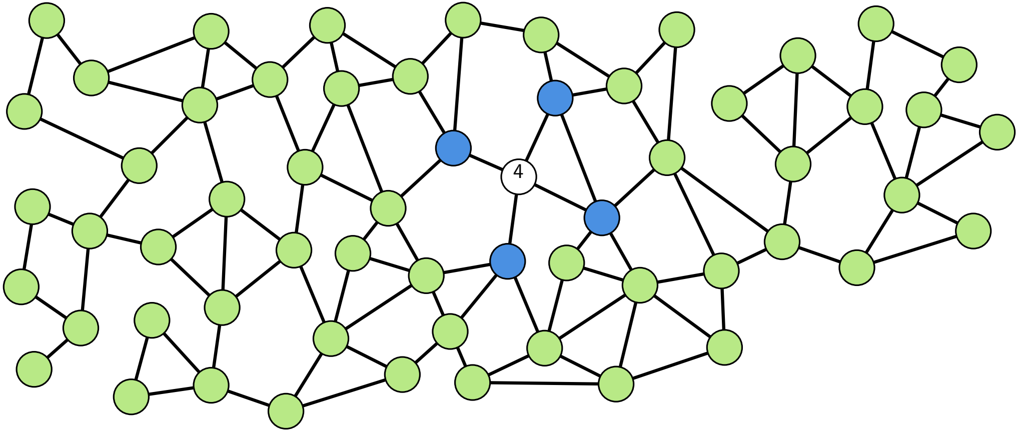

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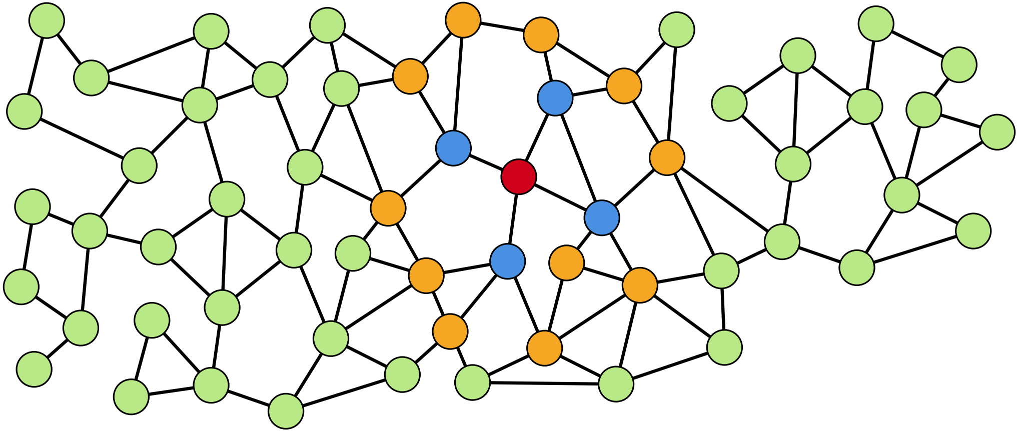

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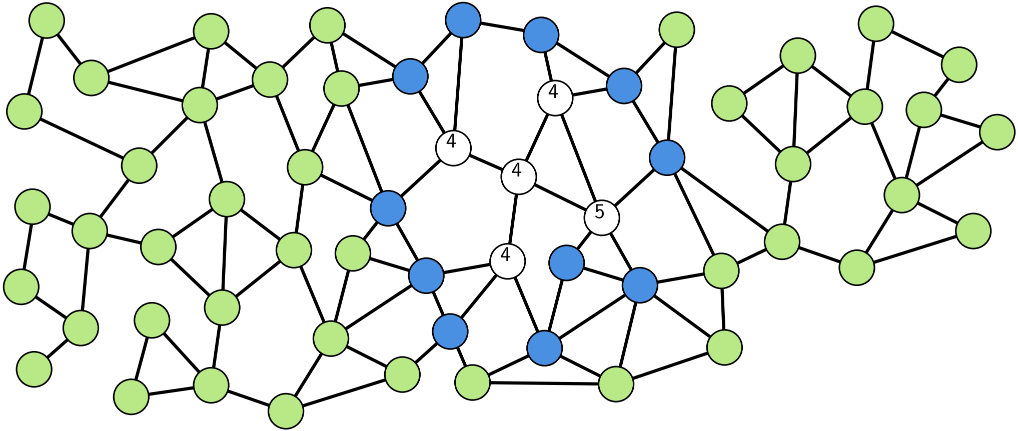

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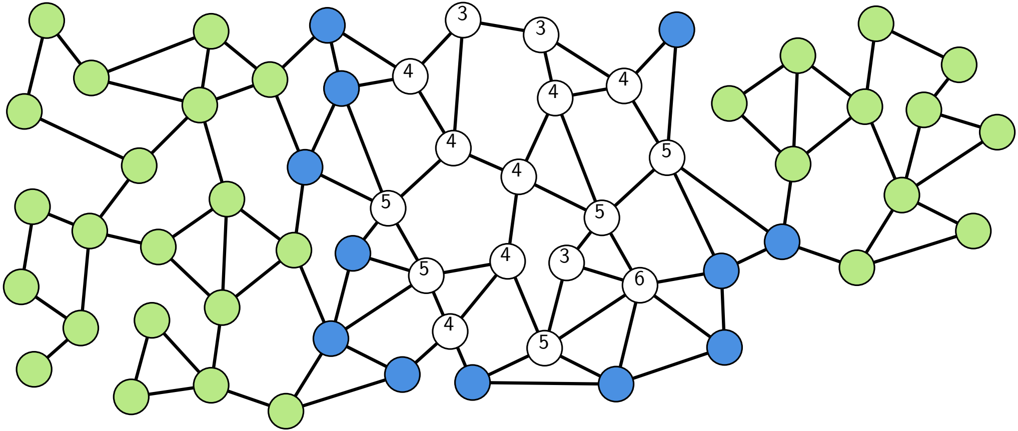

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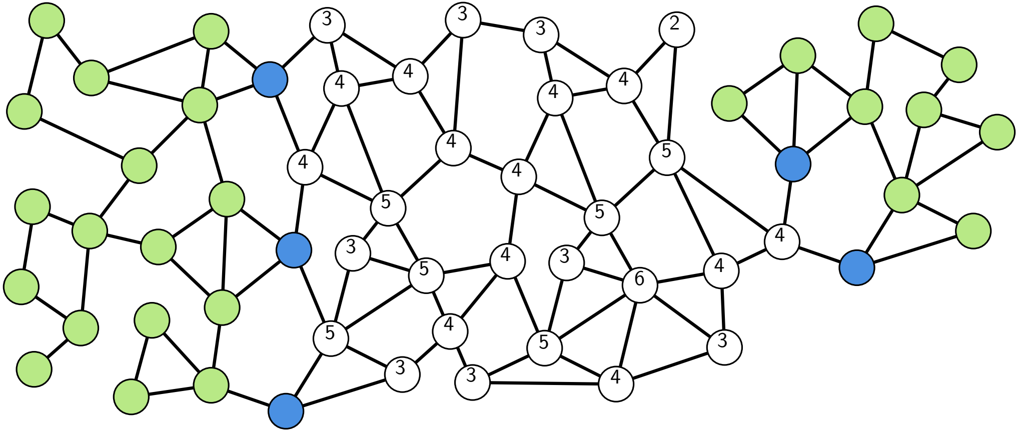

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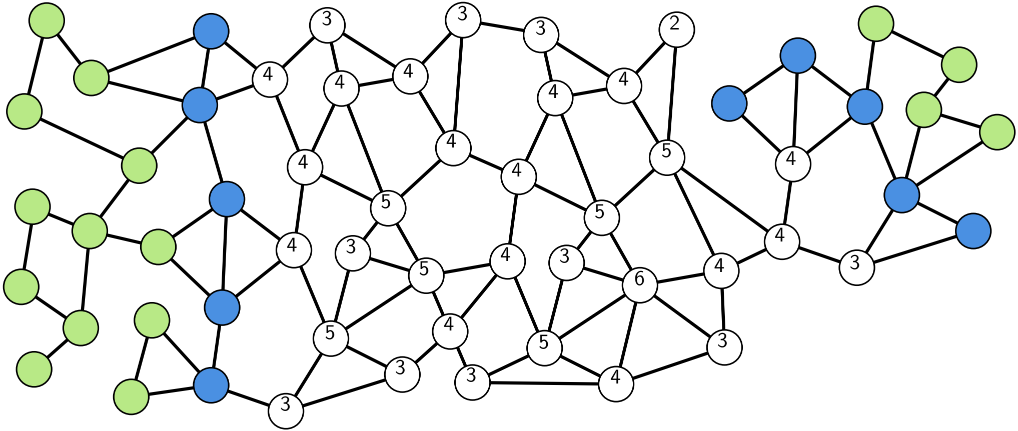

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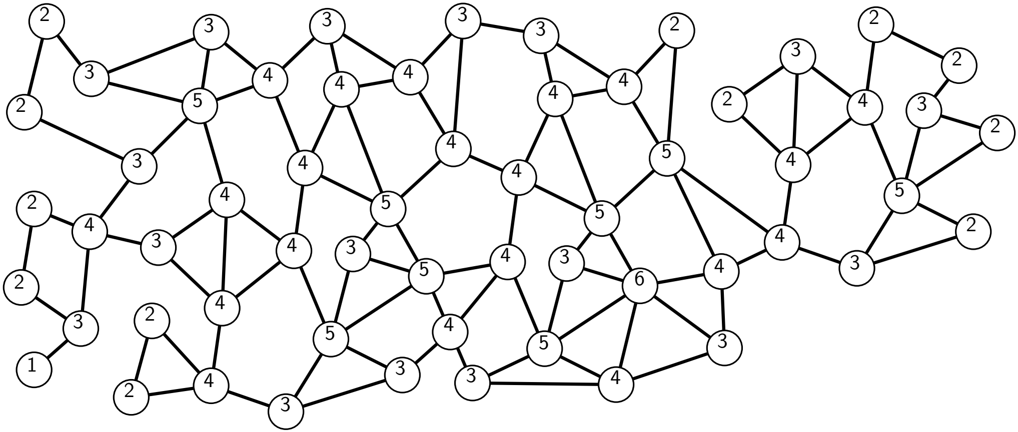

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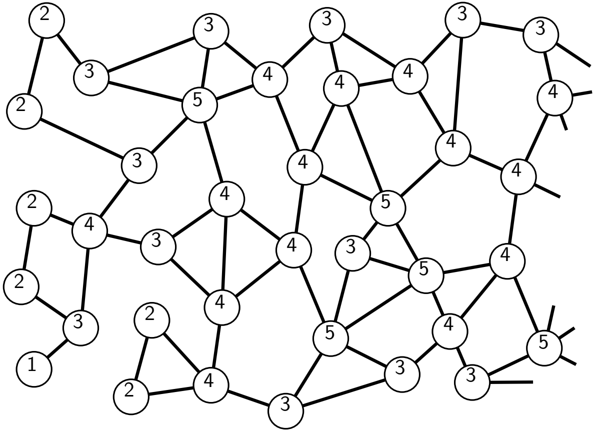

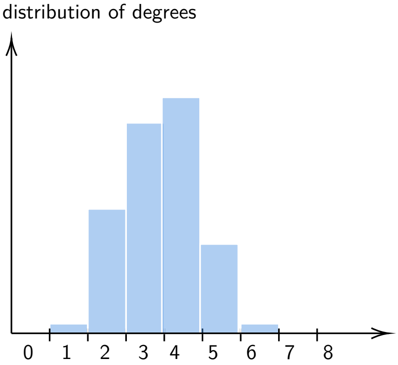

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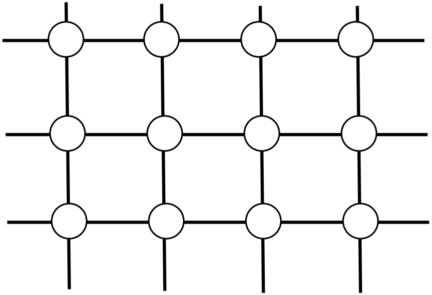
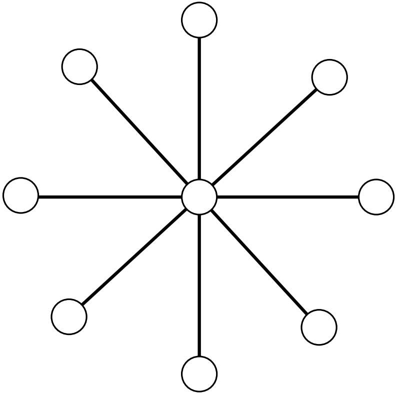
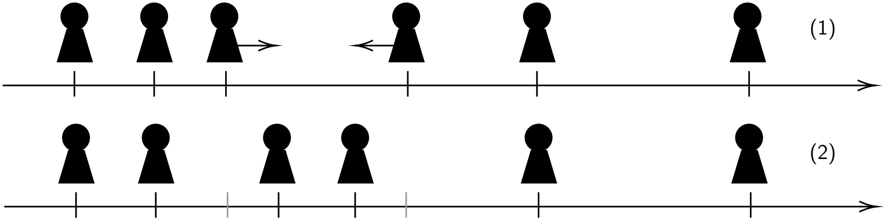




Regular graph vs. star shaped graph
(low variance vs. large variance on D)

Risk Sharing on a Network

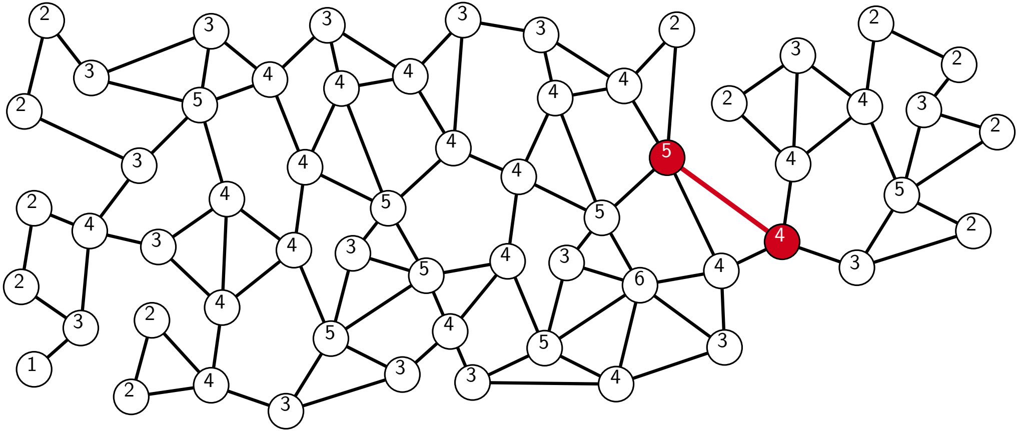


Pigou-Dalton transferts (Dalton (1920)),

$$\boldsymbol{y}^{(1)} \preceq \boldsymbol{y}^{(2)} \longleftarrow \begin{cases} y_i^{(2)} = y_i^{(1)}, \ \forall i \neq j, k \\ y_j^{(2)} = y_j^{(1)} + h, \\ y_k^{(2)} = y_k^{(1)} - h, \ y_j^{(2)} > y_j^{(1)} \end{cases}$$

see martingale property of mean-preserving spread, $Y^{(2)} = Y^{(1)} + Z$, where $\mathbb{E}[Z|Y^{(1)}] = 0$.

Risk Sharing on a Network



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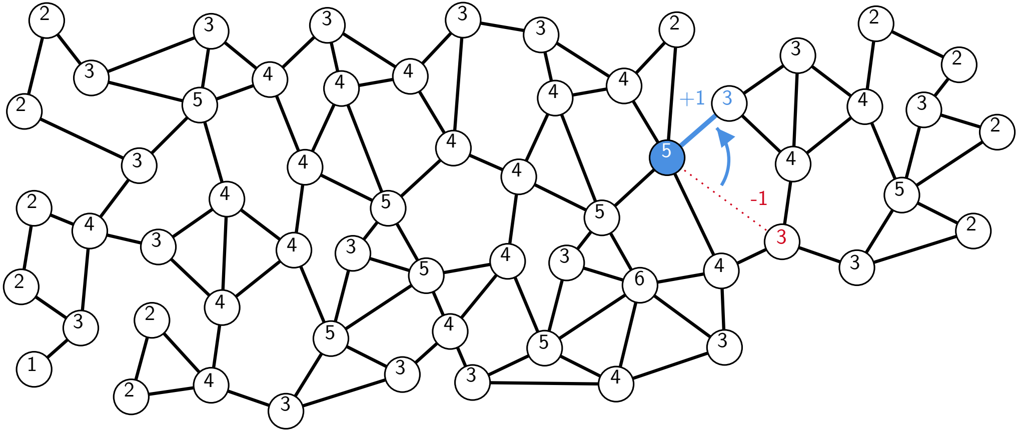
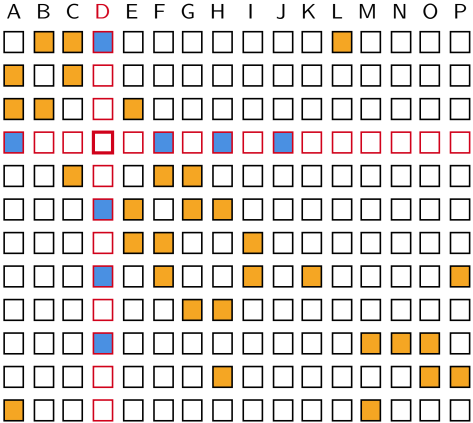
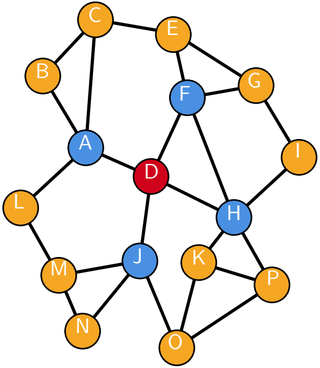


Sharing Risks with Friends





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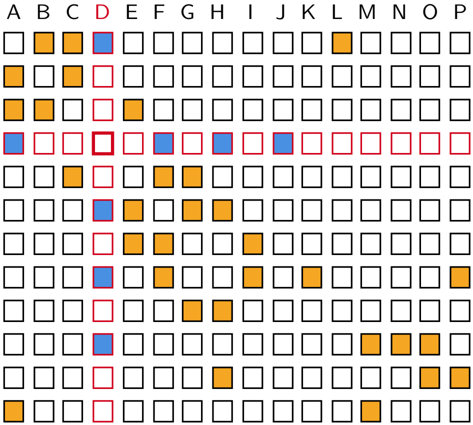

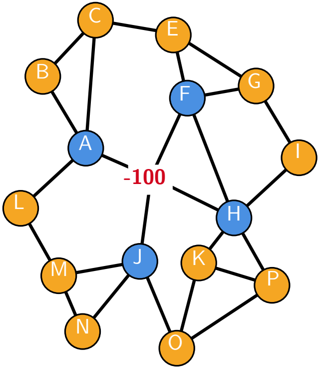

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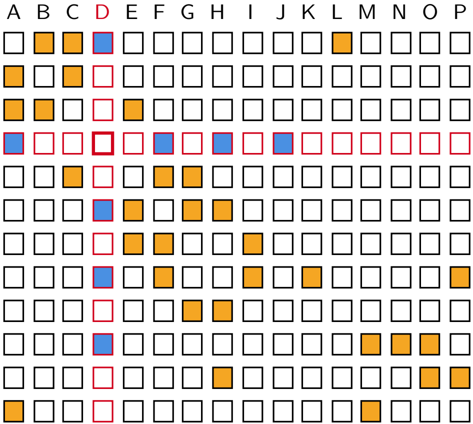

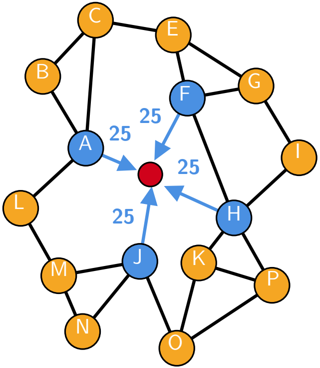

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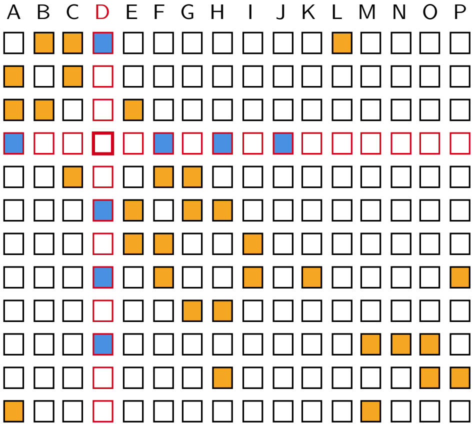

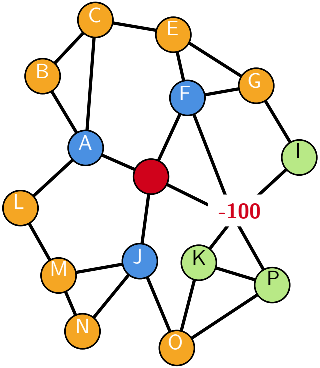

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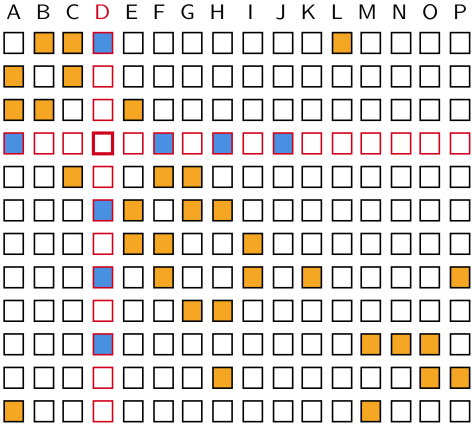

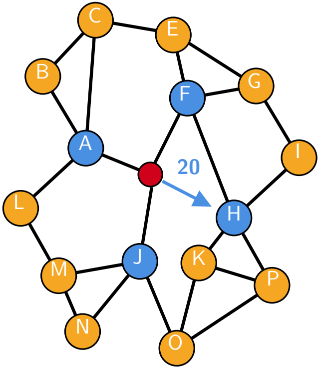

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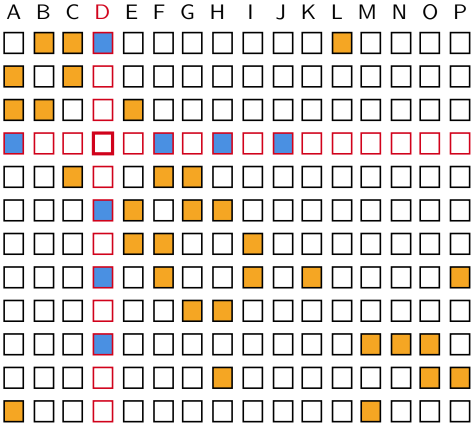

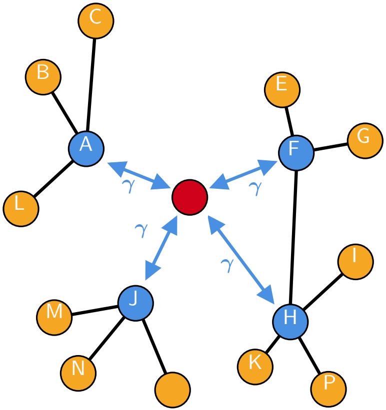

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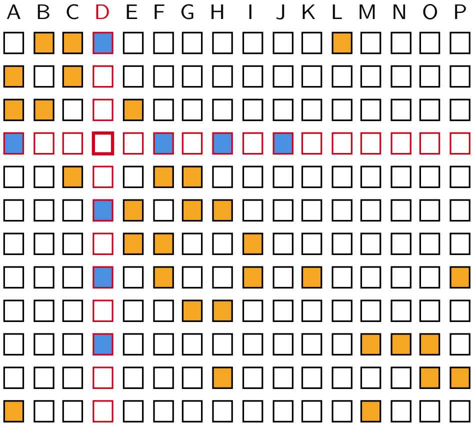

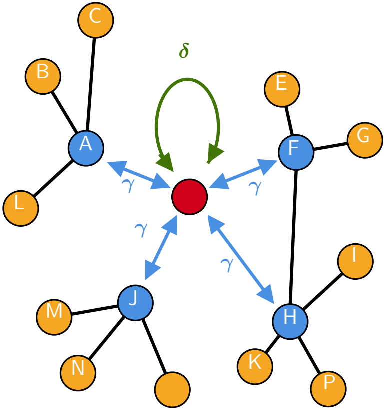

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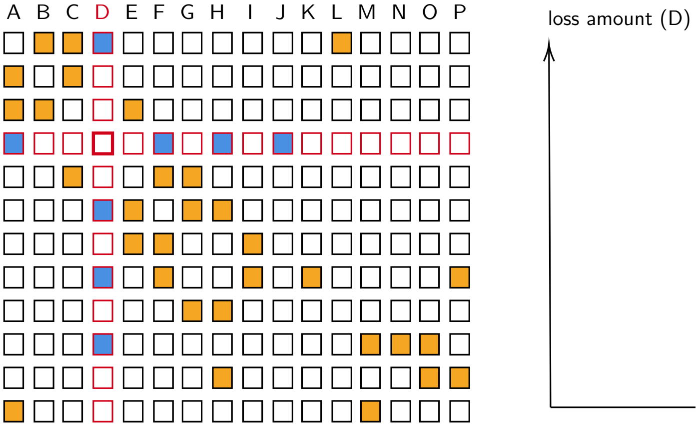

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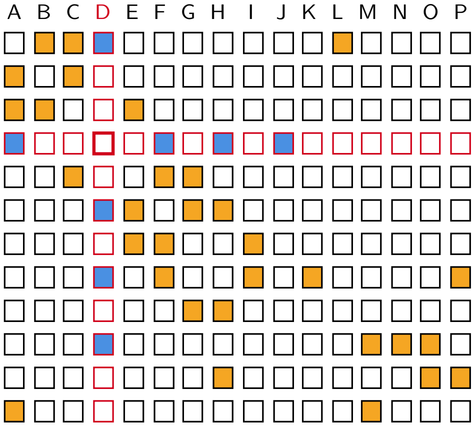

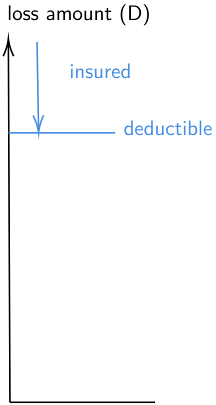

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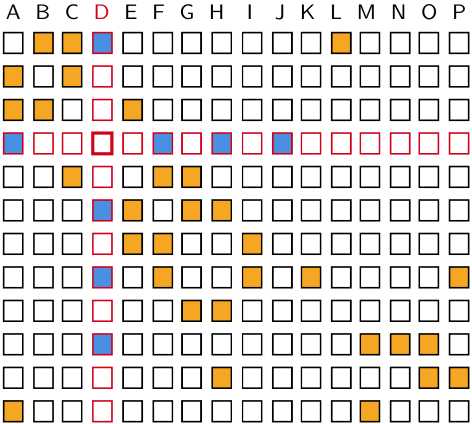

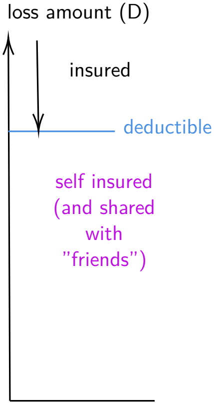

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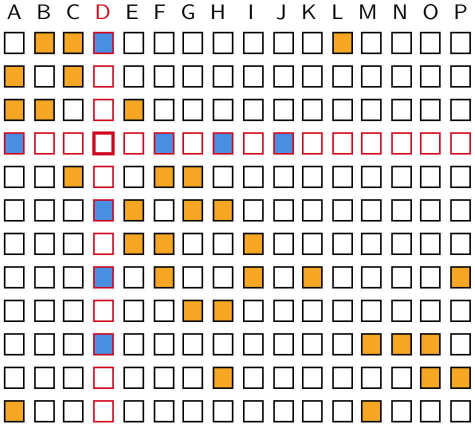

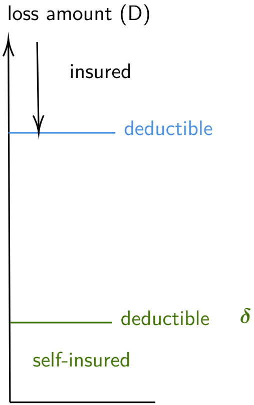

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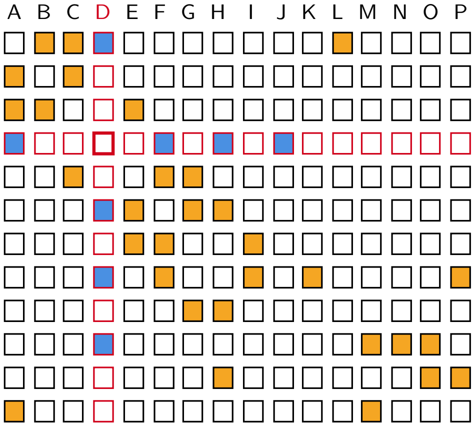

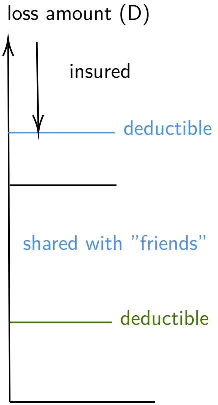

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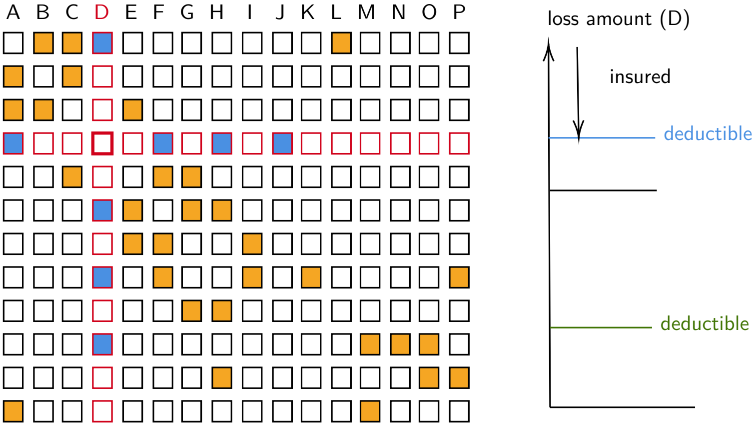

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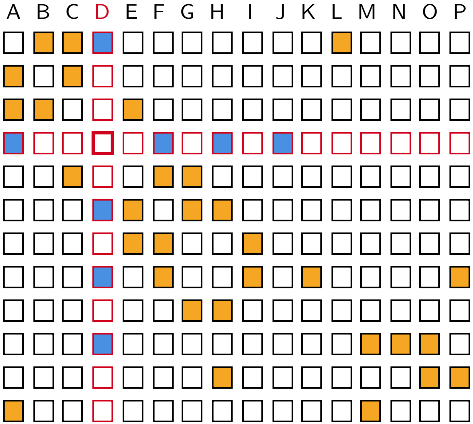

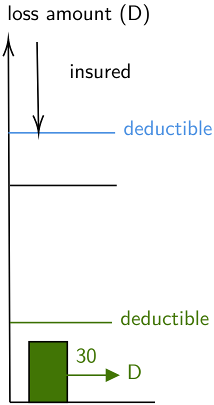

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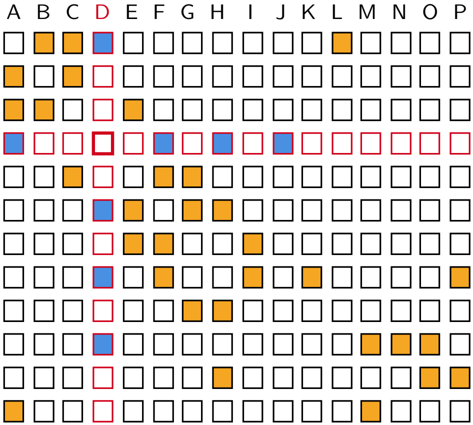

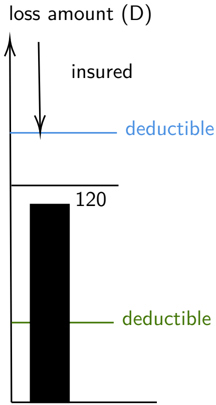

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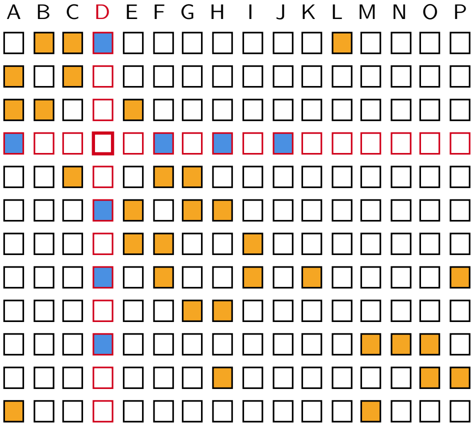

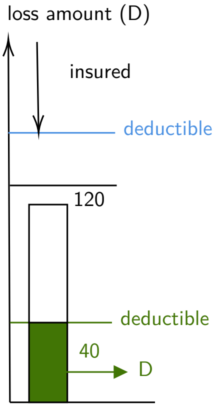

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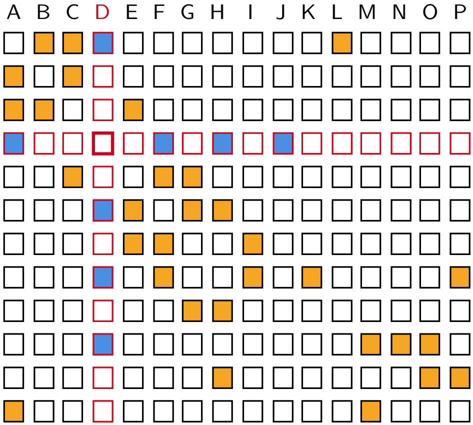

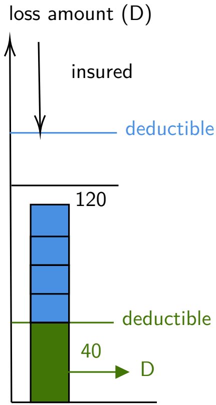

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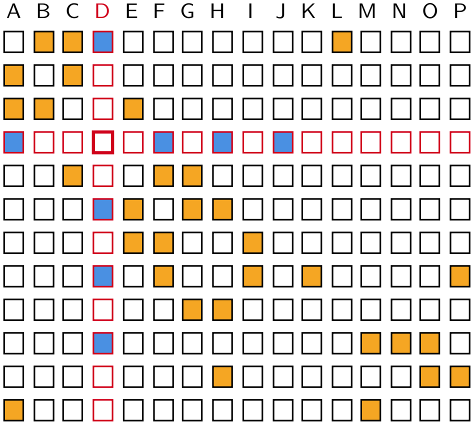

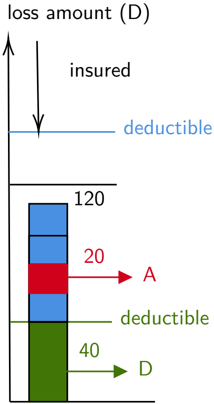

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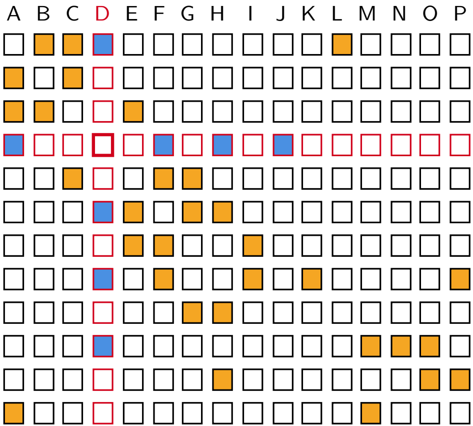
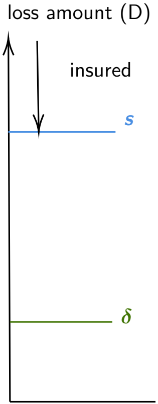
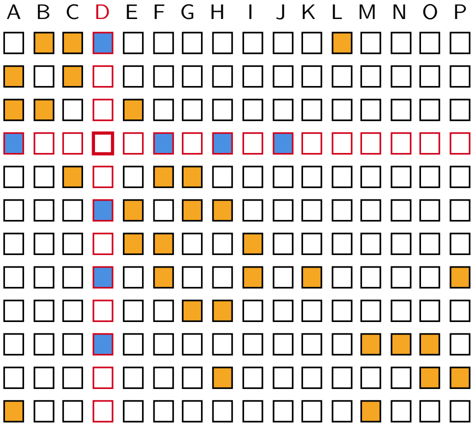
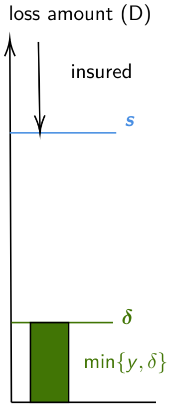


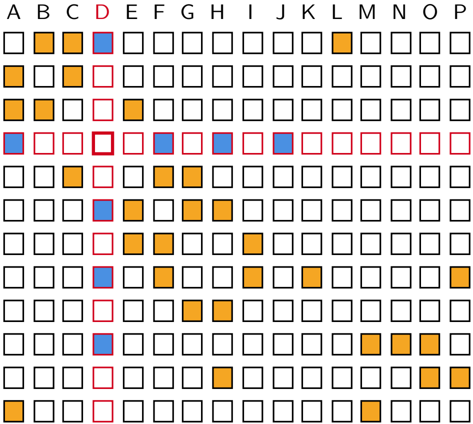


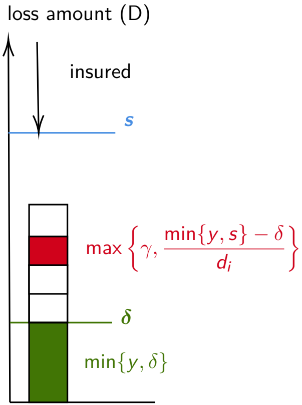
Sharing Risks with Friends





Sharing Risks with Friends

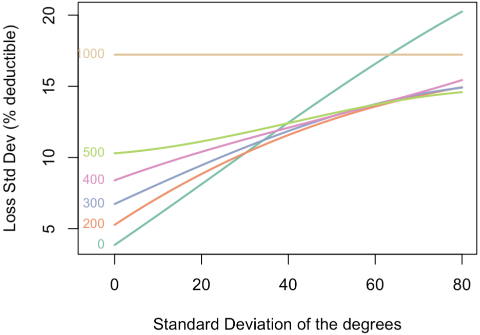




Sharing Risks with Friends

- Y_i loss of insured i, $Z_i = \mathbf{1}(Y_i > 0)$
- $\mathcal{V}_i$ is the set of friends of insured i, $d_i = \text{Card}(\mathcal{V}_i)$
- s deductible of insurance contracts
- γ is the maximum amount shared between i and j (reciprocal contracts)

$$\begin{aligned}\xi_i \quad = \quad & Z_i \cdot \min\{s, Y_i\} \\ & + \sum_{j \in \mathcal{V}_i} Z_j \min\left\{\gamma, \frac{\min\{s, Y_j\} - \delta}{d_j}\right\} \\ & - Z_i \cdot \min\{d_i\gamma, \min\{s, Y_i\} - \delta\}\end{aligned}$$



Optimization of the Risk Sharing Mechanism

$$\begin{cases} \max \left\{ \displaystyle\sum_{(i,j)\in\mathcal{E}} \gamma_{(i,j)} \right\} \\ \text{s.t. } \gamma_{(i,j)} \in [0,\gamma],\ \forall (i,j)\in\mathcal{E} \\ \qquad \displaystyle\sum_{j\in\mathcal{V}_i} \gamma_{(i,j)} \leq s,\ \forall i\in\mathcal{V} \end{cases}$$

Given losses $\boldsymbol{X} = (X_1, \cdots, X_n)$, define contributions $C^\star_{i\to j} = \min\left\{ \dfrac{\gamma^\star_{(i,j)}}{\displaystyle\sum_{i\in\mathcal{V}_j} \gamma^\star_{(i,j)}} \cdot X_j, \gamma^\star_{(i,j)} \right\}$,

and

$$\xi^\star_i = X_i + \sum_{j\in\mathcal{V}_i} [Z_j C^\star_{i\to j} - Z_i C^\star_{j\to i}]$$

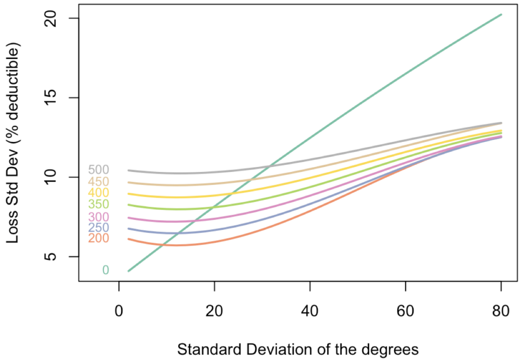
is a risk sharing, called optimal risk sharing.

Sharing Risks with Friends, and Friends of Friends

We can also consider friends of friends

$$\begin{cases} \gamma_1^\star = \text{argmax}\left\{ \displaystyle\sum_{(i,j)\in\mathcal{E}^{(1)}} \gamma_{(i,j)} \right\} \\ \text{s.t. } \gamma_{(i,j)} \in [0,\gamma_1],\ \forall(i,j)\in\mathcal{E}^{(1)} \\ \qquad \displaystyle\sum_{j\in\mathcal{V}_i^{(1)}} \gamma_{(i,j)} \leq s,\ \forall i \end{cases}$$

$$\begin{cases} \gamma_2^\star = \text{argmax}\left\{ \displaystyle\sum_{(i,j)\in\mathcal{E}^{(2)}} \gamma_{(i,j)} \right\} \\ \text{s.t. } \gamma_{(i,j)} \in [0,\gamma_2],\ \forall(i,j)\in\mathcal{E}^{(2)}_{\gamma_1^\star} \\ \qquad \displaystyle\sum_{j\in\mathcal{V}_i^{(1)}} \gamma^\star_{1:(i,j)} + \sum_{j\in\mathcal{V}_i^{(2)}} \gamma_{(i,j)} \leq s,\ \forall i \end{cases}$$



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