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Adaptive Waveform Inversion Using Incomplete Physics, Imperfect Data, and an Incorrect Source

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SUMMARY

Adaptive waveform inversion (AWI) provides a means of performing full-waveform inversion (FWI) that appears to be immune to the effects of cycle skipping. However, the form of the AWI algorithm suggests that it could have increased sensitivity to errors in the assumed source wavelet, to noise in the field data, and to inadequacies in the physics used to simulate wave propagation. We examine each of these for a synthetic model. We show that AWI is in fact less sensitive than FWI to errors in the source wavelet, and is no more sensitive to errors in the data and in the modelling than is FWI. It appears likely that the immunity that AWI displays to cycle skipping also contributes to its reduced sensitivity to errors in the assumed source wavelet.

Introduction

Adaptive waveform inversion (AWI), introduced by Warner and Guasch in 2014, is a form of full-waveform inversion (FWI) that is immune to the effects of cycle skipping. It proceeds initially by making the non-physical assumption that each seismic trace in the observed dataset has been acquired using its own unique source wavelet, and these wavelets are chosen such that the predicted data closely match the observed data. The inversion then proceeds to change the model so that these assumed wavelets evolve towards the true wavelet; as this happens, the predicted data evolve towards the observed data, and the assumed model evolves toward the true model.

If the observed data are noise free, if the physics used to model these observed data is accurate and complete, and if the true source wavelet is accurately known, then both AWI and FWI will recover similar final models provided that the FWI result is unaffected by cycle skipping. However, in field data, none of these assumptions are likely to be entirely achievable. In this paper, we explore how FWI and AWI behave when the observed data are imperfectly acquired, when the assumed physics is incomplete, and when the assumed source wavelet is incorrect. We do this using a simple and well-studied synthetic example so that we can quantify our errors against the true model.

Theory

Given an observed dataset **d** which depends upon the true earth model, and a dataset **p** that has been predicted using a synthetic model, all FWI-like schemes seek to modify the synthetic model so that **p** becomes more like **d**. Conventional FWI and AWI each attempt to achieve this is different ways:

$$\text{FWI:} \quad \mathbf{d} - \mathbf{p} \;\rightarrow\; \text{zero}$$
$$\text{AWI:} \quad \mathbf{d} \,/\, \mathbf{p} \;\rightarrow\; \text{unity}$$

Both methods use a least-squares formulation. FWI operates to minimise the difference between **d** and **p** directly, whereas AWI seeks, at least conceptually, to drive the ratio of the two datasets to unity. In practice, this ratio must be stabilised to avoid division by small noise-dominated values, and the resulting division occurs trace-by-trace, frequency-by-frequency. A stabilised division in the frequency domain, formulated as part of a least-squares inversion, is simply the recipe for designing a Wiener filter. AWI then designs Wiener filters that map one dataset into the other. Since the wave equation is linear in the source wavelet, these Wiener filters also map the locally assumed source wavelet into the true wavelet, and they will become zero-lag delta functions for the true model.

If both **d** and **p** are perfect, and if both these inversion schemes are driven to their global minima, then both AWI and FWI will produce the same result since they are ultimately solving the same equation. However, if the methods are both solved by local inversion schemes, then the paths that they will each follow in their attempt to reach that global minimum will not normally be the same. In practice, this means that the path that FWI follows can become trapped in local cycle-skipped minima, while the path that AWI follows does not see a cycle-skipped result as a minimum in the objective function.

If instead, **d** and/or **p** are imperfect in some way, such that there will always remain irreconcilable differences between them whatever model the inversion is able to find, then it is not clear that AWI and FWI will arrive at the same end point, since they each use a different measure of goodness of fit between **p** and **d**. There are three principle types of imperfection that we examine here:

- the source wavelet used in the simulation is not correct,
- the observed data contain noise,
- the predicted data use incorrect or at least incomplete physics in their simulation.

We will here assume a high-quality starting model since we are only interested in this study in how data imperfections affect AWI and FWI, and not in how they respond to imperfect starting models.

Incorrect Source

Figure 1 shows two low-frequency source wavelets. These were each determined from the same OBC field dataset using two different methods. The bandwidth of both sources is similar, and their nominal zero times and polarities are the same. We have used source (a) to generate synthetic data, and then attempted to invert those data using both the true source and the incorrect source (b). We have examined the performance of both FWI and AWI under these conditions. For this test, the forward and inverse modelling were identical, and there was no noise in the data.

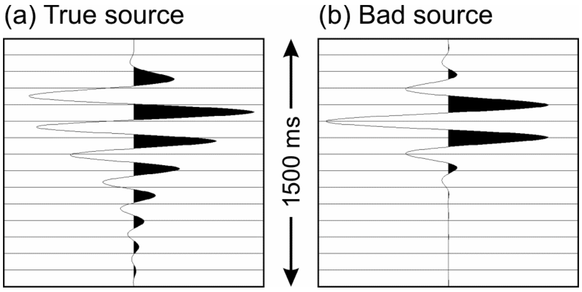

Figure 1 *(a) Wavelet used for modelling.*
(b) Wavelet used for inversion.

In an effort to make these tests as realistic as possible, we limit the total number of iterations to those that we would normally expend when inverting a 3D field dataset, and we limit the temporal bandwidth to 4–12 Hz, and the maximum source-receiver offset to 7 km. The inversion does not honour absolute amplitudes, and we retain the various heuristics and stabilisations within the code that we would use to invert field data. We use a simple approximate diagonal Hessian to precondition the gradient, and otherwise use steepest descent.

Figures 2(a) and (b) show the true and the start models. The latter provides an accurate starting point, and ensures that the results from FWI are not cycle skipped when using the true source. Figures 2(c) and (d) show the results of inverting these data using the true source applied by FWI and AWI respectively. The results are not identical, but are extremely similar; they differ most at the edges and base of the model where there is less-than-ideal data coverage. These figures serve as a benchmark for realistic inversion, and show that FWI and AWI differ minimally under these ideal circumstances.

Figures 2(e) and (f) show analogous inversion results for FWI and AWI when using the incorrect source shown in Figure 1(b). Both inversions are degraded by this change, but the degradation to the FWI result is much larger than for AWI. This is not the result that we had expected, and it appears to be an additional benefit provided by AWI. It is clear that the FWI result shows some of the effects of cycle skipping, but here these are being produced by errors in the wavelet rather than by errors in the start model. The clearest example of the effects of cycle skipping in the FWI result can be seen in the spurious low-velocity dark-blue region in the interior of the shallow high-velocity fault blocks in the centre of the model. Here, the velocity should have increased and it has decreased. Since AWI is immune to cycle skipping, it is able to overcome these effects, and can recover a high-quality final result despite the large errors in the assumed source wavelet.

Imperfect data and incomplete physics

Real field data always contain noise, and in a commercial context, real inversions always use incomplete physics in their forward simulations. The latter is done for reasons of economy, and because it is not yet possible to invert effectively for all the parameters that must be known in order to simulate the full physics properly. Figure 3(a) shows the perfect data used for the study discussed above; Figure 3(b) shows data for the same set of shot records, but with additional noise, and with modified physics. These modifications include elastic rather than acoustic propagation, a density model that is not deterministically related to p-wave velocity, and a modification of density contrast at the seabed which changes the amplitudes of water-bottom multiples. Detailed examination of Figure 3 will reveal that these changes produce quite significant changes in some portions of the shot records.

Figures 2(g) and (h) show the FWI and AWI results, produced by inverting these noisy data, using the simplified physics represented by Figure 3(a). In both cases, the noise and the errors in wave propagation, especially those associated with the seabed, produce significant degradation of the resultant velocity models. These changes in outcome are different in character but similar in magnitude between AWI and FWI. The RMS match to the true model is slightly better here for AWI than for FWI, but in more extensive tests we see no consistent pattern, with both methods succeeding and failing to about the same degree. AWI then is not, unsurprisingly, able to overcome inadequacies in the physics of simulation, but neither does it appear to be more sensitive to this and other forms of noise than is conventional FWI.

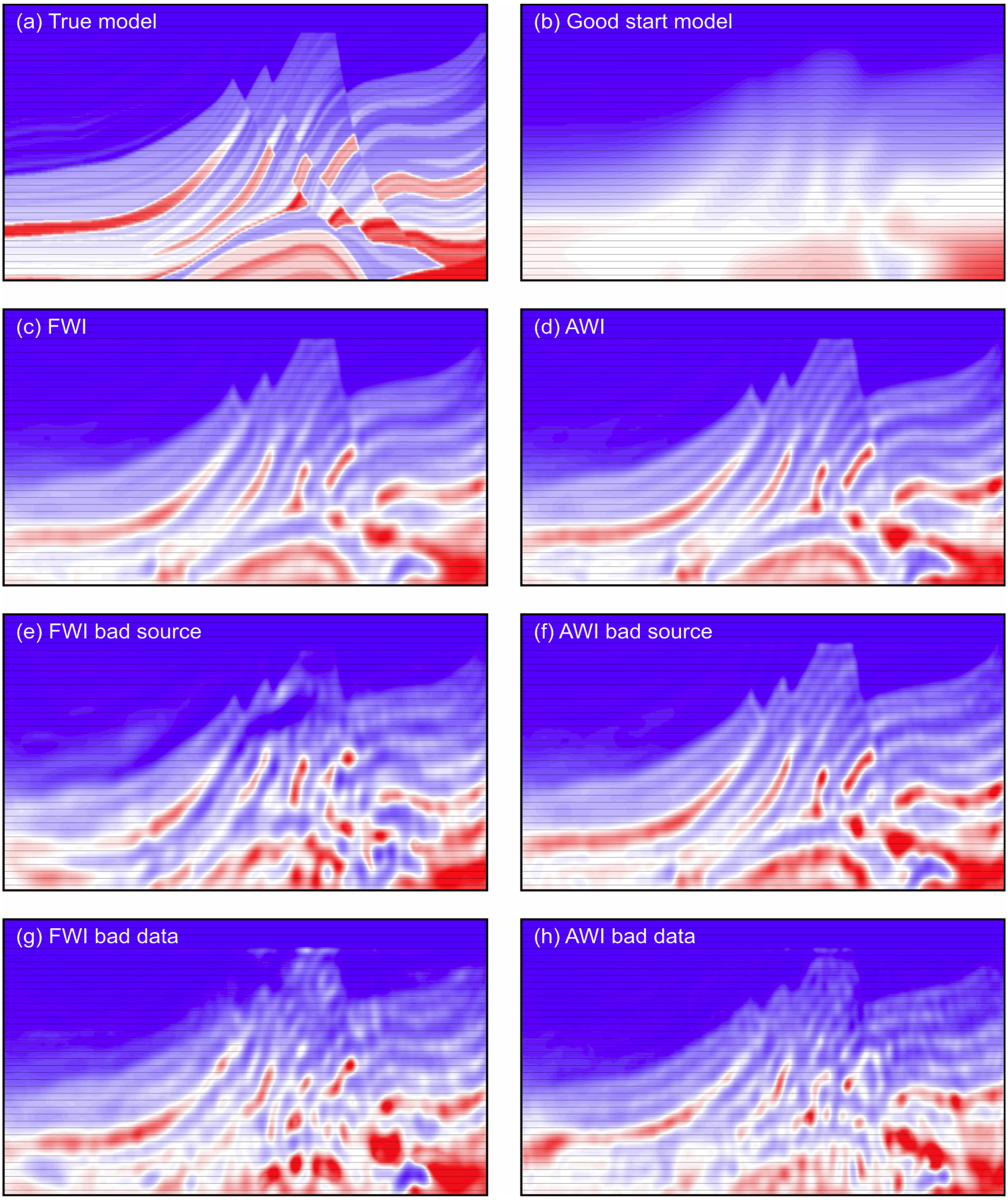

Figure 2 *(a) True model. (b) Accurate starting model. (c) FWI model recovered in ideal circumstances. (d) AWI recovered model. (e) FWI model recovered using wrong source. (f) AWI model recovered using wrong source. (g) FWI model recovered using noisy data and incomplete physics in the inversion. (f) AWI model recovered using noisy data and incorrect physics.*

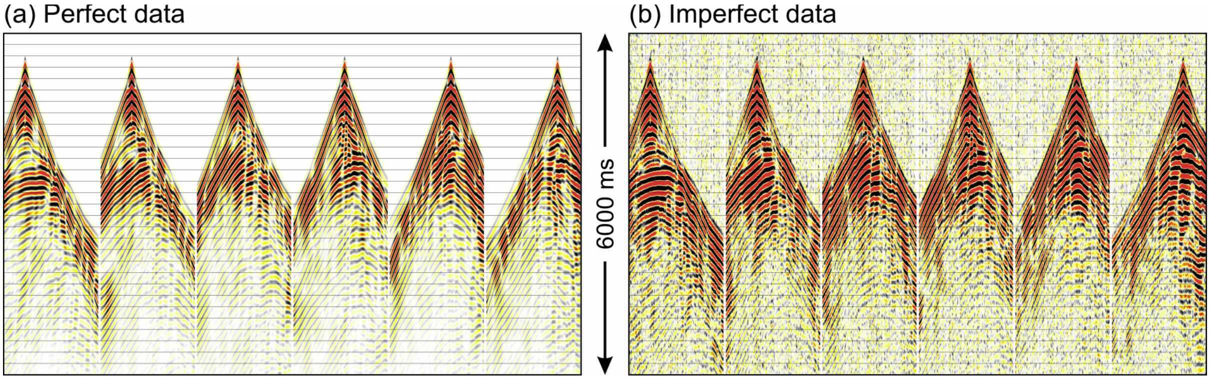

Figure 3 *Shot records spanning the model. (a) Perfect acoustic data with a simple density model. (b) Noisy elastic data with a complicated density model and reflectivity changes at the seabed.*

Discussion and conclusions

AWI works by assuming initially a different source wavelet for every data trace; it sets up the inversion such that the model is changed so that these diverse sources all evolve to become identical to the source that was really used to acquire the field data. This approach ensures that AWI is immune to the effects of cycle skipping. However, since AWI appears to depend critically upon matching the real source, it might be suspected that AWI would be more sensitive than FWI if the wavelet assumed for this real source was not correct. We have shown here that this is not the case.

The immunity of AWI to cycle skipping appears to make it more robust to inadequacies in the source estimation than is FWI. In practice, we take advantage of this, beginning inversion using AWI and an approximate source wavelet and approximate starting model. When a reasonable fit to the observed data is obtained, the current model can be used directly to invert for the true source, and inversion can be completed using FWI and the now-corrected source wavelet.

The results of both AWI and FWI are sensitive to differences in the observed and predicted data that cannot be reconciled by changing the model. These differences include noise in the field data and inadequacies in the sophistication of the physics that is used in the wavefield simulation. The former are inevitable, and the latter are a practical consequence of the cost and difficulties of modelling and inverting fully visco-elastic, arbitrarily anisotropic, three-dimensional, field data.

Since AWI utilises a less-direct method of inversion than does FWI, and since in general inversion methods based upon ratios often converge less readily than do equivalent methods based upon differences, we might expect that AWI would be more troubled than is FWI by inadequacies in data and simulation. We have shown here that this too is not the case. AWI and FWI are both impacted by data and simulation inconsistencies, but AWI appears to be no more sensitive in this than does FWI. In some circumstances, AWI fairs better than FWI in this respect, and in some it fairs less well, and on occasion either or both methods can be catastrophically misled by these effects. We continue to develop our generic understanding of AWI so that we can both predict and avoid such effects.

Acknowledgements

AWI and Adaptive Waveform Inversion are trademarks of Sub Salt Solutions Limited. AWI is the subject of GB patent application number 1319095.4, published as GB2509223.

References

Warner, M. and Guasch, L. [2014] Adaptive Waveform Inversion – FWI without cycle skipping: *76th EAGE Conference & Exhibition*, Extended Abstracts, Amsterdam.