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Bootstrapped Waveform Inversion: Long-wavelength Velocities from Pure Reflection Data

G. Yao* (Sub Salt Solutions Limited) & M. Warner (Imperial College London)

SUMMARY

Conventional FWI cannot successfully invert pure reflection data to recover the long-wavelength velocity model. We present an FWI methodology that can solve this problem, and demonstrate its success inverting data from a simple but difficult-to-invert synthetic model. The method proceeds by modifying the FWI objective function, and by interleaving a migration-like form of FWI with a tomography-like form. Starting from a constant-velocity, this FWI approach produces a full-bandwidth velocity model that accurately depth migrates the reflection data.

Introduction

Conventional full-waveform inversion (FWI) of 3D surface field data has proven to be effective when it has been used in one of two modes:

- applied to datasets that contain significant refracted energy from the target depth, to obtain useful velocity-model updates at all wavelengths ranging from the dimensions of the model down to about half the seismic wavelength;
- applied to datasets in which only sub-critical reflected energy is scattered from or passes through the target region, to obtain useful velocity-model updates that lie only within the seismic bandwidth; this approach is unable to recover the longer-wavelength portion of the velocity model.

However the commercial utility of the first of these can be limited because the maximum depth of penetration of refracted energy using conventional acquisition geometries seldom exceeds two to three kilometres. The utility of the second can be limited because the scale lengths of the updates that it provides are typically only of marginal use in depth migration, and this second approach provides only an incomplete solution to the generation of an accurate full-bandwidth velocity model.

In contrast to both these approaches, in this paper we demonstrate the use of an FWI formulation that is able to recover the long-wavelength portion of the velocity model using only pure-reflection data. We demonstrate the method beginning from a constant velocity model; we do not use travel-time tomography or other image-domain or CDP-based form of velocity analysis to build a starting model. We refer to the method as “Bootstrapped Waveform Inversion” because it builds the velocity model from scratch using only waveform inversion, bootstrapping its way to the final model without initial input from other methods. We demonstrate this approach here using a simple-but-difficult-to-solve synthetic problem. The methodology works on field data; we will present field examples in Madrid.

Theory

A simple and often-repeated theoretical analysis suggests that it is not possible to use FWI to recover long-wavelength velocity anomalies from sub-critical reflection data with limited bandwidth and limited source-receiver offset. That analysis is incomplete, and this conclusion is not correct.

Figure 1(a) illustrates the essence of this analysis. It shows a single point scatterer, *A*, within an otherwise homogeneous background model, imaged by a single source and receiver. The angle θ is the scattering angle at the anomaly; θ is 0° for a zero-offset reflection, and 180° for a horizontally travelling refracted arrival. A simple analysis shows that it is only possible to recover the longest wavelengths at the position of this scatterer, for finite-bandwidth data, if θ approaches 180°. This conclusion is correct. However, the apparently related conclusion, that it is thus not possible to obtain long-wavelength velocity models from FWI applied to limited-offset reflection data, is not correct.

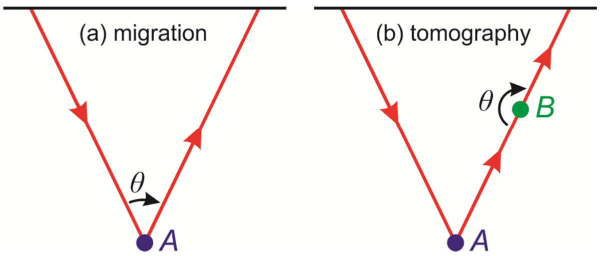

Figure 1 *(a) Scattering by a single scatterer A. If θ is small, then only the short-wavelength model is recoverable; this is depth migration. (b) Scattering by a second scatterer B. Now θ is large, and the long-wavelength model is recoverable; this is tomography.*

Figure 1(b) illustrates a modified analysis. This shows a model with two point scatterers. The first acts as a sub-critical reflector, *A*. For this scatterer, θ is much less than 180°; without it there would be no reflections, and no predicted data. There is now though a second scatterer, *B*, and this plays a different role. For this scatterer, θ is 180°, and it does therefore meet the criterion for the tomographic recovery of the long-wavelength velocity model; now however the model will be recovered at the position of *B* rather than at the position of the original reflector *A*.

Reflection-based FWI then, and indeed any other method of building a macro-velocity model from reflection data, does not generate a long-wavelength velocity model at the reflector. Rather, it builds this velocity model above the reflector using the tomographic potential of transmitted arrivals that have also been sub-critically reflected deeper within the model. The long wavelengths within this velocity model are not limited by source-receiver offset nor by the data bandwidth. The former may limit accuracy, and the latter may limit short-wavelength resolution, cause problems of cycle skipping, or lead to other forms of local minima in the objective function of conventional FWI, but neither bandwidth nor offset prevent the recovery of the long-wavelength velocity model in principle.

The strategy for building a velocity model from pure reflection data via FWI should now be clear. First we must migrate reflections into the starting velocity model to create reflectors that will be capable of producing reflections in subsequent iterations. Conventional least-squares FWI, suitably parameterised, can readily achieve this first step (Yao & Jakubowicz, 2012). However, if the starting velocity model is inaccurate, then these reflectors are likely to be at the wrong depth, have the wrong geometry, and be poorly focused. In a second step therefore, we must use this heterogeneous model to generate predicted reflections, compare these predictions appropriately to the observed data, and use the mismatch to update the long-wavelength portion of the model. This is also a form of FWI, but one that utilises its tomographic rather than its migration-like properties.

This second step is more difficult than the previous one since any change in the long-wavelength velocity model will necessarily change the absolute travel times of reflections, and so will tend to worsen the overall data mismatch. This can be overcome by using a form of adaptive waveform inversion (Warner & Guasch, 2014) that is insensitive to absolute travel times. Having updated the long-wavelength velocity model, we then remigrate the field data using FWI, reforming the reflectors. We proceed with this alternating scheme until the model is sufficiently accurate for conventional FWI to converge. In practice, we must take various precautions to ensure that the alternating scheme converges, that it does not become trapped in local minima, that it is insensitive to any mismatch in absolute reflection amplitudes, and that it does all this using a reasonable number of iterations.

Example

Here we demonstrate one version of such a bootstrapped reflection inversion scheme applied to a simple yet difficult-to-invert synthetic model. Figure 2(a) shows the velocity model that we wish to recover. It contains two horizontal reflectors, together with a shallow velocity anomaly. The synthetic field data (Figure 2b) consist of a split-spread, moving across the top of the model, with a maximum offset of 3000 m, well-sampled in both source and receiver domains. The field data contain only primary reflections from the two reflectors together with their first-order interbed multiple. The direct arrival has been muted in the field data, and there are no refracted arrivals. Free-surface multiples arrive after the end of the shot record. The shallow velocity anomaly is sufficiently smooth that it does not produce significant reflected energy within the bandwidth of the seismic data.

This model is difficult to invert using pure-reflection data if the long-wavelength background velocity model is unknown. An ideal inversion should be able to recover the velocity above the upper reflector, the velocity between the two reflectors, the depth of both reflectors, and the location, shape and magnitude of the shallow velocity anomaly. We would not expect to be able to obtain the velocity below the lower reflector since no energy penetrates this portion of the model. In addition, since sub-critical reflection amplitudes in field data depend strongly upon both density and velocity contrasts, and are also influenced by elastic effects, attenuation and short-wavelength anisotropy variations, at least some of which will be unknown, any realistic inversion should not assume that absolute reflection amplitudes contain useful information about the p-wave velocity model.

Figure 2(c) shows a smoothed version of the true velocity model. This model necessarily has the correct long-wavelength velocity model. If such a model, which is not readily available for field data, is used to begin conventional FWI, then an accurate model can be recovered (Figure 2d). In this instance, conventional FWI has migrated the reflection data into the velocity model at the correct depth with the correct geometry, and has subsequently used the transmitted portion of this reflected energy to improve the recovery of the shallow velocity anomaly through the tomographic properties of transmission FWI. A result analogous to that shown in Figure 2(d) can only be obtained for field data if some other method is first used to generate an accurate long-wavelength starting model.

Figure 2(e) shows a second starting model; this does not match the true velocity above the upper reflector, and the long-wavelength model is not correct. If conventional FWI is now applied using this inaccurate starting model, then the outcome is as shown in Figure 2(f). Continued iteration with this model, or minor changes to the FWI methodology, do not improve this outcome. In this case, which often pertains for field data, the initial migration of the reflection data performed by FWI does not produce reflectors at the correct depth or with the correct geometry. The tomographic properties of FWI cannot then be properly utilised. The results of conventional FWI in this case are approximately those of a least-squares iterated reverse-time depth migration, converted from reflectivity into velocity, and performed using an incorrect velocity model.

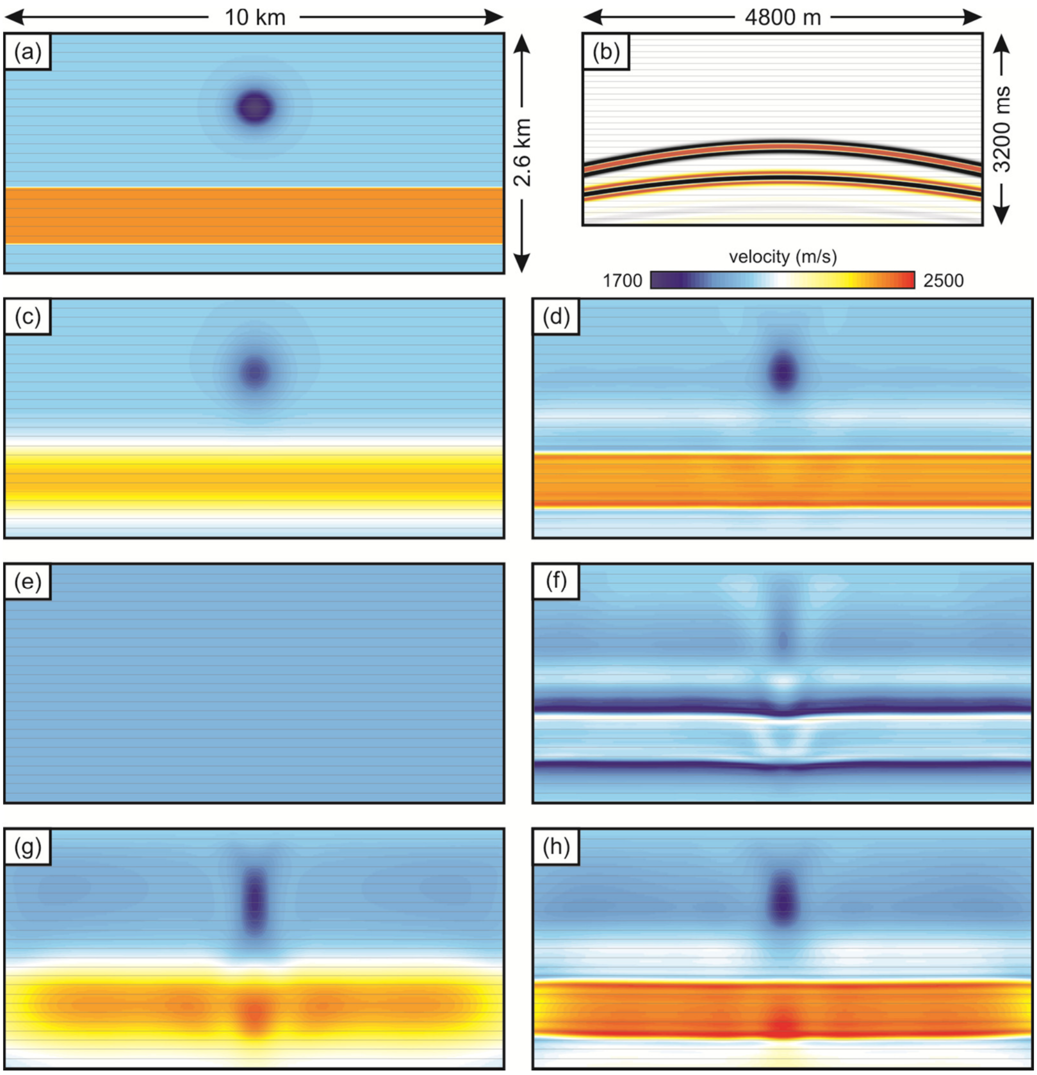

Figure 2 *A simple but difficult-to-invert velocity model. (a) True model. (b) Shot record showing only reflections. (c) True model after smoothing. (d) Model recovered by conventional FWI starting from smoothed model – this result is not possible for field data. (e) Constant-velocity starting model – incorrect everywhere. (f) Model recovered by conventional FWI starting from constant velocity – the long-wavelength model is not recovered. (g) Initial model recovered by bootstrapped reflection FWI. (h) Final model recovered by bootstrapped reflection FWI – close to the ideal result in (d).*

Figures 2(g) and (h) show the results of bootstrapped reflection FWI, starting from the inaccurate model in Figure 2(e), obtained using the alternating scheme described above, run to a maximum offset of 2400 m, followed by conventional reflection FWI to an offset of 3000 m. We used 3 iterations of FWI in a migration mode, followed by 3 iterations in a tomographic mode, to perform one alternating step, and performed 30 alternating steps in total in order to reach the model shown in Figure 2(g). From this point, we continued using conventional reflection FWI only, applying a further 50 iterations to reach the result shown in Figure 2(h). This represents two to three times the number of iterations that we would expect to use in order to solve this problem if refracted data were available.

Discussion and Conclusion

Clearly the result produced by bootstrapped reflection FWI (Figure 2h) is vastly superior to the result produced using conventional FWI applied to reflections (Figure 2f). The former is able to recover the long-wavelength velocity model successfully, while the latter performs little more than an expensive reverse-time migration of the input data. The result is not of course perfect; in this model, data from just two reflectors is being inverted, and the total source-receiver offset is limited. The resolution in the vertical direction is then necessarily limited since changes to the model that leave the average slowness unaltered along near-vertical ray paths produce only marginal changes in the observed data.

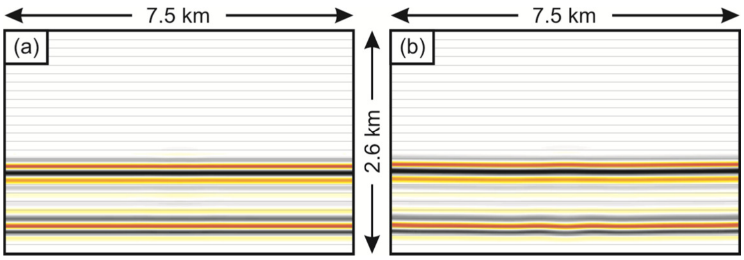

Figure 3 *True-amplitude reverse-time depth migration: (a) using the true model, (b) using the bootstrapped model.*

One measure of the quality of the recovered model is to use it to migrate the reflection data. Figure 3 shows the migration of finite-offset data using the true velocity model and using the model recovered by bootstrapped reflection FWI. The image is shown for the central portion of the model where it is not influenced by edge effects.

Using the bootstrapped-FWI model, the maximum error in the absolute depth of the lower reflector is 12 m; this is produced principally by the incomplete resolution of the shallow velocity anomaly. The maximum error in the absolute amplitude of the migrated image is 3% which is also related to the shallow anomaly. The source wavelet used here has a maximum frequency of about 10 Hz, which has a wavelength of 250 m so that the spatial resolution of the velocity model should be around 125 m. Note however that resolution concerns the ability to distinguish two discrete anomalies rather than the ability to place a single anomaly in its correct position which can be done much more accurately.

In summary, we have shown that it is possible to configure a full-waveform inversion scheme so that it can recover the long-wavelength macro-velocity model from pure sub-critical reflection data using conventional surface acquisition geometries. Conventionally configured FWI, using a simple conventional objective function, fails completely to solve this problem. However, using a modified form of FWI, which we have here referred to as “bootstrapped waveform inversion”, in which a migration-like form of FWI is inter-leaved with a tomography-like form of FWI using a modified objective function, can fully solve the problem leading to models that will accurately migrate the reflection data. This type of FWI methodology opens up the possibility of applying full-waveform techniques to reflection data at all depths without the need first to build an accurate starting model.

References

Warner, M., and L. Guasch, 2014, Adaptive Waveform Inversion – FWI without cycle skipping: 76th EAGE Conference & Exhibition, Extended Abstracts, Amsterdam.

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