An approach for multisource 3-D marine CSEM modelling in the frequency-domain

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SUMMARY

Geophysical multi-source electromagnetic modelling is usually performed solving a linear system of equations for each right-hand side. This approach can be prohibitively expensive when the system of equations has a large number of degrees of fredom and sources. We present an alternative method for multi-source forward modelling of marine CSEM data. For the discretization of the electric field equation we use edge finite elements which naturally satisfy the material interface conditions for the electric field and avoid the presence of spurious modes in the solution. For solving the system of equations we use the software package MUMPS to factorize the system of equations. A solution for each right-hand side is then obtained using the same factorization.

INTRODUCTION

Marine CSEM modelling in the frequency domain requires the solution of a system of equations with complex coefficients. A significant effort has been devoted to the development of different strategies to achieve an accurate and stable solution. Traditionally the linear system is solved using iterative methods generally of the Krylov family of solvers. Nevertheless such methods struggle to find a solution unless efficient preconditioning is applied. Another important aspect is the fact that marine CSEM modelling is a multi-source problem in nature i.e. it is necessary to find the solution of a linear system for multiple right-hand sides. This is specially important for survey design and CSEM data inversion. Direct methods based on matrix factorization are suitable for multi-source problems in the sense that a single decomposition is sufficient to solve as many right-hand sides as necessary. Moreover even for linear systems with high condition numbers such as the ones arising from direct electric field discretization, direct methods can achieve a solution. Nonetheless it is important to note that direct methods are usually avoided due to the fact that memory requirements can limit the size of models used in simulations. In recent years, however, interest on direct methods has gained a new impetus in part due to the increasing availability of affordable powerful hardware, in part due to the improvement of methods that can effectively make problems with a huge number of degrees (thus requiring a huge amount of memory) tractable, and in part because direct methods work better than other methods. This recent trend on the development of efficient direct methods is gaining more attention of the geophysical community (Oldenburg et al., 2008) which usually has to deal with large and poor-conditioned systems of linear equations and multi-source problems for survey data modelling and inversion. For the factorization of the system of equations we employed the numerical solver MUMPS (acronym for Multi-Frontal Massively Parallel Solver).

ELECTRIC FIELD EQUATION IN THE FREQUENCY DOMAIN

For a time harmonic dependence $e^{-i\omega t}$ the electric field for iso tropic media is given by:

$$\nabla\times\nabla\times\mathbf{E}-\omega\mu\left(\omega\varepsilon+i\sigma\right)\mathbf{E}=i\omega\mu\mathbf{J}_s \tag{1}$$

where $\mathbf{J}_s$ is the current density generated by an electric dipole source. In the range of frequencies used in marine CSEM data acquisition we can consider the quasi-static limit, hence $\sigma/\omega\varepsilon \gg 1$ and consequently the displacement currents term in equation 1 can be negleted. For a complete formulation and to ensure the uniqueness of solution the perfect electrically conductive boundary condition is considered:

$$\mathbf{n}\times\mathbf{E}\mid_{\partial\Omega}=\mathbf{0} \tag{2}$$

where $\partial\Omega$ is the boundary of the modelling domain. Representing the source current density directly on a grid can present numerical difficulties due to its singular nature at the origin. To overcome this problem, the electric field is decomposed in primary and secondary components: $\mathbf{E}=\mathbf{E}_p+\mathbf{E}_s$. The decomposition of the electric field leads to a system of two equations, one for each resultant component. The primary electric field $\mathbf{E}_p$ is calculated for a simple model with conductivity σ_p, where a closed form of the solution can be achieved, (*e.g.* an homogeneous medium), and the secondary field is calculated in the domain of electrical conductivity anomalies: $\Delta\sigma=\sigma-\sigma_p$.

FINITE ELEMENT DISCRETIZATION

The equation for the secondary field was discretized using edge elements. Using this procedure the absence of spurious modes and the normal discontinuity and tangential continuity of the electric field components at material interfaces are ensured (Jin, 2002). The corresponding system of finite element equations for the secondary field equation is derived using the weighted residual method or Galerkin criterium (Zienkiewicz and Taylor, 2005). The vector residual equation for the secondary field is defined by:

$$\mathbf{r}=\nabla\times\nabla\times\mathbf{E}_s-i\omega\mu\sigma\mathbf{E}_s-i\omega\mu\Delta\sigma\mathbf{E}_p \tag{3}$$

The Galerkin criterium enforces the condition:

$$R_k=\sum_{e=1}^{N_e}\int_{\Omega^e}\mathbf{N}^e_{i(k)}\cdot\mathbf{r}\,dV=0 \tag{4}$$

were R_k is the integral of the residual weighted by $\mathbf{N}_k$ over the kth edge, N_e is the total number of elements and $i(k)$ represents the local numbering of the edges on each element. This process is know as the assemble of the system of equations. The evaluation of the integrals over each element Ω_e results in the element system of equations:

$$\left[K^e - i\omega\mu\sigma M^e\right]E^e - i\omega\mu\Delta\sigma M^e E_p^e = R^e \tag{5}$$

where E^e and E_p^e is the electric field discretized in the element edges. The sub-matrices K^e and M^e are given by:

$$K^e = \int_{\Omega^e} \left(\nabla \times \mathbf{N}_i^e\right) \cdot \left(\nabla \times \mathbf{N}_j^e\right) dV \tag{6}$$

$$M^e = \int_{\Omega^e} \mathbf{N}_i^e \cdot \mathbf{N}_j^e dV \tag{7}$$

The assemble of the system of equations leads to a large, sparse, complex symmetric and linear system of equations:

$$AE = s \tag{8}$$

were A is the matrix of complex coefficients resulting from the assemble, E is the electric field discretized over the grid and s is the source vector.

SOLUTION OF THE SYSTEM OF EQUATIONS

The system of equations 8 is ill-conditioned (van Rienen, 2001) and consequently iterative methods fail unless special preconditioning is utilized. Moreover one must bear in mind that marine CSEM is a multi-source problem in nature, *i.e.* for data inversion or survey simulation it is necessary to solve the system of equations 8 for several right-hand sides. For those reasons using a direct solver would be desirable in the sense that the condition number would not affect the achievement of a solution and with a single decomposition it is possible to solve as many linear systems as necessary.

In the past few years has been a considerable and growing effort to obtain scalable matrix factorization algorithms for sparse and linear systems of equations on parallel architectures. One of the most successful and promising direct factorization solvers is the package MUMPS (Amestoy et al., 2000). MUMPS uses a multifrontal method with an out-of-core strategy allowing the extension of the available memory for the calculation of the factors and allows real, complex, single or double precision arithmetics.

MUMPS multi-frontal technique is based on LU or the LDL^T direct factorization of the matrix depending on its symmetry.

COMPUTATIONAL IMPLEMENTATION

To test our algorithm we considered a model discretized using a $137 \times 137 \times 37$ irregular grid representing a computational domain of $8 \times 8 \times 3$ km. The sea-water layer is 1 km thick. The grid utilized was finer near the source positions, and the coarser regions respected the Nyquist discretization criterium to avoid aliasing. Figure 1 shows the resistivity model:

$$\rho(z) = \begin{cases} 0.33\ \Omega m, & \text{if } z > 0\ m \\ 1.0\ \Omega m, & \text{if } -200 < z < 0\ m \\ 10.0\ \Omega m, & \text{if } -400 < z < -200\ m \\ 1.0\ \Omega m, & \text{if } z < -400\ m \end{cases} \tag{9}$$

We considered 169 sources, spaced 500 m apart in each direction, and 30 m above sea-floor. The source frequency was 1 Hz. The multi-source solution was obtained in 6 hours approximately. The results where compared with the ones obtained by using a 1-D forward modelling code based on the Chave and Cox (1982) solution. In figures 2 to 4 are displayed the solutions achieved using each code for a source located at $x = 500\ m$, $y = 500\ m$ and $z = 30\ m$. A good agreement between the solutions obtained with each code can be observed. We tested our algorithm on a desk workstation with two quad-core Xeon processors and 14 Gb RAM, running OS-X 10.5.

CONCLUSIONS

We presented a method for multi-source CSEM modelling in the frequency domain. As shown a factorization method can have advantages when a limited number of sources is used. In six hours we obtained a solution for 169 sources, a result that an iterative solver would struggle to achieve on a single machine. Nevertheless the model size continues to be a limitation, and the simulation of models with realistic dimensions may require at least the use of a small cluster of machines.

ACKNOWLEDGMENTS

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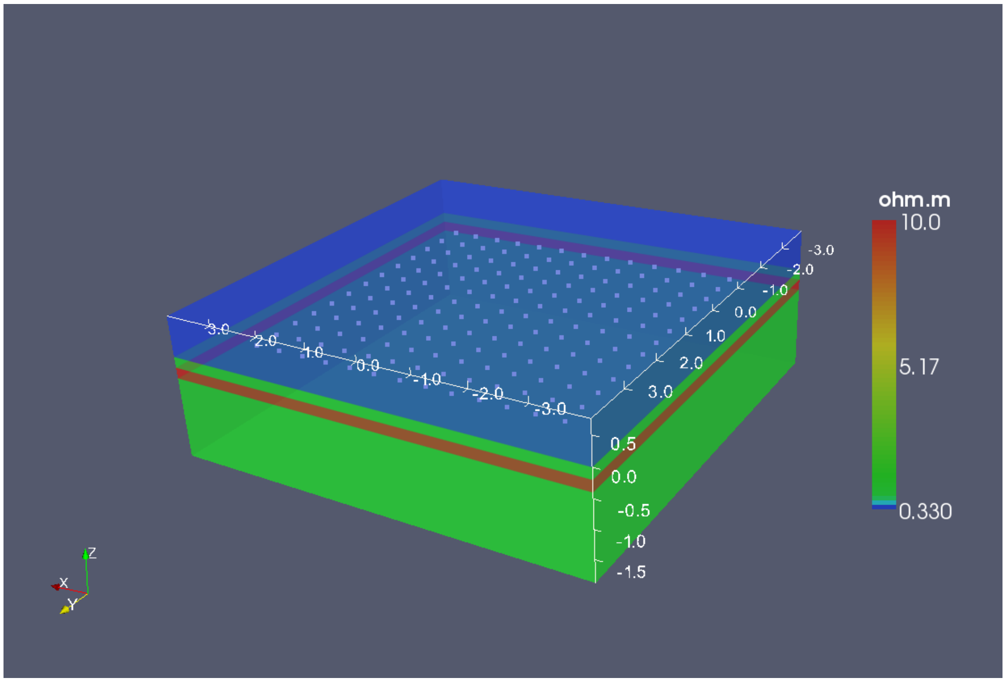

Figure 1: Display of the resistivity model used to test the numerical scheme. The white dots represent the source positions. The distance units are in kilometers.

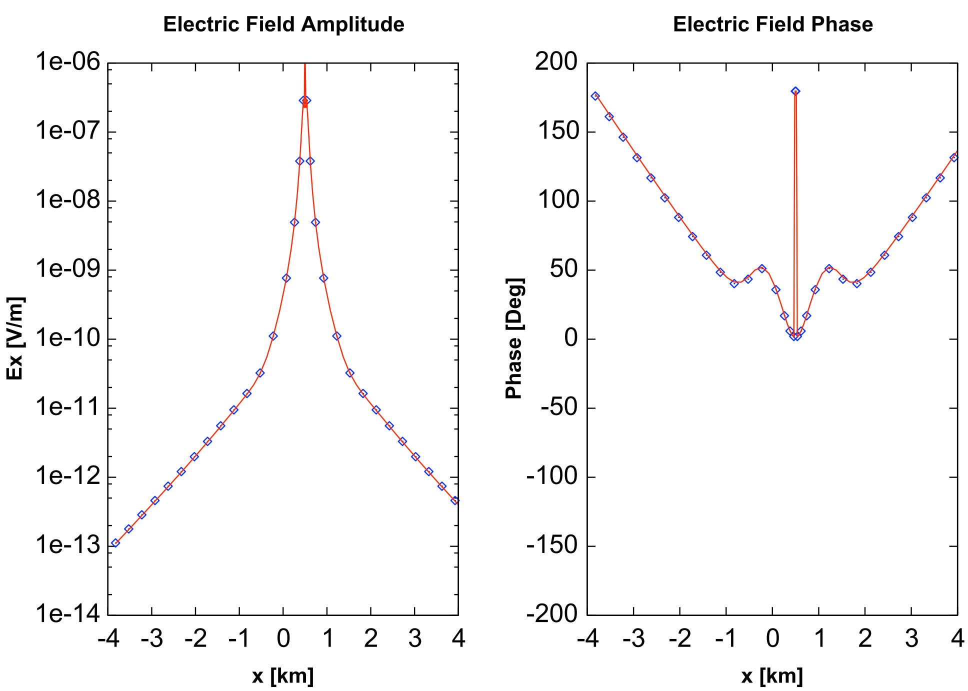

Figure 2: Comparation of the electric field x component computed using the FEM code (circles) and the 1D code (solid lines) in the inline direction at $y = 0$ m for a signal of 1 Hz.

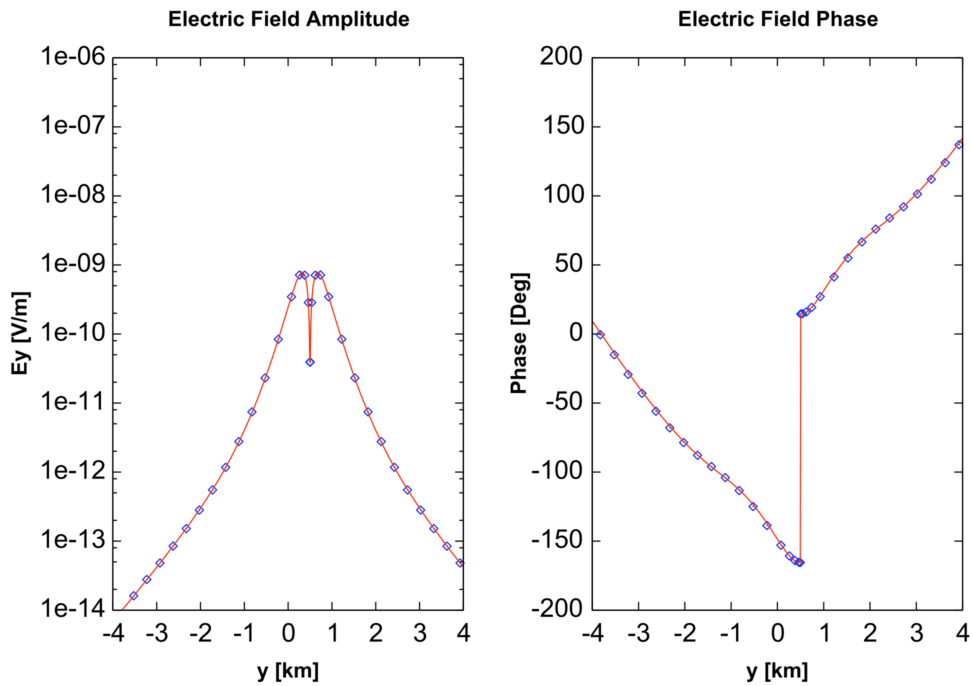

Figure 3: Comparation of the electric field y component computed using the FEM code (circles) and the 1D code (solid lines) in the cross-line direction at $x = 350\ m$, for a signal of 1 Hz.

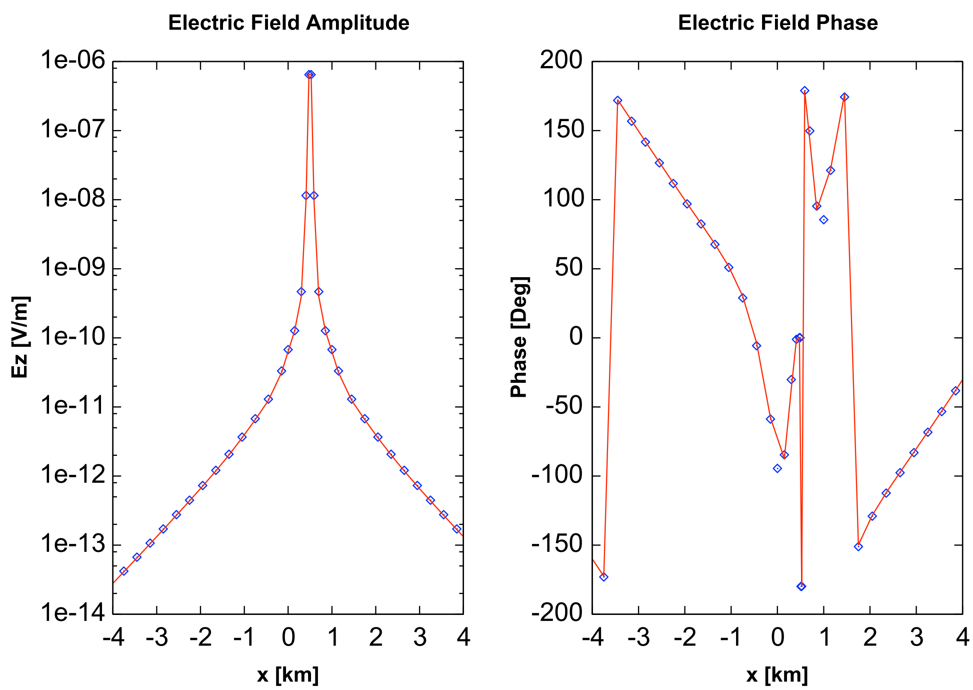

Figure 4: Comparation of the electric field z component computed using the FEM code (circles) and the 1D code (solid lines) in the inline direction $y = 0\ m$ for a signal of 1 Hz.

EDITED REFERENCES

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