Barcelona '10

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A Strategy for Waveform Inversion without an Accurate Starting Model

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SUMMARY

A key limitation of waveform inversion as currently implemented is the need for a starting model of high accuracy or field data with low frequencies. Here we present a new approach - staged waveform inversion - designed to mitigate this need and thereby permit the application of waveform inversion to a much wider range of datasets.

Overview

Waveform inversion seeks to recover the best-fit high-resolution velocity model for a given seismic dataset. It is formulated as a localised least-squares minimisation of the misfit between observed and modelled waveforms. One of the key shortcomings of the method is the necessity of having either a highly accurate starting velocity model or of having extremely low-frequency arrivals present in the field data (Virieux and Operto 2009). This requirement, and the difficulty of meeting it in practice, means that the technique is rather limited in the range of acquisition geometries, problems and target depths that it can address successfully. In this paper, we describe a new approach – *staged waveform inversion* – that appears able to circumvent this requirement, and hence make waveform inversion amenable to a much larger suite of targets and datasets.

To converge, localised waveform inversion requires that its starting-model data match major events in the target dataset to within half a wave-cycle at the lowest useable frequency (Beydoun and Tarantola 1988). Consequently, if we proceed conventionally, and treat the field data as the target data, the inversion will not succeed when beginning from a poor "cycle-skipped" starting model. In staged waveform inversion, we instead proceed by inverting one or more intermediate target datasets that are constructed so that they lie partway between the starting-model data and the true field data. These intermediate data inversions serve to steer the model away from a nearby cycle-skipped local minimum to a position where the field data itself can be successfully inverted to the correct global minimum.

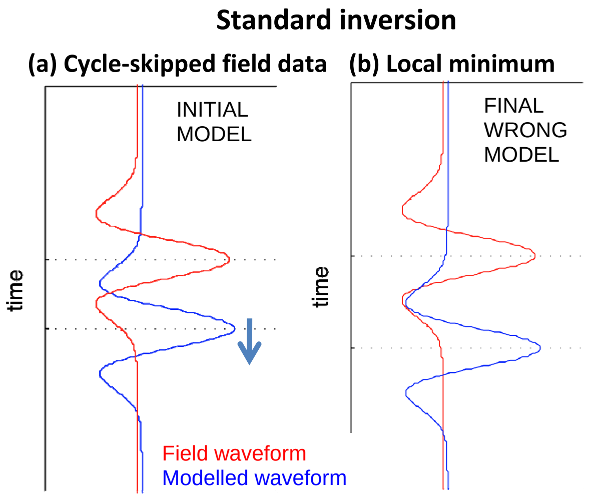


Figure 1 *Cycle skipping and staged inversion*

(a) Field data and starting-model data are more than half a cycle out of phase;

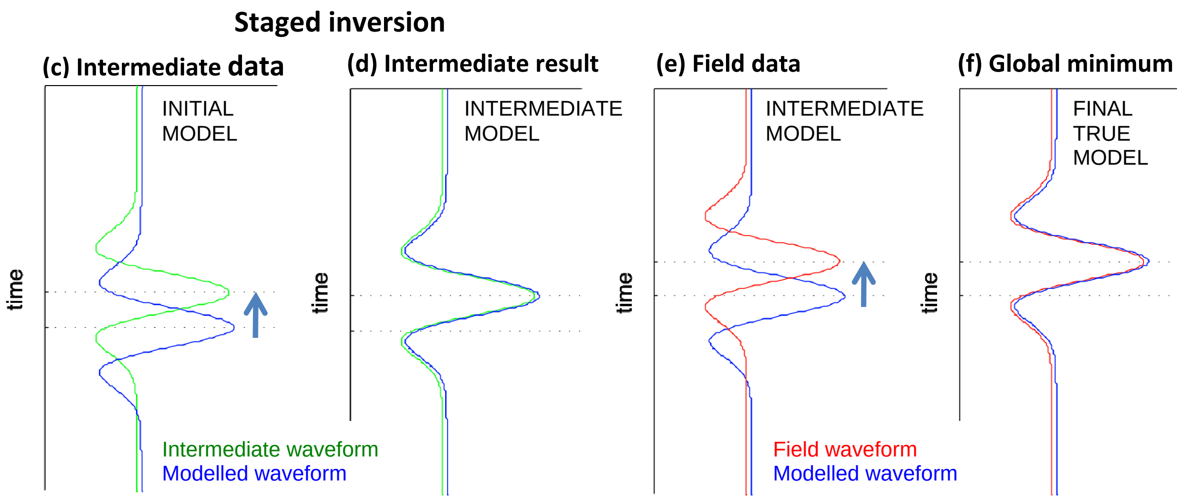
(b) Local inversion moves the model and the model data in the wrong direction;

(c) Intermediate and starting-model data are less than half a cycle out of phase;

(d) Local inversion moves the model in the right direction towards the true model;

(e) Field and modelled data are now less than half a cycle out of phase;

(f) Continued inversion using field data recovers the true model.



Staged waveform inversion in the frequency domain

When the starting model data is cycle-skipped at the lowest useable frequency, conventional wavefield inversion converges to a local minimum – whether carried out in the time domain or frequency domain. Whilst staged waveform inversion may be formulated in either of these domains, the latter is more straightforward. In the frequency domain, we start at the lowest useful frequency in the data and invert initially only for phase. We design an intermediate dataset that has its phase partway between the *unwrapped* phase of the field data and the starting-model data. At low frequencies, it is generally straightforward to unwrap the phase unambiguously; if there are parts of the data where the phase cannot be unambiguously unwrapped, then we exclude those from the initial inversion. After iterating using the intermediate dataset at the lowest frequency, we move to the field data, and thereafter continue using only the field data at successively higher frequencies. If the starting-model data is skipped by several cycles, then we invert a succession of intermediate datasets.

Single-frequency, phase-only, waveform inversion minimises the least-squares misfit function $E = \Sigma |\mathrm{u} - \mathrm{d}|^2$ where the summation is over all sources and receivers, $\mathrm{u} = A\exp\{i\theta\}$ are the amplitude-normalised model data, and $\mathrm{d} = A\exp\{i\phi\}$ are the target data. In conventional waveform inversion, the target dataset is the field data, but in staged inversion, the target dataset will initially be the intermediate dataset. Expressing the misfit function in terms of the phase difference, gives $E = \Sigma 4A^2 \sin^2 \frac{1}{2}(\theta - \phi)$. Because of the sinusoidal form of this misfit, localised inversion shifts the model waveform only towards the nearest cycle of the target waveform. As a result, modelled phase must be within $\pm\,\pi$ of the phase of the target data for convergence to the global minimum. We achieve this by setting the phase of the intermediate dataset to be $\phi = \theta - a\,(\theta - \psi)$ where ψ is the phase of the field data, and a is chosen so that $-\pi < a(\theta - \psi) < \pi$. Figure 2 shows the behaviour of the misfit function in conventional and staged inversion.

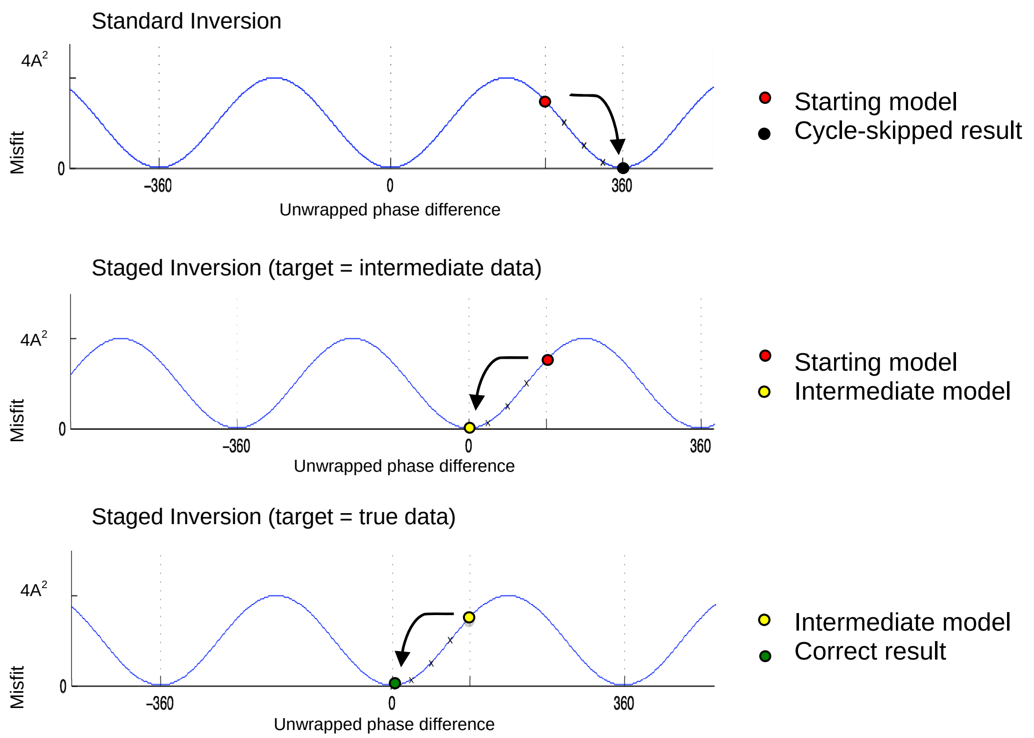


Figure 2 *Misfit as a function of unwrapped phase difference for a single data residual with modelled phase more than* $\pm\,\pi$ *away from the field phase. In conventional inversion the modelled phase shifts towards the nearest local minimum. In staged inversion, the modelled phase shifts, in two or more stages, towards the correct global minimum.*

Staged *vs* conventional waveform inversion a cross-hole example

As a test case, we use a synthetic cross-hole survey with a chequerboard true model and inversion frequencies restricted to between 250Hz and 500Hz – a class of problems shown to be often intractable by conventional means (Pratt et al., 2002). The configuration used is shown in Figure 3. The background velocity is 2650 m/s and the anomalies are ±7.5%. The phase difference contour plots show where the 250Hz data is cycle-skipped relative to a homogeneous starting model.

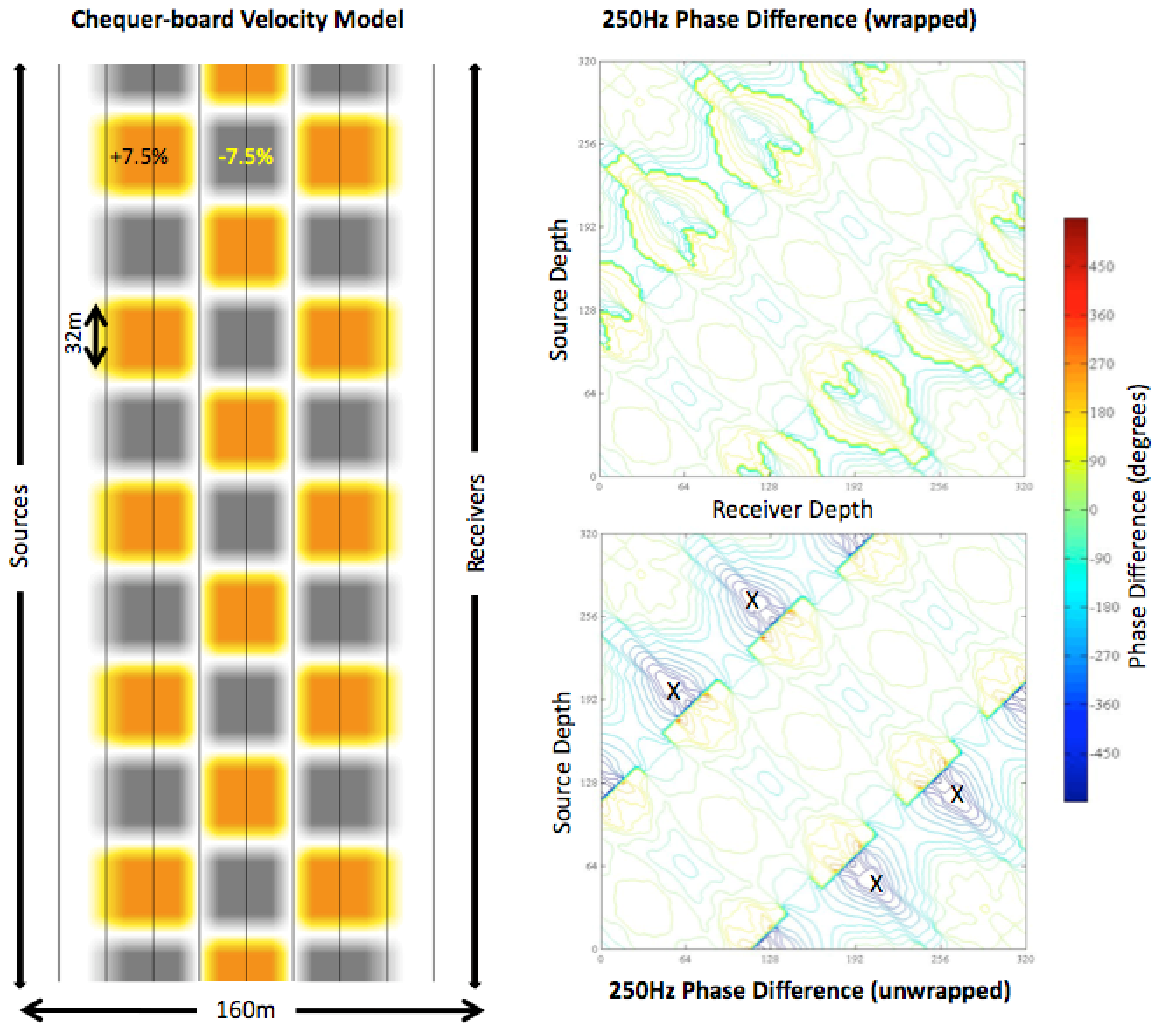


***Figure 3**. Data from a chequer-board model, left, was inverted using conventional and staged waveform inversion with a realistic lowest frequency, for a cross-hole survey, of 250 Hz and a homogenous starting model. Wrapped and unwrapped phase differences between the 250Hz synthetic field data and starting-model data are shown on the right.*

The longer-offset data in this example are cycle skipped at the lowest frequencies, and in small parts of the dataset around the points marked X in Figure 3, they are double cycle skipped. Consequently, conventional waveform inversion fails for this example, Figure 4a. Staged waveform inversion was applied to this problem, Figure 4. The intermediate target dataset was generated by halving the unwrapped phase differences shown in Figure 3, and was retained for ten iterations, before switching to the 250 Hz true data. The results in Figure 4 show that the simplest implementation of staged inversion was sufficient here to provide successful convergence whereas conventional waveform inversion could not. We took no special care to deal with the double-wrapped cycle skips, which suggests that the methodology is robust to small errors in the phase unwrapping.

(a) Standard inversion Final result **(b) Staged inversion Intermediate result** **(c) Staged inversion Final result**

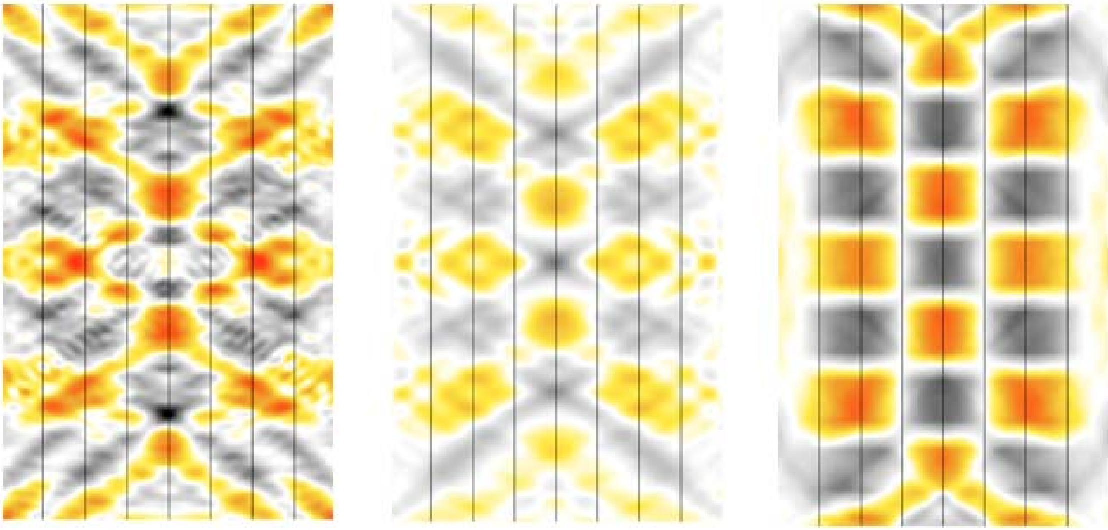


Figure 4 *Inversion results from the central area of the velocity model. (a) The failed result of inverting the true data directly from the 'cycle-skipped' homogenous starting model. (b) The recovered intermediate model, which arises from inverting the intermediate target dataset. (c) The result of using the intermediate model – not cycle-skipped at 250Hz – to invert the true data. The final result is a successful recovery of the chequerboard model with accurately estimated velocities.*

Conclusions

Staged waveform inversion appears to provide a simple and robust methodology for inverting seismic data in the absence of an unusually accurate starting model or of unusually low frequencies in the field data. The method can be applied in both the time and frequency domains. As we have implemented it in the frequency domain, it relies upon the ability to unwrap the phase difference between the field data and the data generated by the starting model – this is usually most easily achieved by unwrapping each dataset separately. We have only to unwrap the phase at the lowest frequency present, with good signal-to-noise ratio, in the field data.

Unwrapping low frequencies is relatively straightforward, and is normally possible if the field data have been acquired such as to allow successful and unambiguous spatial interpolation at the lowest frequencies – the vast majority of datasets are acquired with such a geometry. We are currently applying this methodology to surface seismic and to 3D field data, and if it proves effective in those applications, the approach will open up the benefits of waveform inversion to a much wider range of problems and datasets.

Acknowledgements

This work was sponsored by the FULLWAVE III consortium consisting of BIS, BG, BP, Chevron, CGGVeritas, ConocoPhillips, ENI, Maersk, Nexen, OHM, and Rio Tinto, under the ITF programme. The data were processed and displayed using the Landmark ProMAX system.

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