3D wavefield tomography: Problems, opportunities and future directions

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Wavefield tomography, otherwise known as full-waveform inversion, of two-dimensional seismic data, has become a well-established technique over the past decade, with impressive recovery of realistically complex synthetic models being reported by several groups. However, despite its proven potential, its uptake to tackle real-world exploration and production problems has been rather limited. In our view, this has been principally because the increased spatial resolution, accuracy, and other benefits that the method brings are only genuinely realised for field data when the method is extended to deal with three-dimensional velocity structure, three-dimensional reflection geometry, and a three-dimensional array of sources and receivers. Since the real world is always three-dimensional, very-accurate two-dimensional solutions to three-dimensional problems are nearly always illusory – the higher is the spatial resolution of the method, and the more accurate is the physics of wave propagation that is employed, then the more significant will be the errors that are introduced by neglect of the third dimension. In essence, there is little utility to be gained from a model that is highly resolved in two dimensions, but that is not at all resolved in the third, and where structure from the missing third dimension is mapped incorrectly onto the 2D plane.

There are of course some other significant impediments to wider uptake, but in our view these are relatively straightforward to overcome. Principal among these are the necessity to acquire data at the lowest possible frequencies, the requirement for very-long source-receiver offsets, the difficulty of applying existing methods to only the sub-critical reflected portion of the data, and perhaps also the difficulty of obtaining an effective starting model. As we move fully into three dimensions, there is also the desirability of acquiring data that is well-sampled in azimuth, and the consequent problem of dealing properly with azimuthal anisotropy. In principle, all of these additional problems, except the last, can be solved by straightforward if sometimes costly modifications of conventional acquisition systems. In particular, the problem of the starting model and of inverting pure reflection data, largely disappear if sufficiently low frequencies and sufficiently long offsets are present in the field data.

If the principal problem is then to extend the methods of wavefield tomography fully into three dimensions, then that extension is straightforward in principle but is a computationally daunting prospect in practice. There are two reasons for this. If we consider a 2D model measuring $n \times n$ cells, and a 3D model measuring $n \times n \times n$, then increasing the dimensionality from two to three increases the number of cells in the model by n which will increase both the likely compute time and required memory by a factor of n, and it increases the number of sources with which we have to deal also by a factor of n. If therefore we approach the 3D problem in the most straightforward way, then the required run time will be proportional to n to the power of 6 ($\sim n \times n$, sources, $\sim n \times n \times n$ model size, and a time ~ n to propagate waves across the model). Since, in useful problems, n is a likely to be at least a few hundred to a thousand, the computational scale of the problem becomes unfeasibly large both in compute resource and in total memory.

Notwithstanding the scale of the problem, a number of groups are tackling the full 3D problem. We have tackled the problem using finite differences in the frequency domain. We have implemented the forward solver and the tomographic inverter, and have applied this to both 3D synthetic and 3D field data. To date, we have inverted just one 3D field dataset; this was acquired using conventional, parallel, 3D swaths, surface-streamers and dual sources. We are about to invert two OBC datasets that have a more-complete range of azimuths and longer offsets. Our method is fully three-dimensional and uses the two-way isotropic heterogeneous visco-acoustic wave equation.

Conventionally, solvers in the frequency domain have used LU decomposition to factorise the matrix equations that result from applying finite-difference operators to the wave equation. This approach has the great advantage that, once the matrix has been factorised, solutions for individual sources are relatively inexpensive to compute. In practice, most such methods have used a second-order rotated operator that minimises the bandwidth of the matrix while simultaneously producing results that have similar accuracy to fourth-order schemes. Matrix reordering schemes, such as nested dissection, are also typically used to minimise matrix infill during factorisation. In three dimensions, LU decomposition requires use of a very large memory for the storage of the factors, and nested dissection is significantly more complicated and less efficient for the multiply-banded 3D matrices. In addition, the computational effort for the forward and backward substitution that is required to solve for new sources once the matrix has been factorised is also significant. Consequently, we have chosen to attack the 3D problem differently.

In many branches of computational physics, large, sparse matrix equations are solved iteratively rather than directly. In this approach, an approximate solution for the wavefield is first obtained by solving using a related, *preconditioning* matrix. This preconditioner is chosen such that it is related to the true matrix, but is much faster to invert. The approximate solution from the preconditioner is then substituted into the true matrix equation to discover the effective source for which it actually provides a correct solution. The difference between this effective source and the true source provides an error which can then be treated as the actual source for the next iteration. Once the error has decreased sufficiently, the solution to the original matrix equation has been found.

It is well known that the wave equation is difficult to precondition effectively; many widely-used preconditioners work only poorly, or fail entirely, when applied to the wave equation. We precondition this system hierarchically. At the lowest level, we use a simple one-way propagator to propagate the wavefield from one plane of cells in the model, sweeping this propagator successively in six directions, up-down, back-front, left-right. These one-way waves are damped such that energy is typically propagated about ten cells. This preconditioner has proven sufficiently accurate and stable, that a simple update, without ensuring orthogonality to previous updates, is sufficient to obtain a rapidly convergent solution; removing the need for orthogonality reduces the total memory required. At the next level, we use a multi-grid approach, first solving the wave equation on a coarse grid that is then used as an approximate solution, preconditioning the solution on a finer grid. Grid coarsening is by a factor of two, and may proceed through more than one level of coarsening. At the highest level, we use a block-iterative scheme, solving first for the wave equation within separate, overlapping sub-domains. These solutions act as preconditioner for the full solution over the full domain. This combined system has proven to be stable, fast and reliable. During tomography, we modify the accuracy of the iterative solution according to context, solving more accurately as we learn more about the velocity model.

The second innovation that we have made in 3D is to introduce composite sources. In conventional tomography, every physical source is treated independently. In practice, we have found that it is possible to make composite sources, combining the results from several point sources into a single large composite source. These composite sources are used in the tomography, and provided that the composites are built intelligently, the results obtained are near-identical to the results obtained when using the separate point sources explicitly. We are still exploring the optimum way in which to build the composites. For surface streamer data, we are typically able to combine about 20 point sources into one composite, a saving of a factor of 20 in CPU effort.

When combined with the iterative solver, the use of composite sources makes full 3D tomography practical on realistically sized datasets. To do this, we use clusters of compute nodes, each with multiple cores. Ideally, we use as many nodes as we have composite sources, solving for each source in parallel under MPI, and using POSIX threads locally to solve each source across the local cores. Where there are more composite sources than nodes, we solve in parallel by using each core to solve a separate source. Where there are more composite sources than total cores available, we solve both successive sources sequentially on each core. The principal constraint on the performance of this tomographic scheme is the speed of local memory access when using multiple cores which can be mitigated by using multiple, fast, memory controllers. For typical datasets, the method is not communications bound. The software that implements this scheme has been released to our sponsors, and will be made more widely available from end 2010.

Over the next three years, we intend to extend our existing 3D software to the visco-elastic domain, to combine it with controlled-source electromagnetics to undertake joint inversion of seismic and CSEM datasets, and to add a limited degree of anisotropy. The latter is likely to be important in azimuthal 3D acquisition since the kinematics of the problem can be strongly affected by azimuthal anisotropy, and isotropic wavefield tomography will tend to invent spurious heterogeneity to explain the kinematics. It is possible to add global elliptical anisotropy that is kinematically correct if the ellipsoid is aligned with the coordinate axes. This is obtained at no additional computational cost by rescaling the axes. It is also possible to add, reasonably cheaply, for some particular types of anisotropy, arbitrary variation around the model, provided that the principal axes remain aligned with the coordinate axes. Adding completely general anisotropy however is computationally expensive, and is almost certainly not justified in the vast majority of circumstances, not least because the observed data do not constrain the anisotropy in any detail, and the detailed anisotropy is not known *a priori*.

Beyond this, the key questions to be answered are:

1. what is the most cost-effective way to acquire the data needed for effective wavefield tomography?
2. how can we best generate and record very low frequencies?
3. how can we best generate a starting model?
4. how can wavefield tomography take maximum advantage of more-conventional seismic processing?
5. can the technique be made to work usefully without recording long-offset transmitted waves from the depth of interest?

At present, our partial answers to these questions are:

1. use large numbers of stand-alone ocean-bottom instruments (not cables) deployed beneath conventional surface-acquired 3D surveys;
2. use ocean-bottom recording and include large, deep, clustered guns within otherwise standard airgun arrays;
3. use conventional velocity analysis on pre-stack 3D time-migrated data and/or wide-angle travel-time tomography applied to broad-bandwidth data where the wavelength of the highest-frequency data is equal to the size of the Fresnel-zone for the lowest-frequency data;
4. work in the time domain, perhaps with an adaptive finite-element approach rather than frequency-domain finite differences, and integrate the processing into the iterative tomography, processing both the field and predicted data identically as tomography proceeds;
5. possibly – by using low-frequency broad-band data to obtain a starting model, repeatedly smoothing the recovered velocity model between tomographic iterations to remove the reflections, and re-introducing the reflectors back into the model by mixing the scaled, integrated, migrated, conventionally processed reflection data back into the velocity model at each step.