Constrained waveform inversion in salt-affected datasets

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SUMMARY

We have developed an accurate and robust methodology that is capable of inverting seismic data in salt-affected environments, to obtain a highly resolved velocity model, without the use of travel-time tomography or explicit salt flooding, in circumstances where conventional full-waveform inversion (FWI) and similar approaches fail. A reflection-driven inversion, in combination with total variation (TV) constraints applied to the adaptive waveform inversion (AWI) objective function, provides the building blocks required to recover both long and short-wavelength velocity models in such environments.

INTRODUCTION

Full-waveform inversion in the presence of salt is challenging for three principal reasons:

1. lack of significant refracted energy traveling through target depths which restricts the inversion to utilize mostly reflected data,
2. complex salt geometries with significant velocity contrasts with the surrounding subsurface model,
3. deep targets that are susceptible to cycle-skipping created by the long paths between sources and receivers in combination with poor-quality starting models.

To date, FWI in salt-affected areas has been principally used to add minor detail to models that were already built using other methods. Conventional model building can be time consuming, can involve a great deal of expert interaction, and is not always successful for difficult datasets.

Total variation (TV) penalties have been used previously as a constraint upon waveform-inversion objective functions to preserve edges in complex models (Anagaw and Sacchi, 2011). This type of constraint is particularly relevant in regions with strong evaporite-sediment contrasts, where FWI updates otherwise tend to contain strong edge-related artifacts and spurious oscillations. Unlike conventional salt flooding, appropriate TV constraints do not require that the interior of salt bodies are homogeneous, and they can deal correctly with heterogeneity inside salt.

Since we are attempting to solve a problem in which the macro-velocity model is not already well-developed, we must prevent model evolution becoming trapped at local minima. Consequently the optimization problem to be solved must be insensitive to cycle-skipping. Adaptive waveform inversion (Warner and Guasch, 2014) utilizes adjoint-state methods and an objective function that does not contain cycle-skipped local minima. Waveform reconstruction inversion (WRI) provides a similarly effective approach, substituting the PDE constraint by a penalty term in the functional itself, which has the effect of relaxing the PDE constraint by computing an augmented wavefield that matches the data and approximates a solution of the wave equation at the same time (van Leeuwen and Herrmann, 2013). WRI is most straightforward to implement in the mono-frequency domain, whereas AWI is fundamentally a time-domain approach that requires finite bandwidth. In this paper, we use adjoint-state methods; the use of WRI to solve a similar problem is explored by our senior author in a companion submission.

THEORY

FWI, AWI and other related waveform inversion methods can be defined as PDE-constrained optimization problems of the form:

$$\min_{\mathbf{m},\mathbf{u}} \frac{1}{2}\|F(\mathbf{m},\mathbf{u},\mathbf{d})\|^2 \quad \text{s.t.} \quad \mathbf{Au}=\mathbf{s} \tag{1}$$

where $F(\mathbf{m},\mathbf{u},\mathbf{d})$ is a function of the model parameters $\mathbf{m}$, predicted $\mathbf{u}$ and observed data $\mathbf{d}$, $\mathbf{A}$ is the modeling operator and $\mathbf{s}$ is the source. Conventional FWI defines F as

$$F(\mathbf{u},\mathbf{m},\mathbf{d}) = \sum_{all\ data} \frac{1}{2}\|\mathbf{u}(\mathbf{m})-\mathbf{d}\|^2 \tag{2}$$

where the sum is over the whole dataset. This definition of the mismatch between observed and predicted data creates cycle-skipped local minima because of the oscillatory nature of the seismic data. In contrast, AWI imposes similarity between the two datasets using a different criteria

$$F(\mathbf{u},\mathbf{m},\mathbf{d}) = \sum_{traces} \frac{1}{2}\frac{\|\mathbf{Tw}\|^2}{\|\mathbf{w}\|^2} \tag{3}$$

where $\mathbf{w}$ is a convolutional Wiener filter that matches one trace in one dataset into the equivalent trace in the other, the sum is over all traces, and $\mathbf{T}$ is a penalty function which monotonically increase with lag, penalizing energy at far lags in the filters, forcing them to focus at zero lag, where the filter becomes an identity transformation. Because of the non-oscillatory nature of the penalty, AWI is immune to cycle-skipping. Additionally, the AWI gradient direction enhances the contribution of reflections in the inversion because its associated adjoint source is more sensitive to the existence and mis-positioning of reflected energy than is the gradient obtained from the residual data that is used as the adjoint source in conventional FWI.

Adding constraints to the model parameters makes the inverse problem more well-posed. For example, a box constraint $C = \{\mathbf{m} : m_i \in [b,B]\}$ will force the updated models to have values only in the $[b,B]$ range. Total variation constraints can be similarly imposed on the model parameters to reduce undesired oscillations and other unconstrained noise in the model

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updates. If a two-dimensional model is represented as N_1 by N_2 , then we define the TV norm as:

$$\|\mathbf{m}\|_{TV} = \frac{1}{2}\sum_{ij}\sqrt{(m_{i+1,j}-m_{i,j})^2+(m_{i,j+1}-m_{i,j})^2} \quad (4)$$

where it is trivial to extend this to 3D. We then impose the constraint that the magnitude of (4) be smaller than a certain value τ. Now the constraints imposed on the model updates can be written as

$$\mathbf{m}^{n+1} = \mathbf{m}^n + \Delta\mathbf{m} \quad \text{s.t.} \quad m_i \in [b,B],\ \|\mathbf{m}\|_{TV} \leq \tau \quad (5)$$

APPLICATION

We here show two models to demonstrate the performance of this inversion strategy. The first is a simple-yet-difficult-to-invert homogeneous model containing one deep high-velocity layer with a smooth low-velocity anomaly above it. To decouple the geometry acquisition footprint from the recovered low-velocity anomaly, the smooth anomaly has been placed off center in the model, Figure 1b. The starting model, Figure 1a, is homogeneous with a velocity of 2500 m/s; this is not the correct background velocity.

We invert only reflected data, and the model contains only two reflectors. The scale lengths of the circular anomaly and the high-velocity layer both lie outside the bandwidth of our source wavelet – that is, their recovery requires that we update the macro model and not merely that we migrate these anomalies at seismic bandwidth into an already-correct long-wavelength model. Conventional FWI will not solve this problem.

A total of 48 sources and 96 receivers are spread at a depth of 20 m across the top of the model, which has a size of 200×200 gridpoints, with a grid spacing of 20 m. The source is a ricker wavelet with a peak at 12 Hz. We inverted the same observed dataset starting from the model in Figure 1a following two approaches:

In the first experiment we minimized the conventional FWI functional defined in (2), while in the second we minimized the AWI functional defined in (3) subject to the TV constraints described in (4). Box constraints were imposed in both inversions to avoid unrealistic velocities in the updated models and to prevent instabilities in the discretized forward modeling operators. Both inversions loop from low to high frequencies starting with a band centered at 3.5 Hz and finishing with a band centered at 11 Hz.

The final model for conventional FWI fails to recover the main characteristics of the target (Figure 1c). This result is essentially an rtm-like image of the reflection data migrated using the starting model which will not properly focus the image, and it include long-wavelength artifacts of the sort that conventional rtm is designed to suppress. Conventional FWI is not capable here of recovering the true background model from pure reflection data.

In contrast, the AWI inversion with TV constraints successfully retrieves both the high-velocity layer and the low-velocity

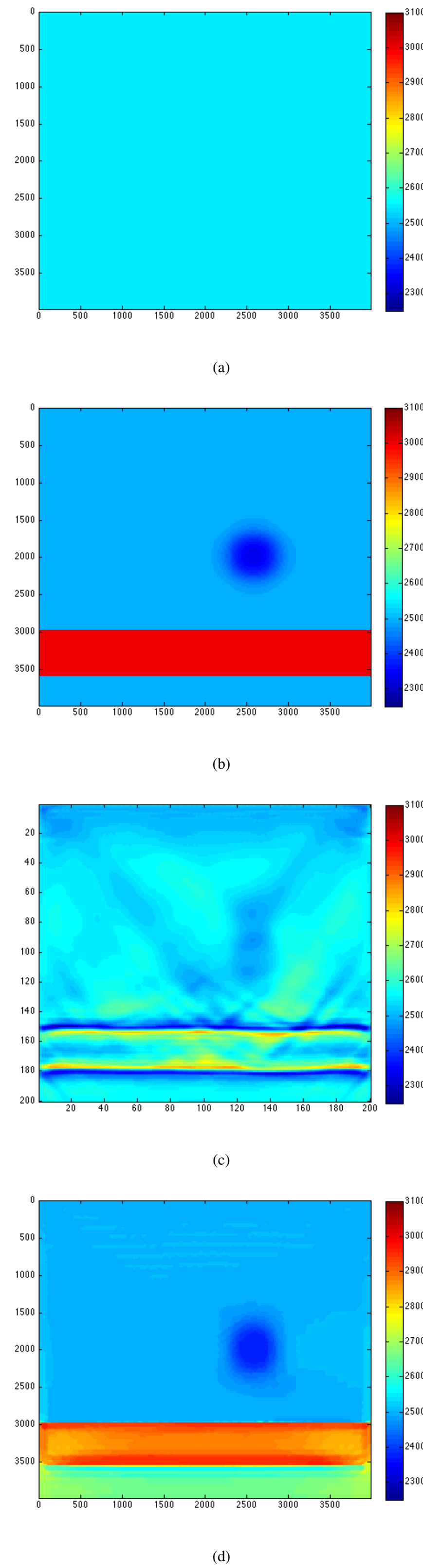

Figure 1: a) Starting model; b) True model; c) Recovered FWI model; d) Recovered reflection-AWI + TV-constrained model. The background velocity in the starting model is 50 m/s faster than the true model; only reflected data are inverted.

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anomaly (Figure 1d). This approach converges to the correct background velocity everywhere above the deepest reflector – there is no information in the dataset about the model below the deepest reflector. The recovered reflectors are sharp, positioned at the correct depth, and contain no artifacts in depth related to the overlying low-velocity anomaly. Although this example is simple, it illustrates clearly the difficulties of applying conventional FWI to pure reflection data to recover the velocity model at all wavelengths, and demonstrates that suitably constrained and configured AWI can solve this problem.

The second model is taken from the central section of the 2004 BP velocity benchmark (Figure 2b). This is a more realistic model; it contains a target salt body surrounded by sediments with a high velocity-contrast between the two, and an irregular bathymetry. This model is hard to invert without an accurate starting model. The aim here is to obtain that accurate model directly from waveform inversion. The starting model for the inversion is a simple one-dimensional gradient (Figure 2a) that approximates a regional velocity increase with depth, and that contains no information about the salt body. Data generated with the starting model are clearly cycle-skipped with respect to the true data (Figures 3 and 4).

In Figure 2c, we show the result of inverting the data using adjoint-state methods with TV constraints. The numbers of sources and receivers are 145 and 290 respectively, and these are spread across the top of the model at a depth of 20 m (sources) and 40 m (receivers). The model is discretized on a 600×150 grid with a grid spacing of 20 m giving a model that is 12 km wide and 3 km deep. The source is a ricker wavelet centered at 12 Hz and, as in the previous experiment, the inverted frequencies range from bands centered at 3.5 Hz to 11 Hz.

The inversion result has converged towards the target model everywhere that there is adequate data coverage; the incomplete recovery of the model at its outer edges is directly related to the lack of data in this area. As a result of the edge-preserving characteristics of the constraints, we are able to properly recover the high-velocity salt body, including the complicated geometry of bottom salt. All major features of the target model are accurately recovered. The evolution of the updates is reminiscent of a manually driven salt-flooding procedure, and our methodology appears to be capable of automatic salt-flooding during the inversion. Extension of this methodology to anisotropic models in three dimensions is straightforward, and is little more demanding than is conventional FWI.

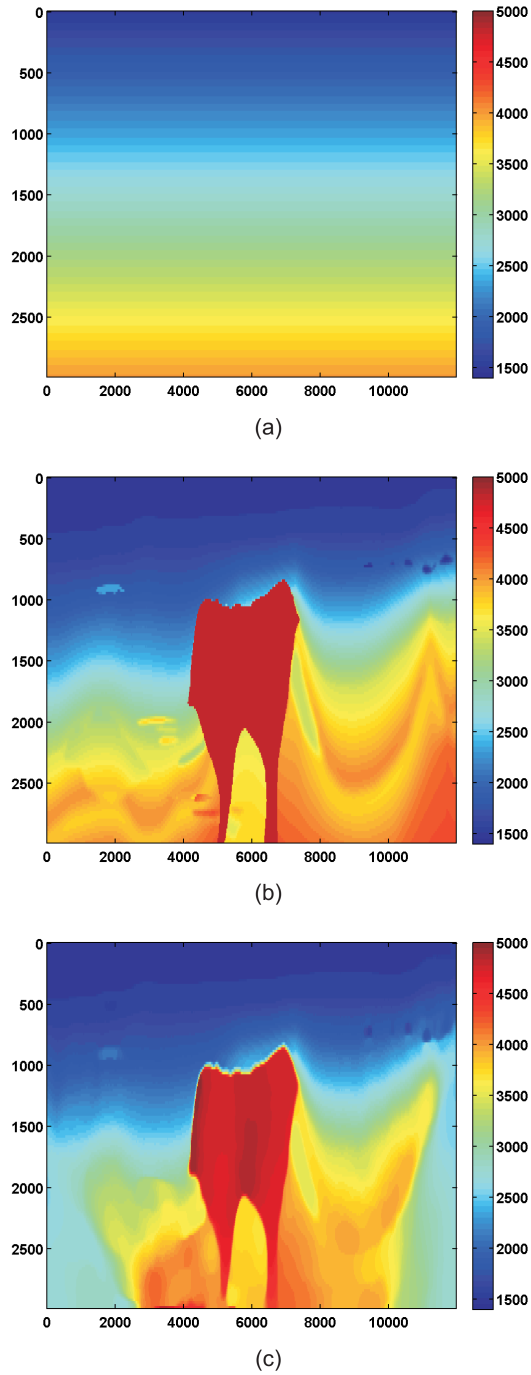

Figure 2: a) Starting velocity model; b) True model; c) Recovered model

CONCLUSIONS

The common approach to velocity model building in the presence of highly anomalous salt bodies involves a significant amount of human interaction. A standard procedure is to pick top salt from migrated images following travel-time tomography above the salt, then apply salt flooding to fill the model below the picked top, followed by a remigration using this new salt-flooded model where the bottom of the salt is expected to have focused so that a second pass of picking will be able to properly define the shape of bottom salt. This can then be followed by sub-salt sediment flooding.

Such an approach has limitations: it can have a high cost in both skilled personnel and time; it assumes *a priori* knowledge of the velocity in the salt; it does not easily allow velocity variation inside the flooded areas; it can be subjective and

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prone to human error; and it can result in ambiguous intermediate results, for example when the salt is connected below the bottom of the model as in the second example shown above. In difficult datasets, this workflow can often involve extensive scenario testing via expensive re-migration, and in important datasets it has not always been ultimately successful.

Here we have demonstrated a largely automated waveform invertion methodology where we solve a single optimization problem driven entirely by an appropriate misfit function that includes constraints on the total variation of the inverted models. Even though FWI has in principle the ability to achieve this same goal, some practical limitations —lack of low frequencies, or unaffordable computational costs of full Hessians— prevent a straightforward application of the method to this type of problem.

In contrast, AWI with TV constraints on the model update does not suffer from these limitations and it clearly improves the quality of the inverted models when compared to their conventional FWI counterparts. Our approach not only enhances the accuracy of the results, but it also increases their spatial resolution in part because the TV constraints promote edge-preservation in the images.

ACKNOWLEDGEMENTS

The work presented in this abstract is a tribute to its main author, Ernie Esser, who passed away in tragic circumstances early this year. He was a young and very promising scientist with whom the co-authors had the privilege to collaborate. One of his favorite quotes illustrates best his passion, optimism and courage in life:

`We don't know what we are doing,`
`but we are doing it!`

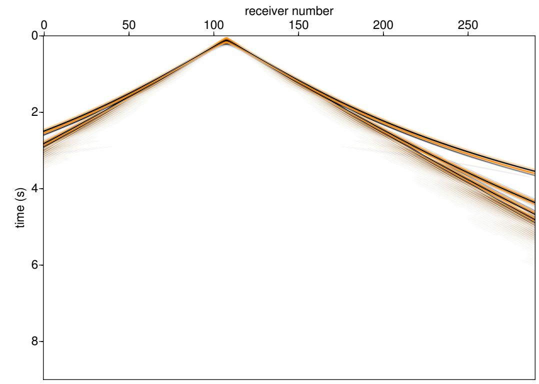

Figure 3: Data predicted using the starting model shown in Figure 2a

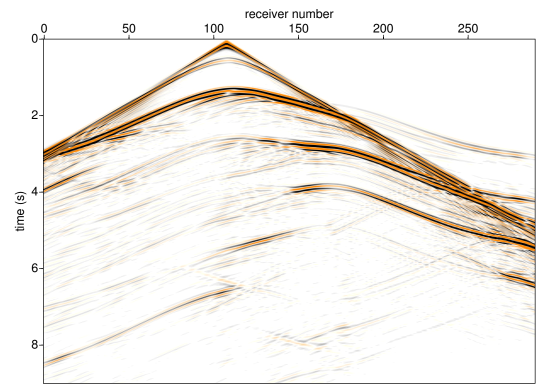

Figure 4: Observed data corresponding to the target model shown in Figure 2b

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EDITED REFERENCES

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