Time vs frequency for 3D wavefield tomography

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Unlike the situation in two-dimensions, where direct factorisation of the matrix equations makes frequency-domain methods much faster than explicit solution in the time-domain, the computational resources required for practical wavefield tomography in 3D can be rather similar in the two domains. We have developed and optimised schemes that undertake wavefield tomography using explicit time stepping in the time domain and that iteratively solve the matrix equations of the implicit problem in the frequency domain.

We have applied these two methods systematically to the same suite of problems. In the frequency domain, the principal advantages are that the initial tomographic updates for lowest frequencies are often seen more quickly, and spatial resolution can be better at the highest frequencies. In the time domain, one of the principal advantages is that it is possible to mute and/or weight the field data in time, and consequently the method can be made to work more effectively with difficult datasets. In practice, both approaches are useful, and both should be available within a comprehensive suite of inversion tools.

Introduction

In two dimensions, full-wavefield tomography in the frequency domain can make use of direct factorisation of the matrix equations, allowing wavefields for multiple shots to be calculated very quickly. Iterative methods for solving the matrix equations are not competitive. There is also no such comparable benefit when working in the time domain for the 2d case. Thus, 2d frequency domain appears to be much more efficient for tomography than the time domain.

In 3d, however, factorisation of the frequency domain matrix equations becomes prohibitive, both for memory requirements and CPU resources. Consequently, iterative methods can become competitive when solving the matrix equations in the frequency domain, since they do not even require storage of the matrix, and can potentially perform a hundred or so matrix-vector operations as quickly as a direct method can perform one backsubstitution. Also, explicit time-stepping in the time domain becomes competitive, even with a few thousand time steps.

We have developed and optimised finite difference schemes for 3d wavefield tomography in both the frequency domain and time domain. The relative merits of each domain can therefore be discovered when consistently applied to the same range of test cases.

Methodology

Wavefield tomography, in both time and frequency domains, usually has three main intensive stages:

1. *Forward modelling* – given an existing model of the domain, calculate the wavefields throughout the domain from each shot, and record the responses at the detector locations.
2. *Backpropagation (and cross-correlation)* – take the mismatch between modelled and actual detector responses and backpropagate through the domain. Calculate a gradient for the model update by cross-correlating the forward wavefields with the backpropagated wavefields.
3. *Step calculation* – change the model some sufficiently small distance along the gradient direction, re-run the forward calculations with the updated model, and use the new detector responses to work out some optimal distance to move the model along the gradient direction.

For the wavefield calculations, the scheme in the frequency domain makes use of a 27-point stencil having an optimal combination of rotated and non-rotated second order finite difference operators.

In the time domain, the explicit time-stepping is second order, while the spatial discretisation uses a 37-point stencil formed from an optimal combination of rotated and non-rotated fourth order finite difference operators. One of the crucial components of the time domain scheme is an innovative method to avoid storage of the forward wavefields for cross-correlation during backpropagation.

The tomography usually progresses from lower starting frequencies, which give poor spatial resolution but better initial large-scale corrections to the starting model, up to higher frequencies, which correct the model at a higher spatial resolution.

Initial Considerations

Discussion of some of the benefits of frequency vs time can begin before any examples are tested.

Time domain benefits include:

- muting/windowing, or time-weighting, of field data;
- no matrix inversion ⇒ no iteration convergence balancing, nor matrix factorisation overheads.

Frequency domain benefits include:

- clear-cut progression through a few specific, increasing frequencies;
- including attenuation is straightforward.

3D Test Cases

Test case 1: simple two-channel model

This model comprises a domain with two channels that cross it at different depths, with different strikes. The upper channel has some wiggles in it, while the bottom channel is perfectly straight (see figure 1). The 1d background velocity range goes from 2000m/s at the top to 3000m/s at the bottom, with perturbations to this background (and the channels) added throughout the domain.

Tomographic inversion proceeded from 4Hz to 24Hz, in 2Hz intervals, with 5 iterations at each frequency (so a total of 55 iterations). The shortest wavelength (in the 2000m/s region) at the highest frequency (24Hz) was six cells. There was no windowing or weighting in the time domain.

Figure 2 shows the results of tomography, starting from the 1d background velocity model, using both frequency domain (top) and time domain (bottom) for a number of slices at different depths through the two channels (including within the region where the two channels overlap).

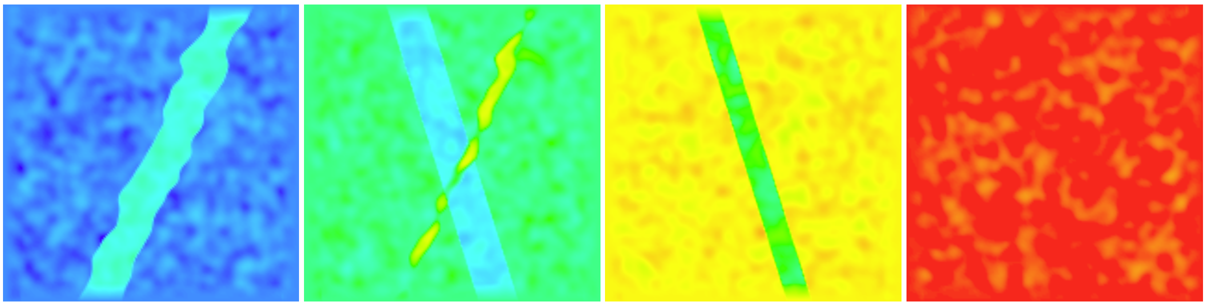

Figure 1 *Two-channel test-case true model at different depths – from left to right: near top of upper channel; bottom of upper channel & top of lower channel; towards bottom of the lower channel; underneath the lower channel.*

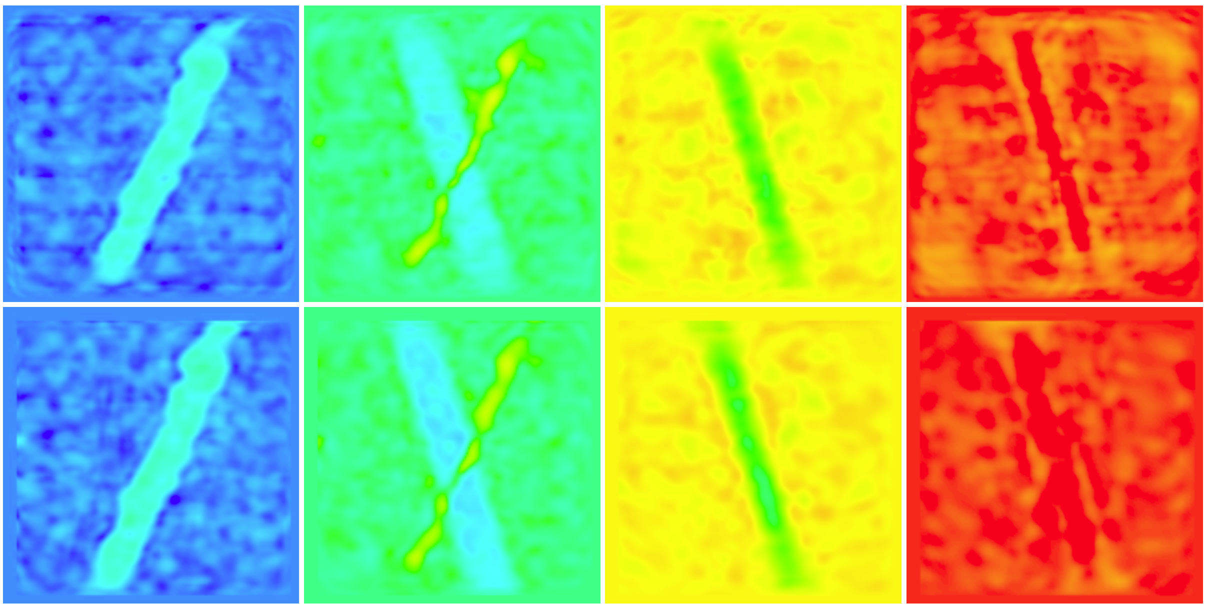

Figure 2 *Comparison of tomography for simple two-channel test-case using frequency domain (top) and time domain (bottom), showing results at same depths as figure 1.*

As can be seen, frequency and time domain are capable of producing comparable results, picking out both of the channels quite adequately at the different depths. Also, particularly towards the top of the domain (near the sources and receivers), some of the smaller perturbations to the background model can be extracted. Both show some significant shadowing below the lower channel.

Figure 3 shows the required computational time for the tomographic iterations. The three most immediate observations are: (i) the overall timing is roughly comparable, (ii) the iterations in the time domain have no significant dependence on the frequency, and (iii) the frequency domain iterations clearly become much more expensive for the highest frequency. The latter is due to the multi-grid iterative solver which allows lower frequencies to be solved, initially, on a much coarser grid, hence quickly giving a very good starting approximation for the finest mesh. The highest frequencies, though, cannot make use of a coarser grid, so take significantly longer to converge.

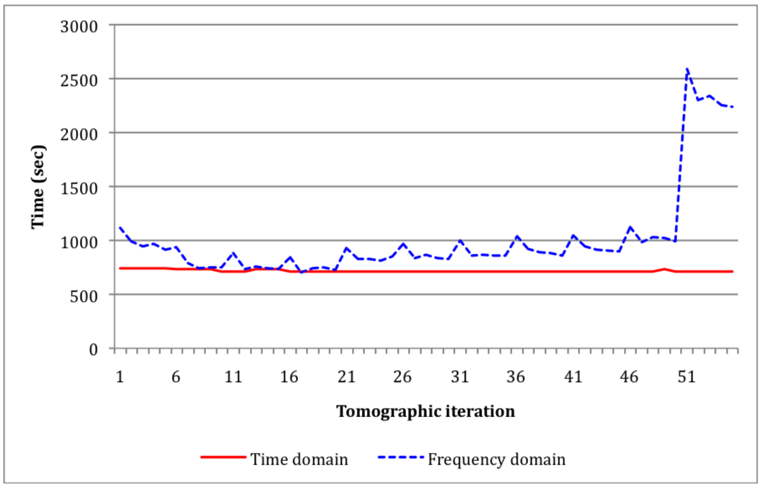

Figure 3 *Comparison of computation time in the frequency domain (dashed blue) and time domain (solid red), for each of the 55 tomographic iterations (run on a single machine with 8 cores).*

Test case 2: Marmousi extended to 2.5d

This model comprises the well-known 2d Marmousi model, but extended into the third dimension in the simplest way – often known as 2.5d. This also allows some degree of comparison between 2d and 3d tomography, for example, to investigate what extent of cross-line receivers are required to give satisfactory results (since a receiver in 2d acts rather like a long line of receivers across a 3d domain). Figure 4 shows the velocity model for a vertical slice through the domain.

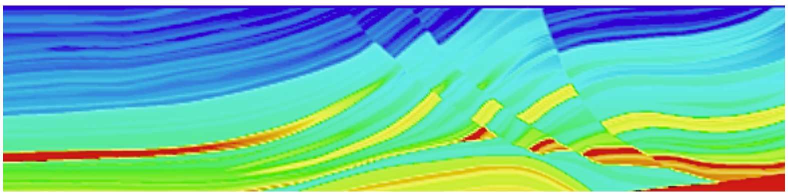

Figure 4 *A vertical slice through the 2.5d Marmousi test case*

The shot-receiver geometry used to examine this case is very simple: a line of shots across the length of the domain at the top of the model, with a band of receivers also along the domain, but with some width either side of the line of shots. The change in the width of this band, as already mentioned, allows some investigation of the receiver coverage that may be required when transitioning to a 3d domain (see fig.8).

Figures 5 to 7 compare results after 25 tomographic iterations (five at each of 4, 5, 6, 8 & 10Hz) in frequency & time domains, starting from a smoothed version of the original model. Note the banding in the frequency domain when using only 31 shots (fig.5), which becomes smoothed out with 93 shots (fig.6), but is also not present in the time domain results using 31 shots (fig.7).

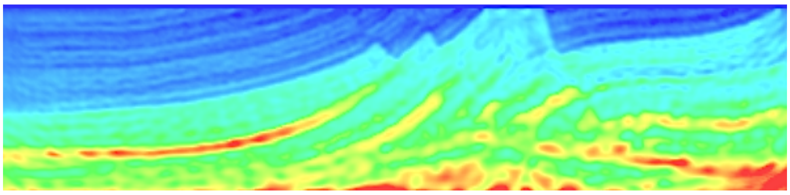

Figure 5 *A vertical slice through the frequency domain tomography results using only 31 shots*

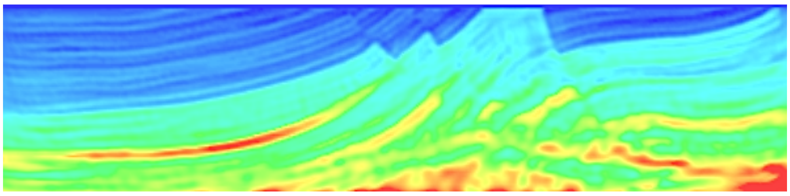

Figure 6 *A vertical slice through the frequency domain tomography results using 93 shots*

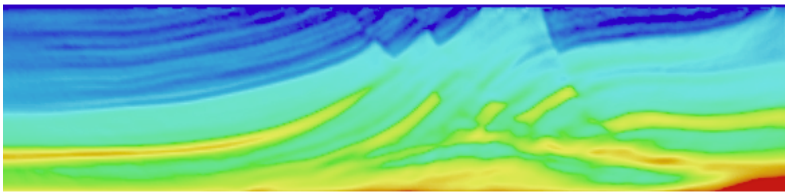

Figure 7 *A vertical slice through the time domain tomography results using only 31 shots*

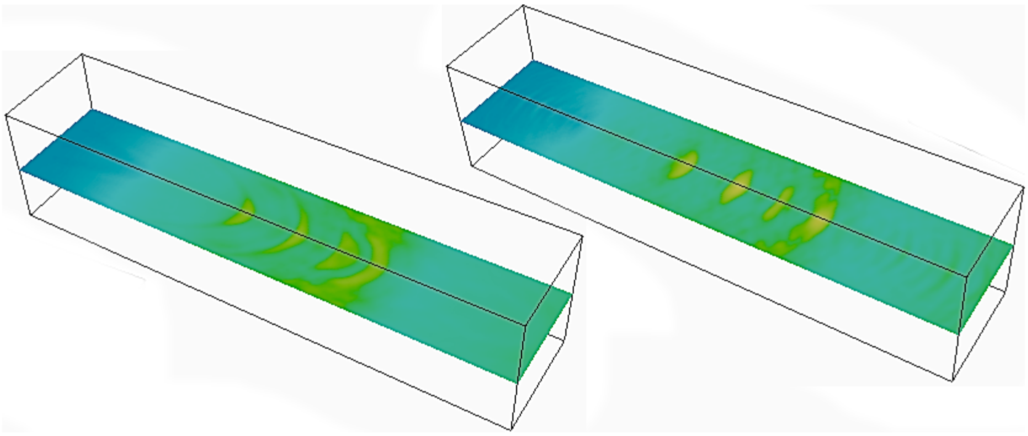

Figure 8 *Tomography with varying width of receiver band – horizontal plane cut through domain. Left: one line of receivers along domain length; right: receivers also extend across half domain width.*

The computational finite difference grid contained nearly 9 million cells (424x143x147), and the time required for the 31 shot case was about 26 hours in the time domain, and nearly 17 hours in the frequency domain, run on a small cluster of 8 nodes with dual quad-core Intel Xeons (Harpertown).

Conclusions

Frequency domain and time domain tomography in 3d are both capable and competitive, each having their own strengths that can be applicable in different situations.

Strengths of the time domain: (i) ability to mute/window or weight the data in time; (ii) much smaller memory requirements (compared to direct factorisation), or no need to balance convergence parameters (compared to iterative solver); (iii) easier to QC data during setup and tomographic run; (iv) can give better performance with shot spacings that would be inadequate for frequency domain.

Strengths of the frequency domain: (i) attenuation is straightforward to include correctly; (ii) memory overhead for detector response data is much lower (this may be significant for very large number of detectors); (iii) early iterations at lower frequencies are often quicker, so the initial direction of tomographic updates may well be seen sooner.