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A Phase-unwrapped Solution for Overcoming a Poor Starting Model in Full-wavefield Inversion

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SUMMARY

We present a new phase-unwrapped full-wavefield inversion (FWI) methodology for applying the technique to seismic data directly from a poor or simple starting model in an automated, robust manner. The well-known difficulty that arises with a poor starting model is a 'cycle-skipped' relationship between predicted and observed data at useable inversion frequencies. The local minimum convergence of cycle-skipped data is one of the root causes for inaccurately recovered models in practical applications of FWI. Further it is why practical applications to date have focussed on favourable datasets possessing very low frequencies and an accurate velocity model already known prior to applying FWI.

Here we tackle the cycle-skipping problem by inverting the lowest useable frequency of the data using an unwrapped phase-only objective function. We minimise a smooth, phase-unwrapped residual, extracted from the data by exploiting the spatial continuity existing between adjacent traces. The majority of field datasets acquired today are spatially well enough sampled to be manipulated in this way. An application to highly cycle-skipped synthetic data from the Marmousi model shows the benefit of applying phase-unwrapped inversion to a dataset prior to starting conventional FWI.

Introduction

One of the primary goals of full-wavefield seismic inversion (FWI) is to provide an accurate high-resolution model of subsurface p-wave velocity to be used within a depth-migration workflow. FWI begins from a starting model which forms an initial best estimate for the p-wave velocity parameter. This model is updated iteratively in a linearised local inversion scheme that seeks to minimise the mismatch between the modelled data that it predicts and the recorded seismic data that it seeks to explain. Where successful applications have been possible, the result has been a remarkable uplift in accuracy and resolution over the starting model and in the subsequent depth-migrated image.

Since FWI is a local inversion scheme, it will only find the global minimum misfit if the starting model is sufficiently close to the true model. The key cause of misconvergence in FWI is the presence of 'cycle-skipped' local minima surrounding the global minimum. These occur when the modelled and recorded wavefields are initially separated in time by more than half a wave-cycle at the lowest useable frequency, and through a reduction in data mismatch they end up misaligned by an integer number of cycles. This well-known problem is conventionally overcome by requiring (very) low frequencies in the field data, and by using a (very) accurate starting velocity model. This greatly limits the utility of the technique, and it affects the confidence that can be placed on the final result.

In this paper, we demonstrate a methodology that overcomes the problems of cycle skipping by inverting the data using a phase-unwrapped objective function. The scheme is straightforward to incorporate prior to conventional FWI, and may be implemented in either the time or frequency domain. The scheme is conceptually simplest in the frequency domain, and we use that domain to present it here. Our methodology can also determine whether a particular starting model and dataset are adequate for FWI, and indicate when the inversion has misconverged.

Methodology

An inversion scheme is characterised by the objective function E that it seeks to minimise when it updates the starting model. In conventional, frequency-domain, single-frequency, least-squares FWI, the objective function utilises the L_2 norm of the wavefield residual as the measure of data mismatch:

$$E = \sum_{s,r} | u(s,r) - d(s,r) |^2 \tag{1}$$

where u and d are the predicted and observed Fourier-transformed wavefields respectively. Minimising this objective function, after normalising amplitudes, will shift the phase of u to the phase value of d occurring within the same 2π wave cycle. We note that this implies that the quantity being minimised is the principal value of the phase residual between u and d. This quantity, lying between $-\pi$ and $+\pi$, may be written as:

$$\varphi = \text{phase}\,(ud^*) \tag{2}$$

The value of φ is controlled by the arrivals that dominate the data at the given frequency. Conventional FWI is based on the assumption that, at the lowest inversion frequency, these arrivals are not cycle-skipped.

In Figure 1(a) we show phase residual φ as a function of source and receiver position for a single moderately low frequency (3.125Hz) calculated by generating "observed" data through the 2D Marmousi model, and "predicted" data using a simple 1D starting model. For these data, φ takes a value close to zero at zero-offset, and its magnitude generally increases with offset. Sharp boundaries between red and blue correspond to places where the magnitude of φ exceeds π; at these boundaries the data are cycle skipped, and beyond them FWI will change the model in the wrong direction. If we were able to unwrap the phase, then in principle, we could define an objective function that was not cycle skipped, and begin FWI much further from the true model. That is, we wish to derive and invert for a new parameter φ' expressed as:

$$\varphi' = \text{phase}(ud^*) + 2n\pi \qquad (3)$$

where n is the number of cycle-skips between the predicted and observed data. However, Figure 1(a) also demonstrates that phase unwrapping is not a clear-cut procedure. Consider the points marked 1 and 2. If we attempt to unwrap the phase between them, in one dimension, following path A, then we will remove a phase jump of 2π as we cross the cycle-skip boundary. But if we follow path B, then we will not cross this boundary, and we will reach point 2 without having removed a jump of 2π. Phase unwrapping therefore does not appear to be a well-defined operation. In 3D data, the problem becomes more acute, as the phase is now a four-dimensional function of source and receiver x and y positions, and there are now many paths between two points. This behaviour prevents us from counting sudden 2π transitions to determine the value of n even in noise-free data.

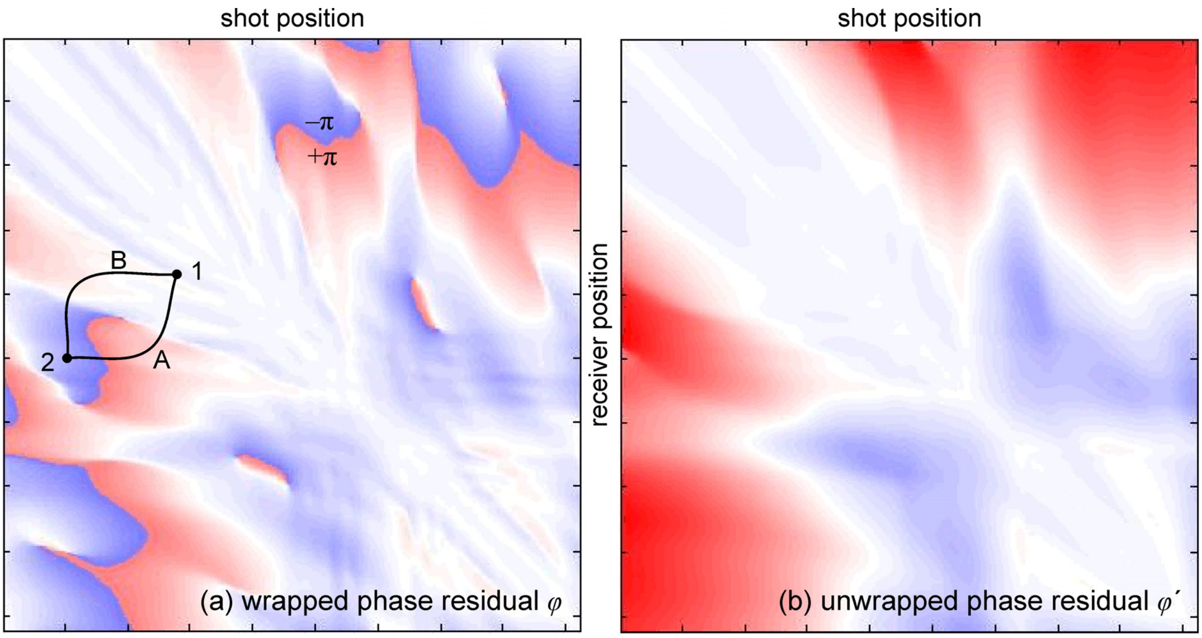

Figure 1 *(a) Wrapped phase residual φ, and (b) unwrapped phase residual φ', for a single low-frequency calculated from the Marmousi model and the simple 1D starting model shown in Figure 2.*

To unwrap φ, we instead specify a system of equations that φ' should satisfy for spatial continuity, and the resulting unwrapped phase residual follows the dominant spatially-coherent arrivals at the given frequency through space. Spatial unwrapping works as follows: pairs of spatially adjacent points are defined which cover the data space. We define a parameter g to be the principal value of the difference in φ between any pair of adjacent points. Then to determine the unwrapped phase residual φ' we must solve the over-determined set of linear equations:

$$\varphi'(s_i, r_i) - \varphi'(s_j, r_j) = g \qquad (4)$$

where s and r are source and receiver numbers, and i and j represent any pair of adjacent points.

We can solve this linear system, including various regularisation and preconditioning, to obtain a least-squares solution that is continuous in space. This smooth, unwrapped phase residual is shown in Figure 1(b). This plot shows the phase residual over a range that is larger than ±π, and it contains no cycle-skipped boundaries or other anomalies; it represents a smooth version of Figure 1(a) with cycle skipping removed. Now to invert φ', we use the phase-only objective function outlined by Shin and Min (2006). This separates phase completely from amplitude, and may be used to invert either wrapped or unwrapped phase. Our new objective function is thus simply:

$$E = \sum_{s,r} \varphi'(s,r)^2 \qquad (5)$$

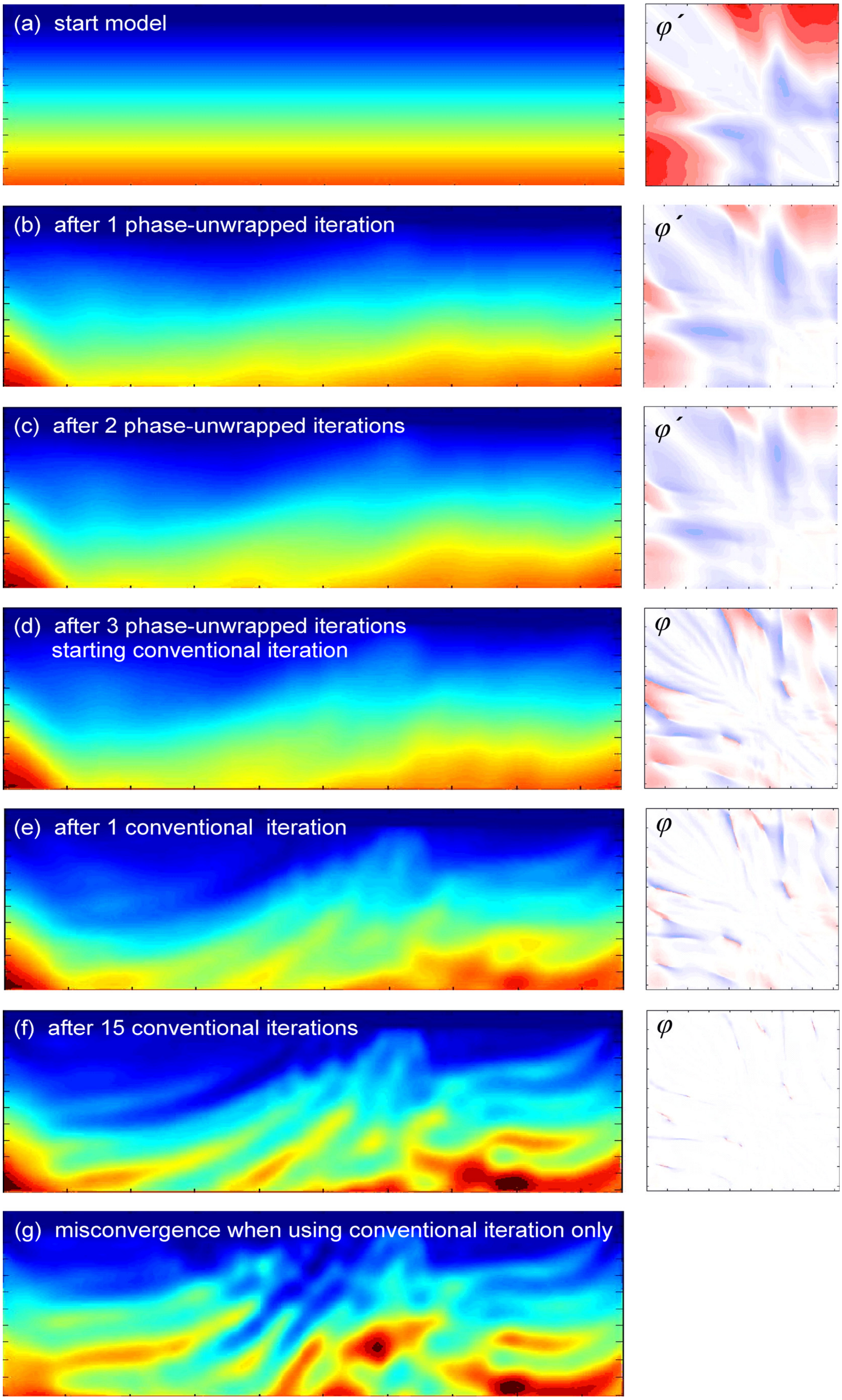

Figure 2 *FWI velocity models and corresponding wrapped φ and unwrapped φ' phase residuals.*

To invert data using a poor starting model, without low frequencies, we do the following. We calculate the forward wavefield from the starting model at the lowest useable frequency in the data. We use that to calculate the principal value φ of the phase residual for all pairs of sources and receivers. We use φ to calculate g as a function of source and receiver number, and set up the system of linear equations, given in (4). We precondition and/or regularise these equations, and solve for φ' with any standard least-squares technique – the matrix that we solve has only two elements per row, so the system is extremely sparse. We then run FWI using an unwrapped phase-only objective function for several iterations. As FWI proceeds, the value of φ' decreases, and after some iterations, φ is no longer cycle skipped. At that point, model updates generated by φ and φ' will be essentially the same, and we move to conventional inversion using a conventional data residual.

Results

Figure 2 shows the scheme in action. Here we invert data from the well-understood 2D Marmousi model starting from a very poor initial velocity model from which conventional FWI cannot proceed. Figures 2(a) – (d) show the scheme using unwrapped phase. At this stage, the data are significantly cycle skipped, but the inversion still heads in the right direction towards the global minimum.

By Figure 2(d), the data are no longer cycle skipped. There are still however phase discontinuities in visible in Figure 2(d); these relate to reflected energy that is not yet being generated by the inverted model, they do not represent orthodox cycle skipping. In Figure 2(e), the scheme moves to a conventional one, and proceeds through further iterations to improve the model. The final result shows a model that is globally converged for this single, low-frequency inversion. If we subsequently move on to higher frequencies, then the model will continue to improve, and there is no need to unwrap the phase again at these higher frequencies.

In contrast to the sequence in Figure 2(a) to (f), Figure 2(g) shows how conventional FWI fails to invert the same data starting from the same model. In a conventional scheme, the very first iteration heads in the wrong direction, and the final model contains strong artefacts from which no amount of iteration or data massage can rescue it.

Conclusions

We present a new phase-unwrapped FWI methodology for applying the technique to seismic data directly from a poor or simple starting model in an automated, robust manner. A poor starting model gives rise to cycle-skipped data and the local minimum convergence of cycle-skipped data is one of the root causes for inaccurately recovered models in practical applications of FWI.

Here we avert the problem of cycle-skipped local minima by inverting the lowest useable frequency of the data using an unwrapped phase-only objective function. We minimise a smooth, phase-unwrapped residual, extracted from the data by exploiting the spatial continuity existing between adjacent traces. The majority of field datasets acquired today are spatially well enough sampled to be manipulated in this way.

In the synthetic example given, the phase-unwrapped inversion of cycle-skipped data heads in the right direction improving a very poor starting model. After several iterations the updated model is sufficiently accurate to be used as a starting model for conventional FWI with a conventional objective function. The phase-unwrapped iterations are rigorously quality-checked by ensuring the unwrapped residual is being reduced towards zero consistently across space.

References

Shin, C. and Min, D. [2006] Waveform Inversion using a logarithmic wavefield. *Geophysics*, **71**, R31-R42.