# Proof of Proposition 1

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**Proposition 1:**
*If for one box $B$ in mass space we can prove that there are at most $n$ solutions, using only the criteria "no solutions" and "at most one solution". And if for one choice of the masses $m_0 \in B$ we can prove that there are exactly $n$ solutions. Then for all choices of masses in $B$ we also have exactly $n$ solutions.*

**Idea of the proof:**

When the program ends, it outputs a subdivision of the real space into boxes with no zero and boxes with at most one zero. By assumptions, we know that there are only such boxes (no boxes with at most two or three zeros), and we know that we have $n$ boxes with at most one solution. If we take one such box $C$ in real space where there is at most one zero, we know that for $m_0$ there will be exactly one zero in that box, because we only have $n$ such boxes and $m_0$ has $n$ solutions. We want to prove that for all masses in $B$ we will also have exactly one zero in $C$.

We also know that $g'$ is non zero on this box, which means that we cannot have zeros of multiplicity higher than one. So for $m_0$, the curves $f_1 = 0$ and $f_2 = 0$ should look like on the left of figure 1.

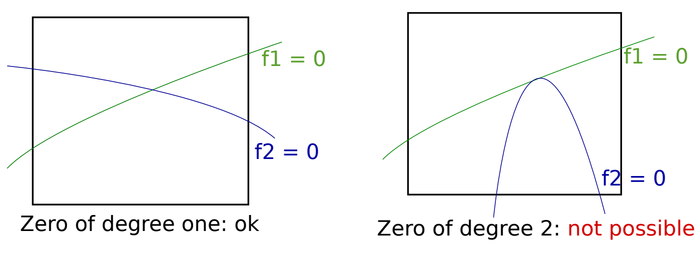


Figure 1: Curves $f_1 = 0$ and $f_2 = 0$ in a box with $g'$ non zero

Moreover, $f_1$ and $f_2$ are continuous in $m$, so the lines $f_1 = 0$ and $f_2 = 0$ move continuously when $m$ varies. If we had $m_1$ in $B$ with no zero in $C$, we would have to go from the situation on the left of figure 2 to the situation on the right. This means that at one point $m$ between $m_0$ and $m_1$ we are in the central situation ($g'$ is non zero on the closed box $C$ so it is also non zero on a bigger open box $C'$ containing $C$).

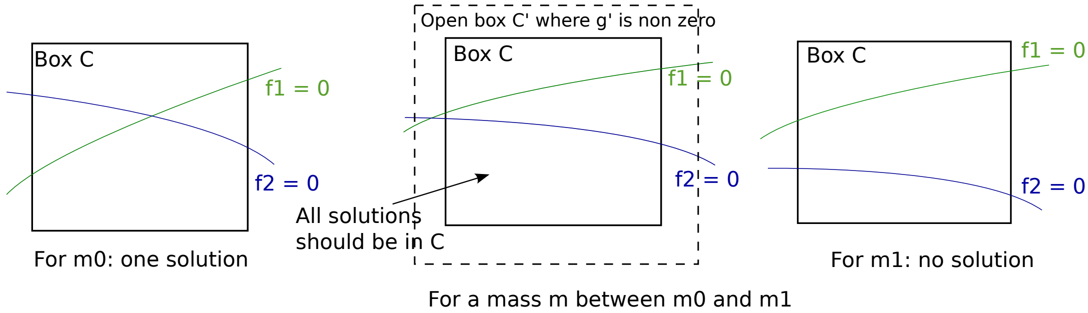


Figure 2: What happens between $m_0$ and $m_1$

But this situation is impossible because we know that the solution can only be inside $C$ (the boxes containing at most one solution are isolated, so there is no solution outside the box if we are close enough). We can then conclude that for all masses in $B$ the curves $f_1 = 0$ and $f_2 = 0$ cross in $C$, which means that we have exactly one solution in $C$ for all masses, and so $n$ solutions over $\mathbb{R}^2$.