Proof of Proposition 1

Alice Pellet-Mary

August 9, 2014

Proposition 1:
If for one box B in mass space we can prove that there are at most n solutions, using only the criteria "no solutions" and "at most one solution". And if for one choice of the masses $m_0 \in B$ we can prove that there are exactly n solutions. Then for all choices of masses in B we also have exactly n solutions.

Idea of the proof:

If we take one box C in real space where there is at most one zero, we know that for m_0 there will be exactly one zero in that box, because we only have n such boxes and m_0 has n solutions. We also know that g' is non zero on this box, which means that we cannot have zeros of multiplicity higher than one. So for m_0, the curves $f_1 = 0$ and $f_2 = 0$ should look like on the left of figure 1.

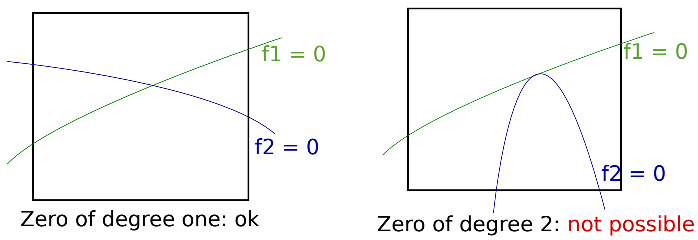

Figure 1: Curves $f_1 = 0$ and $f_2 = 0$ in a box with g' non zero

Moreover, f_1 and f_2 are continuous in m, so the lines $f_1 = 0$ and $f_2 = 0$ move continuously when m varies. If we had m_1 in B with no zero in C, we would have to go from the situation on the left of figure 2 to the situation on the right. This means that at one point m between m_0 and m_1 we are in the central situation (g' is non zero on the closed box C so it is also non zero on a bigger open box C' containing C).

But this situation is impossible because we know that the solution can only be inside C (the boxes containing at most one solution are isolated, so there is no

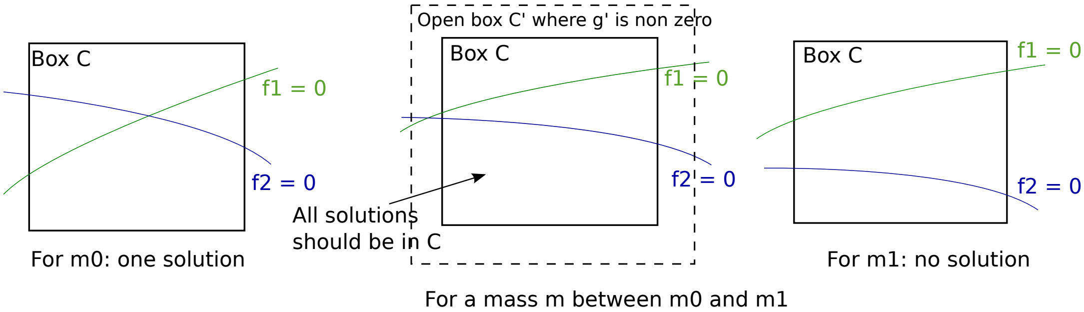

Figure 2: What happens between m_0 and m_1

solution outside the box if we are close enough). We can then conclude that for all masses in B the curves $f_1 = 0$ and $f_2 = 0$ cross in C, which means that we have exactly one solution in C for all masses, and so n solutions over $\mathbb{R}^2$.