EED 3016 CONTROL SYSTEMS

Assoc.Dr. Güleser K Demir
Assist. Dr. Hatice Doğan

CONTROL	SYSTEM	ENGINEER
Regulates a mechanical or scientific process	is a combination of components that act together and perform a certain objective.	understands and controls the materials and forces of nature for the benefit of humankind

A control system is an interconnection of components forming a system configuration that will **provide a desired system response.**

Control system engineers are **concerned with understanding and controlling** segments of their environment, often called **systems**, to provide useful economic products for society.

1. INTRODUCTION

Control applications are all around us:

- A self-guided vehicle
- The rockets fire
- Robotic arms
- Air-conditioning systems
- …
- …

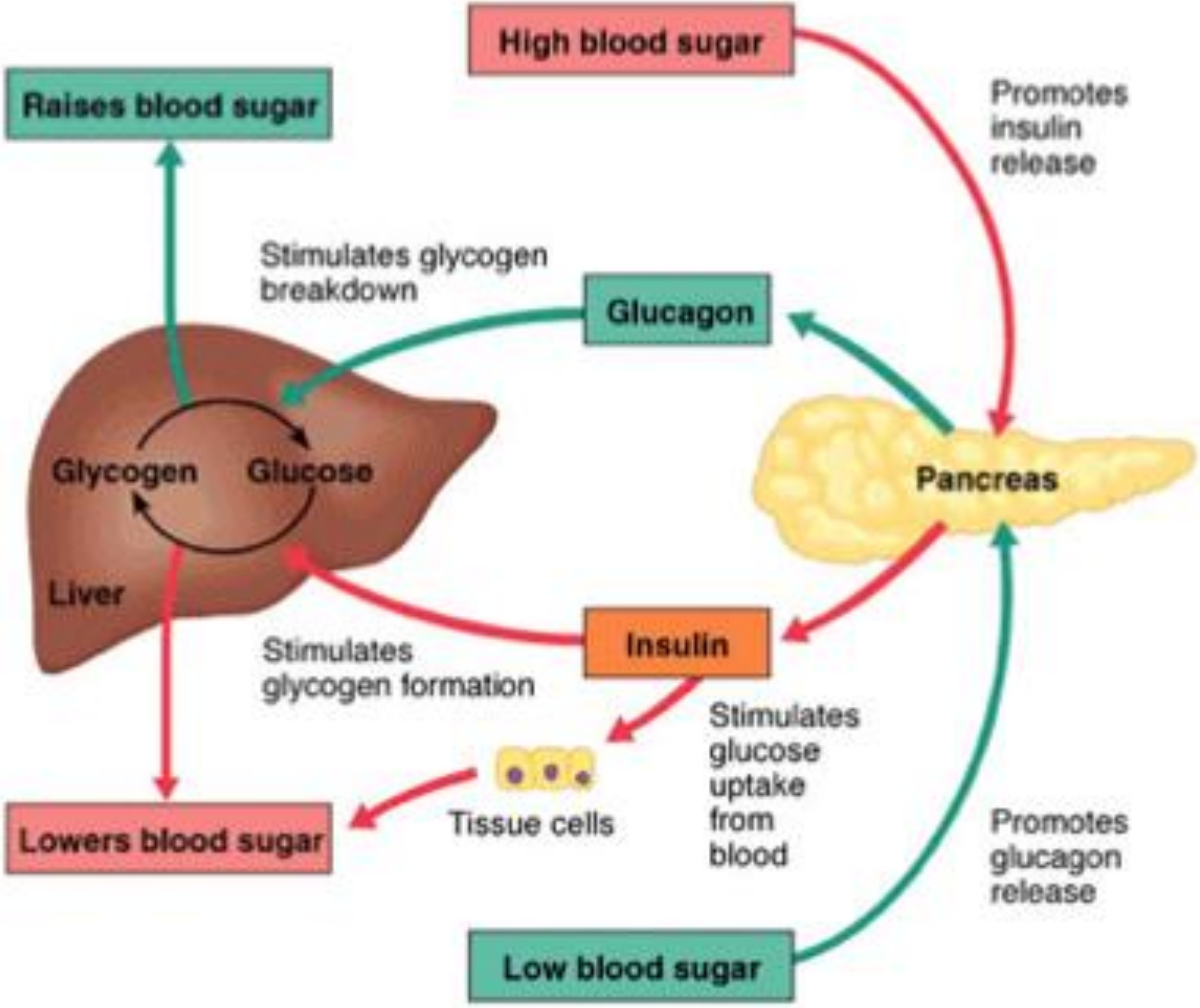
Automatically controlled systems that we can create

1. INTRODUCTION

Control applications also exist in nature:

- Regulating blood sugar by the pancreas
- Tracking moving objects by our eyes
- Adjusting our heart rate by increasing adrenaline in time of fight
- …

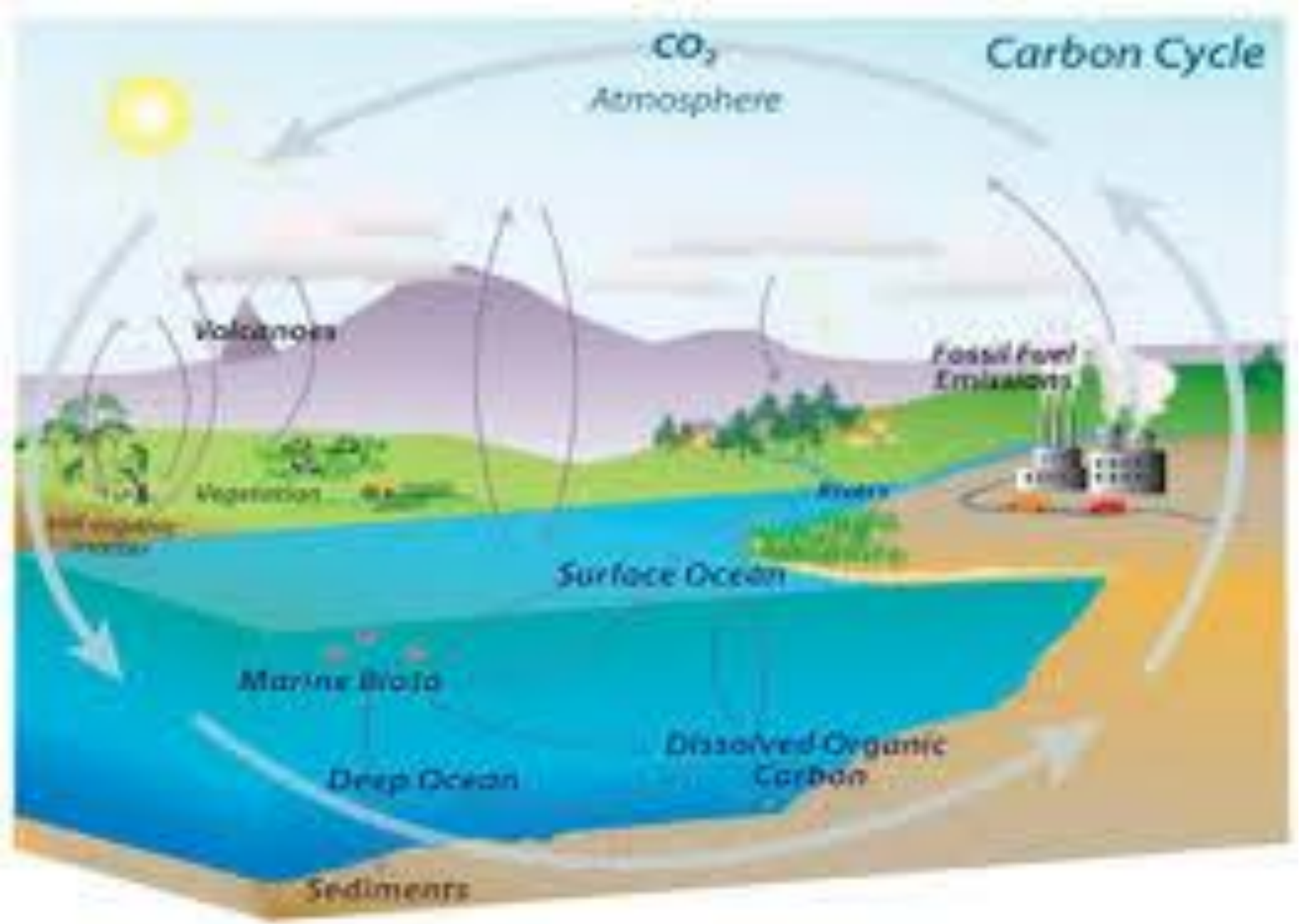
Automatically controlled systems in our own bodies



1. INTRODUCTION

Even the nonphysical world appears to be automatically regulated.

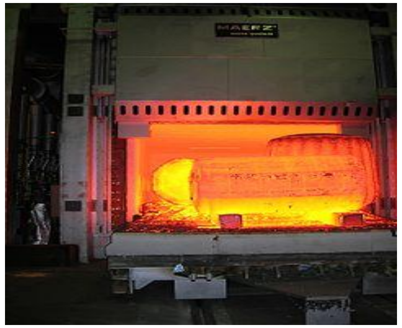
- In quality evaluation services
- In environmental systems such as global carbon cycle or microbial ecosystems
- In financial systems such as markets, supply and service chains
- …



CONTROL SYSTEM DEFINITION

A control system consists of subsystems and processes (or plants) assembled for the purpose of controlling the outputs of the processes.

PLANT: Furnace

Flow of fuel →  Heat →

Aim: Regulating the temperature of a room by controlling the heat output from the furnace

Subsystems: Fuel valves, Fuel-valve actuators, Thermostats which act as sensors, measure the room temperature

Input; Desired response :
Desired room temperature → [valves | controller | thermostats | furnace] → Output; Response :
Actual room temperature

Input; stimulus / Desired response → Control system → Output; response / Actual response

A control system provides an output or response for a given input or stimulus

ADVANTAGES OF CONTROL SYSTEMS

Power amplification

- For example, a radar antenna, positioned by the low-power rotation of a knob at the input, requires a large amount of power for its output rotation.

Remote control

- For example, a remote controlled robot arm can be used to pick up material in a radioactive environments.

Convenience of input form

- For example, in a temperature control system, the input is a position on a thermostat. the output is heat. Thus, a convenient position input yields a desired thermal output.

Compensation for disturbances.

- The system must be able to yield the correct output even with a disturbance. For example, if wind forces the antenna from its commanded position, the system must be able to detect the disturbance and correct the antenna’s position.

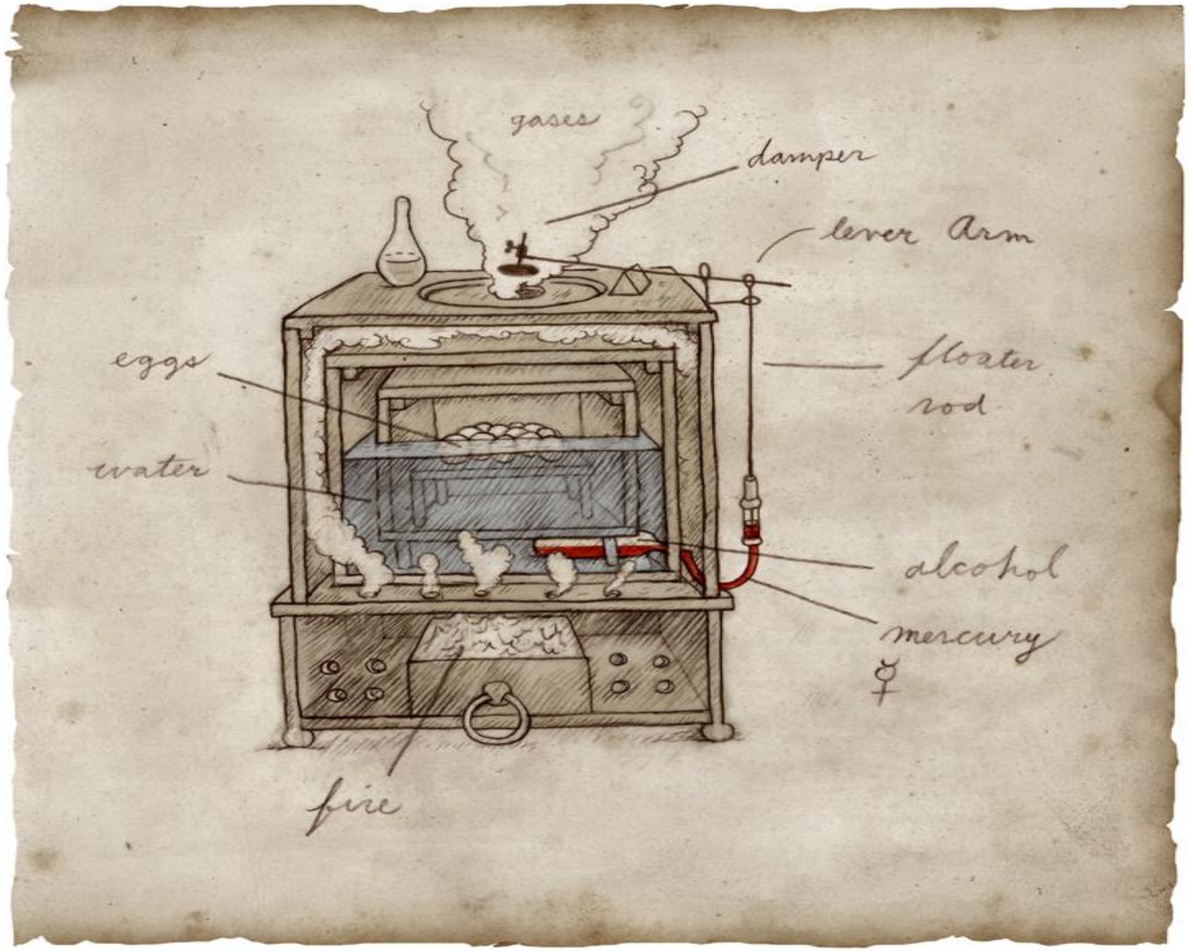
1.2 A HISTORY OF CONTROL SYSTEMS

Liquid-level control

- Around 300 B.C. … Water clocks, oil lamp, etc.

Steam pressure and temperature controls

- Regulation of steam pressure began around 1681 with Papin's invention of the safety valve.
- Also in the 17 th century Drebbel invented a purely mechanical temperature control system for hatching eggs.



CORNELIS DREBBEL'S OVEN:

In this artist's rendering, Drebbel's circulating oven incubates eggs. The oven consists of three nested metal boxes atop an enclosed fire. Fumes from the fire rise up through vents into the outer box, and out of an opening at its top. The alchemist's early thermostat sits in the water just touching the center box. It is an L-shaped glass tube filled with alcohol and topped off by mercury. A metal rod floats in the quicksilver. When the heated alcohol expands, it pushes up the quicksilver, and the rod rises in the tube. If the heat of the fire rises beyond a set point, the floating rod elevates high enough to lower the damper, cutting off the air that feeds the fire. The fire would then diminish, as would the temperature in the center box. If the heat falls too far, so does the rod, with its lever opening the vent to admit air.

Water Level Float Regulator

- 1765 Polzunov…

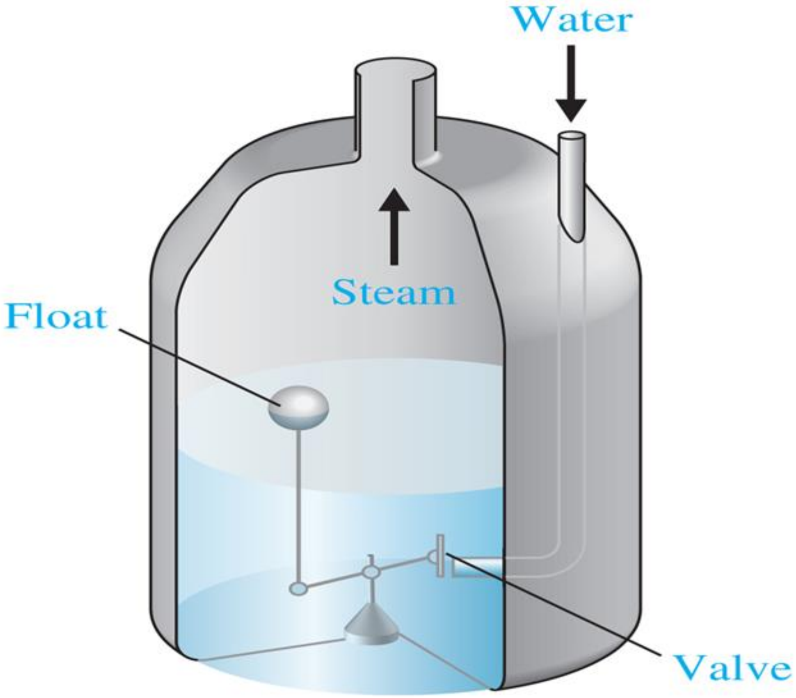
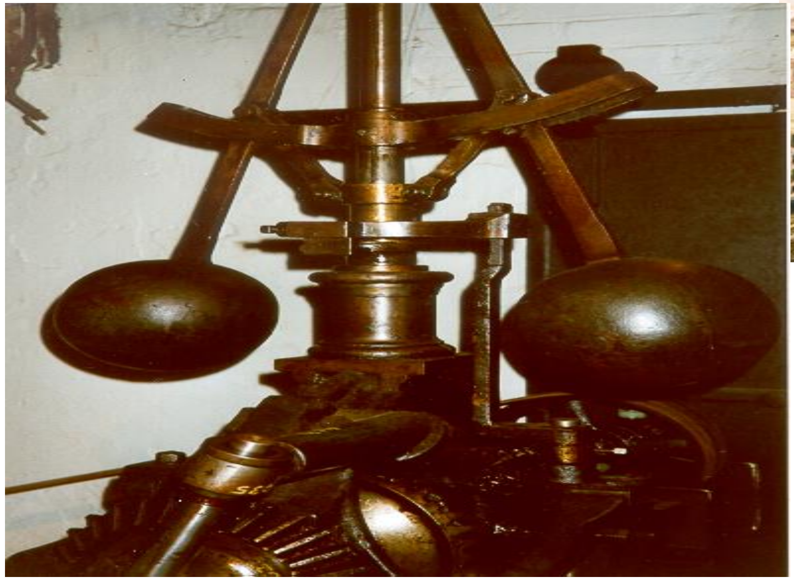
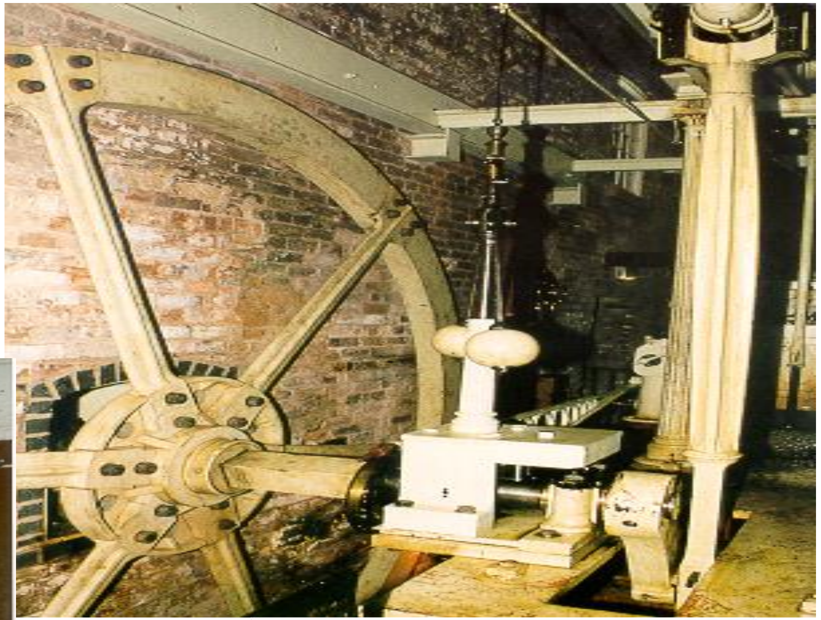
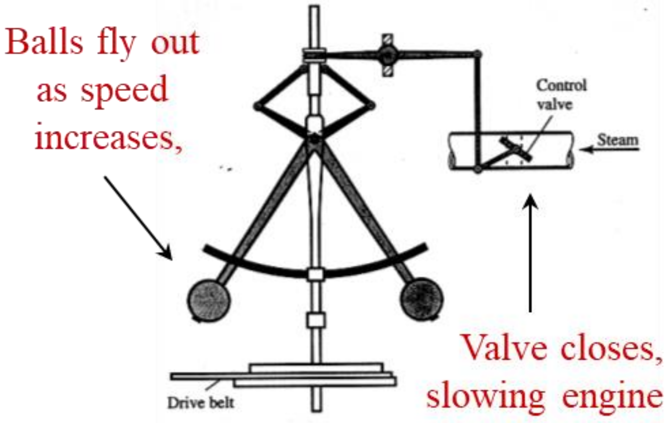


Figure: 01-06

Steam Engine







Flyball governor used on a steam engine in a cotton factory near Manchester. It regulates the speed of a water wheel driven by the flow of the river.

Invented! by J. Watt 1788 to reject the load disturbances also

STEAM ENGINE
J. WATT 1736-1819

- 1769 James Watt's Steam Engine and Governor

- 1868 J. Clerk Maxwell formulates a mathematical model for governor control of a steam engine

- 1927 Harold Black discovers and patents the feedback amplifier

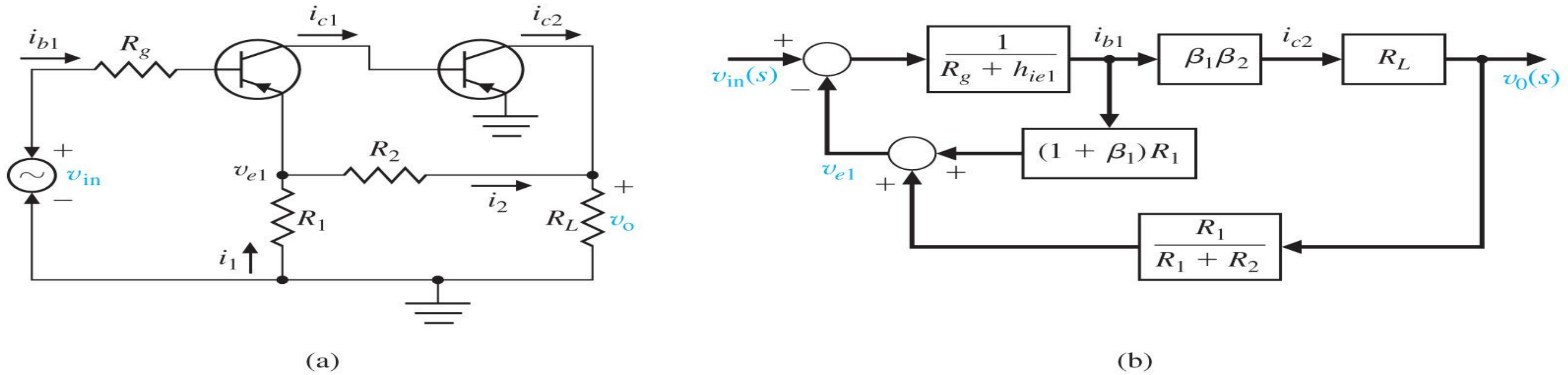
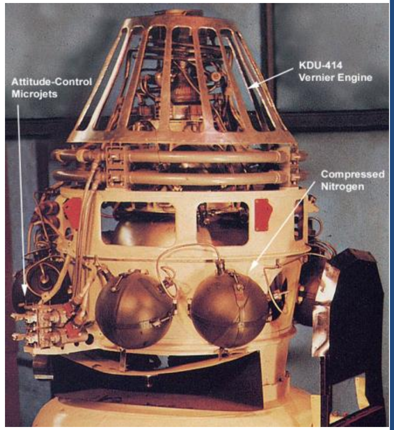


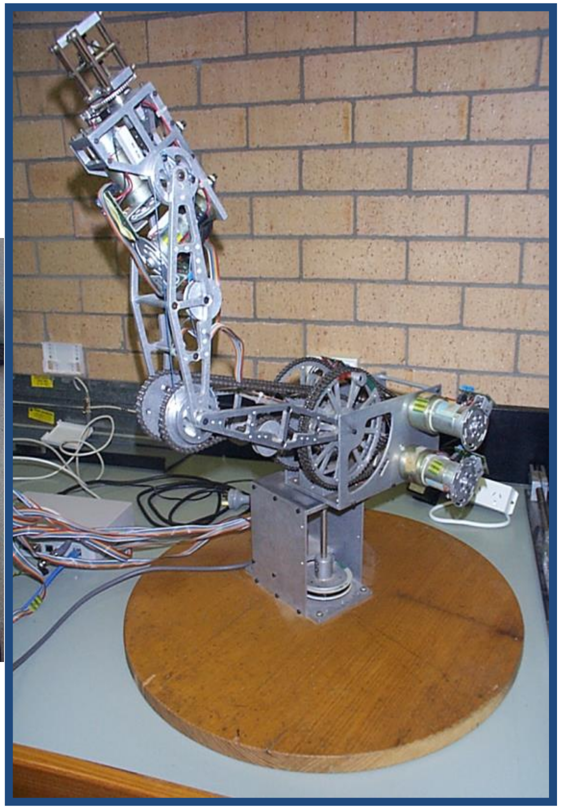
Figure: 02-74-41abP2.24

- 1927 Hendrik Bode analyzes feedback amplifiers

- 1932 Nyquist develops methods for analyzing feedback amplifier stability

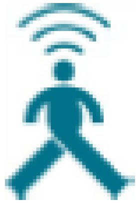
- 1940s Norbert Wiener leads gun positioning effort; becomes an engineering discipline
- 1950s Increased use of Laplace transform, s-plane, root locus
- 1960s Sputnik, highly accurate control systems for space vehicles, missiles and robotics





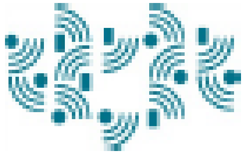
- 1980s Routine use of digital computers as control elements
- 1990s Feedback control in automobiles, automation, planetary exploration

Modern Control Application Areas



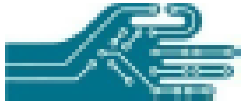
Mobile Internet

Automation of knowledge work

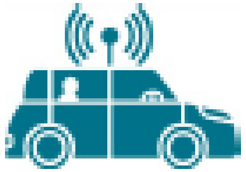


The Internet of Things

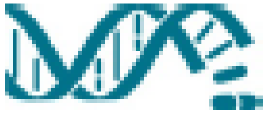
Cloud technology



Advanced robotics



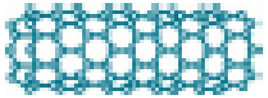
Autonomous and near-autonomous vehicles



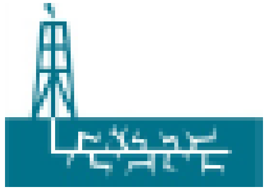
Next-generation genomics

Energy storage

3D printing



Advanced materials



Advanced oil and gas exploration and recovery

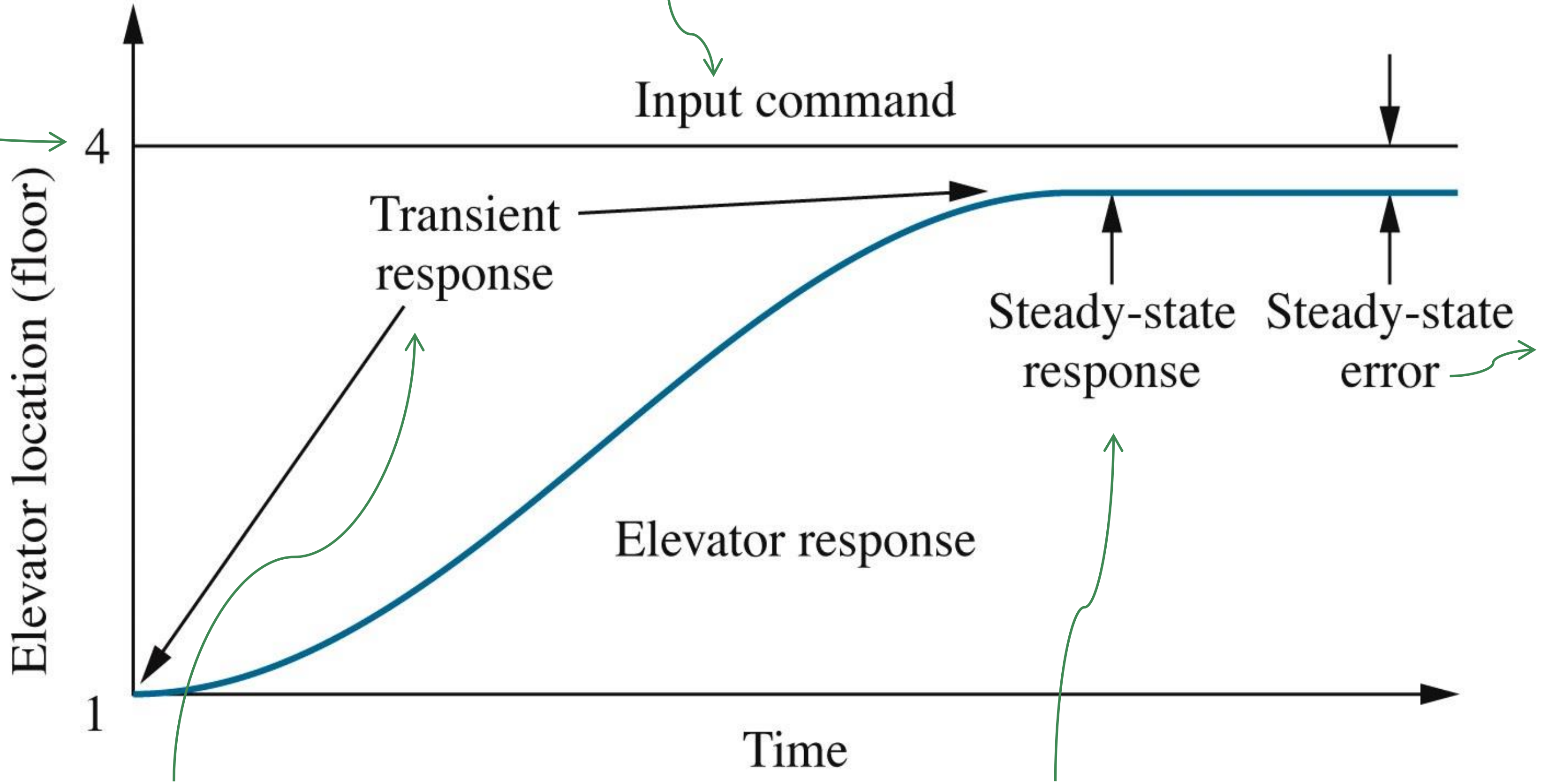
Renewable energy

SOURCE: McKinsey Global Institute analysis

1.3 RESPONSE CHARACTERISTICS

Represents what we would like the output to be after the elevator has stopped. (desired output)

Fourth button of an elevator is pushed



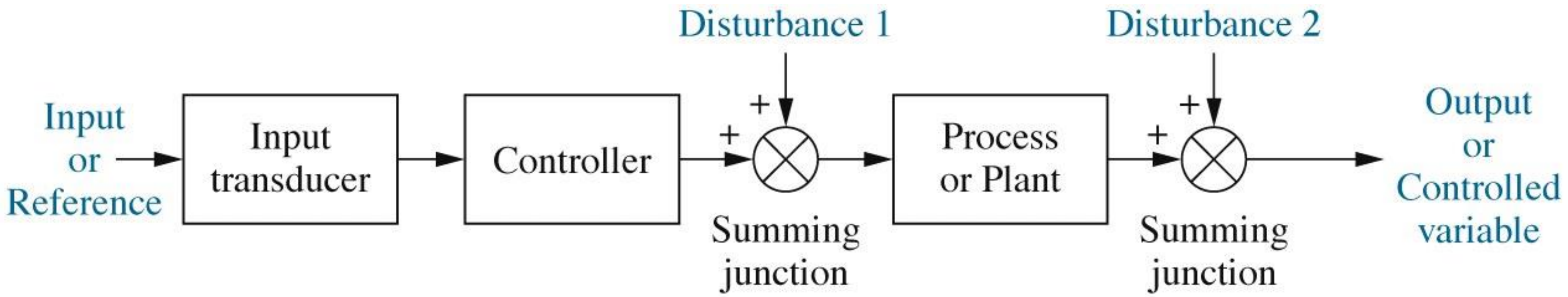
The difference between input and actual output is steady-state error.

Physical entities cannot change their state (such as position or velocity) instantaneously. Thus, the elevator location changes gradually.

After the transient response, a physical system approaches its steady-state response, which is its approximation to the desired response.

OPEN LOOP SYSTEMS

Disturbance means the interruption of a settled and peaceful condition.

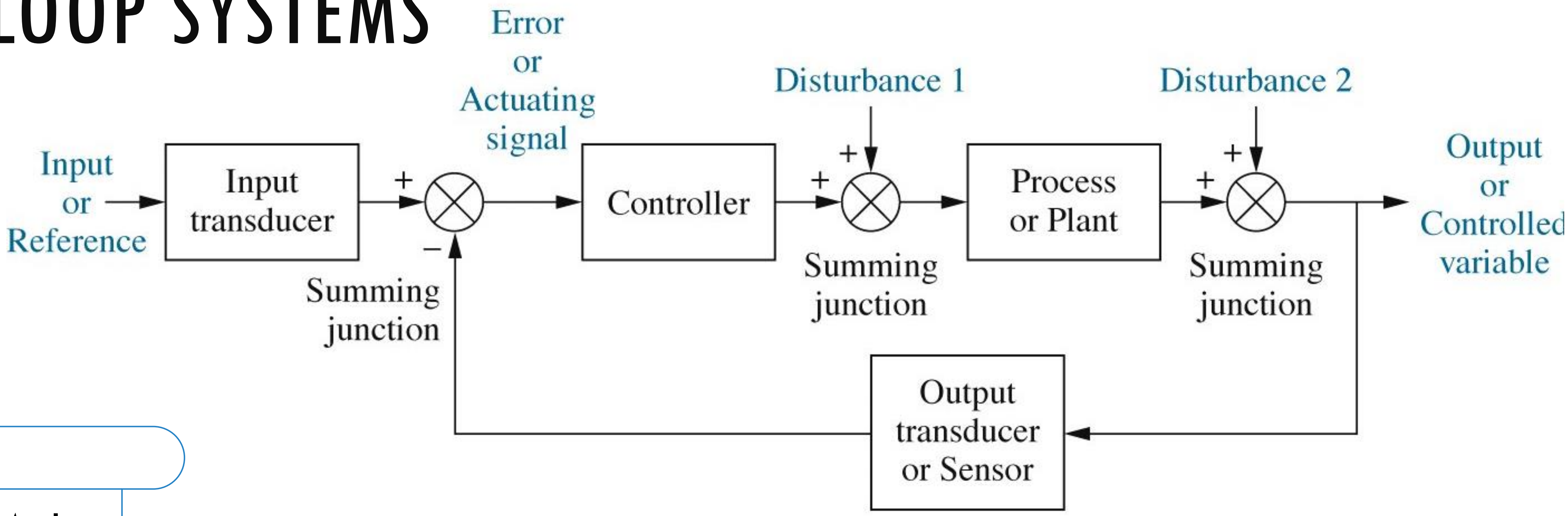


Open-loop systems do not correct for disturbances and are simply commanded by the input.

An example of Open-loop system: Toaster

The device is designed with the assumption that the toast will be darker the longer it is subjected to the heat. The toaster does not measure the color of the toast; it works only for a pre-specified duration. It does not correct/change the duration of heat for the fact that the toast is rye, white or sourdough, nor does it correct for the fact that toast comes in different thicknesses.

CLOSED LOOP SYSTEMS

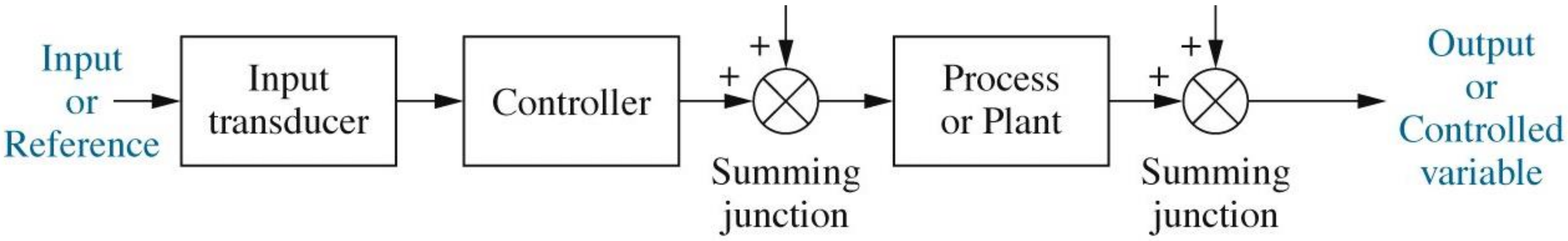


<u>Objective:</u> Make the system output and reference(/input) as close as possible, i.e. make the error as small as possible

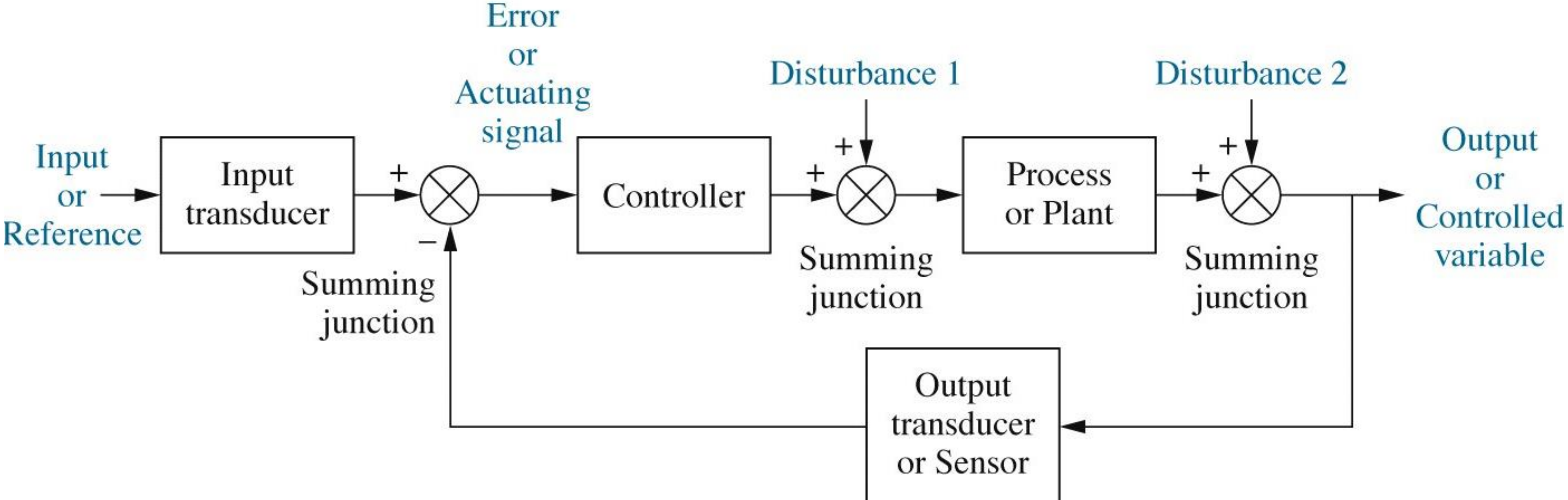
Closed-loop toaster system

It is more complex and expensive than open loop toaster systems. The device has to measure both color (through light reflectivity) and humidity inside the toaster oven. The heat duration is adjusted automatically for different type of breads.

<u>Morphy Richards Toaster (The browning control)</u>



(*a*)



Systems that perform measurement and correction are called closed-loop, feedback control systems.

Systems that do not have this property of measurement and correction are called open-loop systems.

CLOSED LOOP SYSTEMS VS OPEN LOOP SYSTEMS

- Higher accuracy
- Less sensitive to noise, disturbances, and changes in the environment
- Transient response and steady state error can be controlled more conveniently
- More complex
- Expensive

- Simple
- Inexpensive
- Disturbances cause errors, and the output may be different from what is desired.
- Recalibration is necessary from time to time.

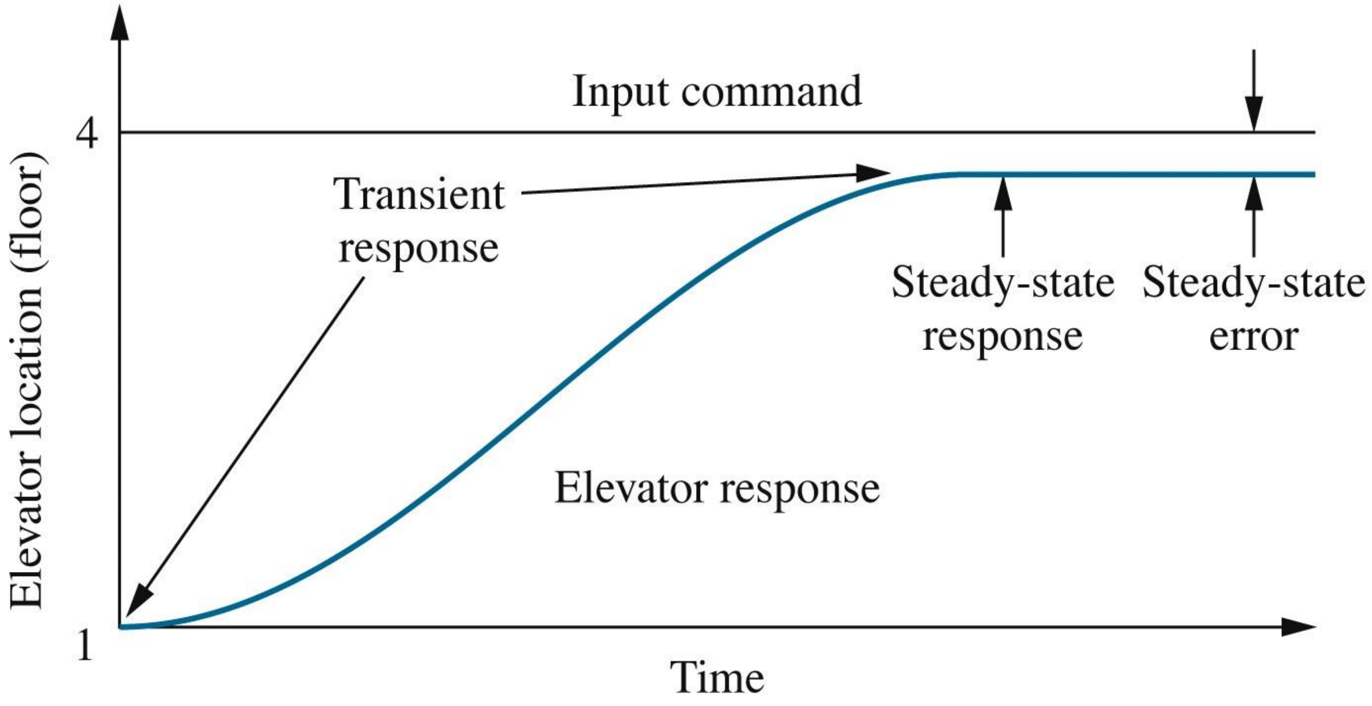
ANALYSIS AND DESIGN OBJECTIVES

What are our focal points in analysis and design of control system?

1. Producing the desired transient response
2. Reducing steady-state error
3. Achieving stability
4. Satisfying other design considerations such as cost, sensitivity, etc.

ANALYSIS AND DESIGN OBJECTIVES

1. Producing the desired transient response



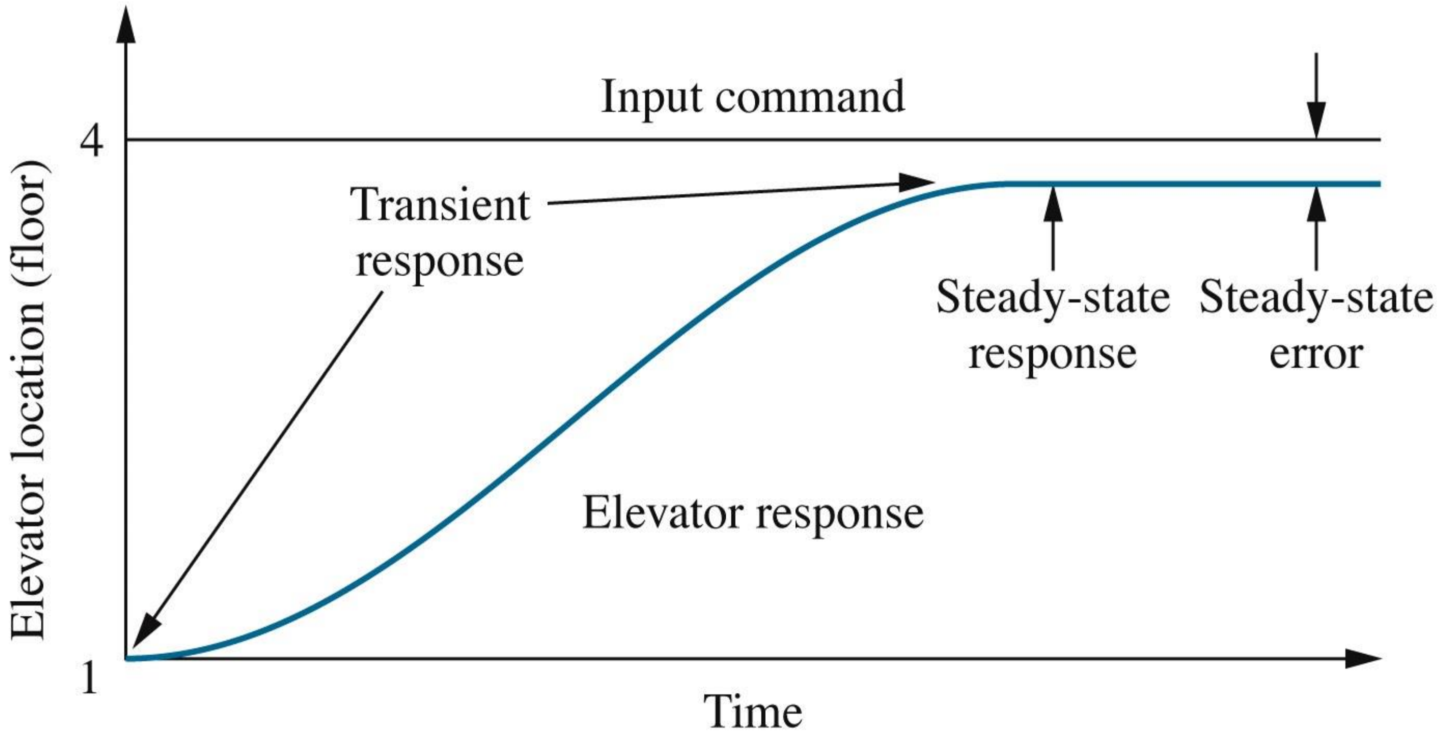
Transient response is also important for structural reasons: Too fast response can cause permanent physical damage.

In an elevator example;

- Slow transient response → passengers impatient
- Rapid transient response → passengers uncomfortable
- Oscillation at the 4th floor more than a second → disconcerting feeling

ANALYSIS AND DESIGN OBJECTIVES

2. Reducing steady-state error



In an elevator example;

- The steady state response → An elevator stopped near 4th floor.
- Our concern → The accuracy of the steady-state response
- An adequate accuracy (low error) → easy exit for the passengers

ANALYSIS AND DESIGN OBJECTIVES

3. Achieving Stability

Total response= Natural response + Forced response

Homogeneous solution.
Depends only on the system,
not the input.

Particular solution.
Depends on the input.

For a control system to be useful → the natural response must
(i) eventually approach zero, or
(ii) oscillate.

If the natural response grows without bound → the system
cannot be controlled,
is called **instable** and
can self-destruct itself (physical damage).

In an elevator example; the elevator would crash through the floor or exit through the ceiling!

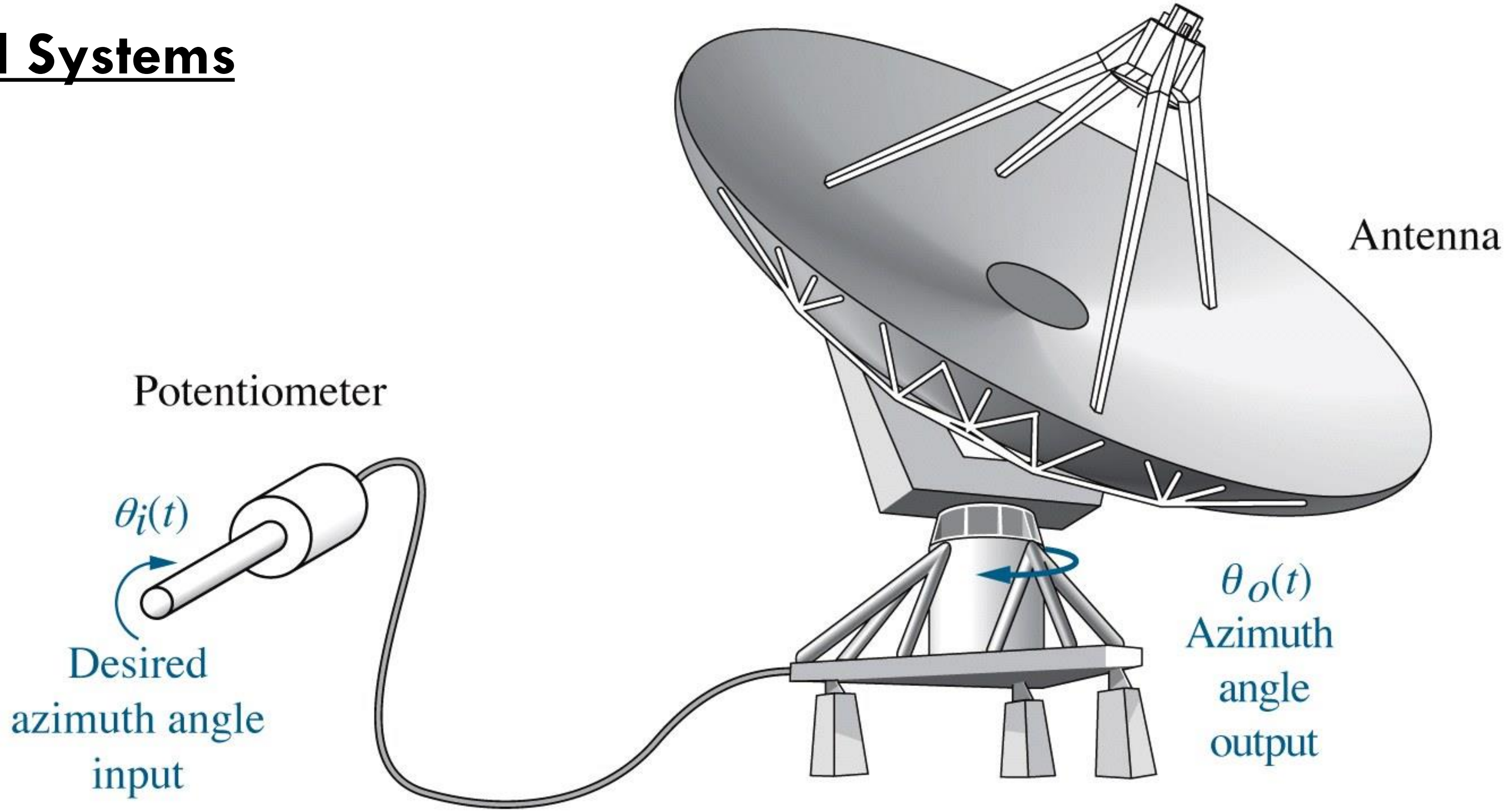
ANALYSIS AND DESIGN OBJECTIVES

4. Other considerations

- Factors affecting hardware selection (power requirements, sensors for accuracies, etc.)
- Finance
- Robustness to system parameter changes

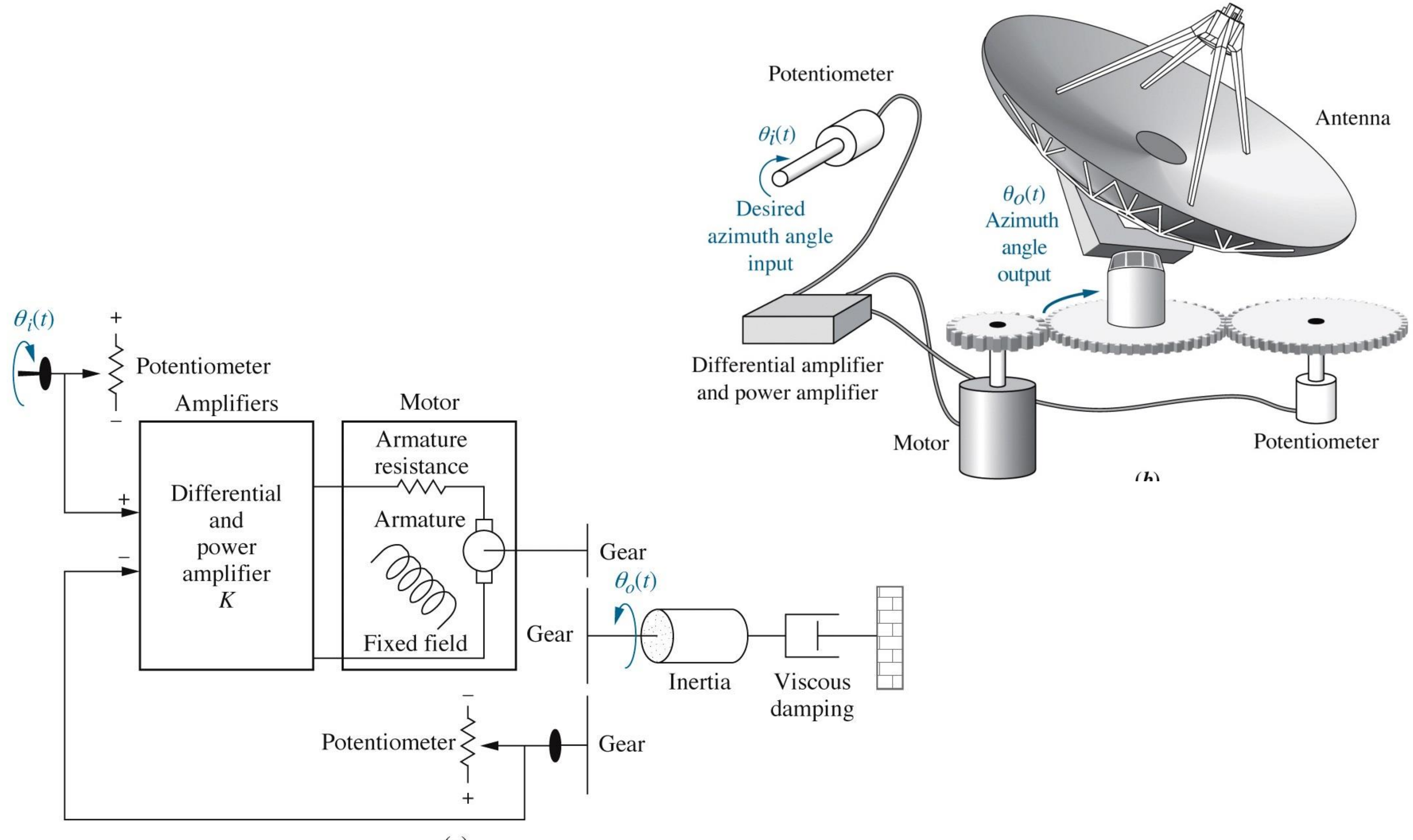
EXAMPLE: ANTENNA AZIMUTH

An Introduction to Position Control Systems

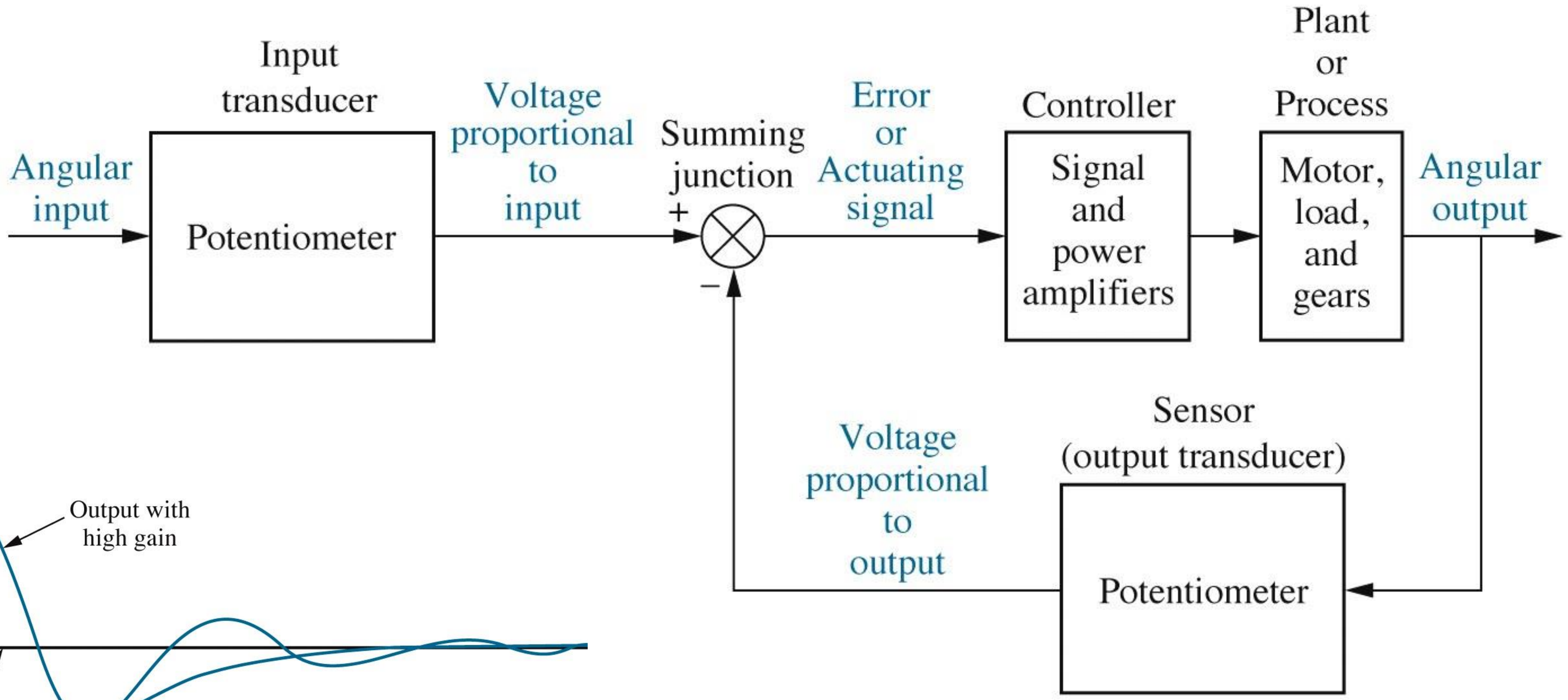


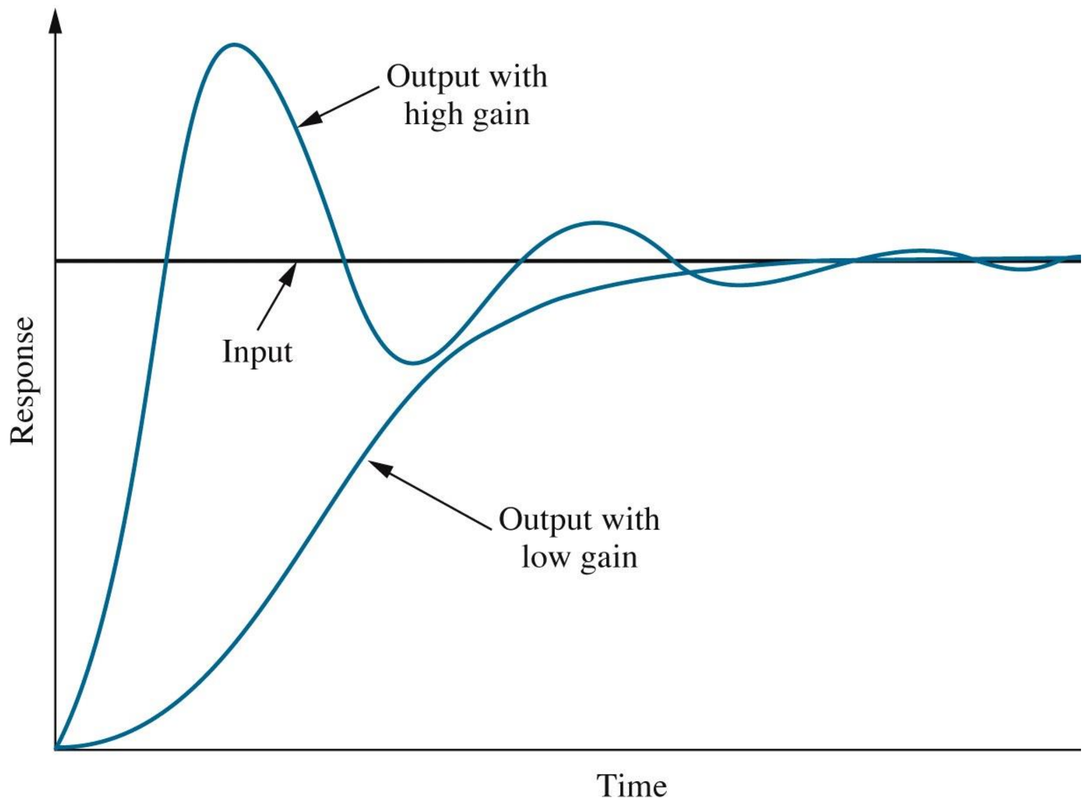
(*a*)

The purpose of this system:
To have the azimuth angle output of the antenna, $\theta_O(t)$, follow the input angle of the potentiometer, $\theta_I(t)$.



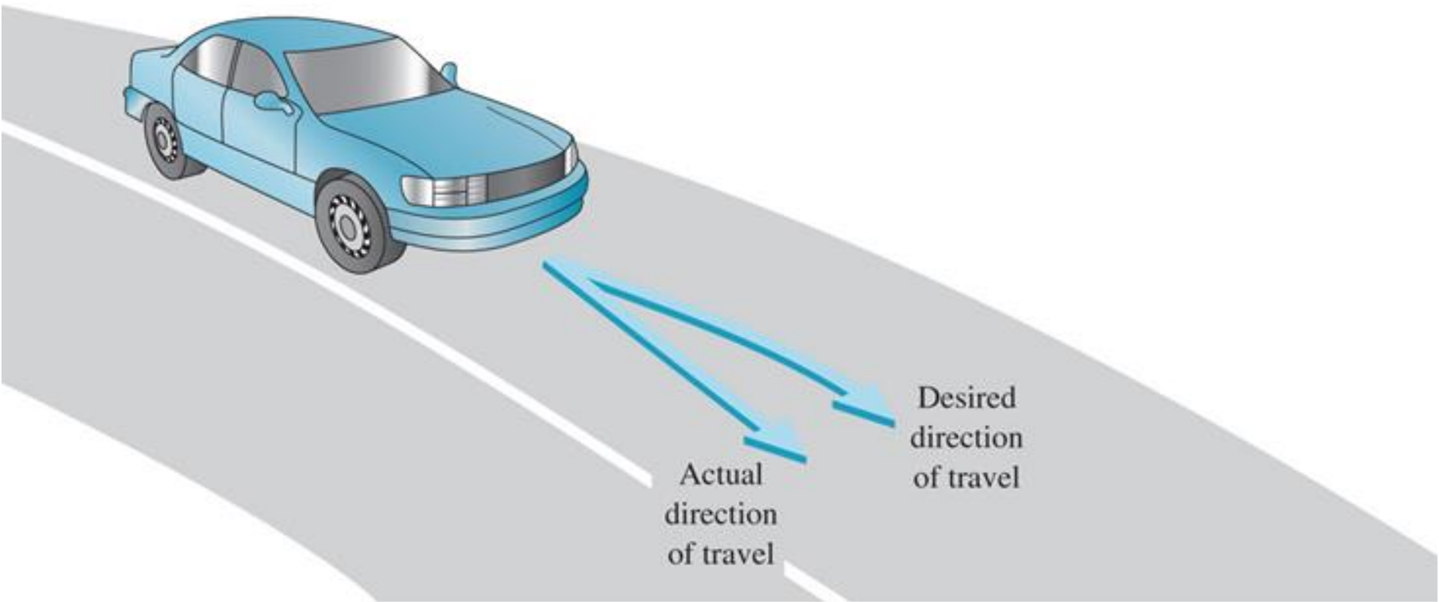
$\theta_i(t)$
+
Potentiometer
–
Amplifiers
Motor
Armature
resistance
+
Differential
and
power
amplifier
K
Armature
–
Fixed field
Gear
$\theta_o(t)$
Gear
Inertia
Viscous
damping
–
Potentiometer
Gear
+
Potentiometer
$\theta_i(t)$
Desired
azimuth angle
input
Antenna
$\theta_O(t)$
Azimuth
angle
output
Differential amplifier
and power amplifier
Motor
Potentiometer

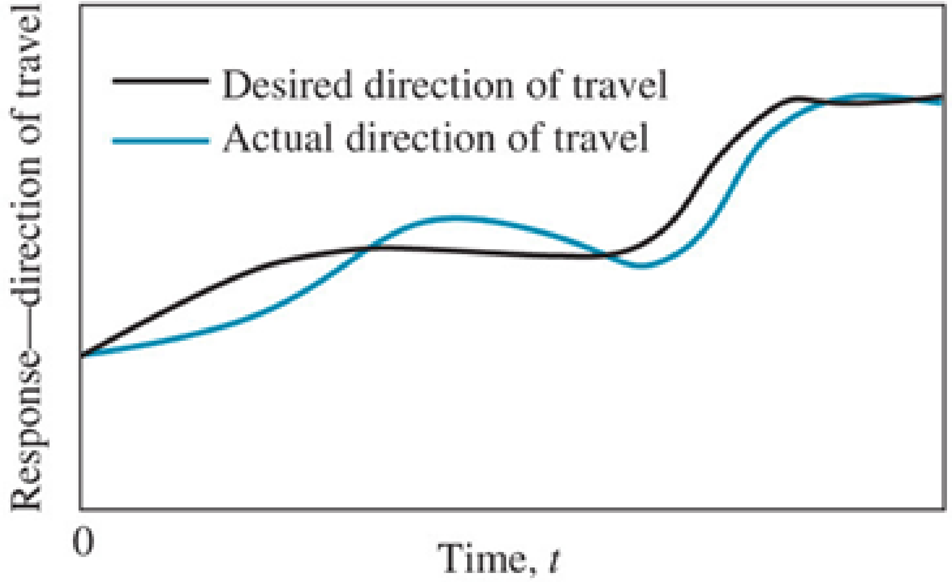

Input
transducer
Potentiometer
Angular
input
Voltage
proportional
to
input
Summing
junction
+
−
Error
or
Actuating
signal
Controller
Signal
and
power
amplifiers
Plant
or
Process
Motor,
load,
and
gears
Angular
output
Sensor
(output transducer)
Potentiometer
Voltage
proportional
to
output


Output with
high gain
Input
Output with
low gain
Response
Time

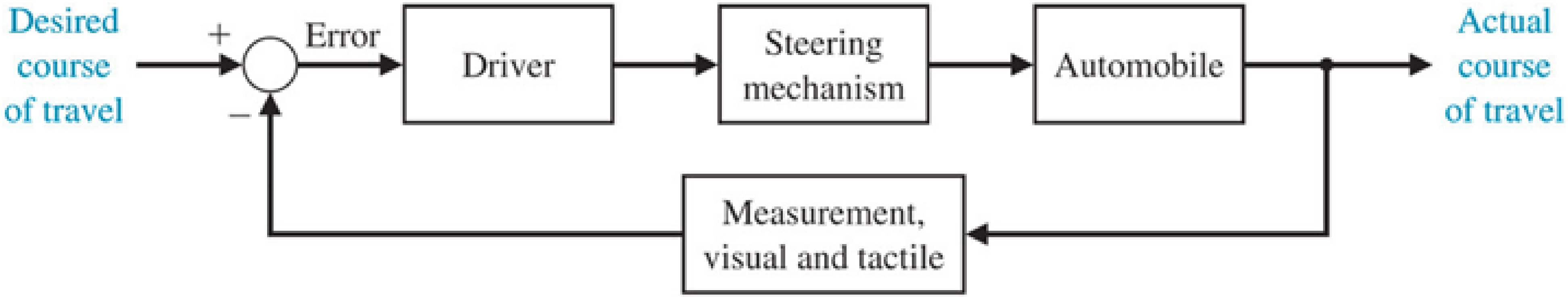
EXAMPLE: AUTOMOBILE STEERING CONTROL SYSTEM

The driver uses the difference between the actual and the desired direction of travel to generate a controlled adjustment of the steering wheel.



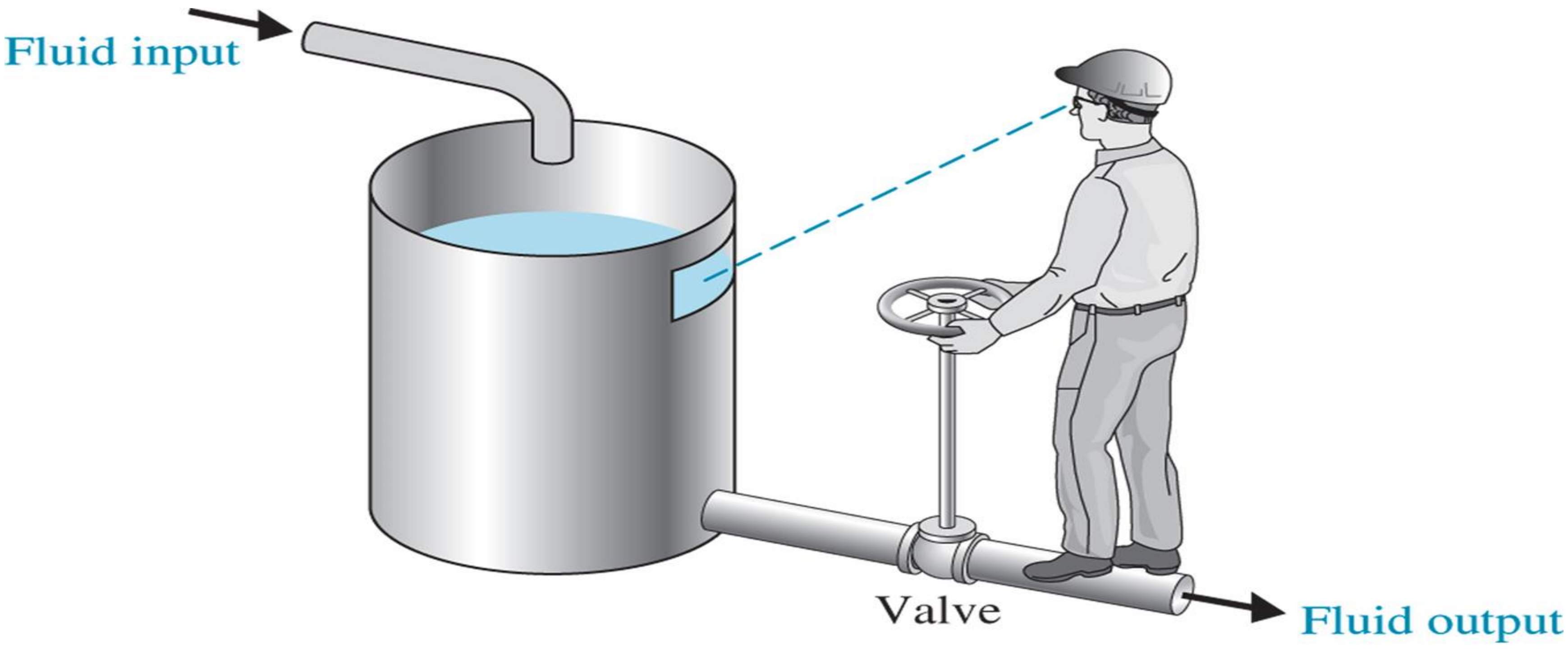


(c)



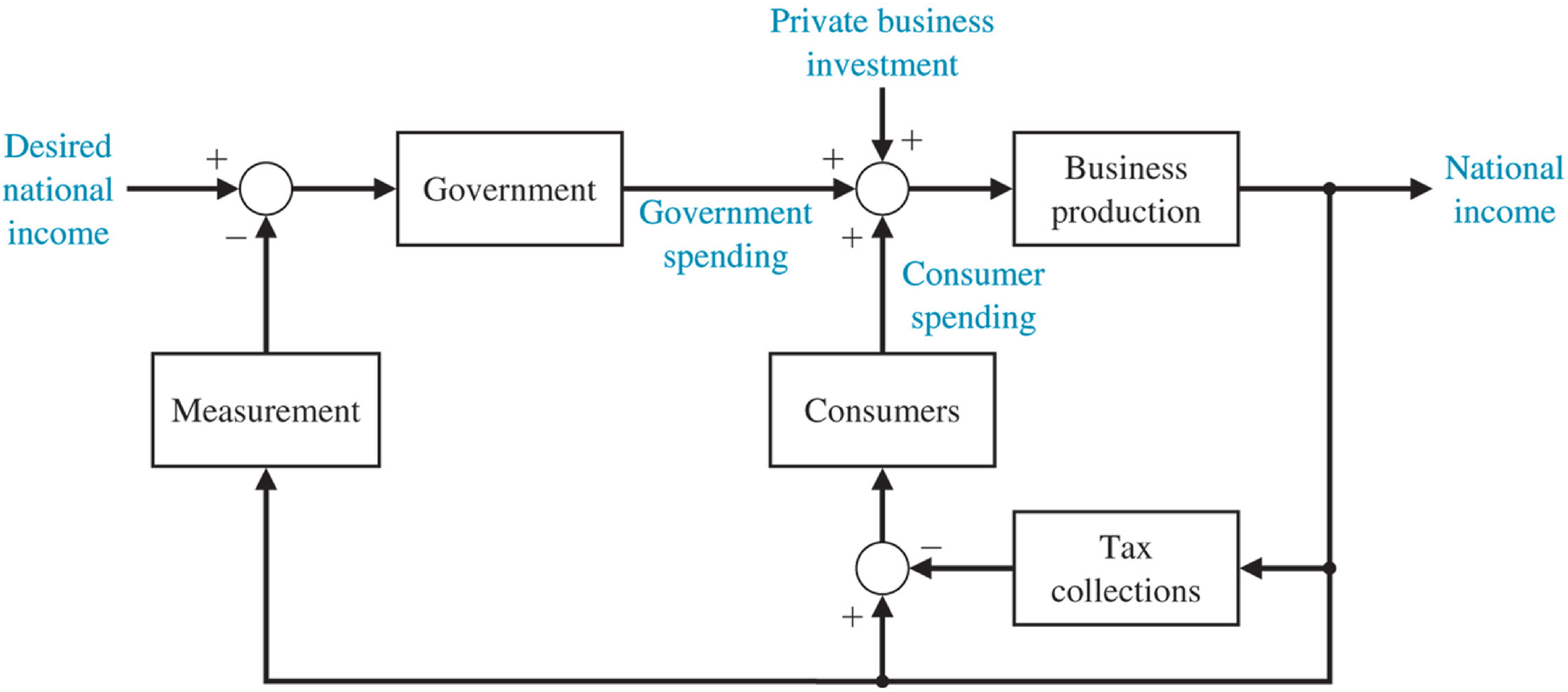
EXAMPLE: REGULATING THE LEVEL OF FLUID

- A basic, manually controlled closed-loop system for regulating the level of fluid in a tank is shown in figure below.
- The input is a reference level of fluid that the operator is instructed to maintain. (This reference is memorized by the operator.)
- The power amplifier is the operator, and the sensor is visual.
- The operator compares the actual level with the desired level and opens or closes the valve (actuator), adjusting the fluid flow out.



EXAMPLE: NATIONAL INCOME FEEDBACK CONTROL

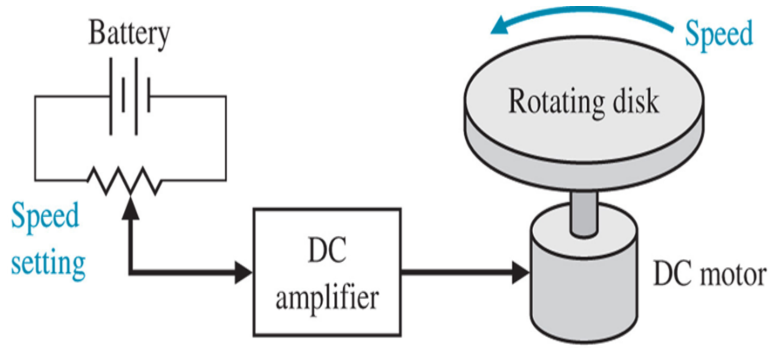
A simple model of the national income feedback control system is shown in figure below.



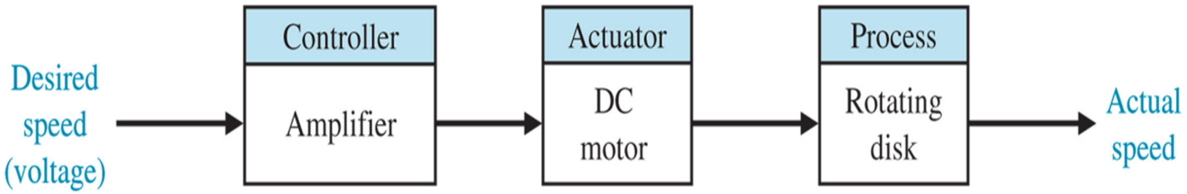
EXAMPLE: ROTATING SPEED CONTROL

Our goal is to design a system for rotating disk speed control that will ensure that the actual speed of rotation is within a specified percentage of the desired speed

<u>Open-loop (without feedback)</u> control of the speed of a rotating disk

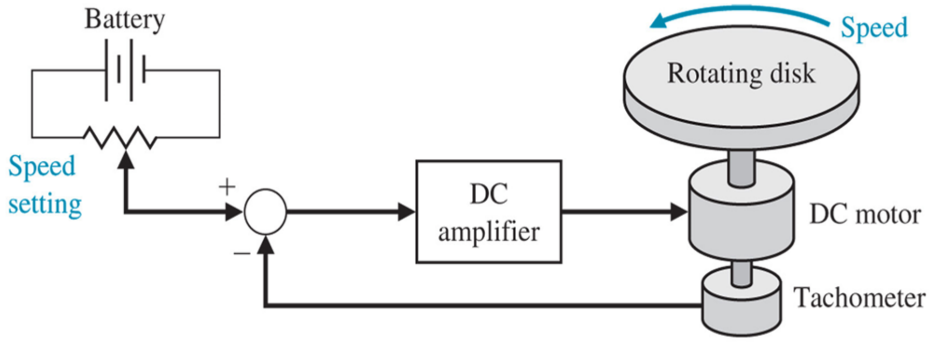


(a)

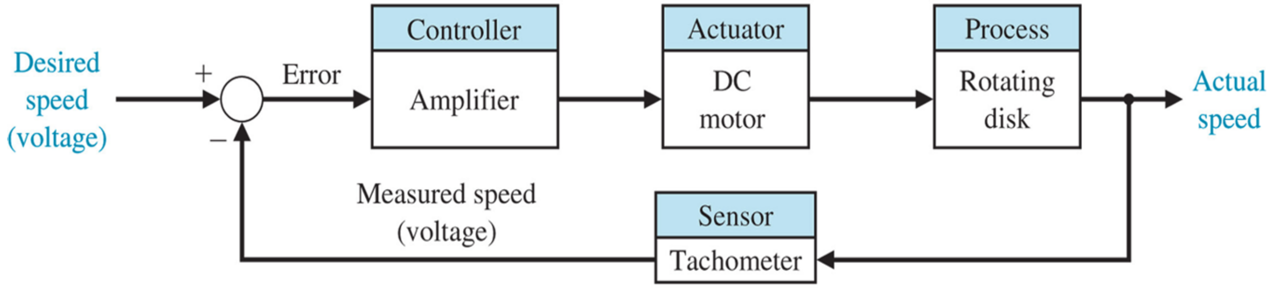


(b)

<u>Closed-loop control</u> of the speed of a rotating disk.



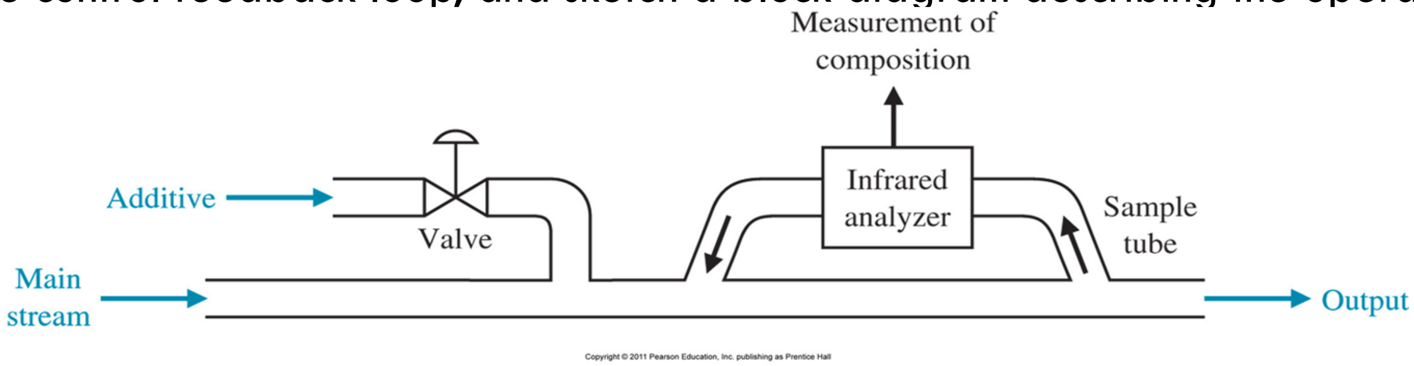
(a)



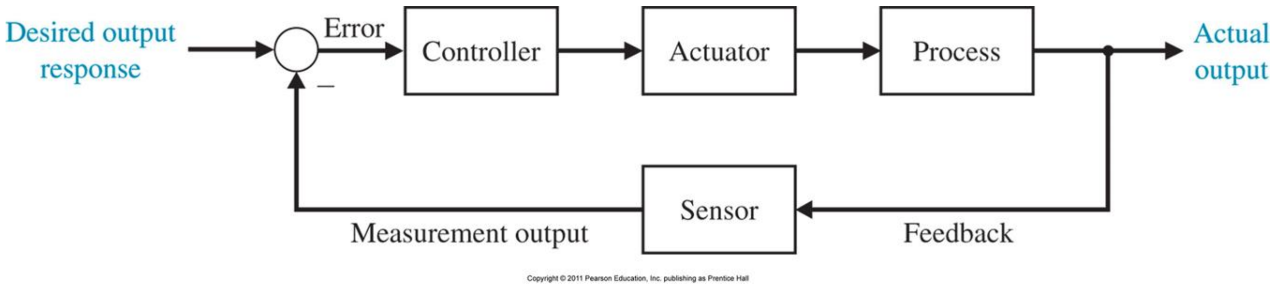
(b)

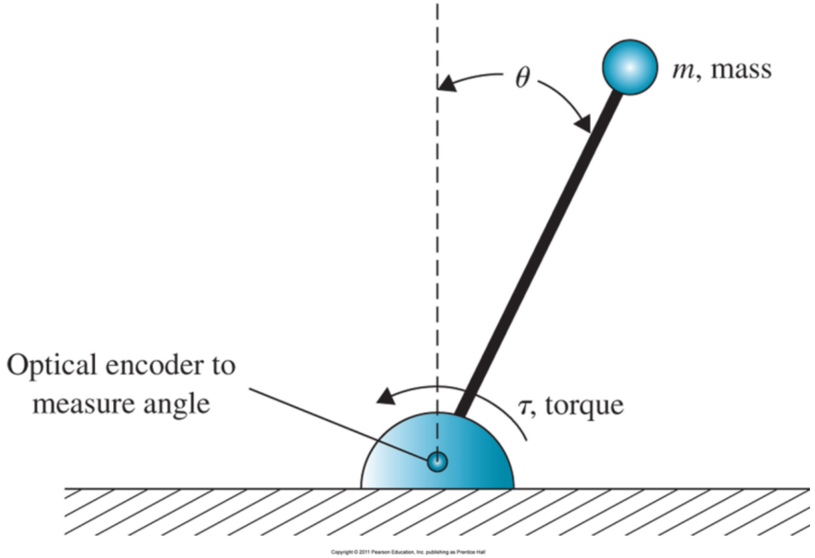
DRILL QUESTIONS

In a chemical process control system, it is valuable to control the chemical composition of the product. To do so, a measurement of the composition can be obtained by using an infrared stream analyzer, as shown in the figure below. The valve on the additive stream may be controlled. Complete the control feedback loop, and sketch a block diagram describing the operation of the control loop.

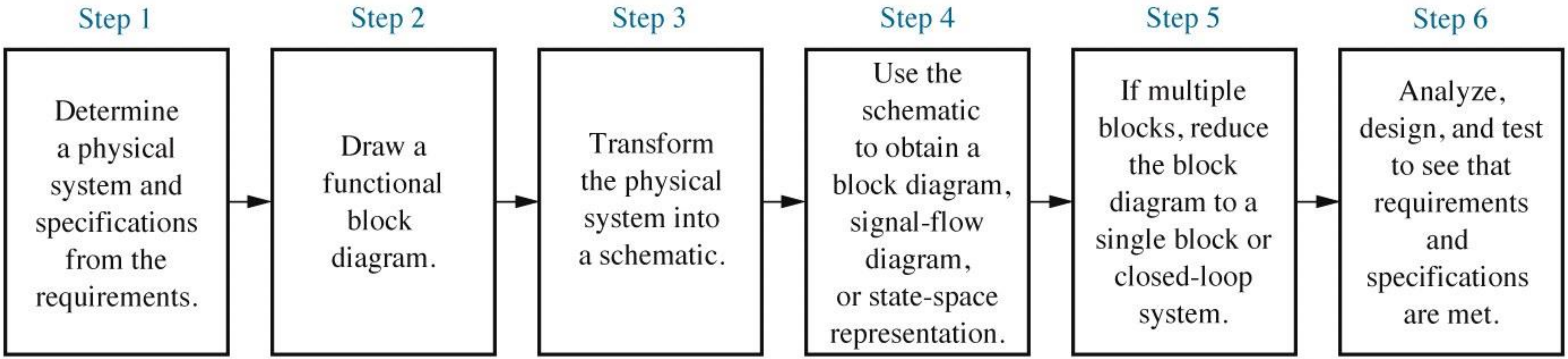


Consider the inverted pendulum shown in the figure. Sketch the block diagram of a feedback control system using the figure below as the model. Identify the process, sensor, actuator, and controller. The objective is keep the pendulum in the upright position, that is to keep $\theta = 0$, in the presence of disturbances.





1.6 THE DESIGN PROCESS STEPS



MODELING IN THE FREQUENCY DOMAIN

To understand and control complex systems, one must obtain quantitative **mathematical models.**

Develop mathematical models applying the fundamental physical laws of science and engineering.

- Transfer functions in the frequency domain
- State equations in the time domain

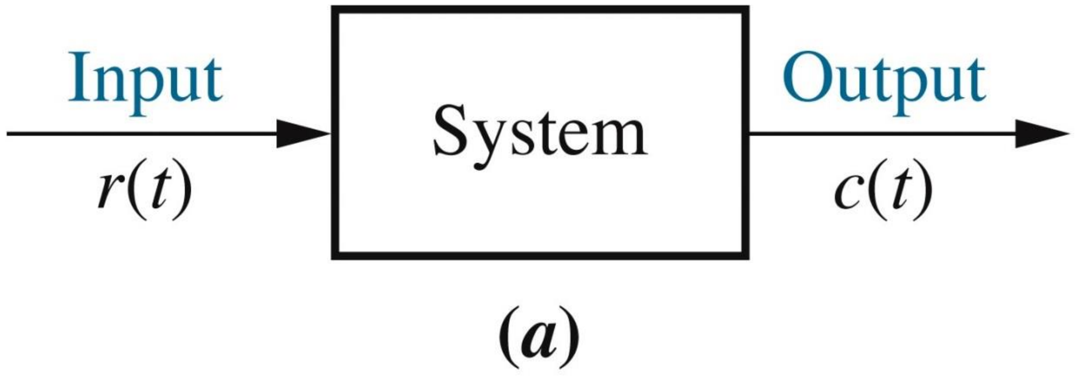


Figure 2.1a

2.2 LAPLACE TRANSFORM **REVIEW**

- Differential equations for modeling
  - Difficult to represent as a block diagram
  - Computationally complex

- Laplace Transform for modeling
  - Easy to represent as a block diagram
  - Algebraic relationship

The Laplace transform is defined as

$$\mathcal{L}\{f(t)\} = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

where $s = \sigma + jw$, a complex variable.

The inverse Laplace transform is

$$\mathcal{L}^{-1}[F(s)] = f(t)u(t) = \frac{1}{2\pi j}\int_{\sigma-jw}^{\sigma+jw} F(s)e^{st}ds$$

where $u(t)$ is the unit step function.

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n + 1}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}[\int_{0-}^{t} f(\tau)d\tau] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

PARTIAL FRACTION EXPANSION

- To find inverse Laplace transform a complicated function, we can convert it into sum of simpler terms for which we know the Laplace transform.

$$F(s) = \frac{N(s)}{D(s)}$$

Roots of the denominator of F(s) can be

- real and distinct (case1)
- real and repeated (case 2)
- Complex or imaginary (case 3)

CASE 1. ROOTS OF DENOMINATOR OF F(S) ARE REAL AND DISTINCT

$$F(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+p_1)(s+p_2)\ldots(s+p_m)\ldots(s+p_n)}$$

$$= \frac{K_1}{s+p_1} + \frac{K_2}{s+p_2} + \cdots + \frac{K_m}{s+p_m} + \cdots + \frac{K_n}{s+p_n}$$

$$K_m = \left.\frac{N(s)(s+p_m)}{(s+p_1)(s+p_2)\ldots(s+p_m)\ldots(s+p_n)}\right|_{s=-p_m}$$

Problem: *Given the following differential equation, solve for y(t) if all initial conditions are zero. Use the Laplace transform.*

$$\frac{d^2y}{dt^2} + 12\,\frac{dy}{dt} + 32y = 32\,u(t)$$

$$s^2Y(s) + 12sY(s) + 32Y(s) = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2+12s+32)} = \frac{32}{s(s+4)(s+8)} = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+8}$$

$$K_1 = \frac{32}{(s+4)(s+8)}\bigg|_{s=0} = 1$$

$$K_2 = \frac{32}{s(s+8)}\bigg|_{s=-4} = -2$$

$$K_3 = \frac{32}{s(s+4)}\bigg|_{s=-8} = 1$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$$y(t) = (1 - 2e^{-4t} + e^{-8t})u(t)$$

CASE 2. ROOTS OF DENOMINATOR OF F(S) ARE REAL AND REPEATED

$$F(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+p_1)^r (s+p_2) \ldots (s+p_n)}$$

$$= \frac{K_1}{(s+p_1)^r} + \frac{K_2}{(s+p_1)^{r-1}} + \cdots + \frac{K_r}{s+p_1} + \frac{K_2}{s+p_2} \ldots + \frac{K_n}{s+p_n}$$

$$K_i = \frac{1}{(i-1)!} \frac{d^{i-1} F_1(s)}{ds^{i-1}} \bigg|_{s=-p_1}$$

where $\quad F_1(s) = (s+p_1)^r F(s)$

Problem: *Find the inverse Laplace transform of*

$$F(s) = \frac{2}{(s+2)^2(s+1)}$$

$$F(s) = \frac{2}{(s+2)^2(s+1)} = \frac{K_1}{s+1} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$$

$$K_1 = \left.\frac{2(s+1)}{(s+2)^2(s+1)}\right|_{s=-1} = 2$$

$$K_2 = \left.\frac{2(s+2)^2}{(s+2)^2(s+1)}\right|_{s=-2} = -2$$

$$K_3 = \left.\frac{-2}{(s+1)^2}\right|_{s=-2} = -2$$

$$F(s) = \frac{2}{s+1} - \frac{2}{(s+2)^2} - \frac{2}{s+2}$$

$$f(t) = (2e^{-t} - 2te^{-2t} - 2e^{-2t})u(t)$$

CASE 3. ROOTS OF DENOMINATOR OF F(S) ARE COMPLEX OR IMAGINARY

$$F(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+p_1)(s^2+as+b)\ldots}$$

$$= \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots$$

$$= \frac{K_1}{(s+p_1)} + \frac{K_2(s+\alpha)}{(s+\alpha)^2+\beta^2} + \frac{B_1\beta}{(s+\alpha)^2+\beta^2}$$

$$f(t) = (K_1 e^{-p_1 t} + K_2\, e^{-\alpha t}\cos\beta t + B_1 e^{-\alpha t}\sin\beta t)u(t)$$

2.3 THE TRANSFER FUNCTION

Consider nth order --- linear -- time-invariant differential equation

output

input

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

Take Laplace Transform

$$a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \cdots + a_0 C(s) + initial\ condition\ terms\ involving\ c(t)$$
$$= b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \cdots + b_0 R(s) + initial\ condition\ terms\ involving\ r(t)$$

Assume all initial conditions are zero

$$a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \cdots + a_0 C(s) = b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \cdots + b_0 \mathrm{R}(s)$$

Obtain transfer function G(s)

$$\boxed{\frac{C(s)}{R(s)} = G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0}}$$

Problem: Find the transfer function, G(s)=C(s)/R(s), corresponding to the differential equation

$$\frac{d^3c}{dt^3} + 3\frac{d^2c}{dt^2} + 7\,\frac{dc}{dt} + 5c = \frac{d^2r}{dt^2} + 4\frac{dr}{dt} + 3r$$

Answer: $G(s) = \frac{C(s)}{R(s)} = \frac{s^2+4s+3}{s^3+3s^2+7s+5}$

Problem: Find the differential equation corresponding to the transfer function $G(s) = \frac{2s+1}{s^2+6s+2}$

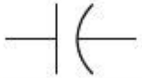
Answer: $\frac{d^2c}{dt^2} + 6\,\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$

Problem: Find the ramp response for a system whose transfer function is $G(s) = \frac{s}{(s+4)(s+8)}$

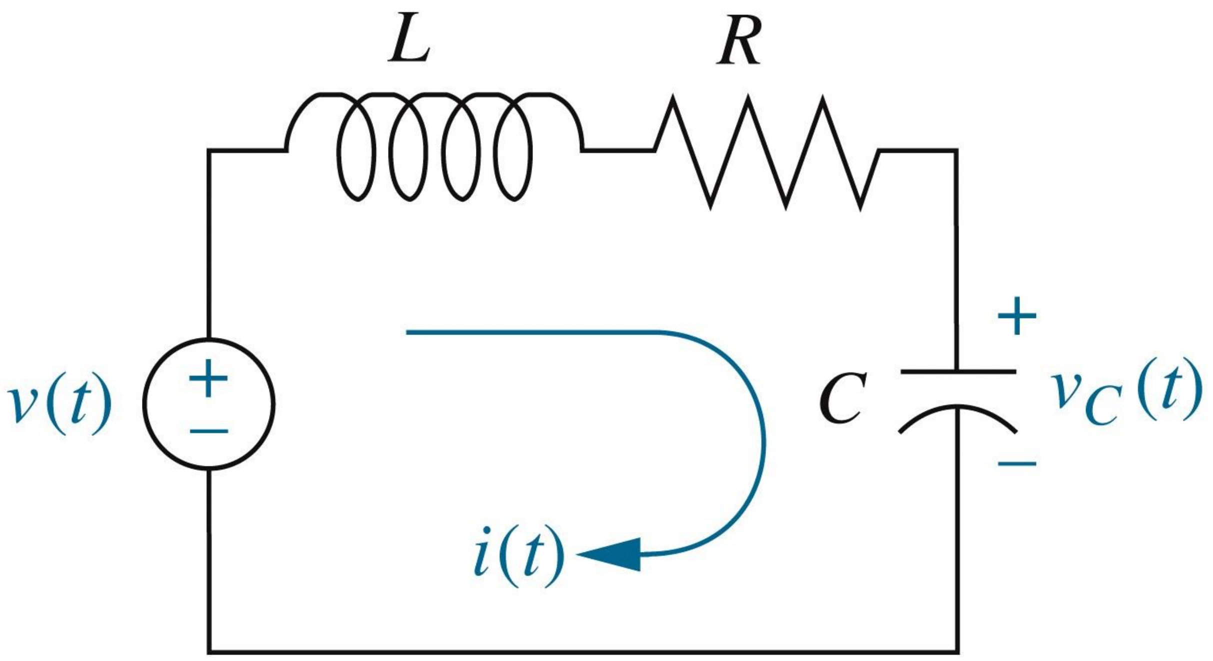
Answer: $c(t) = \left(\frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}\right)$

2.4 ELECTRIC NETWORK TRANSFER FUNCTIONS

TABLE 2.3 Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

TRANSFER FUNCTION – VIA THE DIFFERENTIAL EQUATION



$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

Using $i(t) = dq(t)/dt$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = v(t)$$

$$\boldsymbol{q(t) = Cv_c(t)}$$

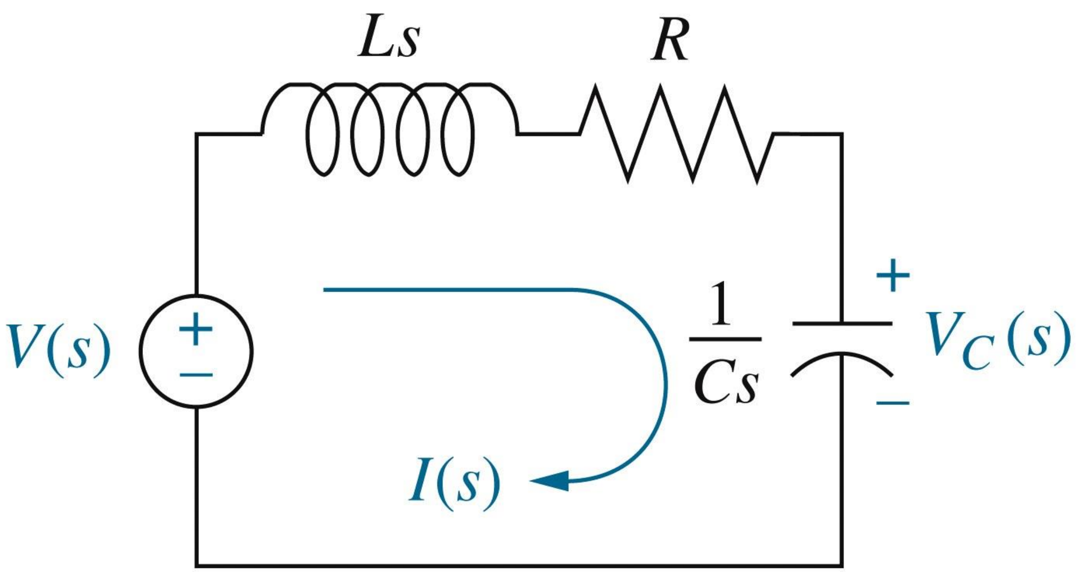
$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = v(t)$$

$$(\mathrm{LC}s^2 + RCs + 1)V_c(s) = V(s)$$

Assume zero initial conditions to obtain transfer function !

$$\boxed{\frac{V_c(s)}{V(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

TRANSFER FUNCTION –
VIA THE TRANSFORM METHODS

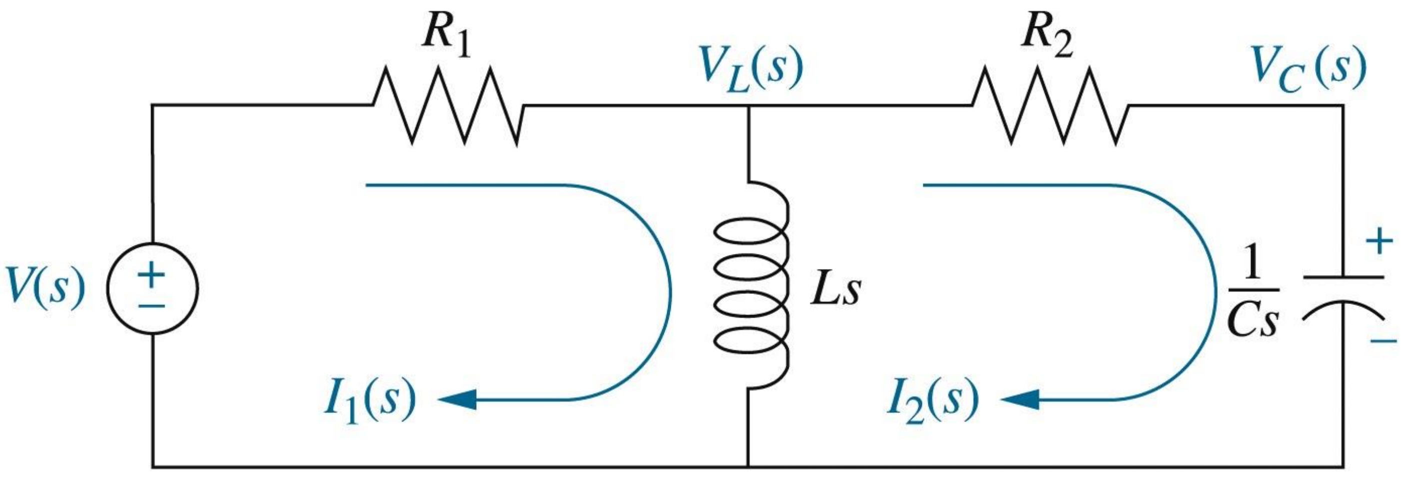


$$\left(Ls + R + \frac{1}{Cs}\right) I(s) = V(s) \Rightarrow \frac{I(s)}{V(s)} = \frac{1}{Ls + R + \frac{1}{Cs}}$$

$$V_c(s) = I(s)\frac{1}{Cs} \Rightarrow I(s) = CsV_c(s)$$

$$\boxed{\frac{V_c(s)}{V(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

TRANSFER FUNCTION – VIA THE TRANSFORM METHODS



(b)

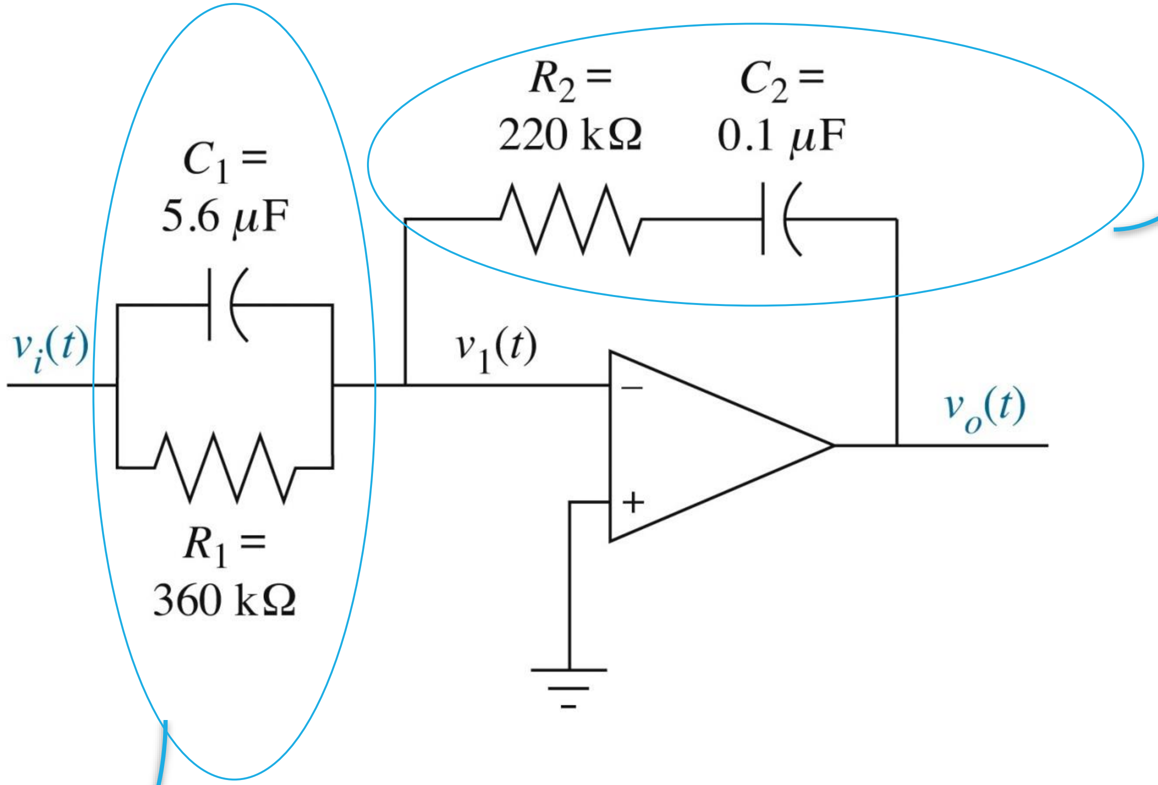
$$\frac{V_L(s) - V(s)}{R_1} + \frac{V_L(s)}{Ls} + \frac{V_L(s) - V_C(s)}{R_2} = 0$$

$$CsV_C(s) + \frac{V_C(s) - V_L(s)}{R_2} = 0$$

$$G_1 = 1/R_1 \qquad G_2 = 1/R_2$$

$$\frac{V_C(s)}{V(s)} = \frac{\frac{G_1 G_2}{C} s}{(G_1 + G_2)s^2 + \frac{G_1 G_2 L + C}{LC} s + \frac{G_2}{LC}}$$

TRANSFER FUNCTION – VIA THE TRANSFORM METHODS



$$Z_2(s) = R_2(s) + \frac{1}{C_2 s} = 220 \text{ x } 10^3 + \frac{10^7}{s}$$

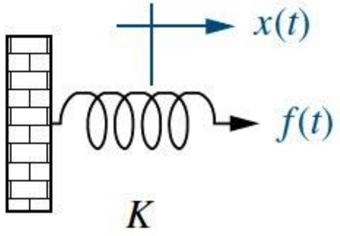
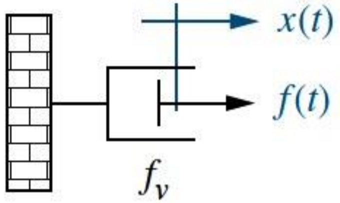
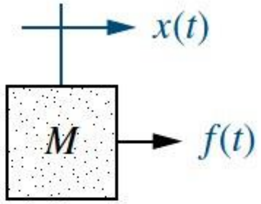
$$\frac{V_0(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)}$$

$$\frac{V_0(s)}{V_i(s)} = -1.232\,\frac{s^2 + 45.95s + 22.55}{s}$$

$$Z_1(s) = \frac{1}{C_1 s + \frac{1}{R_1}} = \frac{1}{5.6 \text{ x}10^{-6} s + \frac{1}{360 \text{ x } 10^3}} = \frac{360 \text{ x } 10^3}{2.016s + 1}$$

This circuit can be used as a PID controller. It improves the performance of a control system. Details later…

2.4 TRANSLATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

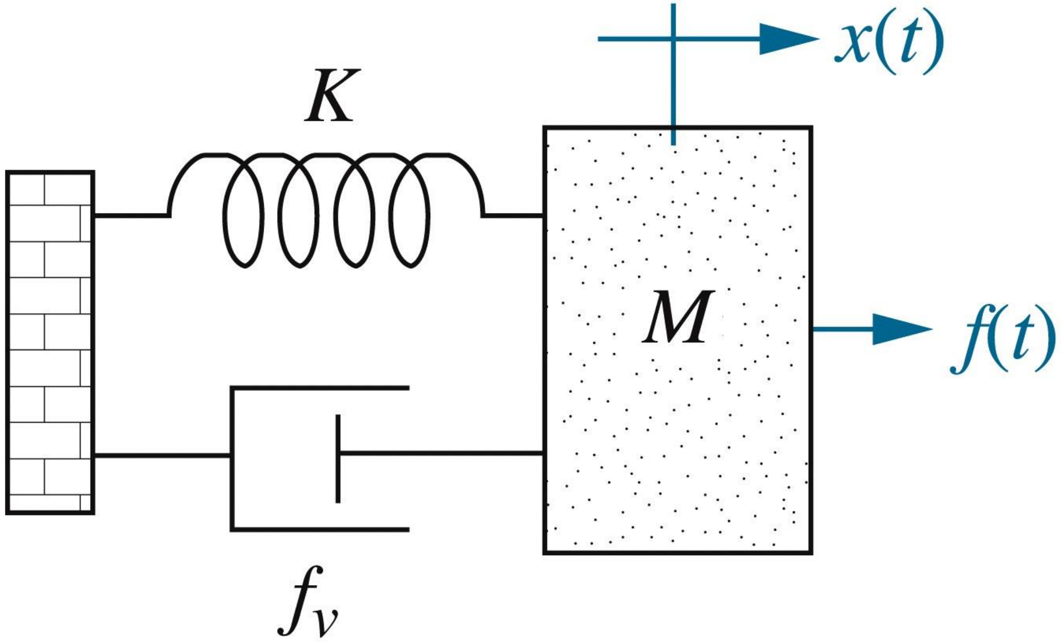
Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

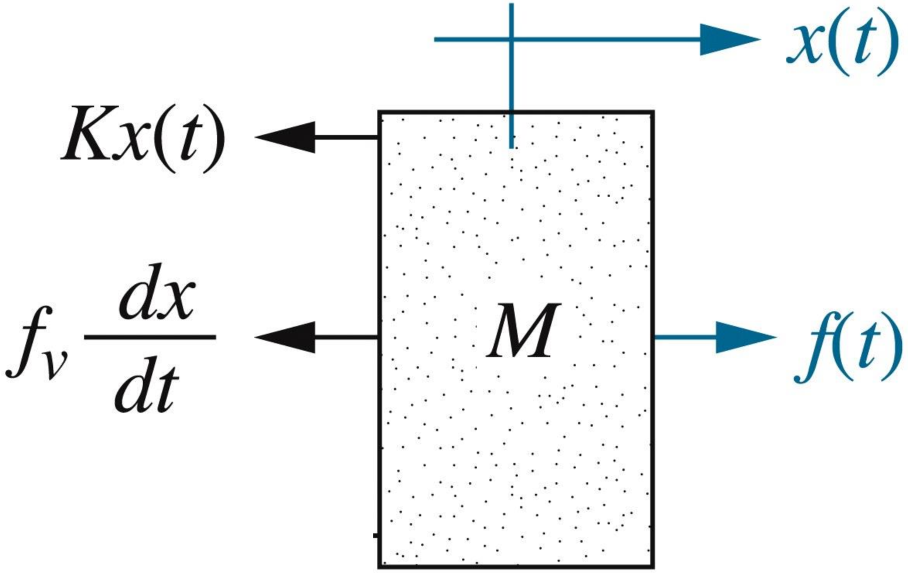
Analogies between electrical and mechanical systems:

Force ⟺ Voltage
Velocity ⟺ Current
Displacement ⟺ Charge

Spring ⟺Capacitor
Viscous damper ⟺ Resistor
Mass ⟺ Inductor

TRANSFER FUNCTION





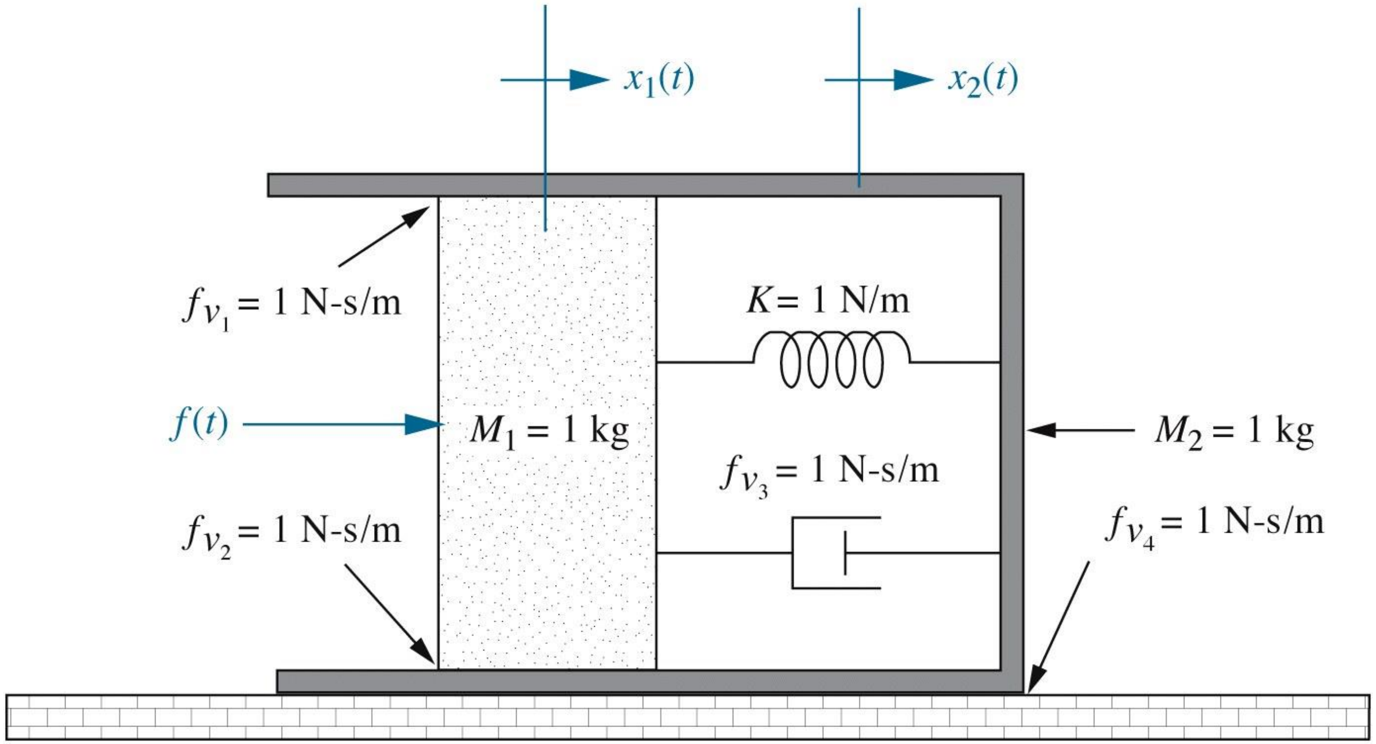
$$f_{net} = M\frac{d^2x(t)}{dt^2}$$

$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dt} + Kx(t) = f(t)$$

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

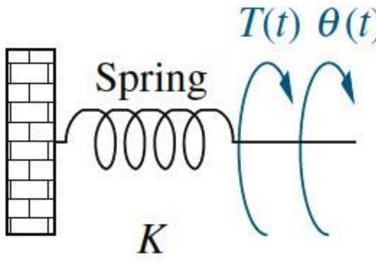
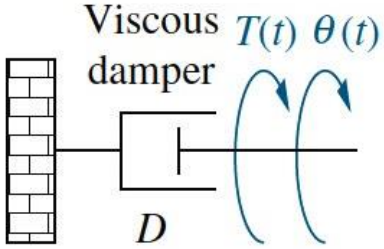
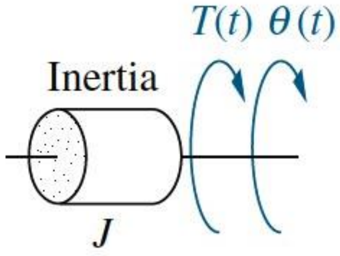
Problem: *Find the transfer function, $G(s)=X_2(s)/F(s)$, for the translational mechanical system shown in below.*



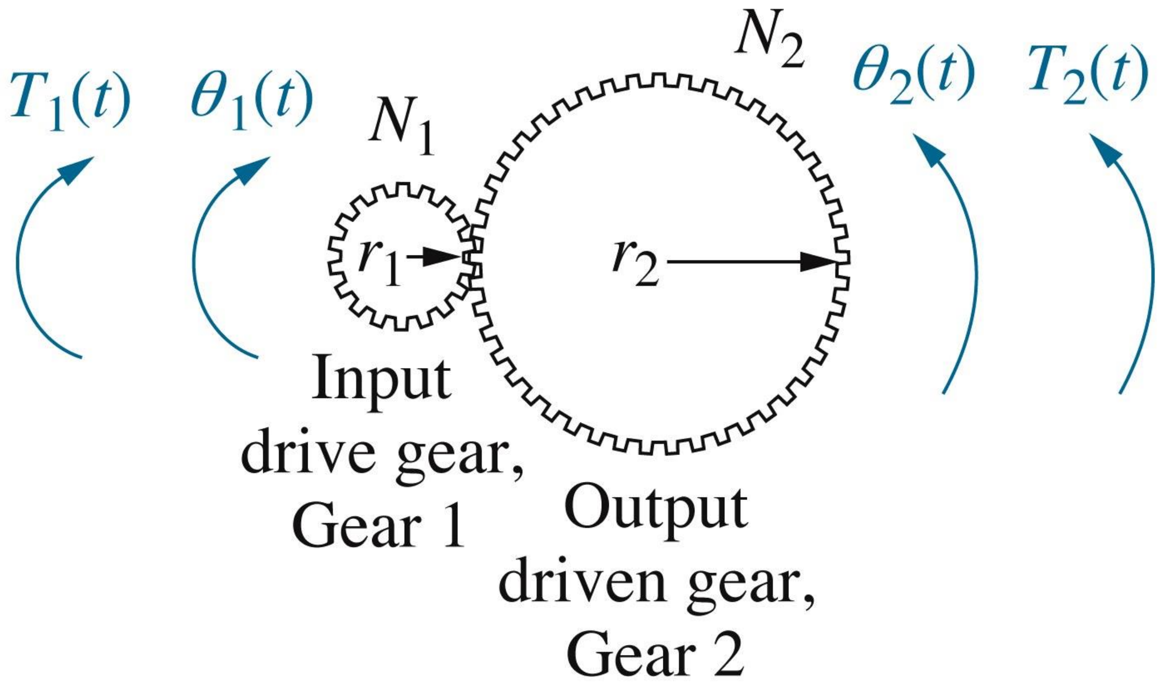
Answer:

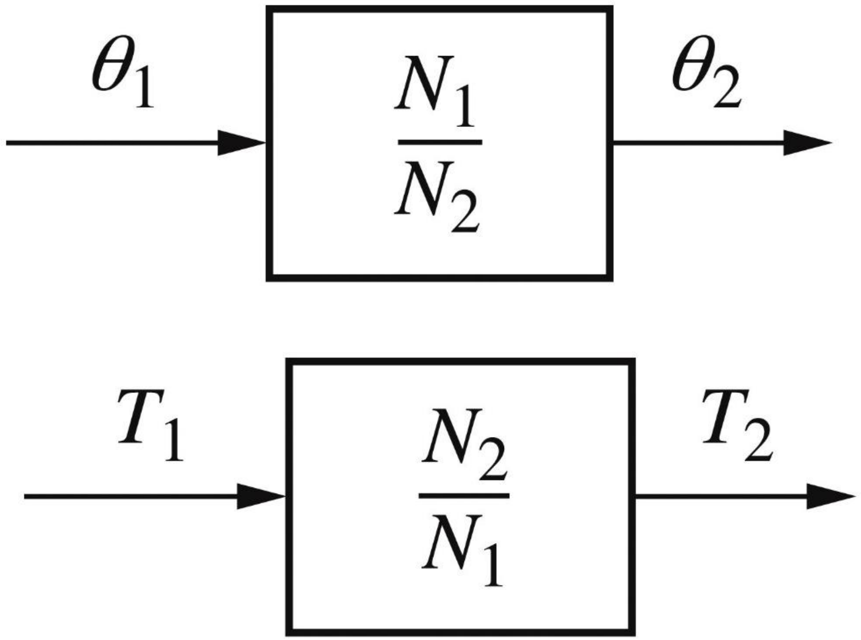
$$G(s) = \frac{3s+1}{s(s^3+7s^2+5s+1)}$$

2.5 ROTATIONAL MECHANICAL SYSTEM TRANSFER FUNCTIONS

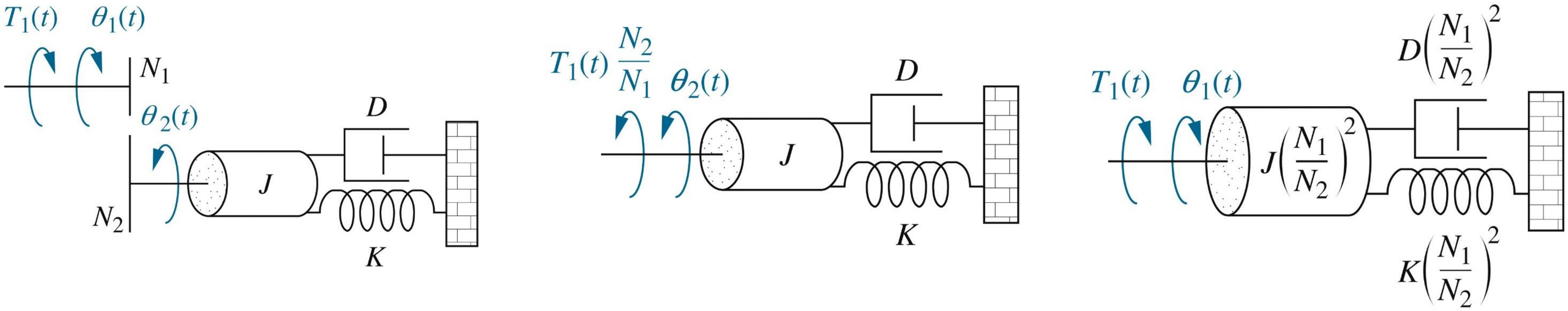
Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

2.7 TRANSFER FUNCTIONS FOR SYSTEMS WITH GEARS





Gears allow you to match the drive system and the load
---a trade-off between speed and torque.

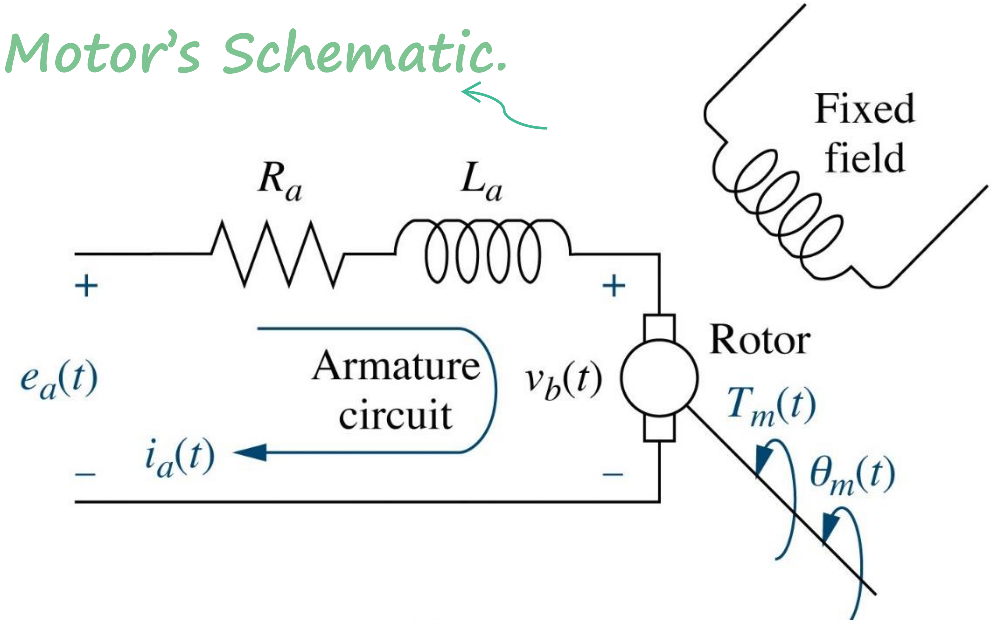


Rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio

$$\left(\frac{\begin{array}{c}Number\ of\ teeth\ of\ gear\\ on\ destination\ shaft\end{array}}{\begin{array}{c}Number\ of\ teeth\ of\ gear\\ on\ source\ shaft\end{array}}\right)^2$$

2.8 ELECTROMECHANICAL SYSTEM TRANSFER FUNCTIONS

Motor's Schematic.



$$v_b(t) = K_b \frac{d\theta_m}{dt} \qquad V_b s = K_b s \theta_m(s)$$

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)$$

$$T_m(s) = K_t I_a(s)$$

$$\frac{(R_a + L_a s) T_m(s)}{K_t} + K_b s \theta_m(s) = E_a(s)$$

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s)$$

$$\frac{(R_a + L_a s)(J_m s^2 + D_m s)\theta_m(s)}{K_t} + K_b s \theta_m(s) = E_a(s)$$

Neglect

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K}{s(s+\alpha)}$$

2.10 NONLINEARITIES

A linear system possesses two properties: Superposition and homogeneity

The output response of a system to the sum of inputs is the sum of the responses to the individual inputs.

$$r_1(t) \xrightarrow{yield} c_1(t)$$
$$r_2(t) \xrightarrow{yield} c_2(t)$$
$$r_1(t) + r_2(t) \xrightarrow{yield} c_1(t) + c_2(t)$$

Multiplication of an input by a scalar yields a response that is multiplied by the same scalar.

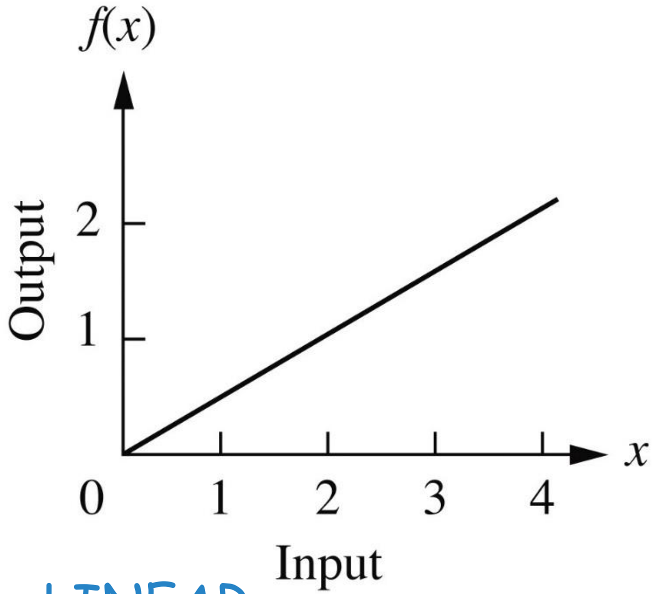
$$r_1(t) \xrightarrow{yield} c_1(t)$$
$$Ar_1(t) \xrightarrow{yield} Ac_1(t)$$

Linear models are useful because

- They are easy to compute, understand and visualize.
- They give predictable outputs, in time and over iterations
- The analysis of linear theory is complete, developed and efficient
- Linear differential equations are easy to solve!

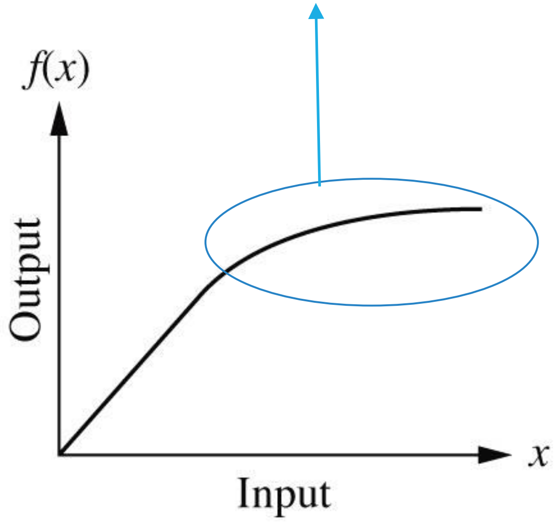
Motor does not respond at very low input due to frictional forces

Gears do not fit tightly exhibit backlash. The input moves over a small range without the output responding.



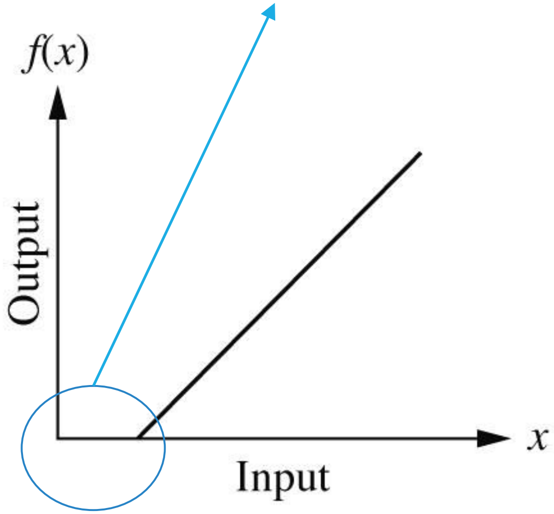
LINEAR

Amplifier saturation



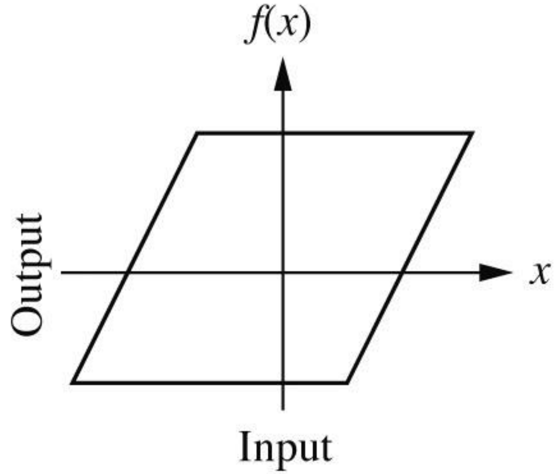
NONLINEAR

Motor dead zone



NONLINEAR

Backlash in gears



NONLINEAR

Make a linear approximation to a nonlinear system.

- Simplifies the analysis and design
- Used as long as the results yield a good approximation to reality
- Analysis of a system represented by nonlinear partial differential equations is extremely difficult and requires heavy computations
- There is no general analytic method available for solving nonlinear systems

2.11 LINEARIZATION

Recognize the nonlinear component

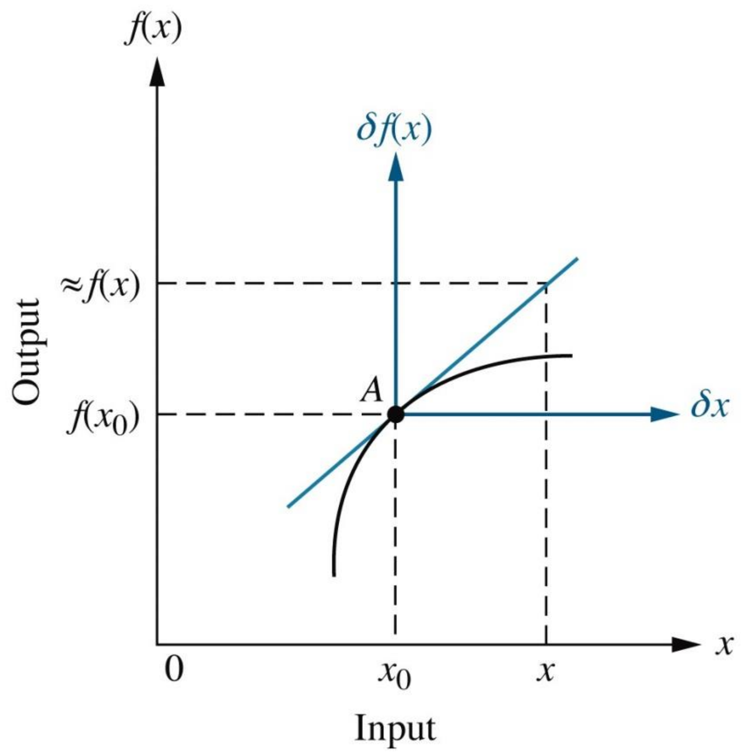
- Write the nonlinear differential equation

Linearize a nonlinear differential eq.

- Linearize it <u>for small-signal inputs</u> about the steady-state solution when the small signal input is equal to zero.
- Steady-state solution is called <u>**equilibrium**</u>

Take the Laplace transform of the linearized diff. eq., assuming zero initial conditions.

- Separate input and output variables and form transfer function.



$$[f(x) - f(x_0)] \approx m_a(x - x_0)$$

$$\delta f(x) \approx m_a \delta x$$

$$f(x) \approx f(x_0) + m_a(x - x_0) \approx f(x_0) + m_a \delta x$$

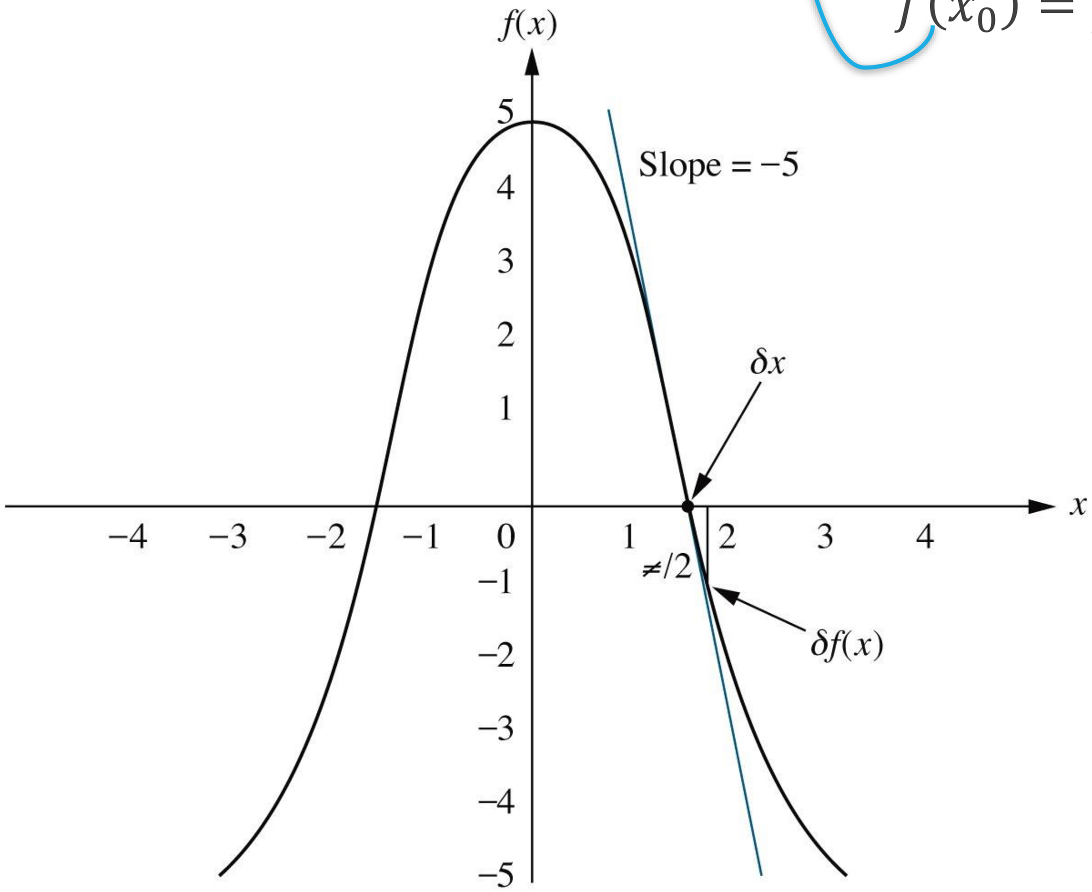
LINEARIZING A FUNCTION

Example: Linearize $f(x) = 5\cos x$ about $x = \pi/2$

$$f(x) \approx f(x_0) + m_a(x - x_0) \approx f(x_0) + m_a \delta x \Longrightarrow \quad \boldsymbol{f(x) = -5\ \delta x}$$

$$f(x_0) = f\left(\frac{\pi}{2}\right) = 5\cos\frac{\pi}{2} = 0$$

$$\frac{df}{dx} = -5\sin x \quad \Rightarrow \left.\frac{df}{dx}\right|_{x=\pi/2} = -5$$



Taylor series expansion:

Ignore higher-order terms

$$y = f(x) = f(x_0) + (x - x_0)\left.\frac{df}{dx}\right|_{x=x_0} + \cancel{\frac{(x-x_0)^2}{2!}\left.\frac{d^2 f}{dx^2}\right|_{x=x_0}} + \ldots$$

$$y = f(x_o) + \left.\frac{df}{dx}\right|_{x=x_0}(x - x_0) = y_0 + m(x - x_0)$$

$$y - y_0 = m(x - x_0)$$

$$\Delta y = m\Delta x$$

LINEARIZING A DIFFERENTIAL EQUATION

Example: Linearize $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + \cos x = 0$ about $x = \pi/4$

$x = \delta x + \pi/4$

$$\frac{d^2(\delta x + \pi/4\,)}{dt^2} + 2\frac{d(\delta x + \pi/4\,)}{dt} + \cos(\delta x + \pi/4\,) = 0 \Rightarrow \boxed{\frac{d^2\delta x}{dt^2} + 2\frac{d\delta x}{dt} - \frac{\sqrt{2}}{2}\delta x = -\frac{\sqrt{2}}{2}}$$

$$\frac{d^2\delta x}{dt^2} \qquad \frac{d\delta x}{dt}$$

Linearized Diff. Eq.

Solve it for δx ***then obtain*** $x = \delta x + \pi/4$

$$[f(x) - f(x_0)] \approx \left.\frac{df(x)}{dx}\right|_{x_0} (x - x_0)$$

$$\cos\left(\delta x + \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = \left.\frac{d\cos x}{dx}\right|_{\frac{\pi}{4}} \delta x = -\sin\left(\frac{\pi}{4}\right)\delta x$$

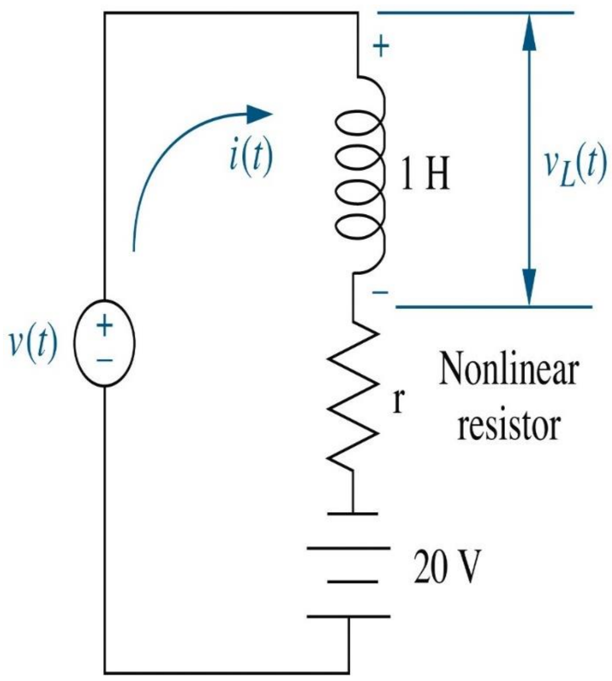
$$\cos\left(\delta x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\delta x$$

Notes:

- *Linearized diff. eq. is non-homogeneous though the non-linear diff. eq. is homogeneous!*
- *The additional term can be thought of as an input.*
- *Since the roots of the equation are positive, the homogeneous solution of linearized eq. grows without bound. Thus this system linearized around $\frac{\pi}{4}$ is not stable.*

TRANSFER FUNCTION--NONLINEAR ELECTRICAL NETWORK

Example: Find the transfer function $V_L(s)/V(s)$ for the electrical network which contains a nonlinear resistor whose voltage-current relationship is defined by $i_r = 2e^{0.1v_r}$. Also, v(t) is a small-signal source.



$$i_r = 2e^{0.1v_r} \Rightarrow v_r = 10\ \ln\frac{1}{2}i_r$$

$$L\frac{di}{dt} + 10\ln\frac{1}{2}i - 20 = \mathrm{v(t)}$$

$$L\frac{d\delta i}{dt} + 10\left(\ln\frac{1}{2}i_0 + \frac{1}{i_0}\delta i\right) - 20 = \mathrm{v(t)}$$

$$\frac{d\delta i}{dt} + 0.677\delta i = \mathrm{v(t)}$$

$$\delta i(s) = \frac{V(s)}{s + 0.677}$$

$$v_L(t) = L\frac{d(i_0 + \delta i)}{dt} = L\frac{d\delta i}{dt}$$

$$V_L(s) = Ls\delta i(s) \Rightarrow \frac{V_L(s)}{V(s)} = \frac{s}{s + 0.677}$$

Linearize this diff. equation.

(Set small-signal source, v(t), equal to zero and find the steady-state current (/equilibrium point), i_o. Then linearize non-linear function around i_o by using 1st order taylor expansion)

$$L\frac{di_0}{dt} + 10\ln\frac{1}{2}i_0 - 20 = 0$$

$$\Rightarrow i_0 = 2e^2 \Rightarrow i_o = 14.78\text{amps}$$

$$L\frac{d(i_0 + \delta i)}{dt} + 10\ln\frac{1}{2}(i_0 + \delta i) - 20 = \mathrm{v(t)}$$

$$\ln\frac{1}{2}(i_0 + \delta i) - \ln\frac{1}{2}i_0 = \left.\frac{d(\ln 0.5i)}{di}\right|_{i_0}\delta i$$

$$\ln\frac{1}{2}(i_0 + \delta i) - \ln\frac{1}{2}i_0 = \frac{1}{i_0}\delta i$$

$$\ln\frac{1}{2}(i_0 + \delta i) = \ln\frac{1}{2}i_0 + \frac{1}{i_0}\delta i$$