Block Diagram Algebra

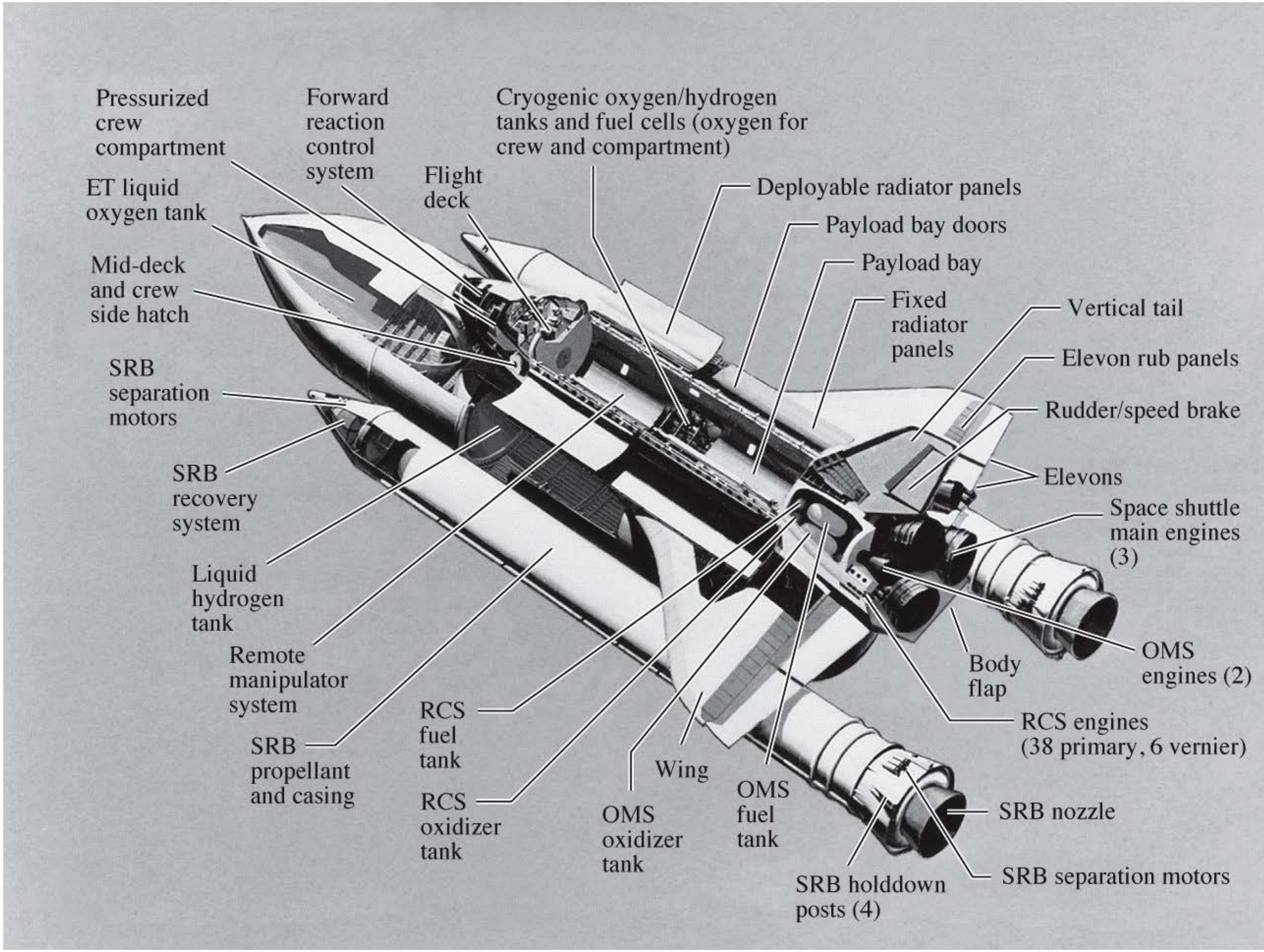
5.1 INTRODUCTION

- *Many systems are composed of multiple subsytems, as in the figure.*

- *A graphical tool* can help us to ***visualize the model*** of a system and ***evaluate the mathematical relationships between their subsytems***, using their transfer functions.

We will now

- examine some common topologies for interconnecting subsytems and
- derive the single transfer function representation.

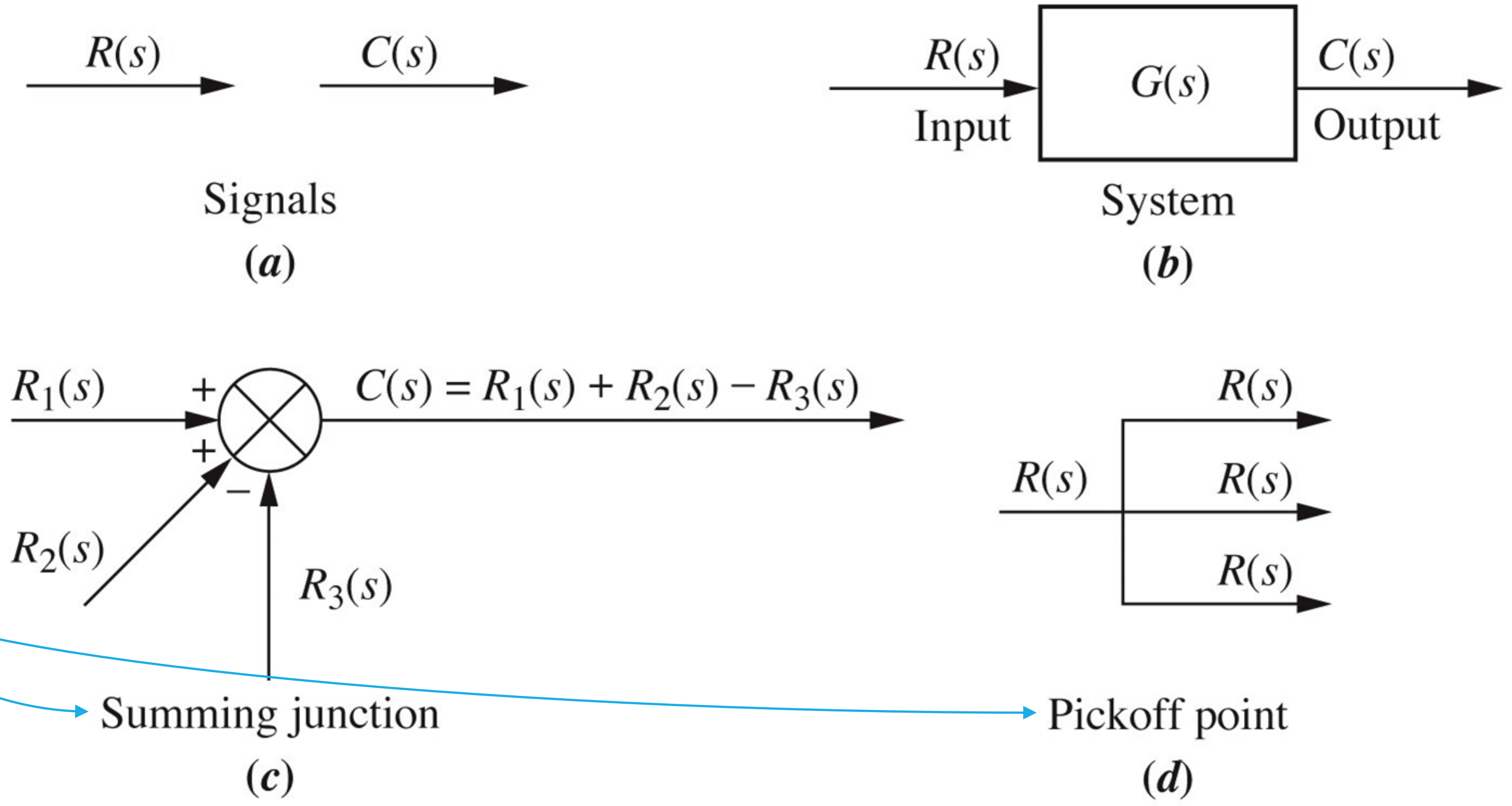


5.2 BLOCK DIAGRAMS

It represents the ***mathematical relationships*** between the elements of the system.

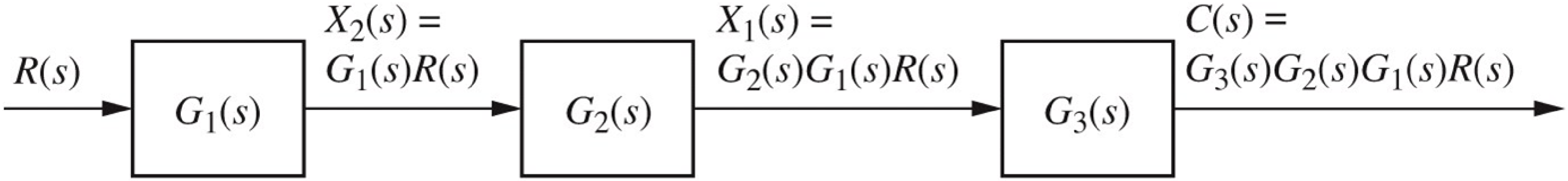
The ***transfer function*** of each component is placed ***in box***, and the ***input-output relationships*** between components are indicated by ***lines and arrows***.

New schematic elements:
Summing junction
and pickoff points.



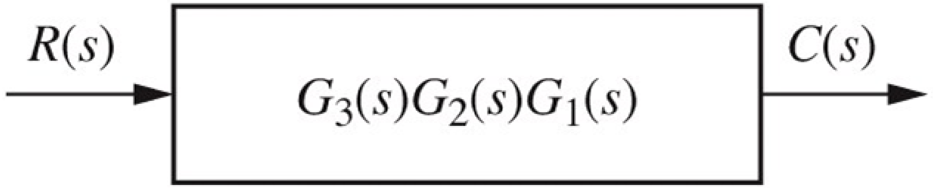
CASCADE FORM

Cascaded subsytems :



(*a*)

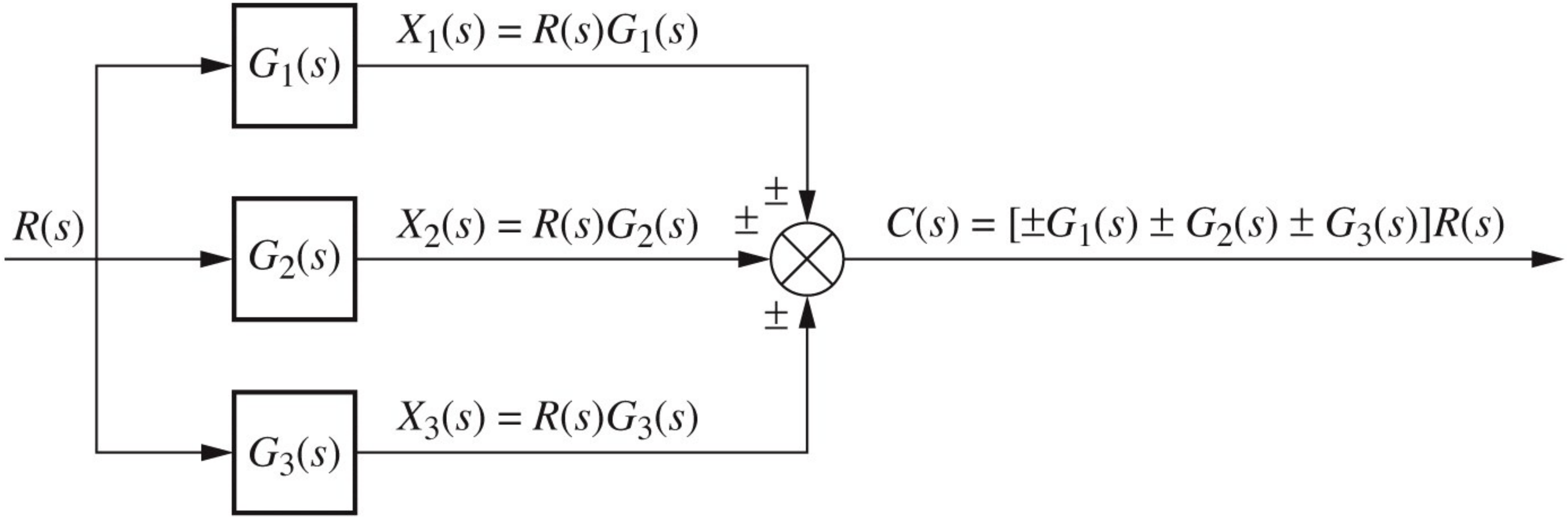
Equivalent transfer function :



(*b*)

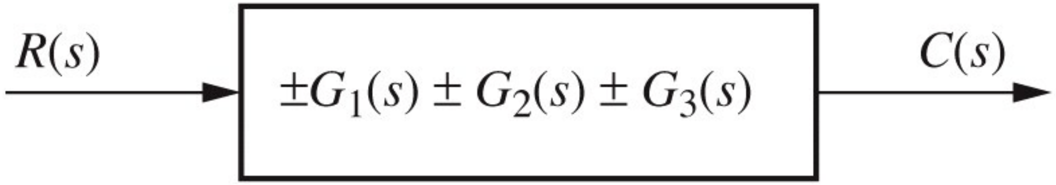
PARALLEL FORM

Paralle subsytems :



(*a*)

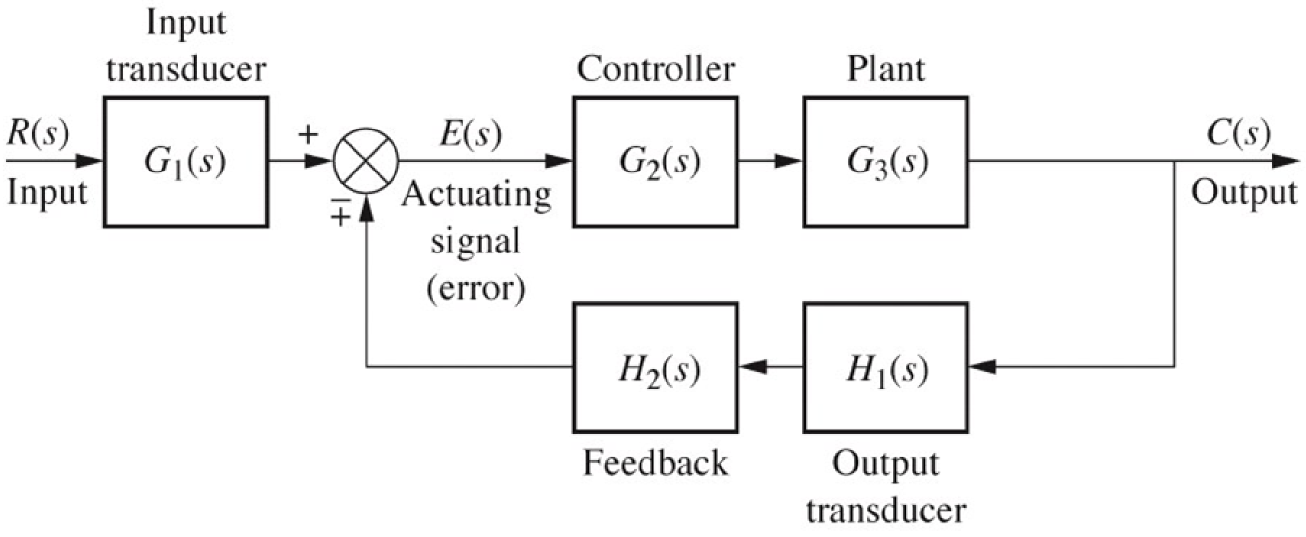
Equivalent transfer function :



(*b*)

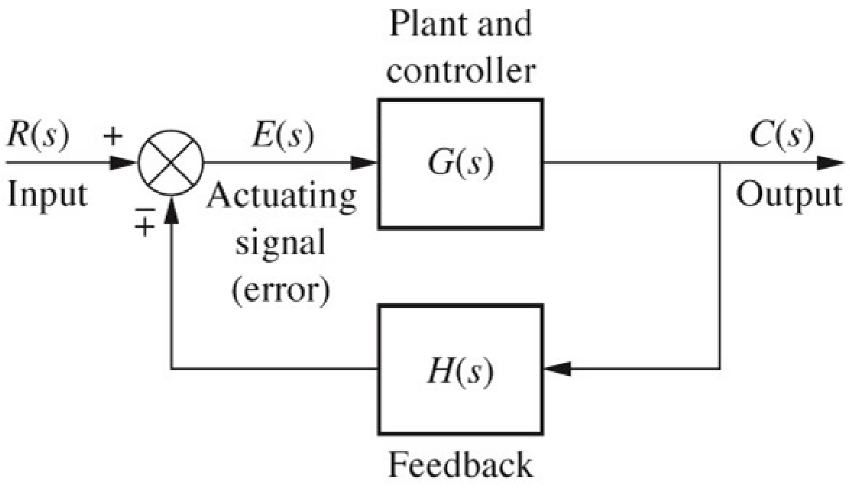
FEEDBACK FORM

Feedback control system:



(*a*)

Simplified model:



(*b*)

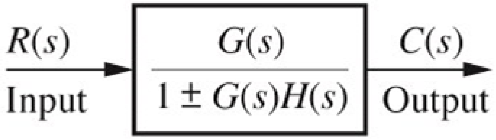
$$C(s) = E(s)\,G(s)$$

$$E(s) = R(s) \mp C(s)H(s)$$

$$C(s) = \big(R(s) \mp C(s)H(s)\big)\,G(s)$$

$$\big(1 \pm H(s)G(s)\big)\,C(s) = G(s)\,R(s)$$

Equivalent transfer function :

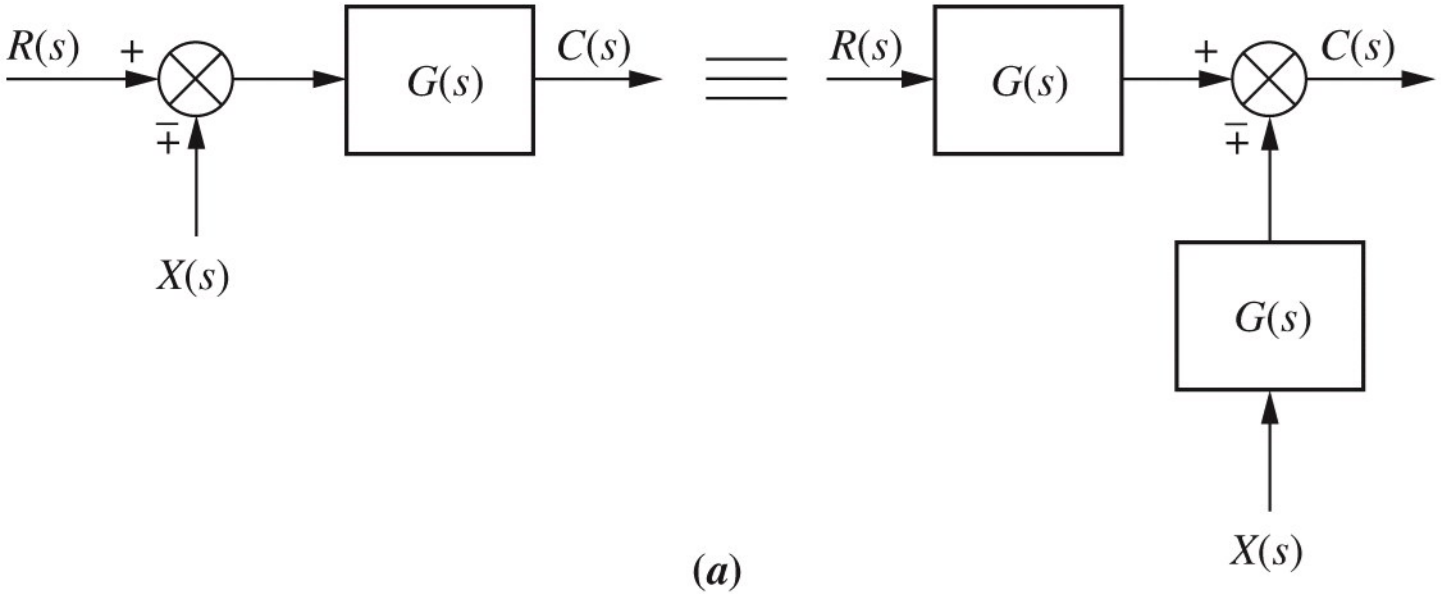


(*c*)

$$\boxed{C(s) = \frac{G(s)}{\big(1 \pm H(s)G(s)\big)}R(s)}$$

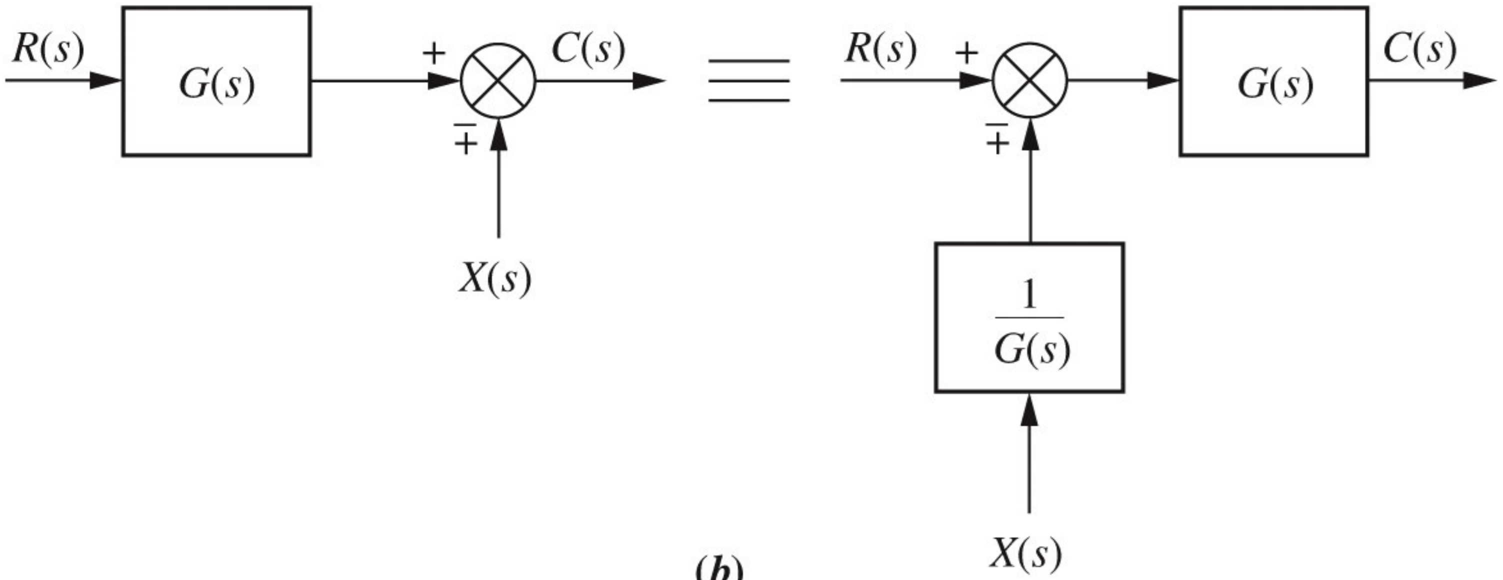
MOVING BLOCKS TO CREATE FAMILIAR FORMS

Moving a block to the left past a **summing junction**



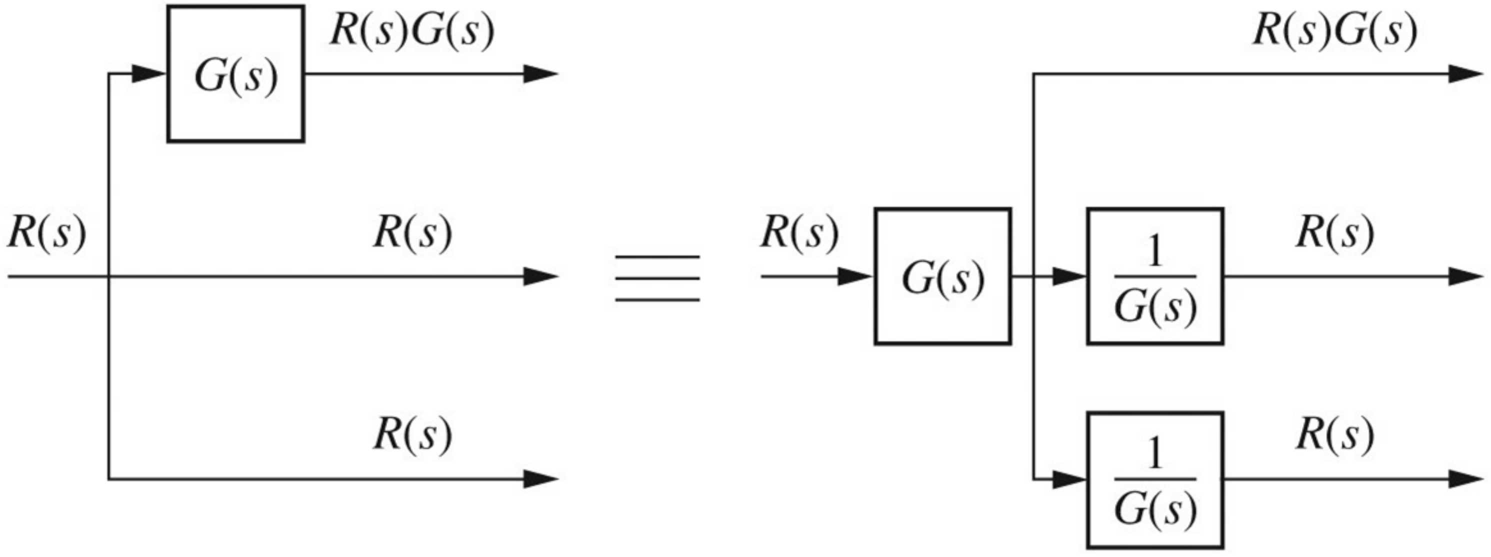
(*a*)

Moving a block to the right past a **summing junction**



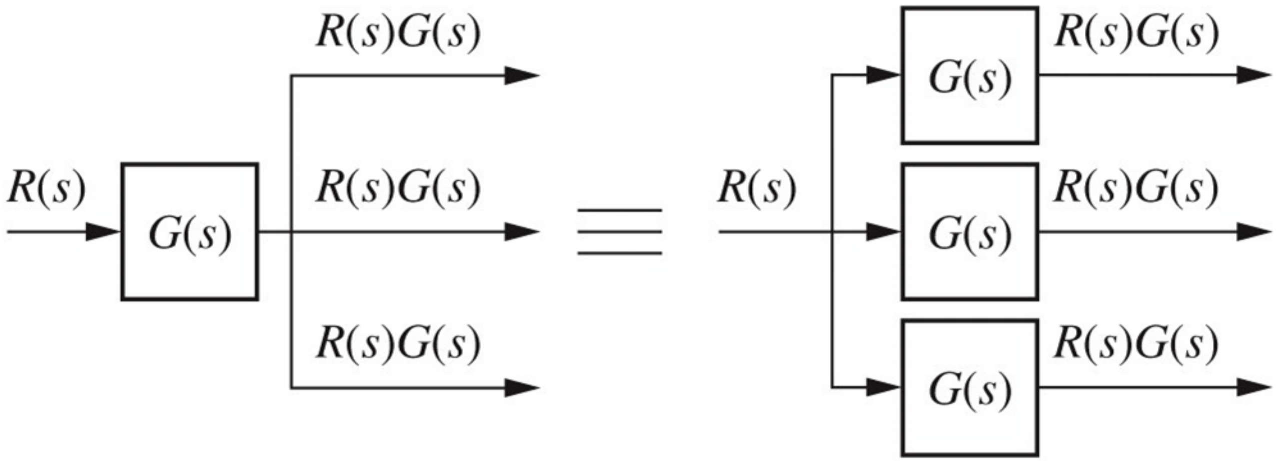
(*b*)

Moving a block to the left past a **pickoff point**



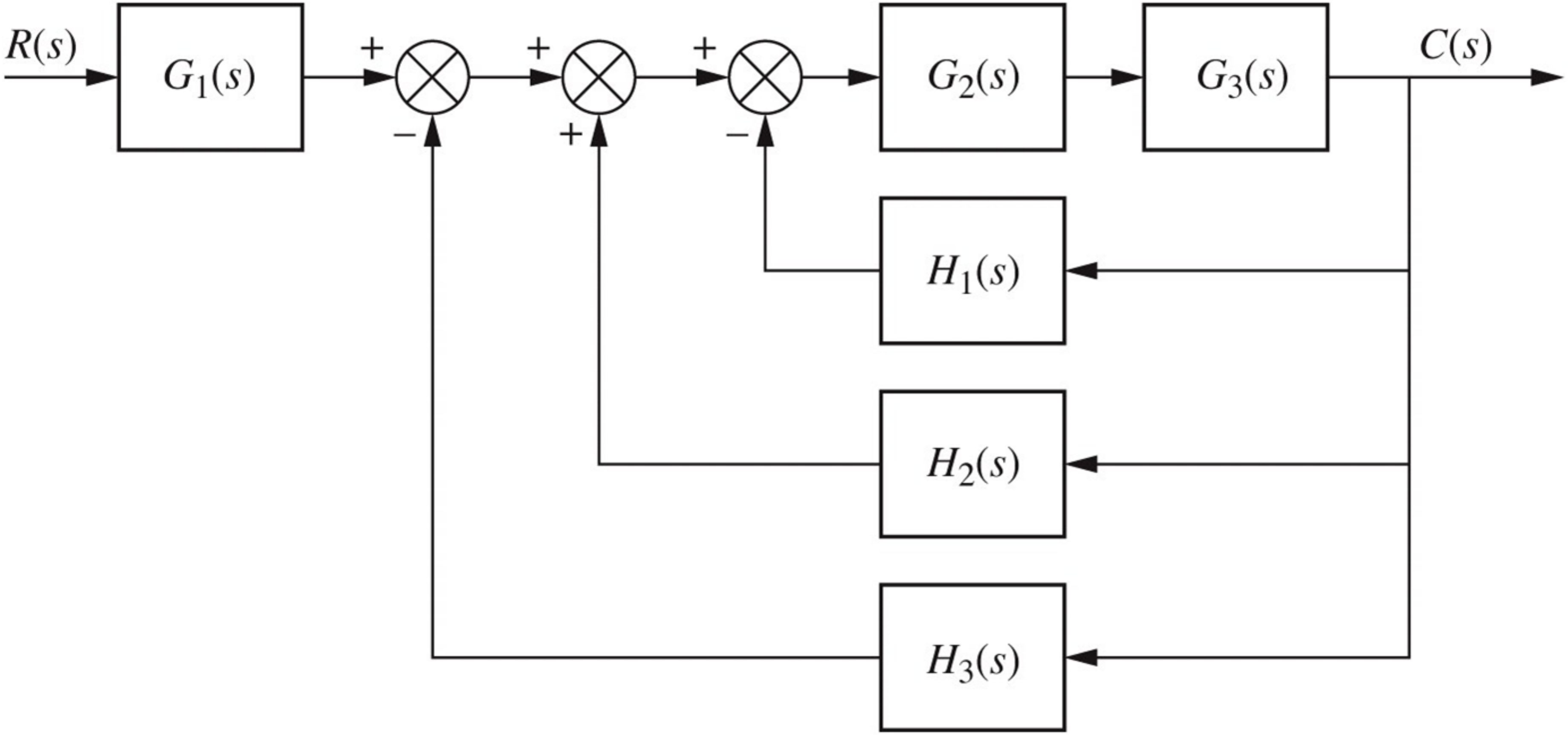
(a)

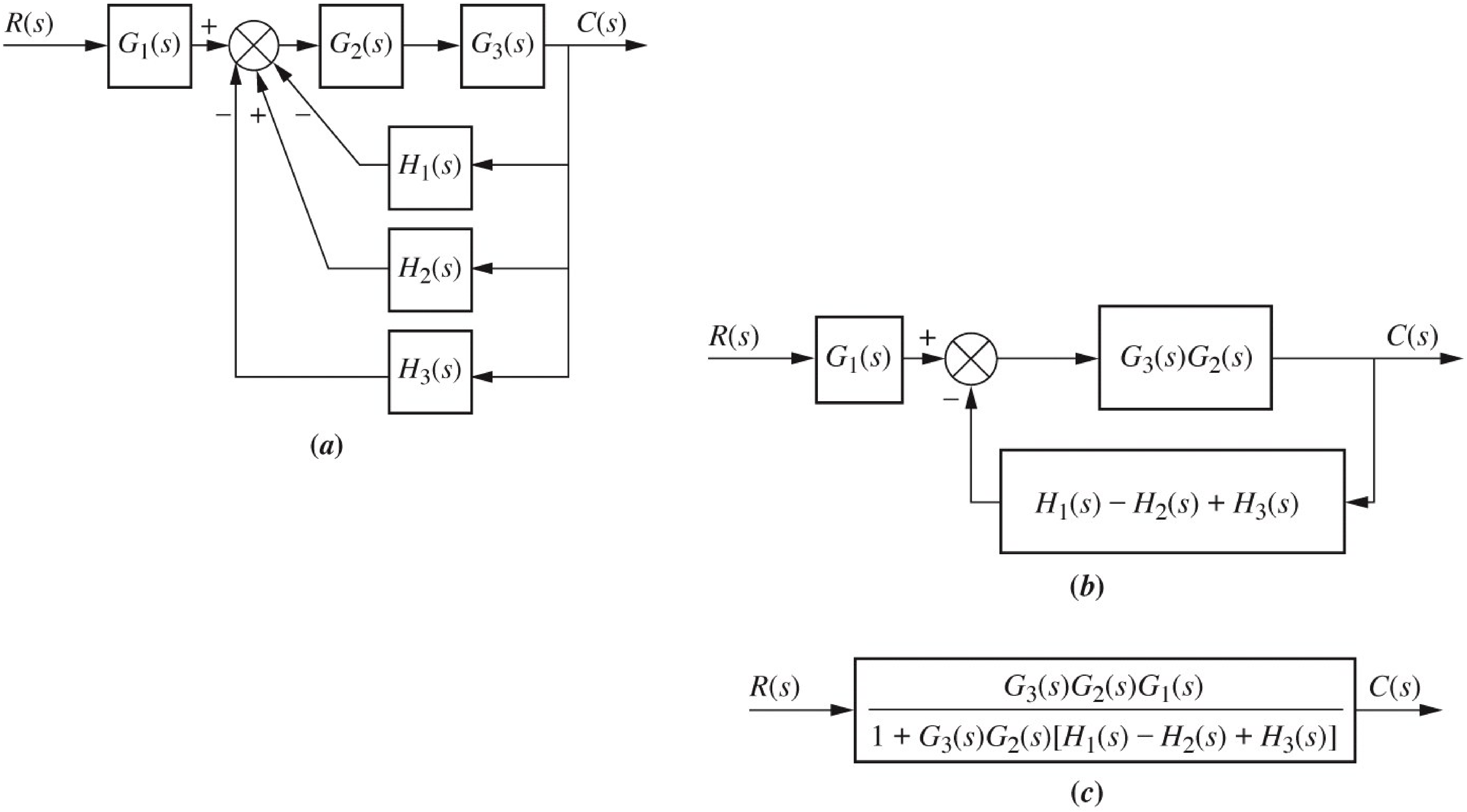
Moving a block to the right past a **pickoff point**



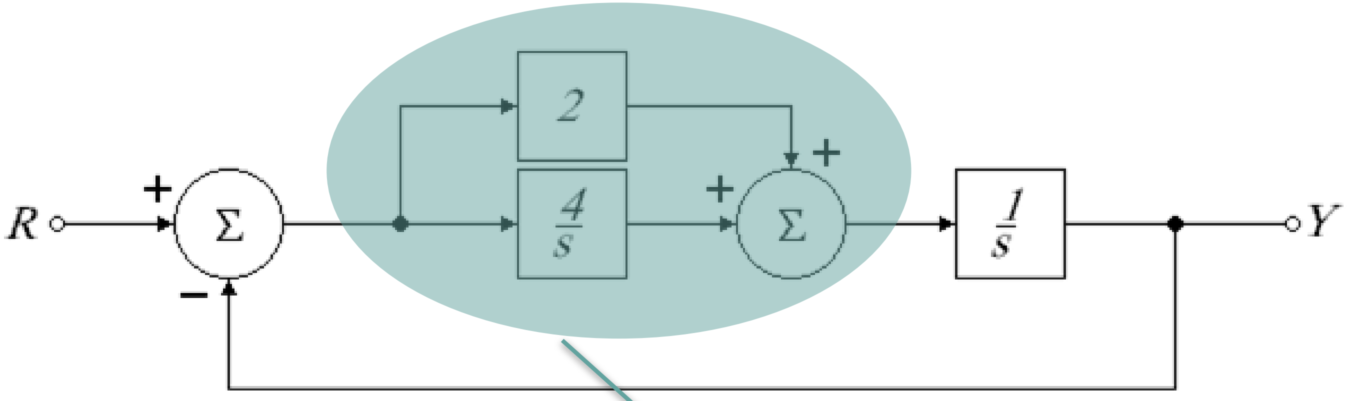
(b)

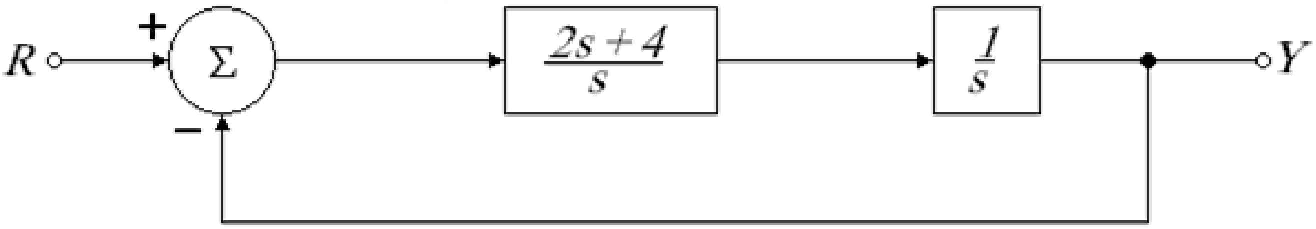
Problem: *Reduce the block diagram to a single transfer function*




R(s)
G1(s)
G2(s)
G3(s)
C(s)
H1(s)
H2(s)
H3(s)
(a)
R(s)
G1(s)
G3(s)G2(s)
C(s)
H1(s) − H2(s) + H3(s)
(b)
R(s)
G3(s)G2(s)G1(s) / 1 + G3(s)G2(s)[H1(s) − H2(s) + H3(s)]
C(s)
(c)

Problem: *Reduce the block diagram to a single transfer function*

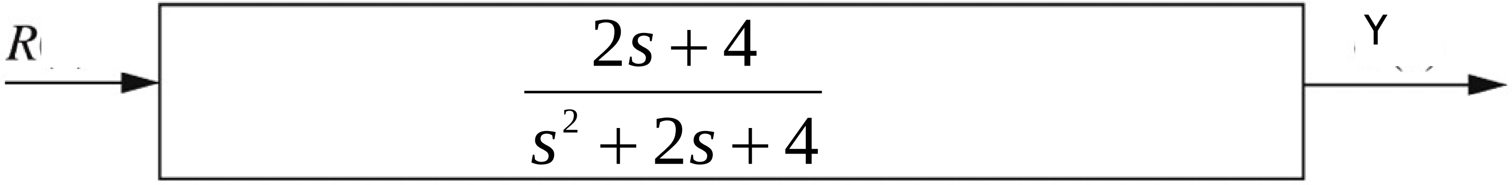




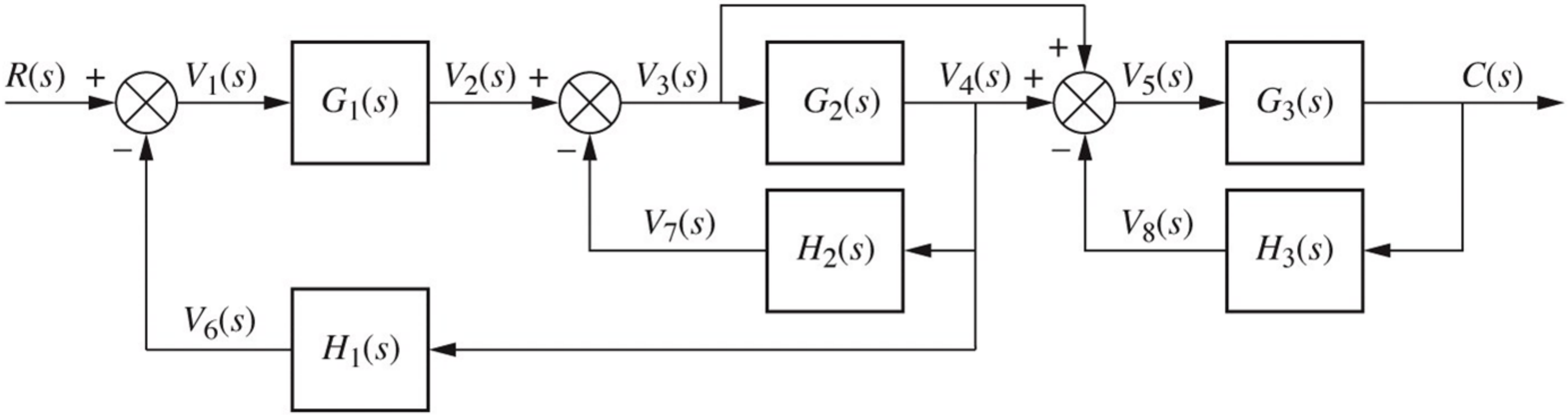
$$T(s) = \frac{Y(s)}{R(s)}$$

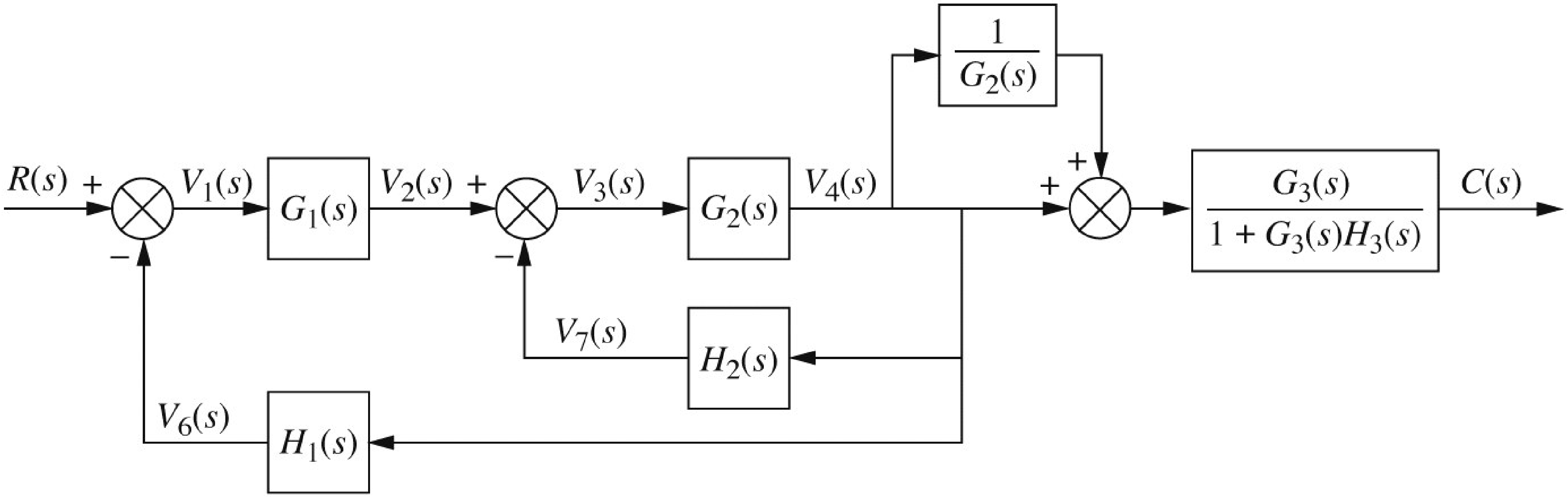
$$T(s) = \frac{\dfrac{2s+4}{s^2}}{1+\dfrac{2s+4}{s^2}}$$

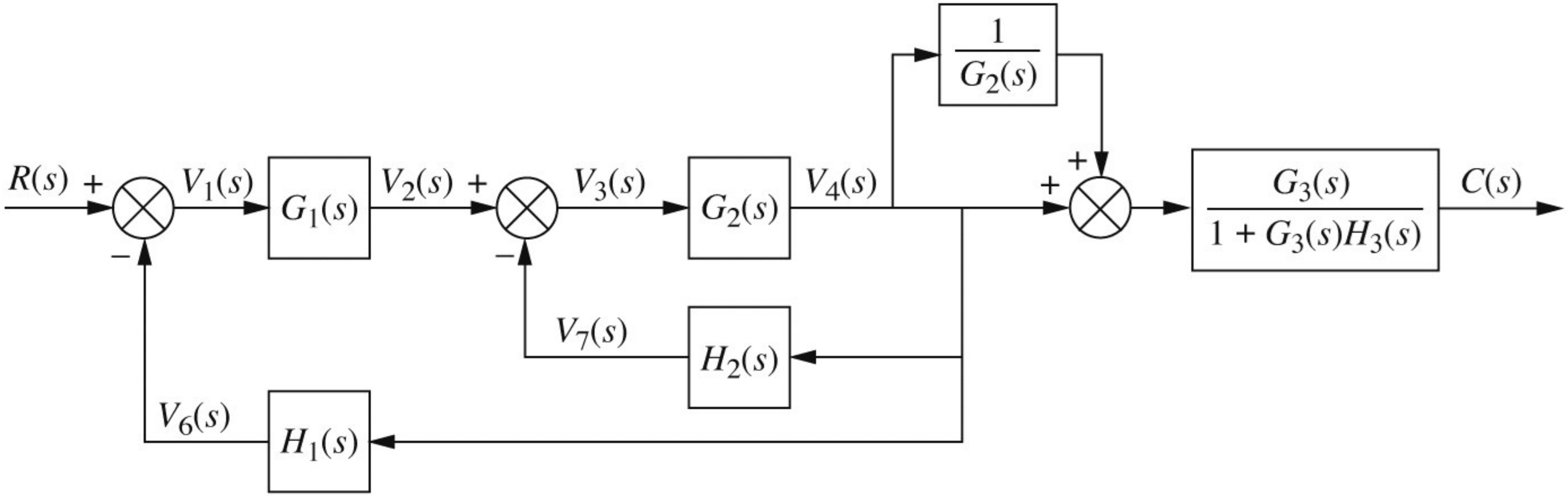
$$T(s) = \frac{2s+4}{s^2+2s+4}$$



Problem: *Reduce the block diagram to a single transfer function*

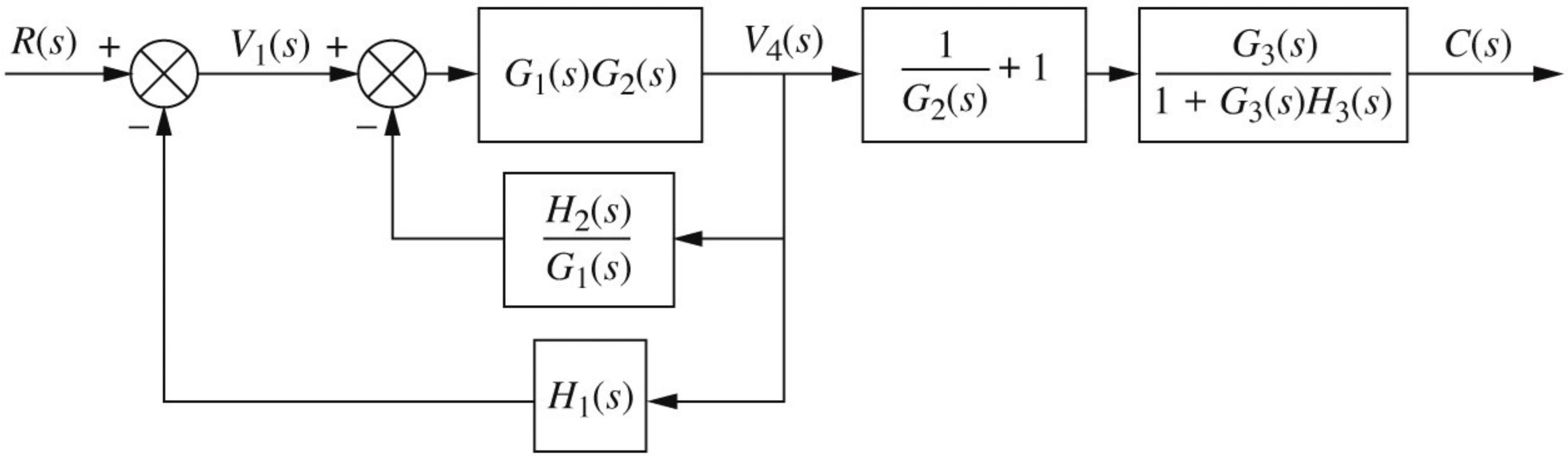






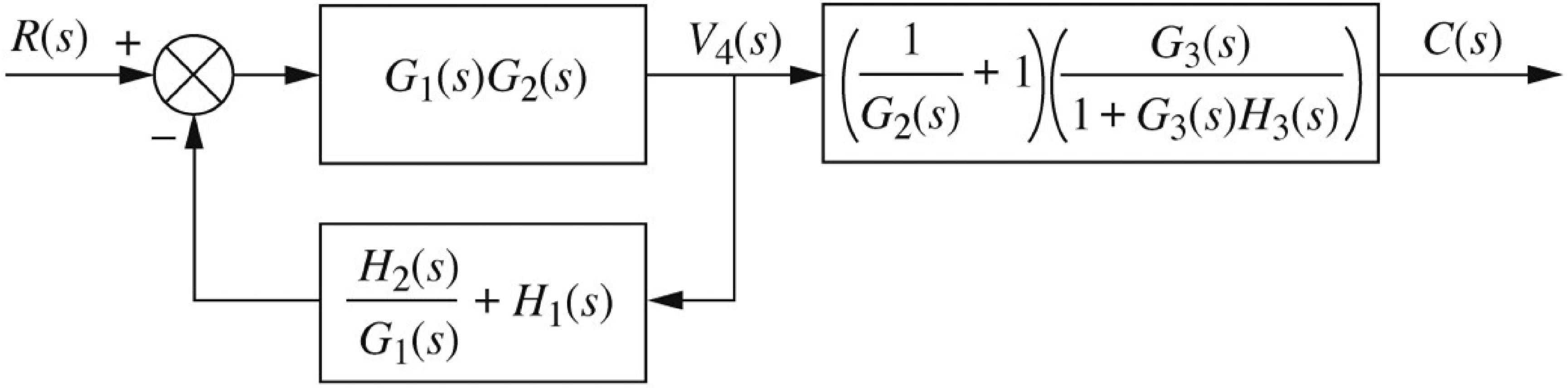
$R(s)$
$V_1(s)$
$G_1(s)$
$V_2(s)$
$V_3(s)$
$G_2(s)$
$V_4(s)$
$\frac{1}{G_2(s)}$
$\frac{G_3(s)}{1 + G_3(s)H_3(s)}$
$C(s)$
$V_7(s)$
$H_2(s)$
$V_6(s)$
$H_1(s)$

(a)

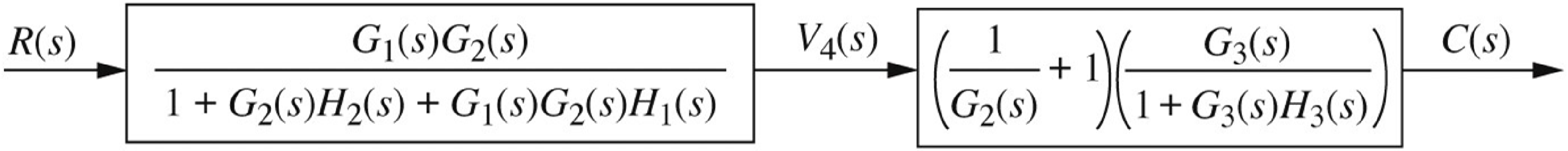


$R(s)$
$V_1(s)$
$G_1(s)G_2(s)$
$V_4(s)$
$\frac{1}{G_2(s)} + 1$
$\frac{G_3(s)}{1 + G_3(s)H_3(s)}$
$C(s)$
$\frac{H_2(s)}{G_1(s)}$
$H_1(s)$

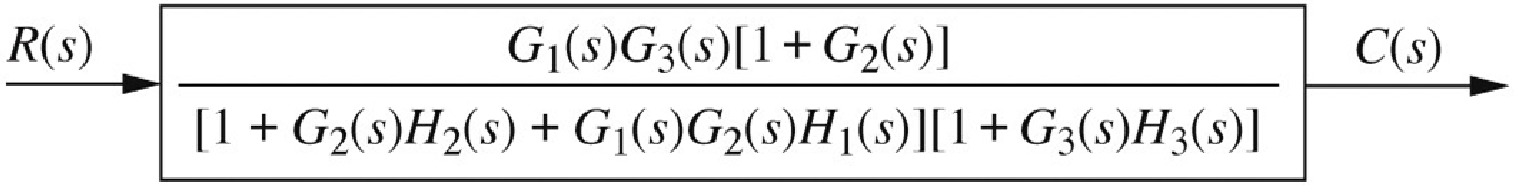
(b)


$R(s)$
+
−
$G_1(s)G_2(s)$
$V_4(s)$
$\left(\frac{1}{G_2(s)}+1\right)\left(\frac{G_3(s)}{1+G_3(s)H_3(s)}\right)$
$C(s)$
$\frac{H_2(s)}{G_1(s)}+H_1(s)$

(*c*)

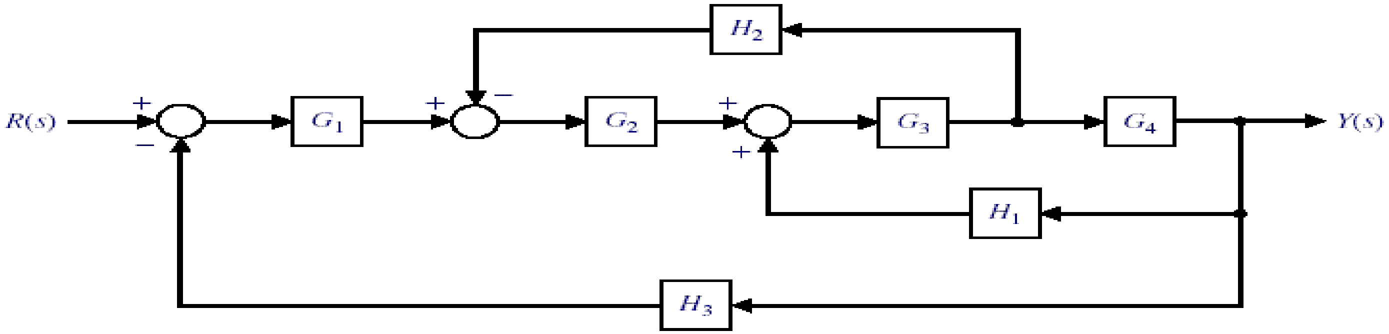

$R(s)$
$\frac{G_1(s)G_2(s)}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)}$
$V_4(s)$
$\left(\frac{1}{G_2(s)}+1\right)\left(\frac{G_3(s)}{1+G_3(s)H_3(s)}\right)$
$C(s)$

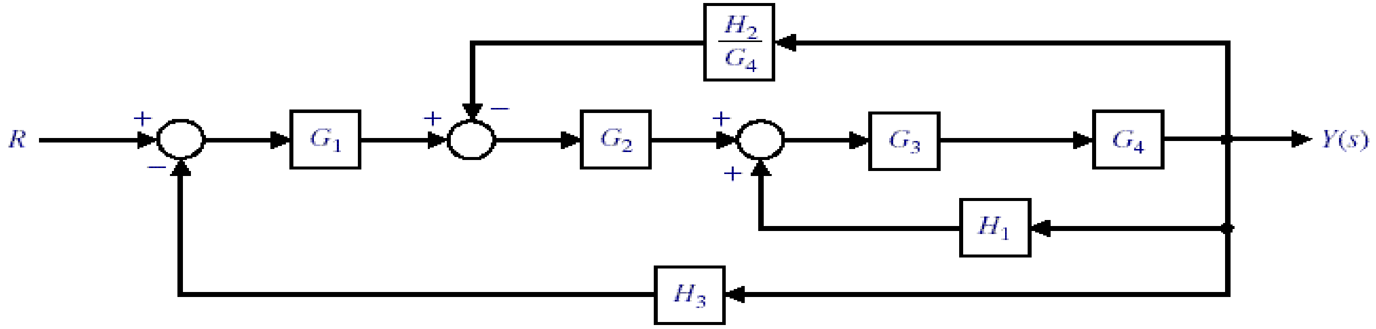
(*d*)


$R(s)$
$\frac{G_1(s)G_3(s)[1+G_2(s)]}{[1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)][1+G_3(s)H_3(s)]}$
$C(s)$

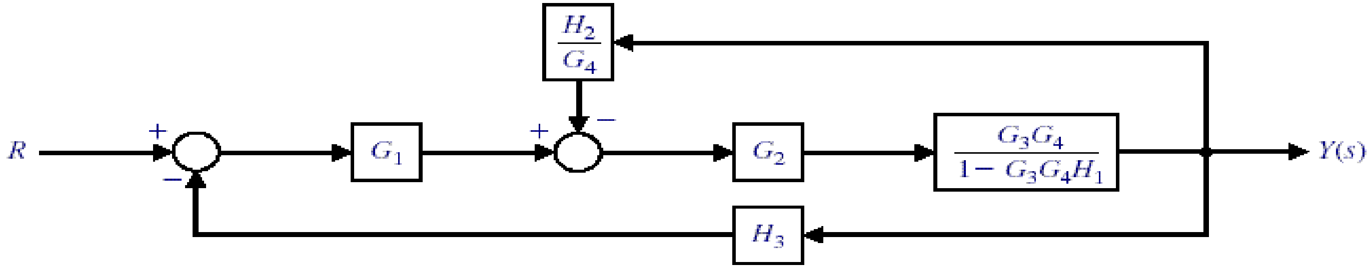
(*e*)

Problem:
Reduce the block diagram to a single transfer function

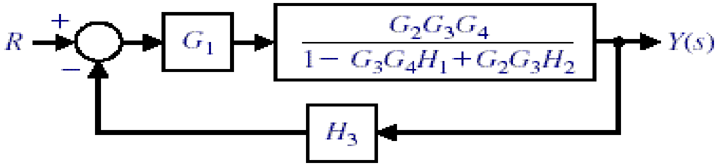




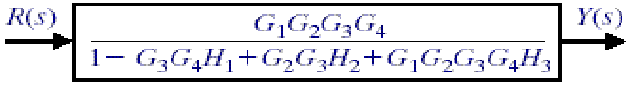
(a)



(b)



(c)



(d)