Continuous linear time-frequency transforms in the analysis of fusion plasma transients

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Introduction - Conclusions

Continuous linear time-frequency transforms [1,2] have a long history and continued popularity in the analysis of fusion plasma transients**.** Uncertainty estimation and application in complex analysis methods require discretion.

Conclusions:

- Invariance properties make short-time Fourier transform (STFT) and analytical wavelet transform (CWT) **ideal for analyzing transients with short-lived harmonic components**.
- STFT and CWT use the same type of basis functions. **Choice between them is determined by the required invariance properties**.
- Uncertainty estimation of real and imaginary parts is straightforward. However, **Gaussian error propagation breaks down** for non-linear derived quantities (e.g. **amplitude and phase**) and **low energy components**.
- STFT and CWT can form **basis for advanced methods**: coherence, bicoherence, mode number determination, ... Care must be taken to **preserve the invariance properties**.
- Averaging is critical for cross-transform based methods: **number of quasi-independent averages has to be uniform, quasi-stationarity of the averaged interval** is important.
- **Bicoherence requires frequency shift invariance** making **STFT** a good choice for the transform.

Standard methods - definitions

Linear time-frequency transform:

$$Tx(u,\xi) = \int_{-\infty}^{\infty} x(t) g_{u,\xi}^{*}(t)\, dt$$

Continuous analytical wavelet transform (CWT):

$g_{u,\xi}(t) = \frac{1}{\sqrt{s}} \Psi\left(\frac{t-u}{s}\right)$, where $s = \frac{\nu}{\xi}$, and Ψ is the Morlet wavelet.

$P_T x(u,\xi) = |Tx(u,\xi)|^2$ ⟶ Scalogram

Short-time Fourier transform (STFT):

$g_{u,\xi}(t) = e^{i\xi t} g(t-u)$, where g(t) is the Gaussian function.

$P_T x(u,\xi) = |Tx(u,\xi)|^2$ ⟶ Spectrogram

Time-frequency coherence [3]:

$$COH_{x,y}(u,\xi) = \frac{\left|\left\langle Tx(u,\xi)\overline{Ty(u,\xi)}\right\rangle\right|}{\sqrt{\left\langle P_T x(u,\xi)\right\rangle \left\langle P_T y(u,\xi)\right\rangle}}$$

<.> Stands for averaging
Without averaging: $COH_{x,y}(u,\xi) = 1$
With averaging independent values: $COH_{x,y}(u,\xi) \to 0$

Time-frequency minimum coherence [3]:
For longer averaging, the coherence estimate slowly approaches the real value from above, but the standard deviation of the estimate is higher for lower real coherence values. Thus choosing the minimum calculated coherences, we are able to separate the coherent and incoherent structures with reasonable temporal resolution.

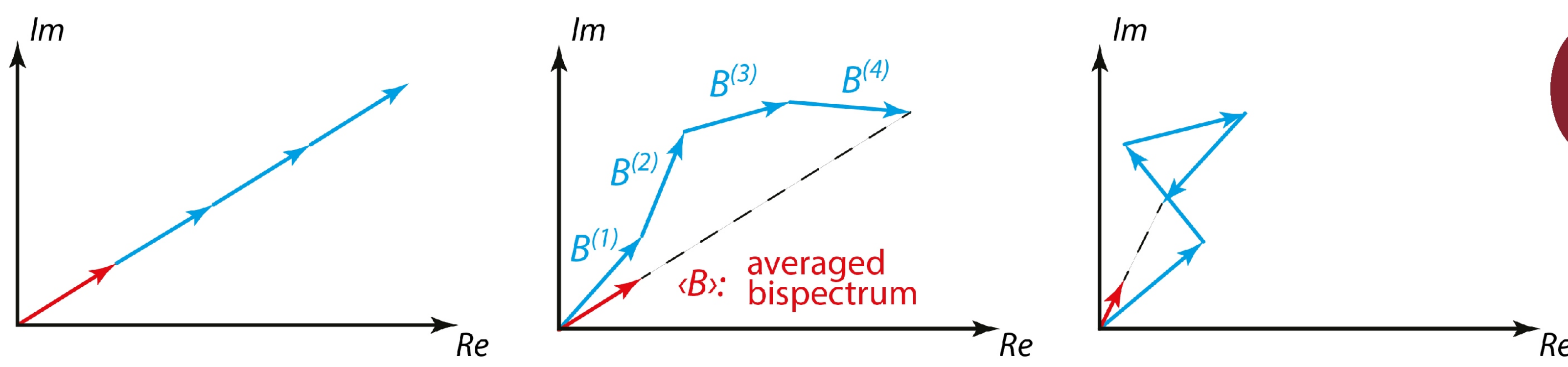
Toroidal mode numbers [4]:
Cross-phase: $\Theta_{x,y}(u,\xi) = \arg\left(Tx(u,\xi)\overline{Ty(u,\xi)}\right)$

For $\Phi_{x,y}$ being the relative position of the probes:

$$Q_n(u,\xi) = \sum_{x,y} \left\| \Theta_{x,y}(u,\xi) - n\Phi_{x,y} \right\|_{2\pi}^2$$

Averaging in transients

Process of **averaging of cross-transforms** (in e.g. coherence and bicoherence) is **analogous to random walk on the complex plane**.
Time shift invariant averaging by **smoothing with a kernel having invariance of the transform**.
Quasi-stationarity is highly important to avoid biasing the result.



(a) No phase changes *(b) Small phase jitter* *(c) Large phase jitter*

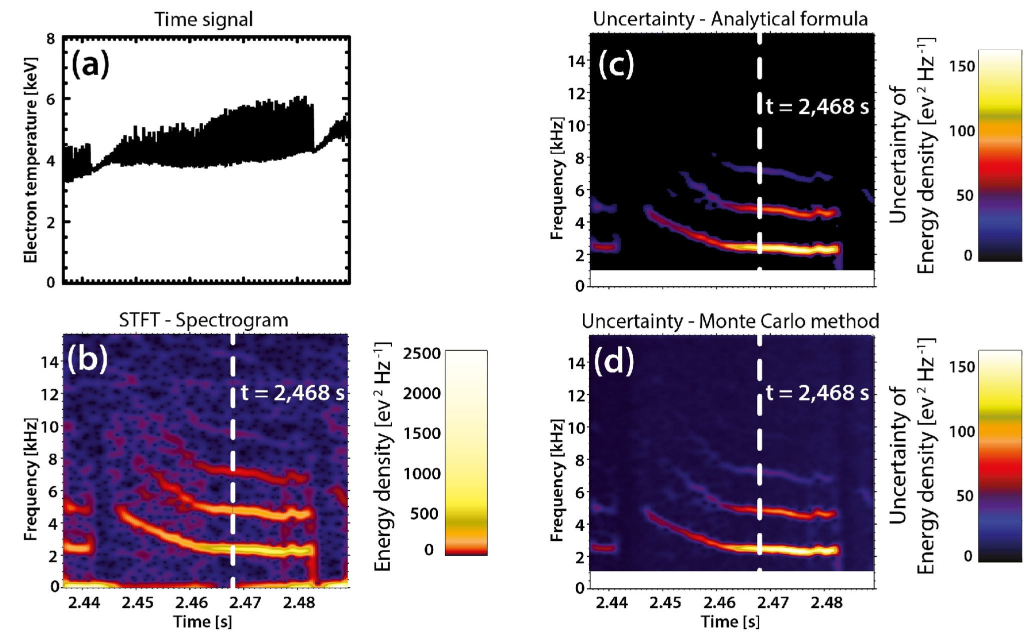
Uncertainty estimation

Uncertainty of real and imaginary part is analogous to Fourier transform [5]

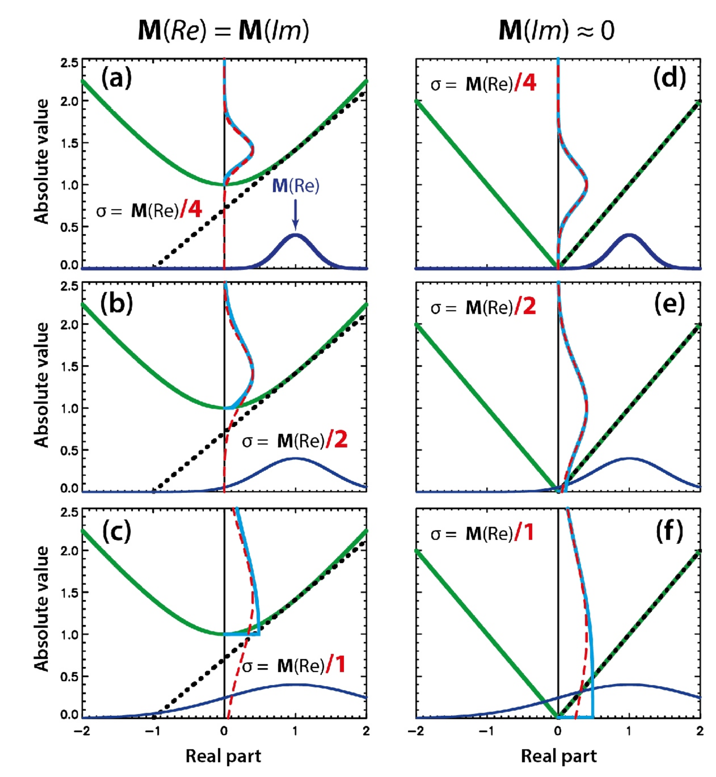
$$\Delta\Re\{F(u)\}^2 = \\ = \sum_{x=0}^{N-1} \left(\frac{\partial \Re\{F(u)\}}{\partial f_x}\right)^2 \Delta f_x^2 = \frac{1}{N^2} \sum_{x=0}^{N-1} \cos(2\pi ux/N)^2\, g_x^2\, \Delta f_x^2$$

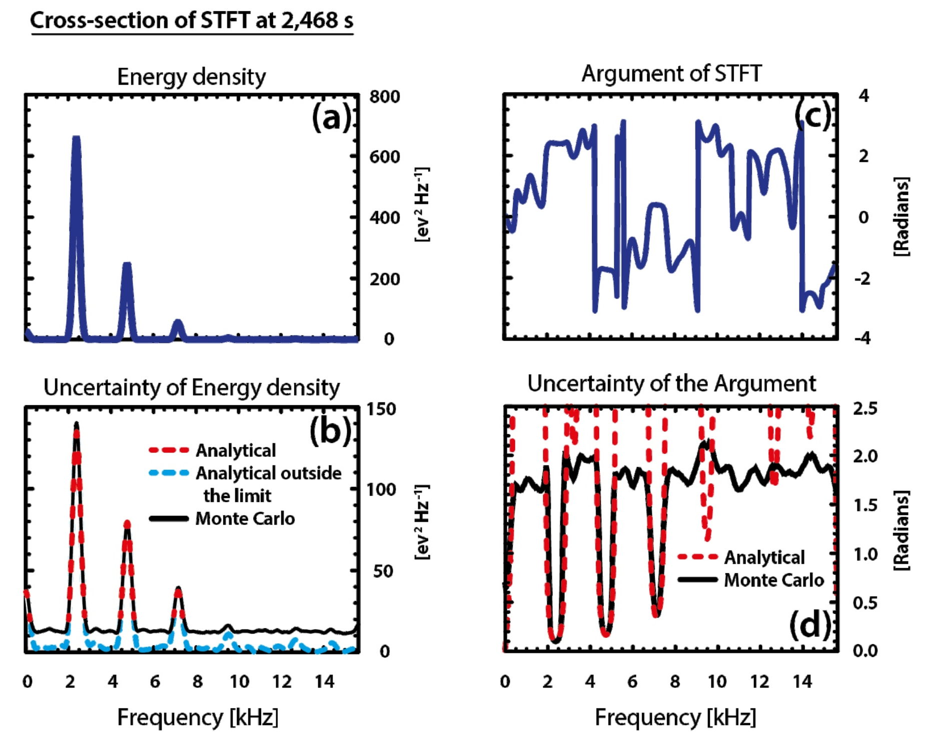
$$\Delta\Im\{F(u)\}^2 = \frac{1}{N^2} \sum_{x=0}^{N-1} \sin(2\pi ux/N)^2\, g_x^2\, \Delta f_x^2$$

Uncertainty of amplitude and phase can be calculated with **Gauss error propagation for high energy modes**. Otherwise the distorted distribution must be taken into account.



Example of mode characterization is shown for ASDEX ECE signals [6].





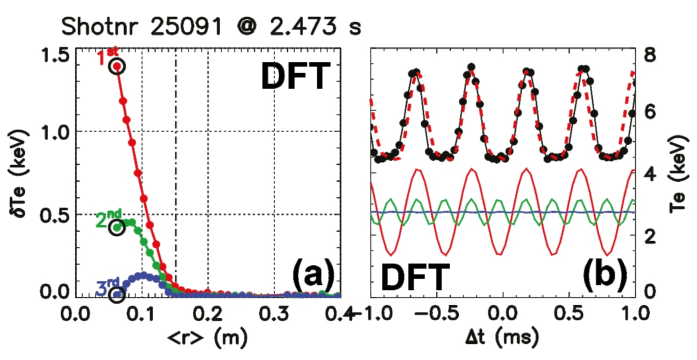


Figure courtesy of Marco Sertoli [6]

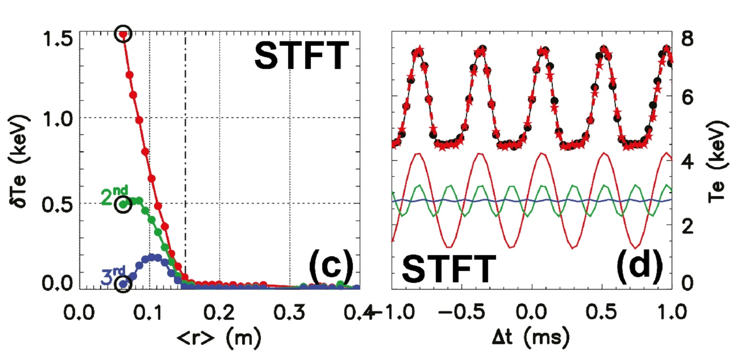


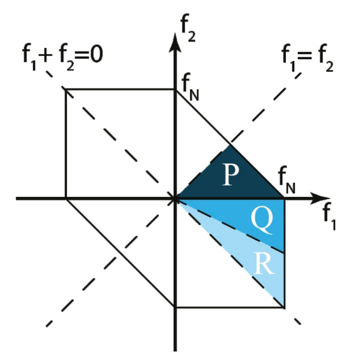
Figure courtesy of Marco Sertoli [6]

Time-frequency bicoherence

Bicoherence detects phase coupling between two frequencies and their sum. Can be adapted to transients by using **STFT as a transform**. (Frequency invariance is important!)

$$b(f_1,f_2,t) = \frac{|B(f_1,f_2,t)|}{\mathbb{E}\left[|STFT(f_1,t)STFT(f_2,t)|^2\right]^{1/2} \mathbb{E}\left[|STFT(f_1+f_2,t)|^2\right]^{1/2}}$$

$$B(f_1,f_2,t) = \mathbb{E}[STFT(f_1,t)STFT(f_2,t)STFT^*(f_1+f_2,t)]$$

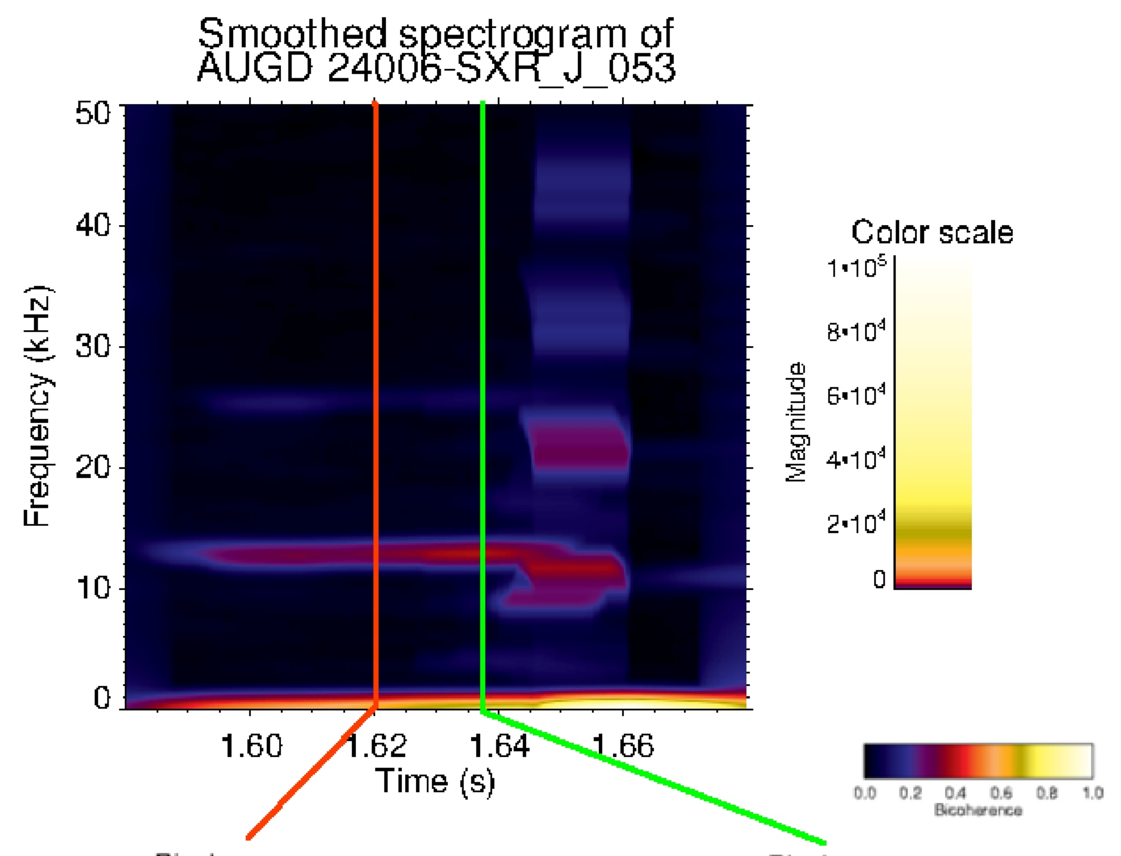


The definition two parts, the absolute value of the averaged bispectrum in the numerator and the normalizing factor in the denominator both containing products of the STFT of the signal.

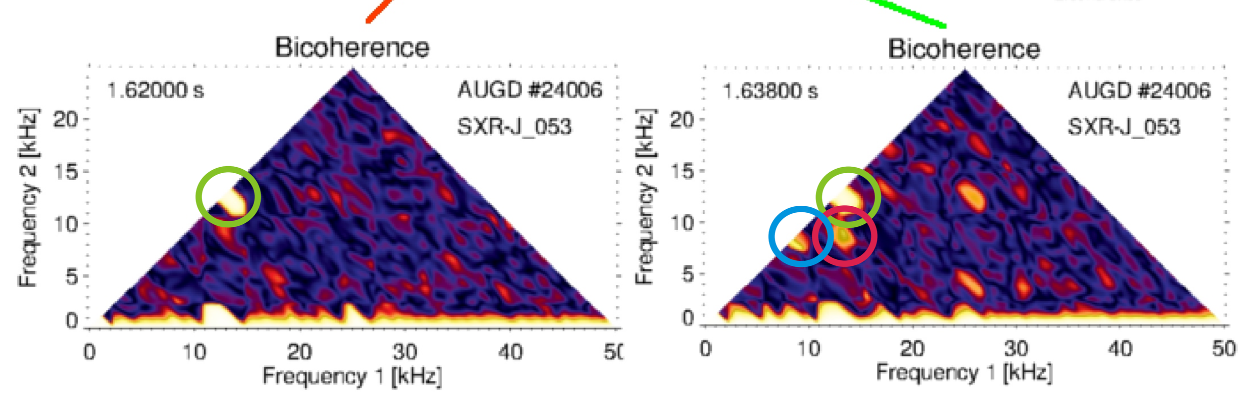
Due to symmetries, enough to plot for region "P" [7].

Example of time-frequency bicoherence is shown for ASDEX SXR signals [8].

Time region effected by strong transient well recognized on smoothed spectrgram - few effective averages **bias bicoherence towards 1**.



Quasi-stationary intervals selected for analysis:
1. **Dominant kink mode shows bicoherence with its harmonic.**
2. **Dominant kink mode shows bicoherence with low frequency sawtooth precursor** (LFSP). **LFSP also has harmonics.**



Acknowledgements

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References

[1] D. Gabor, Journal of the Institute of Electric Engineers 93, 429 (1946)
[2] S. Mallat, A wavelet tour of signal processing, Academic Press (2009)
[3] G. Pokol, et. al., ECA 34A, P5.129 (2010)
[4] G. Pokol, et al., AIP Conference Proceedings 993, 215 (2008)
[5] L. Horvath, et. al., Short Time Fourier Transform in NTI Wavelet Tools, Technical Report BME-NTI-593/2012, https://deep.reak.bme.hu/documents/5 (2012)
[6] M. Sertoli, et. al., Nuclear Fusion 53, 053015 (2013)
[7] Y.C. Kim, E.J. Powers, Plasma Science IEEE Transactions 7, 120 (1979)
[8] G. Papp, et. al., Plasma Physics and Controlled Fusion 53, 065007 (2011)