Continuous linear time-frequency transforms in the analysis of fusion plasma transients

G. I. Pokol[1], L. Horvath[1], N. Lazanyi[1,2], G. Papp[1,3], G. Por[1], V. Igochine[2], ASDEX Upgrade team[2]

[1] *Institute of Nuclear Techniques, Budapest University of Technology and Economics, Association EURATOM, Budapest, Hungary*

[2] *MPI fur Plasmaphysik, Euratom-Association, Garching, Germany*

[3] *Department of Applied Physics, Nuclear Engineering, Chalmers University of Technology and Euratom-VR Association, Gothenburg, Sweden*

Introduction There is a long history of the use of continuous linear time-frequency transforms in the analysis of transients detected in fusion plasma devices [1]. Despite the fact that numerous alternative methods of time-frequency analysis were proposed during the years, Fourier transform based solutions are still the standard method to approach transient wave-like phenomena. The reason for this continued popularity is that these linear time-frequency transforms do not produce any disturbing interference patterns between the time-frequency atoms, which are eigenfunctions of linear and quasi-linear theories [2]. This paper concentrates on continuous transforms that are time-shift invariant, thus ideal for the analysis of transient signals.

The two well-known types of continuous linear time-frequency transforms, namely the short-time Fourier transform (STFT) and the continuous wavelet transform with analytical wavelets (CWT), differ basically in their invariance properties, which determines their optimal field of use. Uncertainty estimation of transform values and derived quantities, like energy density distributions and phases, is also addressed shortly. Finally, the paper presents some advanced methods based on the time-frequency transforms that have been implemented in the recently developed NTI Wavelet Tools package [3] with practical fusion plasma applications. A mode number determination routine is a main feature that is based on fitting mode phases [4]. Time-frequency coherence and transfer functions are introduced briefly, and time-frequency bicoherence is discussed concentrating on consequences of the invariance properties of the transforms used.

Continuous linear time-frequency transforms and their uncertainty Continuous linear time-frequency transforms can be derived from a more general family produced by smoothing the Wigner-Ville distributions of the signals [5]. Their linearity makes them unique in the sense that no non-linear interference patterns are produced between components of composite signals. Two examples are the STFT and the CWT differing only in their invariance properties: the STFT is frequency-invariant, while CWT is scale invariant – both are time-invariant, too. They are well-suited for studying transient signals, and their invariance properties determine their specific field of use: If it is need to resolve fluctuations with frequencies extending throughout many orders of magnitude, CWT should be used with logarithmic frequency axis. On the other hand, STFT is most often used to study fluctuations in a limited frequency range because of its easier interpretation.

Uncertainty of transform values is of central importance if one wants to correctly interpret the results. For the real and imaginary parts of STFT and CWT this can be performed in a way similar to the uncertainty estimation of Fourier transform [3]. Care must be taken, however, when the uncertainty of the energy density distributions (or amplitudes) and phases are considered. These are non-linear functions of the real and imaginary parts of the transforms and for low expected value and large uncertainty the linearity condition behind the Gaussian error propagation breaks down as illustrated in the bottom plot of Fig. 1. Green curve is the absolute value as function of the real part while the imaginary part is kept fix. The real part has a distribution with variance increasing towards the bottom shown by the dark blue curve. The distribution of the amplitude calculated by Gauss error propagation is shown by red dashed curves, and the real one calculated by transformation of the distribution function by light blue curves. Black dashed line indicates the linear fit used by the Gauss error propagation formula. Deviation of the red dashed line from the light blue on indicates breakdown of Gaussian error propagation and a significant deviation form Gaussian distribution.

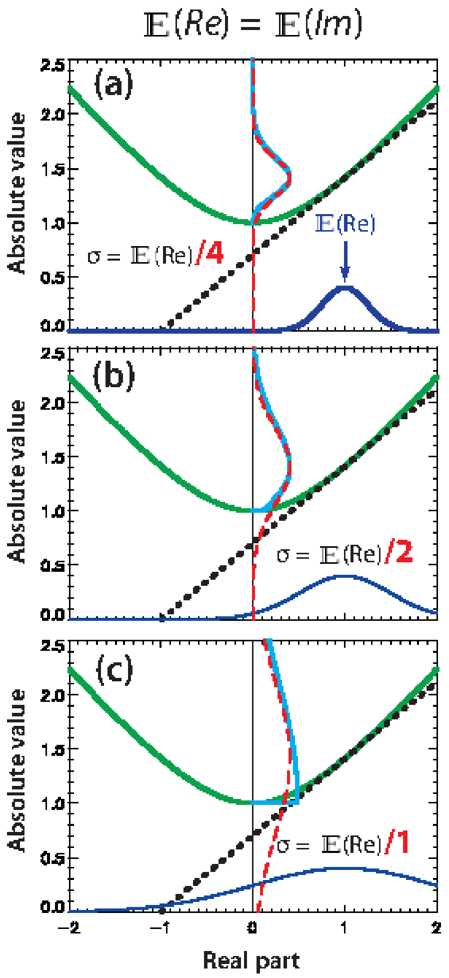

Figure 1: *Illustration of the error propagation on the absolute value of the Fourier transform.*

In report [3] a threshold in the ratio of terms of series expansion of the transformed distribution function for the applicability of Gaussian error propagation is proposed. Amplitude and phase estimation of STFT was used e.g. to reconstruct the changing core mode properties from electron cyclotron measurements at the ASDEX Upgrade tokamak [6].

Applications in transient analysis The results of linear continuous time frequency transforms can be used to derive physical quantities of transient modes, however, one should be careful to preserve the invariance properties of the transforms throughout the analysis. This means that the necessary averaging shall be carried out by convolution smoothing by a frequency-invariant kernel for STFT and a scaled kernel for CWT.

Using this averaging procedure a number of advanced signal processing methods were developed, including mode number fitting [4, 7], wavelet coherence and wavelet minimum coherence [8], time-frequency transfer function and STFT bicoherence. In the present paper, I am concentrating on the STFT bicoherence but pointing out general considerations whenever applicable.

There are many considerations general to all the above mentioned methods: all of them involve application of operations defined for stationary signals for short-time averages of the transforms. However, bicoherence is unique in the sense that it compares two different frequencies typically of the same signal.

$$b(f_1,f_2,t) = \frac{|B(f_1,f_2,t)|}{\mathbb{E}\left[|STFT(f_1,t)STFT(f_2,t)|^2\right]^{1/2}\mathbb{E}\left[|STFT(f_1+f_2,t)|^2\right]^{1/2}}. \tag{1}$$

The definition of Eq. (1) consists of essentially two parts, the absolute value of the averaged bispectrum $B(f_1,f_2,t) = \mathbb{E}[STFT(f_1,t)STFT(f_2,t)STFT^*(f_1+f_2,t)]$ in the numerator and the normalizing factor in the denominator both containing products of the STFT of the signal. The normalizing factor ensures that the bicoherence values are always between 1 and 0, but the more important part is the numerator which is to detect phase coupling between two frequencies and their sum [9]. Similarly to the wavelet coherence, key factor for interpretation is the number of quasi-independent averages of the complex bispectrum values. Attempts have been made to design a time-frequency bicoherence method based on CWT with frequency-invariant averaging kernel [10]. However, this makes interpretation very cumbersome, as for low frequencies the number of quasi-independent averages tends to zero, producing an artifact of high bicoherence. The scale invariant filtering used in wavelet coherence [8] cannot be used here, as it would blur the time localization in a complicated way. The solution is to use STFT as a basis of time-frequency bicoherence, as it is already frequency-invariant.

STFT bicoherence is shown in Fig. 2 for the sawtooth precursor which has been studied in paper [7] with ordinary bicoherence. In the present analysis frequency-invariant averaging of 30 consecutive quasi-independent bispectrum values was performed, which produced a smooth evolution of the bicoherence values. Time-varying bicoherence is a function of three variables (frequency 1, frequency 2 and time), so Fig. 2 shows only two time points. At the first time instance, the dominant $(n,m) = (1,1)$ kink mode is already strongly present at 13 kHz, and shows high bicoherence with its second harmonic at 26 kHz. This harmonic also has

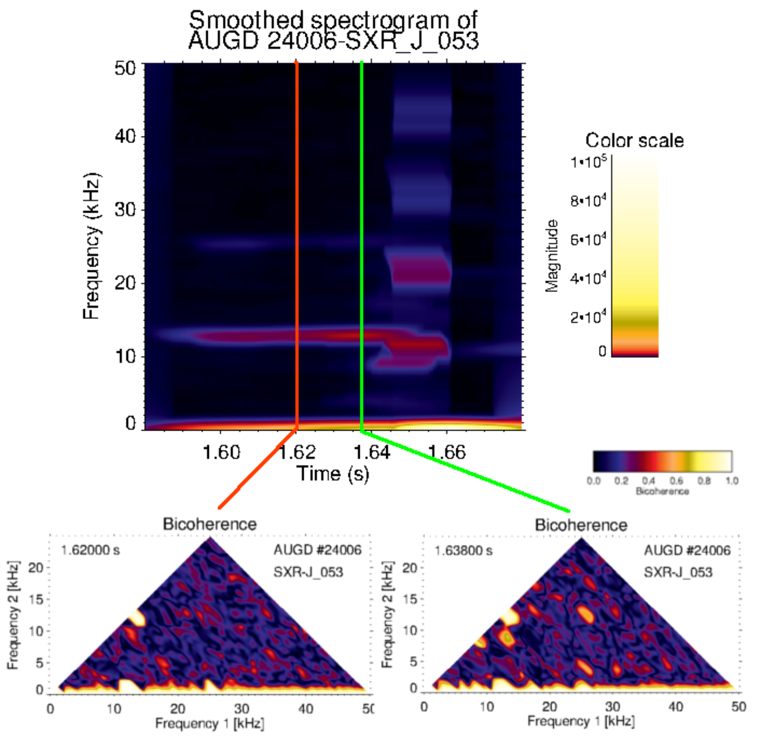

Figure 2: *STFT bicoherence of sawtooth precursor oscillations: top plot shows the smoothed spectrogram, while bottom plots show STFT bicoherence at two time points.*

high coherence with the basic frequency. At the second time instance the low frequency sawtooth precursor (LFSP) studied in paper [7] starts to gain energy at 9 kHz, and has high bicoherence with the dominant mode. Closer to the sawtooth crash bicoherence shows high values in extended regions, however here the signal components change rapidly within the averaged time interval which invalidates our results. It is important to keep in mind that if the signal components are not quasi-stationary within the averaged time interval in the sense that a few short-lived bursts dominate, then the averaging has practically no effect and the bicoherence approaches 1. The smoothed spectrogram plot of Fig. 2 allows easy recognition of such problems. The artifact below 2 kHz is a result of STFT not being able resolve low frequencies with period time in the order of window size; this region should be simply excluded from the interpretation. Interpretation of time-frequency coherence of two signals is completely analogous in that sense.

Conclusions Continuous linear time-frequency transforms (STFT and CWT) offer a basis to design powerful tools for analyzing transient signals. Care must be taken, however, to preserve the invariance properties of the transforms throughout the processing chain and in the end evaluate the results in view of the size of the total smoothing introduced. Even advanced methods, like bicoherence, could be adopted for these transforms, however, the necessary smoothing limits its applicability in case of rapidly changing transient signals.

Acknowledgements This work was partly funded by the European Commission under Association Contract between EURATOM and HAS. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] V. Dose, et. al., Physics of Plasmas **4**, 323 (1997)

[2] D. Gabor, Journal of the Institute of Electric Engineers **93**, 429 (1946)

[3] L. Horvath, et. al., Short Time Fourier Transform in NTI Wavelet Tools, Technical Report BME-NTI-593/2012, https://deep.reak.bme.hu/documents/5 (2012)

[4] G. Pokol, et. al., AIP Conference Proceedings **993**, 215 (2008)

[5] S. Mallat, A wavelet tour of signal processing, Academic Press (2009)

[6] M. Sertoli, et. al., Nuclear Fusion **53**, 053015 (2013)

[7] G. Papp, et. al., Plasma Physics and Controlled Fusion **53**, 065007 (2011)

[8] G. Pokol, et. al., ECA **34A**, P5.129 (2010)

[9] Y.C. Kim, E.J. Powers, Plasma Science IEEE Transactions **7**, 120 (1979)

[10] T. Dudok de Wit, The URSI Review on Radio Science 1993-1995 **CPT-96**, 3369 (1996)