Diffraction of light by ultrasonic waves

A. Simankovich D. Dedkov

Moscow Institute of Physics and Technology

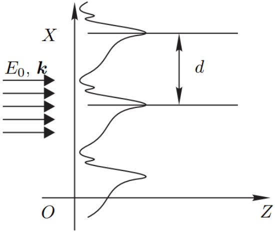
Abstract

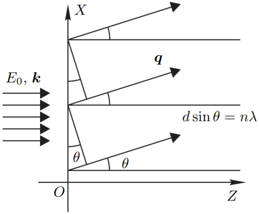
Acoustic waves in liquids cause density changes with spacing determined by the frequency and the speed of the sound wave. TODO TODO To determine the speed of sound in various liquids at room temperature To determine the compressibility of the given liquids

Introduction

Diffraction on periodic structures

TODO: CENTERING





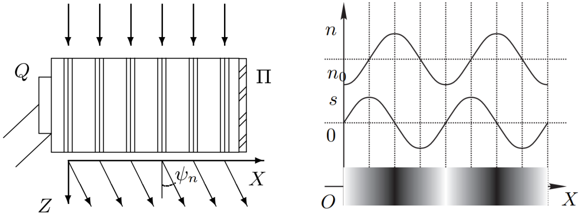
General theory shows that when a plane light wave is normally incident on the grating, the diffracted light has maximum at diffraction angles:

$$d \sin \psi_m = m\lambda, \ (m = 1, 2, ...)$$

$< WAT >$ However, especially the methods of obtaining such structures that are of practical interest.
$< \backslash WAT >$

Diffraction by ultrasonic waves

TODO: CENTERING



One example of such system is **ultrasonic waves grating**. Acoustic waves in liquids cause density changes with spacing determined by the frequency and the speed of the sound wave. Local changes in the water density lead to a change in the refractive index $n \approx n_0(1 + \cos \frac{2\pi}{\Lambda} x)$, where Λ - ultrasonic wavelength.

This forms a **phase diffraction grating**:

$$\varphi(x) = knL = \varphi_0(1 + a \cos \frac{2\pi}{\Lambda} x). \tag{1}$$

Velocity of the ultrasonic wave

If ν is the frequency of the $< WAT >$ crystal $< \backslash WAT >$, the velocity v of ultrasonic wave in the liquid is given by:

$$v = \nu \Lambda. \tag{2}$$

Thus, by measuring the angle of diffraction θ_n, the order of diffraction n, the light wavelength, the length of ultrasonic wave in the liquid can be determined and then, knowing the frequency of sound wave, its velocity v can be obtained.

Phase grating spatial structure observation

Observation of the spatial structure of the phase grating is complicated. The central problem is that phase grating intensity is **constant**. Indeed, complex transmittance has the following form:

$$t(x) = e^{i\varphi(x)},$$

with intensity

$$I(x) = |f_0(x)|^2 = 1.$$

$< WAT >$By the way$< \backslash WAT >$, there are some methods that allow you to observe the spatial structure.

Dark-field method

One of the most popular methods is called **dark-field method**. The idea behind is filtering the central maximum with using the screen, we get:

$$f_0(x) = e^{im\cos\Omega x} \approx 1 + im\cos\Omega x \overset{\textbf{filtering}}{=} im\cos\Omega x.$$

Then the intensity of the filtered light is the following:

$$I_f(x) = m^2\cos^2\Omega x = \frac{m^2}{2}(1 + \cos 2\Omega x) \neq 1.$$

$\cos 2\Omega x$ implies that distance between interference lines is half of grating period.
Another methods, such as **phase constrast method**, is based on the transition from a phase grating to an amplitude one.

Phase and Amplitude Grating with Uniform Beam

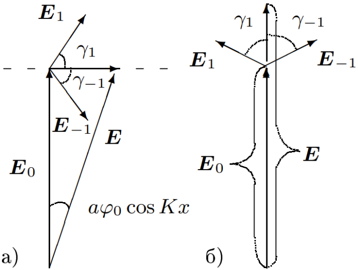


Figure: Phase (a) and amplitude (b) grating vector diagrams

The grating modulation functions are respectively (i - matters):

$$t(x) = e^{im\cos\Omega x} \approx 1 + \frac{im}{2}e^{i\Omega x} + \frac{im}{2}e^{-i\Omega x},$$

$$t(x) = 1 + m\cos\Omega x = 1 + \frac{m}{2}e^{i\Omega x} + \frac{m}{2}e^{-i\Omega x}.$$

Shifting the screen

In other words, we can see interaction of three waves with the following modulations: $1, \frac{im}{2}e^{i\Omega x}, \frac{im}{2}e^{-i\Omega x}$. This gives the following waves equations:

$$e^{ikz}, \frac{im}{2}e^{ik(z\cos\psi + x\sin\psi)}, \frac{im}{2}e^{ik(z\cos\psi - x\sin\psi)} \quad \left(\sin\psi = \pm\frac{\Omega}{k}\right)$$

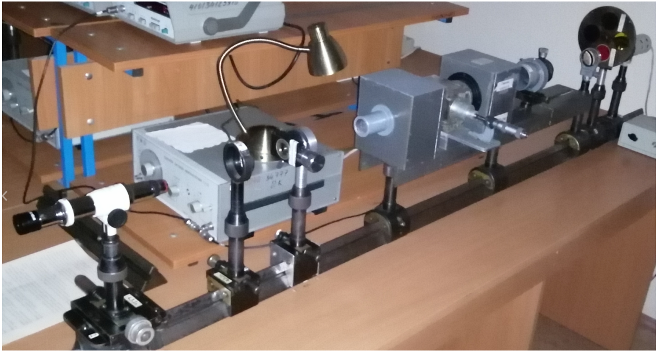
It follows from the above that there is a possibility of the phase-amplitude grating transition. Indeed, let the screen plane shift by Δz, this will result in the phase shift:

$$\text{Phase shift} = k\Delta z(1 - \cos\psi).$$

If $\Delta z = \frac{\pi}{2} + 2\pi n$, then the central wave E_0 has been rotated by $\frac{\pi}{2}$ and phase-amplitude transition has been occurred.

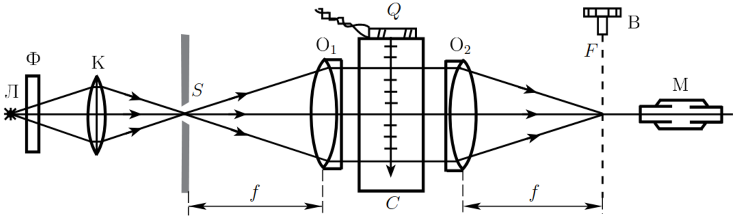
Measurements and Results

Experimental Setup



- $f = (30 \pm 0.1)$ cm – focal length of lenses
- $\lambda = (640 \pm 20)$ nm – light wave length (red)
- $\nu \in [1.0; 5.0]$ MHz – ultrasonic generator frequency range

Experimental Setup



- L - light source
- Φ - light filter
- K - condenser
- O_1, O_2 - lenses
- Q - cuvette
- B - measurement screw
- M - microscope

Light source L illuminates slit S through Φ and K. Parallel light beam passes through cuvette Q. Ultrasonic signal from generator is supplied to quartz piezoelectric plate. Light interacts with ultrasonic wave, resulting in interference pattern in focal plane of O_2. Pattern can be observed in microscope.

Observing diffraction lines

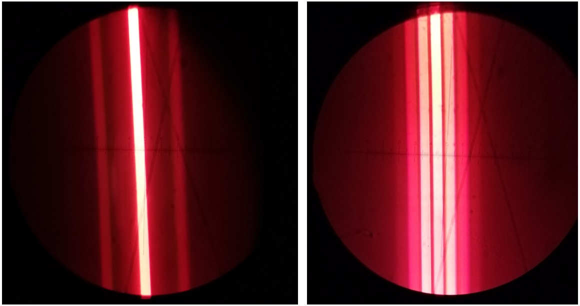


Figure: Diffraction lines in microscope

As described above, we can see interference pattern on different frequencies. We use glass with thin notches and measurement screw B to determine distances between diffraction lines.

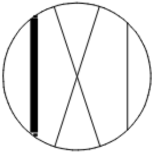


Figure: Notches scheme

Cuvette length variation

Using screw on cuvette we can change it's length. We setup generator frequency to achive well-recognizable interference pattern ($\nu = 1.16$ MHz). In that case wave becomes standing. Taking into account that changing cuvette length to $\Lambda/2$ again gives us standing wave, we estimate:

$$\Lambda/2 = 700 \text{ mkm} \Rightarrow v = \Lambda\nu = 1600 \text{ m/s}.$$

Diffraction maxima

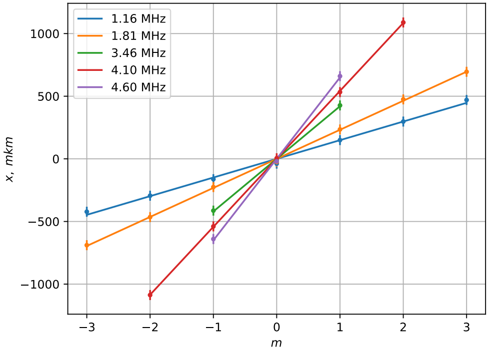


Figure: Maxima positions for different frequencies

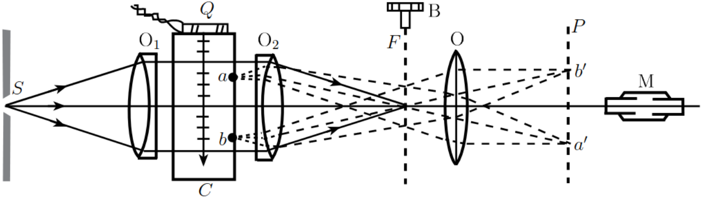
Slope coefficient $k = \frac{dx}{dm}$.
From (1) and (2):

$$v = \frac{f\lambda\nu m}{x_m} = \frac{f\lambda\nu}{k}.$$

After evaluating k for every frequency and taking average:

$$v = (1480 \pm 30)\ \text{m/s}.$$

Experimental Setup



- S - slit
- O_1, O_2 - lenses
- Q - cuvette
- B - measurement screw
- O - auxiliary lens
- P - clear-image plane
- M - microscope

Comparing with previous setup, we add auxiliary lens O. It creates clear image of objects in cuvette Q in plane P. We use wire to cut off central maximum in O_2 focal plane. Interference pattern can be observed in microscope.

Dark-field calibration

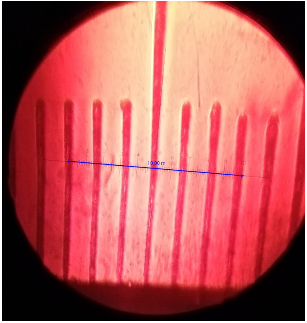


Figure: Calibration ruler in microscope

To determine distance between interference lines we calibrate microscope scale using glass with millimeter notches. Scale coefficient $\gamma = 0.6$ mm/div.

Observing dark-field lines

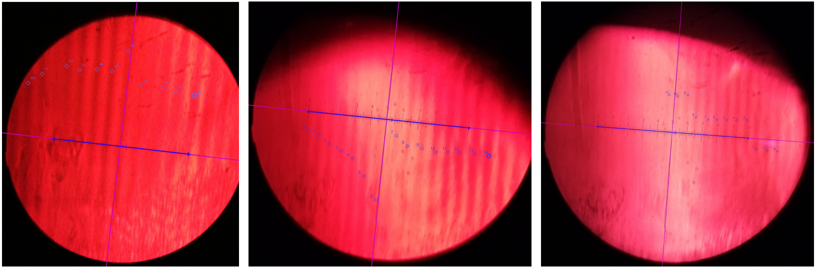


Figure: Dark-field maxima in microscope

Picture is continuous before central maximum is cut off. When it's blocked we can observe interference pattern.
Interference lines on pictures are indexed. Microscope scale is marked in blue.

Dark-field maxima

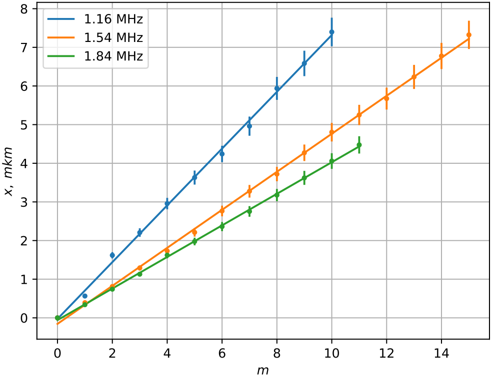


Figure: Maxima positions for different frequencies

From (8) we obtain

$$\Lambda = 2\frac{x_m}{m} = 2k.$$

Therefore:

$$v = 2k\nu.$$

Evaluating k for all frequencies and taking average:

$$v = (1574 \pm 12) \text{ m/s}.$$

Acknowlegements