

ITEA automatic report

ITEA_summarizer

Wednesday 9th June, 2021, 14:07

Automatic report created by *ITEA_summarizer* package. This report makes usage of several methods to automatically inspect and explain the final expression found in the evolutionary process performed by the ITEA algorithm.

Descriptive statistics of the data

Showing descriptive statistics for 5 (from a total of 8) features contained on the given data. The features were selected based on the absolute final importance.

	AveBedrms	MedInc	AveOccup	AveRooms	Latitude
count	13828.000000	13828.000000	13828.000000	13828.000000	13828.000000
mean	1.097533	3.876745	3.128660	5.436556	35.651238
std	0.445688	1.903102	12.646130	2.449446	2.134064
min	0.333333	0.499900	0.692308	0.888889	32.550000
25%	1.006623	2.568575	2.432189	4.459802	33.940000
50%	1.049552	3.538750	2.819702	5.232422	34.270000
75%	1.100283	4.756600	3.282093	6.058566	37.720000
max	25.636364	15.000100	1243.333333	141.909091	41.950000

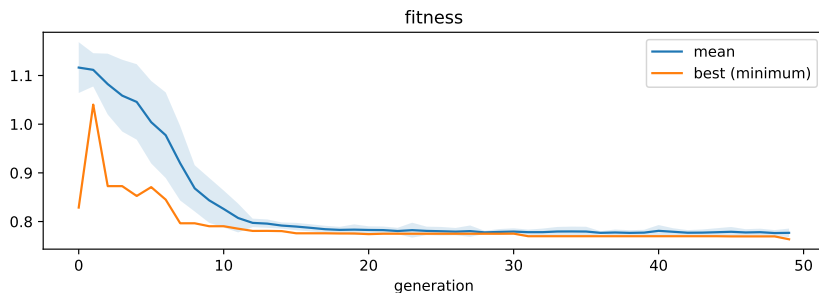
Algorithm Hyper-parameters

The following hyperparameters were used to execute the algorithm. If the `random_state` parameter was set to a integer value, then it is possible to repeat the exact execution by using the parameters listed below.

```
expolim : (-2, 2)
gens : 50
max_terms : 5
popsize : 50
random_state : 42
simplify_method : simplify_by_var
verbose : 10
tfuncs : [log, sqrt.abs, id, sin, cos, exp]
```

Evolution convergence

The algorithm took 208.859 seconds to completely run. Below are the plots for the average fitness of the population and the best individual fitness for each generation.



Best expression

The best expression corresponds to the expression with best fitness on the last generation before the evolution ends. Not necessarily it will be the simpler or the global optimum of the evolution. The final expression is a regressor with fitness of 0.76354, and the number of

IT terms is 5. Below is an representation of the expression:

$$\begin{aligned}
 ITEXpr = & \underbrace{\beta_0 \cdot \log\left(\frac{AveBedrms^2 \cdot AveOccup^2 \cdot Longitude^2}{MedInc^2 \cdot AveRooms^2 \cdot Latitude^2}\right)}_{\text{term 0}} \\
 & + \beta_1 \cdot \log\left(\underbrace{\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms \cdot AveBedrms \cdot AveOccup \cdot Latitude^2}}_{\text{term 1}}\right) \\
 & + \beta_2 \cdot \log\left(\underbrace{\frac{AveRooms \cdot AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2}}_{\text{term 2}}\right) \\
 & + \beta_3 \cdot \log\left(\underbrace{\frac{MedInc^2 \cdot HouseAge^2}{AveRooms^2 \cdot AveBedrms^2 \cdot AveOccup \cdot Latitude^2 \cdot Longitude^2}}_{\text{term 3}}\right) \\
 & + \beta_4 \cdot \log\left(\underbrace{\frac{MedInc \cdot Population^2}{HouseAge^2 \cdot AveRooms^2 \cdot AveBedrms^2 \cdot Latitude^2}}_{\text{term 4}}\right) + I_0
 \end{aligned} \tag{1}$$

Best expression metrics

In the next page is reported a table containing the coefficients for the previous expression, as well as some metrics calculated for each term individually:

- **coef:** coefficient of each term (or coefficients, if the itexpr is an instance of IT-Expr_classifier);
- **coef stderr:** the standard error of the coefficients;
- **disentang.:** mean pairwise disentanglement between each term when compared with the others;
- **M.I.:** mean continuous mutual information between each term when compared with the others;
- **pred. var.:** variance of the predicted outcomes for each term when predicting the training data.

	coef	func	coef stderr	disentang.	M.I.	pred. var.
term 0	0.618	log	0.019	0.124	0.266	0.804
term 1	3.283	log	0.057	0.365	0.507	12.562
term 2	0.191	log	0.01	0.320	0.480	0.154
term 3	-1.459	log	0.03	0.328	0.495	5.245
term 4	-0.18	log	0.008	0.180	0.405	0.165
term 5	-33.276	intercept	0.655	0.000	0.000	0.000

Partial derivatives

$$\begin{aligned}
& \frac{\partial}{\partial \text{MedInc}} \text{ITExpr} \tag{2} \\
&= \underbrace{-2\beta_0 \cdot \log' \left(\frac{\text{AveBedrms}^2 \cdot \text{AveOccup}^2 \cdot \text{Longitude}^2}{\text{MedInc}^2 \cdot \text{AveRooms}^2 \cdot \text{Latitude}^2} \right)}_{\text{term 0}} \left(\frac{\text{AveBedrms}^2 \cdot \text{AveOccup}^2 \cdot \text{Longitude}^2}{\text{MedInc}^3 \cdot \text{AveRooms}^2 \cdot \text{Latitude}^2} \right) \\
&+ \underbrace{2\beta_1 \cdot \log' \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge} \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2} \right)}_{\text{term 1}} \left(\frac{\text{MedInc} \cdot \text{HouseAge} \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2} \right) \\
&+ \underbrace{-1\beta_2 \cdot \log' \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc} \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right)}_{\text{term 2}} \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc}^2 \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right) \\
&+ \underbrace{2\beta_3 \cdot \log' \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2} \right)}_{\text{term 3}} \left(\frac{\text{MedInc} \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2} \right) \\
&+ \underbrace{1\beta_4 \cdot \log' \left(\frac{\text{MedInc} \cdot \text{Population}^2}{\text{HouseAge}^2 \cdot \text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{Latitude}^2} \right)}_{\text{term 4}} \left(\frac{\text{Population}^2}{\text{HouseAge}^2 \cdot \text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{Latitude}^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial \text{HouseAge}} \text{ITE} \text{Expr} \tag{3} \\
&= 1\beta_1 \cdot \log' \left(\underbrace{\frac{\text{MedInc}^2 \cdot \text{HouseAge} \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2}}_{\text{term 1}} \right) \left(\frac{\text{MedInc}^2 \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2} \right) \\
&+ -2\beta_2 \cdot \log' \left(\underbrace{\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc} \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2}}_{\text{term 2}} \right) \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc} \cdot \text{HouseAge}^3 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right) \\
&+ 2\beta_3 \cdot \log' \left(\underbrace{\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2}}_{\text{term 3}} \right) \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2} \right) \\
&+ -2\beta_4 \cdot \log' \left(\underbrace{\frac{\text{MedInc} \cdot \text{Population}^2}{\text{HouseAge}^2 \cdot \text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{Latitude}^2}}_{\text{term 4}} \right) \left(\frac{\text{MedInc} \cdot \text{Population}^2}{\text{HouseAge}^3 \cdot \text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{Latitude}^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial AveRooms} ITEXpr \tag{4} \\
&= \underbrace{-2\beta_0 \cdot \log' \left(\frac{AveBedrms^2 \cdot AveOccup^2 \cdot Longitude^2}{MedInc^2 \cdot AveRooms^2 \cdot Latitude^2} \right) \left(\frac{AveBedrms^2 \cdot AveOccup^2 \cdot Longitude^2}{MedInc^2 \cdot AveRooms^3 \cdot Latitude^2} \right)}_{\text{term 0}} \\
&+ \underbrace{-1\beta_1 \cdot \log' \left(\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms \cdot AveBedrms \cdot AveOccup \cdot Latitude^2} \right) \left(\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms^2 \cdot AveBedrms \cdot AveOccup \cdot Latitude^2} \right)}_{\text{term 1}} \\
&+ \underbrace{1\beta_2 \cdot \log' \left(\frac{AveRooms \cdot AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2} \right) \left(\frac{AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2} \right)}_{\text{term 2}} \\
&+ \underbrace{-2\beta_3 \cdot \log' \left(\frac{MedInc^2 \cdot HouseAge^2}{AveRooms^2 \cdot AveBedrms^2 \cdot AveOccup \cdot Latitude^2 \cdot Longitude^2} \right) \left(\frac{MedInc^2}{AveRooms^3 \cdot AveBedrms^2 \cdot AveOccup \cdot Latitude^2 \cdot Longitude^2} \right)}_{\text{term 3}} \\
&+ \underbrace{-2\beta_4 \cdot \log' \left(\frac{MedInc \cdot Population^2}{HouseAge^2 \cdot AveRooms^2 \cdot AveBedrms^2 \cdot Latitude^2} \right) \left(\frac{MedInc \cdot Population^2}{HouseAge^2 \cdot AveRooms^3 \cdot AveBedrms^2 \cdot Latitude^2} \right)}_{\text{term 4}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial AveBedrms} ITEpr \tag{5} \\
& = 2\beta_0 \cdot \log' \left(\underbrace{\frac{AveBedrms^2 \cdot AveOccup^2 \cdot Longitude^2}{MedInc^2 \cdot AveRooms^2 \cdot Latitude^2}}_{\text{term 0}} \right) \left(\frac{AveBedrms \cdot AveOccup^2 \cdot Longitude^2}{MedInc^2 \cdot AveRooms^2 \cdot Latitude^2} \right) \\
& + -1\beta_1 \cdot \log' \left(\underbrace{\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms \cdot AveBedrms \cdot AveOccup \cdot Latitude^2}}_{\text{term 1}} \right) \left(\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms \cdot AveBedrms^2 \cdot AveOccup \cdot Latitude^2} \right) \\
& + 1\beta_2 \cdot \log' \left(\underbrace{\frac{AveRooms \cdot AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2}}_{\text{term 2}} \right) \left(\frac{AveRooms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2} \right) \\
& + -2\beta_3 \cdot \log' \left(\underbrace{\frac{MedInc^2 \cdot HouseAge^2}{AveRooms^2 \cdot AveBedrms^2 \cdot AveOccup \cdot Latitude^2 \cdot Longitude^2}}_{\text{term 3}} \right) \left(\frac{MedInc^2 \cdot HouseAge^2}{AveRooms^2 \cdot AveBedrms^3 \cdot AveOccup \cdot Latitude^2 \cdot Longitude^2} \right) \\
& + -2\beta_4 \cdot \log' \left(\underbrace{\frac{MedInc \cdot Population^2}{HouseAge^2 \cdot AveRooms^2 \cdot AveBedrms^2 \cdot Latitude^2}}_{\text{term 4}} \right) \left(\frac{MedInc \cdot Population^2}{HouseAge^2 \cdot AveRooms^2 \cdot AveBedrms^3 \cdot Latitude^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial Population} ITE_{\text{Expr}} \tag{6} \\
&= 2\beta_2 \cdot \log' \left(\underbrace{\frac{AveRooms \cdot AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2}}_{\text{term 2}} \right) \left(\frac{AveRooms \cdot AveBedrms \cdot Population \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2} \right) \\
&+ 2\beta_4 \cdot \log' \left(\underbrace{\frac{MedInc \cdot Population^2}{HouseAge^2 \cdot AveRooms^2 \cdot AveBedrms^2 \cdot Latitude^2}}_{\text{term 4}} \right) \left(\frac{MedInc \cdot Population}{HouseAge^2 \cdot AveRooms^2 \cdot AveBedrms^2 \cdot Latitude} \right)
\end{aligned}$$

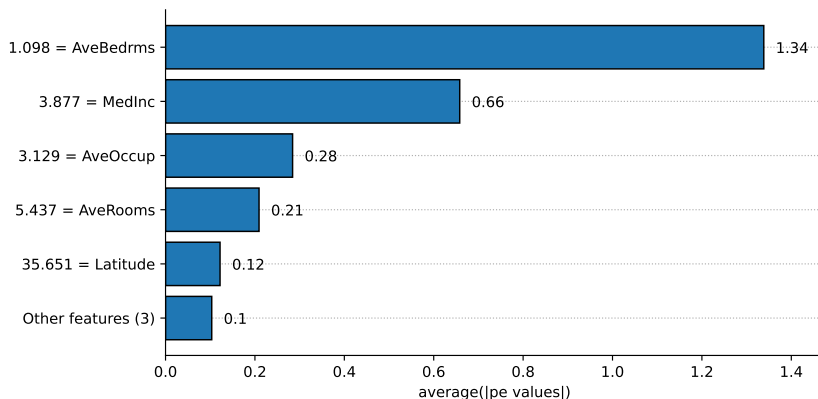
$$\begin{aligned}
& \frac{\partial}{\partial AveOccup} ITEpr \\
& = 2\beta_0 \cdot \log' \left(\underbrace{\frac{AveBedrms^2 \cdot AveOccup^2 \cdot Longitude^2}{MedInc^2 \cdot AveRooms^2 \cdot Latitude^2}}_{\text{term 0}} \right) \left(\frac{AveBedrms^2 \cdot AveOccup \cdot Longitude^2}{MedInc^2 \cdot AveRooms^2 \cdot Latitude^2} \right) \\
& \quad + -1\beta_1 \cdot \log' \left(\underbrace{\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms \cdot AveBedrms \cdot AveOccup \cdot Latitude^2}}_{\text{term 1}} \right) \left(\frac{MedInc^2 \cdot HouseAge \cdot Longitude^2}{AveRooms \cdot AveBedrms \cdot AveOccup^2 \cdot Latitude^2} \right) \\
& \quad + -1\beta_2 \cdot \log' \left(\underbrace{\frac{AveRooms \cdot AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup \cdot Longitude^2}}_{\text{term 2}} \right) \left(\frac{AveRooms \cdot AveBedrms \cdot Population^2 \cdot Latitude}{MedInc \cdot HouseAge^2 \cdot AveOccup^2 \cdot Longitude^2} \right) \\
& \quad + -1\beta_3 \cdot \log' \left(\underbrace{\frac{MedInc^2 \cdot HouseAge^2}{AveRooms^2 \cdot AveBedrms^2 \cdot AveOccup \cdot Latitude^2 \cdot Longitude^2}}_{\text{term 3}} \right) \left(\frac{MedInc^2 \cdot HouseAge^2}{AveRooms^2 \cdot AveBedrms^2 \cdot AveOccup^2 \cdot Latitude^2 \cdot Longitude^2} \right)
\end{aligned} \tag{7}$$

$$\begin{aligned}
& \frac{\partial}{\partial \text{Latitude}} \text{ITExpr} \tag{8} \\
&= \underbrace{-2\beta_0 \cdot \log' \left(\frac{\text{AveBedrms}^2 \cdot \text{AveOccup}^2 \cdot \text{Longitude}^2}{\text{MedInc}^2 \cdot \text{AveRooms}^2 \cdot \text{Latitude}^2} \right)}_{\text{term 0}} \left(\frac{\text{AveBedrms}^2 \cdot \text{AveOccup}^2 \cdot \text{Longitude}^2}{\text{MedInc}^2 \cdot \text{AveRooms}^2 \cdot \text{Latitude}^3} \right) \\
&+ \underbrace{-2\beta_1 \cdot \log' \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge} \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2} \right)}_{\text{term 1}} \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge} \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^3} \right) \\
&+ \underbrace{1\beta_2 \cdot \log' \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc} \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right)}_{\text{term 2}} \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2}{\text{MedInc} \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right) \\
&+ \underbrace{-2\beta_3 \cdot \log' \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2} \right)}_{\text{term 3}} \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^3 \cdot \text{Longitude}^2} \right) \\
&+ \underbrace{-2\beta_4 \cdot \log' \left(\frac{\text{MedInc} \cdot \text{Population}^2}{\text{HouseAge}^2 \cdot \text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{Latitude}^2} \right)}_{\text{term 4}} \left(\frac{\text{MedInc} \cdot \text{Population}^2}{\text{HouseAge}^2 \cdot \text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{Latitude}^3} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial \text{Longitude}} \text{ITE} \text{Expr} \\
&= \underbrace{2\beta_0 \cdot \log' \left(\frac{\text{AveBedrms}^2 \cdot \text{AveOccup}^2 \cdot \text{Longitude}^2}{\text{MedInc}^2 \cdot \text{AveRooms}^2 \cdot \text{Latitude}^2} \right) \left(\frac{\text{AveBedrms}^2 \cdot \text{AveOccup}^2 \cdot \text{Longitude}}{\text{MedInc}^2 \cdot \text{AveRooms}^2 \cdot \text{Latitude}^2} \right)}_{\text{term 0}} \\
&+ \underbrace{2\beta_1 \cdot \log' \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge} \cdot \text{Longitude}^2}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2} \right) \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge} \cdot \text{Longitude}}{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{AveOccup} \cdot \text{Latitude}^2} \right)}_{\text{term 1}} \\
&+ \underbrace{-2\beta_2 \cdot \log' \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc} \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right) \left(\frac{\text{AveRooms} \cdot \text{AveBedrms} \cdot \text{Population}^2 \cdot \text{Latitude}}{\text{MedInc} \cdot \text{HouseAge}^2 \cdot \text{AveOccup} \cdot \text{Longitude}^2} \right)}_{\text{term 2}} \\
&+ \underbrace{-2\beta_3 \cdot \log' \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2} \right) \left(\frac{\text{MedInc}^2 \cdot \text{HouseAge}^2}{\text{AveRooms}^2 \cdot \text{AveBedrms}^2 \cdot \text{AveOccup} \cdot \text{Latitude}^2 \cdot \text{Longitude}^2} \right)}_{\text{term 3}}
\end{aligned} \tag{9}$$

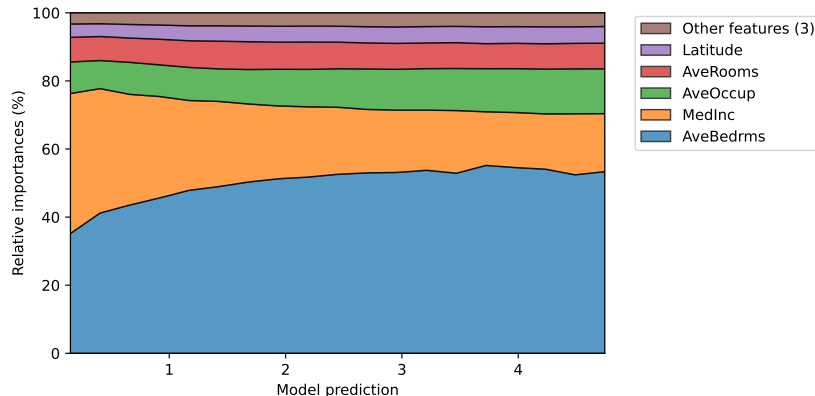
Average partial Effects

Feature importances with Average Partial Effects. This method attributes the importance to the i -th variable by calculating the average of the partial derivative w.r.t. i , evaluated for all data in the training set.



Normalized partial Effects

Feature importances with Normalized Partial Effects. To create this plot, first, the output interval is discretized. Then, for each interval, the partial effect of all samples in the training set that results in a prediction within the interval are calculated. Finally, they are normalized in order to make the total contribution be 100%.



Partial Effects at the Means

Partial Effects plots created by fixing the co-variables at the means and evaluating the model's output when only one variable changes. For simplicity, at most 5 variables are selected to create the plot (the 5 most important variables considering their Average Partial Effects).

