A Number-Guessing Game

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Introduction

On the TV show "The Price Is Right," a player tries to win a new car by guessing its price. The price consists of a known number of digits (usually 5). Possibilities (hints) for each digit are revealed to the player. If the player fails to guess at least one correct digit in its correct place during a turn, the player loses. A player who eventually guesses all digits correctly wins the car.

For example, suppose that the price is $12,345. Hint sets for each digit are shown in **Figure 1**; the underlined numerals are the correct digits.

	C_1 $\Downarrow$	C_2 $\Downarrow$	C_3 $\Downarrow$	C_4 $\Downarrow$	C_5 $\Downarrow$	(Answer: 12,345)
					7	
				6	6	
			5	5	5	
		4	4	4	4	
	3	3	3	3	3	
	2	2	2	2	2	
	1	1	1	1	1	
First Guess:	2	$\underline{2}$	2	2	2	
Second Guess:	$\underline{1}$		$\underline{3}$	5	$\underline{5}$	
Third Guess:				$\underline{4}$		$\Longrightarrow$ Win
				1,3,6		$\Longrightarrow$ Lose

Figure 1. Hint sets for the price $12,345.

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The hint set for the ten thousands digit is $\{1, 2, 3\}$, meaning that the price is between \$10,000 and \$39,999; and similarly for the other hint sets.

Suppose that the first guess is \$22,222. The player is informed that (just) the second digit is correct, hence knows that the 2s for the other digits are incorrect and can use this information to increase the probability of winning. Suppose that the second guess is \$12,355. The player learns that the first, third, and fifth digits are now also correct; only the fourth digit is still unknown. The possibilities, from its hint set, are reduced to 1, 3, 4, and 6. The player wins the car with a guess of 4 but otherwise loses.

A larger hint set for each digit decreases the chance of winning. However, as we will see, increasing the number of unknown digits need not decrease the chance of winning.

Suppose we somehow know in advance that 1 and 2 are the most likely numerals for the first and second digits, respectively. Would it be better to choose 1 and 2 in the first guess, or to choose 1 in the first guess and to save 2 for the second guess (if there is one)? We ddiscuss optimal strategies for when the numerals are equally likely and when they are not.

Hint Set Numerals Equally Probable

When there are only a few digits, the chance of winning can be computed easily; but even when the numerals in the hint sets are equally probable, the expression for the chance of winning rapdily becomes complicated as the number of digits increases. Let $P(W)$ be the probability of winning, with $|C_i|$ the number of hints in hint-set C_i for digit i. For equally likely numerals, we have:

Example 1. If $|C_1| = m \geq |C_2| = n$, then

$$P(W) = \begin{cases} \frac{3}{mn}, & \text{if } m \geq n \geq 2; \\ \frac{2}{m}, & \text{if } m \geq 2, n = 1. \end{cases}$$

Example 2. If $|C_1| = m \geq |C_2| = n \geq |C_3| = k$, then

$$P(W) = \begin{cases} \frac{2}{m}, & \text{if } m \geq 2, n = k = 1; \\ \frac{2}{m}, & \text{if } m = n = 2, k = 1; \\ \frac{5}{2m}, & \text{if } m \geq 3, n = 2, k = 1; \\ \frac{6}{mn}, & \text{if } m \geq n \geq 3, k = 1; \\ \frac{7}{mnk}, & \text{if } m = n = k = 2; \\ \frac{9}{mnk}, & \text{if } m \geq 3, n = k = 2; \\ \frac{11}{mnk}, & \text{if } m \geq n \geq 3, k = 2; \\ \frac{13}{mnk}, & \text{if } m \geq n \geq k \geq 3. \end{cases}$$

One reason why the expression becomes complicated is that when the number of hints is smaller than the number of digits, the player may get a "free pass" (free guess) automatically. For example, suppose that $|C_1| = |C_2| \geq 3$ and $|C_3| = 2$. If the first guess gets just the first digit right, the player can guess the third digit correctly on the second guess (since there are only two possibilities) and either win on that turn or at least assure a third turn.

So we start with the situation where the number of hints is always at least as great as the number of digits to be guessed.

Theorem 1. *Suppose that that there are n digits, that the numerals in the hint sets for every digit are equally probable, and that $|C_i| \geq n$ for $1 \leq i \leq n$. Then*

$$P(W) = \frac{a_n}{|C_1| \times |C_2| \times \cdots \times |C_n|}, \qquad \textbf{(1)}$$

where

$$a_n = \sum_{i=0}^{n-1} \binom{n}{i} a_i \qquad \text{and} \qquad a_0 = a_1 = 1. \qquad \textbf{(2)}$$

Proof: By induction, as follows.

(i) Suppose that $n = 1$, that is, there is only one digit. The player either wins or loses on the first guess. So, $P(W) = 1/|C_1|$, or $a_1 = 1$.

(ii) Suppose that the result in **(1)** holds for cases where there are $n - 1$ or fewer digits, and we want to show that the result holds in the case of n digits as well. To win, the player needs to get at least one correct digit in the first turn; without loss of generality, let it the nth digit. This situation occurs with probability

$$P(\text{correct } n\text{th digit in the first turn}) = \prod_{i=1}^{n-1} \frac{|C_i| - 1}{|C_i}| \times \frac{1}{|C_n|}.$$

After that first turn, there can be at most $n - 1$ unknown digits, and for each there is one fewer possibility (because of the known wrong guess on the first turn). Because we assume that **(1)** holds when there are $n - 1$ or fewer digits, the probability that the player wins when only the first $n - 1$ digits are left is

$$\frac{a_{n-1}}{\prod_{i=1}^{n-1}(|C_i| - 1)}.$$

This second event is conditional on the first, so the probability of their intersection is

$$P(\text{correct } n\text{th digit in the first turn and player wins}) = \frac{a_{n-1}}{\prod_{i=1}^{n} |C_i|}.$$

Because there are n different ways of getting only one digit right in the first turn, by following the same derivation we have

$$P(\text{one digit right in the first turn and wins}) = \frac{1}{\prod_{i=1}^{n} |C_i|} \times \binom{n}{1} a_{n-1}.$$

Similarly, we can derive

$$P(k \text{ digits right in the first turn and wins}) = \frac{1}{\prod_{i=1}^{n} |C_i|} \times \binom{n}{k} a_{n-k},$$

with $k = 1, \ldots, n$. Thus, the probability of winning when there are n digits is

$$P(W) = \frac{1}{\prod_{j=1}^{n} |C_j|} \times \sum_{k=1}^{n} \binom{n}{k} a_{n-k};$$

and if we let $i = n - k$, we have

$$P(W) = \frac{1}{\prod_{j=1}^{n} |C_j|} \times \sum_{i=0}^{n-1} \binom{n}{i} a_i. \qquad \square$$

To give readers a brief idea of the a_ns, **Table 1** shows the values of a_n for $n = 1, \ldots, 10$.

Table 1.

The values of a_n for $n = 1, \ldots, 10$.

n	1	2	3	4	5	6	7	8	9	10
a_n	1	3	13	75	541	4683	47293	545835	7087261	102247563

For the values in the table, a_n/n^n is a decreasing function of n; this is true in general. The proof is pretty straightforward and is left as an exercise for the reader. Note that, in addition to **(2)**, the a_ns can also be derived via *generatingfunctionology* [Wilf 1990, 146–147; Sloane and Plouffe 1995], where a_n is the coefficient of the term z^n by expanding $1/(2 - e^z)$ into a power series. The a_ns are known as the *ordered Bell numbers* and can be interpreted as the total number of distinct rational preferential arrangements available to a person faced with n distinguished decisions; for further interpretations, see Gross [1962] and Mor and Fraenkel [1984].

When the size of each hint set is not smaller than the number of digits, a_n is actually the number of different possible ways of winning. For example, suppose that there are two digits and each digit has at least two hints. Let T and F denote the cases where a digit is guessed correctly and incorrectly, respectively. Because at least one digit needs to be correct in the first turn, there are only three successful possibilities in the first turn: (T, T), (T, F), and (F, T). Both (T, F) and (F, T) lead to a second (and last!) guess, so each corresponds to only one possible way for winning. In other words, there are only three different choices in the first guess that can lead to a win, or, $a_2 = 3$.

Similarly, suppose that there are three digits and each digit has at least three hints. If the result of the first guess is (T, F, F), (F, T, F), or (F, F, T), then in each situation just one digit is correct and we are reduced to the case above of exactly three ways to win. If the result of the first guess is (T, T, T), (T, T, F), (T, F, T), or (F, T, T), there is only one way to win—guess the remaining digit on the one remaining turn. Adding all possibilities, there are $3 \times 3 + 4 \times 1 = 13$ different ways to win, as shown in **Table 1**. Similar interpretations can be applied to cases of more than three digits.

We would expect that the probability of winning does not increase if more digits are added, provided that the k added digits satisfy

$$\frac{a_{n+k}}{a_n} \leq \prod_{i=1}^{k} |C_{n+i}|. \tag{3}$$

This result follows directly from **Theorem 1**, as does:

Theorem 2. *Suppose that $|C_i| \geq n+k$ for $1 \leq i \leq n+k$ and that the numerals in the hint sets are equally probable. If $\prod_{i=1}^{k} |C_{n+i}| \geq a_{n+k}/a_n$, then*

$$P(W|C_1, \ldots, C_n) \geq P(W|C_1, \ldots, C_n, C_{n+1}, \ldots, C_{n+k}),$$

where $P(W|C_1, \ldots, C_i)$ is the probability of winning with hint sets $C_1, \ldots, C_i$ for digits $1, \ldots, i$ respectively.

Intuitively, it seems that increasing the number of digits would reduce the probability of winning. However, this is not true. Since $a_n \geq n a_{n-1}$, adding one more digit can actually increase the probability of winning, provided that $|C_1| = \cdots = |C_{n-1}| = n$ and the new digit has $|C_n| = n$ hints as well. In general, even when $|C_i| \geq n+k$ for all $i = 1, \ldots, n+k$, there is still no guarantee that $P(W|C_1, \ldots, C_n) \geq P(W|C_1, \ldots, C_{n+k})$ holds. The following examples demonstrate this claim:

Example 3. Suppose there are 3 or 4 digits and every digit has 5 hints, then adding more digits can increase the probability of winning. Similar patterns appear in the case where $|C_i| = 6$ for all i.

- $|C_i| = 5$

No. of digits	1	2	3	4	5
$P(W)$	0.2	0.12	**0.104**	**0.12**	**0.1731**

- $|C_i| = 6$

No. of digits	1	2	3	4	5	6
$P(W)$	0.1667	0.0833	0.0602	**0.0579**	**0.0696**	**0.1004**

The cases where the number of hints for some digits is smaller than the number of digits is similar, except that some winning combinations never appear.

Example 4. Suppose there are 5 digits and 4 hints for each digit, then there will be no fifth guess and the number of all possible winning combinations is $a_5 - 5! = 541 - 120 = 421$. Thus the probability of winning is $421/4^5 \approx 0.4111$. In general, if $|C_i| = n - 1$ for $1 \le i \le k$ and $|C_i| \ge n$ for $k + 1 \le i \le n$, then

$$P(W|C_1, \ldots, C_n) = \frac{a_n - k \times (n-1)!}{\prod_1^n |C_i|}.$$

The cases where $|C_i| \le n - 2$ are similar, and we can use a_n to deduce the number of winning combinations that never appear. For example, in the "Price is Right" example, the fact that the numbers of hints for the first two digits are smaller than 5 rules out 126 of the 541 winning combinations, so the probability of winning is $(541 - 126)/2520 \approx 0.1647$. (Confirming the details of this claim is left as an exercise for the reader.)

Example 5. Suppose that each of the n digits has only two equally likely possibilities. The probability of winning (with optimal guessing strategy) is $1 - 1/2^n$, an increasing function of n.

Hint Set Numerals Not Equally Probable

When the numerals in hint sets for some digits are not equally probable, we need to decide whether the optimal strategy is to choose all the probable answers in the first turn or to save some of these digits for later turns.

Let π_i denote the probability distribution of digit i. Thus, $\pi_i = (1/2, 1/4, 1/4)$ indicates that the chance of the first hint being correct in digit i is $1/2$, the second hint being correct is $1/4$, and the third hint being correct is $1/4$.

Two Digits

Example 6. Suppose there are two digits: $|C_1| = 2$ and $|C_2| = 3$. Also, let $\pi_1 = (4/5, 1/5)$ and $\pi_2 = (1/2, 1/4, 1/4)$. Assume that τ_1 is the best strategy, according to which the numerals with probability $4/5$ from C_1 and $1/2$ from C_2 are chosen during the first turn. Also, let τ_2 be the best strategy, in which $4/5$ is chosen from C_1 and $1/4$ is chosen from C_2 during the first turn. Thus,

$$P(W|\tau_1) = \frac{4}{5}\cdot\frac{1}{2} + \frac{4}{5}\cdot(1-\frac{1}{2})\cdot\frac{1/4}{1-1/2} + (1-\frac{4}{5})\cdot\frac{1}{2}\cdot\frac{1/5}{1/5} = \frac{7}{10}$$

and

$$P(W|\tau_2) = \frac{4}{5}\cdot\frac{1}{4} + \frac{4}{5}\cdot(1-\frac{1}{4})\cdot\frac{1/2}{1-1/4} + (1-\frac{4}{5})\cdot\frac{1}{4}\cdot\frac{1/5}{1/5} = \frac{13}{20}.$$

Similarly, $P(W|\tau_3) = 11/20$ if τ_3 is the best strategy, in which $1/5$ is chosen from C_1 and $1/2$ is chosen from C_2 during the first turn. Also, $P(W|\tau_4) = 7/20$ if τ_4 is the best strategy starts with $1/5$ and $1/4$ from C_1 and C_2. Therefore, choosing the most probable numerals in the first guess is the optimal strategy in this situation.

Theorem 3. *If there are only two digits, the optimal strategy is to choose the most probable numerals from both digits in the first guess.*

Proof: Assume that

$$\pi_1 = (p_1, p_2, \ldots),\ \ p_1 \geq p_2 \geq \cdots \quad \text{and} \quad \pi_2 = (q_1, q_2, \cdots),\ \ q_1 \geq q_2 \geq \cdots.$$

Let τ_1 be the best strategy for choosing the numerals with the probability of p_1 from C_1 and q_1 from C_2 in the first turn. Also, let τ_2 be the best strategy for choosing the numerals with the probability of p_i from C_1 and q_j from C_2 with not both $i = 1$ and $j = 1$ in the first turn. Since at most one of i and j can be 1, without loss of generality, let $j \neq 1$. Then $P(W|\tau_1) = p_1q_1 + p_1q_2 + p_2q_1$ and

$$P(W|\tau_2) = \begin{cases} p_1q_1 + p_1q_j + p_2q_j, & \text{if } \ i = 1; \\ p_iq_j + p_iq_1 + p_1q_j, & \text{if } \ i \neq 1. \end{cases}$$

Therefore,

$$P(W|\tau_1) - P(W|\tau_2) = \begin{cases} p_1(q_2 - q_j) + p_2(q_1 - q_j), & \text{if } \ i = 1; \\ (p_1q_1 - p_iq_j) + p_1(q_2 - q_j) + (p_2 - p_i)q_1, & \text{if } \ i \neq 1. \end{cases}$$

Both cases are nonnegative. Also, if either p_1 or q_1 is the only maximum in π_1 or π_2, respectively, choosing the numerals with the probability of p_1 and q_1 during the first turn would be the only optimal strategy. $\square$

Three Digits or More

When there are 3 or more digits, choosing the most probable numerals in the first turn is not necessarily the best strategy. For example, suppose that there are 3 digits and $\pi_1 = \pi_2 = (5/6, 1/12, 1/12)$ and $\pi_3 = (1/3, 1/3, 1/3)$. Let τ_1 be the best strategy for choosing in the first turn the numerals with the probability of $5/6$, $5/6$, and $1/3$ from C_1, C_2, and C_3, respectively. Similarly, let τ_2 be the best strategy, in which $5/6$, $1/12$, and $1/3$ are chosen from C_1, C_2, and C_3 during the first turn. Then, similar to the calculation in **Example 6**, we have

$$P(W|\tau_1) = \frac{283}{432} < P(W|\tau_2) = \frac{355}{432},$$

which indicates that choosing the most probable numerals from all digits in the first run is not generally optimal when there are more than 2 digits.

The optimal strategy is difficult to find when there are 3 or more digits and the hint sets are not equally probable. In particular, there is no guarantee that choosing all the most probable numerals for all digits on a turn is optimal, even when all digits have identical hint sets and probability distributions. For example, suppose that there are 3 digits and $\pi_1 = \pi_2 = \pi_3 = (p_1, p_2, \dots)$, with $p_1 \geq p_2 \geq \cdots$. Let τ_1 be the best strategy that begins with choosing p_1 for all three digits in the first turn. Let τ_2 be the best strategy that begins with choosing p_1 from C_1 and C_2 and p_2 from C_3 on the first turn. Then, from direct calculation, we can show that

$$P(W|\tau_1) - P(W|\tau_2) = (p_1 - p_2)(p_2^2 + 2p_2p_3 - 2p_1p_3).$$

For $p_2 = p_3$, this reduces to

$$P(W|\tau_1) - P(W|\tau_2) = p_2(p_1 - p_2)(3p_2 - 2p_1);$$

that is, τ_1 is better than τ_2 if $3p_2 > 2p_1$. Reader can use this idea to check when one strategy is better than another.

Some Digits Are Known

It Pays to Hold Back

When some digits are known in advance, it is better to use these digits as "free passes" one at a time. For the unknown digits, intuitively, we should hold back our best guesses until the first "nonfree" turns. For example, if there are only two unknown digits (with probability distributions π_1 and π_2) and k known digits, then we use most probable values from π_1 and π_2 in the $(k+1)$st turn. **Example 7** and **Theorem 4** demonstrate this result. The proof of **Theorem 4** is similar to that of **Theorem 3** and can be done via induction, and is thus omitted.

Example 7. Suppose that there are 3 digits and $\pi_1 = \pi_2 = (1/2, 1/3, 1/6)$ (so that for both digits 1 is the most likely numeral) and $\pi_3 = (1, 0, 0)$ (digit 3 is known). Let τ_1 be the best strategy that begins with the guess $(1, 1, 1)$ on the first turn and let τ_2 be the best strategy that begins with the guess $(2, 2, 1)$ on the first turn. Then, from direct calculation,

$$P(W|\tau_1) = \frac{29}{36} < P(W|\tau_2) = \frac{31}{36}.$$

Similar calculation shows that $P(W|\tau_3) = 30/36$ and $P(W|\tau_4) = 28/36$, where τ_3 and τ_4 are the best strategies for selecting $(1, 2, 1)$ and $(3, 3, 1)$ on the first turn.

Theorem 4. *Suppose that there are $k + 2$ digits, that the k digits 3 through $k + 2$ are known, and that the distribution for digit 1 is $\pi_1 = (p_1, p_2, \dots)$ and the distribution for digit 2 is $\pi_2 = (q_1, q_2, \dots)$ (with both in descending order). Then the optimal strategy is to choose a correct numeral for one of the k known digits in the first k guesses, and choose numerals for the two unknown digits according to the orders*

$$\begin{array}{llll} \overbrace{p_2, p_3, p_4, \dots, p_{k+1},}^{\text{any order of } (2,\dots,k+1)} & p_1, & p_{k+2} & \text{for digit 1 and} \\ \underbrace{q_2, q_3, q_4, \dots, q_{k+1},}_{\text{any order of } (2,\dots,k+1)} & q_1, & q_{k+2} & \text{for digit 2,} \end{array}$$

How Much Is It Worth to Have a Digit Revealed?

If there are n digits and one of them is revealed (so that the player is certain about it), how does that affect the the chance of winning?

Theorem 5. *Suppose that there are n digits, the first digit is known in advance, and (for $j = 2, \dots, n$), the probability distributions π_j are uniformly distributed, and $|C_j| \geq n$ for $2 \leq j \leq n$. Then*

$$P(W) = \frac{a_{n-1}}{\prod_{i=2}^{n} |C_i|} + \frac{a_{n-1}}{\prod_{i=2}^{n} |C_i - 1|} \approx \frac{2\, a_{n-1}}{\prod_{i=2}^{n} |C_i|}.$$

Proof: According to **Theorem 4**, we should make sure that our guess on the first turn includes the known digit, so that we will always get a second turn. The two terms of the result correspond to two situations: If we do not get any other digit correct on that first guess, we are in the situation of **Theorem 1** with $n - 1$ digits to guess and one fewer possibility for each digit; otherwise, we are in the situation of **Theorem 1** with $n - 1$ digits to guess.

Having a digit revealed is equivalent to not being able to lose on the first turn, and we see that this advantage approximately doubles the chance of guessing the remaining $n - 1$ digits without that advantage.

We can also compare the chance of winning when one digit is revealed with the original chance of winning:

$$\frac{P(W \mid \text{digit 1 revealed})}{P(W \mid \text{no digit revealed})} \approx \frac{\dfrac{2\, a_{n-1}}{\prod_{i=2}^{n} |C_i|}}{\dfrac{a_n}{\prod_{i=1}^{n} C_i}} = \frac{2a_{n-1}}{a_n}\, |C_1|.$$

Wilf [1990, 146–147] shows that

$$a_n \approx \frac{n!}{2(\ln 2)^{n+1}},$$

so the ratio above is approximately

$$2|C_1| \frac{\dfrac{(n-1)!}{2(\ln 2)^n}}{\dfrac{n!}{2(\ln 2)^{n+1}}} = 2\ln 2\,\frac{|C_1|}{n}.$$

Because we have assumed that all $|C_i| \geq n$, this ratio is at least $2\ln 2 \approx 1.38$. Thus, having a digit revealed increases your chances of winning by at least 38%, and by more if the digit had more than n possibilities.

How Much Is One Extra Guess Worth?

Suppose that instead of being given hints for some digits, we are offered one more turn if we fail to get one new numeral right.

Theorem 6. *Suppose that there are $n \geq 3$ digits, that π_j is uniform, and that $|C_j| \geq n$ for $1 \leq j \leq n$. Let one extra chance be given if we fail to find the answer. Then*

$$\begin{aligned} P(W) &= \frac{b_n}{\prod_{i=1}^{n} |C_i|}, \\ \textit{where} \qquad b_n &= a_n + \sum_{i=0}^{n-1} \binom{n}{i} b_i, \quad b_0 = 1, \quad b_1 = 2. \end{aligned} \tag{4}$$

We give the first few values of b_n in **Table 2**.

Table 2.
The values of b_n for $n = 1, \ldots, 10$.

n	1	2	3	4	5	6	7	8	9	10
a_n	2	8	44	308	2612	25988	296564	3816548	54667412	862440068

To see the value of this extra chance, we can also use *generatingfunctionology* [Wilf 1990], which tells us that

$$b_n \text{ is the coefficient of the term } z^n \text{ in } \frac{1}{(2-e^z)^2}.$$

and that

$$a_n \approx \frac{n!}{2(\ln 2)^{n+1}} \qquad \text{and} \qquad b_n \approx \frac{[n + (1+\ln 2)]n!}{4(\ln 2)^{n+2}} \approx \frac{nn!}{4(\ln 2)^{n+2}}.$$

(See Wilf [1990, 146–147] for details concerning the approximation of a_n, which is astonishingly accurate; the sophisticated technique there, plus some knowledge about Laurent series, extends to the result for b_n.) Consequently, we have

$$\frac{b_n}{na_n} \approx \frac{1}{2\ln 2} \approx 0.721.$$

This result indicates that one extra guess multiplies the probability of winning by approximately n.

Which Is Worth More?

Given the choice between having one digit revealed—which amounts to a free first guess—or being given one extra guess (after a turn on which you fail to guess any digit correctly), which should you choose?

The original probability of winning is

$$P = P(W) = \frac{a_n}{\prod_{i=1}^{n} C_i},$$

and the two conditional probabilities of winning are

$$\begin{aligned} P(W \mid \text{digit 1 revealed}) &\approx \frac{2a_{n-1}}{\prod_{i=2}^{n} |C_i|} = P\,\frac{2a_{n-1}}{a_n}\,|C_1|, \\ P(W \mid \text{one extra guess}) &= \frac{b_n}{\prod_{i=1}^{n} |C_i|} = P\,\frac{b_n}{a_n}. \end{aligned}$$

The ratio is

$$\frac{P(W \mid \text{digit 1 revealed})}{P(W \mid \text{one extra guess})} = \frac{2a_{n-1}}{b_n}\,|C_1|.$$

For large n, this ratio is approximately

$$\frac{2\,\dfrac{(n-1)!}{2(\ln 2)^n}}{\dfrac{nn!}{4(\ln 2)^{n+2}}}\,|C_1| = \frac{4(\ln 2)^2}{n^2}\,|C_1| \approx 1.92\,\frac{|C_1|}{n^2}.$$

Again, we are assuming that $|C_1| \geq n$. Thus, given a choice between having a digit revealed or being given one extra guess, unless there are really an awful lot of possibilities for the digit, you are better off choosing the extra guess!

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About the Authors

[need short biographical sketches and photos]