Lab 6

Cubic spline functions

1. Consider the function: $f(x) = sin(x)$ defined on $[0, 2\pi]$ and the nodes $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

   a) display the value of the function, the value of the cubic natural spline and the value of cubic clamped spline function at $x = \frac{\pi}{4}$.

   b) plot the graphs of the function, of the cubic natural spline and of the cubic clamped spline function.

Least squares approximation

2. The following table list the temperatures of a room recorded during the time interval $[1:00, 7:00]$. Find the best liniar least squares function $\varphi(x) = ax + b$ that approximates the table, using the normal equations. Use your result to predict the temperature of the room at $8:00$. Find the minimum value $E(a, b)$, for the obtained a and b. In the same figure, plot the points (Time, Temperature) and the least squares function.

Time	$1:00$	$2:00$	$3:00$	$4:00$	$5:00$	$6:00$	$7:00$
Temperature	13	15	20	14	15	13	10

3. The vapor pressure P of the water (in bars) as a function of temperature T (in $°C$) is:

T (temperature)	0	10	20	30	40	60	80	100
P (pressure)	0.0061	0.0123	0.0234	0.0424	0.0738	0.1992	0.4736	1.0133

a) Obtain two least squares approximations for the given data, using $polyfit$ for 2 different degrees of the polynomials. Find their values for $T = 45$ using $polyval$. Compute the approximation errors, knowing that the exact value is $P(45) = 0.095848$.

b) Plot the interpolation points, the least squares approximants and the interpolation polynomial, in the same figure.

4. Consider 10 random points in the plane $[0,3]\times[0,5]$ using Matlab function *ginput*. Plot the points and the least squares polynomial of 2nd degree that best fits these points.

Facultative:

5. Consider 12 random points in the interval $[0,10]$. Find the discret least squares approximant of n-th degree for the function $f(x)=x^3$ using the least squares approximation method with weight function $w(x)=1$ and the basis $1,x,x^2,...,x^n$. (The least squares approximant is of the form $\varphi(x)=\sum\limits_{i=1}^{n} a_i g_i(x)$, where $\{g_i,\ i=1,...,n\}$ is a basis of the space and the coefficients a_i are obtained solving the normal equations: $\sum\limits_{i=1}^{n} a_i \langle g_i, g_k\rangle = \langle f, g_k\rangle, \quad k=1,...,n.$). Plot the obtained approximant.