Variant 1

I. Prove that the series $\sum_{n\geq1}\frac{4}{(2n-1)^2}$ and $\sum_{n\geq1}\frac{3}{n^2}$ are convergent, and moreover, they have the same sum

II. Let$f:(-e,\infty)\to R, f(x)=\ln(x+e)$, and let $x_0=0$. Determine the n^{th} Taylor polynomial of f centered at x_0 and study whether the improper integrals $\int_{-e}^{x_0} f(x)dx$ and $\int_{x_0}^{\infty} f(x)dx$ are convergent or divergent.

III. Let$f:R^2\to R, f(x,y)=x^3+y^3+3(x^2-1)y$. Find the local extremum points of f and specify their type

IV. Compute $\iint_M \cos\frac{\pi x^2}{4}dxdy$, where $M=\{(x,y)\in R^2|0\leq y\leq x\leq 1\}$

Variant 2

I. Prove that the series $\sum_{n\geq1}\frac{8}{(2n-1)^3}$ and $\sum_{n\geq1}\frac{7}{n^3}$ are convergent and have the same sum

II. Let$f:(0,\infty)\to R, f(x)=\frac{1}{\sqrt{x}}$ and let $x_0=1$. Determine the n^{th} Taylor polynomial of f centered at x_0 and study whether the improper integrals $\int_{0}^{x_0} f(x)dx$ and $\int_{x_0}^{\infty} f(x)dx$ are convergent or divergent.

III. Let$f:R^2\to R, f(x,y)=(\mathrm{x}+\mathrm{y})^3-3x^2y-3x$ Find the local extremum points of f and specify their type

IV. Compute $\iint_M \cos\frac{xy^2}{4}dxdy$, where $M=\{(x,y)\in R^2|0\leq x\leq y\leq 1\}$

Variant 3

I. Let $(x_n)_{n\in N}$ be a sequence of real numbers such that the series $\sum_{n\geq 1} x_n$ is convergent. Prove that the series $\sum_{n\geq 1}\sqrt{x_n \cdot x_{n+1}}$ is convergent

II. For $\alpha \in R$ study whether the improper integral $\int_1^\infty \frac{x^\alpha}{\sqrt{x^4+1}}$dx is convergent or divergent

III. Let$f: R^2 \to R, f(x,y) = e^{x^2} + (y-1)y^2(y+1)$. Find the local extremum points of f and specify their type

IV. Compute $\iint_A \frac{y}{1+x^2y^2}\,dxdy$, where $A = [0,1]x[0,\sqrt{3}]$

Variant 4

I. Let $(x_n)_{n\in N}$ be a sequence of positive numbers, such that the series $\sum_{n\geq 1} x_n$ is convergent. Prove that the series $\sum_{n\geq 1}\sqrt{\frac{x_n}{n^3}}$ is convergent

II. For $\alpha \in R$ study whether the improper integral $\int_1^\infty \frac{1}{x^\alpha \cdot \sqrt{x^4+1}}$dx is convergent or divergent

III. Let$f: R^2 \to R, f(x,y) = x^2(x^2-1) + e^{y^2}$. Find the local extremum points of f and specify their type

IV. Compute $\iint_A \frac{y}{1+xy}\,dxdy$, where $A = [0,1]x[0,e-1]$

Variant 5

I. Study whether the series $\sum_{n\geq 1} \frac{n}{2^{n-1}}$ converges or diverges, then compute $\lim_{n\to\infty} \sum_{k=n}^{2n} \frac{k}{2^{k-1}}$

II. Let $f: R \to R, f(x) = \frac{e^{2x}+1}{e^x} - x^2$ and let $x_0 = 0$. Determine the Taylor polynomial $T_5(x)$ of f centered at x_0 and study whether x_0 is a local extremum of f.

III. Study the continuity $O_2 = (0,0)$ of the function $f: R^2 \to R$, defined by

$$f(x,y) = \begin{cases} \frac{(x-y)^2}{x^2+y^2}, if\ (x,y) \neq 0_2 \\ 0, \qquad if\ (x,y) = 0_2 \end{cases}$$

V. Compute $\iint_A \frac{2xysin\ y}{(1+x^4)(1+\cos^2 y)} dxdy$, where $A = [0,1]x[0,\pi]$

Variant 6

I. Compute $\lim_{n\to\infty} (1 - \frac{1}{2} + \frac{1}{3} - \cdots + \frac{1}{2n-1} - \frac{1}{2n})$

II. Find all numbers $x\epsilon R$ for which the power series $\sum_{n\geq 1} \frac{(-1)^n}{2^n}(x+1)^n$ converges.

III. Consider the function $f: R^2 \to R$, defined by:

$$f(x,y) = \begin{cases} (x^2+y^2)sin\frac{1}{x^2+y^2}, if\ (x,y) \neq 0_2 \\ 0, \qquad if\ (x,y) = 0_2 \end{cases}$$

Prove that f is partially differentiable with respect to both variables at any point $(x,y)\ \epsilon\ R^2$ and compute $\nabla f(x,y)$

VI. Compute $\iint_A y(1-y)2^{xy}\ dxdy$, where $A = [0,1]x[0,1]$

Variant 7

I. Prove that the series $\sum_{n\geq 1} \frac{(-1)^n}{\sqrt{n^2-n+1}}$ is absolutely convergent, then compute $\lim_{n\to\infty} \sum_{k=n}^{2n} \frac{(-1)^k}{\sqrt{k^2-k+1}}$.

II. Let $f: R \to R, f(x) = x^3 - 2(x + sinx\ cosx)$, and let $x_0 = 0$. Determine the Taylor polynomial $T_4(x)$ of f centered at x_0 and study whether x_0 is a local extremum of f.

III. Let$f: R^2 \to R, f(x, y) = xy(xy - 2) + 1$. Find the local extremum points of f and prove that all of them are global minimum points.

IV. Compute $\iint_M \sqrt{1-\mathrm{y}} \cdot e^{y^2-2y}\, dxdy$, where $M = \{(x,y) \in R^2 | 0 \leq y \leq 1, |x| \leq \sqrt{1-y}\}$

Variant 8

I. Prove that the series $\sum_{n\geq 1} \frac{(-1)^n}{arctg(n^2+n+1)}$ is absolutely convergent, then compute $\lim_{n\to\infty} \sum_{k=n}^{2n} \frac{(-1)^k}{arctg\left(k^2+k+1\right)}$.

II. Let $f: R \to R, f(x) = 2x - (x^3 + \sin 2x) + 1$, and let $x_0 = 0$. Determine the Taylor polynomial $T_4(x)$ of f centered at x_0 and study whether x_0 is a local extremum of f.

III. Let$f: R^2 \to R, f(x, y) = (x + y)(2 - x - y) - 1$. Find the local extremum points of f and prove that all of them are global maximum points.

IV. Compute $\iint_M \sqrt{1-\mathrm{x}} \cdot e^{x^2-2x}\, dxdy$, where $M = \{(x,y) \in R^2 | 0 \leq x \leq 1, |y| \leq \sqrt{1-x}\}$