Babeş-Bolyai University, Faculty of Mathematics and Computer Science

Bachelor, Computer Science, Groups 911-917, Academic Year 2017-2018

Mathematical Analysis
Seminar 4

1. Consider the series
$$\sum_{n\geq 1} \frac{2n-1}{2^n}.$$
Compute its sum and deduce whether the series converges or not.

2. Let $(\alpha_n)_{n\in\mathbb{N}}$ be a sequence of real numbers. Using the Cauchy's general criterion of convergence for series, prove that the following series are convergent:

a) $\displaystyle\sum_{n\geq 1} \frac{\sin(\alpha_n)}{n(n+1)}$;

b) $\displaystyle\sum_{n\geq 1} \frac{\alpha_n}{2^n(1+|\alpha_n|)}$.

3. Prove that the following series are divergent:

a) $\displaystyle\sum_{n\geq 1} \operatorname{arctg} n$;

b) $\displaystyle\sum_{n\geq 1} \cos(n\pi/6)$;

c) $\displaystyle\sum_{n\geq 1} \sin n$.

4. Study if the following series are convergent or divergent:

a) $\displaystyle\sum_{n\geq 1} \frac{e^n}{n+3^n}$; b) $\displaystyle\sum_{n\geq 1} \frac{1}{n^2-\ln n+\sin n}$;

c) $\displaystyle\sum_{n\geq 1} \frac{\sqrt{n+1}}{1+2+\cdots+n}$; d) $\displaystyle\sum_{n\geq 1} \frac{2^n\cdot n!}{n^n}$;

e) $\displaystyle\sum_{n\geq 1} \frac{5^{n/2}}{n2^n}$; f) $\displaystyle\sum_{n\geq 1} (\arctan n)^n$;

g) $\displaystyle\sum_{n\geq 1} \frac{n^2}{2^{n^2}}$; h) $\displaystyle\sum_{n\geq 1} \frac{(n+1)^n}{n^{n+2}}$;

i) $\displaystyle\sum_{n\geq 1} \ln\left(1+\frac{1}{n}\right)$; j) $\displaystyle\sum_{n\geq 2} \frac{1}{n\ln n}$.

5. Let $\sum_{n\geq 1} x_n$ be a convergent series with nonnegative terms. Study which of the following series are convergent:

a) $\displaystyle\sum_{n\geq 1} \frac{x_n}{1+x_n}$, b) $\displaystyle\sum_{n\geq 1} x_n^2$, c) $\displaystyle\sum_{n\geq 1} \sqrt{x_n}$, d) $\displaystyle\sum_{n\geq 1} \frac{\sqrt{x_n}}{n}$.