Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: Primăvara 2017

Laboratory 4. First Order Nonlinear Differential Equations

1. We consider $x' = 1 - x^2$.

a) Find its constant solutions.

b) Find the solution $\phi(t, \eta)$ of this differential equation that satisfies $x(0) = \eta$, where $\eta \in \mathbb{R}$ is fixed. Denote it with $\phi(t, \eta)$. Notice that $\phi(t, 1) = 1$ and $\phi(t, -1) = -1$ but the formula returned by Maple for $\phi(t, \eta)$ after using the command **dsolve** is not defined neither for $\eta = 1$ nor for $\eta = -1$.

c) In your notebooks represent simultaneously various integral curves of this differential equation. This means that you have to represent the graph of many solutions, as, for example: $\phi(t, -1.3)$, $\phi(t, -1.2)$, $\phi(t, -1.1)$, $\phi(t, -0.7)$, $\phi(t, -0.5)$, $\phi(t, 0.4)$, $\phi(t, 0.9)$, $\phi(t, 1.1)$, $\phi(t, 1.2)$, $\phi(t, 1.3)$. Of course you can use Maple. When using Maple, for better results, first represent simultaneously for initial values $\eta \in (-1, 1)$. Then for initial values $\eta > 1$ but very close to 1, and for positive values of t (on the interval $(0, 2)$, for example). Then for initial values $\eta < -1$ but very close to -1, and for negative values of t (on the interval $(-2, 0)$, for example). Use Maple to understand better the shape of the graphs on large intervals.

d) Study the validity of the proposition "Each nonconstant solution is strictly monotone."

e) Study the validity of $\lim\limits_{t \to \infty} \phi(t, \eta) = 1$ for all $\eta > -1$.

f) Also, in your notebooks represent the phase portrait of $x' = 1 - x^2$ and confirm the answers to d) and e).

2. Find the general solution of $x' = 1 - tx^3$. Set **infolevel[dsolve]** to 3 to see what is going on. So, there is no algorithm available to Maple to find its general solution. Not even for writing it in terms of some special functions. Using **dfieldplot** represent the corresponding direction field. Using **DEplot** represent the numerical solution that satisfies, for example $x(0) = 0$. Add other initial conditions to represent more numerical solutions simultaneously.

3. Using **dfieldplot** represent the direction field in a box that contains the origin, for each of the differential equations. We know that these directions are tangent to the integral curves. What kind of curves could be the integral curves of each differential equation?

Find the general solution of each differential equation and see if your intuition is confirmed. Remember that $y = mx$ is the equation of a line, $xy = a$ is the equation of a hyperbola, $x^2 + y^2 = r^2$ is the equation of a circle, while $a^2x^2 + b^2y^2 = 1$ is the equation of an ellipse.

a) $y'(x) = \frac{y(x)}{x}$; b) $y'(x) = -\frac{y(x)}{x}$; c) $y'(x) = \frac{-x}{y(x)}$; d) $y'(x) = \frac{-2x}{y(x)}$.

4. Using **contourplot** find the level curves in a box that contains the origin of the scalar functions of two variables $H_1(x, y) = xy$, $H_2(x, y) = x^2 + y^2$ and, respectively, $H_3(x, y) = 2x^2 + y^2$. Relate these with the previous exercise. Notice that the differential equation found performing the calculations in $\frac{d}{dx}H(x, y(x)) = 0$, is the equation whose integral curves are the level curves of H. Find in this way the corresponding differential equation for each function H_1, H_2, H_3 written above.

5. Find the differential equation whose integral curves are the level curves of $H(x, y) = x^2 + 4y^2$.

6. Represent the level curves of $H(x, y) = y^2 - \cos x$ in a sufficiently small box that contains the origin. Notice that the level curves are closed. We will use this property during the Lecture.