

BFS

BFS

Input:

G : graph
s : a vertex

Output:

accessible : the set of vertices that are accessible from s
prev : a map that maps each accessible vertex to its predecessor on a path from s to it

Algorithm:

```
Queue q
Dictionary prev
Dictionary dist
Set visited
q.enqueue(s)
visited.add(s)
dist[s] = 0
while not q.isEmpty() do
    x = q.dequeue()
    for y in Nout(x) do
        if y not in visited then
            q.enqueue(y)
            visited.add(y)
            dist[y] = dist[x] + 1
            prev[y] = x
        end if
    end for
end while
accessible = visited
```

DFS

DFS - connected components

Input:

G : directed graph

Output:

comp : a map that associates, to each vertex, the ID of its strongly connected component

Subalgorithm DF1(Graph G, vertex x, Set& visited, Stack& processed)

```
for y in Nout(x) do
    if y not in visited then
        visited.add(y)
        DF1(y)
    end if
end for
processed.push(x)
```

Algorithm:

```
Stack processed
Set visited

for s in X do
    if s not in visited then
        visited.add(s)
        DF1(G, s, visited, processed)
    end if
end for

visited.clear()
Queue q
int c = 0
while not processed.isEmpty() do
    s = processed.pop()
    if s not in visited then
        c = c + 1
        comp[s] = c
        q.enqueue(s)
        visited.add(s)
        while not q.isEmpty() do
            x = q.dequeue()
            for y in Nin(x) do
                if y not in visited then
                    visited.add(y)
                    q.enqueue(y)
                    comp[y] = c
                end if
            end for
        end while
    end if
end while
```

Dijkstra

Dijkstra

```
def dijkstra(g, s):
    prev = {}
    q = PriorityQueue()
    q.add(s, 0)
    d = {}
    d[s] = 0
    visited = set()
    visited.add(s)
    printDijkstraStep(s, None, q, d, prev)
    while not q.isEmpty():
        x = q.pop()
        for y in g.parseNout(x):
            if y not in visited or d[y] > d[x] + g.cost(x, y):
                d[y] = d[x] + g.cost(x, y)
                visited.add(y)
                q.add(y, d[y])
                prev[y] = x
        printDijkstraStep(s, x, q, d, prev)

    return (d, prev)
```

Bellman Ford

Bellman Ford

```
def bellman(g, s):
    w = [{s : 0}]
    path = [{s : (0,)}]
    print "k=%s" % 0
    print "w=%s" % w[0]
    print "path = %s" % path[0]
    for k in range(1, len(g.parseX())):
        w.append({})
        path.append({})
        for y in w[k-1]:
            for x in g.parseNout(y):
                if ((x not in w[k]) or
                    (w[k][x]>w[k-1][y]+g.cost(x, y) )):
                    w[k][x] = w[k-1][y]+g.cost(y, x)
                    path[k][x] = path[k-1][y]+ (x,)

    print "k=%s" % k
    print "w=%s" % w[k]
    print "path = %s" % path[k]
```

Prim

Prim

Input:

G : directed graph with costs

Output:

edges : a collection of edges, forming a minimum cost spanning tree

Algorithm:

PriorityQueue q

Dictionary prev

Dictionary dist

edges = \emptyset

choose s in X arbitrarily

vertices = {s}

for x in N(x) do

 dist[x] = cost(x, s)

 prev[x] = s

 q.enqueue(x, d[x]) // second argument is priority

while not q.isEmpty() do

 x = q.dequeue() // dequeues the element with minimum value of priority

 if x \notin vertices then

 edges.add({x, prev[x]})

 for y in N(x) do

 if y not in dist.keys() or cost(x,y) < dist[y] then

 dist[y] = cost(x, y)

 q.enqueue(y, dist[y])

 prev[y] = x

 end if

 end for

 end if

end while

Kruskal

Kruskal

```
def kruskal(graph):  
    for vertice in graph['vertices']:  
        make_set(vertice)  
  
    minimum_spanning_tree = set()  
    edges = list(graph['edges'])  
    edges.sort()  
    for edge in edges:  
        weight, vertice1, vertice2 = edge  
        if find(vertice1) != find(vertice2):  
            union(vertice1, vertice2)  
            minimum_spanning_tree.add(edge)  
    return minimum_spanning_tree
```

A*

A*

Input:

G : directed graph with costs
 s, t : two vertices
 $h : X \rightarrow \mathbb{R}$ the estimation of the distance to t

Output:

$dist$: a map that associates, to each accessible vertex, the cost of the minimum

cost walk from s to it

$prev$: a map that maps each accessible vertex to its predecessor on a path from s to it

Algorithm:

```
PriorityQueue q
Dictionary prev
Dictionary dist
q.enqueue(s, h(s))
dist[s] = 0
found = false
while not q.isEmpty() and not found do
    x = q.dequeue()
    for y in Nout(x) do
        if y not in dist.keys() or dist[x] + cost(x,y) < dist[y] then
            dist[y] = dist[x] + cost(x, y)
            q.enqueue(y, dist[y]+h(y))
            prev[y] = x
        end if
    end for
    if x == t then
        found = true
    endif
end while
```

Floyd

Floyd

let dist be an $V \times V$ array of minimum distances initialized to infinity
let next be an $V \times V$ array of vertex indices initialized to null

```
procedure FloydWarshallWithPathReconstruction ()  
  for each edge (u,v)  
    dist[u][v] ← w(u,v) // the weight of the edge (u,v)  
    next[u][v] ← v  
  for k from 1 to |V| // standard Floyd-Warshall implementation  
    for i from 1 to |V|  
      for j from 1 to |V|  
        if dist[i][j] > dist[i][k] + dist[k][j] then  
          dist[i][j] ← dist[i][k] + dist[k][j]  
          next[i][j] ← next[i][k]
```

```
procedure Path(u, v)  
  if next[u][v] = null then  
    return []  
  path = [u]  
  while u ≠ v  
    u ← next[u][v]  
    path.append(u)  
  return path
```


DAG

DAG

Input:

G : directed graph

Output:

sorted : a list of vertices in topological sorting order, or null if G has cycles

Algorithm:

```
sorted = emptyList
Queue q
Dictionary count
for x in X do
    count[x] = indeg(x)
    if count[x] == 0 then
        q.enqueue(x)
    endif
endfor
while not q.isEmpty() do
    x = q.dequeue()
    sorted.append(x)
    for y in Nout(x) do
        count[y] = count[y] - 1
        if count[y] == 0 then
            q.enqueue(y)
        endif
    endfor
endwhile
if sorted.size() < X.size() then
    sorted = null
endif
```

DAG DFS

Input:

G : directed graph

Output:

sorted : a list of vertices in topological sorting order, or null if G has cycles

Subalgorithm TopoSortDFS(Graph G, Vertex x, List sorted, Set fullyProcessed, Set inProcess)

```
    inProcess.add(x)
    for y in Nin(x)
        if y in inProcess then
            return false
        else if y not in fullyProcessed then
            ok = TopoSortDFS(G, y, sorted, fullyProcessed, inProcess)
            if not ok then
                return false
            endif
        endif
    endfor
    inProcess.remove(x)
    sorted.append(x)
    fullyProcessed.add(x)
    return true
```

Algorithm:

```
sorted = emptyList
fullyProcess = emptySet
inProcess = emptySet
for x in X do
    if x not in fullyProcessed then
        ok = TopoSortDFS(G, x, sorted, fullyProcessed, inProcess)
        if not ok then
            sorted = null
            return
        endif
    endif
endif
```

Hamiltonian Cycle

Hamiltonian Cycle

```
def hamiltonianCycle(g):
    '''Returns a Hamiltonian cycle in g, if one exists, as a list of vertices,
    with the first and the last vertex on the list being equal (the length of
    the returned list will be n+1).
    Returns None if no Hamiltonian cycle exists in g.'''

    start_vertex = None

    for x in g.parseVertices():
        start_vertex = x
        break

    sol = []

    dfs(g, [ start_vertex ], sol)

    if( sol == [] ):
        return None

    return sol

def dfs(g, current_path, solution):
    '''
    the dfs function will put the hamiltonean cycle in solution if one exists
    '''
    current_node = current_path[-1]

    if( len( current_path ) == g.nrofVertices() ):
        if( current_path[ 0 ] in g.parseEdgeOut( current_node ) and solution
        == [] ):
            for it in current_path:
                solution.append( it )
            solution.append( current_path[ 0 ] )

    return
```