Kalman Filter implementation

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Contents

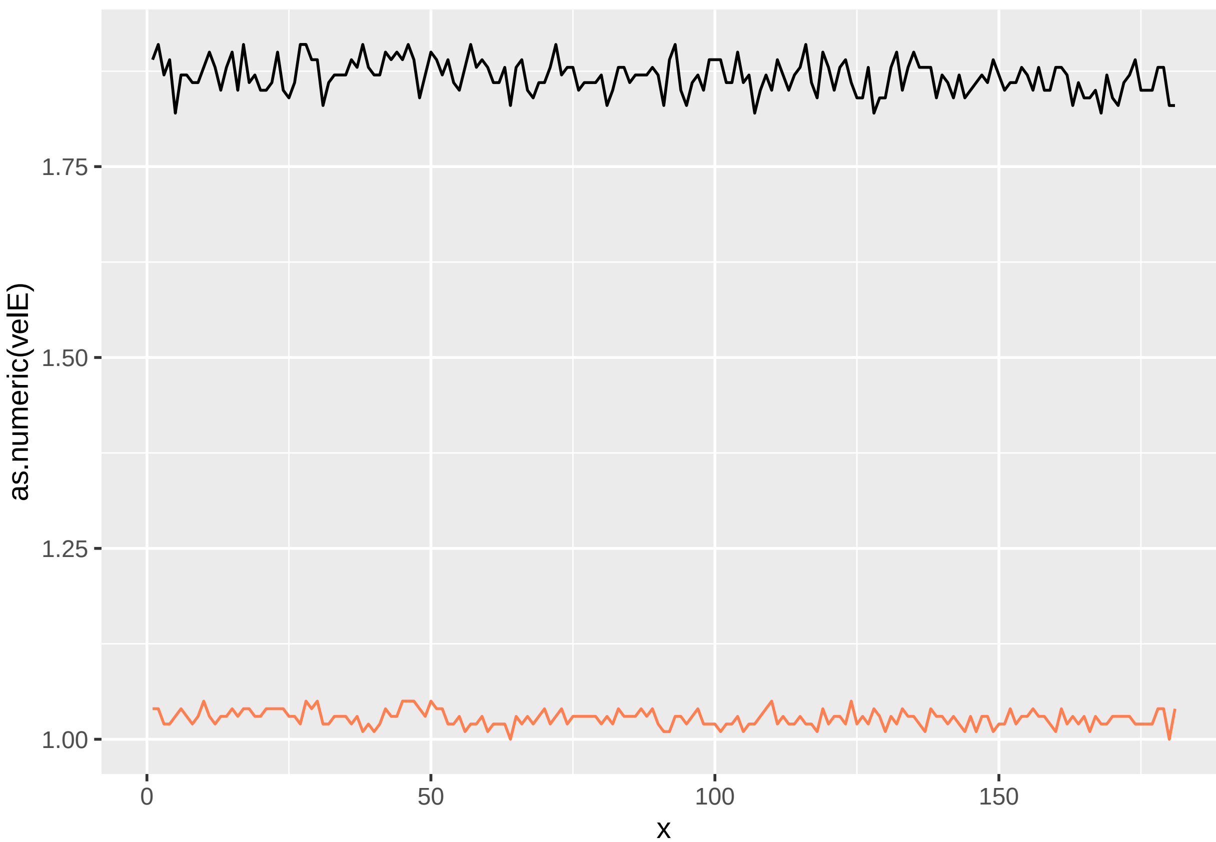
1 Experimental data

```
X2022_03_10_17_20_36 <- read_csv("2022-03-14-15-27-49.csv")
AE <- X2022_03_10_17_20_36[ 120: 300, ]
AE <- cbind( AE$`657 [m/s]`, AE$`076 [m/s]`)
colnames(AE) <- c("velE", "velA")
AE <- as.data.frame(AE)
velA <- as.numeric(AE$velA)
auto.arima(velA)  # it should behave as white noise: ARIMA(0,0,0)
```

```
## Series: velA
## ARIMA(0,1,3)
##
## Coefficients:
##           ma1     ma2      ma3
##       -0.8728  0.0403  -0.1289
## s.e.   0.0758  0.0942   0.0692
##
## sigma^2 estimated as 0.0001056:  log likelihood=568.93
## AIC=-1129.85   AICc=-1129.63   BIC=-1117.08
```

1.1 Graphic

```
x <- index(AE)
g.dlm <- ggplot(data =AE, aes(x= x, y= as.numeric(velE)), color= "navy") + geom_line()+
  geom_line(aes( y=as.numeric(velA)), color = "coral")
g.dlm
```



2 Simulated data

Also the punctual distribution over time were simulated for point A for the turbulent flow

```
PTdata <- read.csv("Puntuals 180s - Full 1.csv", dec=",", header=TRUE, stringsAsFactors=FALSE)
PT <- as.data.frame(cbind(PTdata$X.1[4:362]*10^(-16), PTdata$X.3[4:362]*10^(-16), PTdata$X.5[4:362]*10^
colnames(PT) <- c("velA05", "velC05", "velC06", "velA06")
row_odd <- seq_len(nrow(PT)) %% 2              # Create row indicator
#row_odd
PTu <- as.data.frame(PT[row_odd == 0, ] )

mmA <- as.data.frame((PTu$velA05+ PTu$velA06)/2)
colnames(mmA) <- "mitjana"
```

The simulated values remain lower than real distribution this time. However, this is caused by the asymmetry of the system's solution. Therefore, the data used for the KF is the obtained computing the mean between the different y axis.

3 Kalman Filter

3.1 SMM

We build our SSM through and ARIMA model. The best ARIMA model was found using R's package called `auto.arima` and was a MA(2) with:

```
velA_mit <- mmA$mitjana[59:178] - 1.0566  # we should encode 1.0566 as mean(mmA$mitjana[59:178])
auto.arima(velA_mit)
```

```
## Series: velA_mit
## ARIMA(0,0,2) with zero mean
##
## Coefficients:
##          ma1     ma2
##       0.8505  0.2416
## s.e.  0.0837  0.0837
##
## sigma^2 estimated as 7.435e-05:  log likelihood=400.76
## AIC=-795.53   AICc=-795.32   BIC=-787.16
```

I.e. 0.8505, 0.2416 as the MA(2) parameters.

3.2 Applying the KF

We build our SSM matrices using

```
m1.dlm <- dlmModARMA( ma= c(0.8504, 0.2416))
m1.dlm
```

```
## $FF
##      [,1] [,2] [,3]
## [1,]    1    0    0
##
## $V
##      [,1]
## [1,]    0
##
## $GG
```

```
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    0    0    1
## [3,]    0    0    0
##
## $W
##        [,1]      [,2]       [,3]
## [1,] 1.0000 0.8504000 0.24160000
## [2,] 0.8504 0.7231802 0.20545664
## [3,] 0.2416 0.2054566 0.05837056
##
## $m0
## [1] 0 0 0
##
## $C0
##       [,1]  [,2]  [,3]
## [1,] 1e+07 0e+00 0e+00
## [2,] 0e+00 1e+07 0e+00
## [3,] 0e+00 0e+00 1e+07
```

And we apply the KF storing the result at `A3mean`

```
##MA(2) to shifted mean velA centered (afterwards we'll include the mean)
## it is simply z_t = x_t - \mu
model.filteredA3 <- dlmFilter(velA-mean(velA), m1.dlm)
A3mean <-  model.filteredA3$f + mean(velA)  # this is a little tricky; I'll comment it later
```

4 Results

```
experimentalFilter  <- data.frame(x = index(velA[60:179]),
                                  values = c(velA[58:177], # already shifted!
                                             A3mean[59:178],
                                             mmA$mitjana[59:178]),
                        Data = c(rep("True velocity point A", 120),
                                 rep("Kalman Filtered using MA(2) w/mean", 120),
                                 rep("Simulated velocity point A", 120)
                                 ))
ggplot(experimentalFilter, aes(x, values, col= Data, linetype=Data))+geom_line(size=0.5)+
  scale_color_manual(values= c("blue",  "deepskyblue", "navy")) +
  scale_linetype_manual(values=c("solid", "dotdash","dashed"))+
  theme(panel.background = element_rect(fill = "white", colour = "grey50"))+
  xlab("Time (s)") + ylab("Velocity (m/s)")
```

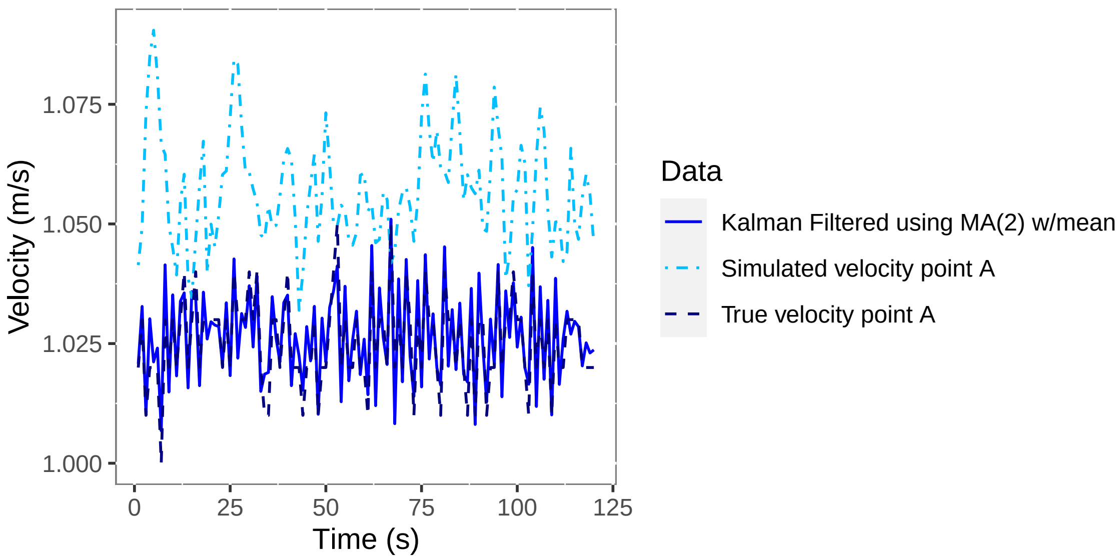


Figure 1: Experimental and simulated flow velocity for the measurement points over time and filtered data using the Kalman Filter with the simulated data set update. Experimental configuration with diameter of 0.04m, and simulated data for the configuration of turbulent flow with 0.04m diameter.