

# Appendix

## A Privacy Analysis

We analyze the security of GCFGAE under the semi-honest model [Brickell and Shmatikov, 2005], that is, all participants and the coordinator strictly follow the procedure of GCFGAE and never collude with others to peek at any participant’s privacy. Specifically, we examine each step of GCFGAE’s training to ensure that no graph privacy defined in subsection 3.2 is breached.

The PSI protocol is used to compute the overlapping vertices for each pair of participants before running GCFGAE. As proved in [De Cristofaro and Tsudik, 2010], no graph privacy except for the existence of overlapping vertices is disclosed during the running of the PSI protocol.

In the stage of Laplacian matrix construction, the intermediate data transmitted between participants and the coordinator includes  $\mathbf{D}_{k,ov}$  and  $\mathbf{D}_{c,ov}$ . The former is passed by secret sharing. The latter is the sum of all  $\mathbf{D}_{k,ov}$ s from which no one can deduce  $\mathbf{D}_{k,ov}$  unless only one participant is in model training, which is impractical. Therefore, the Laplacian matrix construction is secure under the semi-honest model.

During the forward propagation of GCFGAE, the intermediate data transmitted between participants and the coordinator in steps 1 and 2 is the initial weight matrices  $\mathbf{W}^{(0)} \sim \mathbf{W}^{(L-1)}$  which contain no participant’s graph privacy. In steps 3 and 4,  $\mathbf{H}_{k,ov}^{(0)}$  and  $S_{k,ov}$  are sent from each participant to the coordinator. First, they are encapsulated as secrets. Second,  $S_{k,ov}$  includes only the intermediate terms in the computation of the embedding vectors of overlapping vertices. Therefore, steps 3 and 4 do not compromise the privacy of any participant. In step 5, a participant can deduce no information about other participants’ overlapping vertices from  $\mathbf{H}_{k,ov}^{(0)}$  unless only two participants are involved in model training, a situation that is rarely encountered in the real world. Furthermore, according to the analysis in [Chen et al., 2021; Duddu et al., 2020], a participant cannot deduce any raw information of other participants from the embedding vectors transmitting between participants and the coordinator without auxiliary information such as a participant’s subgraph. Therefore, step 5 is secure under

the semi-honest adversary assumption. In steps 6 and 7, the intermediate data transmitted between participants and the coordinator is  $\mathbf{H}_k^{(L-1) \prime}$ . It does not disclose any participant’s graph privacy for the same reason as step 5. The reconstructed  $\hat{\mathbf{A}} \neq \mathbf{A}$  so the coordinator is unaware of the true topology of each participant’s local graph. In summary, the forward propagation of GCFGAE is secure against semi-honest adversaries.

During the backward propagation of GCFGAE, the intermediate data transmitted between participants and the coordinator in steps 1 and 2 includes  $\left(\left(\frac{\partial \hat{\mathbf{A}}}{\partial \mathbf{H}^{(L-1)}}\right)_k\right)_i$  and  $\left\langle \frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(L-1)}} \right\rangle$ . The former is only a partial term in equation 7 for the computation of  $\frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(L-1)}}$ . Since another term  $\left(\frac{\partial \mathcal{L}_k}{\partial \mathbf{A}_k}\right)_i$  in equation 7 is computed by each participant locally, an adversary is unable to infer gradient matrix  $\frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(L-1)}}$  even he has the plain text of  $\left(\left(\frac{\partial \hat{\mathbf{A}}}{\partial \mathbf{H}^{(L-1)}}\right)_k\right)_i$ . The latter is encapsulated as a secret. Therefore, Steps 1 and 2 leak no participant’s graph privacy. The intermediate data sent by the coordinator to each participant in step 3 is the  $\left(\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(L-1)}}\right)_k$  corresponding to participant  $P_k$  and extracted from  $\left(\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(L-1)}}\right)$ . According to equation 8,  $\left(\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(L-1)}}\right)$  is computed based on three terms where  $\frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(L-1)}}$  is in ciphertext. Therefore, it is impossible for the coordinator or any adversary to obtain any participant’s  $\left\langle \frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(L-1)}} \right\rangle$ . In step 4, the  $\left(\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-1)}}\right)_k$  sent by each participant to the coordinator is encapsulated as a secret. In step 5, no participant can deduce other participant’s  $\left(\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-1)}}\right)_k$  from the sum  $\left(\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-1)}}\right)$ . Step 6 is executed by each participant locally. Therefore, the three steps bear no privacy leakage. The intermediate data transmitted between participants and the coordinator in steps 7 and 8 is the encapsulated secret  $\left\langle \left(\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(L-2)}}\right)_{ov} \right\rangle$ . Hence, it discloses no participant’s graph privacy. Step 9 is conducted locally. In summary, the backward propagation of GCFGAE is also secure against semi-honest adversaries.

From the above analysis, we conclude that GCFGAE can protect the graph privacy of all participants under the semi-honest model.

## B Consistency Analysis

We verify the consistency between GCFGAE and the centralized GAE step by step.

In the stage of Laplacian matrix construction, under the scenario of horizontal federated graph learning shown in Figure 1, participants share overlapping vertices but no overlapping links. Therefore, the  $\mathbf{D}_{c,ov}$  computed in steps 1 and 2 through the collaboration of participants and the coordinator is complete for overlapping vertices and contains no redundant term. Secret sharing does not affect the correctness of the computation. In step 3, because participants share only overlapping vertices but not overlapping links, the Laplacian matrix  $\tilde{\mathbf{A}}_{nov}$  encoding the adjacency relationships between two participants' non-overlapping vertices is a zero matrix, that is,  $\tilde{\mathbf{A}}_{nov} = [0]$ . Therefore, the local degree matrices  $\mathbf{D}'_k$ s form a complete global degree matrix  $\mathbf{D}$  when they are placed in their correct positions in  $\mathbf{D}$ , that is,  $\mathbf{D} = \|\mathbf{D}'_k$ . Furthermore, when all local Laplacian matrices  $\tilde{\mathbf{A}}_k$ s computed based on  $\mathbf{D}_{k,ov}$ s and  $\mathbf{D}'_k$ s are combined with  $\tilde{\mathbf{A}}_{nov}$ , we have  $(\|\tilde{\mathbf{A}}_k\| \tilde{\mathbf{A}}_{nov} = \|\tilde{\mathbf{A}}_k = \tilde{\mathbf{A}}$ . Consequently, the Laplacian matrices computed by GCFGAE and the centralized GAE are identical.

During the forward propagation of GCFGAE, all participants share the same weight matrices  $\mathbf{W}^{(0)} \sim \mathbf{W}^{(L-1)}$ . In step 1, the local first-layered embedding matrix  $\mathbf{H}_k^{(0)}$  is computed according to the same equation 1 as the centralized GAE. In steps 2 to 5, first, the part in  $\mathbf{H}_k^{(0)}$  corresponding to non-overlapping vertices, denoted as  $\mathbf{H}_{k,nov}^{(0)}$ , is equivalent to that in the centralized GAE's  $\mathbf{H}^{(0)}$  because there is no link between each participant's non-overlapping vertices and between a participant's overlapping vertices and any other participant's non-overlapping vertices in the scenario of shown in Figure 1. Second, for the overlapping vertices, the local embedding matrix  $\mathbf{H}_{k,ov}^{(0)}$  of each participant is correctly constructed, given that set  $S_{k,ov}$  correctly records the duplicated terms in the computation of overlapping vertices' embedding vectors and these terms are correctly deduced according to equation 5 in step 4. In other words, the embedding vectors in  $\mathbf{H}_{k,ov}^{(0)}$  are identical to those computed for the overlapping vertices in the centralized GAE. Therefore, the embedding vectors in  $\mathbf{H}_k^{(0) \prime}$  are equivalent to those corresponding to the same vertices in participant  $P_k$  in the first-layered embedding matrix  $\mathbf{H}^{(0)}$  computed by the centralized GAE. The same conclusion applies to the computation of  $\mathbf{H}_{k,ov}^{(1)}$  to  $\mathbf{H}_{k,ov}^{(L-2)}$ . In step 6, the local reconstructed adjacency matrix  $\hat{\mathbf{A}}_k$  is computed according to the same equation as that used for computing  $\hat{\mathbf{A}}$  in the centralized GAE. In step 7, the embedding vectors in  $\mathbf{H}^{(L-1) \prime}$  which is constructed by combining all  $\mathbf{H}_k^{(L-1) \prime}$  are equivalent to those computed in the centralized GAE for the same rea-

son stated above. The global loss  $\mathcal{L}$  is not directly computed, but its gradients with respect to embedding and weight matrices are computed in the backward propagation and are proved to be correct in the next paragraph. In summary, the forward propagation of GCFGAE is consistent with that of the centralized GAE.

During the backward propagation of GCFGAE, the gradients of the global loss  $\mathcal{L}$  with respect to the  $(L-1)$ th-layered embedding matrix  $\mathbf{H}^{(L-1)}$ , as shown in equation 8, is computed according to the chain rule of matrix calculus in steps 1 to 3. The direct computation of  $\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(L-1)}} = \sum_k \frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(L-1)}}$  overcounts the gradients of  $\mathcal{L}$  with respect to the embedding vectors of overlapping vertices in  $\mathbf{H}^{(L-1)}$ . Therefore, by subtracting  $\sum_i \left( \left( \frac{\partial \mathcal{L}}{\partial \tilde{\mathbf{A}}_{nov}} \right)_i \left( \frac{\partial \tilde{\mathbf{A}}_{nov}}{\partial \mathbf{H}^{(L-1)}} \right)_i \right)$  from them, we can deduct the duplicated terms from the sum. Additionally, the gradients of  $\mathcal{L}$  with respect to the embedding vectors of non-overlapping vertices, that is,  $\sum_i \left( \left( \frac{\partial \mathcal{L}}{\partial \tilde{\mathbf{A}}_{nov}} \right)_i \left( \frac{\partial \tilde{\mathbf{A}}_{nov}}{\partial \mathbf{H}^{(L-1)}} \right)_i \right)$ , should be appended. The complex computation is used to ensure that the  $\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(L-1)}}$  computed by GCFGAE is equivalent to that computed by the centralized GAE. In steps 4 to 6, the gradients of the global loss  $\mathcal{L}$  with respect to the  $(L-1)$ th-layered weight matrix  $\mathbf{W}^{(L-1)}$  computed according to equations 9 and 10 is equivalent to that computed by the centralized GAE because only one global  $\mathbf{W}^{(L-1)}$  is shared between all participants. In steps 7 to 8, the equivalence between the gradients  $\left( \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-2)}} \right)_{c,ov}$  computed by GCFGAE for the  $(L-2)$ th-layered embedding vectors of overlapping vertices and that computed by the centralized GAE is guaranteed by selecting and aggregating the proper gradients in  $\left( \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-1)}} \right)_k$  according to the chain rule. In step 9, since  $\left( \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-2)}} \right)_{c,ov}$  is properly computed and the correctness of steps 4 to 6 is explained above, the computation of  $\left( \frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-2)}} \right)$  by GCFGAE is equivalent to that computed by the centralized GAE. In summary, the backward propagation of GCFGAE is consistent with that of the centralized GAE.

From the above analysis, we conclude that GCFGAE is consistent with the centralized GAE in federated graph model training.

## C Communication and Space Complexity

The communication complexity of the Laplacian matrix construction is  $O(\sum_k n_{k,ov})$ , where  $n_{k,ov}$  is the number of overlapping vertices of participant  $P_k$ , which is spent on the transmission of degree matrices  $\mathbf{D}_{k,ov}$ s. The intermediate data transmitted during the forward propagation includes the embedding matrices  $\mathbf{H}_k^{(l)}$ ,  $\langle \mathbf{H}_{k,ov}^{(l)} \rangle$ ,  $\mathbf{H}_{c,ov}^{(l)}$  and  $\mathbf{H}_k^{(0) \prime}$  of each layer  $l$ . Therefore, the communication complexity of the forward propagation is  $O(I_r \sum_k (d_e n_{k,ov} L + n_k d_e))$ , where  $n_k$  is the number of vertices of participant  $P_k$ 's local graph,  $d_e$  is the dimension of a vertex's embedding vector,  $I_r$  is the number of iterations,  $L$  is the number of lay-

ers in GCFGAE. The communication complexity of the forward propagation is linear to the number of overlapping vertices rather than the total number of vertices of a participant's local graph. The communication complexity of the backward propagation is  $O((\sum_k (n_k^2 d_e + n_k d_e + n_{k,ov} d_e L) + n_W L) I_r)$ , where  $n_W$  is the number of elements of the weight matrix  $\mathbf{W}$ . The complexity is quadratic to the total number of vertices of a participant's local graph because some gradient matrices transmitted between participants and the coordinator, such as  $(\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L-1)}})_k$ , is in the size of  $O(n_k^2)$ . The overall communication complexity of GCFGAE is  $O(I_r \sum_k (d_e n_{k,ov} L + n_k d_e) + (\sum_k (n_k^2 d_e + n_k d_e + n_{k,ov} d_e L) + n_W L) I_r)$ , which can be reduced to  $O(n^2)$  where  $n = \sum_k n_k$  is the sum of the number of vertices in all participants' graphs because  $d_e, L, I_r \ll n$  and  $n_k, n_{k,ov}, n_W < n$ .

The space complexity of the Laplacian matrix construction is  $O(\sum_k (n_k^2 + n_k d_e))$ , which is used to store the adjacency matrix  $\tilde{\mathbf{A}}_k$  and the attribute matrix  $\mathbf{X}_k$  of participant  $P_k$ . The space complexity of the forward propagation of GCFGAE is  $O(\sum_k (n_k^2 + n_k d + L n_w + L n_k d_e + L n_{k,ov} d_e) + n^2)$ , which is used to store the adjacency matrix  $\tilde{\mathbf{A}}_k$ , attribute matrix  $\mathbf{X}_k$ , parameter matrix  $\mathbf{W}_k$  and embedding matrix  $\mathbf{H}_k$  of participant  $P_k$  and the duplicated term set  $S_{k,ov}$  and the global reconstructed matrix  $\hat{\mathbf{A}}$ . The space complexity of the backward propagation is  $O(n^2 + n d_e + \sum_k (n_k^2 + n d_e) + \sum_k (L n_k d_e) + \sum_k (n_k^2 + n_k d + L n_w) + \sum_k (L n_k d_e))$ , which is used to store the global reconstructed matrix  $\hat{\mathbf{A}}$ , the global embedding matrix  $\mathbf{H}^{(l)}$  and the matrices  $\frac{\partial \mathcal{L}_k}{\partial \mathbf{H}^{(l)}}$  and  $(\frac{\partial \mathcal{L}}{\partial \mathbf{H}^{(l)}})_k$  computed by participant  $P_k$  for the  $l$ th layer, and the weight matrix  $\mathbf{W}_k$  in participant  $P_k$ . Therefore, the overall space complexity of GCFGAE is  $O(n^2 + n d_e + \sum_k (n_k^2 + n_k d + L n_w + L n_k d_e + L n_{k,ov} d_e) + n^2)$ , which can be reduced to  $O(n^2)$  because  $d_e, L \ll n$  and  $n_k, n_W < n$ .

## D Experimental Results

The raw numerical results for Figures 5 to 7 are presented in Tables 1 to 6. Additionally, the running time and convergence rate of GCFGAE and the baselines are depicted in Figure 1. Although limited GPU resources hinder GCFGAE's running time, it runs much faster than SAPGNN. Furthermore, GCFGAE exhibits a fast convergence speed.

## References

[Brickell and Shmatikov, 2005] Justin Brickell and Vitaly Shmatikov. Privacy-preserving graph algorithms in the semi-honest model. In *Advances in Cryptology - ASIACRYPT 2005*, 11th International Conference on the Theory and Application of Cryptology and Information Security, Chennai, India, December 4-8, 2005, Proceedings, 2005.

Network	GCFGAE		GAE	
	NMI	ARI	NMI	ARI
Cora	0.5409	0.4640	0.5409	0.4640
Citeseer	0.2538	0.1668	0.2538	0.1668
Amazon	0.8856	0.8011	0.8856	0.8011
DBLP	0.4045	0.3215	0.4045	0.3215
Lastfm	0.6134	0.5772	0.6134	0.5772

Table 1: Numerical results of the consistency experiment

Network	GCFGAE	SAPGNN	FedAvg+GAE	SDGAE
Cora	0.5409	0.2396	0.3143	0.2357
Citeseer	0.2538	0.1725	0.1449	0.1418
Amazon	0.8856	0.6501	0.7974	0.6244
DBLP	0.4045	0.2108	0.2995	0.2064
Lastfm	0.6134	0.3125	0.5357	0.3087

Table 2: NMI values in the accuracy experiment (2 participants)

Network	GCFGAE	SAPGNN	FedAvg+GAE	SDGAE
Cora	0.5409	0.0511	0.3552	0.1405
Citeseer	0.2538	0.0658	0.1994	0.0562
Amazon	0.8856	0.3494	0.6516	0.2350
DBLP	0.4045	0.1439	0.2832	0.0339
Lastfm	0.6134	0.2251	0.4657	0.2021

Table 3: NMI values in the accuracy experiment (10 participants)

Number of participants	GCFGAE	SAPGNN	FedAvg+GAE	SDGAE
2	0.8856	0.6501	0.7974	0.6244
4	0.8856	0.5271	0.7484	0.4062
6	0.8856	0.4443	0.7023	0.3113
8	0.8856	0.3556	0.6799	0.2361
10	0.8856	0.3494	0.6516	0.2350

Table 4: NMI values in the accuracy experiment (Amazon)

Number of participants	GCFGAE	SAPGNN	FedAvg+GAE	SDGAE
2	0.6134	0.3125	0.5357	0.3087
4	0.6134	0.2779	0.4992	0.2623
6	0.6134	0.2625	0.4846	0.2538
8	0.6134	0.2510	0.4695	0.2297
10	0.6134	0.2251	0.4657	0.2021

Table 5: NMI values in the accuracy experiment (Lastfm)

Network	GCFGAE				SAPGNN			
	CO(lg MB)		SO(lg MB)		CO(lg MB)		SO(lg MB)	
	2	10	2	10	2	10	2	10
Cora	5.96	5.36	3.37	3.36	8.73	8.16	6.75	6.62
Citeseer	6.07	5.41	3.45	3.42	8.75	8.19	6.90	6.77
Amazon	5.69	5.01	3.34	3.32	8.49	7.64	6.56	6.42
DBLP	5.40	4.74	3.33	3.31	8.15	7.45	6.44	6.31
Lastfm	5.17	4.83	3.32	3.31	8.06	7.65	6.06	5.96

Table 6: Numerical results of the communication and space overhead experiments

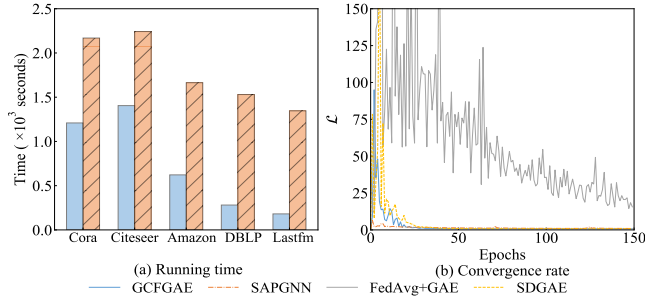


Figure 1: Running time and convergence rate of the models

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