

Artículo bonito que queremos que publiquen

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Abstract

Blah, blah, blah...

Keywords:

1. Introduction

Having a system of information on traffic conditions and the use of roads by vehicles seems a key point in the current context. With a increasingly informed population, provided with ubiquitous communication devices, which are commonly used by about 90% of the population, obtaining information about the traffic in any of the nearly 20,000 kilometers of Spanish roads would mean to optimally manage a communications network vital for a high percentage of users.

In this work, a system based on bluetooth (BT) device discovery is proposed, so that an information system on the traffic status is obtained, as well as a valuable set of data suitable to be used in a time-series forecasting process.

Both, the data-collecting and the time-series forecasting processes are necessary steps to achieve our ultimate goal: to have information about traffic flows that occur or will occur in a certain area, allowing to optimally manage motion decisions by citizens.

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Therefore, from the viewpoint of the transport management various needs have been found:

- A versatile and autonomous data collection and monitoring device is needed.
- It is also necessary to collect traffic data in real time.
- Once the data has been collected, it has to be properly processed.
- And finally, a system that allows sharing data and information with those who make decisions about mobility is needed, both from the institutional and personal points of view.

In order to fulfill these requirements, the BT device discovery used for this work is able to catch waves emitted by different technological components. The components can be the ones embedded in vehicles (handsfree, gps), accessories that the users incorporate to their vehicles, as well as mobile phones, tablets or laptops. The main data being collected is the MAC address of the device BT card. This is an unique identifier for each device, so that passing vehicles can be identified. The intrusiveness is minimal since one-way encryption algorithms are used to hide the real MAC addresses stored. And, it is also safe from the point of view of data privacy given that this piece of information can not be attached to any given person.

The increasingly large amount of data collected, related to passing BT devices, is the starting point that allows to calculate statistics, to study several indicators about the use of vehicles, and to perform time-series forecasting by the monitored area population.

The rest of the paper is organized as follows: In Section ?? current technologies to monitor the traffic that passes through a certain area is summarized. Section ?? details the goals of this paper. In Section ??, the Intelify device is presented. In Section ?? several analysis and statistics are reported from the data obtained. Finally, we present some conclusions and future work (Section 5).

2. Preliminaries

2.1. Traffic detection technologies

Traffic detection technologies can generally be classified into two groups: intrusive and nonintrusive.

Intrusive detection technologies are installed on/within the roadway, requiring lane closures. Using this type of technology is inherently more hazardous and is generally more time consuming, especially for temporary traffic data collection. This technology has a number of drawbacks:

- Installation requires pavement cut.
- Improper installation decreases pavement life.
- Installation and maintenance require lane closure.
- Detection accuracy may decrease when design requires detection of a large variety of vehicle classes.
- Poor pavement condition can dramatically shorten the life span of intrusive sensors.

Non-intrusive technologies are traffic detection sensors that cause minimal disruption to normal traffic operations during installation, operation and maintenance compared to conventional detection methods. They can also be deployed more safely than conventional detection methods, since they are located adjacent to the roadway and require minimal interaction with traffic flow. They can be classified in to big groups: active technologies (microwave radar, ultrasonic and laser radar), and passive technologies (infrared, acoustic and video image processing).

Figure 1 shows a classification of information systems according to the intrusiveness of the technology.

Main technologies currently used in traffic monitoring include pneumatic tubes, loop detectors, floating vehicles, and automatic recognition systems, among others.

Manual counts is the most traditional method. In this case trained observers gather traffic data that cannot be efficiently obtained through automated counts e.g. vehicle occupancy rate, pedestrians and vehicle classifications. The most common equipments used are tally sheet, mechanical count boards and electronic count board systems.

Passive and active infra-red sensors are based on detecting the presence, speed and type of vehicles using the infrared energy radiating from the detection area. The main drawbacks are the performance during bad weather, and limited lane coverage.

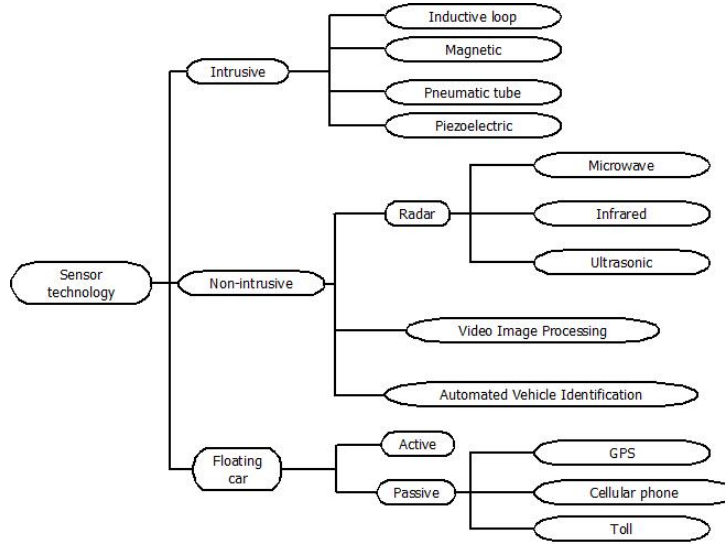


Figure 1: Information systems classification, according to the intrusiveness of the technology.

Microwave radar can detect moving vehicles and speed (Doppler radar). It records count data, speed and simple vehicle classification and is not affected by weather conditions. *Ultrasonic sensors* devices emit sound waves to detect vehicles by measuring the time for the signal to return to the device. They can be affected by temperature or bad weather.

Pneumatic road tubes are placed across the road lanes to detect vehicles from pressure changes that are produced when a vehicle tyre passes over the tube. The pulse of air that is created is recorded and processes by a counter located on the side of the road. The main drawback of this technology is that it has limited lane coverage and its efficiency is subject to weather, temperature and traffic conditions. This system may also not be efficient in measuring low speed flows. *Piezoelectric sensors* are very similar to pneumatic road tubes, although the principle is to convert mechanical energy into electrical energy. Indeed, mechanical deformation of the piezoelectric material modifies the surface charge density of the material so that a potential difference appears between the electrodes. The amplitude and frequency of the signal is directly proportional to the degree of deformation. This system can be used to measure weight and speed.

Magnetic loops (inductive, magnetic, or video processing based) may be

used temporarily or permanently, the latter being the more usual. It is the most conventional technology used to collect traffic data. The loops are embedded in roadways in a square formation that generates a magnetic field. The information is then transmitted to a counting device placed on the side of the road. This has a generally short life expectancy because it can be damaged by heavy vehicles, but is not affected by bad weather conditions. This technology has been widely deployed over the last decades. However, the implementation and maintenance costs can be expensive.

The use of so-called *floating vehicles* consists on a vehicle provided with sensors to collect information while driving on a predefined route. This active device data collection is one of the most popular among operators of roads, used especially for the collection of travel time and for loop detector calibration. Depending on the level of automation in the data collection, the cost can vary.

In some areas, such as electronic toll or transit systems, *automatic vehicle identification systems* (AVI) are also widely used. These sensors are non-exhaustive data sources to identify tags located in vehicles, such as in payment-systems without stopping. The system detects the pass, and the data is sent to the server where it is processed in order to carry out an event (pay toll, opening in the fence, etc).

The main disadvantage of these systems is that they are unable to identify vehicles detected, in order to obtain origin/destination matrixes. Just the number of vehicles and their type can be calculated, but does not allow to obtain moves flow, nor to determine whether a certain vehicle passes repeatedly. In addition, its high cost makes it unprofitable covering secondary roads with them, so they are often located on major roads.

Finally, the *automatic recognition* technology has experienced an increase in recent years due to its ability to detect individual vehicles without relying on in-vehicle systems. Video image detection is a good example: video cameras record vehicle numbers, type and speed by means of different video techniques, e.g. trip line and tracking. Furthermore, they are used for automatic detection of incidents on the road. That is the main advantage over previous information systems. However, system reliability might not be the best, as the system can be sensitive to meteorological conditions. Moreover, these systems are very costly compared to the previous ones. Finally, from

a privacy point of view, the Spanish Data Protection Agency¹ considers the car license plate as a personal data, so that it would require the user consent.

2.1.1. Commercial products

There are different companies working in the traffic information area using approaches similar to the presented in this work.

- **Bit Carrier** ?: It offers a traffic management system based in BT to count people and commercial routes (pathsolver). Its technology was implanted in highways managed by Abertis for traffic control and monitoring. Actually it has a 150 devices network in Catalonia, so it allows count the traffic times of 200.000 persons each day.
- **Trafficnow** ?: Another BT system product. A pilot experience has been implanted in Vigo.
- **Traffax Inc** ?: It is a company that also has used BT for calculating origin-destination and transport time matrixes.
- **Savari Networks** ?: It offers the commercial product StreetWAVE for traffic monitoring to know in real time the traffic status.
- **TrafficCast** ?: They have developed prediction models in different cities based on different technologies, such as cameras, BT and RFID included in the vehicles.

The proposal presented in this work have some common features with the previous approaches, offering similar functionalities but with reduced cost.

2.2. Time series forecasting

Time series forecasting has been a major field of research in the area of statistics ? as well as operational research ?. In the latest years numerous methods have been emerged with the objective of modeling and/or forecasting time series by means of linear and nonlinear models. Linear methods have been widely used to model time series, among them stand out the exponential smoothing methods ??, simple exponential smoothing (SES), Holt's linear methods, some variations of the Holt-Winter's methods, and State

¹Agencia de Proteccion de Datos

space models ?. However, the ARIMA methods ?, which also belongs to linear methods, established a border line between traditional and modern methods. ARIMA methods integrate autoregressive (AR) and moving average (MA) models in a three-stage iterative cycle consisting of: identification of the time series, estimation of the parameters of the model, and verification of the model.

Nevertheless, these linear time series forecasting were insufficient in many real applications, leading to the development of nonlinear time series forecasting. Nonlinear models include regime-switching models, which comprise the wide variety of existing threshold autoregressive (TAR) models ? as: self-exciting (SETAR) models ?, smooth transition (STAR) models ?, and continuous-time (CTAR) models ?. Nevertheless, as pointed out by Clements ?, current nonlinear method's main problems are the followings: they use to develop very complex models; they do not perform in a robust way; and, worst of all, they are difficult to use. De Gooijer ? also concludes that future research on nonlinear models should include, among others, the search for easy to use software.

On the other hand, time series forecasting has been faced with soft computing approaches, as the ones by Samanta ? and Zhu ?, who developed methods based on cooperative particle swarm optimization. Works like ? and ? proposed fuzzy time series models for forecasting, and Yu and Huarng ? applied ANNs for training and forecasting in their fuzzy time series model. Models such as support vector regression ? and fuzzy expert system ? were proposed for the electricity demand forecasting, among others.

ANNs have been also successfully applied to time series and recognized as an important tool for forecasting.

The work by Tang ? concluded that neural networks not only could provide better long-term forecasting but also did a better job than ARIMA models with short series of input data. Furthermore, contrary to the traditional linear and nonlinear time series models, ANNs are nonlinear data-driven approaches with more flexibility and effectiveness in modeling for forecasting ?. Jain and Kumar determined in their work ? that the ANN models were able to produce more accurate forecasts than traditional models because they do not presuppose any functional form of the model to be developed and they do not depend on the assumptions of linearity.

There exist numerous works of different application areas where ANNs are used to forecast time series. The work by Arizmendi ? obtained accurate predictions of the airborne pollen concentrations using ANNs. Zhang and

Hu [10] employed ANNs, and Rivas et al. [11] RBFNs, for forecasting British pound and US dollar exchange rates. Bezerianos et al. [12] employed RBFNs for the assessment and prediction of the heart rate variability.

Specifically inside the ANNs, the use of RBFs as activation functions for them and its application to time series forecasting were firstly considered by Broomhead and Lowe in 1988 [13]. Afterwards, new works by Carse and Fogarty [14], and Whitehead and Choate [15] focused on the prediction of time series.

In later works, Harpham and Dawson [16] studied the effect of different basis functions on an RBFN for time series prediction. Moreover, Du [17] used time series with an encoding scheme for training RBFNs by GAs. Both the architecture (numbers and selections of nodes and inputs) and the parameters (centers and widths) of the RBFNs were represented in one chromosome and evolved simultaneously by GAs so that the selection of nodes and inputs could be automatically achieved.

Previous works found in literatur can also be classified according to the prediction horizon. Thus, forecasting can be divided into short-term, medium-term, and long-term. Generally, forecasting is trended to short-term prediction such as one-step ahead prediction, since longer period prediction (medium-term or long-term) is more difficult, and sometimes may not be reliant because of the error propagation [18]. Thus, neural network models have been traditionally applied in short-term forecasting [19]. For instance, the work by Perez-Godoy [20] applied a hybrid evolutionary cooperative-competitive algorithm for the design of RBFNs to the short-term and even medium-term forecasting of the extra-virgin olive oil price.

As mentioned, another problem that emerges working with time series is the correct choice of the lags considered for representing the series. The relationship involving time series historical data defines a d -dimensional space where d is the minimum dimension capable of representing such a relationship. Takens' theorem [21] establishes that if d is sufficiently large is possible to build a state space using the correct time lags and if this space is correctly rebuilt also guarantees that the dynamics of this space is topologically identical to the dynamics of the real systems state space.

In order to tackle the lags selection problem, an evolutionary method that performs a search for the minimum number of dimensions, Time-delay Added Evolutionary Forecasting (TAEF), is presented in [22]. The methodology is inspired in Takens' theorem and consists of an iterative hybrid model composed of an ANN combined with a genetic algorithm (GA). In [23] the

evolutionary selection of lags is divided into two stages: first, the optimal dimension of the reconstructed phase space is determined by the false nearest neighbors algorithm and then a near-optimal set of time lags is found with a genetic algorithm for a fuzzy inference system.

There are some methods that carry out an automatic search of the relevant lags. QIEHI algorithm [?], for instance, is an evolutionary hybrid intelligent method which is composed of an ANN and a modified evolutionary algorithm to search the minimum dimension to determine the characteristic phase for time series. Another hybrid methodology composed of a modular morphological ANN with an evolutionary algorithm that searches for the best time lags is described in [?]. In [?] a study on the selection not only of the lags but also of the exogenous features with classical feature selection algorithms as pre-processing stage is performed. The lag selection is performed as a postprocessing stage in [?] with a sensitivity computation of the output to each time lag.

As can be observed, the approaches in the literature consider the lags selection as a pre- or post-processing or as a part of the learning process but, instead of together, in hybrid processes with two or three stages. On the contrary, our goal is to address the selection of the lags which represent the series (with any type of correlation) jointly with the design process.

For this reason, we consider coevolutionary algorithms a good mechanism to simultaneously solve these problems.

Cooperative coevolution, introduced by Potter [?], consists of identifying the natural decomposition of a problem into subcomponents. Then, a population of individuals per subproblem is created and evolved by means of collaboration with individuals from others populations. There are many possible methods for choosing representatives with which to collaborate: random collaboration [?], best collaboration [?] (the most widely used in the methods of the literature), complete collaboration, and mixed collaboration [?]. Another important point is the collaboration credit assignment method, i.e., the way an individual is being set a fitness when multiple collaborators are selected. There are three common methods: maximum, average, and minimum, although it has been proved to be significantly better using maximum method than using minimum or average [?].

Cooperative coevolution has been employed for tasks like function optimization [?], multi-objective evolutionary optimization [?], instance selection [?], and feature selection [?], among others. Cooperative coevolution has also been used in order to train ANNs, such as the cooperative coevolutionary approach

for designing neural network ensembles ? and RBFNs ?.

It is possible to find coevolution applied to forecasting tasks as in ? where coevolution with immune network, evolving the structure and parameters of the neural network, is applied for predicting short-term load of a city in eastern China. The work by Qian-Li ? proposes a coevolutionary recurrent neural network for the multi-step-prediction of chaotic time series estimating the proper parameters of phase space reconstruction and optimizing the structure of recurrent neural networks by coevolutionary strategy.

??? SERIES TEMPORALES

3. Data-collecting

Identifying the MAC address of BT devices passing by the roads has being achieved using Intelify (see Figure 2), a hardware solution with a low power consumption and a high detection range.



Figure 2: Intelify device with a connected USB 3G dongle.

Intelify is a small autonomous computer that can be installed in any area to be monitored. It is an autonomous unit, provided by several sensors so that it can discover what is happening in its surroundings (like the flow of people and vehicles). At the same time the environment is been scanned, Intelify is able to send the information to a central server for further processing and interpretation. Table 1 shows main features of the device.

This hardware device is based on technology developed by Ciudad 2020 ??, and the services it offers are based on a net of Intelify devices. Using such an structure, it has the capacity to discover information about the physical

Dimensions	113x163x30mm
LEDs	Power 3G activity Ethernet activity
Networking	Ethernet Wireless Bluetooth 3G
USB ports	City Analytics Antenna 3G dongle
Other ports	RS-232 VGA
Power	18v - 1200mA external jack - 5.5mm internal jack - 2.1mm
Network connections	Ethernet RJ45 3G USB Modem
Antennas	City Analytics USB antenna wireless antenna
Microphone	noise sensor
Temperature	main board temperature sensor with extrapolation
Box	1.5mm aluminum box external use possible
Operating System	Debian 6.0 Squeeze

Table 1: Main features of the Intelify device.

environment and help with decision making to any kind of organization based on people flow and behavior.

Valuable information about tourism, trade and mobility can be gathered through the deployment of autonomous devices around a city. A specific example is the service offered in [1]. It offers information about foot traffic through Cordoba city center.

The cost of this solution is 1000 euros per device, including maintenance of remote computer, communications using a 3G telephony service and storage and data management.

Data accuracy is very representative, compared against other technologies. In [1] it was obtained an a priori error estimation of 8.5% of detections.

??????Descripcin de los datos recopilados

4. Experiments and results

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In this section the analysis of collected data during the monitoring period (November 8 to December 9) to obtain statistics and so study the use of vehicles is carried out.

Specifically, the following subsections report information about the total number of vehicles detected by each node, on weekdays or holidays, information on traffic density by time range on individual movements, and the average speed on a section delimited by two consecutive nodes.

Finally, since November 14 2012 a general strike was held, we will study how the strike affected traffic in the metropolitan area of Granada (Spain), by comparing total number of devices detected that day (November 14), and the following day (November 15).

4.1. Total number of vehicles detected (weekdays and holidays)

The first analysis consisted in calculating the number of devices detected by each node.

In total, 773,845 BT devices have been detected by the six nodes. As shown in Table 2, nodes located in the Sierra Nevada Highway (A44, nodes 4 and 5) have collected a higher number of data, while the node located in a side street (node 6) has detected the smallest number of devices.

Node Id.	N. of devices detected
1	31408
2	45032
3	33165
4	358494
5	297874
6	7872

Table 2: Number of BT devices detected by each node.

Node Id.	N. of devices detected
1	2149
2	2804
3	2832
4	32182
5	24166
6	1269

Table 3: Total number of BT devices detected by each node (only on non-working days).

4.2. Total vehicles detected on non-working days

To compare the traffic intensity between working and non-working days, the number of pass on holidays and non-working days have been obtained.

Table 3 shows how the number of detected devices lowers by all nodes on non-working days, compared to the number of detections on weekdays. Nodes located in the Sierra Nevada Highway still collected much more data than the remainder, due to the traffic this road supports on holidays.

4.3. Traffic density on the road by time range

Traffic density can be calculated taking into account the total number of detected devices by time range.

Figure 3 shows higher density on all nodes, at peak times or out of work and school.

4.4. Total detections by time range

Additionally we can calculate for each node, the number of detected devices by time range, without differentiating whether the device is the same or

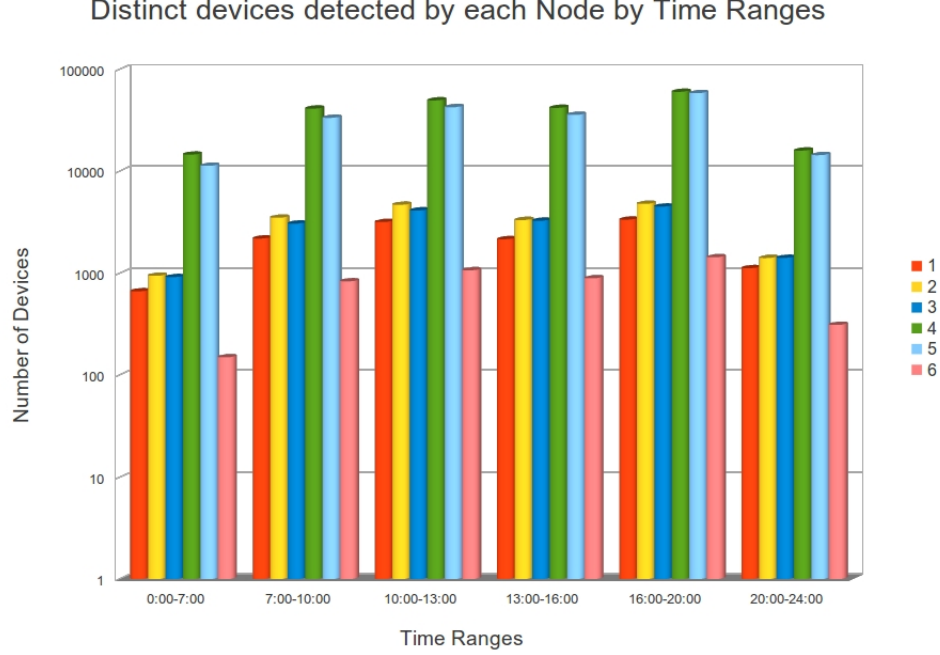


Figure 3: For each node, the total number of different detected devices by time range is shown. Figure is shown in logarithmic scale.

not (repeated passes). Thus, repeated passes of the same vehicle are counted.

As in the previous case, a greater traffic density can be observed on all nodes, at peak times or out of work and school. (see Figure 4).

4.5. Number of individual vehicles detections

We can take advantage of the proposed system's ability to identify BT devices. Thus, it can be detected whether vehicles pass by different nodes.

Figure 5 shows a large number of vehicles that pass repeated times (up to 10 times) by some of the nodes (mainly those located in the A44). Even it can be seen that nodes 4 and 5 detect about 1,000 vehicles passing more than 25 times repeated. On the other nodes, over 25 repetitions of the same device have been detected only around 120 times.

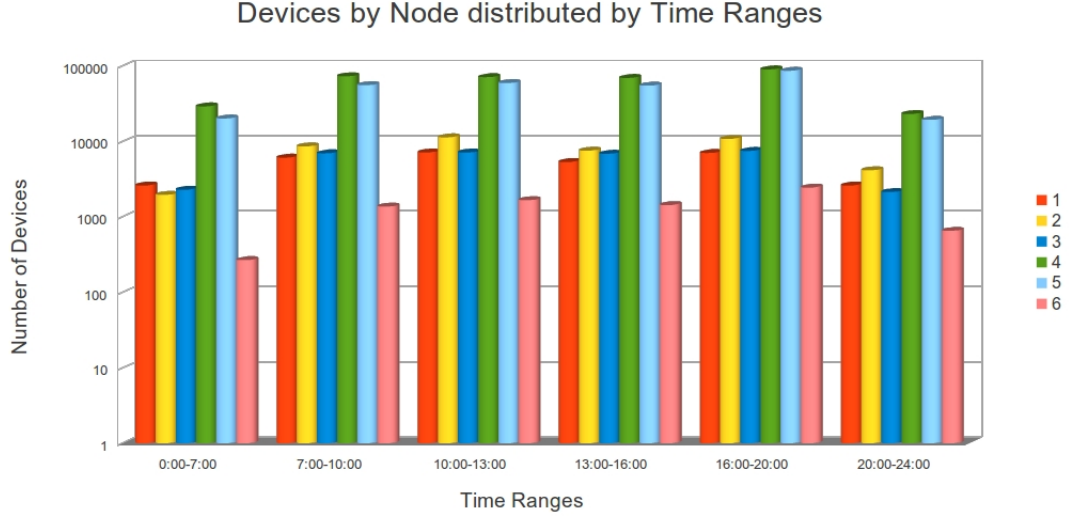


Figure 4: For each of the six nodes, the total number of detected devices by time range is shown. Figure is shown in logarithmic scale.

4.6. Complexity of displacement

To study the complexity of displacements, the number of vehicles that have passed through two nodes, 3 nodes and up to 6 nodes were calculated. Table 4 also shows the average number of times that vehicles have passed through 2, 3, 4, 5 or 6 nodes.

The above information is complemented with Figure 6, that shows how many cars pass by only one node, two nodes, three nodes, etc.

As expected, most of the BT devices rarely passed by all nodes, while most of devices pass only by one or two of nodes (their displacements are focused on a small part of the monitored area).

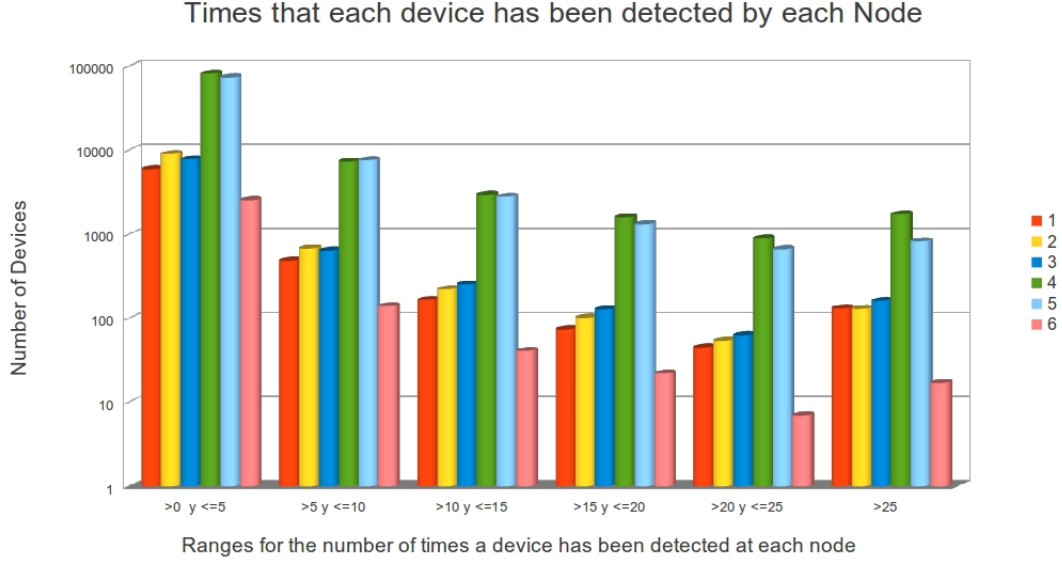


Figure 5: For each of the six nodes, the total number of detected devices N times (repeated occurrences of the same device) are shown. Figure is shown in logarithmic scale.

4.7. Effect of Nov-14 strike on zone traffic

Right in the middle of the monitoring period in Spain was held a day of general strike (November 14, 2012), which has been reflected in the number of detected devices (cars) on the nodes.

The effect of the general strike in the traffic of the monitored area has been analyzed and shown in Table 5 as the number of detected devices on November 14 and the very next day.

Table 8 shows a lowest number of detected vehicles the day of the strike that on the following day (working day in which the activity should be normal in the area).

4.8. Analysis of vehicles speed between two consecutive nodes

Finally, taking two consecutive nodes, located on the A44 highway, average speeds in the section bounded by nodes 4 (located at km 119.550) and 5 (located at km 123.250) can be calculated. This highway section where the study takes place is of 3700 meters long. Actually, we can calculate the

No. of nodes	No. of devices	Total number of passes	Mean \pm std. dev.
1	72989	165033	2.26 ± 31.16
2	53947	425667	7.89 ± 11.48
3	8125	131570	16.19 ± 24.71
4	1359	39241	28.88 ± 140.82
5	254	8603	33.87 ± 59.51
6	61	3731	61.16 ± 94.78

Table 4: Total number of vehicles that have passed through two nodes, 3 nodes and up to 6 nodes, and average number of times that vehicles have passed through 2, 3, 4, 5 or 6 nodes. In some cases the deviations are high because some devices have a very high number of occurrences for some nodes.

Node	Total number of passes (nov-14)	Total number of passes (nov-15)
2	1841	2722
3	891	1169
4	10807	16942
5	831	4017
6	946	1419

Table 5: Comparison of the number of passes for each node between the general strike day (November 14) and the very next day. Node 1 results are not reported because the hardware device suffered a power supply problem for a couple of days at that time.

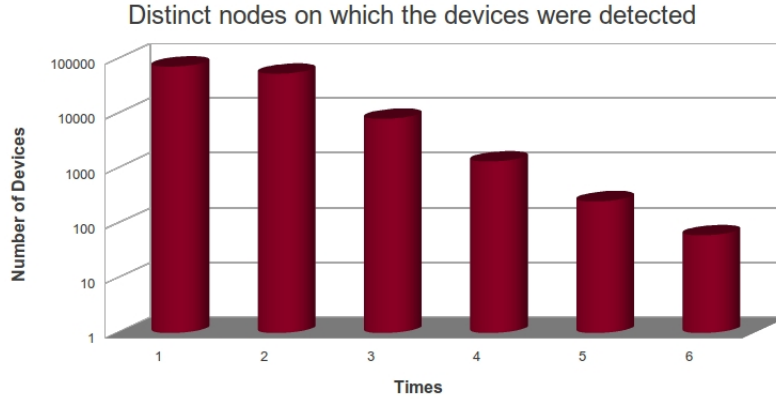


Figure 6: Figure shows how many cars pass by only one node, two nodes, three nodes, etc. Figure is shown in logarithmic scale.

average speed in the global section, not the speed at which each vehicle has at each instant within the section.

In that section, the speed is limited to 100 km/h. However, although most of the vehicles respect this limit, a lot of cars exceed this limitation.

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Finally, in order to determine the accuracy of the forecast method applied to time series data, many measures have been proposed. Most textbooks recommended the use of the Mean Absolute Percentage Error (MAPE) ? and it was the primary measure in the M-competition ?. Other works recommended other measures such as Geometric Mean Relative Absolute Error (GMRAE), Median Relative Absolute Error (MdRAE), and Median Absolute Percentage Error (MdAPE) ??. Later, the MdRAE, sMAPE (Symmetric Mean Absolute Percentage Error), and sMdAPE (Symmetric Median Absolute Percentage Error) were proposed ?.

Nevertheless, Hyndman and Koehler in their work ? determined that all measures mentioned before were not generally applicable since they can be infinite or undefined and can produce misleading results. Therefore, they proposed a new measure suitable for all situations: the Mean Absolute Scaled Error (MASE), which is less sensitive to outliers, less variable on small samples, and more easily interpreted.

In ? and ?, it can be found a description of different error measures. Among all of them, in this work we use the followings:

Speed range (km/h)	No. of passes
$v \leq 60.0$	1495
$60.0 \leq v \leq 70.0$	2585
$70.0 \leq v \leq 80.0$	7421
$80.0 \leq v \leq 90.0$	16339
$90.0 \leq v \leq 100.0$	20144
$100.0 \leq v \leq 120.0$	14384
$120.0 \leq v \leq 140.0$	5434
$v \geq 140.0$	1326

Table 6: Average speeds (globally) in the section bounded by nodes 4 and 5.

- Mean Absolute Error (MAE):

$$MAE = mean(|e_t|) \quad (1)$$

- Mean Absolute Percentage Error (MAPE):

$$MAPE = mean(|p_t|) \quad (2)$$

- Median Absolute Percentage Error (MdAPE):

$$MdAPE = median(|p_t|) \quad (3)$$

- Symmetric Median Absolute Percentage Error (sMdAPE):

$$sMdAPE = median(200 |Y_t - F_t| (Y_t + F_t)) \quad (4)$$

- Mean Absolute Scaled Error (MASE):

$$MASE = mean(|q_t|) \quad (5)$$

where Y_t is the observation at time $t = 1, \dots, n$; F_t is the forecast of Y_t ; e_t is the forecast error (i.e. $e_t = Y_t - F_t$); $p_t = 100e_t/Y_t$ is the percentage error, and $q_t = \frac{e_t}{\frac{1}{n-1} \sum_{i=2}^n |Y_i - Y_{i-1}|}$

5. Conclusions