

# Towards a Multiobjective Evolutionary Approach to Inventory and Routing Management in a Retail Chain

A.I. Esparcia-Alcazar and A. Martínez-García  
S2 Grupo, Spain  
Email: {aesparcia,amartinez}@s2grupo.es

P. García-Sánchez, J.J. Merelo and A.M. Mora  
Depto. Arquitectura y Tecnología de Computadores  
University of Granada, Spain  
Email: {pgarcia,jmerelo,amorag}@geneura.ugr.es

**Abstract**—In this work we address the problem of inventory and routing management in a retail chain. This involves the minimisation of two contradicting objectives, inventory holding costs and transportation costs, but which can be compounded in to a single one, the global costs. In previous work we addressed this using a single objective evolutionary algorithm but the duality inherent in the problem prompts us to consider a multiobjective approach; the aim is to determine what advantages each can bring. A number of experiments are carried out on several simulated and one real retail chain.

## I. INTRODUCTION

Inventory and Routing Problems (IRPs) are the most interesting and challenging extensions to the classical vehicle routing problem (VRP). In these problems inventory control and routing decisions have to be made simultaneously. IRPs share in general two characteristics: (1) either the supplier or the customer has a limited storage capacity and (2) inventory holding costs may be incurred, either at the supplier's or customer's end, which must be taken into account [1]. IRPs are more recent (dating back to the late 1970's, see for instance [2]) than the Vehicle Routing Problem (VRP) and differ from it in several aspects. Firstly, in the VRP the customers place orders and the delivery company serves them on any given day; in the IRP problem, however, there are no customer orders and the delivery company decides how much to deliver to which customers each day [3]. Another difference is the planning horizon, which typically spans a single day for the VRP, being longer for the IRP. Typically, an IRP aims to find an answer to three issues: (1) When to deliver to each customer (2) How much to deliver to a customer each time it is served and (3) Which delivery routes to use.

In general, IRPs are defined on a graph  $G = (V, E)$ , where  $V = \{S, 1, \dots, n\}$  is the set of vertices and  $E$  is the set of edges (or arcs). Vertex  $S$  represents the supplier and vertices  $1 \dots n$  represent the customers. A travel time  $t_{i,j}$  and a cost  $c_{i,j}$  are associated with edge  $(i, j) \in E$ . A time window  $[a_i, b_i]$  and a service time  $s_i$  are associated with each vertex  $i \in V$ . The capacity of each vehicle is denoted by  $Q$ . The inventory holding cost at the supplier is  $h_S$  and at customer  $i$  is  $h_i$ . The length of the planning horizon is denoted by  $H$ . Vehicles are denoted by  $k \in K$ . Other variables that can be considered are the initial inventories at supplier and customers and rates of usage of goods.

Many variants of the IRP have been described in the

literature, e.g. [4], [5], some of which have referred to the specific case of retail chains [6]. Among these versions some have been addressed using metaheuristics such as PSO [7], tabu search [8] or evolutionary algorithms [9]. We will follow this latter work and address here a variant of the Inventory Routing Problem for the real case of a Spanish cosmetics retail chain, which was first introduced in [10]. In this problem variant the customers are the shops in the chain and the deliverer is a central depot, all of them belonging to the same company. Added features of the problem, as imposed by the company, are as follows:

- The planning horizon  $H$  is one week, weekend excluded.
- Inventory costs are considered for the whole planning horizon and only at the shops' end,  $h_S = 0$ .
- Each shop  $i$  has a set of *admissible delivery frequencies*,  $F_i$ , denoting the number of times a week they can be served.
- For a given delivery frequency only certain combinations of days, or *delivery patterns*, are admissible. The set of admissible patterns  $\mathcal{P}_{adm}$  is common to all shops.
- The load is measured in units called *roll containers*. A roll container is a sort of tall cage with wheels that can store many kinds of items. Hence, although there are thousands of different items we will consider them only as part of a homogeneous, discrete load.
- For each shop and delivery frequency, the chain management has established an amount to deliver, expressed in roll containers. The question "how much to deliver" is thus simplified.
- The delivery fleet is homogeneous and capacitated (i.e. all vehicles used have the same finite capacity  $Q$ ). The number of vehicles (i.e. the size of  $K$ ) is not restricted. This is a consequence of the transport being subcontracted.
- All routes start and finish in the depot; each shop must be served once a day maximum.
- The service time is zero for the depot and the same value  $s$  for the shops. There are no time windows.

The inventory holding costs,  $c_h$ , are thus calculated as follows:

$$c_h = \sum_{i \in V} h_i \quad (1)$$

and the transportation (delivery) costs,  $c_t$ ,

$$c_t = \sum_H \sum_{k \in K} \sum_{(i,j) \in E} c_{i,j} x_{ijk} \quad (2)$$

where the flow variable  $x_{ijk}$  equals 1 if edge  $(i,j)$  was used by vehicle  $k$  and 0 otherwise. The global costs are calculated as the sum of the inventory costs and the transportation costs,

$$c_g = c_h + c_t \quad (3)$$

and the goal is to *minimise*  $c_g$ .

The key issue in our problem is that the delivery frequency determines the size of the deliveries, which in turn determine the inventory cost at the shop's end. A low frequency of delivery implies a large size of deliveries and a high inventory cost, and vice-versa. On the other hand, a low delivery frequency also means low transportation costs. Hence, the minimisation of inventory and transportation costs are contradicting, independent objectives, although it is true that they are both measured in the same (currency) units and that the global cost is calculated by simply adding them.

The purpose of this work is to explore the interest of applying a multiobjective approach to this seemingly monoobjective problem. The rationale behind this is that the search space will be explored in a different way and this may lead to novel results. This problem has been previously addressed [9] employing a two-level methodology in which each level aimed at minimising each objective. The top level used an evolutionary algorithm to obtain delivery patterns for each shop on a weekly basis so as to minimise the inventory costs, while the bottom level used various metaheuristics to solve the Capacitated Vehicle Routing Problem (CVRP) for every day in order to obtain the minimum transportation costs associated to a particular set of patterns.

Here we have carried out an extensive series of experiments using the eight instances used in that work, plus three new ones (see Table IV). These correspond to 10 simulated geographical layouts found frequently in the literature, plus that of a real retail chain. The set of restrictions, consisting of the characteristics of the vehicles employed and the working hours of the drivers (see Table V), plus the parameter configuration of the evolutionary algorithm those used in [9]. In the multiobjective approach we have used the NSGA-II algorithm [11] because it is considered one of the best algorithms in the state of the art.

The rest of the paper is structured as follows. The next section is devoted to briefly introduce the main concepts of multiobjective optimization. Delivery patterns are explained in Section III. The methodology is described in Section IV. Section V describes the experimental setup with the simulated data, with results contained in Section VI. Section VII describes the experiments with real geographical data. Finally, Section VIII presents the conclusions and outlines future areas of research.

TABLE I. THE ADMISSIBLE PATTERNS AND THEIR RESPECTIVE FREQUENCIES. THE 1 REPRESENTS THAT THE SHOP IS SERVED ON THAT DAY, THE 0 THAT IT IS NOT. AS A CONSEQUENCE OF THE BUSINESS LOGIC, WE WILL ONLY CONSIDER 11 PATTERNS OUT OF THE 31 THAT ARE POSSIBLE.

Pattern Id.	Freq. (days)	Mon	Tues	Wed	Thurs	Fri
5	2	0	0	1	0	1
9	2	0	1	0	0	1
10	2	0	1	0	1	0
11	3	0	1	0	1	1
13	3	0	1	1	0	1
17	2	1	0	0	0	1
18	2	1	0	0	1	0
21	3	1	0	1	0	1
23	4	1	0	1	1	1
29	4	1	1	1	0	1
31	5	1	1	1	1	1

## II. MULTIOBJECTIVE OPTIMIZATION

Multicriteria or Multiobjective Optimization Problems (MOP) [12] are those where several objectives have to be simultaneously optimized. These problems do not have a single best solution, one that is better than any other with respect to every objective. In fact, frequently improving the solution for one objective implies a worsening for another. Thus in a MOP there is a set of solutions that are better than the rest considering all the objectives; this set is known as the *Pareto set* (PS). They try to approach the ideal set of solutions which is usually represented graphically as the *Pareto front* (PF). These are related to an important concept, the *dominance*, defined as follows ( $a$  dominates  $b$ ):

$$a \prec b \text{ if :} \\ \forall i \in 1, 2, \dots, k \mid C_i(a) \leq C_i(b) \wedge \exists j \in 1, 2, \dots, k \mid C_j(a) < C_j(b) \quad (4)$$

where  $a \in X$  and  $b \in X$  are two different decision vectors of  $n$  values, and every  $C$  is a cost function (one per objective). If it intends to minimize the cost and Equation 4 is true, then  $b$  is *dominated* by  $a$ .

Hence, the solutions in the PS are known as *non-dominated solutions*, while the remainder are known as dominated solutions. Since none of the solutions in the PS is absolutely better than the other non-dominated solutions, all of them are equally acceptable as regards the satisfaction of all the objectives. These concepts can be extended in [13].

## III. ADMISSIBLE PATTERNS

A delivery pattern is a set of days in the week in which a shop can be served. It can be represented by  $d$  bits, with a bit set to one meaning that the shop is visited that day and nil that it is not. For instance, pattern 21, i.e. 10101 in binary, corresponds to deliveries on Monday, Wednesday and Friday. The total number of possible patterns is  $2^d - 1$ , in our case  $2^5 - 1 = 31$  (obviously pattern 0 does not make sense, as it would involve not serving any day). However, not all patterns are acceptable from a business point of view. For instance, no patterns of frequency 1 (serving one day a week) are admissible, because this would involve big deliveries which would interfere with the shop staff's main activity of selling. Furthermore, delivery patterns that involve serving only Monday and Tuesday are not admissible, as most sales take place towards the end of the week. Hence, for a pattern

$p_i$  to be valid it must be contained in the set of admissible patterns,  $p_i \in \mathcal{P}_{adm}$  or as given in Table I.

As a pattern has an associated delivery frequency and only a limited number of frequencies are available to each shop, this in turn means that the number of patterns available to a shop can be very low. For instance, a shop whose admissible frequencies are 4 and 5 has only three possible patterns: 23, 29 and 31.

#### IV. METHODOLOGY

Following [9] we have employed an evolutionary algorithm (see details in Table III) with each individual being a set of patterns  $\mathcal{P}$  represented as a vector of length equal to the number of shops to serve,  $(n)$ ,

$$P = (p_1, p_2, \dots, p_n)$$

with  $p_i \in \mathcal{P}_{adm}$ .

To calculate the inventory cost for such an individual we first obtain the inventory cost  $h_i$  for each shop  $i$ . This stems from the shop's pattern, which in turn determines the associated delivery frequency for that shop. With this value we would read the inventory cost for that shop in the corresponding table. An example of the latter is given in Table II.

TABLE II. INVENTORY COST (IN EURO) AND SIZE OF THE DELIVERIES PER SHOP (EXPRESSED IN ROLL CONTAINERS) DEPENDING ON THE DELIVERY FREQUENCY (IN DAYS). MISSING DATA CORRESPONDS TO FREQUENCIES THAT ARE NOT ADMISSIBLE FOR EACH SHOP.

Shop #	Inventory cost (€)					Delivery size (roll containers)				
	Frequency (days)					Frequency (days)				
	1	2	3	4	5	1	2	3	4	5
1	-	-	-	336	325	-	-	-	2	2
2	-	-	-	335	325	-	-	-	2	2
...	...	...	...	...	...	...	...	...	...	...
N	-	311	293	286	284	-	3	2	2	1

For instance, let us assume that shop N was assigned pattern 23, which has a frequency of 4; we would look up in the table the inventory cost for the shop at that frequency, which is 286€. Proceeding in the same way with all shops and adding up the results we would obtain the total inventory cost.

The next step involves the calculation of transport costs, which are obtained by solving the VRP with an algorithm of choice. In our case we will employ the one that yielded better results in [9], namely the Clark and Wright algorithm [14] enhanced with local search. C&W's algorithm is based on the concept of *saving*, which is the reduction in the traveled length achieved when combining two routes. The local search method implemented consists on performing 2-interchanges on the solution obtained by the C&W algorithm. Every possible pair of shops is exchanged, first between shops in the same route and then between shops in different routes. If at any time an invalid route is generated (because the restrictions on time or capacity are violated) the depot is inserted where required

in the route. The best neighbour solution will be the one with a lower associated transport cost.

Because we are dealing with the *capacitated* VRP, we need to know what the demands (i.e. the size of deliveries) of each shop are. These are taken from Table II. In the example given earlier, for shop N at frequency 4 the delivery size is 2 roll containers.

Having established the inventory and routing costs, we need to define what the optimum is. In the single objective case, this is defined as the solution that minimises the global cost, given by Equation (3). In the multiple objective approach, we will deal with two separate cost functions, given by Equations (1) and (2). However, in the latter case we will still use the global cost defined for the single objective case to compare the results between multi and monoobjective solutions. The reason is that the company is interested in spending less, irrespective of where the reduction comes from, and in any case both costs come in the same units (€).

TABLE IV. PROBLEM INSTANCES USED IN THE EXPERIMENTS AND THEIR CHARACTERISTICS. ALL INSTANCES WERE OBTAINED FROM [BRANCHANDCUT.ORG/VRP/DATA/](http://BRANCHANDCUT.ORG/VRP/DATA/) EXCEPT THE LAST ONE, WHICH WAS OBTAINED FROM [WWW.FERNUNI-HAGEN.DE/WINF/TOUREN/INHALTE/PROB-INST.HTM](http://WWW.FERNUNI-HAGEN.DE/WINF/TOUREN/INHALTE/PROB-INST.HTM)

ID	Instance	Distribution	$n$	Eccentricity	Source
A32	A-n32-k5.vrp	uniform	31	0.370	Branchandcut
A33	A-n33-k5.vrp	uniform	32	0.173	Branchandcut
A69	A-n69-k9.vrp	uniform	68	0.124	Branchandcut
A80	A-n80-k10.vrp	uniform	79	0.460	Branchandcut
B35	B-n35-k5.vrp	clusters	34	0.518	Branchandcut
B45	B-n45-k5.vrp	clusters	44	0.128	Branchandcut
B67	B-n67-k10.vrp	clusters	66	0.163	Branchandcut
B68	B-n68-k9.vrp	clusters	67	0.468	Branchandcut
P100	P-n101-k4.vrp	uniform	100	0.017	Branchandcut
X200	c1_2_1.txt	clusters	200	0.043	Uni. Hagen

#### V. EXPERIMENTS WITH SIMULATED DATA

As explained above, we have employed a number of *geographical layouts* available on the web. We have selected our instances so as to achieve the maximum representation on three categories:

- **size**, given by the number of shops,  $n$ ,
- **distribution**. We consider two kinds of distributions: *uniform* and *in clusters*, corresponding to shops that are scattered more or less uniformly on the map or grouped in clusters and,
- **eccentricity**. This represents the distance between the depot and the geographical center of the distribution of shops, normalised by dividing it by the maximum distance between any two points in the geographical distribution. The coordinates of the geographical center are calculated as follows:

$$(x_{gc}, y_{gc}) = \frac{1}{n} \sum_{i=1}^n (x_i, y_i)$$

with  $n$  being the number of shops. An instance with low eccentricity (in practise, less than 0.15) would

TABLE III. CONFIGURATION OF THE TOP-LEVEL EVOLUTIONARY ALGORITHM EMPLOYED (BOTH MONO- AND MULTIOBJECTIVE).

<b>Encoding</b>	The gene $i$ represents the pattern for shop $i$ . The chromosome length is equal to the number of shops ( $n$ ).
<b>Selection</b>	Tournament in 2 steps. To select each parent, we take $tSize$ individuals chosen randomly and select the best. For the single objective algorithm the best 10 individuals of each generation are preserved as the elite.
<b>Evolutionary operators</b>	2 point crossover and 1-point mutation. The mutation operator changes the pattern for 1 shop in the chromosome.
<b>Termination criterion</b>	Terminate when the total number of generations (including the initial one) equals 100.
<b>Fixed parameters</b>	Population size, $popSize = 100$ Tournament size, $tSize = 2$ Mutation probability, $pM = 0.2$ Crossover probability, $pC = 1$

have the depot centered in the middle of the shops while in another with high eccentricity (above 0.35) most shops would be located on one side of the depot.

TABLE V. DATA FOR VEHICLE ROUTING PROBLEM

<b>Vehicle capacity</b>	12 roll containers
<b>Transportation cost</b>	0.6 €/Km
<b>Average speed</b>	60 km/h
<b>Unloading time</b>	15 min
<b>Maximum working time</b>	8h

We chose ten instances with different levels of each category, see Table IV. It must be noted that we are only using the spatial location and not other restrictions given in the bibliography, such as the number of vehicles or the shop demand values. As pointed out earlier, a main characteristic of our problem is that the latter is a function of the delivery frequency, so we had to use our own values for the demands. We also added the set of admissible patterns  $\mathcal{P}_{adm} = \{5, 9, 10, 11, 13, 17, 18, 21, 23, 29, 31\}$  and the inventory costs, an example of which is given in Table II. The inventory, demand and admissible patterns data were obtained from Druni SA, <http://www.druni.es>, a major regional Spanish drugstore chain. Finally, we used the vehicle data given in Table V. Problem data is freely available from our website <http://ourdata.repository.org>

We tested the mono- and multiobjective approaches on each of the ten instances selected, performing ten runs per instance with a termination criterion in all cases of 100 generations.<sup>1</sup>

The results were evaluated on two fronts: the global costs obtained, and the computational time taken in the runs. The latter is important when considering a possible commercial application of the proposed methodology in a decision making environment, where answers to queries must come as fast as possible.

<sup>1</sup>The motivation for such a small number of runs is the high computational expense of some of the instances: the running times ranged from several minutes to several days in the computers employed (PCs with Intel Celeron processor, between 1 and 3GHz, between 256 and 512 MB RAM).

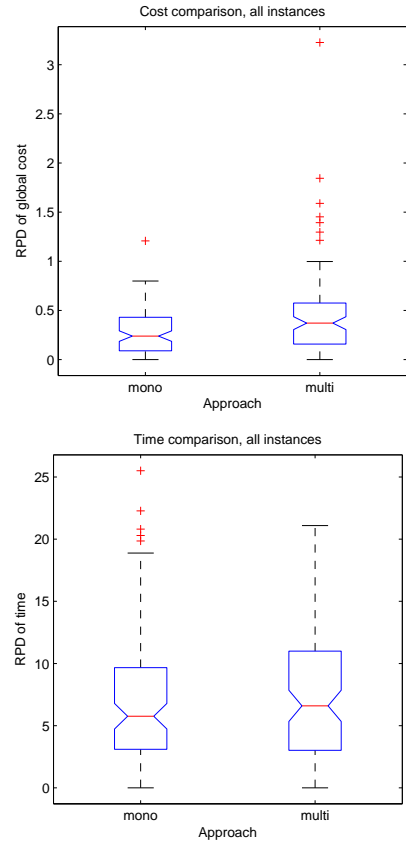


Fig. 1. Boxplots of RPDs of global costs and time for all instances.

## VI. RESULTS WITH SIMULATED DATA

In order to be able to compare results between the different instances we normalised the values of interest by defining the *relative percentage deviation*,  $RPD$ , given by the following expression:

$$RPD = \frac{val - val_{min}}{val_{min}} \times 100 \quad (5)$$

where  $val$  is the value of the variable of interest obtained

by EVITA using one of the two approaches (single or multiobjective) on a given instance. The *RPD* is, therefore, the average percentage increase over the lower bound for each instance,  $val_{min}$ . In our case, the values of interest are the **cost** (global, inventory and transportation) and the **execution time**. The lower bound is the best result obtained for that instance.

We carried out Mann-Whitney-Wilcoxon (MWW) tests for the results yielded by the best individuals for all runs and problem instances, both in single and multiobjective. In the multiobjective case, we define the best individual as that member of the final Pareto front yielding the lowest global cost (as defined for the single objective problem).

Firstly we examined all instances together, both in terms of cost (global, inventory and routing) and time. Next, we grouped the results in several ways, considering two levels in the three different categories:

- Size: small (A32, A33, B35, B45) and large instances (A80, P100, X200)
- Distribution: uniform (A32, A33, A69, A80, P100) and clusters (B35, B45, B67, B68, X200)
- Eccentricity: low (P100, X200, A69, B45) and high (A80, B35, B68)

The result of the MWW tests is that the single objective approach yields the best performance in terms of cost in all cases but one, the low eccentricity group, for which there were no significant differences.

Regarding the computational time, the test did not find any significant differences between the two approaches. Figure 1 shows the resulting boxplots for global cost and time for the ten instances studied. Figure 2 portrays the comparison between methods when the costs of transport and inventory are considered separately. Figure 3 shows the boxplots when different groupings by category are considered.

## VII. EXPERIMENTS WITH REAL GEOGRAPHICAL DATA

For this part we employed the geographical layout of the actual Druni stores. We restricted our problem to the 60 shops in the province of Valencia (Spain), which make up a medium-sized instance when compared to the simulated instances of the previous sections. The distances between shops and between the depot and each shop were calculated using Google Maps and therefore are not Euclidean but driving distances. Figure 6 shows the geographical layout of the shops, taken from <http://maps.google.es/>. The black dot marks the position of the depot. From this map we can see that the instance is not easily classifiable as having a uniform or clustered distribution.

We may also calculate the eccentricity of this distribution by calculating an approximated geographical center. For this, we will project the shops onto a plane and obtain their cartesian coordinates, which we will then use to obtain the center in the same way as for the simulated distributions. The cartesian coordinates of shop  $i$  expressed in kilometers are approximated as follows<sup>2</sup>:

$$y_i = 1.852 \cdot lat_i \quad (6)$$

$$x_i = 1.852 \cdot long_i \cdot \cos(lat_i) \quad (7)$$

<sup>2</sup>We wish to thank A Friend for his help in nautical issues

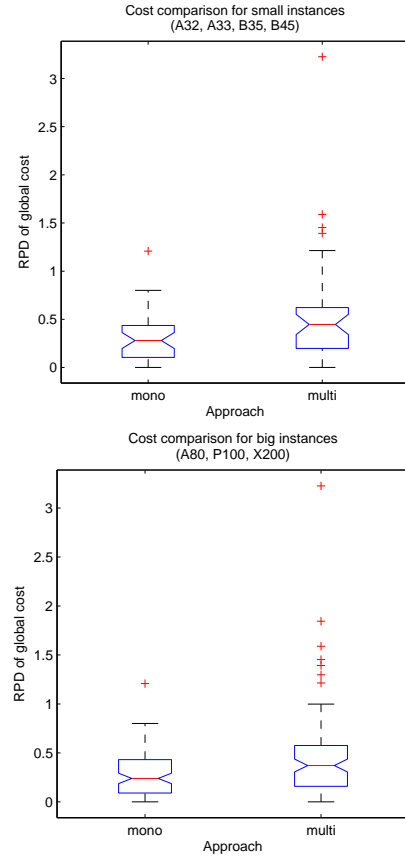


Fig. 3. Boxplots of the RPD of the global costs for large and small instances.

where  $lat_i$  and  $long_i$  are the latitude and longitude in minutes of shop  $i$ , respectively, and 1.852 is used to transform nautical miles into Km (taking into account that one minute of latitude is the length of a nautical mile). Employing this approximation we obtain an eccentricity of either 0.091 (employing Euclidean distance measure) or 0.113 (employing Google Maps driving distance); in both cases and according to the definition given in the previous section we can classify this as a low eccentricity setting, which is not unexpected given that this is a real retail chain.

Depending on their location, we consider seven types of shop, labeled A to G. We also used the list of admissible patterns defined in Section V and the inventory costs given in Table VI. All the problem data, including the table of driving distances, can be downloaded from <http://ourdata.repository.org>.

For this case we tried the following setups:

- 10 runs with a population size of 100 and a termination criterion of 100 generations
- 10 runs with a population size of 100 and a termination criterion of 1000 generations

The results of the MWW test did not show any differences in the three costs (total, inventory, transportation) for the 100/100 case, which is consistent with the simulated instances for the low eccentricity case. However, for the 100/1000 case the tests

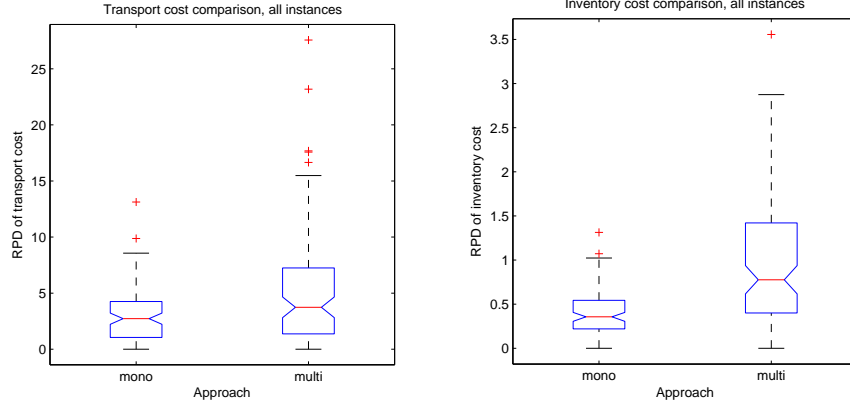


Fig. 2. Boxplots for RPD of the transport (left) and inventory costs (right) for all instances

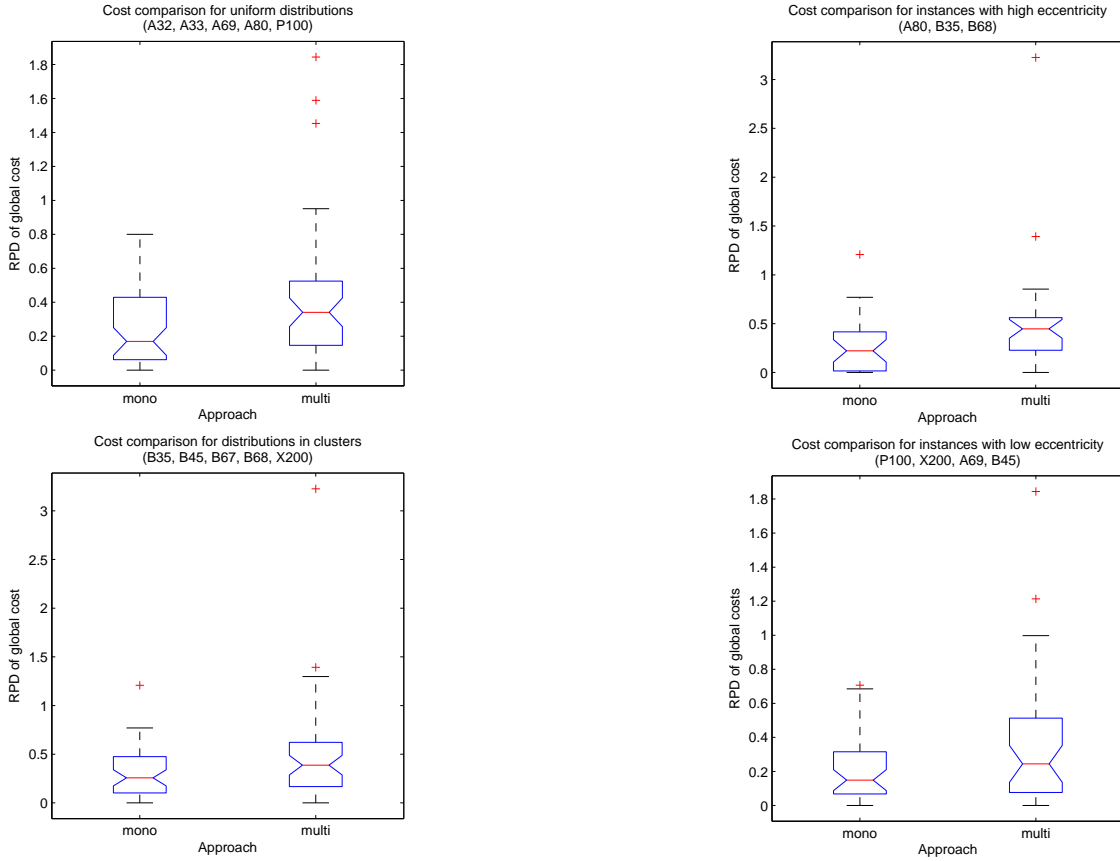


Fig. 4. Boxplots of the RPD of the global costs for instances with uniform and clustered distributions.

Fig. 5. Boxplots of the RPD of the global costs for instances with high and low eccentricity.

revealed lower total and inventory costs for the monoobjective approach.

### VIII. CONCLUSIONS AND FUTURE WORK

A multiobjective approach can also be used on problems that are easily decomposable into two objectives, like the one presented here. The theoretical advantage of working this way would be to explore the space of possible solutions in a

different way, so that in could, in principle, discover solutions that had not been reachable using a monoobjective approach.

However, for the particular problem we have applied to in this paper, we have shown that the multiobjective approach does not yield any advantage over the single objective one, *when the same settings are used for both*. This could be explained by the fact that, due to the specificities of the problem data (the items moved are cosmetic products) inventory costs



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